Fast imaging via *depth* stepping with the *two*-way wave equation

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Fast *two*-way wave equation migration

Motivation : A **stable** and **affordable** two-way wave equation based *depth*-extrapolation migration for *laterally* varying media.

- **Stability** : Evanescent waves
  - Spectral projector
- **Affordability** : Computation cost and memory usage
  - Hierarchically Semi-Separable (HSS) Matrix Representation

Two-way wave-equation migration

$$p_{tt} = v(z,x)^2(p_{xx} + p_{zz})$$
$$p(x,0,t) = 0, \;\; t \leq 0$$
$$\begin{cases} p(x,z,0) = f(x) \\ p_t(x,z,0) = g(x) \end{cases}$$

2D Fourier Transform →

$$\hat{p}_{zz} = \left[-\left(\frac{\omega}{v(x,z)} \right)^2 - D_{xx} \right] \hat{p} \equiv L\hat{p}$$
$$\begin{cases} \hat{p}(x,z,\omega) = q(x,z,\omega) \\ \hat{p}_z(x,z,\omega) = q_z(x,z,\omega) \end{cases}$$

Two way wave-equation as an initial value problem

Unstable with evanescent wave components

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Two-way wave-equation migration

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Spectral Projector $\mathcal{P}$ →

$$\hat{p}_{zz} = \mathcal{P}L\mathcal{P}\hat{p}$$
$$\hat{p}(x,z_n,\omega) = q(x,z_n,\omega)$$
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Two way wave-equation as an initial value problem

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Stabilized initial value problem with spectral projector

L. Auslander and A. Tsao. On parallelizable eigensolvers. Citeseer, 1991.
N. J. Higham. Functions of matrices: theory and computation. Siam, 2008.

Spectral projector

Geophysical	→	Unstable extrapolation operator with propagating and **evanescent** waves	Get rid of all **evanescent** waves $k_z^2 = \left(\frac{\omega}{v(x,z)}\right)^2 - k^2 \geq 0$	Stable extrapolation operator with only propagating waves
		$L = V\Lambda V^*$	→	$\mathcal{P}L\mathcal{P} = V\tilde{\Lambda}V^*$
Mathematical	→	Indefinite matrix with negative and **positive** eigenvalues	$\lambda = -\left(\frac{\omega}{v(x,z)}\right)^2 + k^2 \leq 0$ Set all **positive** eigenvalue to be 0	Definite matrix with only negative eigenvalues

↓

$$\mathcal{P} = (I - \text{sign}(L))/2$$

$$\text{sign}(L) = V\text{sign}(D)V^*$$

C. S. Kenney and A. J. Laub. The matrix sign function. Automatic Control, IEEE Transactions on, 40(8):1330–1348, 1995.

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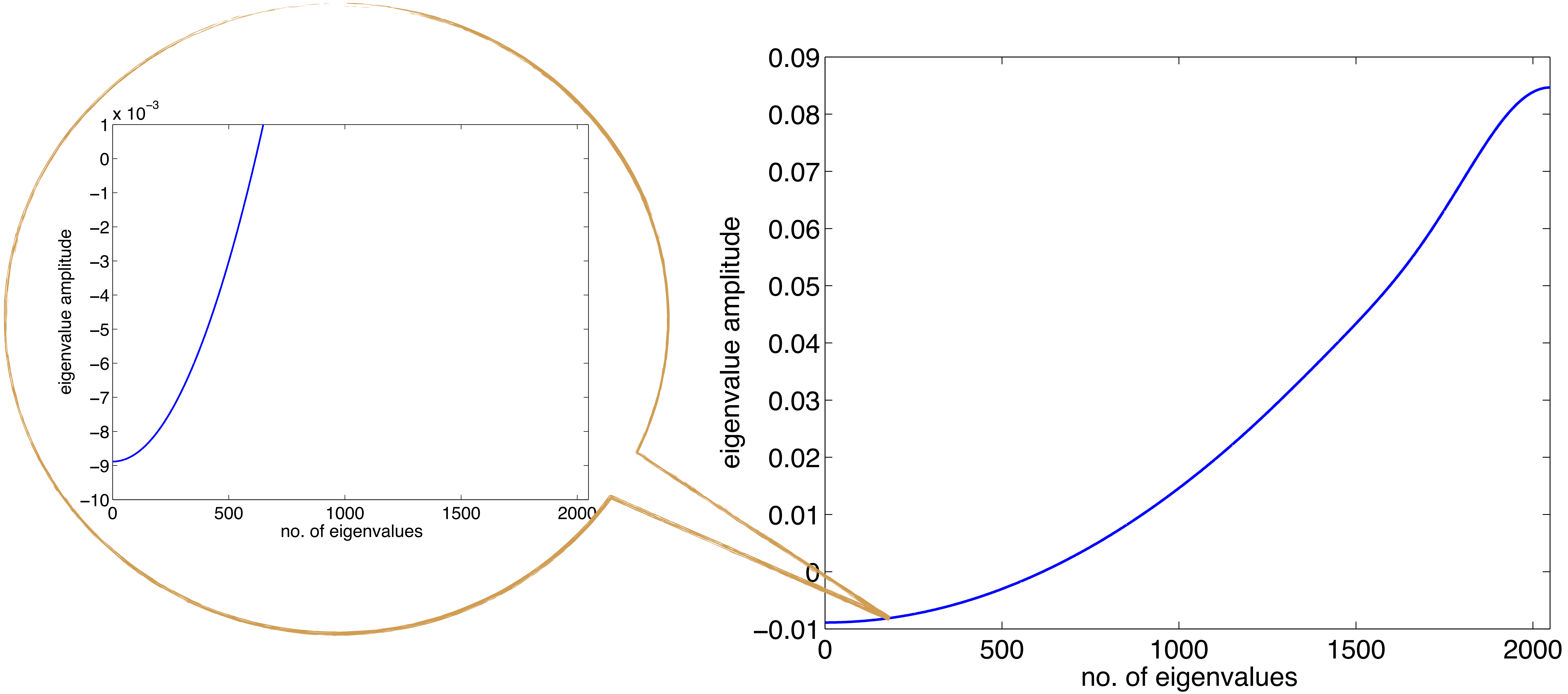
Sign function computation via *polynomial* recursion

Algorithm Solving for sign function via recursion

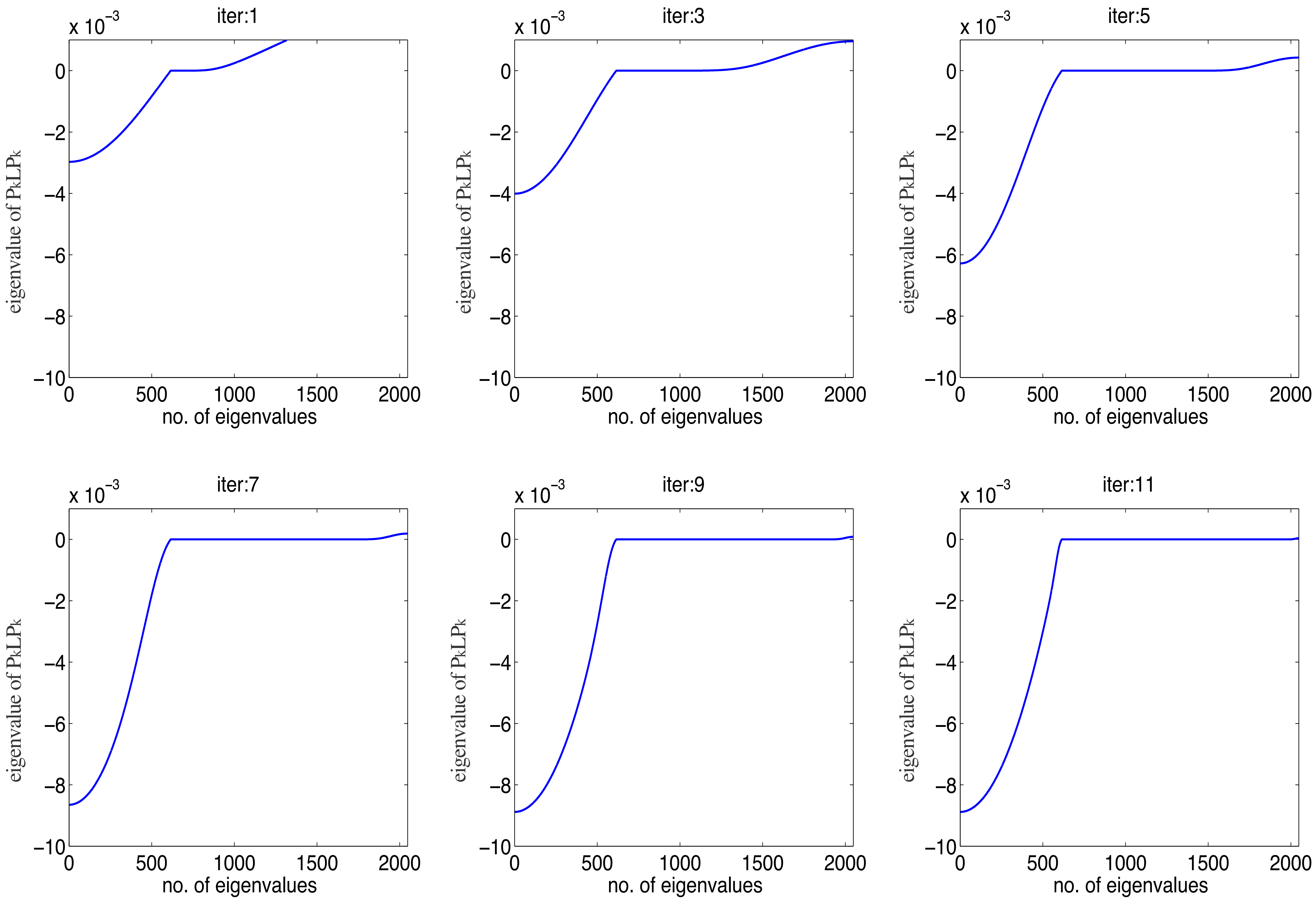
Input: self adjoint matrix L
output: $\text{sign}(L)$

