The bridge from orthogonal to redundant transforms in weighted ℓ_1 minimization

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Seismic Laboratory for Imaging and Modelling
Consortium Meeting

December 3rd, 2013

About myself and time with SLIM

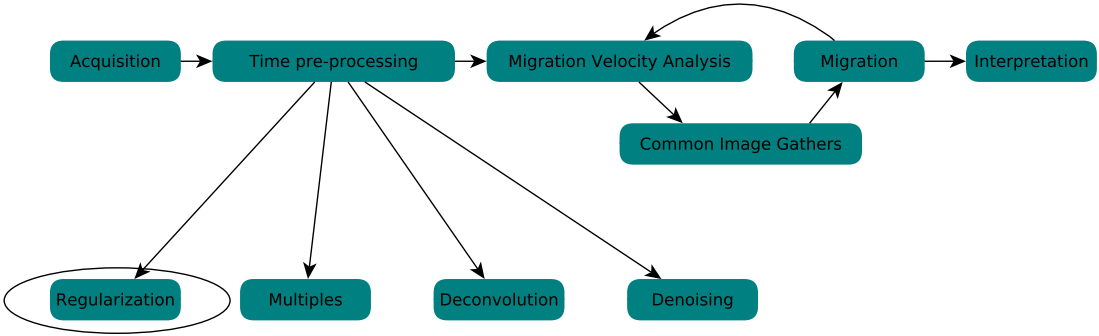
- 2006-2011: Bachelor of Science at University of Calgary, Math and Statistics
- 2011-2013: Master of Science with SLIM, University of British Columbia, Mathematics
- Summer 2013: Internship with Total

Rest of my talk

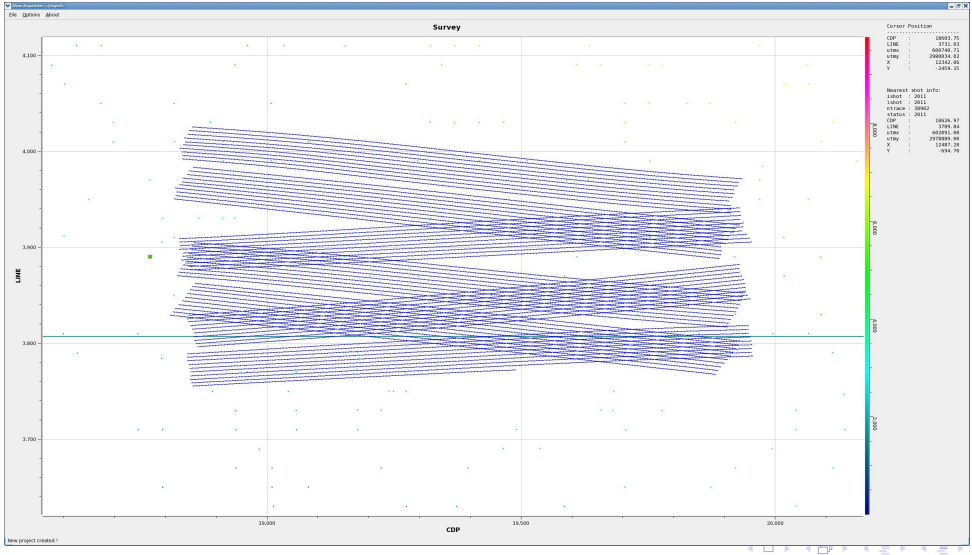
Major messages:

1. The D-RIP framework (by Needell et al and Shidong et al) and it's role in seismic interpolation via ℓ_1-synthesis
2. Analysis and synthesis and choosing the best possible analysis operator
3. Improvement of this framework in weighted ℓ_1-synthesis
4. This framework provides design criteria for sampling geometries and potentially provide optimal jitter sampling schemes.

