SINBAD Consortium
Meeting Fall 2013

Model-space versus *data*-space regularized FWI with the *acoustic* wave equation

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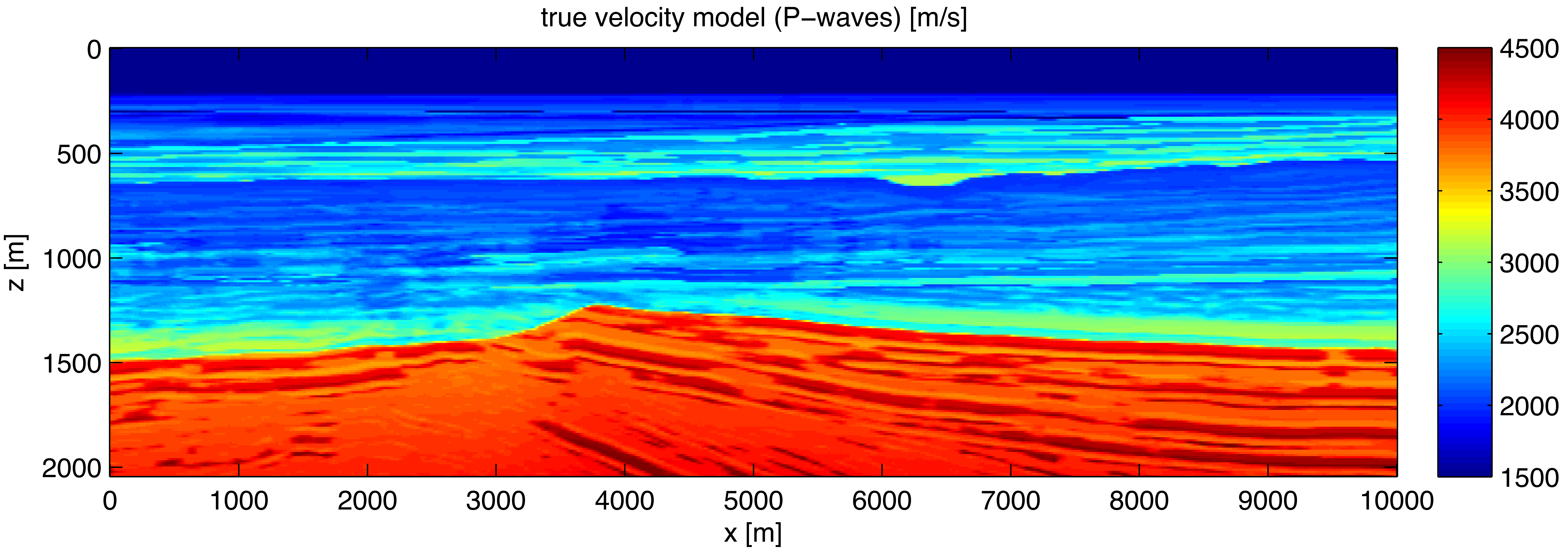
SLIM
University of British Columbia

Motivation

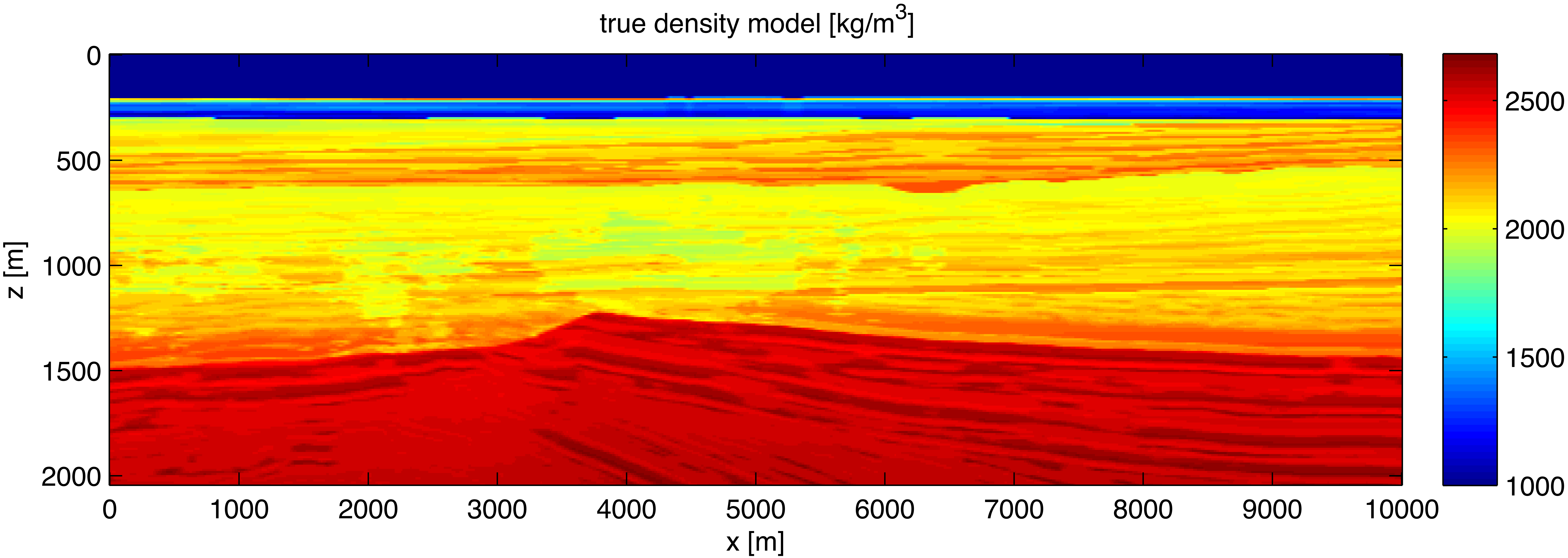
BG group

BG compass model (P-velocity)



Motivation

BG compass model (density)

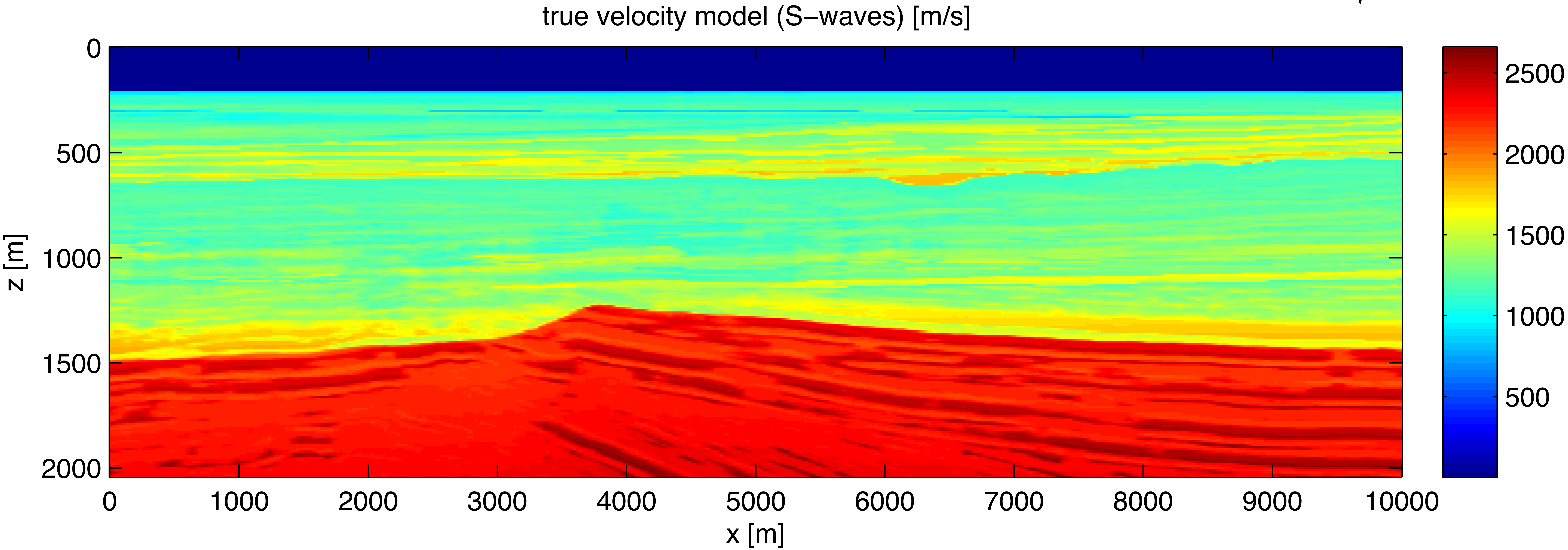


Motivation

BG compass model (S-velocity)

