Extended images in action
(efficient WEMVA via randomized probing)

Rajiv Kumar, Tristan van Leeuwen and Felix J. Herrmann

SLIM
University of British Columbia

Motivation

- *initial model for FWI*

- computation of *full*-subsurface offset volumes is computationally *prohibitively* expensive
(storage & computation time)

- can't form full *E* but *action* on (random) vectors allows us to get information from *all* or *subsets* of *subsurface points*

Extended images

- Given two-way wave equations, source and receiver wavefields are defined as

$$H(\mathbf{m})U = P_s^T Q$$
$$H(\mathbf{m})^* V = P_r^T D$$

where

$H(\mathbf{m})$: discretization of the Helmoltz operator

Q : source

D : data matrix

P_s, P_r : samples the wavefield at the source and receiver positions

$\mathbf{m}$: slowness

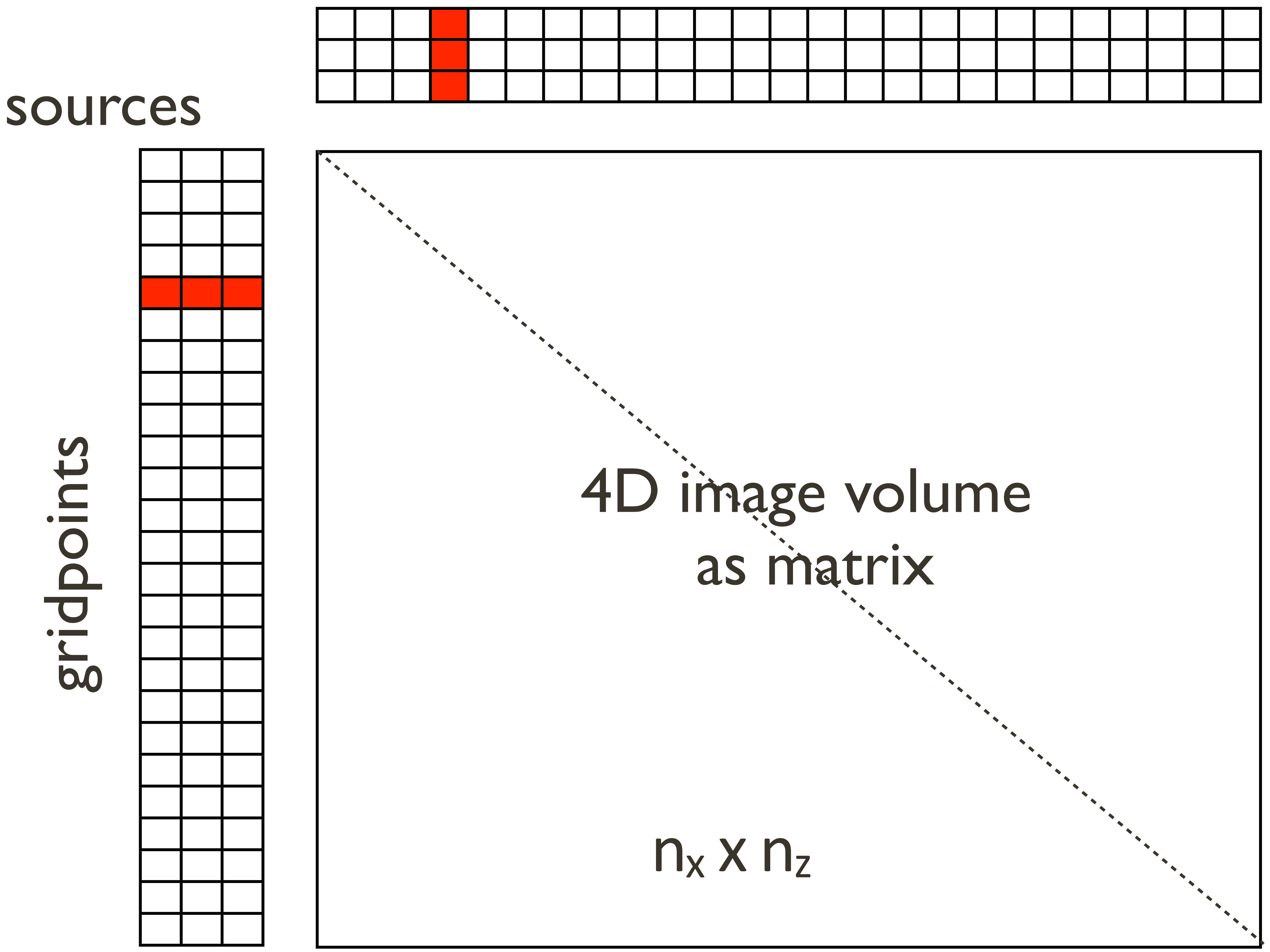
Extended images

- Organize wavefields in monochromatic data *matrices* where each *column* represents a *common* shot gather

- Express *image* volume *tensor* for *single* frequency as a *matrix*

$$E = UV^*$$

Extended images



Extended images

- Too *expensive* to compute *(storage and computational time)*

- Instead, *probe* volume with *tall* matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_l]$

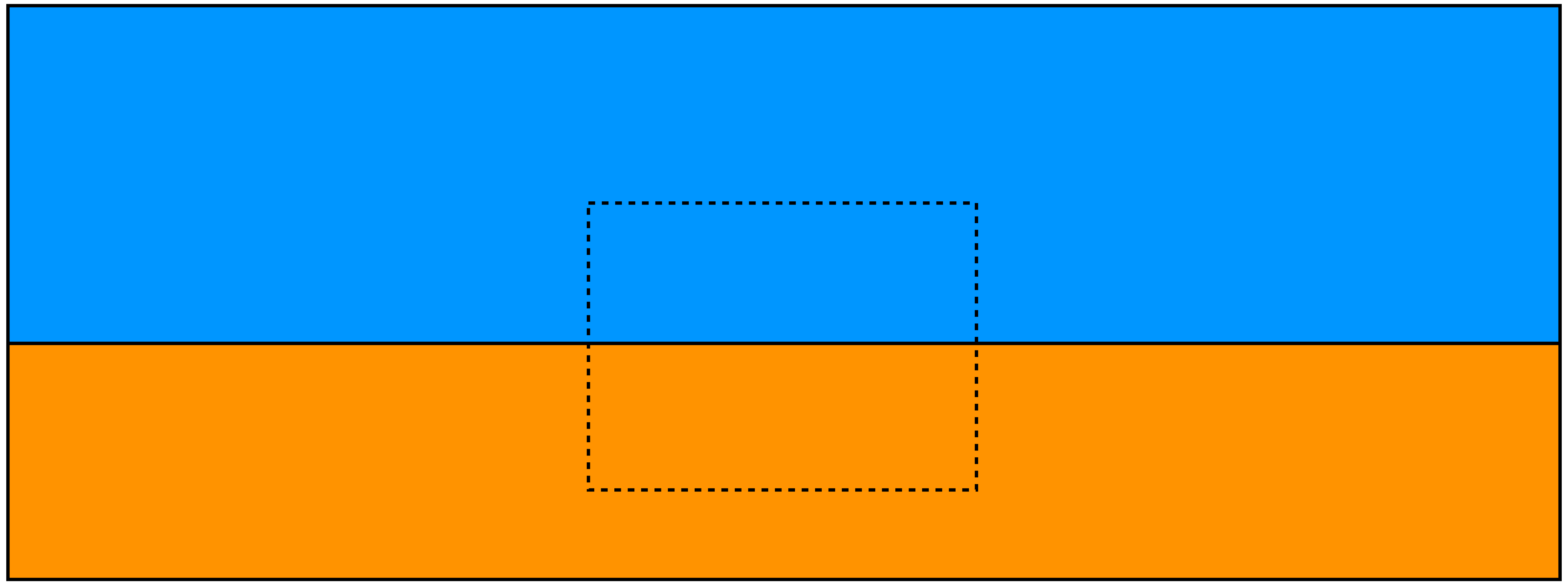
van Leeuwen 2012

$$\widetilde{E} = EW = H^{-1} P_s^T Q D^* P_r H^{-1} W$$

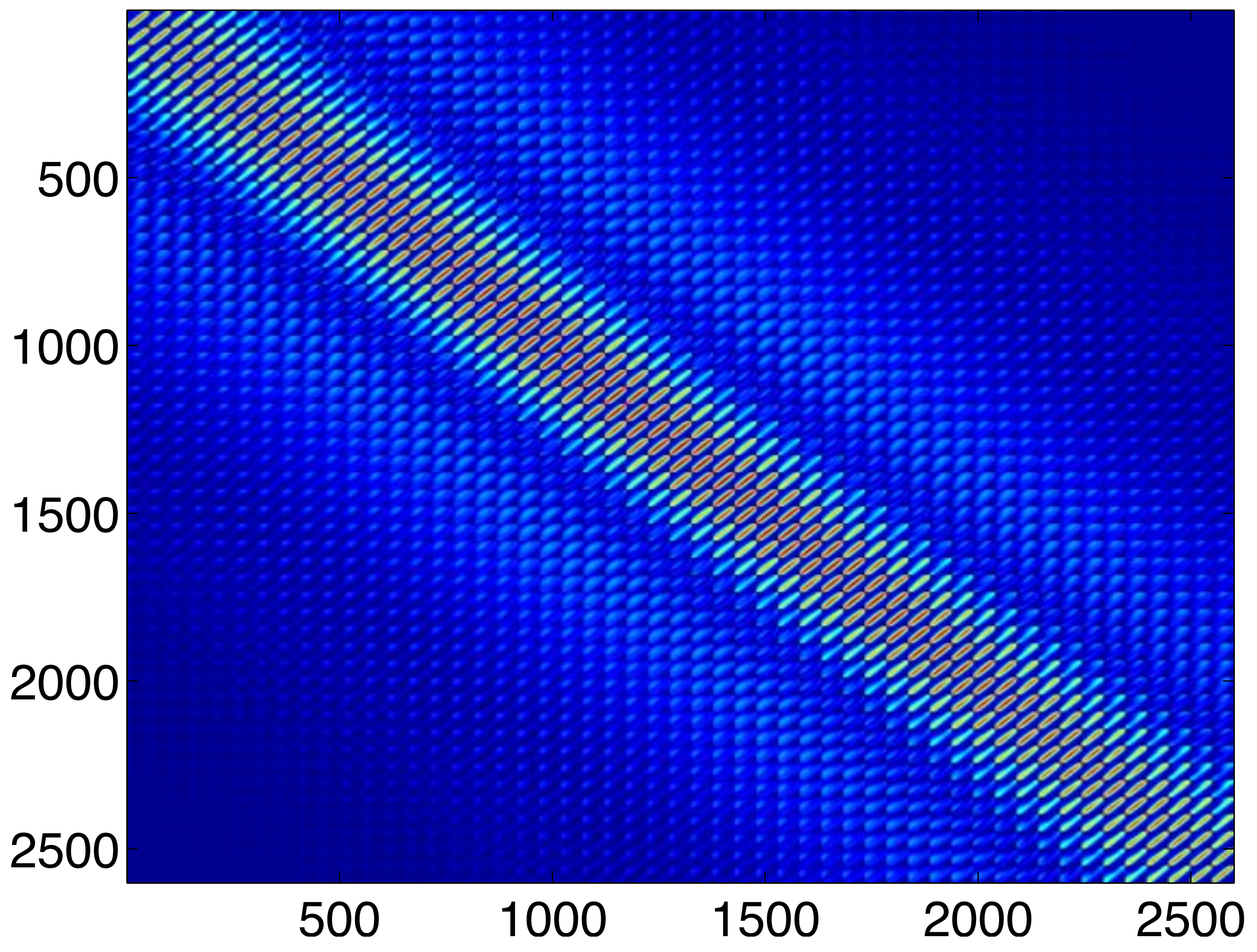
where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents *single* scattering points

