SLIM Seismic Laboratory for Imaging and Modeling

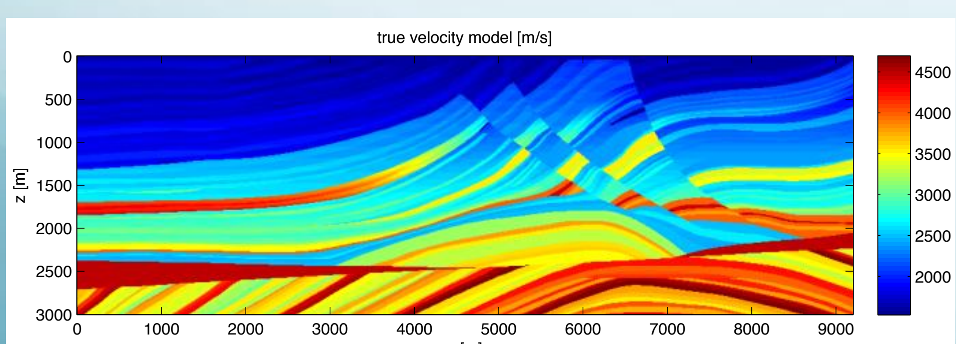
SINBAD Fall 2012 Consortium Meeting

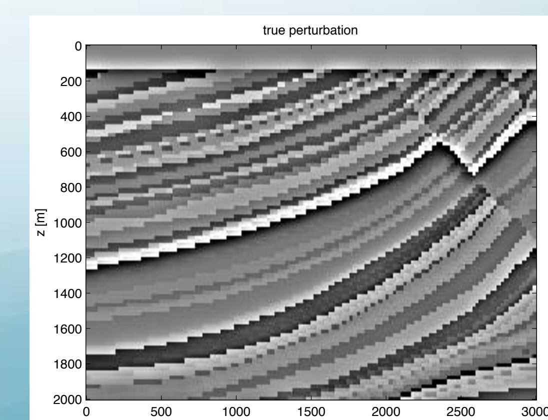
TOTAL

Variance Estimation
Application to FWI

Anaïs TAMALET

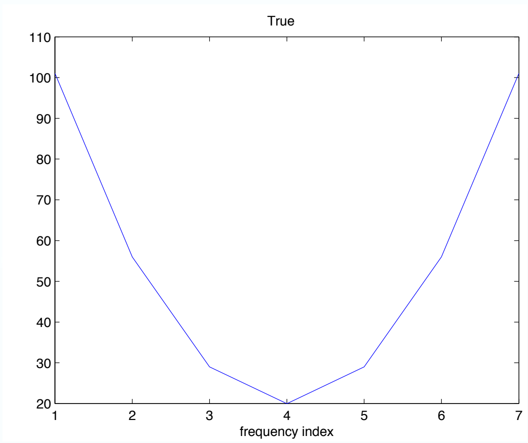
TOTAL Supervisor: Henri CALANDRA
UBC Professor: Felix HERRMANN

