Yet another perspective on extended images

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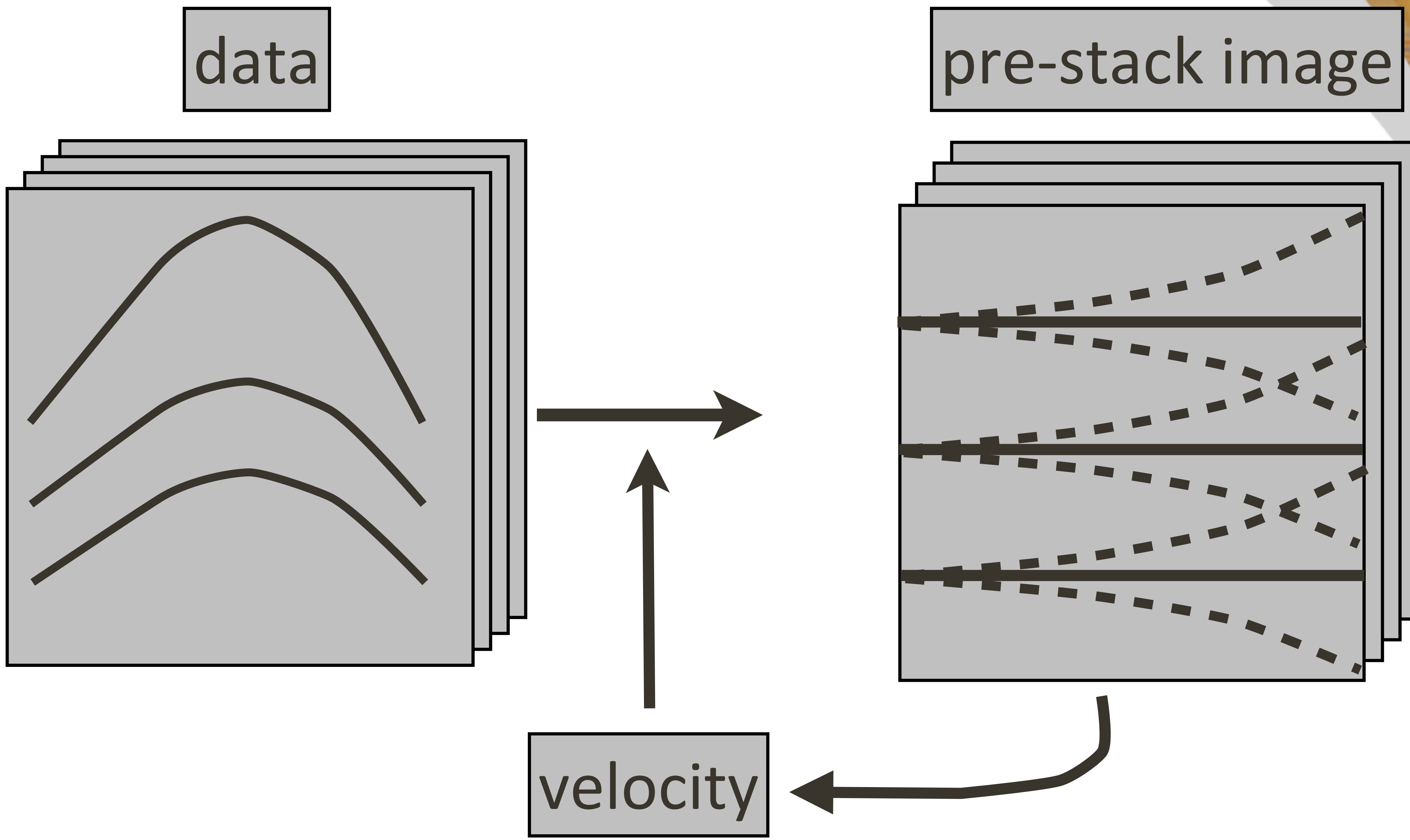
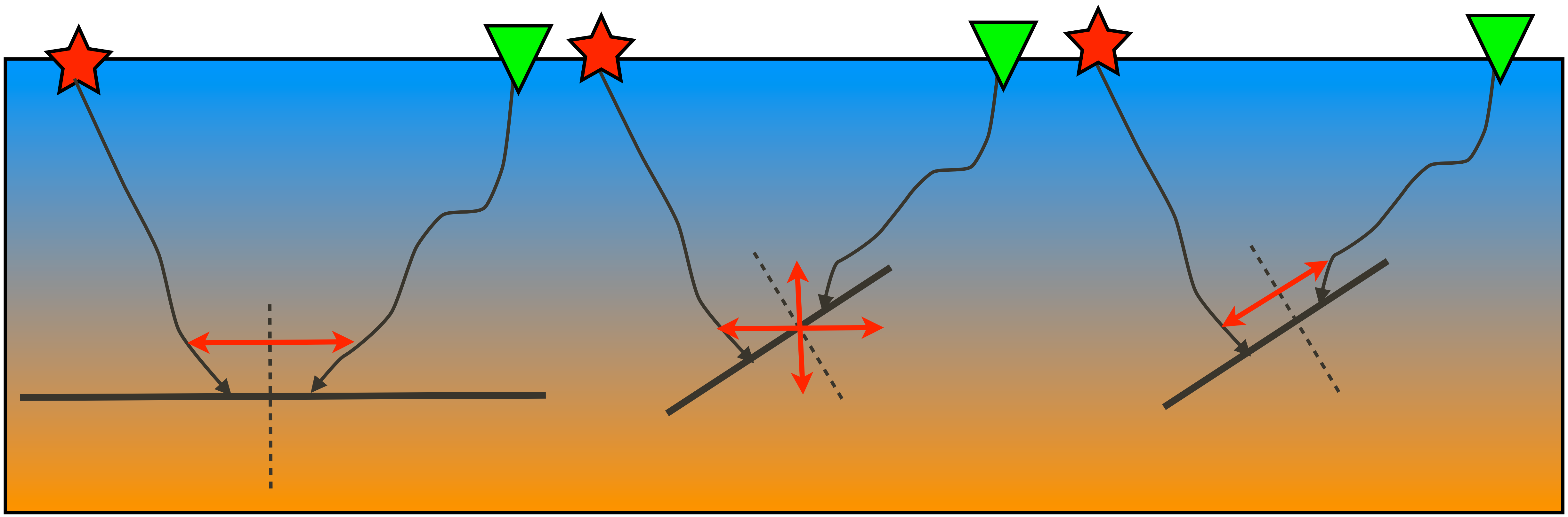
SLIM Consortium meeting

SLIM
University of British Columbia


data
pre-stack image
velocity



horizontal offset

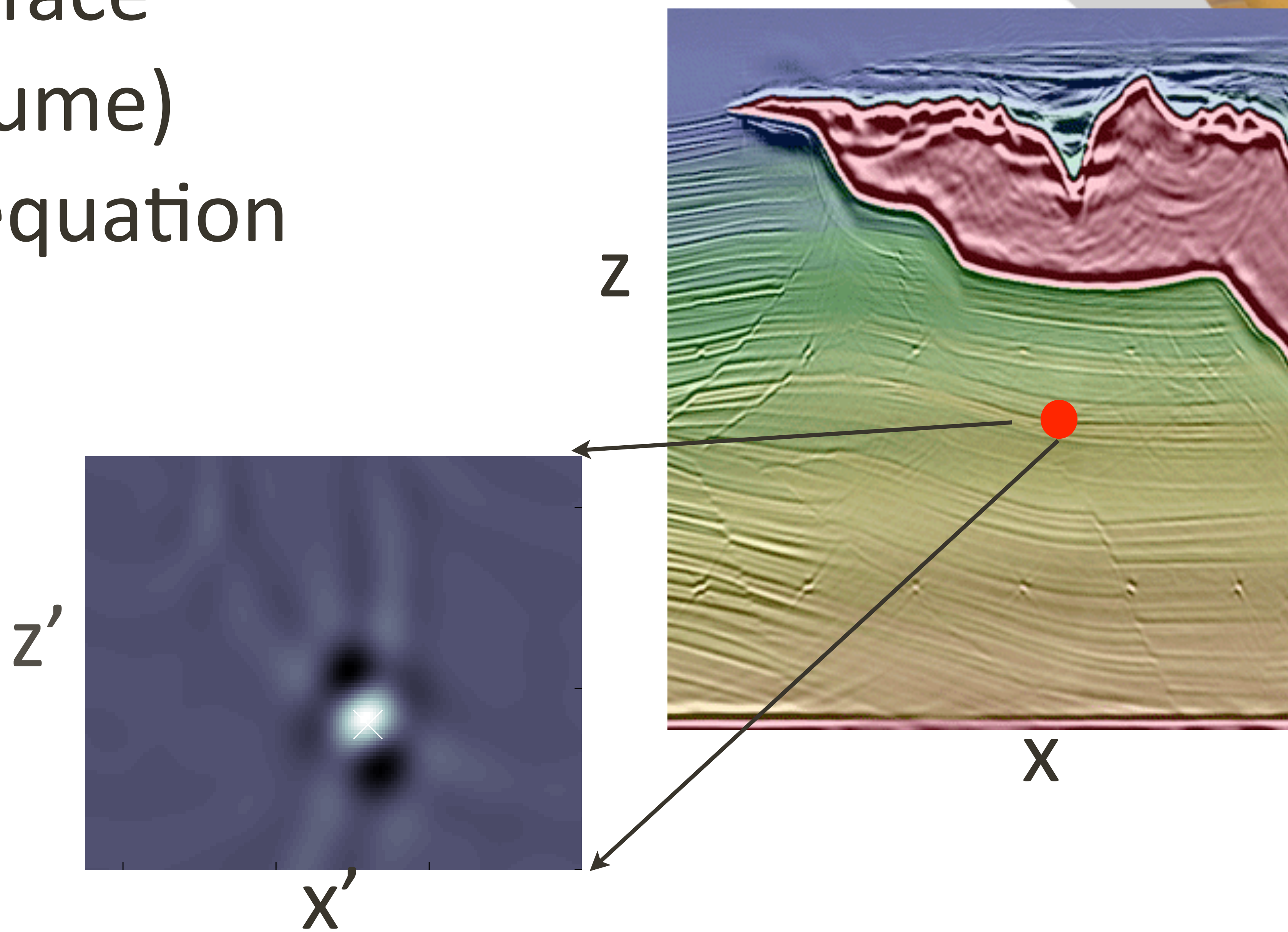
horizontal +vertical offset

all offsets

[Biondo & Symes, '04 ;Sava & Vasconcelos, '11]

•use *all* subsurface offsets (4D volume)

•2-way wave-equation



but.... we can never hope to compute or store such an image volume!

Can we work with the volume *implicitly* ?

Overview

- Anatomy
- Physics
- Computation
- MVA
- Conclusions

Extended images

Correlation of wavefields

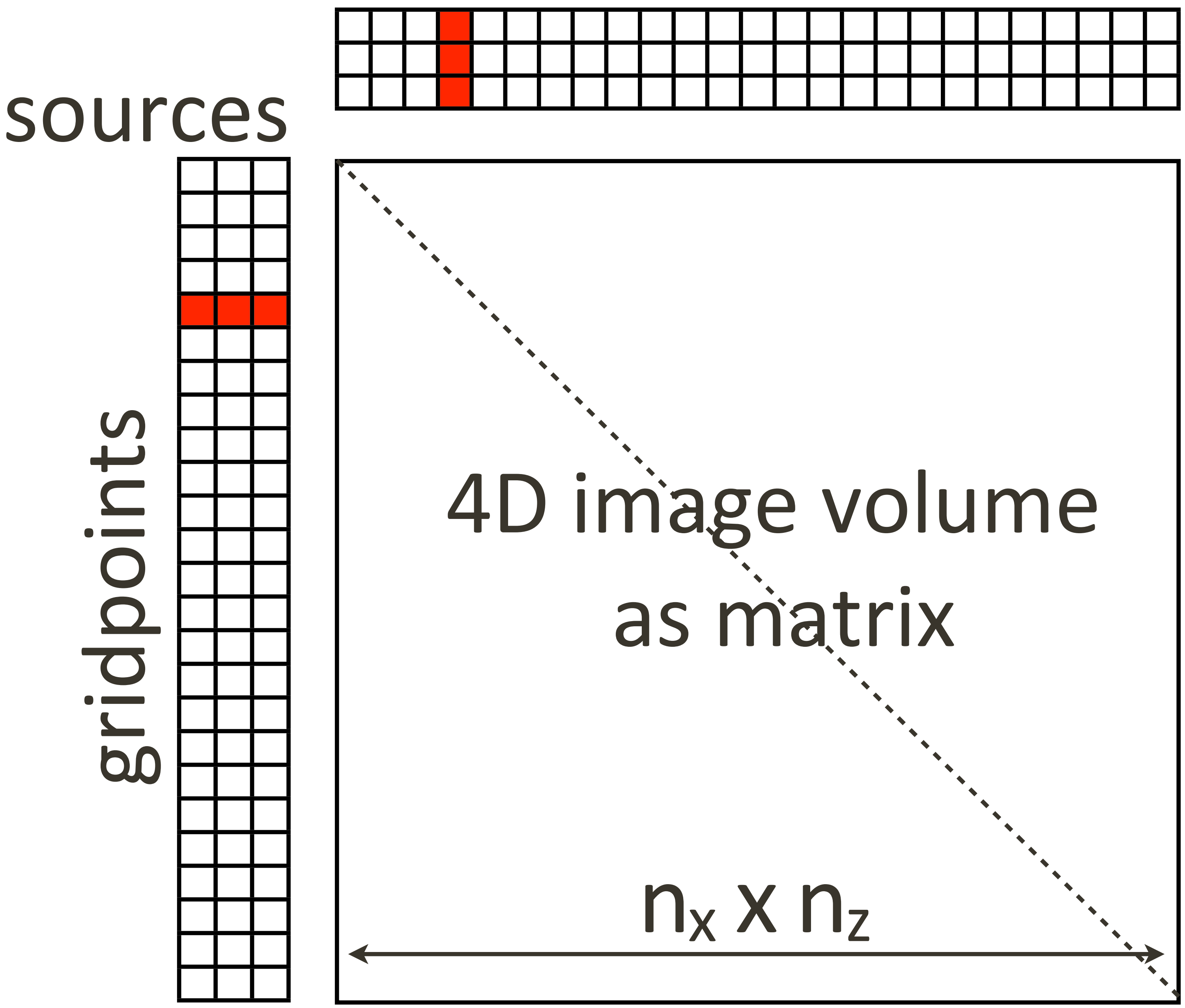
$$e(\omega, \mathbf{x}, \mathbf{x}') = \sum_i v_i(\omega, \mathbf{x}) u_i(\omega, \mathbf{x}')^*$$

in *data-matrix* notation:

