Fast imaging with multiples by sparse inversion

Ning Tu and Felix J. Herrmann

SLIM
University of British Columbia

Lin, Tu, and Herrmann, 2010; Verschuur and Berkhout, 2011; Whitmore et.al., 2010; Liu et.al., 2011

Motivation

- making use of primaries and multiples *simultaneously*
- avoiding imaging *artifacts* from multiples
- looking for a computationally *efficient* approach

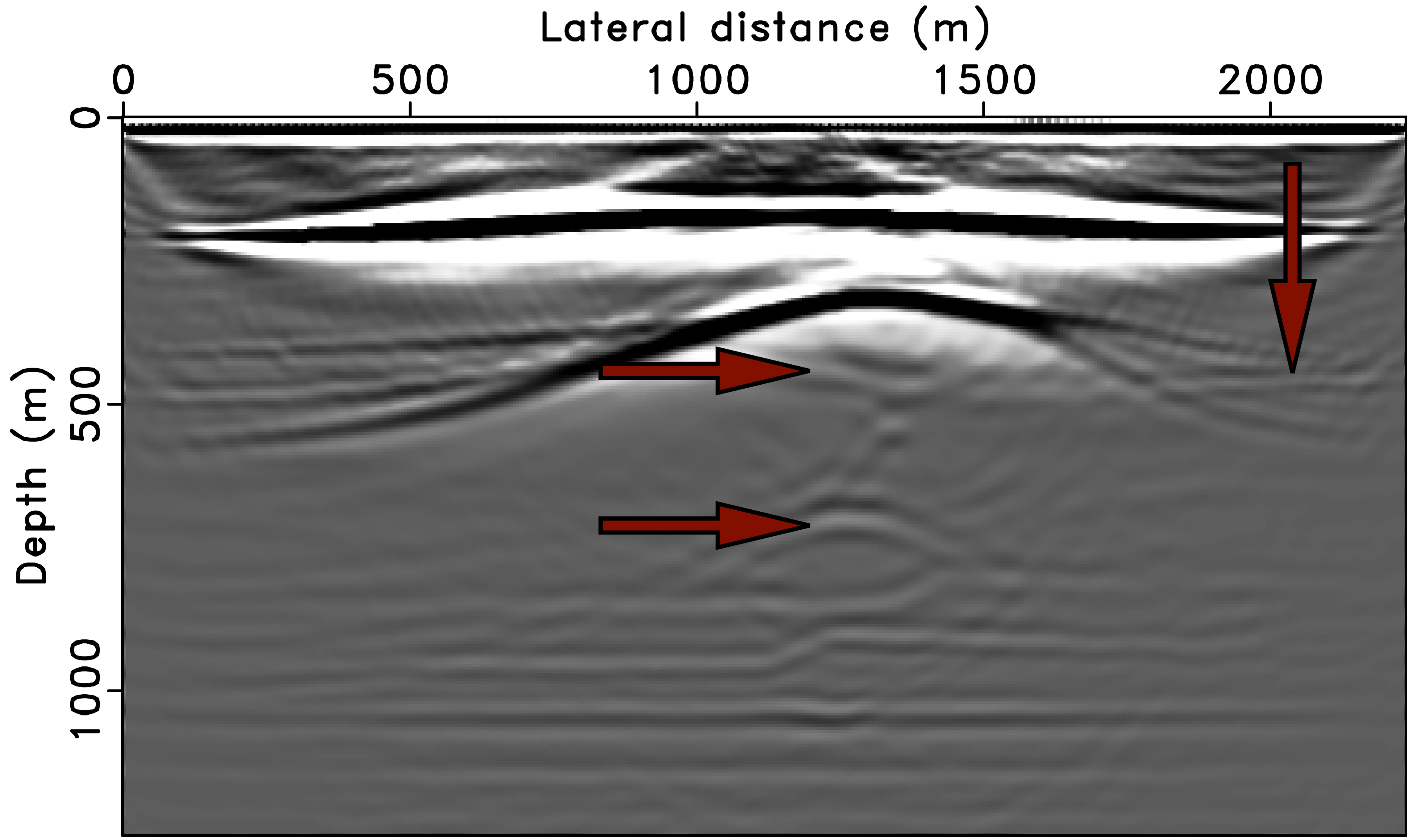
Tu and Herrmann, 2011; Verschuur and Berkhout, 2011; Lu et.al., 2011; Liu et.al., 2011

Primaries & multiples: not "OR" but "AND"

- primaries have a higher signal-to-noise ratio
- multiples provide extra illumination if correctly used
- skip the procedure of separating primaries/multiples

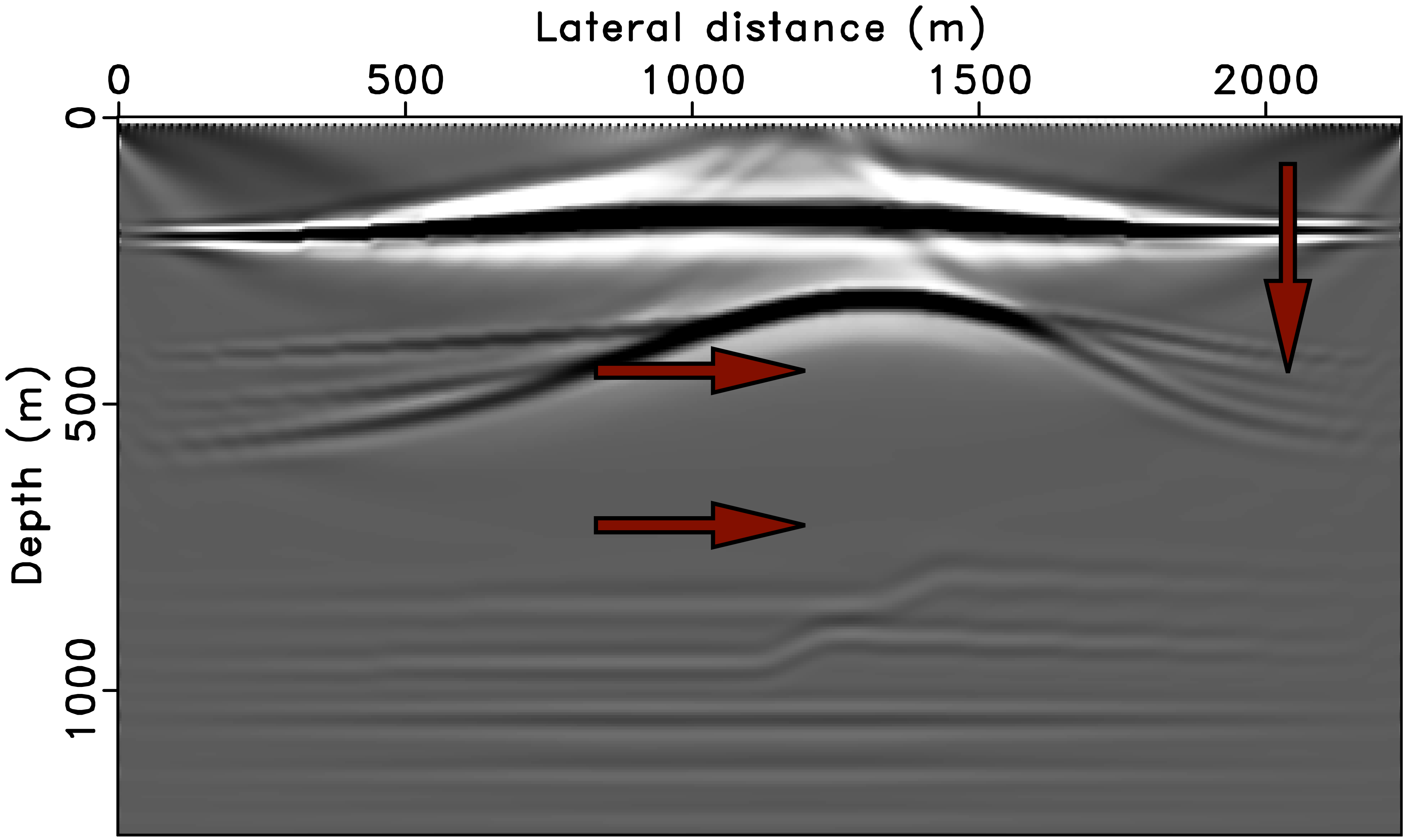
Whitmore et.al., 2010; Liu et.al., 2011

Artifacts from multiples



Reverse time migration of *multiples* using total data as “source”

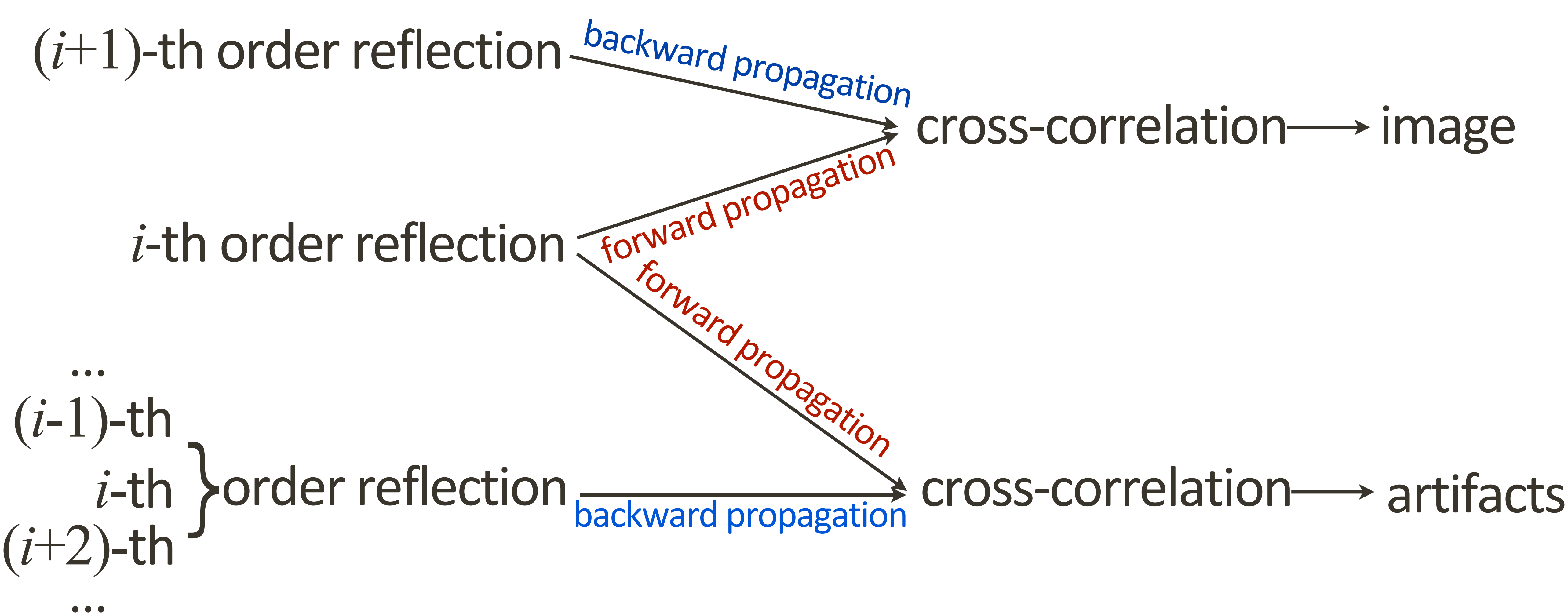
Not there for primaries



Reverse time migration of *primaries*

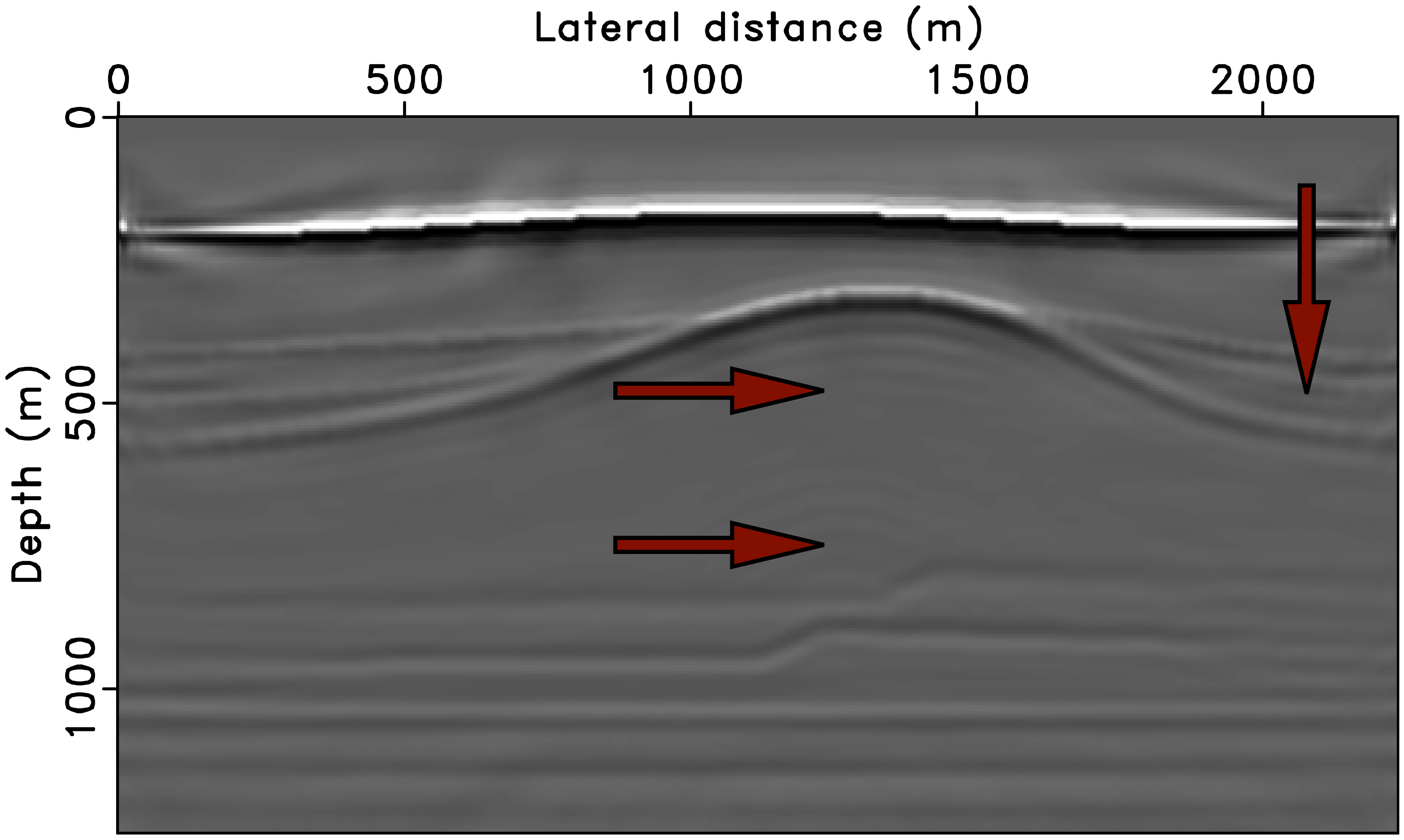
Liu et.al., 2011

When a free-surface presents



Lin et. al., 2010; Tu and Herrmann, 2011a

Want to avoid them?