Extended images

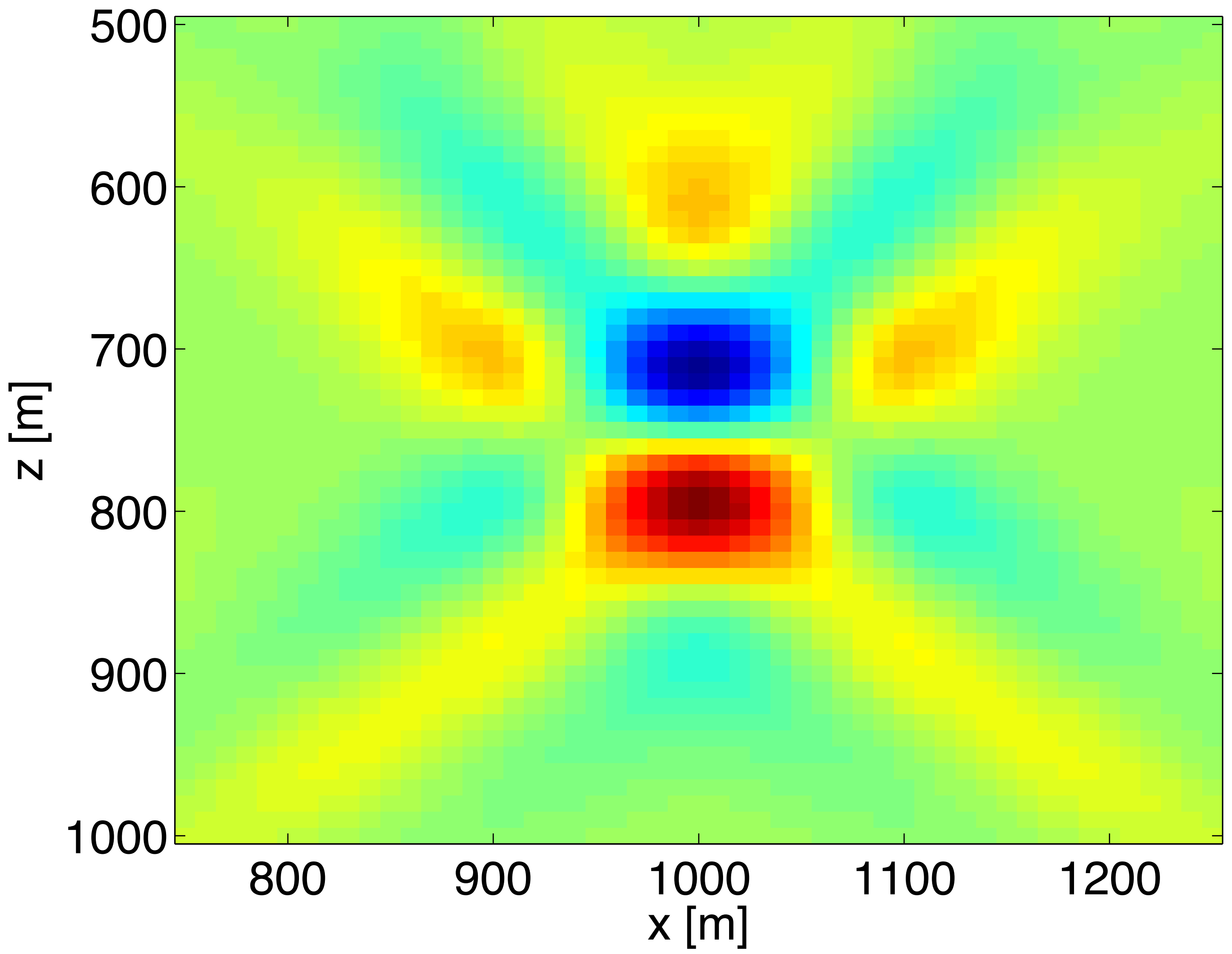
example for *one* layer



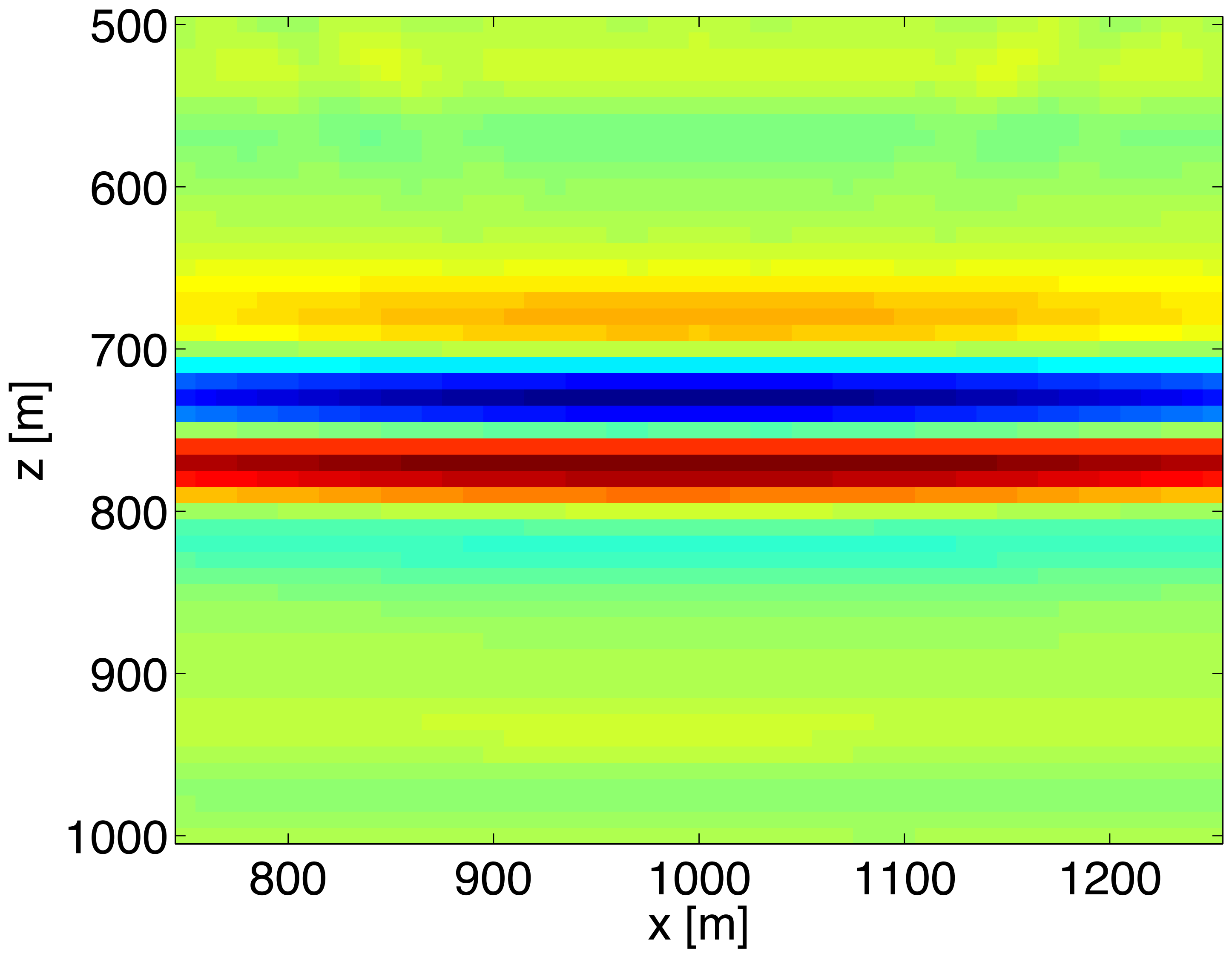
Extended images



full matrix



one column



diagonal

Take-away message

Computational *costs*

Full subsurface offset *extended* images:

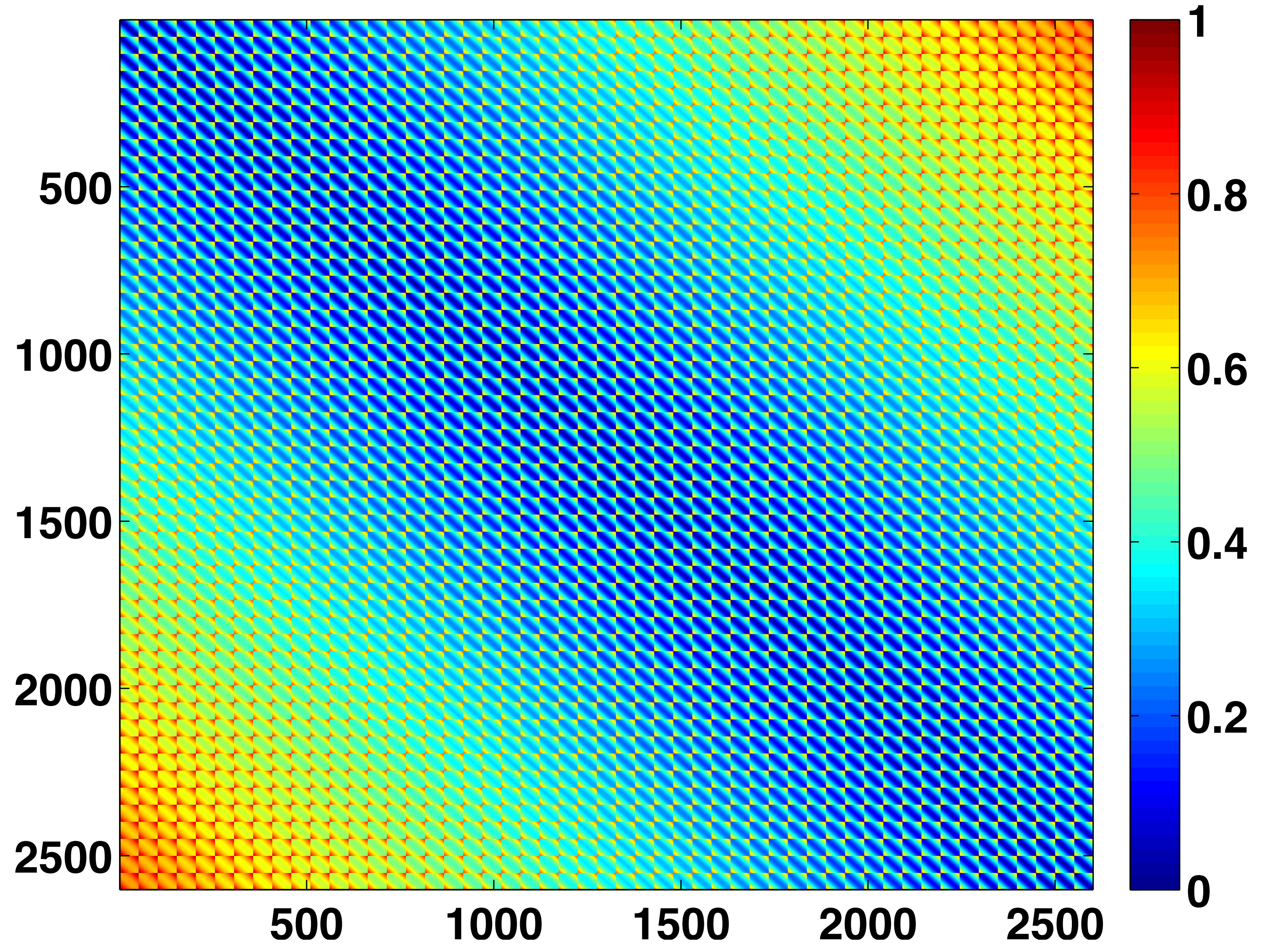
	# of PDE solves	"flops for correlations"
conventional	$2N_s$	$N_s \times N_h$
mat-vecs	$2N_x$	$N_s \times N_r$

N_s - # of sources
N_r - # of receivers
N_h - # of subsurface offsets
N_x - # of sample points

Biondo & Symes, ’04 , Symes 2008, Sava & Vasconcelos, ’11

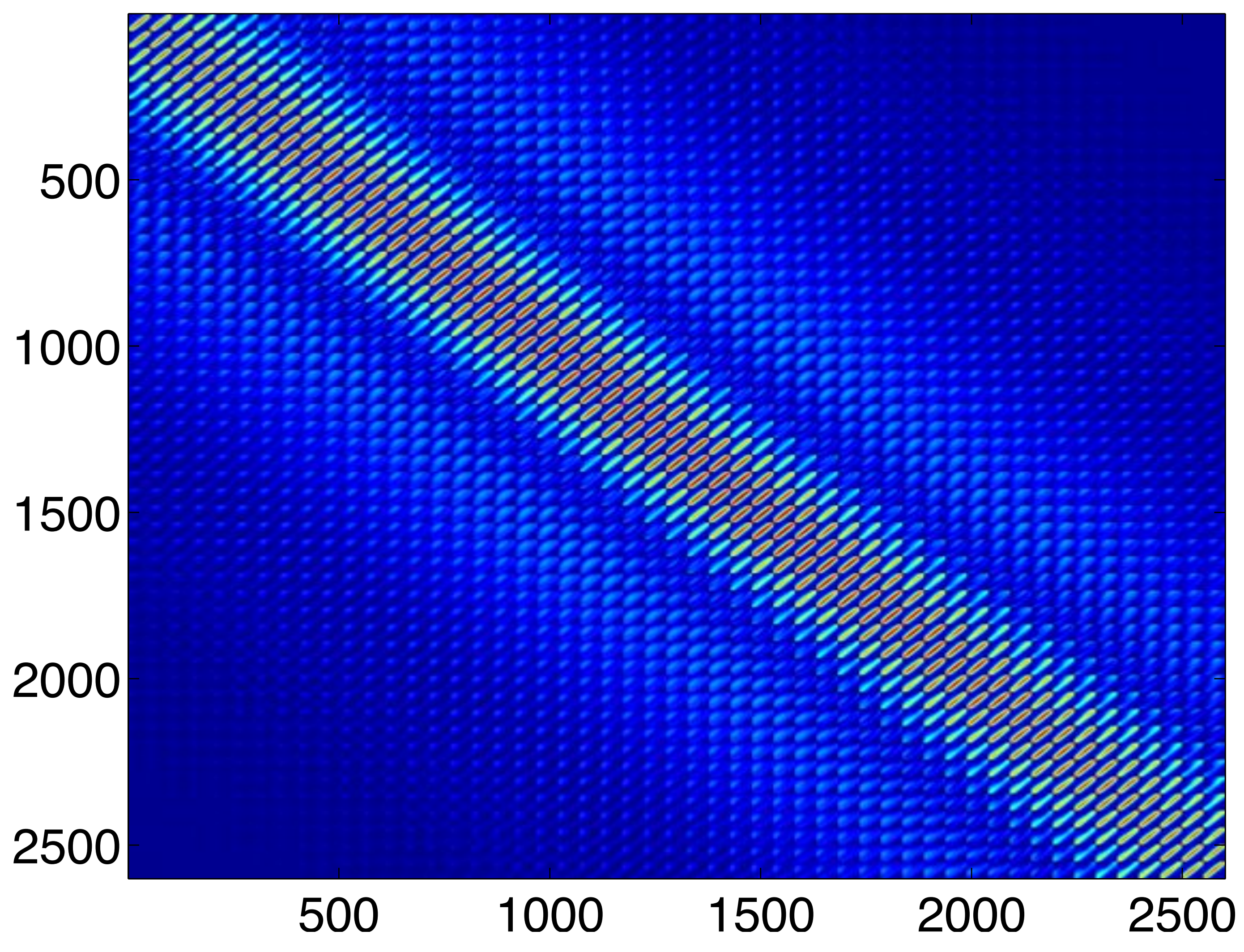
WEMVA

[conventional method]



h

$.*$



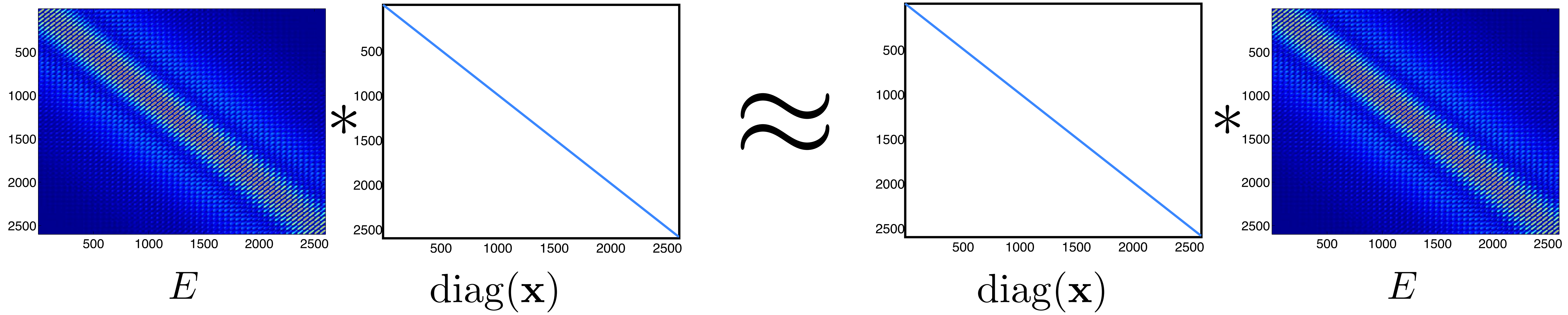
E

$.*$ **stand for element-wise multiplication**

Focusing

[propose method] van Leeuwen 2012

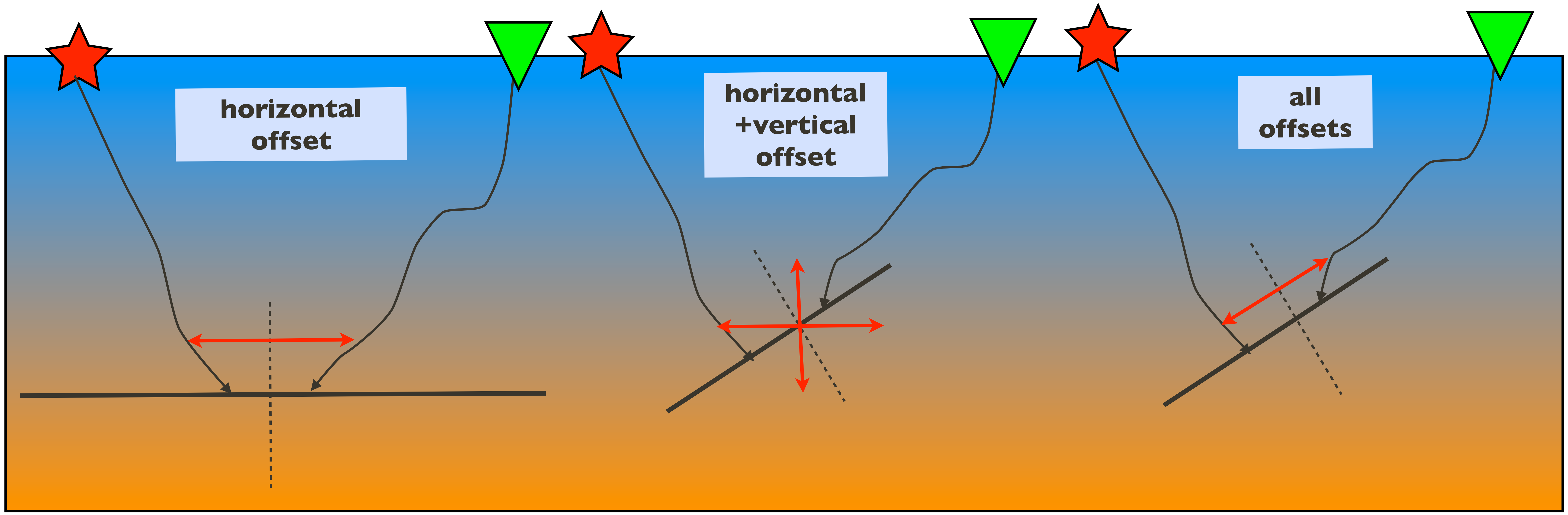
$$E\mathrm{diag}(\mathbf{x}) \approx \mathrm{diag}(\mathbf{x})E$$



$*$ **matrix-matrix multiplication**

Focusing

where $\mathbf{x}$ represents horizontal, vertical or all offset.



