Wavefield reconstruction via SVD-free low-rank matrix factorization

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Motivation

- acquisition challenges
  - missing data
  - noise

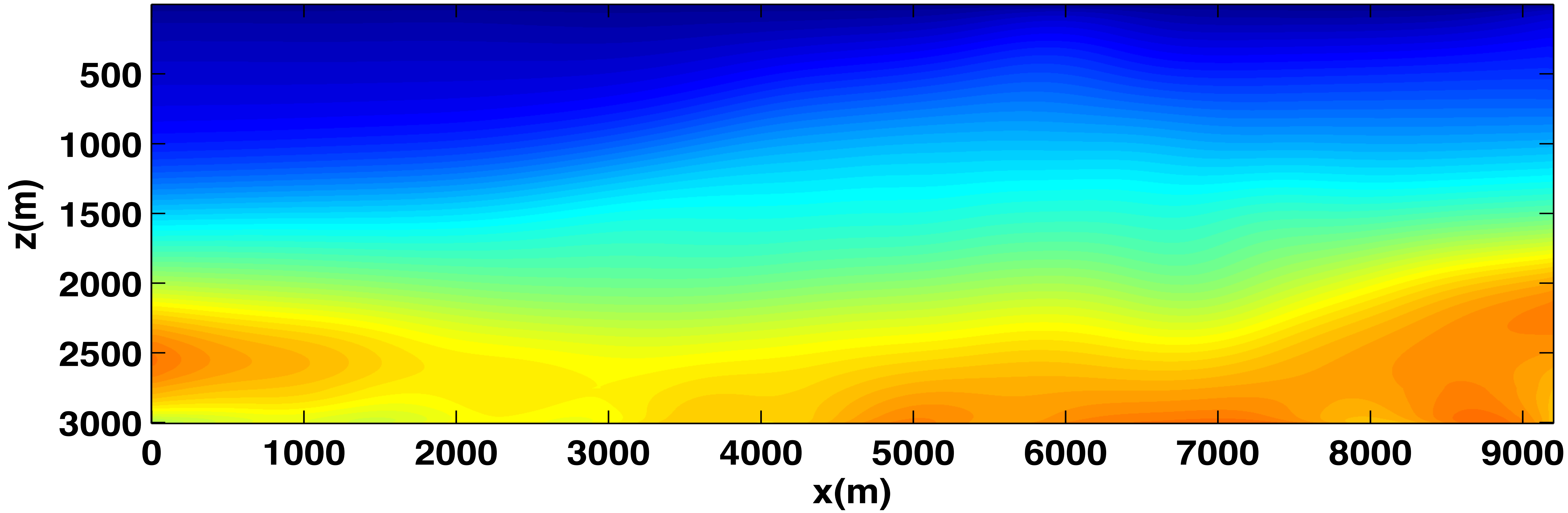
- fully sampled data
  - simultaneous shot based FWI & migration
  - estimation of primaries by sparse inversion & SRME

- exploit *low-rank* structure of seismic data
  - randomized sampling
  - *SVD-free* matrix factorization

[Li et. al. 2012, van Leeuwen and Herrmann 2012]

Full waveform inversion

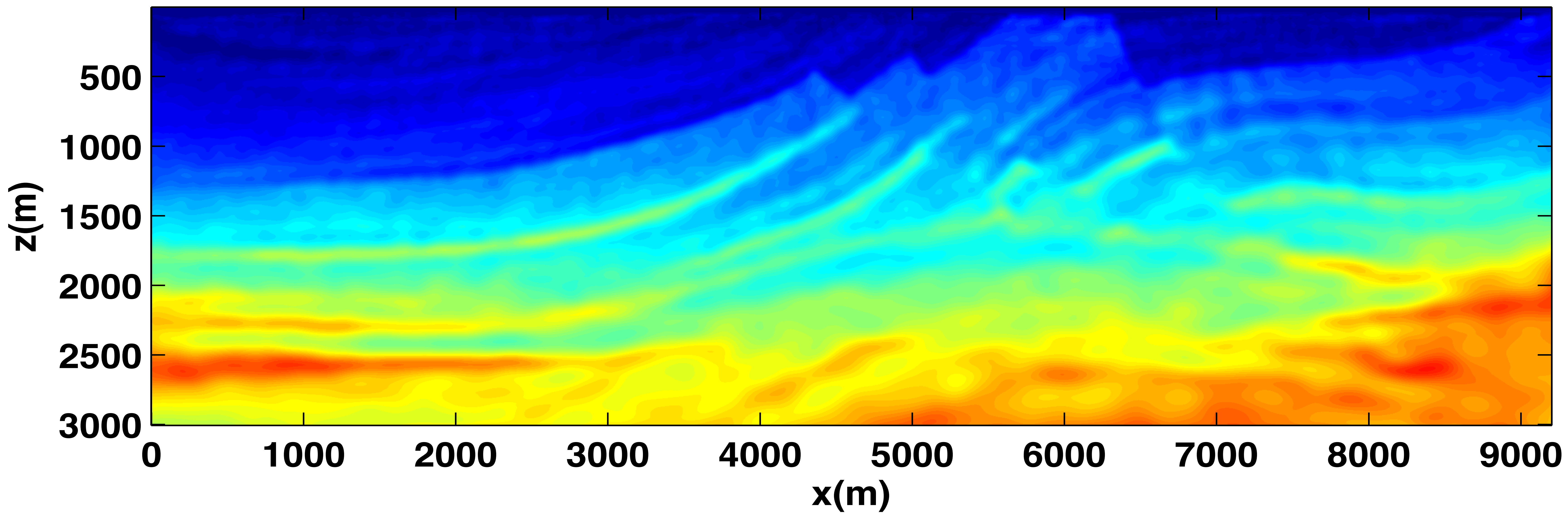
[initial model]



[Li et. al. 2012, van Leeuwen and Herrmann 2012]

Full waveform inversion

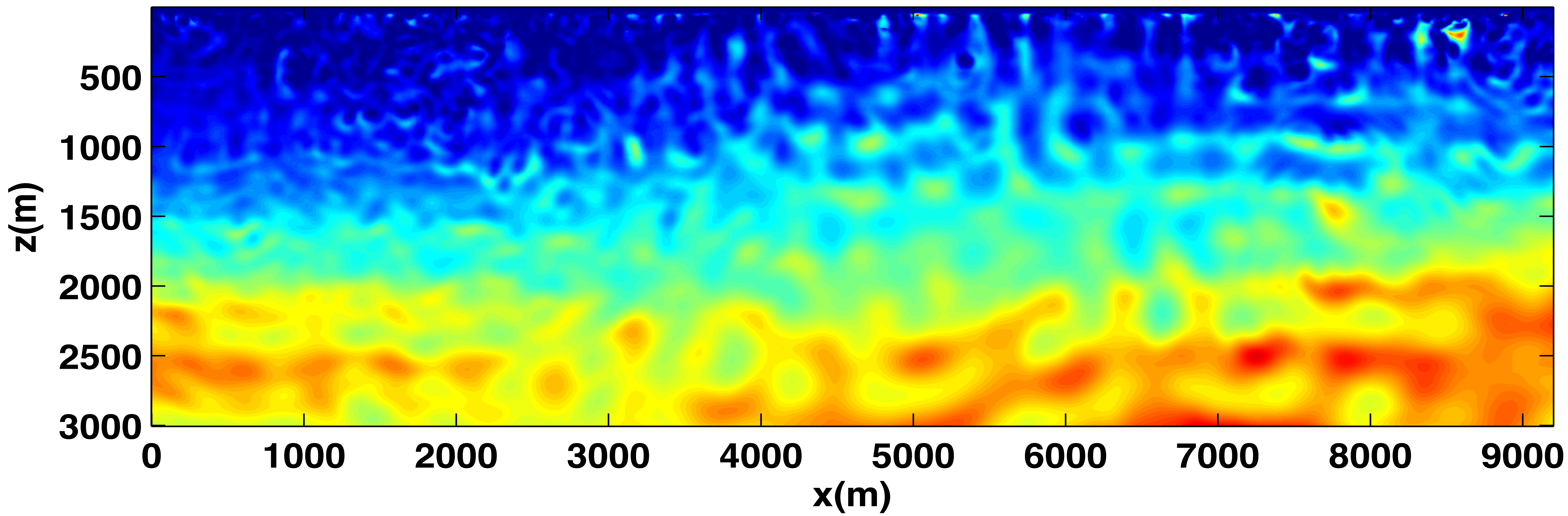
[fully sampled data]



[Li et. al. 2012, van Leeuwen and Herrmann 2012]

Full waveform inversion

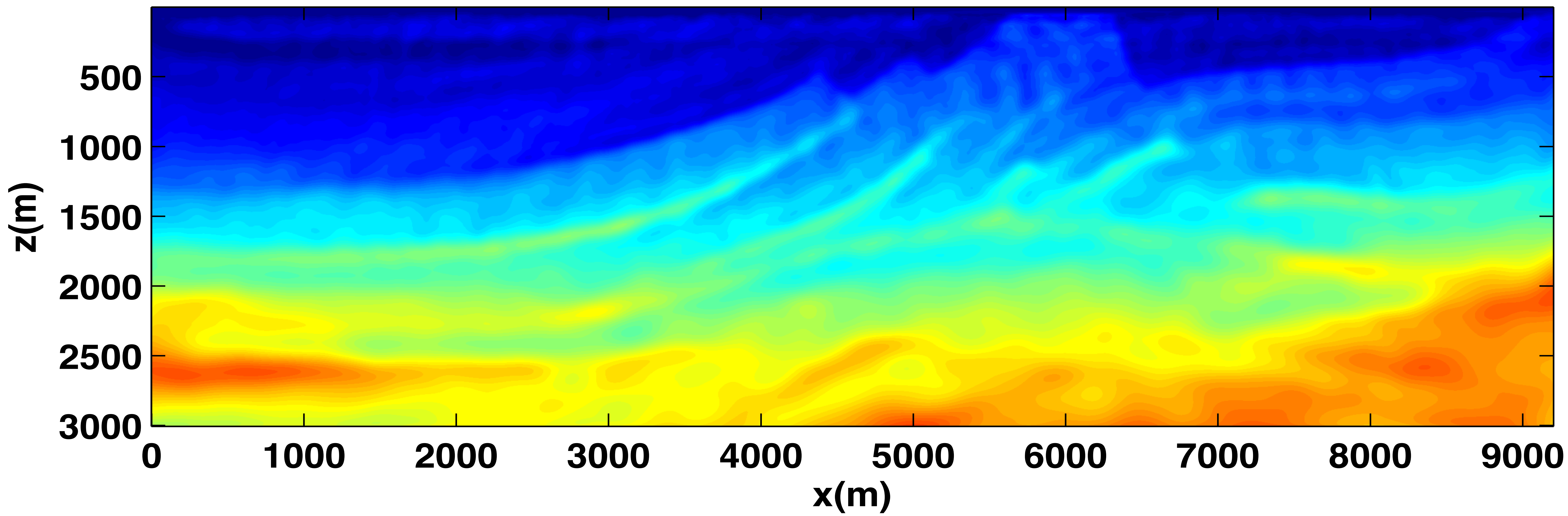
[Subsampled data]



[Li et. al. 2012, van Leeuwen and Herrmann 2012]

Full waveform inversion

[low-rank interpolation]



[1] Oropeza V and M D Sacchi, 2011, Simultaneous seismic data de-noising and reconstruction via Multichannel Singular Spectrum Analysis (MSSA), Geophysics, 76 (3), V25-V32.

[2] Nadia Kreimer, Aaron Stanton and Mauricio D. Sacchi, Tensor completion based on nuclear norm minimization for 5D seismic data reconstruction, Technical report, 2013

Existing work

- SVD computation [1,2]
  - expensive for large scale data
- Overlap windows in space and time [2]
  - heuristic approach
- uniform random subsampling
  - no control on the maximum gap size

[Candes and Donoho 2000, Herrmann 2008]

Compressive sensing
[transform based]

- signal structure
  - *sparse/compressible*
- sampling scheme
  - random missing traces make signal *less sparse* in transform domain
- recovery using *sparsity promoting* scheme

is *sparsity* the only inherent structure in seismic??

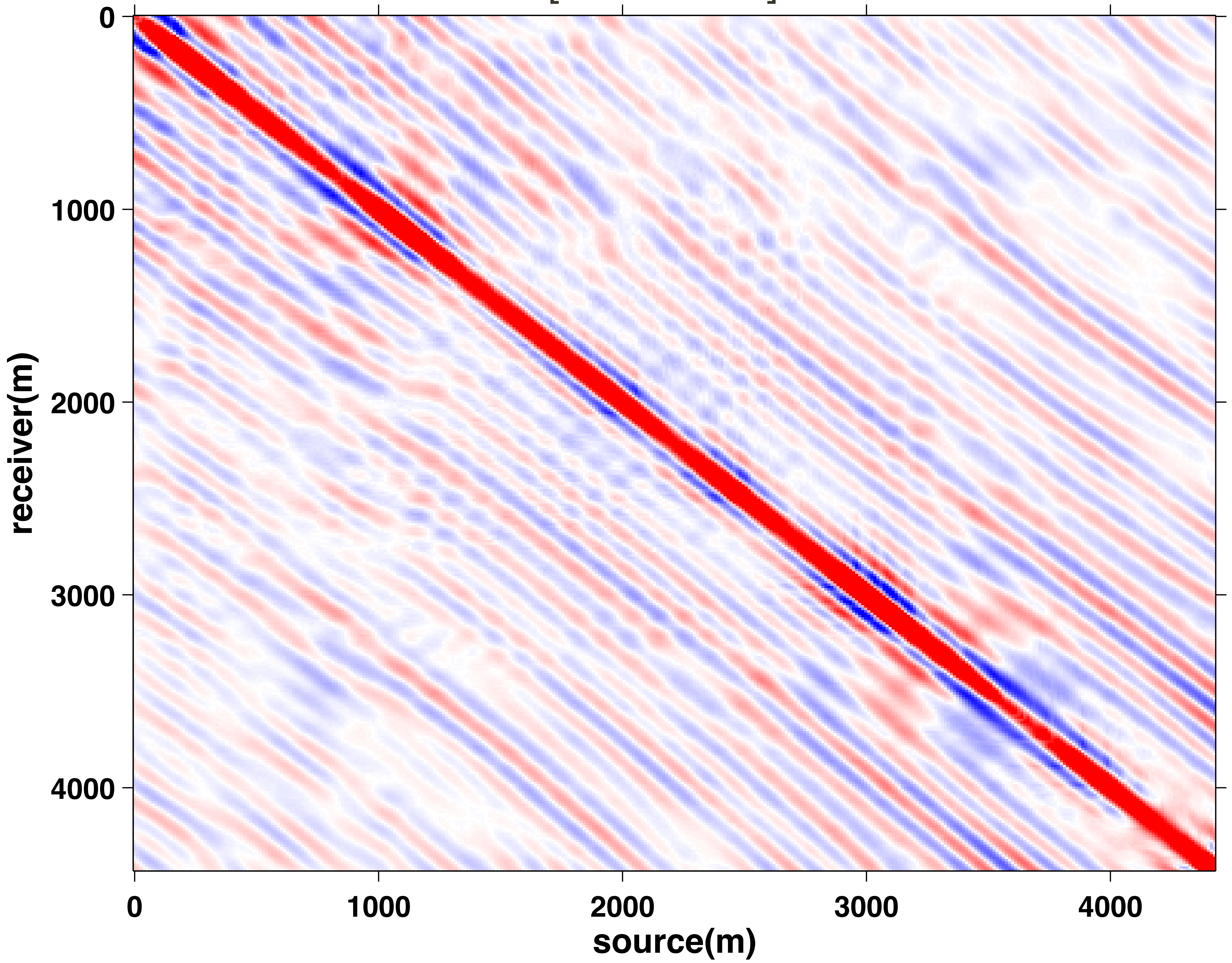
[Candes and Plan 2010, Oropeza and Sacchi 2011]

Matrix completion

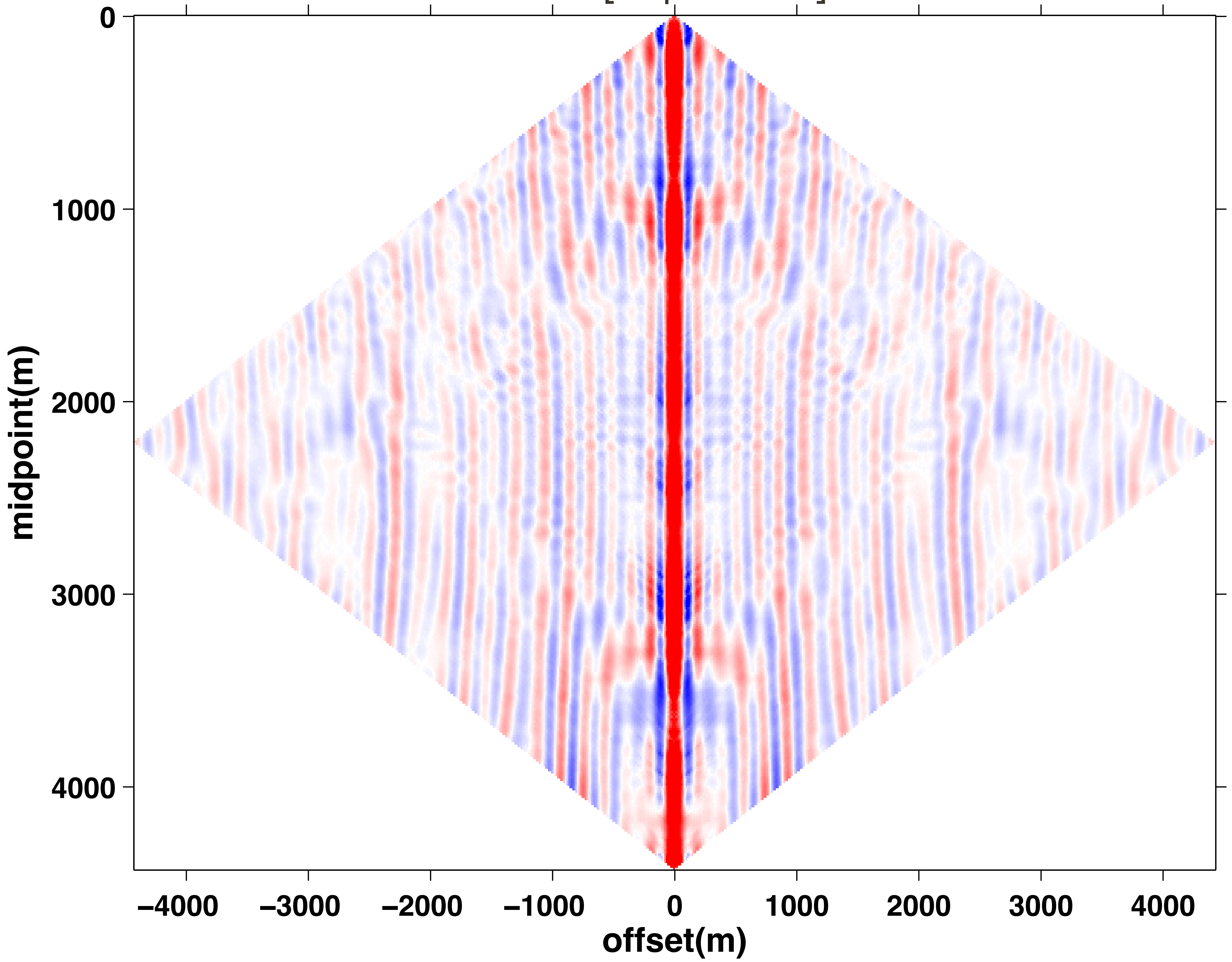
- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Low-rank structure
[2-D acquisition]

