Parallel, matrix-free framework for frequency-domain modelling, imaging and inversion

Tristan van Leeuwen

SLIM
University of British Columbia

Matlab framework

Wave-equation solver
model->wavefield

Modelling operator
model -> data

Migration/de-migration
(Jacobian)
model pert. <-> data pert.

objective function
(model,data)->(misfit,gradient,Hessian)

Misc. linear operators
(interpolation)
vector -> vector

Transforms
(Curvelets, Wavelets)
vector -> vector

Misc. non-linear functions
vector->(scalar,gradient)
vector -> (vector, Jacobian)

SPOT: Matrix-free linear operators
pSPOT: Parallel SPOT
function handles

gradient-based optimization
(L-BFGS, Gauss-Newton)
objective function->solution

linear solvers
(LSQR, SPGl1)
(Matrix, r.h.s)->solution

Full waveform inversion =
objective function +
non-linear optimization

Least-Squares migration =
Jacobian + LSQR

sparsity promoting migration =
Jacobian + SPGL1

Matlab framework

Modelling

```
[D,J] = F(m,Q,model)
```

`D` - data cube as Matlab distributed array

`J` - Jacobian as SPOT operator

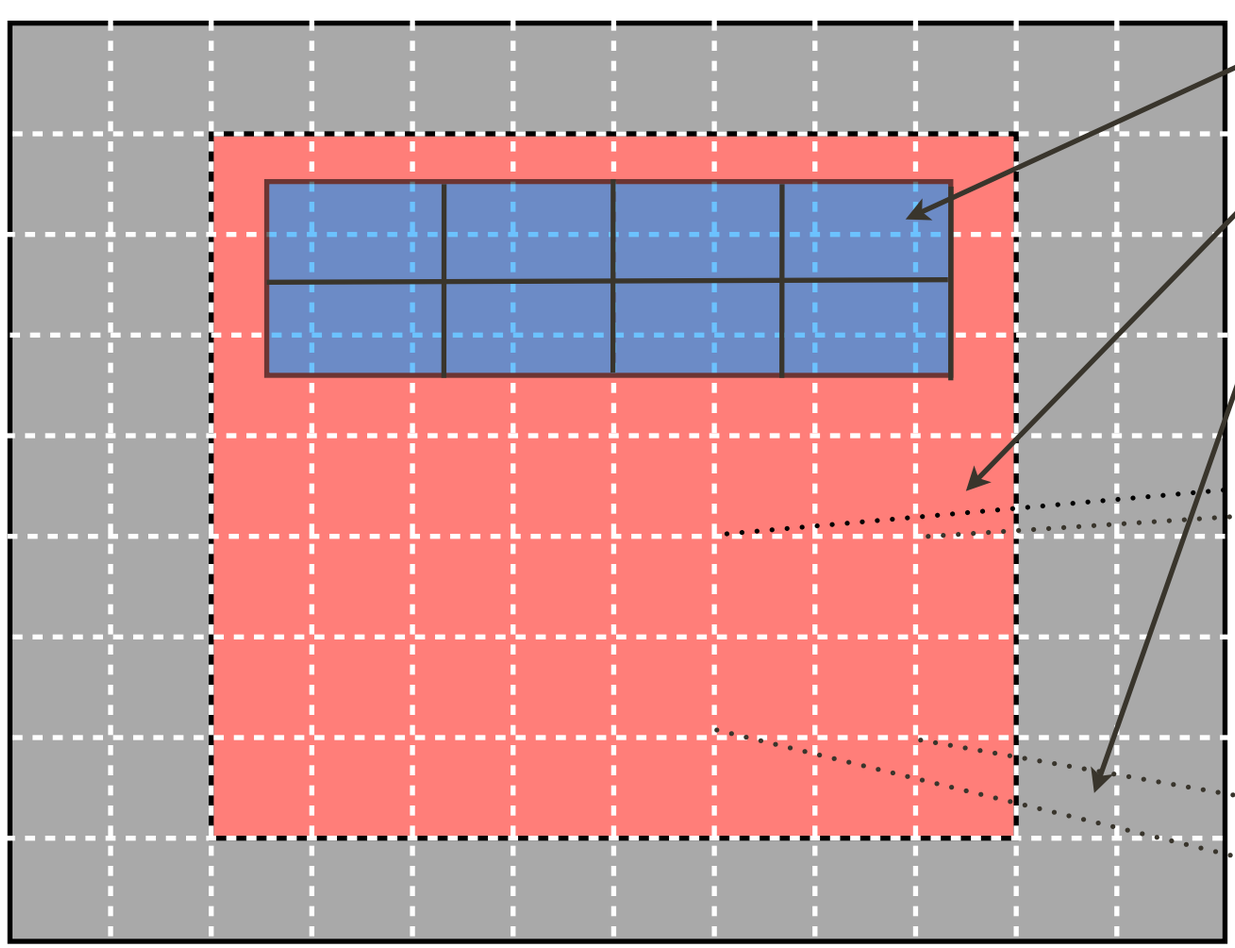
`m` - model as Matlab array

`Q` - source functions [q1, q2, ...]

`model` - struct containing acquisition setup etc.

Matlab framework

Discretization:



Matlab framework

- 9-point mixed-grid stencil [Jo et al '96]
- sponge boundary
- cubic interpolation for source injection and sampling
- Exact adjoint of discretized system
- parallel over frequencies

Matlab framework

1140 x 344 gridpoints, 500 sources, 64 frequencies

# of procs	time [s]	speed up	efficiency
2	36611	1.58	0.79
4	19544	2.96	0.74
8	9623	6.03	0.75
16	4525	12.82	0.80
32	2289	25.34	0.79

Matlab framework

Action of the Jacobian

```
D    = J*dm;
% calls J.multiply(dm,1)
dmt = J'*D;
% calls J.multiply(D,-1)
```

Matlab framework

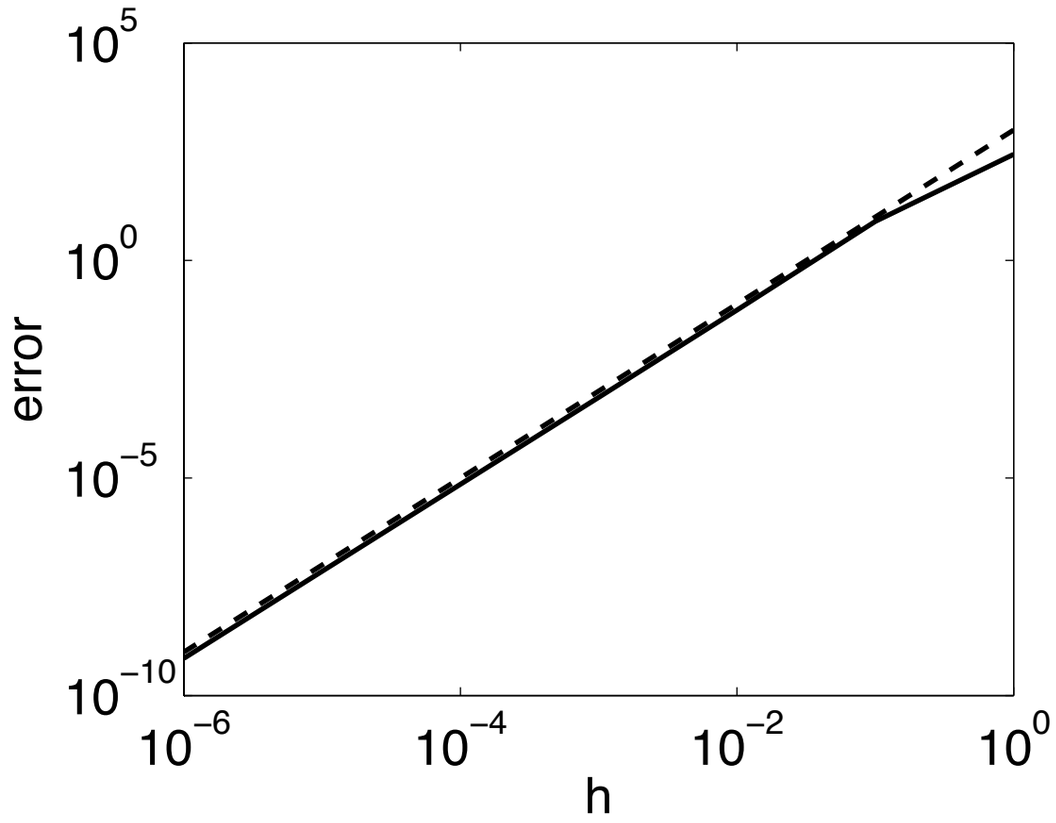
Testing

```
%test Jacobian
[D0,J0] = F(m0,Q,model);
D1      = F(m0+dm,Q,model);
dD      = J0*dm;
norm(D1 - D0 - dD)

%dottest: (Ax)'*b = x'*(A'*b)
(J*dm)'*dD
dm'(J'*dD)
```