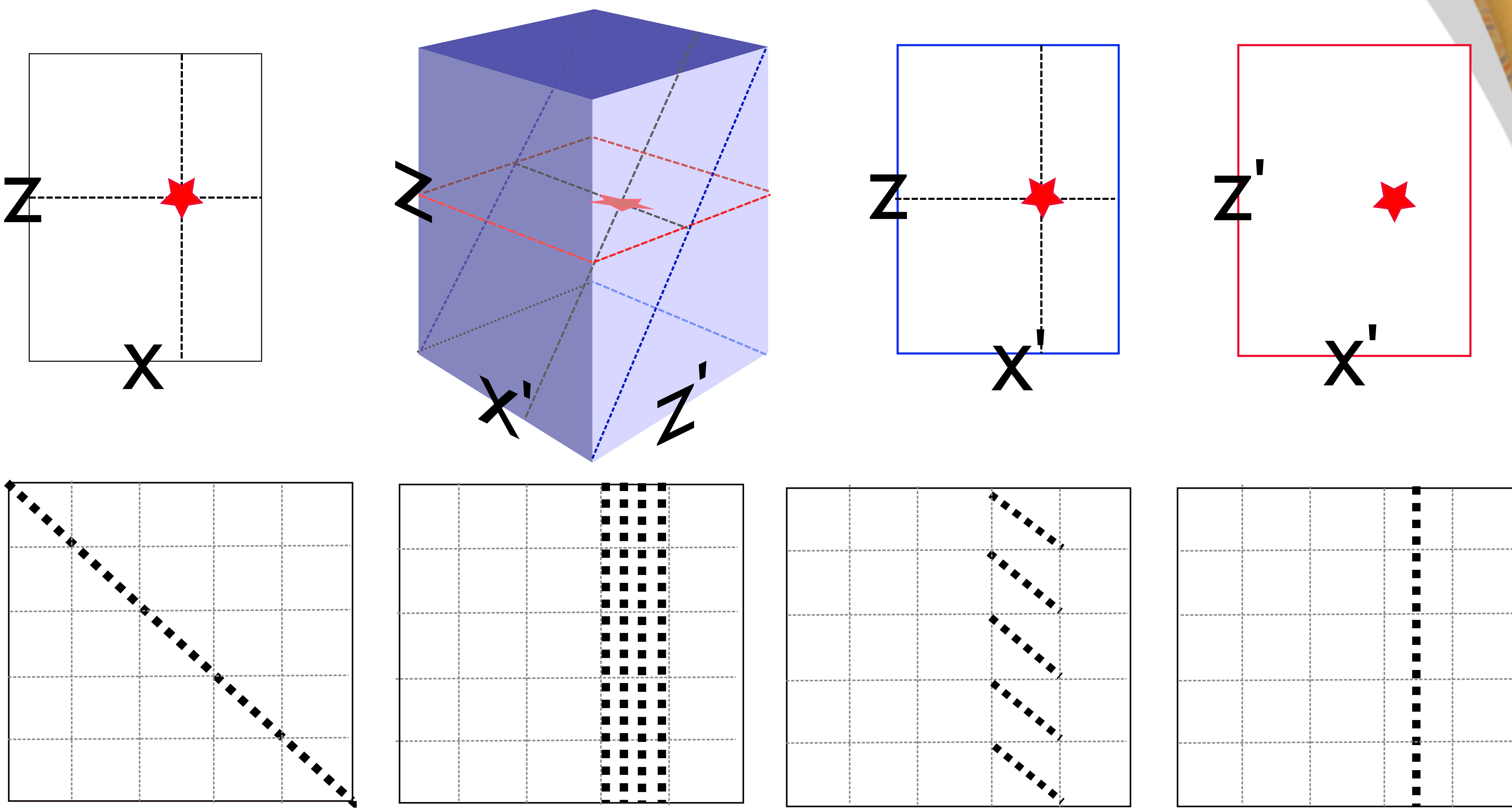
$$E(\omega) = V(\omega)U(\omega)^*$$

imaging condition: $\sum_\omega \mathrm{diag}(E(\omega))$

Extended images

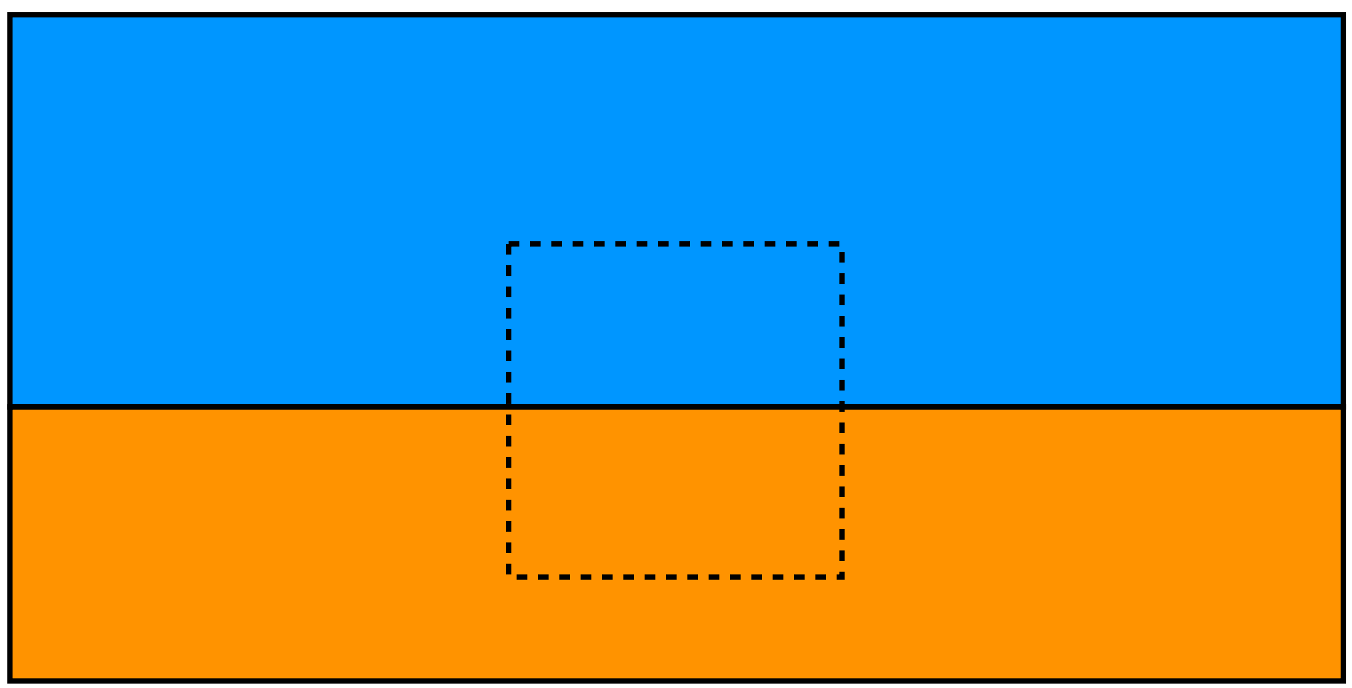


Extended images



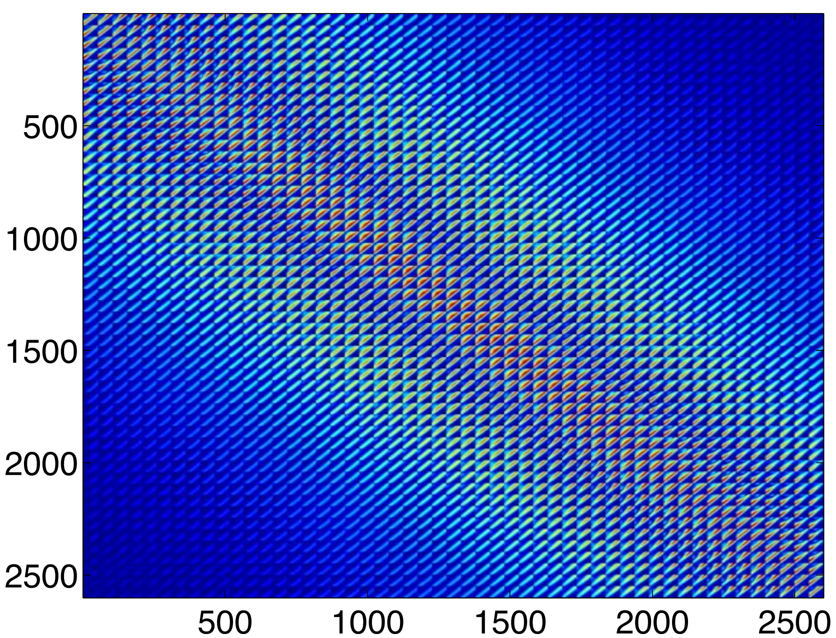
Extended images

example for one layer

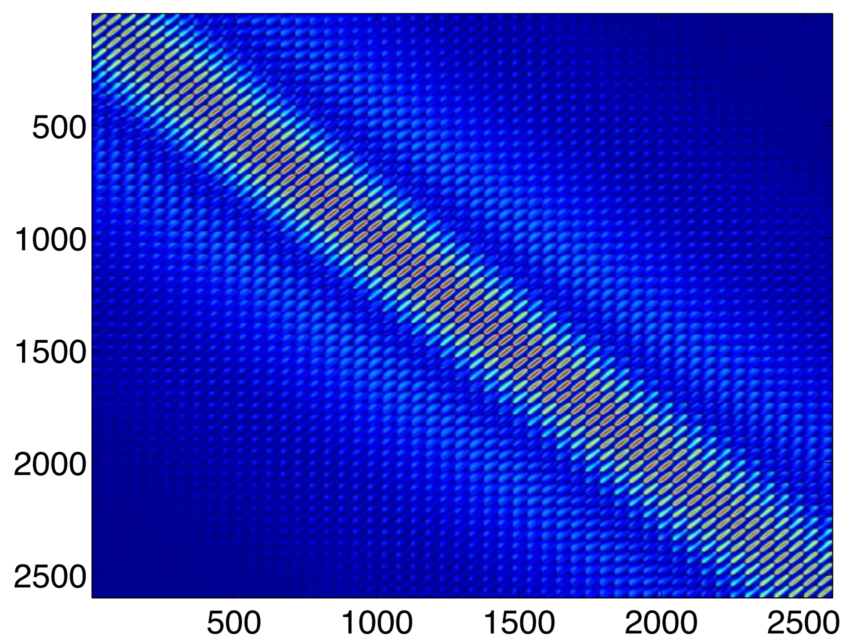


Extended images

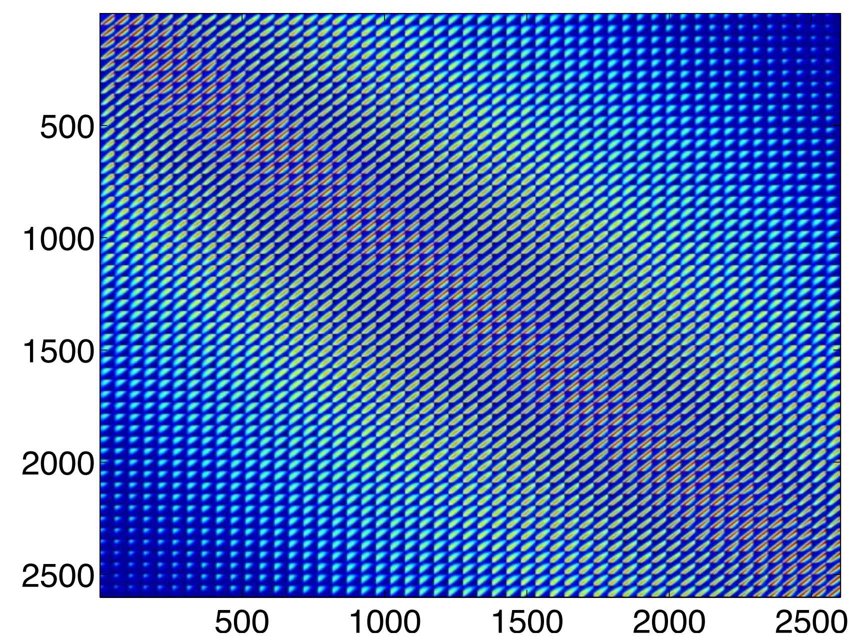
full matrix



low velocity



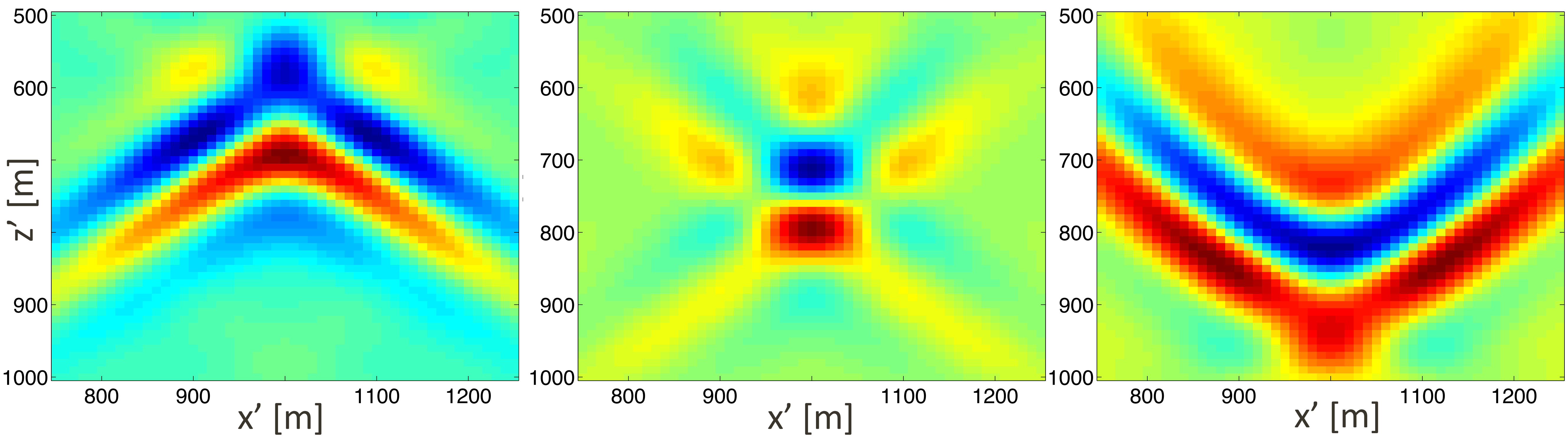
correct velocity



high velocity

Extended images

one column



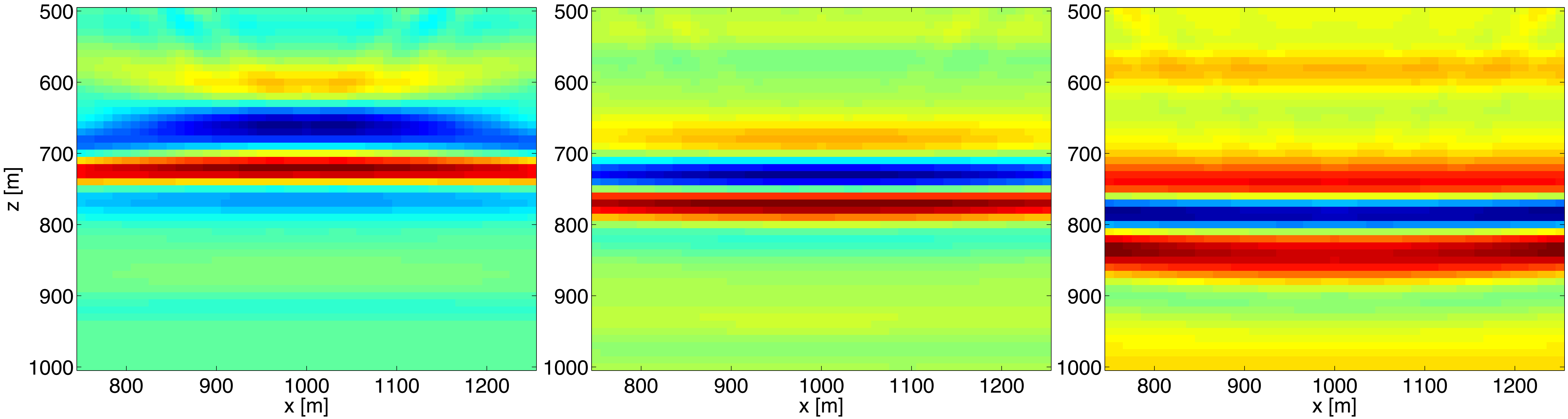
low velocity

correct velocity

high velocity

Extended images

diagonal



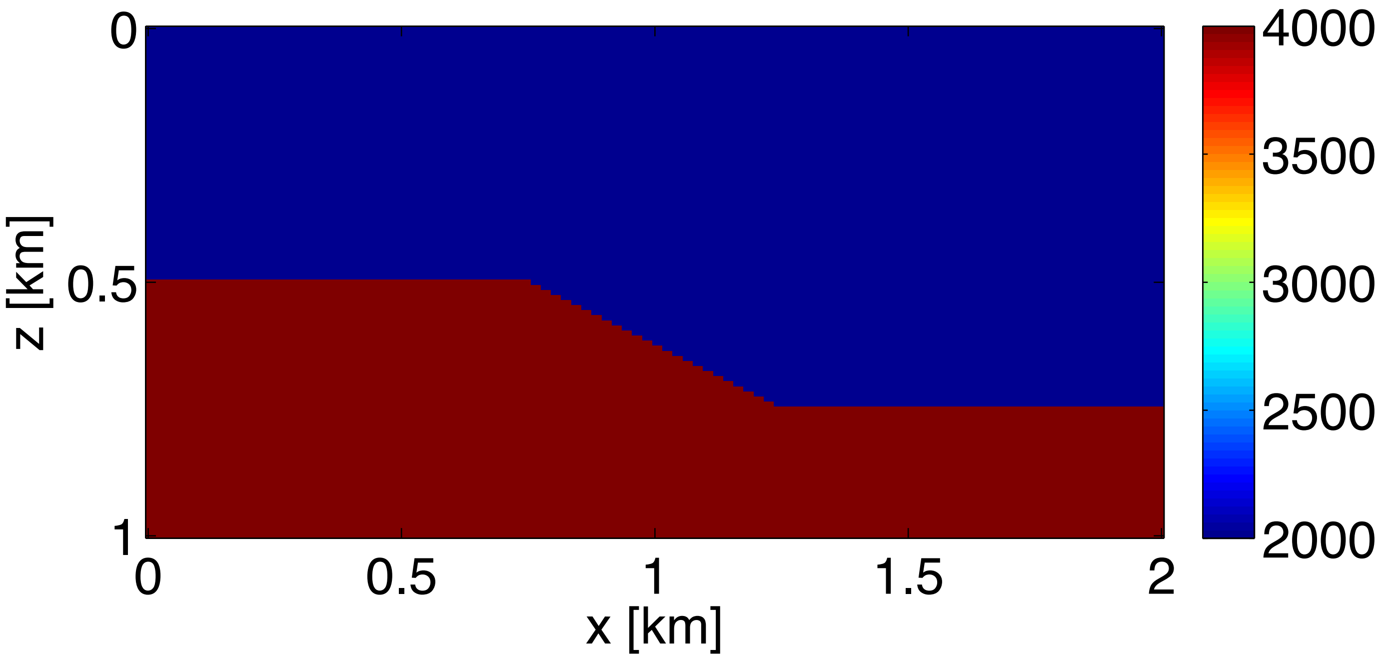
low velocity

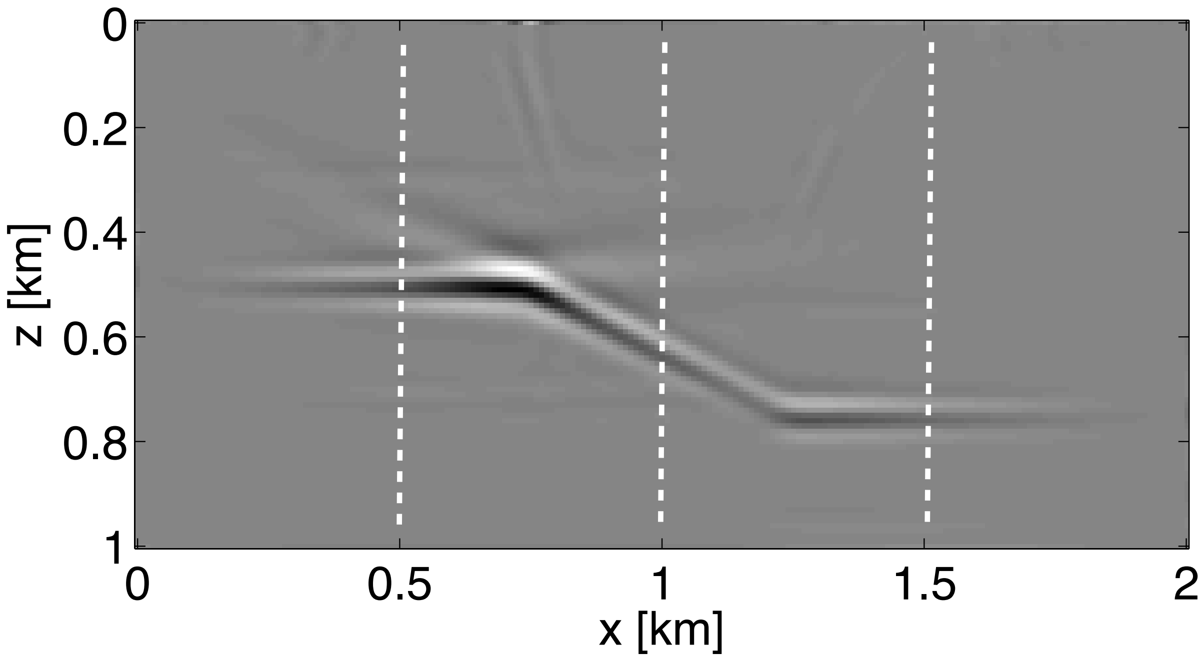
correct velocity

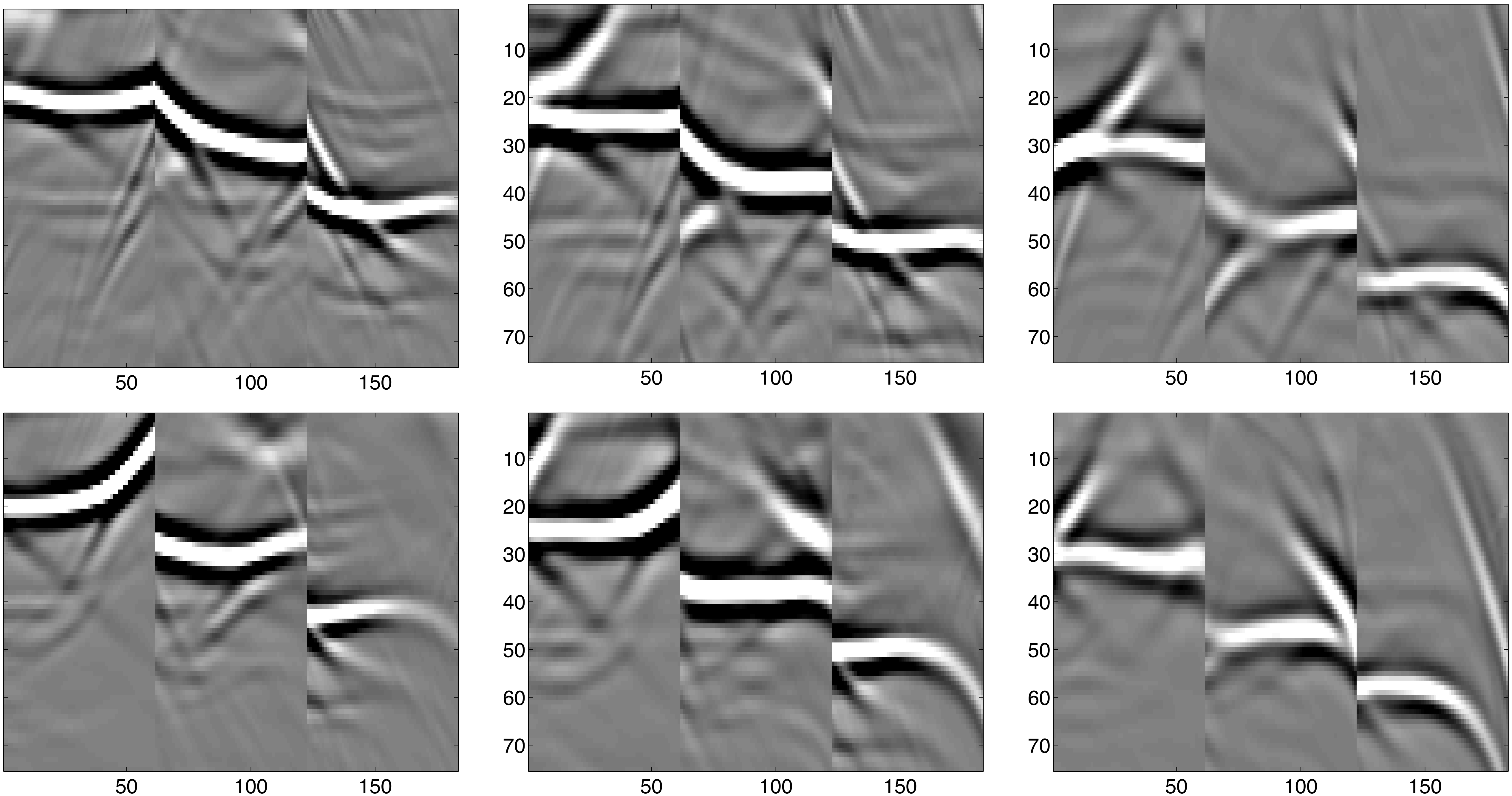
high velocity

Extended images

Example for dipping reflector







dip: 0

dip: 25

low correct high

Double wave-equation

Helmholtz operator: $H = \omega^2 \text{diag}(\mathbf{m}) + \nabla^2$