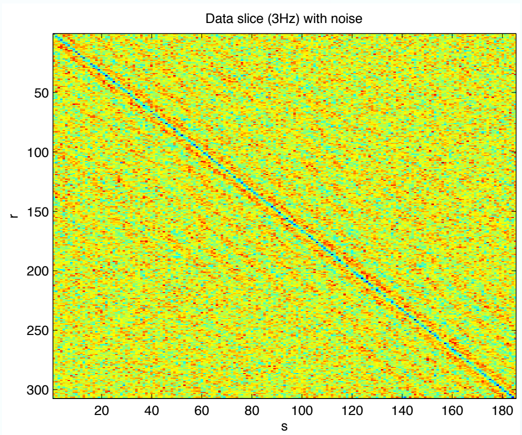


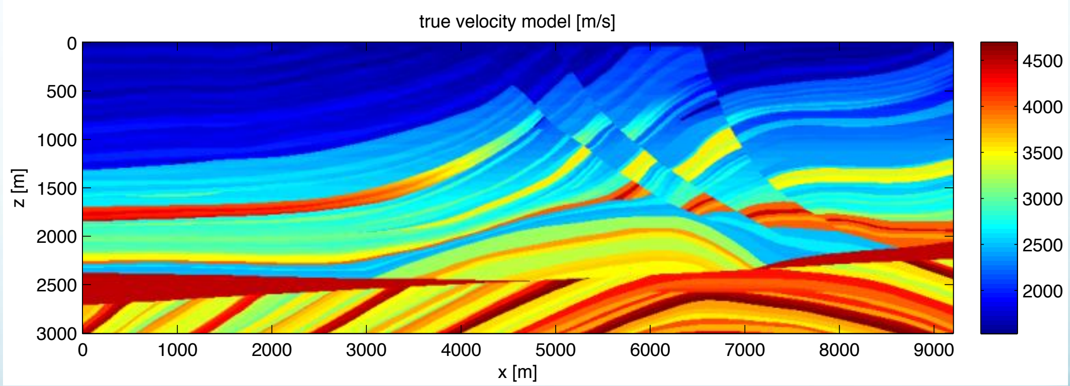


Outline

- Introduction
- Variance Estimation
  - Inverse problem
  - Maximum Likelihood Formulation
  - Variance in Multiple Data Sets
  - Modified Problem
- Application to Full-Waveform Inversion
  - Full-Waveform Inversion
  - Experiments
- Conclusion







Introduction

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- **Company**: TOTAL SA
- **University**: The University of British Columbia (UBC)
- **Laboratory**: Seismic Laboratory for Imaging and Modeling (SLIM)
- **Professor**: Felix HERRMANN
- **TOTAL Supervisor**: Henri CALANDRA
- **Dates**: January 2012 – June 2013

Variance Estimation

Variance Estimation

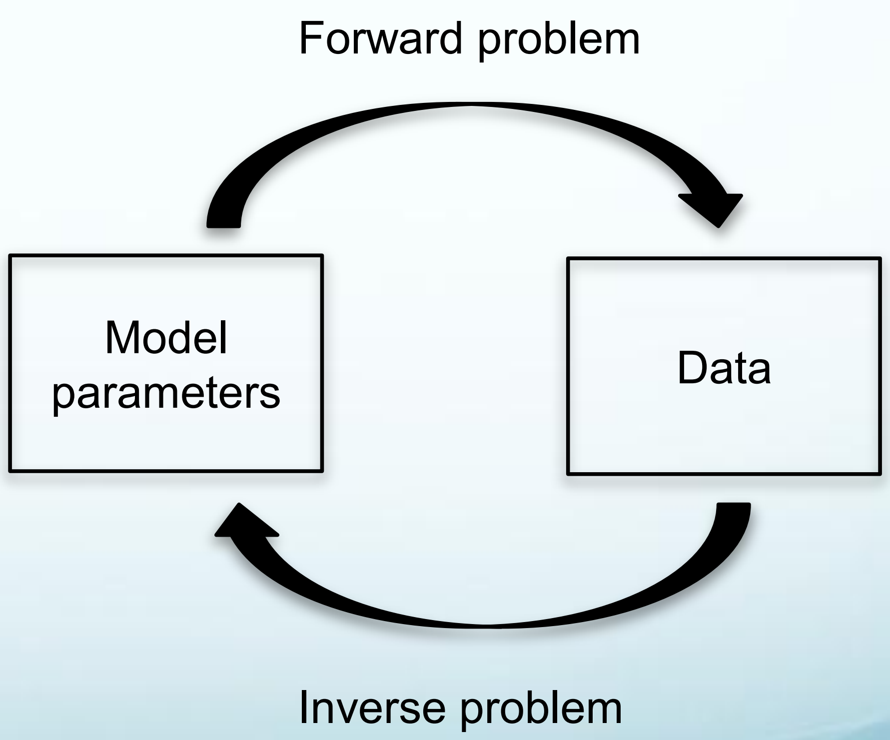
Inverse problem

- We consider inverse problems of the form

$$\min_{x} \rho \left(d - F(x) \right)$$

where

- **ρ** is a twice differentiable function
- **d** denotes the data
- **F** is the forward modeling operator
- **x** is the vector of unknown parameters