Fixed Poisson's ratio 0.25

$$V_s = \sqrt{\frac{1-2\sigma}{2(1-\sigma)}}V_p$$

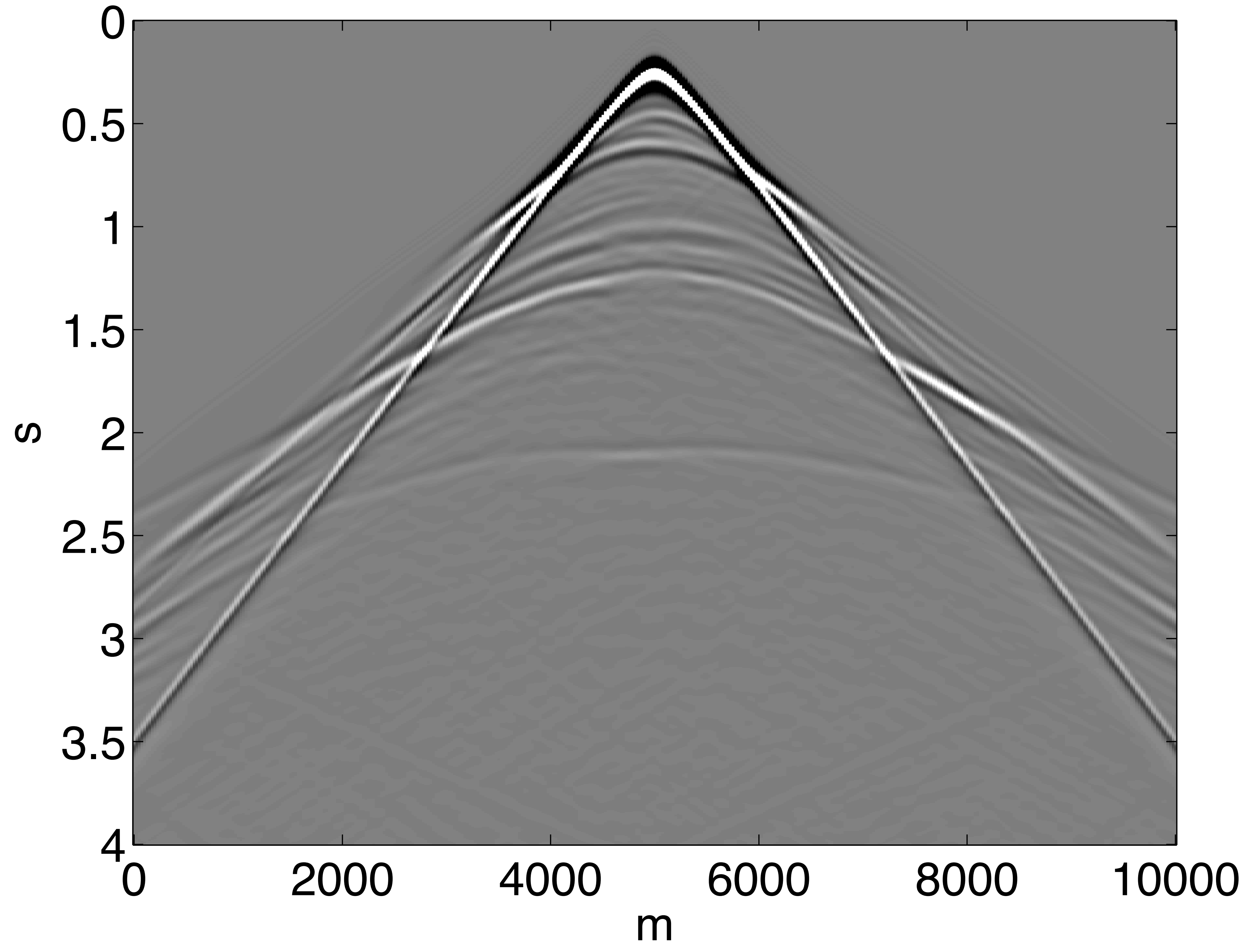


Motivation

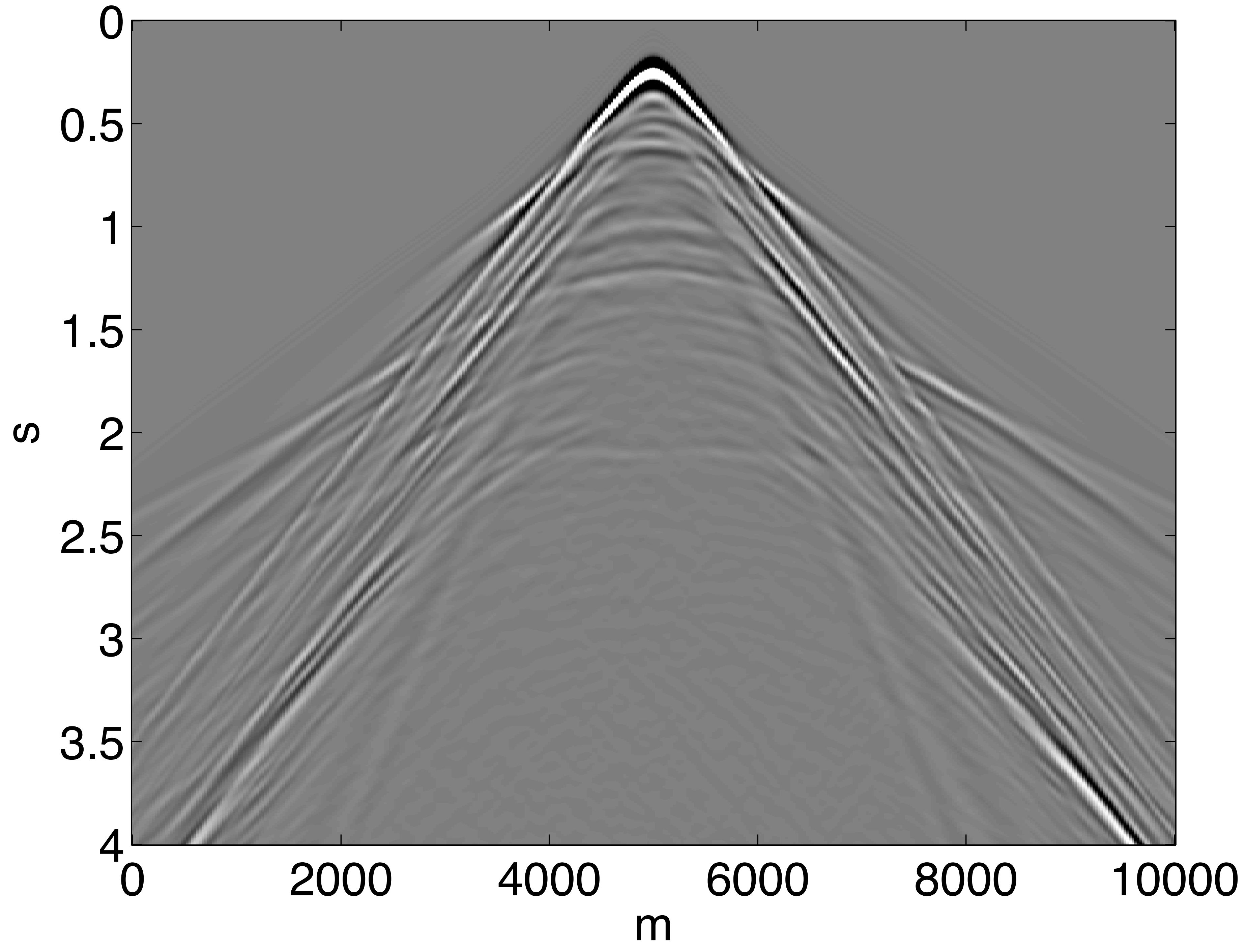
modeling kernel: time domain
finite difference

Thorbecke, 2013

one sample shot in time domain



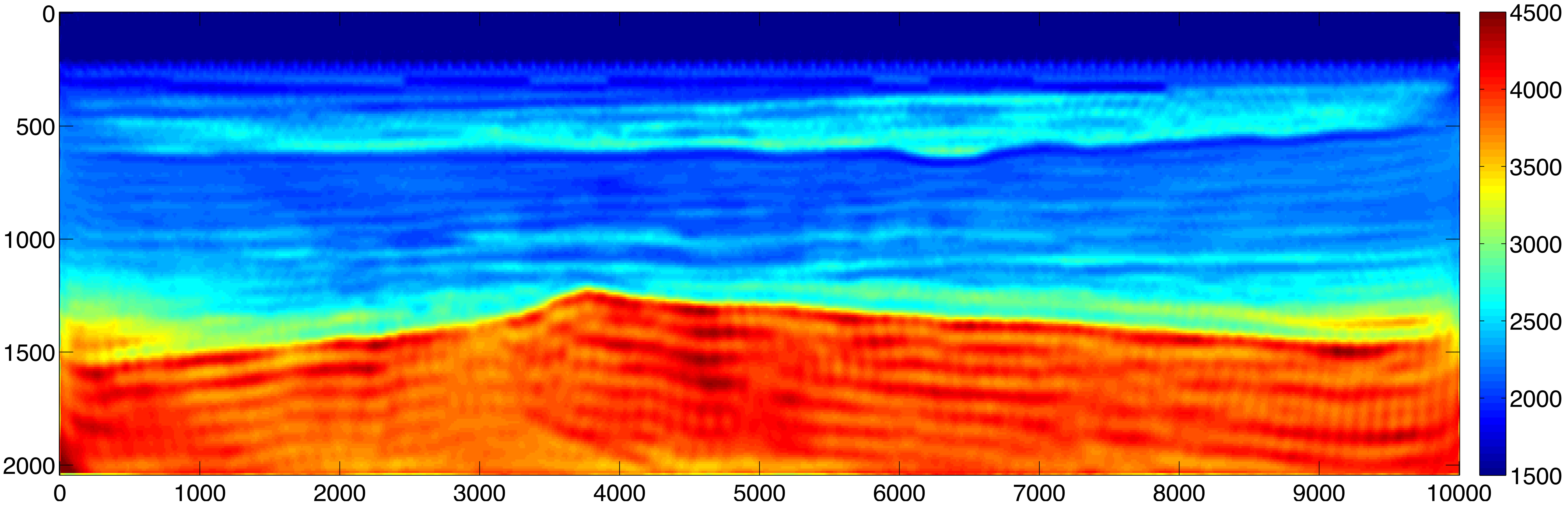
Acoustic



Elastic

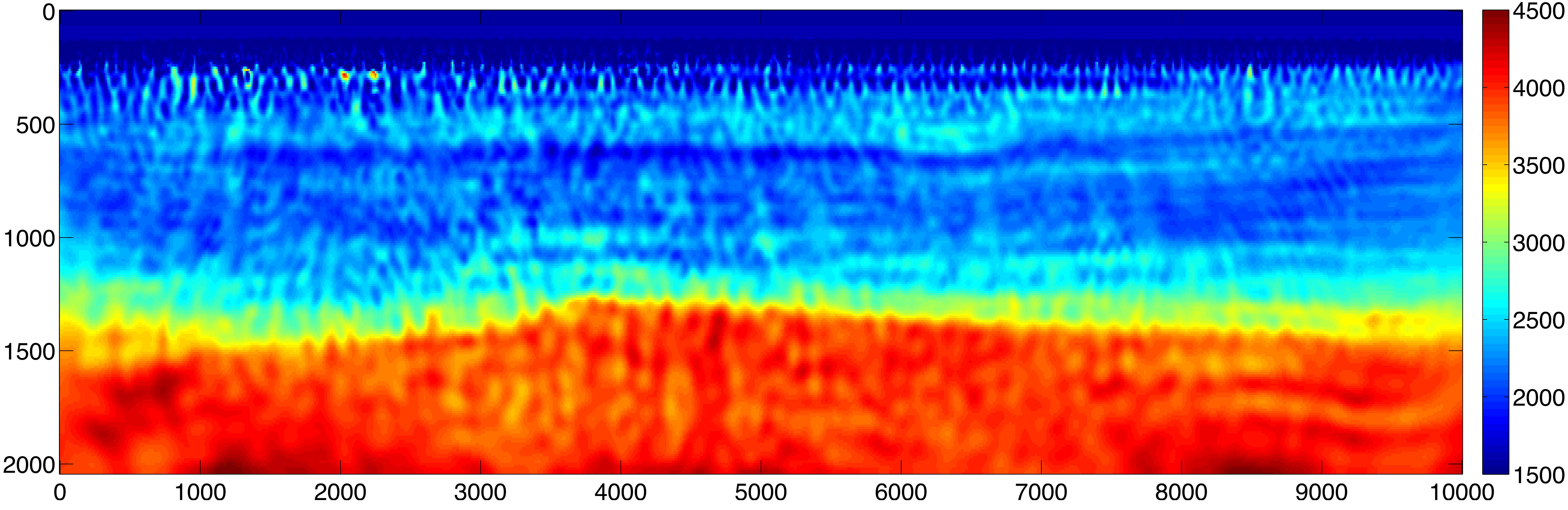
FWI example

standard acoustic Gauss-Newton inversion with *acoustic* data



FWI example

standard acoustic Gauss-Newton inversion with *elastic* data



Question

Can we invert *elastic* data with an *acoustic* modeling kernel?