Fast WEMVA w/ *randomized* probing

- Measure the error in some norm van Leeuwen 2012

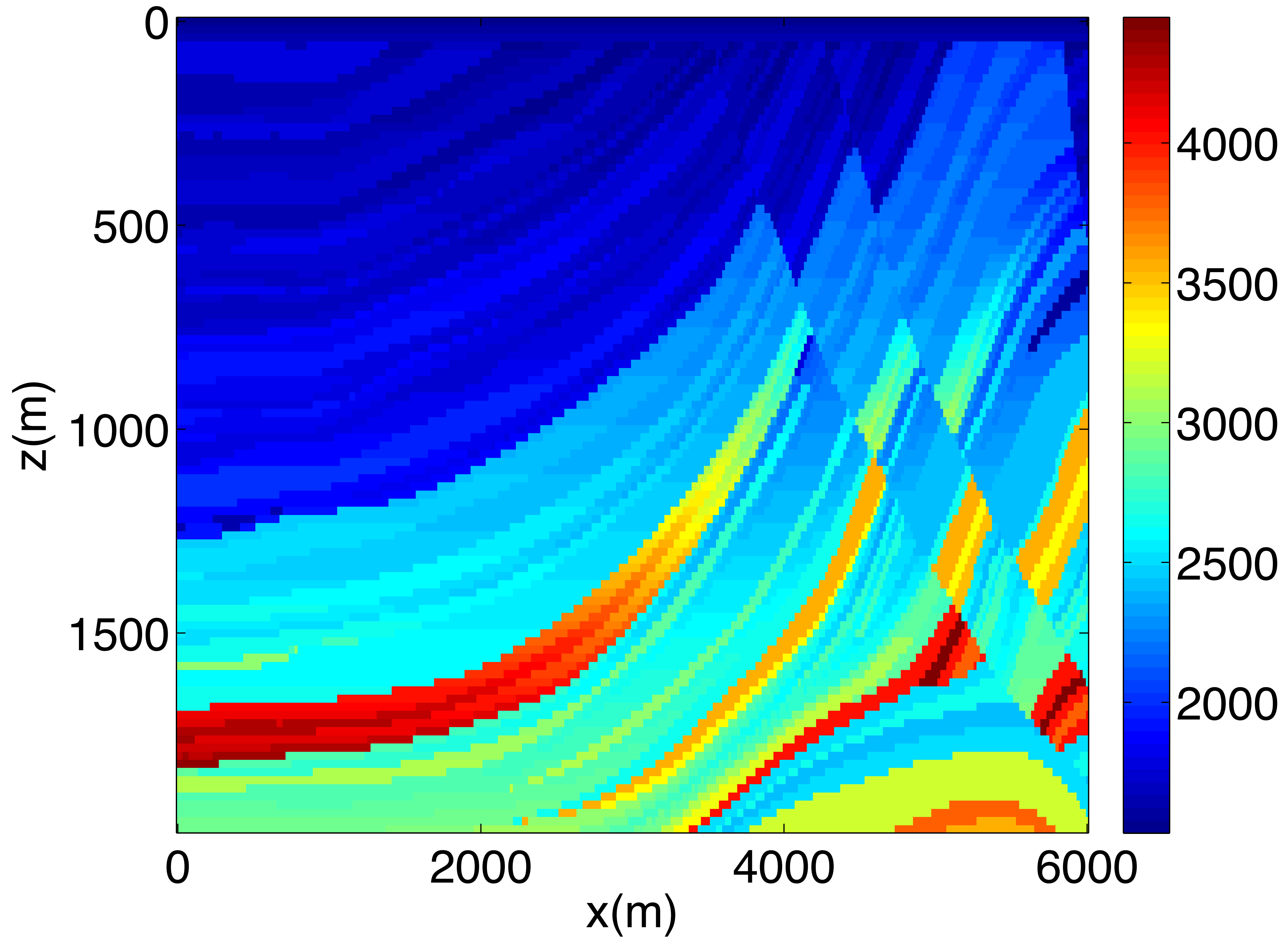
$$\min_{\mathbf{m}} ||E(\mathbf{m})\text{diag}(\mathbf{x}) - \text{diag}(\mathbf{x})E(\mathbf{m})||_?^2$$

- The *Frobenius* norm can be estimated via randomized trace estimation : Avron and Toledo, 2011

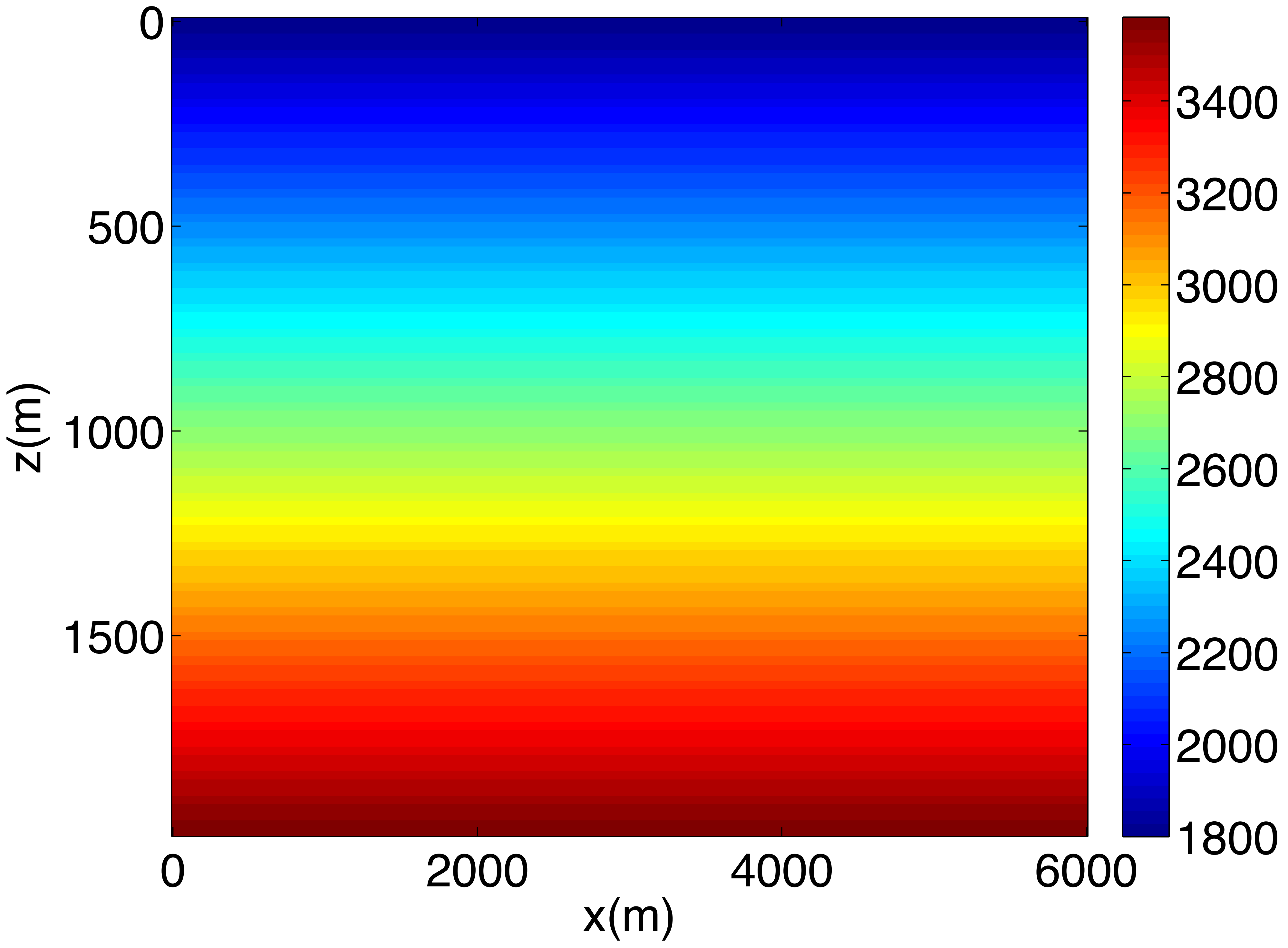
$$\begin{aligned} ||A||_F^2 &= \text{trace}(A^T A) \\ &\approx \sum_{i=1}^{K} \mathbf{w}_i^T A^T A \mathbf{w}_i = \sum_{i=1}^{K} ||A\mathbf{w}_i||_2^2 \end{aligned}$$

where $\sum_{i=1}^{K} \mathbf{w}_i \mathbf{w}_i^T \approx I$

Randomized probing
[reflection]