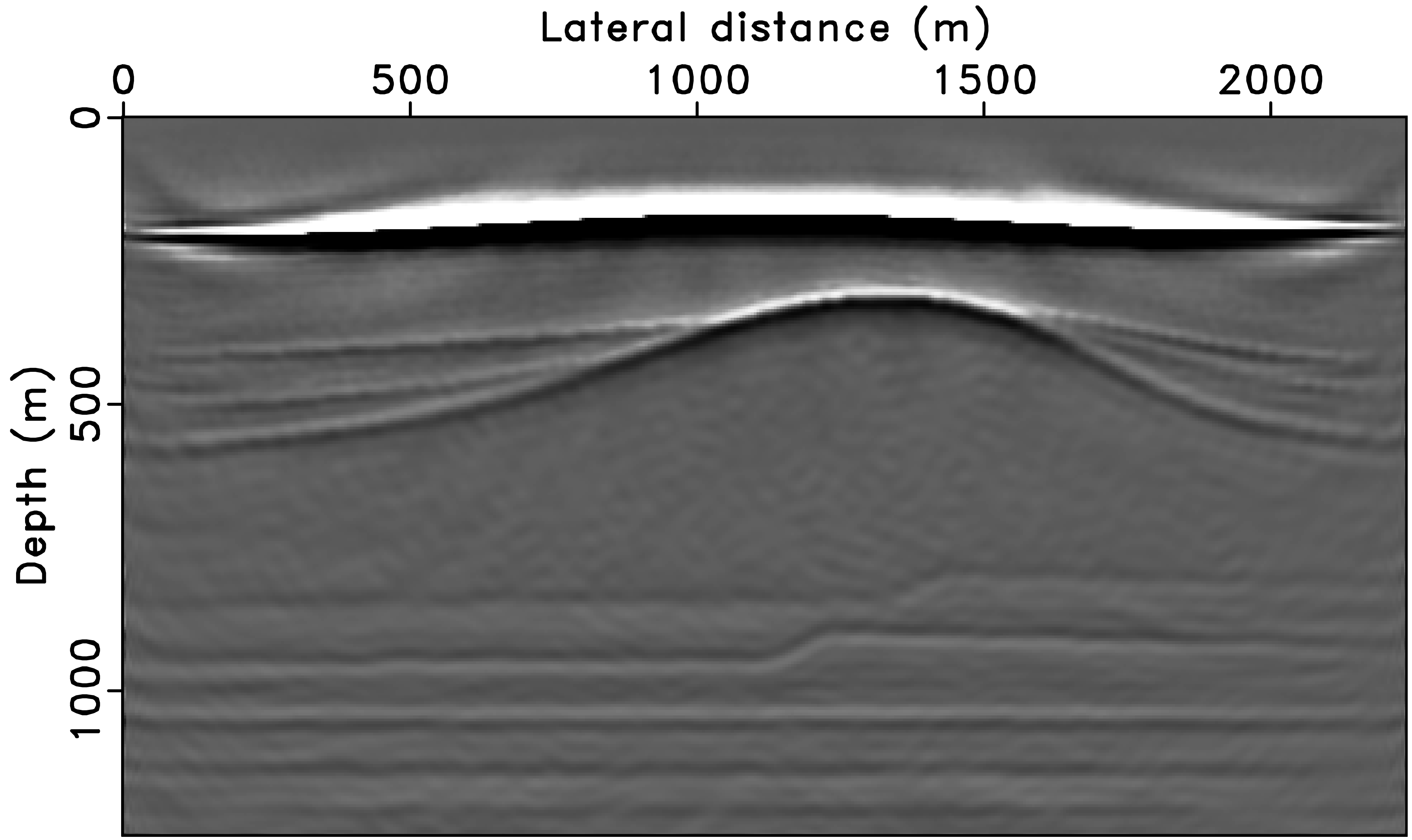
Imaging of *multiples* by *inversion*

Inversion? Sounds expensive...

- repeated evaluations of the Born scattering operator and its adjoint
- each evaluation requires solving 4*(#source)*(#frequencies) PDEs

Sneak peek of our result

[with a computational budget of a single RTM with all data]



Fast imaging of *total up-going wavefield* by sparse inversion

120X speed up compared with the previous result

Method

Incorporating the free surface

Total data and the surface-free Green's function can be related by the SRME formulation:

$$\mathbf{P}_{\omega_\mathrm{i}} = \mathbf{G}_{\omega_\mathrm{i}}(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}})$$

$\mathbf{P}$: total up-going wavefield
$\mathbf{G}$: surface-free Green's function
$\mathbf{Q}$: source wavelet
$\mathbf{R}$: surface reflectivity

Expressed in model space

$$
\begin{aligned}
\mathbf{P}_{\omega_\mathrm{i}} &= \mathrm{vec}^{-1}(\mathbf{F}_{\omega_\mathrm{i}}[\mathbf{m}, \mathbf{I}])(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}) \\
&= \mathbf{D}_\mathrm{r}\mathbf{H}_{\omega_\mathrm{i}}^{-1}[\mathbf{m}](\mathbf{D}_\mathrm{s}^{*}\mathbf{I})(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}) \\
&= \mathbf{D}_\mathrm{r}\mathbf{H}_{\omega_\mathrm{i}}^{-1}[\mathbf{m}](\mathbf{D}_\mathrm{s}^{*}(\mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}})) \\
&\doteq \mathrm{vec}^{-1}(\mathbf{F}_{\omega_\mathrm{i}}[\mathbf{m}, \mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}])
\end{aligned}
$$

$\mathbf{F}$: modelling operator

$\mathbf{m}$: true velocity/density model

$\mathbf{I}$: impulsive source array

$\mathbf{D}$: detection operator at receiver/source locations

Linearization

$$\mathbf{p}_{\omega_\mathrm{i}} = \nabla\mathbf{F}_{\omega_\mathrm{i}}[\mathbf{m}_0, \mathbf{Q}_{\omega_\mathrm{i}} + \mathbf{R}_{\omega_\mathrm{i}}\mathbf{P}_{\omega_\mathrm{i}}]\delta\mathbf{m}$$

$\nabla\mathbf{F}$: Born scattering operator
$\mathbf{m_0}$: background model
$\delta\mathbf{m}$: model perturbation
$\mathbf{p}_{\omega_\mathrm{i}}$: vectorized wavefield

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Stacking over frequencies

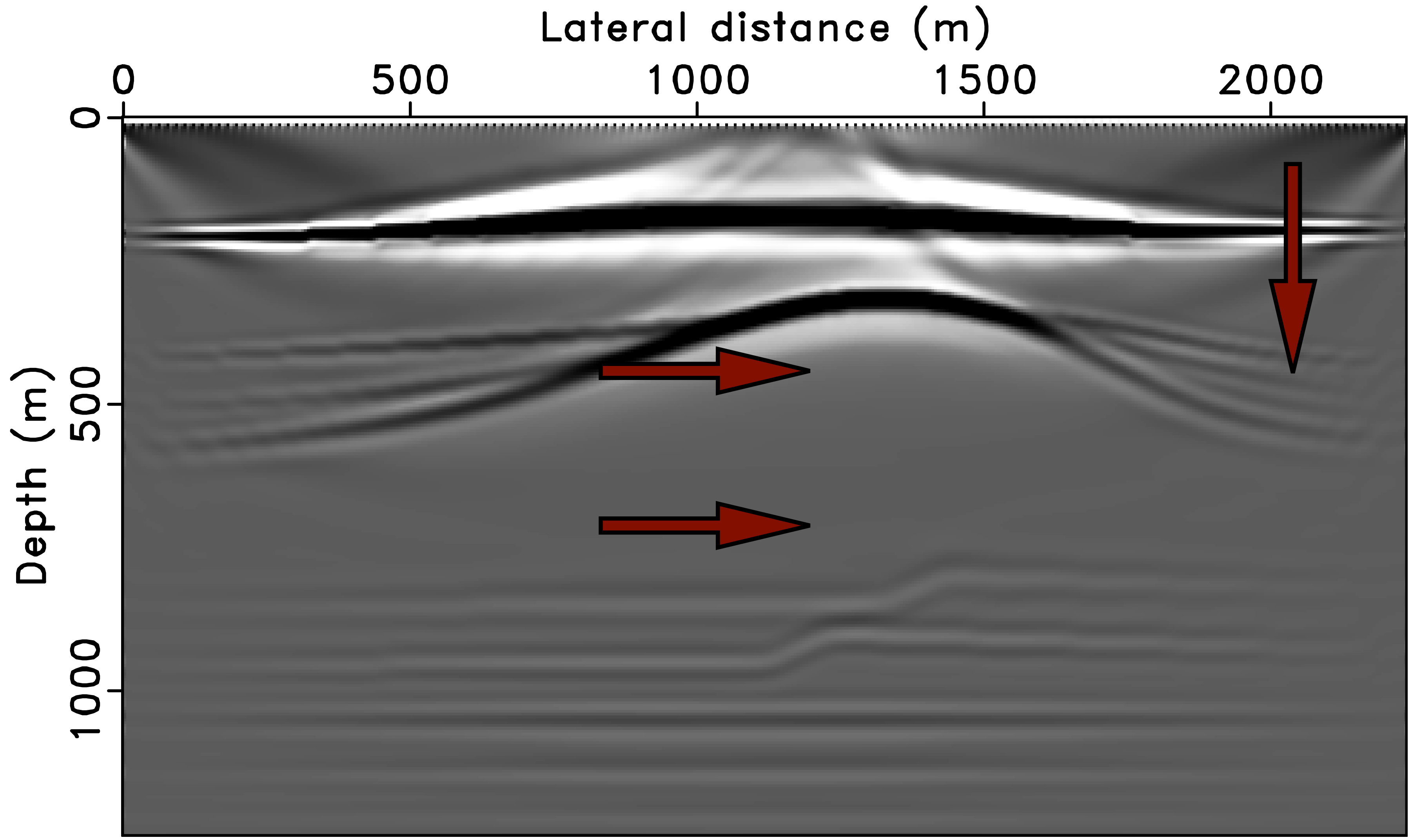
$$\mathbf{p} \quad = \quad \begin{bmatrix} \nabla \mathbf{F}_{\omega_1}(\mathbf{m}_0, \mathbf{Q}_{\omega_i} + \mathbf{R}_{\omega_i}\mathbf{P}_{\omega_i}) \\ \vdots \\ \nabla \mathbf{F}_{\omega_{\mathrm{nf}}}(\mathbf{m}_0, \mathbf{Q}_{\omega_i} + \mathbf{R}_{\omega_i}\mathbf{P}_{\omega_i}) \end{bmatrix} \delta\mathbf{m}$$

$$\doteq \quad \nabla \mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{R}\mathbf{P}]\delta\mathbf{m}$$

$$\delta\mathbf{m} = \nabla \mathbf{F}^{\dagger}[\mathbf{m}_0, \mathbf{Q} + \mathbf{R}\mathbf{P}]\mathbf{p}$$

RTM of primaries

[replace inversion by adjoint]

