Using prior support information in approximate message passing algorithm

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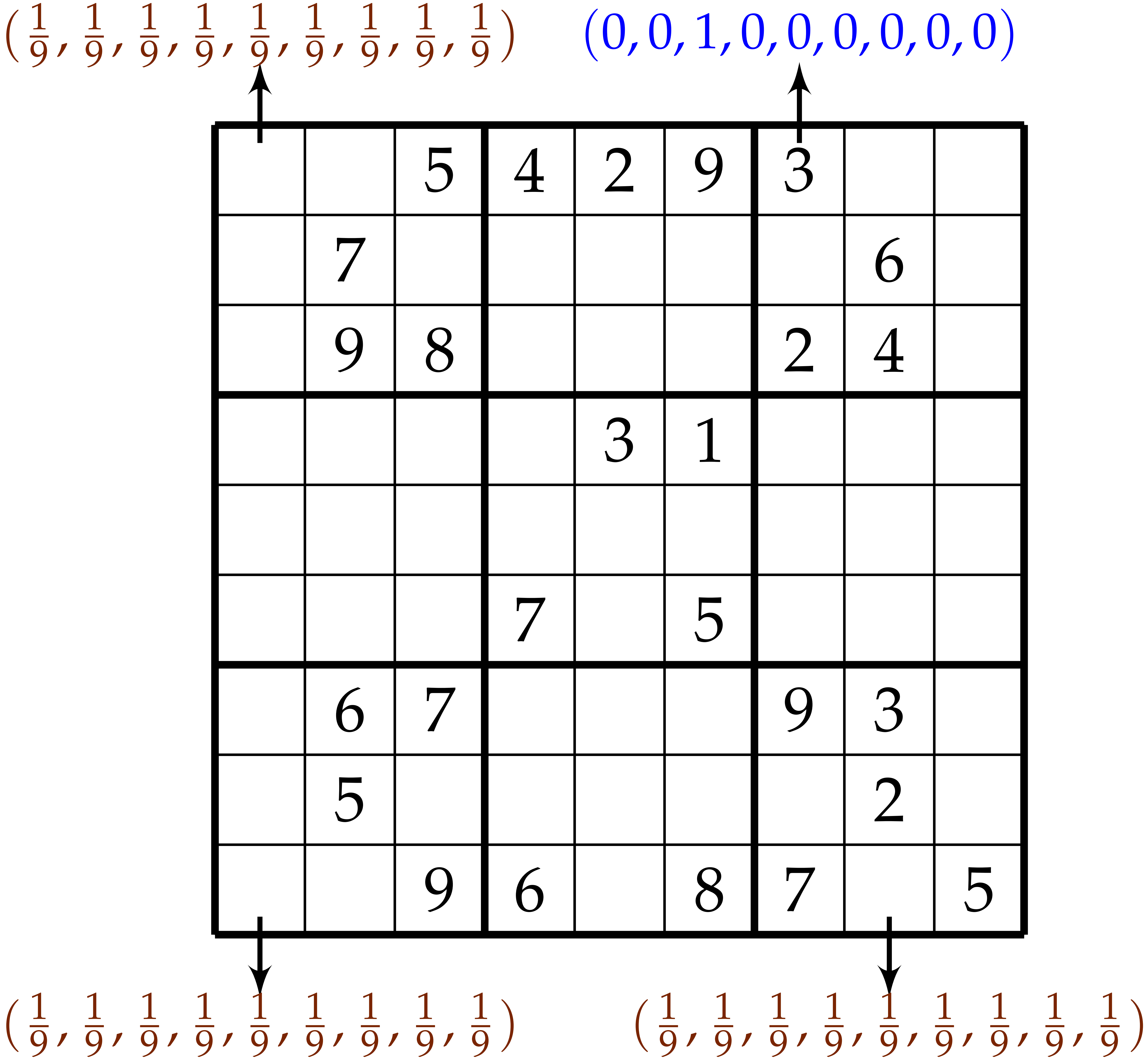
Main messages

- We incorporate prior support information into the approximate message passing (AMP) algorithm which is a fast iterative algorithm for sparse recovery.
- Using weighted AMP as a pre-processor we can improve the results of seismic trace interpolation via ℓ_1 recovery in terms of both accuracy and convergence time.

A simple example



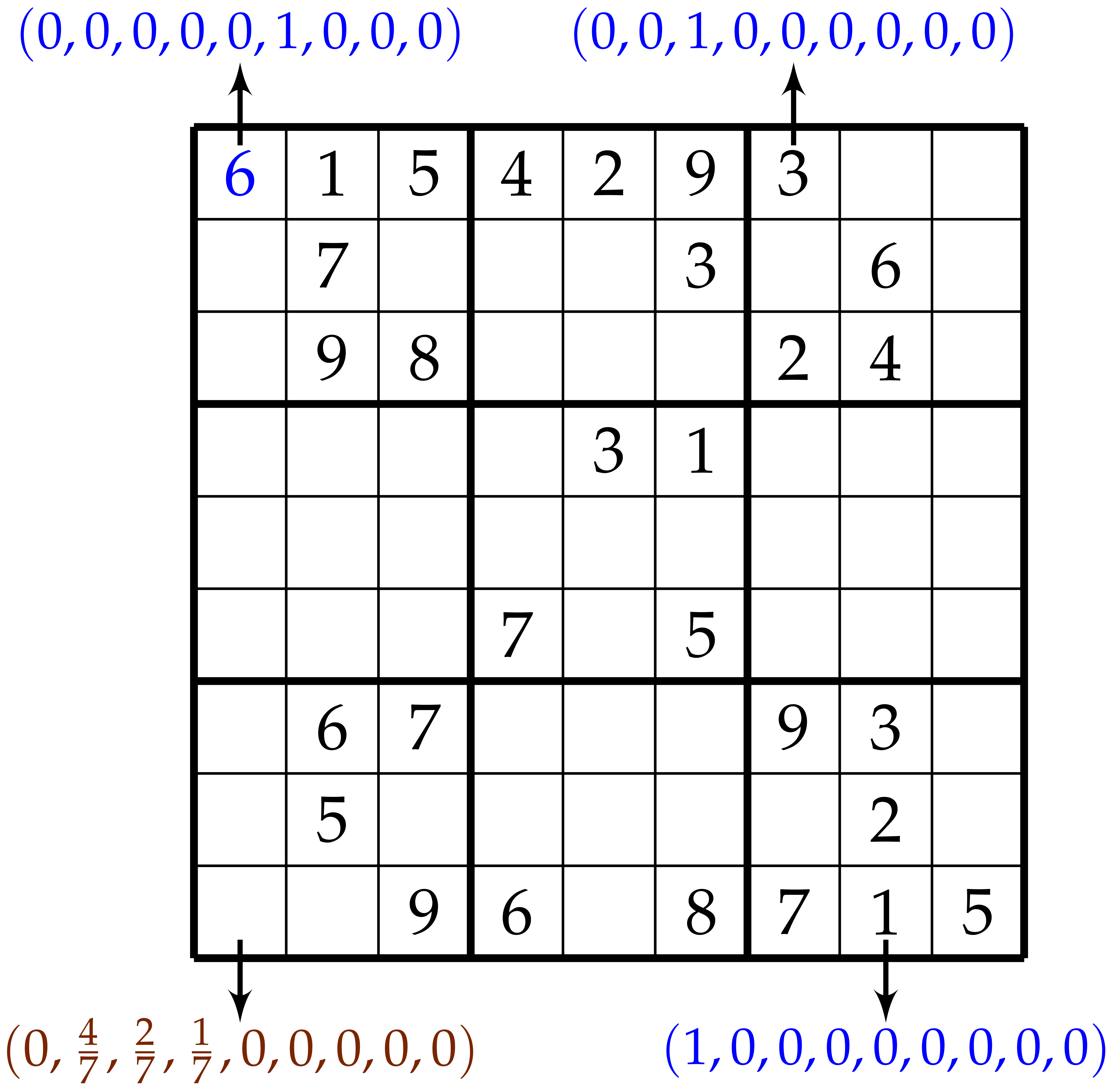
First iteration

$(\frac{1}{2}, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0)$ $(0, 0, 1, 0, 0, 0, 0, 0, 0)$

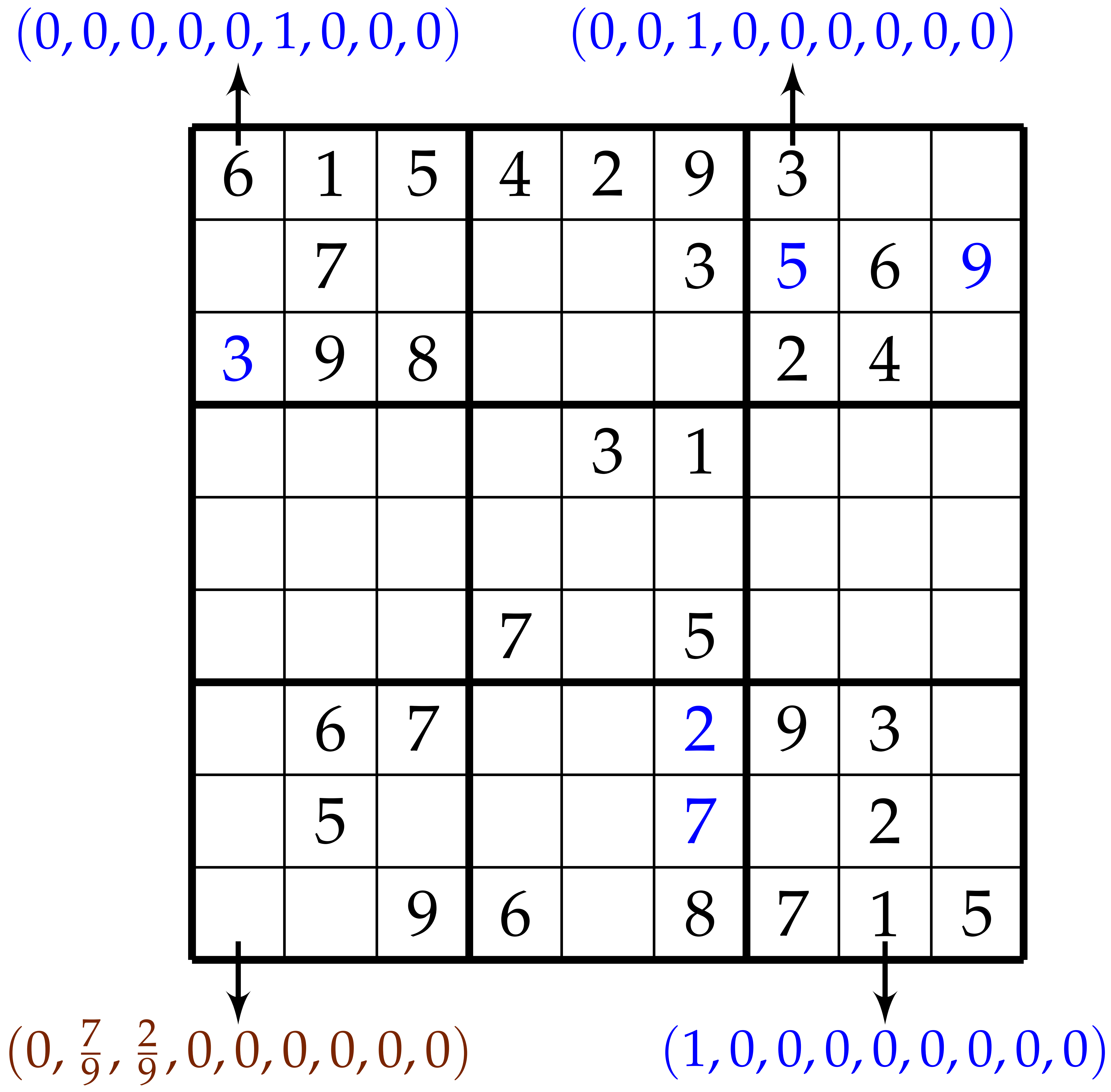
	1	5	4	2	9	3		
	7				3		6	
	9	8				2	4	
				3	1			
			7		5			
	6	7				9	3	
	5						2	
		9	6		8	7	1	5

$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, 0, 0, 0)$ $(1, 0, 0, 0, 0, 0, 0, 0, 0)$

Second iteration

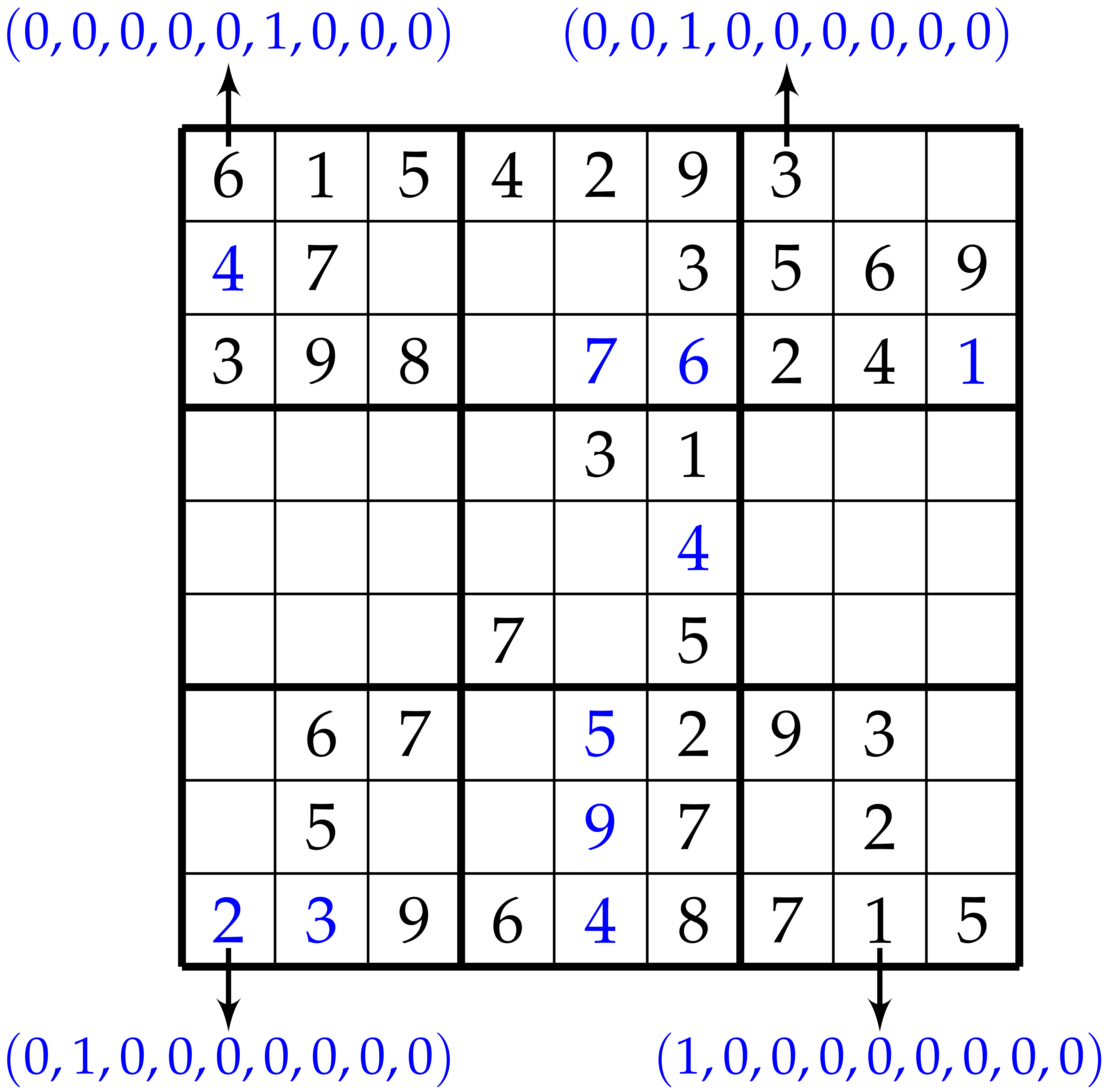


Third iteration



6	1	5	4	2	9	3		
	7				3	5	6	9
3	9	8				2	4	
				3	1			
			7		5			
	6	7			2	9	3	
	5				7		2	
		9	6		8	7	1	5

Fourth iteration



8th iteration

$(0,0,0,0,0,1,0,0,0)$ $(0,0,1,0,0,0,0,0,0)$

6	1	5	4	2	9	3	7	
4	7	2	8	1	3	5	6	9
3	9	8	5	7	6	2	4	1
		6		3	1	4		
					4			
9			7	6	5		8	
8	6	7	1	5	2	9	3	4
1	5	4	3	9	7		2	
2	3	9	6	4	8	7	1	5

$(0,1,0,0,0,0,0,0,0)$ $(1,0,0,0,0,0,0,0,0)$

12th iteration

$(0,0,0,0,0,1,0,0,0)$ $(0,0,1,0,0,0,0,0,0)$

6	1	5	4	2	9	3	7	8
4	7	2	8	1	3	5	6	9
3	9	8	5	7	6	2	4	1
5	8	6	2	3	1	4	9	7
7	2	1	9	8	4	6	5	3
9	4	3	7	6	5	1	8	2
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Example: Revisiting ℓ_1 for randomized acquisition of seismic lines

Consider a seismic line with 64 sources, 64 receivers, and 256 time samples.

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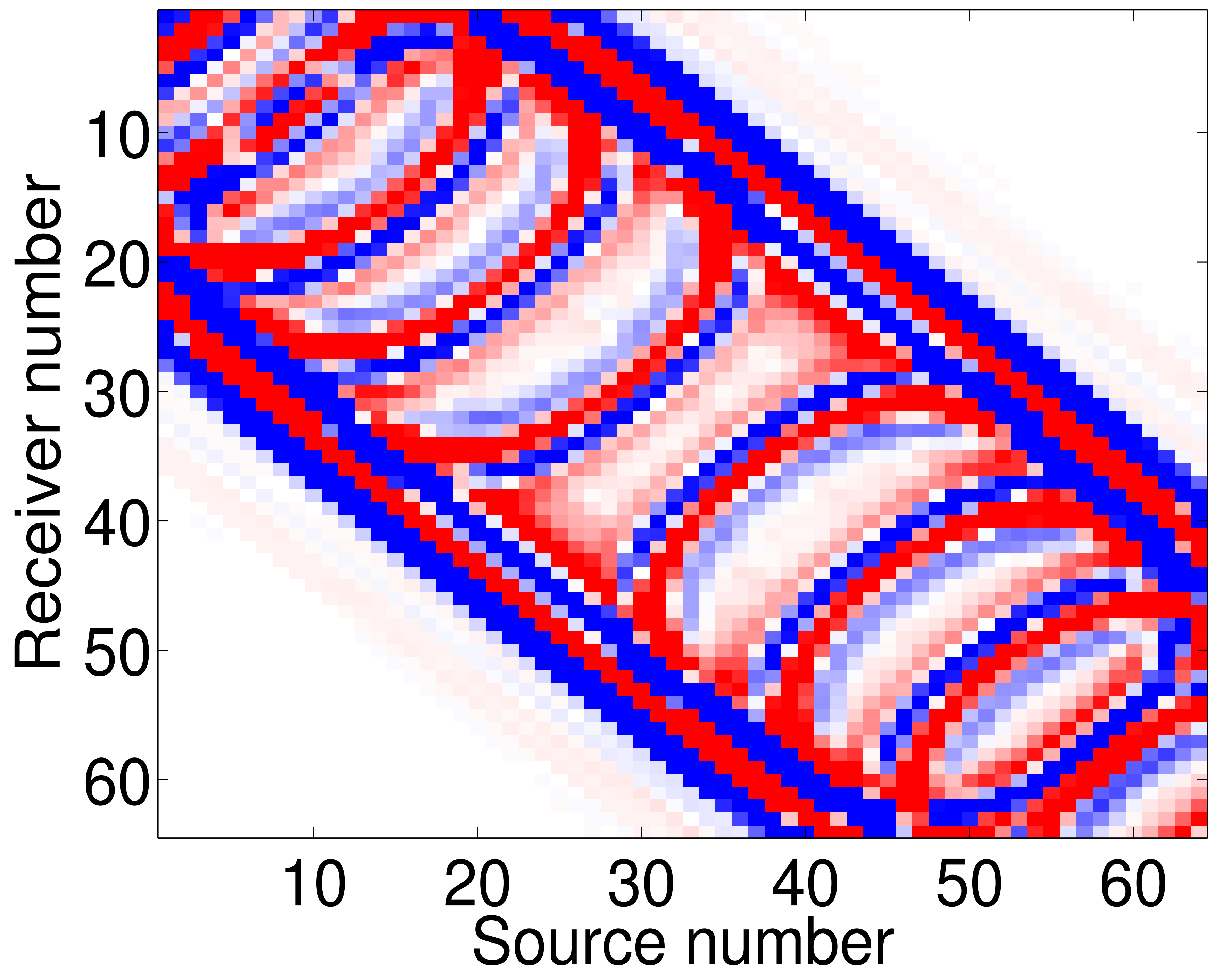
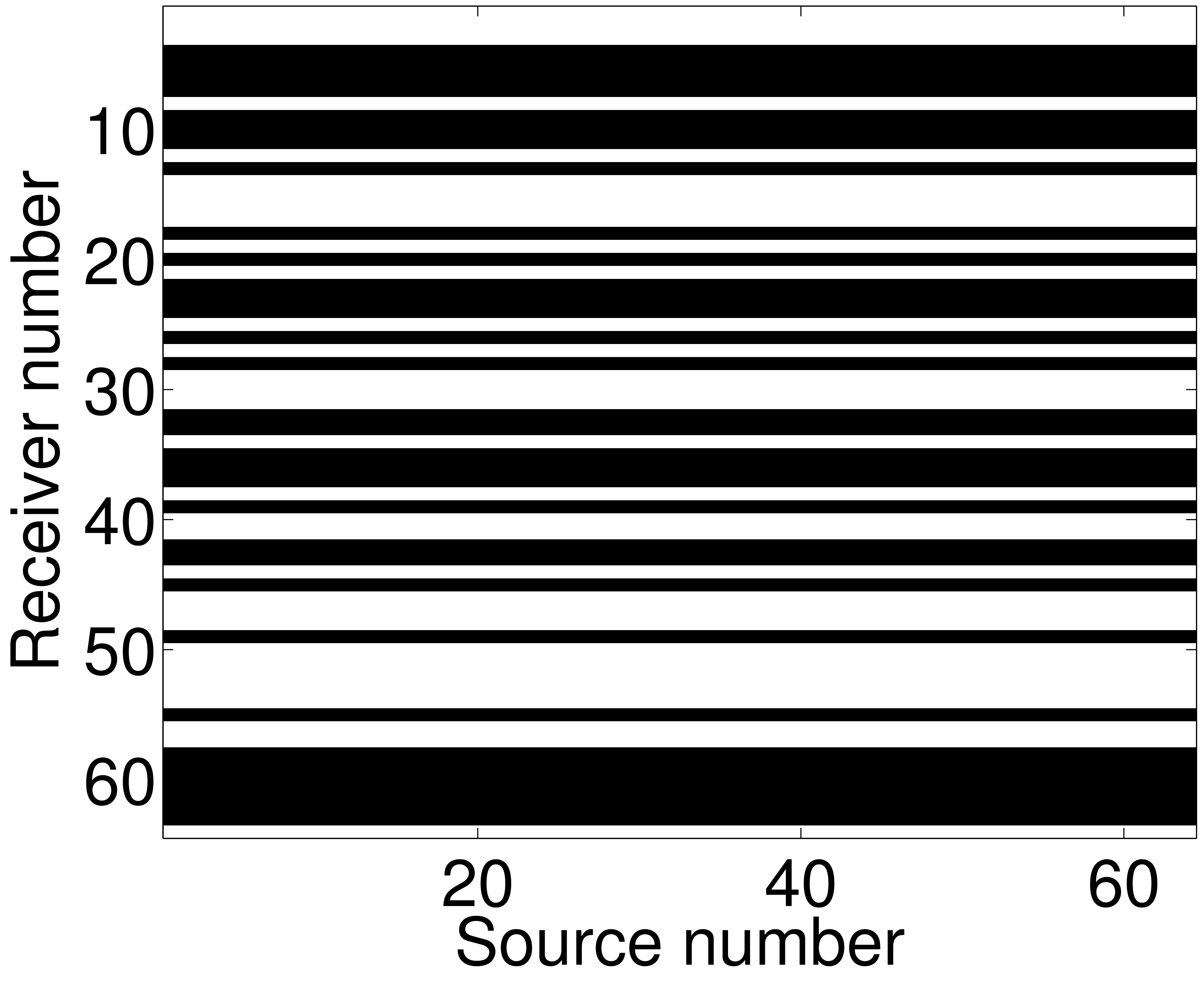