true model

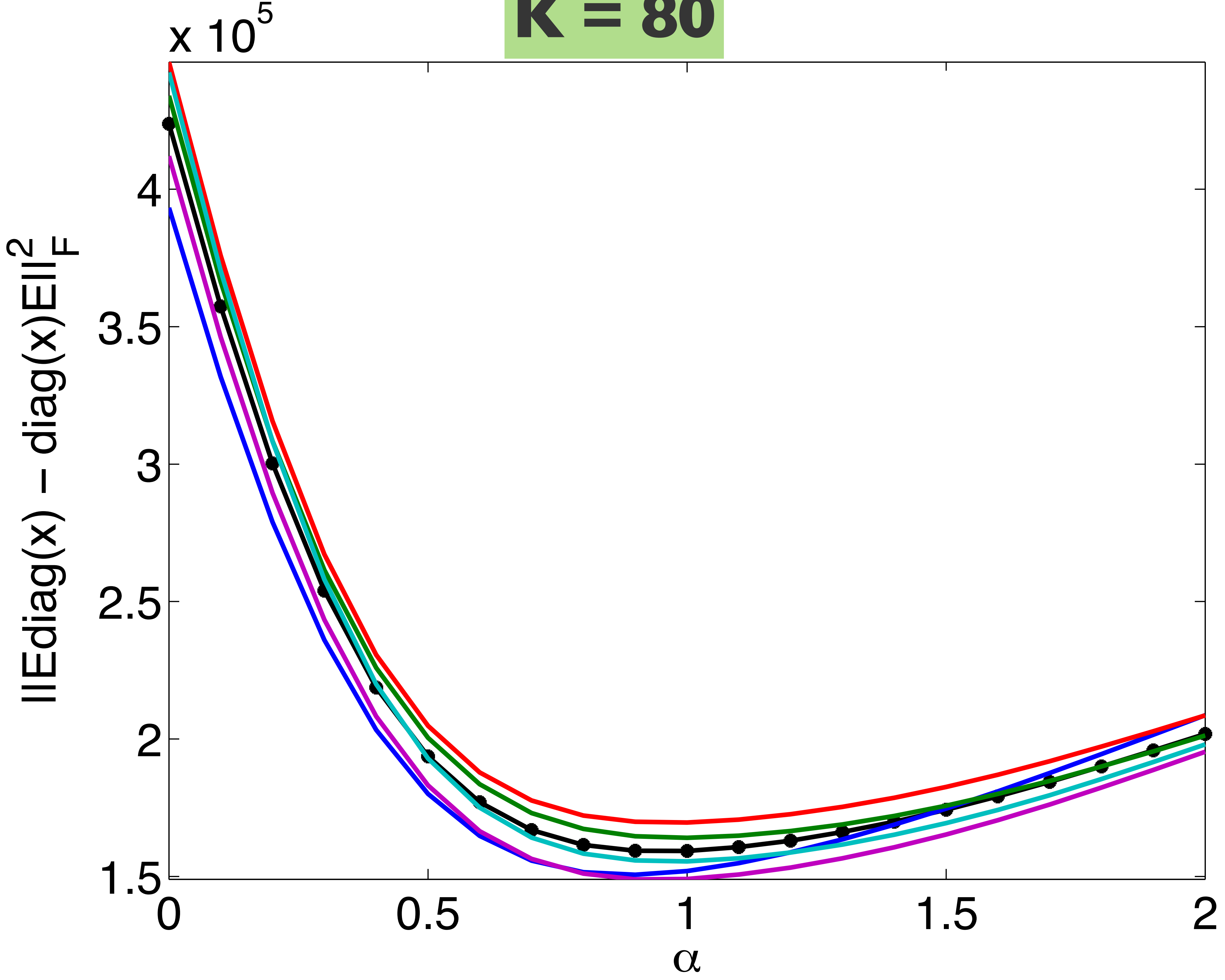


initial model

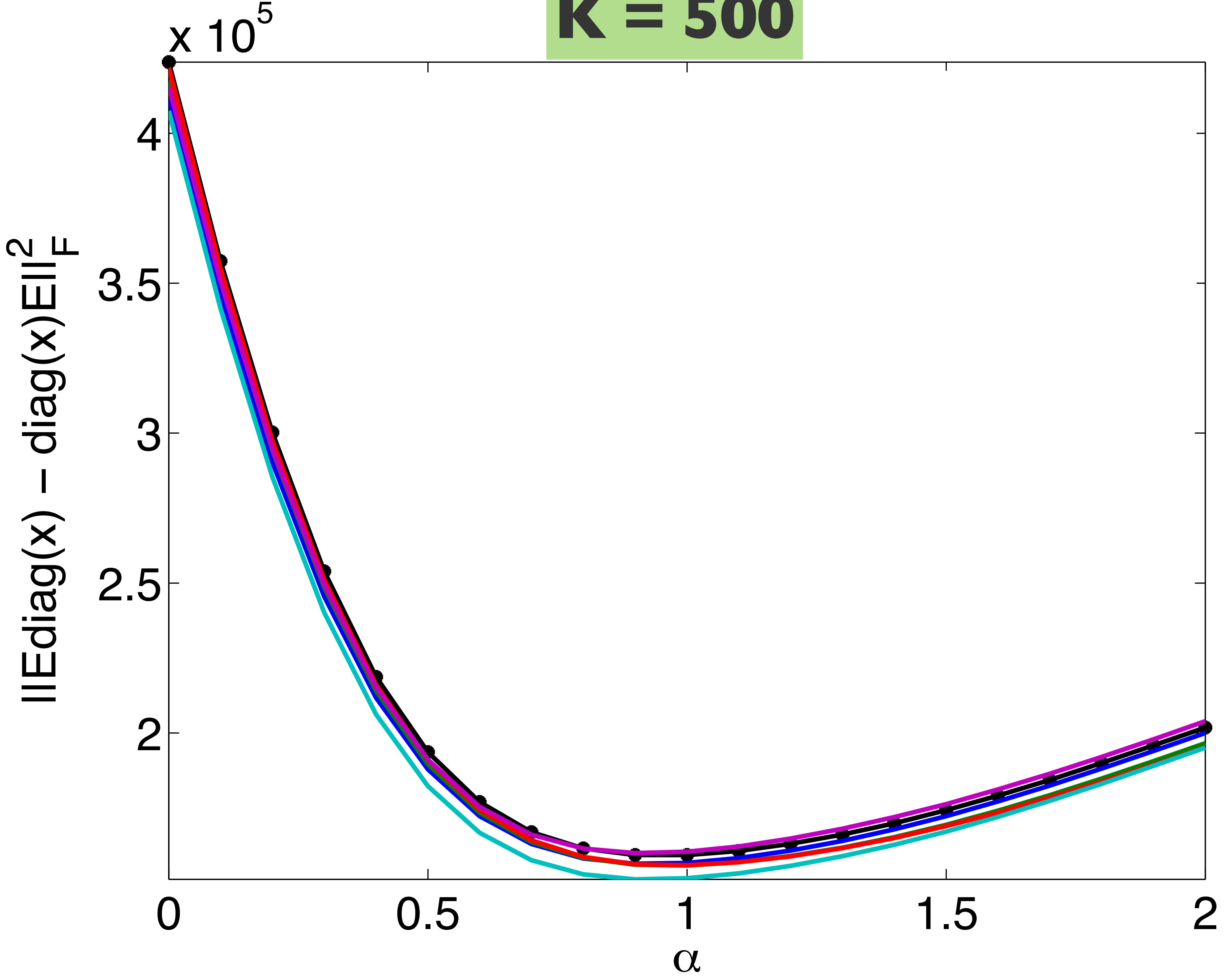
Randomized probing
[reflection]