Question

Can we invert *elastic* data with an *acoustic* modeling kernel?

- data-space student’s T

- model-space sparsity promotion

Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \sum_{i=1}^{K} \|\mathbf{d}_i - \boldsymbol{\mathcal{F}}_i(\mathbf{m})\|_2^2,$$

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
K	batch size
$\mathbf{m}$	unknown medium parameters

SLIM

Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - \boldsymbol{\mathcal{F}}_i(\mathbf{m})),$$

penalty function

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
$\boldsymbol{\mathcal{B}}_i$	data-processing operator
ρ	penalty function
K	batch size
$\mathbf{m}$	unknown medium parameters

Penalty functions

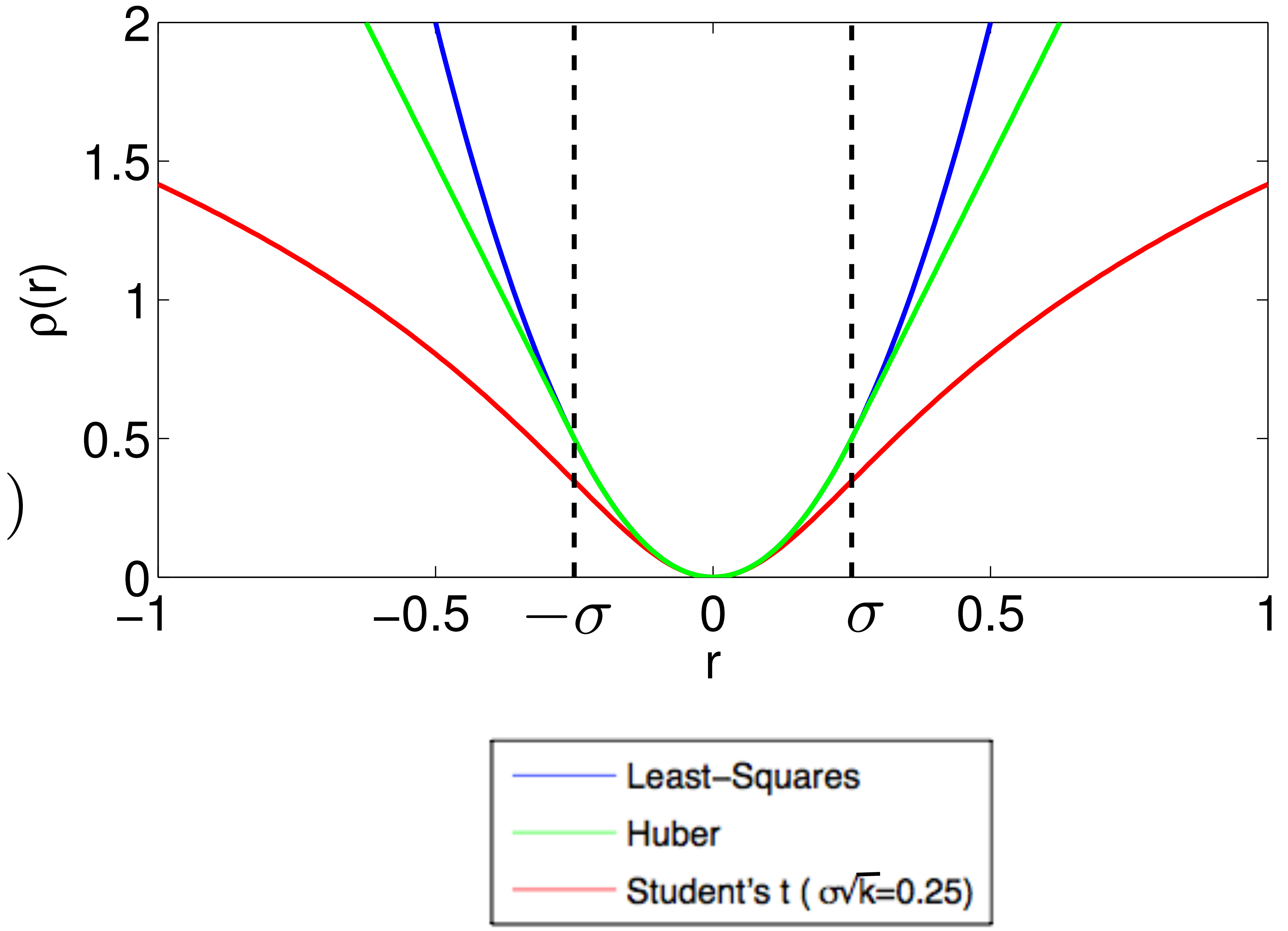
Least-squares
$$\rho(\mathbf{r}) = \sum_i |r_i|^2$$

Huber

Guitton and Symes, 2003
$$\rho(\mathbf{r}) = \sum_{|r_i|\le\sigma} \frac{1}{2\sigma^2}|r_i|^2 + \sum_{|r_i|>\sigma} \left(\frac{1}{\sigma}|r_i| - \frac{1}{2}\right)$$

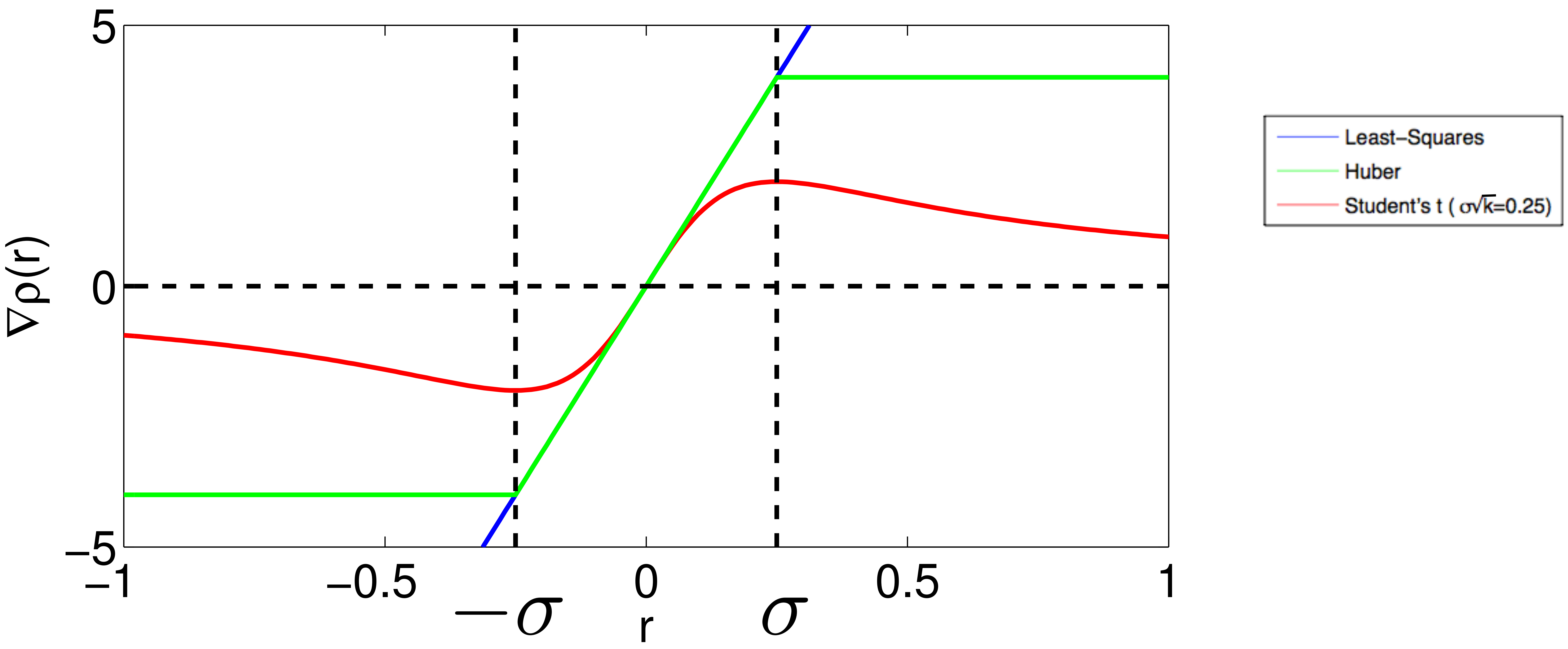
Student's T

Aravkin et al., 2011
$$\rho(\mathbf{r}) = \sum_i \mathbf{log}(1 + |r_i|^2/\sigma^2 k)$$