1. Initialize $S_0 = L/||L||_2$, where $||L||_2$ stands for the 2 norm of matrix L
2. For $k = 1......N$, $S_{k+1} = \frac{3}{2}S_k - \frac{1}{2}S_k^3$

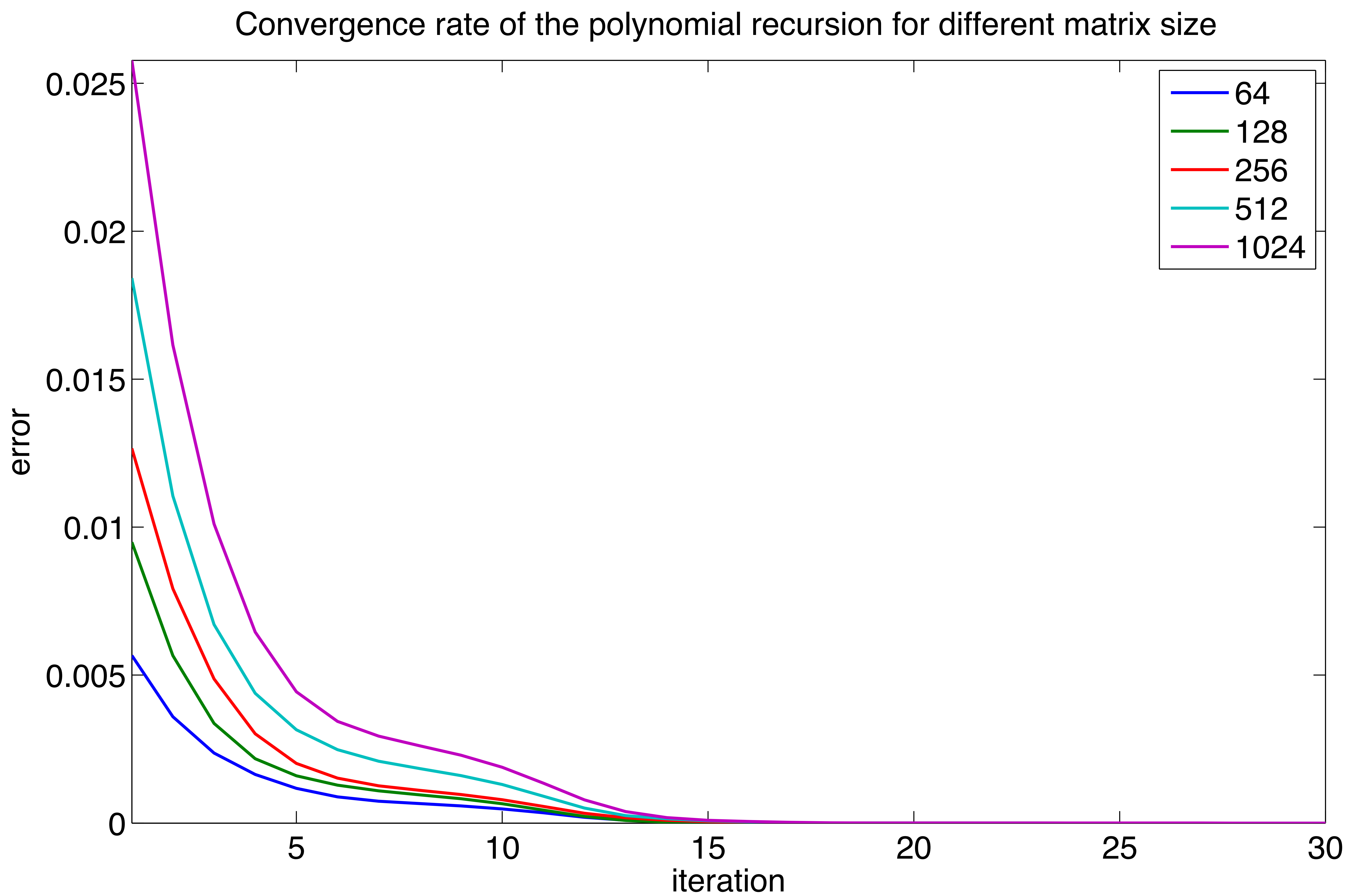
Sign function computation via *polynomial* recursion



Sign function computation via *polynomial* recursion



Sign function computation via *polynomial* recursion



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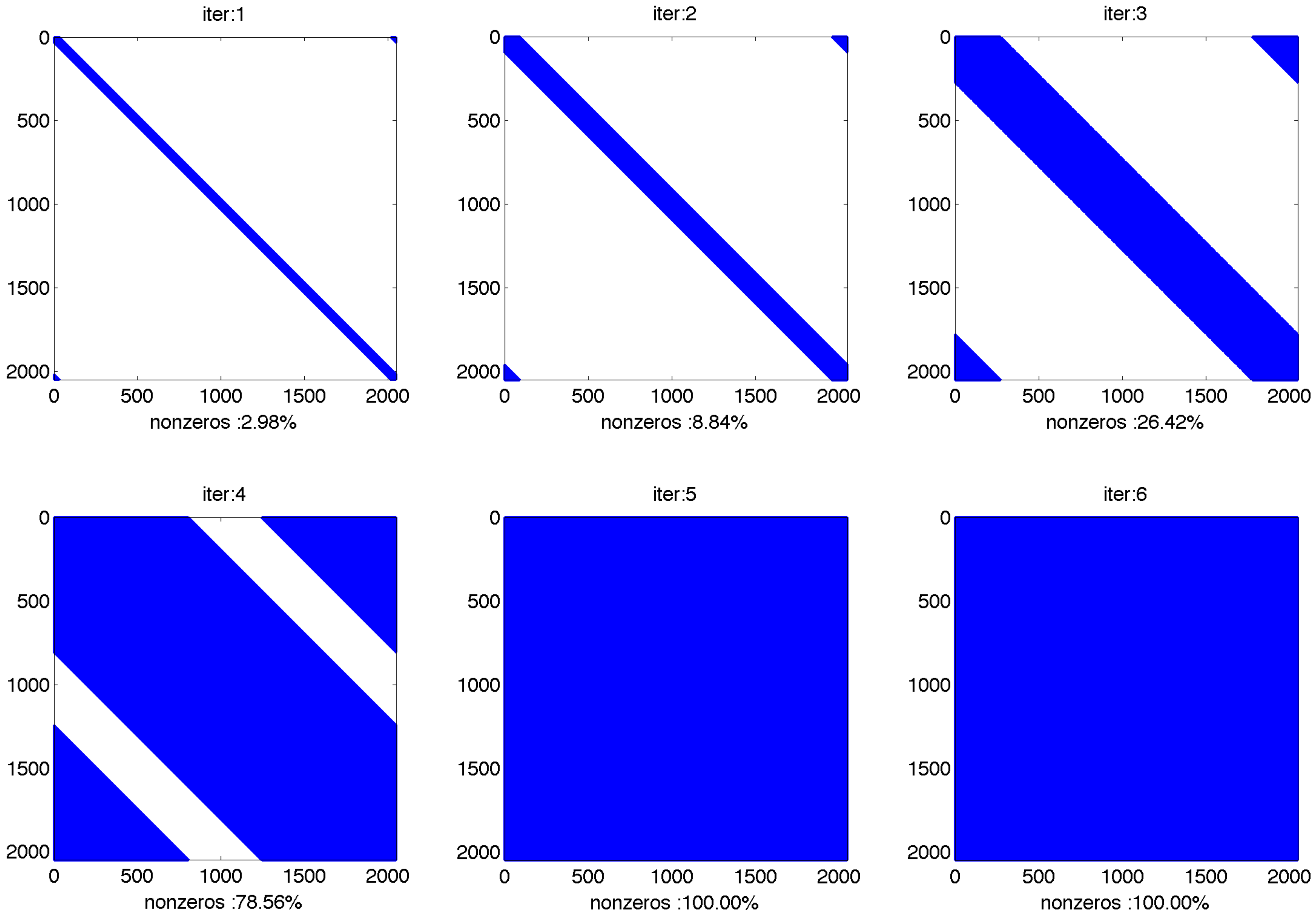
Matrix *sparsity* in *polynomial* recursion