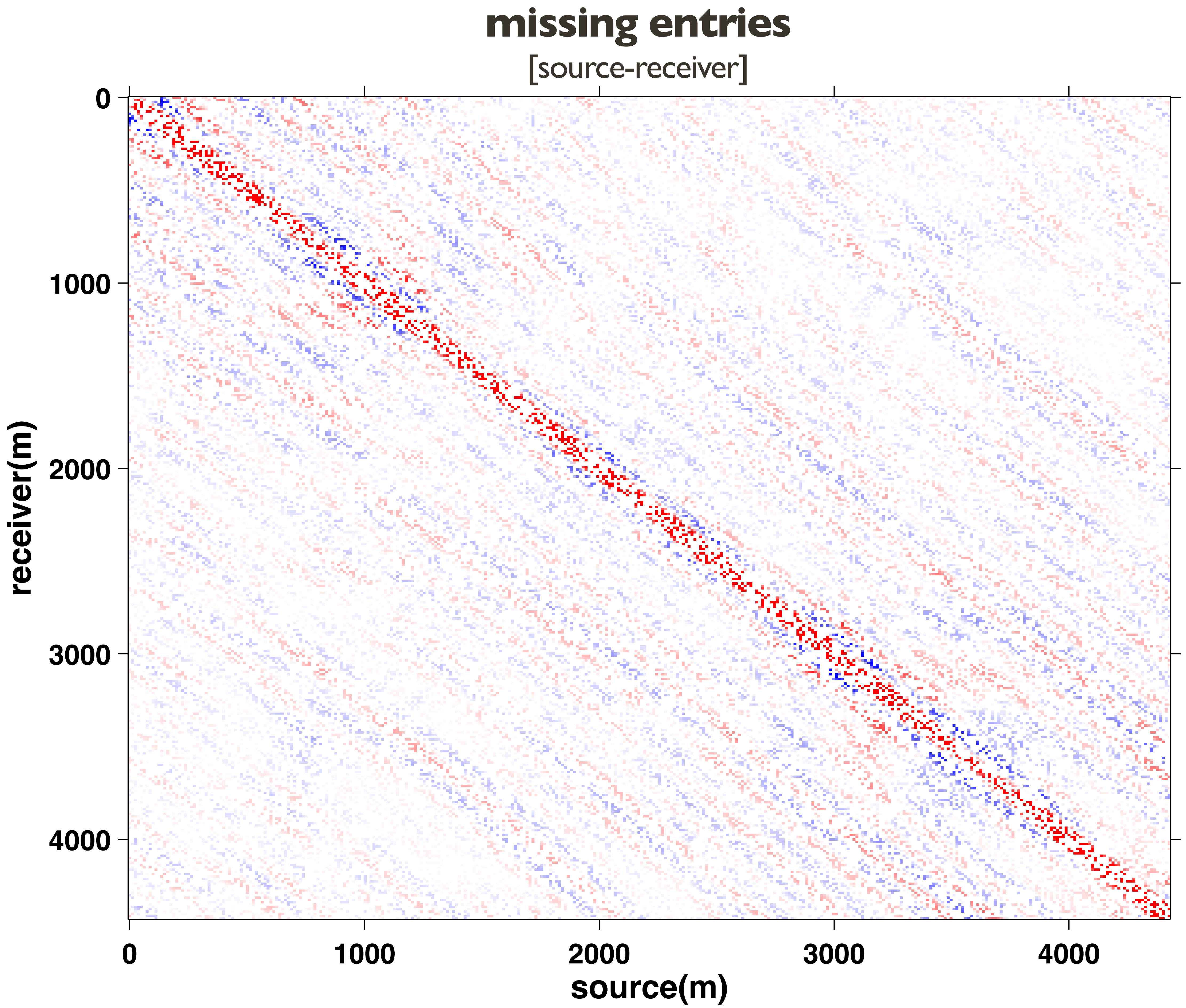
acquisition domain
[source-receiver]

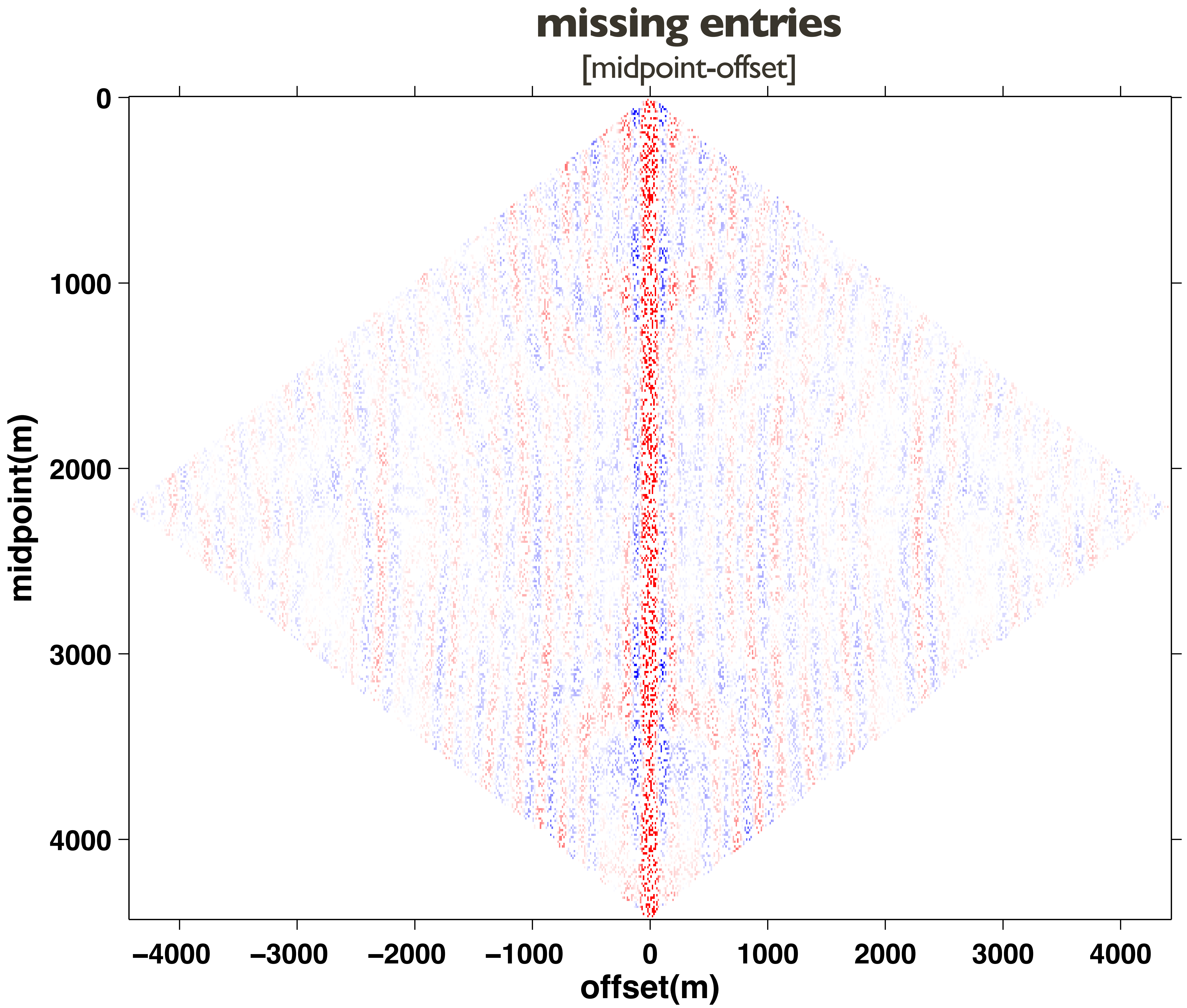


transform domain
[midpoint-offset]

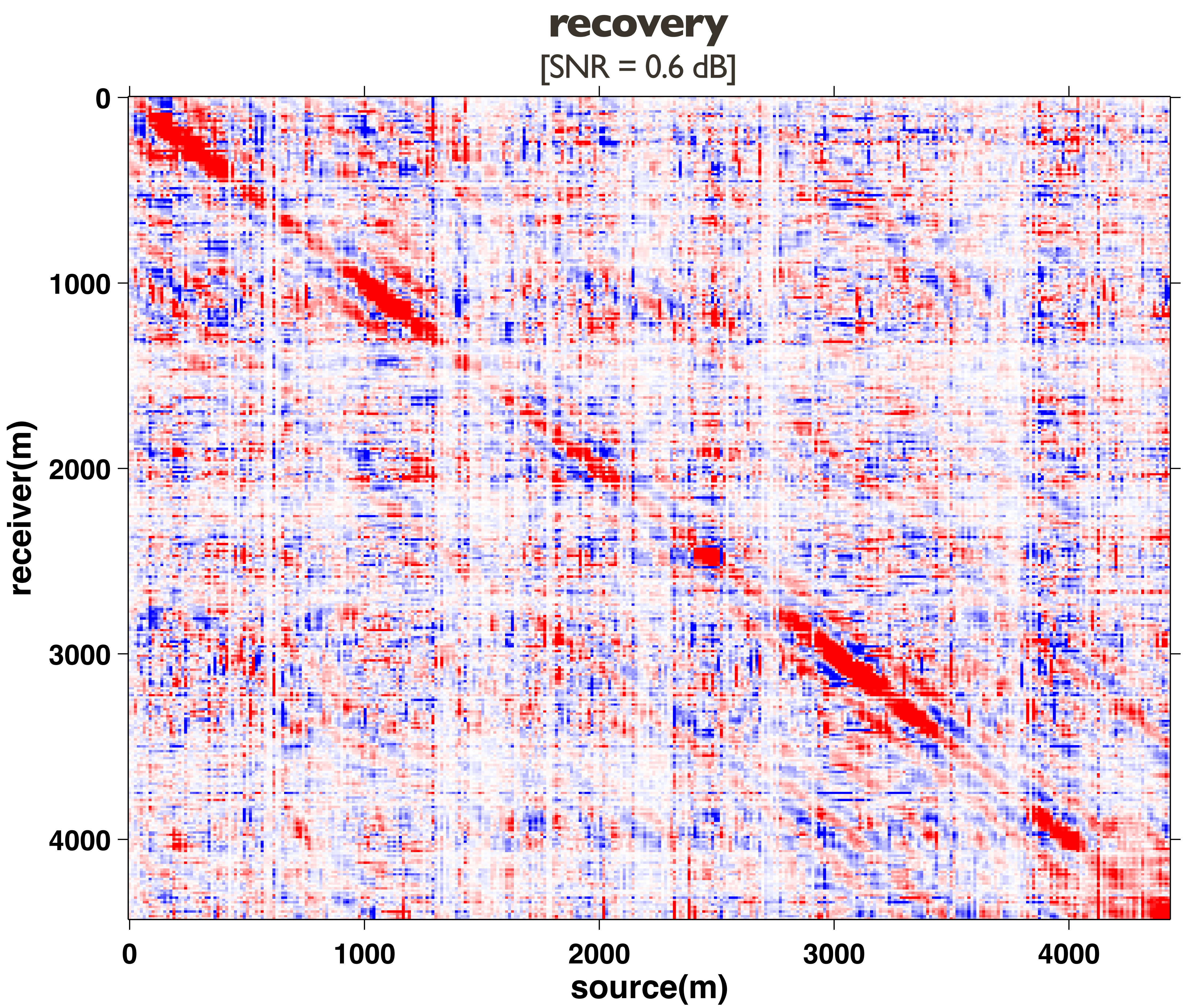


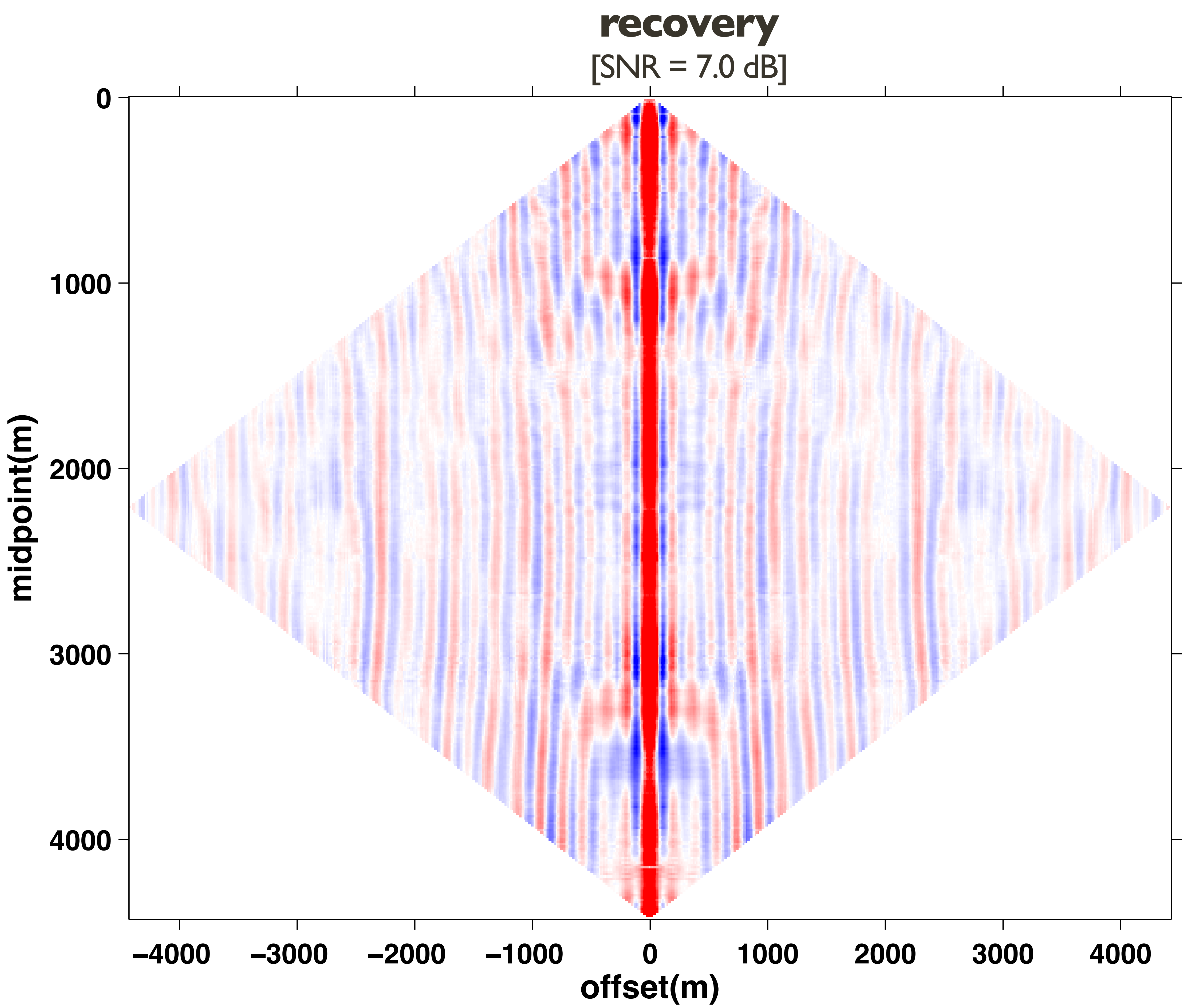
Matrix completion problem



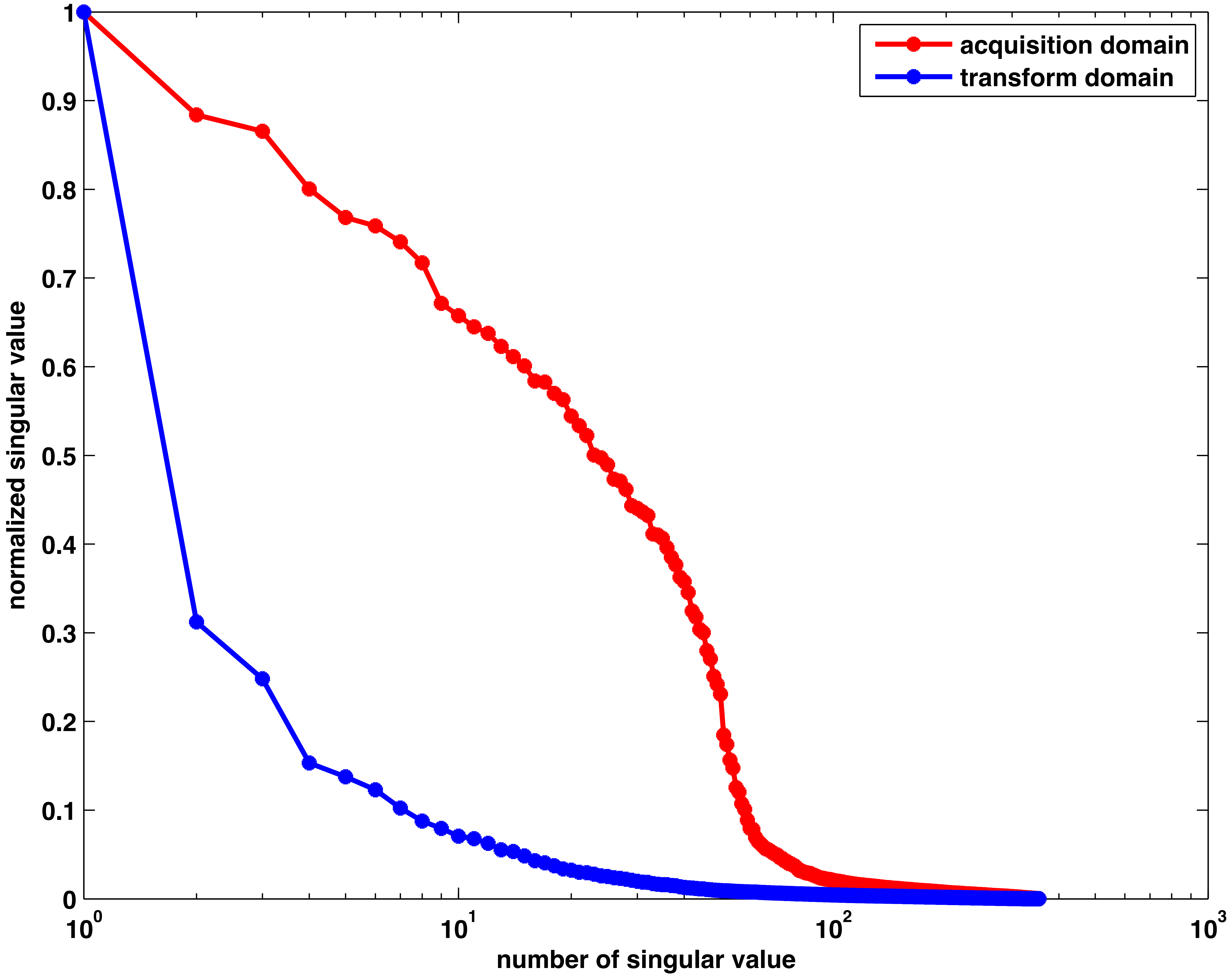


Low-rank interpolation





Singular value decay
[2-D acquisition]



