Latest developments in seismic data recovery

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Premise

Signals in nature including seismic wavefields & sedimentary basins *exhibit* some sort of *structure*

- transform-domain sparsity
- low-rank property

Come up with *new* cost-*effective randomized* sampling *strategies*

- for *land* with *randomized* arrays or *simultaneous* sweeps
- for *marine* with *randomized* time-*dithered simultaneous* sources

Compressive sensing

Compressive sensing *delivers* on this *premise* by coming up

- a *rigorous* theory with *recovery* guarantees
- *constructive* recovery algorithms by *convex* optimization

SINBAD is a world-leader in *adapting* compressive sensing

- seismic-data acquisition (land & marine)
- seismic-data processing (RTM & FWI)

This talk

Weighted one-norm minimization (Felix):

- theoretical *recovery* results
- extension to 3D seismic
- recovery based on curvelet-domain *sparsity* promotion

Nuclear-norm minimization (Sasha):

- *new* solver using factor approximation of *nuclear* norm
- recovery based on *low-rank* promotion

Trace interpolation

Involves an *underdetermined* inverse problem.

Given a seismic line of N_s sources, N_r receivers and N_t time samples, arranged into a vector $\mathbf{f}$ of length $N = N_s N_r N_t$.

Recover a sparse approximation $\tilde{\mathbf{f}}$ from *irregularly* random sampled measurements

$$\mathbf{b} = \mathbf{RMf}$$

where $\mathbf{RM}$ is the sampling operator.

Today's focus

Move to missing trace interpolation for 3D seismic

Work on *frequency* slices with 2D curvelet transform

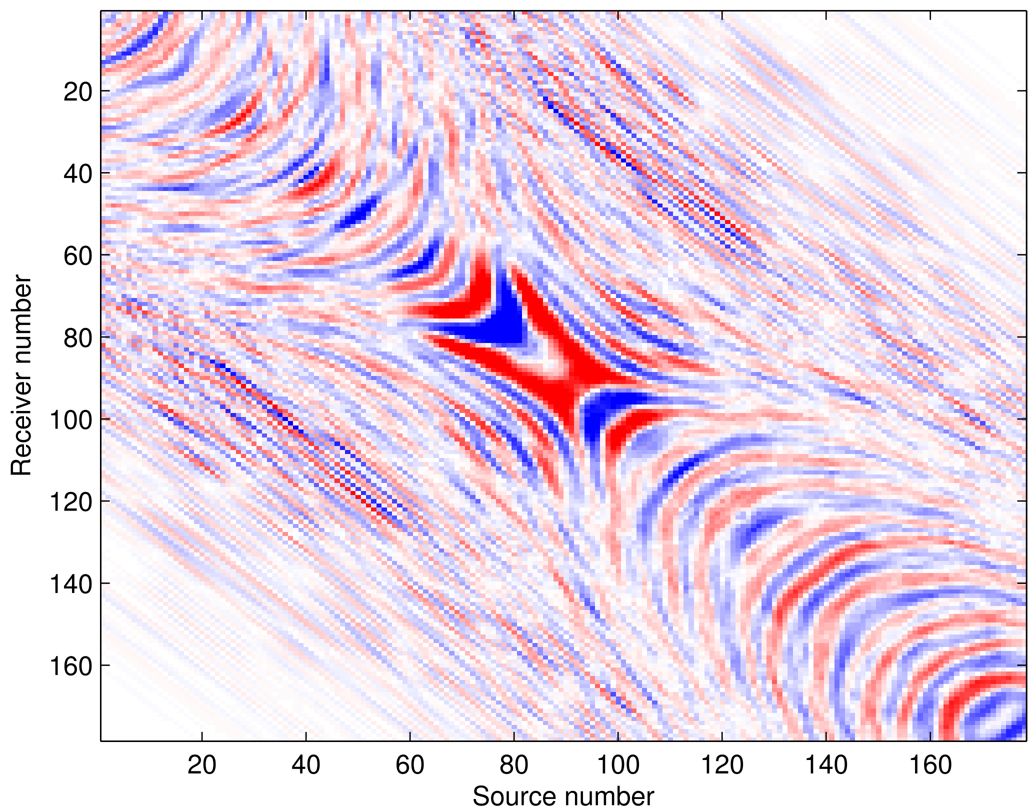
- exploit *correlations* in the support
- use *weighted* one-norm minimization

Opens the way to do 4D interpolation

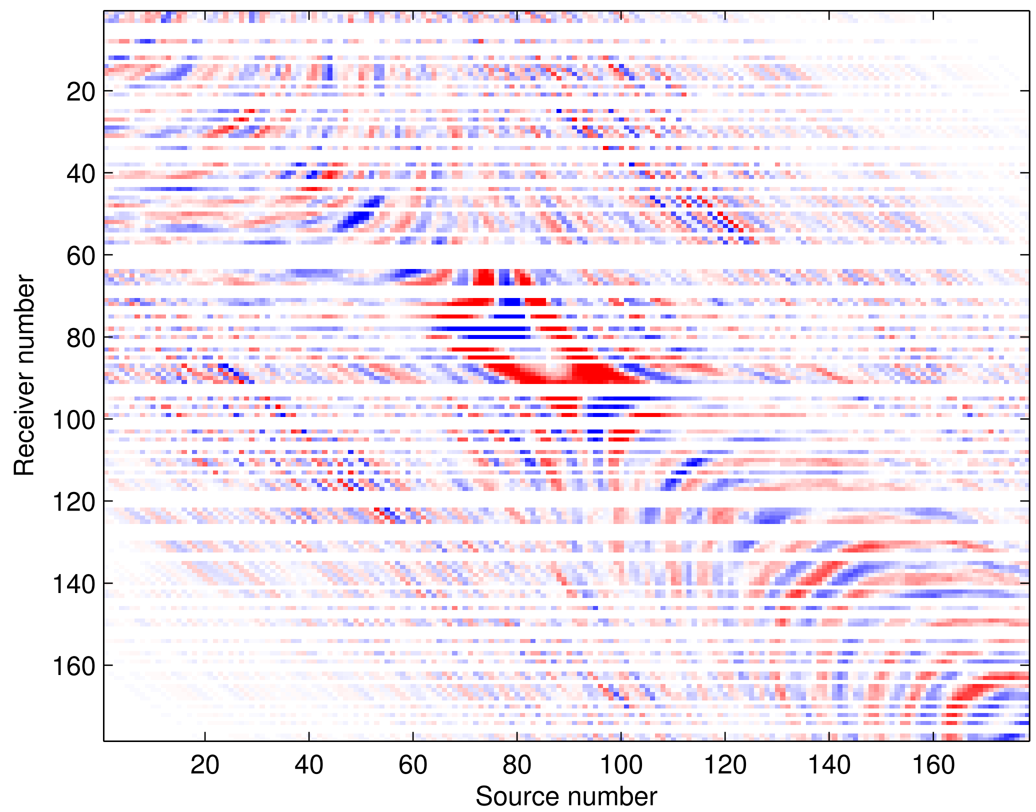
Subsampled traces

Example of a time slice with missing receivers.

$$\mathbf{f}$$



$$\mathbf{b} = \mathbf{RMf}$$



CS and random trace interpolation

When data are *randomly* subsampled & admit a *sparse* representation, the trace-interpolation problem falls under the CS paradigm.

Corresponds to finding the sparsest representation $\tilde{\mathbf{x}}$ of the data in some domain $\mathbf{S}$, by solving

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{u}\in\mathbb{C}^P} \|\mathbf{u}\|_1 \text{ subject to } \mathbf{b} = \mathbf{A}\mathbf{u}$$

where $\mathbf{A} := \mathbf{RMS^H}$.

Compressed Sensing

Candes, Romberg, and Tao; and Donoho proved that CS recovery is stable to model mismatch and robust to noise.

Theorem (CRT'06):

Suppose there exists an $a > 1$, and that A has the RIP with $\delta_{(a+1)k} < \frac{a-1}{a+1}$. Then the sparse approximation $\tilde{x}$ of x can be obtained from the solution to the ℓ_1 minimization problem and obeys

$$\|\tilde{x} - x\|_2 \leq C_0 \epsilon + C_1 \frac{\|x - x_k\|_1}{\sqrt{k}}.$$

* Remark: If x is k-sparse, then the recovery is exact.

CS implications

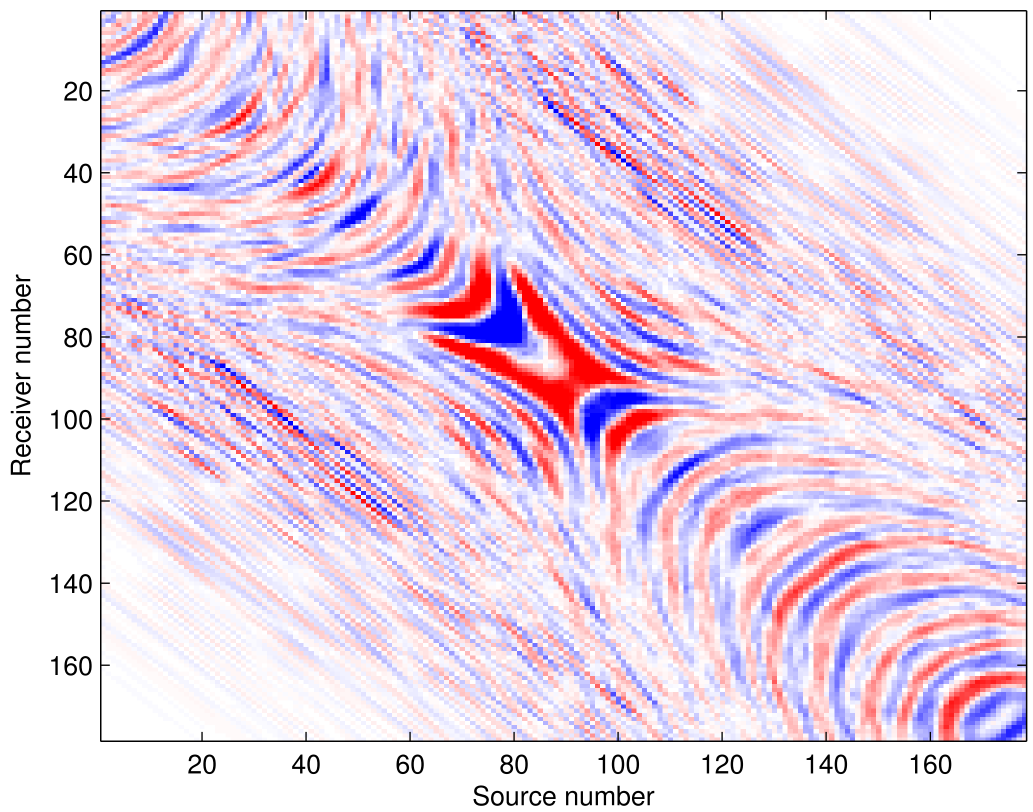
For nice measurement matrices $\mathbf{A}$, CS guarantees exact recovery for signals $\mathbf{f}$ that are *strictly* sparse