source/receiver wavefields:

$$HU = P_s^T Q \qquad H^* V = P_r^T D$$

RTM extended image: $E = VU^*$

yields: $H^* E H = P_r^T D Q^* P_s$

Double wave-equation

$$Le(\omega,\mathbf{x},\mathbf{x}') = \int \mathrm{d}\mathbf{s} \int \mathrm{d}\mathbf{r}\, d(\omega,\mathbf{s},\mathbf{r})\delta(\mathbf{x}-\mathbf{s})\delta(\mathbf{x}'-\mathbf{r})$$

two-way:

$$L = \left[\omega^2/c(z,x)^2 + \partial_x^2 + \partial_z^2\right]\left[\omega^2/c(z',x')^2 + \partial_{x'}^2 + \partial_{z'}^2\right]$$

one-way (DSR):

$$L = \left[\partial_z - \imath\sqrt{\omega^2/c(z,x)^2 + \partial_x^2} - \imath\sqrt{\omega^2/c(z,x')^2 + \partial_{x'}^2}\right]$$

[Claerbout, '84; Stolk & de Hoop '01]

Velocity continuation

since r.h.s. is model-independent:

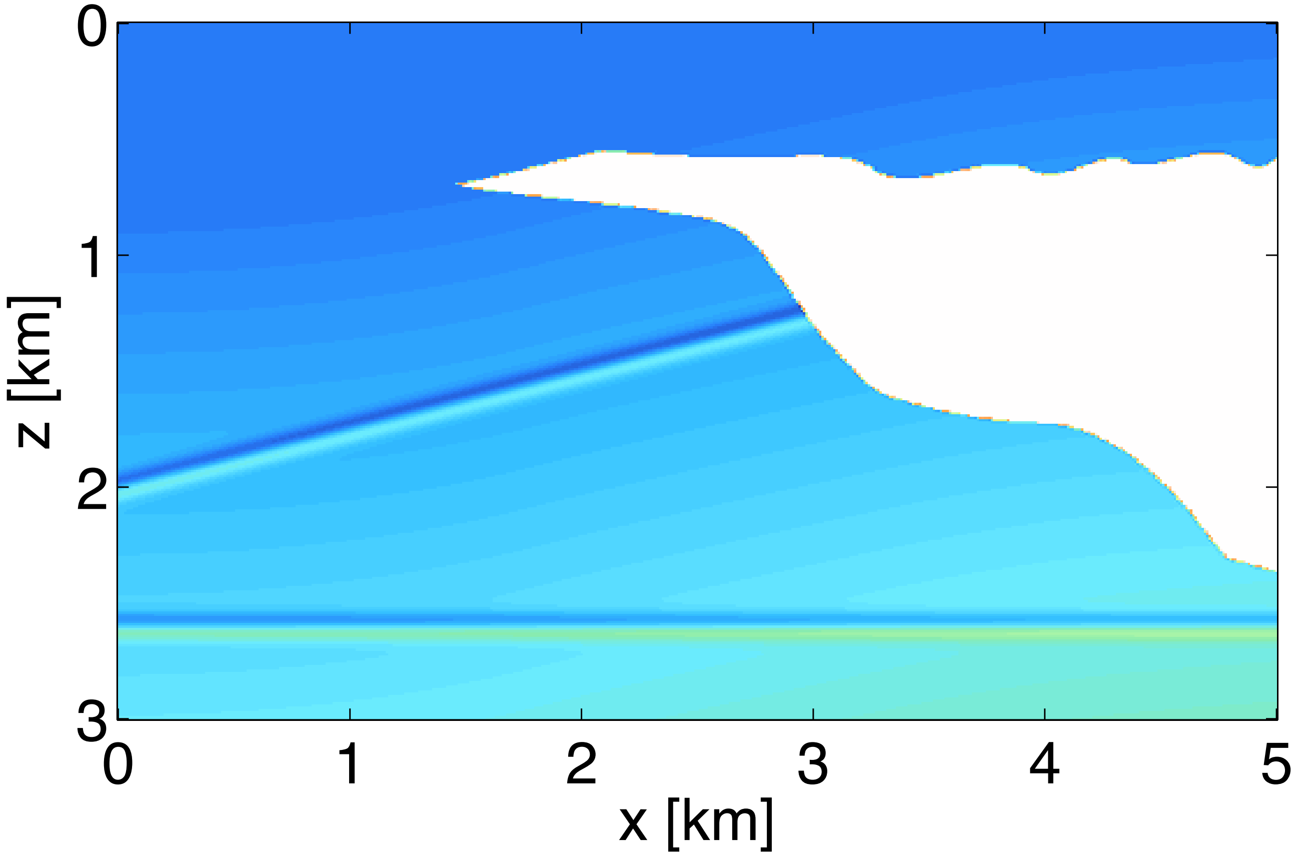
$$H_2^* E_2 H_2 = H_1^* E_1 H_1$$

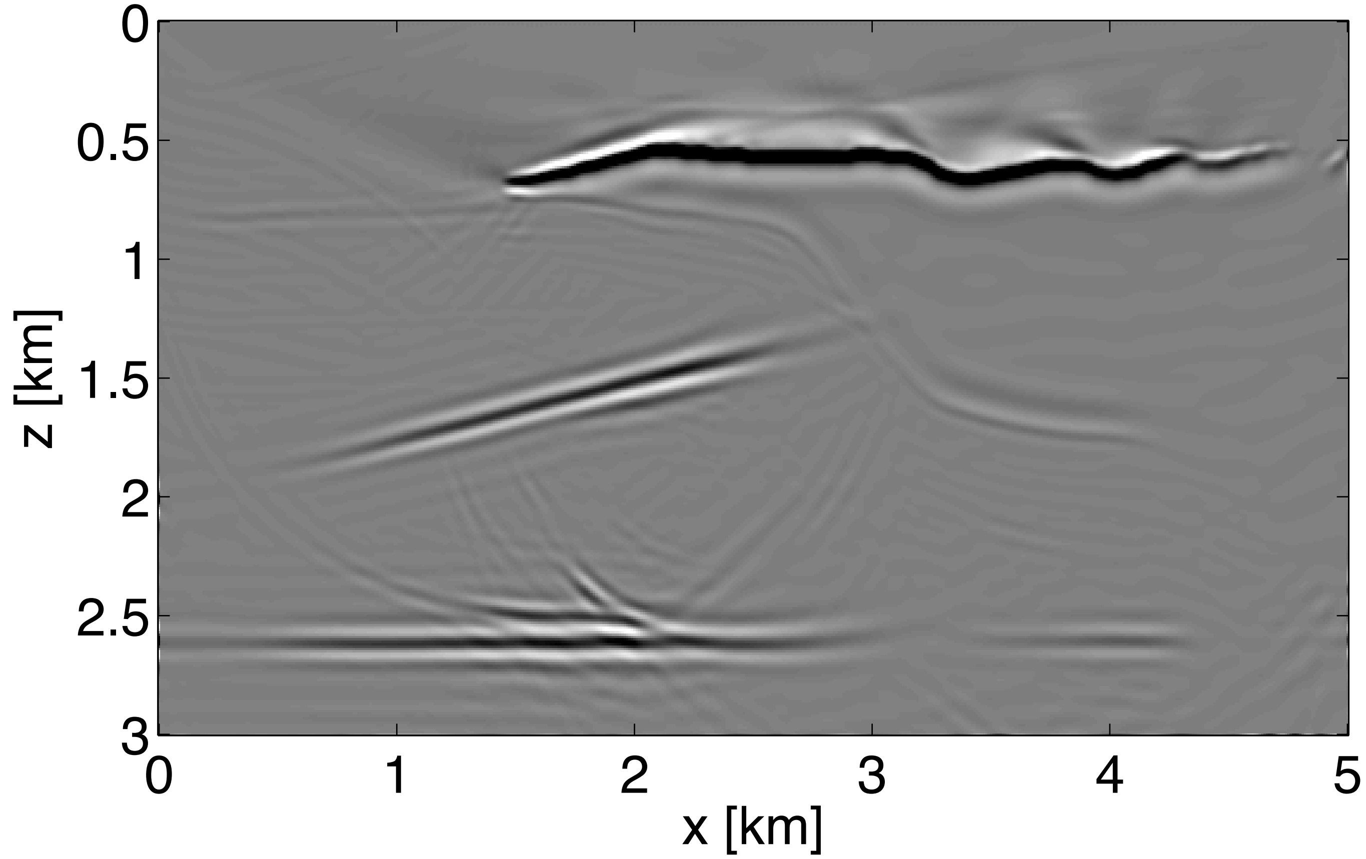
or

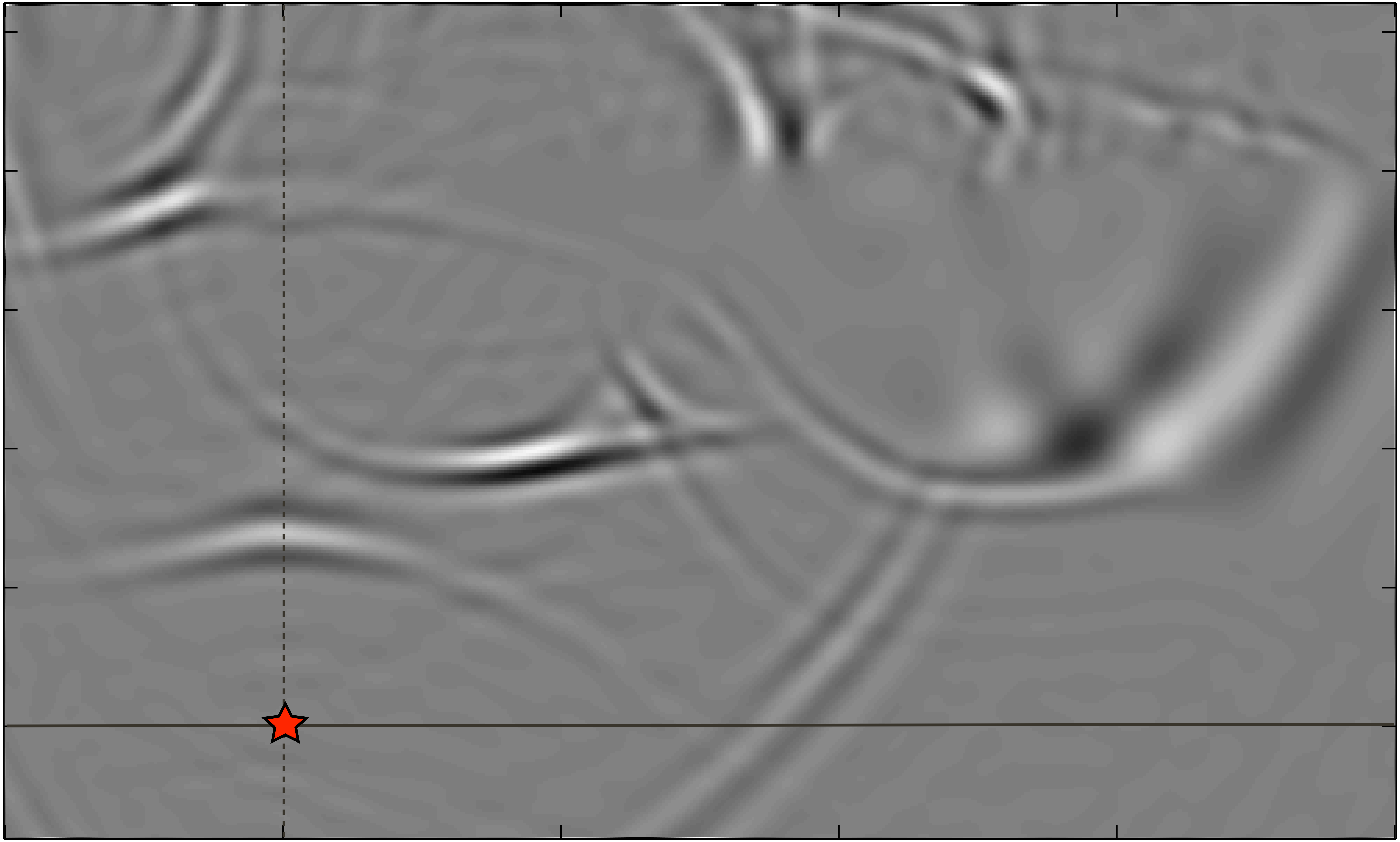
$$E_2 = H_2^{-*} H_1^* E_1 H_1 H_2^{-1}$$

