Fast RTM with *multiples* and *source* estimation

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Main messages

Demonstrate how *linearized* inversion

- can be carried out *efficiently*
- modelling errors can be *mitigated*

by *sparsity*-promotion *accelerated* by *rerandomization*

Demonstrate how *surface*-related *multiples* can be

- *imaged* by including the *upgoing* wavefield as an *areal* source
- used to estimate the *source* function *on the fly*

Disclaimer

Assume that

- *receiver*-side *ghost* has been *removed* by *processing*
- we have access to a *kinematically* correct *background* velocity models

‘*Ideal*’ imaging vs inversion

[w/ *primaries* only]

What are the *advantages* of iterative *inversion* over single-pass RTM *imaging*?

How does *randomized* inversion handle *mundane* modelling *errors*?

Canonical linearized inversion

$$\min_{\delta\mathbf{m},\mathbf{q}} \sum_{i=1}^{n_f} \|\mathbf{d}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \mathbf{Q}(q_i)]\delta\mathbf{m}\|_2^2$$

$\delta\mathbf{m}$: model perturbation

$\mathbf{q}$: source wavelet spectrum

$\mathbf{d}_i$: wavefield

$\nabla\mathbf{F}_i$: demigration operator

$\mathbf{m}_0$: background model

$\mathbf{Q}(q_i)$: source wavefield

Herrmann and Li, 2012
Tu and Herrmann, 2012
Candes et. al., 2006

Sparsity promotion

[w/ simultaneous sources]

BPDN: $\min_{\mathbf{x}} \|\mathbf{x}\|_1$

$$\text{subject to} \sum_{i\in\mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(q_i)]\mathbf{S}^*\mathbf{x}\|_2^2 \leq \sigma^2$$

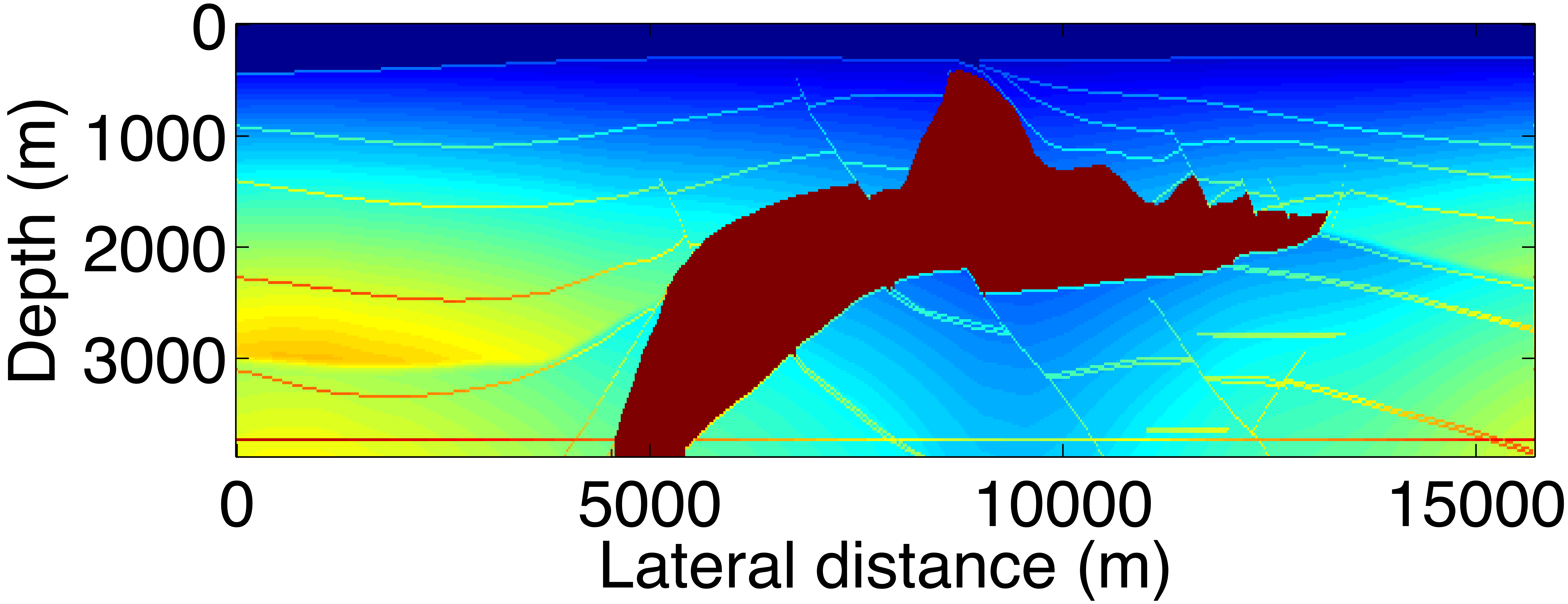
Work w/ *random* frequency subsets

Form *randomized* source aggregates (super shots)

$\mathbf{S}^*$: Curvelet synthesis operator

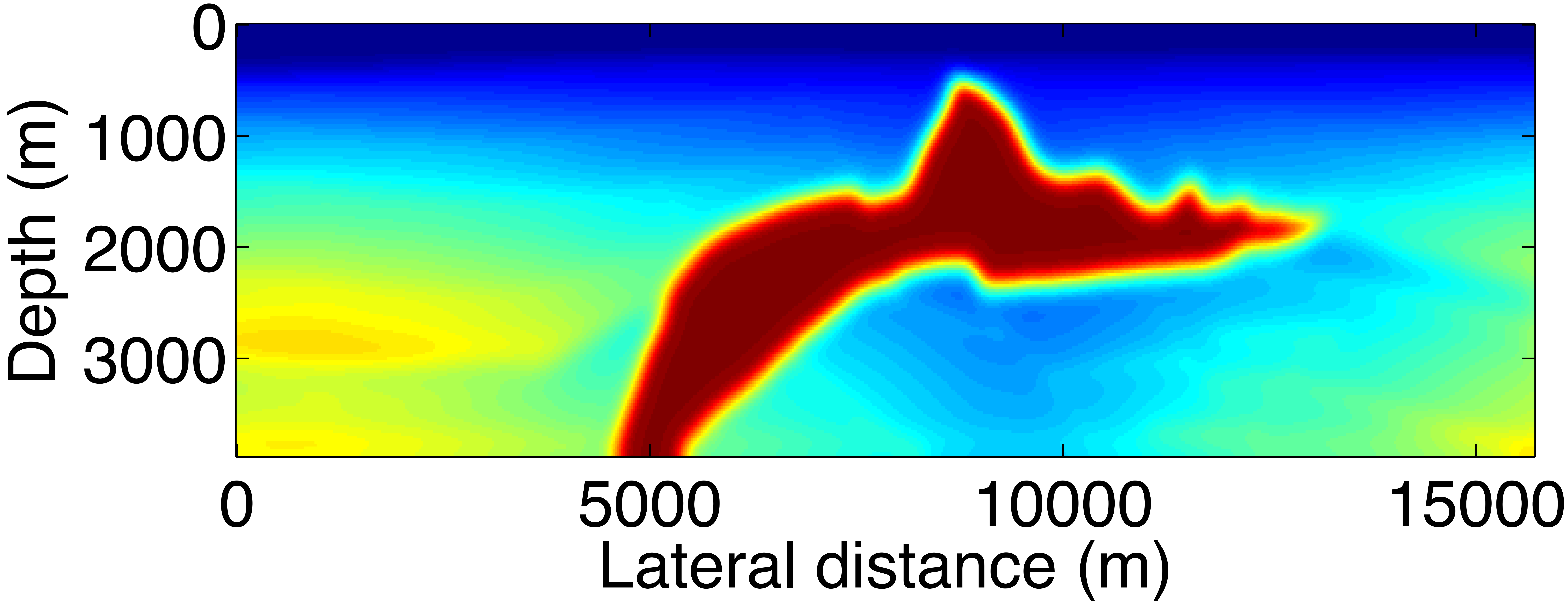
σ : tolerance

True model

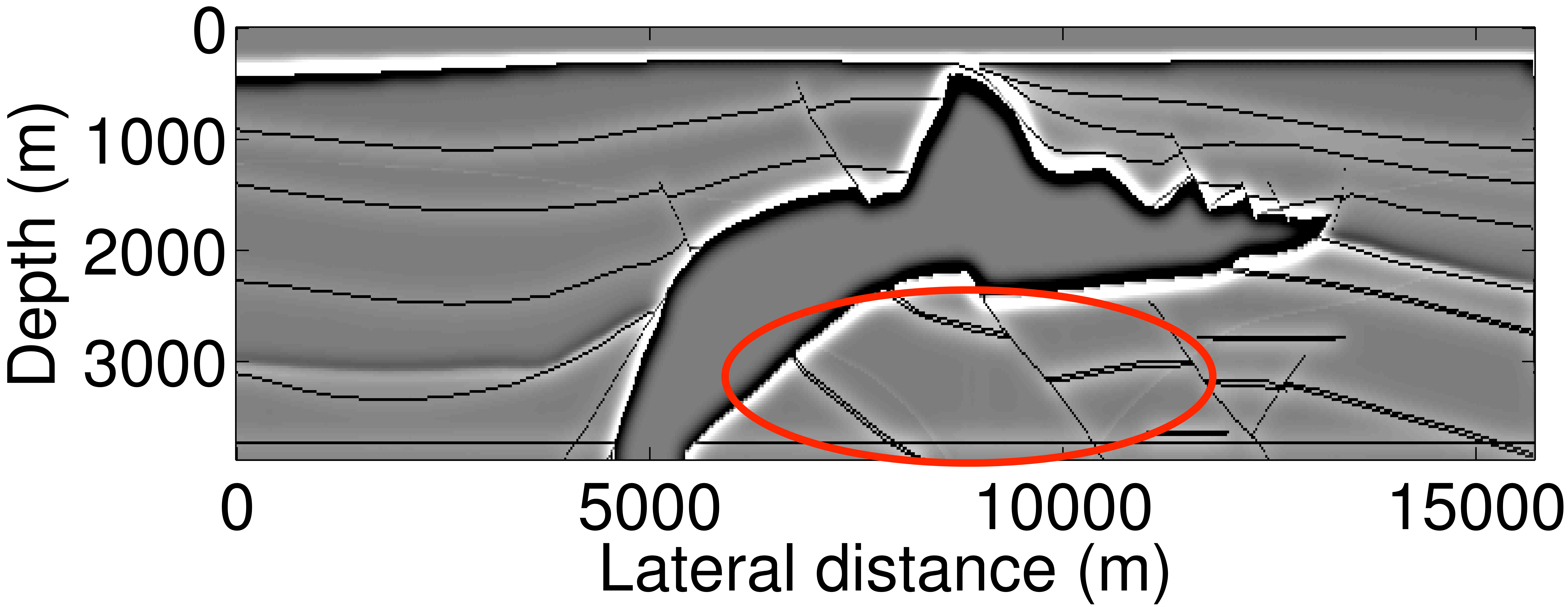


0
1000
2000
3000
Depth (m)
0
5000
10000
15000
Lateral distance (m)

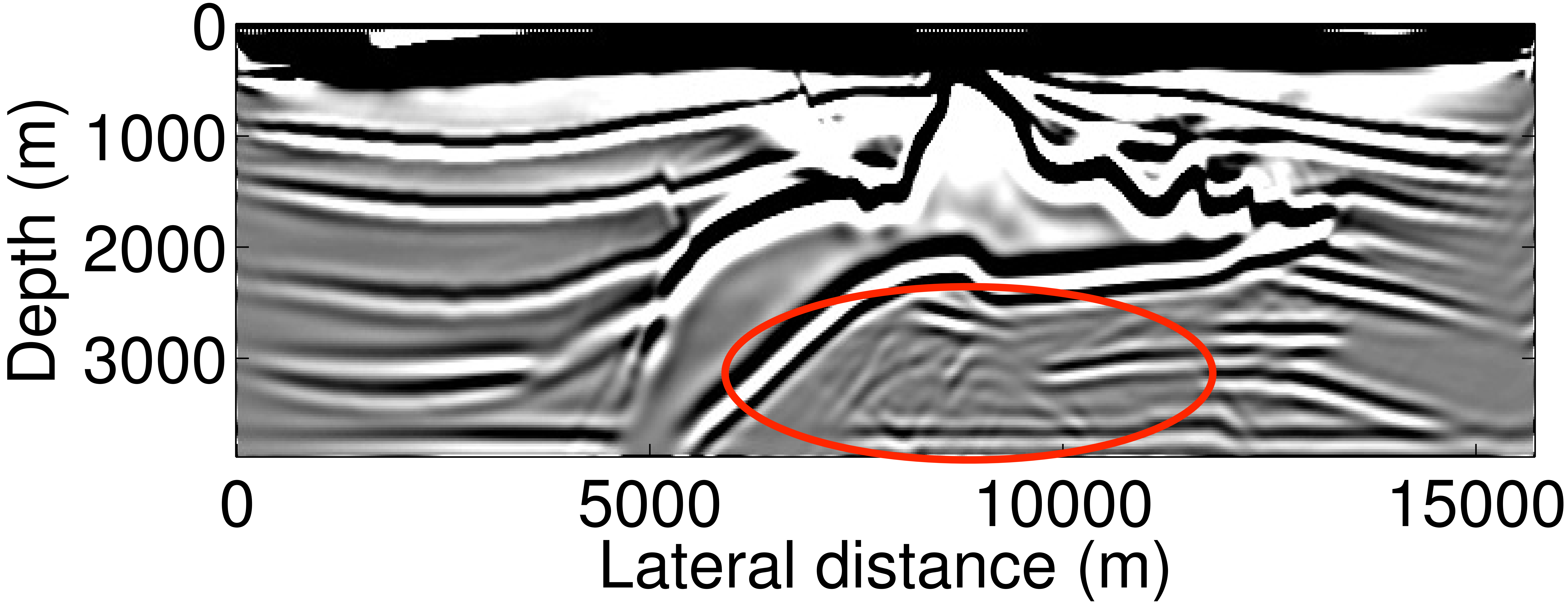
Smooth background model



True model perturbation



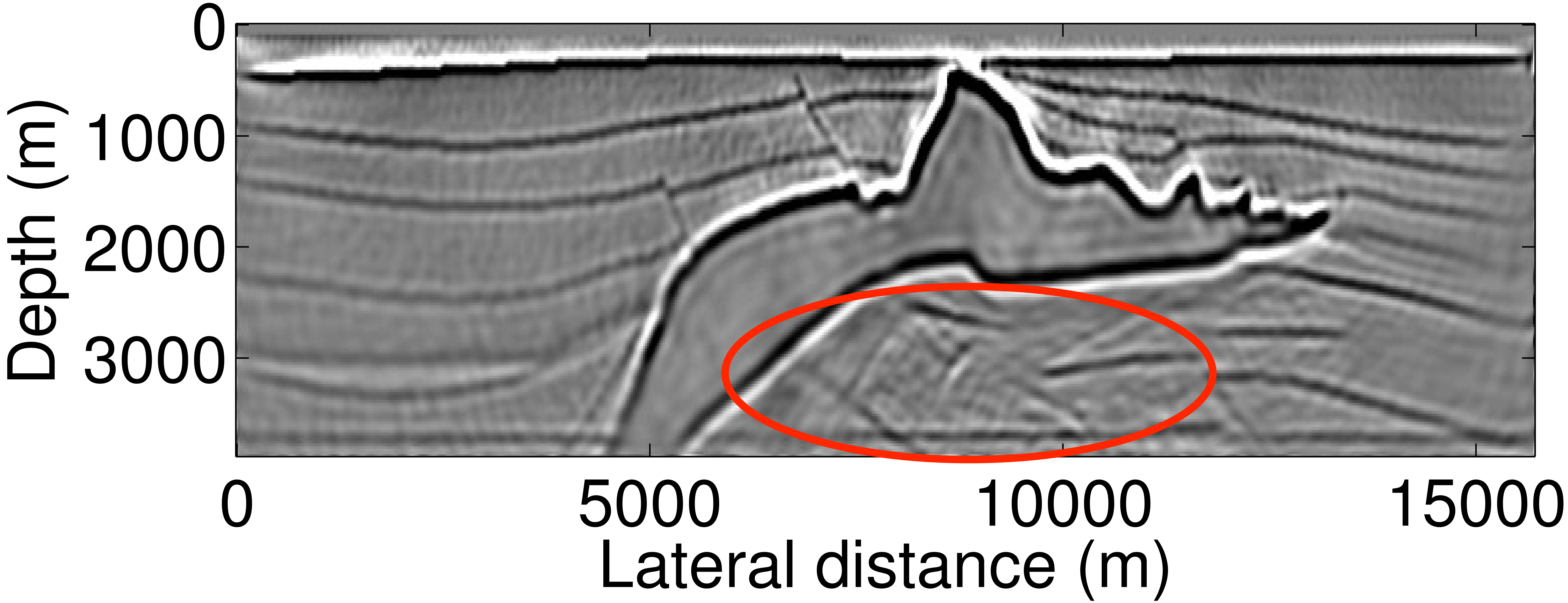
RTM



0
1000
2000
3000
Depth (m)
0
5000
10000
15000
Lateral distance (m)