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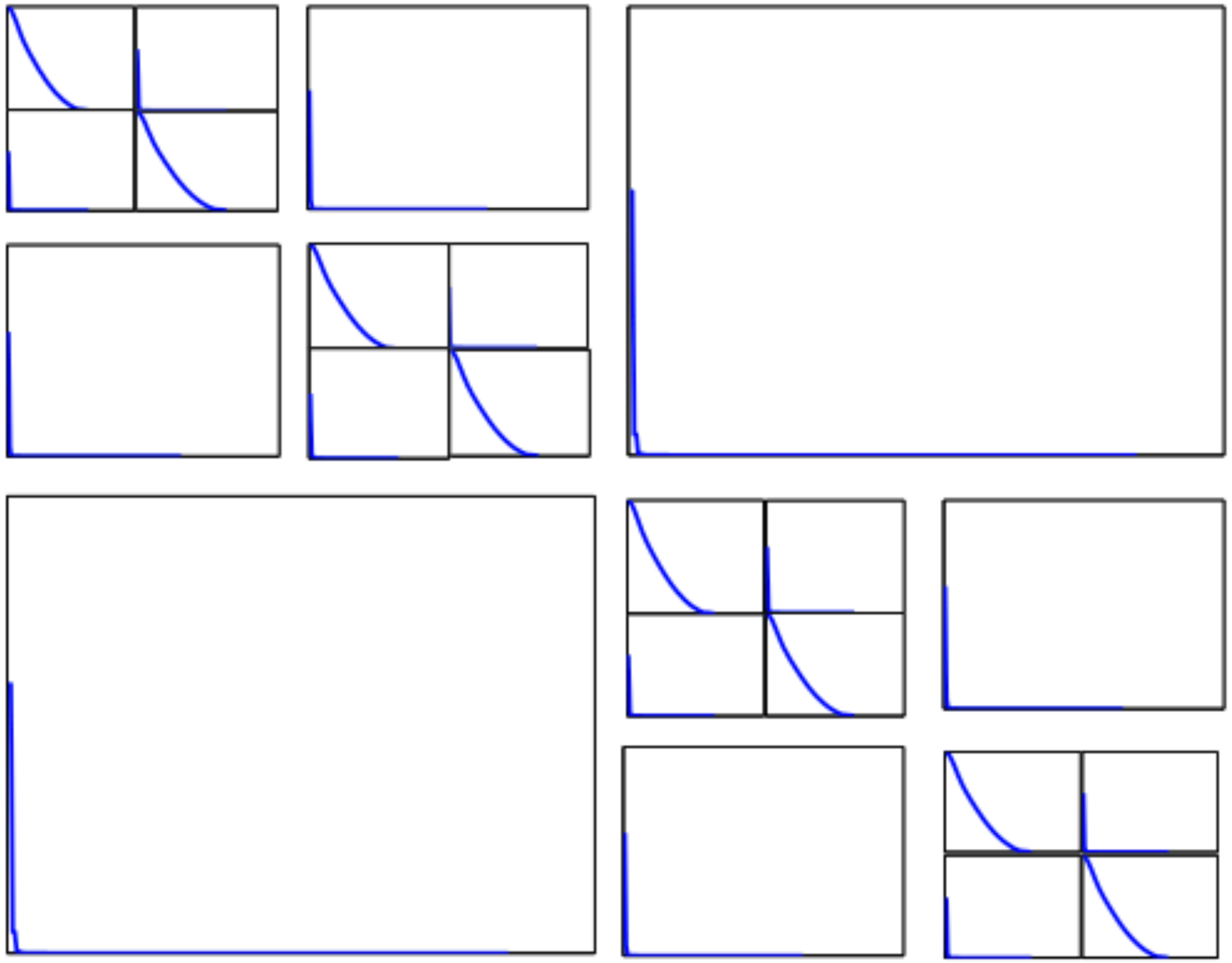
Acceleration of *matrix-matrix* multiplications



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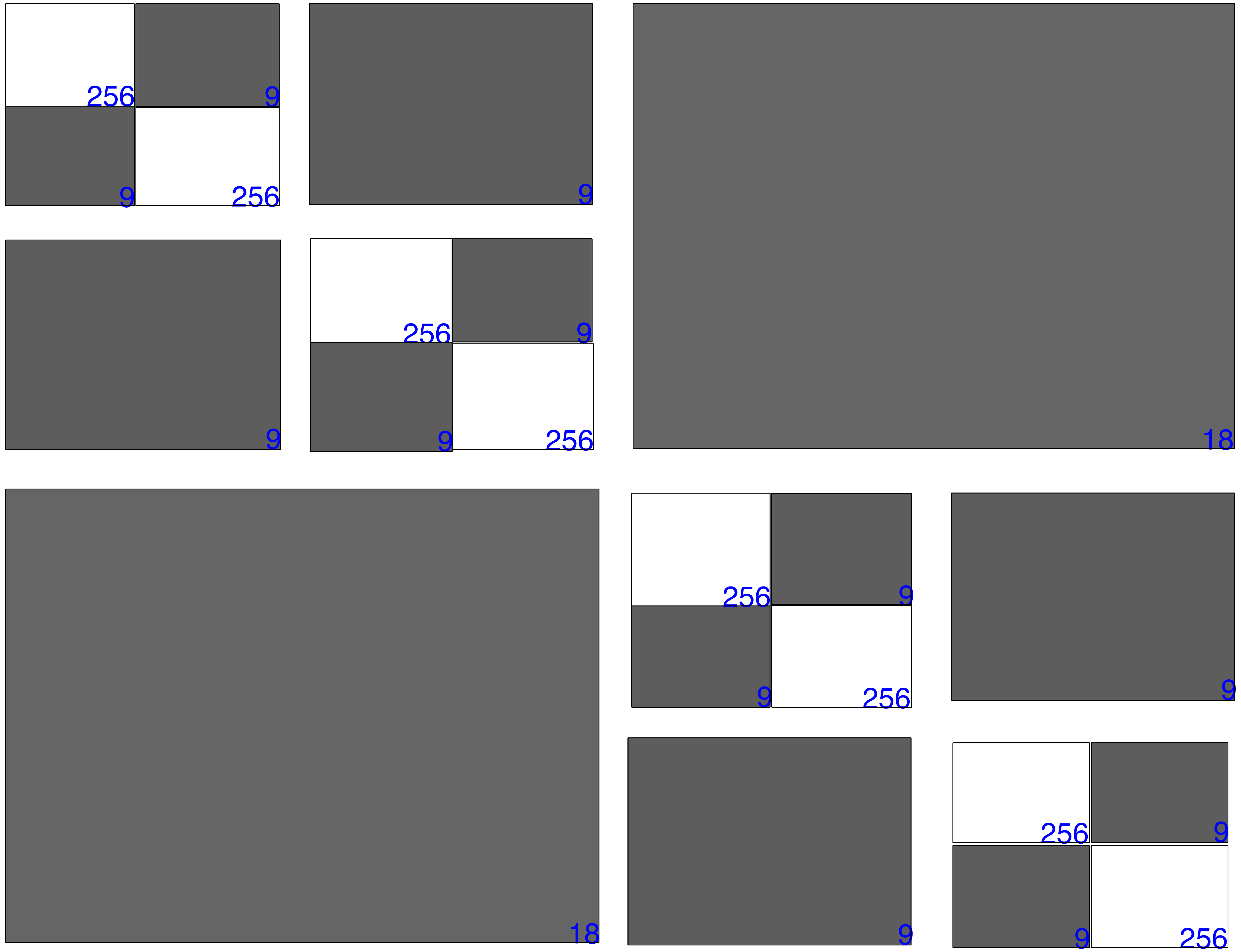
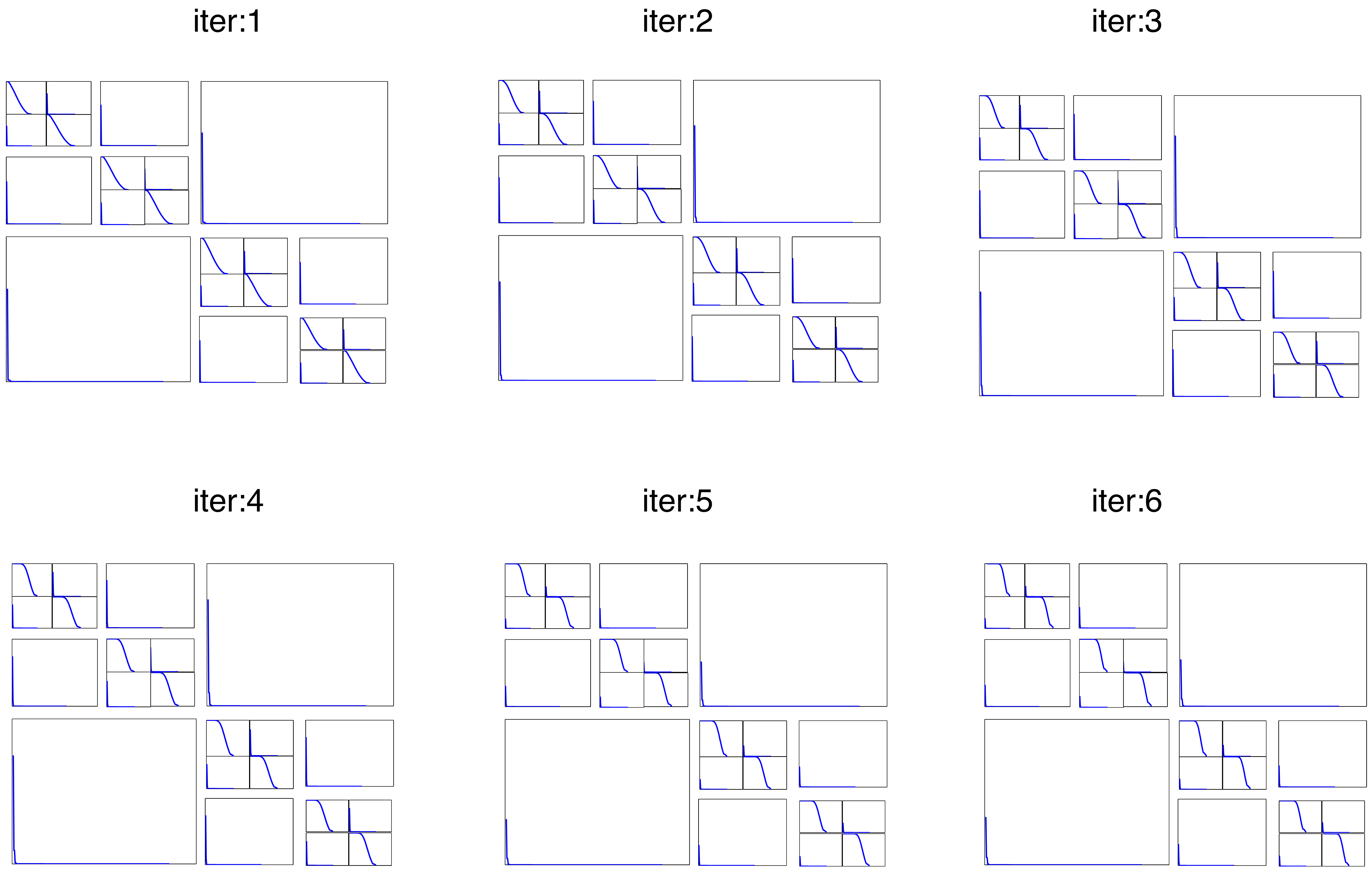