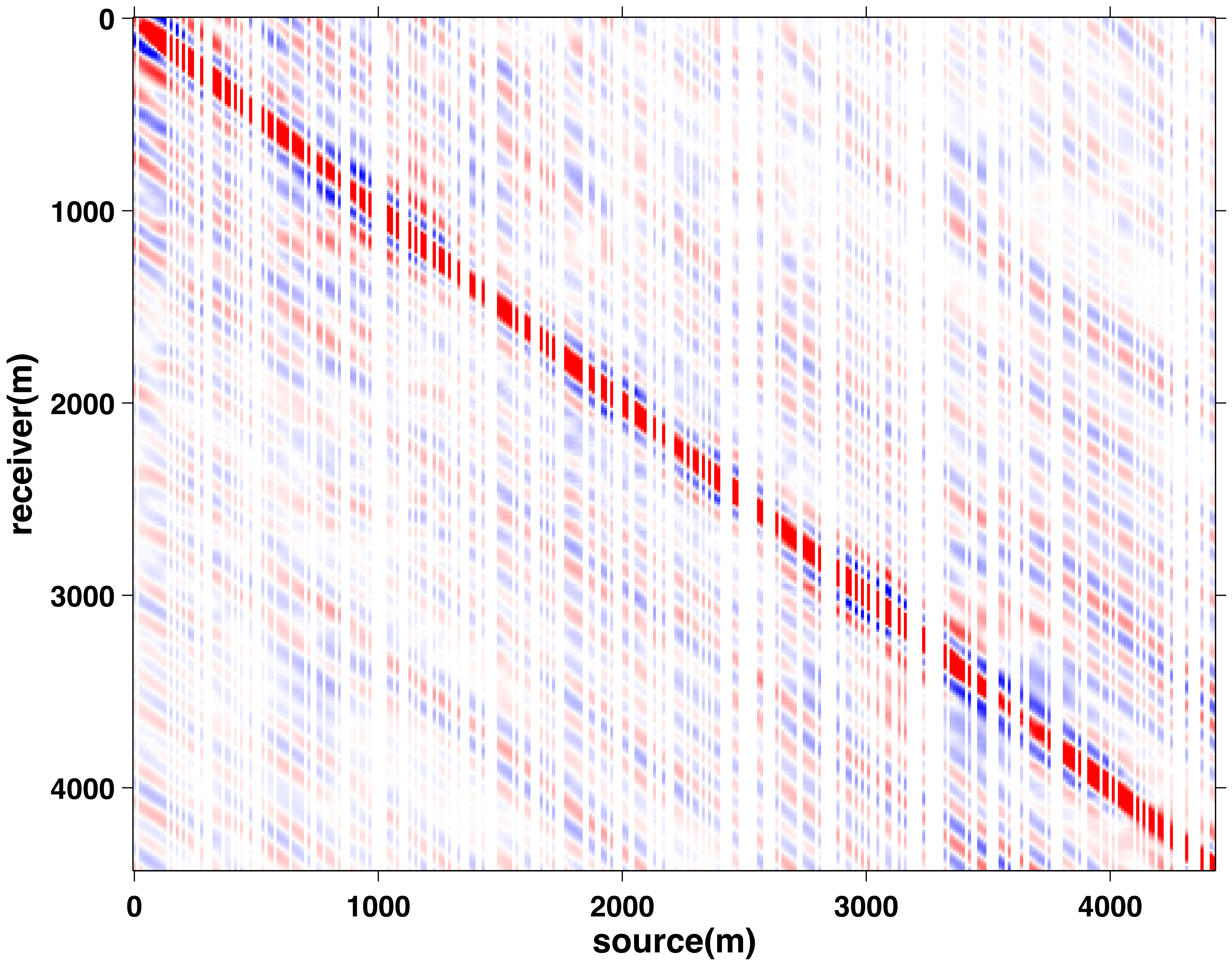
Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

2-D acquisition
[randomized sampling]