Fast inversion

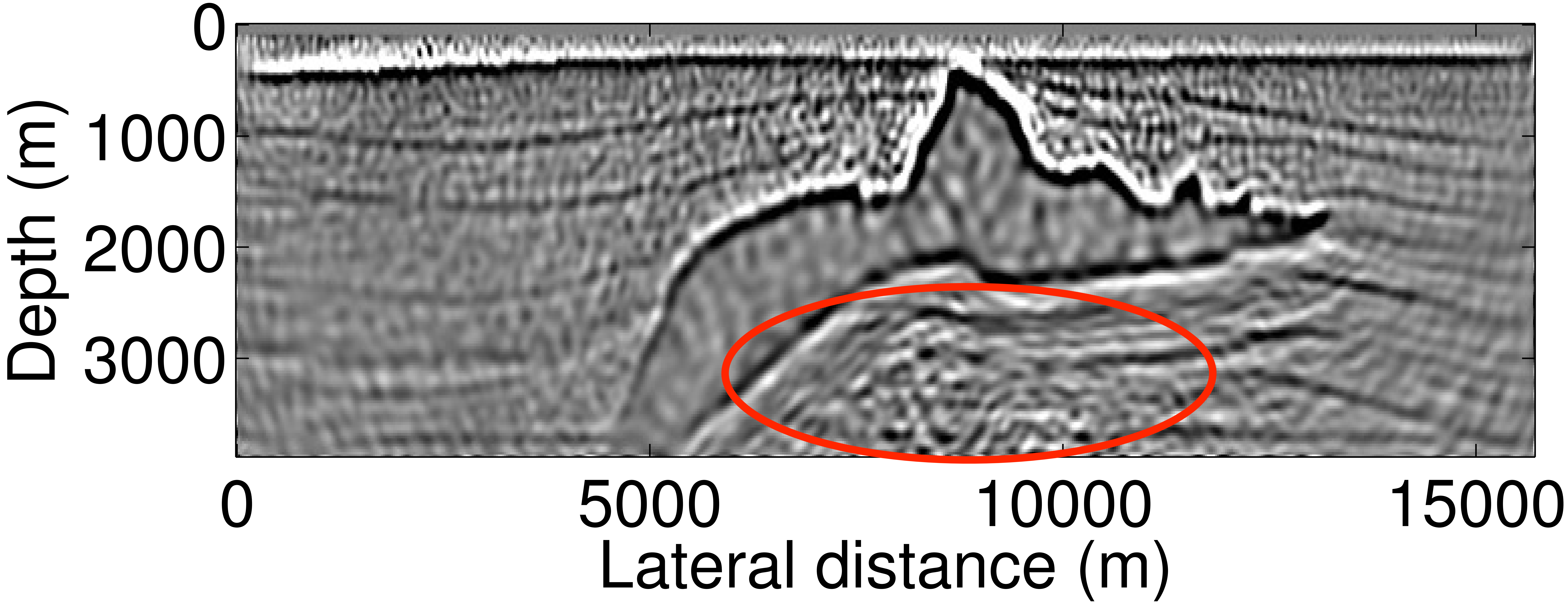
[w/ modeling errors and **w/ rerandomization**]



~1.45X the simulation cost of a single RTM with all data

Fast inversion

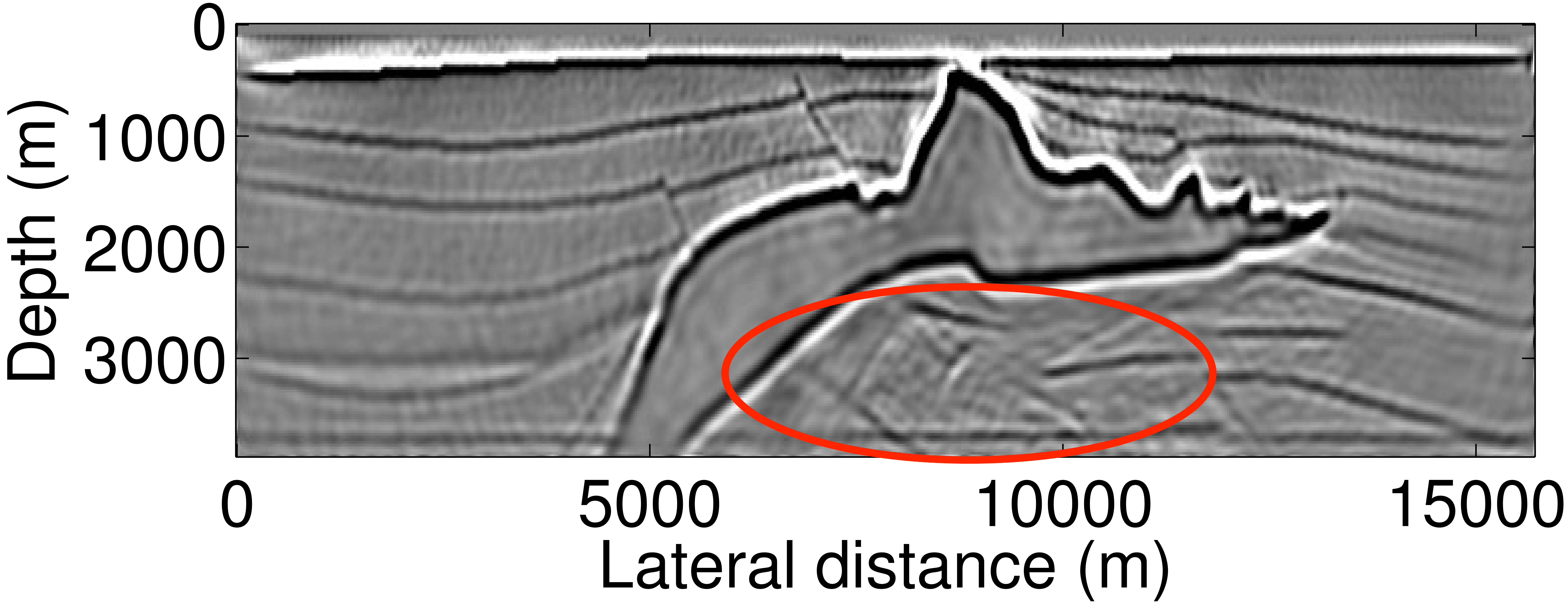
[w/ modeling errors and w/o rerandomization]



~1.45X the simulation cost of a single RTM with all data

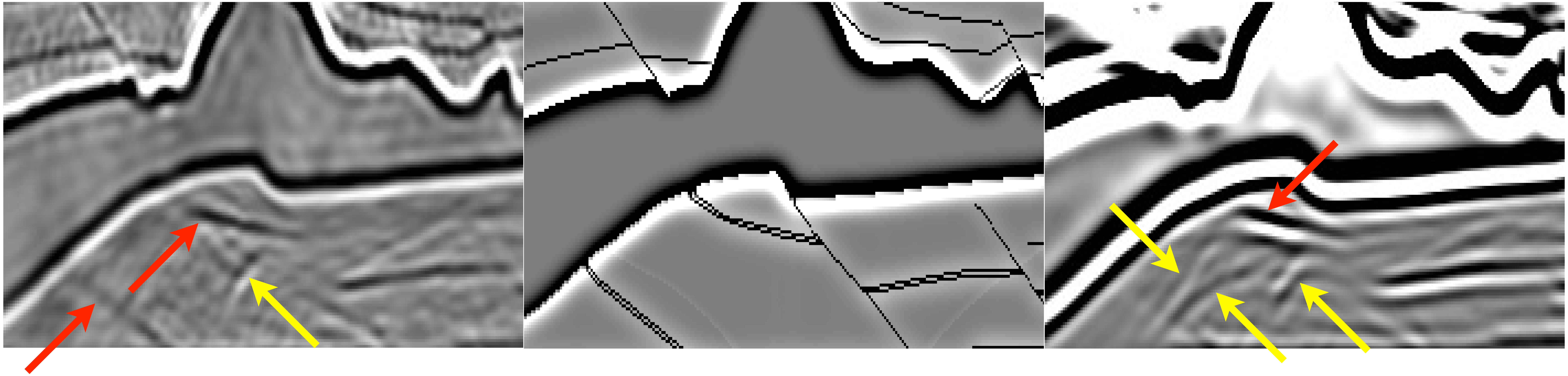
Fast inversion

[w/ modeling errors and w/ rerandomization]



~1.45X the simulation cost of a single RTM with all data

Details

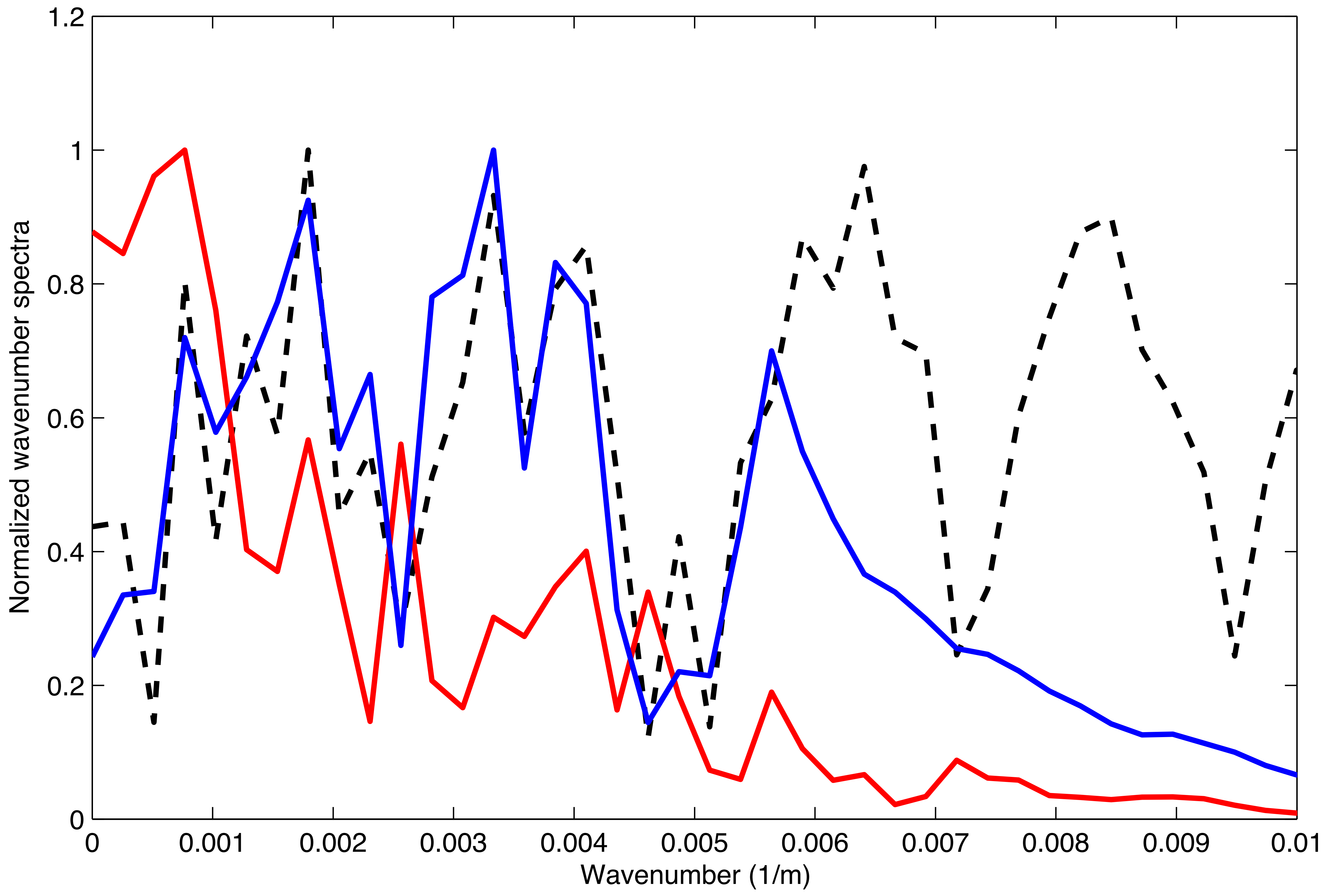


Fast inversion

True perturbation

RTM

Vertical wavenumber contents



Black: true; Blue: inversion image; Red: RTM image

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Imaging vs inversion

What are the *advantages* of iterative *inversion* over single-pass RTM *imaging*?

- restoration of amplitudes for complex geology
- correction for the source & improved spatial resolution
- possibility to image cheaply by working with *randomized* subsets of data

How does *randomized* inversion handle *mundane* modelling errors?

- rerandomization cancels noise buildup on the model & *accelerates* convergence

Can *surface*-related multiples be *ignored*?

Imaging vs inversion

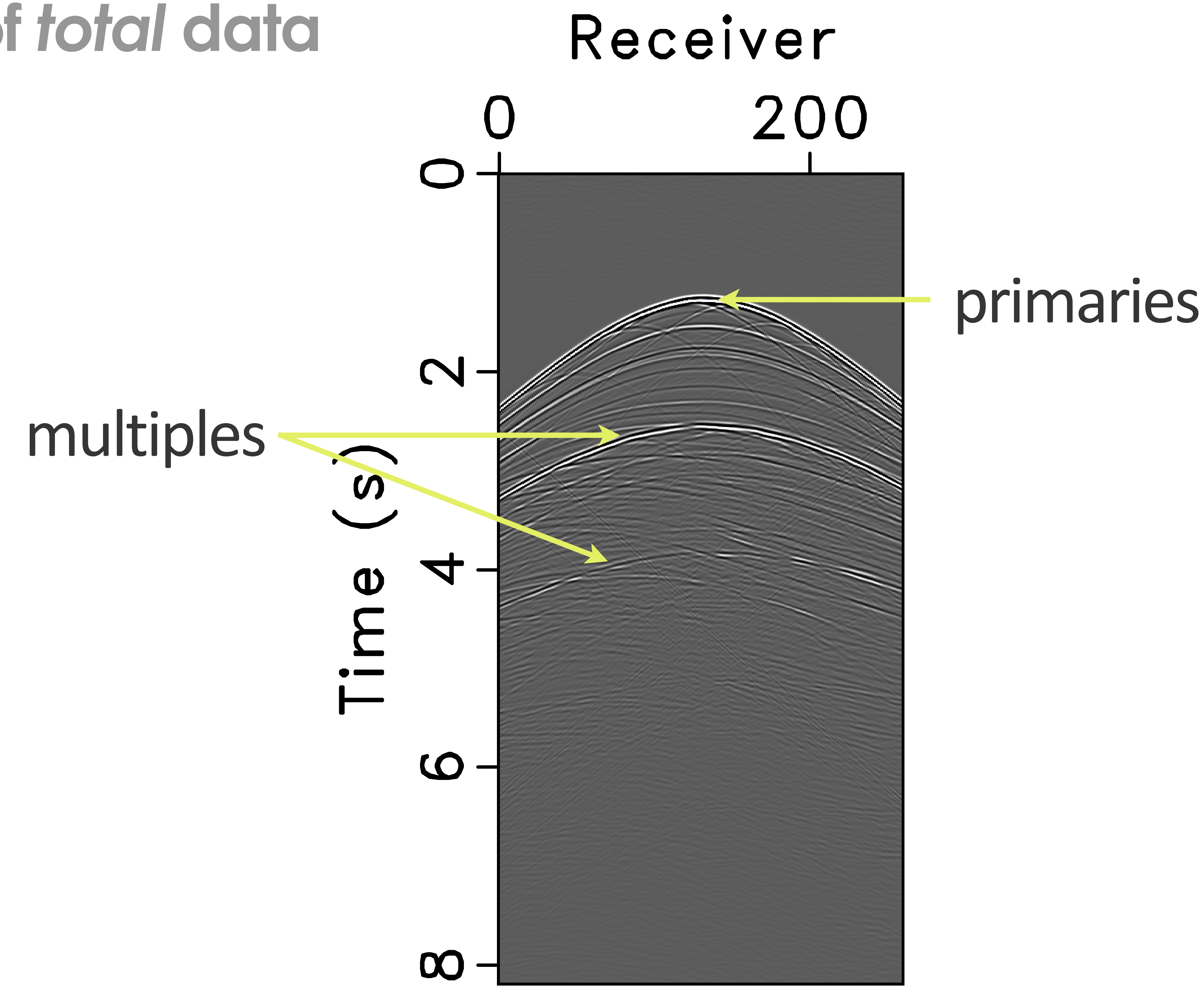
[w/ multiples]

What is the impact if we ignore *surface*-related multiples?