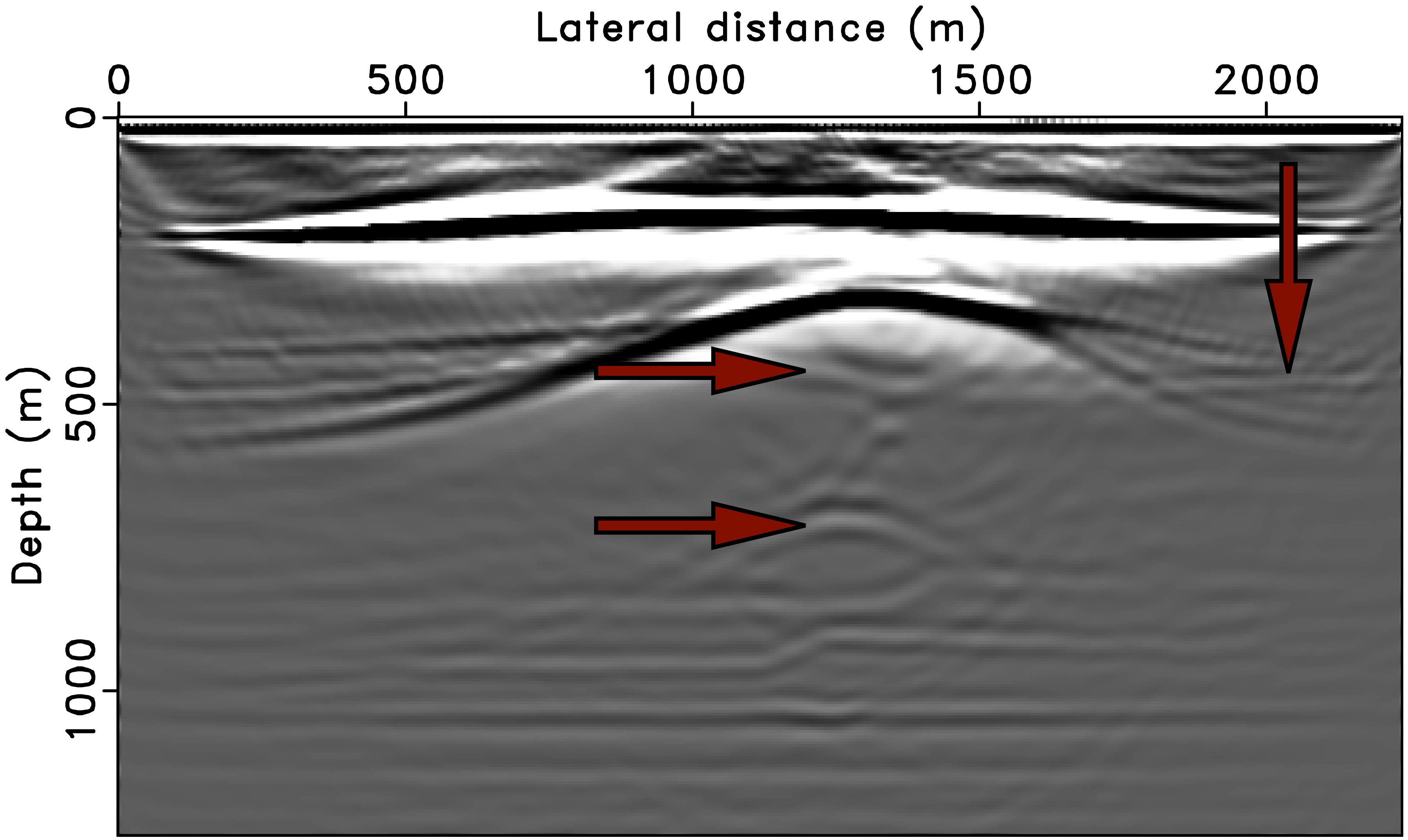


Reverse time migration of *primaries* with all data
number of PDE solves: 36.6 thousand
vertical differentiation is applied

Liu et.al., 2011

RTM of multiples

[replace inversion by adjoint]



Reverse time migration of *multiples* using total data as “source”
number of PDE solves: 36.6 thousand
vertical differentiation is applied

Sparse inversion

We use a sparsity-promotion formulation:

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\mathbf{p} - \nabla\mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

$\mathbf{C}$: curvelet transform

solver: $\mathrm{SPG}\ell_1$

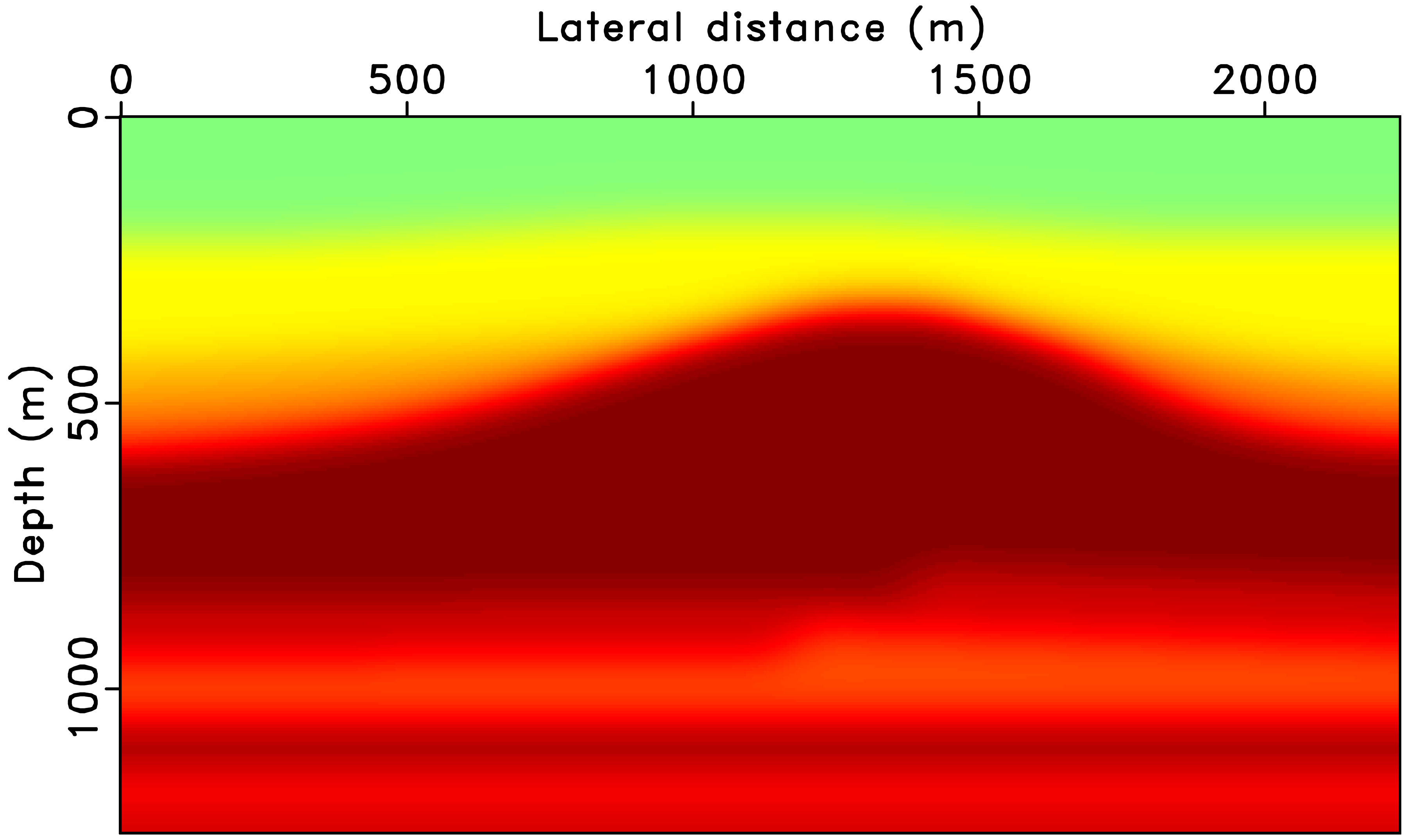
Demonstrative examples

- model grid spacing: 5 meters
- using linearized data:

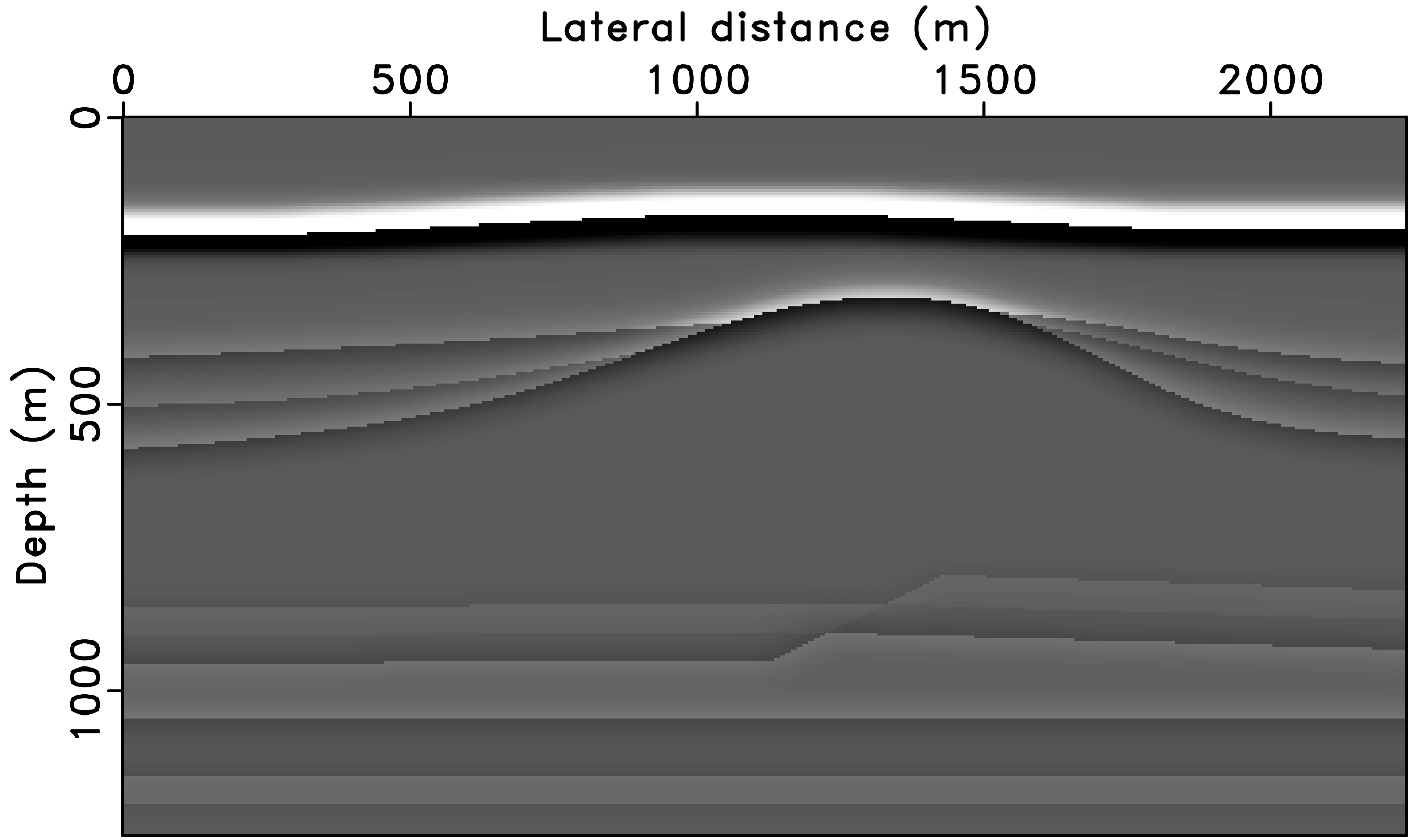
$$\nabla \mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\delta \mathbf{m}$$