Matlab framework

Jacobian test

$$F(\mathbf{x} + h\delta\mathbf{x}) = F(\mathbf{x}) + h\nabla F(\mathbf{x})\delta\mathbf{x} + \mathcal{O}(h^2).$$



Matlab framework

Dottest:

$\langle A\mathbf{x}, \mathbf{y} \rangle$	$\langle \mathbf{x}, A^*\mathbf{y} \rangle$
-9.3405e+08+4.1677e+08i	-9.3405e+08+4.1677e+08i
8.8128e+08-1.6759e+08i	8.8128e+08-1.6759e+08i
3.7126e+08+2.1559e+07i	3.7126e+08+2.1559e+07i
-1.7585e+08-1.7851e+08i	-1.7585e+08-1.7851e+08i

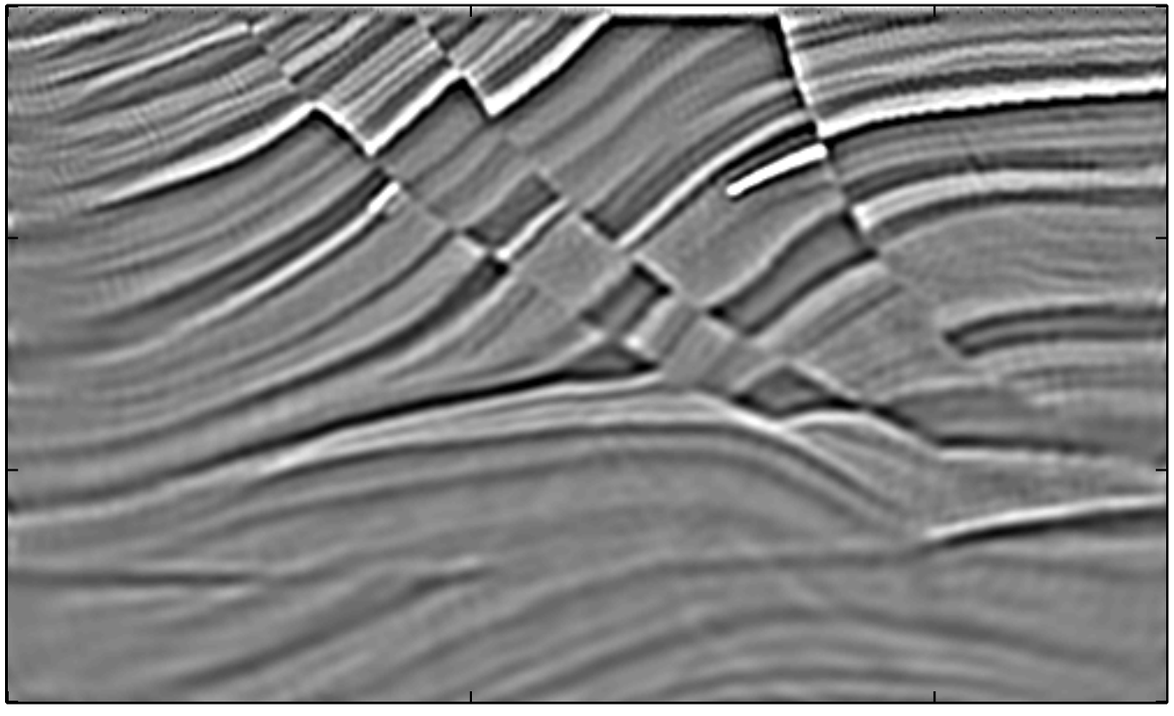
Matlab framework

Least-squares migration

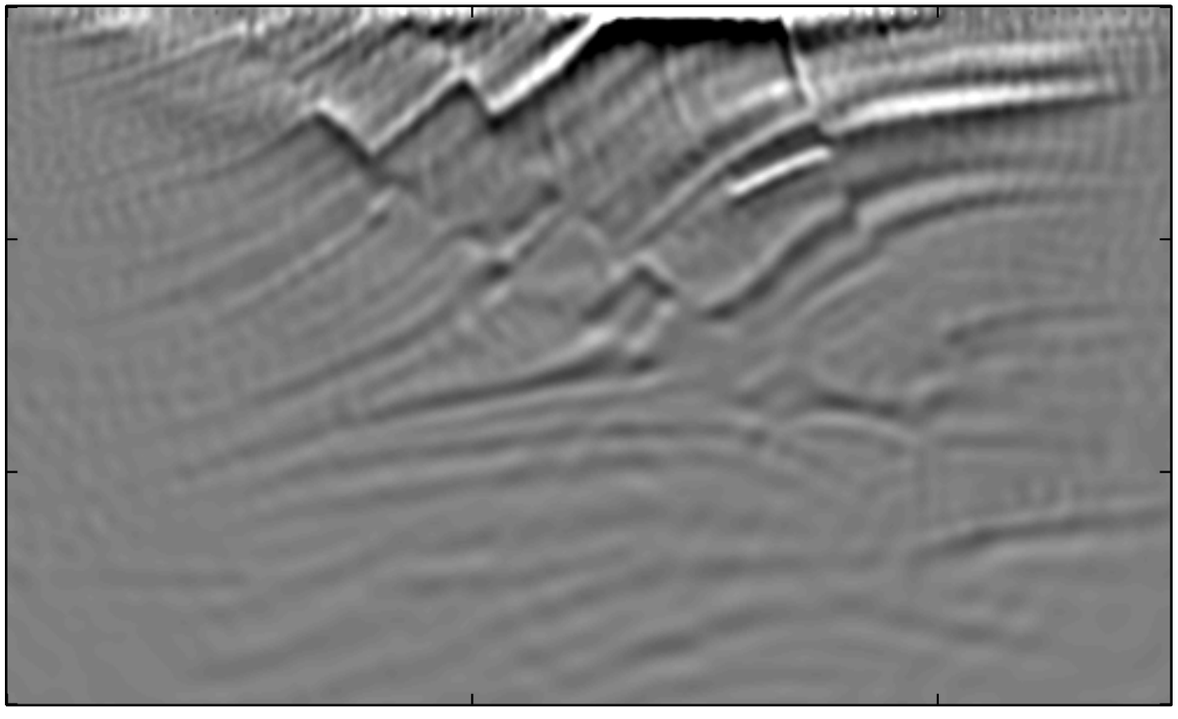
```
% model m0, data D

J   = opDF(m0,Q,model);
dm = LSQR(J,D);
```