- Exact
- different color represents different random realization

K = 80



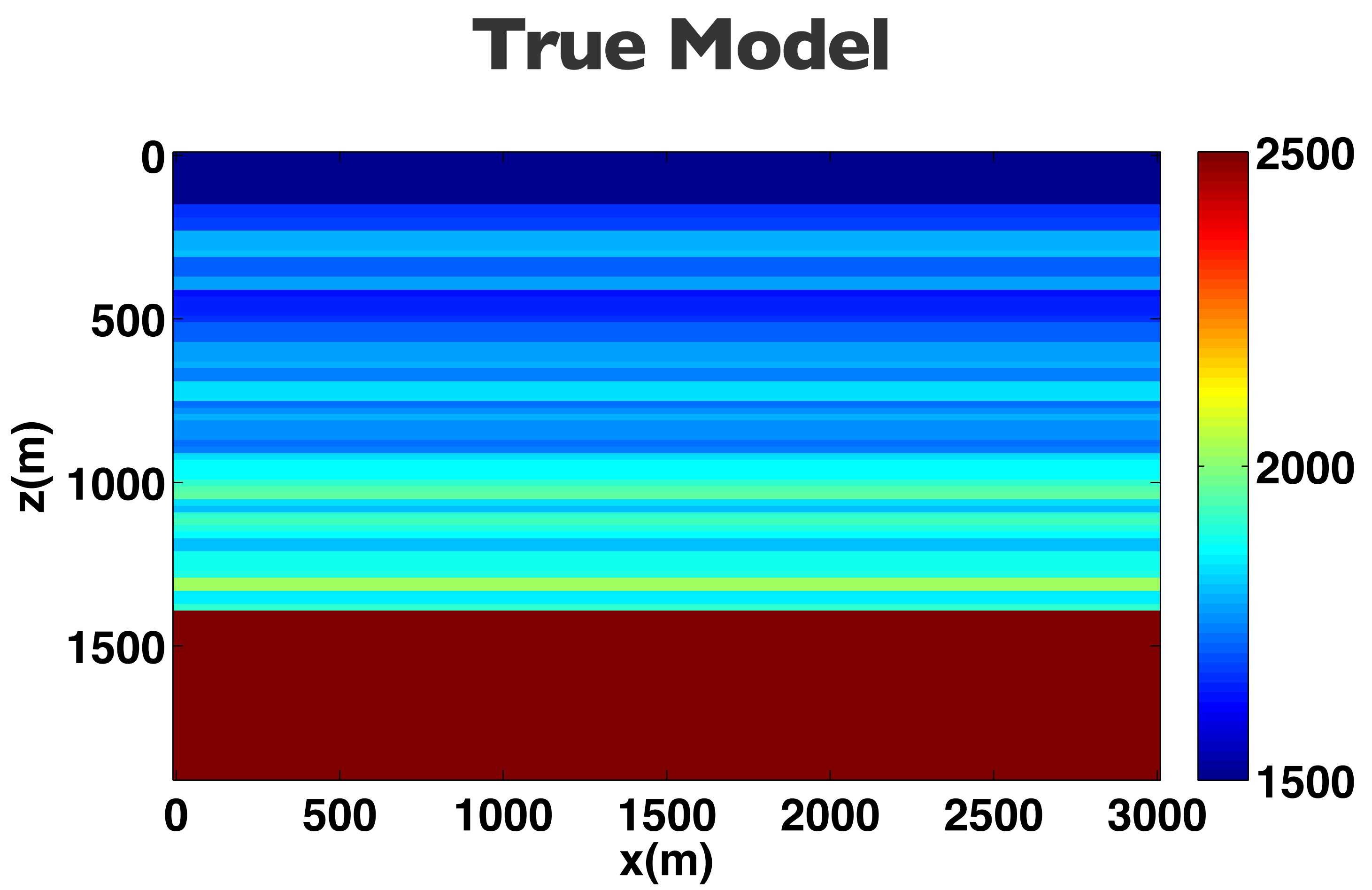
K = 500

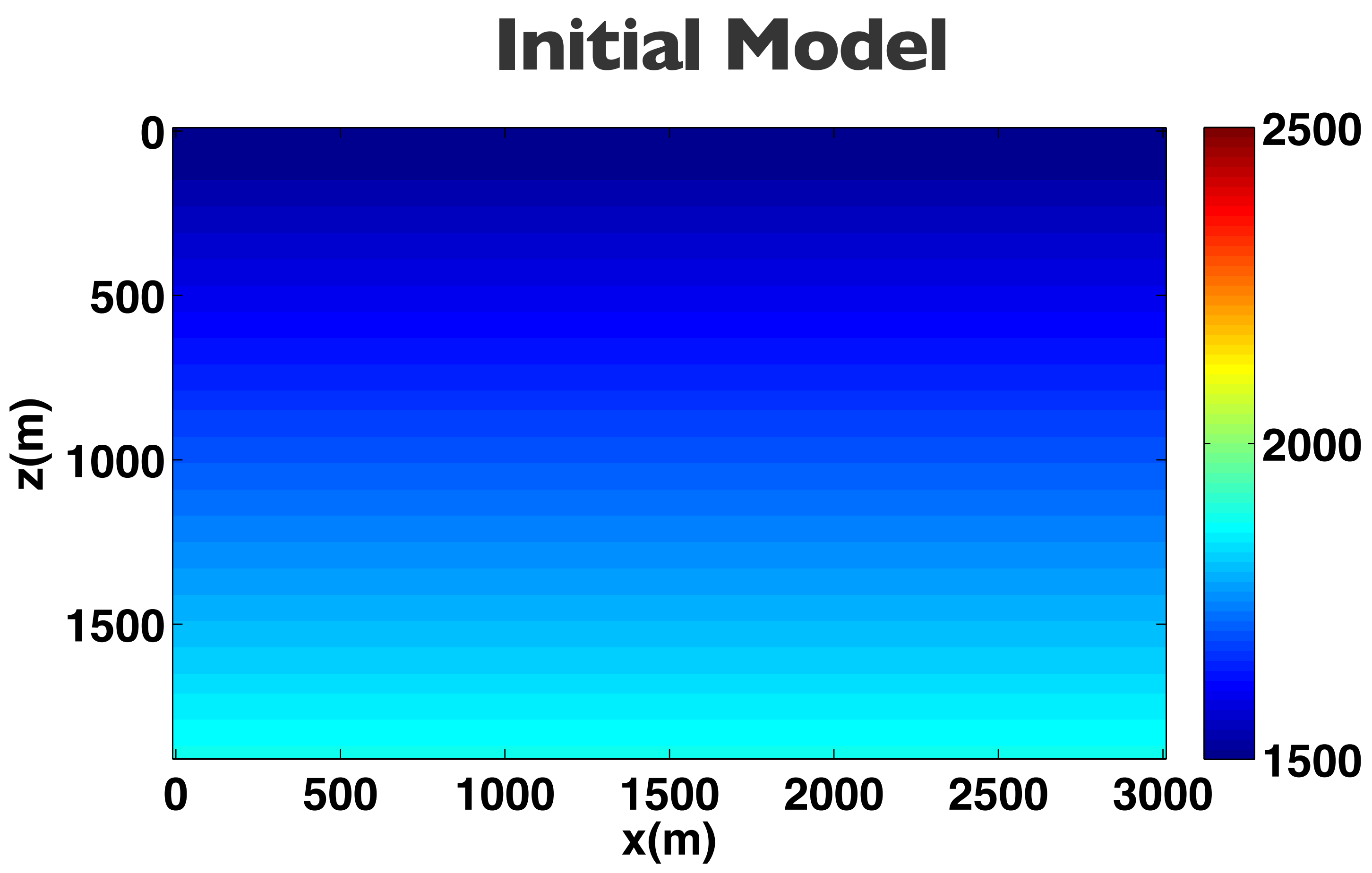


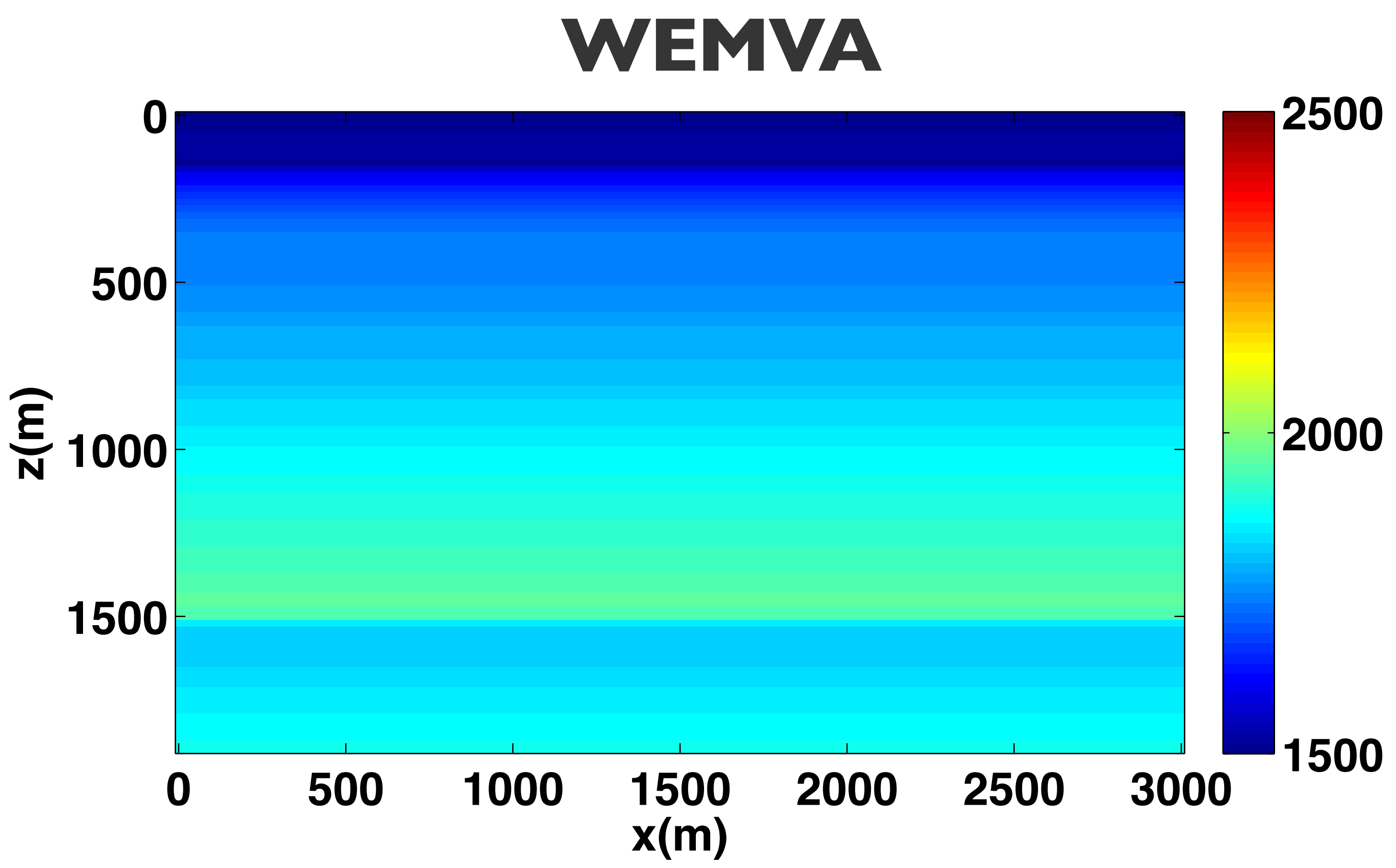
Applications

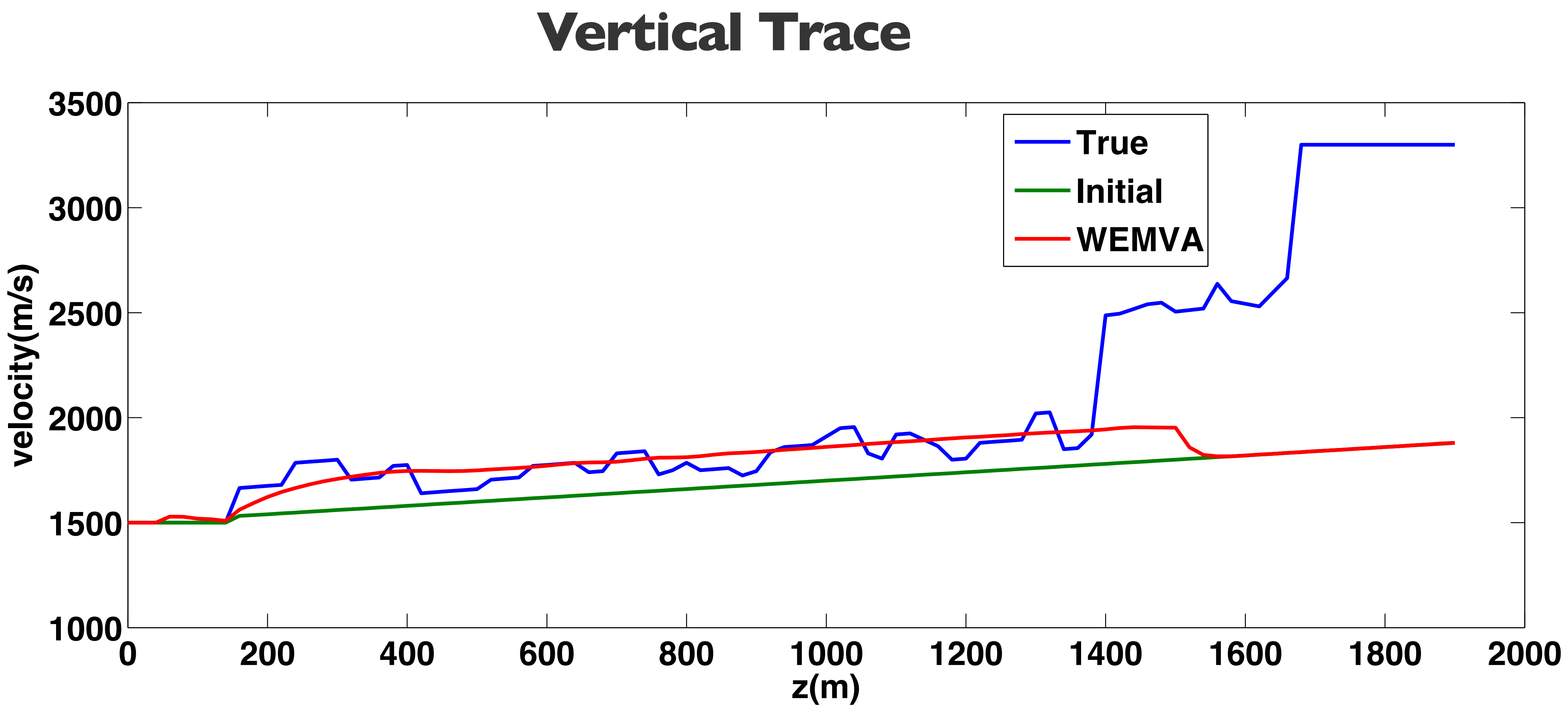
- Reflector model
- Lens model
- Vertical gradient model
- Marmousi model

Reflector model

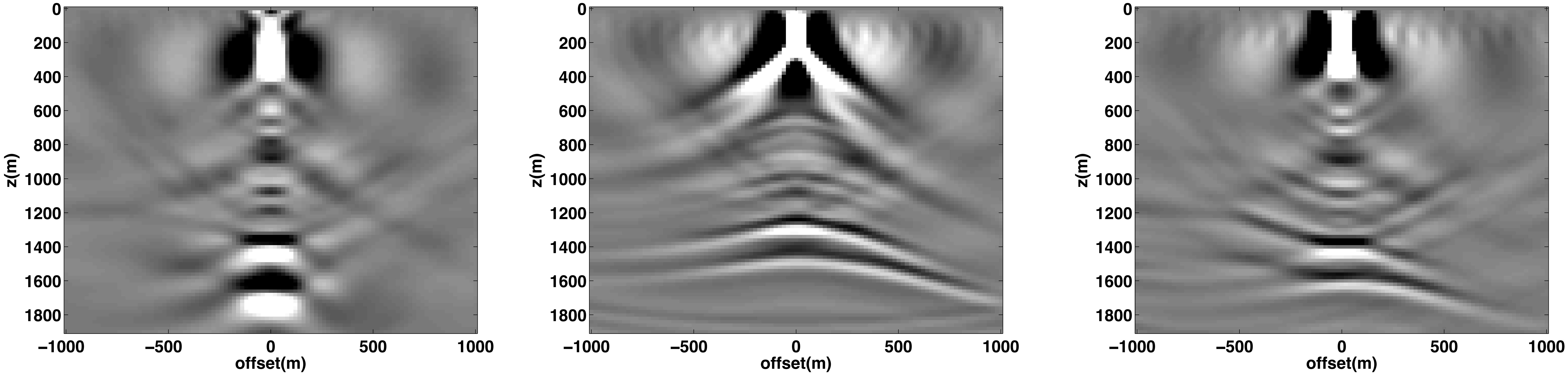








Reflector model
[image gathers]

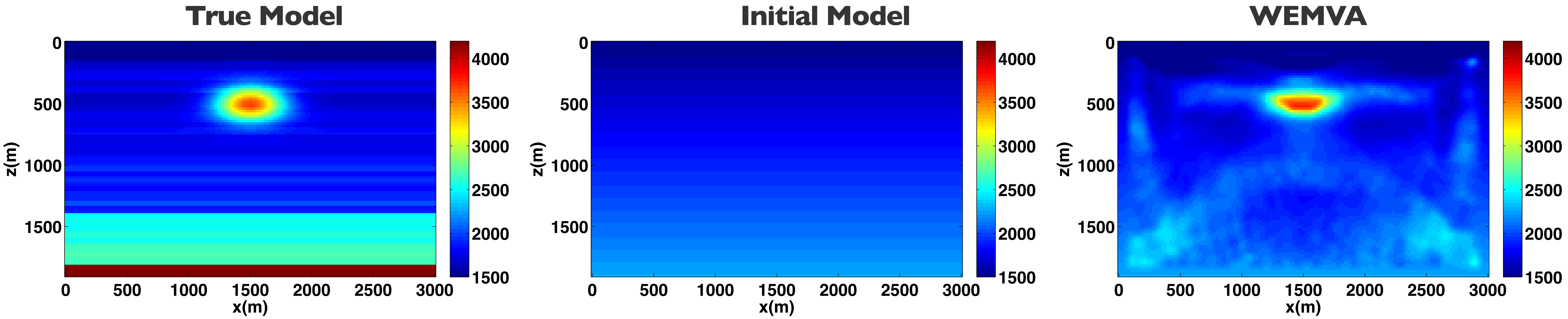


true model

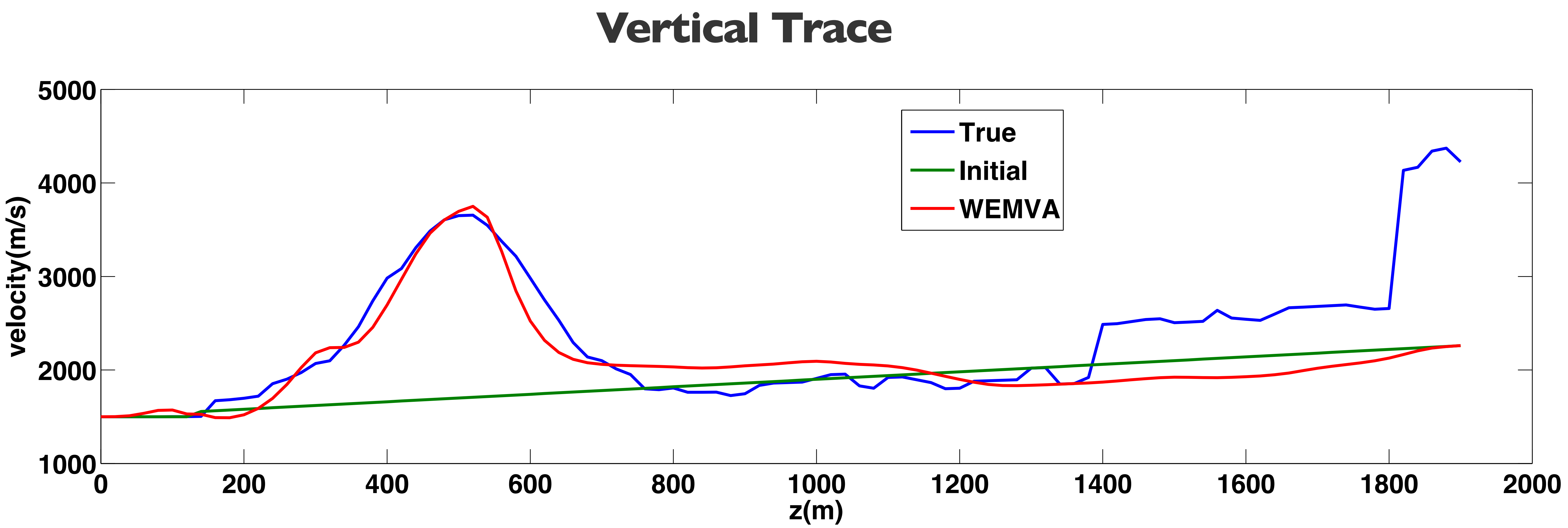
initial model

WEMVA

Lens Model

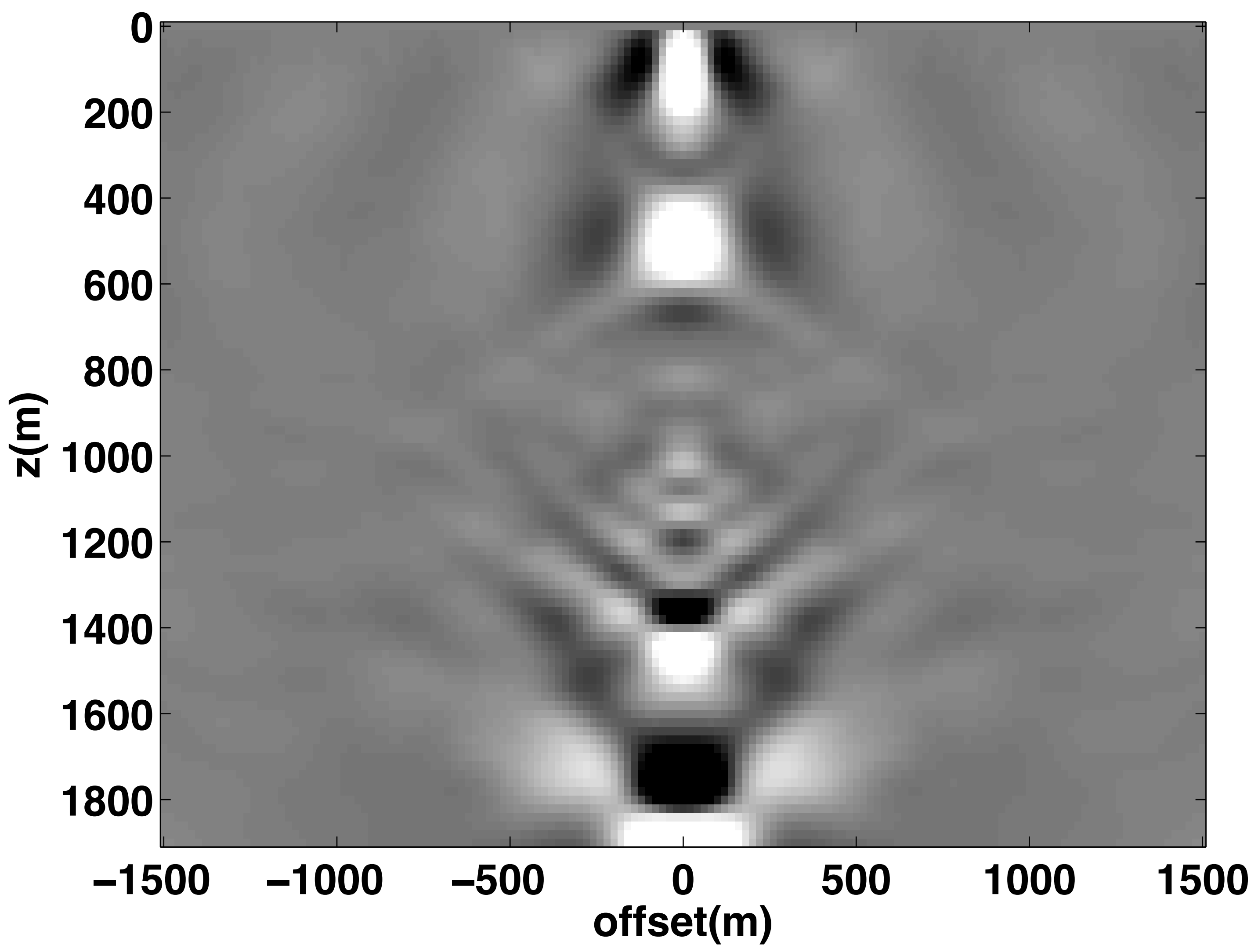


True Model
Initial Model
WEMVA
z(m)
x(m)
0
500
1000
1500
2000
2500
3000
4000
3500
3000
2500
2000
1500

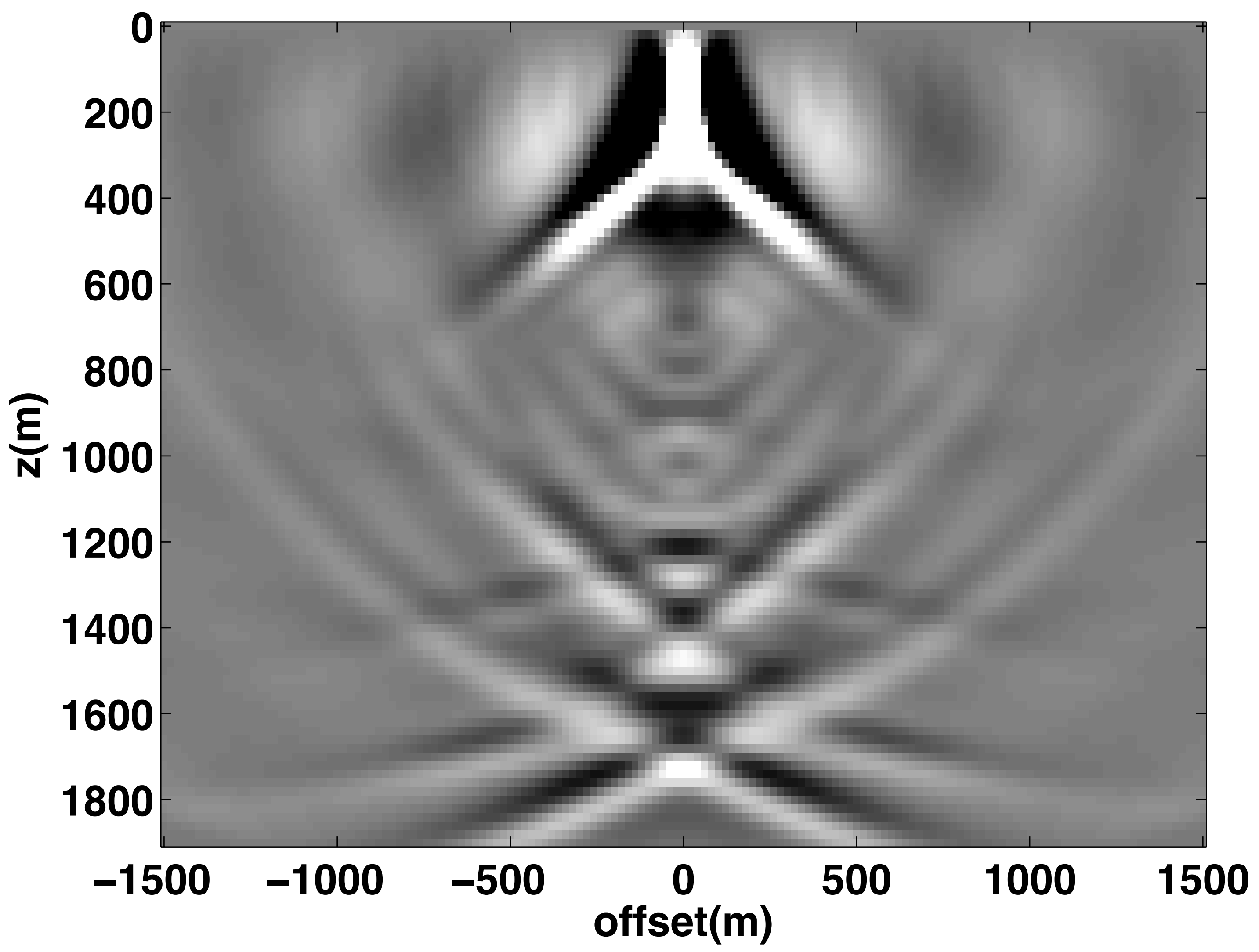


Vertical Trace
velocity(m/s)
z(m)
True
Initial
WEMVA
5000
4000
3000
2000
1000
0
200
400
600
800
1000
1200
1400
1600
1800
2000