- 150 collocated sources/receivers
- 122 frequencies in 0-60Hz range

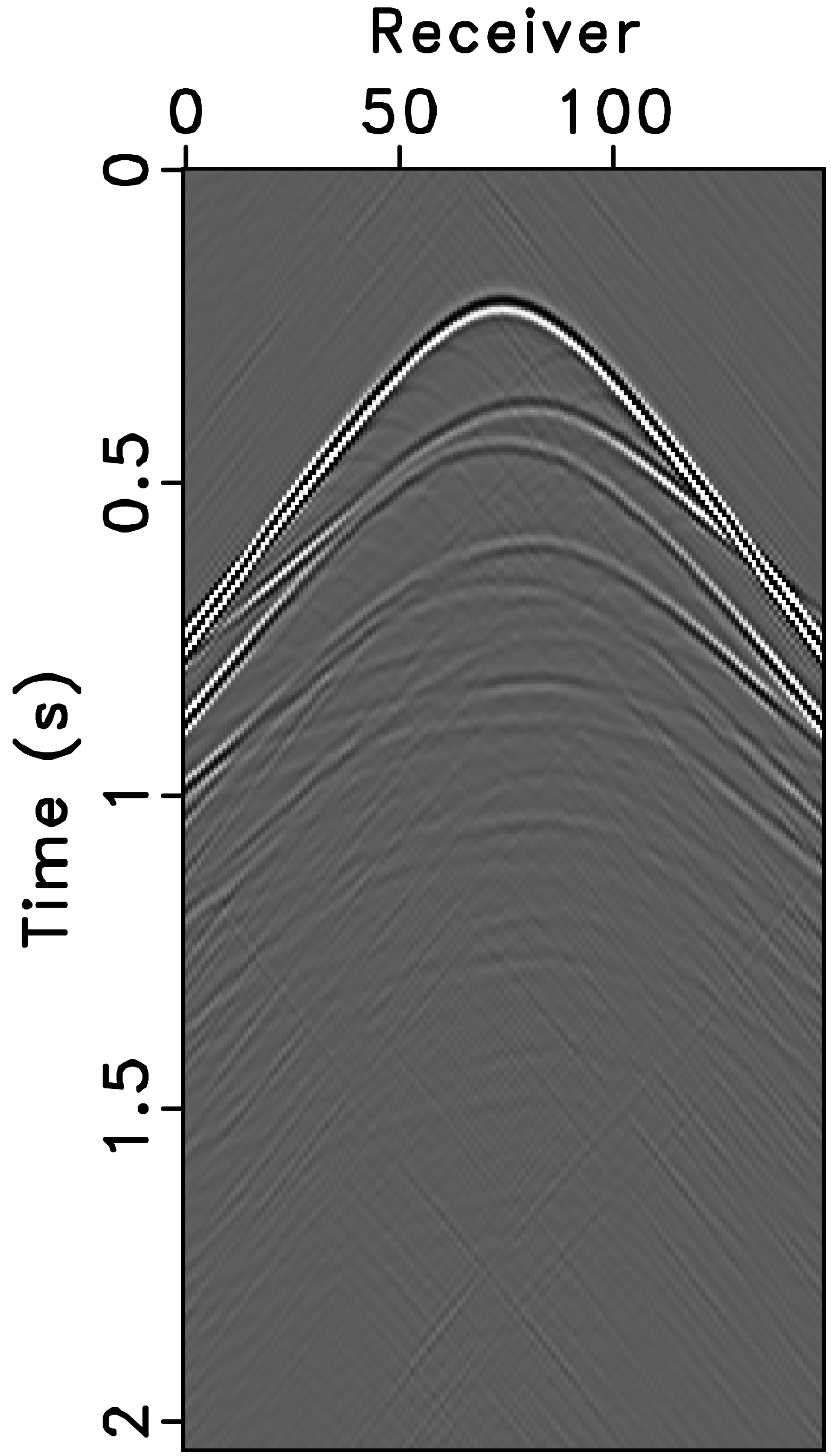
Background model



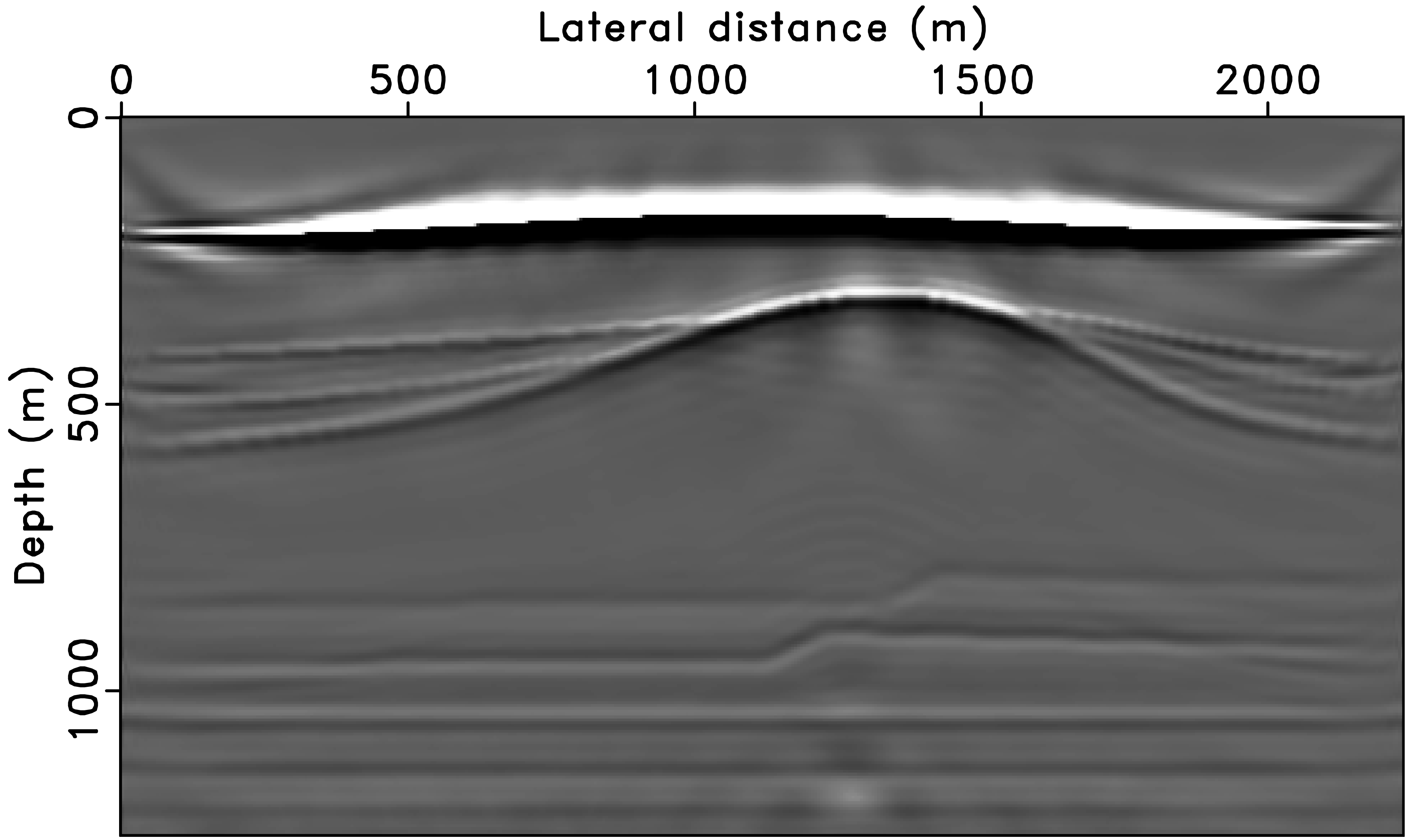
True perturbation



Linearized total data



Result



Inversion of the *total up-going wavefield* using all sequential sources and all frequencies
number of PDE solves: ~4.4 million (by calculation)

Dimensionality reduction

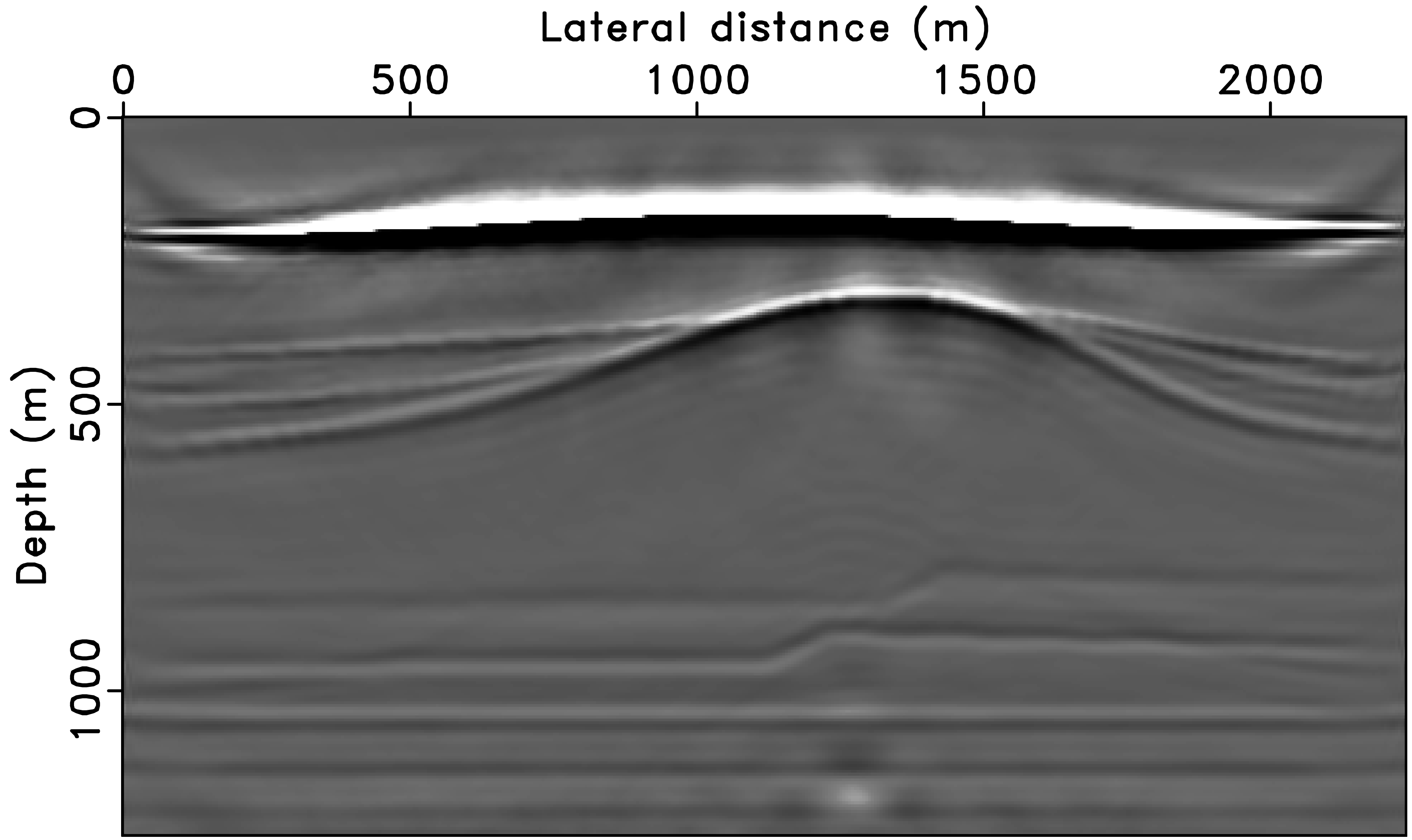
$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \operatorname*{argmin}_{\delta\mathbf{x}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

source: combine sources into a few simultaneous sources, using Gaussian distributed random weights

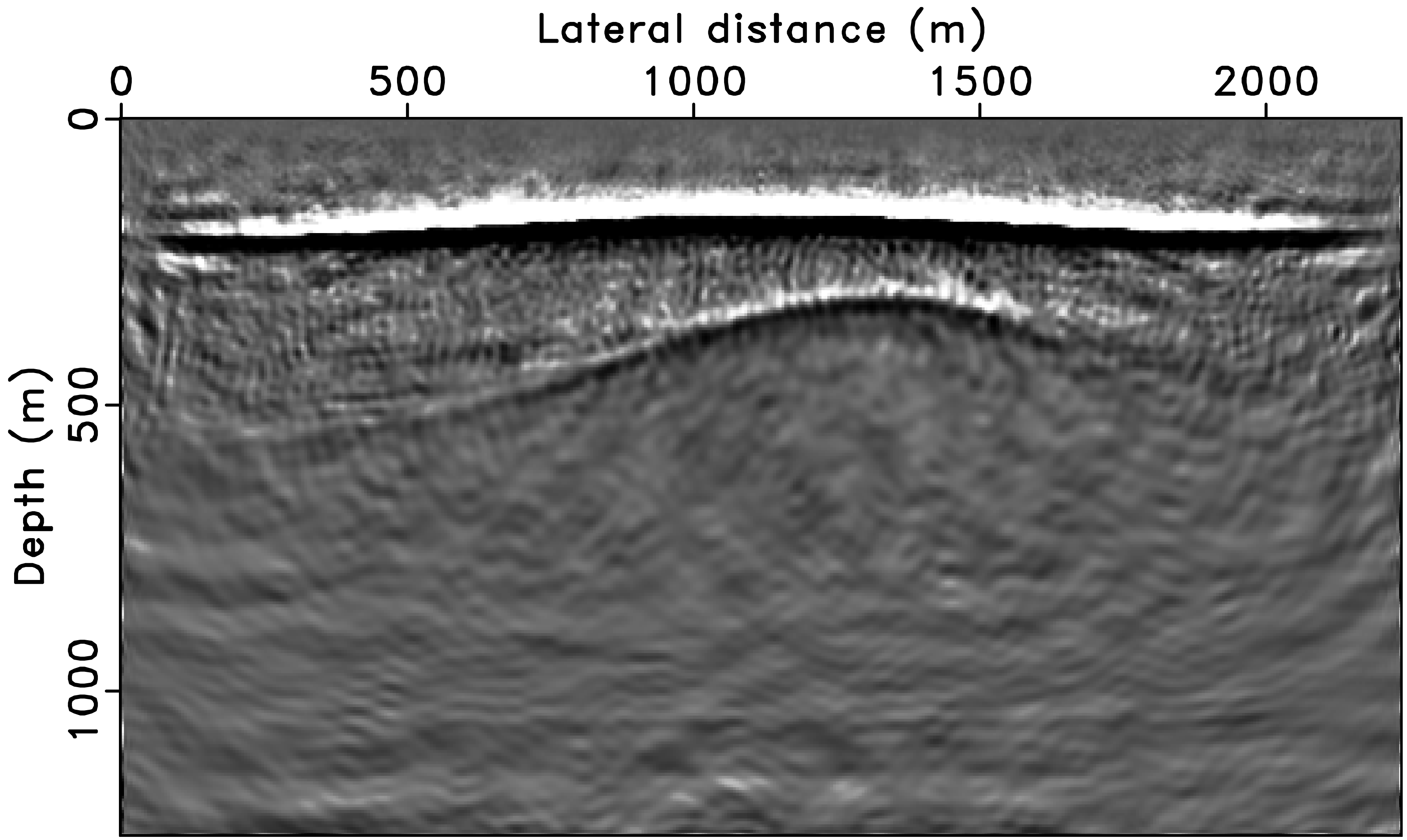
frequency: randomly choose a subset of frequencies

Result with 15x speed-up



Inversion of the total up-going wavefield using *10 simultaneous sources* and all frequencies
number of PDE solves: ~0.3 million