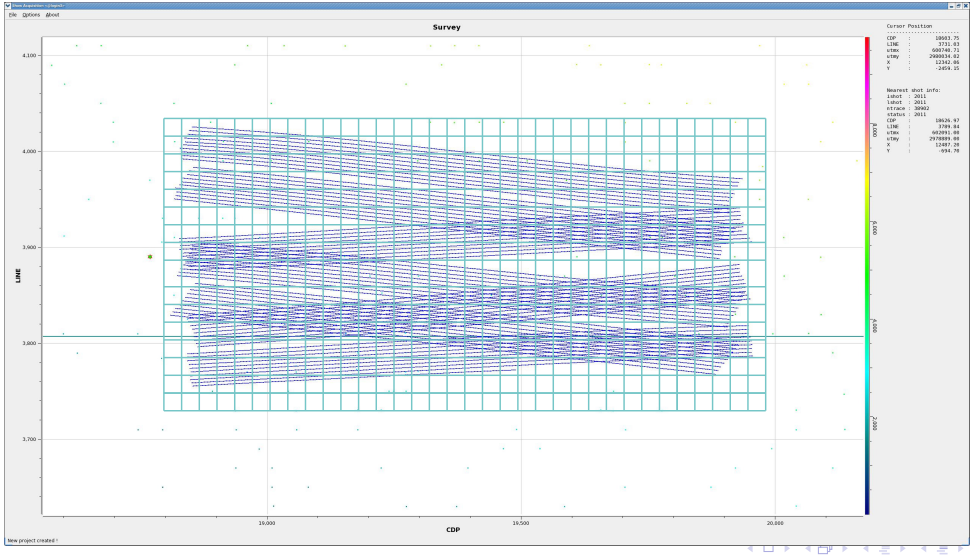
Basic workflow



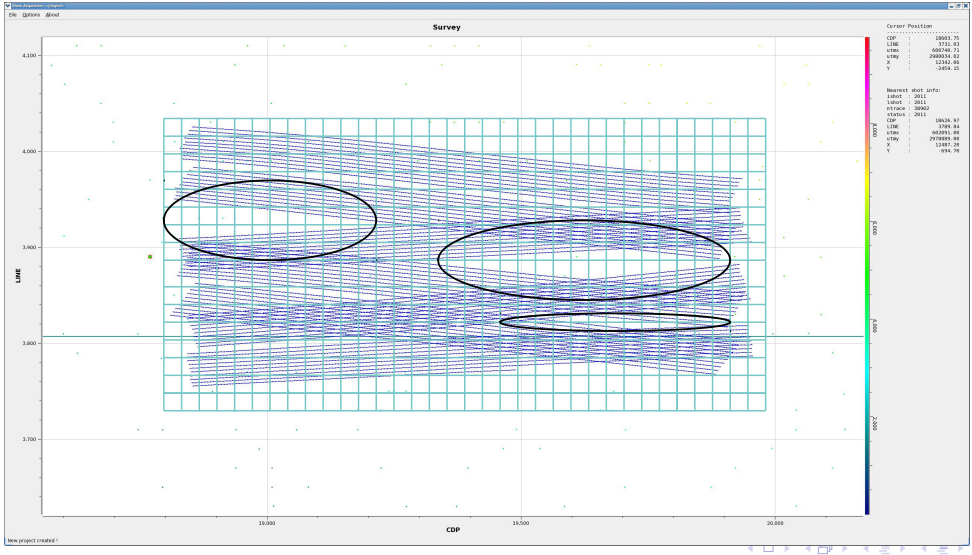
Seismic Acquisition



Data Regularization



Data Regularization



Seismic Wavefield Reconstruction Missing Traces

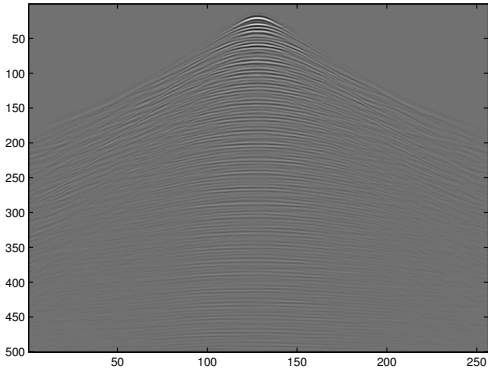


Figure: Shot gather

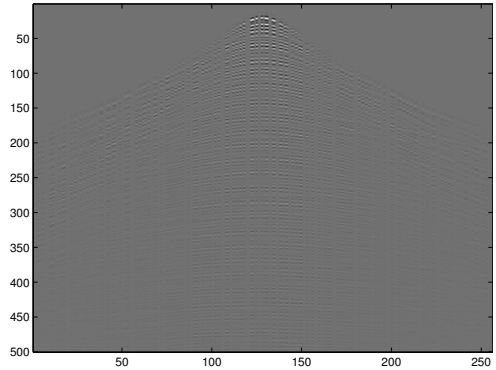


Figure: Shot gather with missing traces

Analysis and Synthesis Formulations

Synthesis: Find a sparse representation of a wavefield whose reconstruction fits the data.

$$\hat{f}_{synthesis} = D \cdot \arg\min_{\tilde{x}} ||\tilde{x}||_{1,w} \text{ subject to } ||AD\tilde{x} - y||_2 \leq \epsilon$$

Analysis: Find a wavefield whose forward transform coefficients are sparse and fit the data.

$$\hat{f}_{analysis} = \arg\min_{\tilde{f}} ||D^{\dagger}\tilde{f}||_{1,w} \text{ subject to } ||A\tilde{f} - y||_2 \leq \epsilon$$

where $D^{\dagger}$ is the pseudoinverse of D.