Variance Estimation

Maximum Likelihood (ML) Formulation

- **Statistical Model**
  - Inverse problems can be formulated as Maximum Likelihood (ML) problems

$$d = F(x) + \epsilon$$

Variance Estimation

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  - i.i.d. Gaussian errors $\epsilon \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$
  - Even though σ^2 is unknown, it does **not affect** the ML formulation in **x**

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⇨ Not true if the data comes from different sources with each group having its **own variance**

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⇨ Not true if the data comes from different sources with each group having its **own variance**

⇨ Need **variance estimation**

Variance Estimation

Variances in Multiple Data Sets

- **Multiple Data Sets**
  - **M experiments** indexed by i, each with its **own** (unknown) **variance** σ_i^2
  - Each experiment yields to N_i **measurements**
  - All experiments share a **common set of primary parameters x**

Variance Estimation

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- **Statistical model**

$$d_i = F_i(x) + \epsilon_i \qquad \epsilon_i \sim \mathcal{N}(0, \sigma_i^2 \mathbf{I})$$

Variance Estimation

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- **Statistical model**

$$d_i = F_i(x) + \epsilon_i \qquad \epsilon_i \sim \mathcal{N}(0, \sigma_i^2 \mathbf{I})$$

- **Joint ML estimation problem**

We want to estimate $\boldsymbol{\sigma^2 = \{\sigma_i^2\}}$ and **x**

$$\min_{x,\sigma^2} g(x,\sigma^2) = \sum_{i=1}^{M} \left(N_i \log(2\pi\sigma_i^2) + \frac{1}{\sigma_i^2} \|d_i - F_i(x)\|_2^2 \right)$$

Variance Estimation

Modified problem

- **Variance Estimation**

With **x fixed**, the **estimate** of $\boldsymbol{\sigma_i}^{\mathbf{2}}$ is given by

$$\widehat{\sigma_i^2}(x) = \frac{1}{N_i}||d_i - F_i(x)||_2^2 \qquad x \text{ fixed}$$

Variance Estimation

Modified problem

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$$\widehat{\sigma_i^2}(x) = \frac{1}{N_i}||d_i - F_i(x)||_2^2 \qquad\qquad x \text{ fixed}$$

- **Modified Problem**

Thus, the problem **for x** becomes

$$\min_x \tilde{g}(x) = \sum_{i=1}^{M}\left(N_i \log(2\pi\widehat{\sigma_i^2}(x)) + \frac{1}{\widehat{\sigma_i^2}(x)}\|d_i - F_i(x)\|_2^2\right)$$