Too much subsampling brings artifacts



Inversion of the total up-going wavefield using *2 simultaneous sources and 15 frequencies*

number of PDE solves: 36.6 thousand

If working alright, it will give us *120X speed-up*

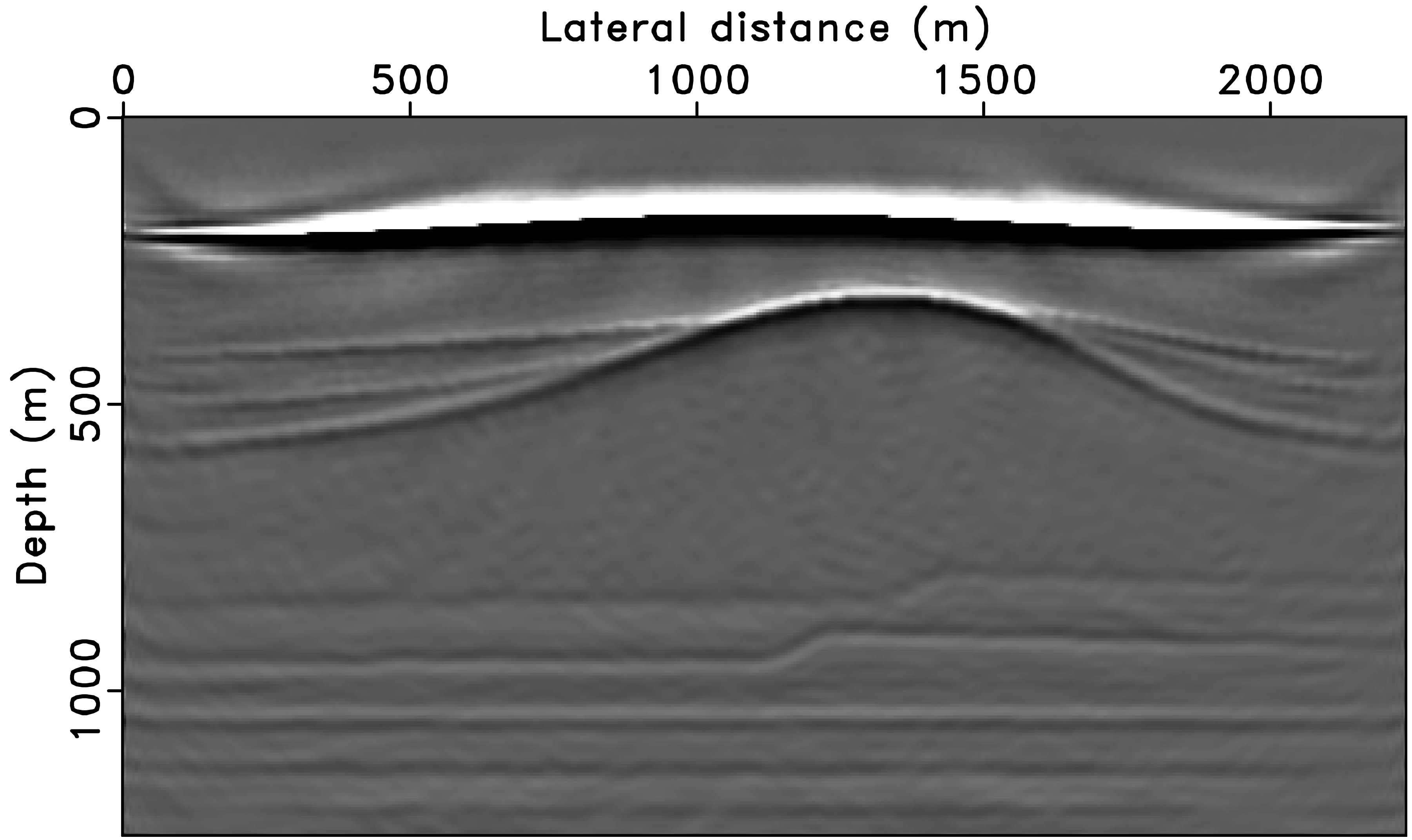
Draw new subsampling operator

- $\mathrm{SPG}\ell_1$ solves a series of subproblems:

$$\operatorname*{argmin}_{\delta\mathbf{x}} ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2$$
$$\text{subject to } ||\delta\mathbf{x}||_1 \leq \tau$$

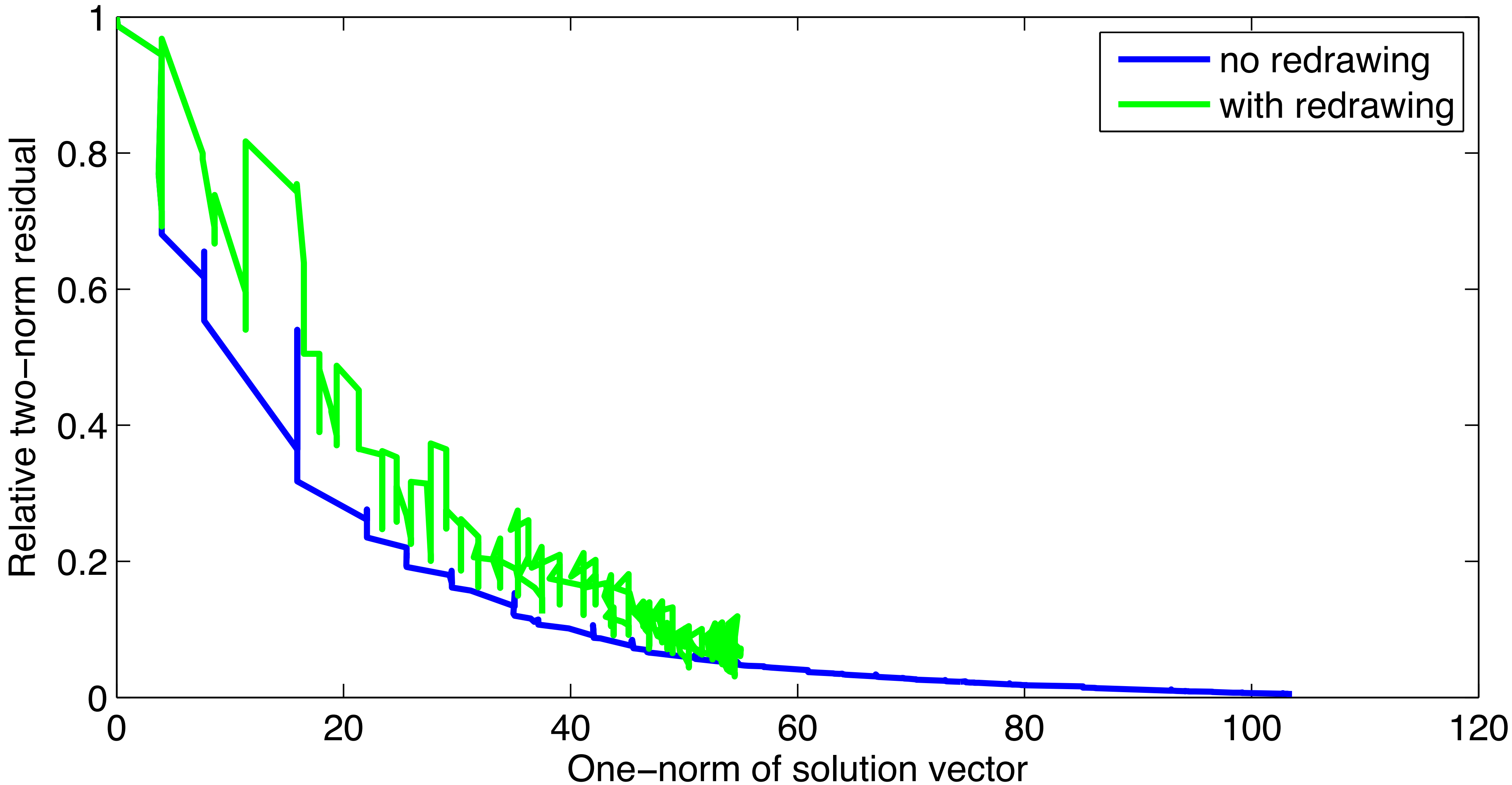
- redraw subsampling operator for each new subproblem

Draw new sim. sources and frequencies --*120X speed-up*

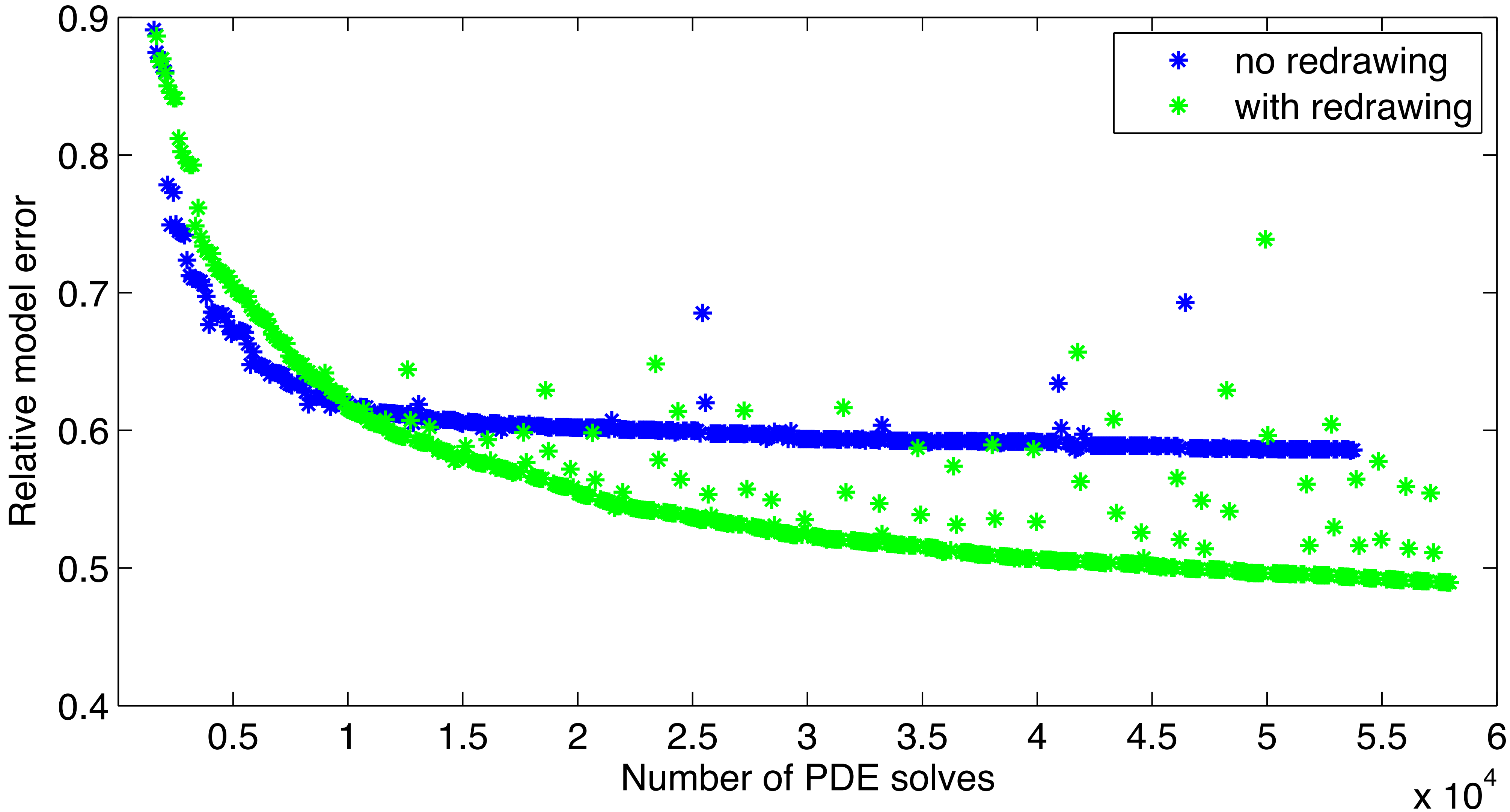


Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand (by calculation)

Solution path

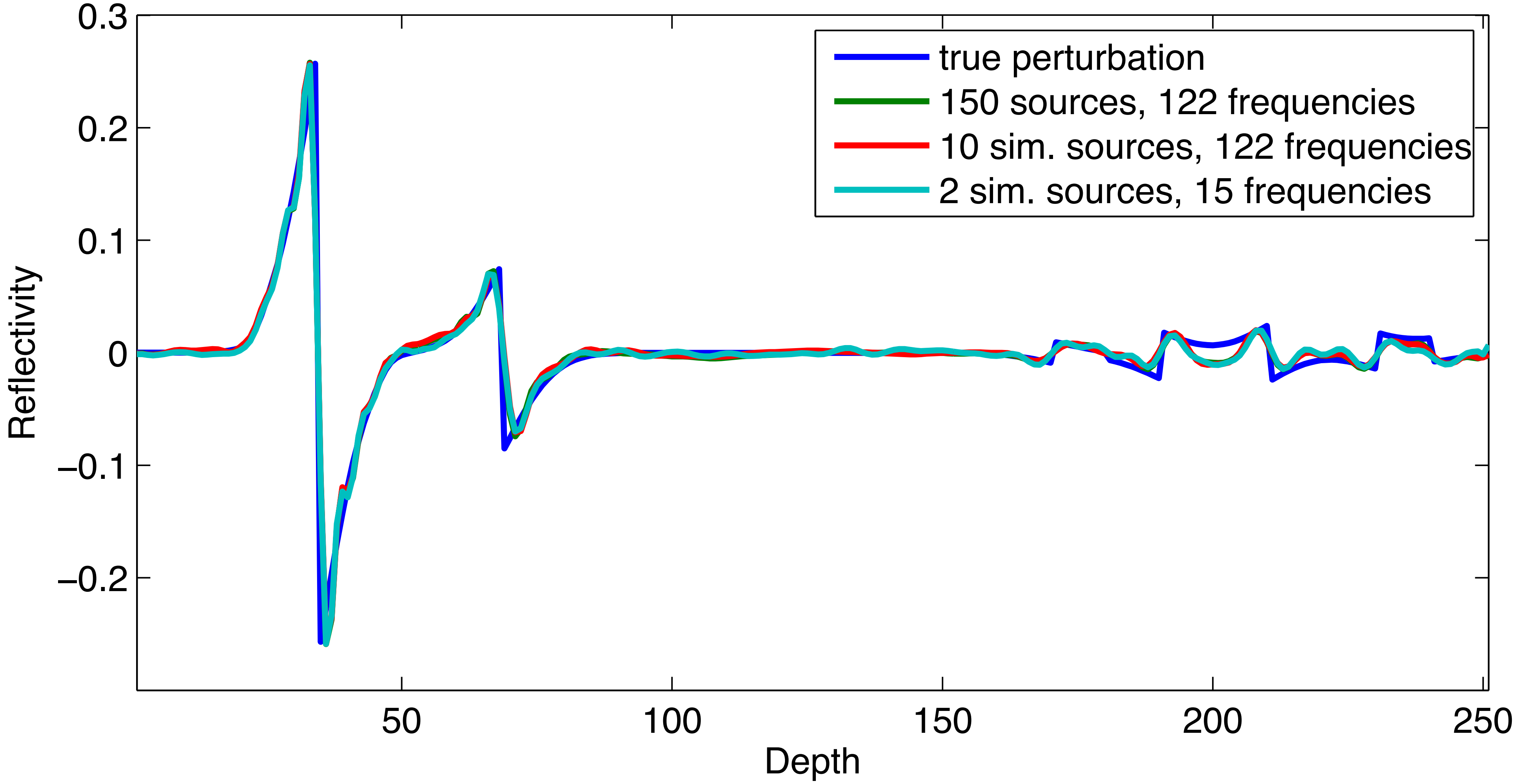


Model error decrease



Note: outliers are intermediate line-search results, not a concern; number of PDE solves in practice has ~50% overhead due to line search, etc.