Seismic Wavefield Reconstruction Example

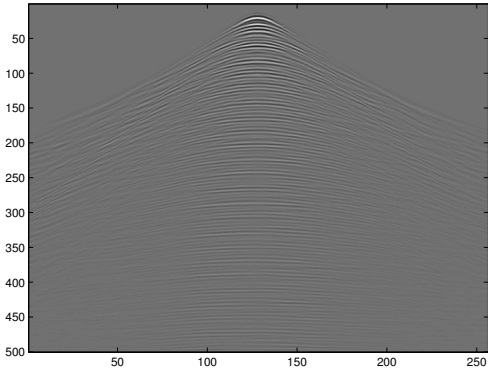


Figure: Shot gather

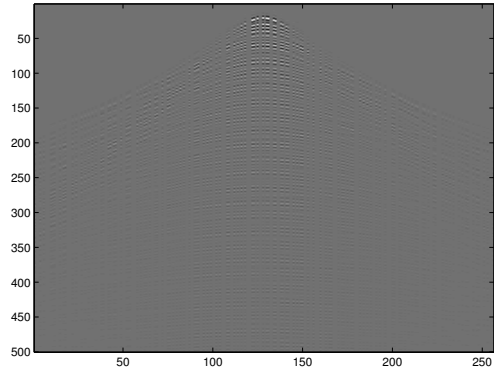


Figure: Shot gather with 'missing' traces

Seismic Wavefield Reconstruction Example

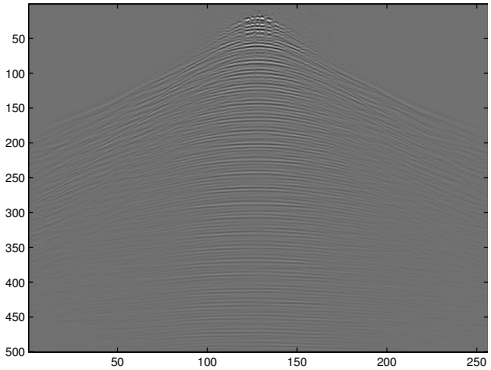


Figure: Synthesis Recovery

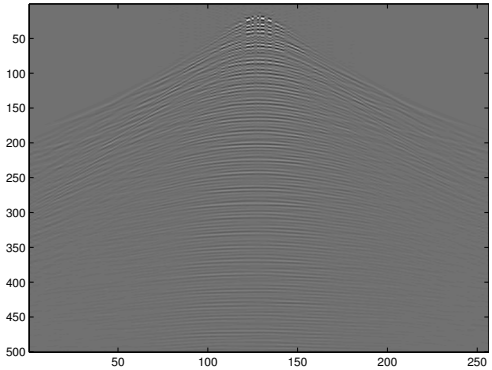
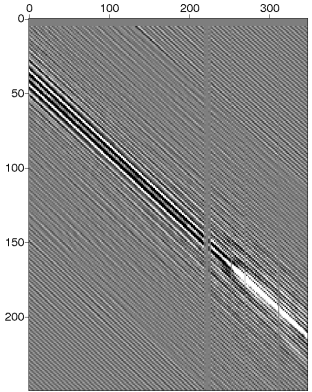
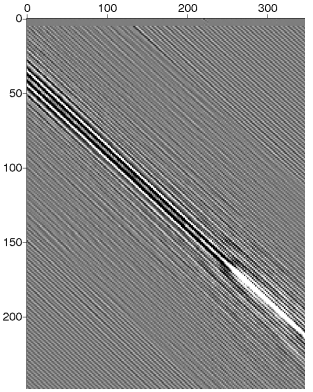


Figure: Analysis Recovery

Seismic Wavefield Reconstruction Example with only missing sources - Frequency 11 hz





General and Optimal dual analysis

Yulong, Tiebin, Shidong Li, 2012 [2][1], Hargreaves and Yilmaz, 2013

But we have an infinite amount of choices of the analysis operator. So choose $\tilde{D}$ such that $\tilde{D}^* f$ is sparse. The natural problem to solve is then:

$$\hat{f}_{general} = \arg\min_{\tilde{f}} ||\tilde{D}^* \tilde{f}||_{1,w} \text{ s.t. } ||A\tilde{f} - y||_2 \leq \epsilon$$

Find an 'optimal' analysis operator:

$$\hat{f}_{optimal} = \arg\min_{\tilde{f}, D\tilde{D}^* = I} ||\tilde{D}^* \tilde{f}||_{1,w} \text{ s.t. } ||A\tilde{f} - y||_2 \leq \epsilon$$

Equivalence of optimal dual ℓ_1-analysis and synthesis

Theorem (Shidong et al, 2012)

$$\hat{f}_{synthesis} = D \cdot \arg\min_{\tilde{x}} ||\tilde{x}||_1 \ \ subject\ to\ \ ||AD\tilde{x} - y||_2 \leq \epsilon$$

$$\hat{f}_{optimal} = \arg\min_{\tilde{f}, D\tilde{D}^* = I} ||\tilde{D}^*\tilde{f}||_1 \ \ s.t.\ ||A\tilde{f} - y||_2 \leq \epsilon$$

$$\hat{f}_{synthesis} = \hat{f}_{optimal}$$

Further still, this theorem can be generalized to the weighted case.