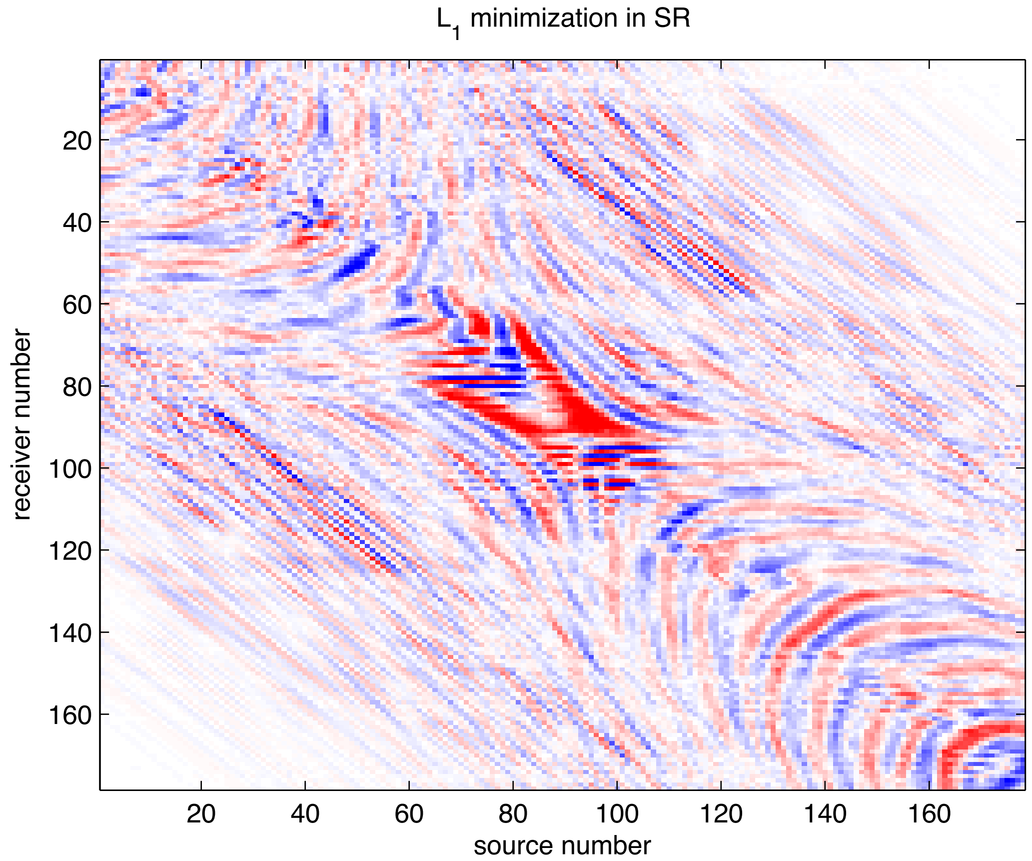
- more sparse (fewer nonzeros) => better recovery
- more randomization (better RIP) => better recovery

Extends to *compressible* signals

- more compressible => better recovery
- *recovery* akin *nonlinear* approximation

CS interpolation (time-slice)





What more can be done?

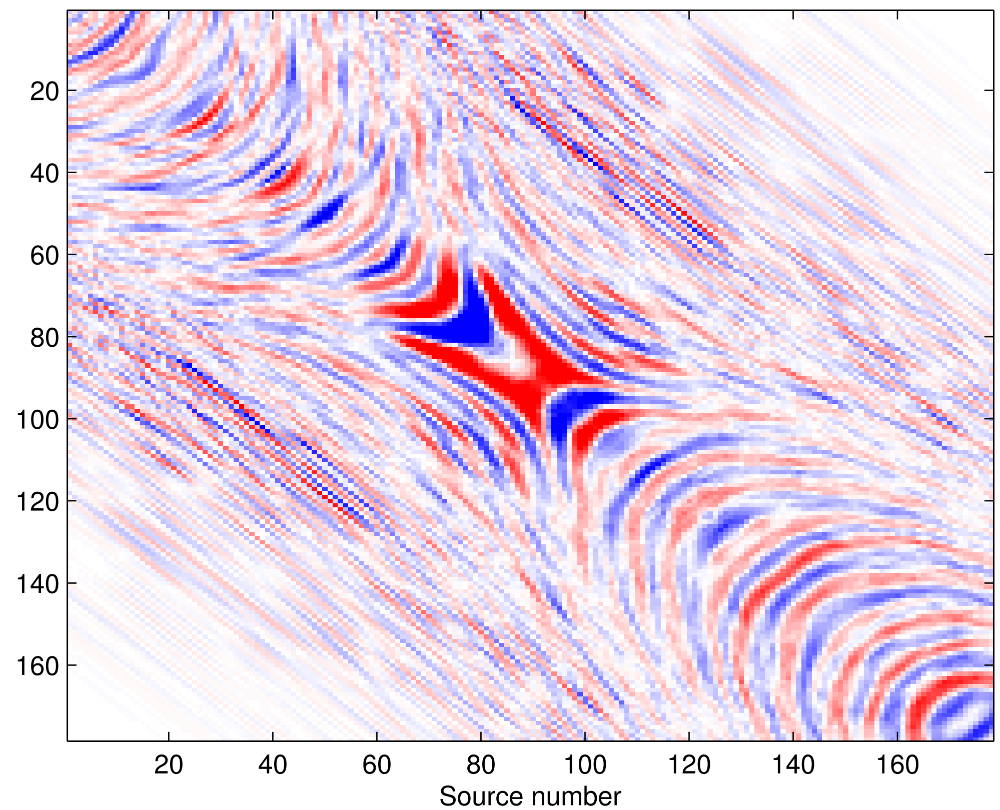
Improve the RIP of the measurement matrix $\mathbf{A}$

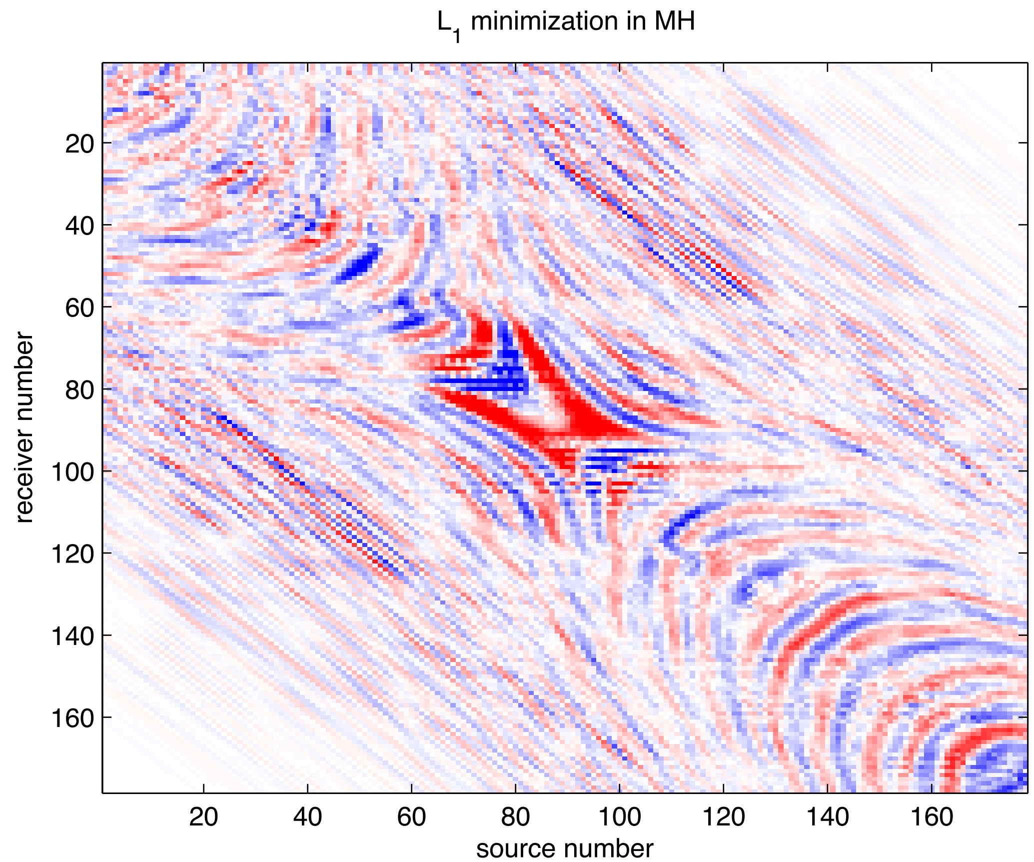
- perform recovery in *other* domains
  - e.g. midpoint-offset, time-midpoint,...

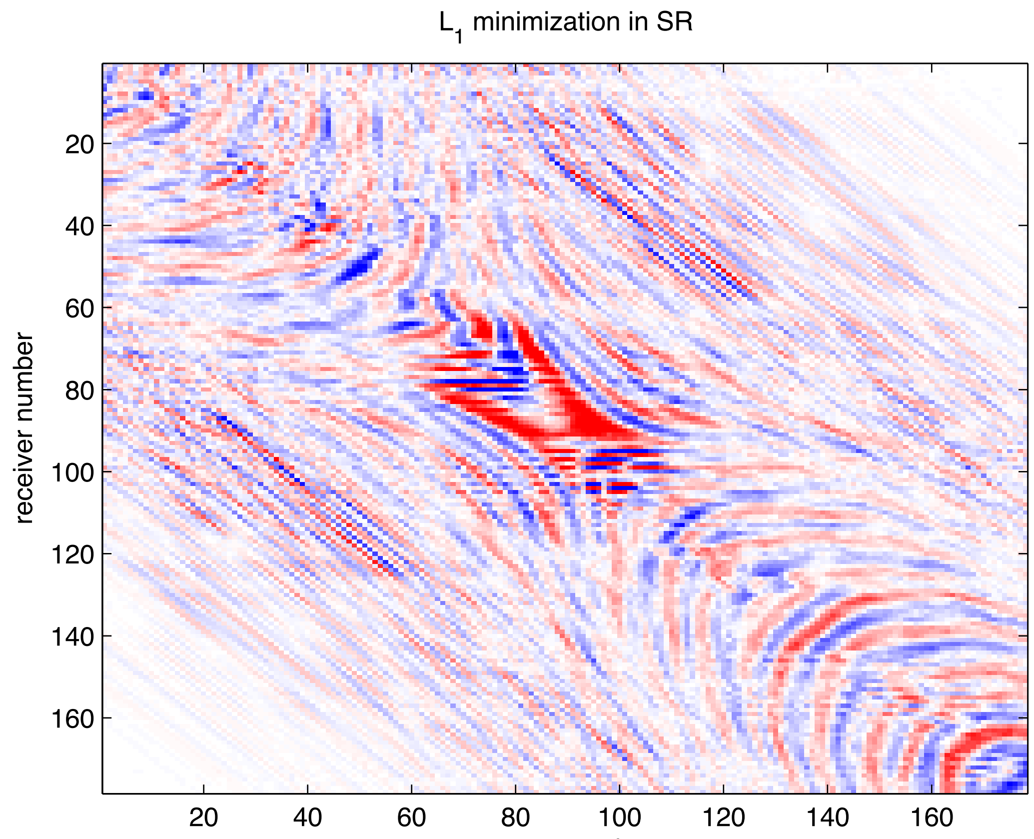
Incorporate *additional* information in the *recovery*

- *frequency* slices of seismic lines are *highly* correlated
  - incorporate *prior support* information in the *recovery*

CS in MH domain







Correlations in seismic data

Frequency slices exhibit considerable *structure* that is shared between *adjacent* frequencies.