(a) Fully sampled time slice f



(b) Mask R

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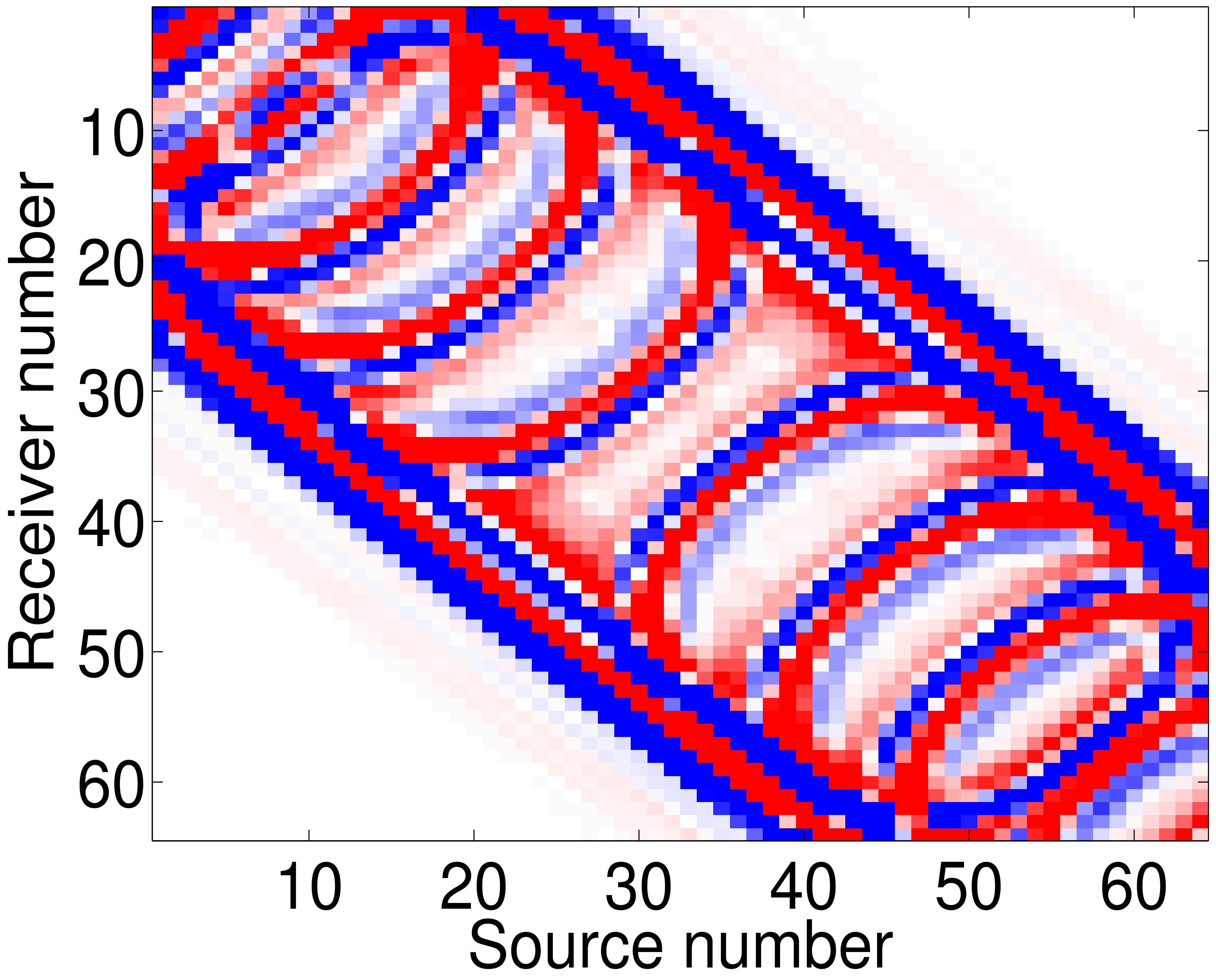

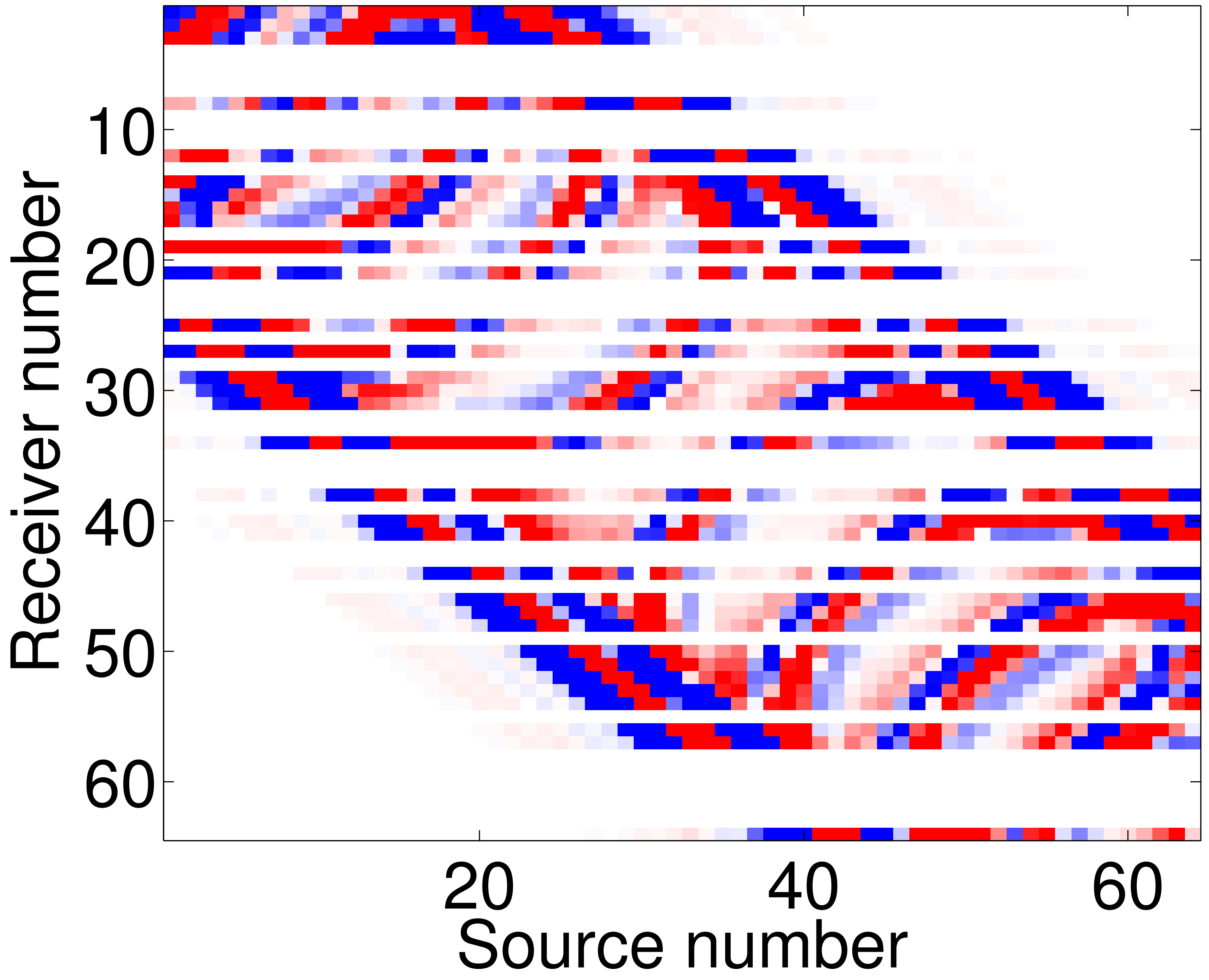

Using ℓ_1 for seismic trace interpolation

We want to recover a high dimensional seismic data volume f by interpolating between a smaller number of measurements $b = RMf$.

Let S be a sparsifying operator that characterizes the transform domain of f, such that $S \in \mathbb{C}^{P \times N}$ with $P > N$. In the case of the redundant curvelet transform $S^H S = \mathbb{I}$.

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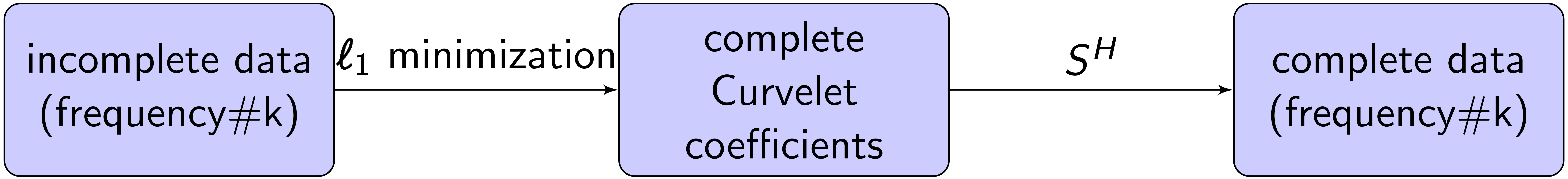
Using ℓ_1 for seismic trace interpolation

To recover f from the measurements $b = RMS^H x$, we solve the ℓ_1 minimization problem

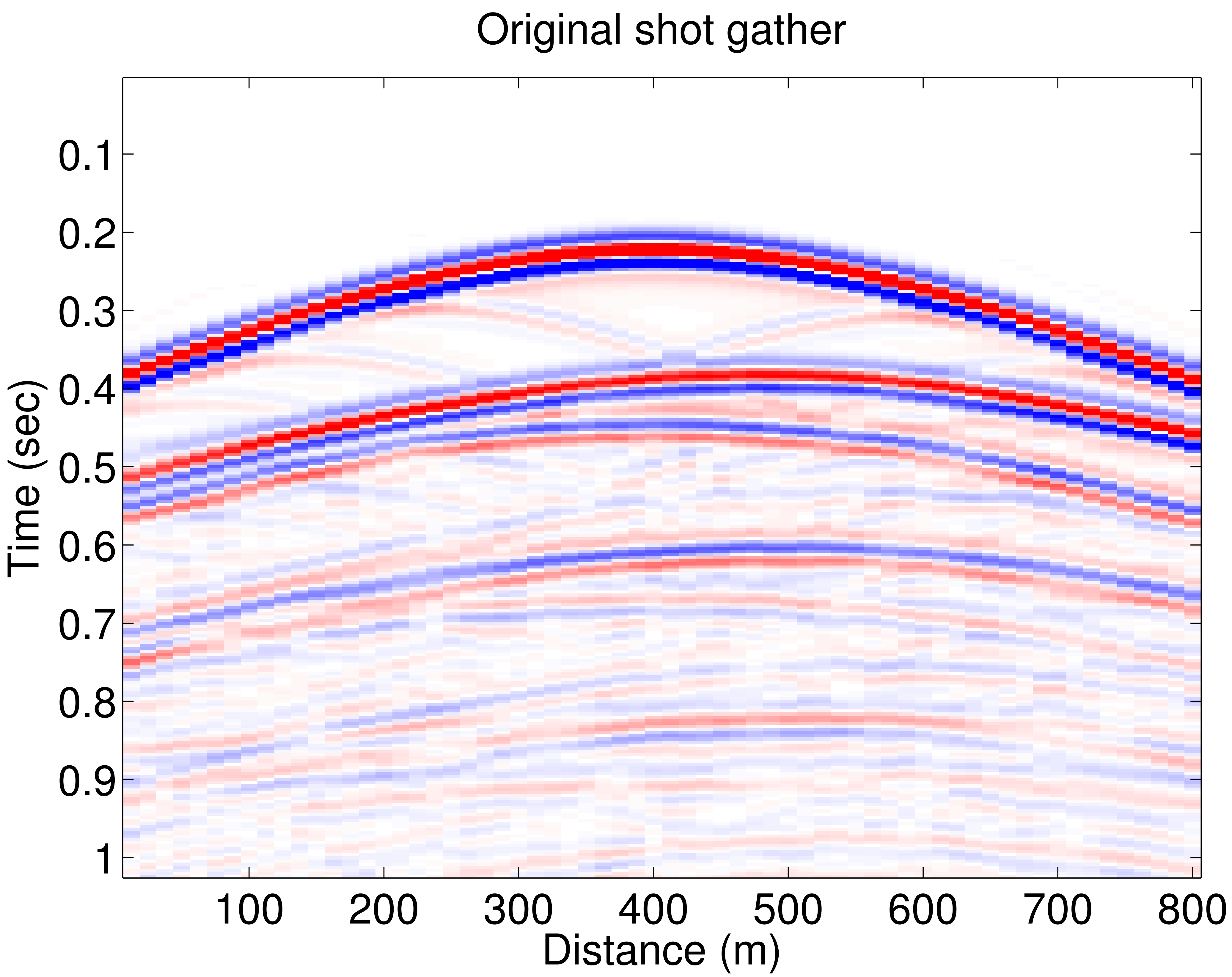
$$x^{\ell_1} := \underset{z \in R^P}{\text{minimize}} ||z||_1 \quad \text{subject to} \quad ||RMS^H z - b||_2 \leq \epsilon,$$

and approximate f by $S^H x^{\ell_1}$.

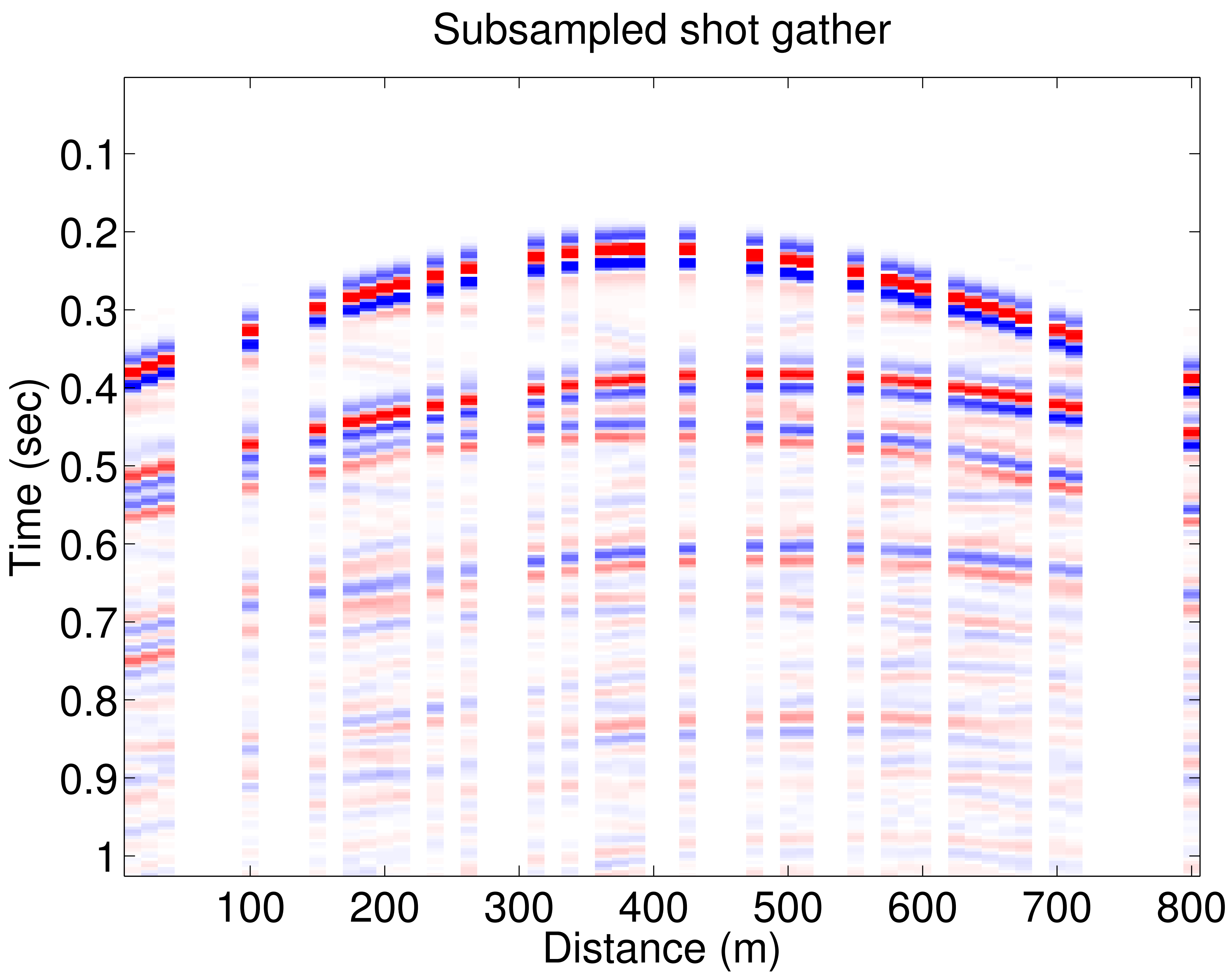


Recovery using ℓ_1 minimization on frequency slices (shotgather # 32)

Shotgather number 32 from the seismic line:

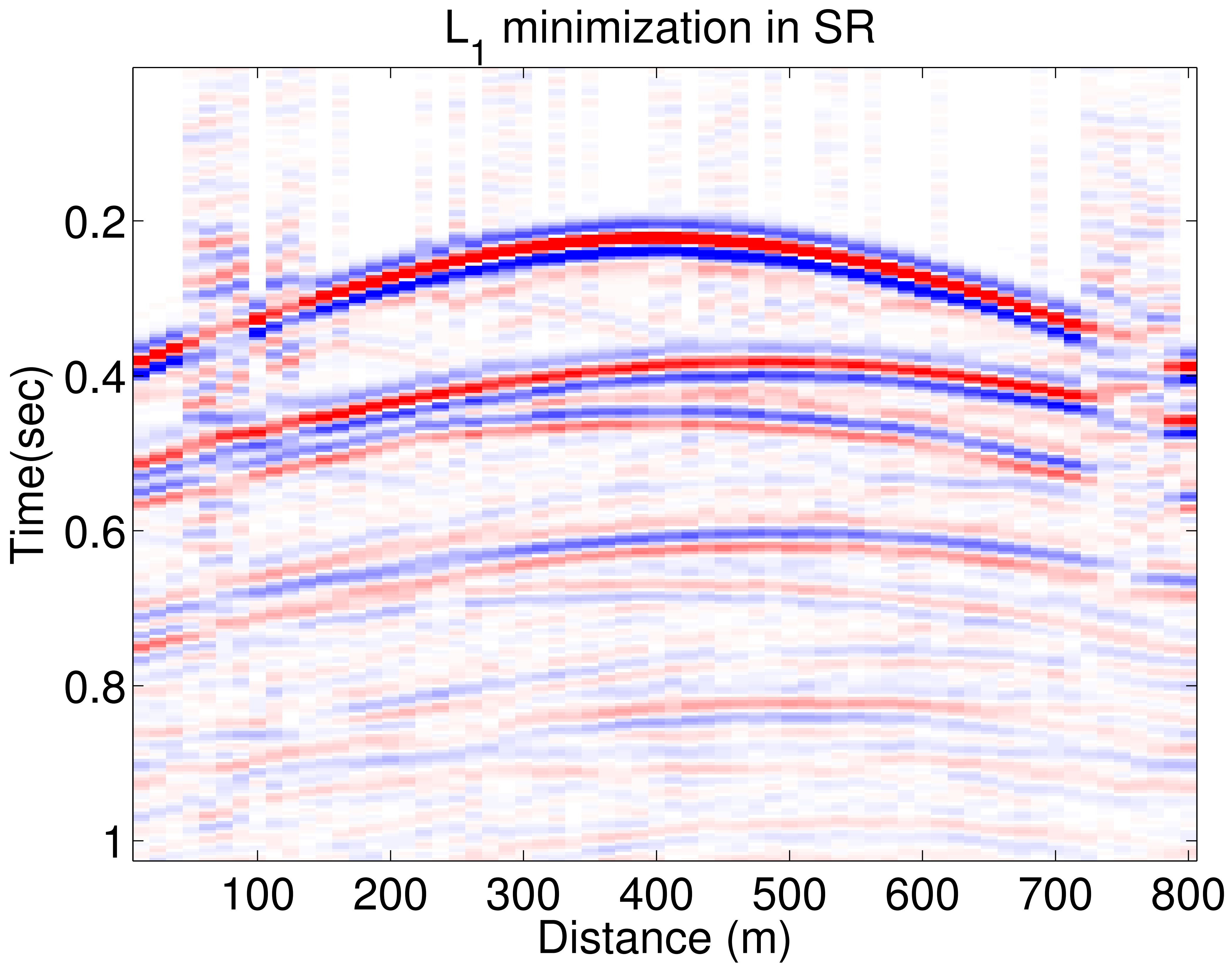


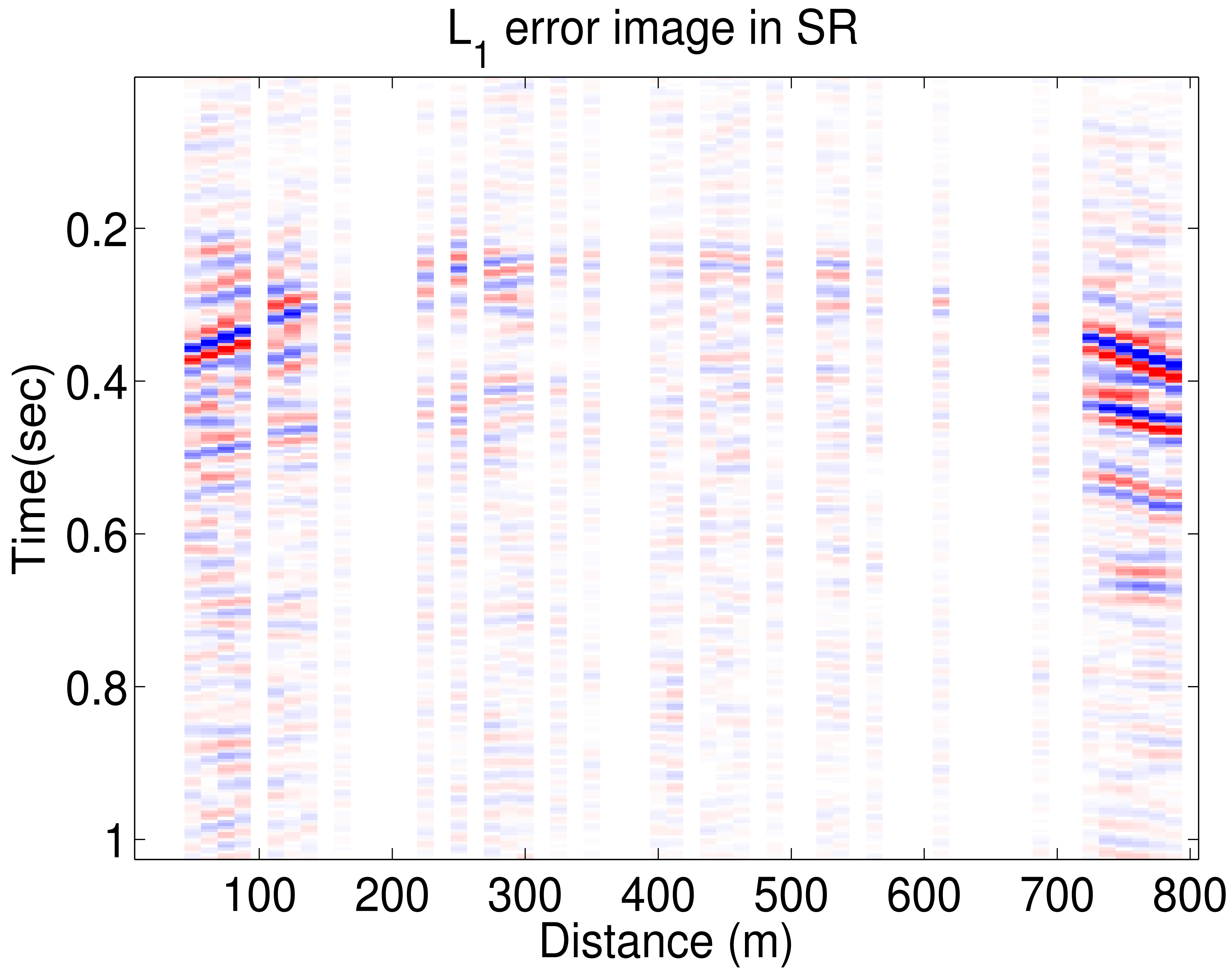
(e)