Recovery conditions

Suppose $y = ADx$

- **Synthesis** (RIP): A condition imposed on the composition AD for error bounds on the recovery of x
- **Analysis** (D-RIP): A condition for recovery imposed on A for error bounds on the recovery of $f = Dx$

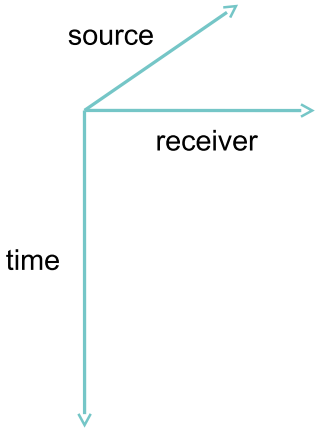
Note: Focus shifts on acquisition geometry

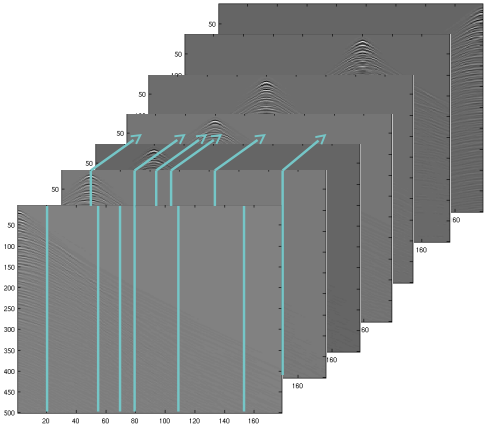
How equivalence helps us

- Traditional CS: AD does not satisfy RIP due to coherency in D, so these conditions don't explain why synthesis does well.
- If A satisfies D-RIP then optimal dual ℓ_1-analysis does well $\rightarrow$ synthesis does well

Mathematical caveat: The D-RIP bounds depend on us knowing the upper frame bound of optimal dual apriori, but this is not possible. However we find this bound to be independent of the signal in practice.

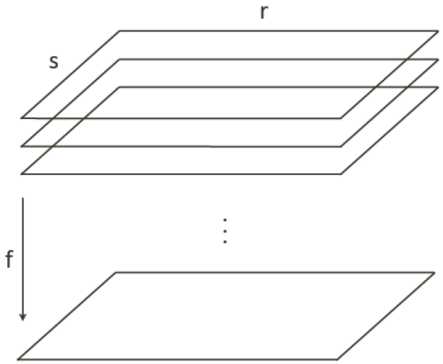
Seismic Wavefield Reconstruction





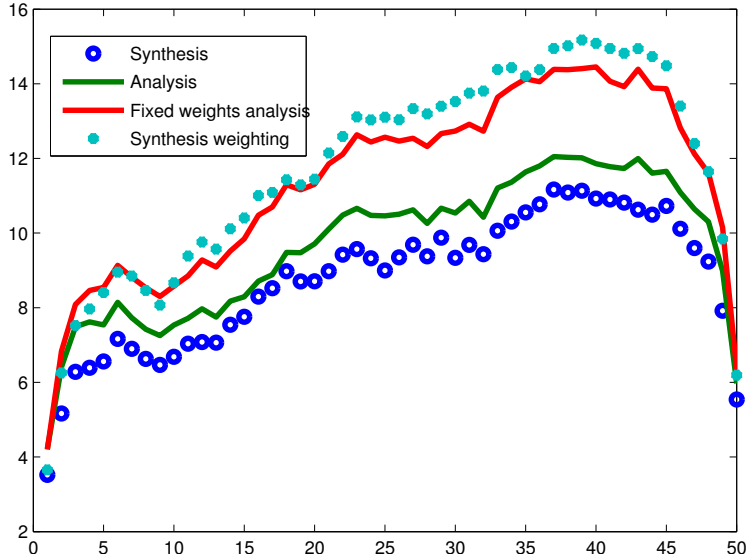
Build support information iteratively

Mansour, Herrmann, Yilmaz 2013[3]



1. Recovery first frequency slice and estimate support
2. Recovery second frequency slice using previous support estimate
3. Update support estimate and continue . . .

Gulf of Suez Experiment



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Acknowledgements

- Özgür Yilmaz, Felix Herrmann, SLIM and their wonderful administrative staff
- My mentors at Total: Fuchun Gao, Biaolong Hua
- Total for their marine datasets

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, and WesternGeco.