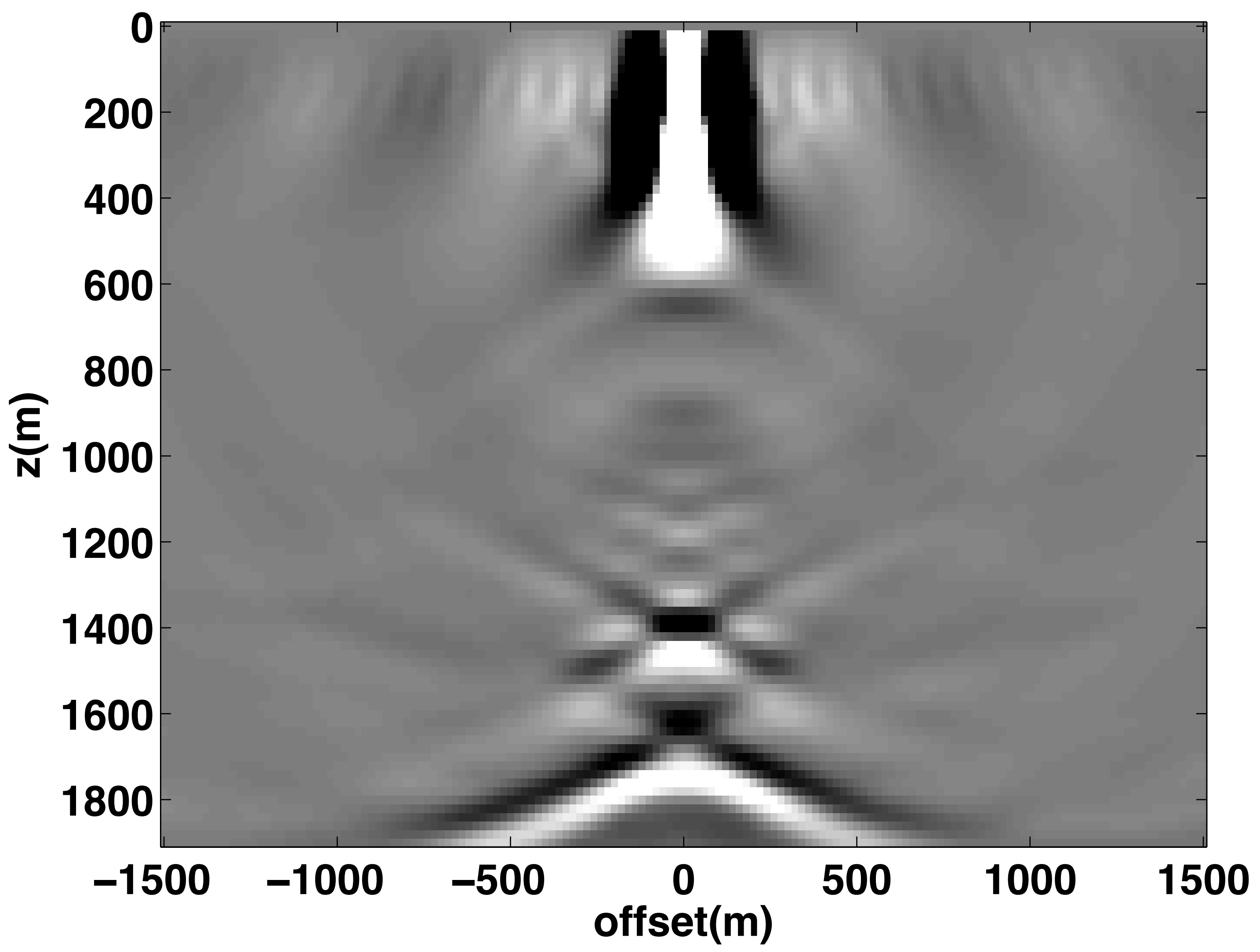
Lens Model
[image gathers]



true model

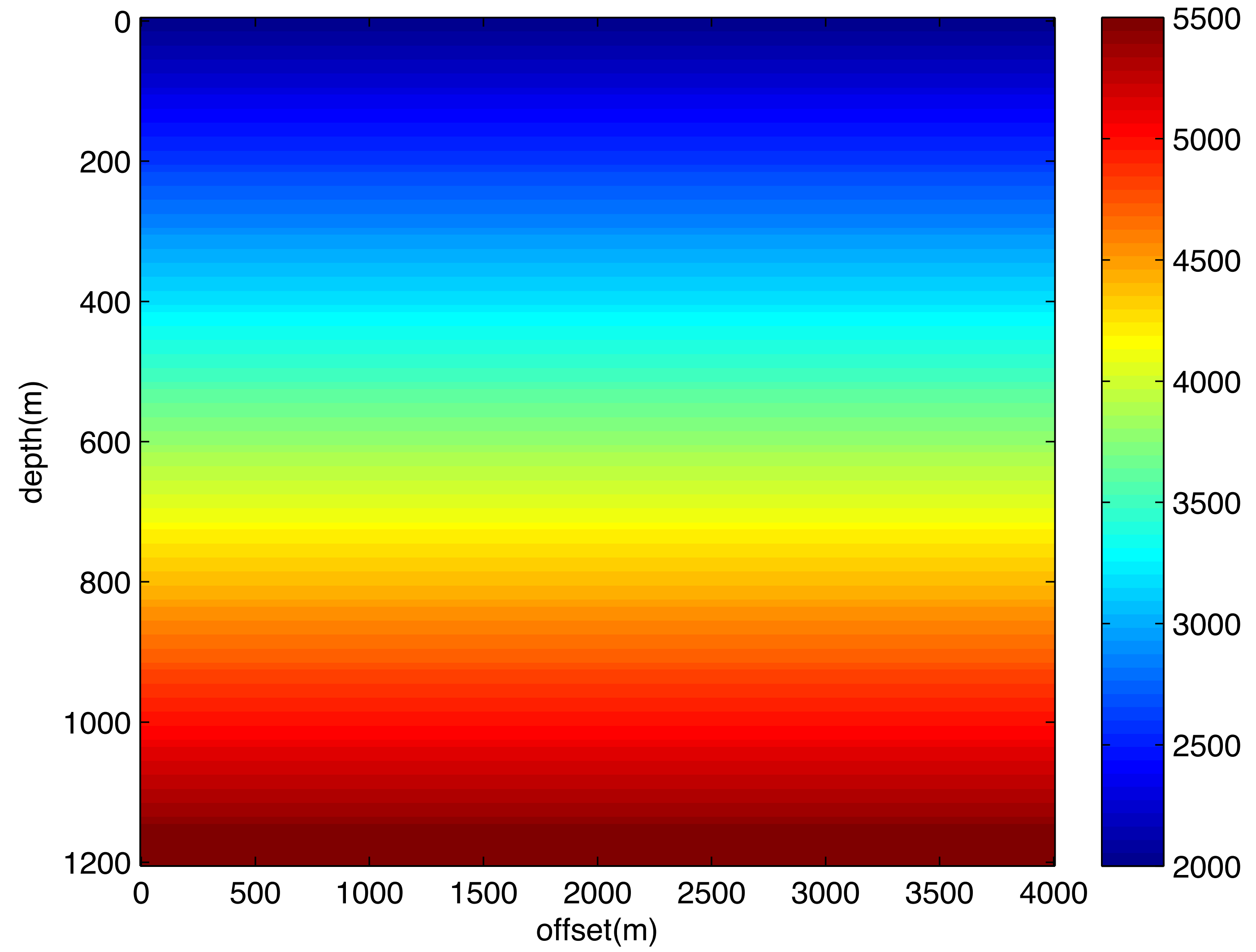


initial model

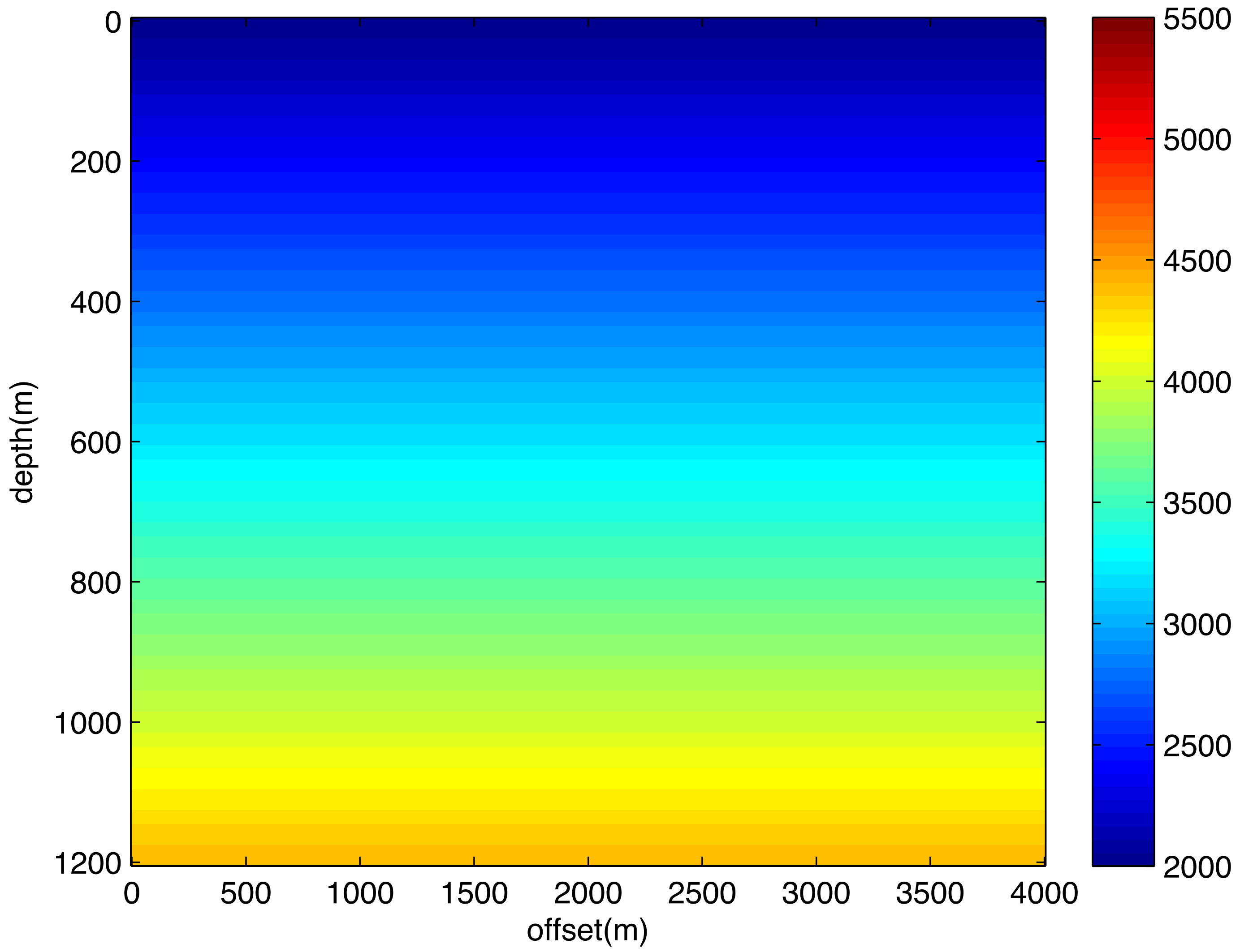


WEMVA

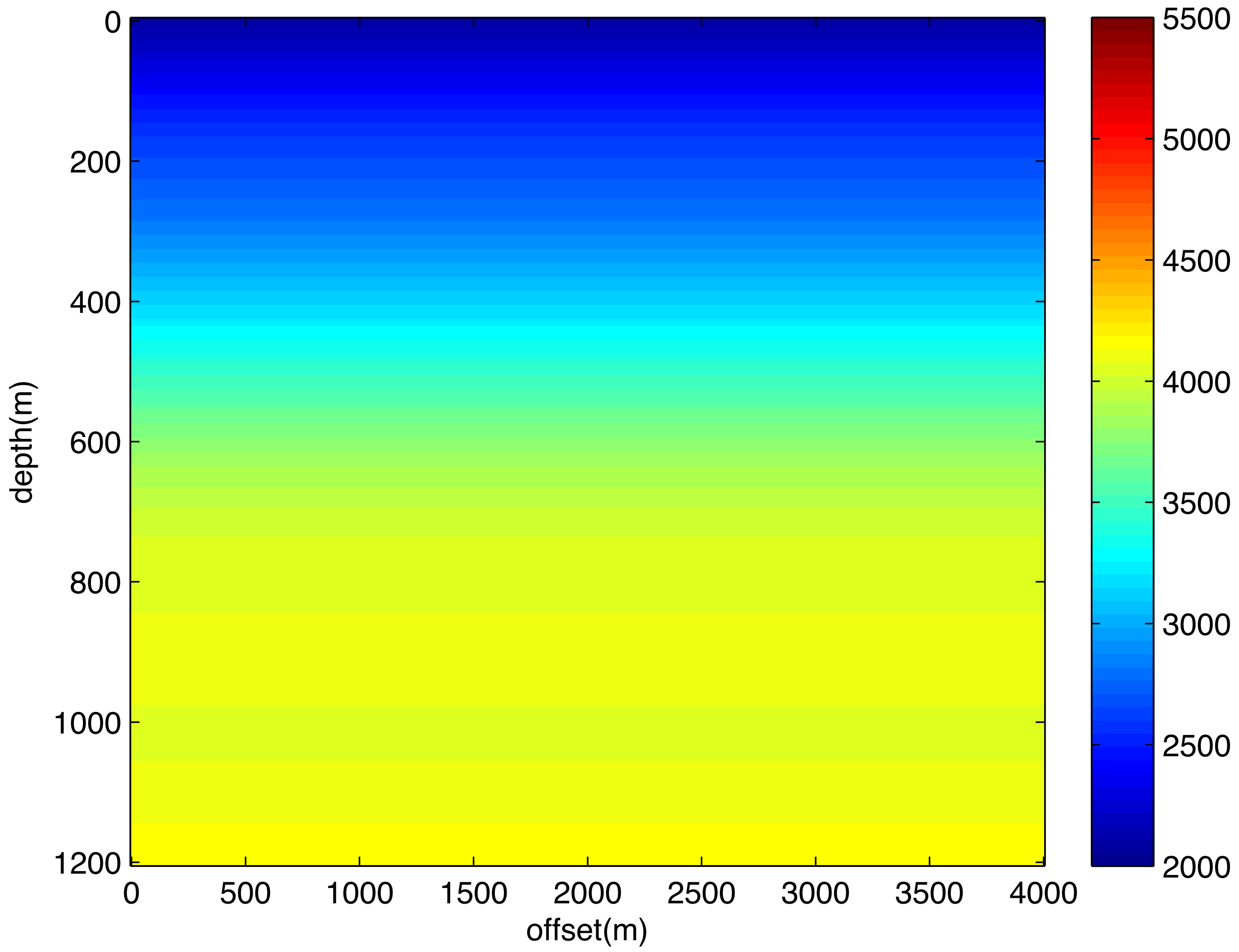
Vertical gradient
[diving waves]