Acceleration of *matrix-matrix* multiplications



J. Xia. On the complexity of some hierarchical structured matrix algorithms. SIAM Journal on Matrix Analysis and Applications, 33(2):388–410, 2012.

Hierarchically semi-separable matrix representation

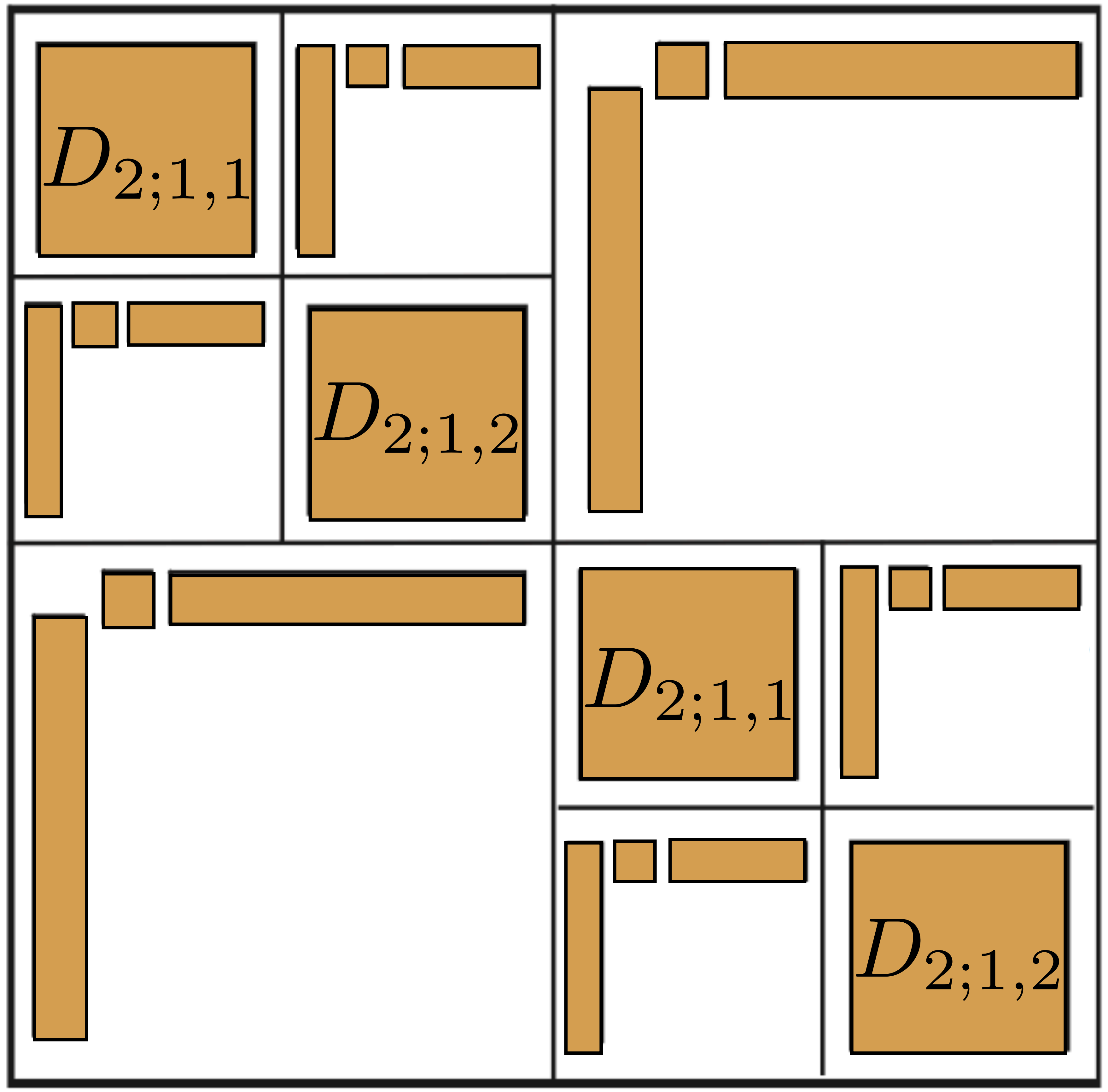
$$A = \begin{pmatrix} A_{1;1,1} & A_{1;1,2} \\ A_{1;2,1} & A_{1;2,2} \end{pmatrix}$$

$$\downarrow$$

$$\mathbf{A} = \begin{pmatrix} D_{1;1,1} & (USV^*)_{1;1,2} \\ (USV^*)_{1;2,1} & D_{1;2,2} \end{pmatrix}$$

$$\downarrow$$

$$\mathbf{A} = \begin{pmatrix} \begin{pmatrix} D_{2;1,1} & (USV^*)_{2;1,2} \\ (USV^*)_{2;2,1} & D_{2;2,2} \end{pmatrix} & (USV^*)_{1;1,2} \\ (USV^*)_{1;2,1} & \begin{pmatrix} D_{2;1,1} & (USV^*)_{2;1,2} \\ (USV^*)_{2;2,1} & D_{2;2,2} \end{pmatrix} \end{pmatrix}$$



Z. Sheng, P. Dewilde, and S. Chandrasekaran. Algorithms to solve hierar- chically semi-separable systems. pages 255–294, 2007.

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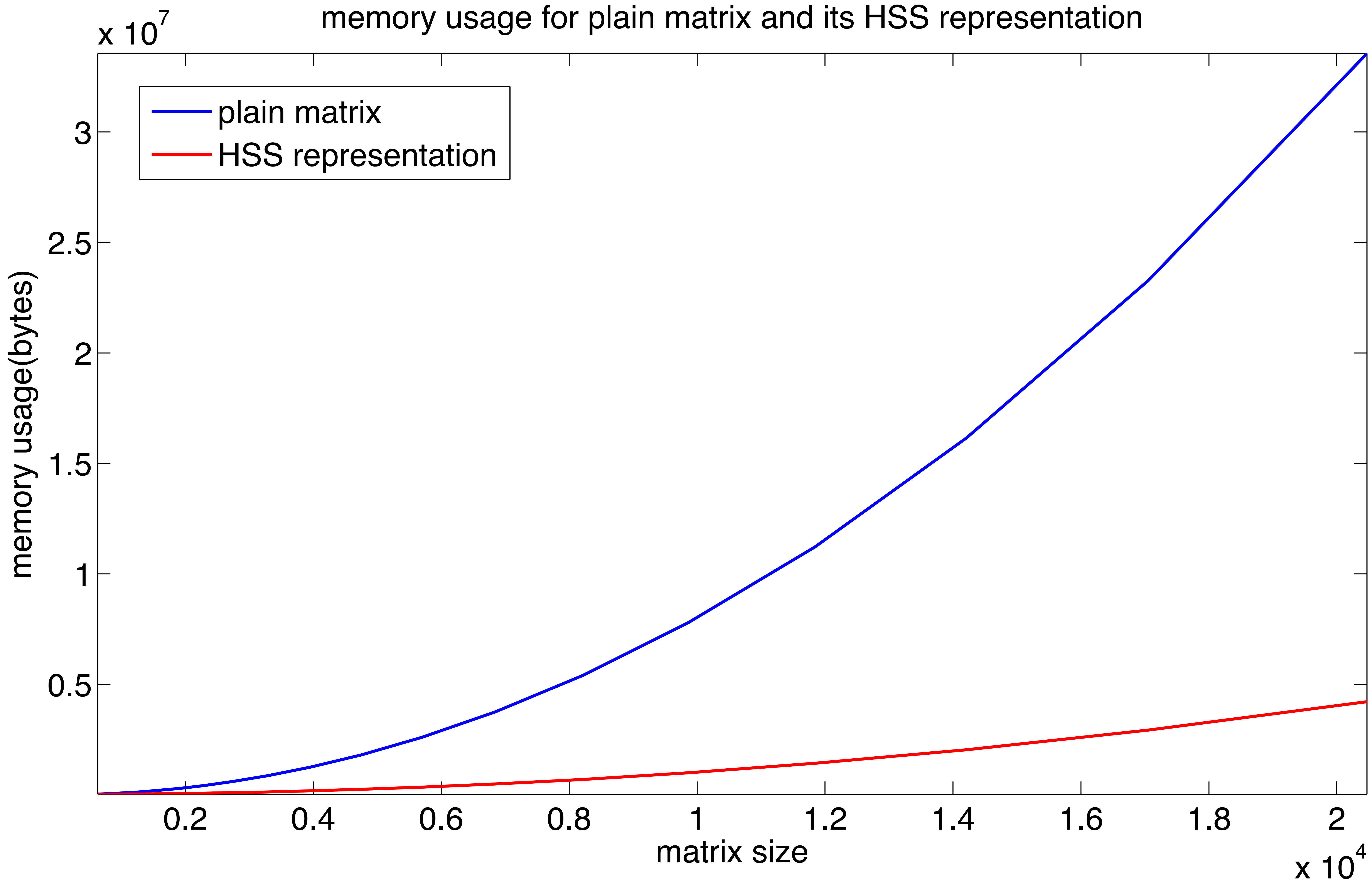
Computation cost of HSS operations