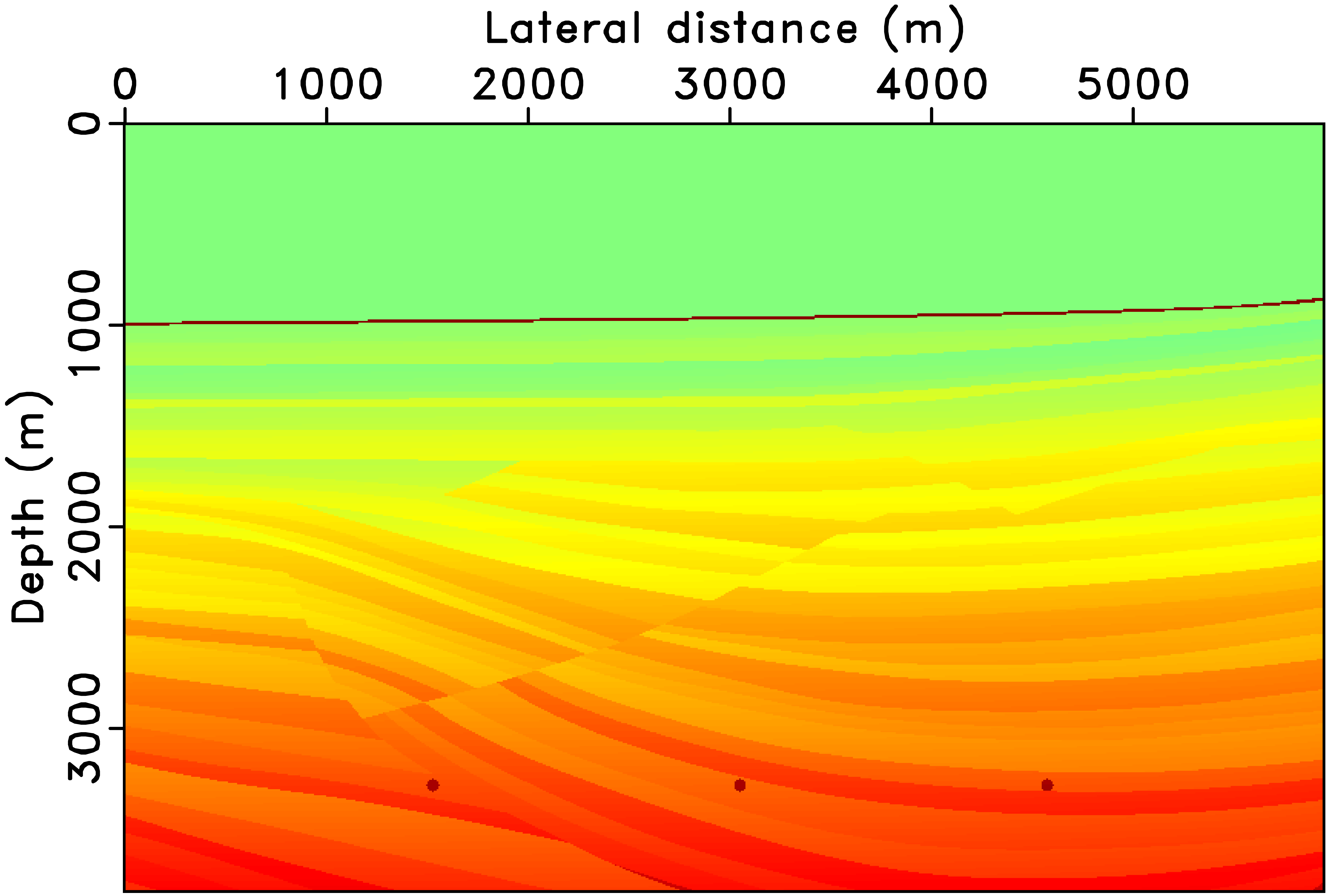
What are the advantages of inversion over RTM imaging?

Are there more potential enemies?