This *shared* structure translates into a *high* degree of *overlap* in the *support* sets of curvelet coefficients.

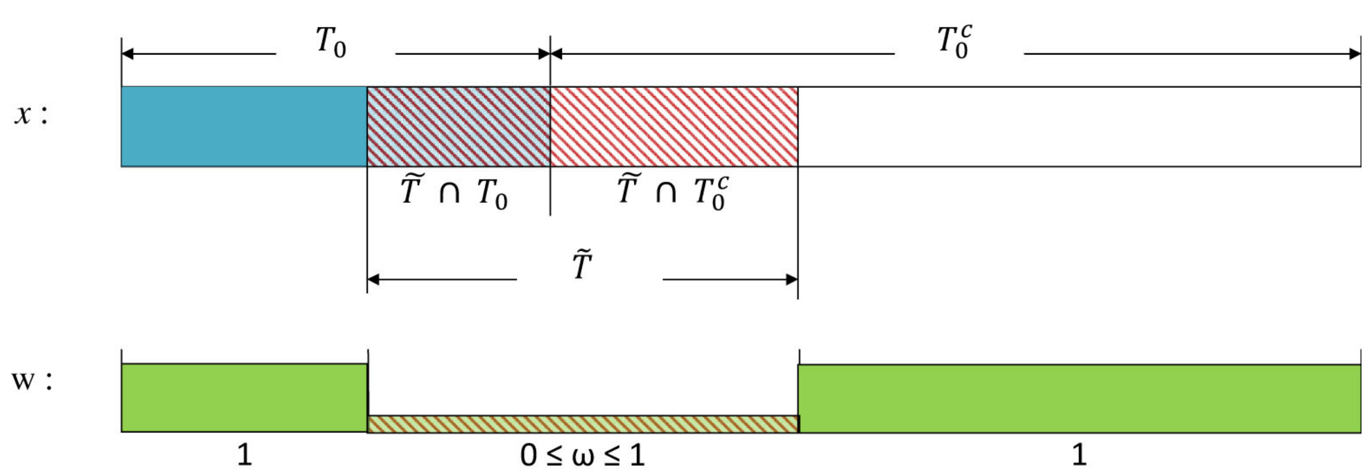
CS
with prior *support* information

Mansour et al. proposed *weighted* one-norm minimization to incorporate *prior support* information.

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{u}} \ \|\mathbf{u}\|_{1,\mathbf{w}} \text{ subject to } \|\mathbf{A}\mathbf{u} - \mathbf{b}\|_2 \leq \epsilon$$

where $\|\mathbf{u}\|_{1,\mathbf{w}} := \sum_i \mathrm{w}_i |u_i|$ is the weighted ℓ_1 norm, and the weights are assigned such that

$$\mathrm{w}_i = \begin{cases} 1, & i \in \widetilde{T}^c, \\ \omega, & i \in \widetilde{T}. \end{cases}$$



CS
with prior *support* information

Let T_0 be the support of x_k and given a *support* estimate $\widetilde{T}$ of size k and *accuracy*

$$\alpha = \frac{|\widetilde{T} \cap T_0|}{|T_0|}$$

Theorem (FMSY'12)

Suppose there exists an $a > 1$, and that A has the RIP with $\delta_{(a+1)k} < \frac{a-\gamma^2}{a+\gamma^2}$. Then the sparse approximation $\tilde{x}$ of x can be obtained from the solution to the weighted ℓ_1 minimization problem and obeys

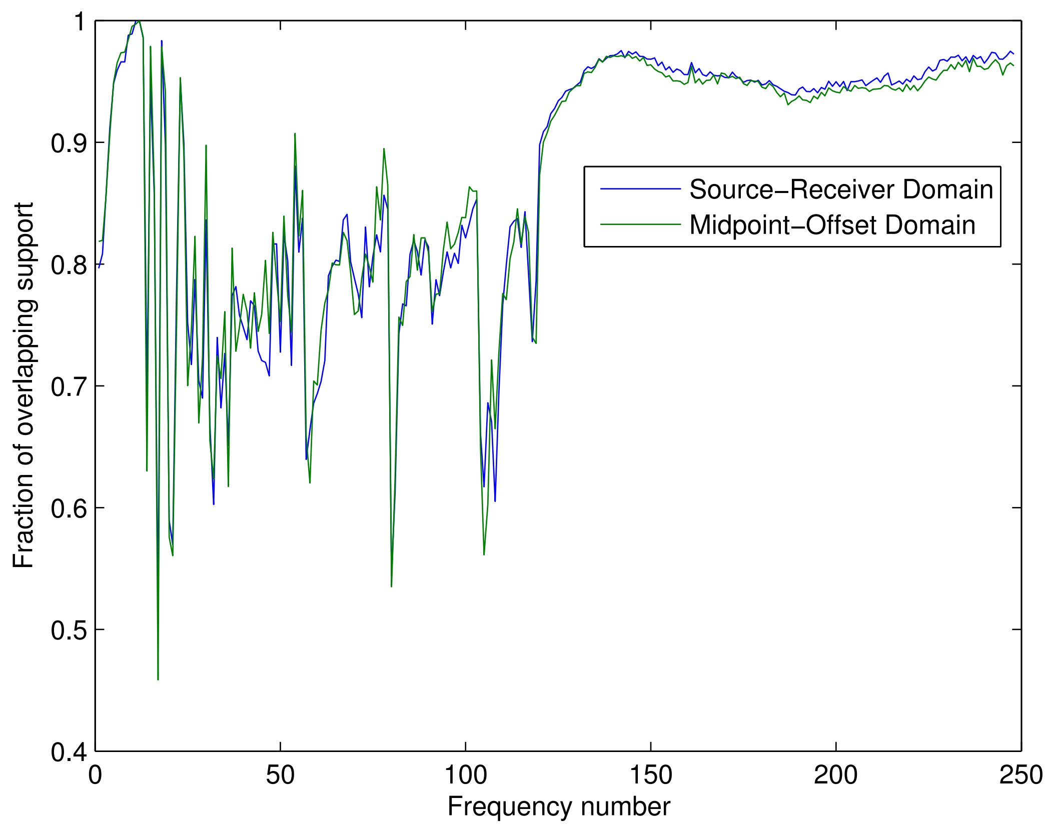
$$\|\tilde{x} - x\|_2 \leq C_0(\gamma)\epsilon + C_1(\gamma)\frac{\omega\|x_{T_0^c}\|_1 + (1-\omega)\|x_{\widetilde{T}^c \cap T_0^c}\|_1}{\sqrt{k}}.$$

Note: $\gamma = \omega + (1-\omega)\sqrt{2-2\alpha}$, where $\alpha = \frac{|\widetilde{T} \cap T_0|}{|T_0|}$.