Operation	Complexity with HSS	Complexity with plain matrix
Matrix-Vector Multiplication	$\mathcal{O}(nk^2)$	$\mathcal{O}(n^2)$
Matrix-Matrix Multiplication	$\mathcal{O}(nk^3)$	$\mathcal{O}(n^3)$
Construct HSS	$\mathcal{O}(nk)$	Not Applicable
Matrix addition	$\mathcal{O}(nk^2)$	$\mathcal{O}(n^2)$
Compression	$\mathcal{O}(nk^3)$	Not Applicable
LU Decomposition	$\mathcal{O}(nk^3)$	$\mathcal{O}(n^3)$
Inverse	$\mathcal{O}(nk^3)$	$\mathcal{O}(n^3)$
Transpose	$\mathcal{O}(nk)$	$\mathcal{O}(n^2)$

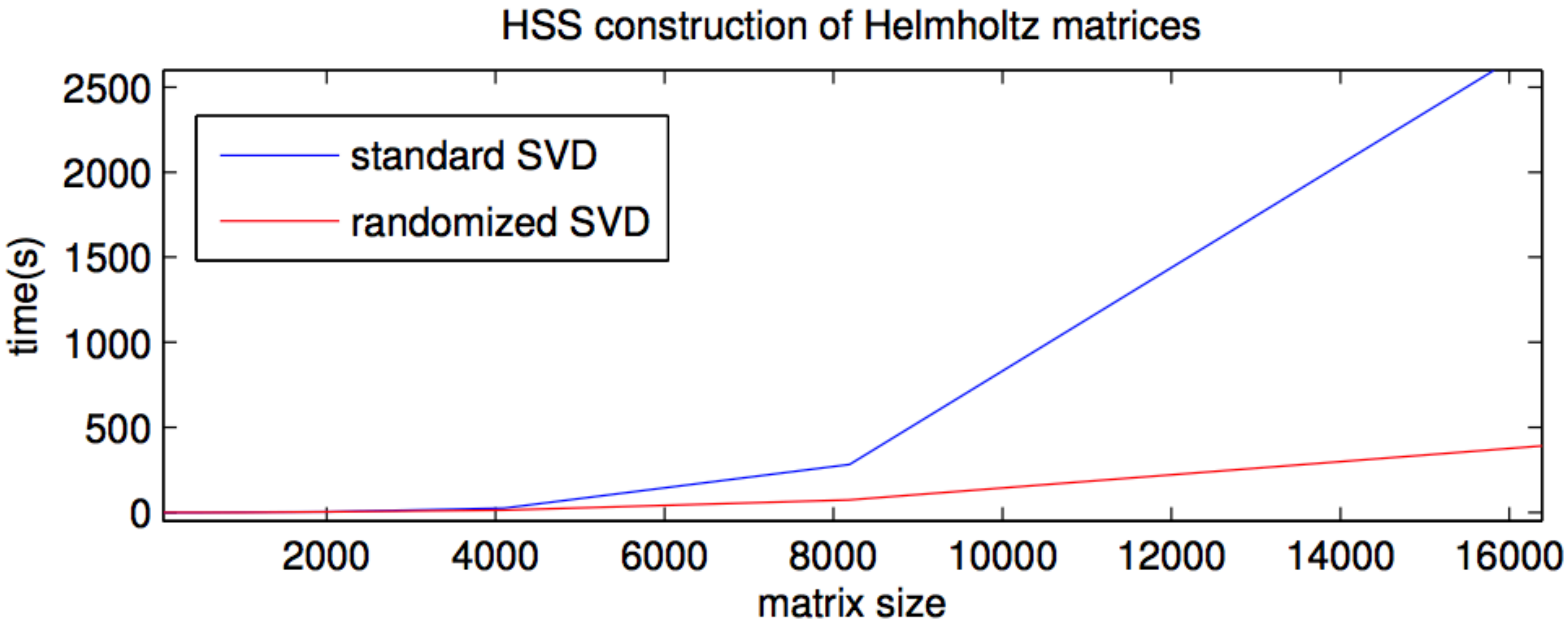
k is maximum rank of the off-diagonal matrices

Memory usage of *HSS* matrix representations

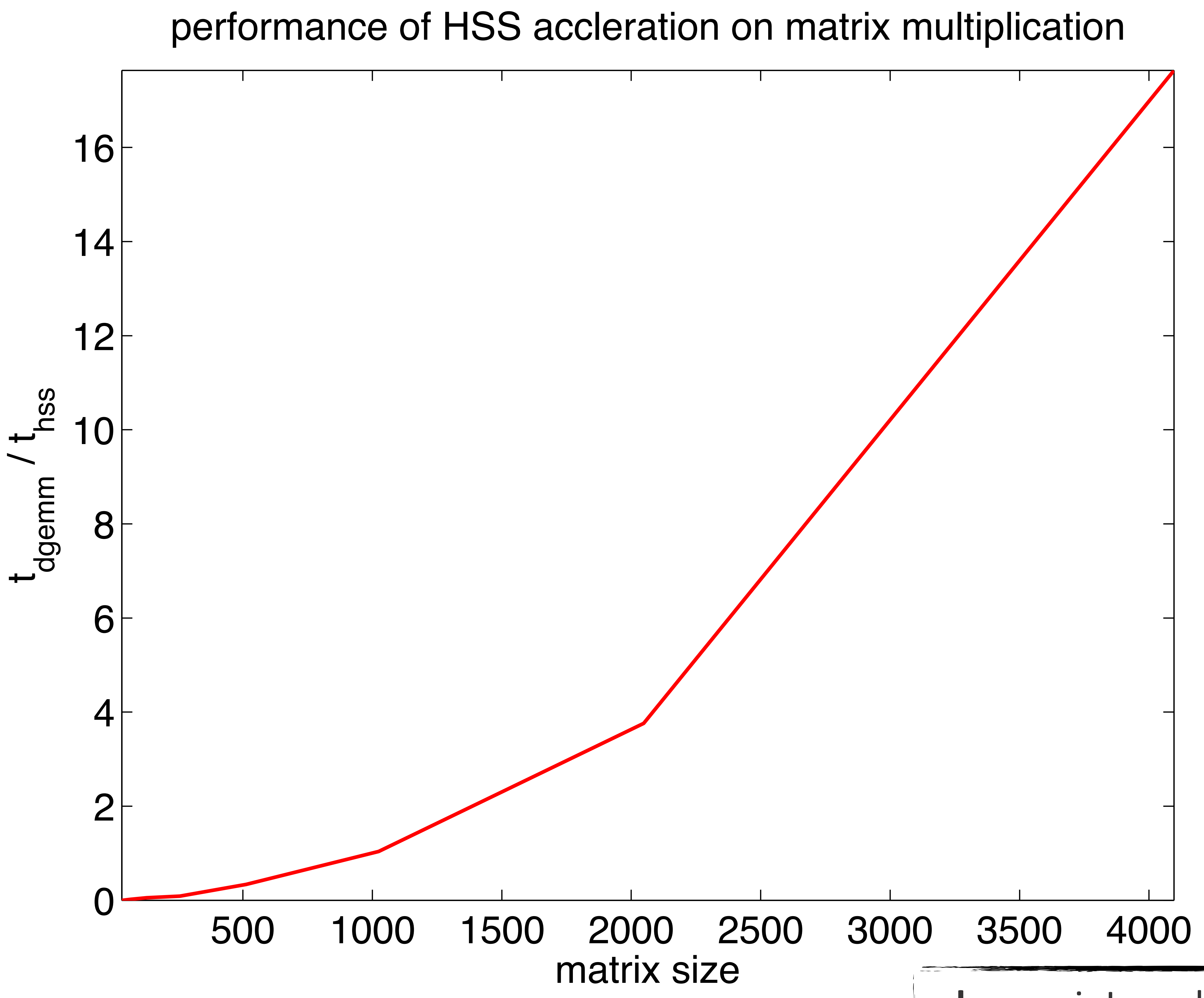


M. Tygert and Rokhlin. A randomized algorithm for approximating the svd of a matrix. 2007. URL https://www.ipam.ucla.edu/publications/ setut/setut_7373.pdf.
Halko, Nathan, Per-Gunnar Martinsson, and Joel A. Tropp. "Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions." *SIAM review* 53.2 (2011): 217-288.
B. Jumah and F. J. Herrmann. Dimensionality-reduced estima- tion of primaries by sparse inversion. 2012. URL https: //www.slim.eos.ubc.ca/Publications/Private/Submitted/ Journal/bander2012dre/bander2012dre.pdf.