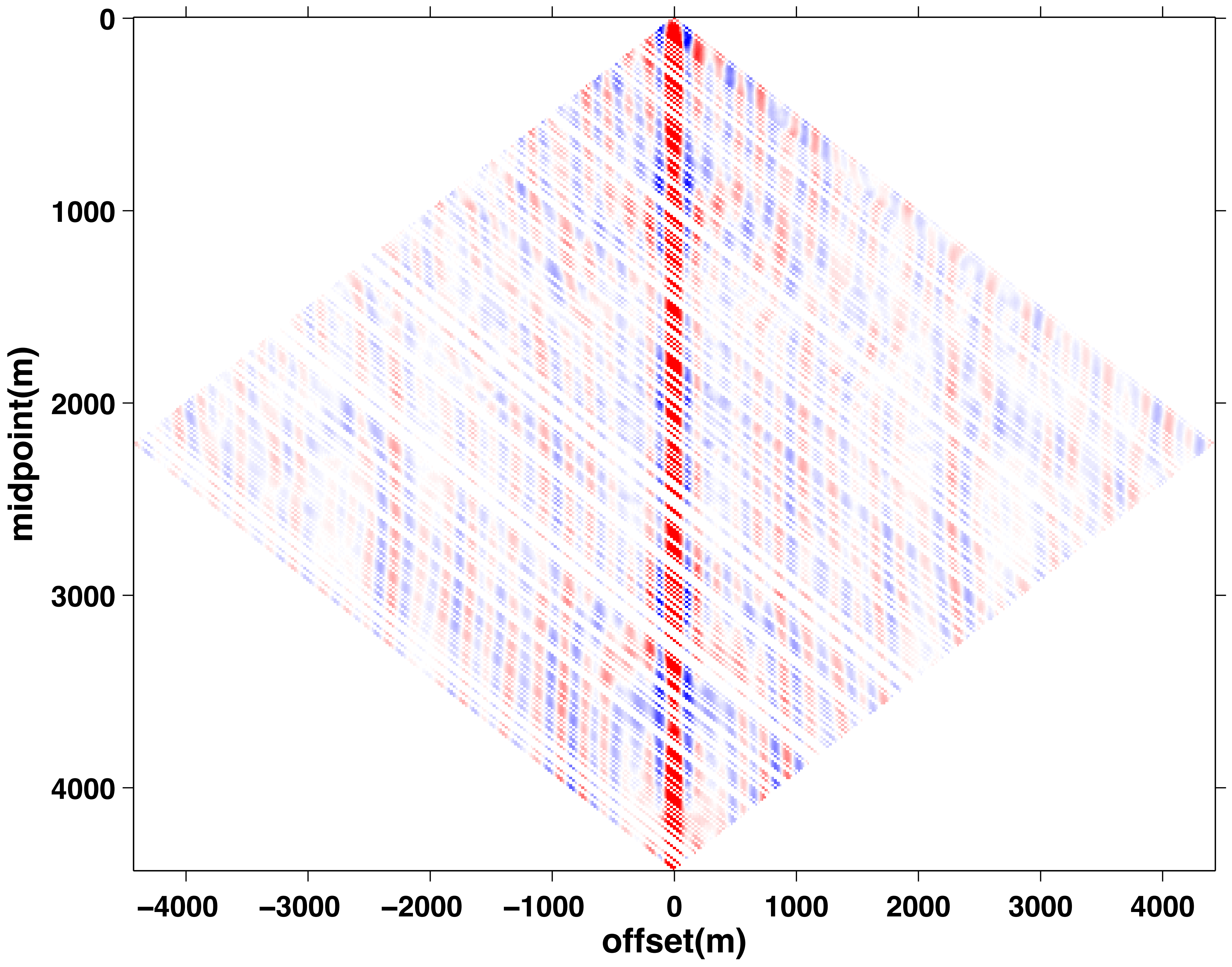
acquisition domain

missing columns *do not* increase rank

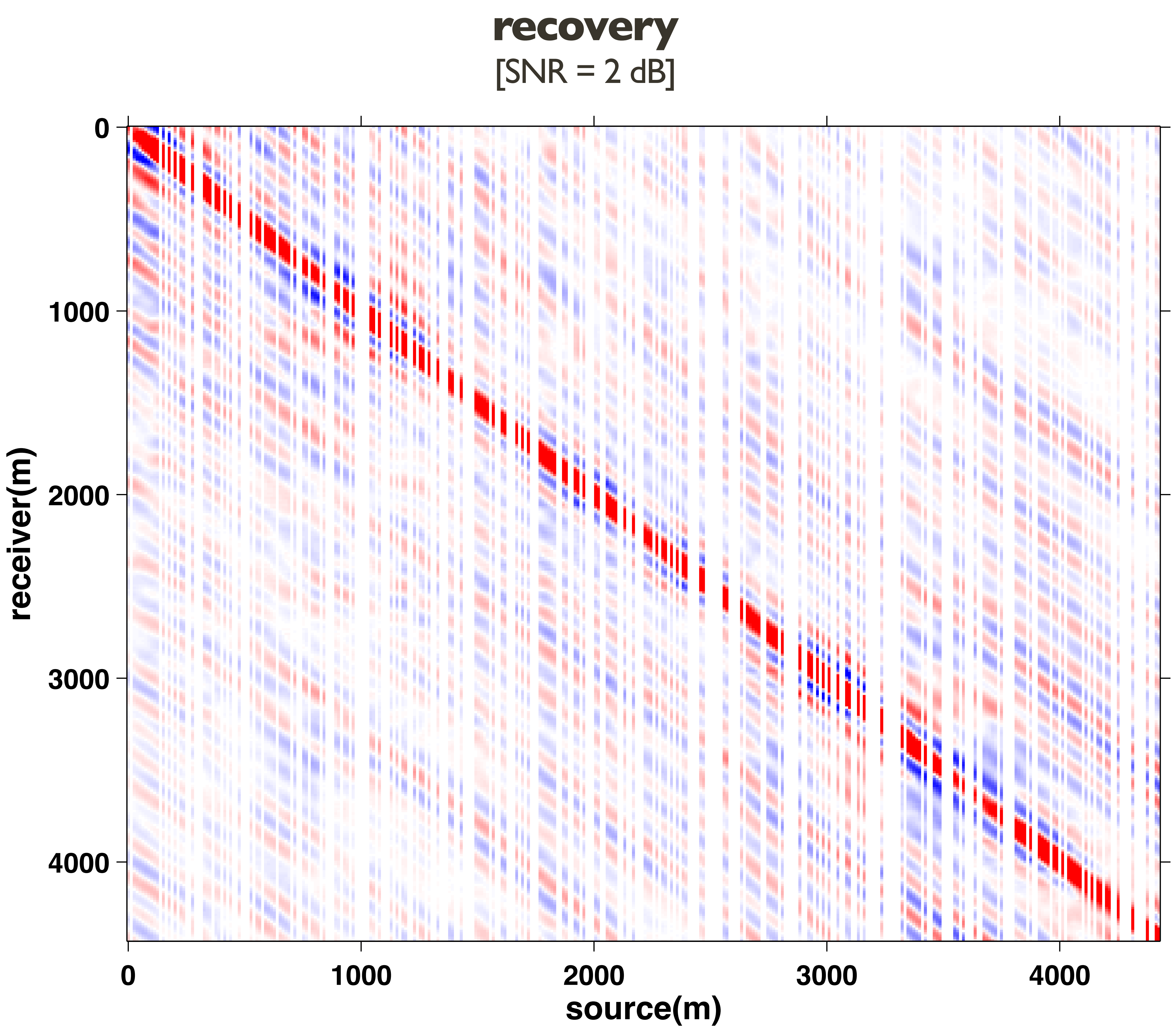


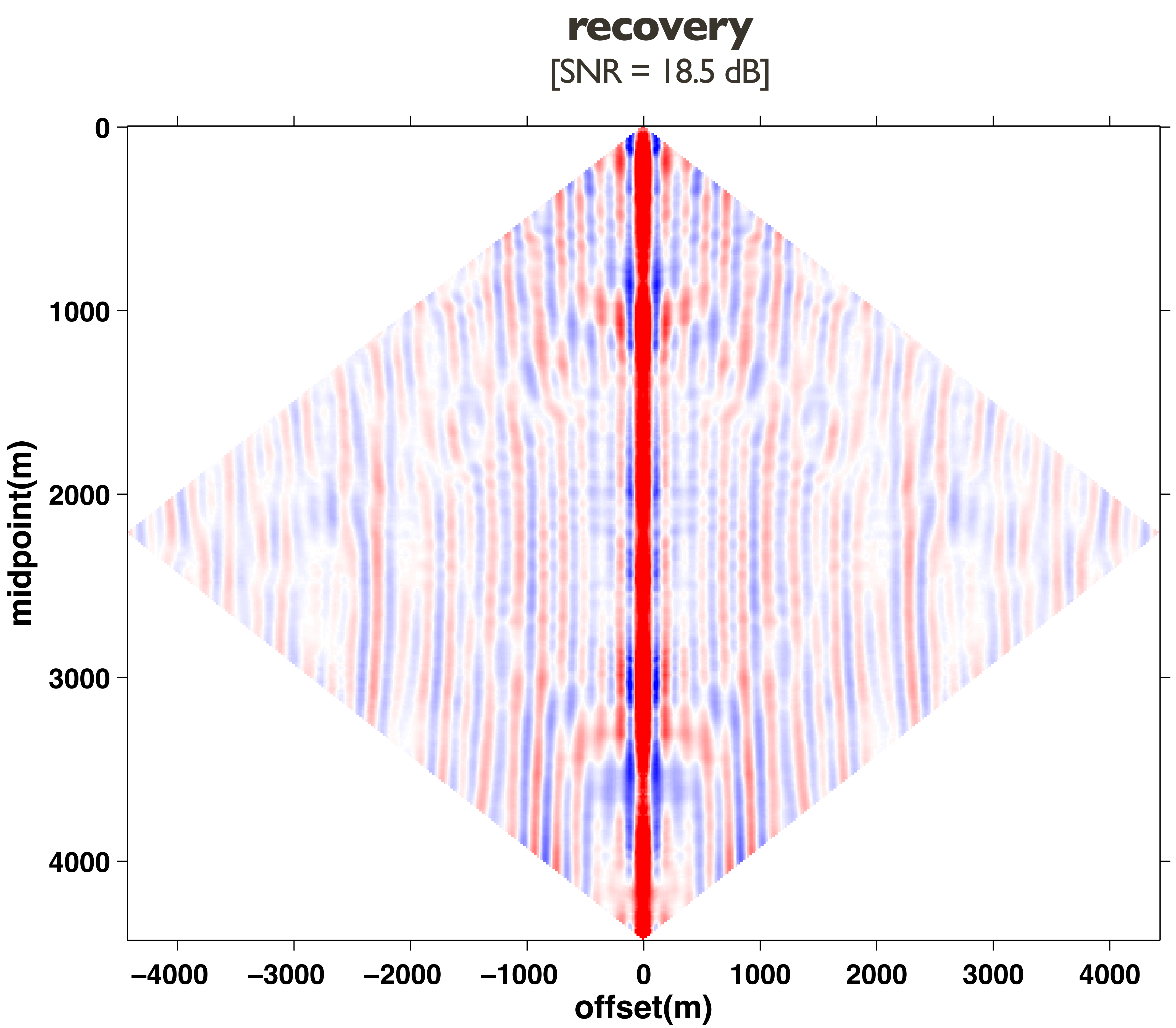
transform domain

missing columns *do* increase rank

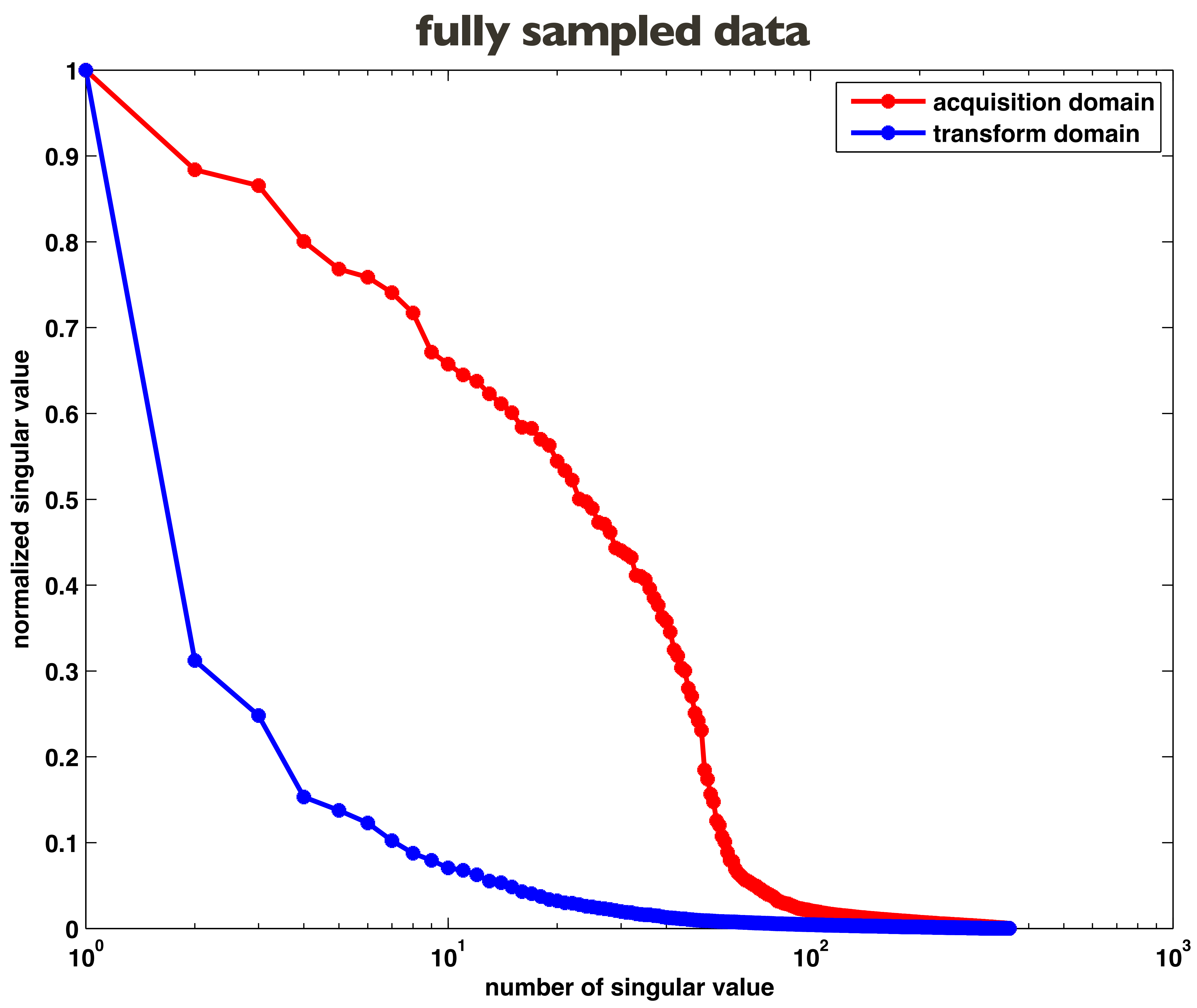


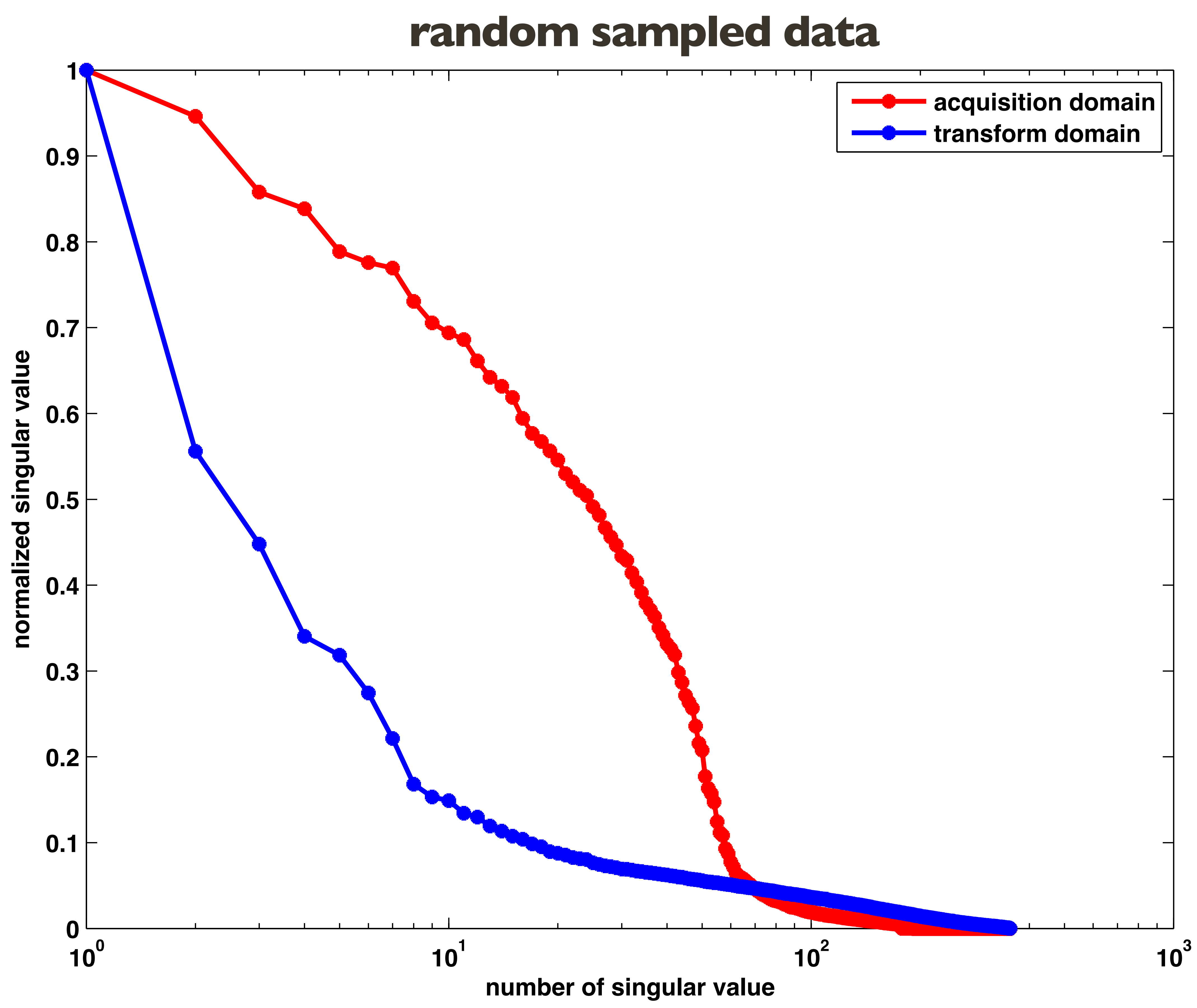
Low-rank interpolation





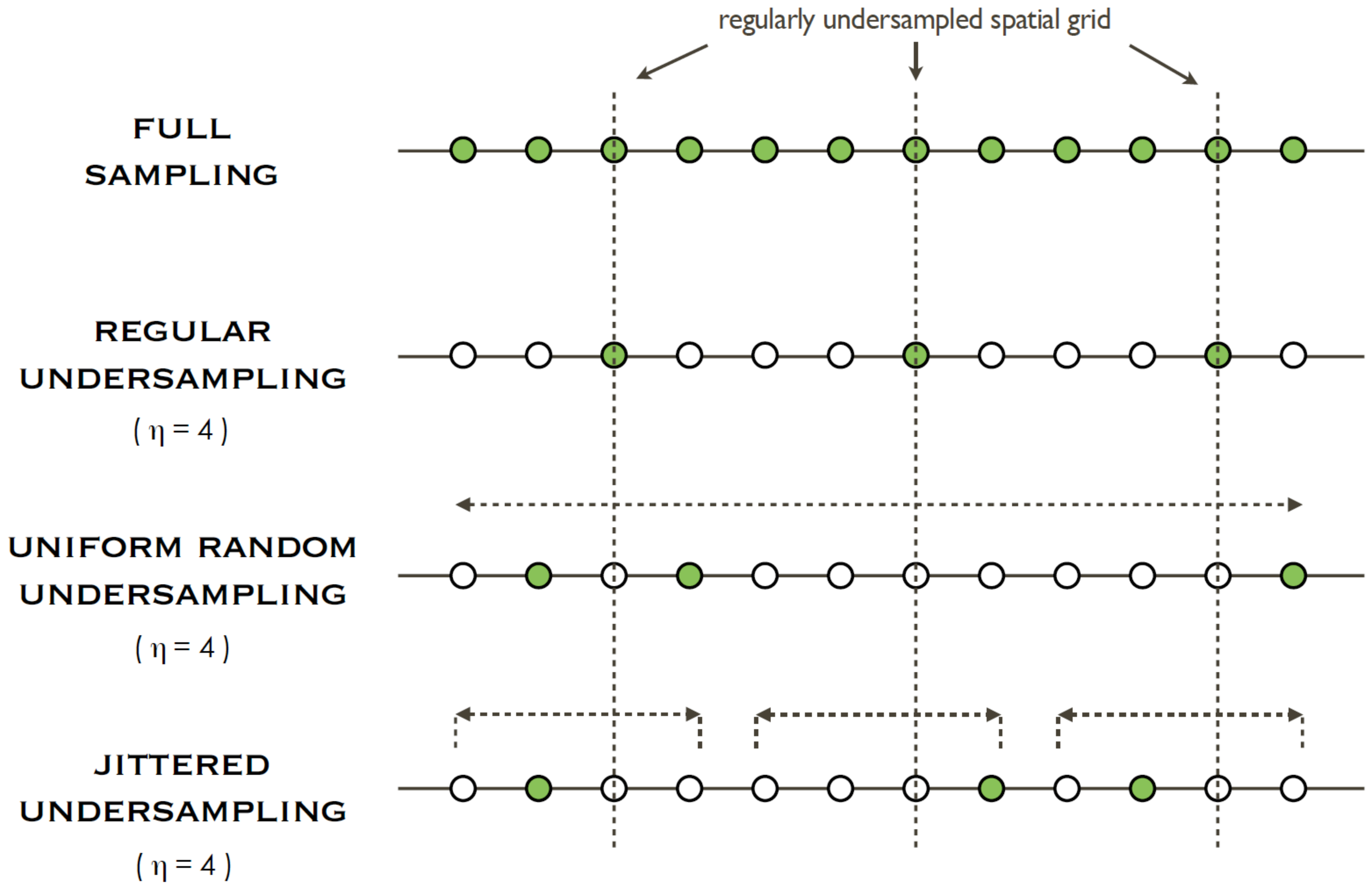
Randomized sampling
[singular value decay]





[Hennenfent et. al. 2008]

Sampling schemes



Matrix completion

- signal structure
  - *low rank/fast decay* of singular values

- sampling scheme
  - missing data *increase* rank in "transform domain"

- recovery using *rank penalization* scheme

Rank minimization

- given a set of measurements $\mathbf{b}$, aim is to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} \quad \text{rank}(\mathbf{X}) \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where $\text{rank}(\mathbf{X}) =$ number of singular values of $\mathbf{X}$

- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator
$\mathbf{M}$: measurement operator
$\mathcal{S}^H$: transform operator

Rank minimization

- prohibitively *expensive*
  - do not know rank value in advance
  - search over all possible values of rank

- instead solve nuclear-norm minimization
  - convex relaxation of rank-minimization **[Recht et. al. 2010]**

[Recht et. al. 2010]

Nuclear-norm minimization

- we want to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t.} \; ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

where λ_i are the *singular* values

Challenges

- requires repeated application of *SVD* for projections

- expensive to compute for large system
  - curse of dimensionality

- can we exploit rank structure "*SVD* free"

[Rennie and Srebro 2005, Lee et. al. 2010, Recht and Re 2011]

Factorized formulation

$$\mathbf{X} \in \mathbb{R}^{n \times m} \quad = \quad \mathbf{L} \in \mathbb{R}^{n \times k} \quad \mathbf{R}^H \in \mathbb{R}^{k \times m}$$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$

[Berg and Friedlander 2008, Aravkin et al. 2012b]

Factorized formulation

- reformulate $(BPDN_\sigma)$ formulation

$$\min_{\mathbf{L},\mathbf{R}} ||\mathbf{L}\mathbf{R}^H||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \quad \text{s.t.} \quad \|\mathbf{L}\mathbf{R}^H\|_* \leq \tau$$

where τ is a rank regularization parameter

[Rennie and Srebro 2005]

Factorized formulation

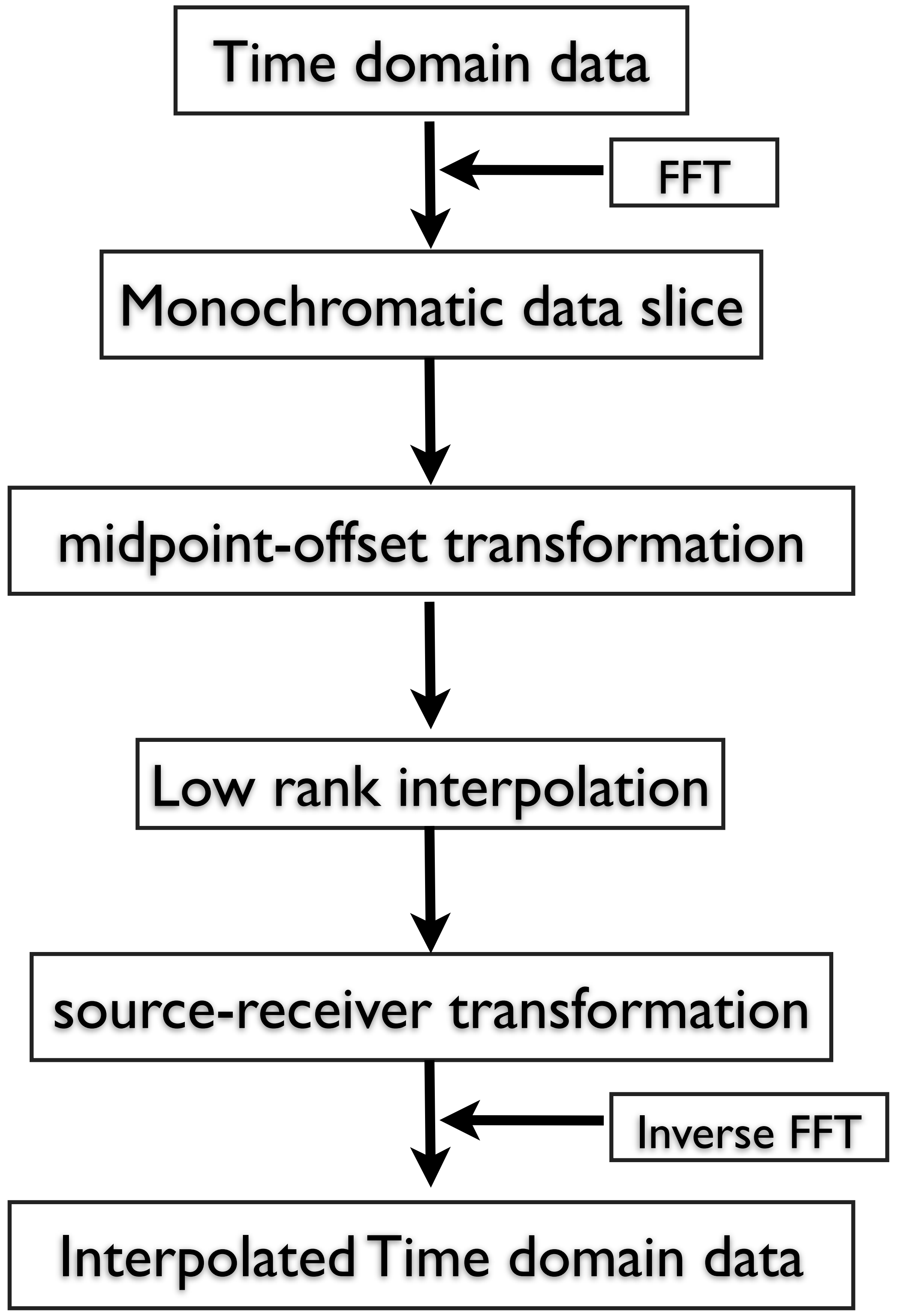
- Upper-bound on nuclear norm is defined as

$$\|\mathbf{L}\mathbf{R}^H\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2$$

where $\|\cdot\|_F^2$ is sum of squares of all entries

- choose k *explicitly* & avoid costly *SVD's*

Interpolation flow chart



Experiments and Results

Case 1 : Uniform random subsampling

Case 2 : Jittered subsampling

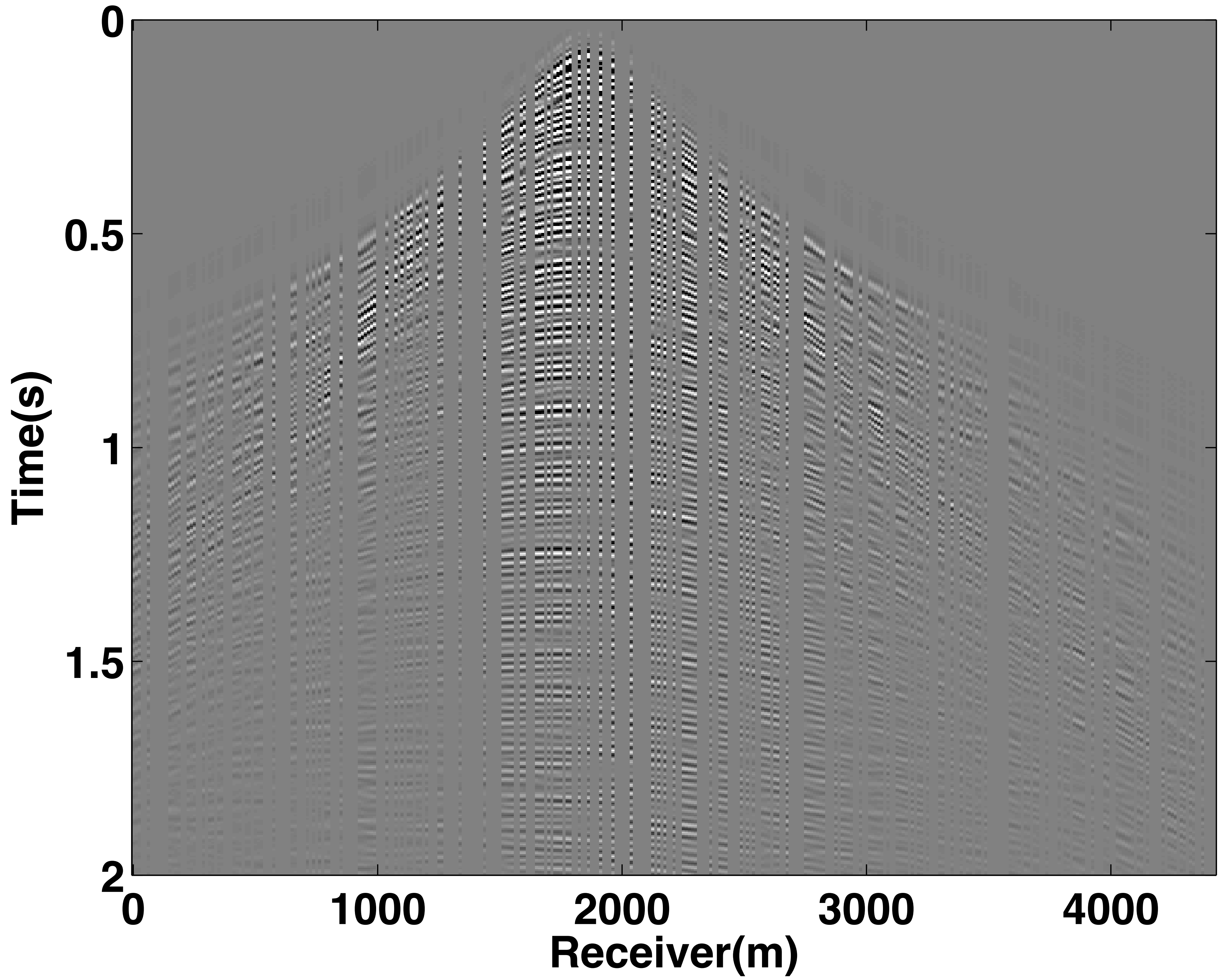
Case 3 : Jittered + Reciprocity

Case 4 : Simultaneous acquisition (Land)