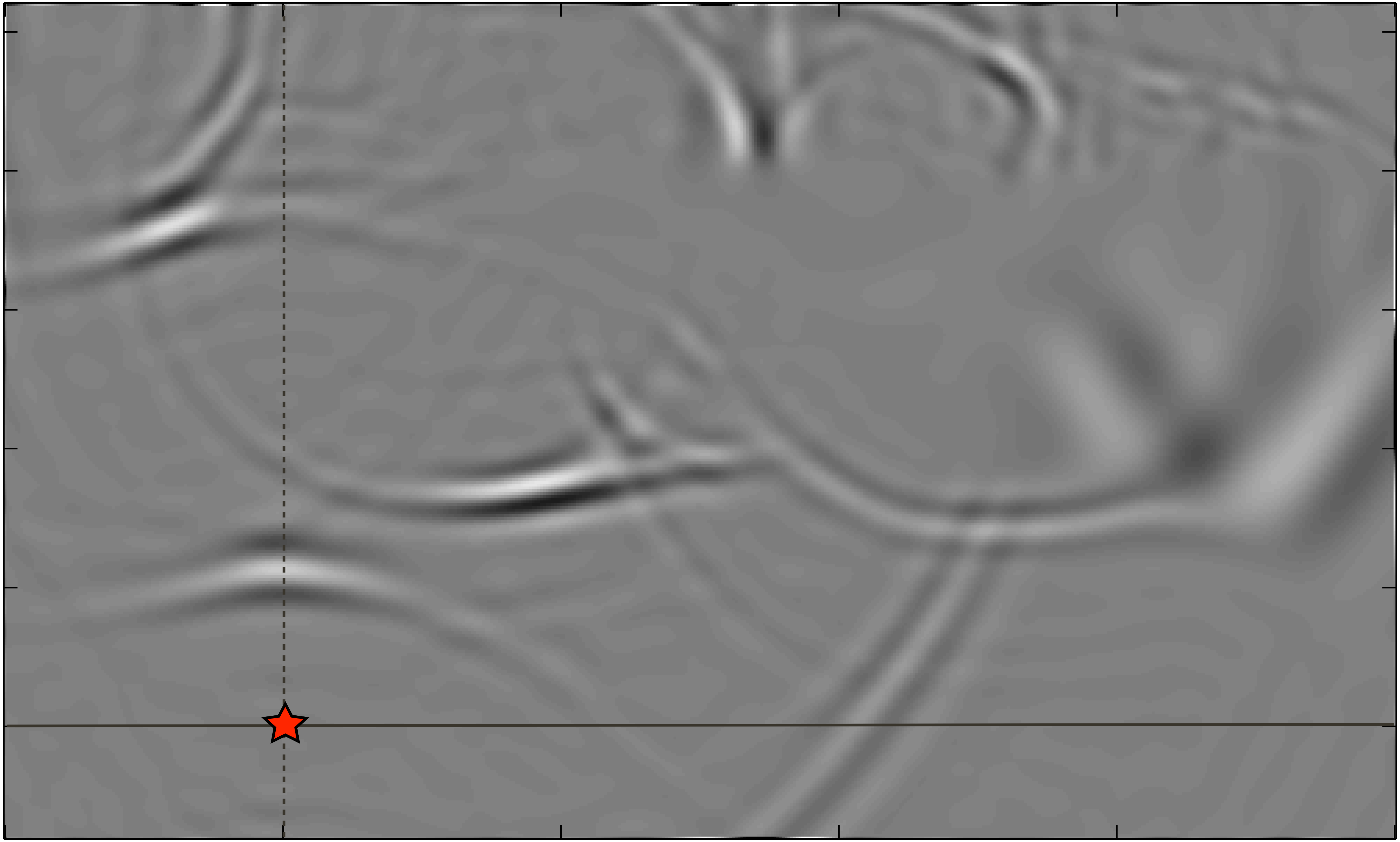
[Duchkov et al, '08]