A shot-gather of *total* data

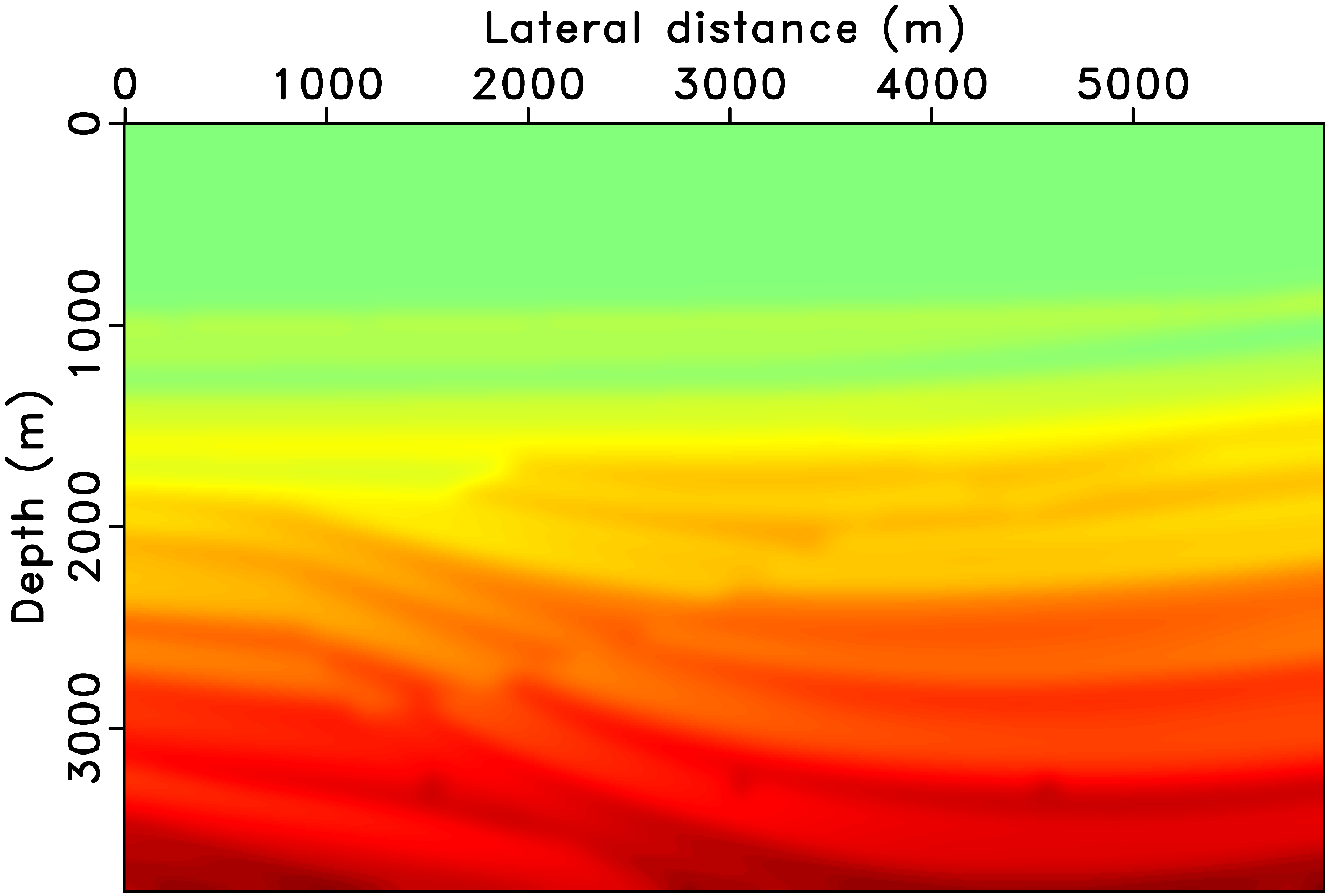


True model

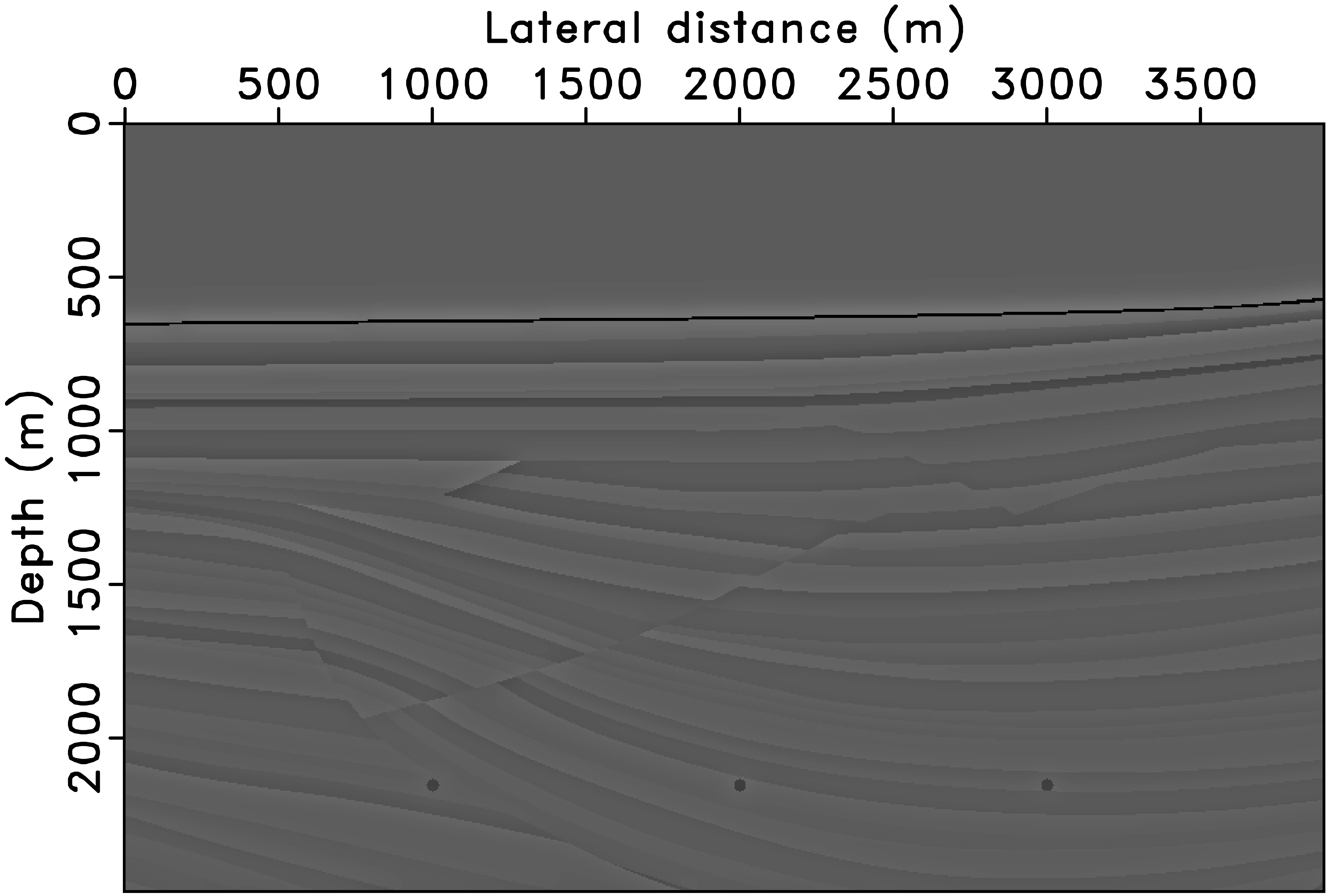


Lateral distance (m)
0
1000
2000
3000
4000
5000
Depth (m)
0
1000
2000
3000

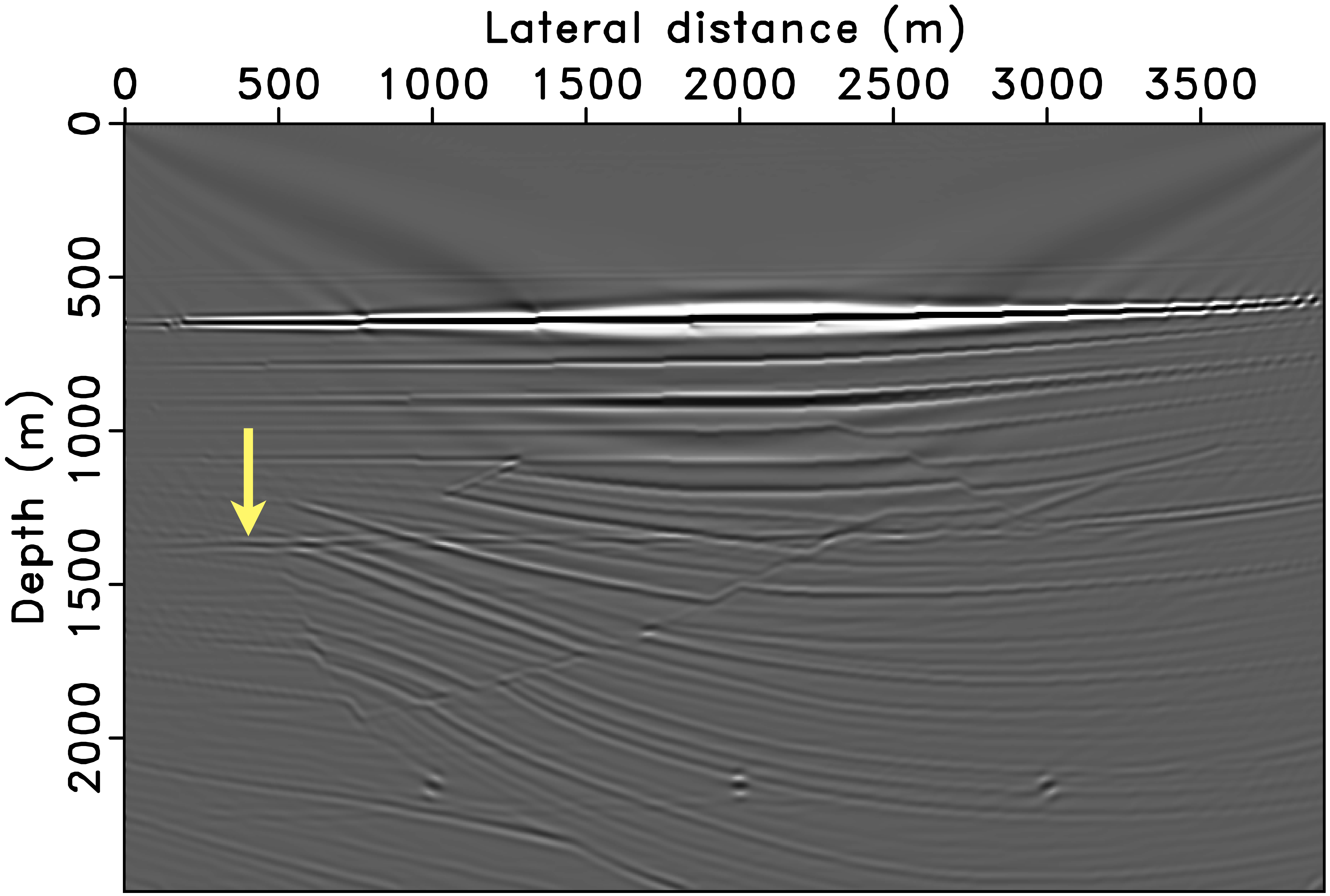
Background model



True model perturbation

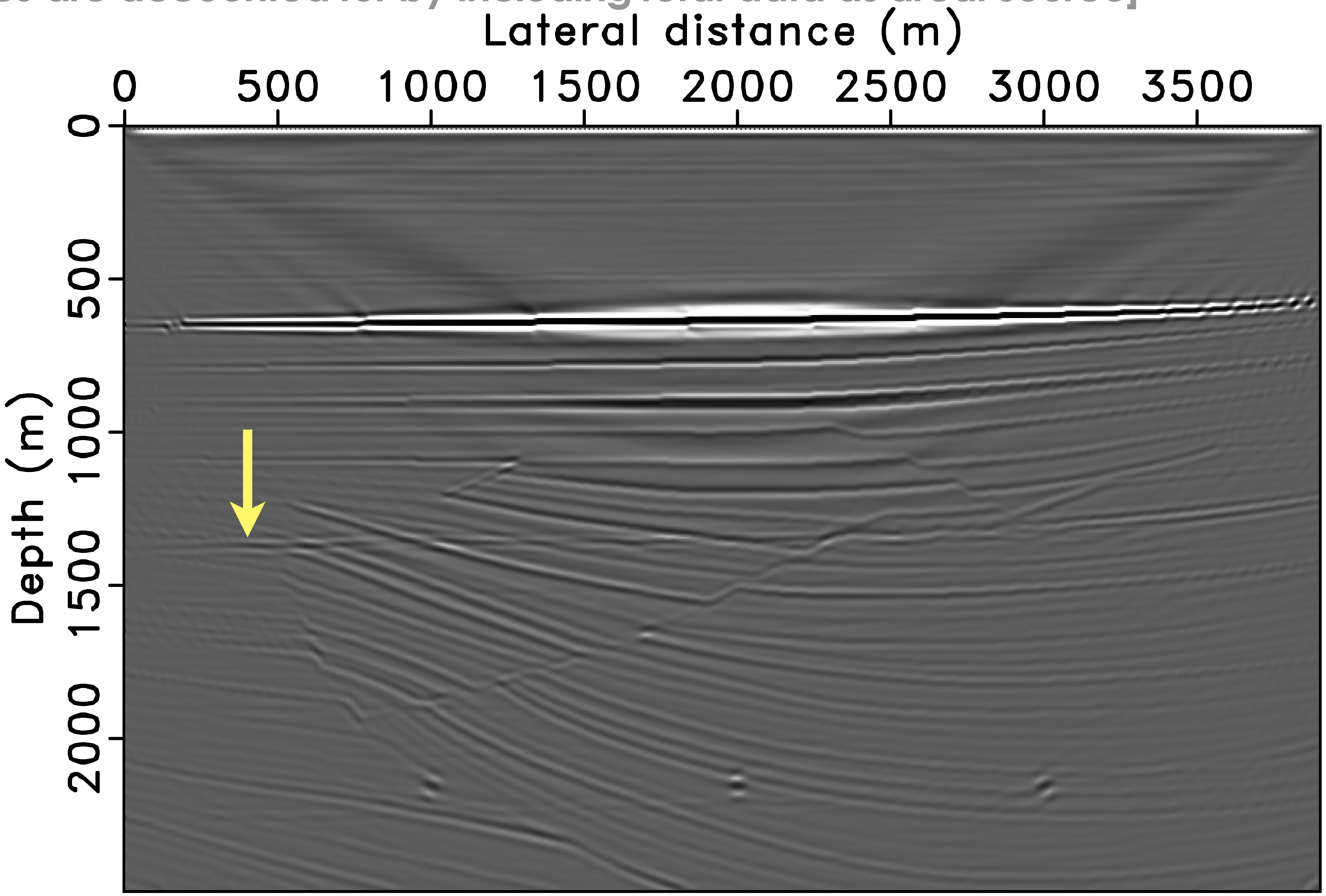


Conventional RTM image



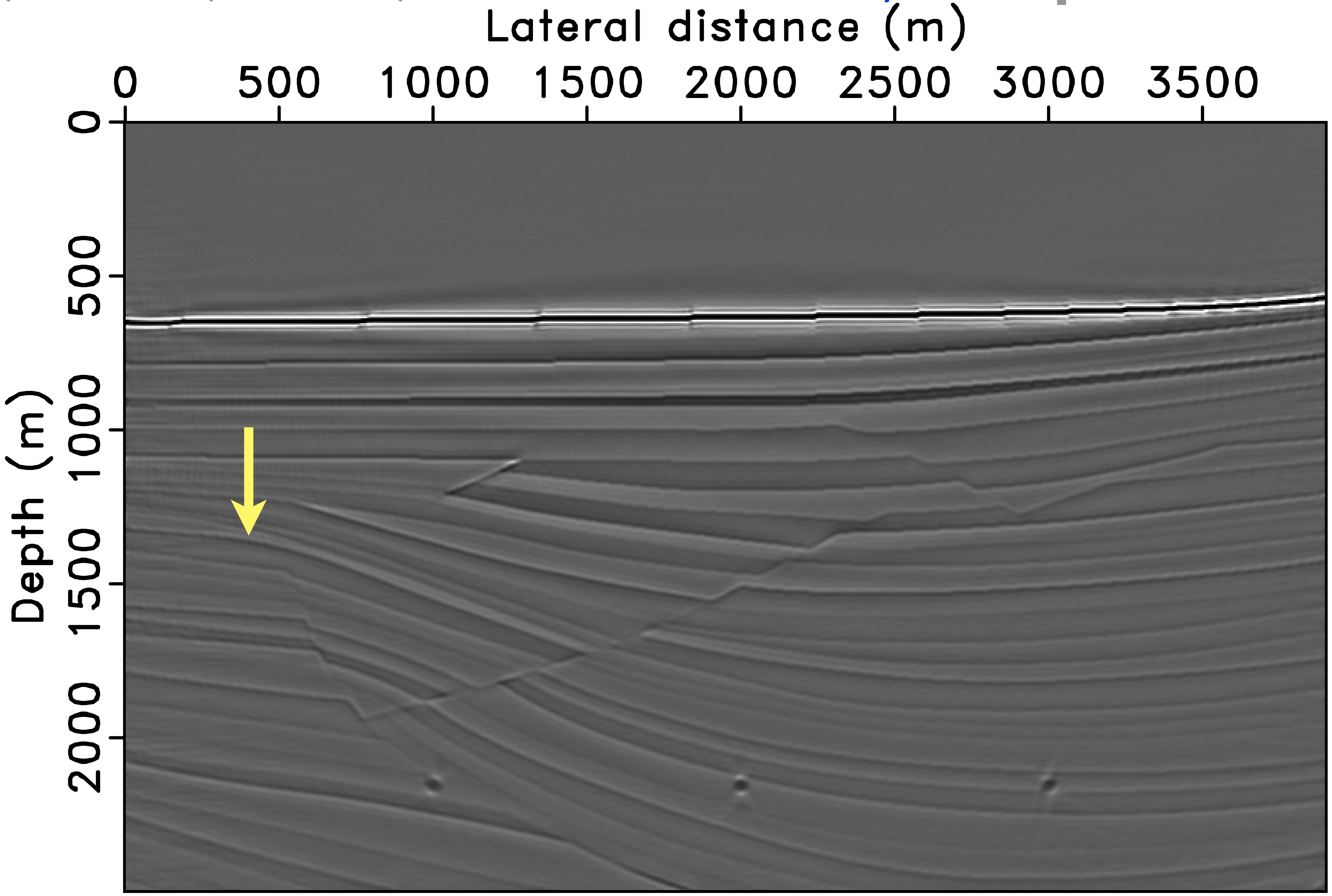
RTM image w/ *total* data

[*multiples* are accounted for by including *total* data as *areal* source]

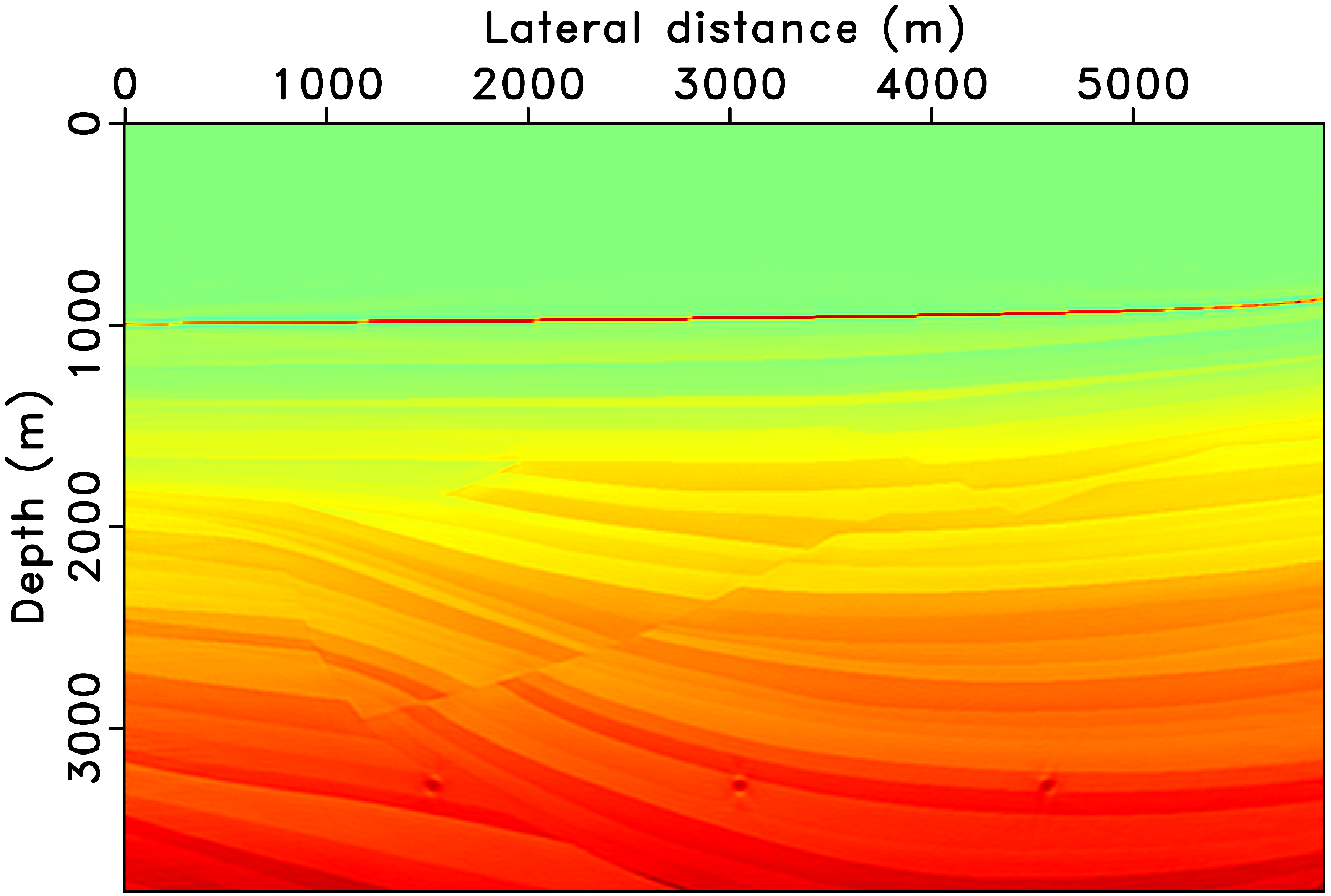


Fast inversion w/ *sparsity* promotion

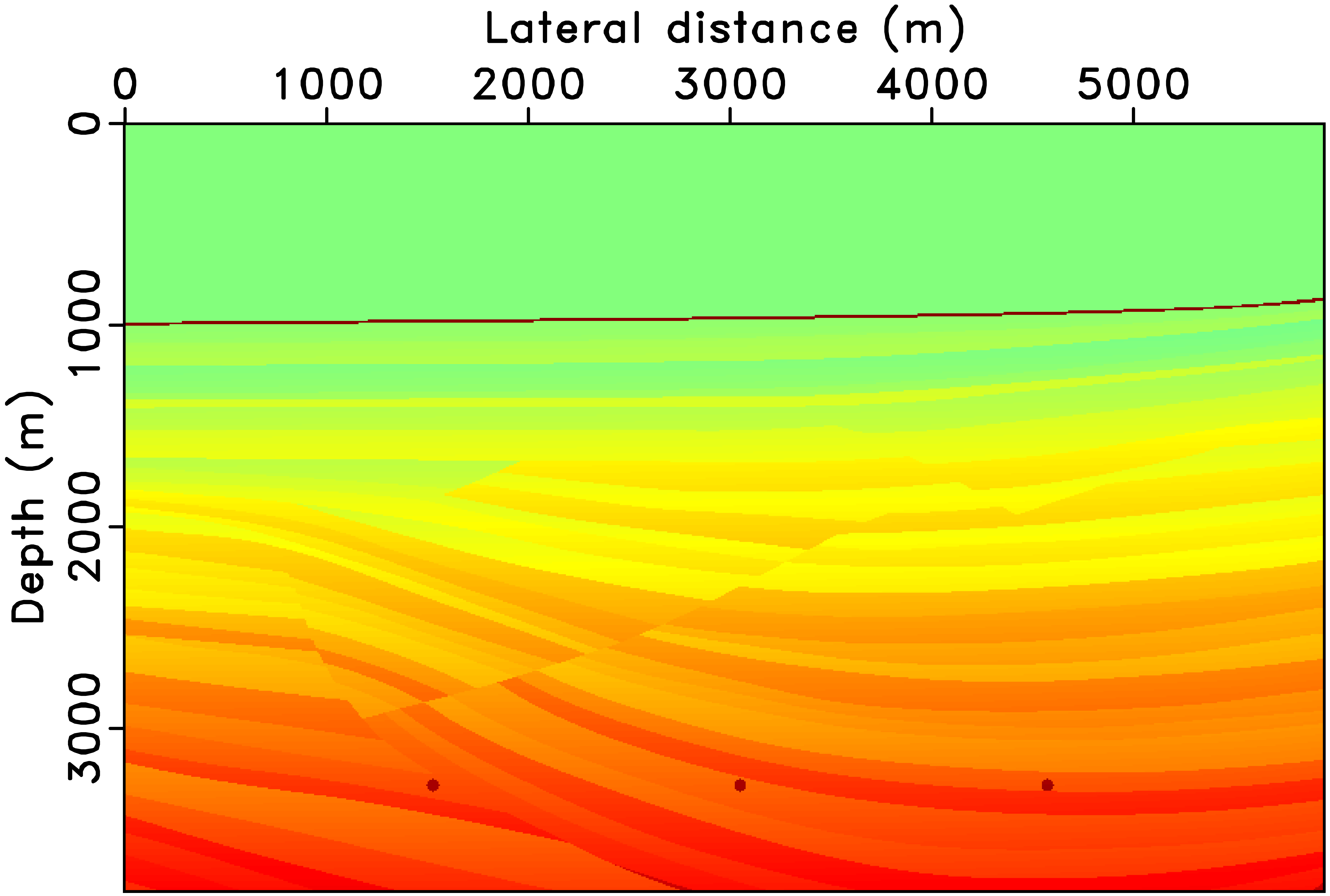
[15 freq., 8 sim. src., ~300 iter., simulation cost ~1 RTM w/ all data]



True-amplitude inversion



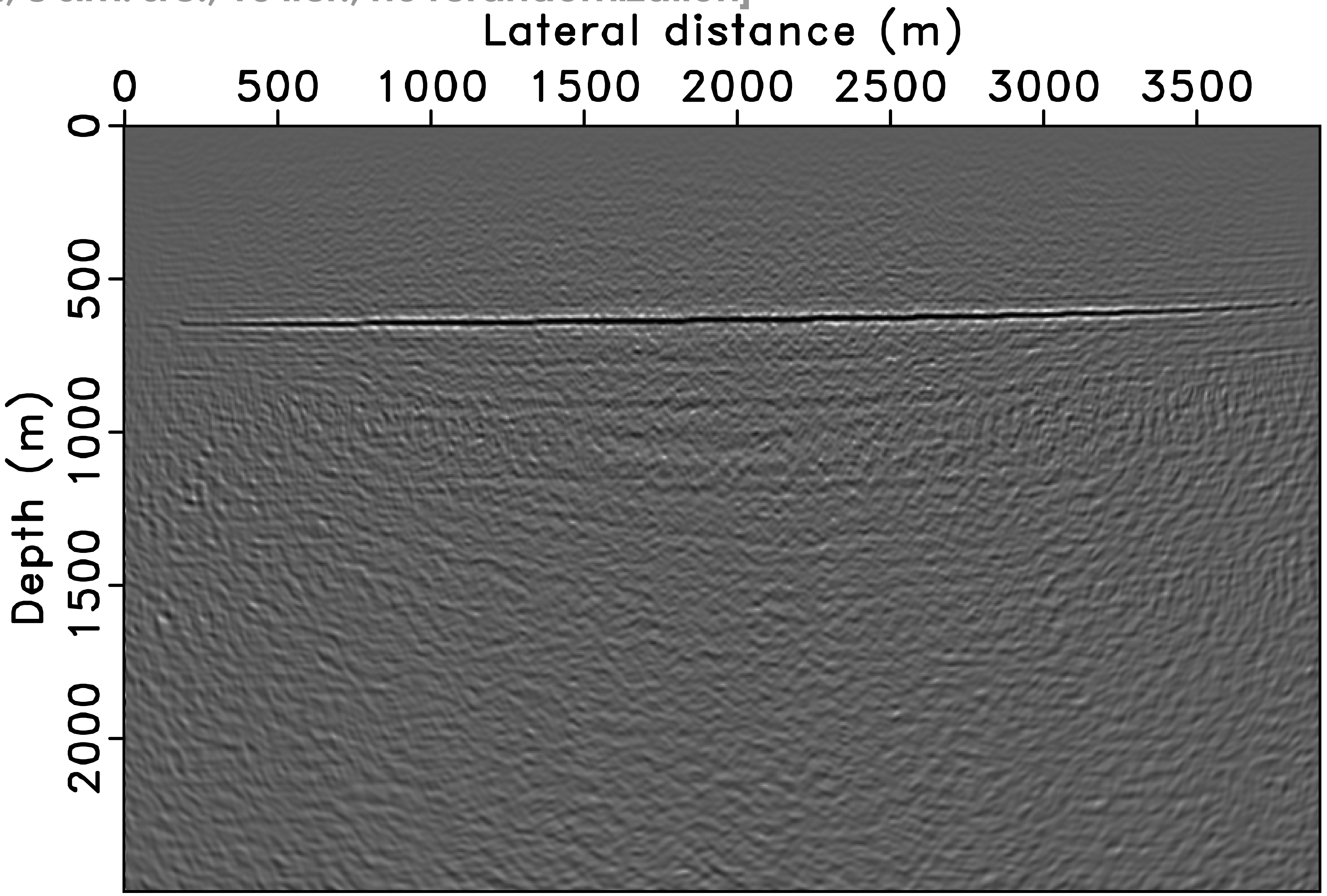
True model



Lateral distance (m)
0
1000
2000
3000
4000
5000
Depth (m)
0
1000
2000
3000

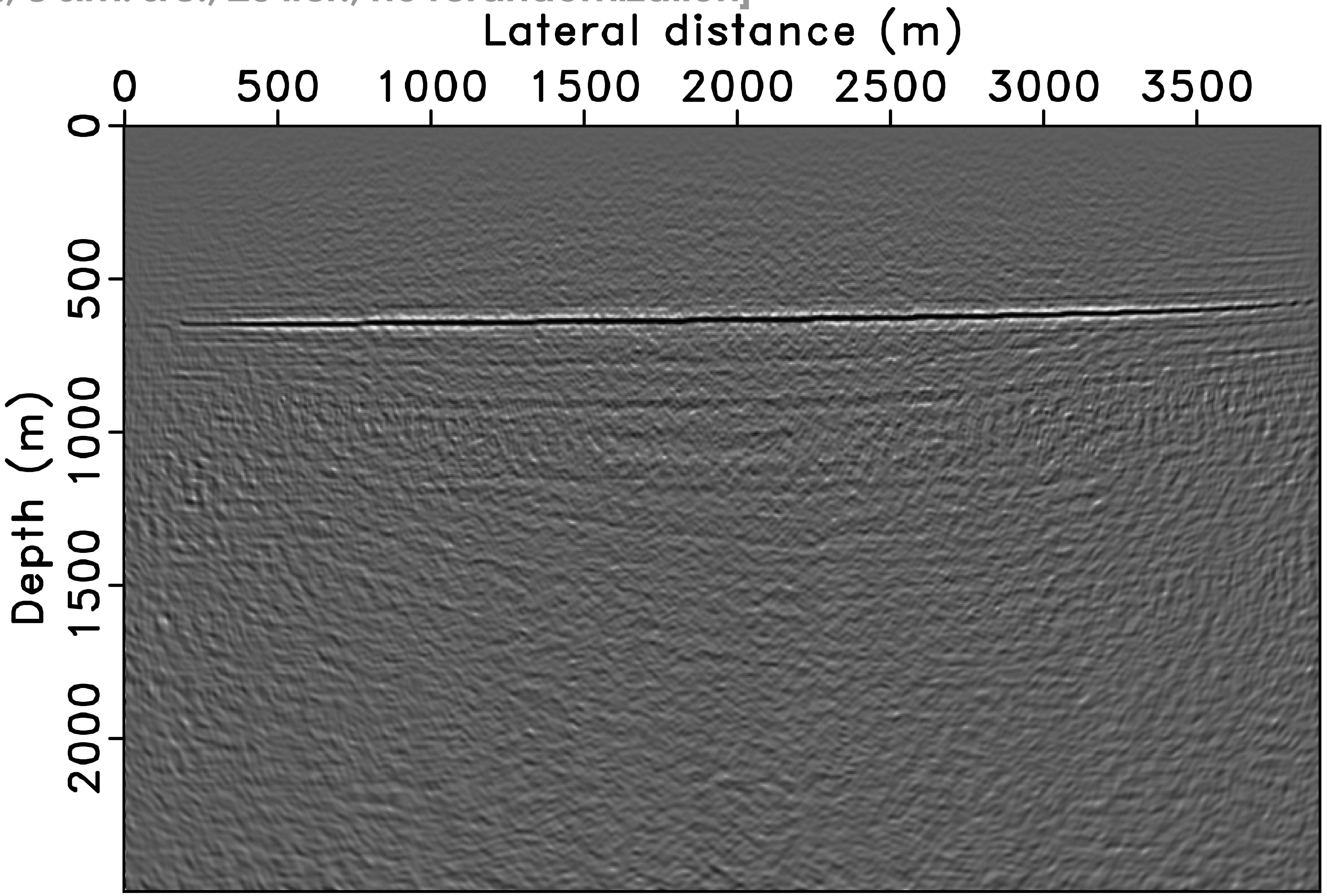
Inversion with L2 solver

[15 freq., 8 sim. src., 10 iter., no rerandomization]