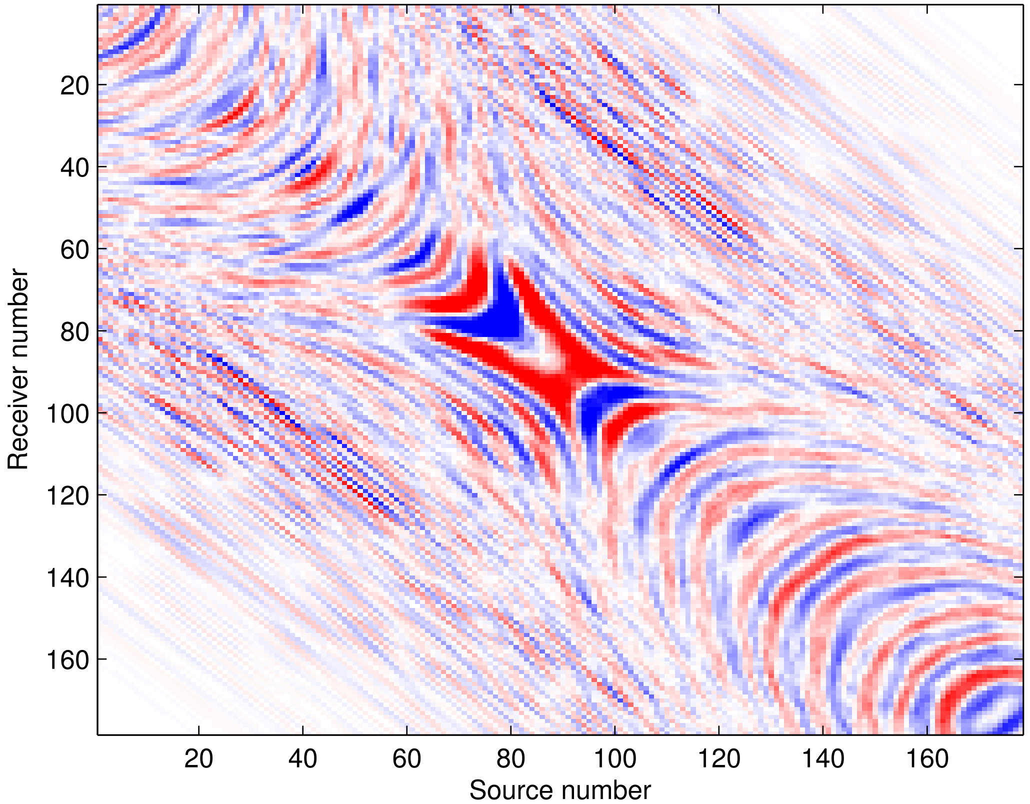
Weighted one-norm implications

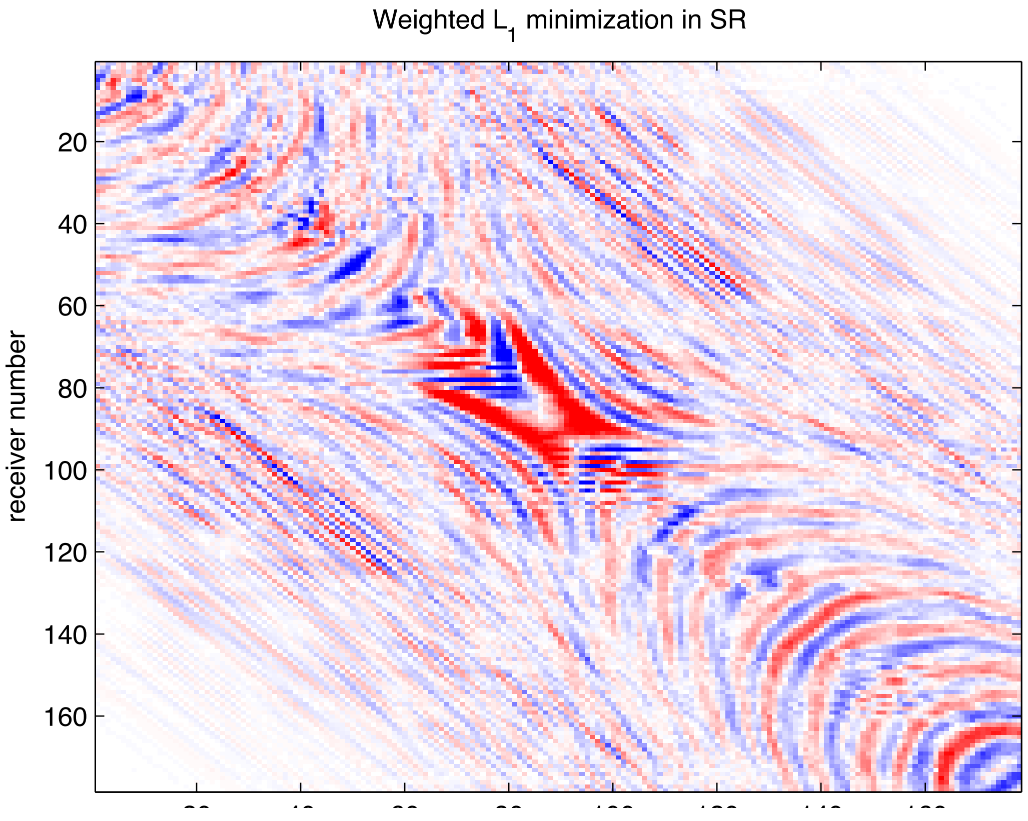
When $\alpha > \frac{1}{2}$, weighted one-norm minimization has better recovery guarantees than standard CS.

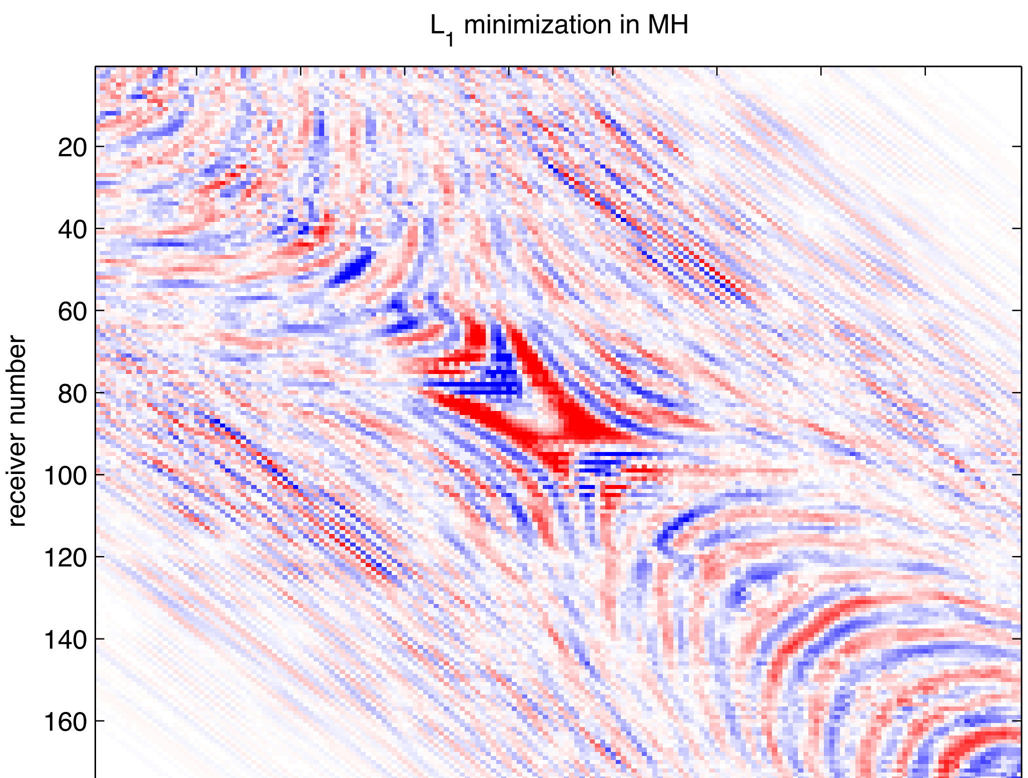
The support sets of the curvelet coefficients of seismic lines make good estimates for adjacent frequency slices.



Weighted L1 recovery
[source-receiver]