- student's T distribution is robust to large outliers and artifacts in the data

Influence functions



Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - \boldsymbol{\mathcal{F}}_i(\mathbf{m})),$$

transforms the residual into a domain where the outliers are localized

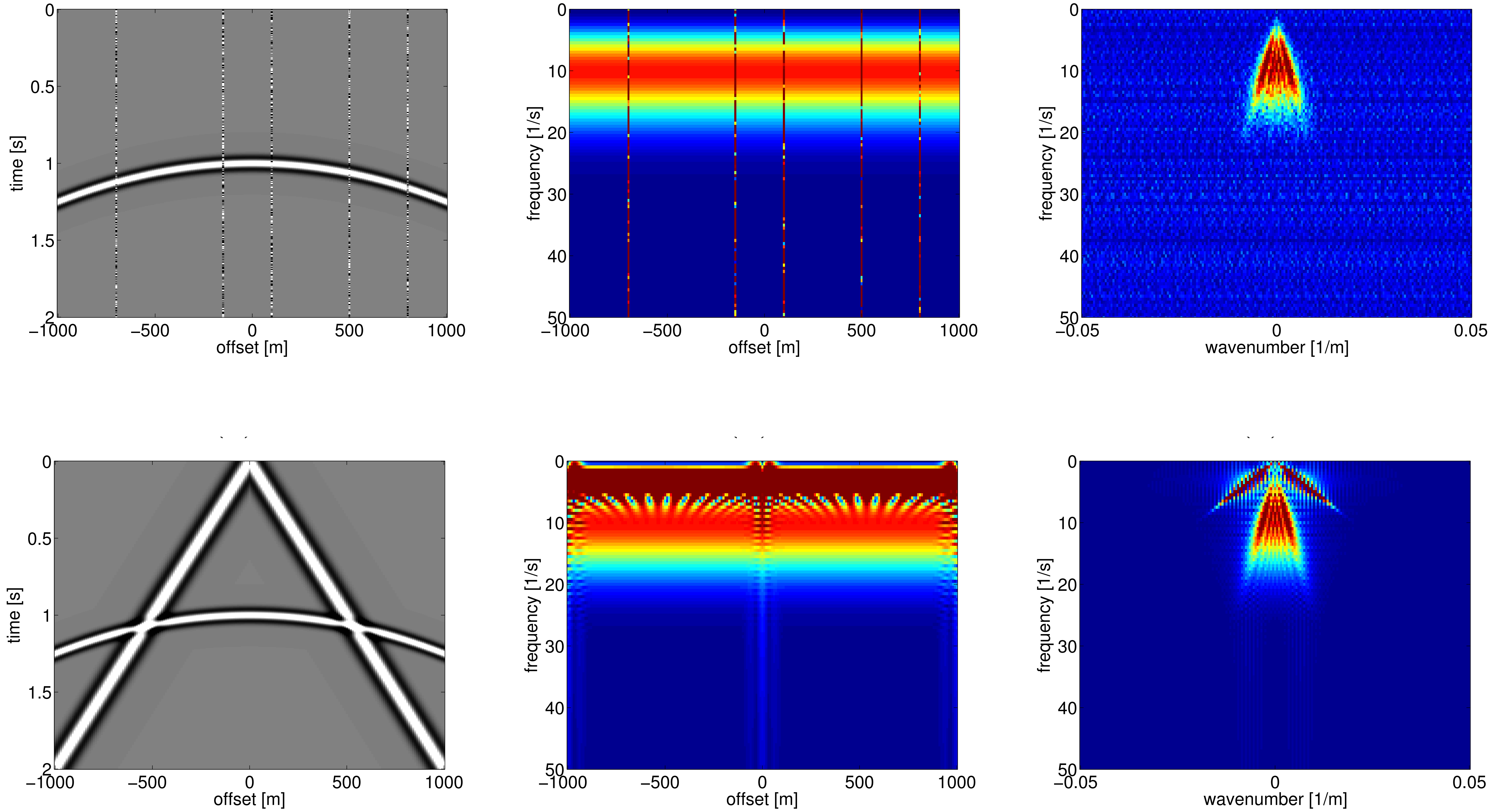
$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
$\boldsymbol{\mathcal{B}}_i$	data-processing operator
ρ	penalty function
K	batch size
$\mathbf{m}$	unknown medium parameters

Outliers in different domains

Van Leeuwen, 2013

bad traces

Spurious events



t-x

F-x

F-K

Example

observed data simulation

- DELPHI Package (delmodc) Thorbecke, 2013

ocean Bottom acquisition

Forward modeling (Elastic)		
101 shots	interval = 100m	depth = 200m
401 receivers	interval = 25 m	depth = 10m
Sampling time	interval = 4ms	total = 4 seconds

Inversion setting

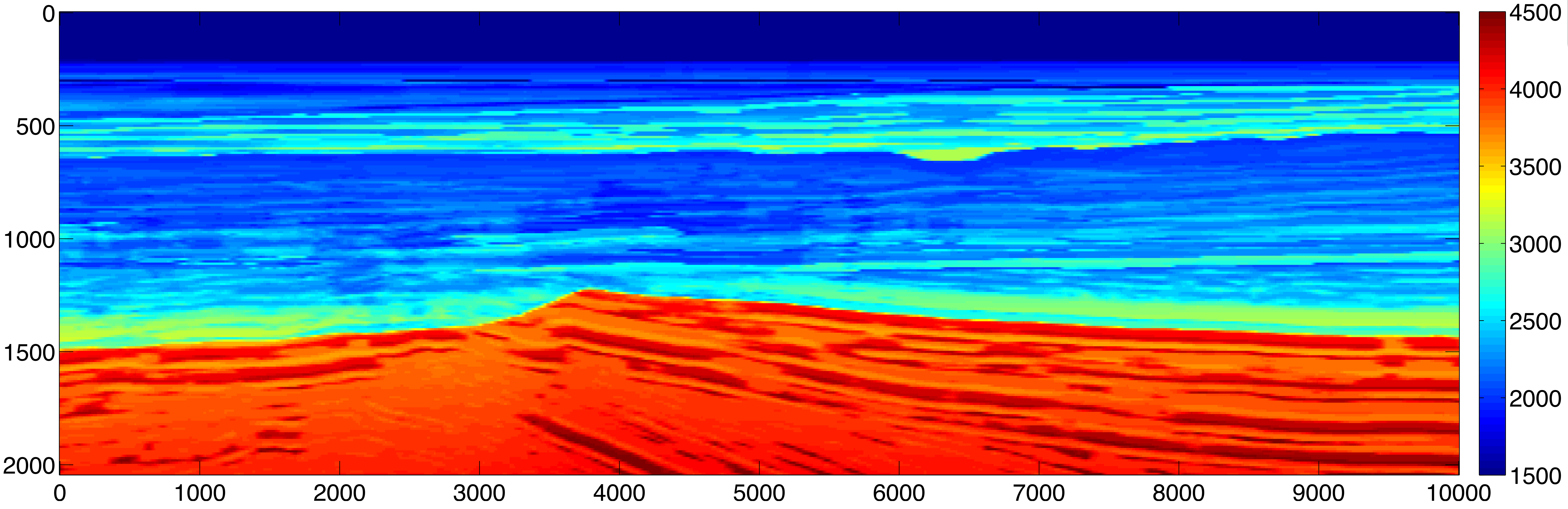
modeling kernel

- Frequency domain Helmholtz (acoustic)

acoustic inversion of elastic data

- 11 frequency band (3-12Hz), each has 3 frequcencies
- compute 10 *quasi-Newton (L-BFGS) updates* for each frequency band
- each L-BFGS update uses *all* shots