(f)

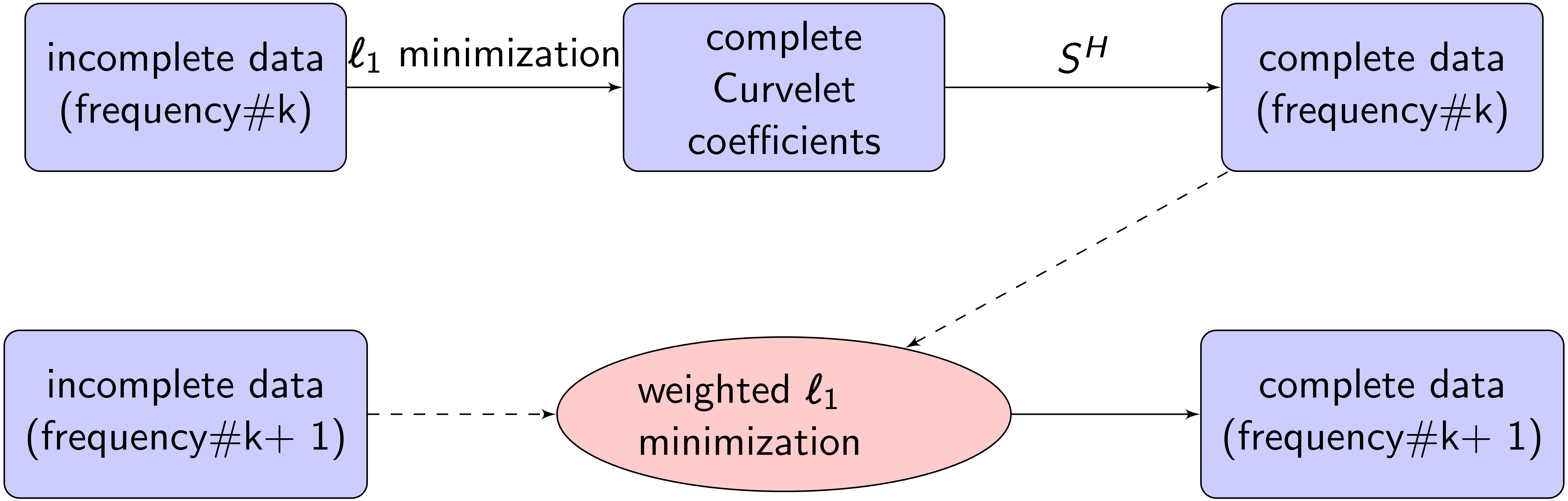
Recovery results: ℓ_1 minimization





Using weighted ℓ_1 for seismic trace interpolation

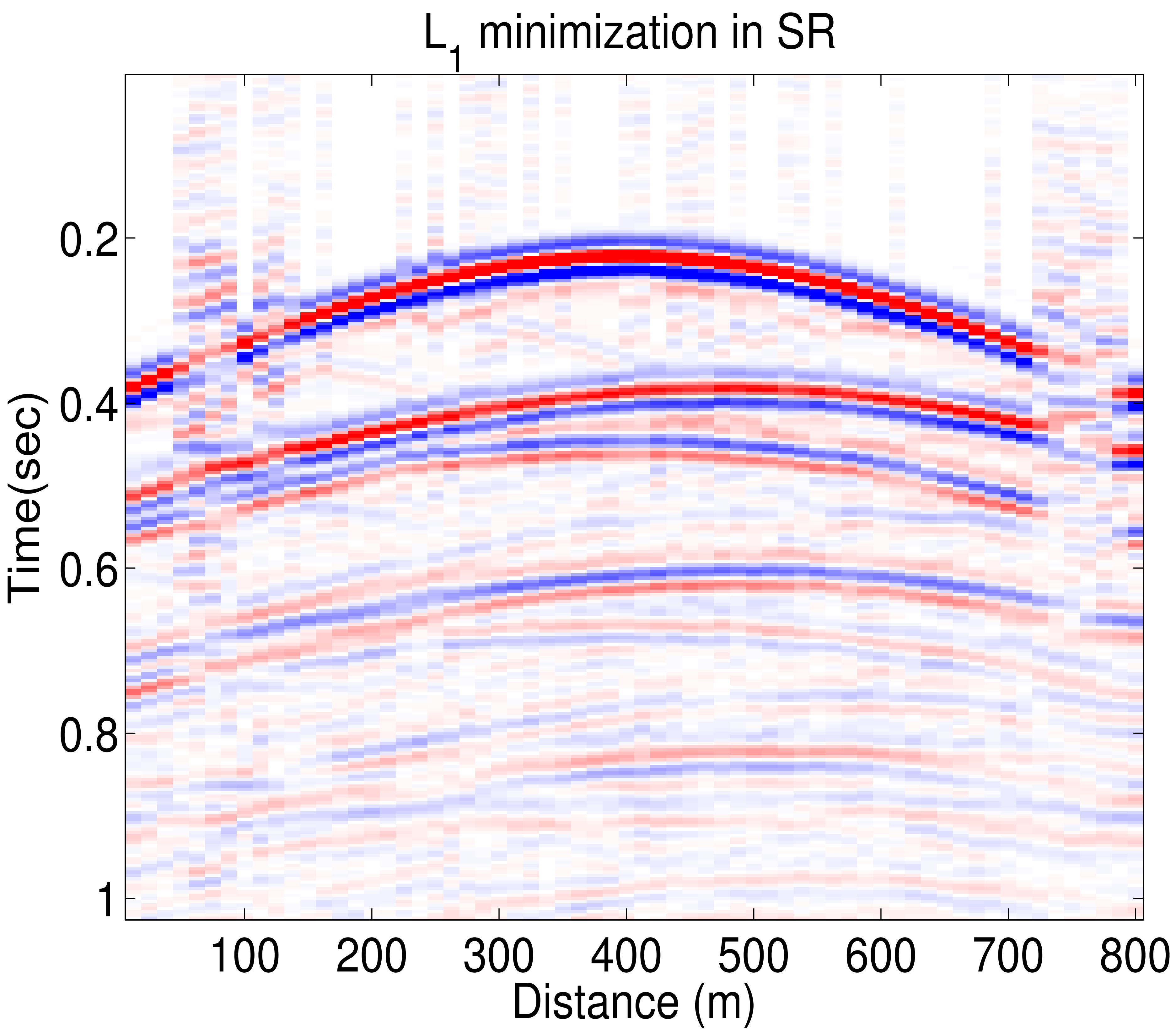
Adjacent frequency slices and have highly correlated curvelet domain support sets. Hence we can use the support set of each slice as an estimate for the support of the next slice.

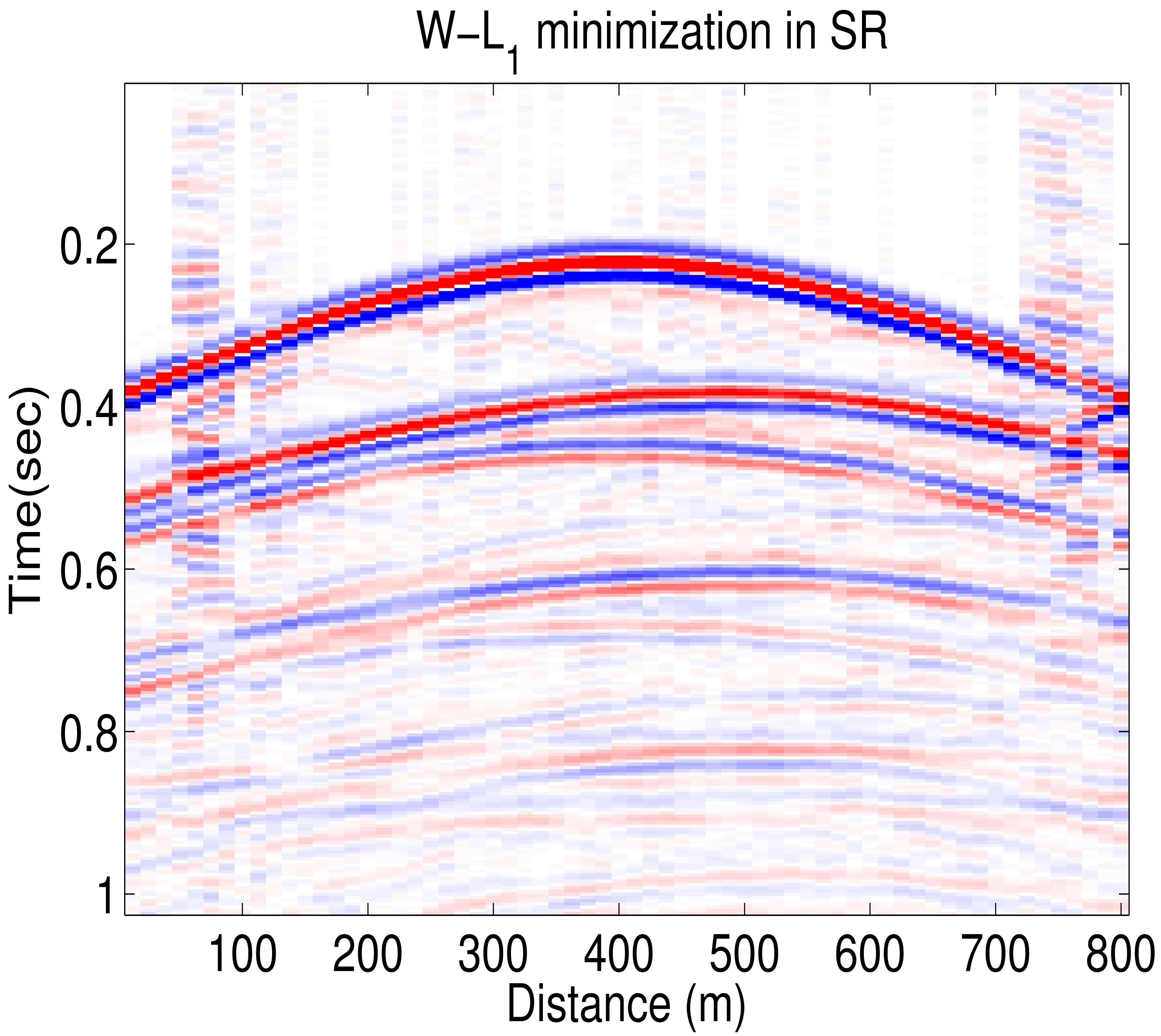


Weighted ℓ_1 for seismic trace interpolation

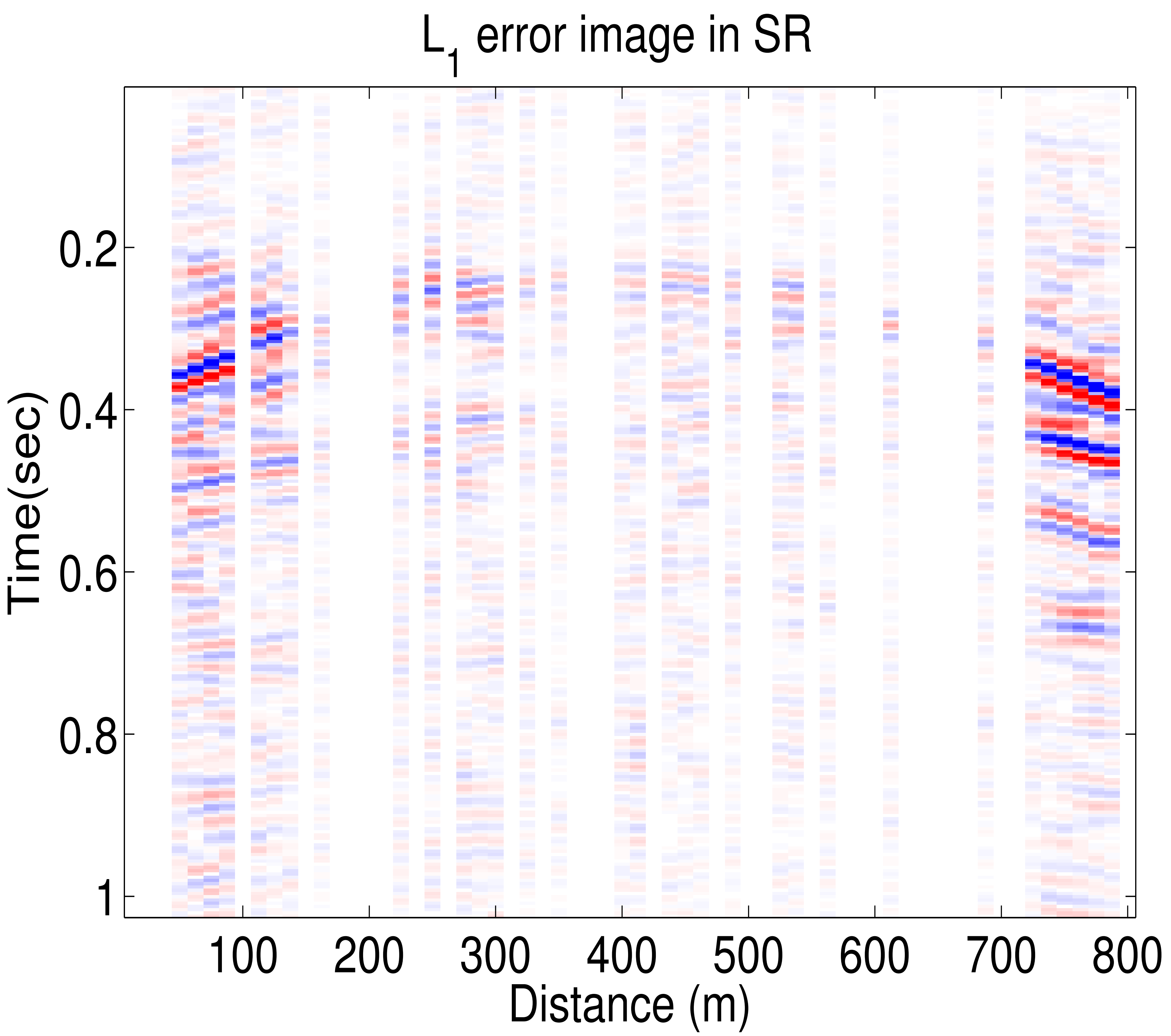
$$x^{w\ell_1} := \underset{z \in R^P}{\text{minimize}} ||z||_{1,\mathtt{w}} \quad \text{subject to} \quad ||RMS^H z - b||_2 \leq \epsilon,$$

$\mathtt{w} \in \{\omega, 1\}^N$ is the weight vector and $||z||_{1,\mathtt{w}} := \Sigma_i \mathtt{w}_i |z_i|$ is the weighted ℓ_1 norm.

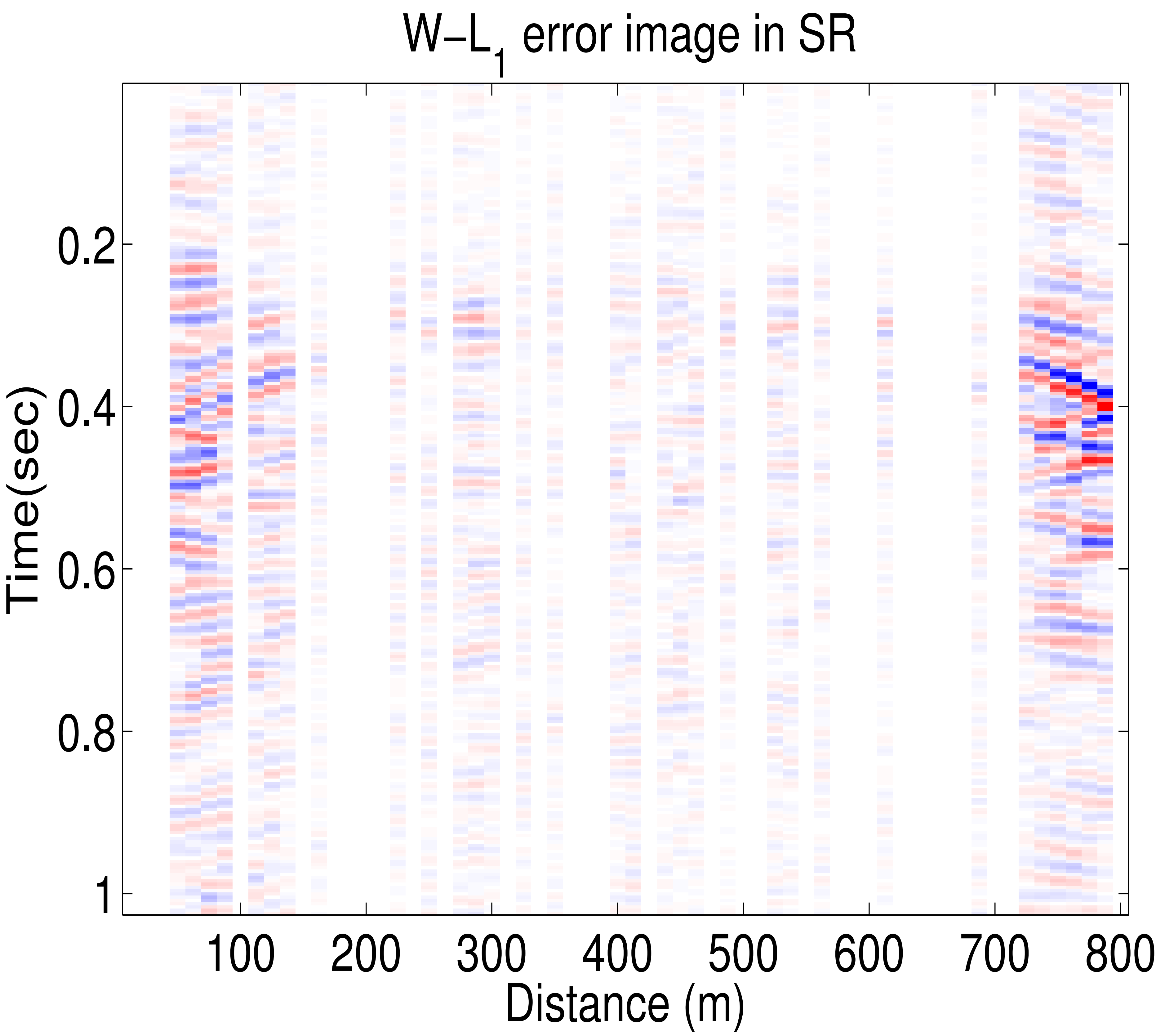




Recovery error: ℓ_1 vs weighted ℓ_1



(k) SNR=8.3 dB



(l) SNR= 10.7 dB

Iterative thresholding algorithms

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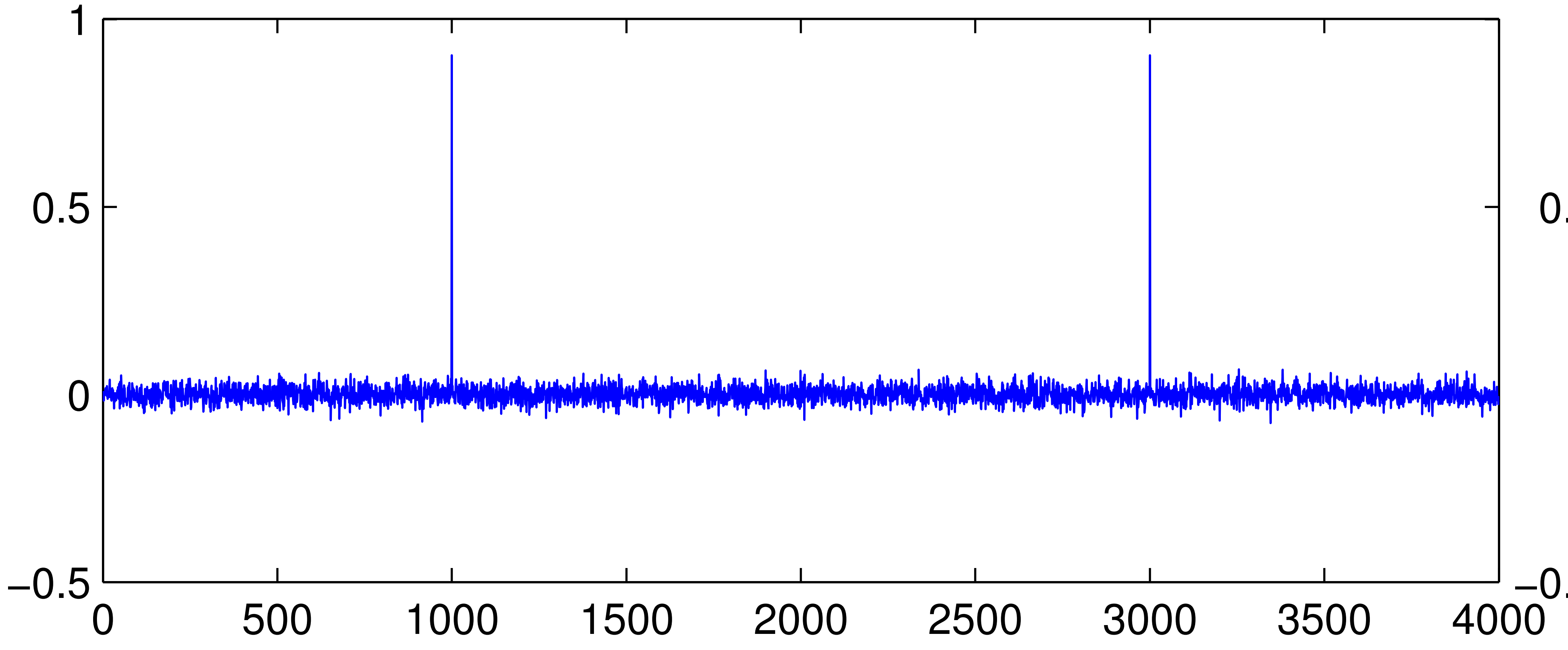
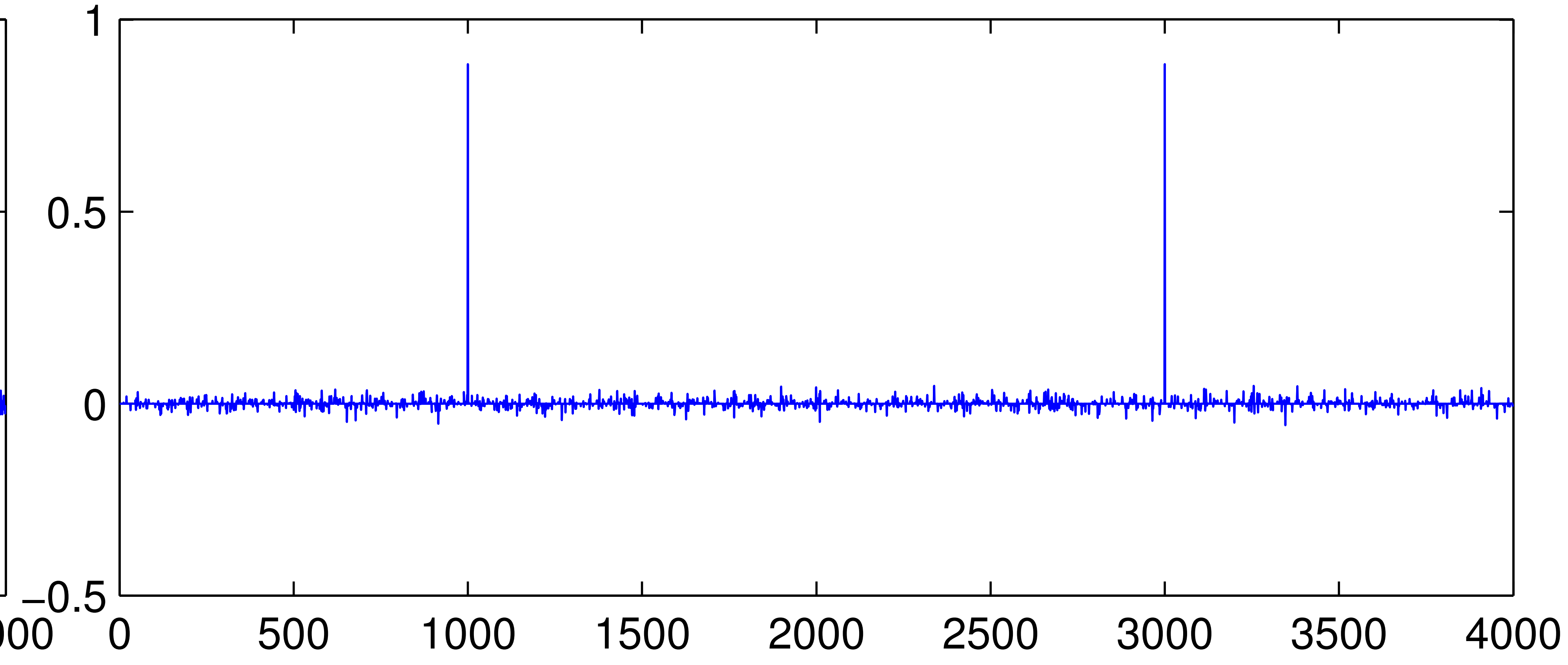
Simple example with a 2-sparse signali Iteration $t = 1$