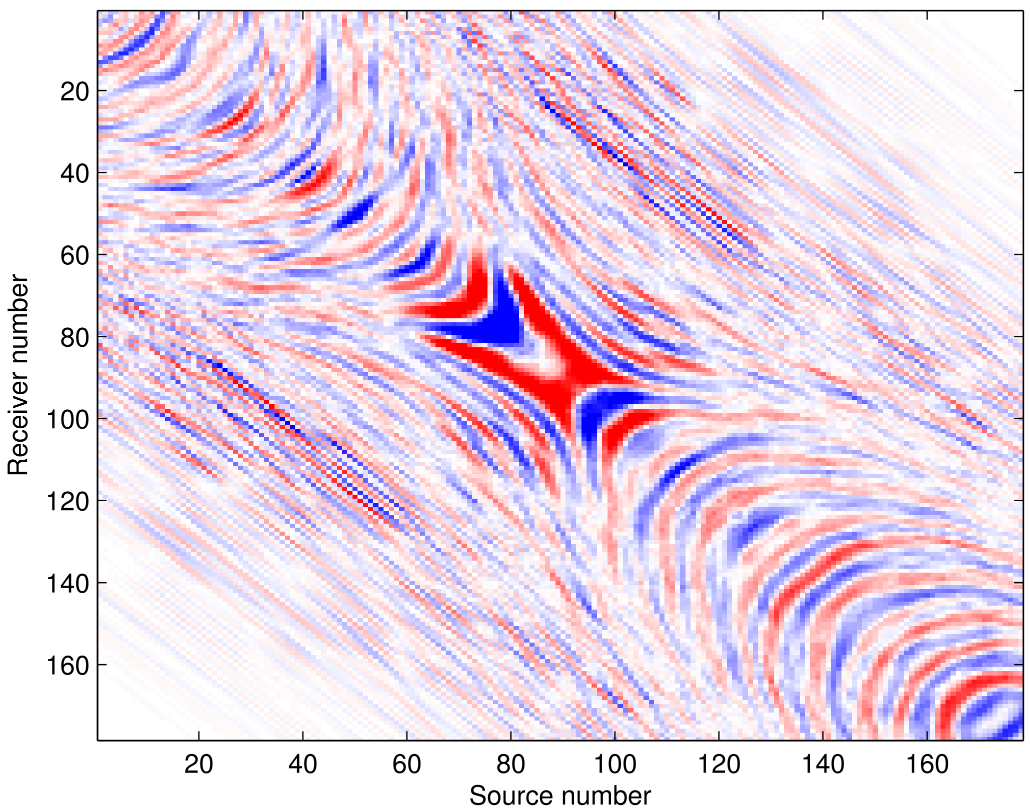


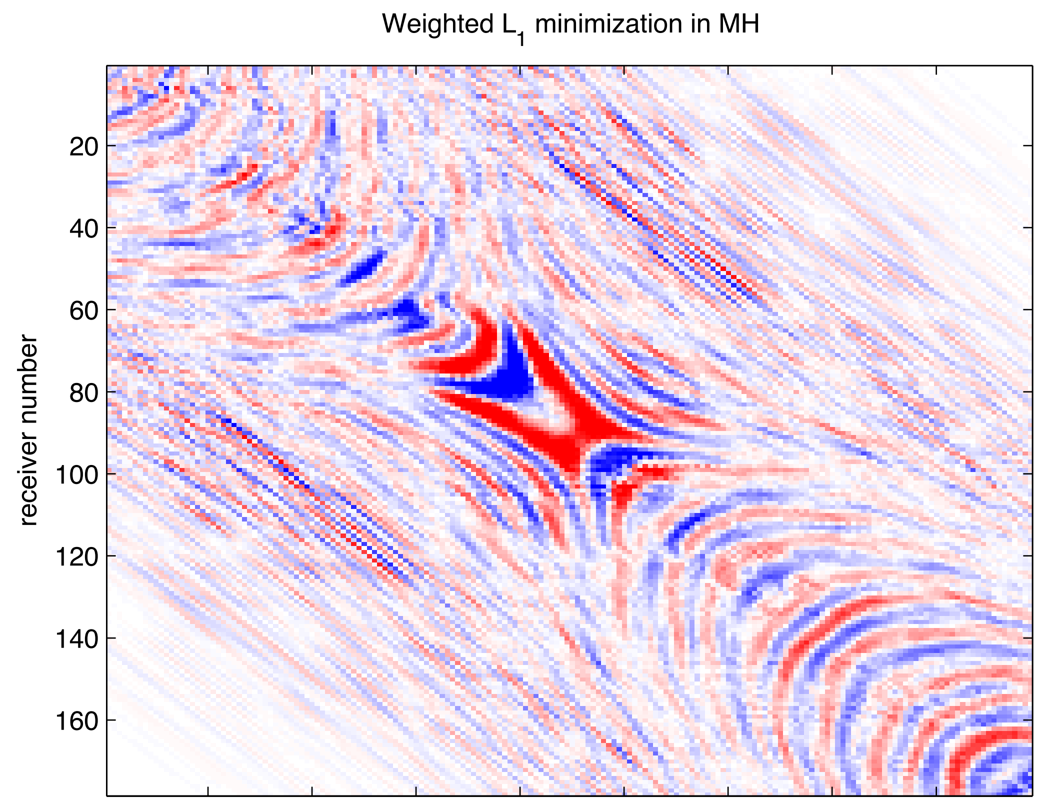


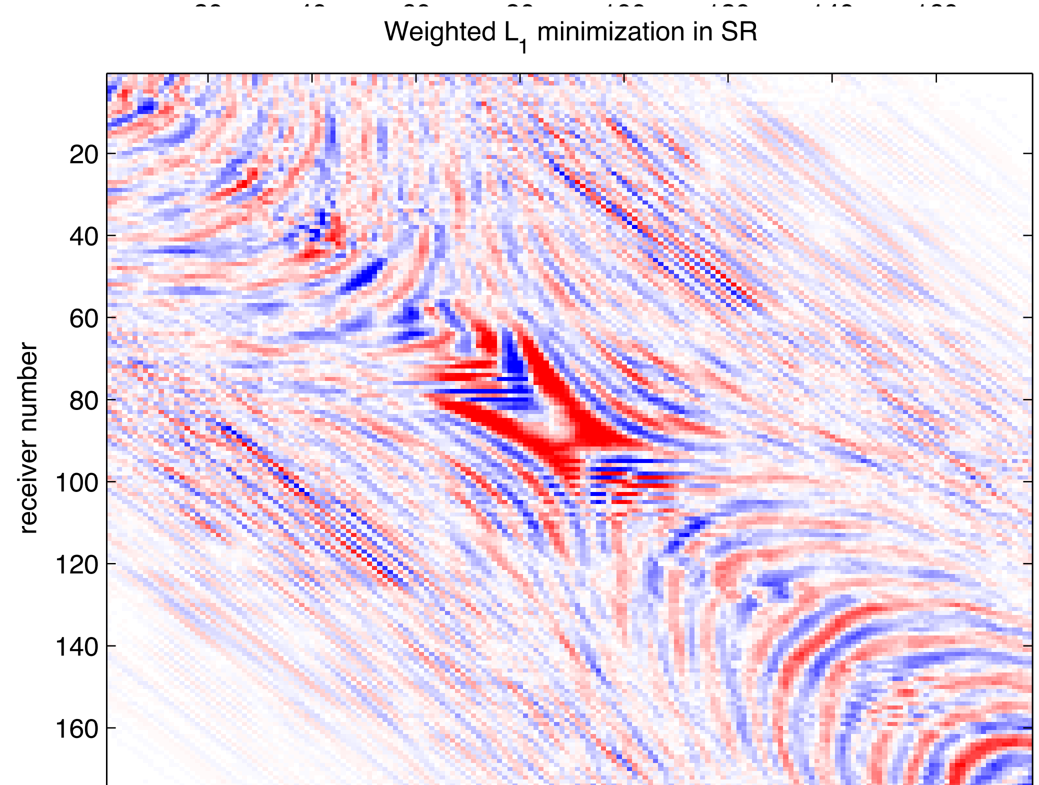


Weighted L1 recovery

[midpoint-offset]







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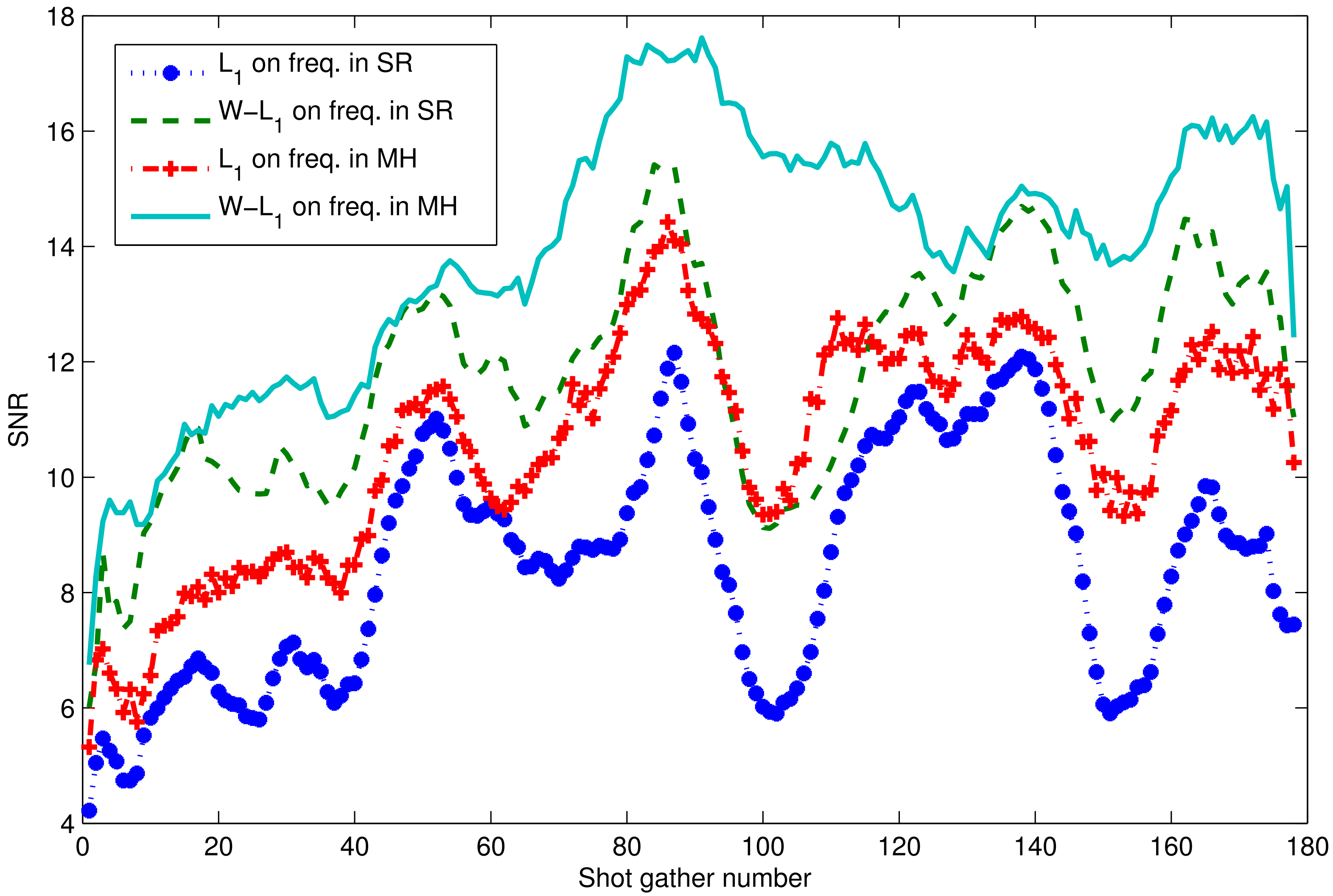
Weighted L1 recovery

Signal-to-noise-ratio (SNR) comparison



Promoting Low rank

- Seismic data is *low rank* in some frequencies/ transformed domains (e.g. midpoint-offset), motivating *low-rank* optimization to denoise/recover missing data.
- Low-rank optimization classically relies on *nuclear* norm and SVD, and hence is prohibitively *expensive*.
- We can work with a new *factorized* formulation of *nuclear* (and other) norms, bringing down the cost
- We formulate and solve the problem in an SPGL1-type setting.

Nuclear Norm

- Given any matrix $X = USV^T$,

  the nuclear norm is $\|X\|_* = \sum(\text{diag}(S))$.
- Just like the 1-norm approximates the 0-norm, so the *nuclear* norm approximates the rank.
- Therefore, to find a low rank solution, solve:

$$\min_X \|X\|_*$$

$$\text{such that } \|b - \mathcal{F}(X)\|_2 \leq \sigma \ .$$

Bring on the Pareto!

$$\min_{X} \|X\|_* \quad \text{such that } \|b - \mathcal{F}(X)\|_2 \le \sigma \ .$$

- We can use SPGL1 to solve such problems if
  - It is easy to project onto $\mathbb{B}_*^\tau := \{X : \|X\|_* \le \tau\}$
  - It is easy to evaluate the *dual* norm.
- *Dual* norm is simply *maximum* singular value (op norm)
- But just computing the nuclear norm requires SVDs. Fortunately, we can use a clever trick...

Factorization Approach

- The *Nuclear* norm has a convenient property:

$$\|X\|_* = \inf_{X=LR^*} \frac{1}{2}\left(\|L\|_F^2 + \|R\|_F^2\right)$$

- We can work with L, R rather than X:

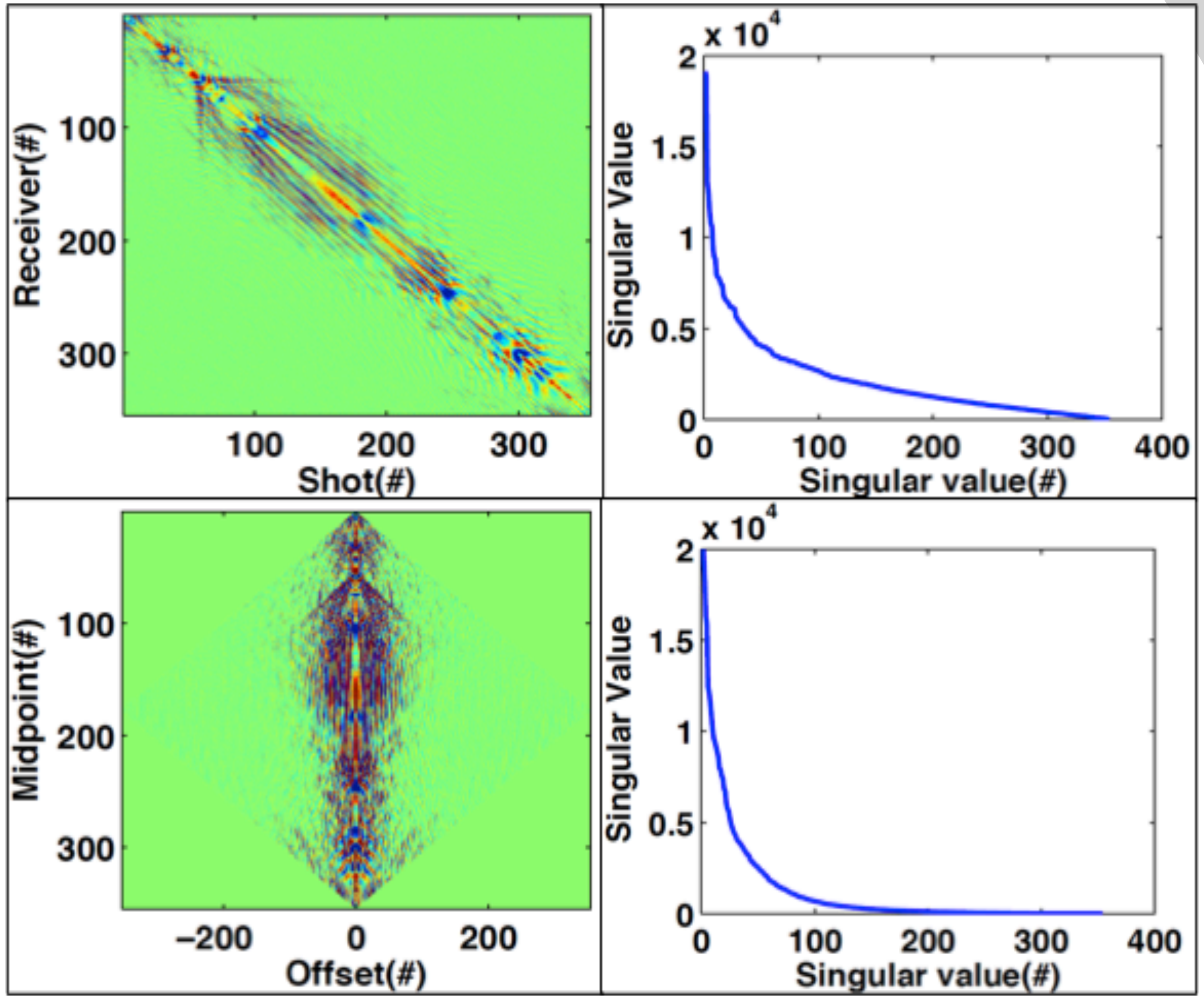
$$\min_{L,R} \frac{1}{2}\left(\|L\|_F^2 + \|R\|_F^2\right)$$

$$\text{such that } \|b - \mathcal{F}(LR^*)\|_2 \leq \sigma \ .$$

- Advantages: no SVD required; trivial projection; potential to use factors L, R downstream.