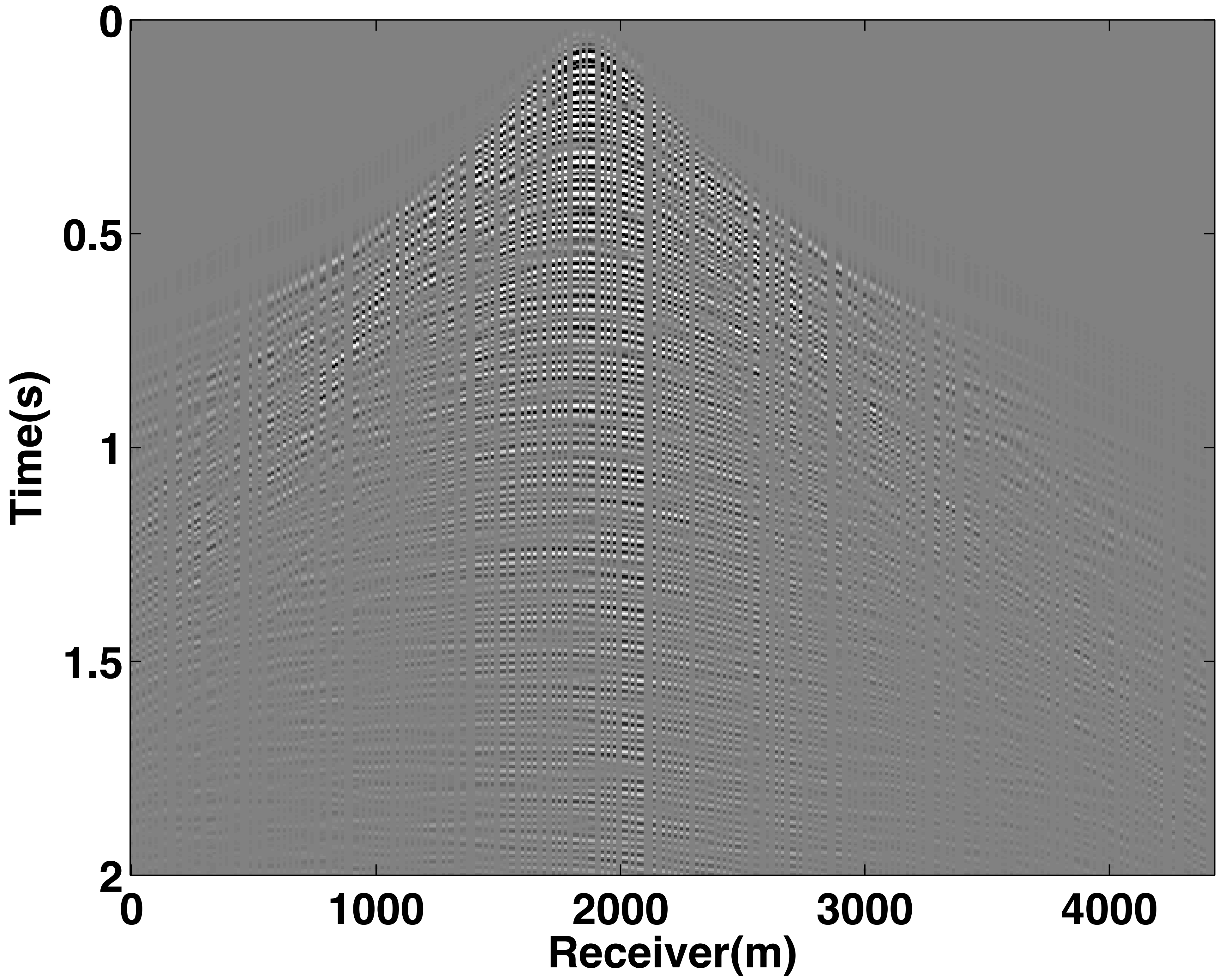
Experiments and Results

- Gulf of Suez
  - 2-D seismic line
  - 50 % missing traces
  - rank adjusted from low to high frequency
  - 150 iterations

Uniform random v/s Jittered



Discrete random

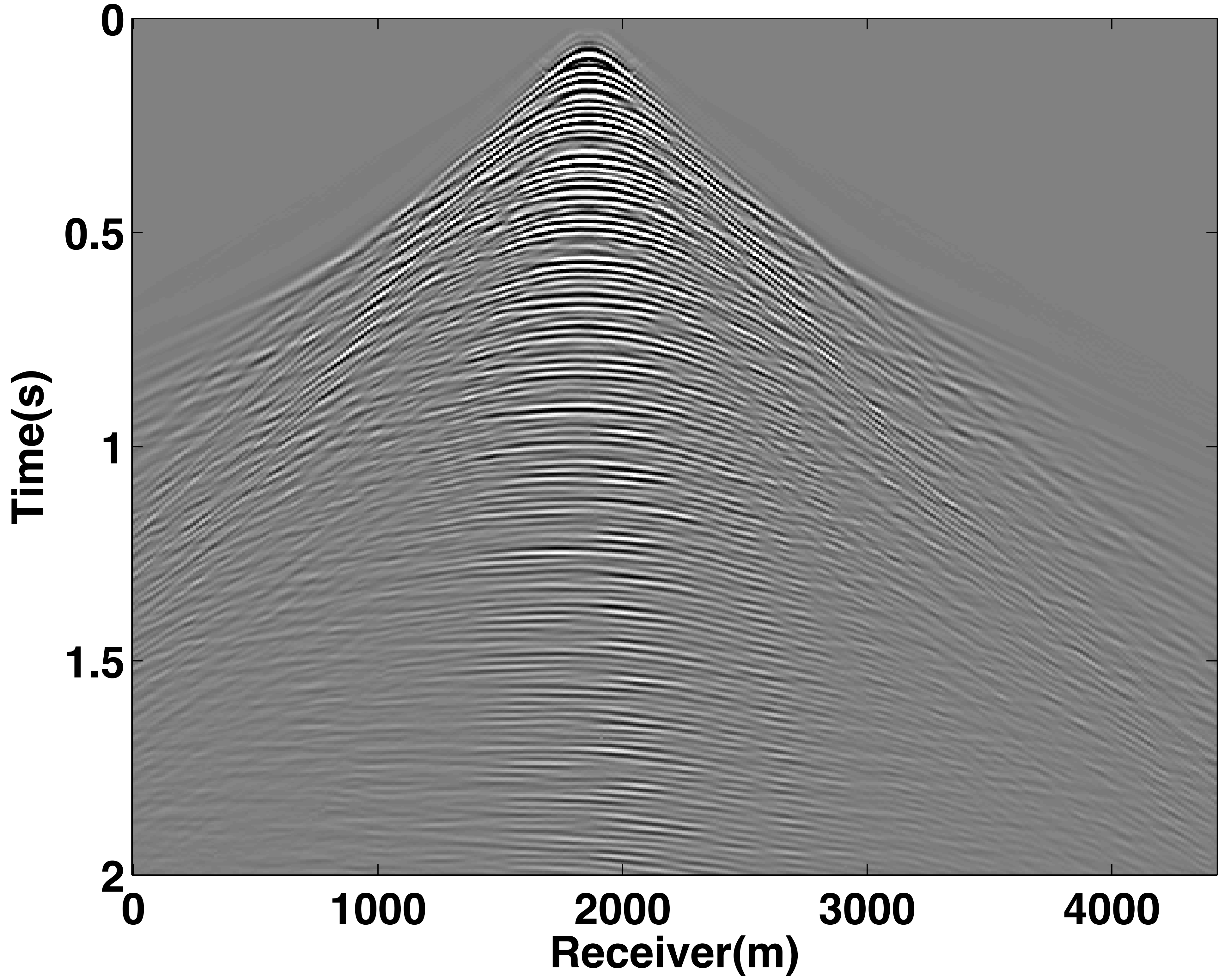


Jittered

Case I

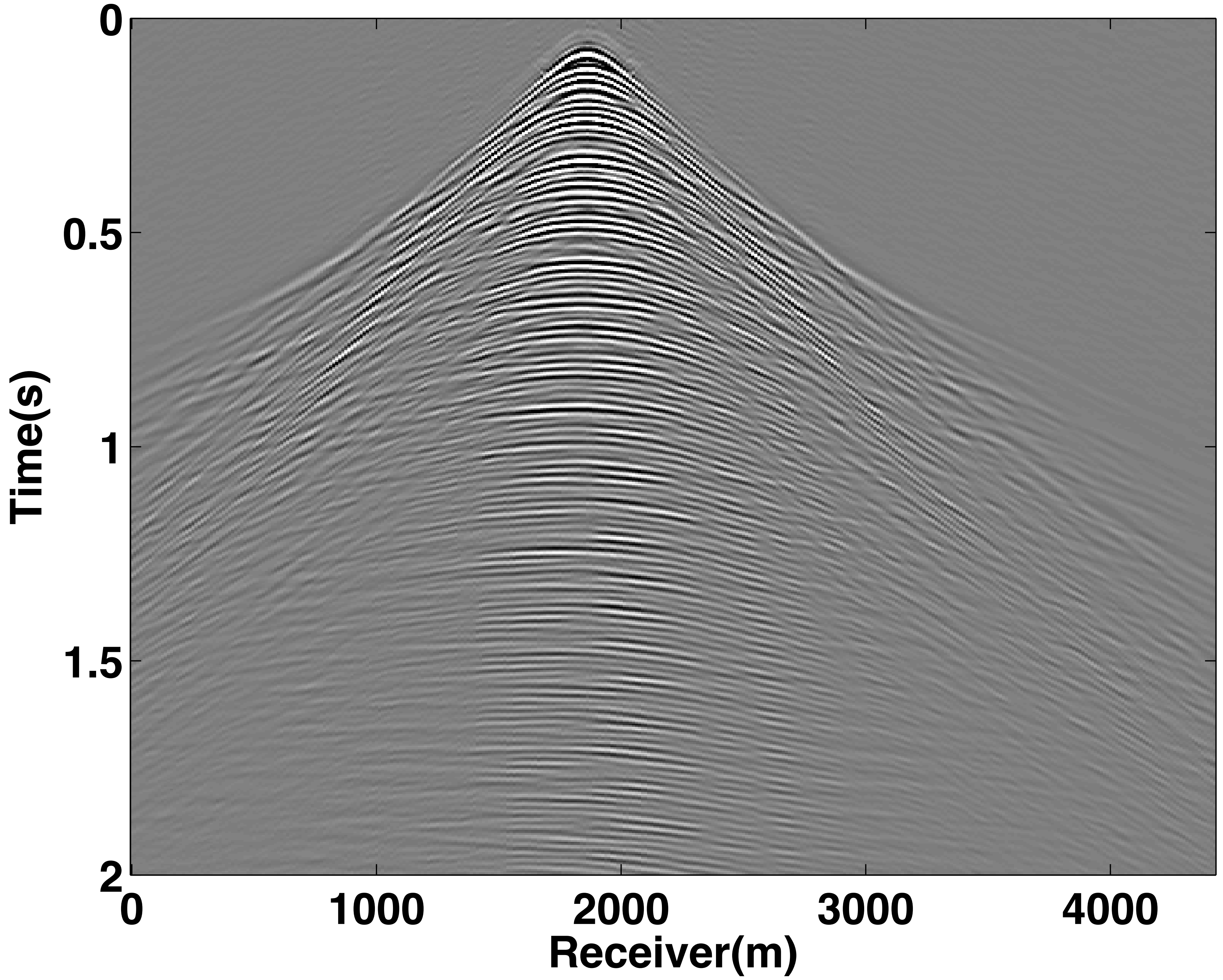
[uniform random subsampling]

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth

SNR = 16.2 dB

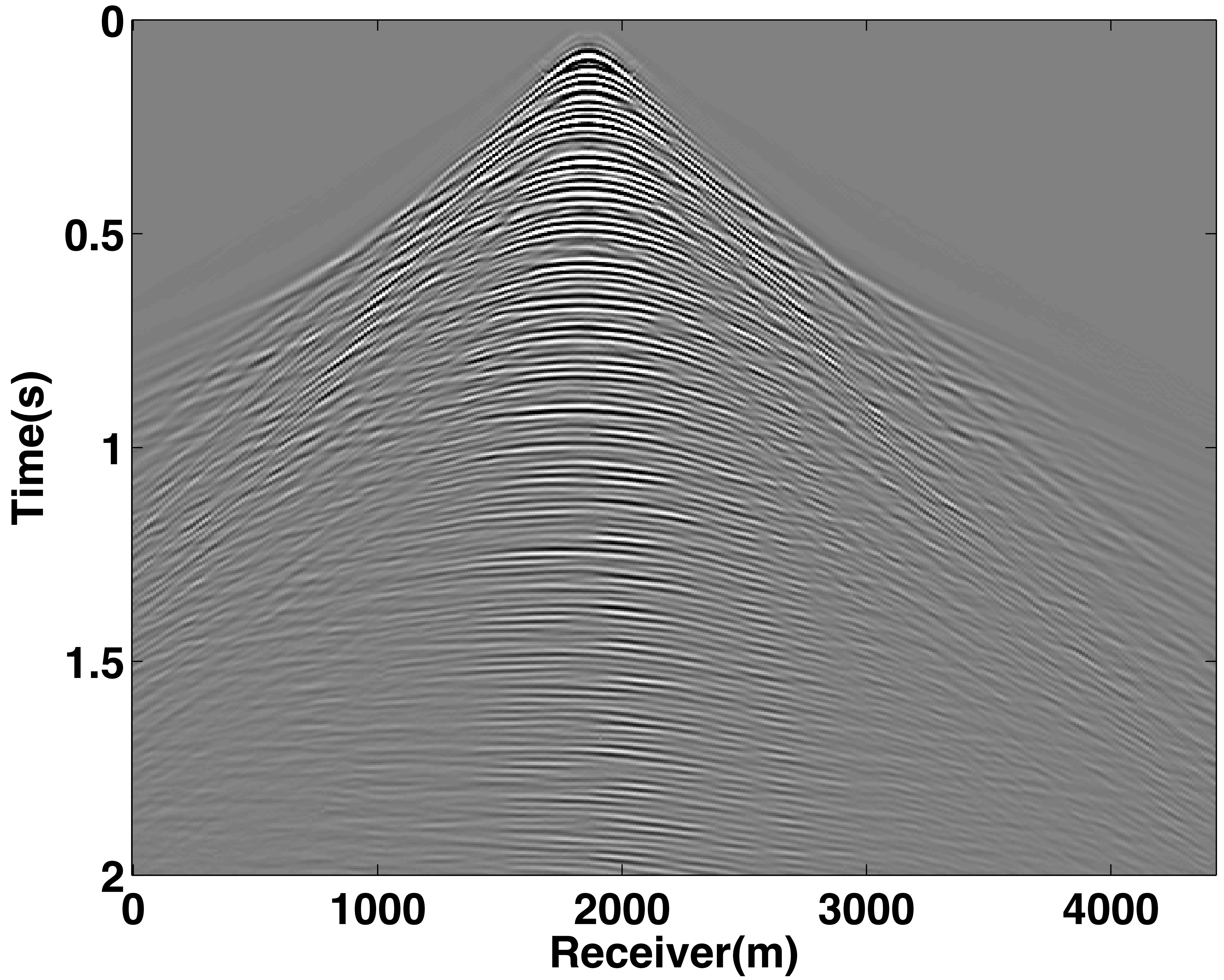


Recovery

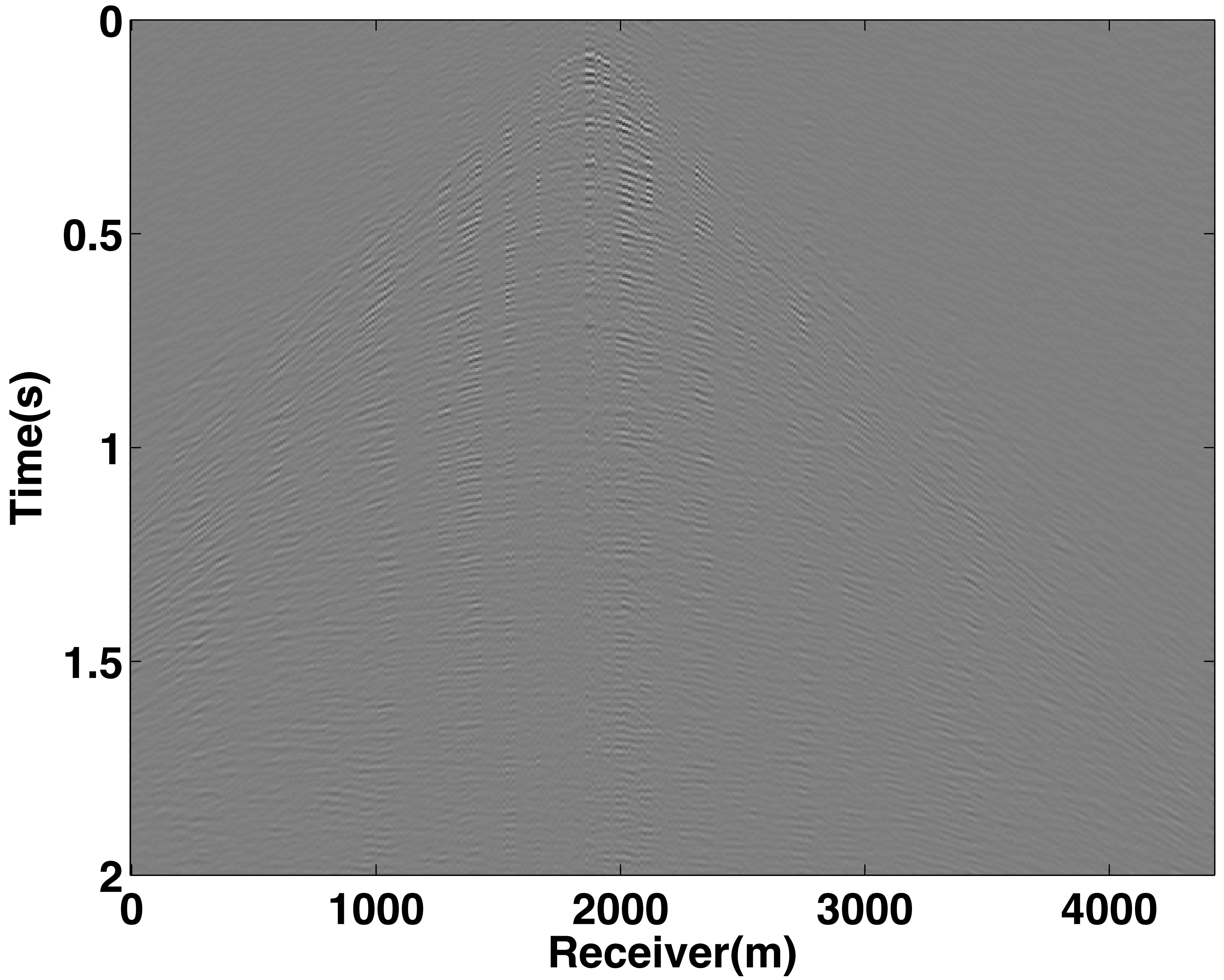
Case I

[uniform random subsampling]