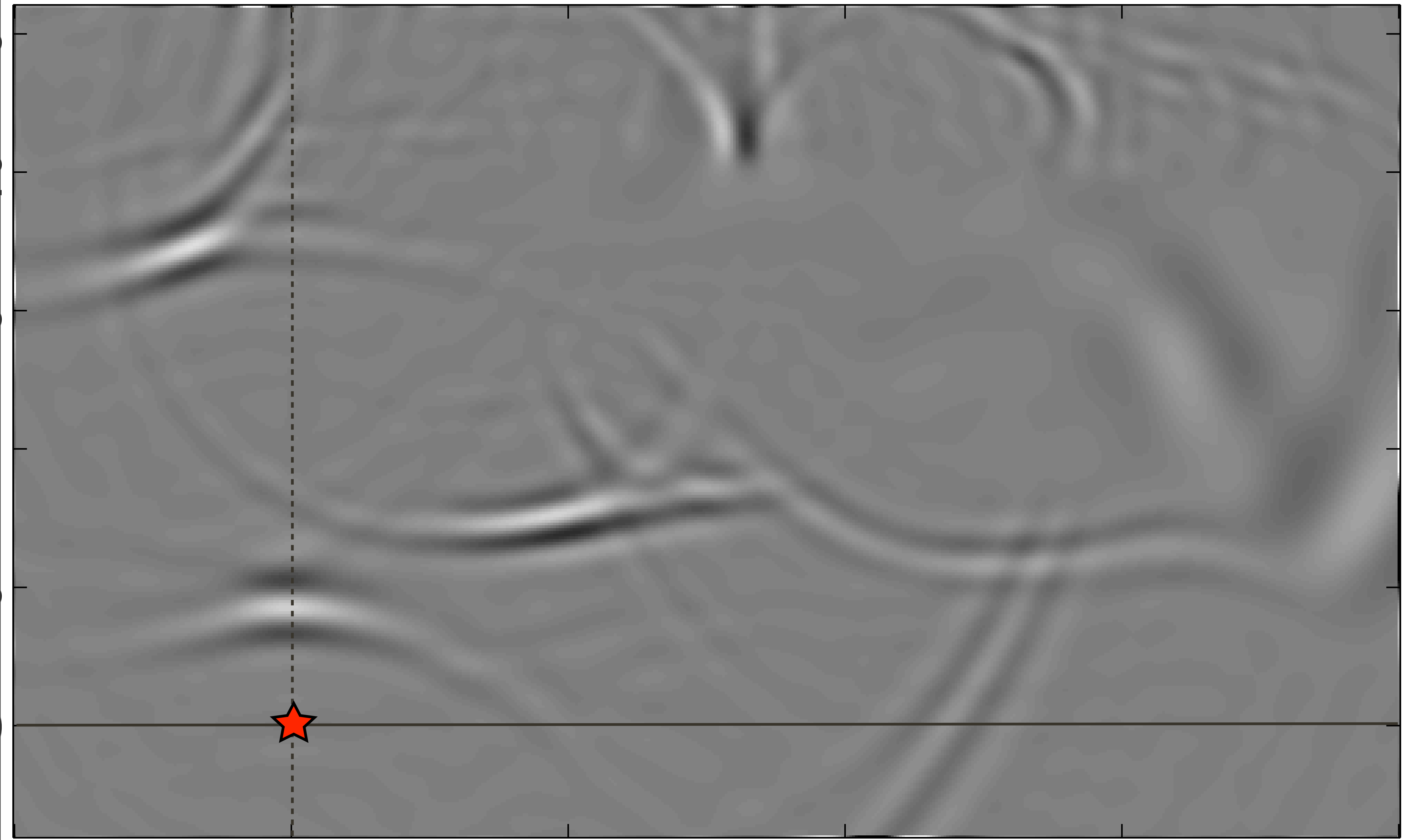
Examples

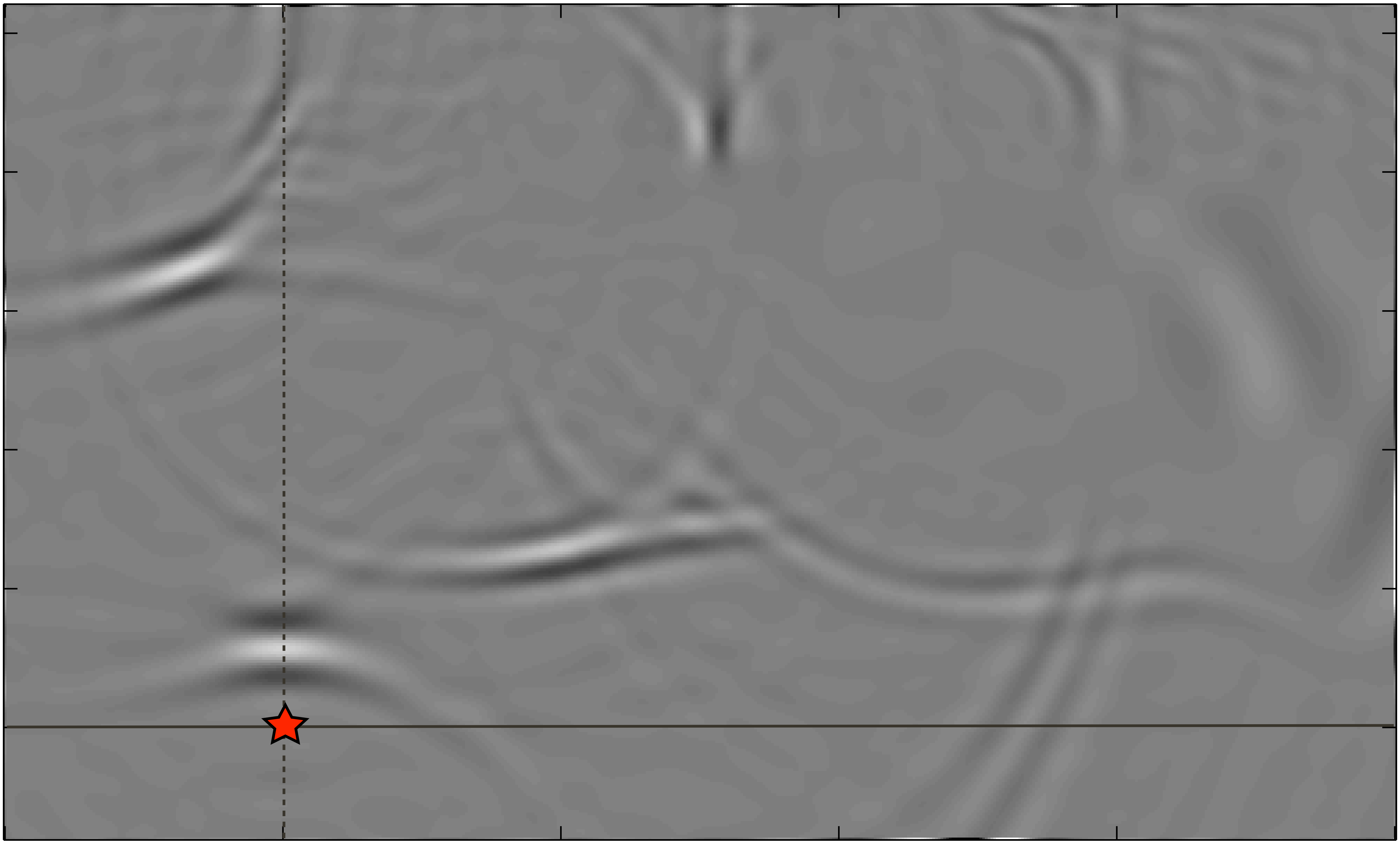


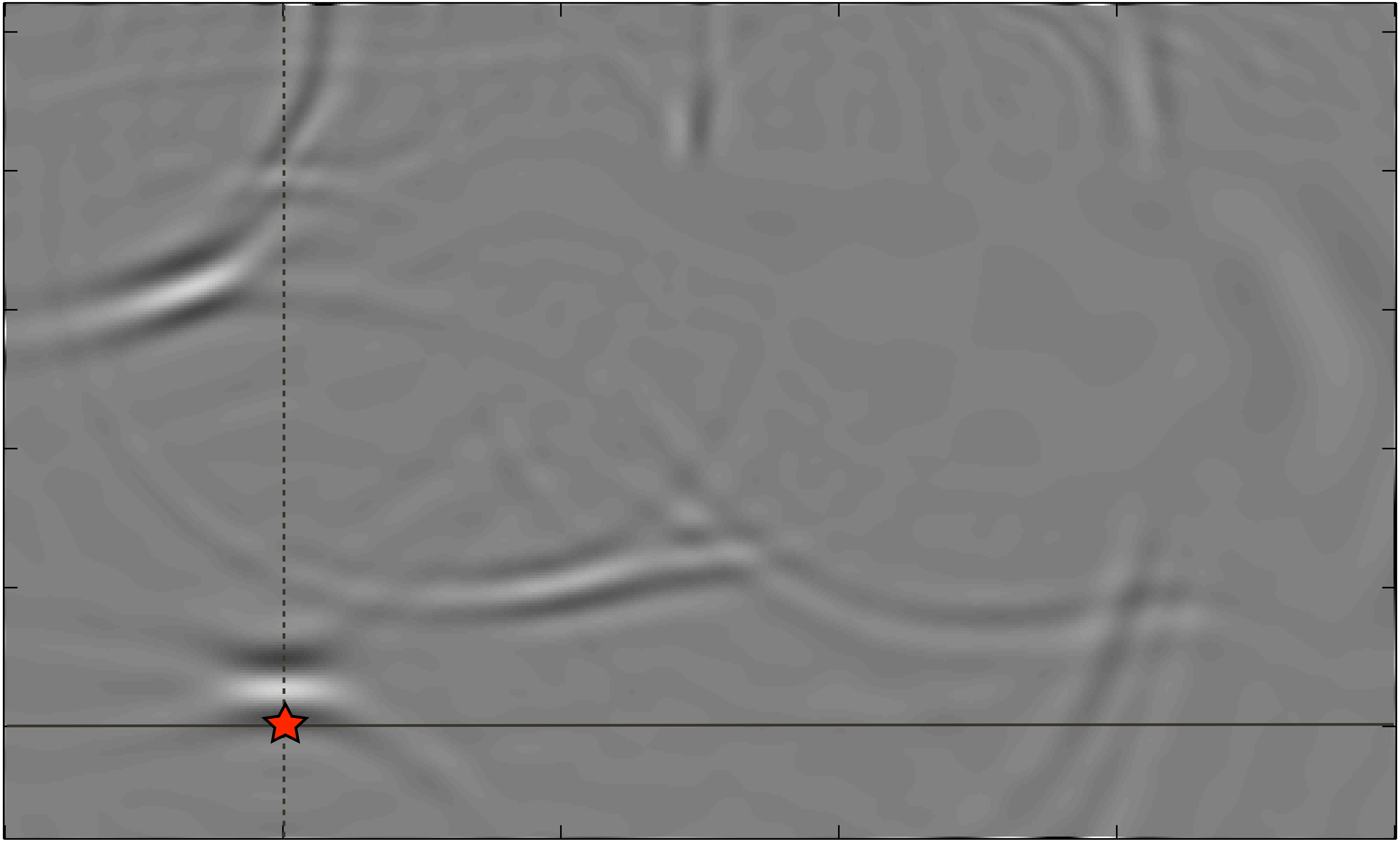


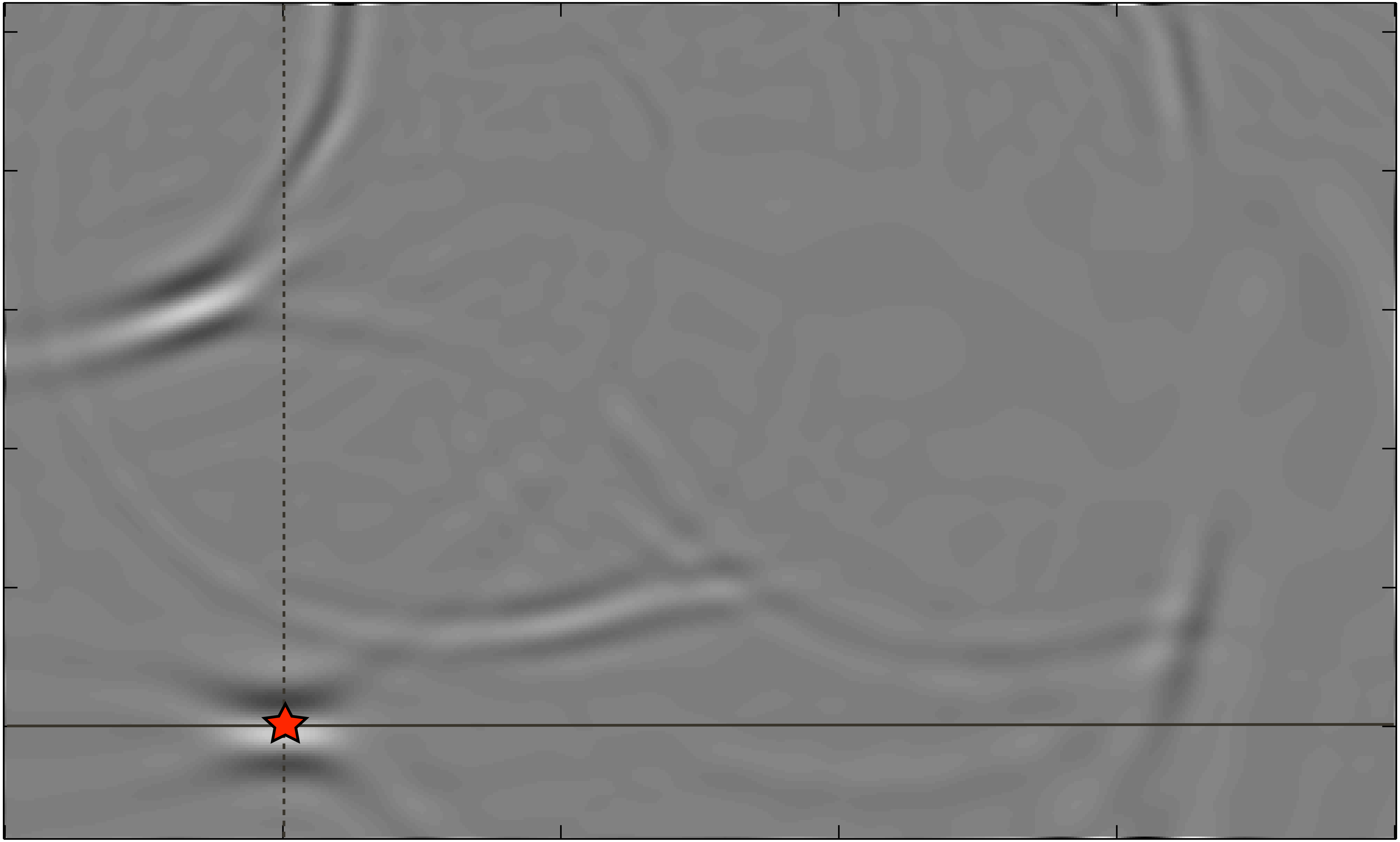


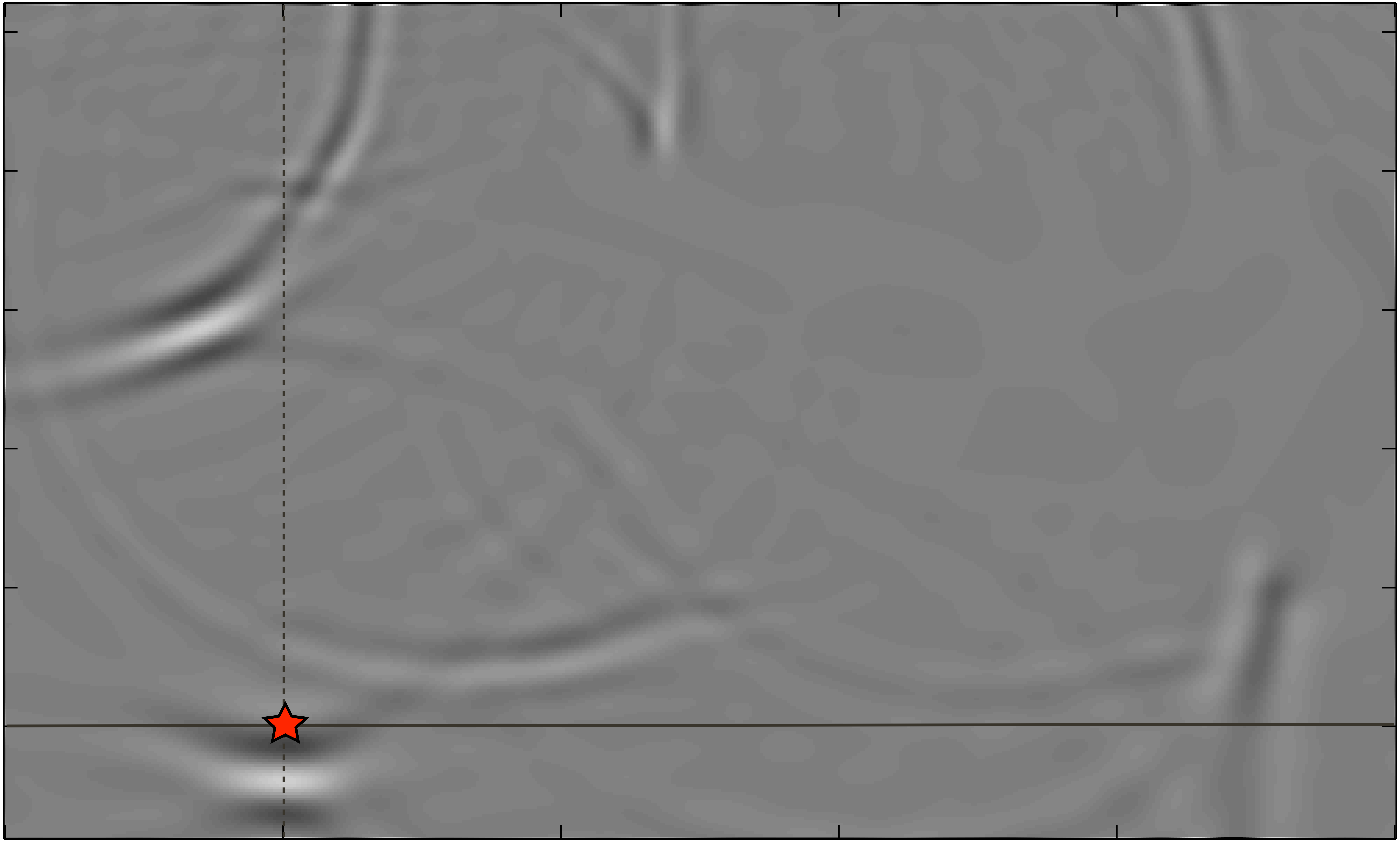


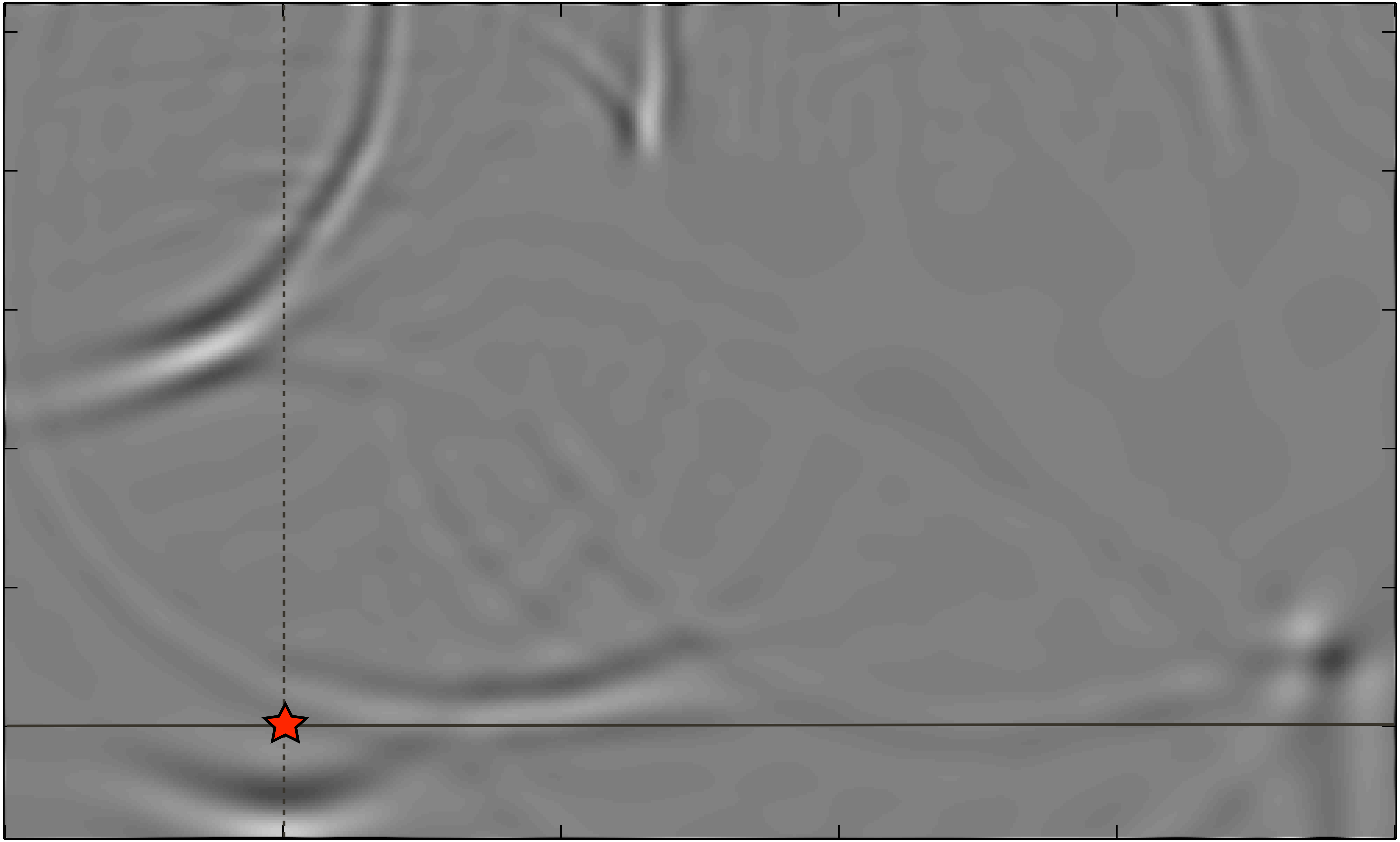