Matlab framework

50 LSQR iterations



correct adjoint



wrong adjoint

Matlab example

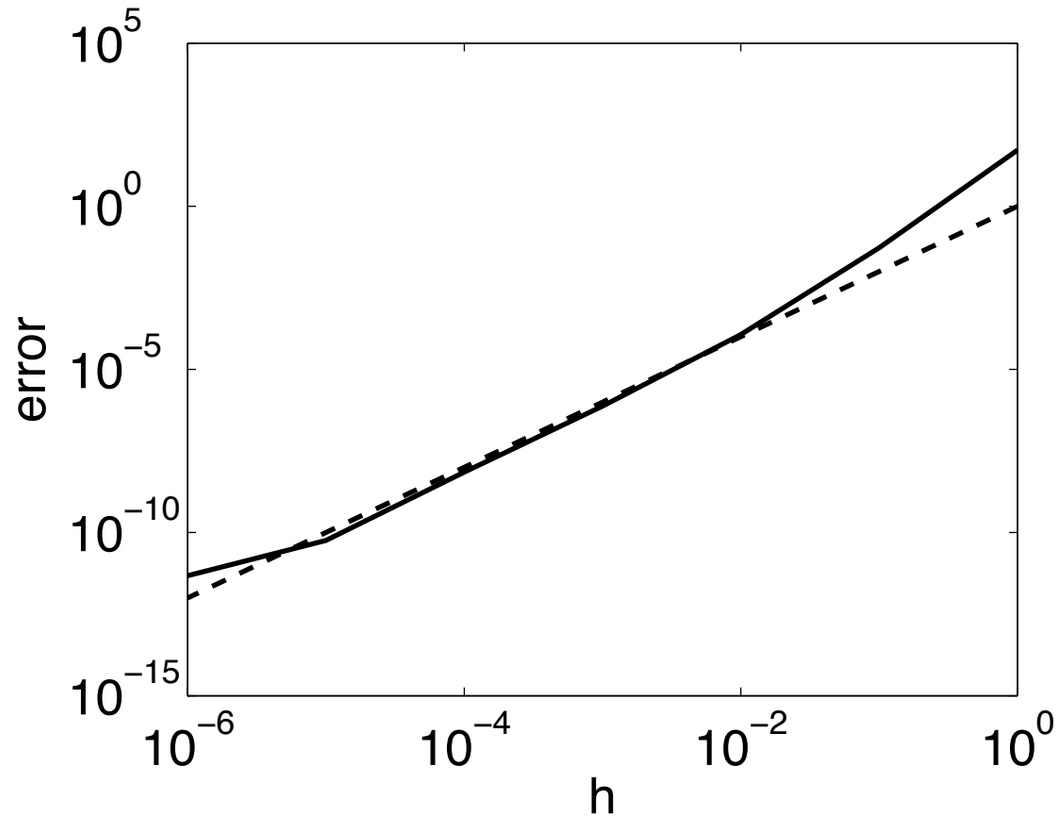
misfit

```
function [f,g] = Phi(m,Q,D,model)

[Dt,Jt]  = F(m,Q,model);
[f,df]   = penalty(Dt-D);
g        = Jt'*df;
```

slide title

gradient test

$$f(\mathbf{x} + h\delta\mathbf{x}) = f(\mathbf{x}) + h\langle \nabla f(\mathbf{x}), \delta\mathbf{x} \rangle + \mathcal{O}(h^2).$$