Soft thresholding:

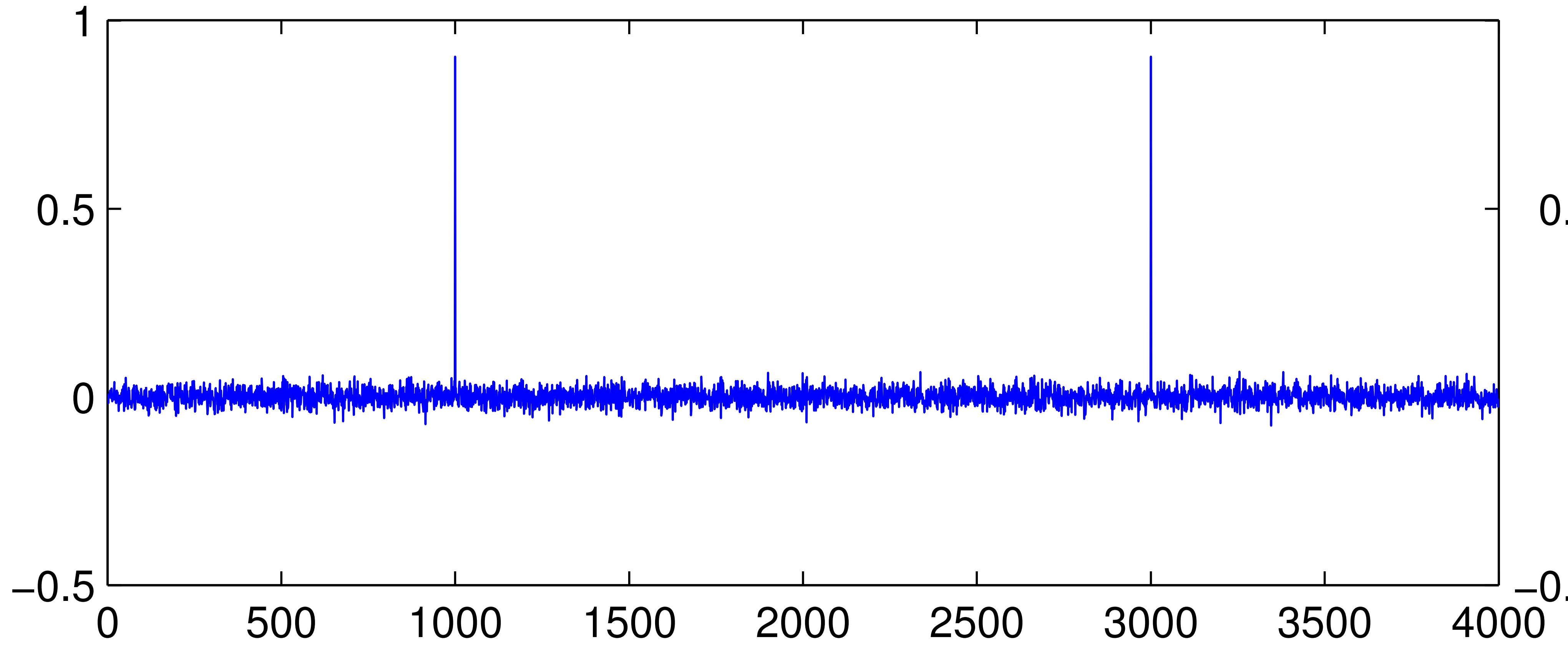
$x^t + A^* z^t$ $\qquad$ $\eta(x^t + A^* z^t; \tau^t)$

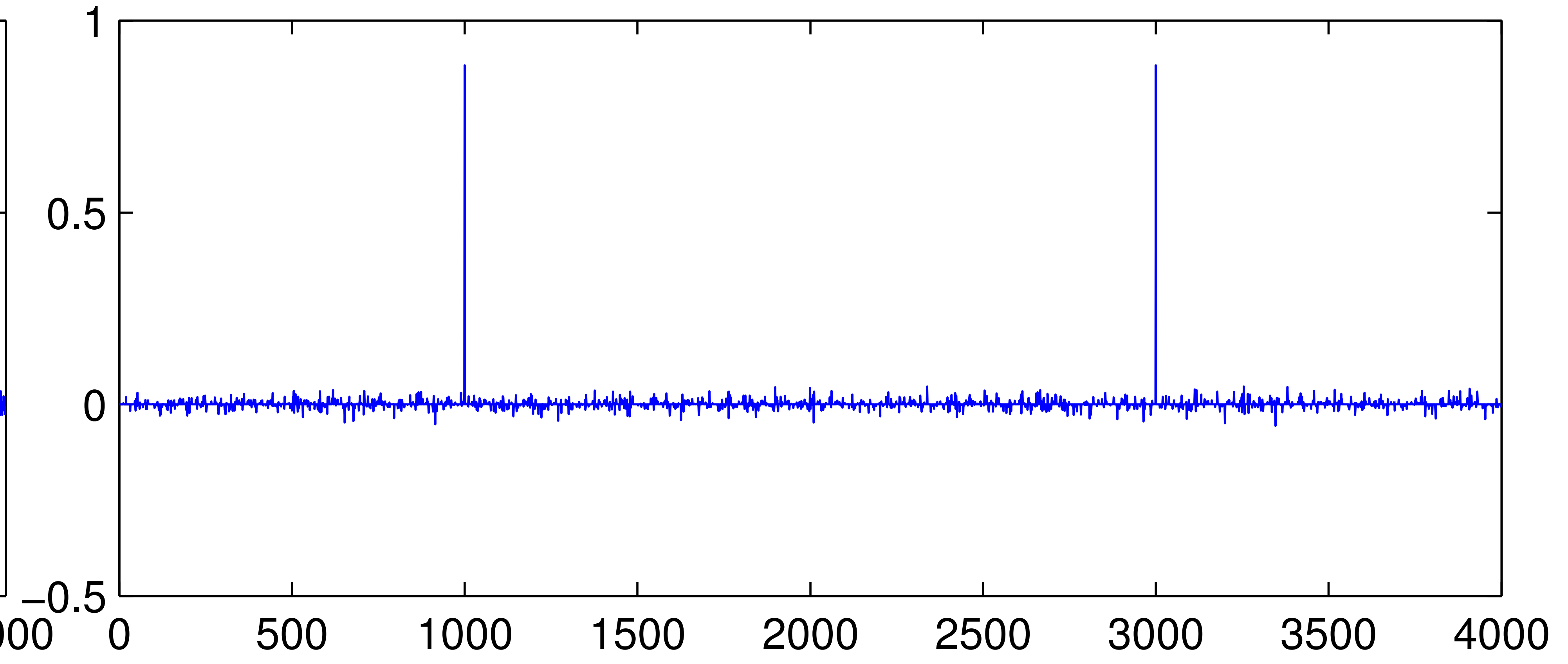




AMP:

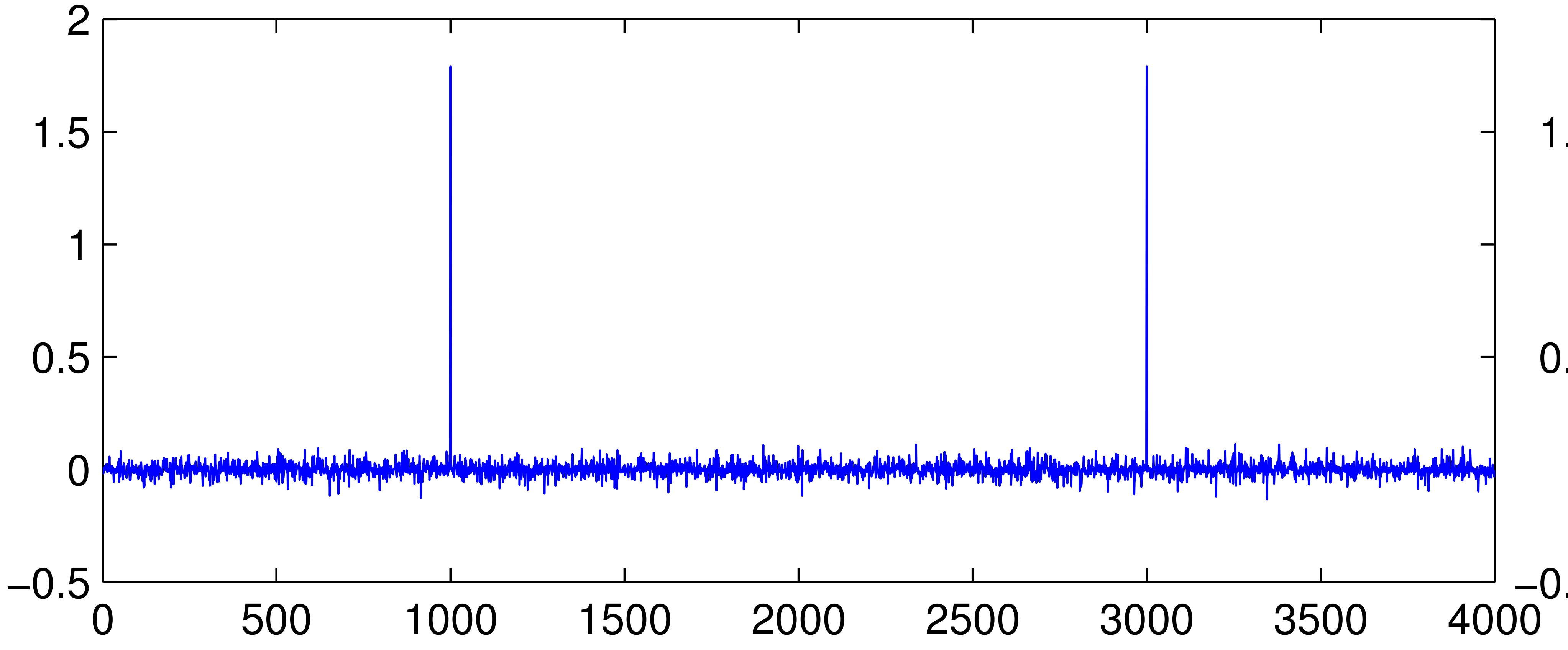
$x^t + A^*(z^t + \alpha^t z^{t-1})$ $\qquad$ $\eta(x^t + A^*(z^t + \alpha^t z^{t-1}); \tau^t)$

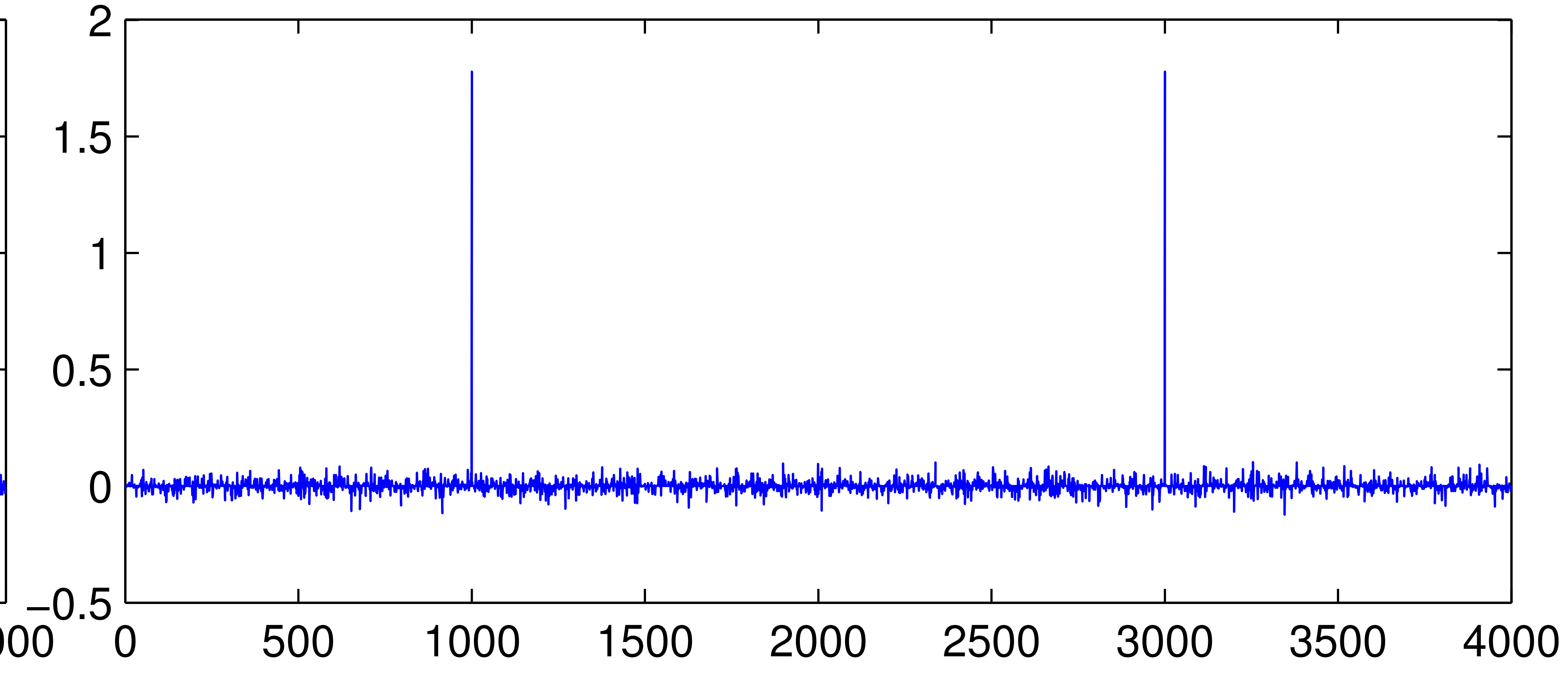




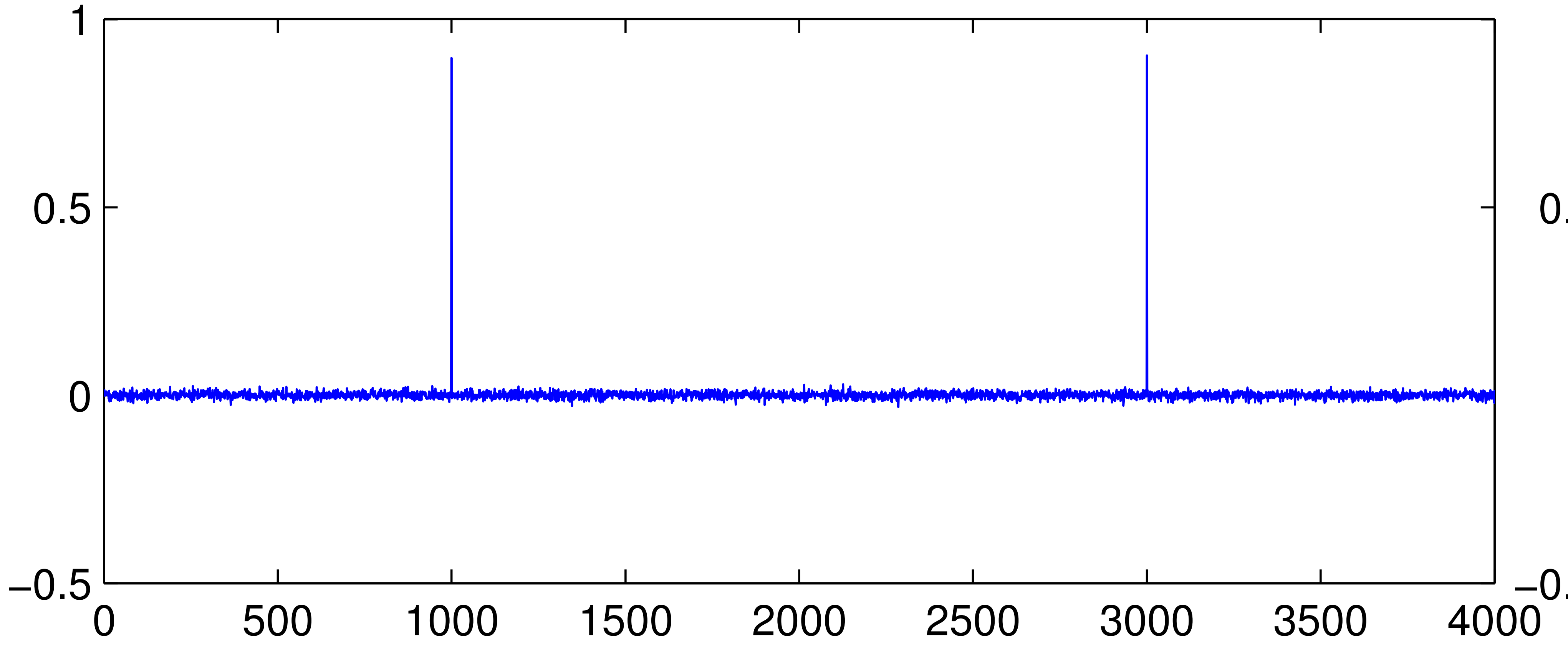
Iteration $t = 2$

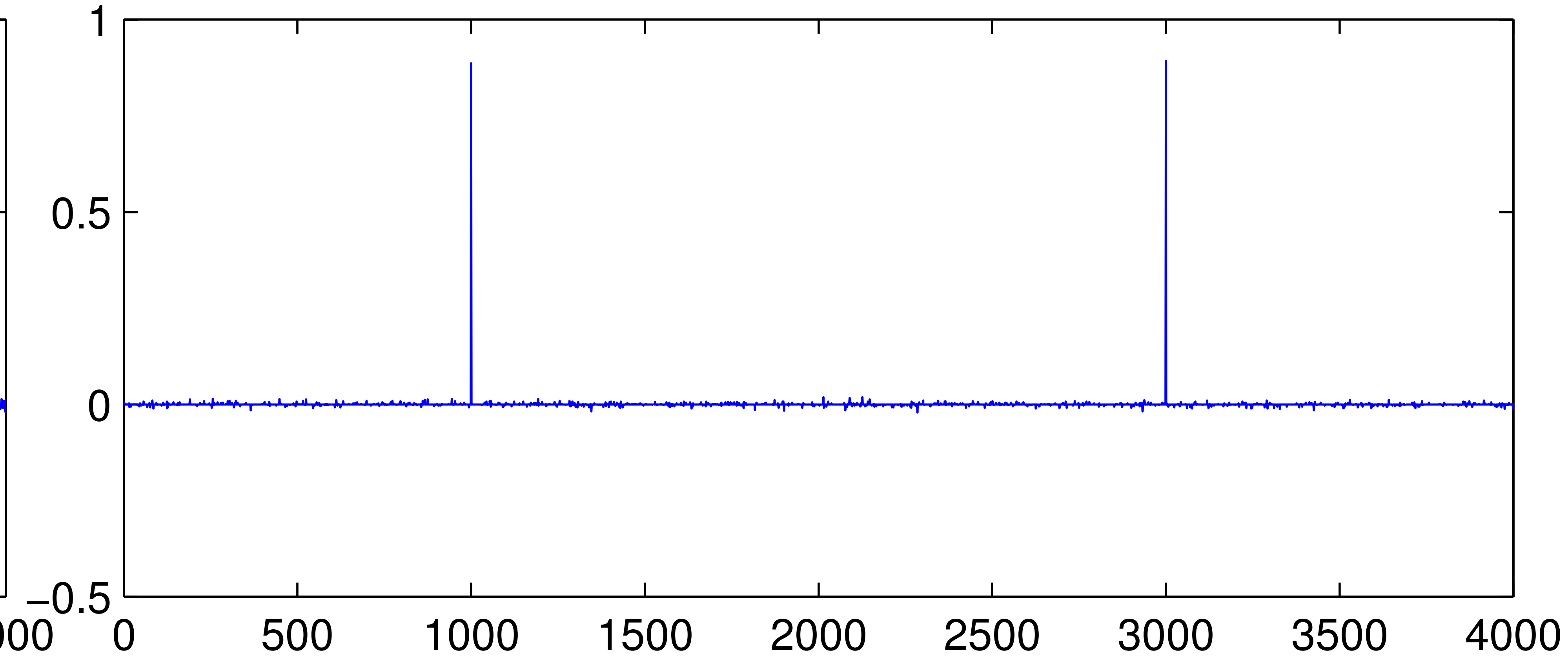
Soft thresholding: $x^t + A^* z^t$ $\qquad$ $\eta(x^t + A^* z^t; \tau^t)$





AMP: $x^t + A^*(z^t + \alpha^t z^{t-1})$ $\qquad$ $\eta(x^t + A^*(z^t + \alpha^t z^{t-1}); \tau^t)$





Iteration $t = 3$

Soft thresholding:

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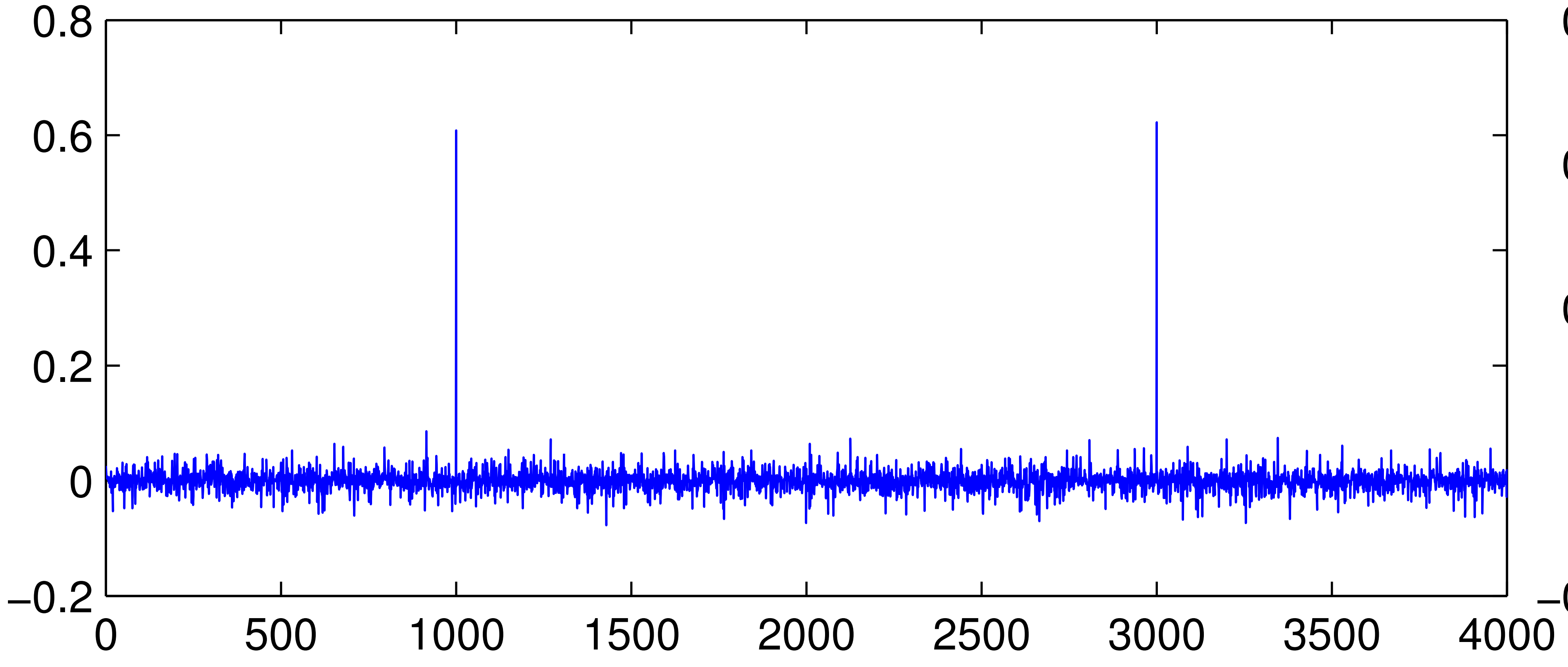