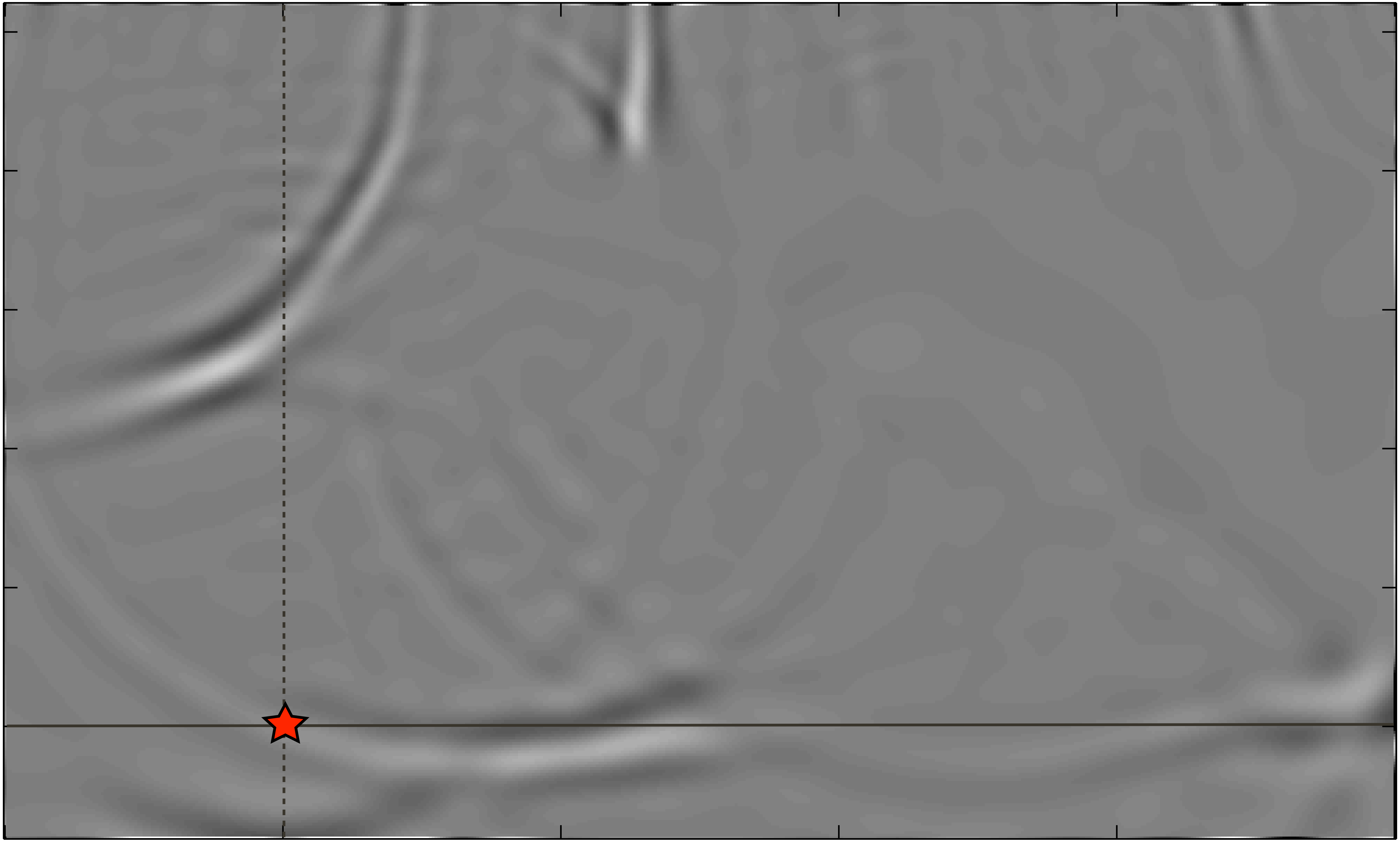


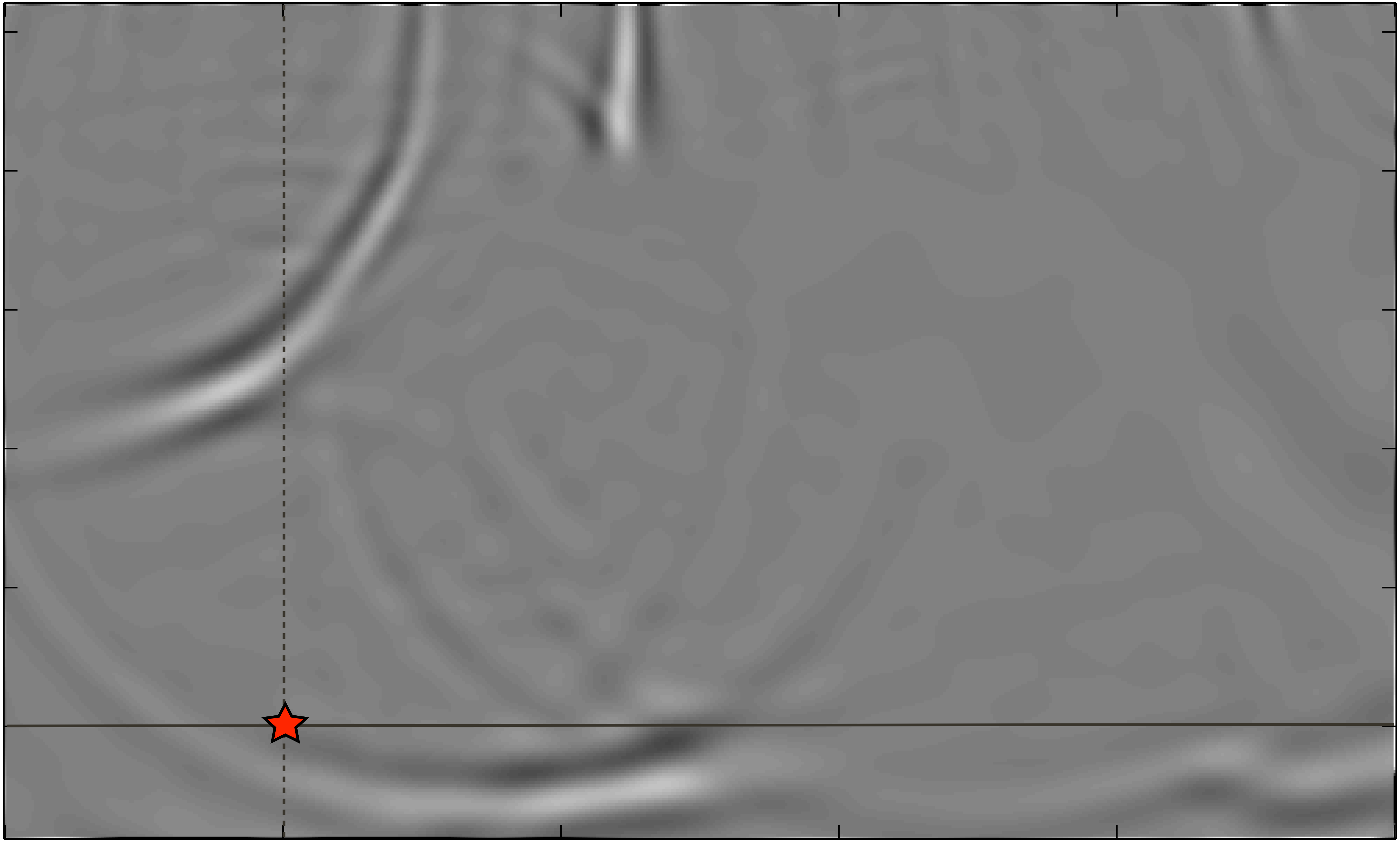


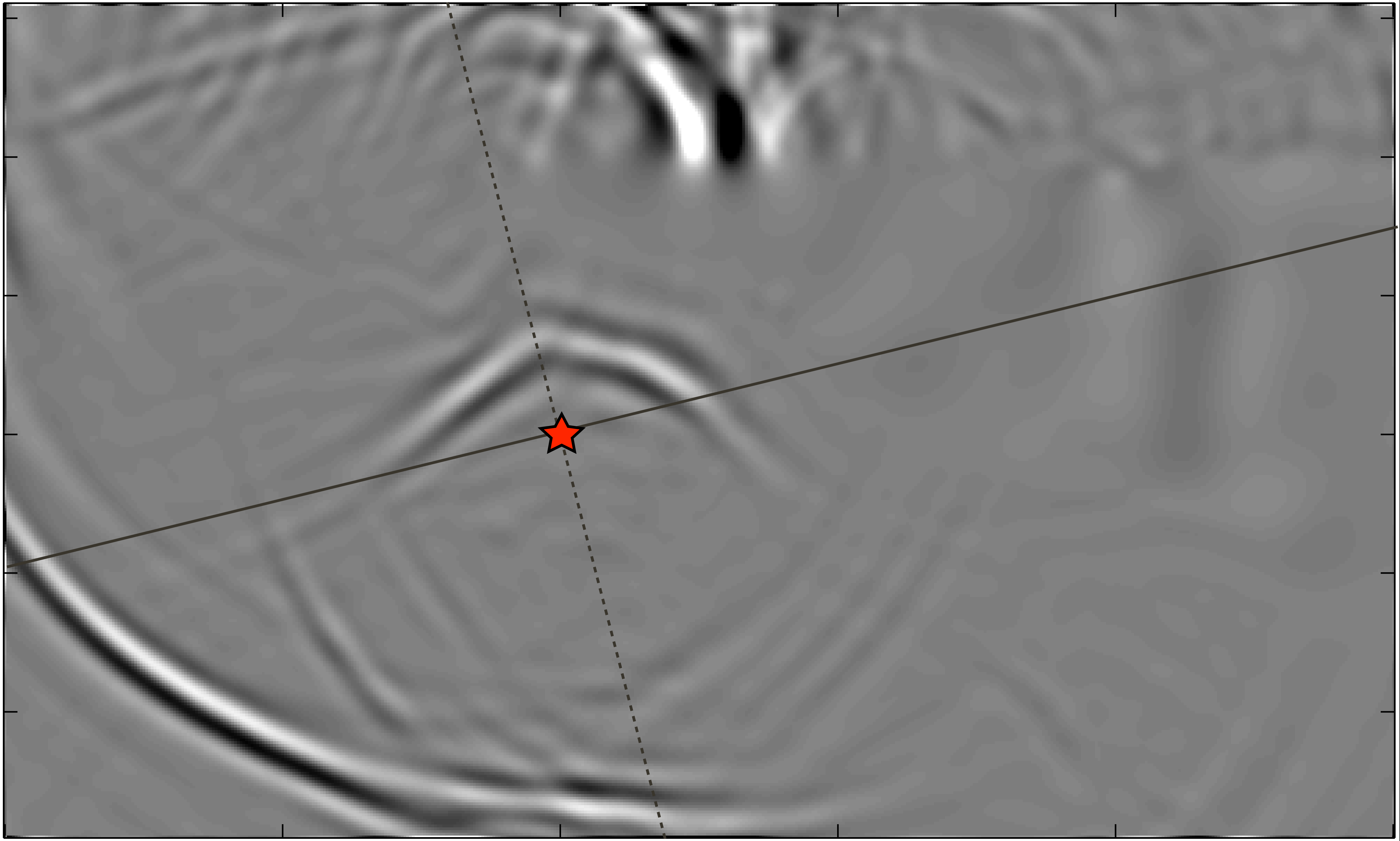


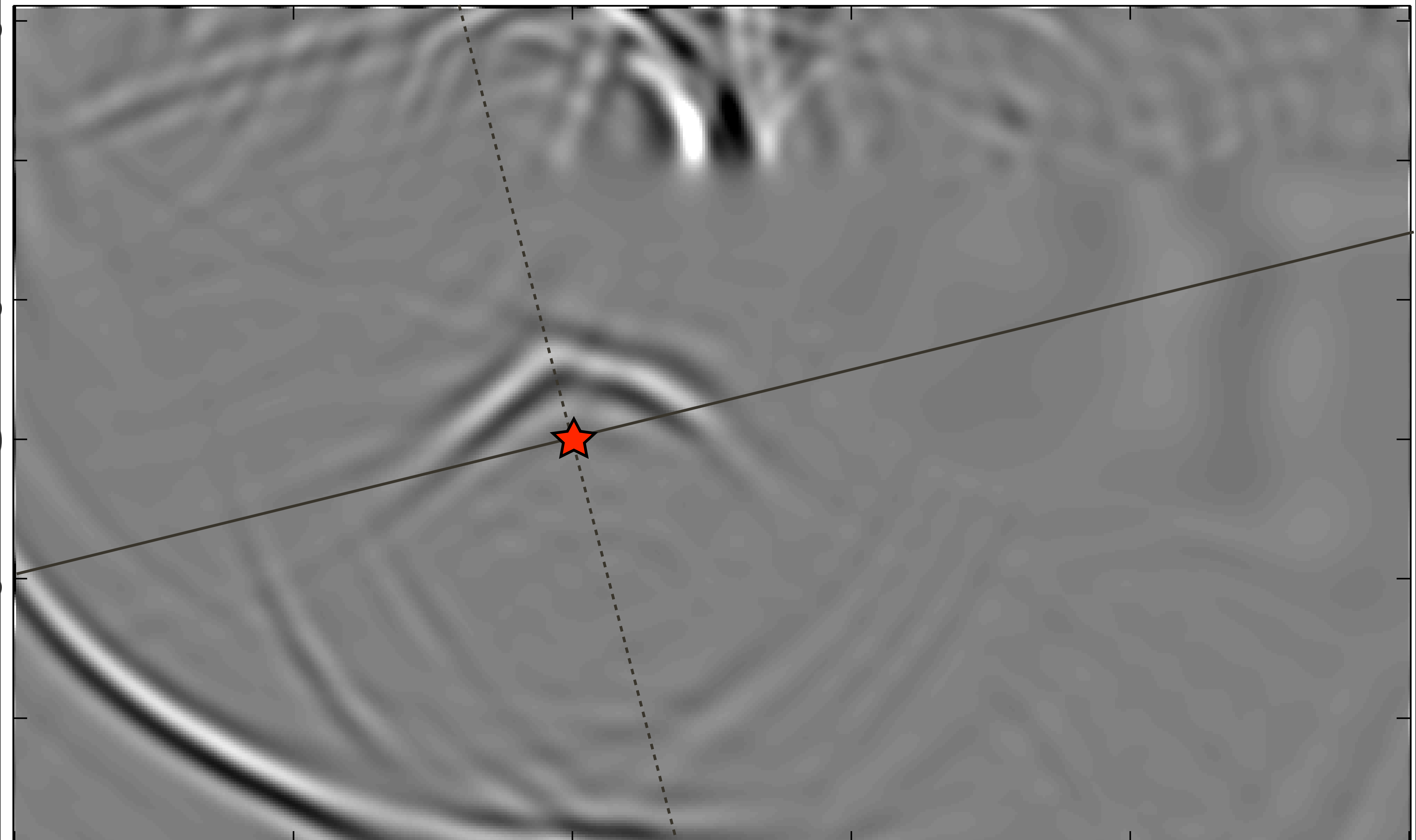


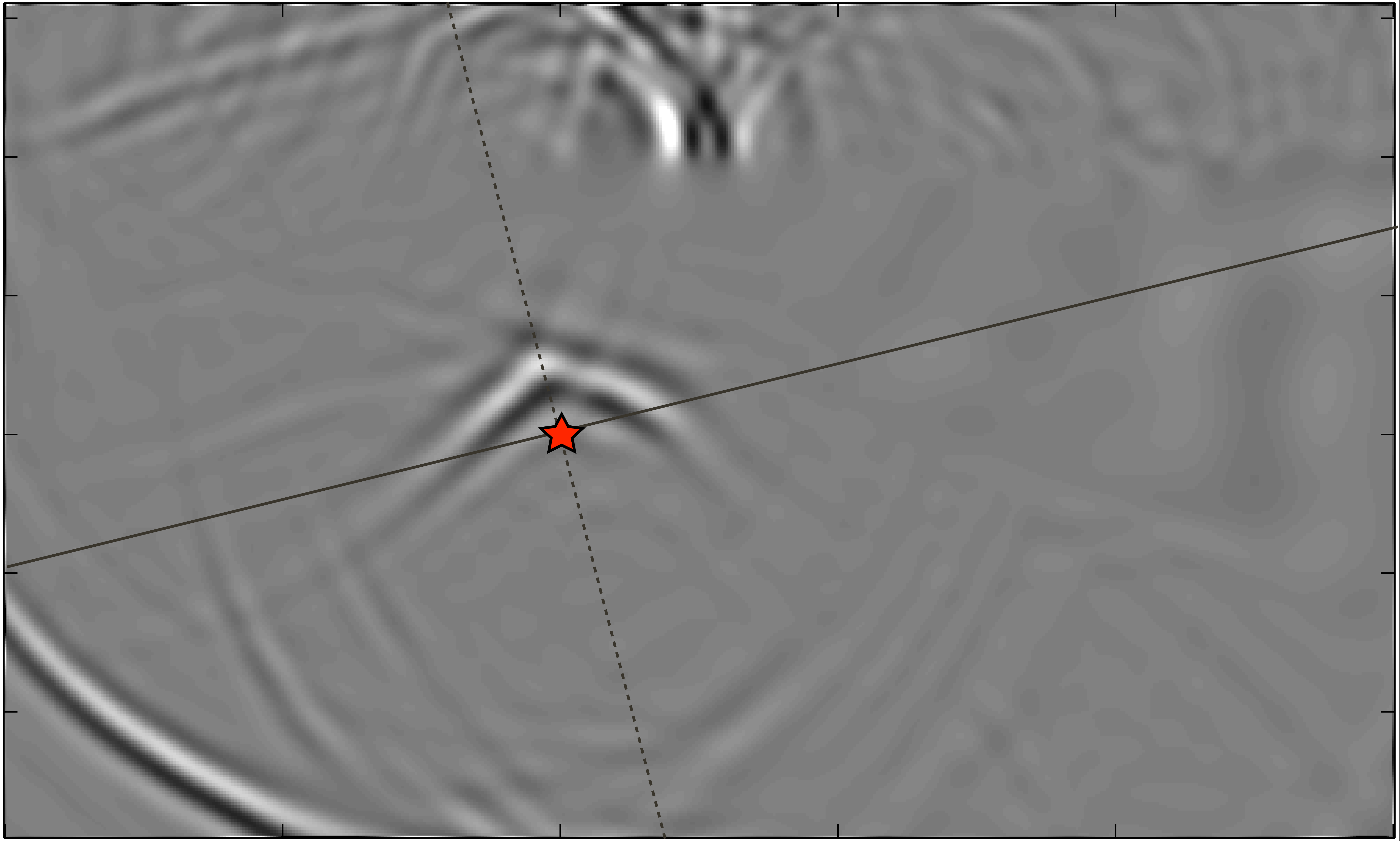


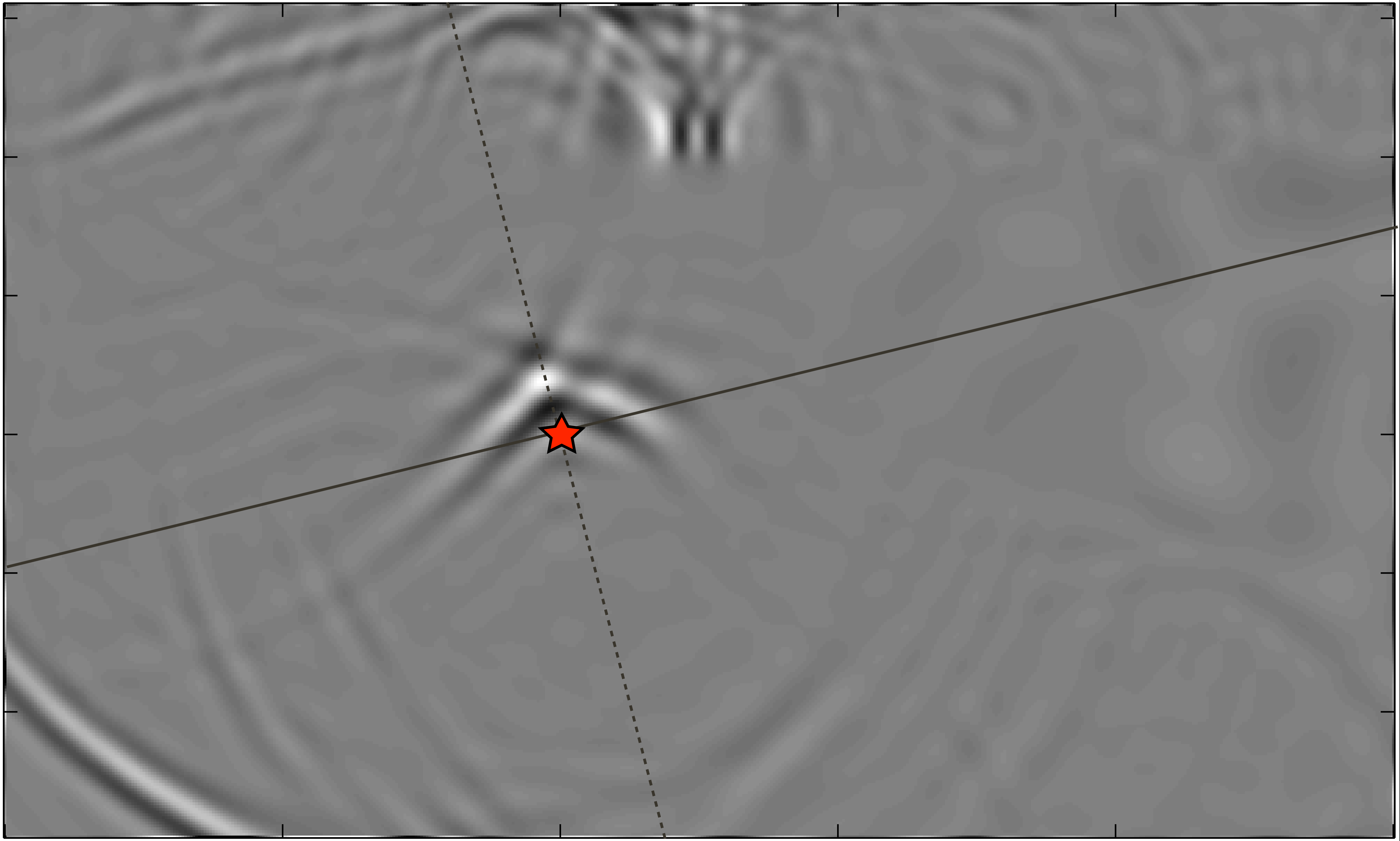