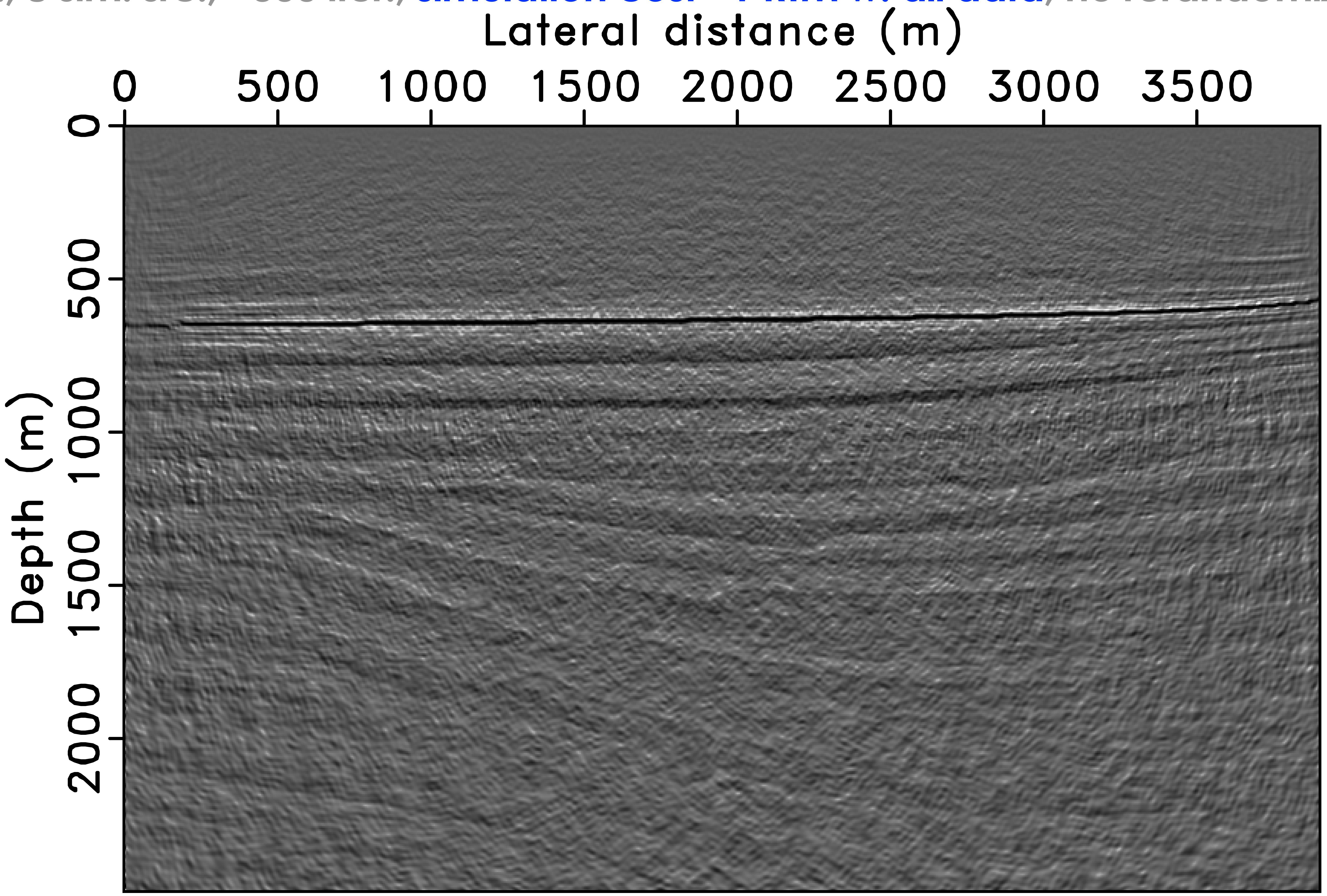
Inversion with L2 solver

[15 freq., 8 sim. src., 20 iter., no rerandomization]



Inversion with L2 solver

[15 freq., 8 sim. src., ~300 iter., simulation cost ~1 RTM w. all data, no rerandomization]



Imaging vs inversion w/ multiples

What is the impact if we ignore surface-related multiples?

- major because of the occurrence of coherent noise

What are the advantages of inversion over RTM imaging?

- remove cross terms from *areal* source

Are there more potential challenges?

- we need to know the source and velocity model
- computational costs could be an issue

Imaging vs inversion

[w/ multiples & source estimation]

Do surface related multiples help with source estimation?

Can we estimate the source during inversion w/ sufficient accuracy?

Does this improve the image?

Aravkin and van Leeuwen 2012
Aravkin et. al, 2013
Tu et. al, 2013

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Formulation

[w/ source estimation on the fly]

$$\min_{\mathbf{x},\mathbf{q}} \sum_{i \in \mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(q_i)]\mathbf{S}^*\mathbf{x}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau$$

- $\mathbf{q}$: frequency spectrum of the source wavelet, which is *unknown*

Wavelet estimation

Given an $\mathbf{x}$, a least-squares solution for $\mathbf{q}$ can be determined:

$$\tilde{q}_i(\mathbf{x}) = \frac{<\nabla\mathbf{F}_i[\mathbf{m}_0,\underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x},\mathbf{d}_i+\nabla\mathbf{F}_i[\mathbf{m}_0,\underline{\mathbf{D}}_i]\mathbf{S}^*\mathbf{x}>}{\|\nabla\mathbf{F}_i[\mathbf{m}_0,\underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x}\|_2^2}$$

By variable projection, we now solve:

$$\min_{\mathbf{x}} \sum_{i\in\mathbb{F}} \|\underline{\mathbf{d}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{Q}}(\tilde{q}_i(\mathbf{x}))]\mathbf{S}^*\mathbf{x}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau$$

using the same root-finding algorithm as standard SPGl1 to determine the value of τ.

Pseudo-code

Input:

total upgoing wavefield $\mathbf{D}$, initial model $\mathbf{m}_0$, tolerance σ

Initialization:

$k \leftarrow 0, \mathbf{x}_k \leftarrow \mathbf{0}, \mathbf{q}_k \leftarrow \mathbf{1}$

while not converged **do**

$k \leftarrow k+1$

$\mathbf{RM} \leftarrow$ Draw new $(\mathbf{RM})$, $\underline{\mathbf{d}} \leftarrow \mathbf{RMd}$, $\underline{\mathbf{Q}}(\mathbf{q}) \leftarrow \text{invvec}(\mathbf{RM}\text{vec}(\mathbf{Q}(\mathbf{q})))$

$\tau_k \leftarrow$ determine from τ_{k-1} and σ by root finding on the Pareto curve

$$\mathbf{x}_k \leftarrow \begin{cases} \text{minimize } \|\underline{\mathbf{d}} - \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{Q}}(\mathbf{q})]\mathbf{S}^*\mathbf{x}\|_2^2 \\ \text{subject to } \|\mathbf{x}\|_1 \leq \tau_k \end{cases}$$

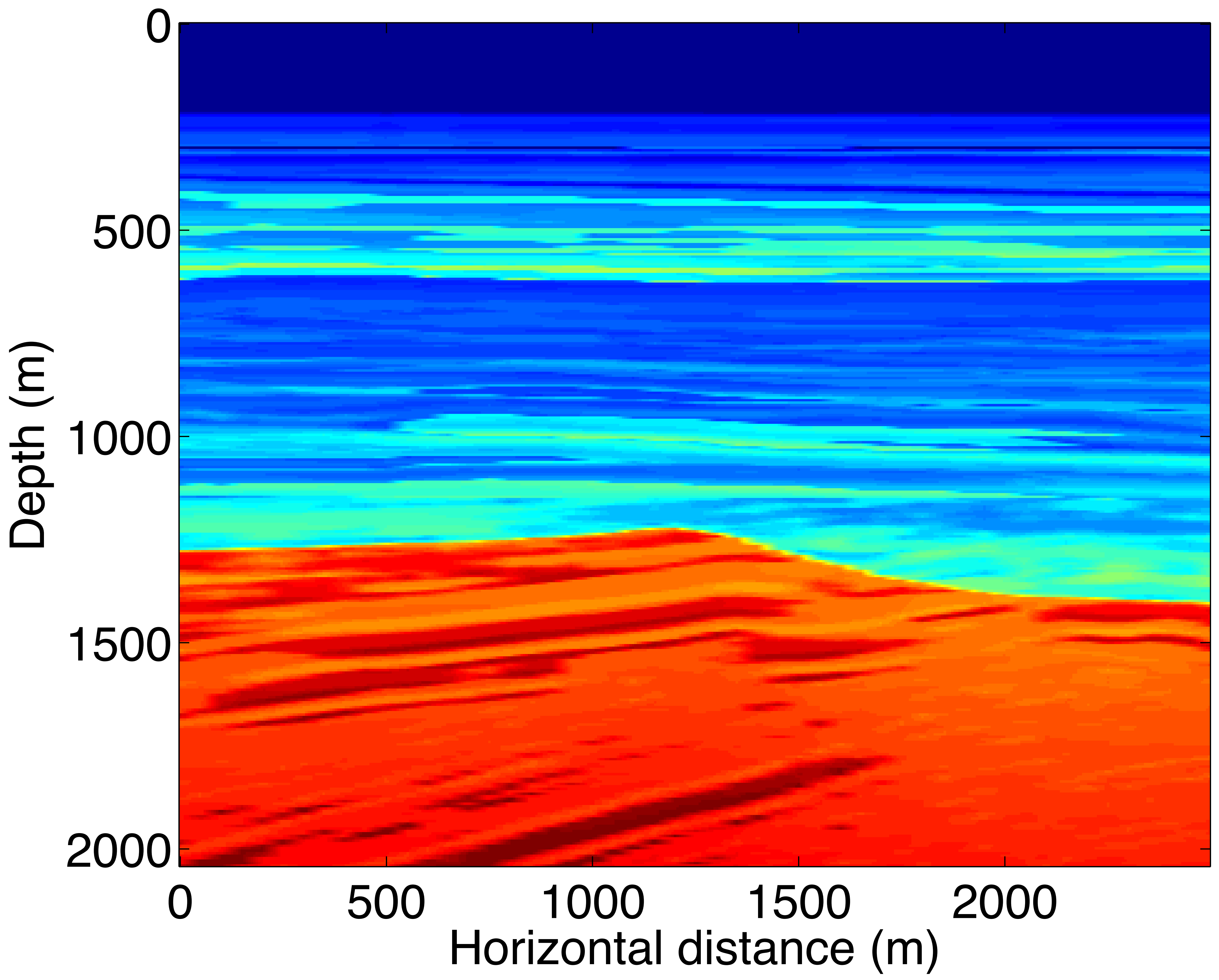
//warm start with $\mathbf{x}_{k-1}$

For each frequency i, compute $q_i(\mathbf{x}_k) = \frac{<\nabla\mathbf{F}_i[\mathbf{m}_0,\underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x},\mathbf{d}_i+\nabla\mathbf{F}_i[\mathbf{m}_0,\underline{\mathbf{D}}_i]\mathbf{S}^*\mathbf{x}>}{\|\nabla\mathbf{F}_i[\mathbf{m}_0,\underline{\mathbf{I}}]\mathbf{S}^*\mathbf{x}\|_2^2}$

end while

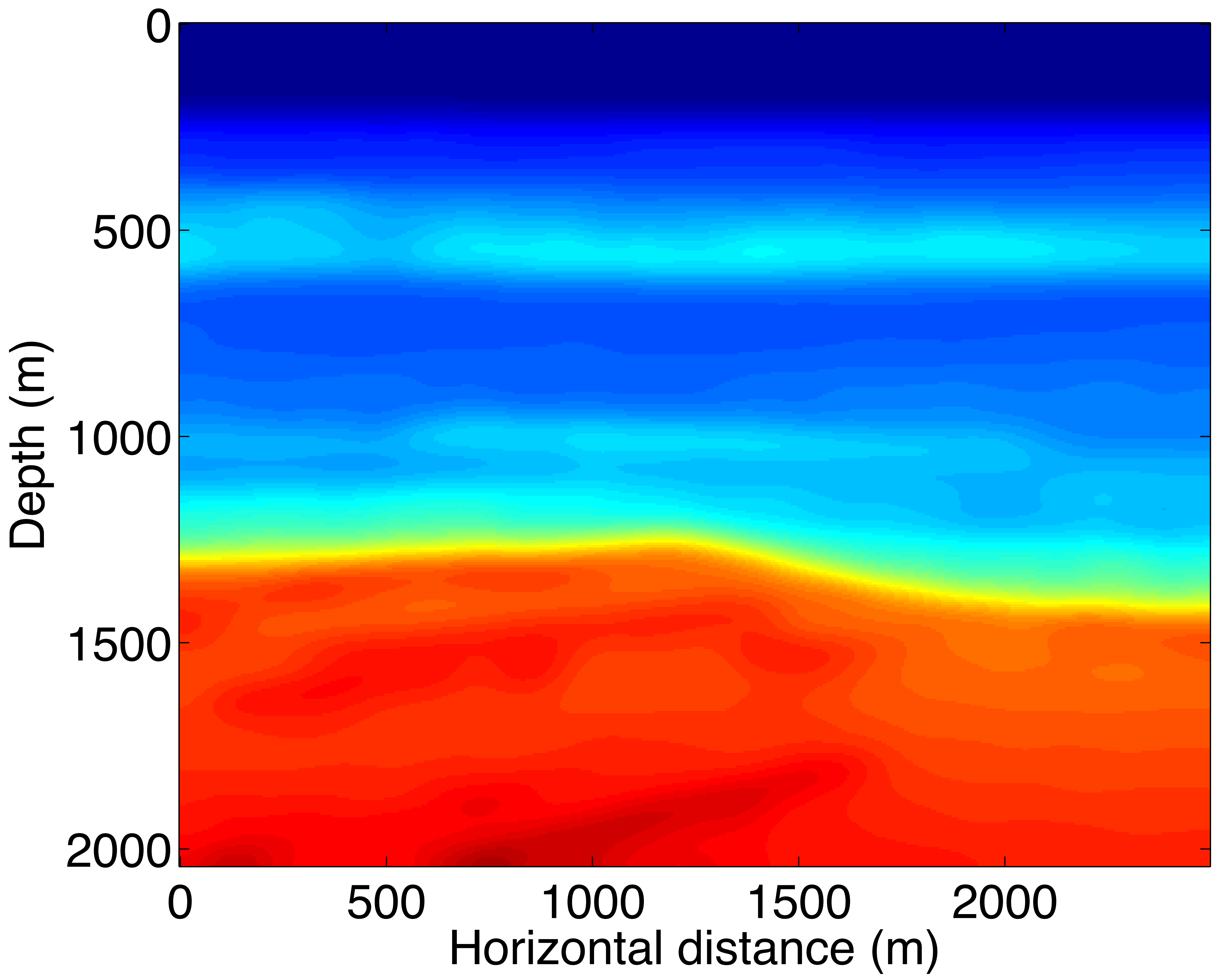
Output: Model perturbation estimate $\delta\mathbf{m} = \mathbf{S}^*\mathbf{x}$

