Parallel 3D FWI with *simultaneous* shots

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Motivation

1. Challenge for 3D FWI:
   1. Data is extremely large (5D data);
   2. Model is large (3D model);
   3. Solving the wave-equation is expensive.
2. Solutions:
   1. Stochastic optimization;
   2. Simultaneous shots;
   3. Randomly selected shots;
   4. Composite shots.

3D full-waveform inversion

Optimization problem:

$$\min_{\mathbf{m}} \varphi(\mathbf{m}) = \sum_{i=1}^{M} f_i(\mathbf{m}) = \sum_{i=1}^{M} \|F_i(\mathbf{m}) - \mathbf{d}_i\|_2^2 = \sum_{i=1}^{M} \|P_i A_i^{-1} \mathbf{q}_i - \mathbf{d}_i\|_2^2$$

where

$$A_i := \Delta + \mathbf{m}\omega_i^2$$

$\mathbf{m}$ - model (squared slowness);
$\mathbf{d}_i$ - i^{th} observed data;
F_i - i^{th} forward operator;
P_i - i^{th} projection operator;
$\mathbf{q}_i$ - i^{th} source.

3D full-waveform inversion

Optimization method:

- Gradient descent method

  search direction: $\mathbf{s} = -\mathbf{g}$.

- Gauss-Newton method

  search direction: $\mathbf{s} = -[\mathbf{J}^*\mathbf{J}]^{-1}\mathbf{g}$.

- Quasi Newton method (l-bfgs)

  search direction: $\mathbf{s} = -\tilde{\mathbf{H}}^{-1}\mathbf{g}$, $\tilde{\mathbf{H}}^{-1}$ is the low-rank approximation of $\mathbf{H}^{-1}$.

Stochastic optimization

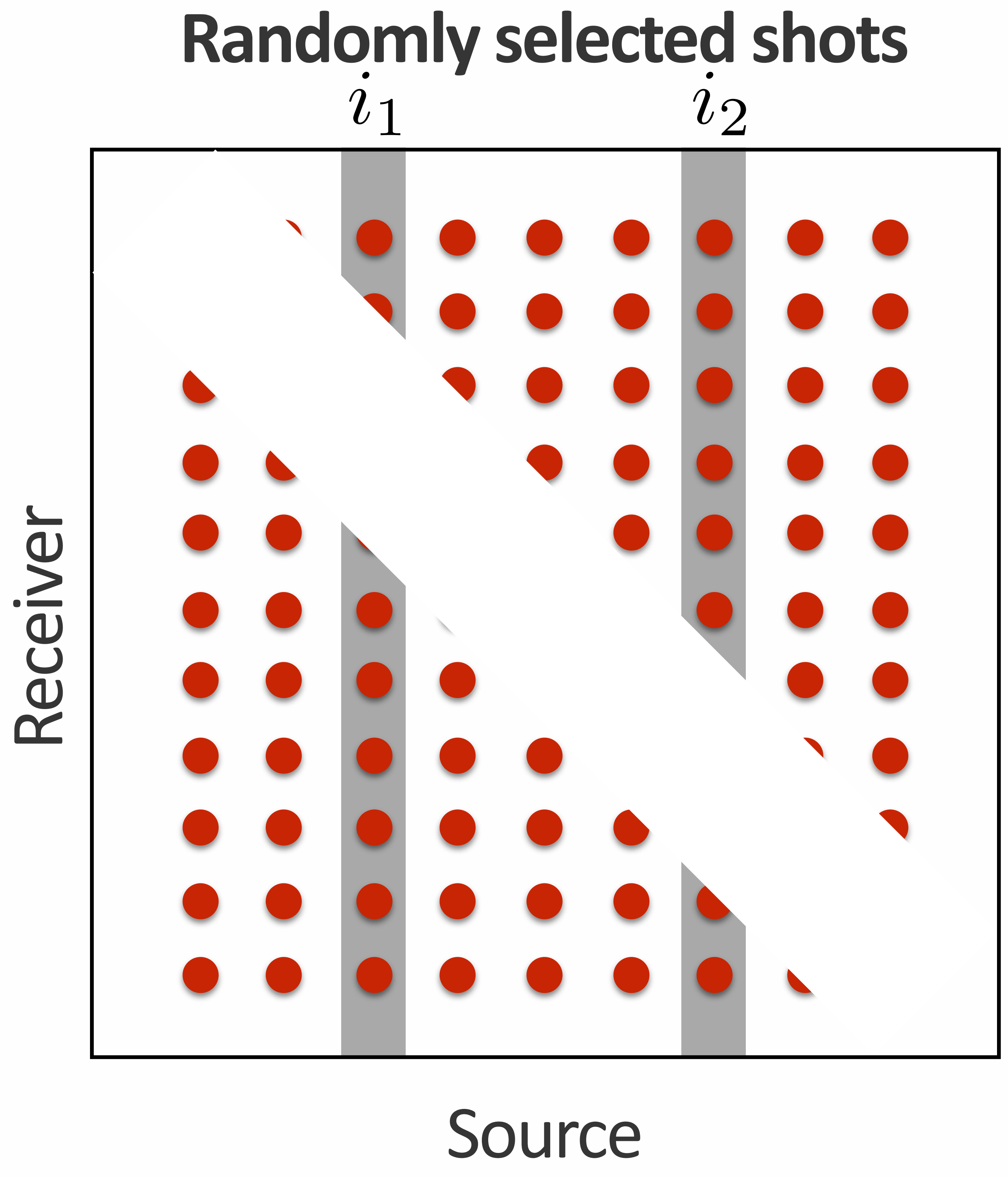
	Deterministic optimization	Stochastic optimization	
Object function	$\min_{\mathbf{m}} \varphi(\mathbf{m}) = \frac{1}{M}\sum_{i=1}^{M} f_i(\mathbf{m})$	$\min_{\mathbf{m}} \overline{\varphi}(\mathbf{m}) = \frac{1}{\lVert\mathcal{I}\rVert}\sum_{i\in\mathcal{I}} f_i(\mathbf{m})$	$\min_{\mathbf{m}} \overline{\varphi}(\mathbf{m}) = \frac{1}{M_s}\sum_{i=1}^{M_s} \tilde{f}_i(\mathbf{m})$
Gradient	$g(\mathbf{m}) = \frac{1}{M}\sum_{i=1}^{M} g_i(\mathbf{m})$	$\overline{g}(\mathbf{m}) = \frac{1}{\lVert\mathcal{I}\rVert}\sum_{i\in\mathcal{I}} g_i(\mathbf{m})$	$\overline{g}(\mathbf{m}) = \frac{1}{M_s}\sum_{i=1}^{M_s} \tilde{g}_i(\mathbf{m})$
Size of batch	M	$\#$ of $\mathcal{I}$	M_s

Michael P. Friedlander, 2012

Tristan van Leeuwen, 2013

Randomly selected shots

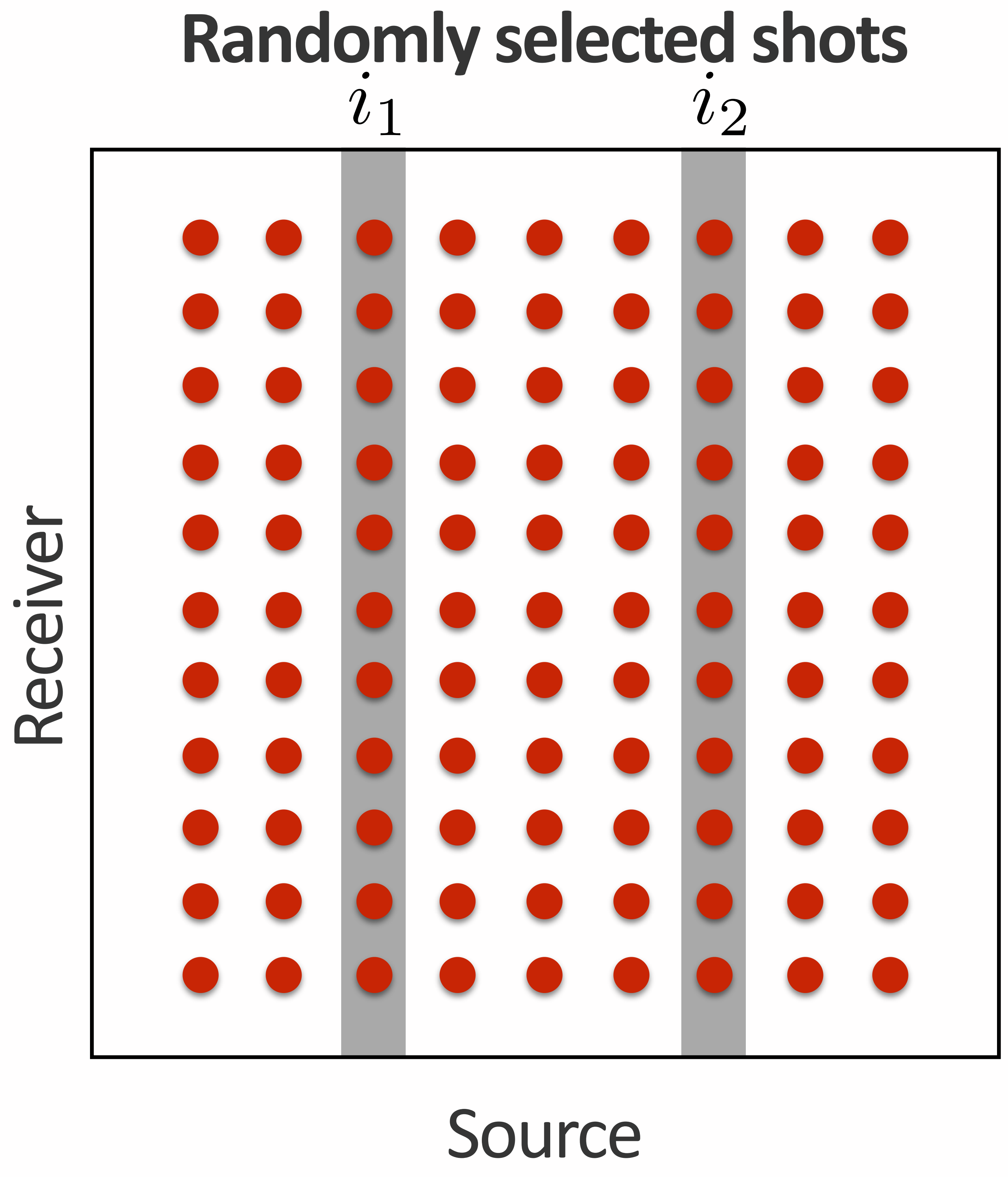
Ocean bottom acquisition:

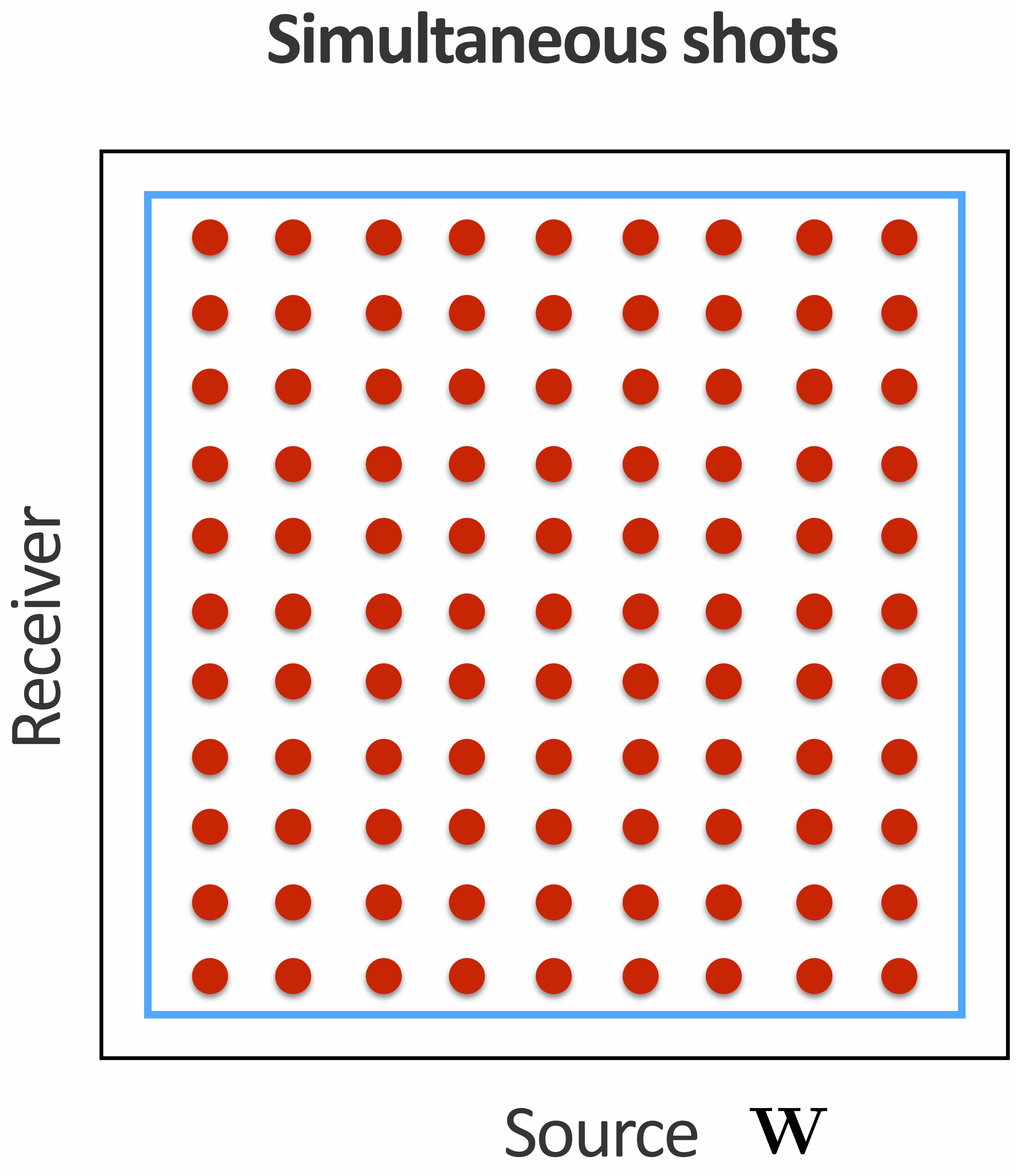


$$\min_{\mathbf{m}} \overline{\varphi}(\mathbf{m}) = \sum_{i \in \mathcal{I}} \|F_i(\mathbf{m}) - \mathbf{d}_i\|_2^2 = \sum_{i \in \mathcal{I}} \|P_i A_i^{-1} \mathbf{q}_i - \mathbf{d}_i\|_2^2$$