Rank Optimization in Midpoint-Offset

- Seismic data have faster singular value decay in midpoint-offset domain
- We recover 50% missing data by solving the rank optimization problem for high (70) and low (20) frequencies.
- nr = ns = 354.



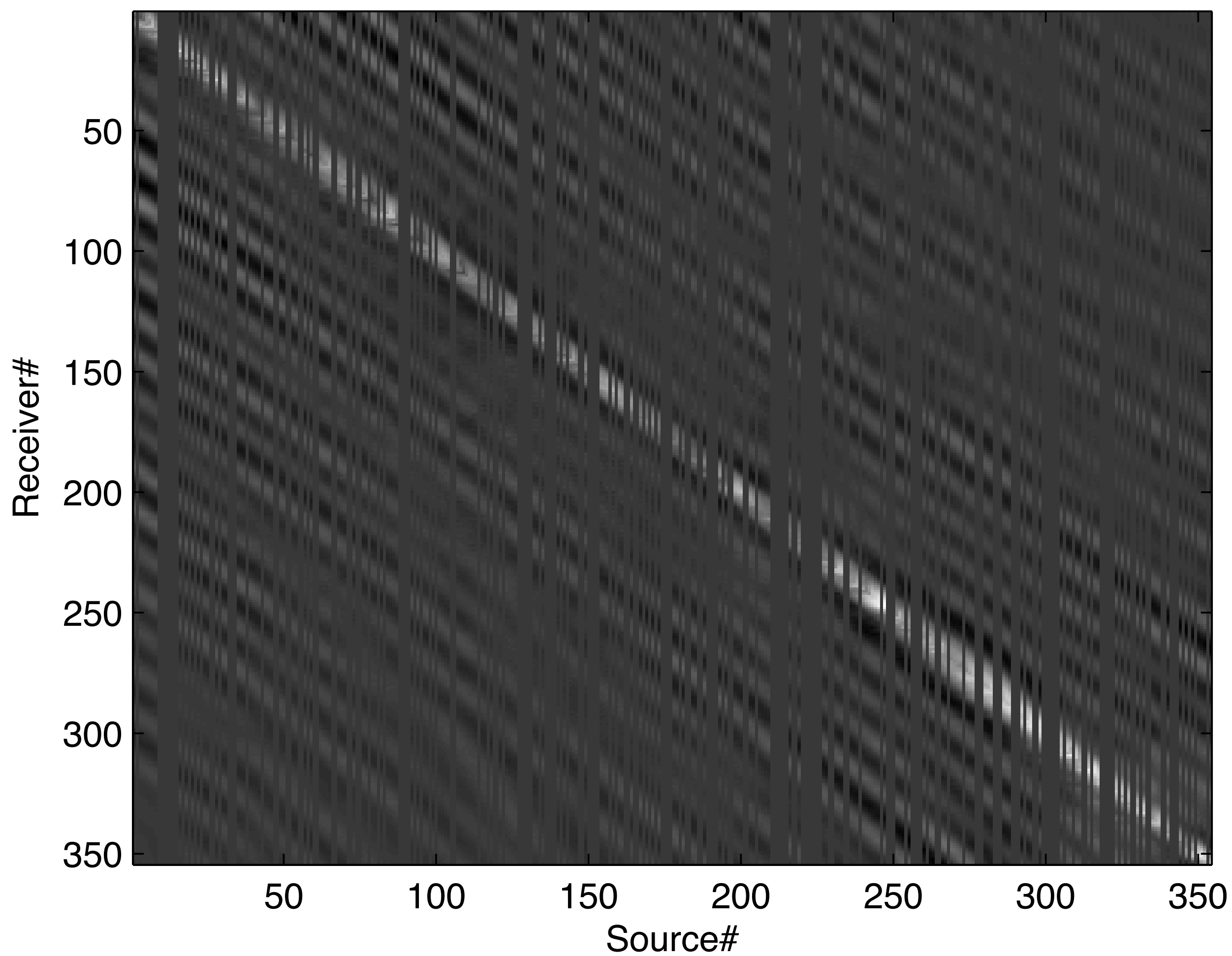
Complete data before and after transformation

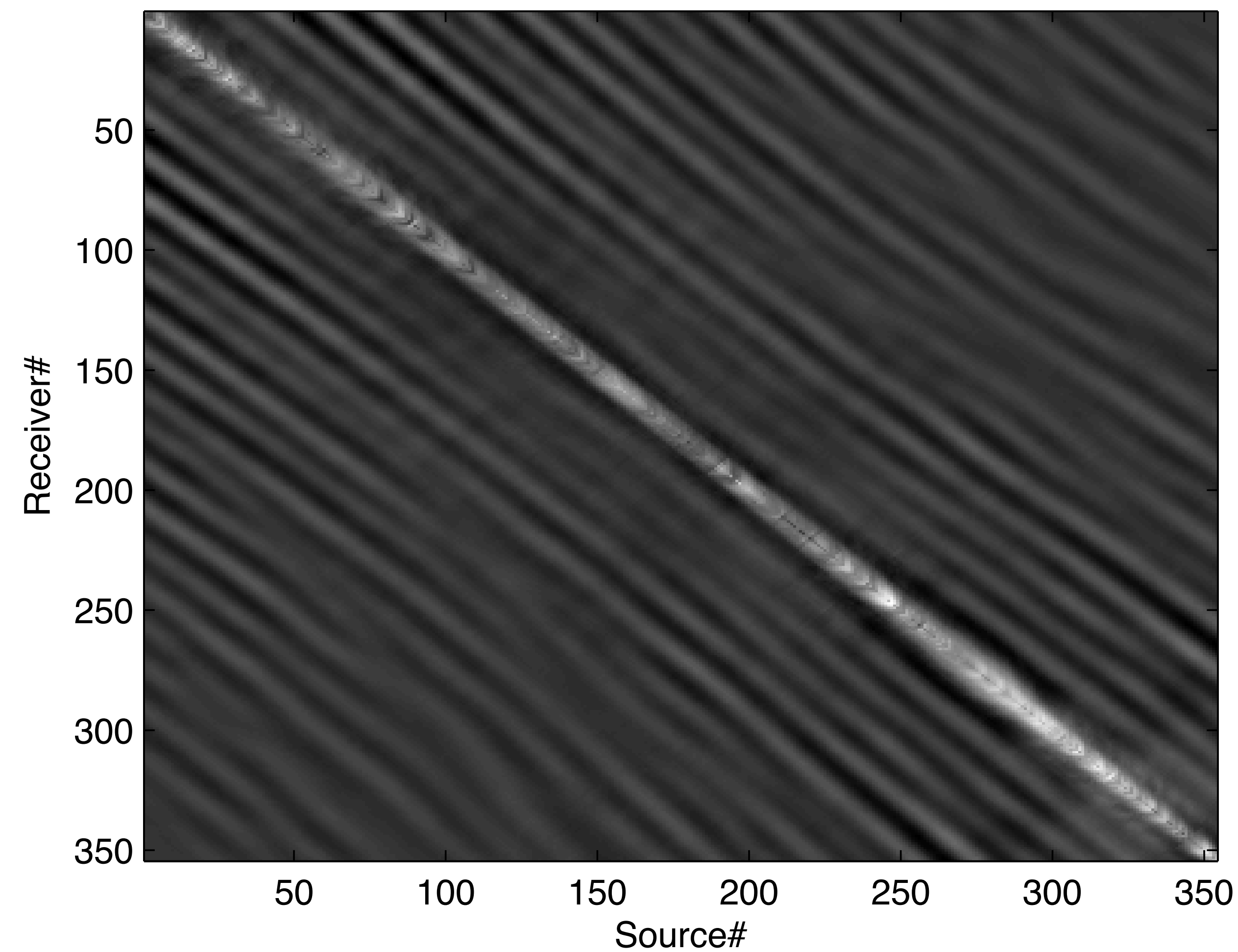
Work flow:

- Convert data with missing traces to M-O domain.
- Initialize L, R factors of pre-selected rank.
- Run rank optimization algorithm (SPGL1+).
- Form dense solution X = LR*
- Convert solution back to source-receiver domain.