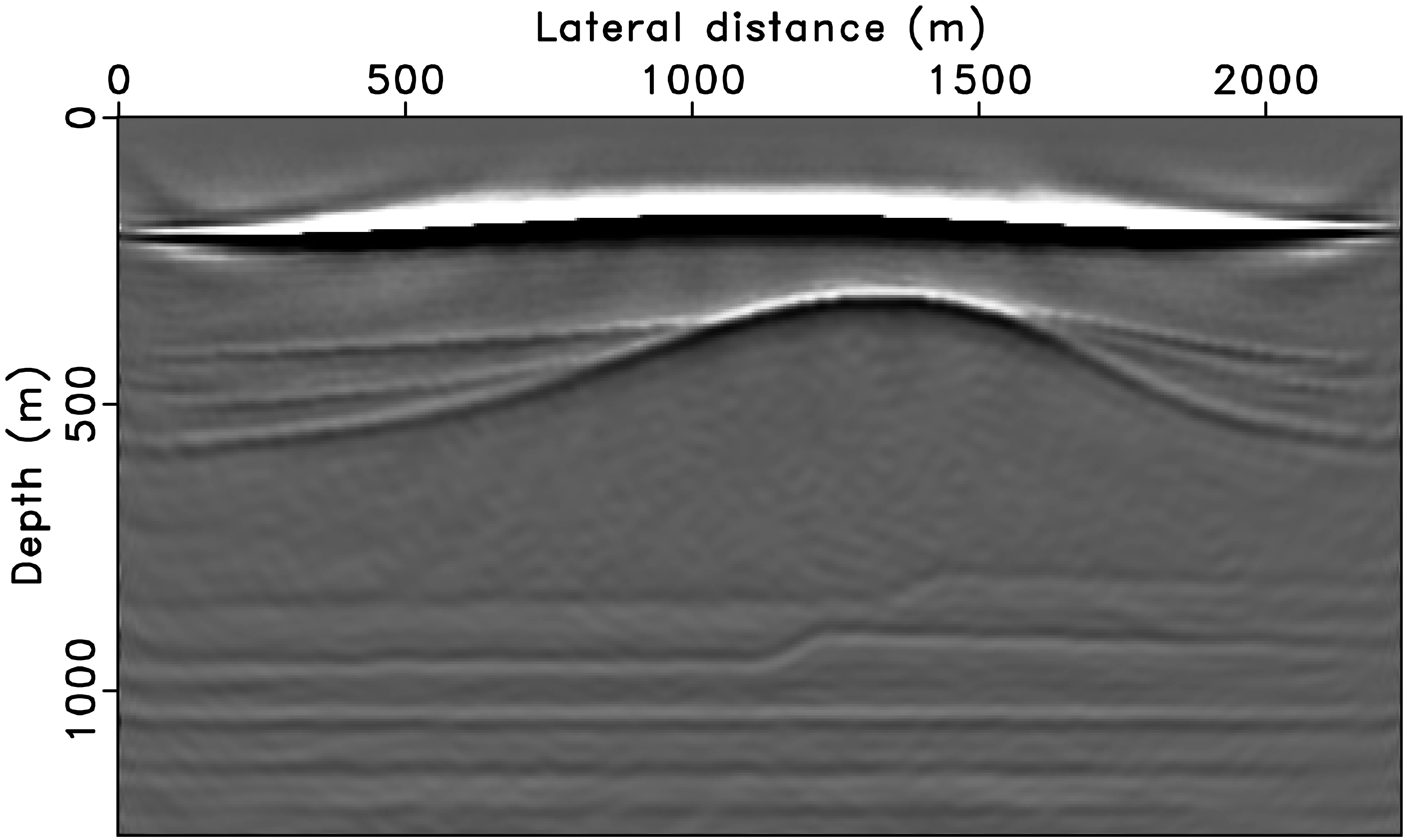
Inversion results



Trace to trace comparison: the 224th trace of model perturbation

Comparison: batch size

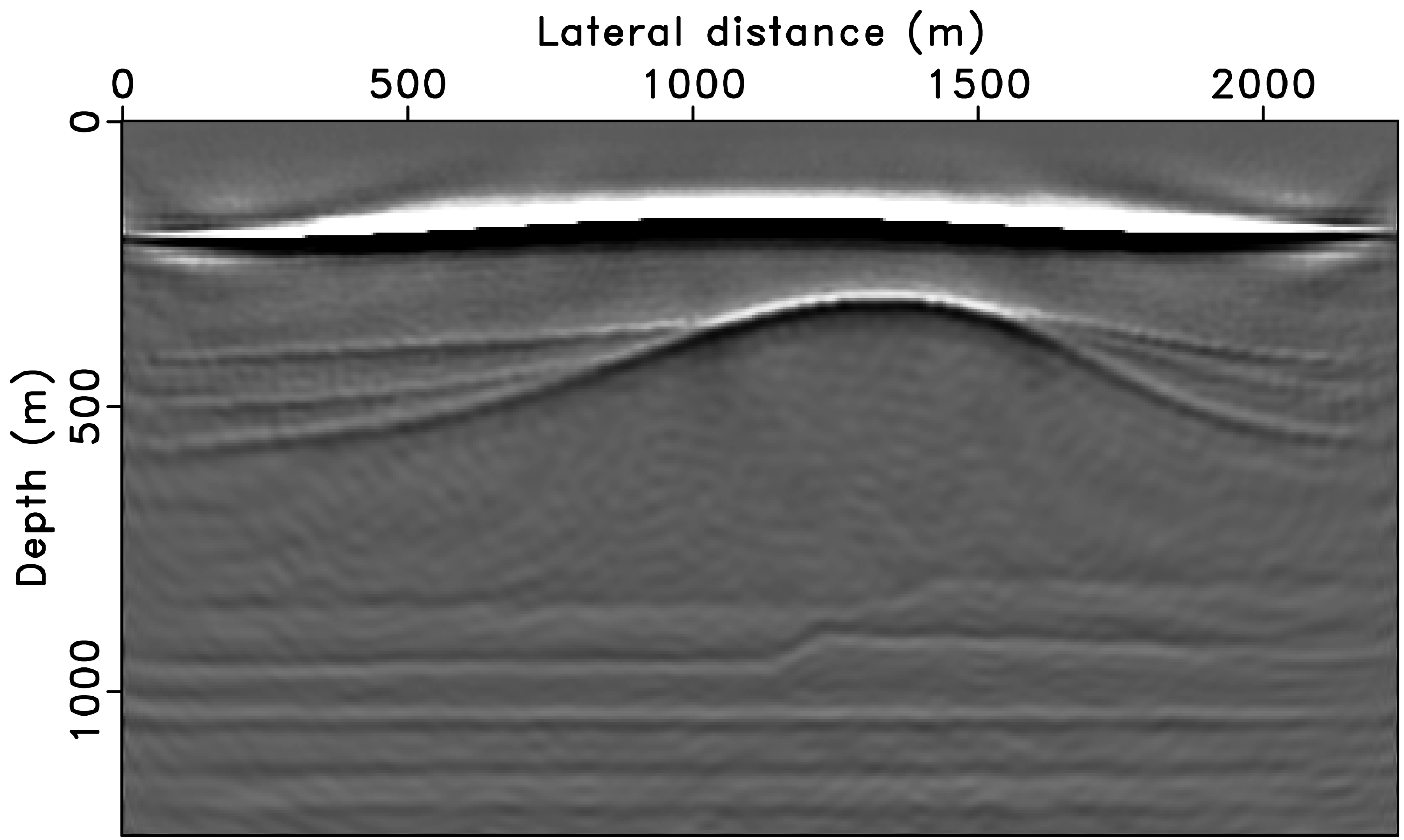
[same budget of PDE solves]



Fast imaging of total data
Batch size: 30 (2 simultaneous sources and 15 frequencies)
Iteration: 305
Number of PDE solves: 36.6 thousand (by calculation)

Comparison: batch size

[same budget of PDE solves]



Fast imaging of total data

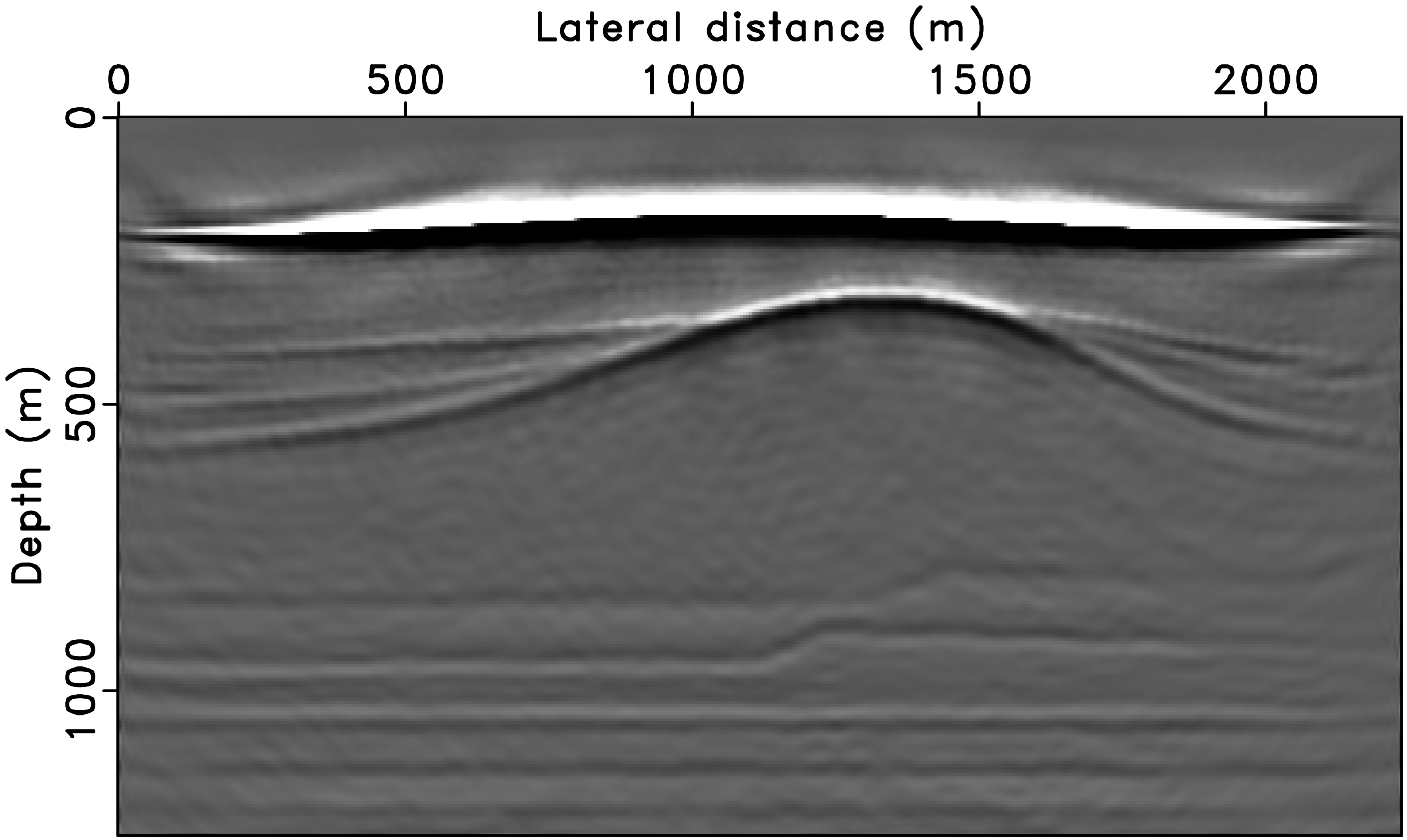
Batch size: 15 (1 simultaneous sources and 15 frequencies)

Iteration: 610

Number of PDE solves: 36.6 thousand (by calculation)

Comparison: batch size

[same budget of PDE solves]