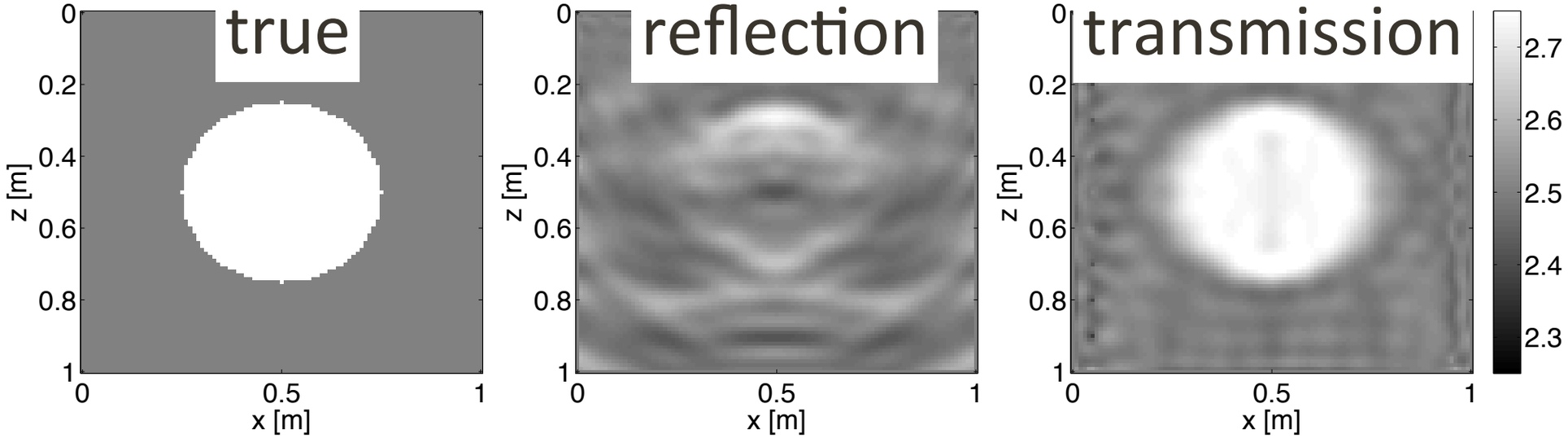


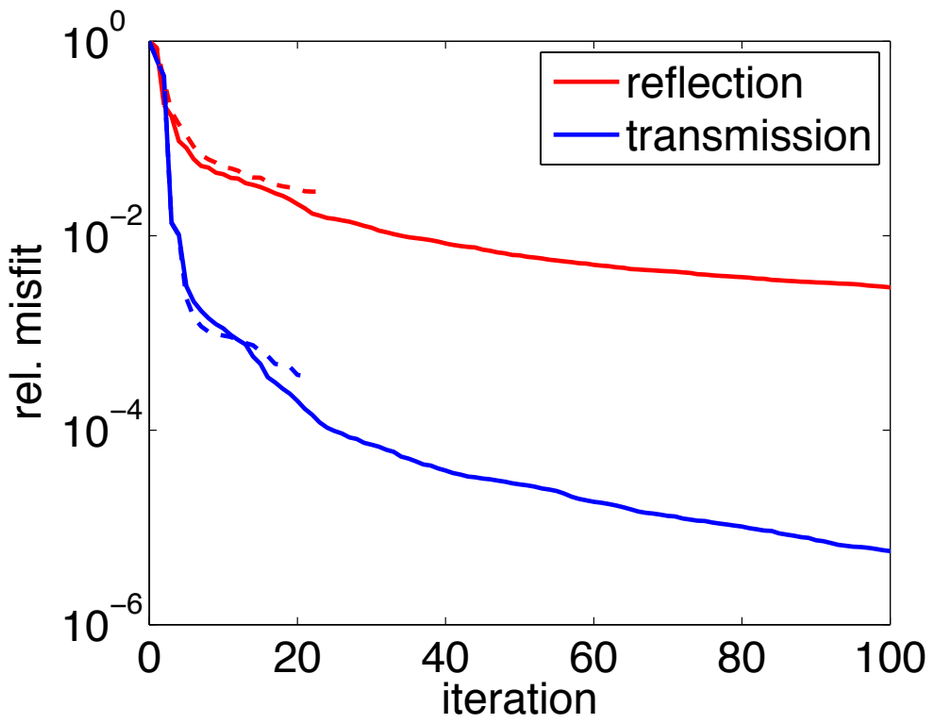
Matlab framework

Optimization

```
% read data D
% define model,Q,m0

% function handle
fh = @(m)misfit(m,Q,D,model);

% optimization
mn = mylbfgs(fh,m0);
```

Matlab framework





Summary

- Flexible and scalable framework
- easy prototyping
- correct Jacobian & adjoint is important
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Acknowledgements

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