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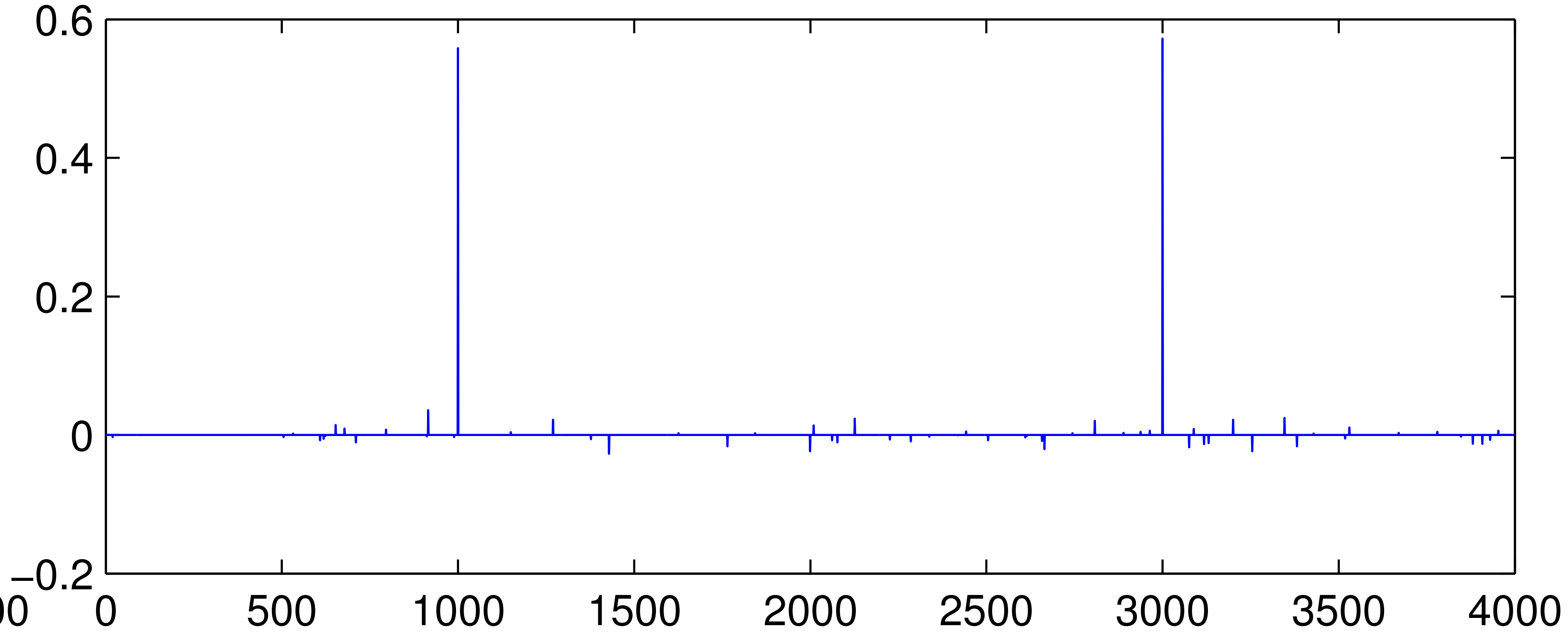

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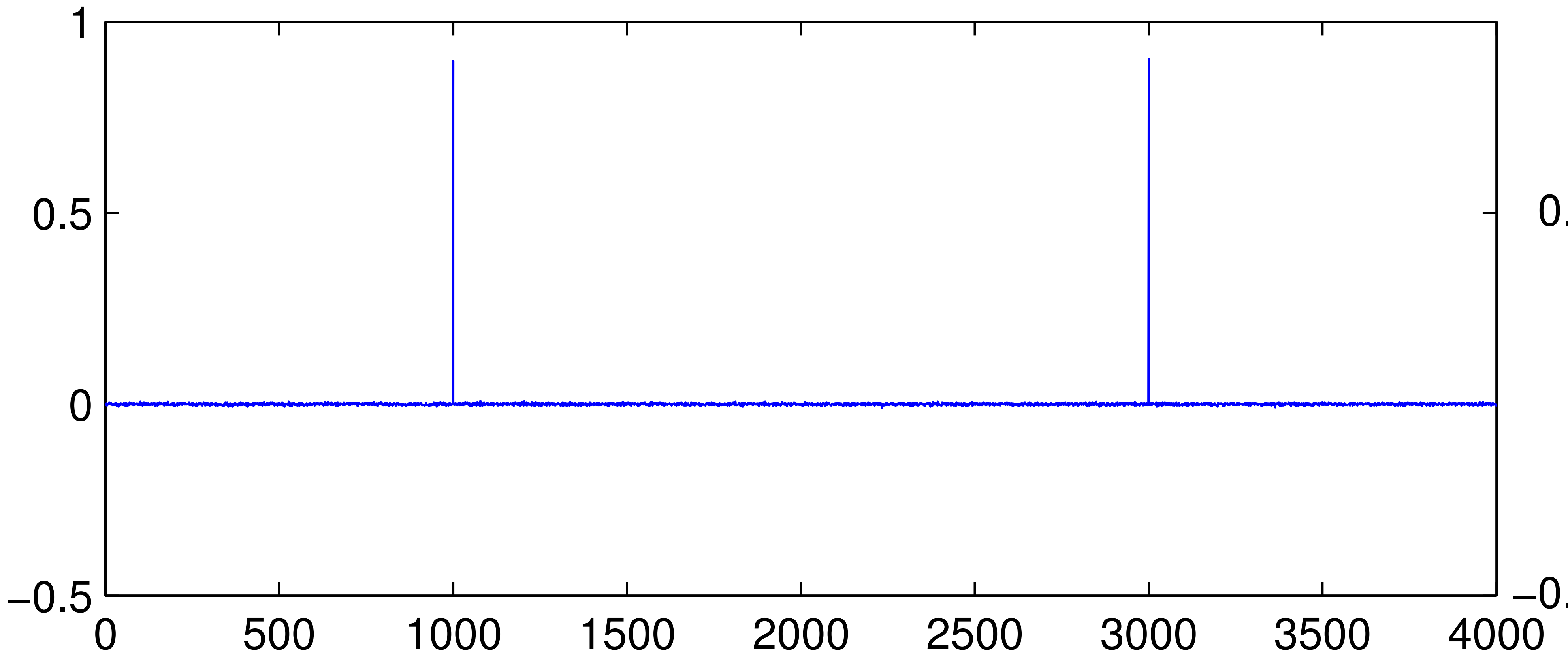

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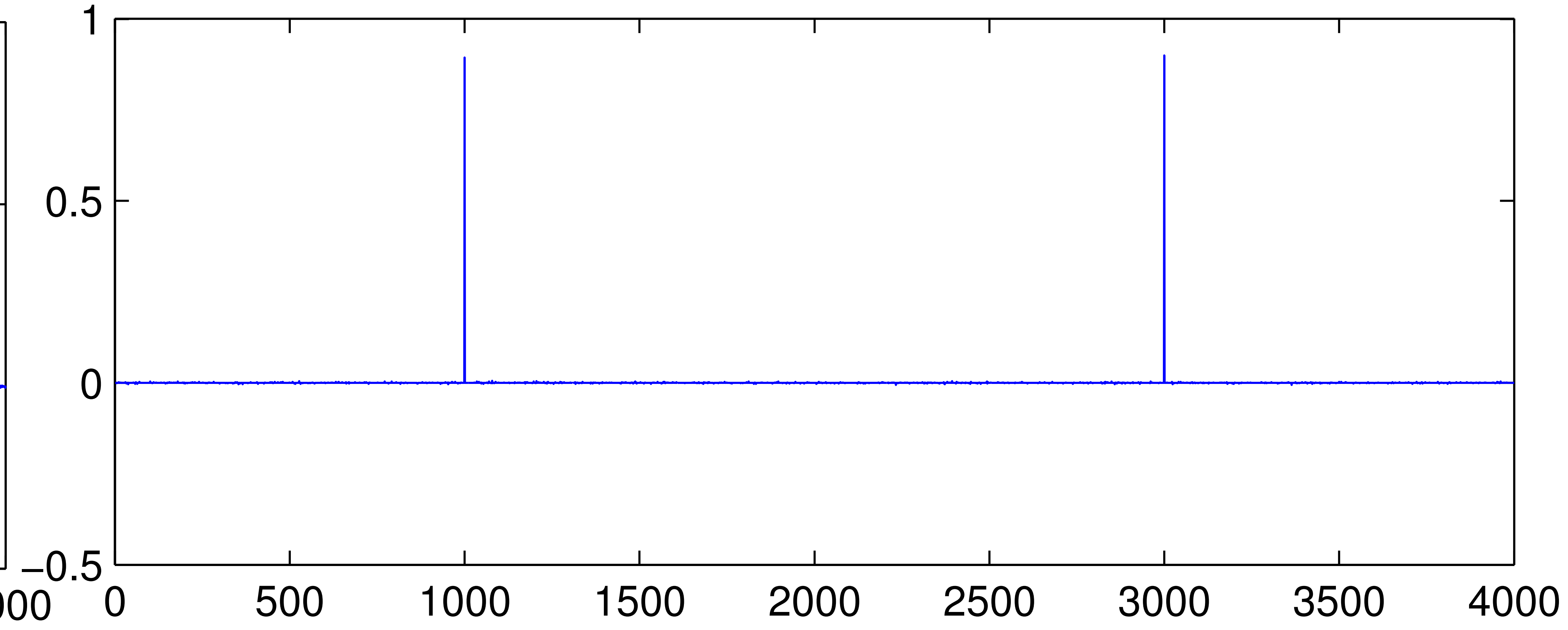

Weighted AMP

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We incorporate this information into the AMP algorithm by the following weighted AMP algorithm:

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How does AMP work?

Assume $[\mathrm{w}_1, ..., \mathrm{w}_\mathrm{N}]^\mathrm{T}$ are the weights we use for the coefficients of the signal s.

Consider the following distribution over variables $s_1, \ s_2, \ \ldots, \ s_N$:

$$\mu(ds) = \frac{1}{Z} \prod_{i=1}^{N} exp(-\beta \mathrm{w}_i |s_i|) \prod_{a=1}^{m} \delta_{\{y_a = (As)_a\}}, \tag{4}$$

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Then $b = RMF_s^H F_s f$, where F_s is a 2-D DFT matrix in the source-receiver domain.

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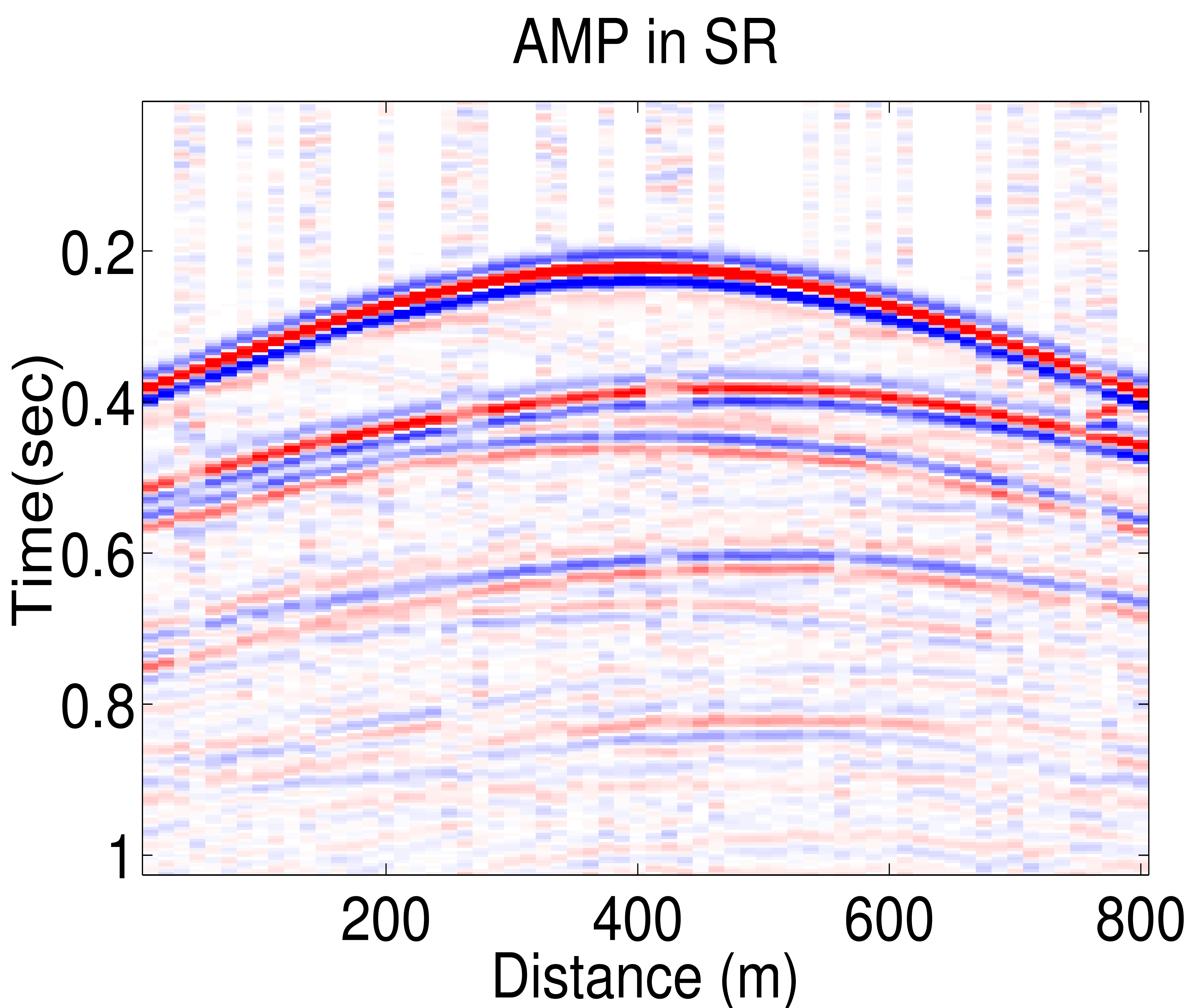
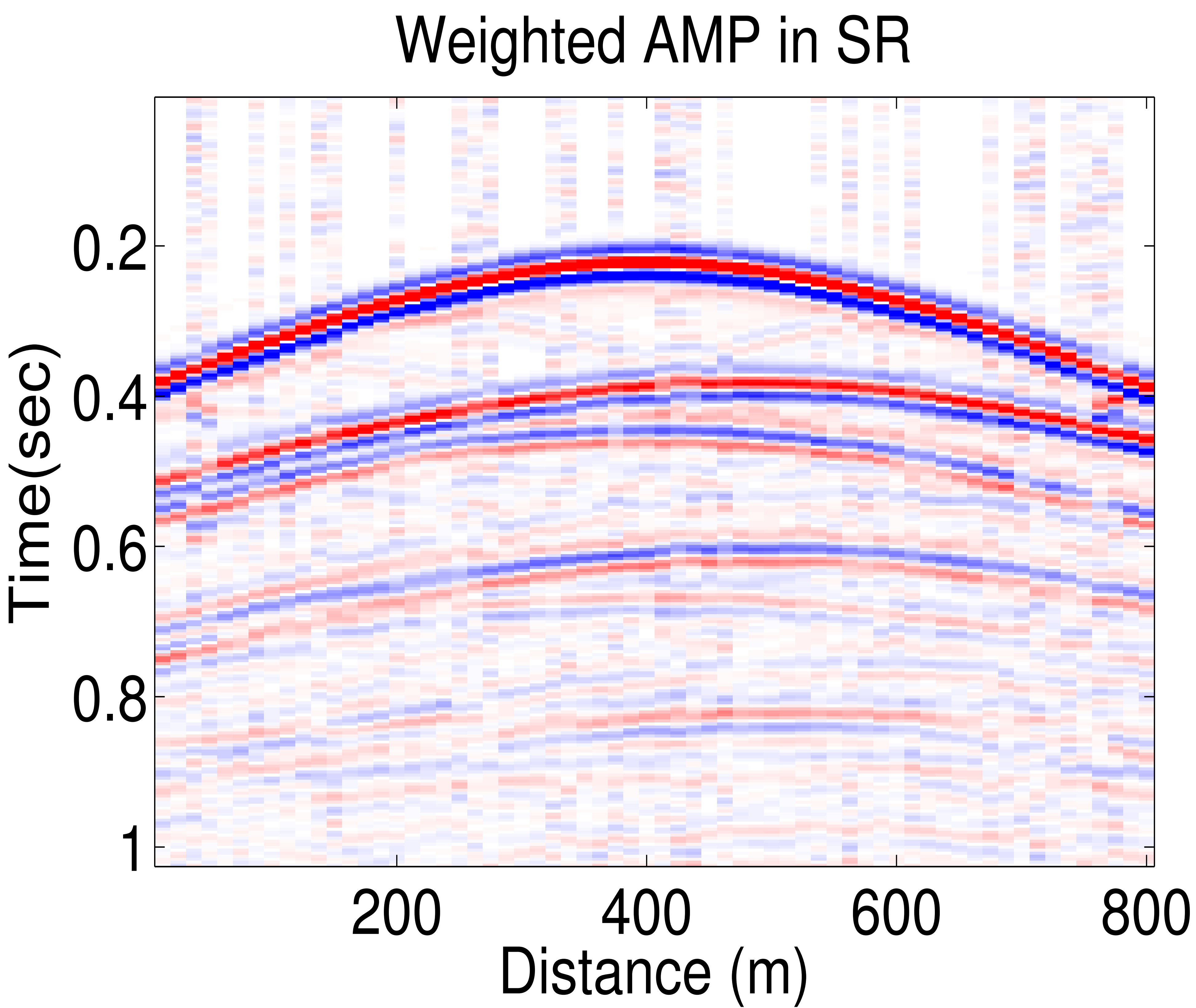
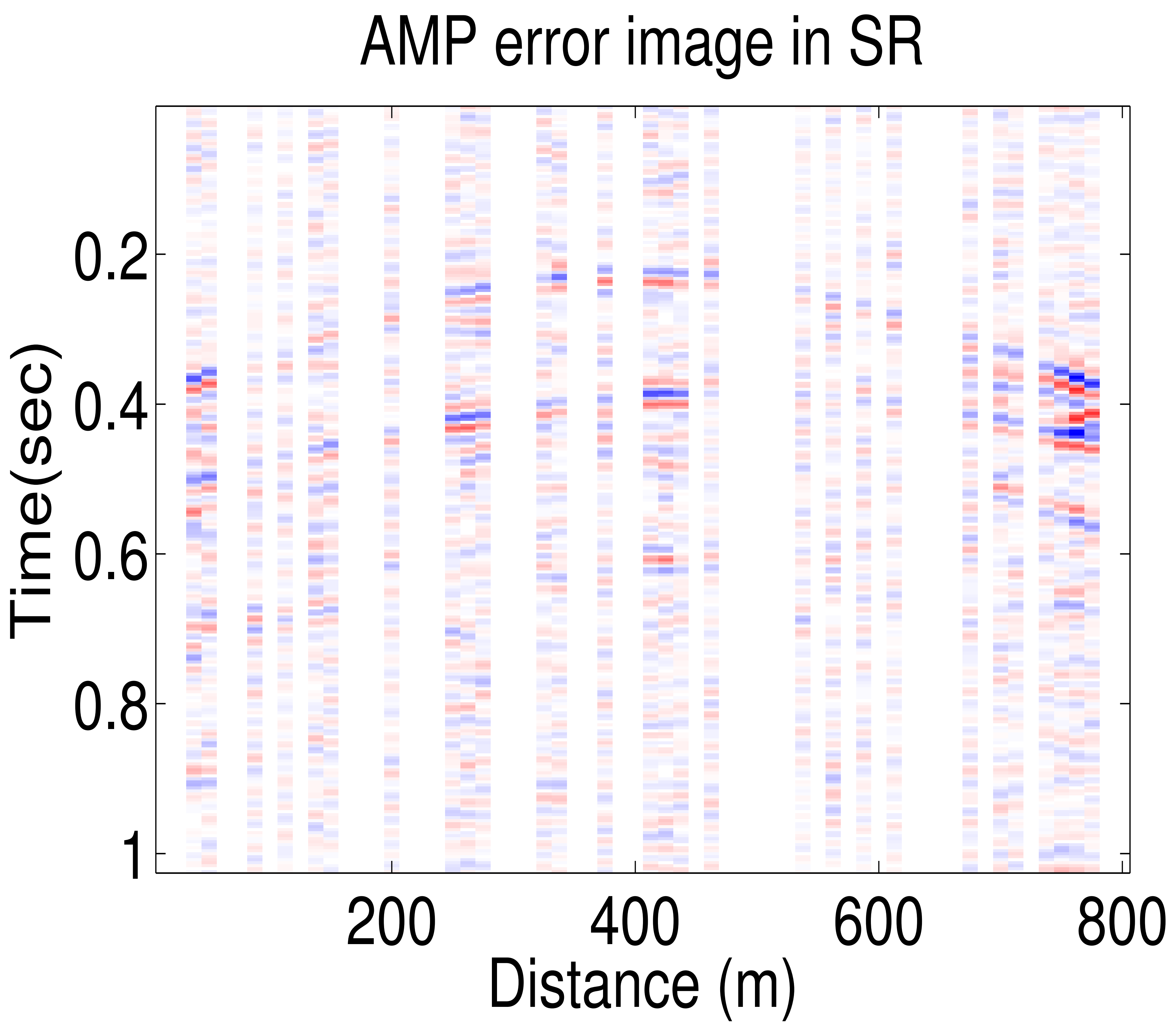
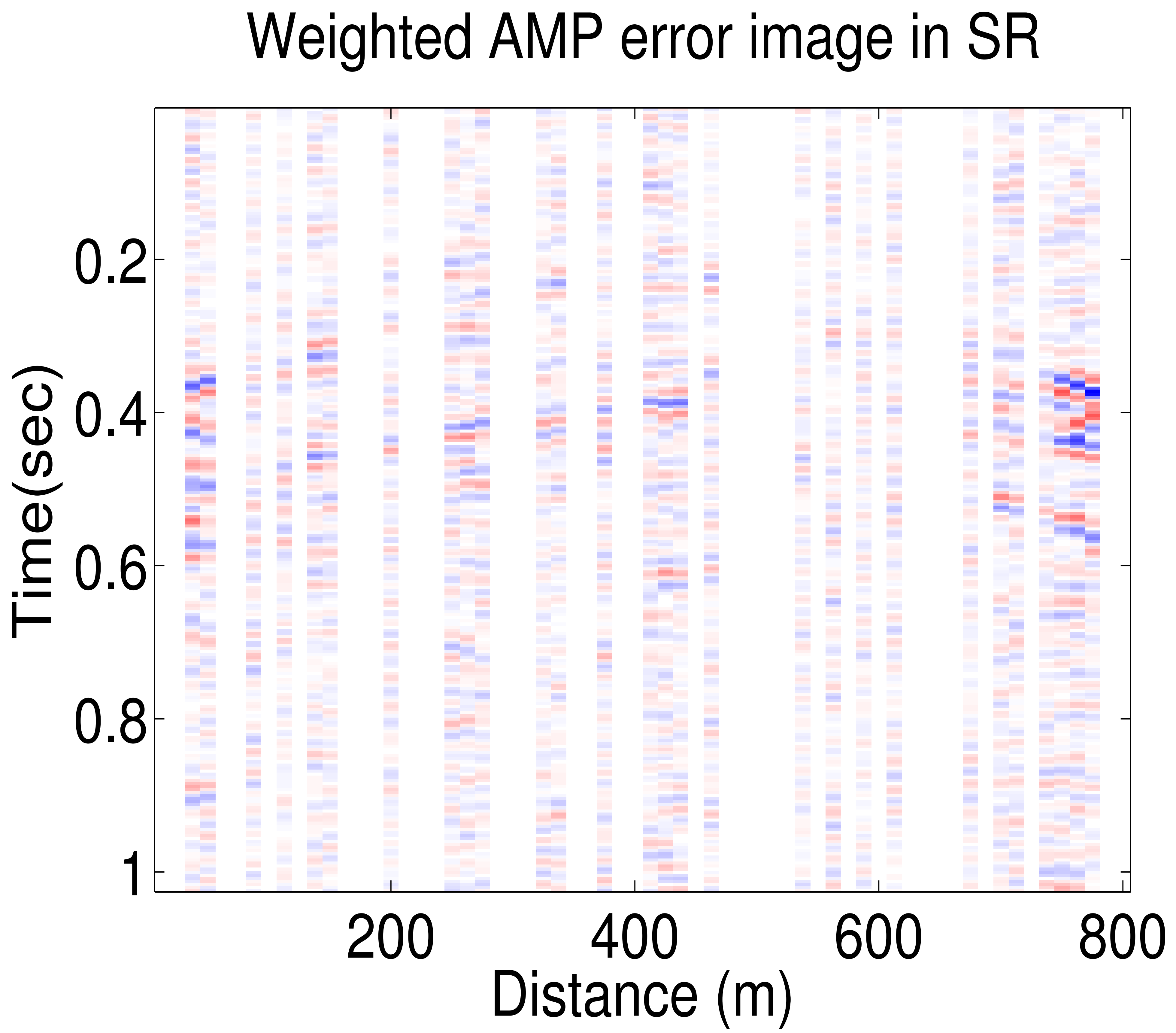




Recovery error: AMP vs weighted AMP



(o) SNR=10.2 dB



(p) SNR= 11.5 dB

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- Let x^* be the approximation obtained by solving the AMP algorithm then we have:

$$
\begin{aligned}
x^* &\approx F_s f \\
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Hence we can use $SF_s^H x^*$ as an approximation for the ℓ_1 solver.