Fast imaging of total data

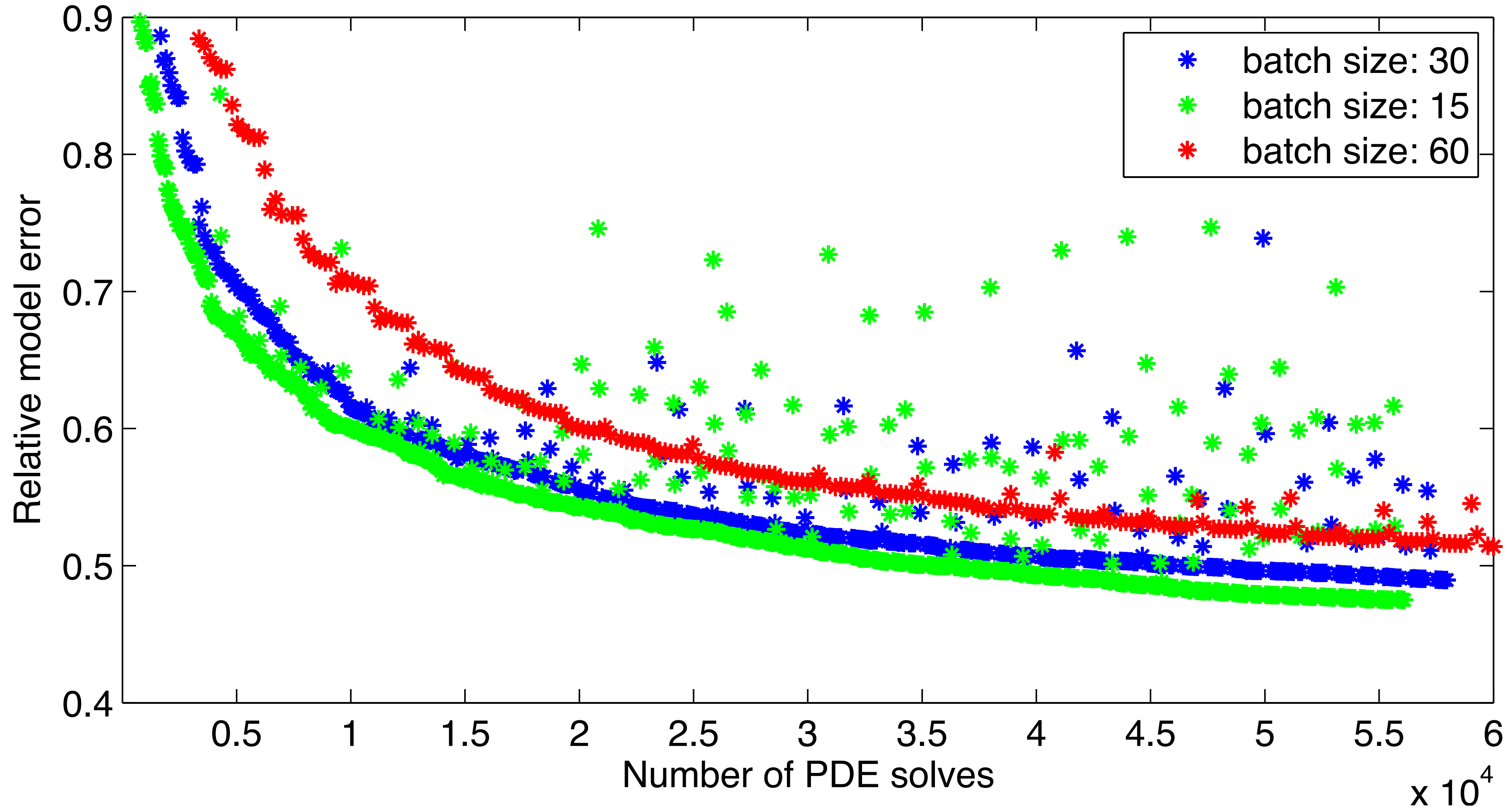
Batch size: 60 (4 simultaneous sources and 15 frequencies)

Iteration: 152

Number of PDE solves: 36.6 thousand (by calculation)

Comparison: batch size

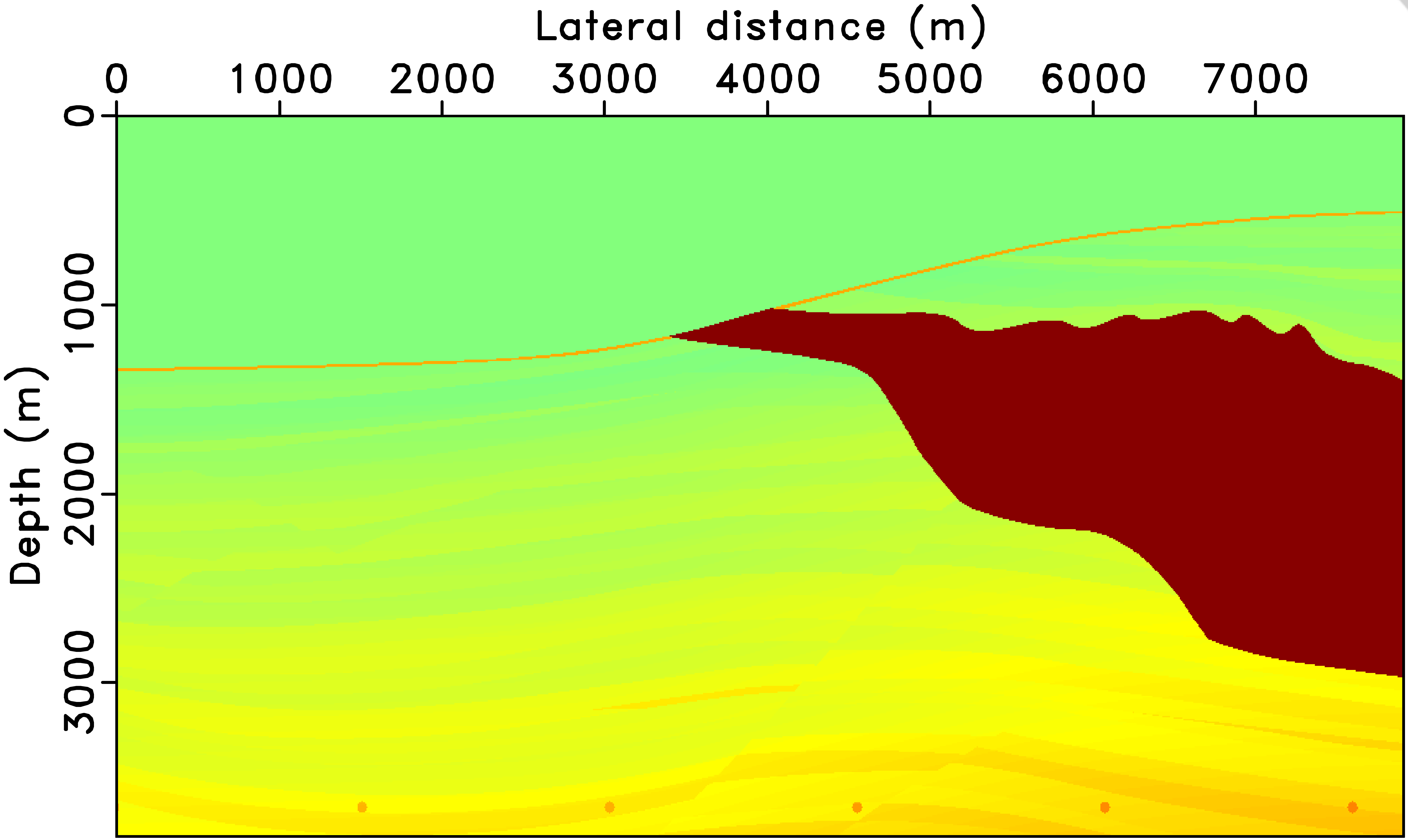
[same budget of PDE solves]



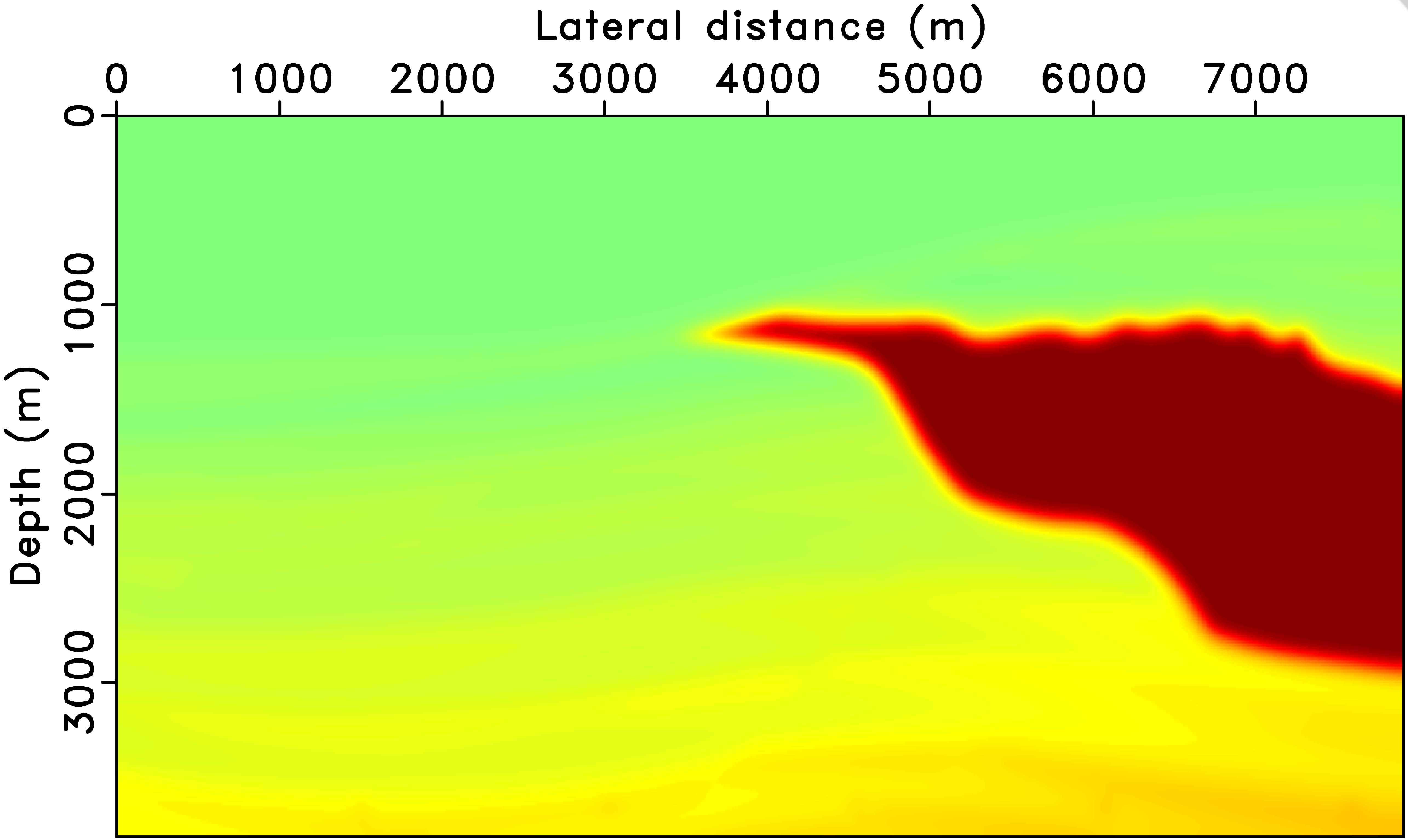
The Sigsbee2B model (cropped)

- model grid spacing: 7.62m
- using linearized data
- 174 sequential sources
- 278 frequencies in 0-34Hz range
- using 8 simultaneous sources and 15 frequencies with redrawing