Simultaneous shots

Ocean bottom acquisition:





$$\min_{\mathbf{m}} \overline{\varphi}(\mathbf{m}) = \sum_{j=1}^{n_f} \|PA_j^{-1}(\mathbf{m})\mathbf{Q}_j\mathbf{W}_j - \mathbf{D}_j\mathbf{W}_j\|_F^2$$

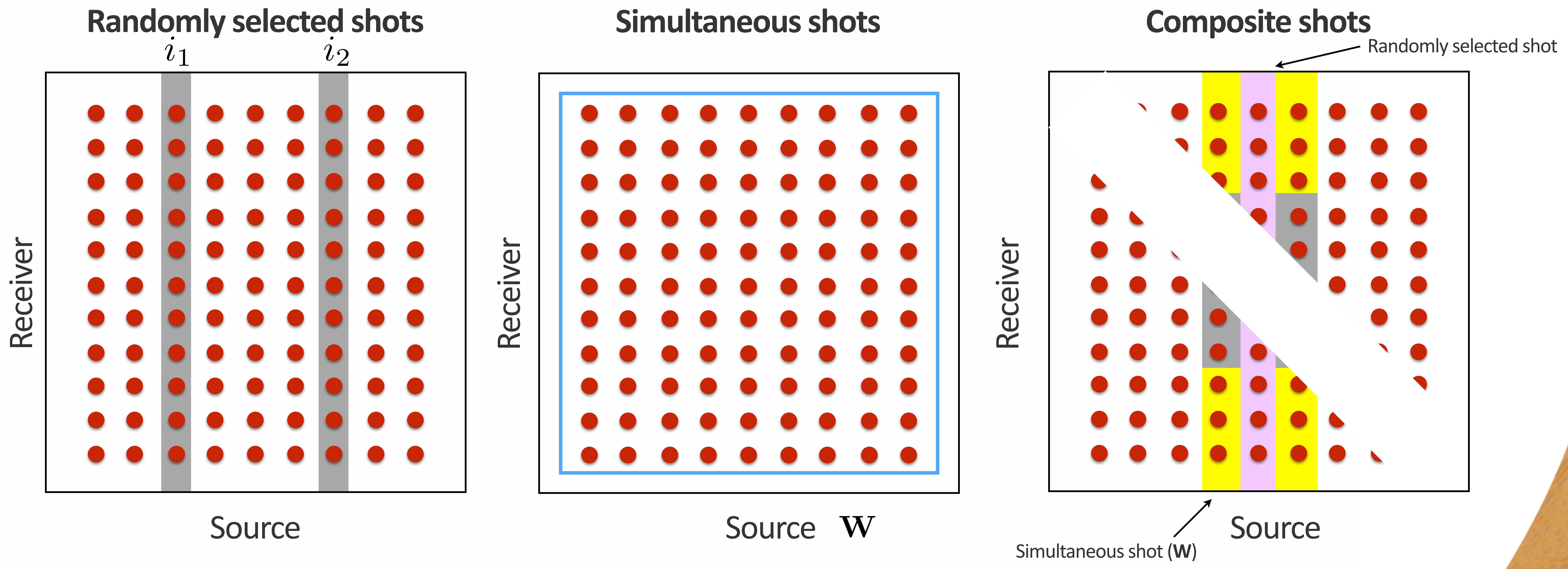
n_f - number of frequencies;
$\|\cdot\|_F$ - Frobenius norm.
$\mathbf{Q}_j = [\mathbf{q}_{j1}, ..., \mathbf{q}_{jn_j}]$;
$\mathbf{D}_j = [\mathbf{d}_{j1}, ..., \mathbf{d}_{jn_j}]$;

Randomly selected shots v.s. *simultaneous* shots

	Advantage	Disadvantage
Randomly selected shots	Can use offset *weighting*	Only use information from *subset* of shots
Simultaneous shots	*All* shots are used	All shots must share *same* receivers

Composite shots

Ocean bottom acquisition:



Composite shots

Optimization problems with composite shots:

$$\min_{\mathbf{m}} \overline{\varphi}(\mathbf{m}) = \sum_{j=1}^{n_f} \sum_{k=1}^{n_{cs}} \|P_k A_j^{-1}(\mathbf{m})\tilde{\mathbf{q}}_k - \mathbf{d}_k\|_2^2$$

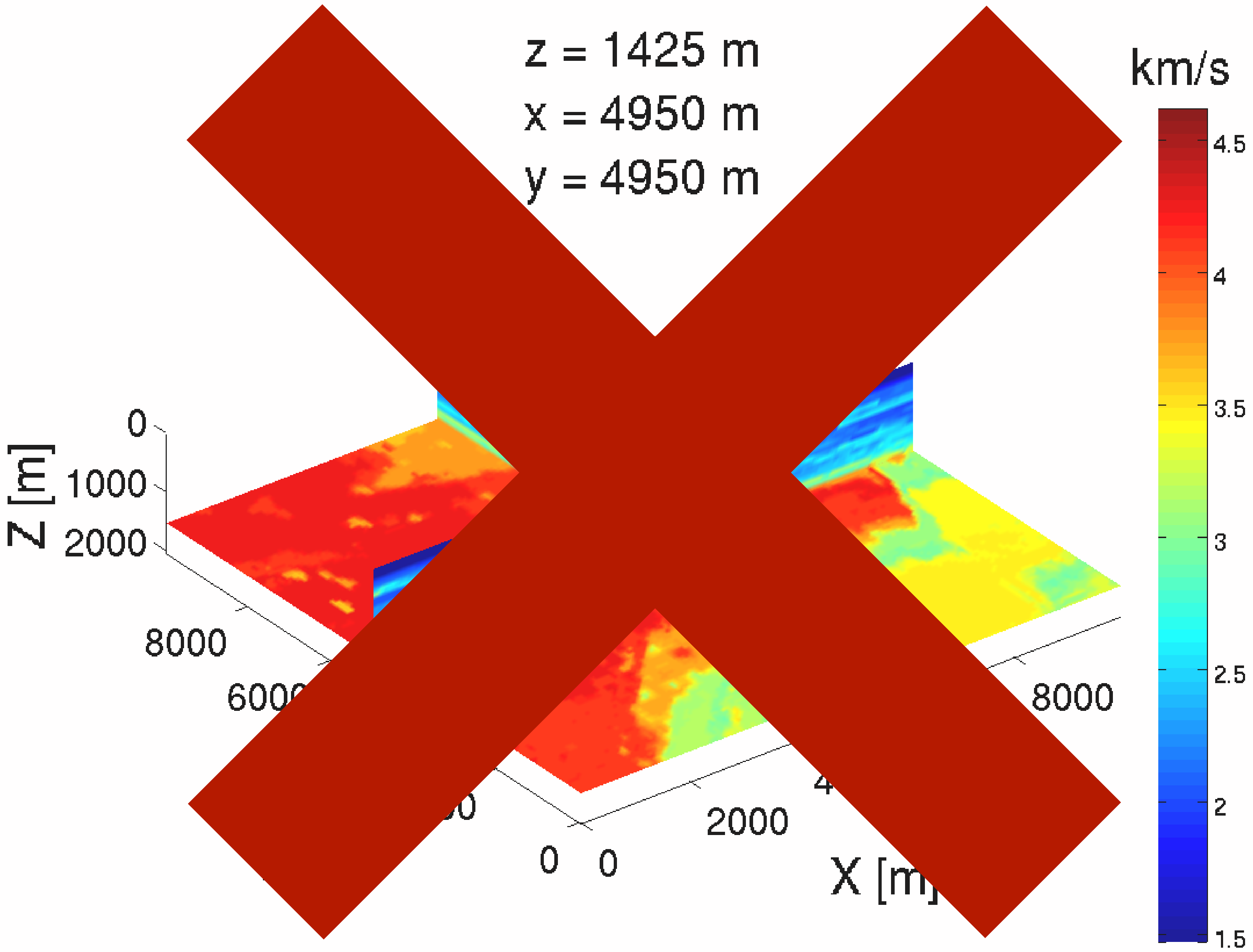
Advantage:

1. Do *not* need *all* shots to share the *same* receivers;

2. Uses *more* information from shots than *randomly* selected shots.

Numerical experiment

BG Model



True Model

Model Infomation:

model size - 28*134*134

2 * 10 *10 km

d - [75 75 75] m

# of shots - 441

# of receivers - 10201

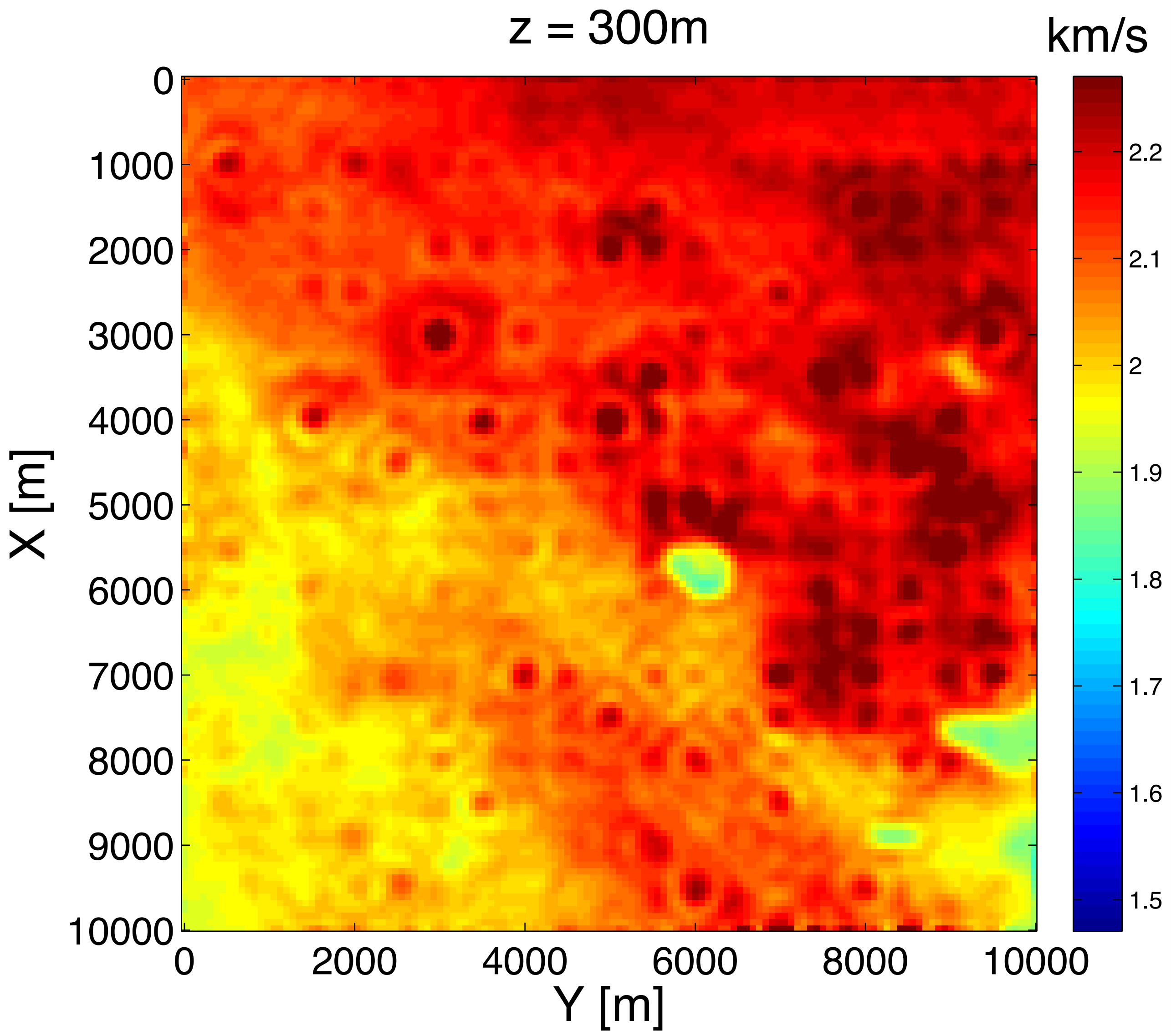
frequency - 3.2 Hz

source depth - 14 m

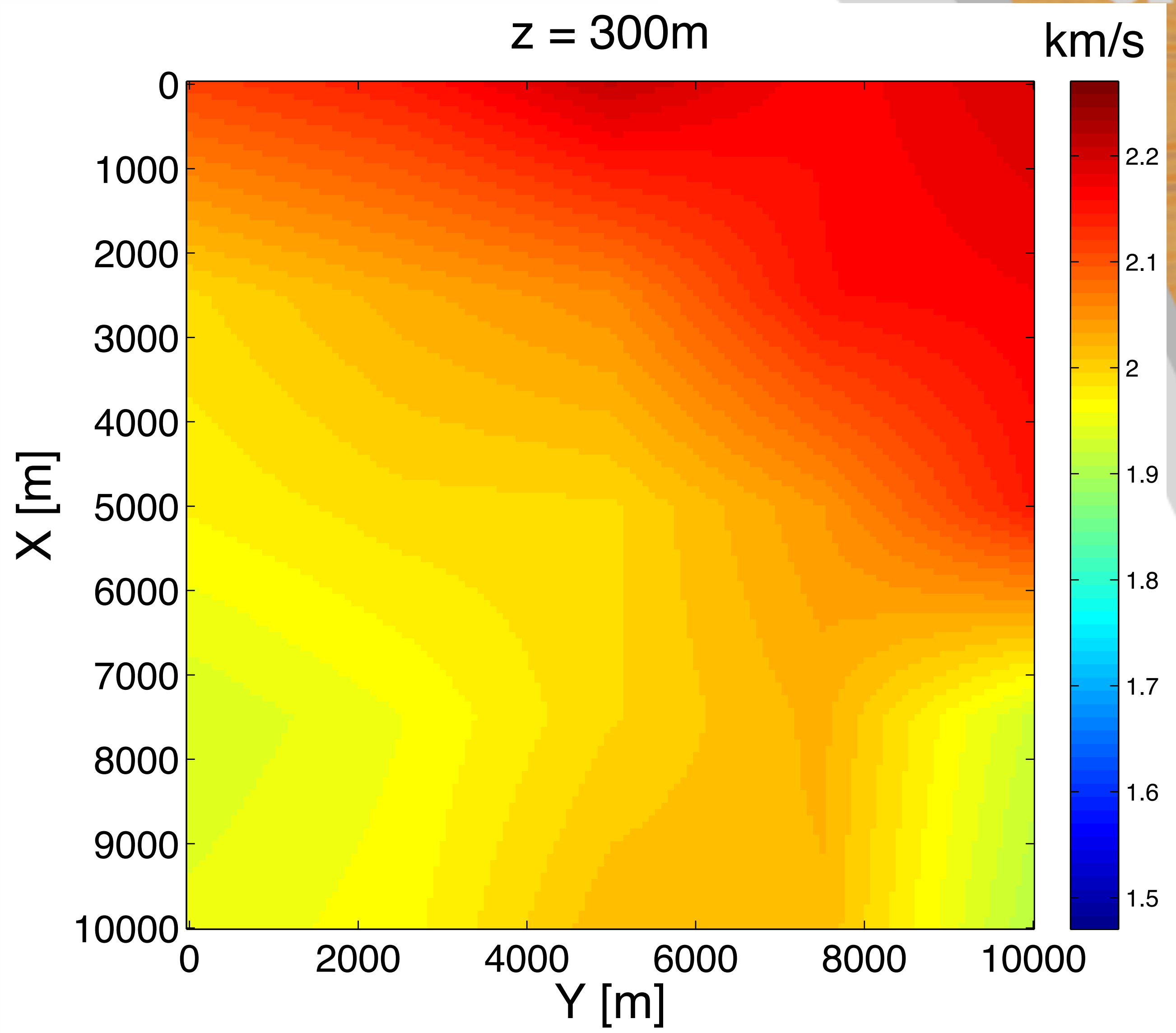
receiver depth - 14 m

l-bfgs - 34 iterations

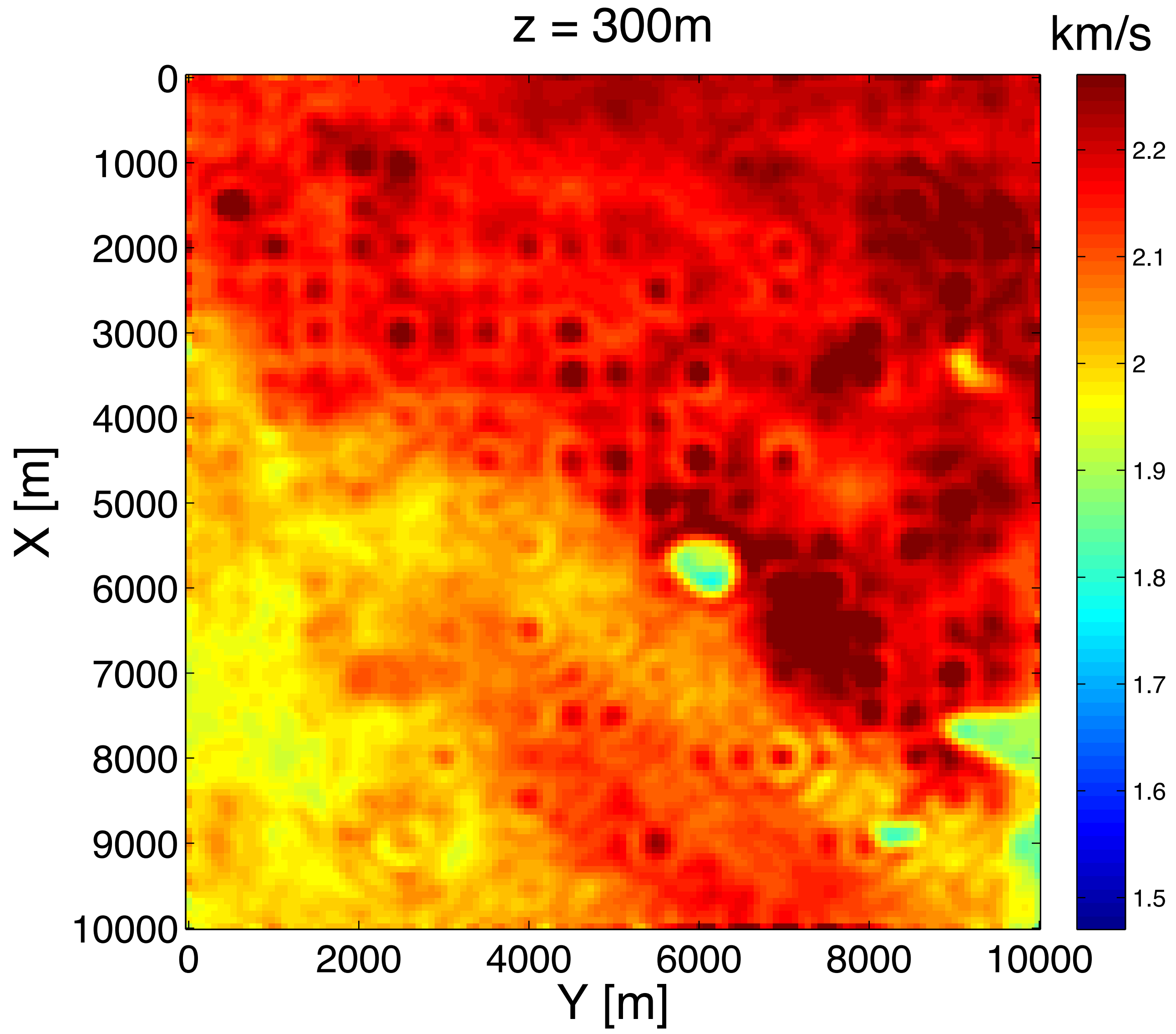
z = 300m



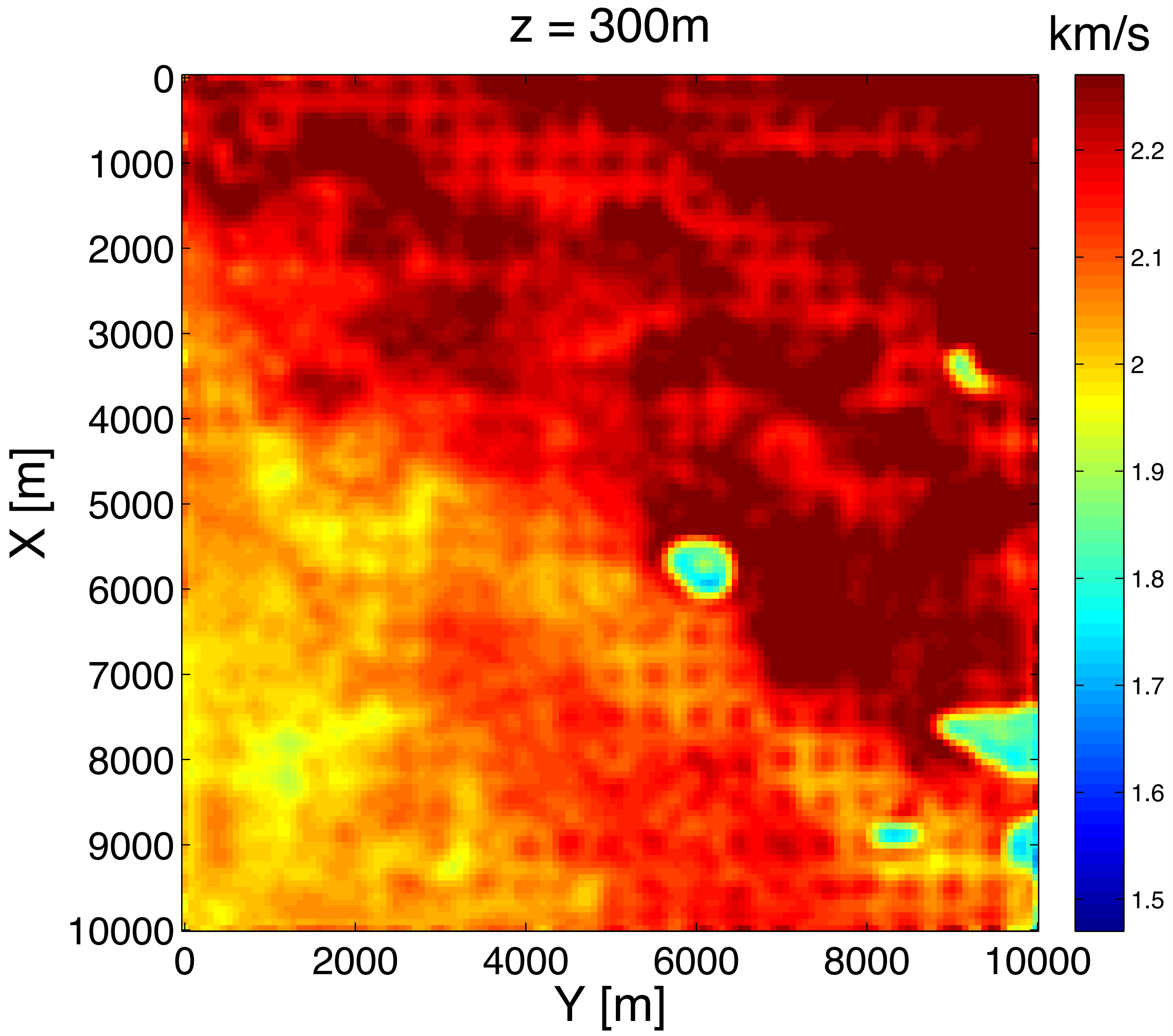
Composite shots



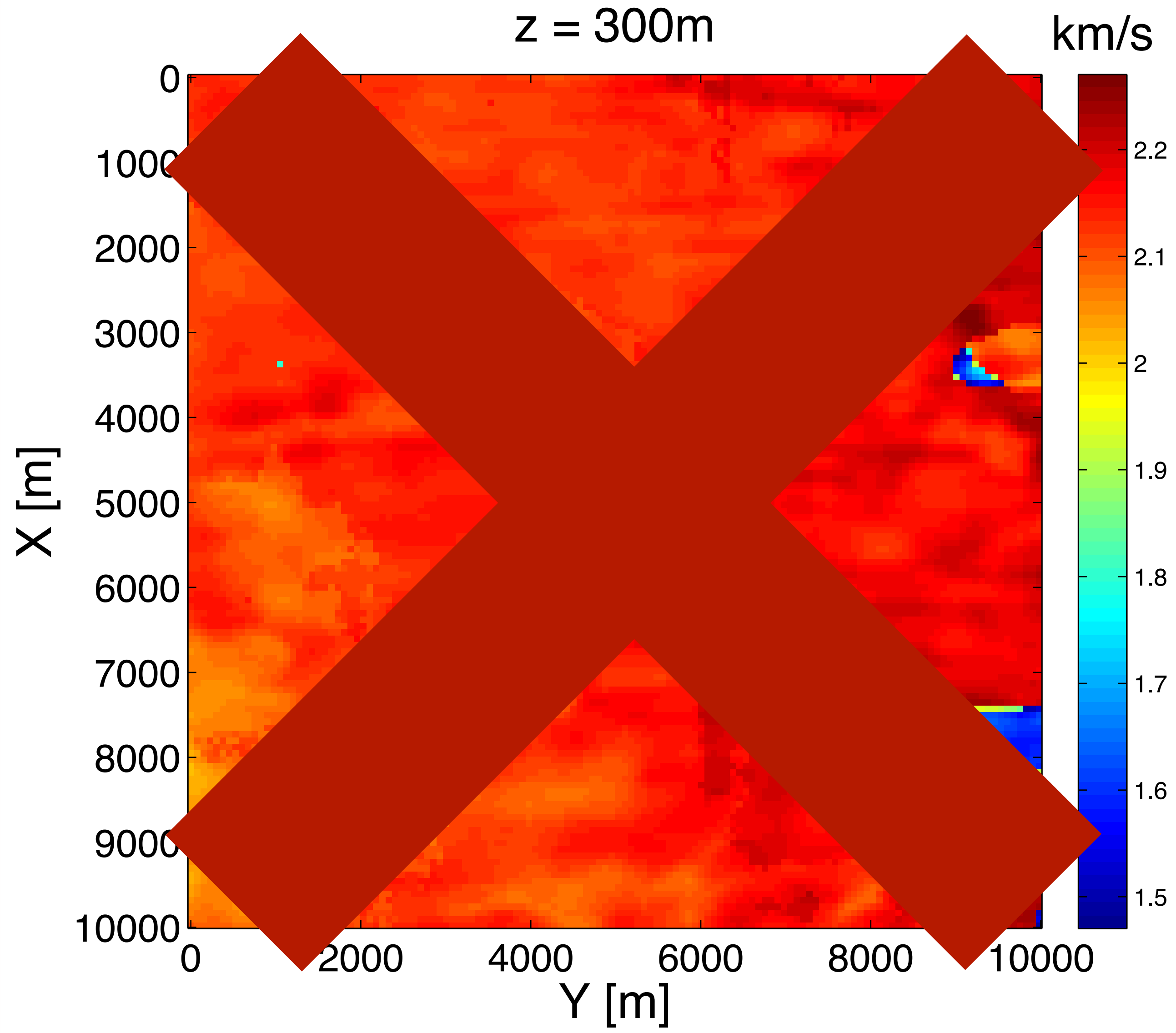
Initial model