True model

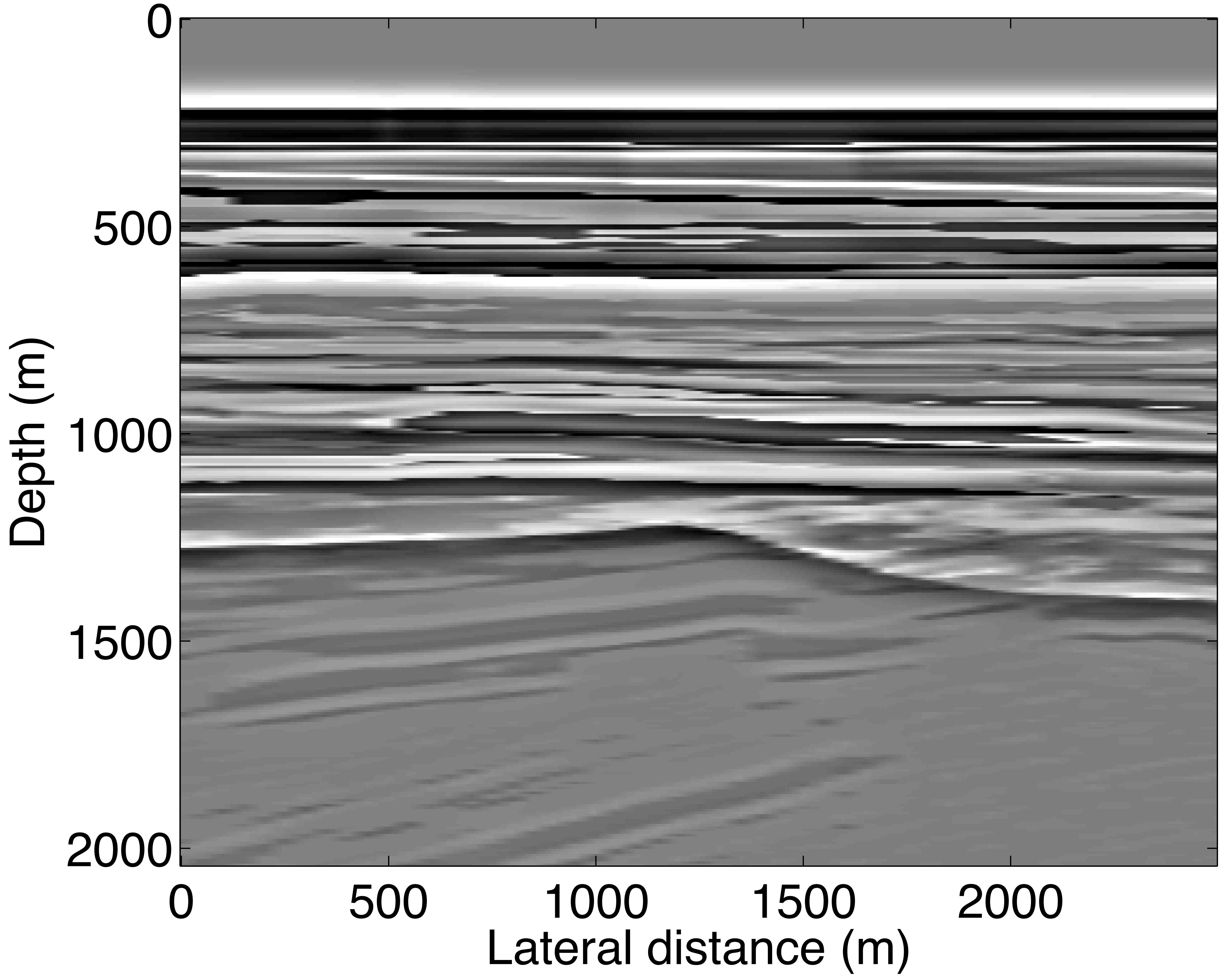


0
500
1000
1500
2000
Depth (m)
0
500
1000
1500
2000
Horizontal distance (m)

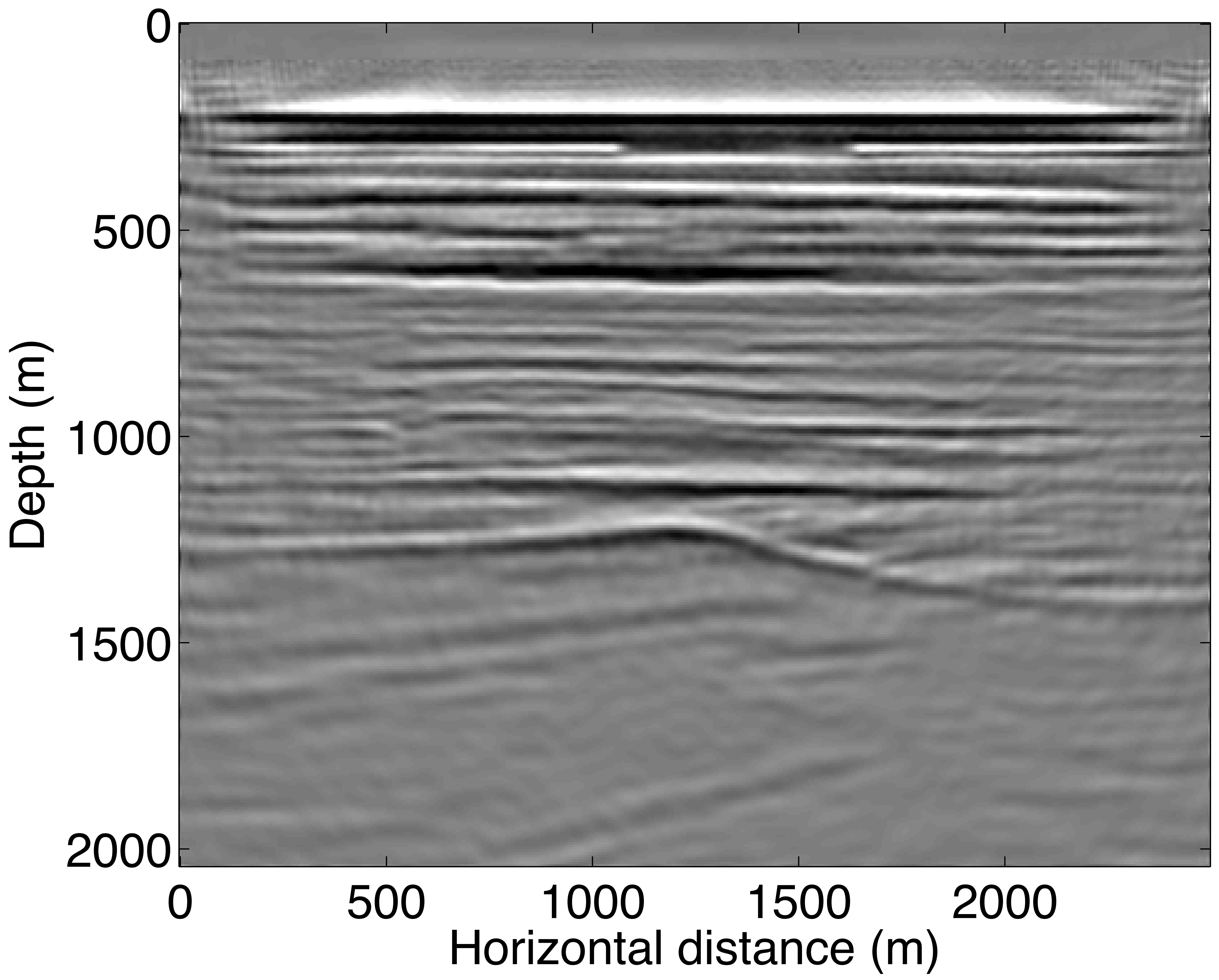
Background model



True model perturbation

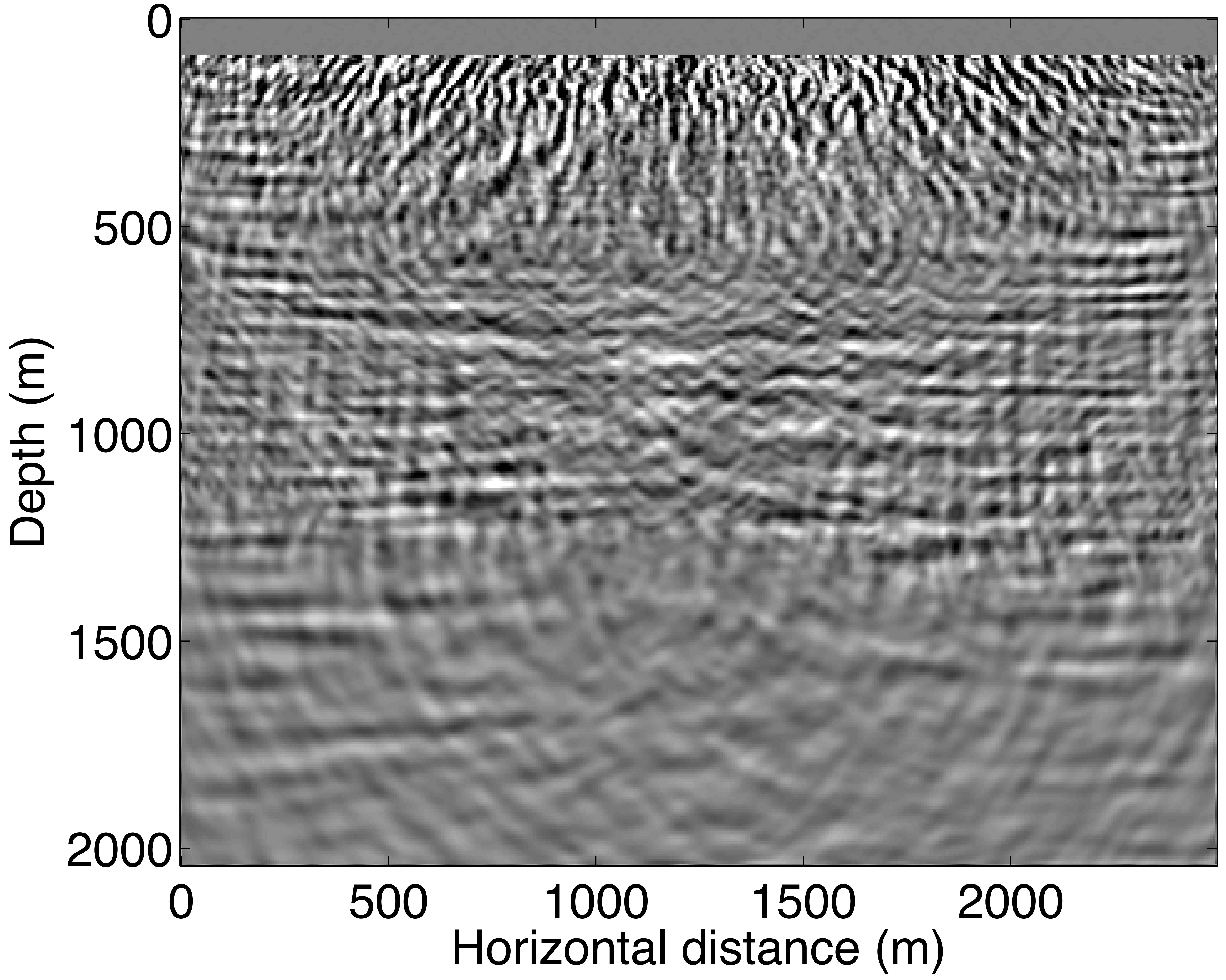


Inversion with the true wavelet



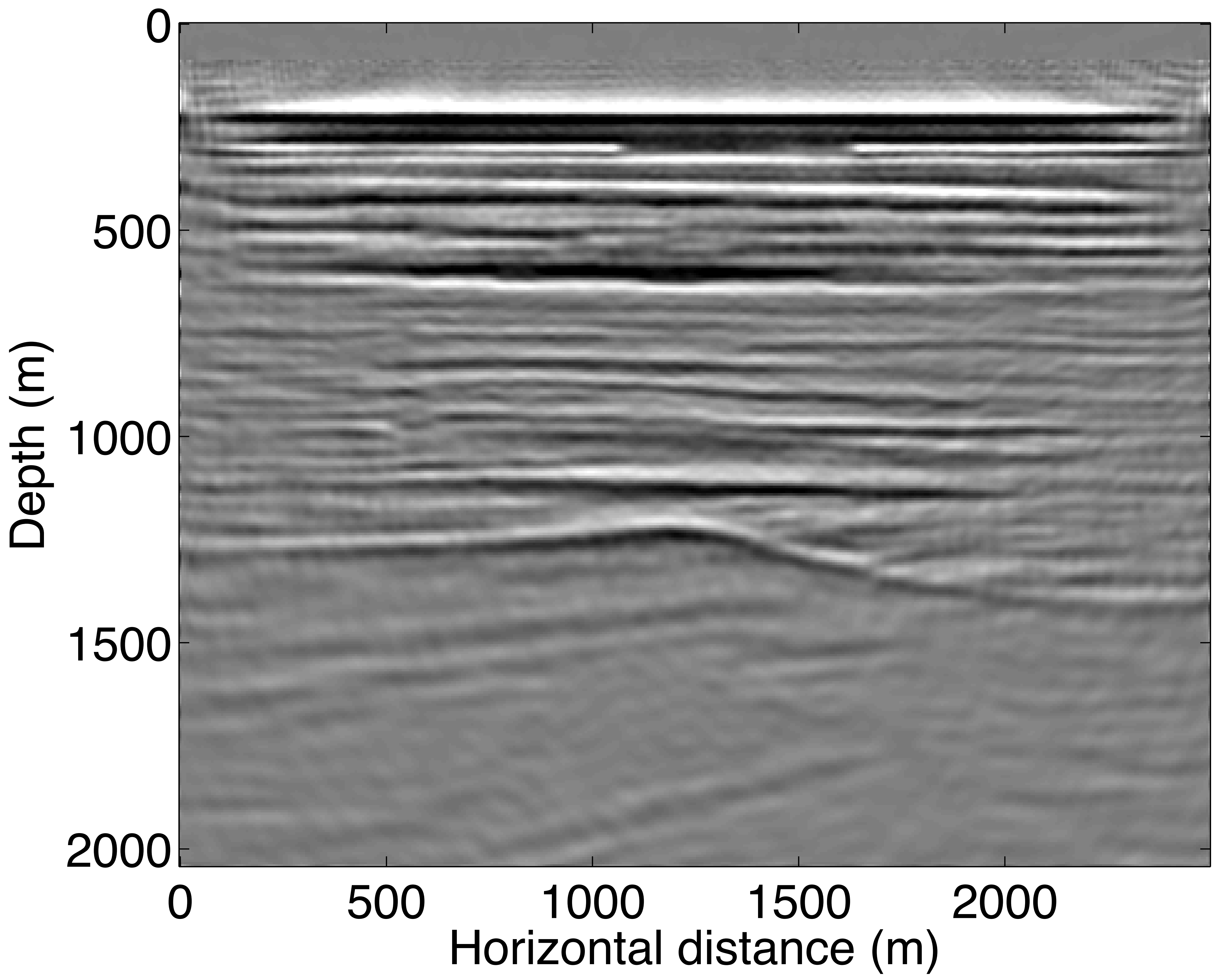
Inversion with a wrong wavelet

[no rerandomization, *primary* data, wavelet simply an impulse at t=0]



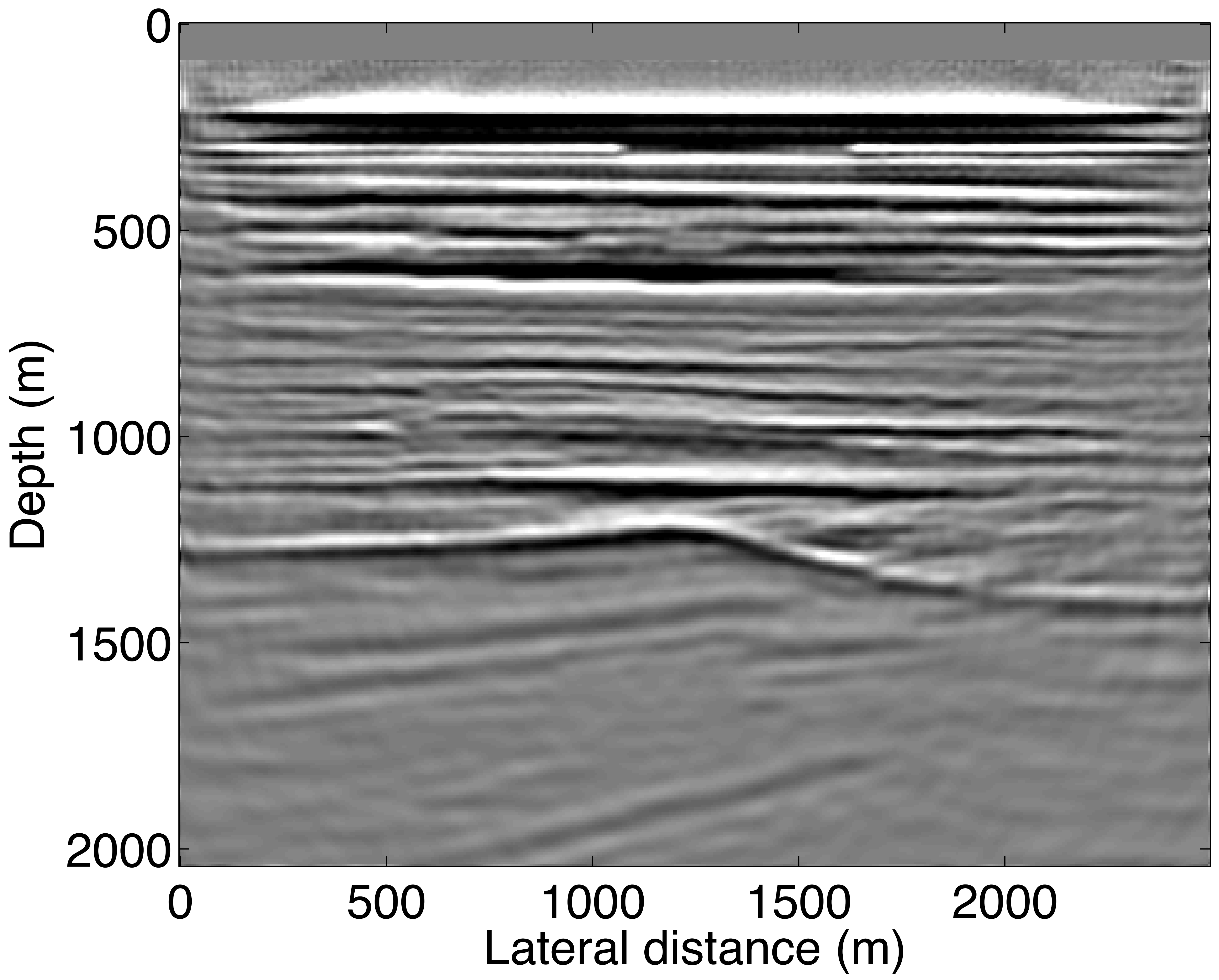
Inversion with source estimation

[no rerandomization, *primary* data, initial wavelet guess simply an impulse at t=0]