Randomized HSS

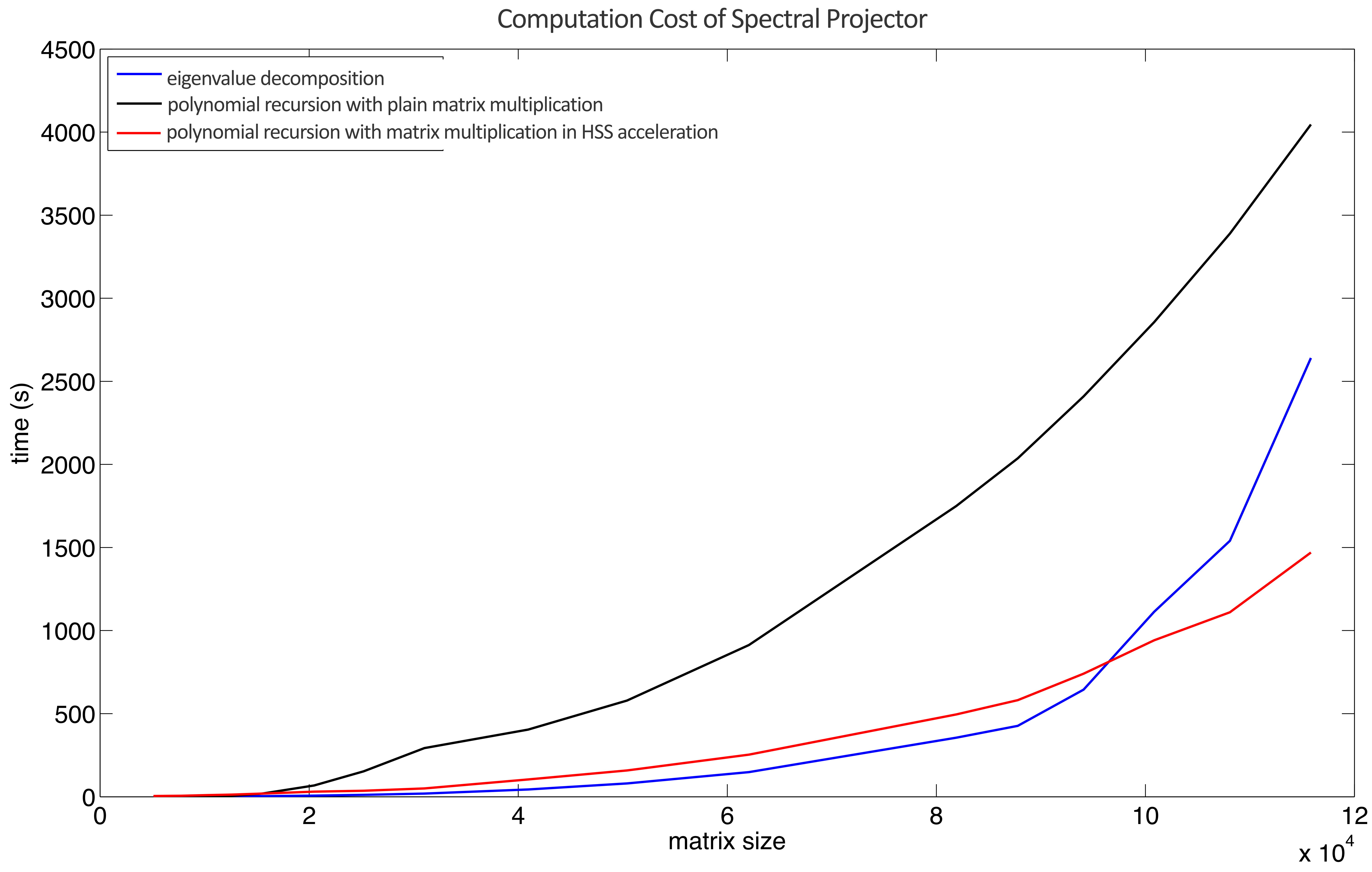


Computation of matrix-matrix multiplication with HSS

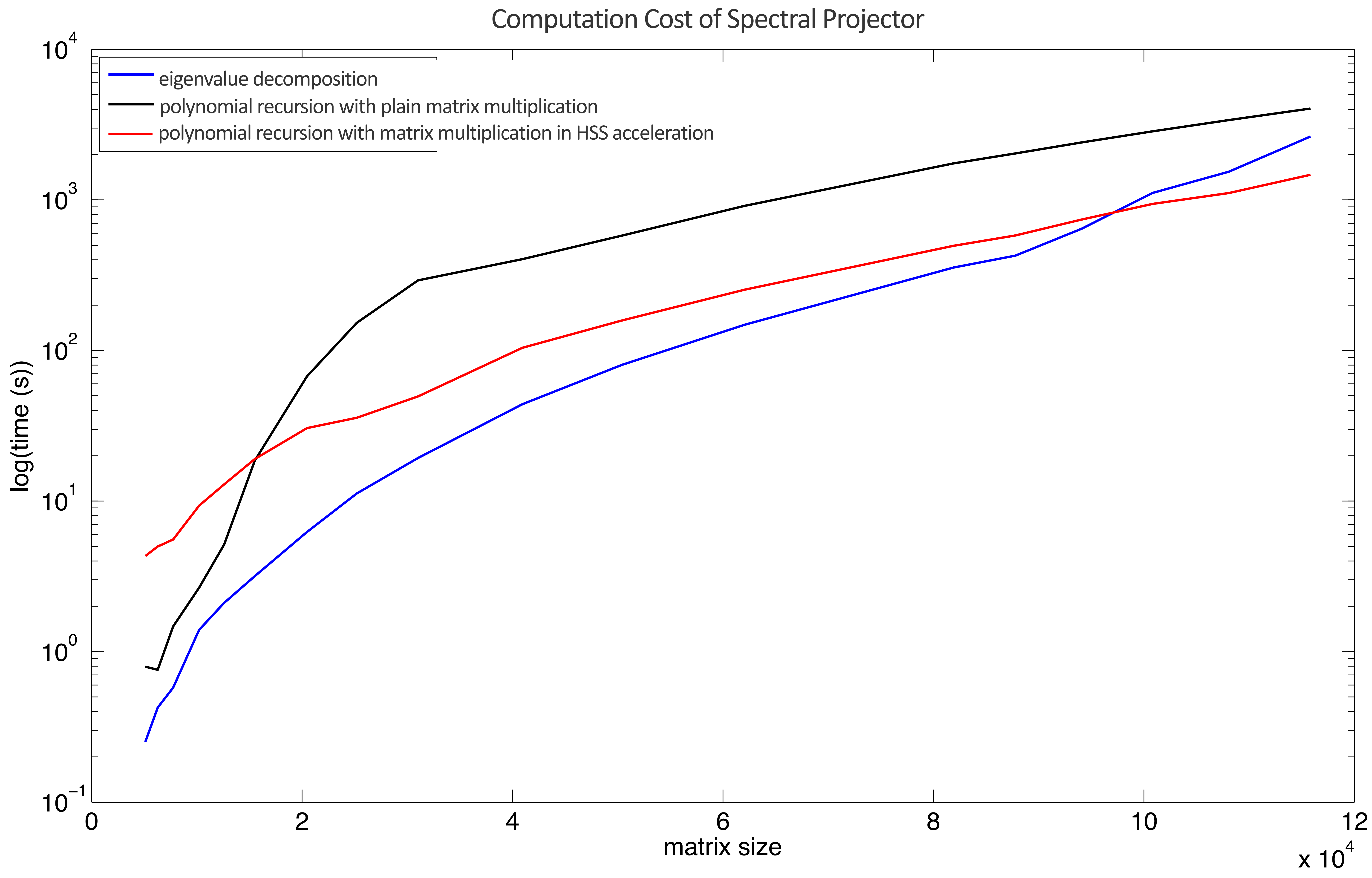


dgemm is Lapack routine for matrix-matrix multiplication

Computation of *spectral* projector with HSS

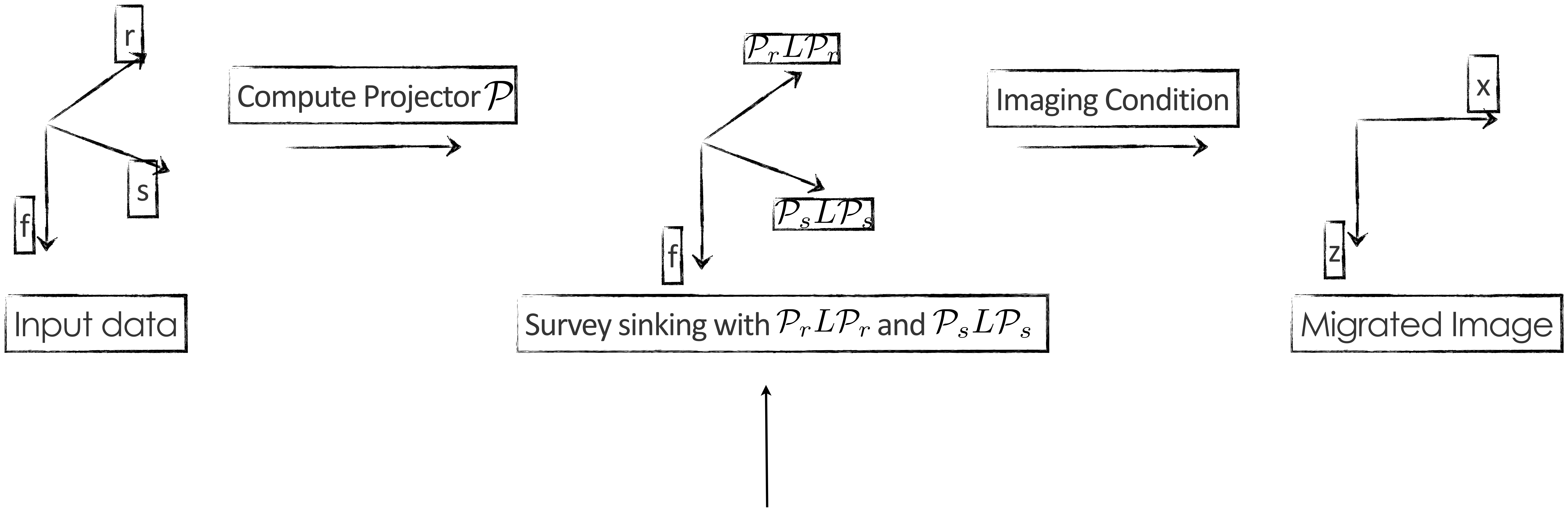


Computation of *spectral* projector with HSS