Randomly selected shots

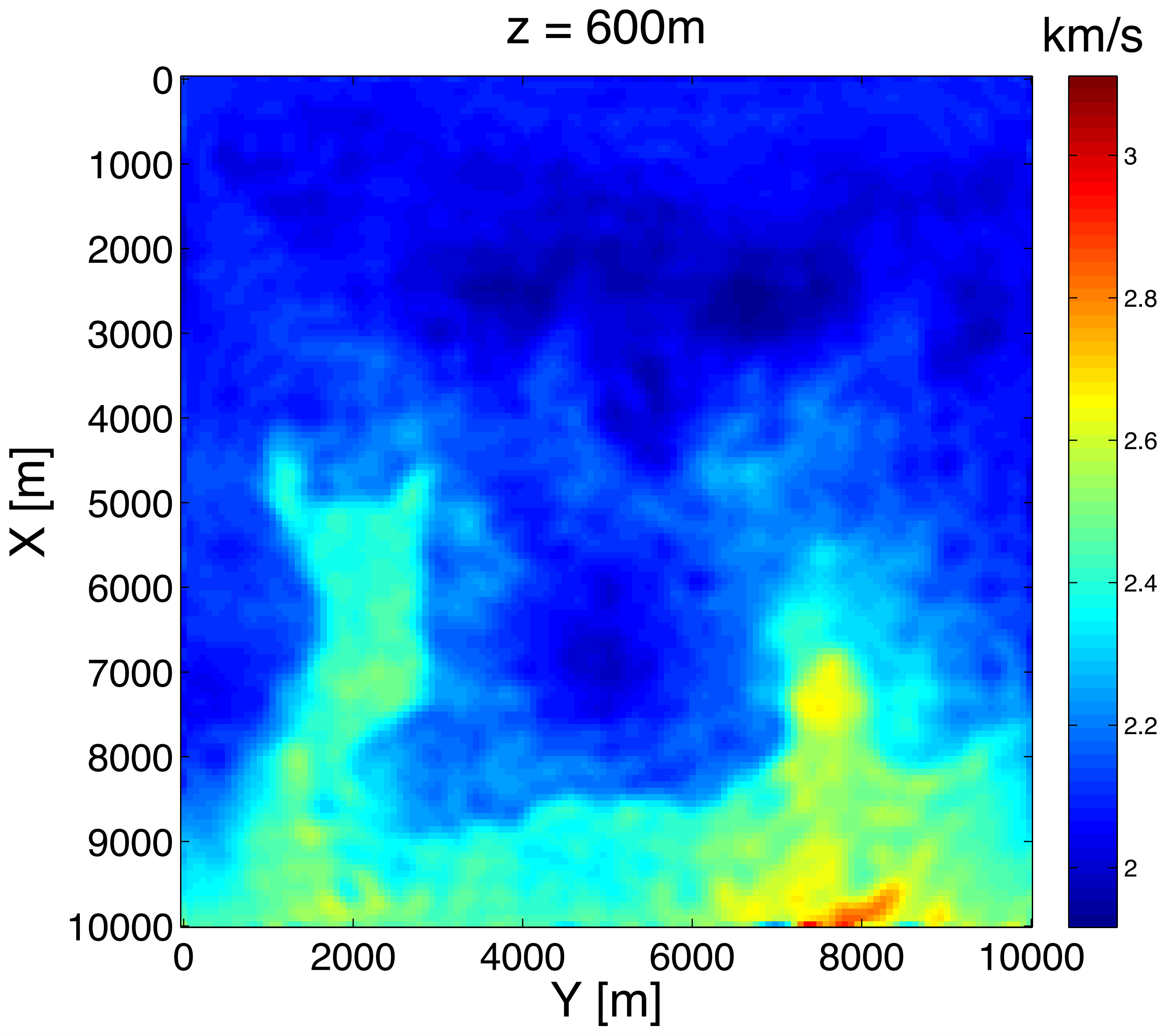


Simultaneous shots

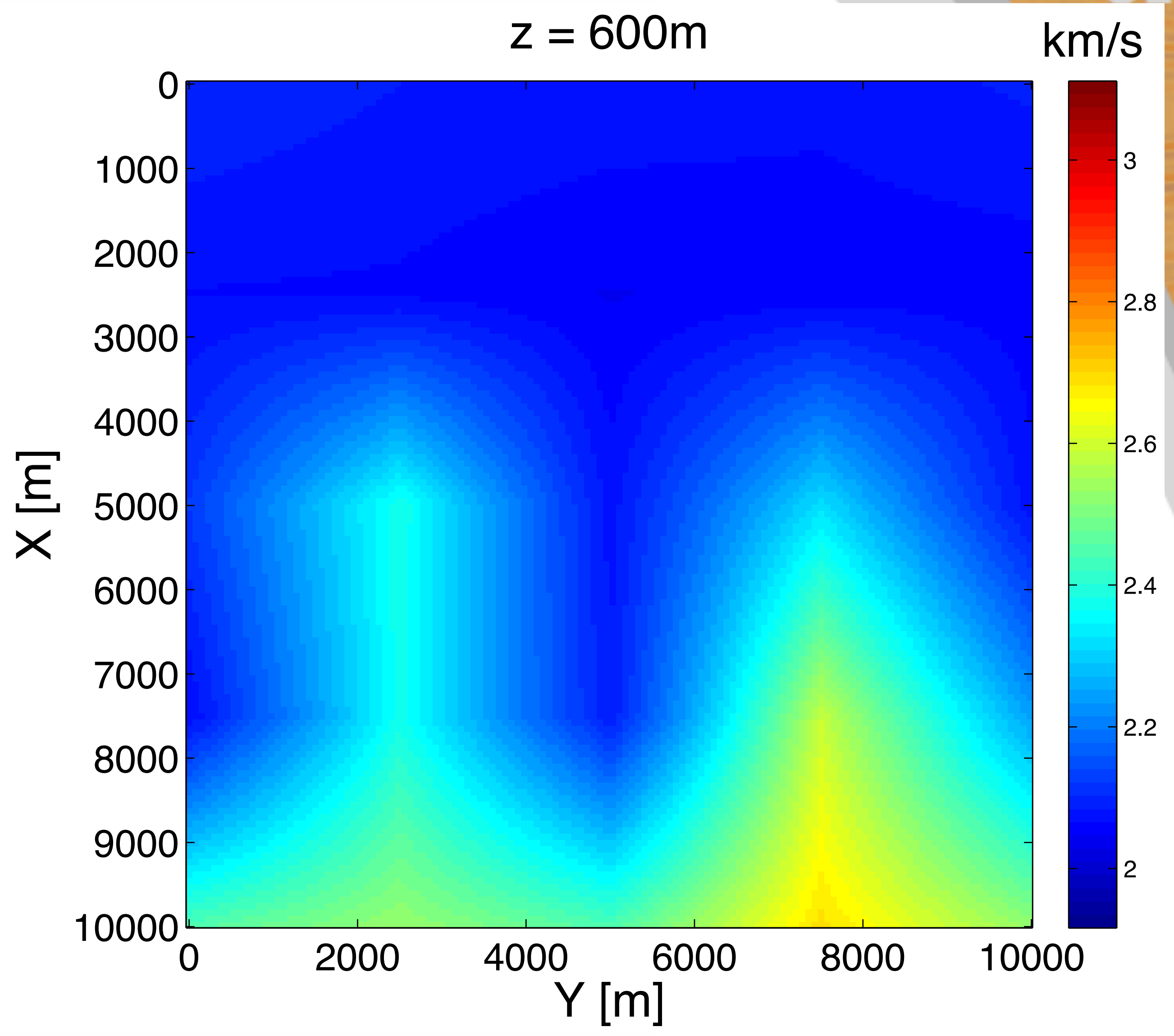


True model

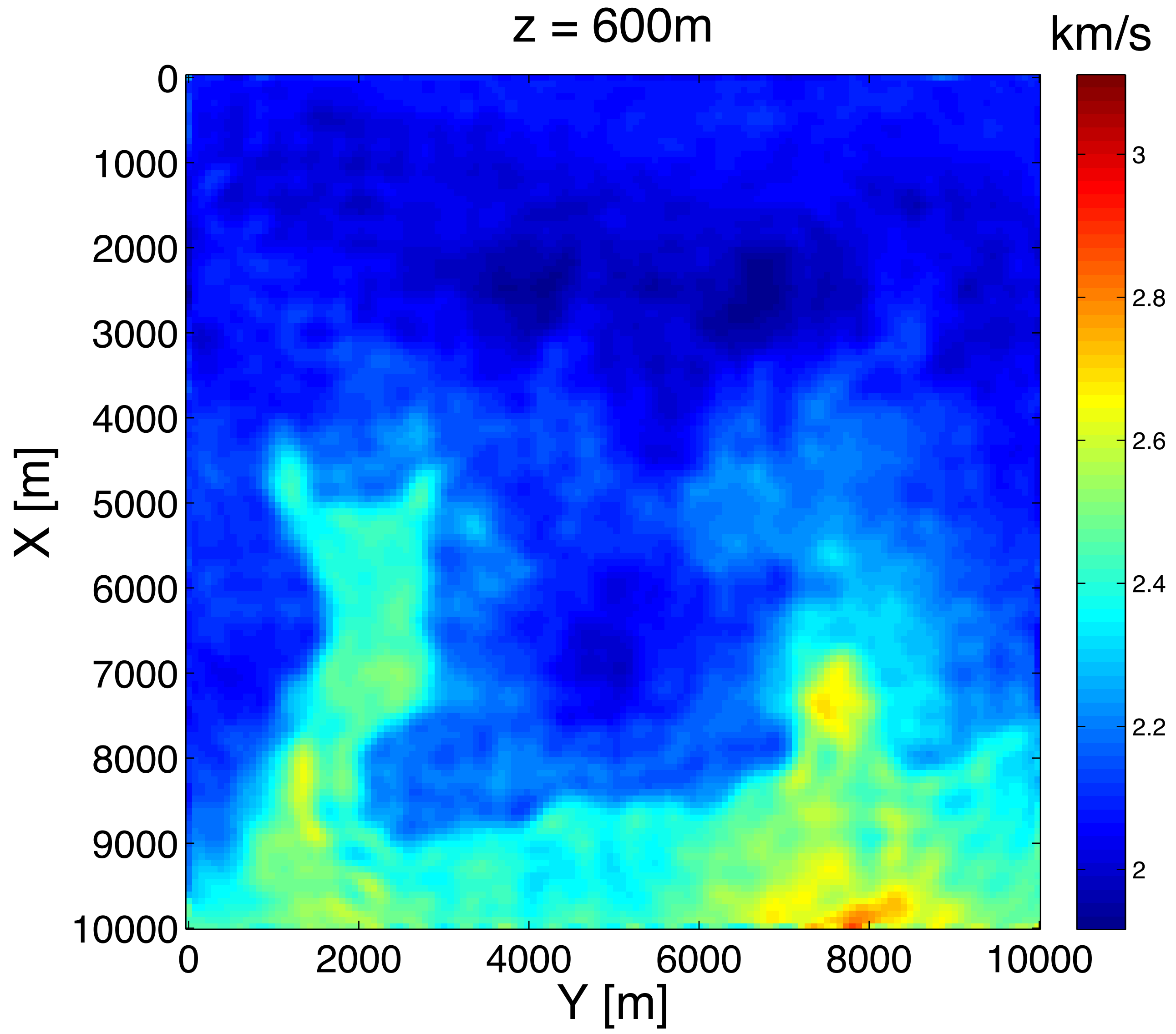
z = 600m



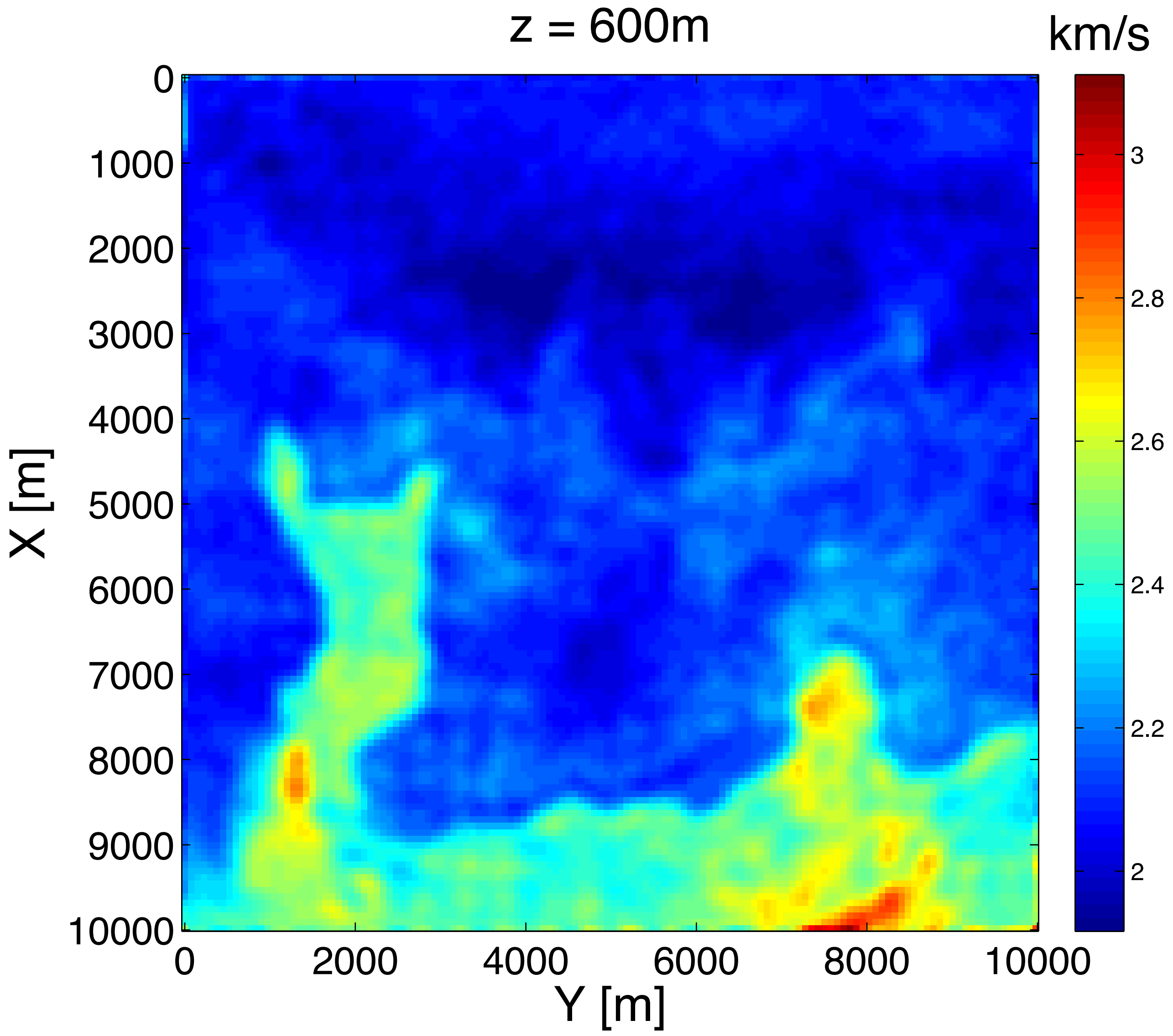
Composite shots



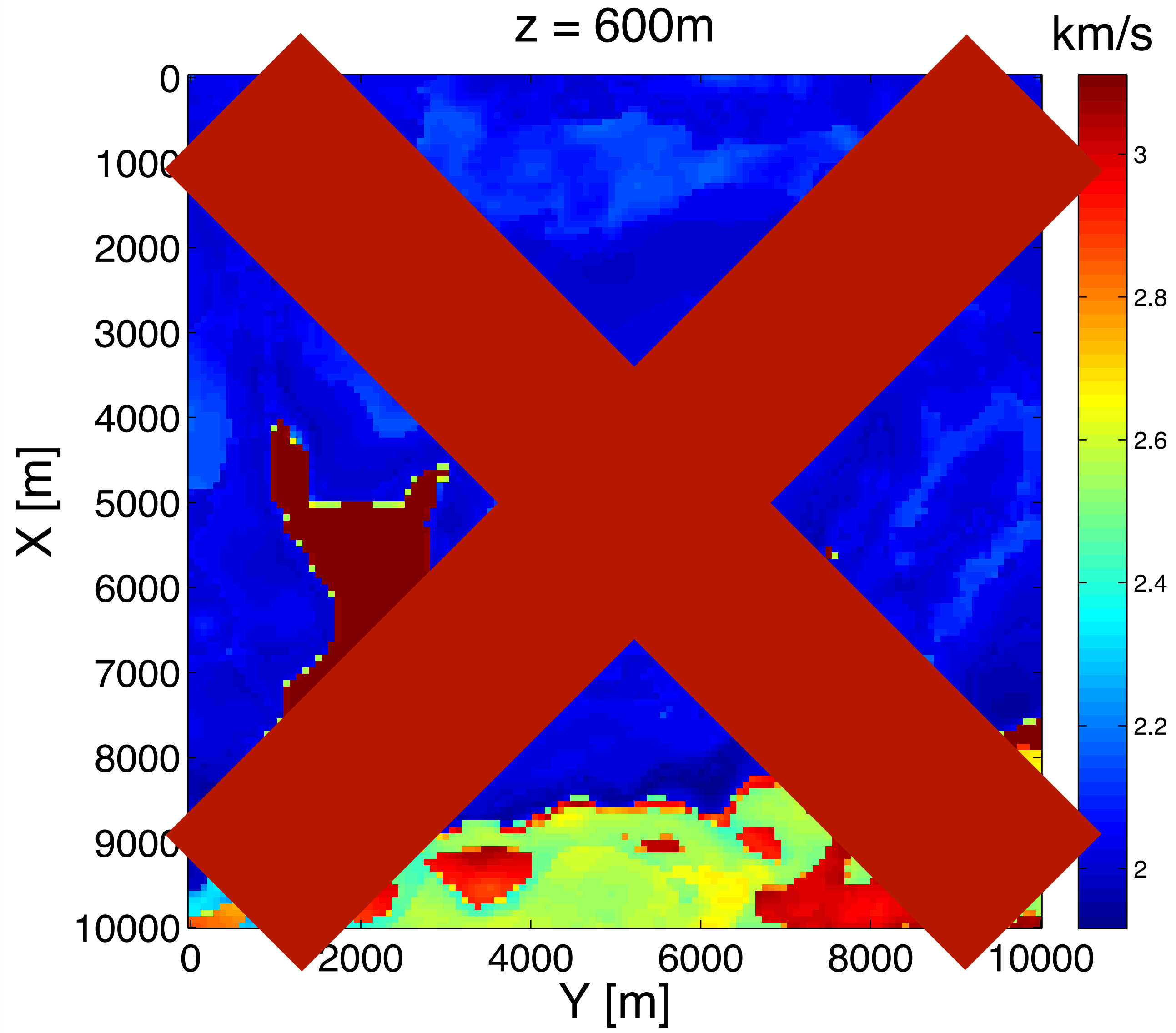
Initial model



Randomly selected shots

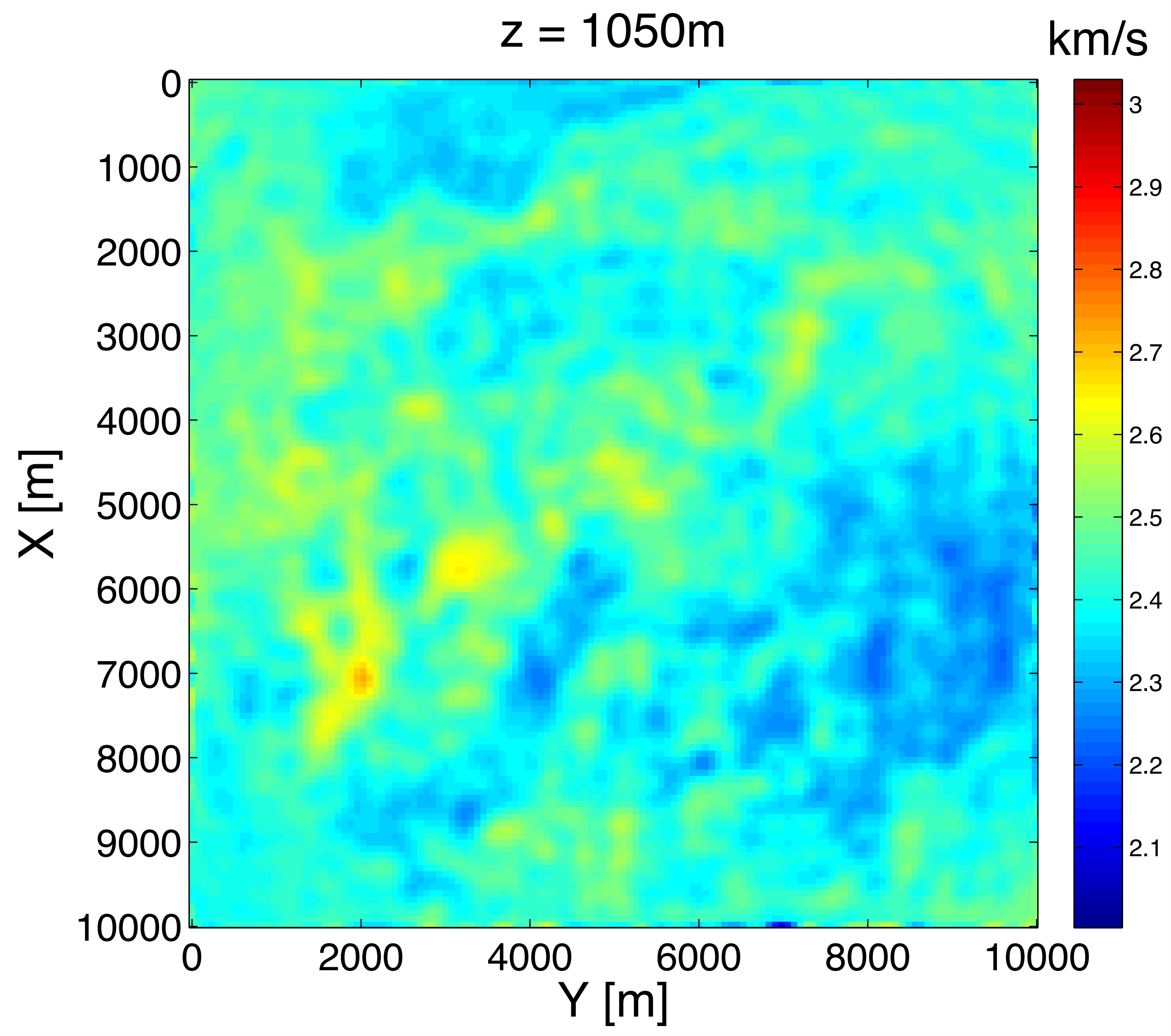


Simultaneous shots

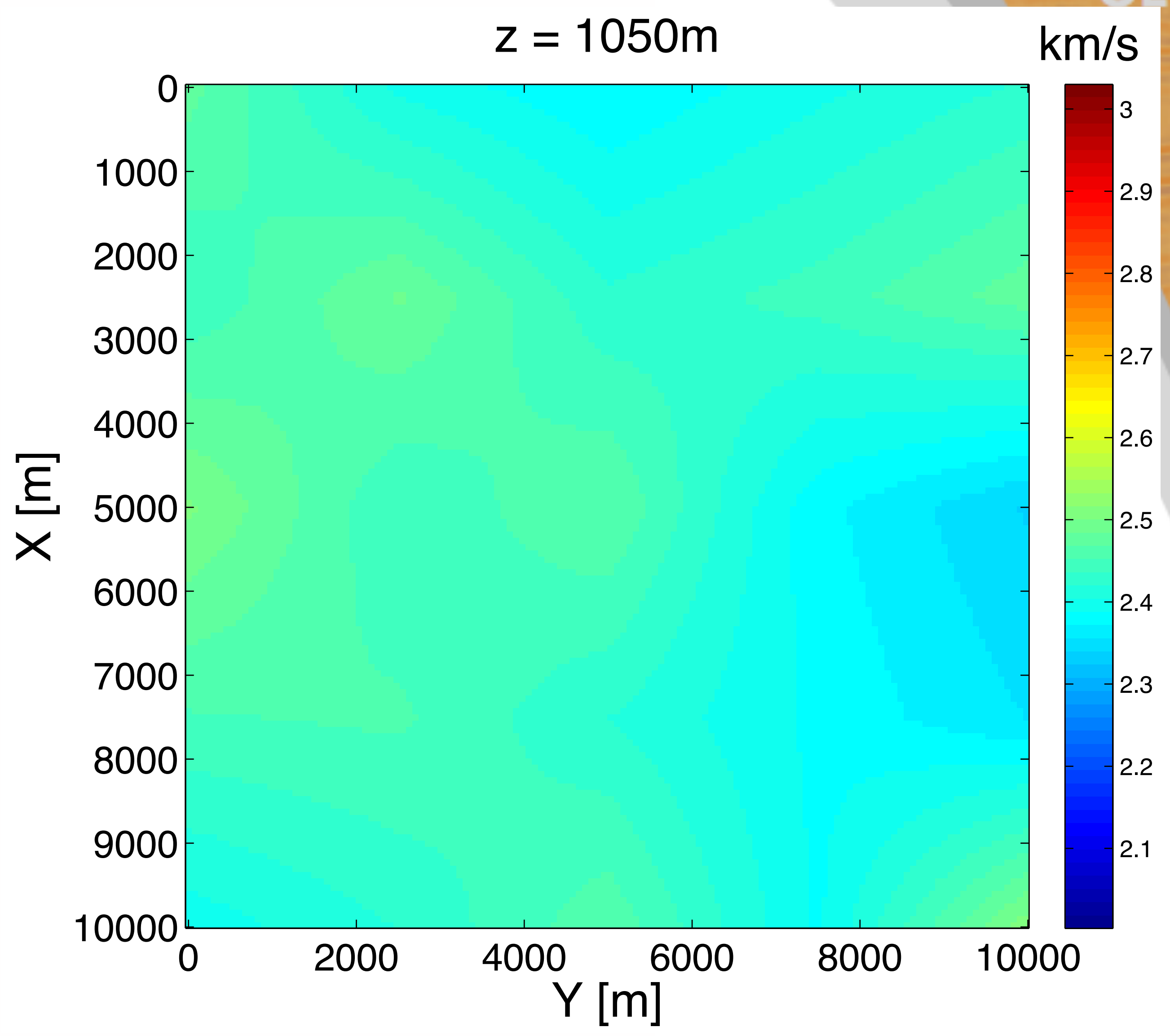