Gulf of Suez Data: Least Squares+Low Rank

Frequency Slice : 20 Hz, Rank : 20





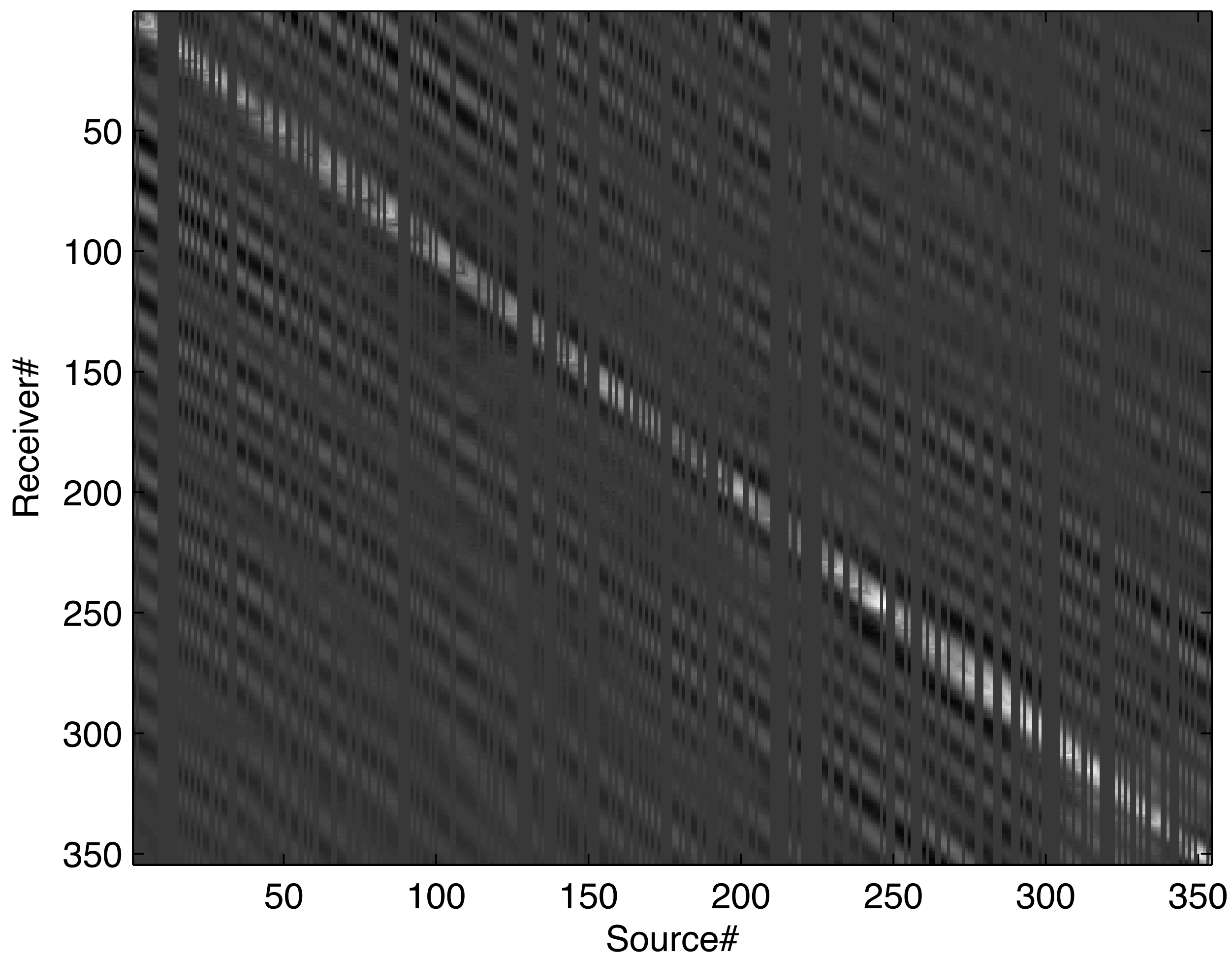
50% Missing data before interpolation

Data after interpolation, **SNR = 41.7 db**

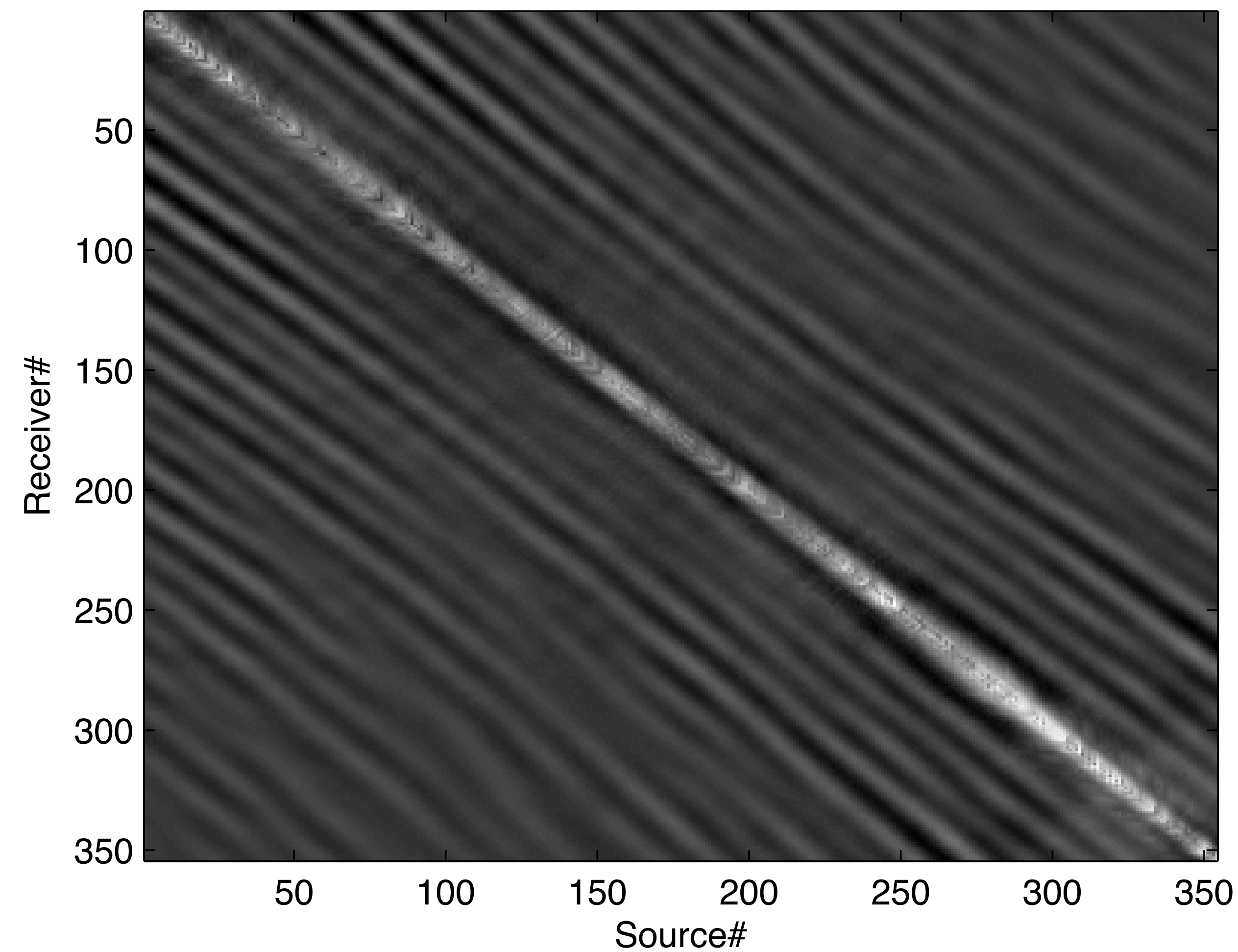
- 150 SPGL1 iterations; sigma = 1e-6, nr = ns = 354.
- Curvelet Sparsity (200 SPGL1 iterations): SNR of 18-20dB