K. Sandberg and G. Beylkin. Full-wave-equation depth extrapolation for migration. Geophysics, 74(6):WCA121–WCA128, 2009.

Accelerated two-way wave-equation migration

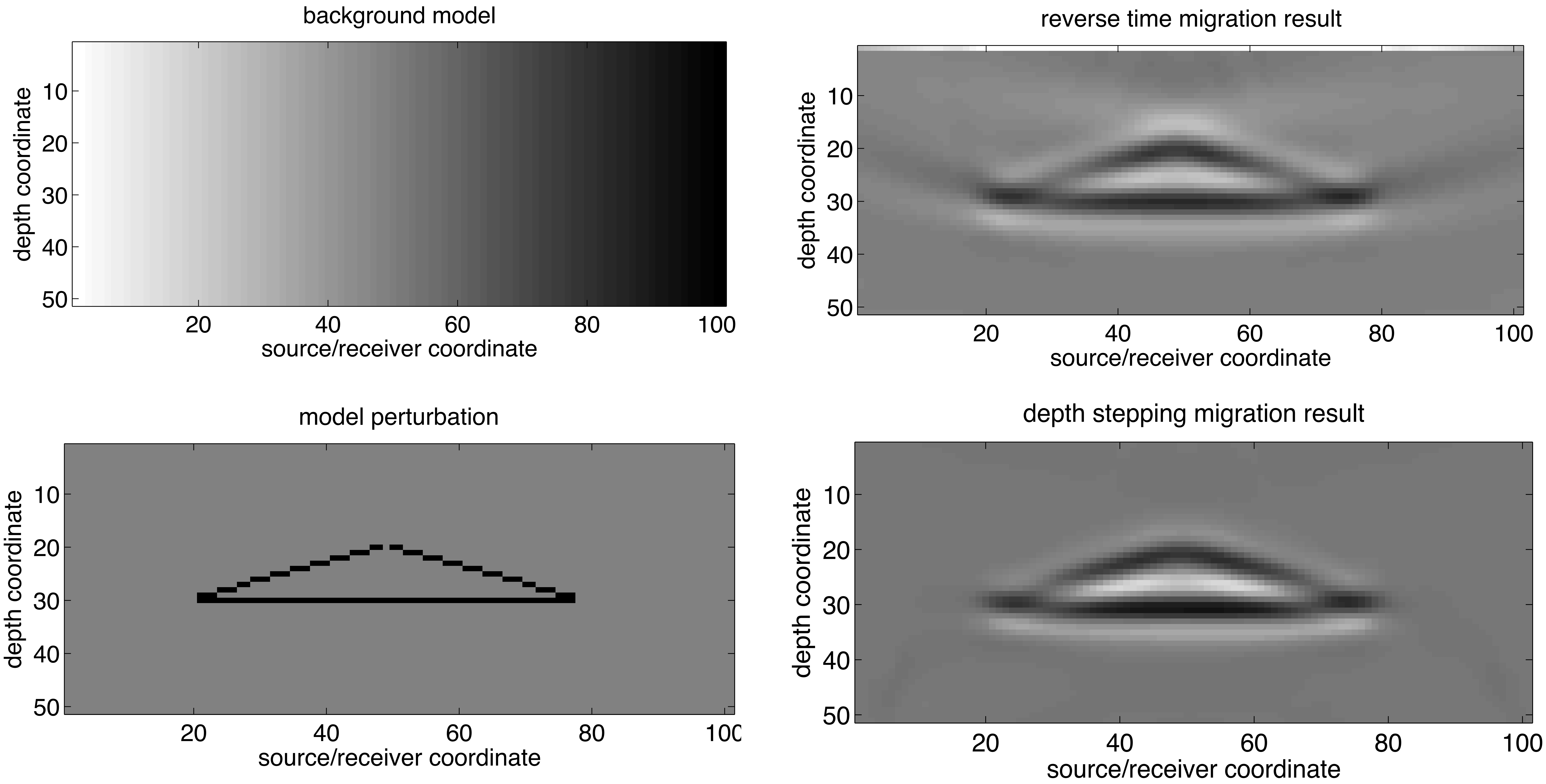


$$\hat{p}_{zz} = \mathcal{P} L \mathcal{P} \hat{p}$$

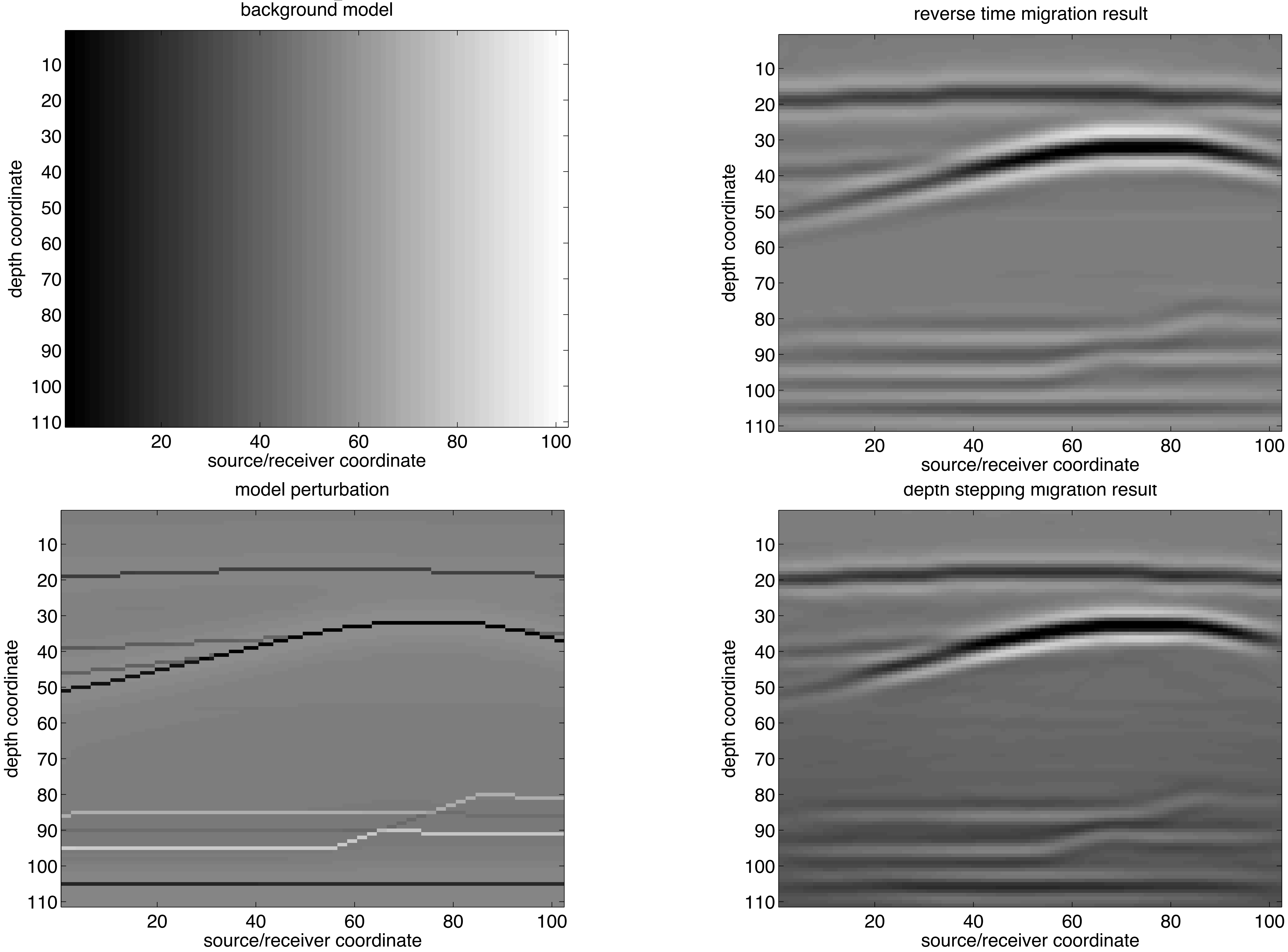
$$\hat{p}(x, z_n, \omega) = q(x, z_n, \omega)$$