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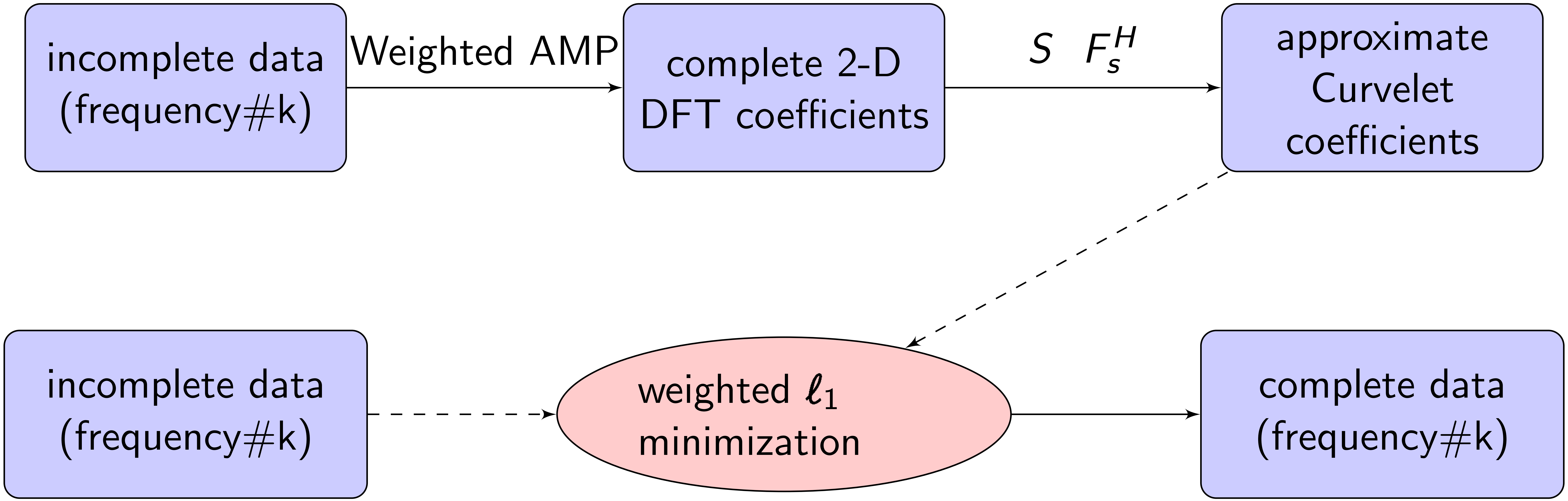
$$\begin{aligned} x^* &\approx F_s f \\ F_s^H x^* &\approx f \\ SF_s^H x^* &\approx Sf \end{aligned} \tag{5}$$

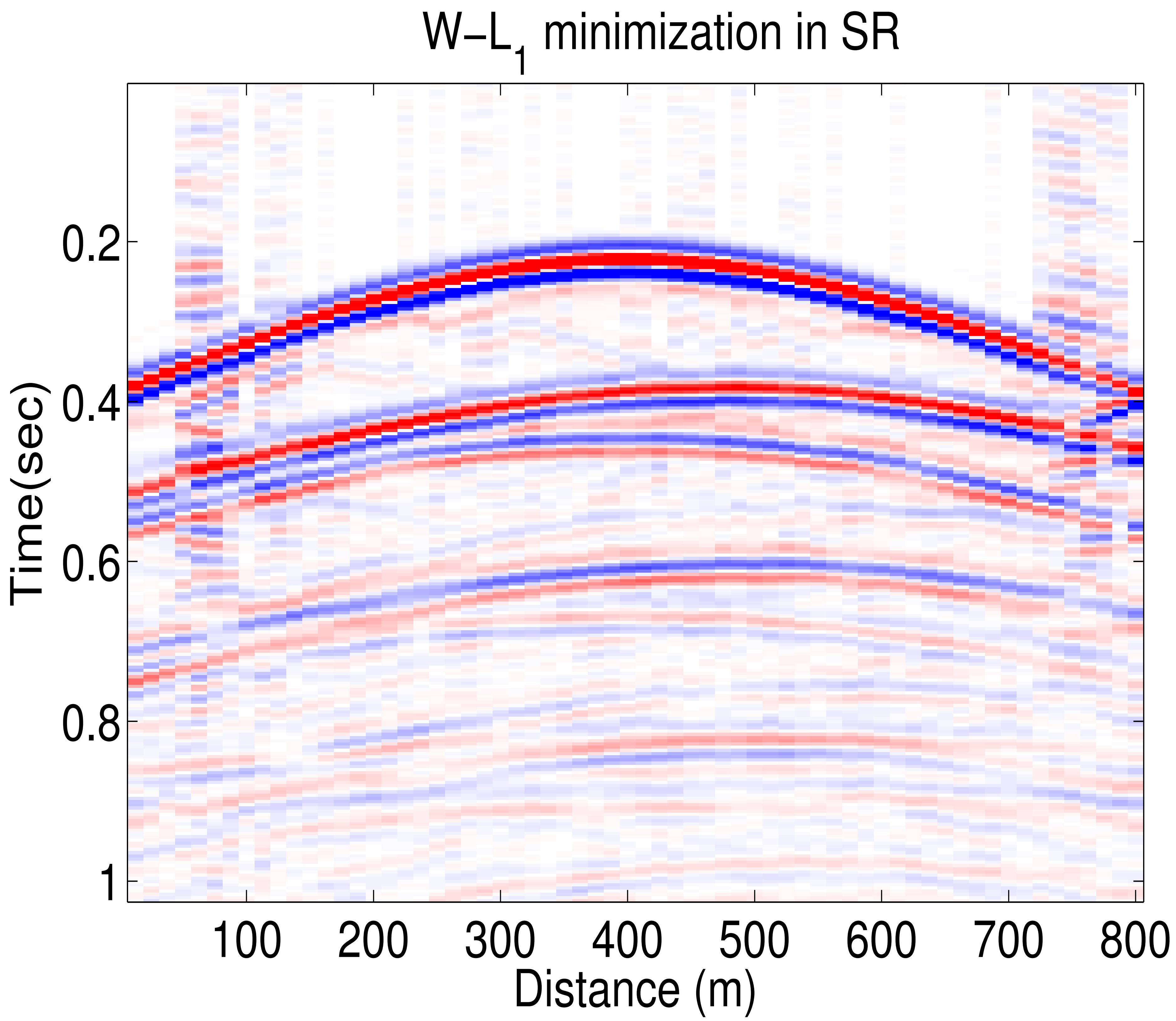
Hence we can use $SF_s^H x^*$ as an approximation for the ℓ_1 solver.

Flowchart of the 2-stage algorithm WAMP+weighted ℓ_1

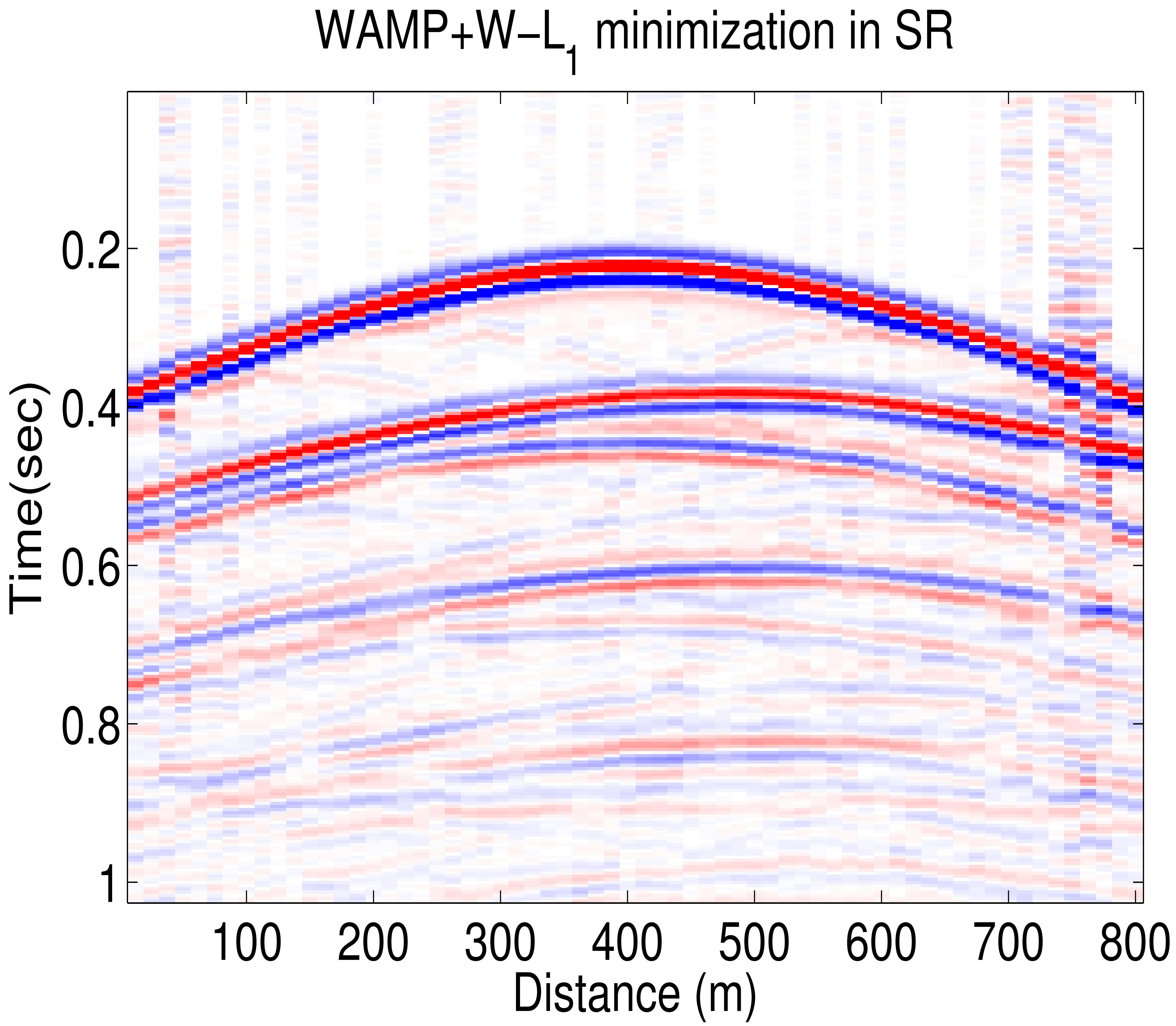


Weighted ℓ_1 vs 2-stage WAMP+weighted ℓ_1

In the 2-stage algorithm, for each frequency slice we first apply a fast WAMP algorithm and use the result to derive new weights for weighted ℓ_1 minimizer with Curvelet coefficients.

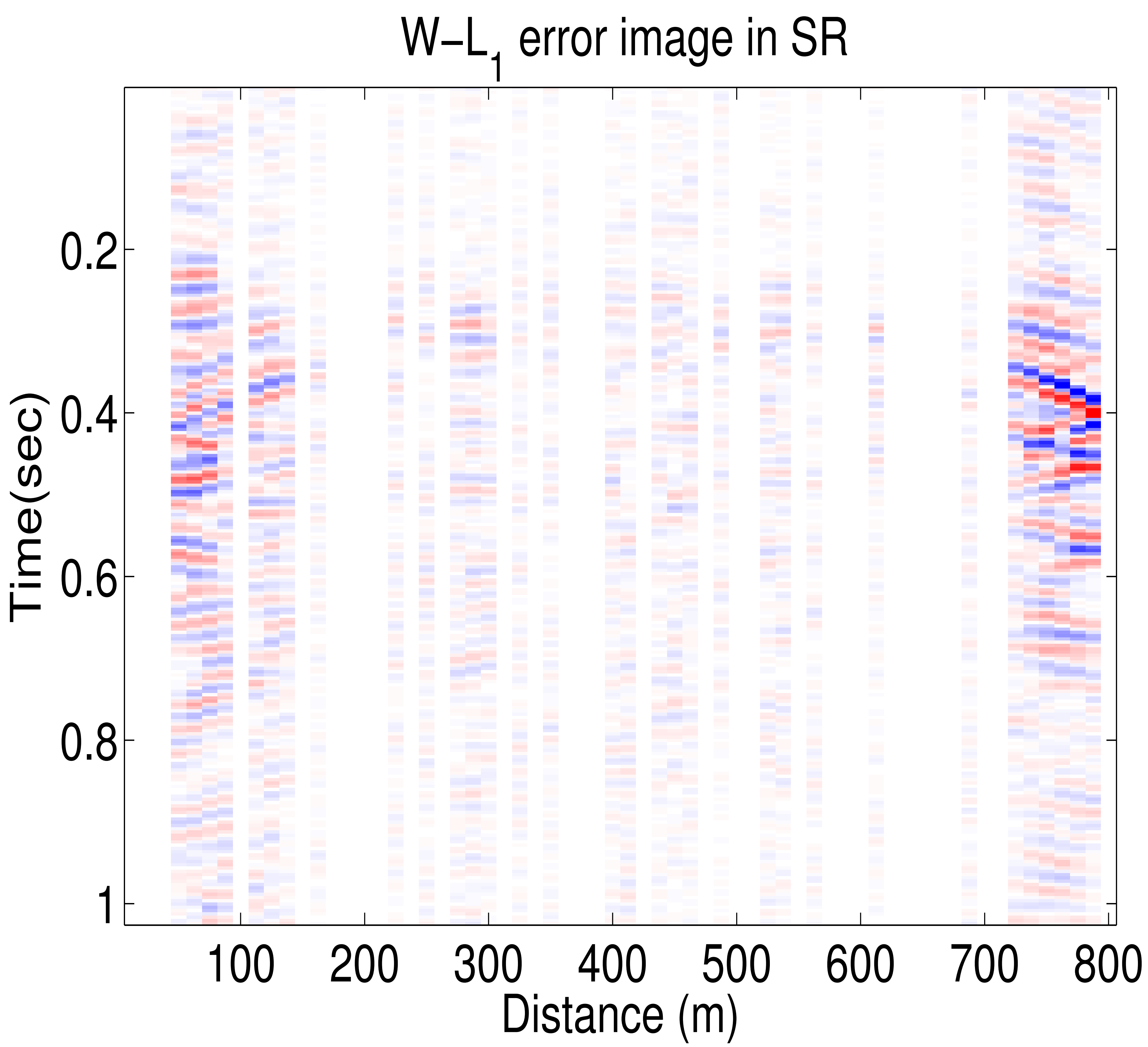


(q)

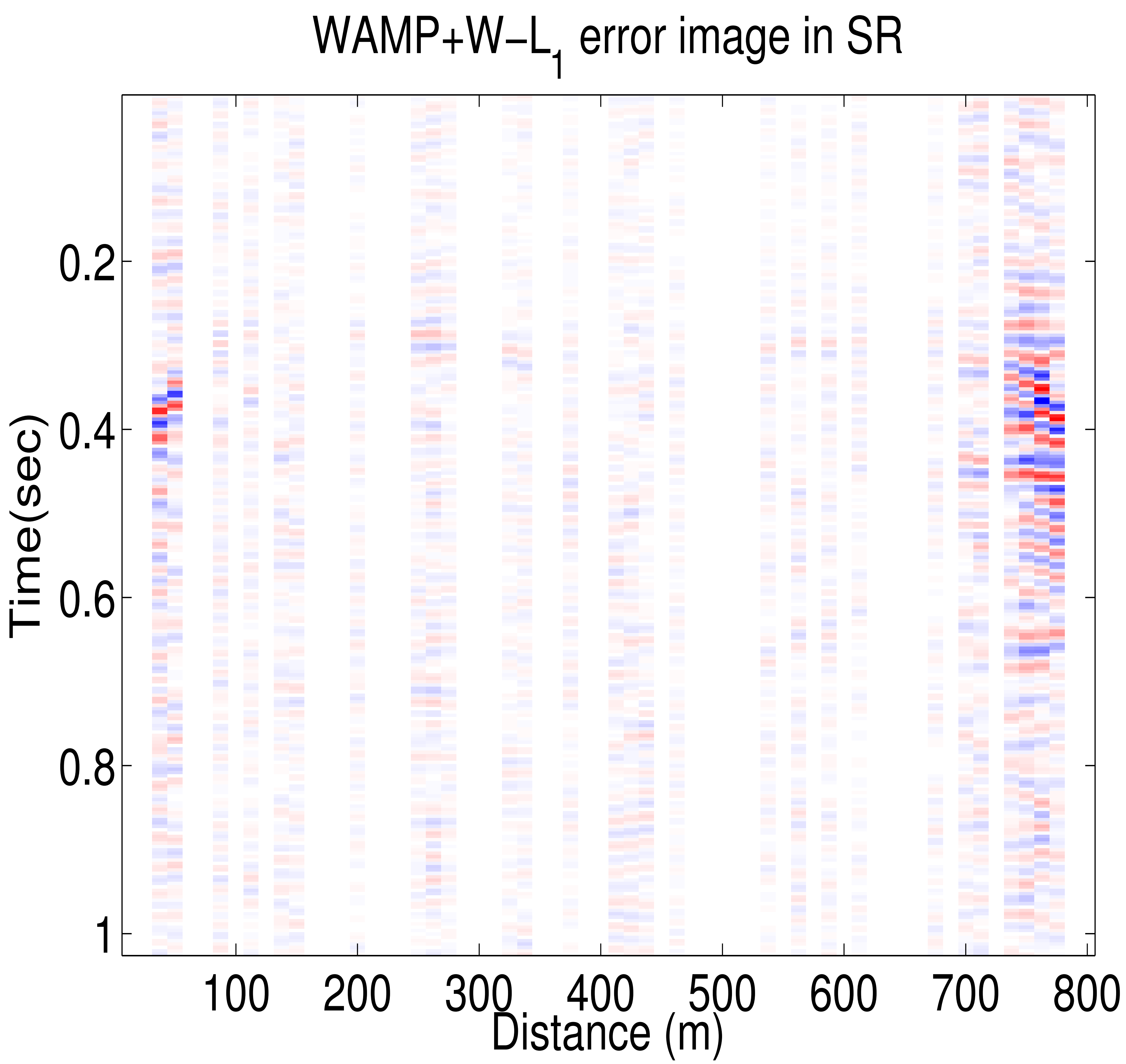


(r)

Recovery error: AMP vs weighted AMP

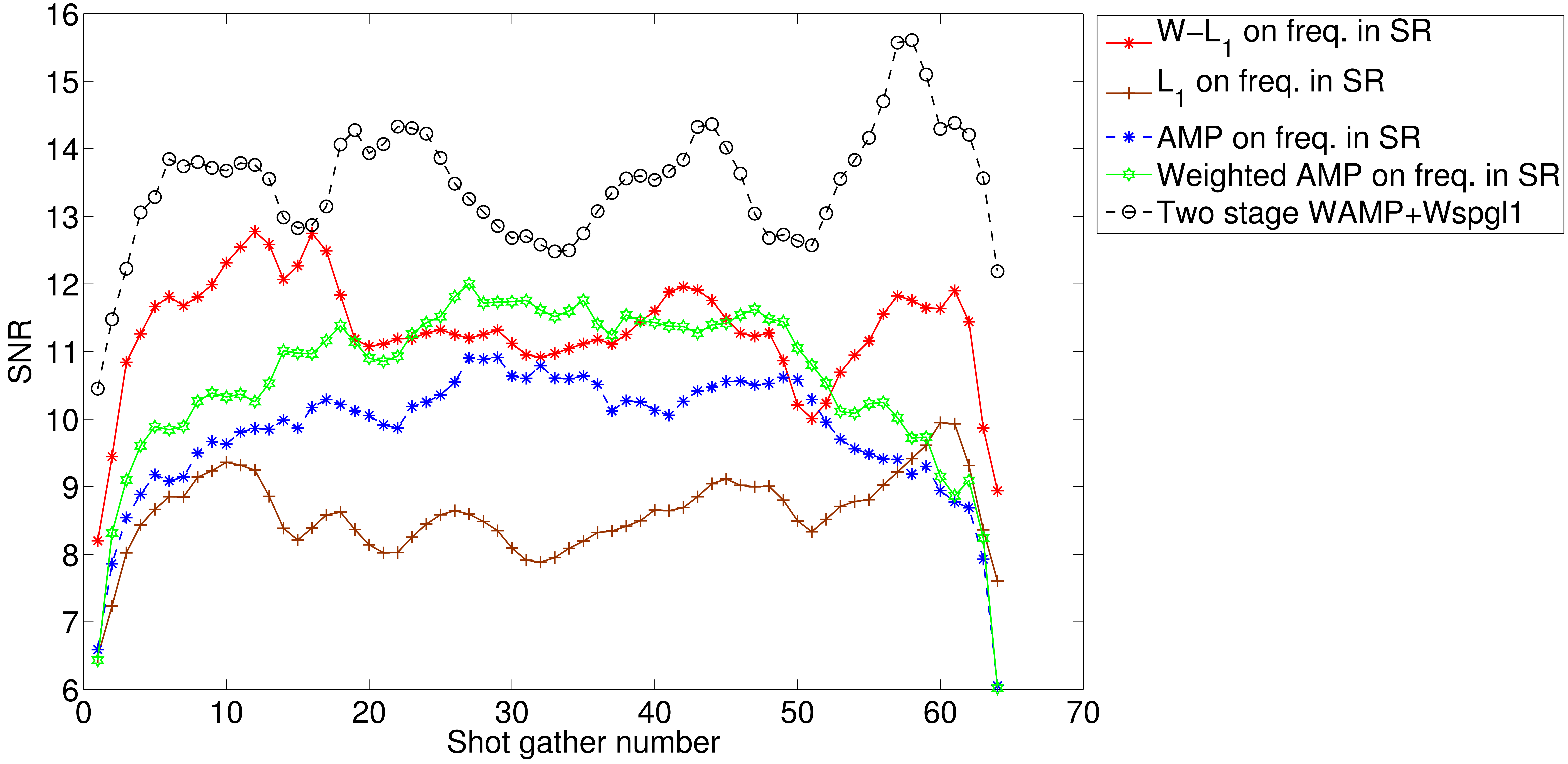


(s) SNR=10.2 dB



(t) SNR= 12.8 dB

Comparison of shotgather SNRs



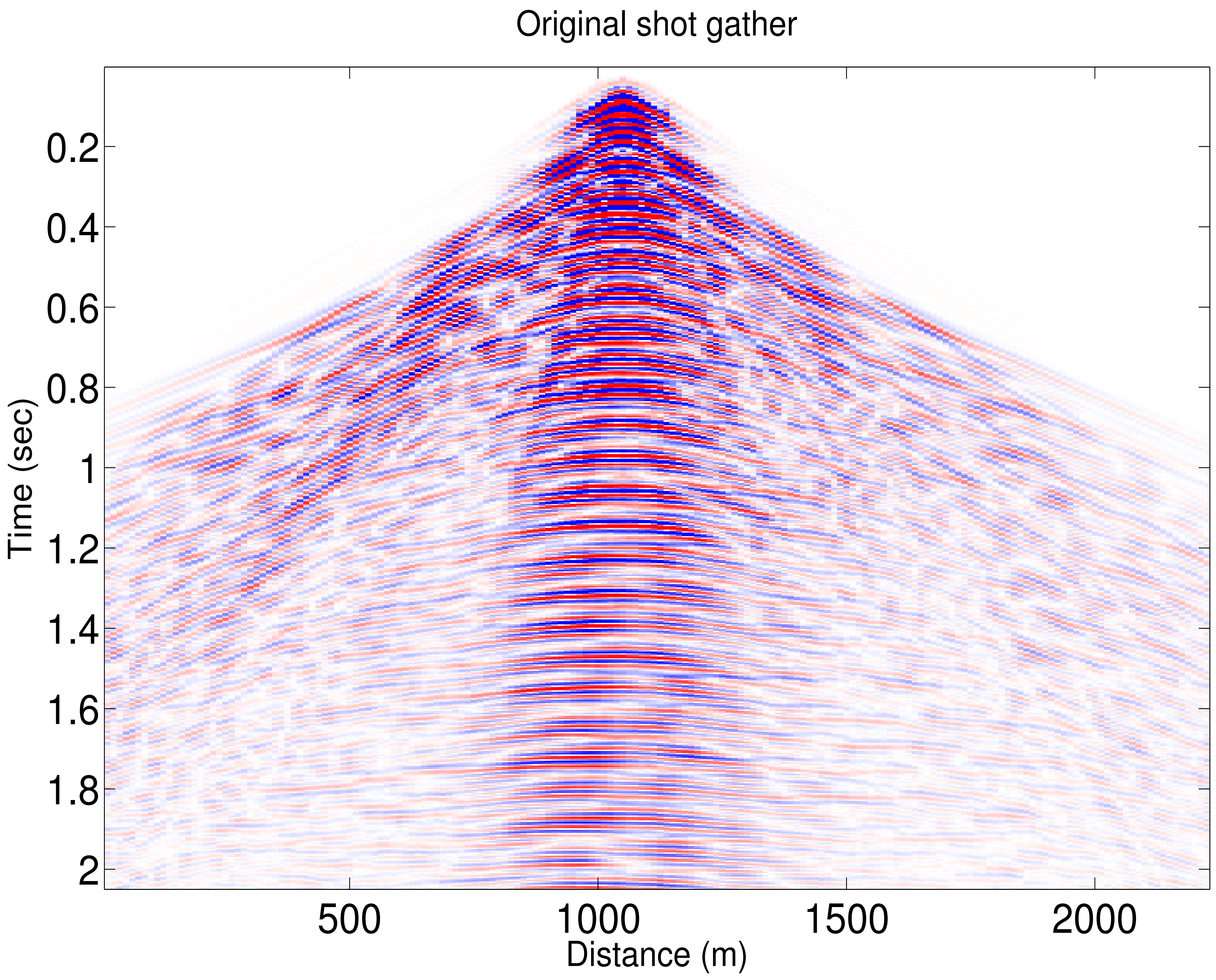
Results on the Gulf of Suez data

In the next slides we show the results of applying these algorithms on a seismic line from the Gulf of Suez.