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth

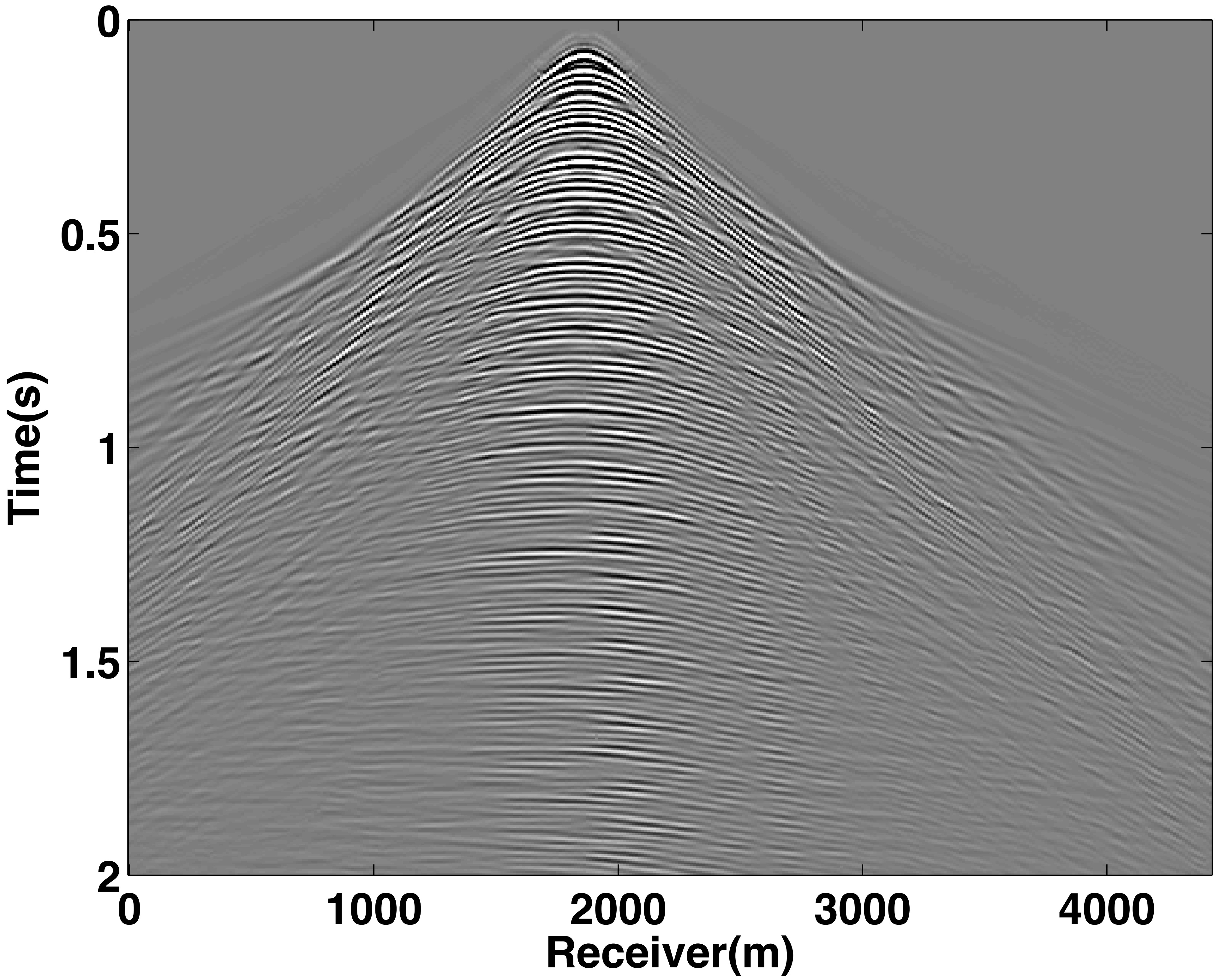


Difference

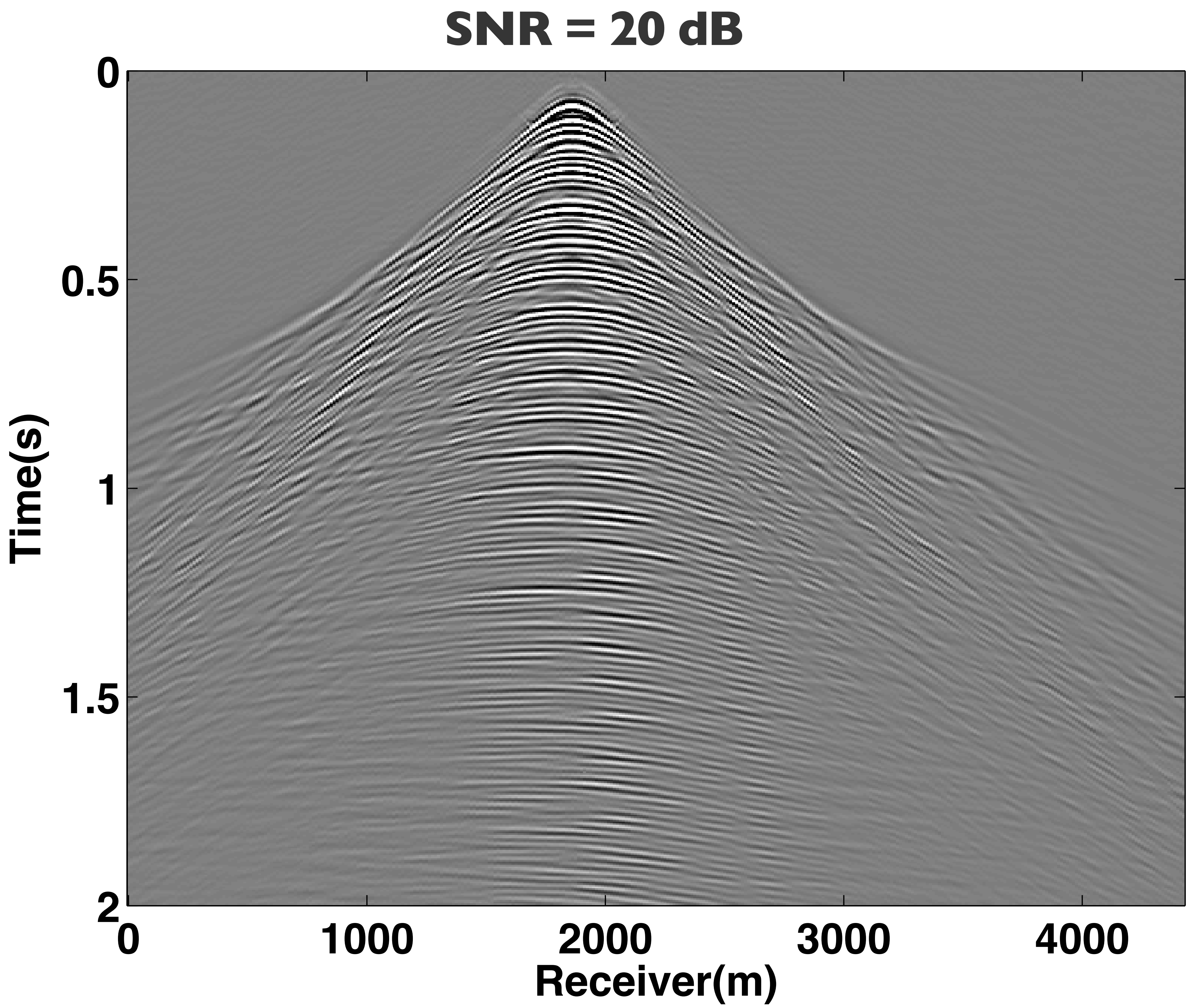
Case 2

[jittered subsampling]

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



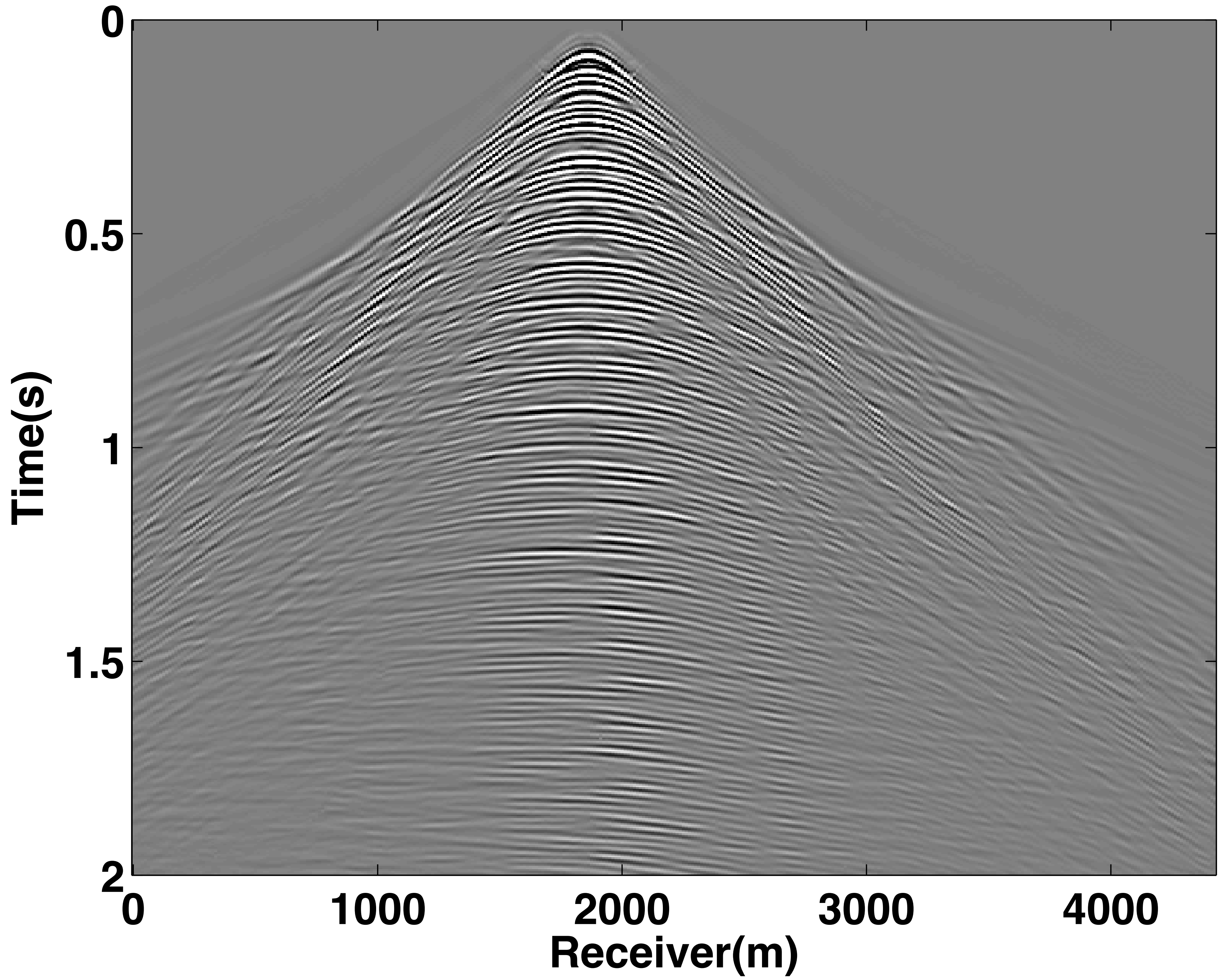
Ground Truth



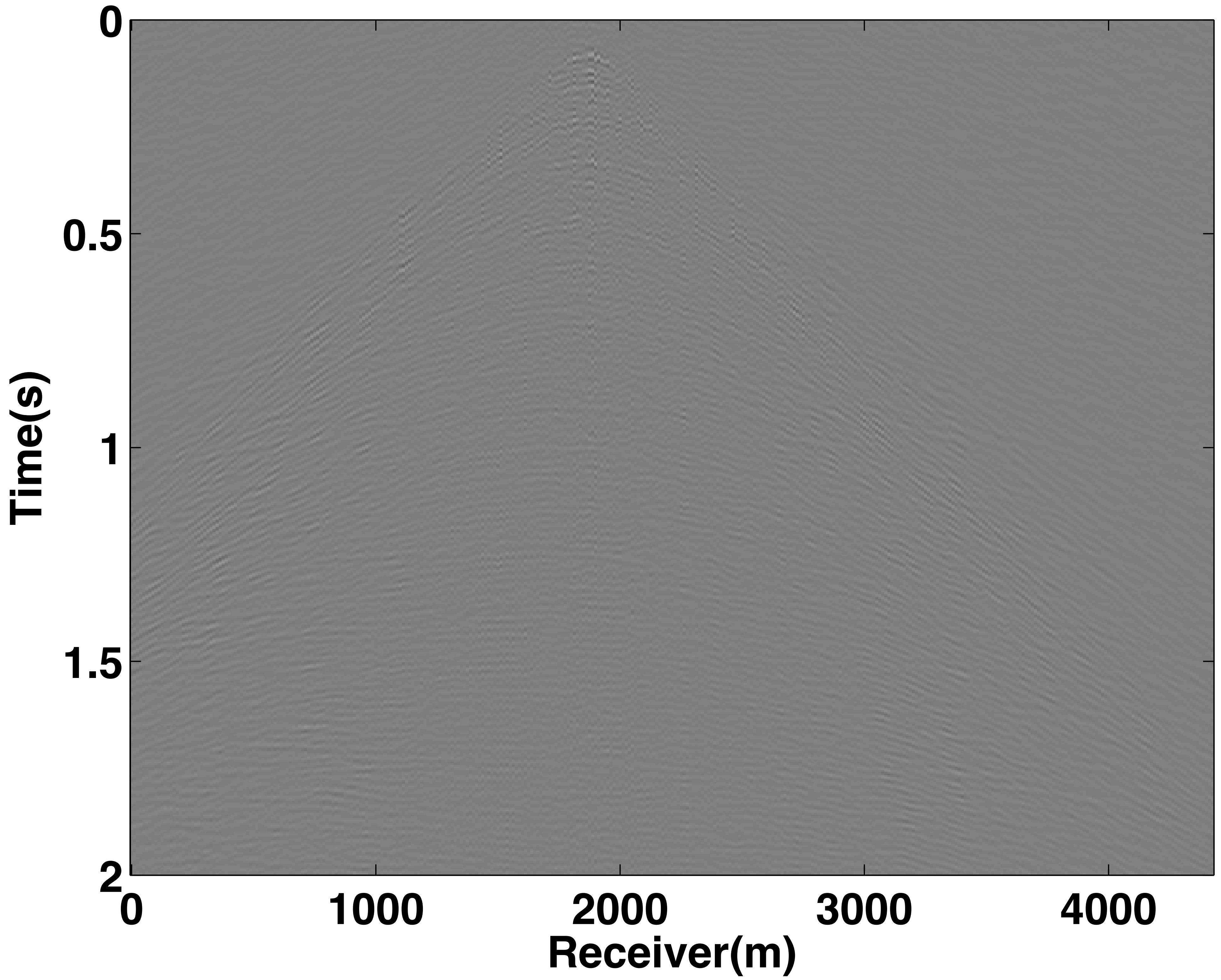
Recovery

Case 2
[jittered subsampling]