true model

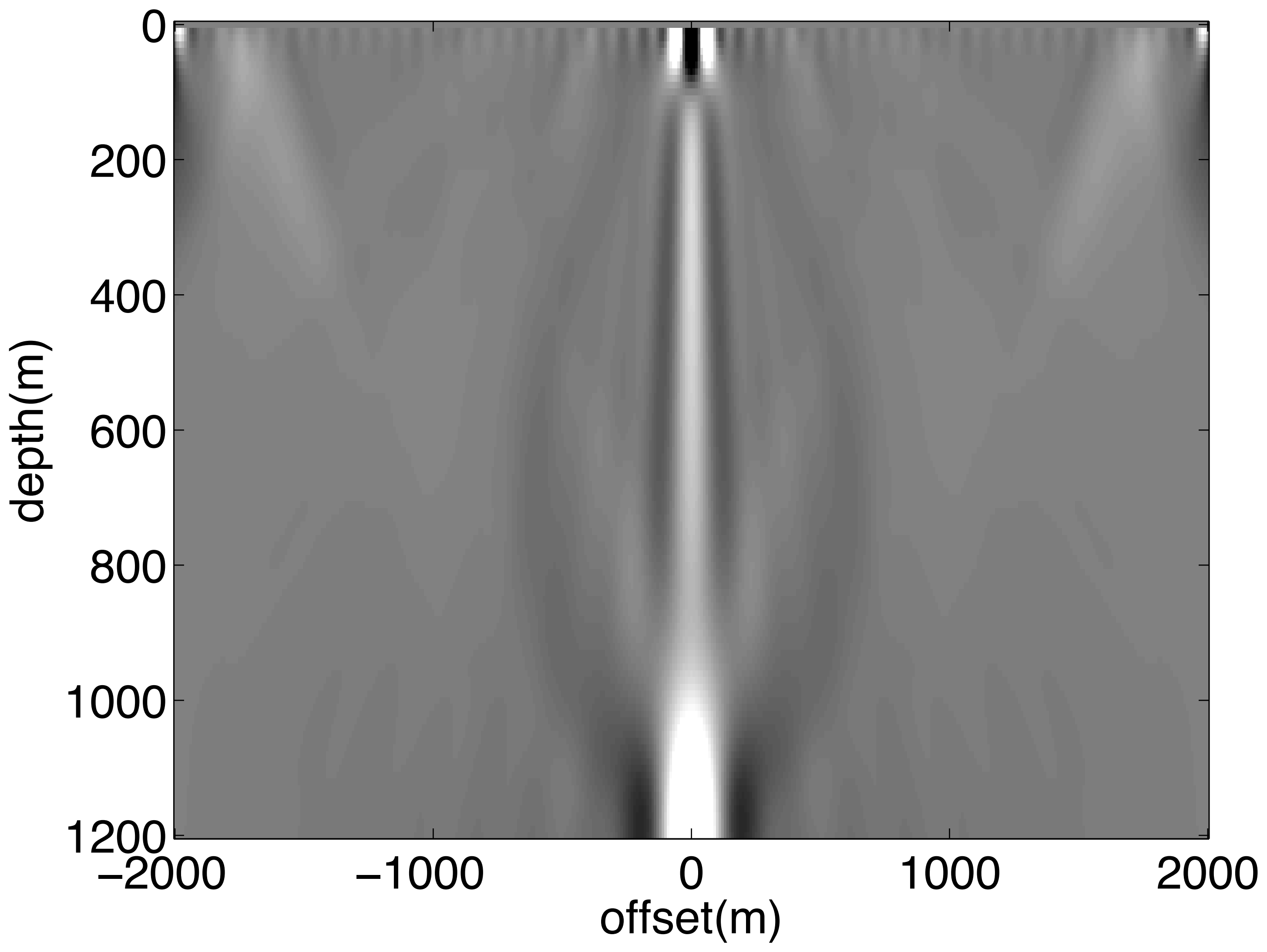


initial model

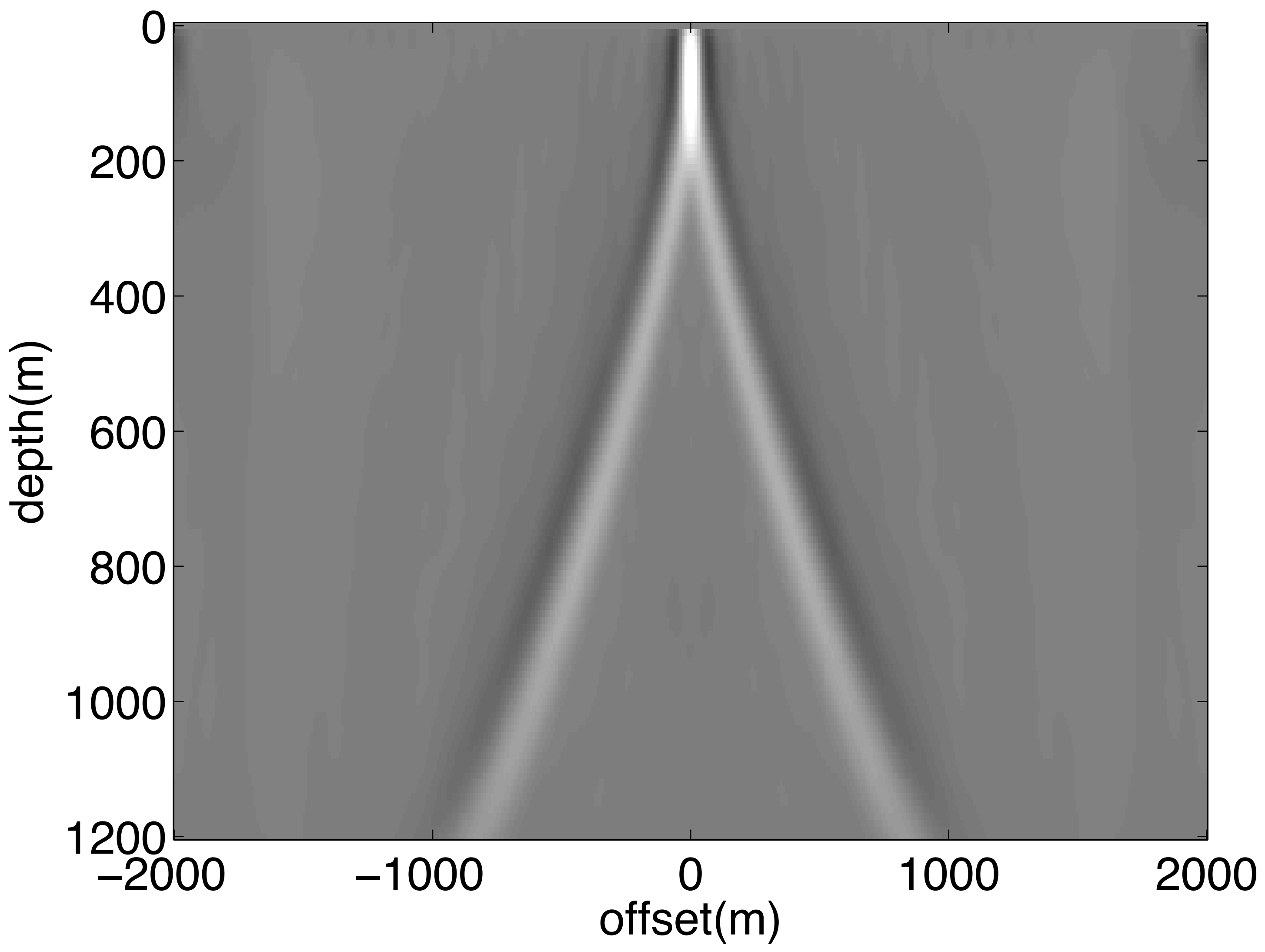


WEMVA

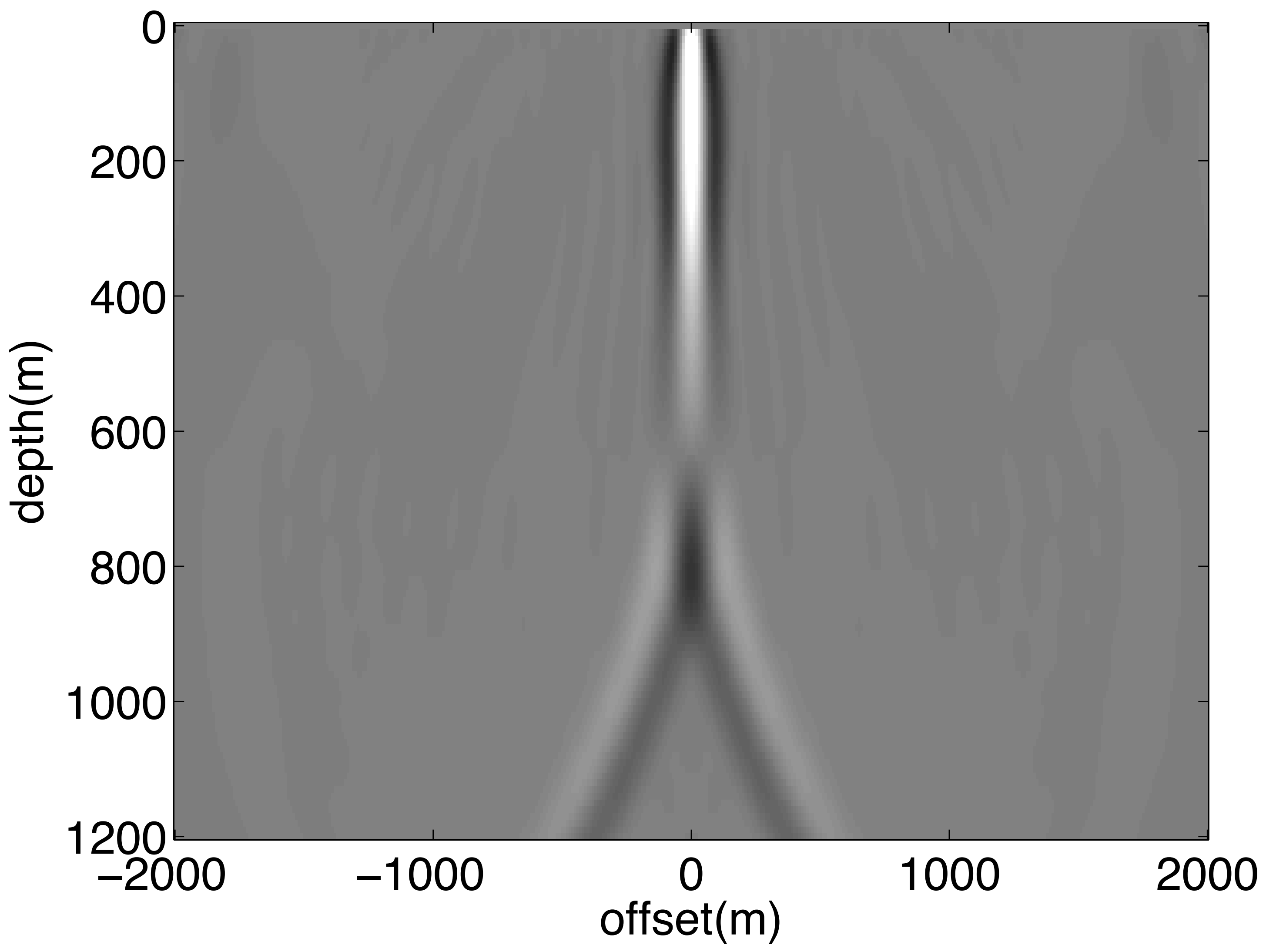
Vertical gradient
[image gathers]