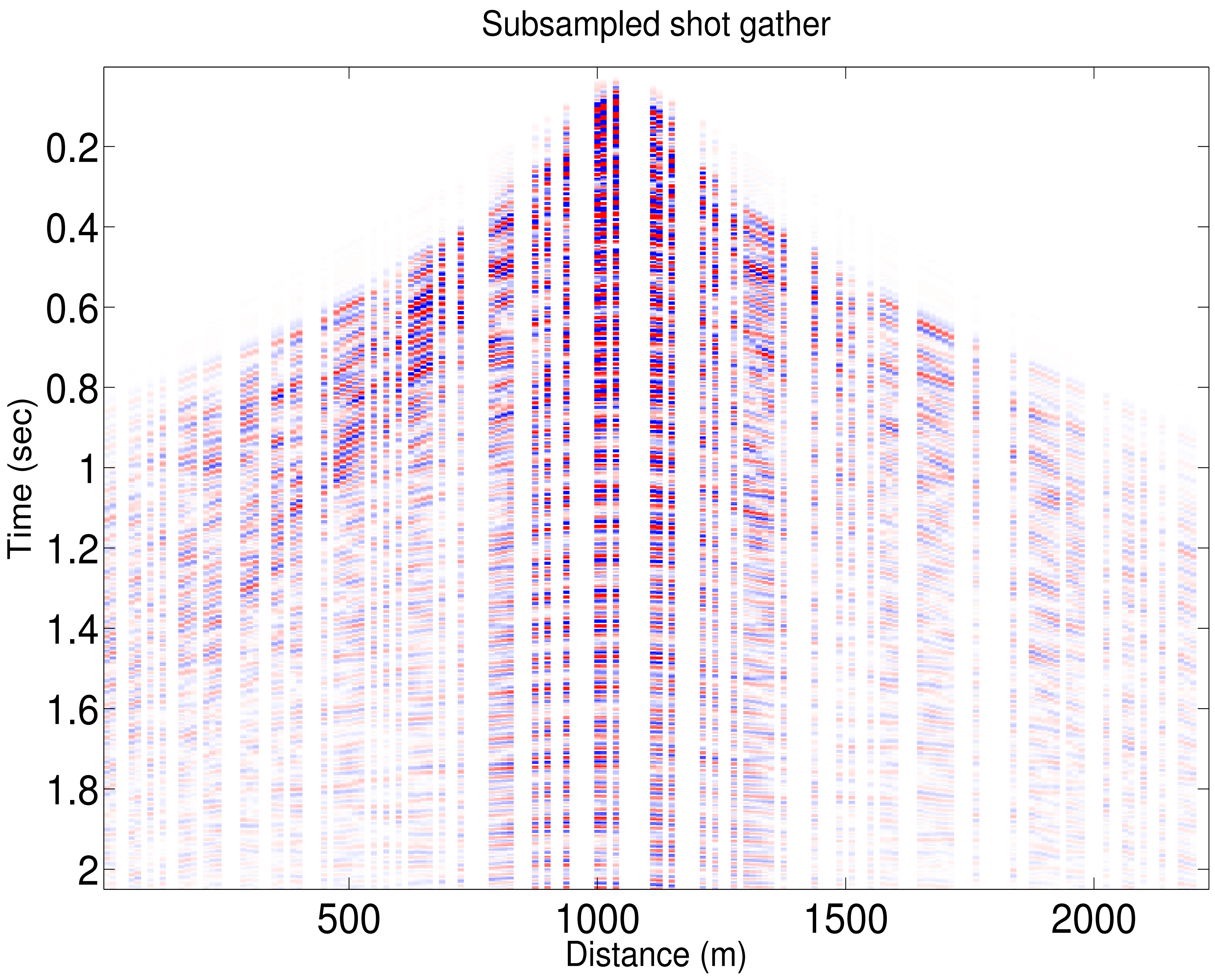
The Seismic line at full resolution has $N_s = 178$ sources, $N_r = 178$ receivers with a sample distance of 12.5 meters, and $N_t = 512$ time samples acquired with a sampling interval of 4 milliseconds. Consequently, the seismic line contains samples collected in a 2s temporal window with a maximum frequency of 125 Hz.

Results on the Gulf of Suez data

Shotgather number 84 from the seismic line:

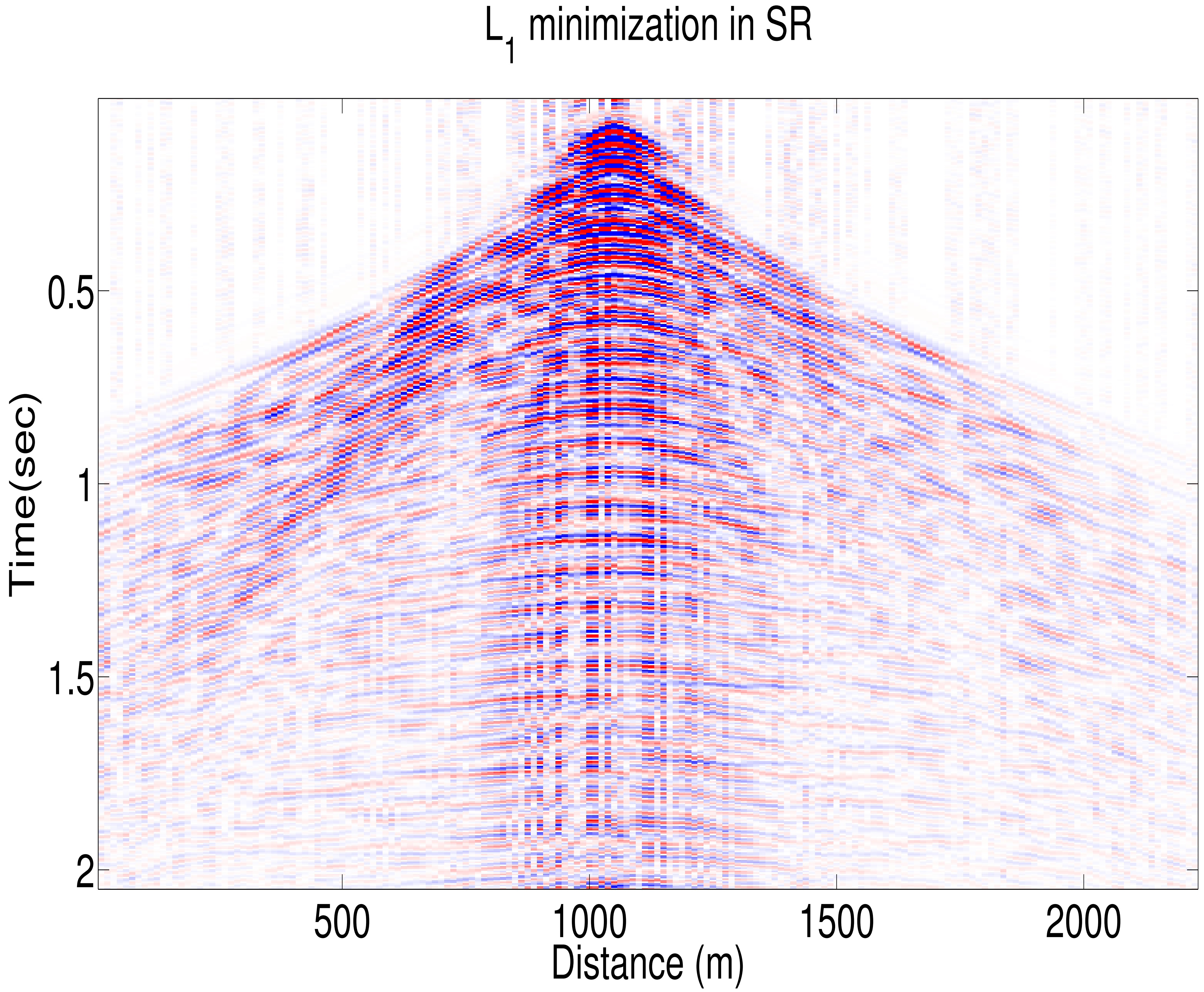


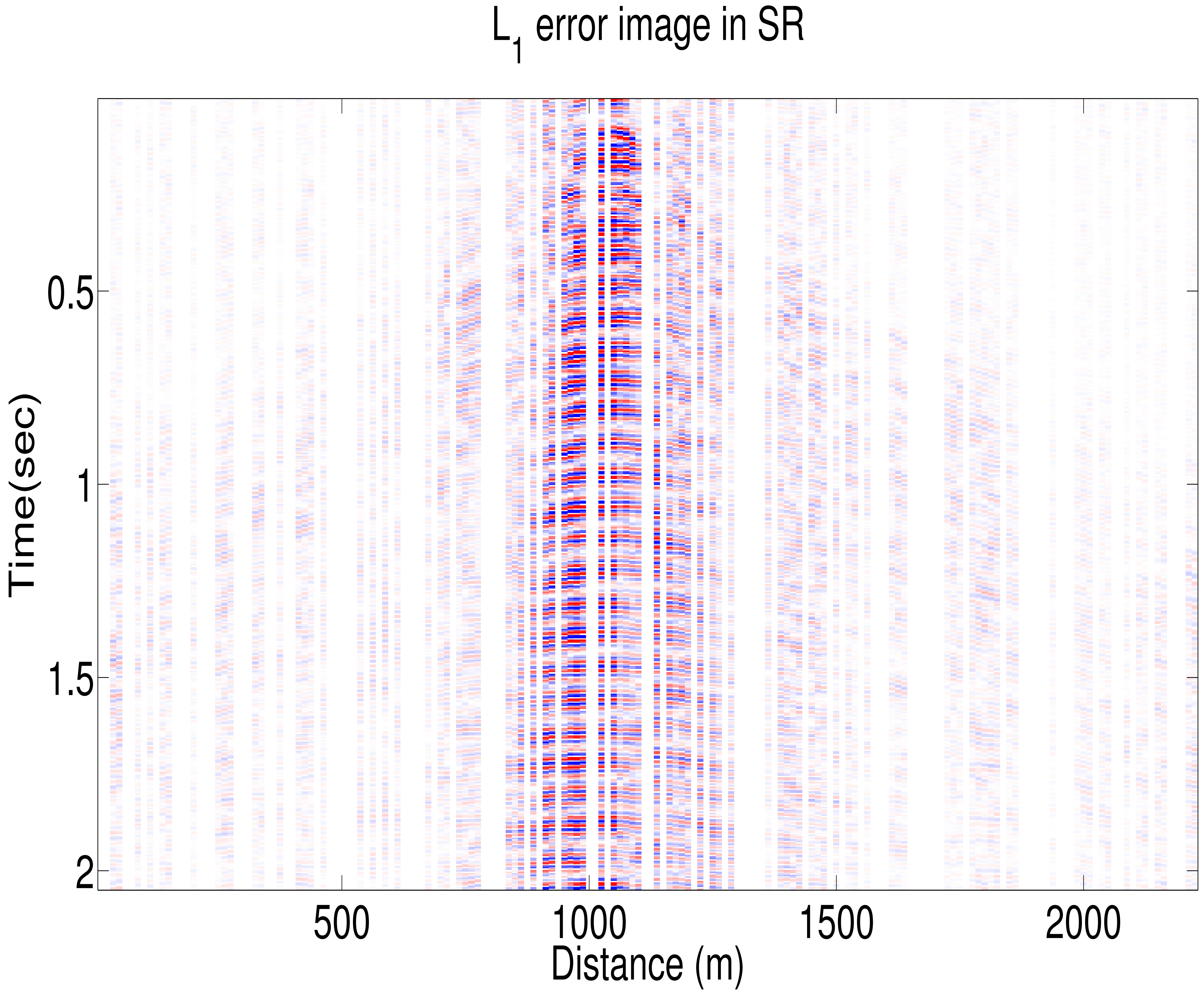
(u)



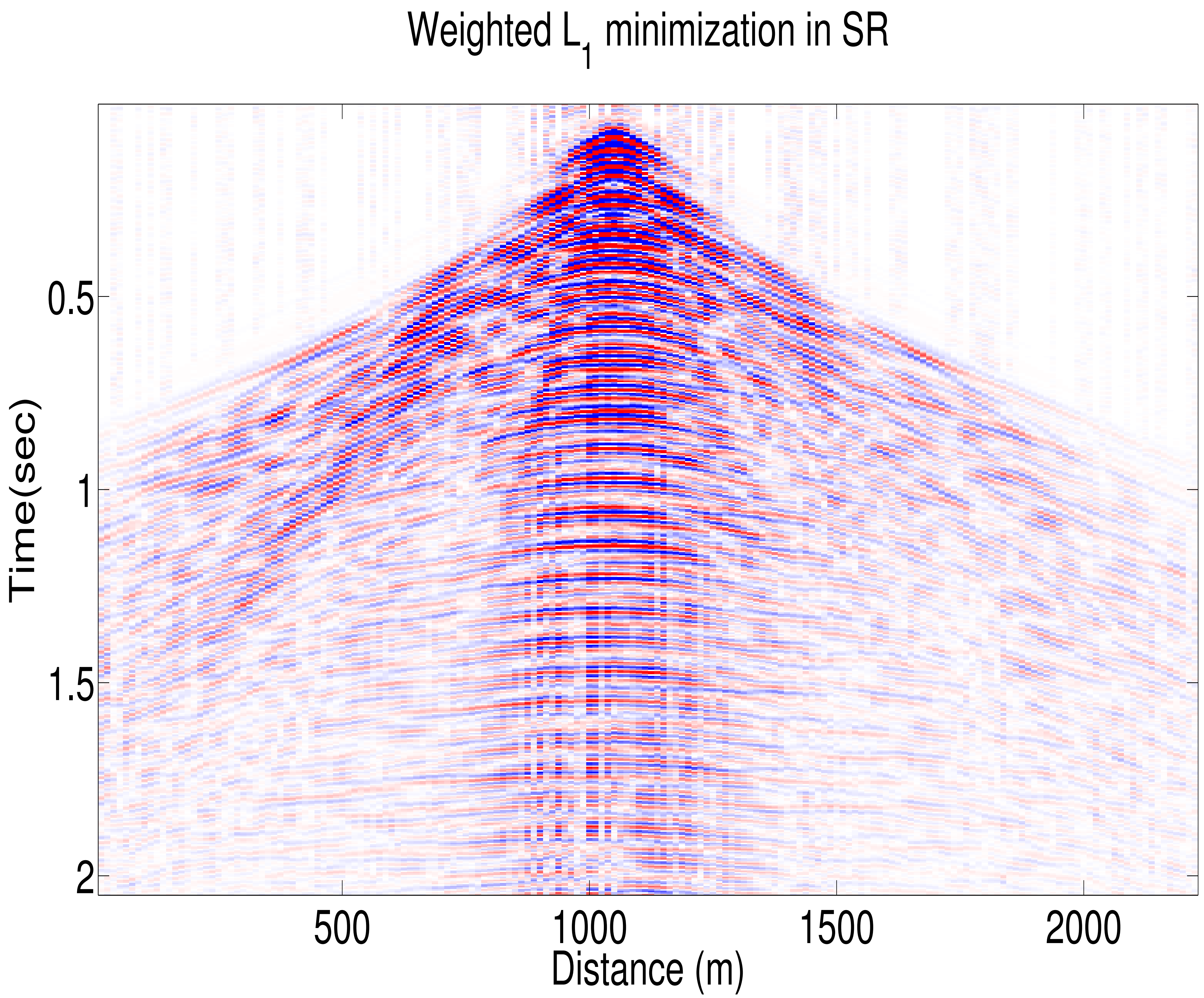
(v)