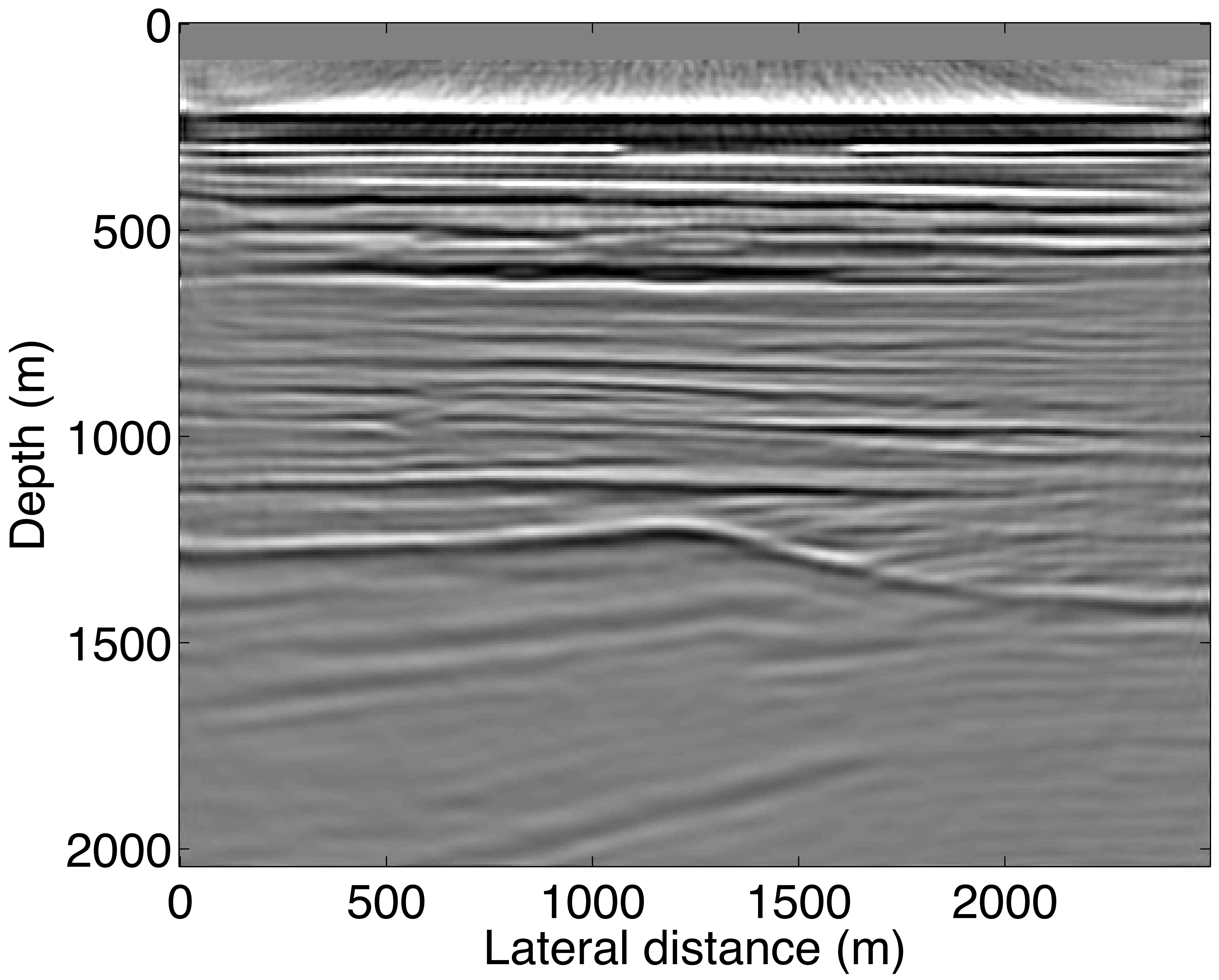
Inversion with source estimation

[with rerandomization, *primary* data, initial wavelet guess simply an impulse at t=0]

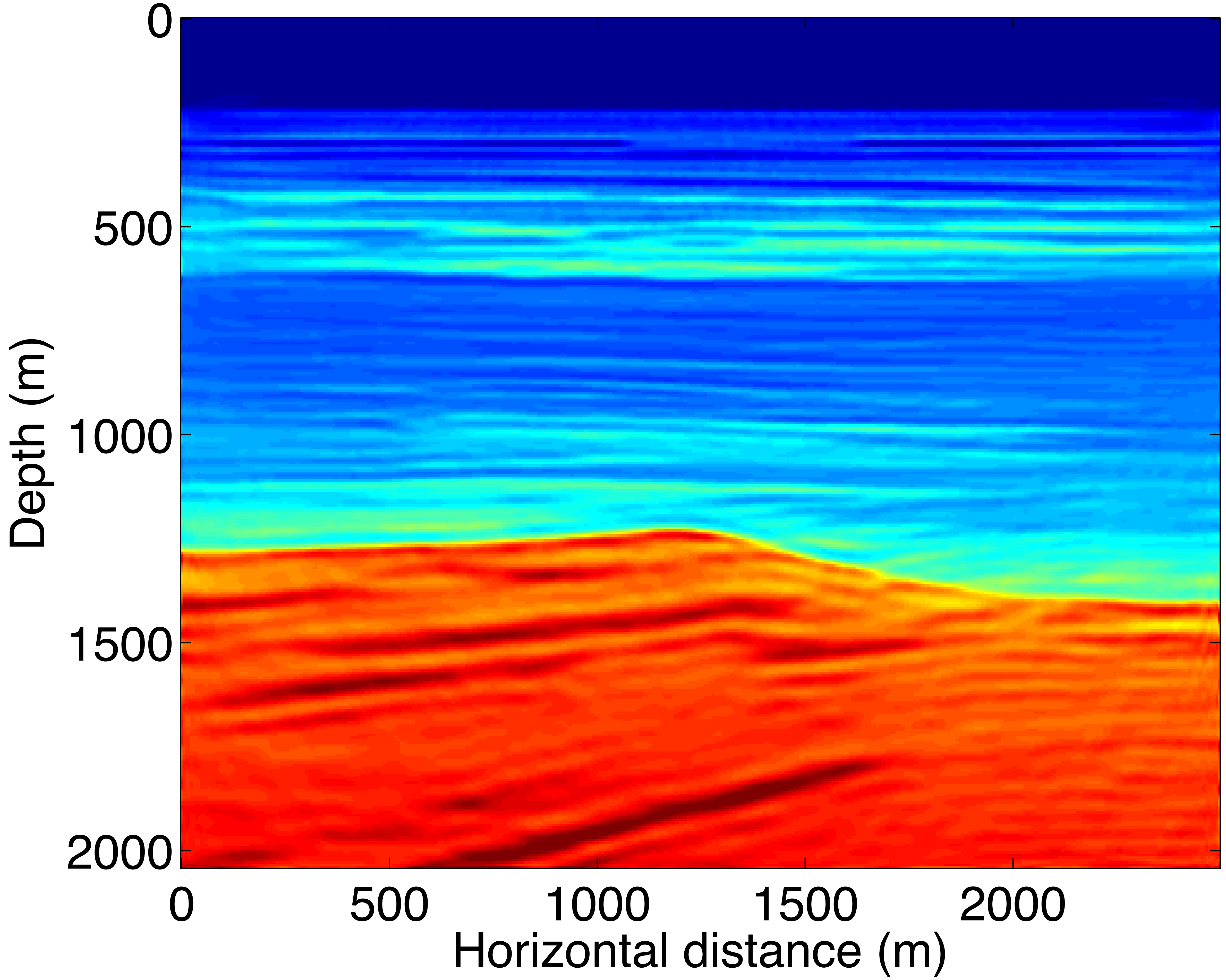


Inversion with source estimation

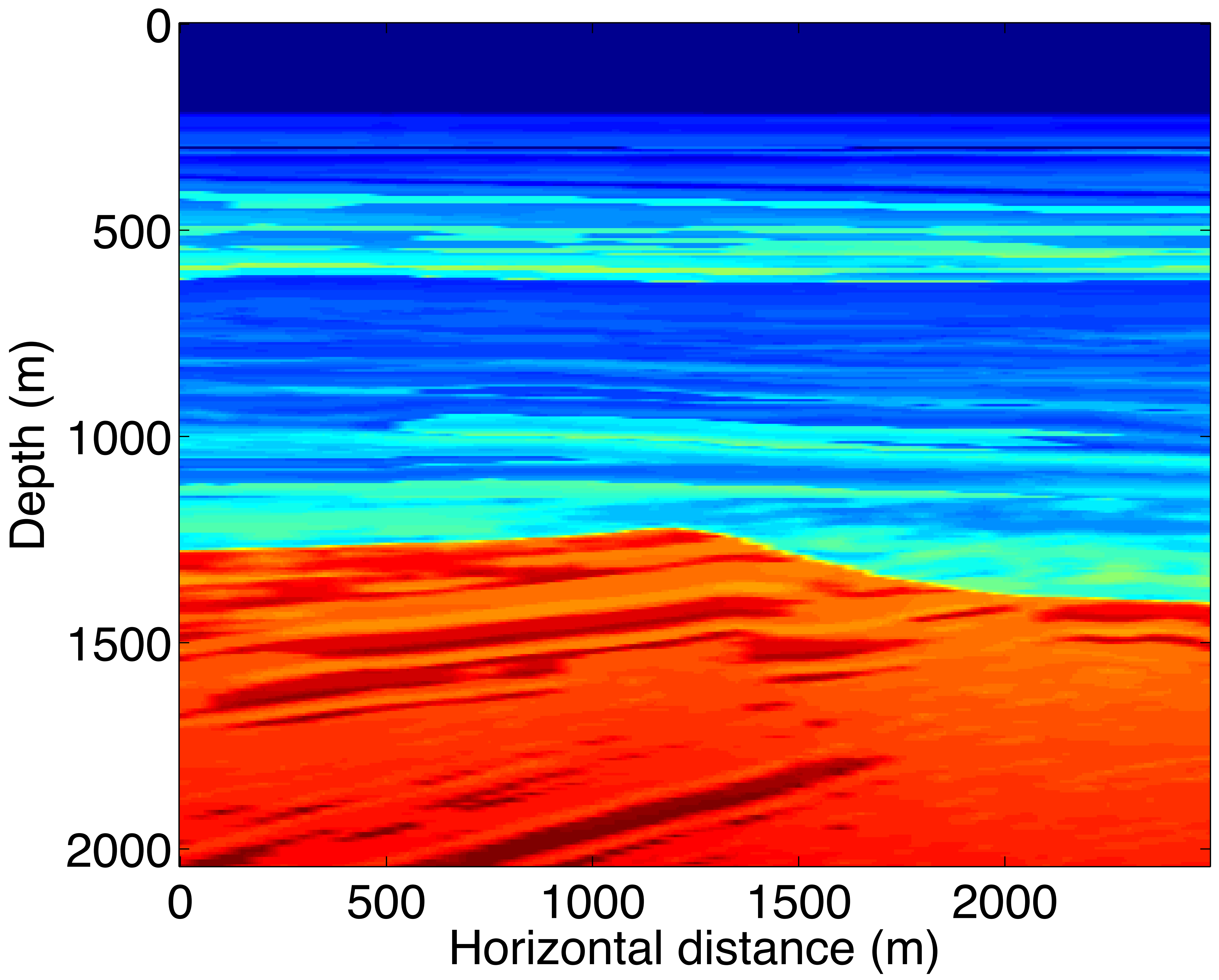
[with rerandomization, *total* data, initial wavelet guess simply an impulse at t=0]



REAL true-amplitude inversion w/o knowledge of the true source [adding inversion result w. multiples back to smooth model, *no* rescaling whatsoever]

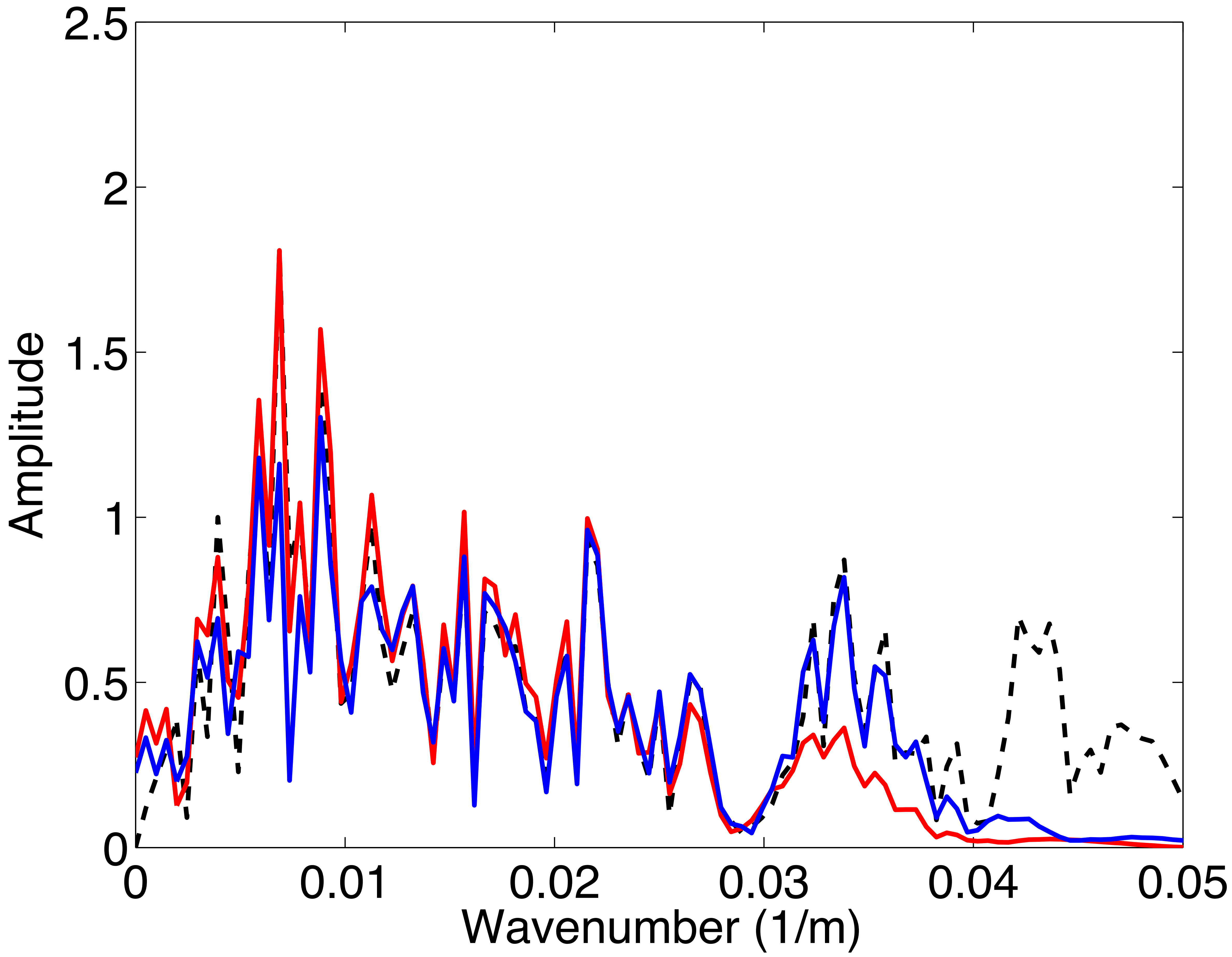


True model



0
500
1000
1500
2000
Depth (m)
0
500
1000
1500
2000
Horizontal distance (m)

Wavenumber contents



Black: true; Blue: w/ multiple; Red: primaries only (rescaled)