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth

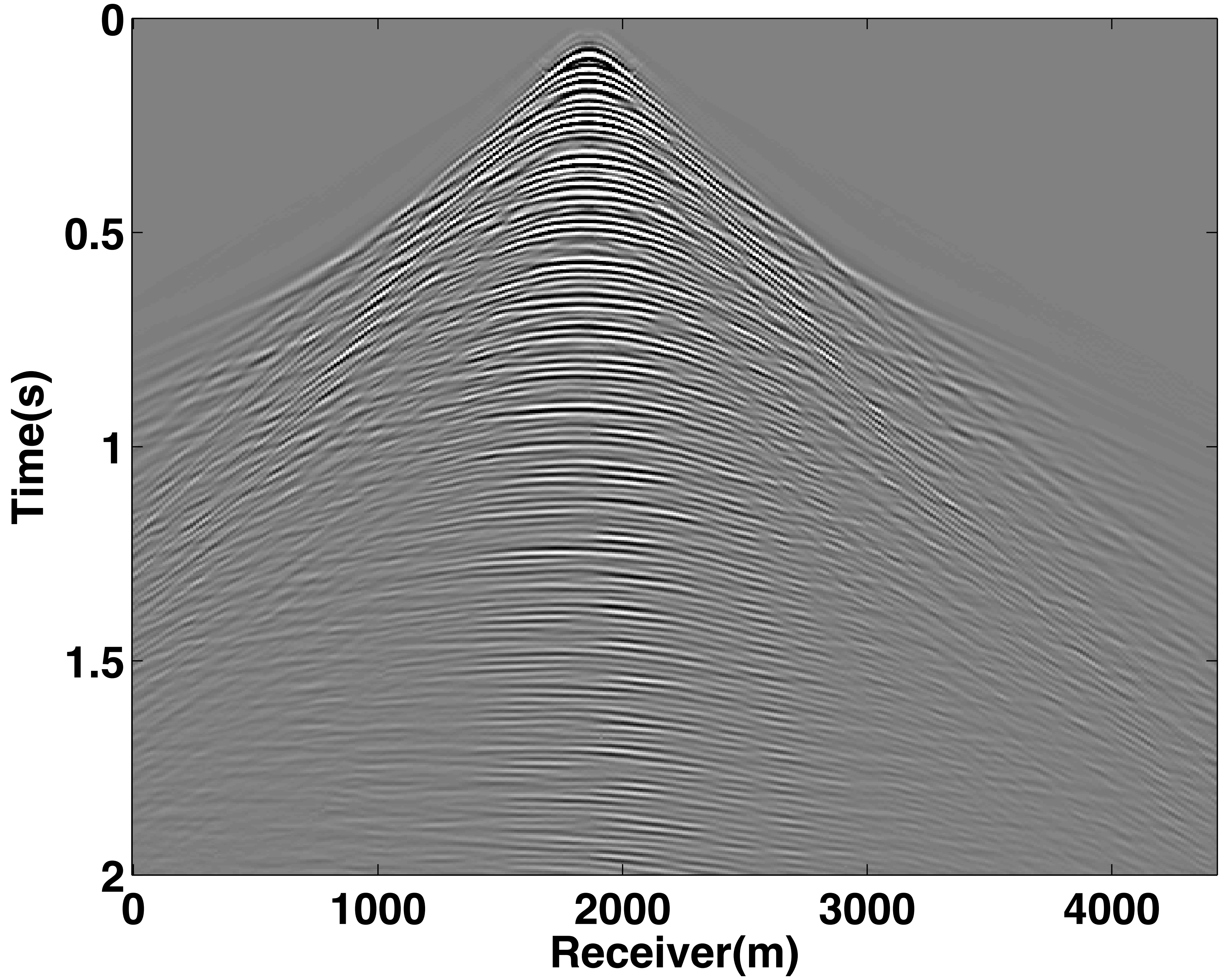


Difference

Case 3

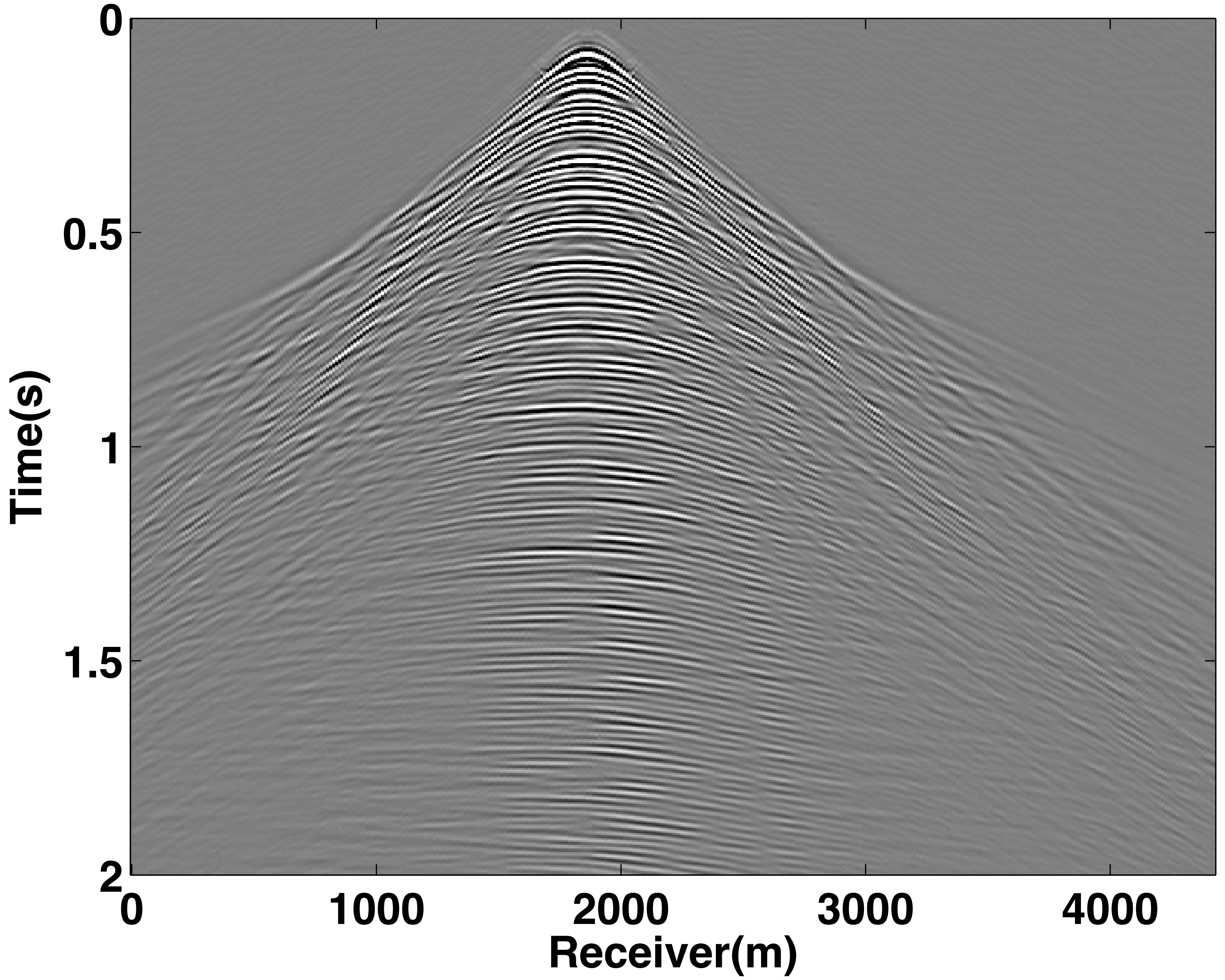
[jittered subsampling + reciprocity]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\frac{(\mathbf{I}+\mathbf{T})}{\mathbf{2}}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth

SNR = 20.6 dB

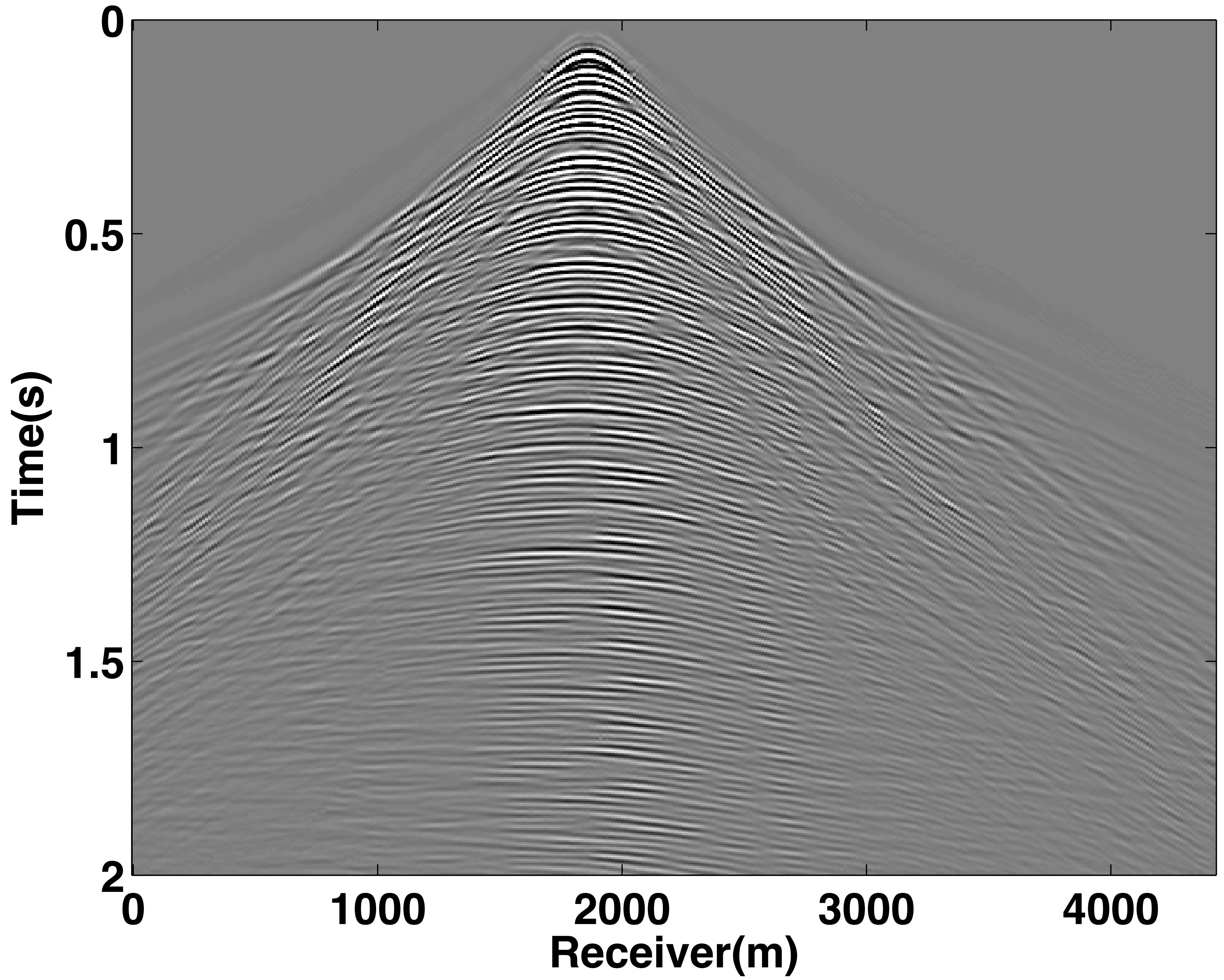


Recovery

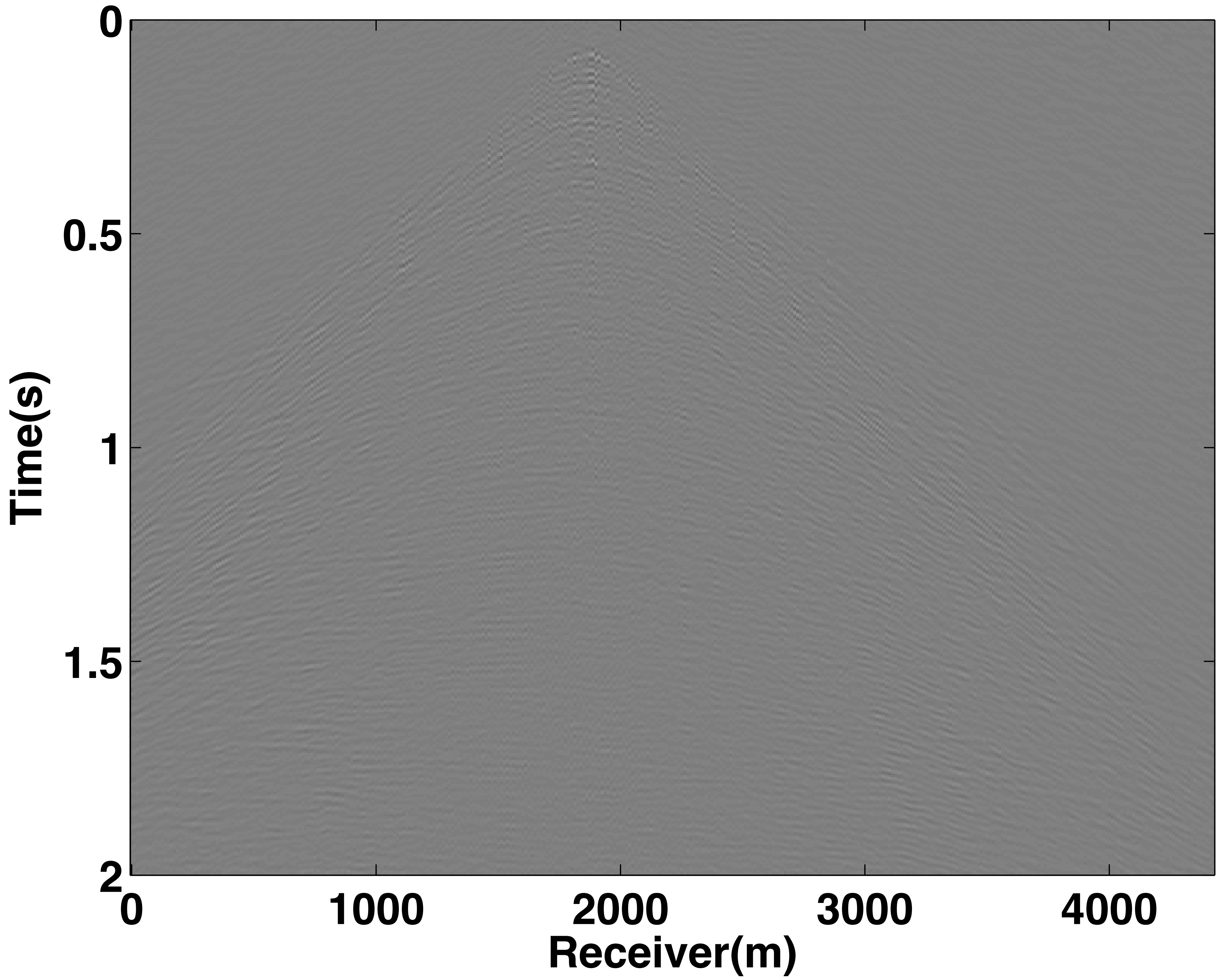
Case 3

[jittered subsampling + reciprocity]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\frac{(\mathbf{I}+\mathbf{T})}{\mathbf{2}}\mathcal{S}^H$$
$$\mathbf{M} = \mathbf{I}$$