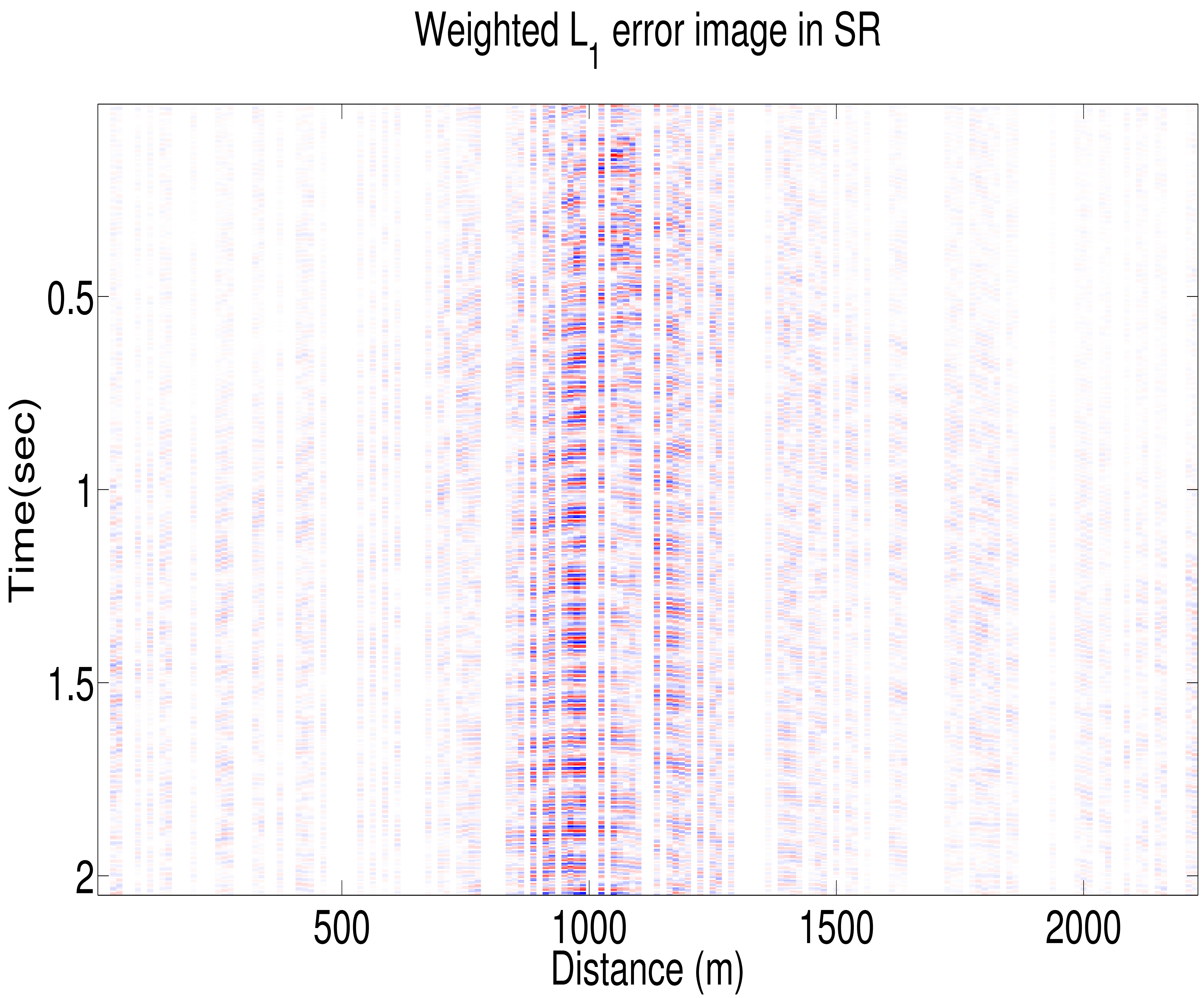
ℓ_1 minimization



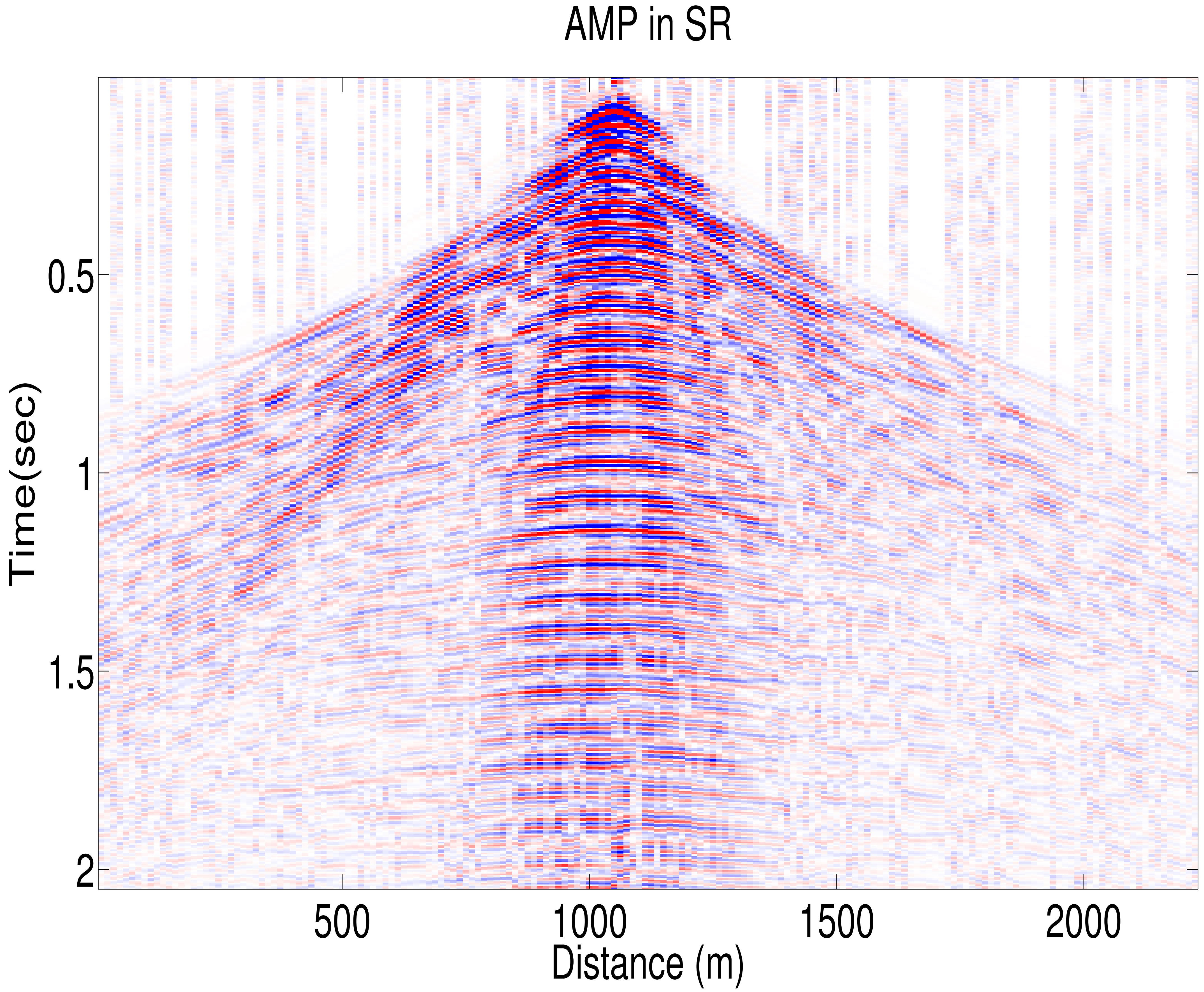


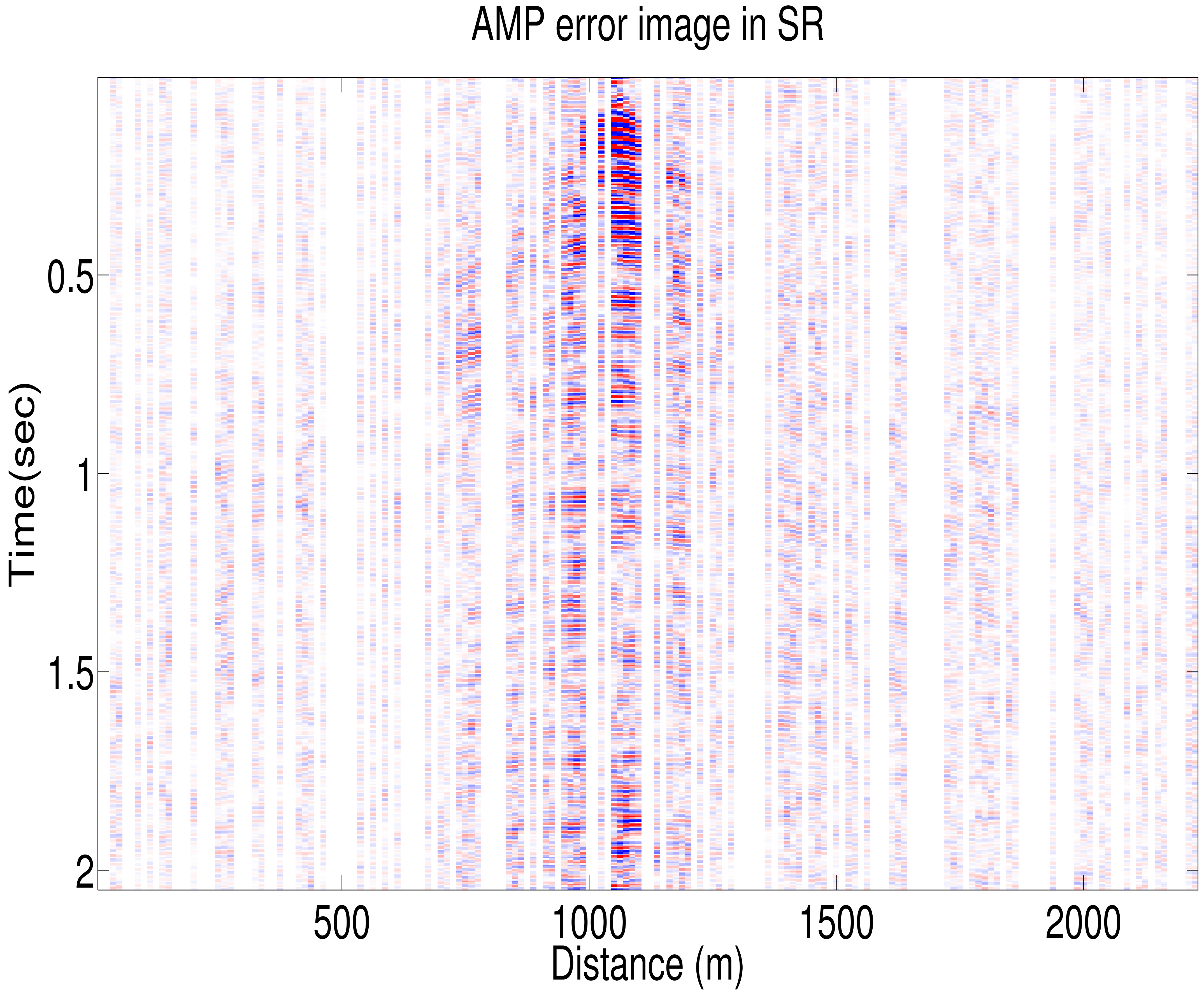
Weighted ℓ_1 minimization



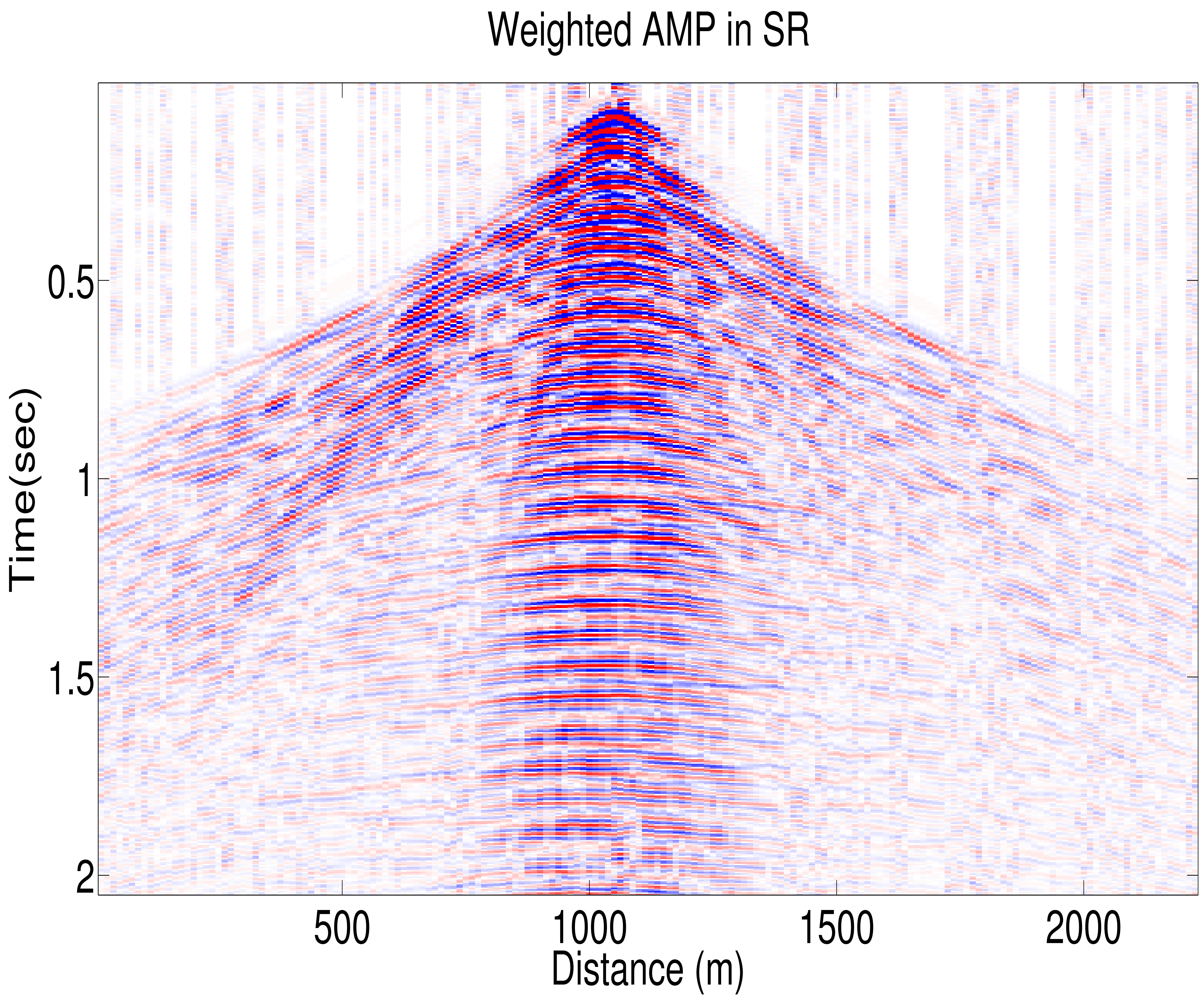


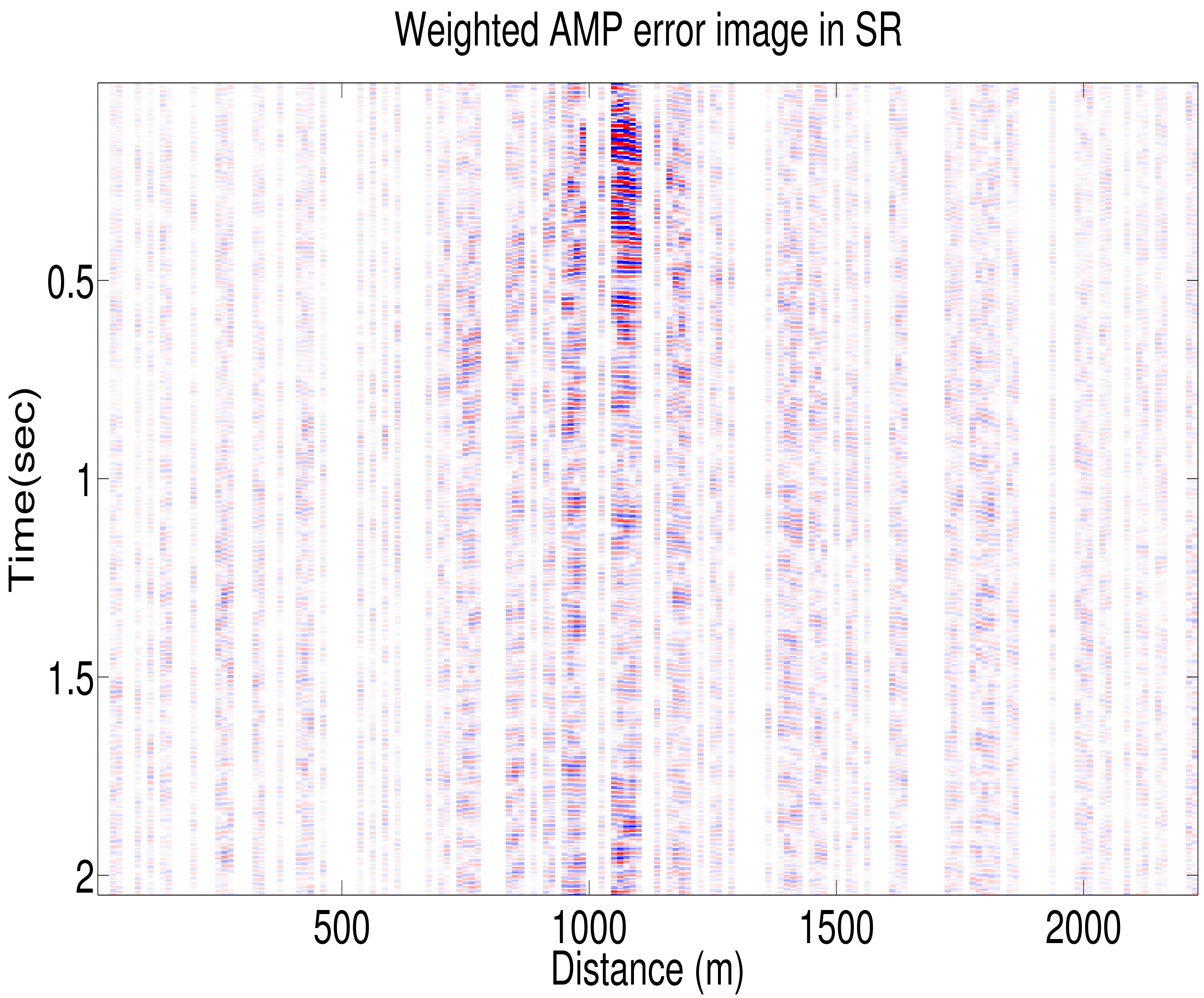
AMP


AMP in SR
Time(sec)
0.5
1
1.5
2
500
1000
1500
2000
Distance (m)

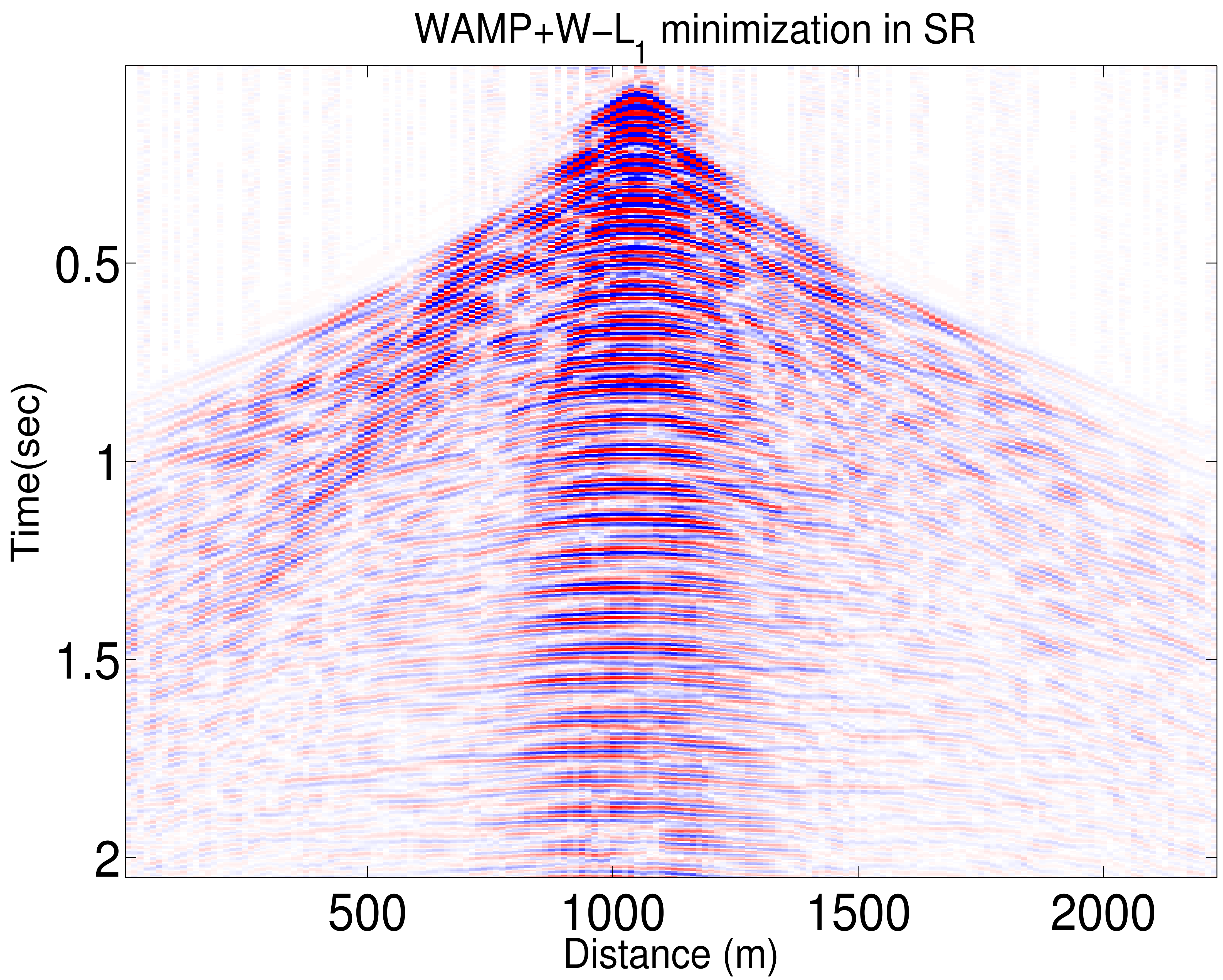

AMP error image in SR
Time(sec)
0.5
1
1.5
2
500
1000
1500
2000
Distance (m)

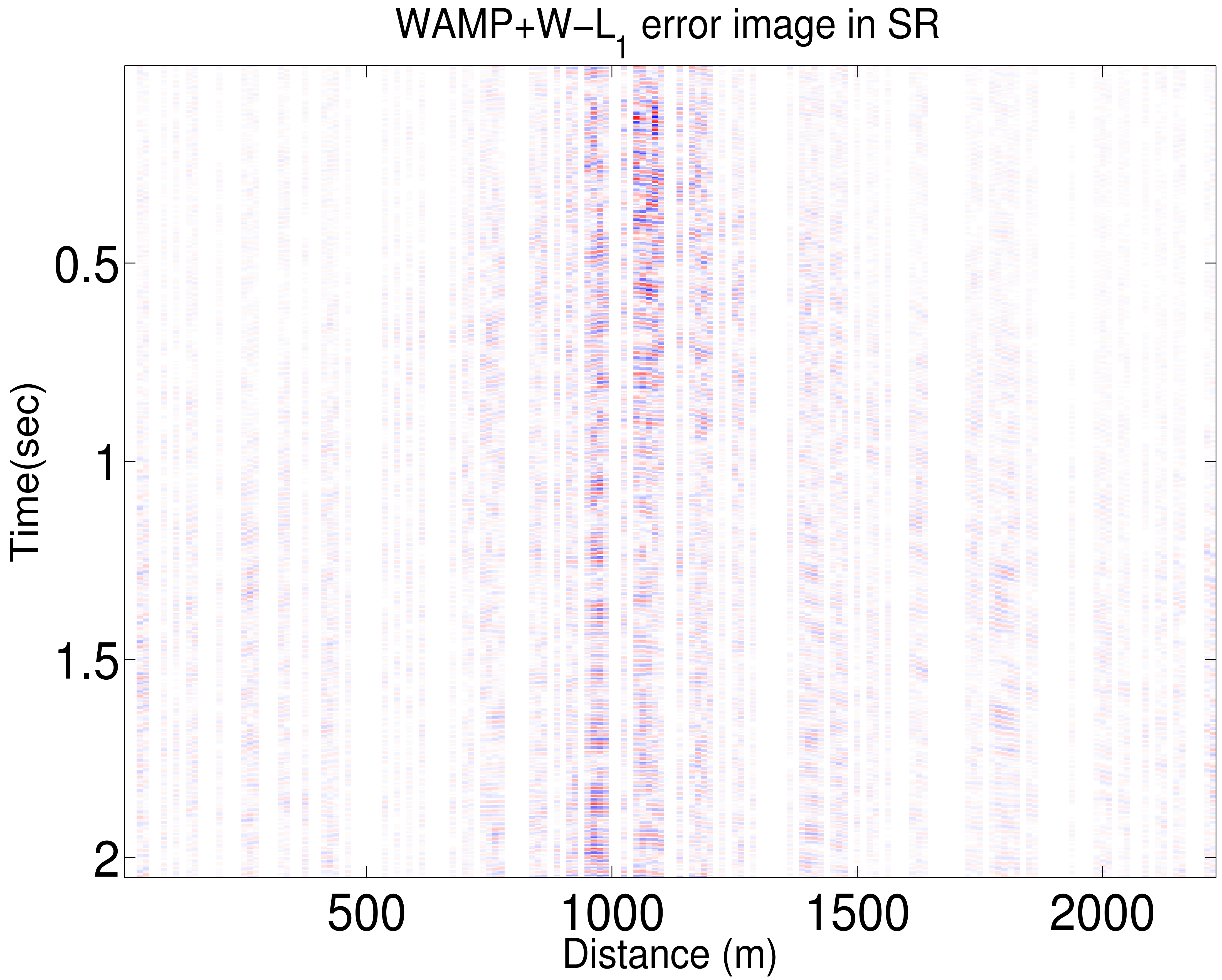
Weighted AMP



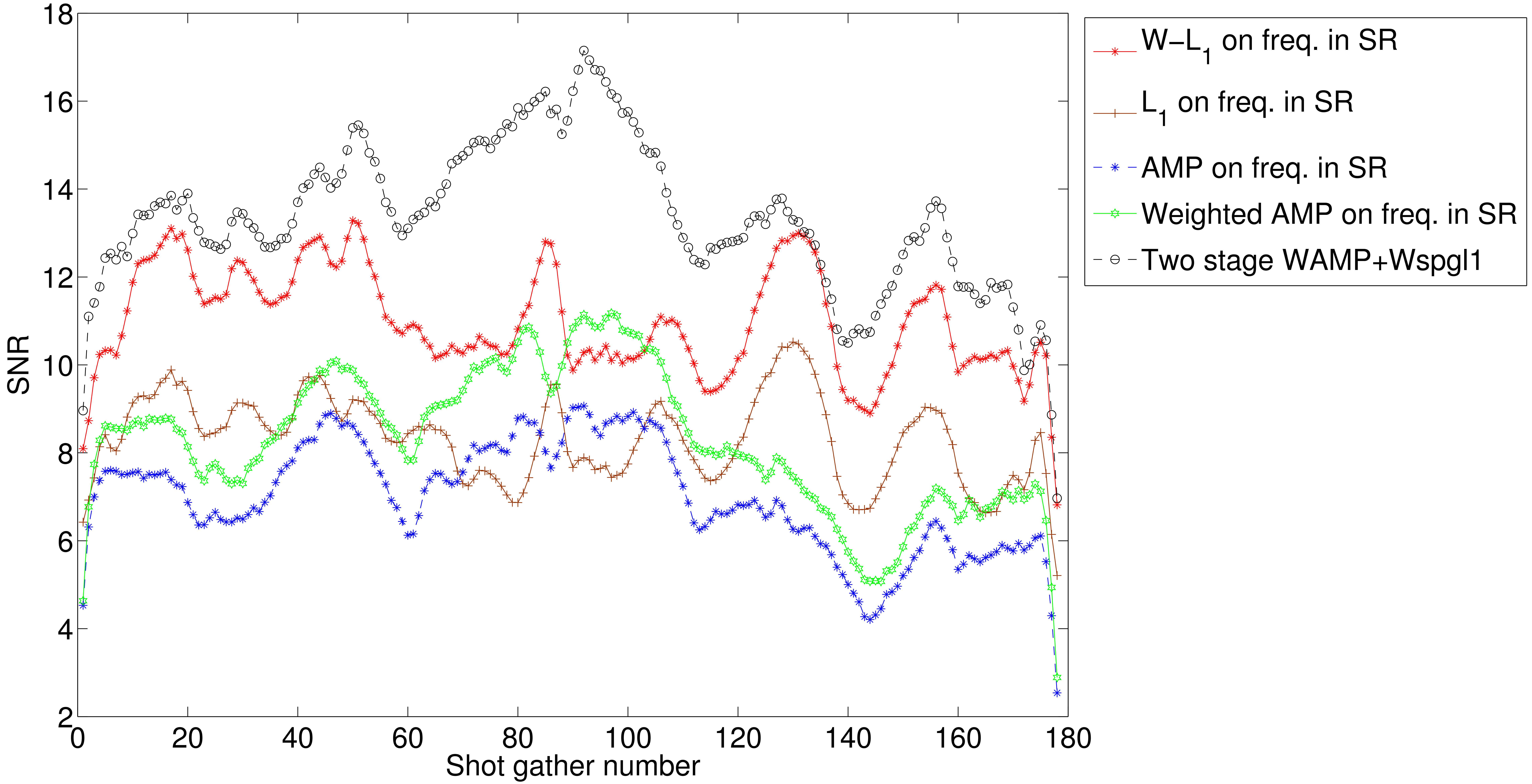


Weighted AMP+Weighted ℓ_1





Shotgather SNRs



SLIM

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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