True model



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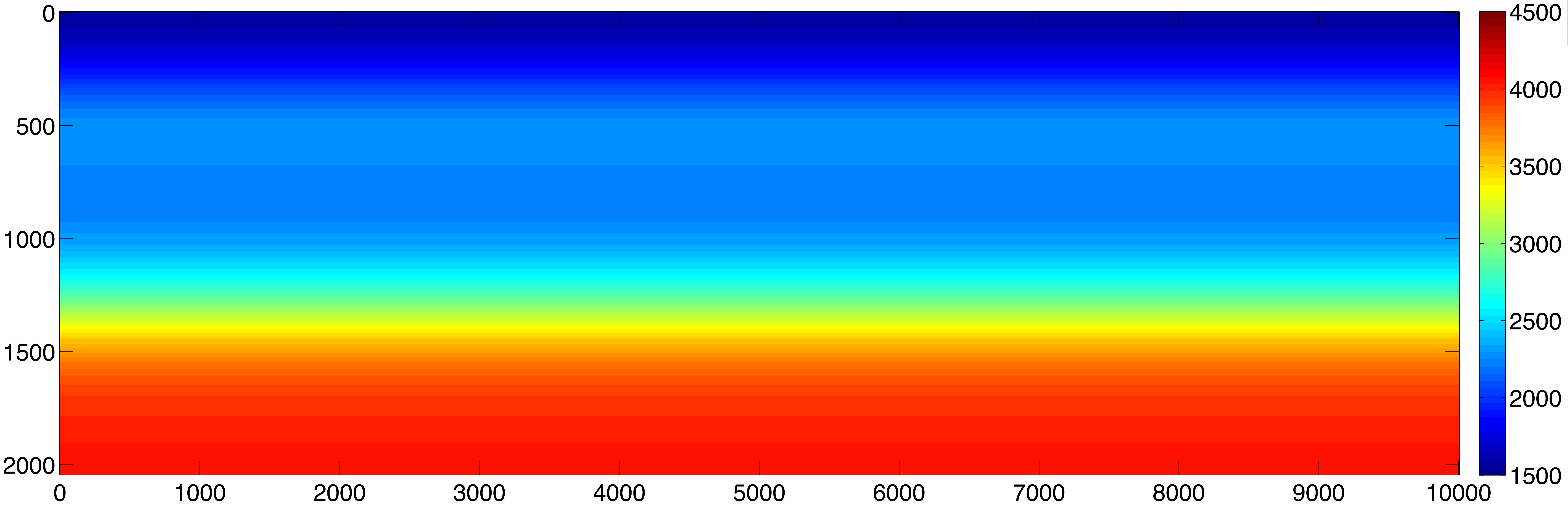
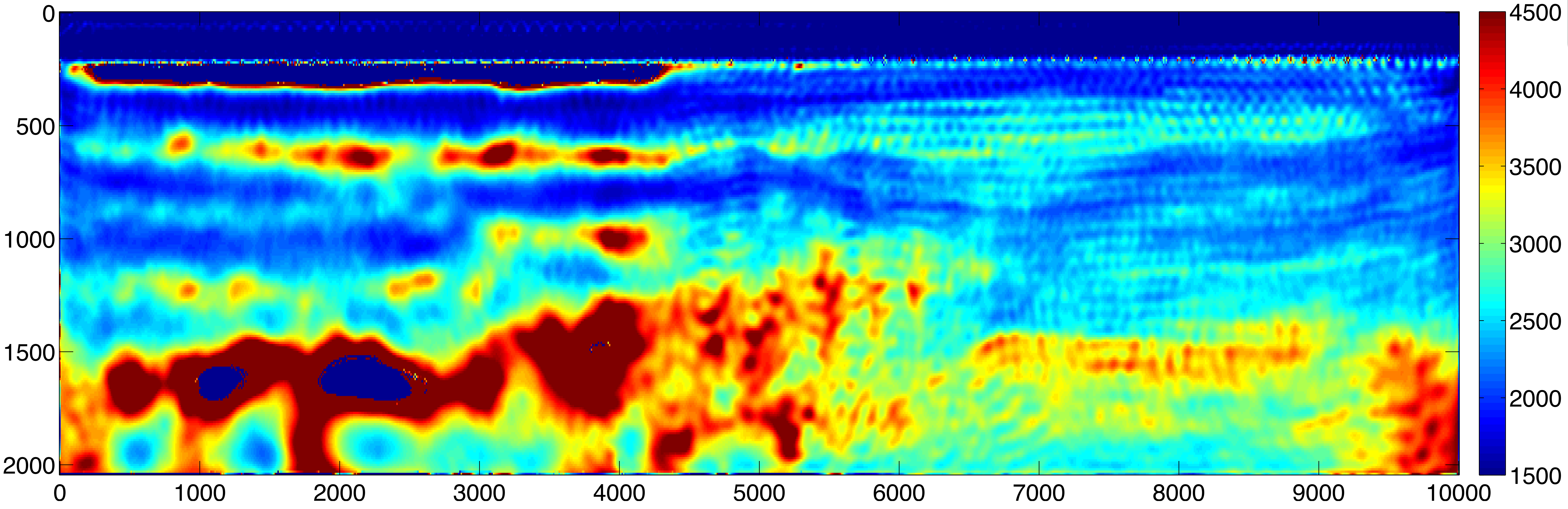
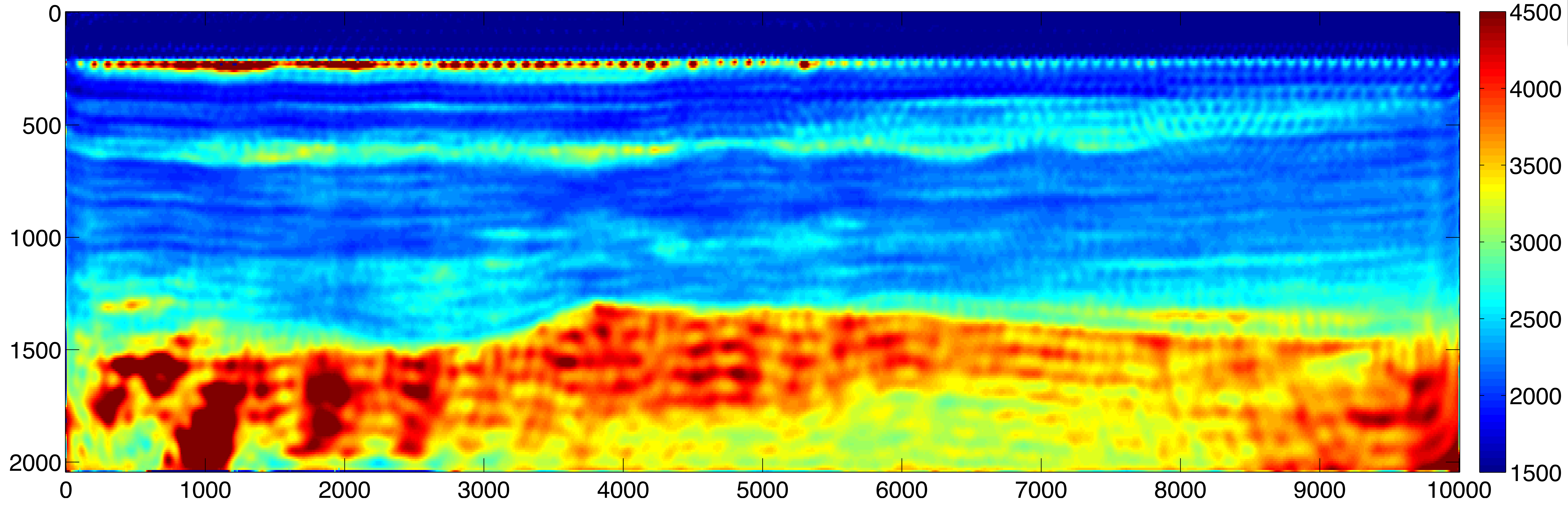
Initial model



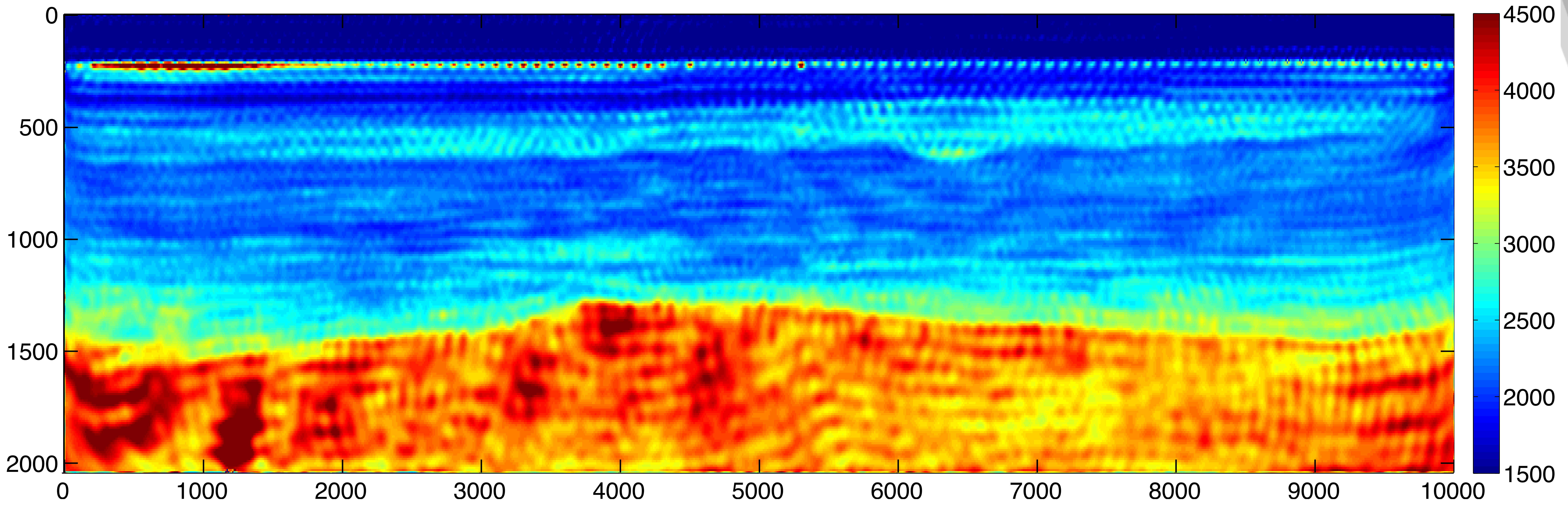
Least-Squares misfit result



Student's T misfit result



Student's T misfit in FK domain



Observations

- It is possible to invert *elastic* data with an *acoustic* modeling kernel, although the result is *not* perfect yet
- Student's T misfit function is insensitive to large outliers
- Appropriate transforms will help localize the outliers, improving the resolution

Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \sum_{i=1}^{K} \|\mathbf{d}_i - \boldsymbol{\mathcal{F}}_i(\mathbf{m})\|_2^2,$$

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
K	batch size
$\mathbf{m}$	unknown medium parameters

Full-waveform inversion

problem formulation in frequency domain

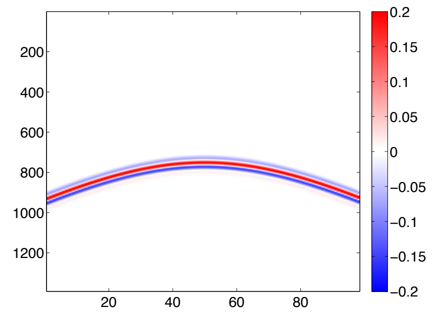
$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \sum_{i=1}^{K} \|\mathbf{d}_i - \boldsymbol{\mathcal{F}}_i(\mathbf{m})\|_2^2 \qquad \text{"} + \quad \mathbf{R}(\mathbf{m})\text{"}$$

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
K	batch size
$\mathbf{m}$	unknown medium parameters
$\mathbf{R}$	regularization

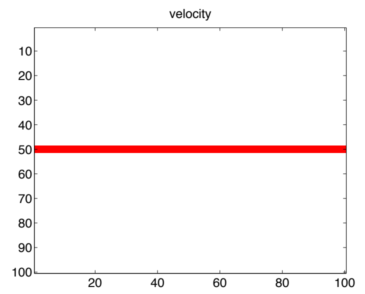
Gauss-Newton update

Gauss-Newton subproblem:

$$\delta\mathbf{m} = \mathop{\arg\min}_{\delta\mathbf{m}} \frac{1}{2}\sum_{i=1}^{K'} \|\delta\mathbf{d}_i - \nabla\boldsymbol{\mathcal{F}}_i[\mathbf{m_0}]\quad \delta\mathbf{m}\|_2^2$$



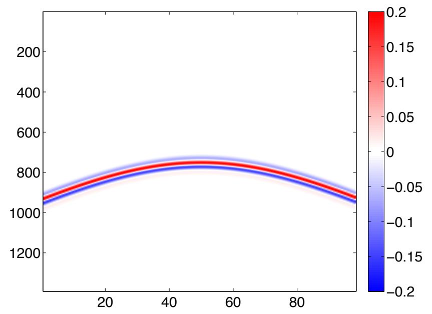
Jacobian operator (born modeling operator)



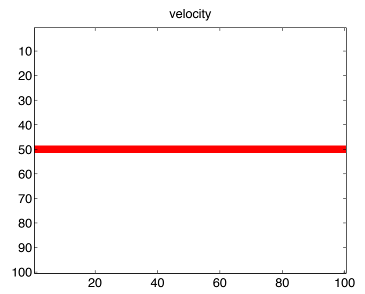
Gauss-Newton update

Gauss-Newton subproblem:

$$\delta\mathbf{m} = \quad \underset{\delta\mathbf{m}}{\arg\min}\ \frac{1}{2}\sum_{i=1}^{K'} \|\delta\mathbf{d}_i - \nabla\boldsymbol{\mathcal{F}}_i[\mathbf{m_0}] \quad \delta\mathbf{m}\|_2^2$$