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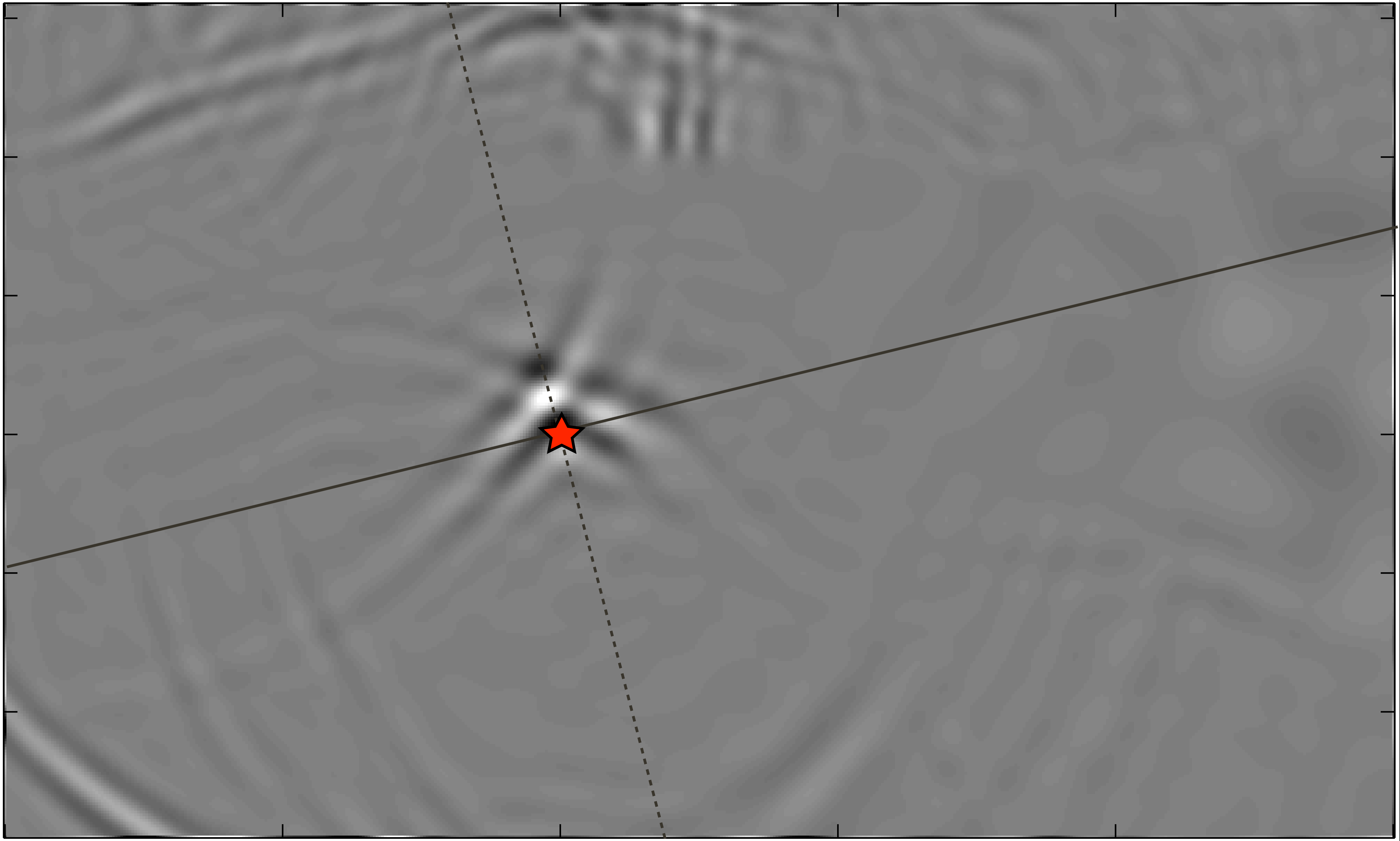
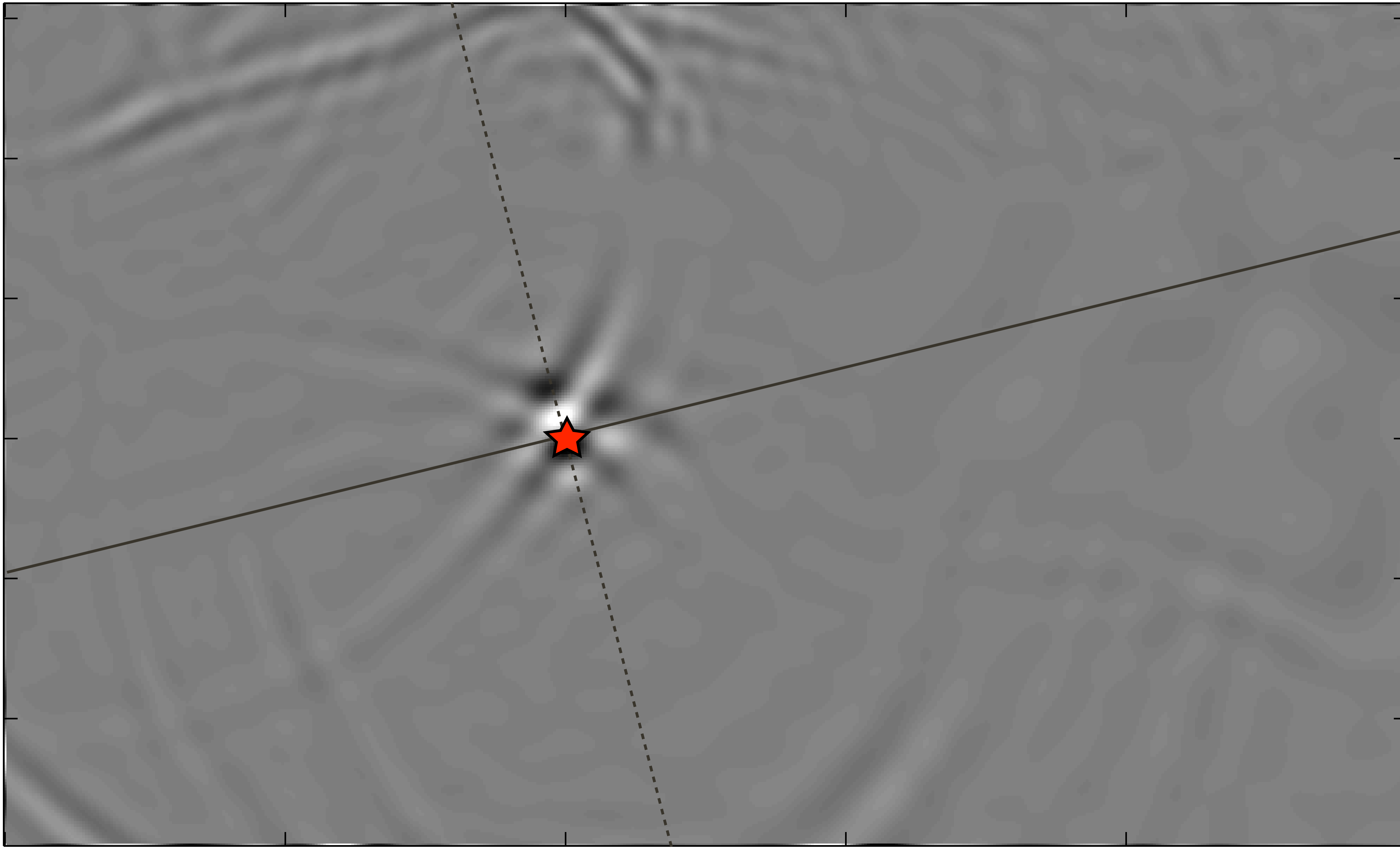
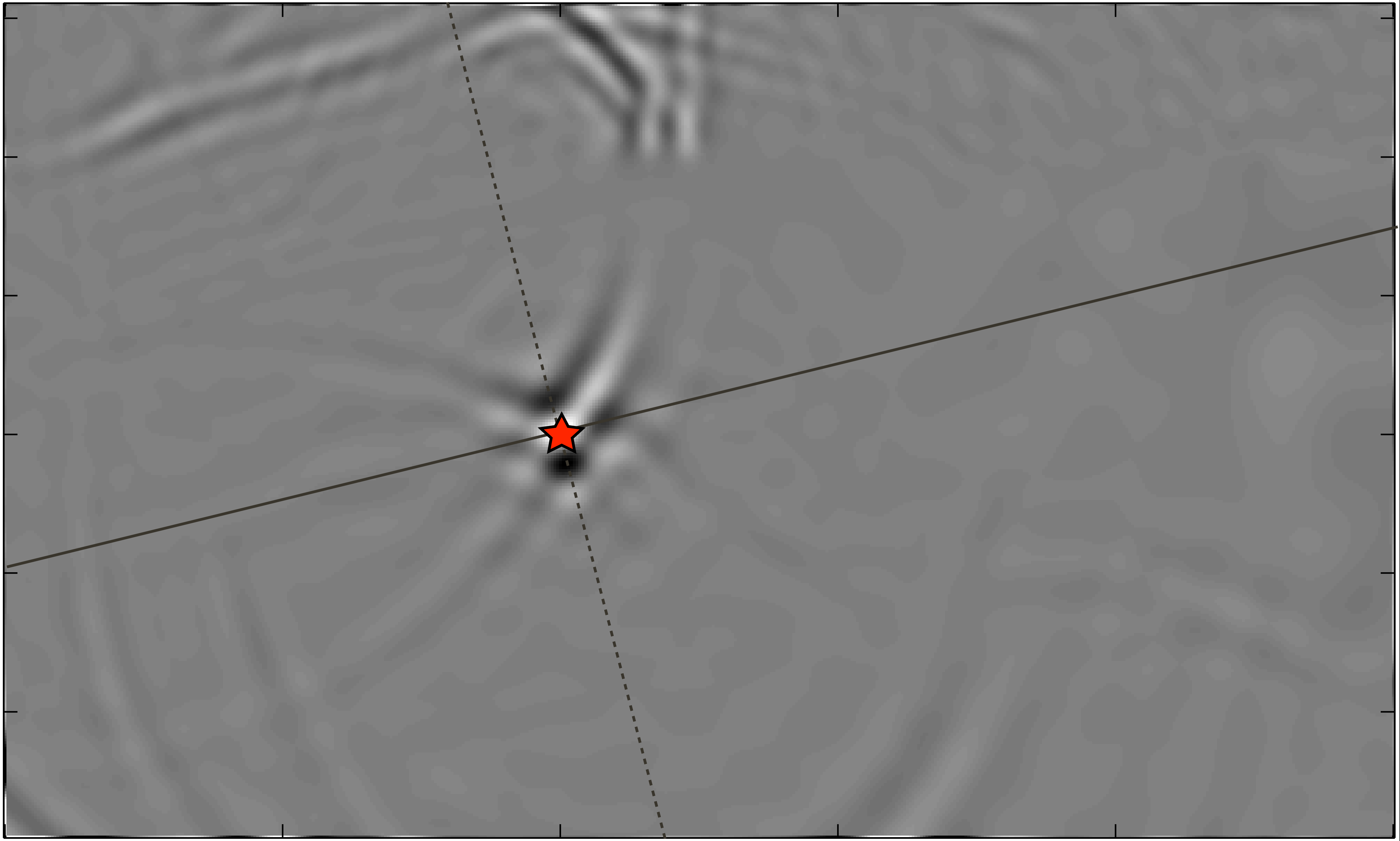
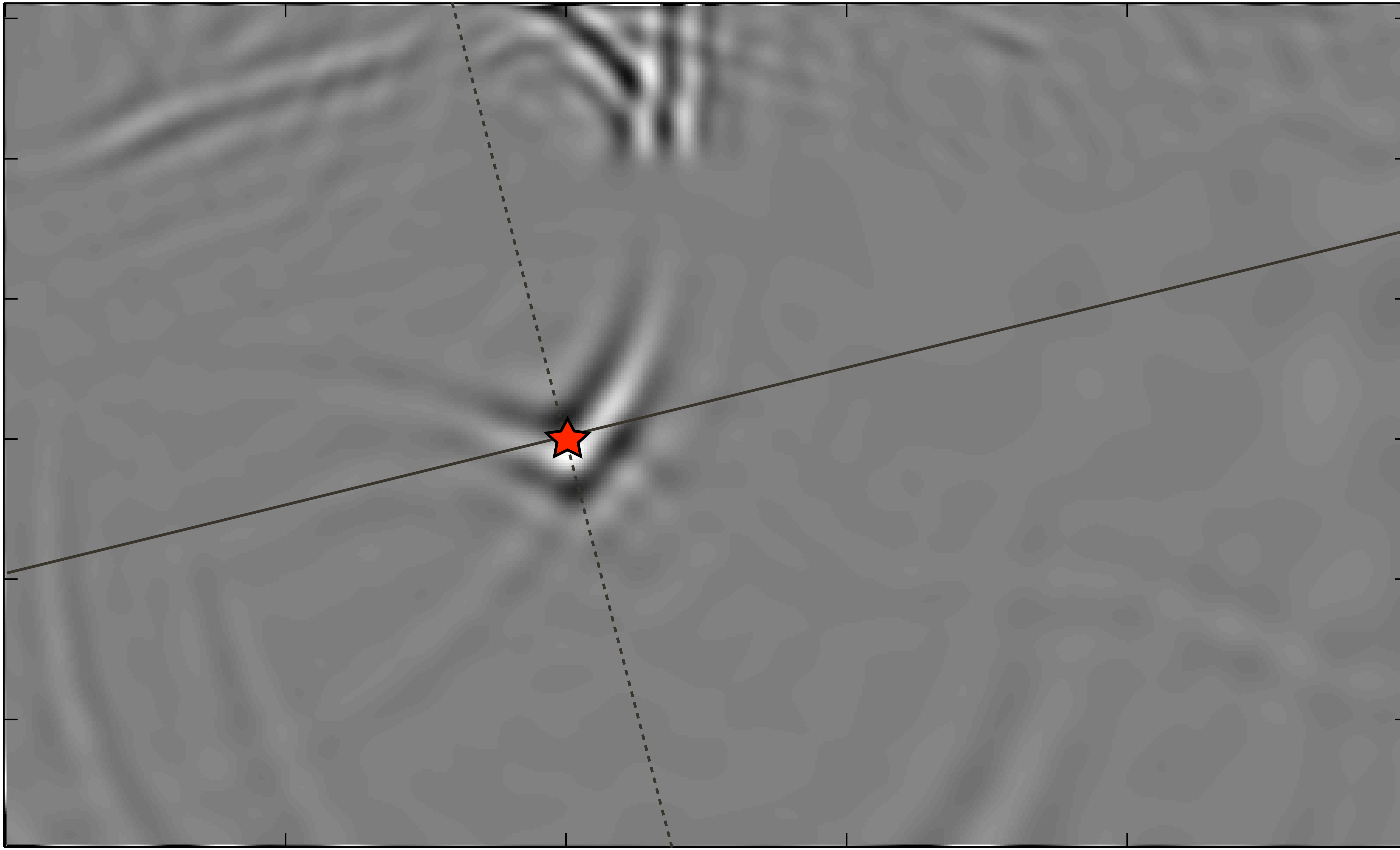
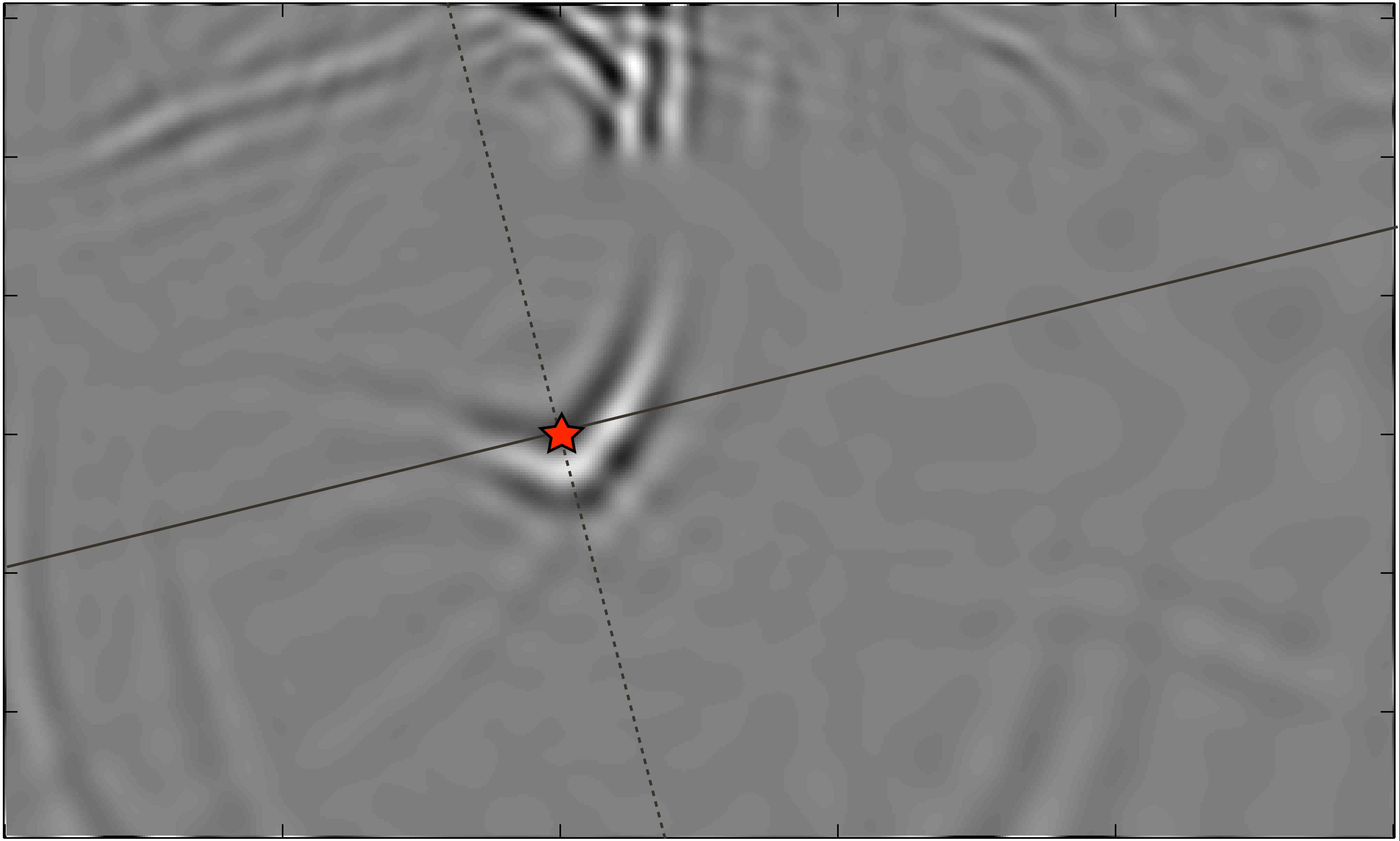
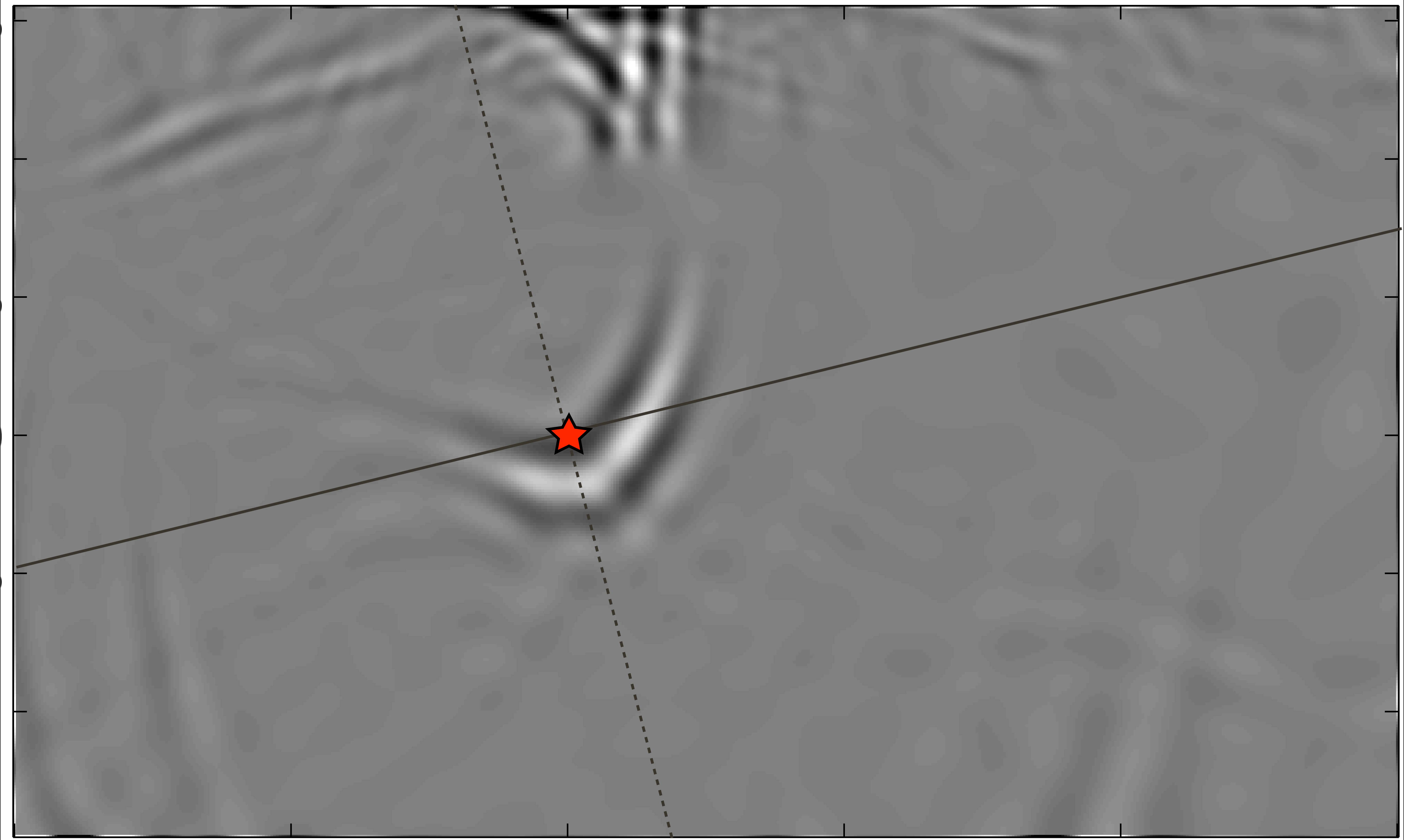