Gulf of Suez Data: Least Squares+Low Rank

Frequency Slice : 20 Hz, Rank : 20



50% Missing data before interpolation

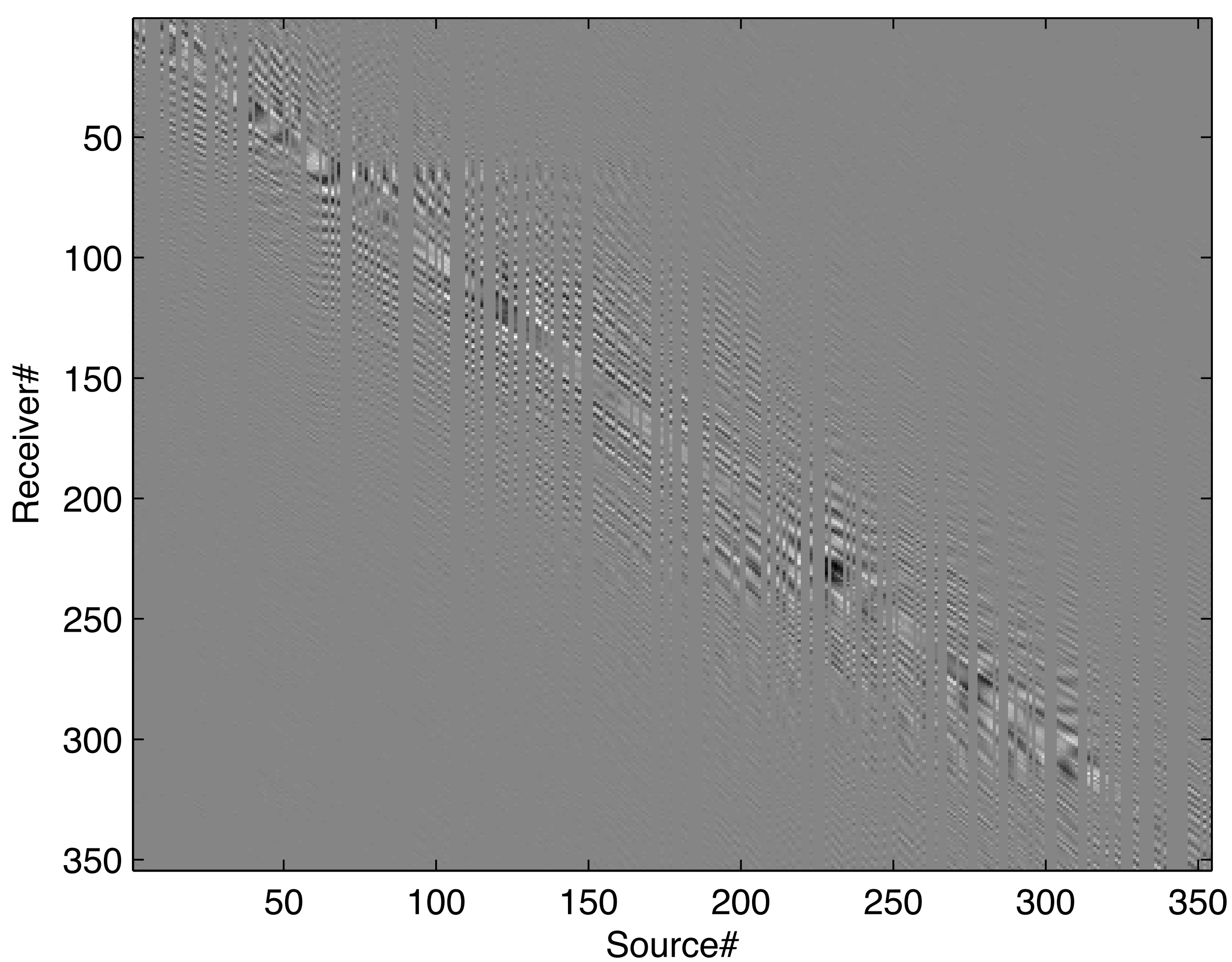


Data after interpolation, **SNR = 42.2 db**

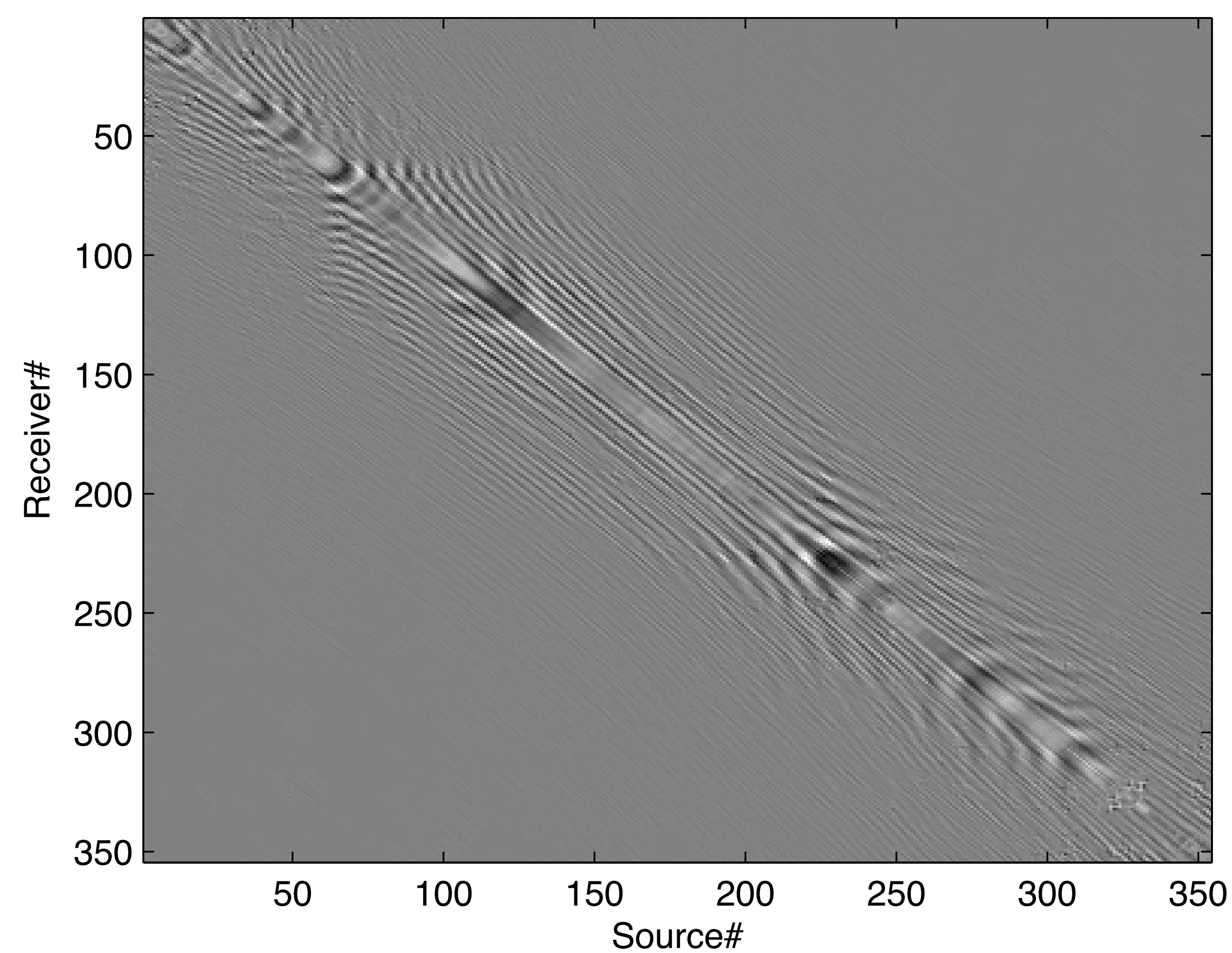
- 150 SPGL1 iterations; sigma = 1e-6, nr = ns = 354.
- Curvelet Sparsity (200 SPGL1 iterations): SNR of 18-20dB

Gulf of Suez: Least Squares + Low Rank

Frequency Slice : 70 Hz, Rank : 20



50% Missing data, before interpolation

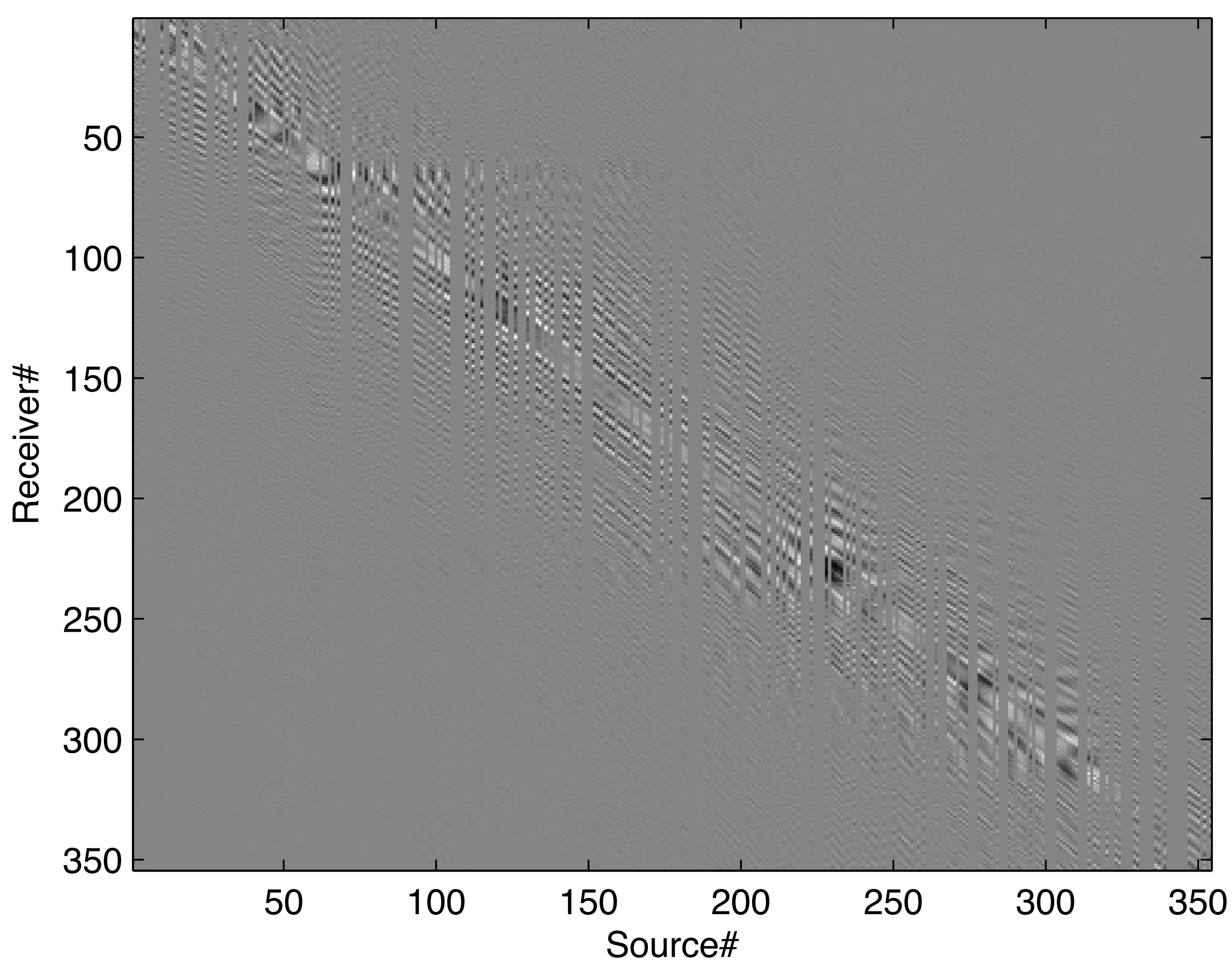


Data after interpolation, **SNR = 22.7 db**

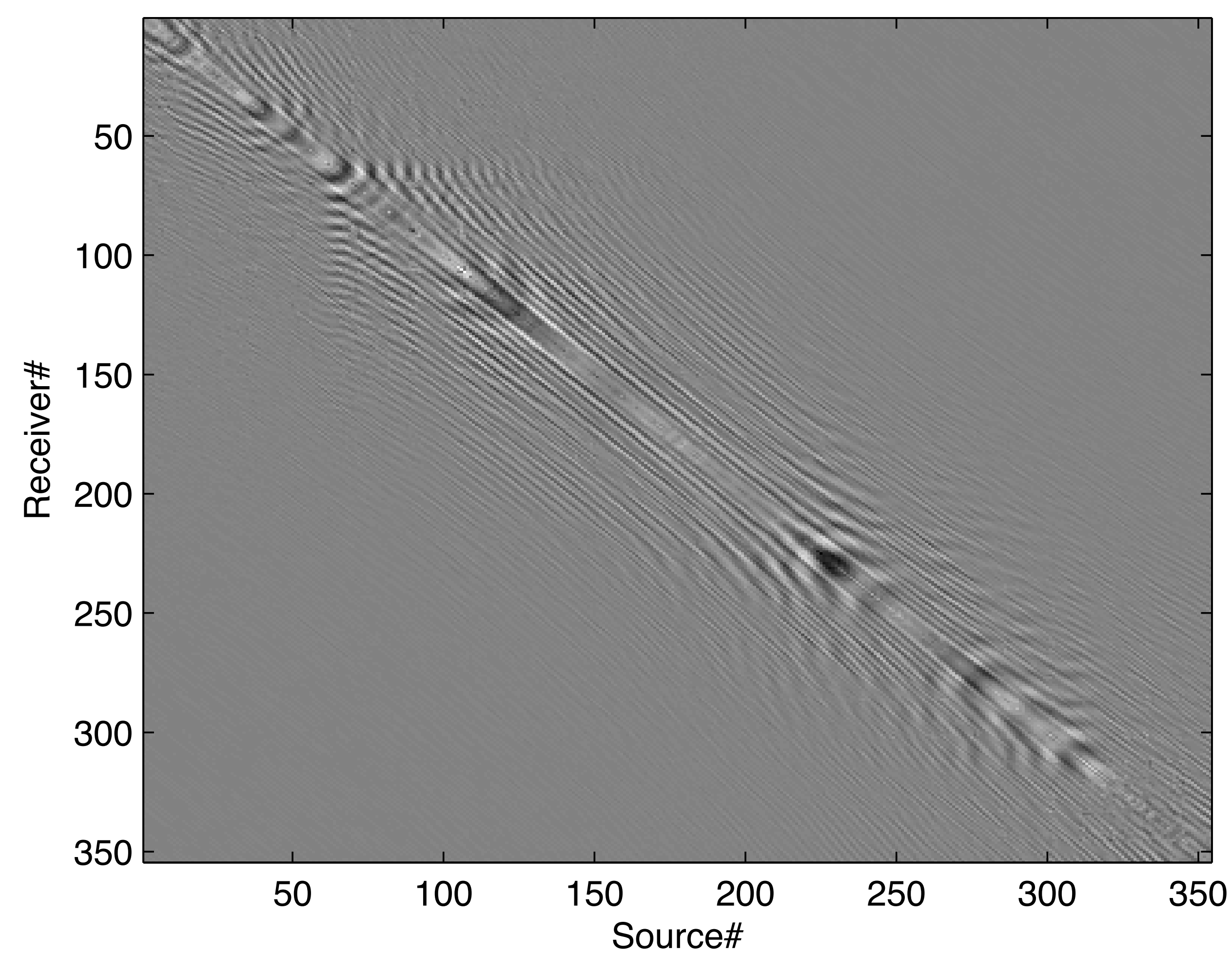
- 150 SPGL1 iterations; sigma = 1e-6, nr = ns = 354.
- Curvelet Sparsity (200 SPGL1 iterations): SNR of 9-10dB

Gulf of Suez: Least Squares + Low Rank

Frequency Slice : 70 Hz, Rank : 40



50% Missing data, before interpolation



Data after interpolation, **SNR = 29.3 db**

- 150 SPGL1 iterations; sigma = 1e-6, nr = ns = 354.
- Curvelet Sparsity (200 SPGL1 iterations): SNR of 9-10dB

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Current/Future work:

- Find transforms and representations that make seismic data *low-rank*.
- Can factorizations $X = LR^*$ in these representations be used directly?
- Combine *low rank* with *robust* penalties (later talk)
- Use other norms that have computational advantages, such as the Max norm.