Ground Truth



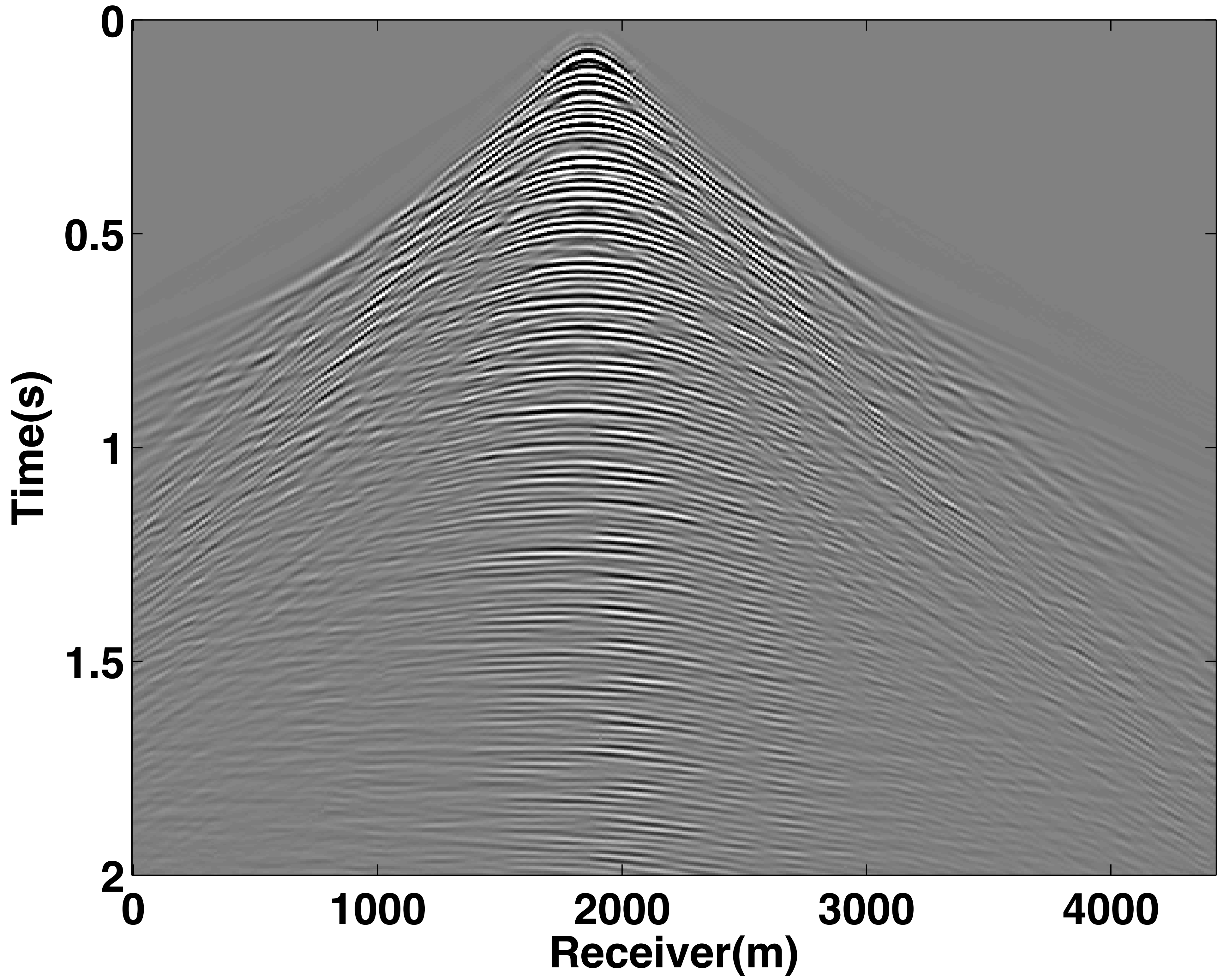
Difference

Case 4

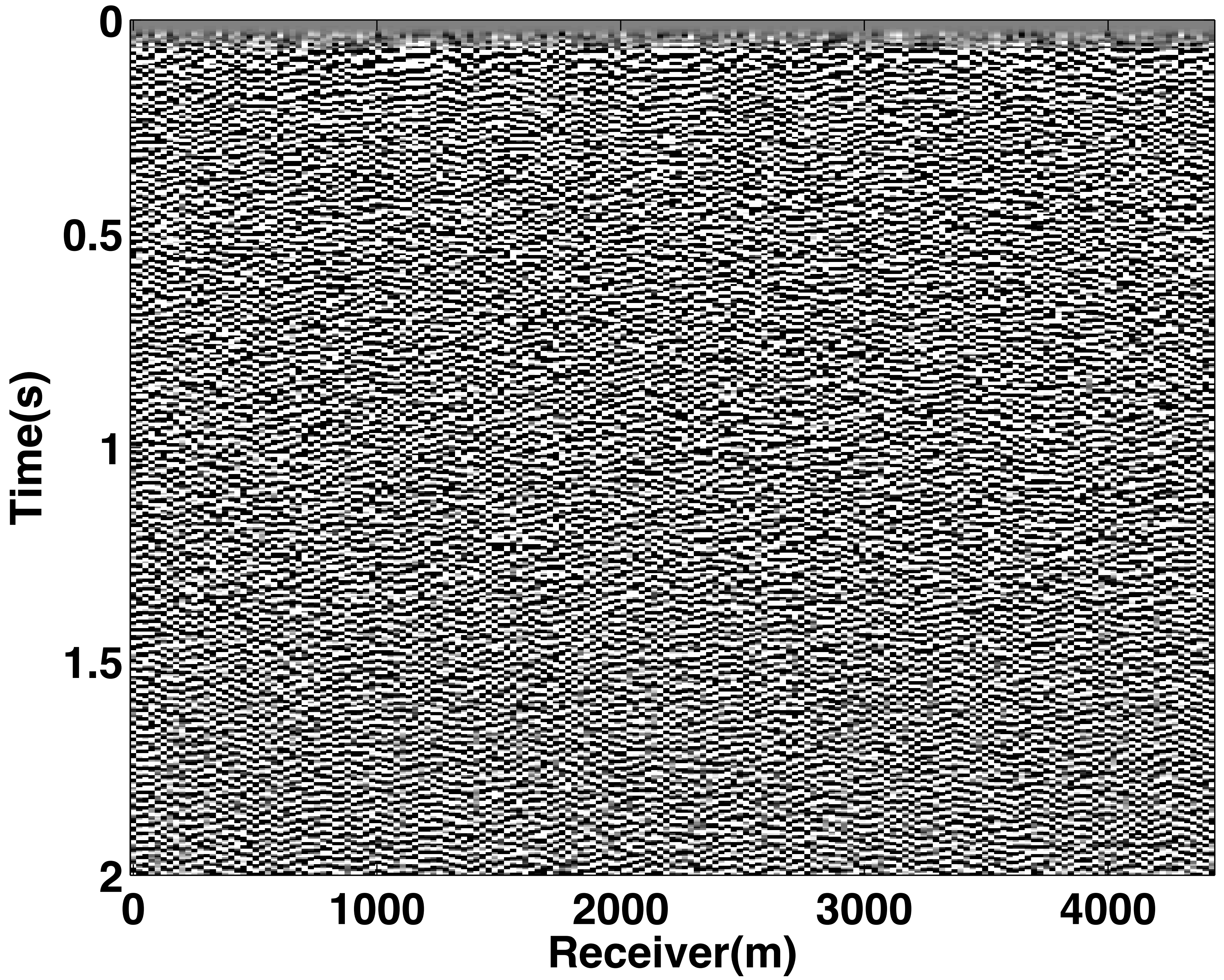
[Simultaneous Source : Land]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$

$$\mathbf{M} \stackrel{\text{def}}{=} [\mathbf{I} \otimes \mathbf{diag}(\eta)\mathcal{F}_{\mathbf{s}}{}^{*}\mathbf{diag}(\mathbf{e}^{\mathbf{i}\theta})\mathcal{F}_{\mathbf{s}} \otimes \mathbf{I}]$$ [Herrmann et. al. 2009]



Ground Truth



Case 4
[Simultaneous Source : Land]

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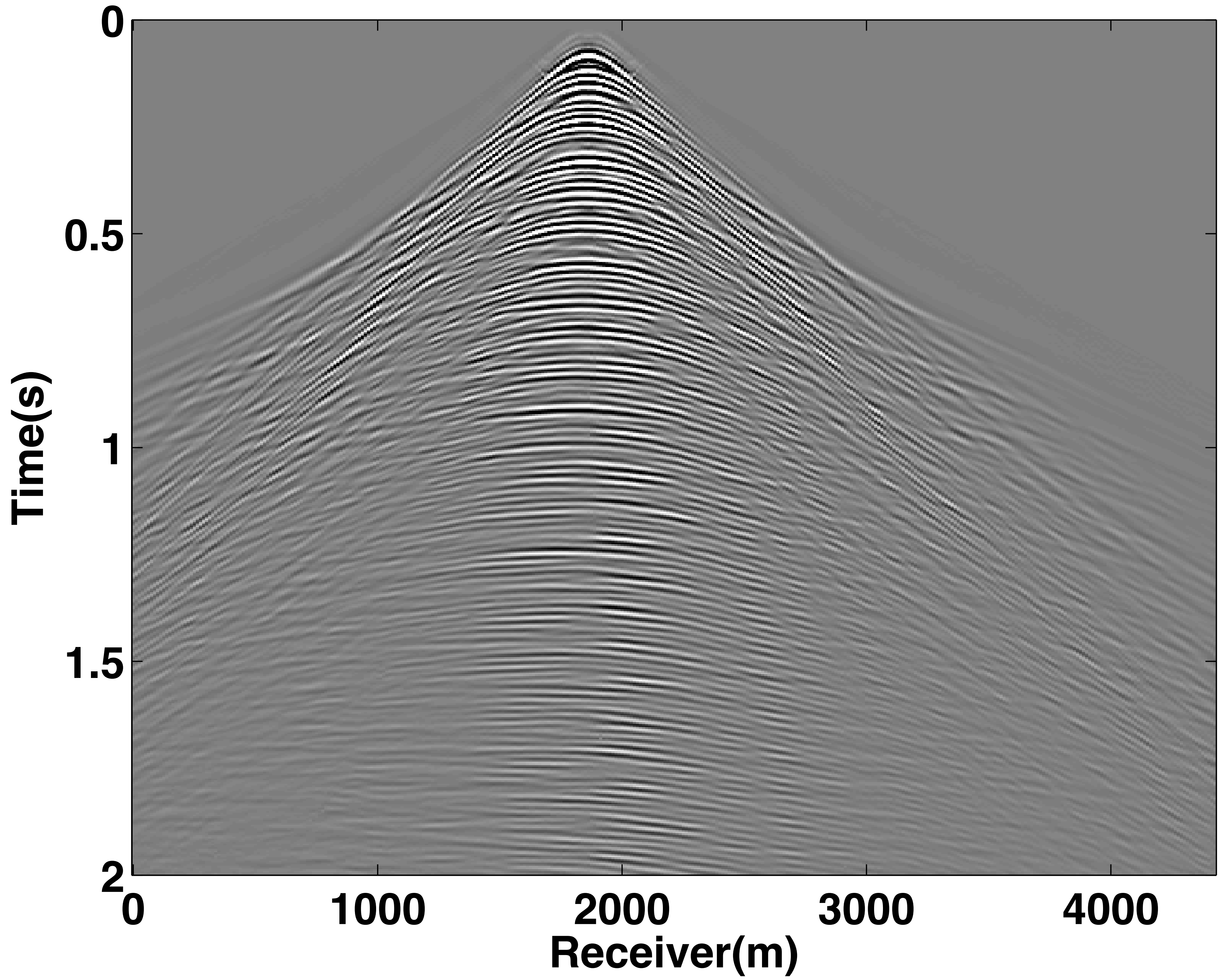
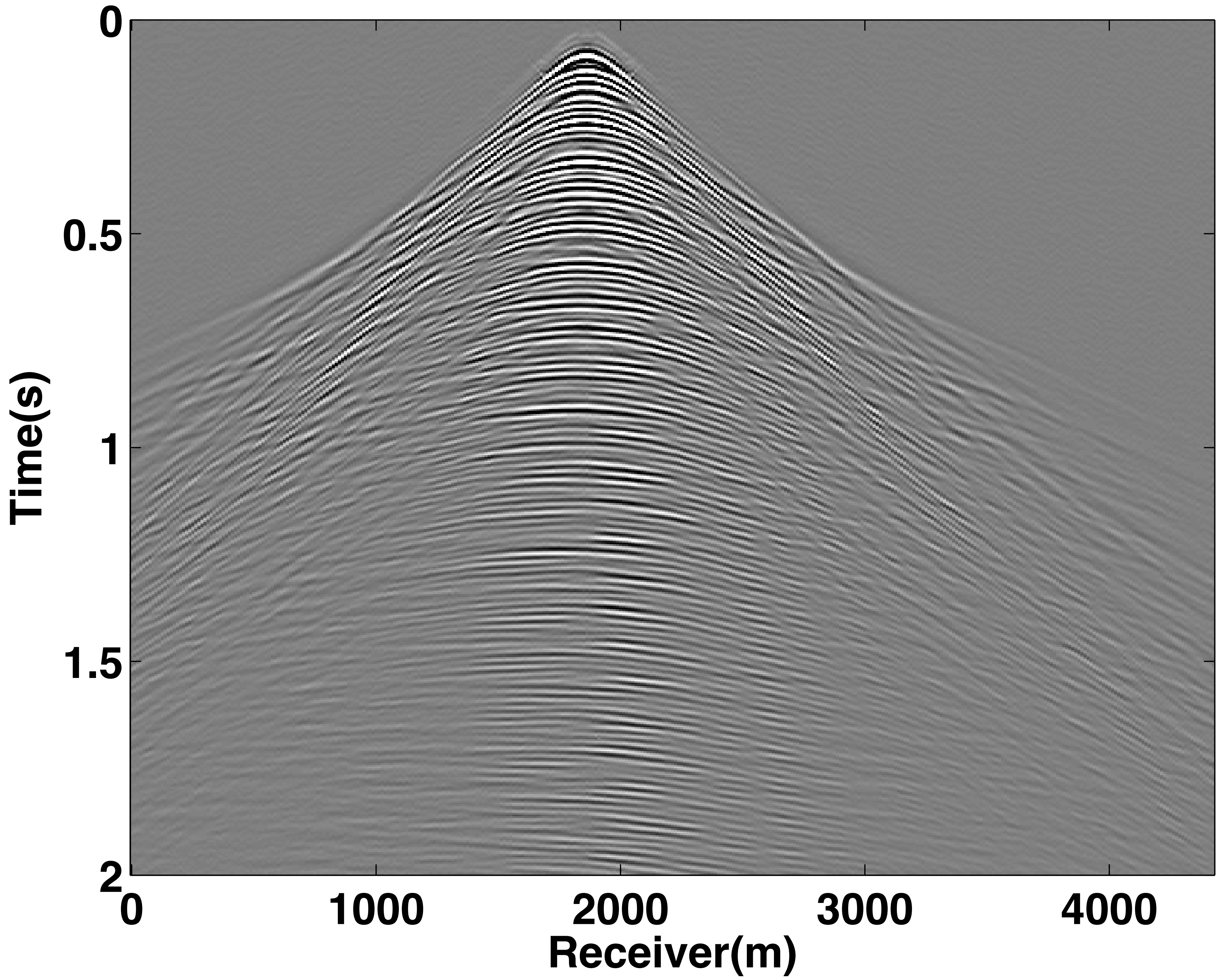


Ground Truth

SNR = 21.2 dB

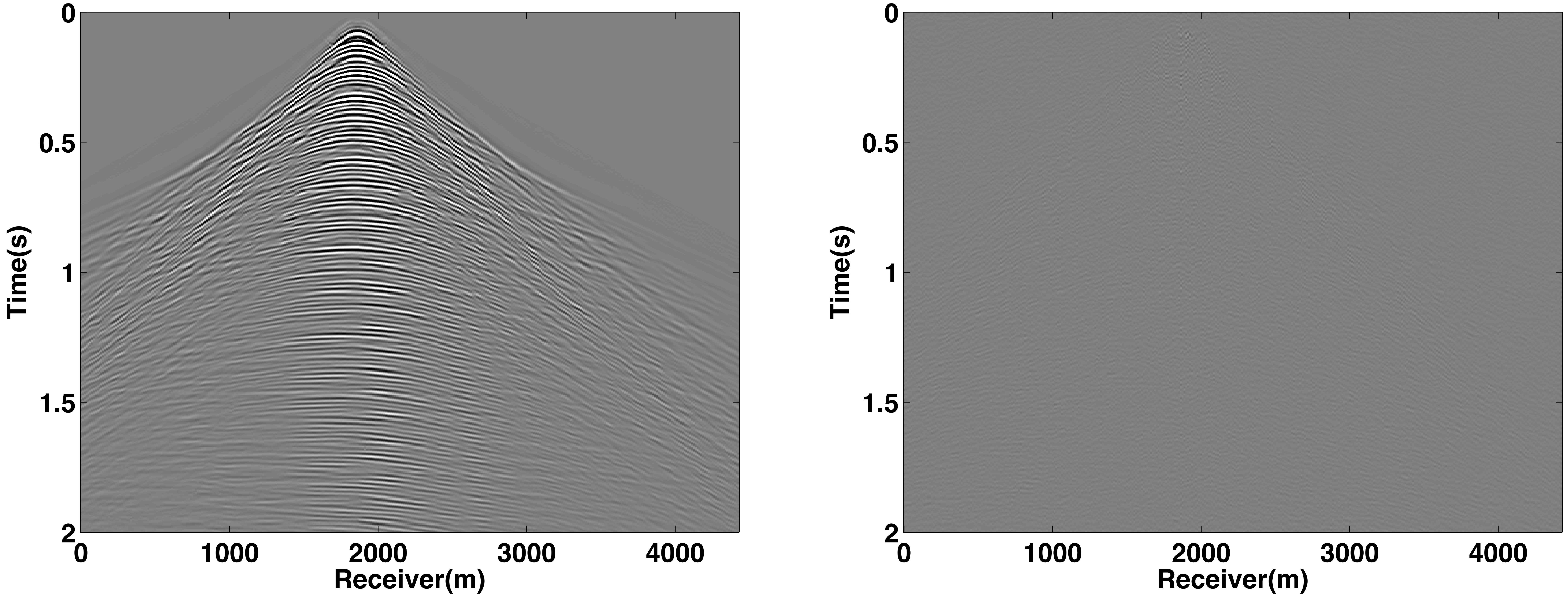


Recovery

Case 4
[Simultaneous Source : Land]

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$

$$\mathbf{M} \stackrel{\text{def}}{=} [\mathbf{I} \otimes \mathbf{diag}(\eta)\mathcal{F}_{\mathbf{s}}{}^{*}\mathbf{diag}(\mathbf{e}^{\mathbf{i}\theta})\mathcal{F}_{\mathbf{s}} \otimes \mathbf{I}]$$ [Herrmann et. al. 2009]

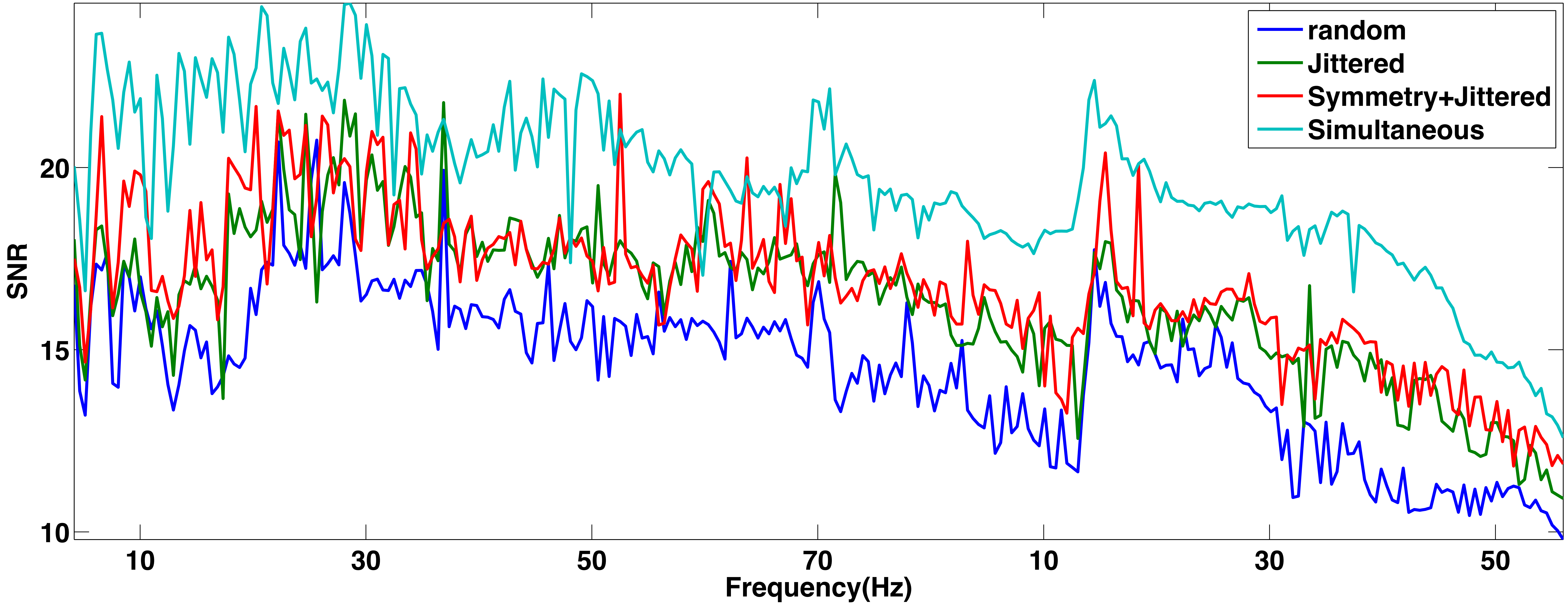


Ground Truth

Difference

Comparison

[uniform random vs jittered vs jittered + reciprocity]



Conclusion

- jittered sampling gives advantage of controlling the gap size
- matrix factorization allows SVD-free low-rank methods that work fast on large data
- low-rank structure holds promise for data recovery and more compact representation

Future Work

- Incorporate jittered sampling in 3D
- High frequency are *not low-rank* in nature, explore *HSS* (Hierarchical semi-separable representation) in 5D
- weighted low-rank interpolation

SLIM

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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