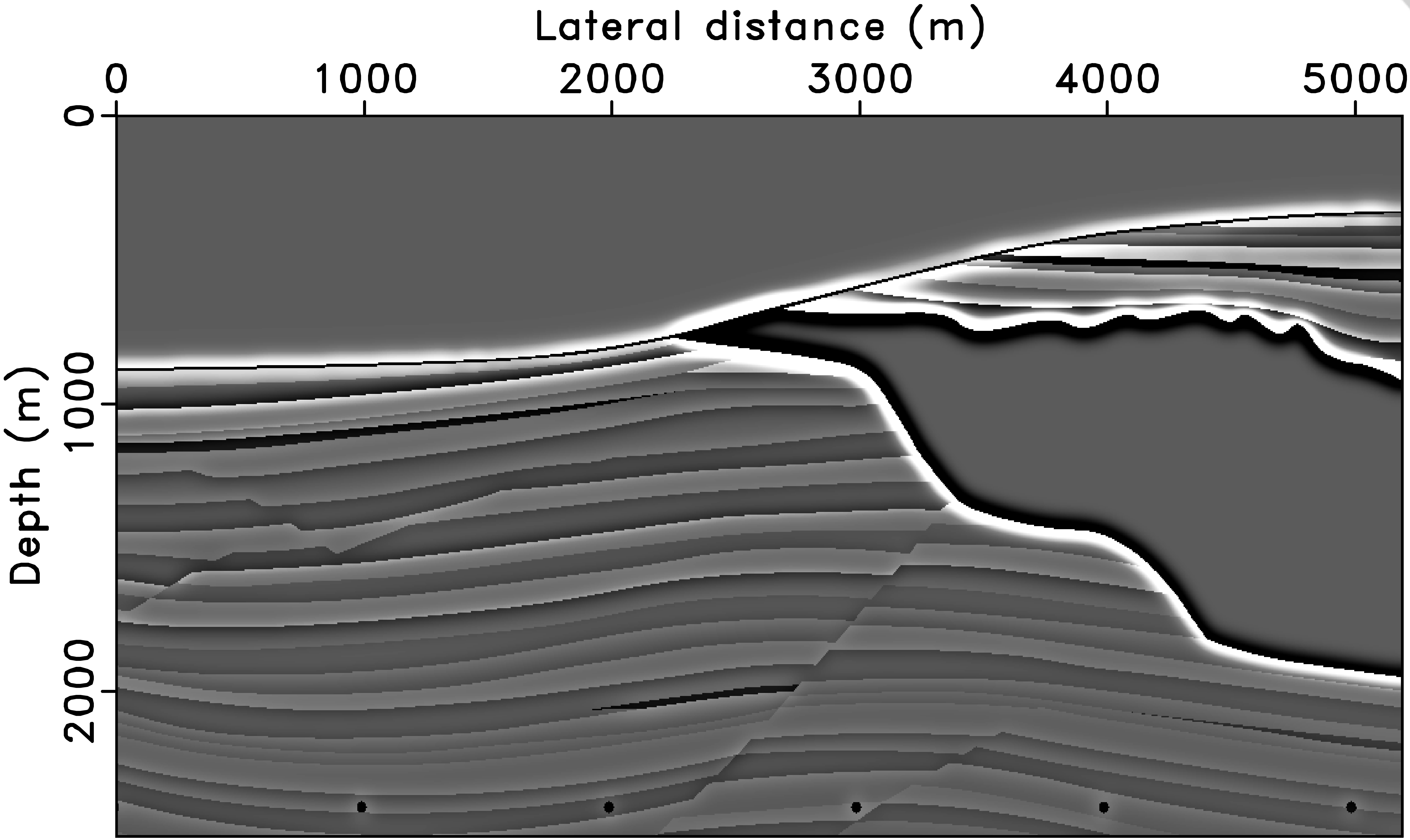
The Sigsbee2B model



Background model

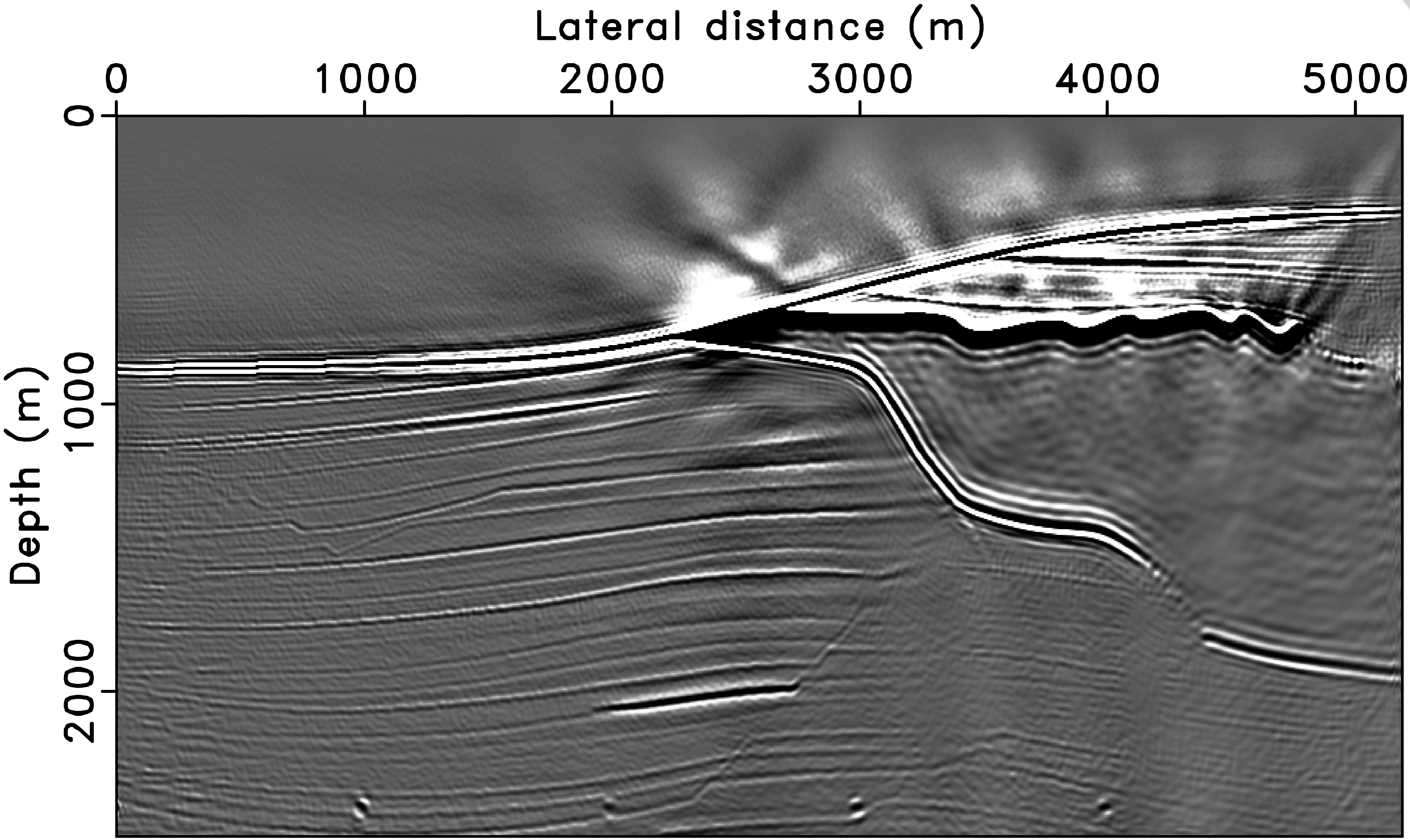


True perturbation



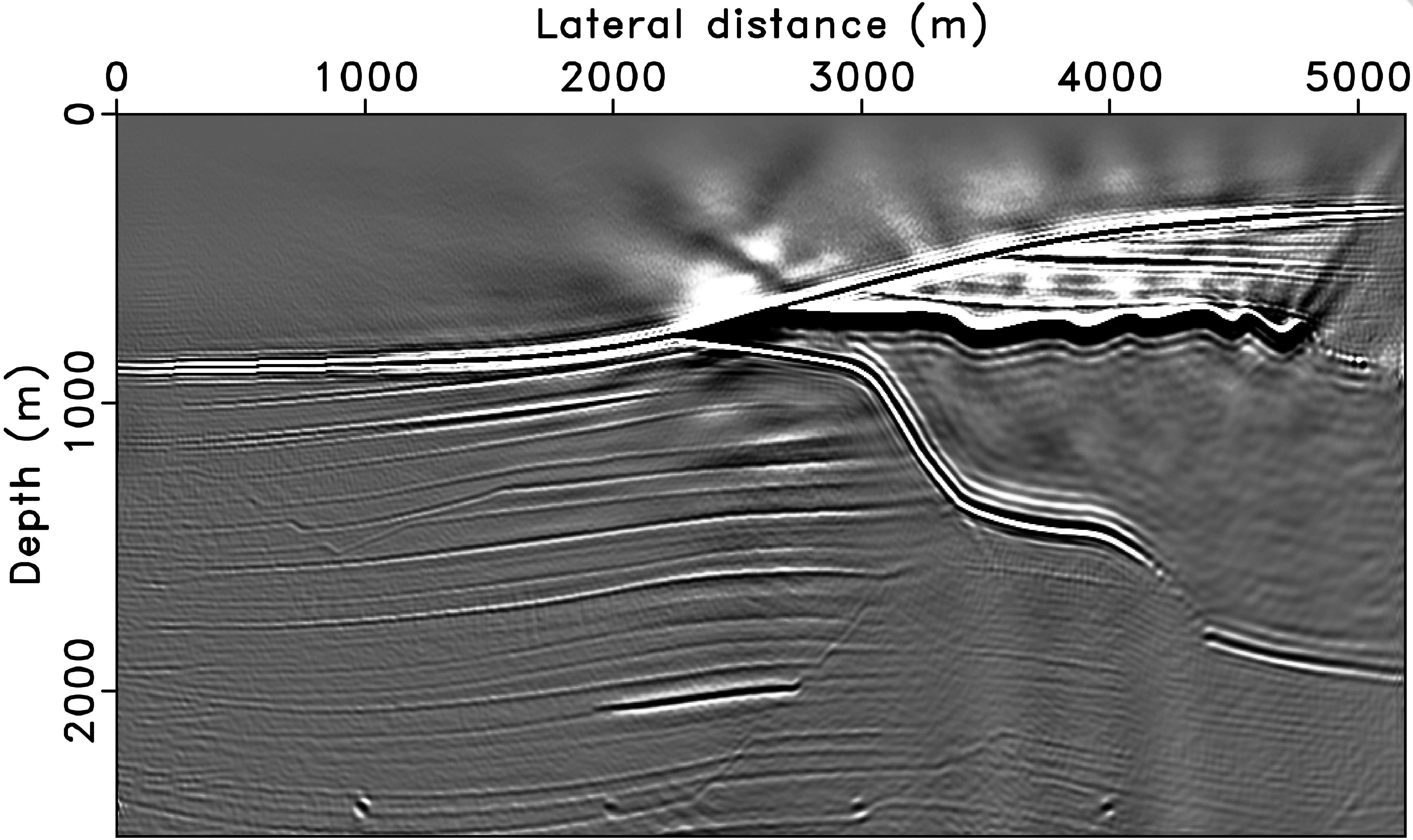
Fast inversion of primaries

[with a computational budget of a single RTM with all data]

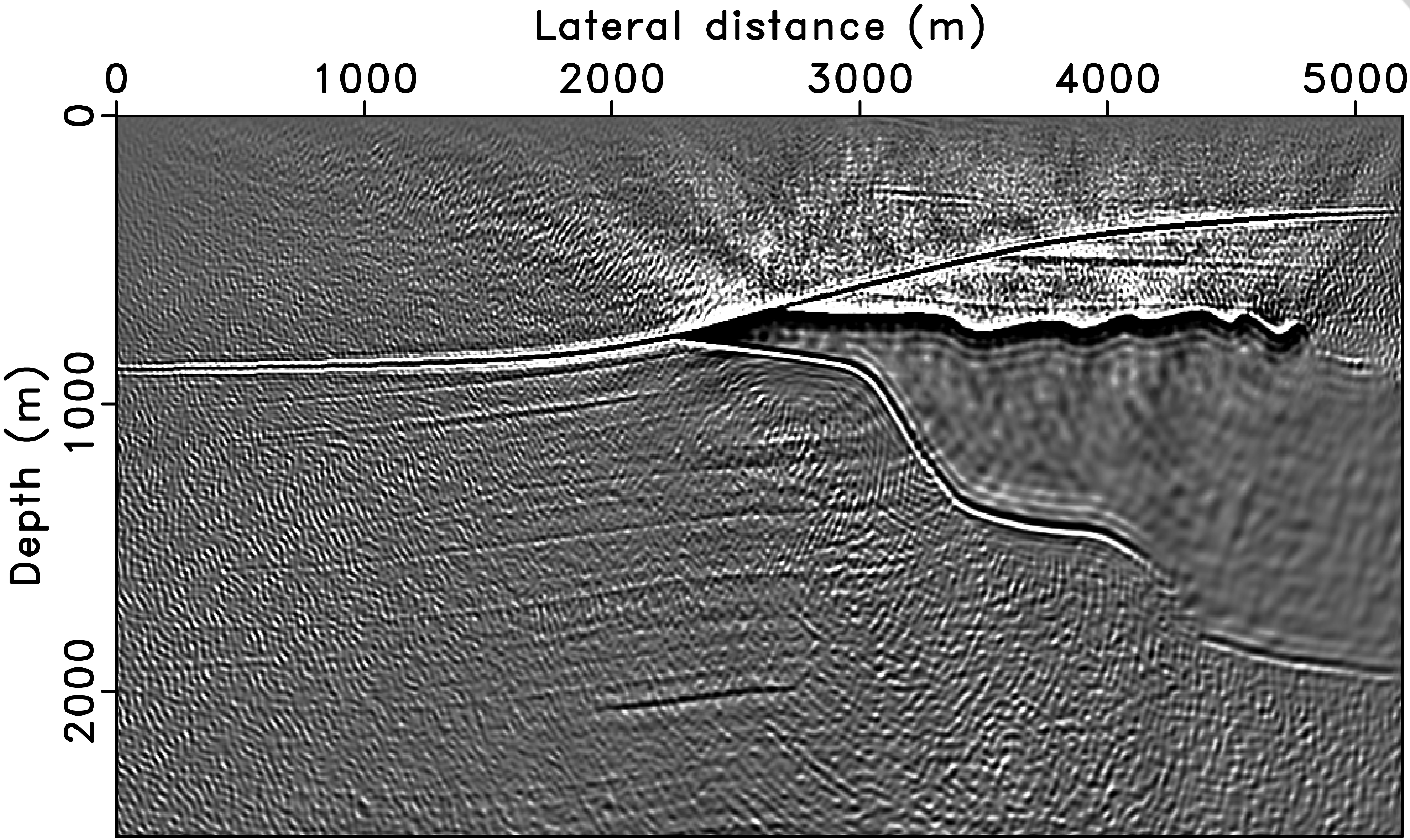


Fast inversion of total data

[with a computational budget of a single RTM with all data]



Ignore the multiples



Conclusions

- An formulation is derived to image the total data based on the SRME formulation.
- Non-causal cross correlations when imaging multiples can be avoided by inversion.
- We greatly speed up the inversion by subsampling and redrawing.

Future work

- take source/receiver ghosts into consideration
- accurate estimation of source wavelet

Acknowledgements

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