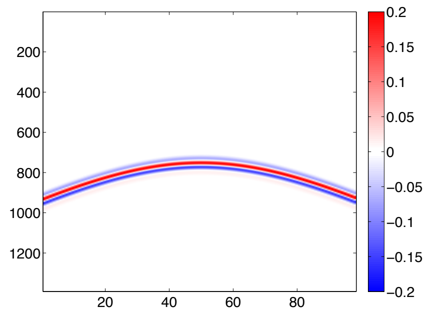


Jacobian operator (born modeling operator)



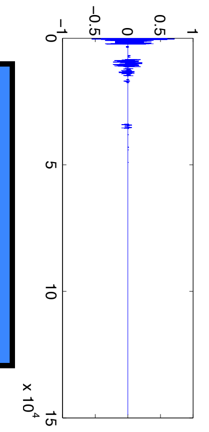
Modified GN subproblem:

$$\delta\mathbf{m} = \mathbf{C^H}\ \underset{\mathbf{x}}{\arg\min}\ \frac{1}{2}\sum_{i=1}^{K'} \|\delta\mathbf{d}_i - \nabla\boldsymbol{\mathcal{F}}_i[\mathbf{m_0}]\mathbf{C^H}\mathbf{x}\|_2^2 \quad \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau$$



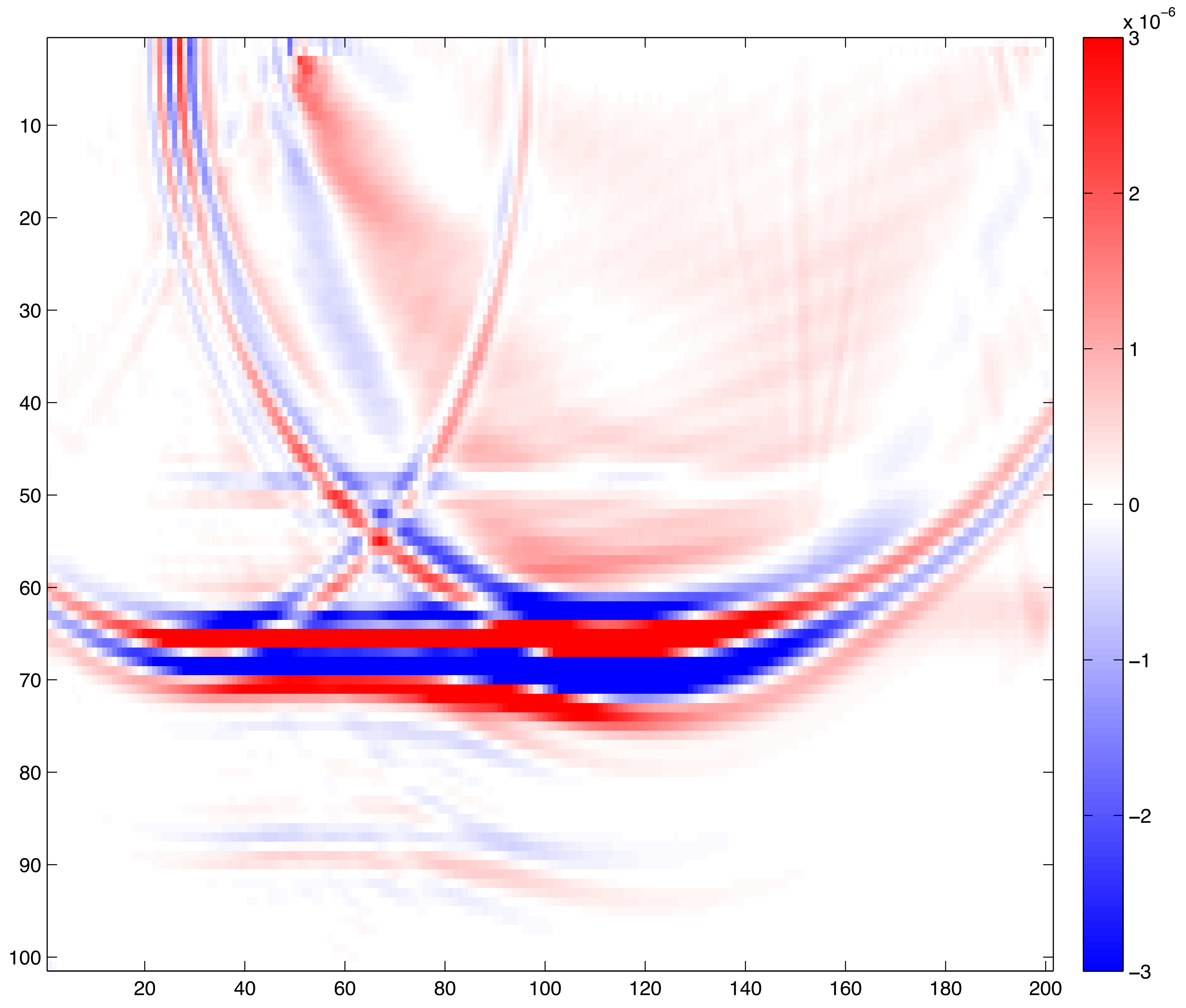
Jacobian operator (born modeling operator)

Sparsifying transform

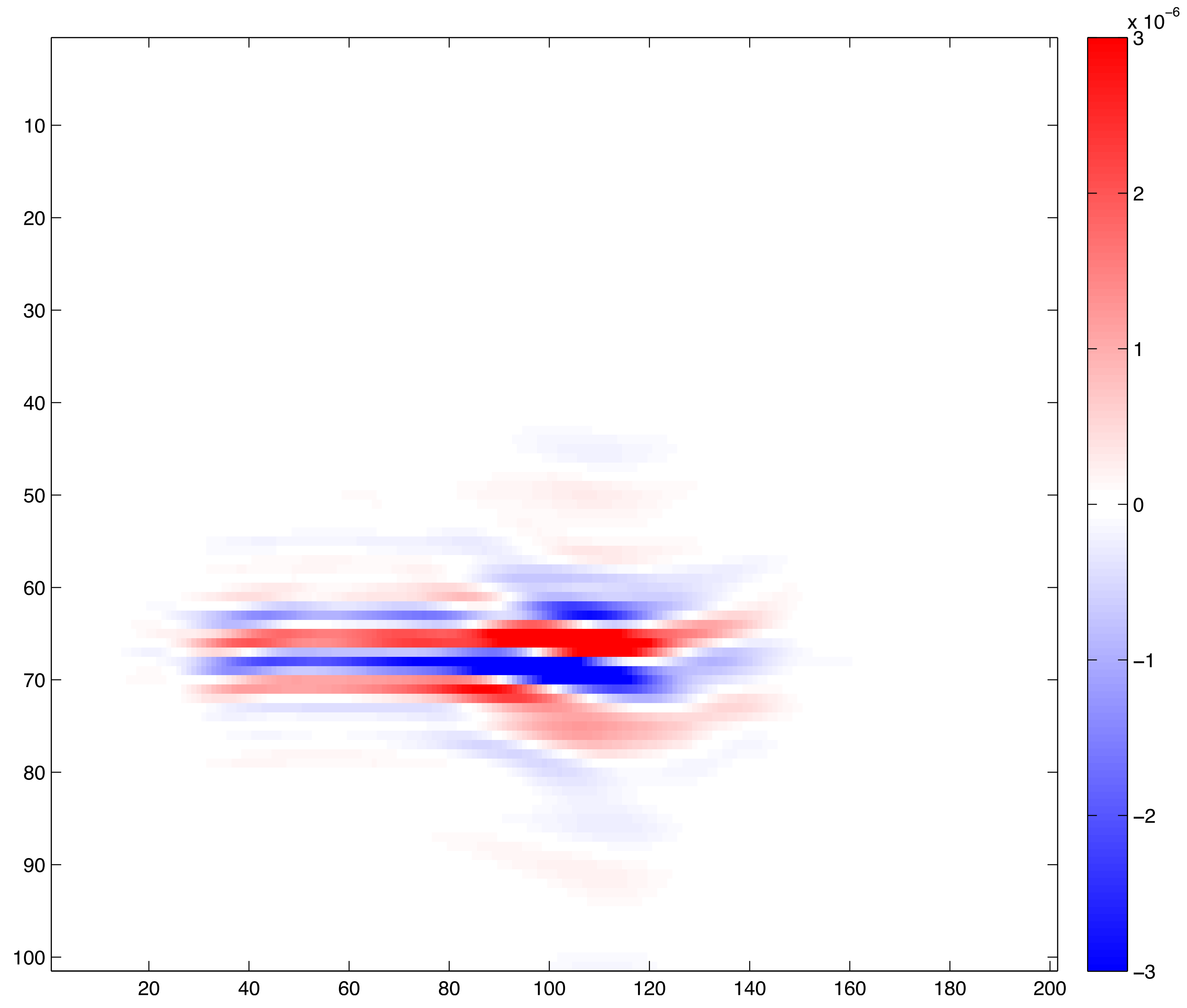


$\mathbf{C^H}$ inverse curvelet transfrom

Gauss-Newton update



GN update w\o curvelet regularization



GN update w\ curvelet regularization

Inversion setting

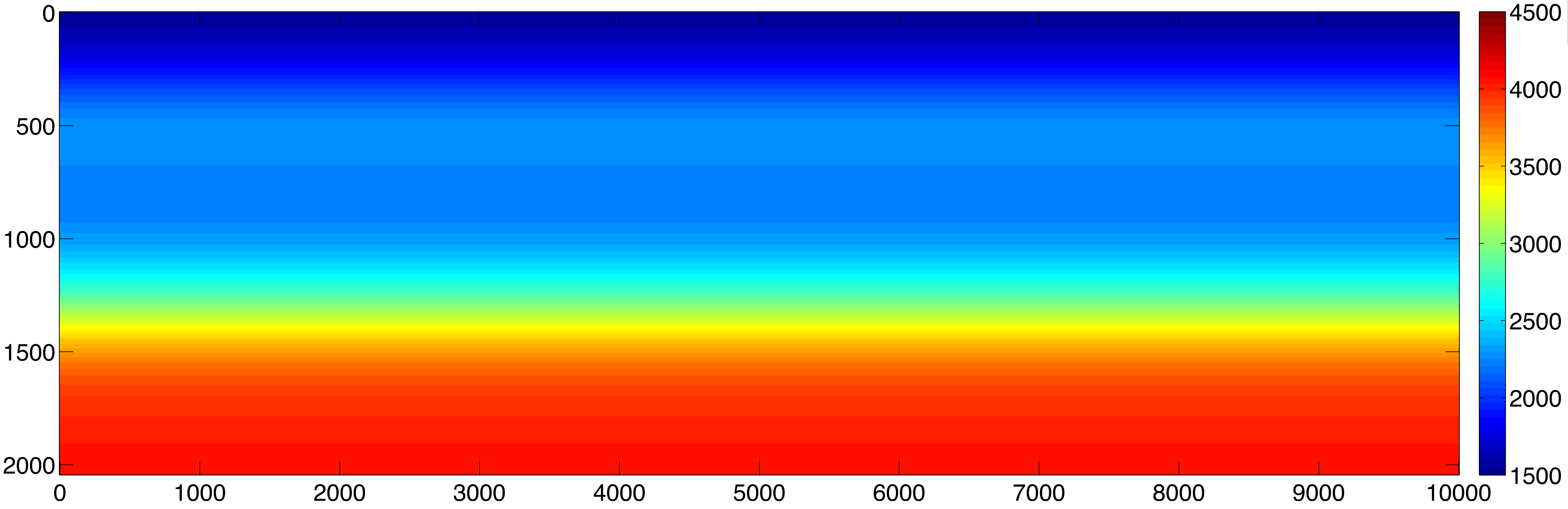
modeling kernel

- Frequency domain Helmholtz (acoustic)