true model



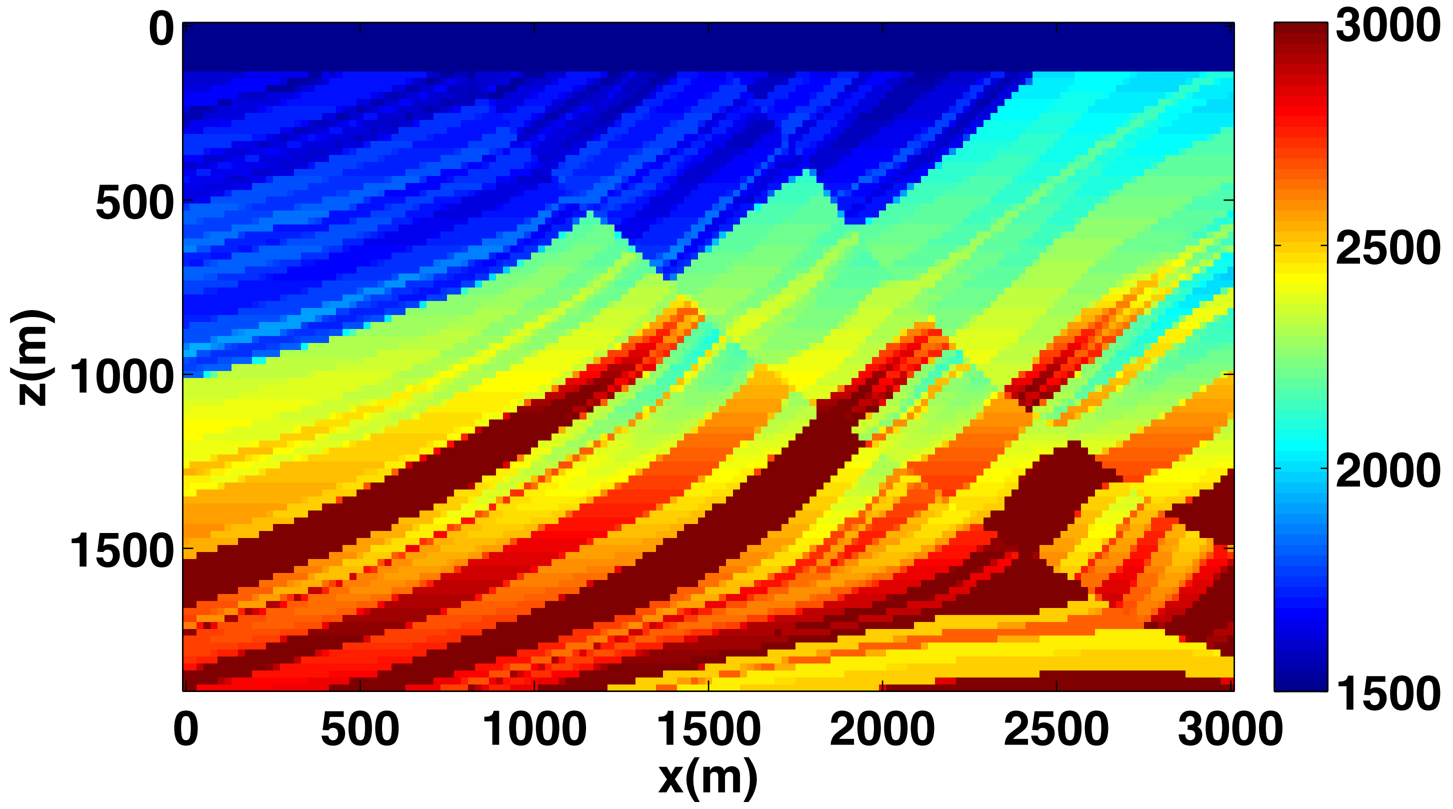
initial model



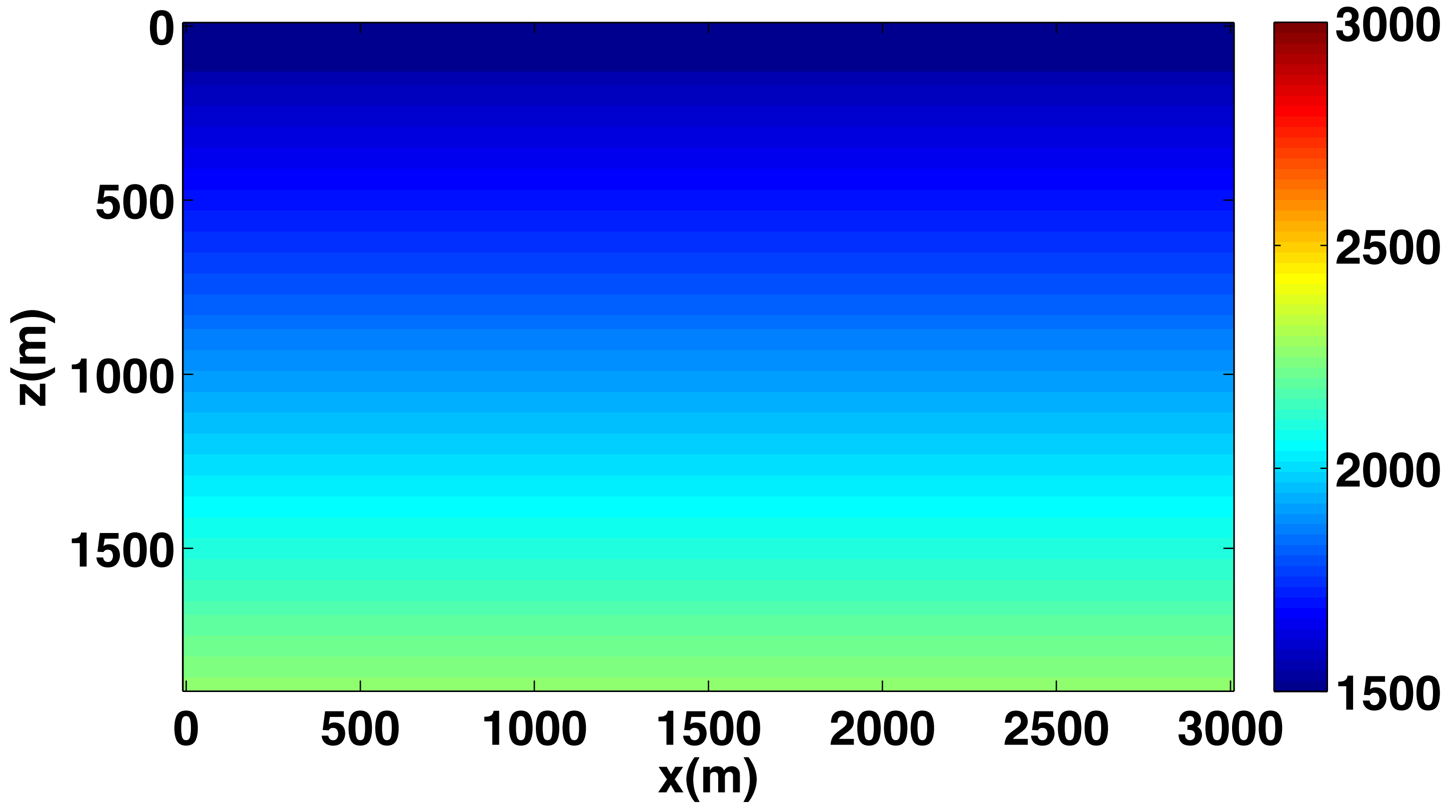
WEMVA

Marmousi Model
[preliminary results]

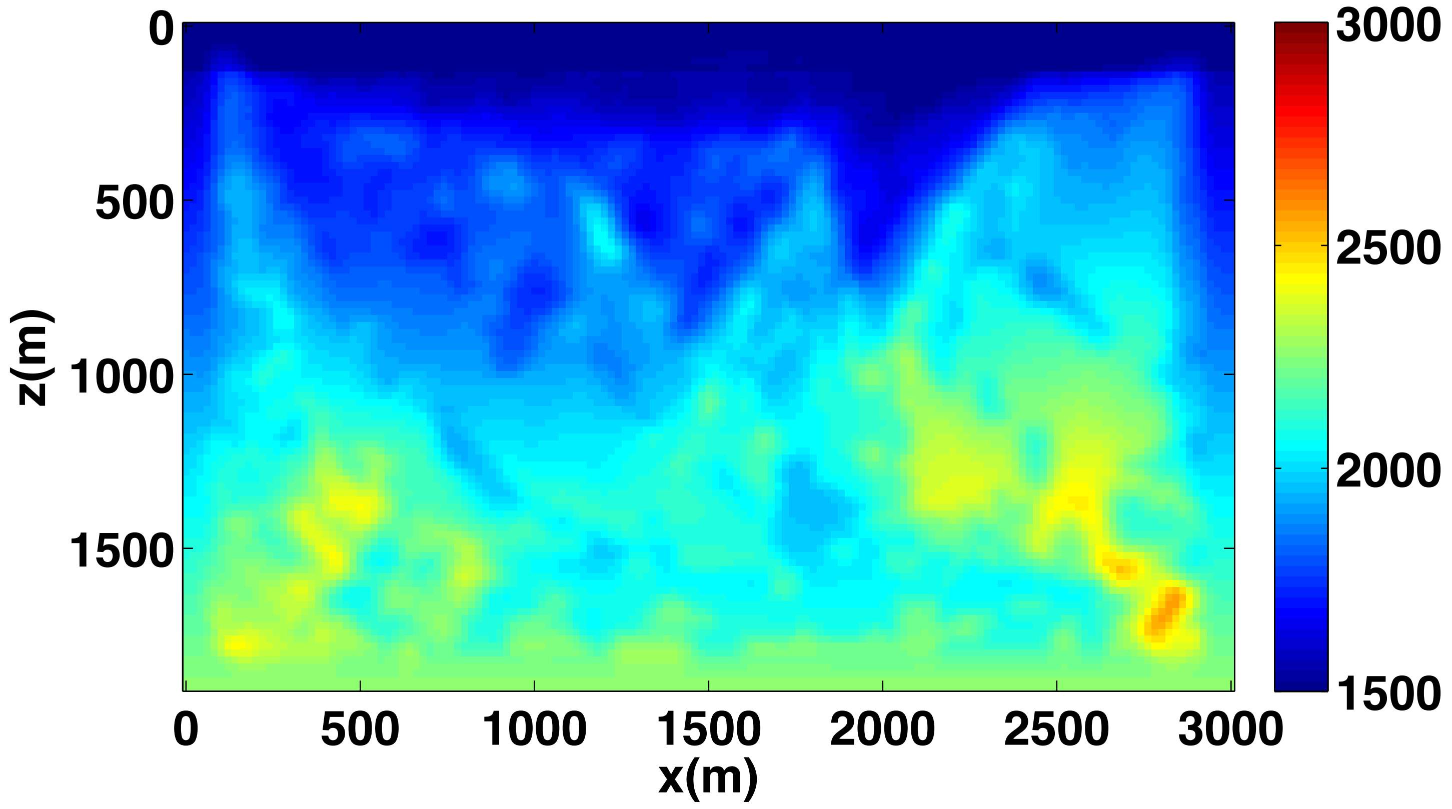
True Model



Initial Model



WEMVA



Conclusions

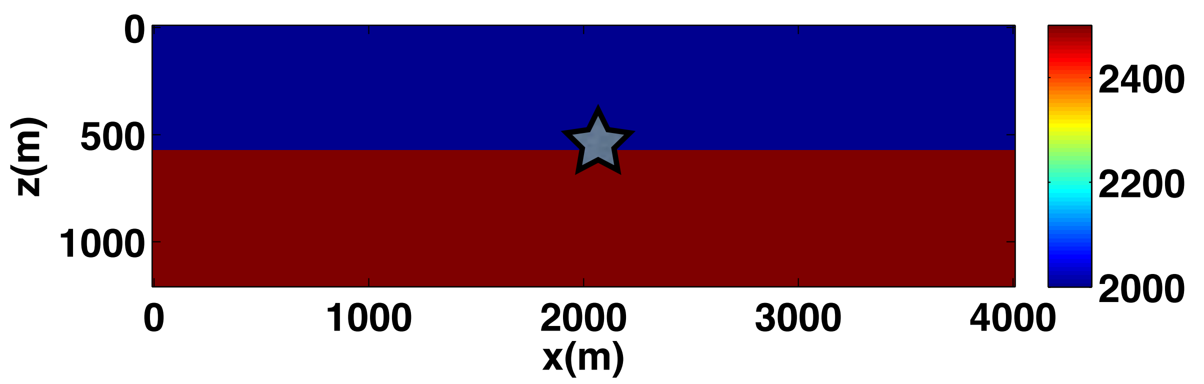
- *probings* allows us to get *offset* information for *all direction*
- *randomized* trace estimation allows us to *compute* WEMVA objective cheaply

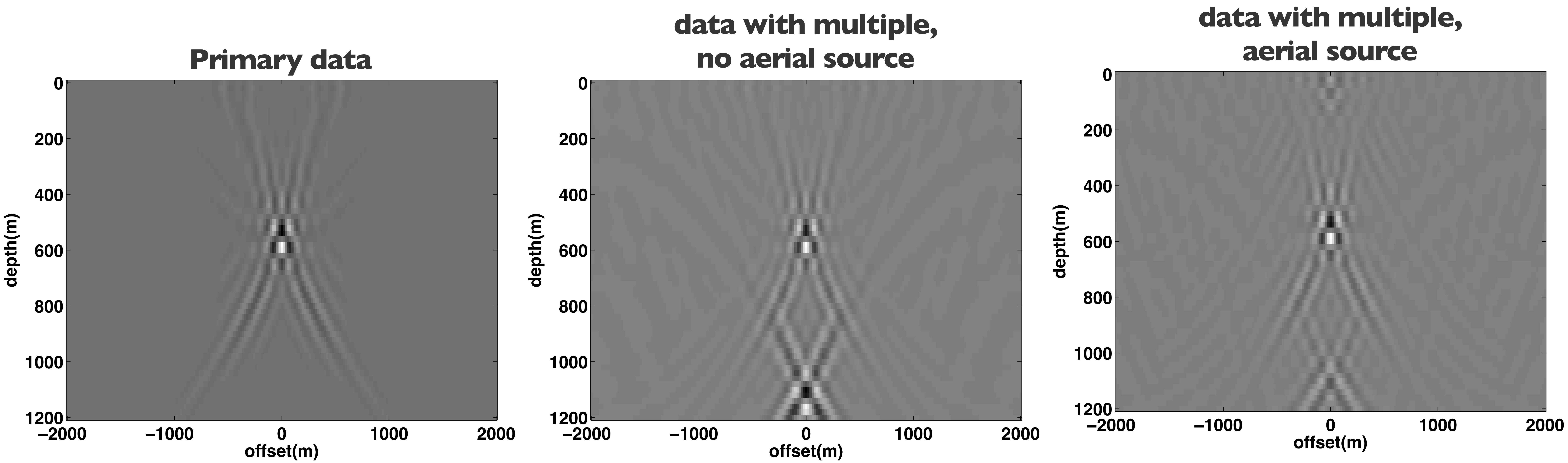
Future Work

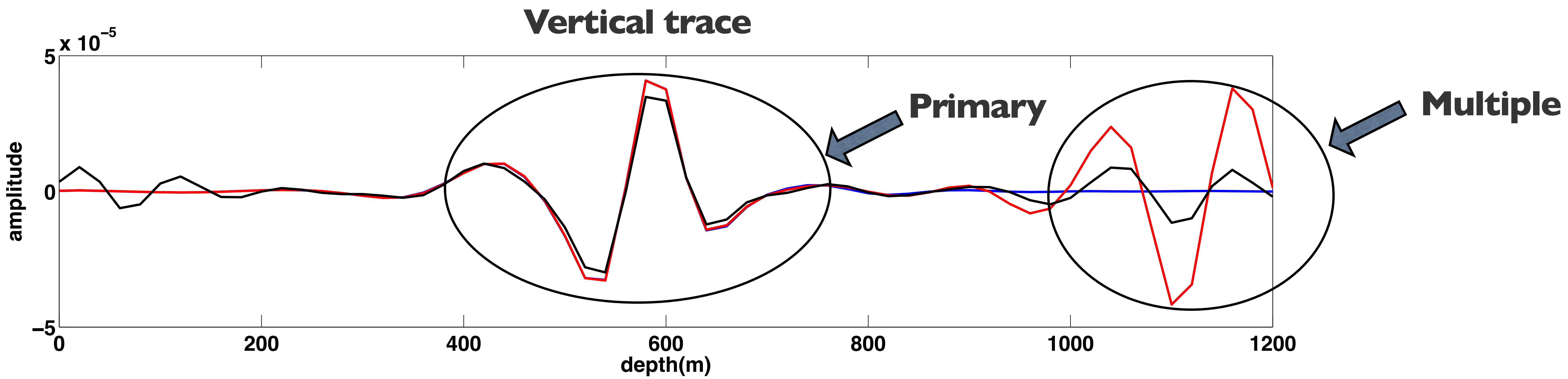
- control *sloppiness* in WEMVA *(frugal method)*
- incorporate the *free-surface multiple* in WEMVA

least-squares *extended* images

with free-surface multiples







SLIM

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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