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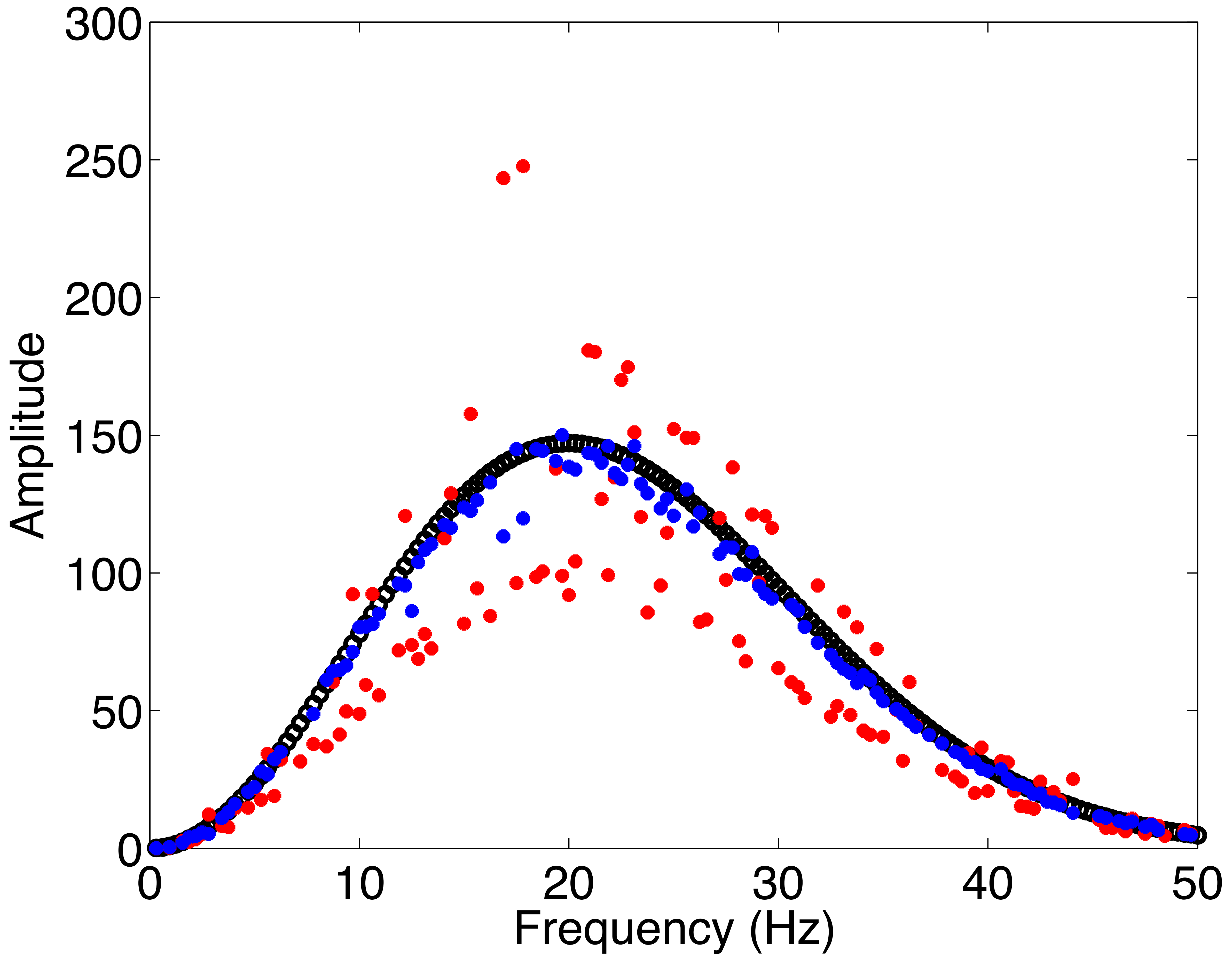
Estimated wavelet: amplitude

[with vs. without using multiples, with rerandomization, with source estimation]



Black: true; Blue: w. multiple; Red: primaries only (rescaled)

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Imaging vs inversion

[w\ multiples & source estimation]

Do surface related multiples help with source estimation?

- yes, because source appears only for the primary data & multiples improve illumination

Can we estimate the source during inversion w/ sufficient accuracy?

- yes, as long we do this on the fly using variable projection

Does this improve the image?

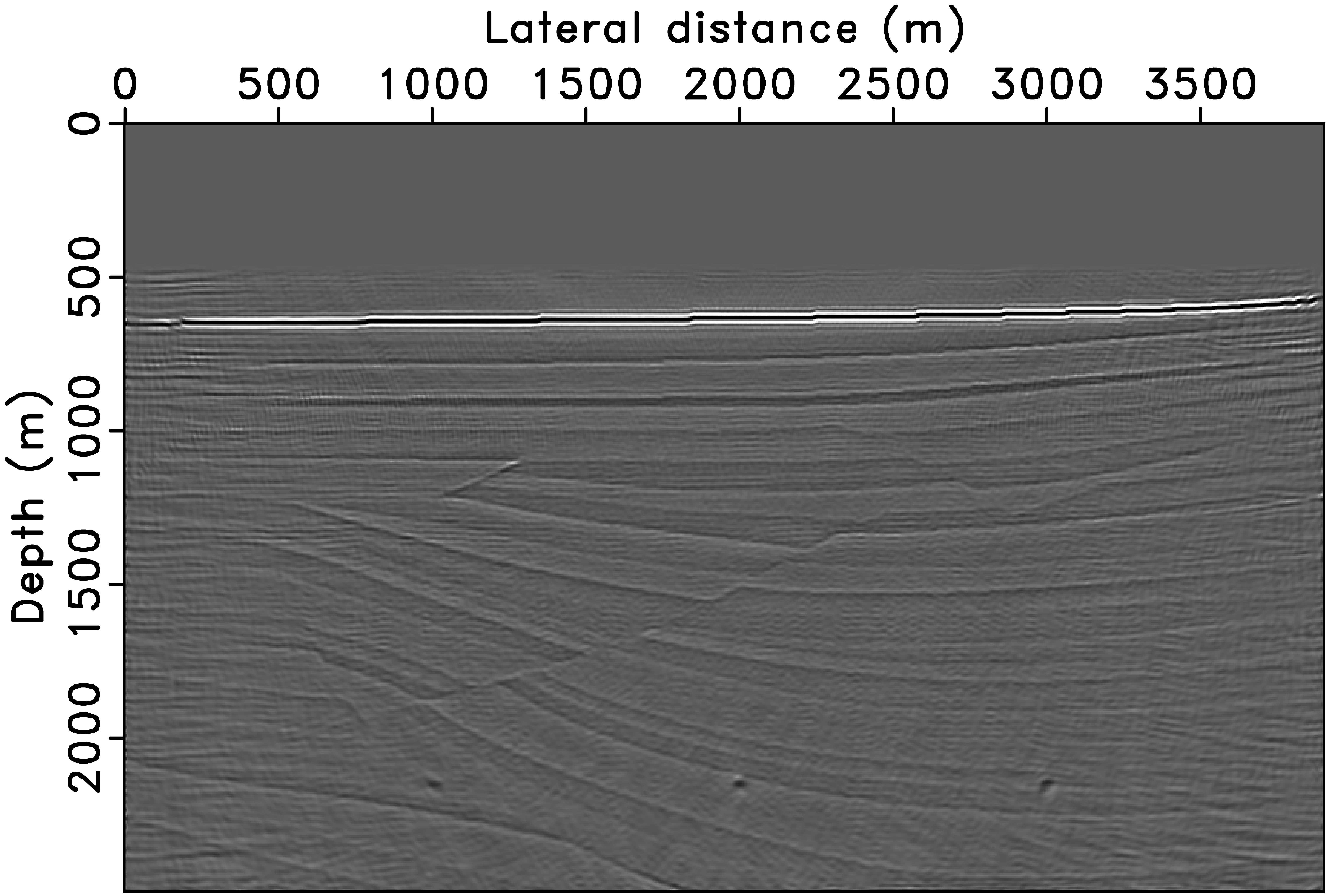
- yes

Ongoing and future work

1. Refining the inversion algorithm w. source estimation for realistic seismic data, to also take care of density variations (the Sigsbee 2B model relies on a high velocity contrast at the ocean bottom to generate strong surface multiples).

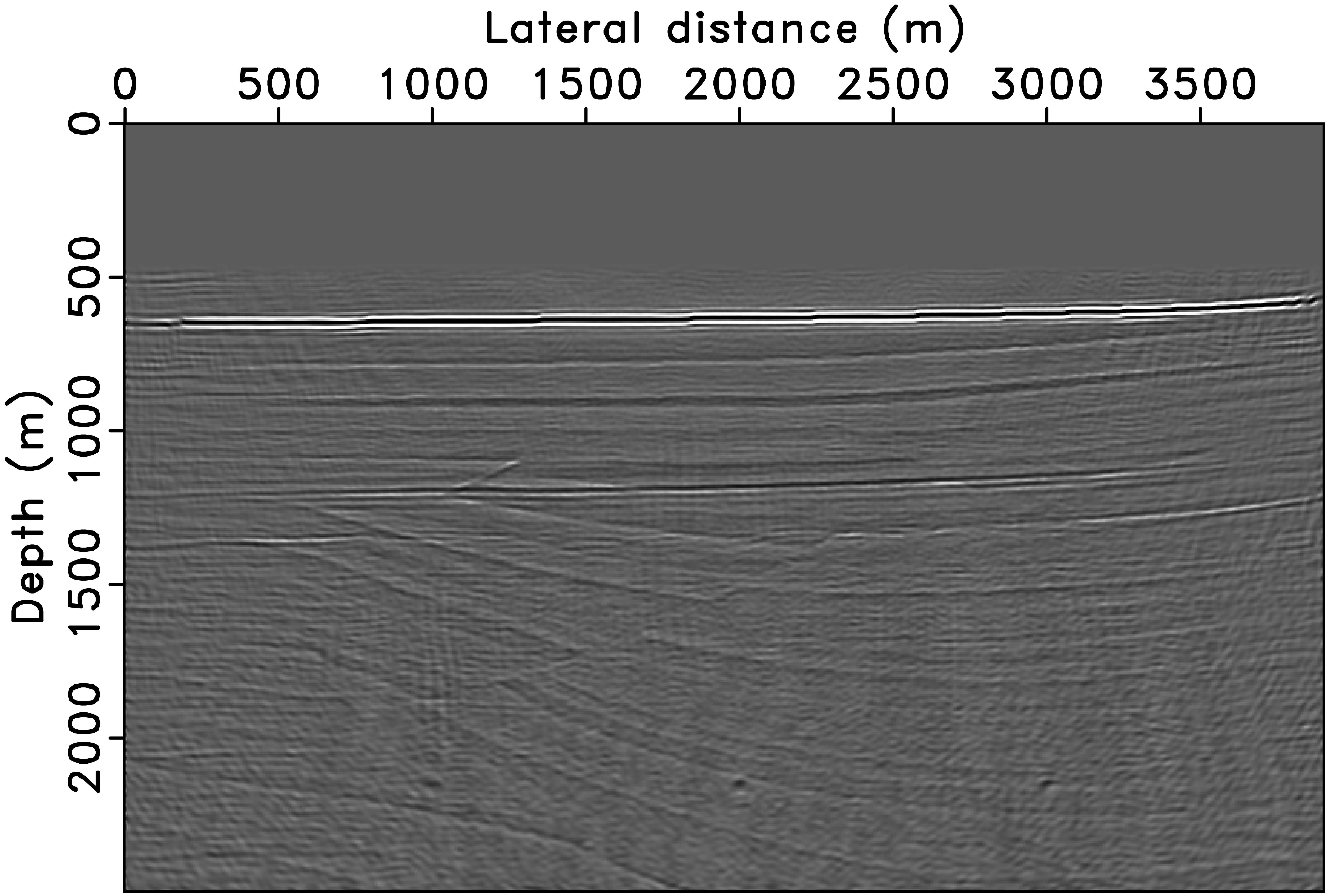
2. Demonstrate the benefits of the algorithm on coarsely sampled seismic data, and where the sources and receivers are not co-located.

Fast imaging w. multiples w. source estimation on data modelled w. free surface



NON-inverse crime

Fast imaging w. multiples w. source estimation on *forward modelling* data [multiples are ignored]



Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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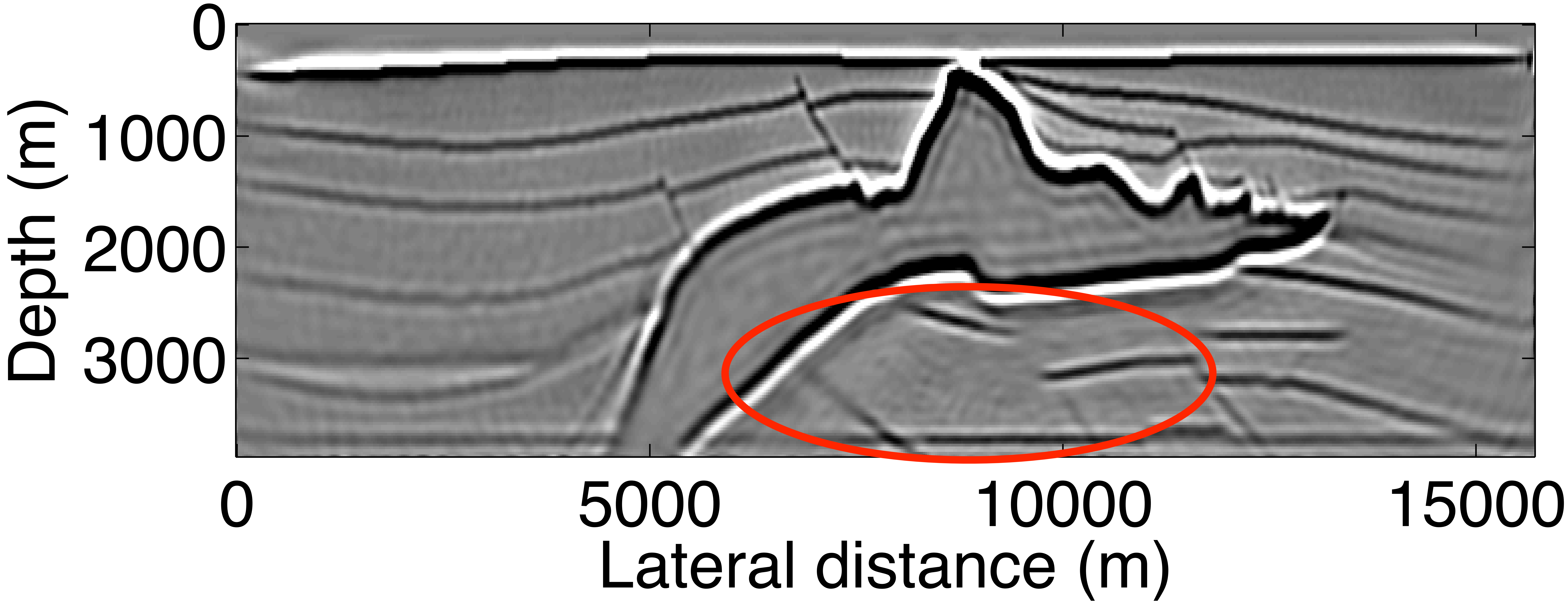
Ning Tu, Aleksandr Y. Aravkin, Tristan van Leeuwen, and Felix J. Herrmann, “Fast least-squares migration with multiples and source estimation”, EAGE technical program Expanded Abstracts, 2013

Slides for discussion

In the first set of examples, we still see some artifacts in the subsalt area of the inverted image. What are those artifacts? We think they should arise from internal multiples due to strong contrast caused by the salt. If we do the same inversion to linearized data (internal multiples free), we do not have those artifacts.

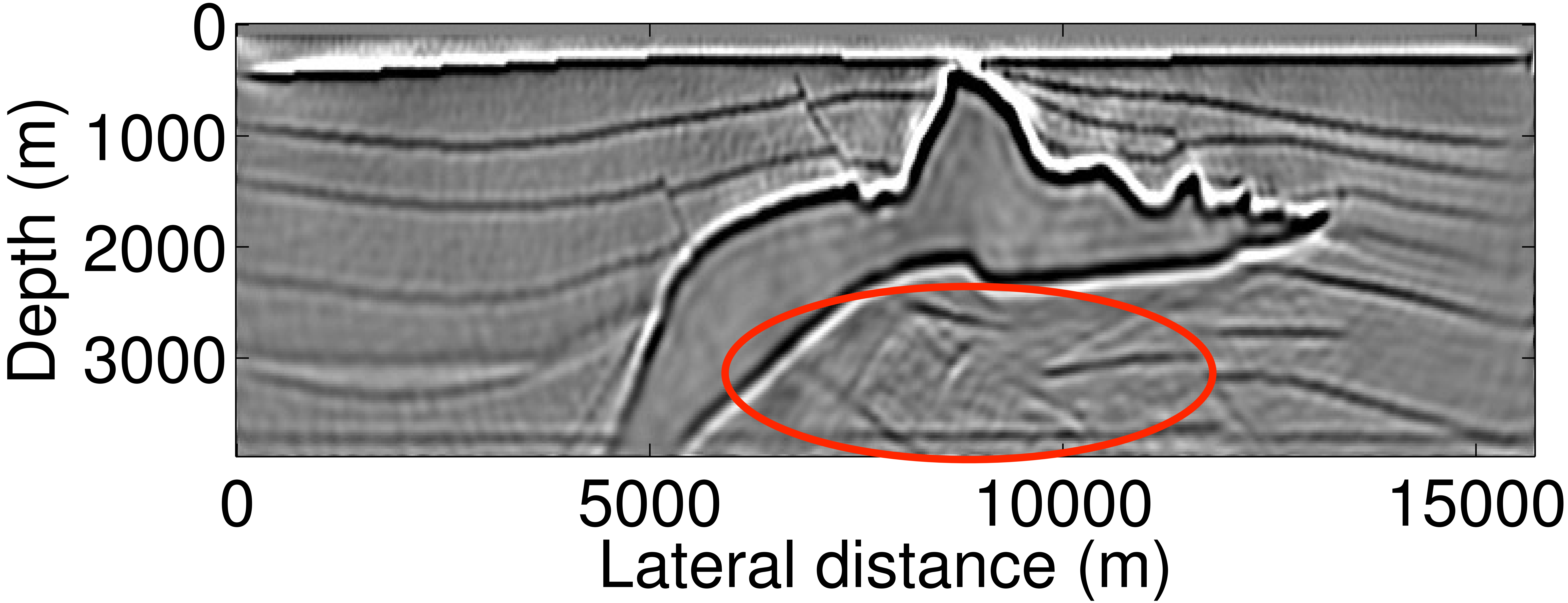
Fast inversion

w/ *ideal* data ***w/*** **rerandomization**



~1.45X the simulation cost of a single RTM with all data

Fast inversion
[w/ modeling errors and **w/ rerandomization**]



~1.45X the simulation cost of a single RTM with all data