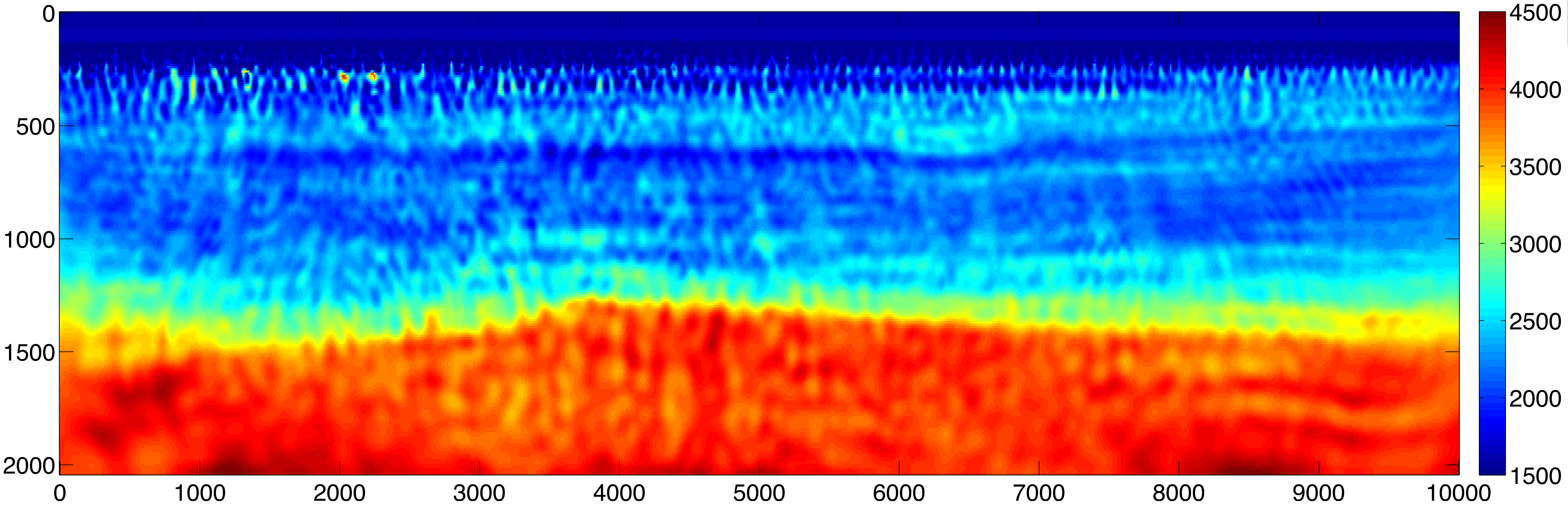
acoustic inversion of elastic data

- 11 frequency band (3-12Hz), each has 3 frequcencies
- compute 5 sparsity-promoting GN *updates* for each frequency band
- each GN update uses *50 randomly selected* shots

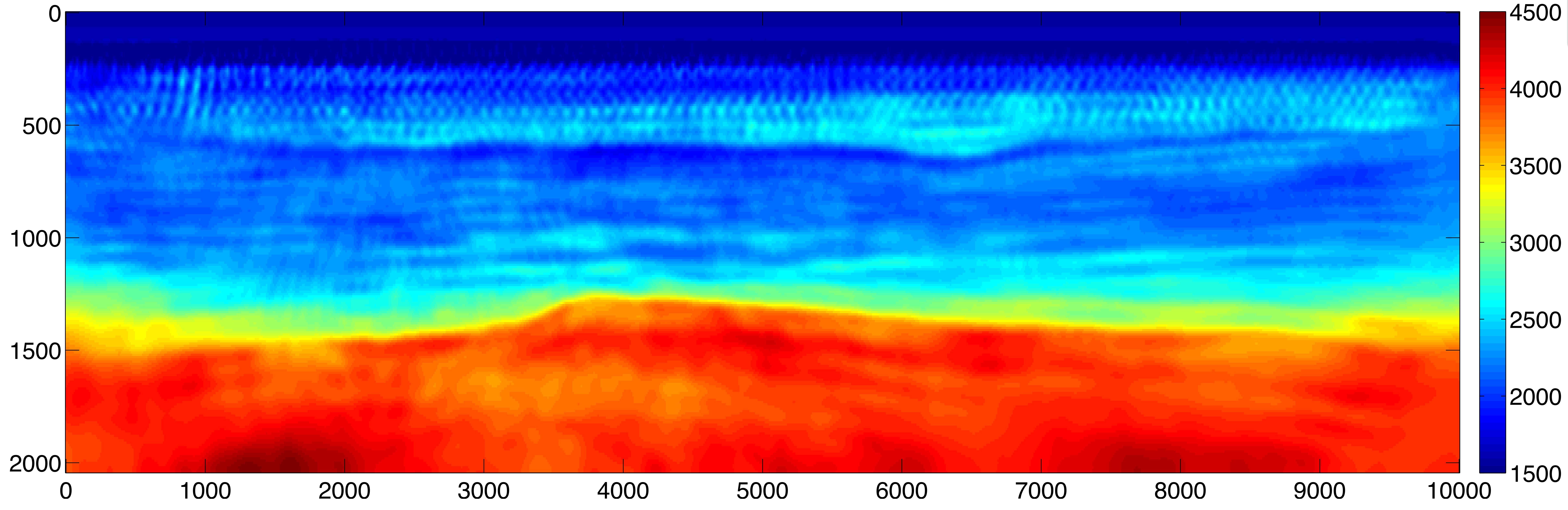
Initial model



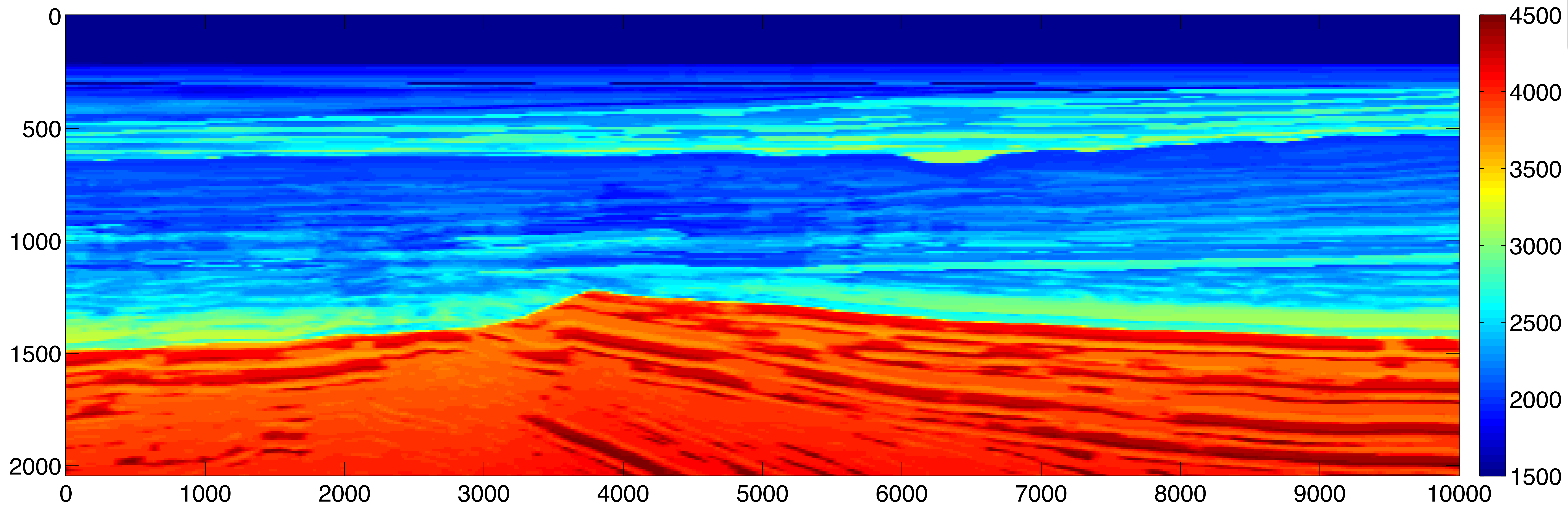
GN inversion w/o *sparsity* promotion



GN inversion w/ sparsity promotion



True model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Observations

- *Curvelet transform* efficiently represents geological models

- *sparsity regularization in Curvelet domain* can significantly suppress model space artifacts