True model

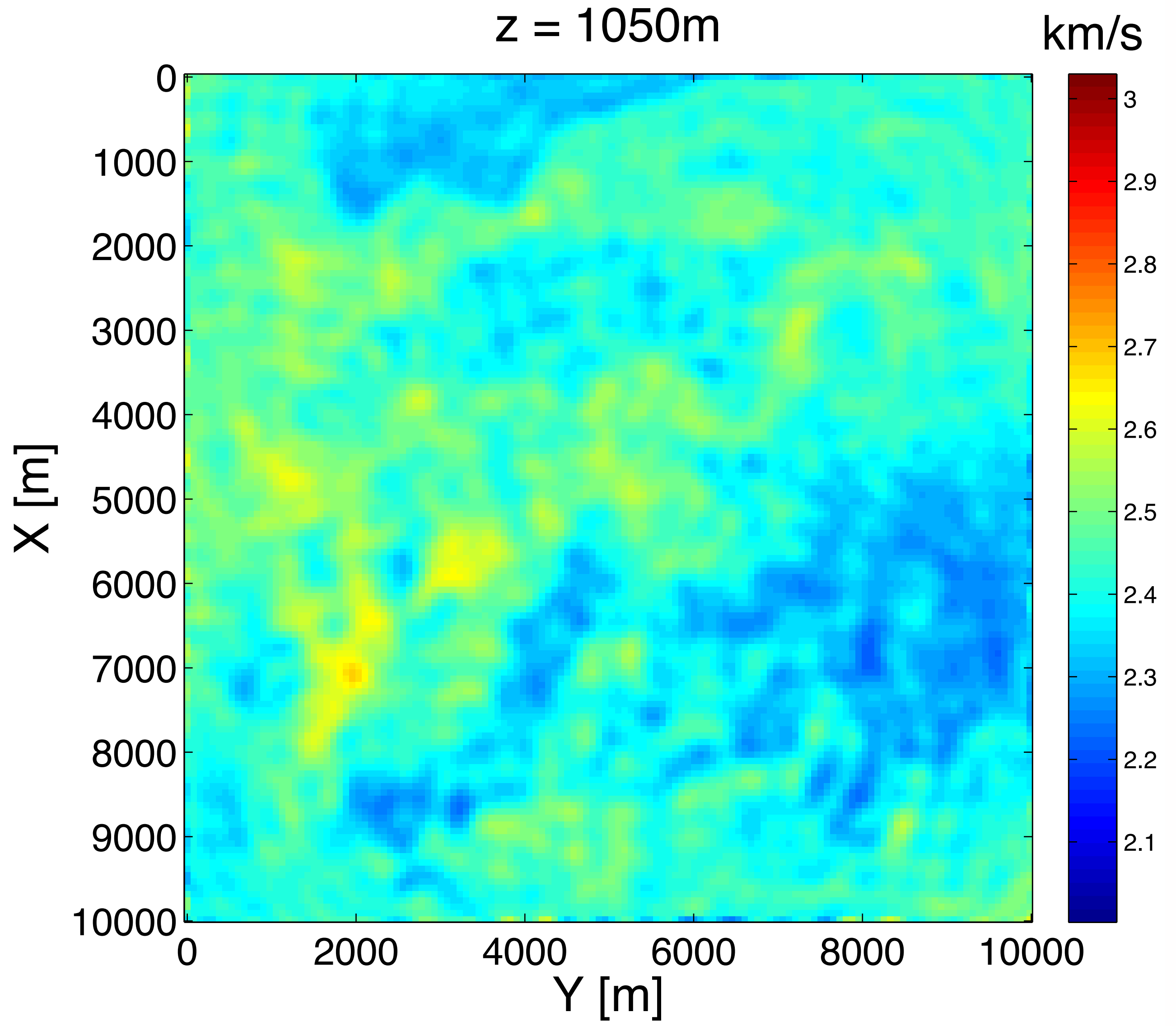
z = 1050m



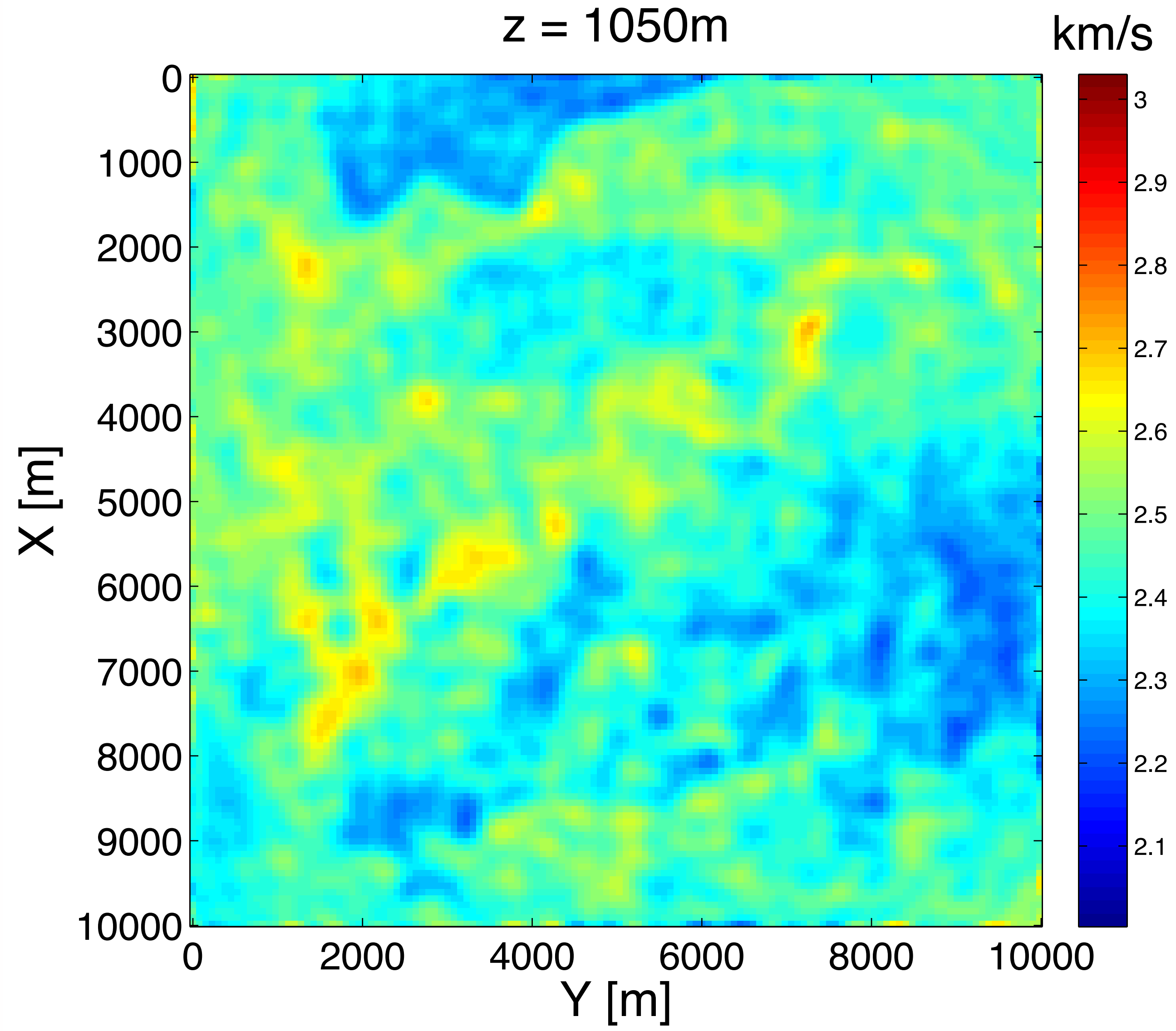
Composite shots



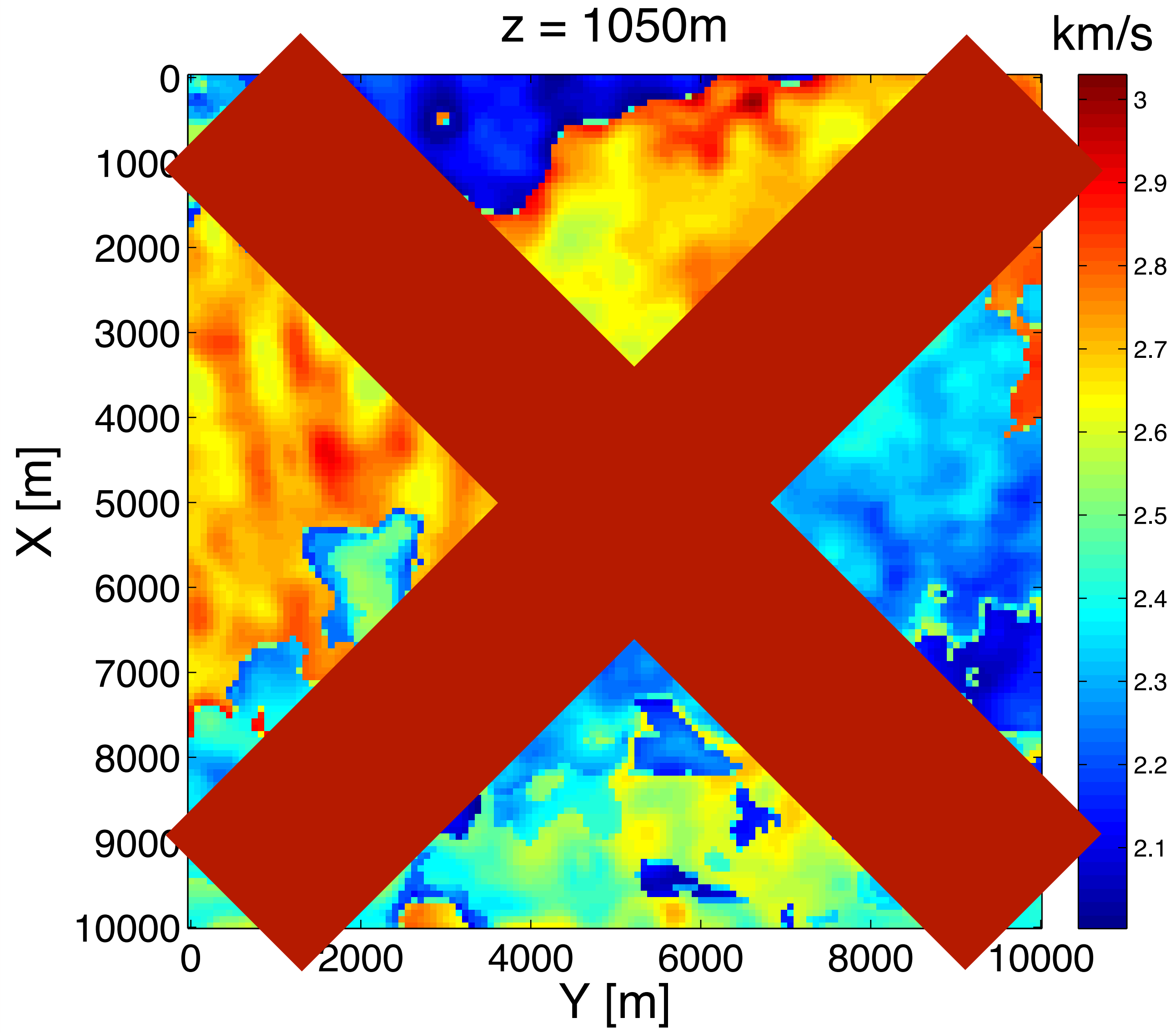
Initial model



Randomly selected shots

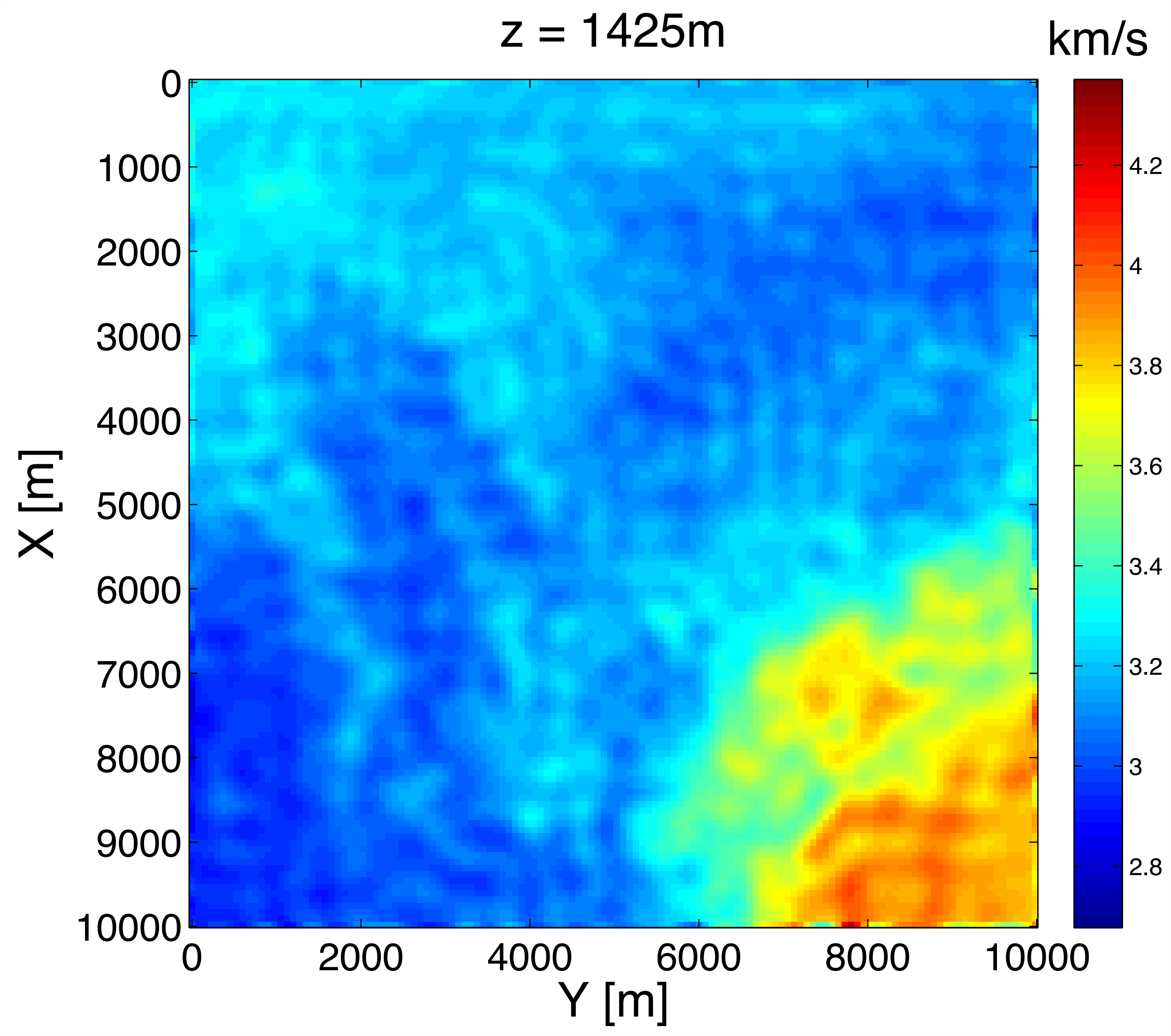


Simultaneous shots

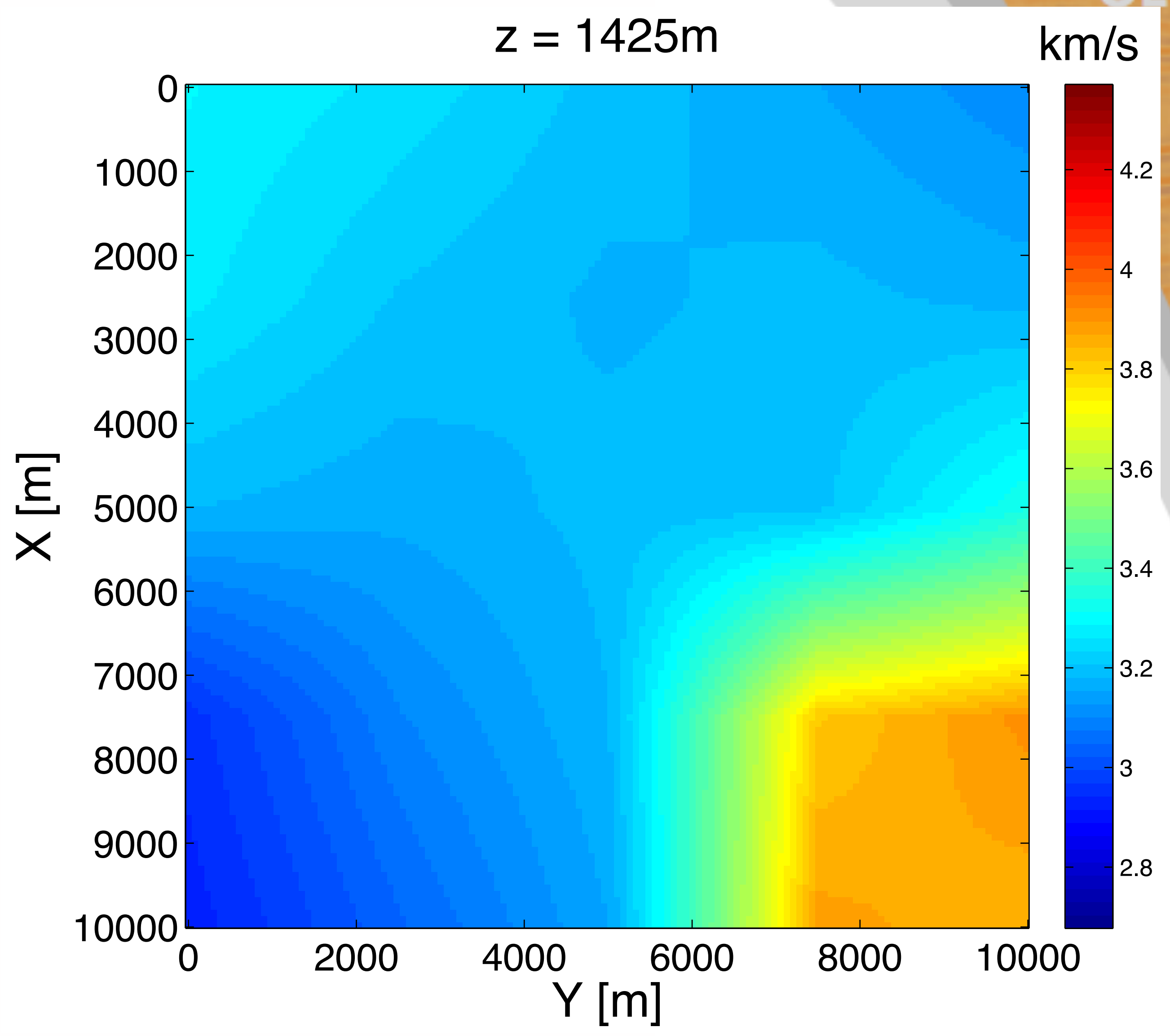


True model

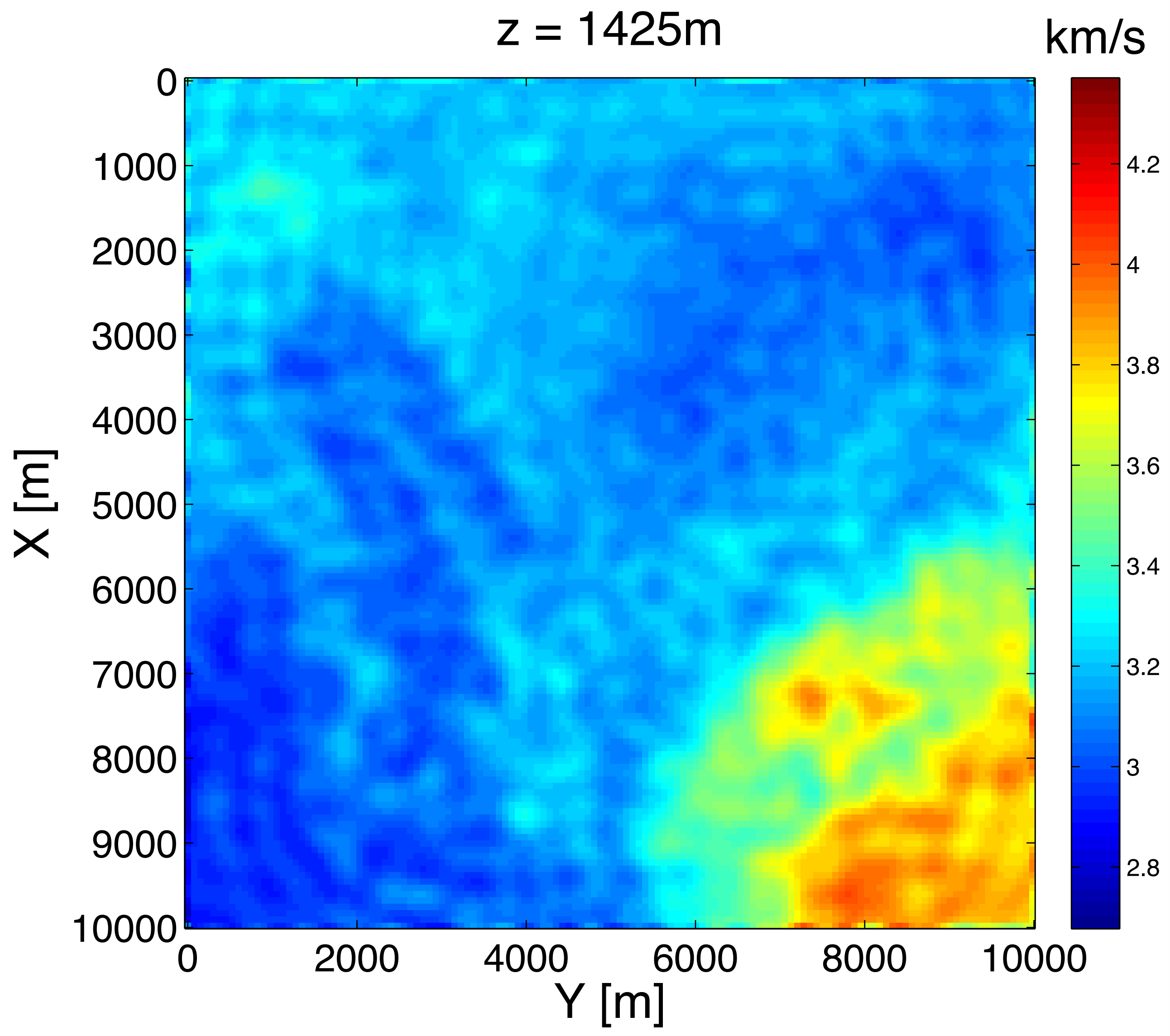
z = 1425m



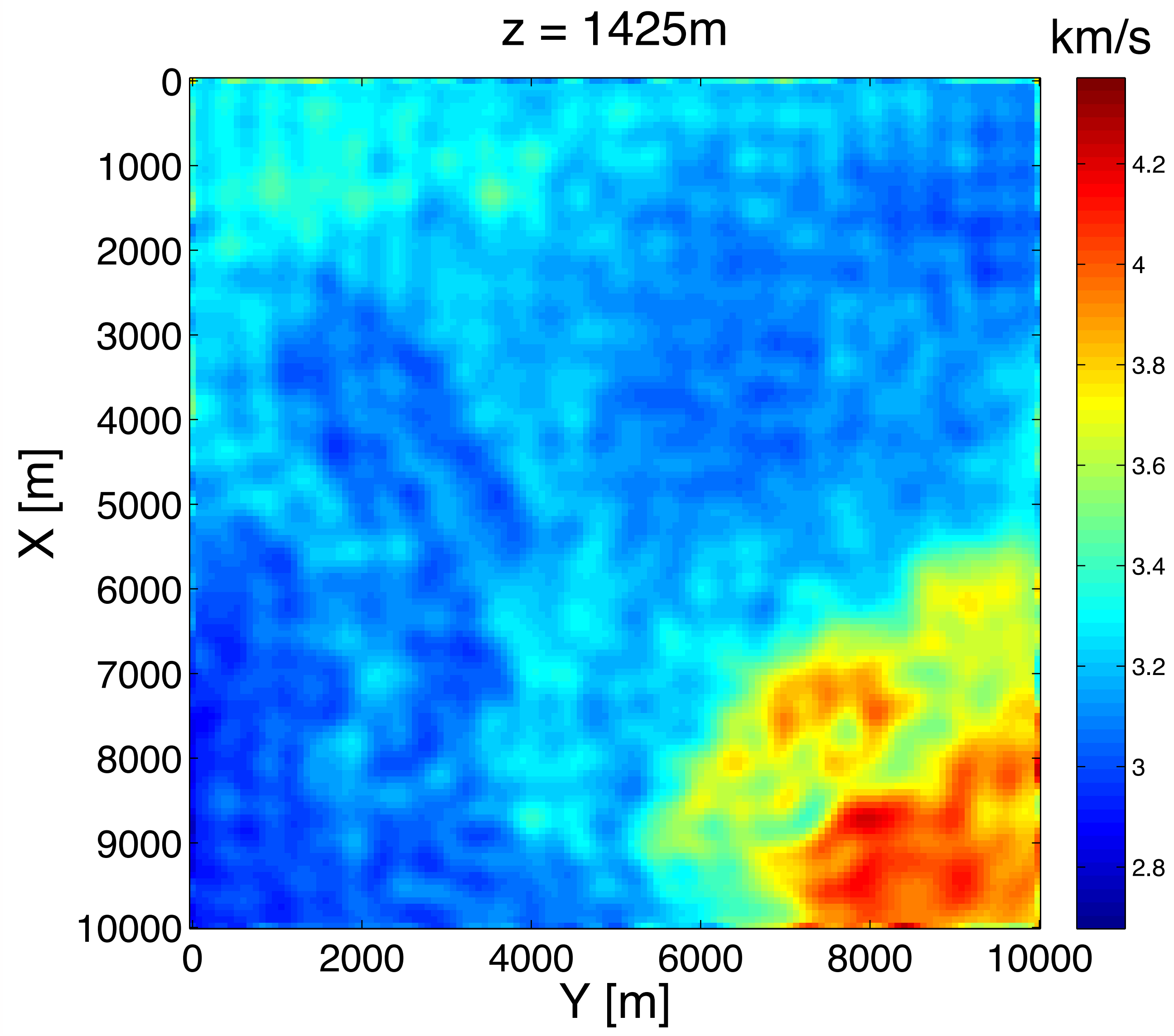
Composite shots