$$\hat{p_z}(x, z_n, \omega) = q_z(x, z_n, \omega)$$

Migration example



Migration example



Conclusions

- Spectral *projectors* computed with *polynomial* recursion using HSS matrix representations provides a *stable* and *affordable* two-way wave equation based depth extrapolation migration for *laterally* varying media.
- The proposed algorithm has both time and memory advantages compared to *reverse* time migration.

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Future work

- Further optimize algorithm performance.
- Implement more proper boundary conditions, absorbing the boundary reflections.
- Apply algorithm to more realistic dataset.
- Explore broader usage of spectral projector.

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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References

L. Auslander and A. Tsao. On parallelizable eigensolvers. Citeseer, 1991. J. Barzilai and J. M. Borwein. Two-point step size gradient methods. IMA Journal of Numerical Analysis, 8(1):141–148, 1988.

E. v. d. Berg and M. P. Friedlander. Joint-sparse recovery from multiple measurements. arXiv preprint arXiv:0904.2051, 2009.

A. Berkhout and D. Verschuur. Imaging of multiple reflections. Geophysics, 71(4):SI209–SI220, 2006.

D. P. Bertsekas. Projected newton methods for optimization problems with simple constraints. SIAM Journal on Control and Optimization, 20(2): 221–246, 1982.

R. H. Byrd, J. Nocedal, and R. B. Schnabel. Representations of quasi- newton matrices and their use in limited memory methods. Mathematical Programming, 63(1-3): 129–156, 1994.

S. Chandrasekaran, P. Dewilde, M. Gu, W. Lyons, and T. Pals. A fast solver for hss representations via sparse matrices. SIAM Journal on Matrix Analysis and Applications, 29(1):67–81, 2006.

J. Chen and X. Huo. Theoretical results on sparse representations of multiple-measurement vectors. Signal Processing, IEEE Transactions on, 54(12):4634–4643, 2006.

J. F. Claerbout. Imaging the earth's interior, volume 316. Citeseer, 1985.

S. F. Cotter and B. D. Rao. Sparse channel estimation via matching pursuit with application to equalization. Communications, IEEE Transactions on, 50(3):374–377, 2002.

S. F. Cotter, B. D. Rao, K. Engan, and K. Kreutz-Delgado. Sparse solutions to linear inverse problems with multiple measurement vectors. Signal Processing, IEEE Transactions on, 53(7):2477–2488, 2005

M. E. Davies and Y. C. Eldar. Rank awareness in joint sparse recovery. Information Theory, IEEE Transactions on, 58(2):1135–1146, 2012.

D. L. Donoho. Compressed sensing. Information Theory, IEEE Transactions on, 52(4):1289–1306, 2006.

Y. C. Eldar and H. Rauhut. Average case analysis of multichannel sparse recovery using convex relaxation. Information Theory, IEEE Transactions on, 56(1):505–519, 2010.

I. J. Fevrier, S. B. Gelfand, and M. P. Fitz. Reduced complexity decision feedback equalization for multipath channels with large delay spreads. Communications, IEEE Transactions on, 47(6):927–937, 1999.

E. M. Gafni and D. P. Bertsekas. Two-metric projection methods for con- strained optimization. SIAM Journal on Control and Optimization, 22 (6):936–964, 1984.

R. Gribonval, H. Rauhut, K. Schnass, and P. Vandergheynst. Atoms of all channels, unite! average case analysis of multi-channel sparse recovery using greedy algorithms. Journal of Fourier analysis and Applications, 14 (5-6):655–687, 2008.

G. Hennenfent, L. Fenelon, and F. J. Herrmann. Nonequispaced curvelet transform for seismic data reconstruction: A sparsity- promoting approach. Geophysics, 75(6):WB203–WB210, 2010. URL http://slim.eos.ubc.ca/Publications/Public/Journals/ hennenfent2010nct.pdf.

F. J. Herrmann. Randomized sampling and sparsity: Getting more informa- tion from fewer samples. Geophysics, 75(6):WB173–WB187, 2010. URL http://link.aip.org/link/?GPY/75/WB173/1.

F. J. Herrmann and X. Li. Efficient least-squares imaging with sparsity promotion and compressive sensing. Geophysical prospecting, 60(4):696– 712, 2012.

F. J. Herrmann, C. R. Brown, Y. A. Erlangga, and P. P. Moghad- dam. Curvelet-based migration preconditioning and scaling. Geophysics, 74:A41, 2009. URL http://slim.eos.ubc.ca/Publications/Public/ TechReports/herrmann08cmp-r.pdf.

References

F. J. Herrmann, M. P. Friedlander, and O. Yilmaz. Fighting the curse of dimensionality: Compressive sensing in exploration seismology. Signal Processing Magazine, IEEE, 29(3):88–100, 2012. ISSN 1053- 5888. URL https://www.slim.eos.ubc.ca/Publications/Public/ Journals/IEEESignalProcessingMagazine/2012/ Herrmann11TRfcd/ Herrmann11TRfcd.pdf.

N. J. Higham. Functions of matrices: theory and computation. Siam, 2008.

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C. S. Kenney and A. J. Laub. The matrix sign function. Automatic Control, IEEE Transactions on, 40(8):1330–1348, 1995.

E. Levitin and B. Polyak. Constrained minimization methods. {USSR} Computational Mathematics and Mathematical Physics, 6(5):1 – 50, 1966. ISSN 0041-5553. doi: http://dx.doi.org/10.1016/0041-5553(66) 90114-5. URL http://www.sciencedirect.com/science/article/ pii/0041555366901145.

T. T. Lin and F. J. Herrmann. Compressed wavefield ex- trapolation. Geophysics, 72(5):SM77–SM93, 2007. URL https://www.slim.eos.ubc.ca/Publications/Public/ Journals/ Geophysics/2007/lin07cwe/lin07cwe.pdf.

T. T. Lin, N. T. Tu, and F. J. Herrmann. Sparsity-promoting migration from surface-related multiples. In 2010 SEG Annual Meeting, 2010.

W. Lyons. Fast algorithms with applications to pdes. PhD thesis, UNIVER- SITY of CALIFORNIA, 2005.

D. Malioutov, M. Çetin, and A. S. Willsky. A sparse signal reconstruction perspective for source localization with sensor arrays. Signal Processing, IEEE Transactions on, 53(8):3010–3022, 2005.

A. M. Popovici. Prestack migration by split-step dsr. Geophysics, 61(5): 1412–1416, 1996.

K. Sandberg and G. Beylkin. Full-wave-equation depth extrapolation for migration. Geophysics, 74(6):WCA121–WCA128, 2009.

M. Schmidt, D. Kim, and S. Sra. 1 projected newton-type methods in machine learning. Optimization for Machine Learning, page 305, 2011.

M. W. Schmidt, E. Berg, M. P. Friedlander, and K. P. Murphy. Optimiz- ing costly functions with simple constraints: A limited-memory projected quasi-newton algorithm. In International Conference on Artificial Intel- ligence and Statistics, 2009.

P. Shen and W. W. Symes. Automatic velocity analysis via shot profile migration. Geophysics, 73(5):VE49–VE59, 2008.

Z. Sheng, P. Dewilde, and S. Chandrasekaran. Algorithms to solve hierar- chically semi-separable systems. pages 255–294, 2007.

J. A. Tropp. Algorithms for simultaneous sparse approximation. part ii: Convex relaxation. Signal Processing, 86(3):589–602, 2006.

M. Tygert and Rokhlin. A randomized algorithm for approximating the svd of a matrix. 2007. URL https://www.ipam.ucla.edu/publications/ setut/setut_7373.pdf.

E. van den Berg and M. P. Friedlander. Probing the pareto frontier for basis pursuit solutions. SIAM Journal on Scientific Computing, 31(2):890–912, 2008. doi: 10.1137/080714488. URL http://link.aip.org/link/?SCE/ 31/890.

E. van den Berg and M. P. Friedlander. Sparse optimization with least- squares constraints. SIAM J. Optimization, 21(4):1201–1229, 2011.

K. Wapenaar and J. L. Grimbergen. A discussion on stability analysis of wave field depth extrapolation. In 1998 SEG Annual Meeting, 1998.

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