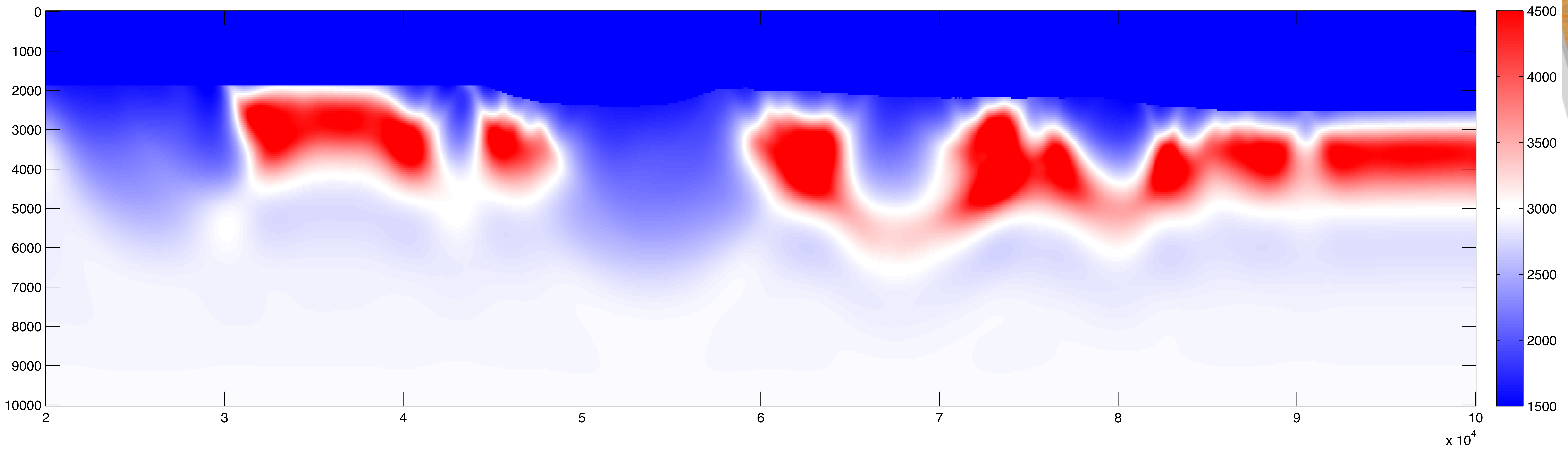
SLIM

Gulf of Mexico data

Chevron blind test

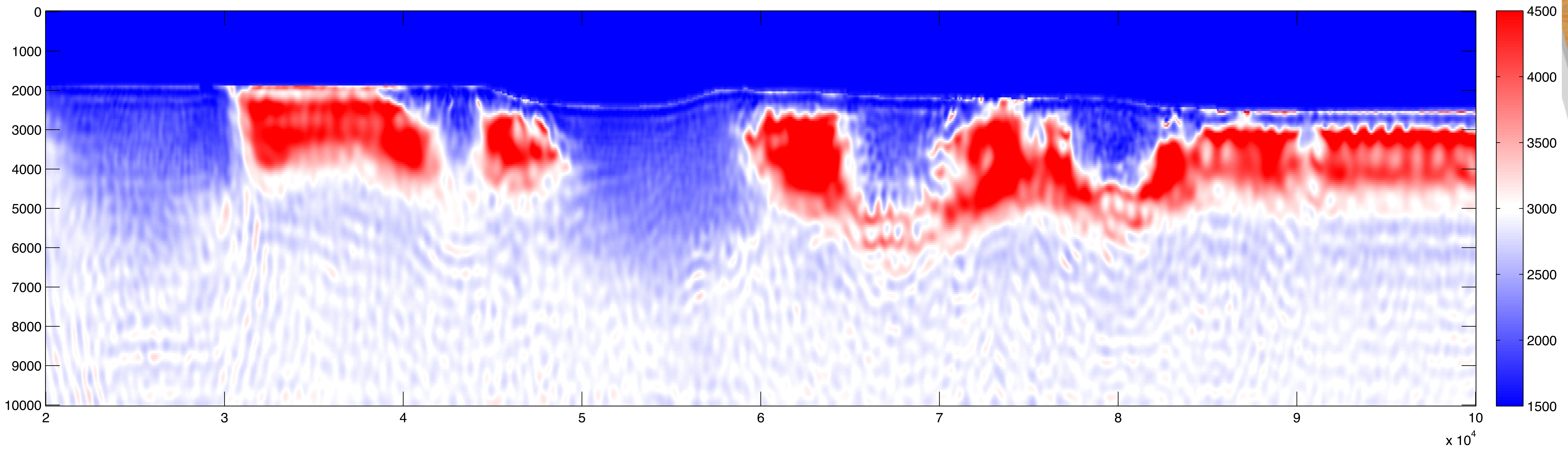
- 3201 shots with interval 25 m
- 801 receivers with interval 25 m, yielding 20km offset
- record time 14s, sample rate 4ms
- free surface
- isotropic elastic

Initial model [ray-based tomography]

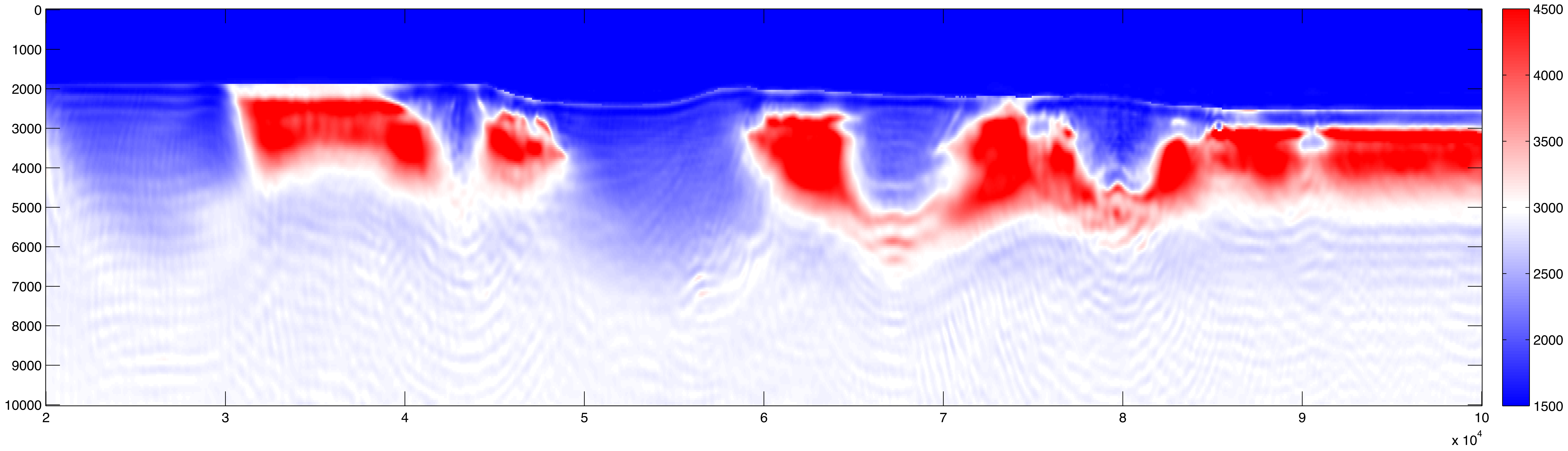


Andrew J. Calvert

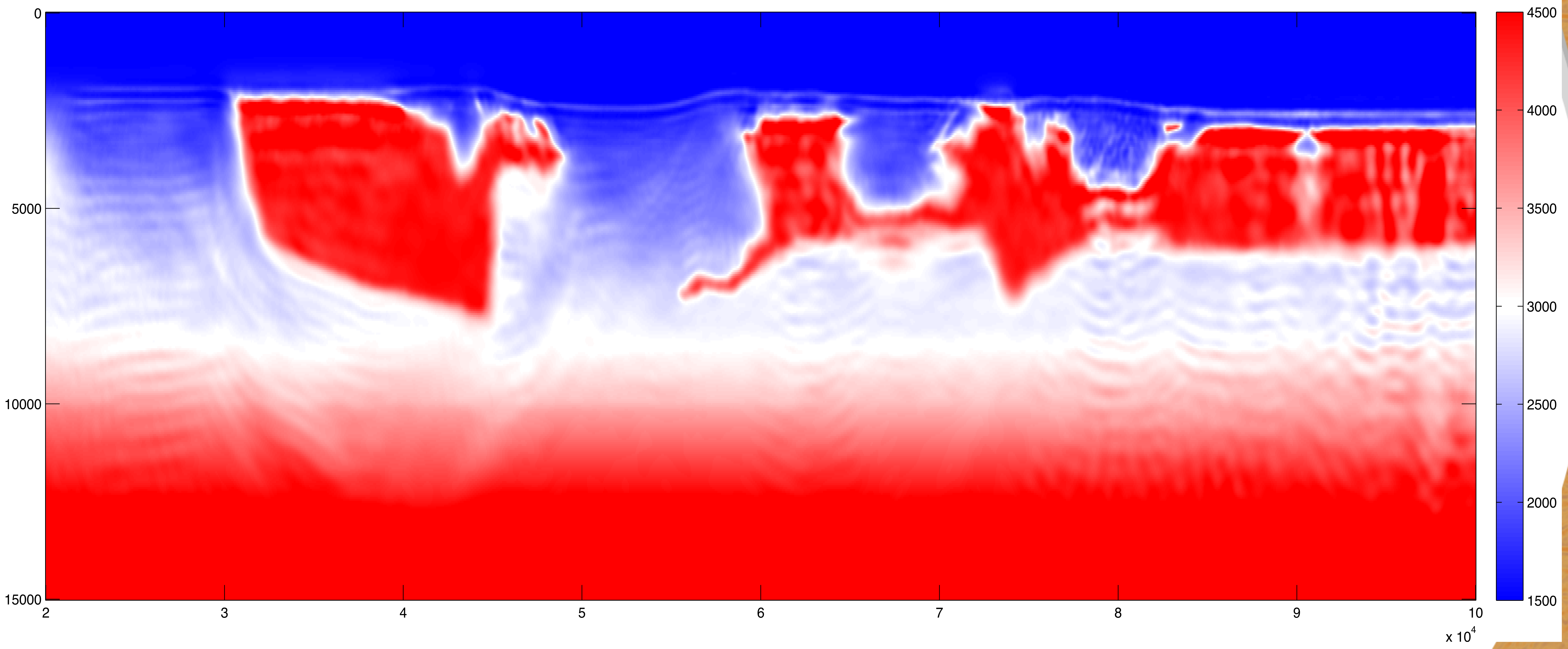
Standard GN FWI without sparsity promoting



Sparsity promoting GN FWI



“Latest” result



Future plan

- a wise way to stop the algorithm
- 3D FWI or Least-squares imaging
- take advantages from both time domain and frequency domain algorithm

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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