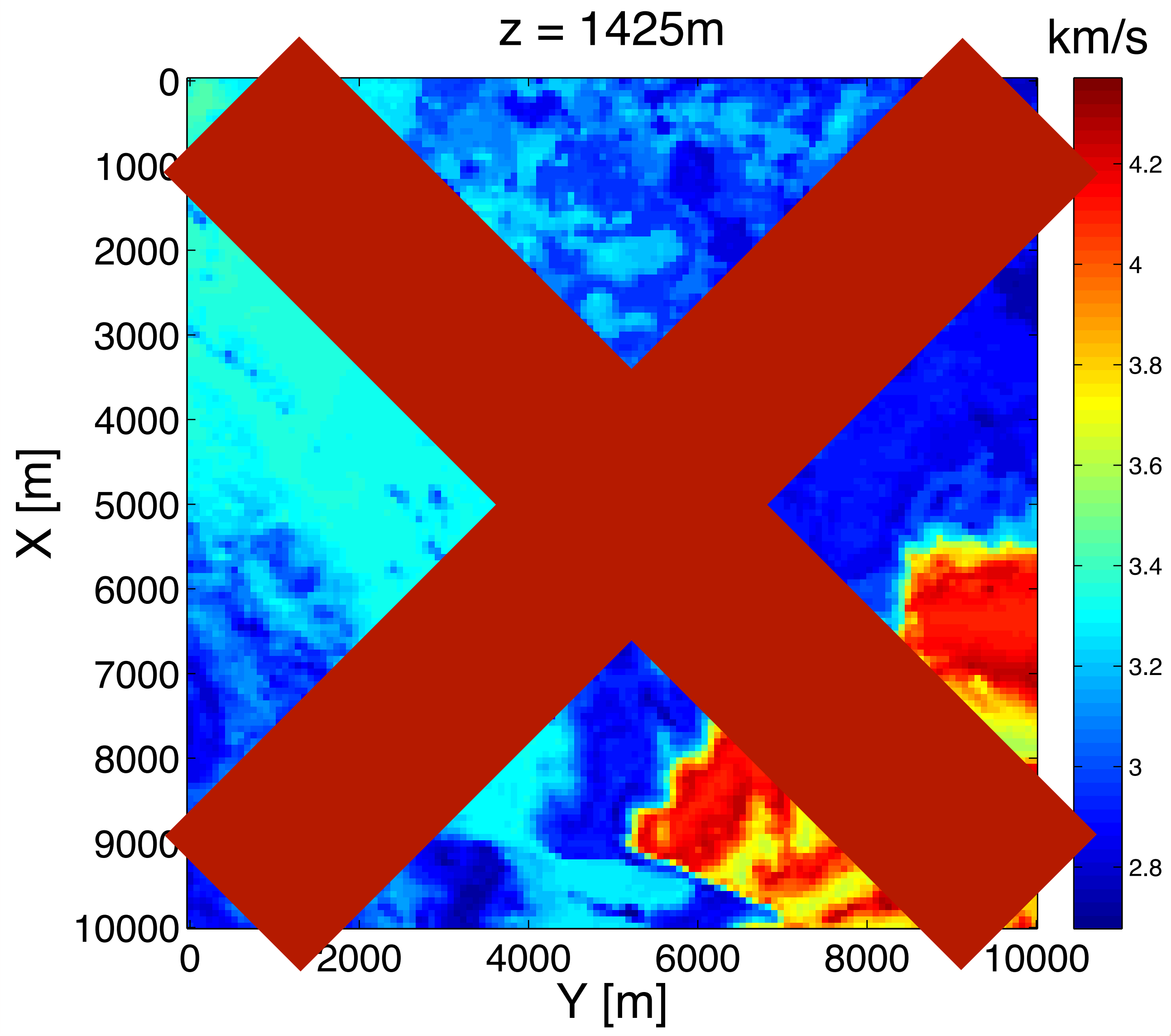
Initial model



Randomly selected shots

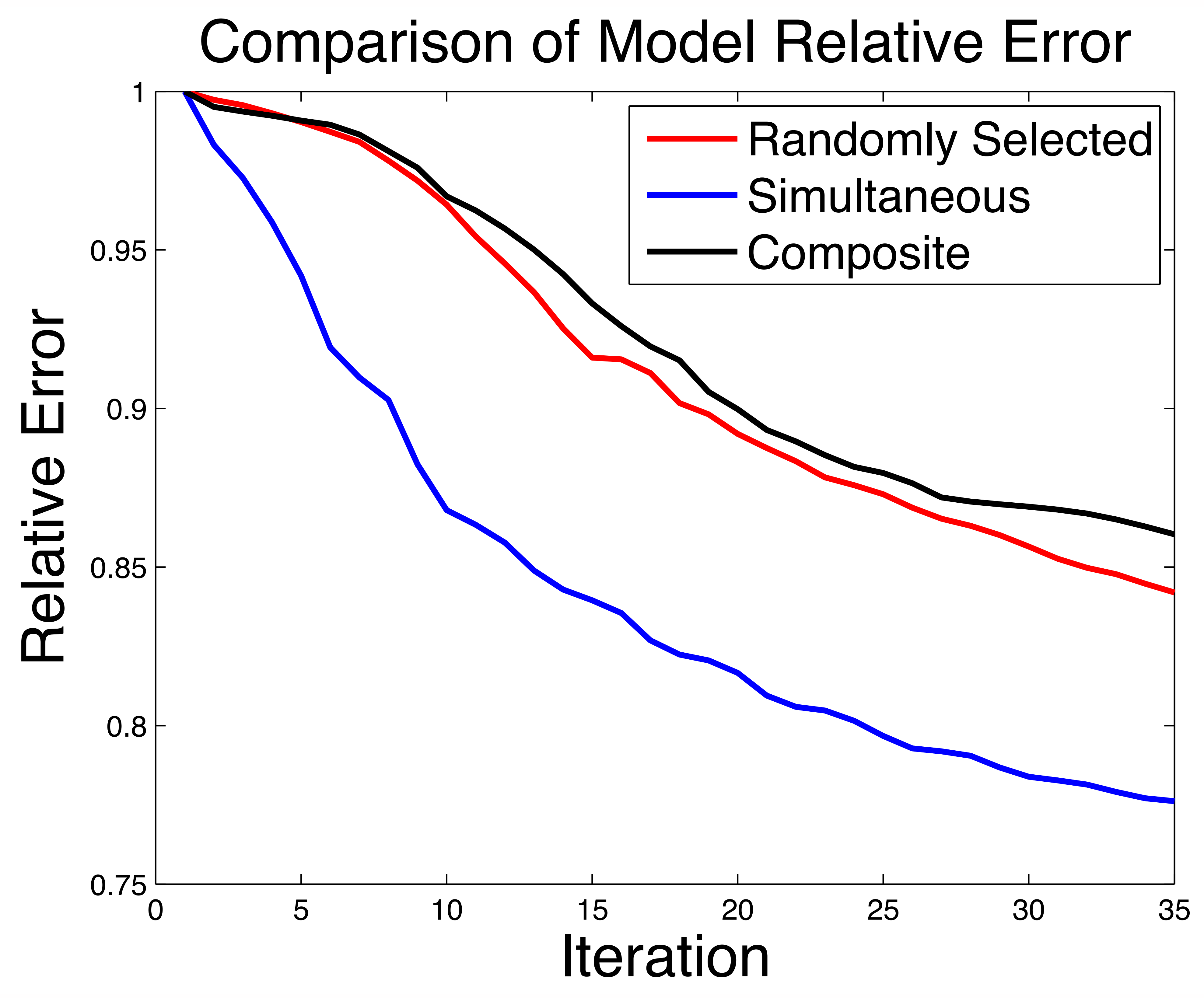


Simultaneous shots

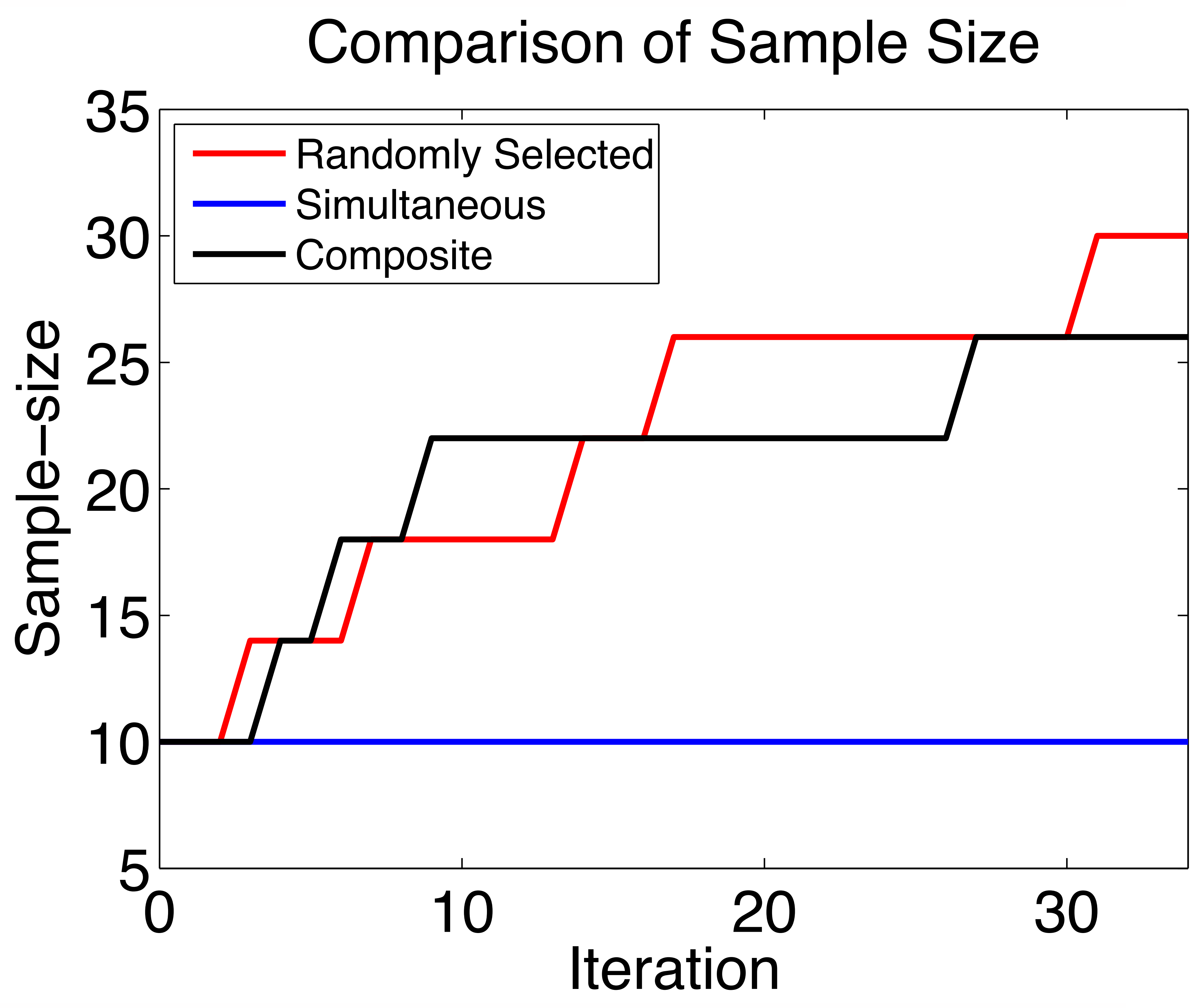


True model

Comparison



Comparison of Model Relative Error
Relative Error
Iteration
1
0.95
0.9
0.85
0.8
0.75
0
5
10
15
20
25
30
35
Randomly Selected
Simultaneous
Composite



Comparison of Sample Size
Sample−size
Iteration
35
30
25
20
15
10
5
0
10
20
30
Randomly Selected
Simultaneous
Composite

Conclusions

1. Applied *stochastic* optimization method where *computational* cost is *reduced* from $\mathcal{O}(n_s)$ to $\mathcal{O}(n_{ss})$.

2. *Simultaneous* shots require *fewer* source experiments (= # PDE solves) to obtain *better* results compared to *other* two methods.

3. We present a *new* sampling method - *composite* shots which
   - can extract more information per *composite* shot
   - allows for *offset* weighting
   - suitable for multi-source vessel marine

Future work

1. Deal with higher frequency data;
2. Study different strategies of composite shots;
3. Adaptive importance sampling;
4. Work with all data (If we have a big machine).

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Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

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