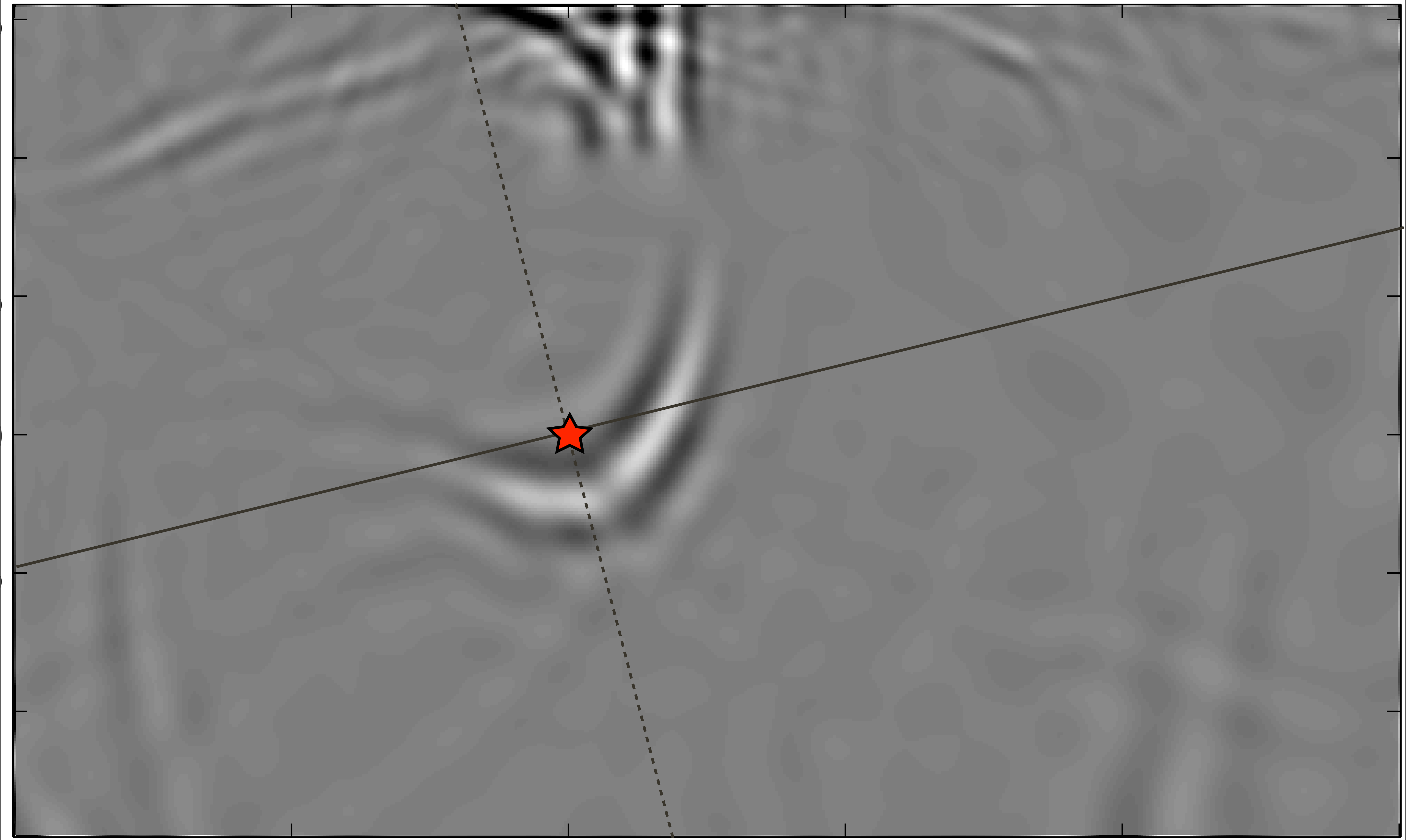












Extended images

- complete image volume too large to form: $(n_x \times n_z)^2$
- instead, probe volume for information via mat-vecs $E\mathbf{y}$
- $\mathbf{y}$ can be interpreted as subsurface source function

Computation

mat-vec with extended image:

$$\mathbf{e} = E\mathbf{y} = H^{-*}P_r^T D Q^* P_s H^{-1}\mathbf{y}$$

- $\mathbf{d} = P_s H^{-1}\mathbf{y}$ *(one subsurface source)*
- $\mathbf{w} = Q^*\mathbf{d}$ *(source weights)*
- $\mathbf{e} = H^{-*}P_r^T(D\mathbf{w})$ *(one source)*

MVA

Focusing in Δx implies a *commutation* relation: $x \cdot f(x, x') = x' \cdot f(x, x')$

or

$$E\text{diag}(\mathbf{x}) = \text{diag}(\mathbf{x})E$$

Measure the error in some norm

$$||E\text{diag}(\mathbf{x}) - \text{diag}(\mathbf{x})E||_?^2$$

[Symes ’08]

MVA

The Frobenius norm can be estimated via randomized trace estimation:

$$||A||_F^2 = \text{trace}(A^T A)$$

$$\approx \sum_{i=1}^{K} \mathbf{w}_i^T A^T A \mathbf{w}_i = \sum_{i=1}^{K} ||A\mathbf{w}_i||_2^2$$

where $\sum_{i=1}^{K} \mathbf{w}_i \mathbf{w}_i^T \approx I$

[Avron & Toledo, '11]

MVA

objective and gradient

$$\phi(\mathbf{m}) = \sum_k \tfrac{1}{2}||R(\mathbf{m})\mathbf{w}_k||_2^2$$

$$\nabla\phi(\mathbf{m}) = \sum_k DR(\mathbf{m}, \mathbf{w}_k)^* R(\mathbf{m})$$

where

$$R(\mathbf{m}) = E(\mathbf{m})\mathrm{diag}(\mathbf{x}) - \mathrm{diag}(\mathbf{x})E(\mathbf{m})$$

$$DR(\mathbf{m}, \mathbf{w}) = \frac{\partial R\mathbf{w}}{\partial \mathbf{m}}$$

MVA

The spectral norm can be estimated by the power method

$$\mathbf{w}_{n+1} = A^T A \mathbf{w}_n / ||\mathbf{w}_n||_2$$

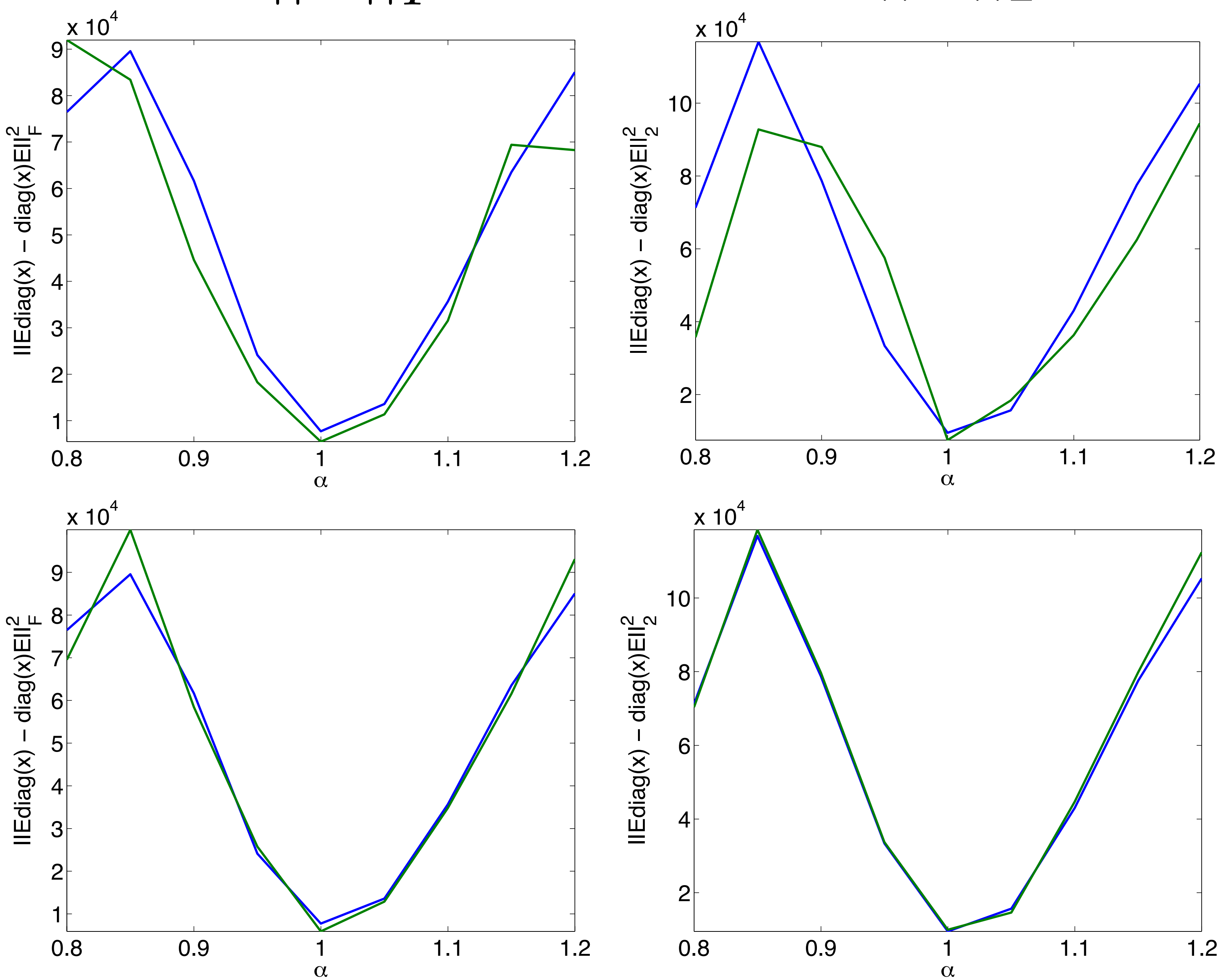
$$||A||_2^2 \approx ||A\mathbf{w}_K||_2^2 / ||\mathbf{w}_K||_2^2$$

MVA

$||\cdot||_F^2$ $||\cdot||_2^2$

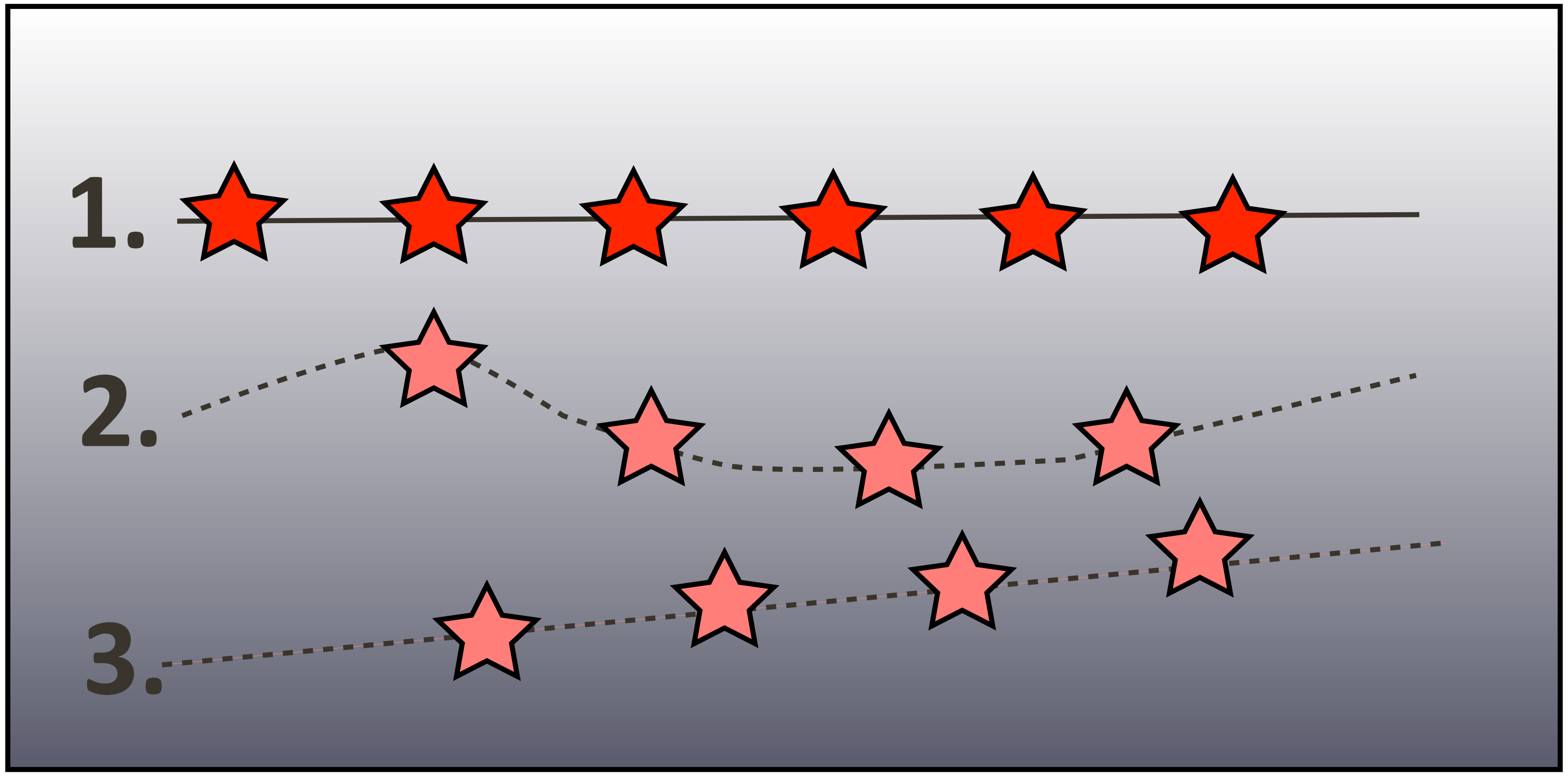
$K = 2$

$K = 10$



MVA

- vectors can be seen as subsurface sources
- implement `layerstripping' approach:



Conclusions

- image volume for *all* offsets easily expressed in terms of data matrices
- two-way equivalent of DSR equation
- cost of MVA proportional # of subsurface (simultaneous) sources (**not** # of sources or subsurface offsets)

Future work

- exploit tensor structure
- automatically detect dip
- velocity continuation
- AVA
- MVA/FWI hybrid

MVA/FWI Hybrid

Combined penalty:

$$\phi(\mathbf{m}) = \alpha||R_{\mathrm{MVA}}(\mathbf{m})||^2 + \beta||R_{\mathrm{FWI}}(\mathbf{m})||^2$$

where

$$R_{\mathrm{MVA}}(\mathbf{m}) = E(\mathbf{m})\mathrm{diag}(\mathbf{x}) - \mathrm{diag}(\mathbf{x})E(\mathbf{m})$$
$$R_{\mathrm{FWI}}(\mathbf{m}) = D - P^T U(\mathbf{m})$$

Estimate norms with mat-vecs

MVA/FWI Hybrid

Introduce scale separation:

$$\phi(\mathbf{m}_0, \delta\mathbf{m}) = \alpha||R_{\mathrm{MVA}}(B_s\mathbf{m}_0)||^2 + \beta||R_{\mathrm{FWI}}(B_s\mathbf{m}_0 + B_r\delta\mathbf{m})||^2$$

where B_s is a basis for smooth models and B_r is a basis for oscillatory models.

[Almomin & Biondo '12]

MVA/FWI Hybrid

$$\phi(\mathbf{m}_0, \delta\mathbf{m}) = \alpha||R_{\text{MVA}}(B_s\mathbf{m}_0)||^2 + \beta||R_{\text{FWI}}(B_s\mathbf{m}_0 + B_r\delta\mathbf{m})||^2$$

1. shallow velocity updates: diving wave tomography
2. deep velocity updates: reflection tomography
3. reflectors: (non-linear) least-squares migration

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