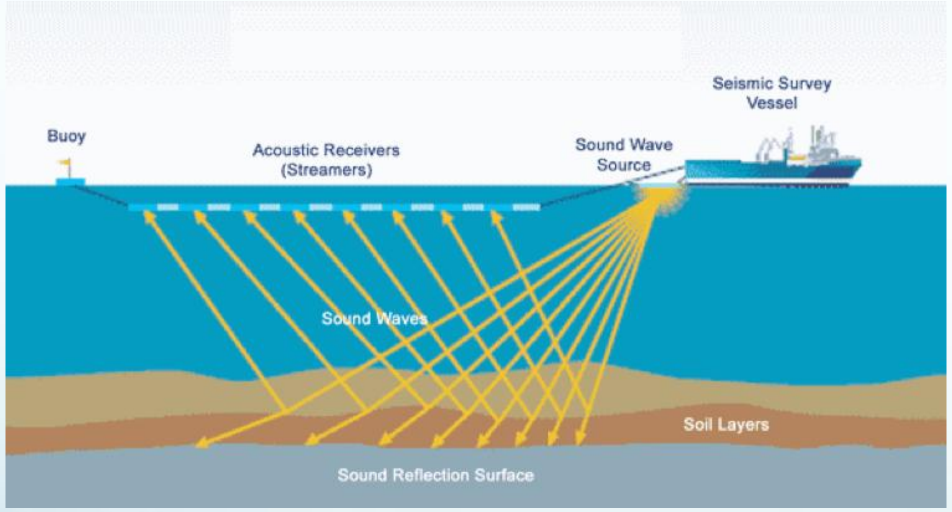
Application to Full-Waveform Inversion

Full-Waveform Inversion (FWI)

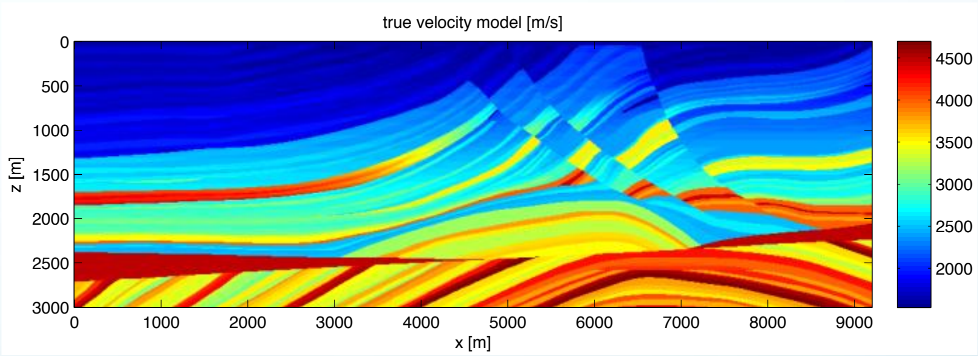
FWI

- Data-fitting procedure based on full-wavefield modeling to extract **medium parameters** from the **seismic data**

Data



Model



Full-Waveform Inversion (FWI)

FWI

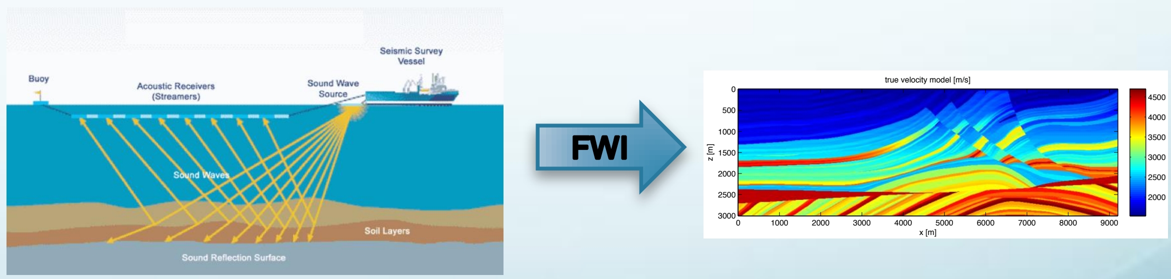
- **Formulation of the problem**

$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \|D - F(\mathbf{m}; Q)\|_F^2$$

where

- D is the data matrix
- F is the forward modeling operator
- Q specifies the source experiments
- **m** is the vector of unknown medium parameters

Data　　　　Model



Experiments

Variance Estimation: Application to FWI

- **Scenario**
  - Data with noise with a **variance varying** according to the **frequency**

Experiments

Variance Estimation: Application to FWI

- **Scenario**
  - Data with noise with a **variance varying** according to the **frequency**
- **Notation**
  - **Each experiment** corresponds to **1 frequency**

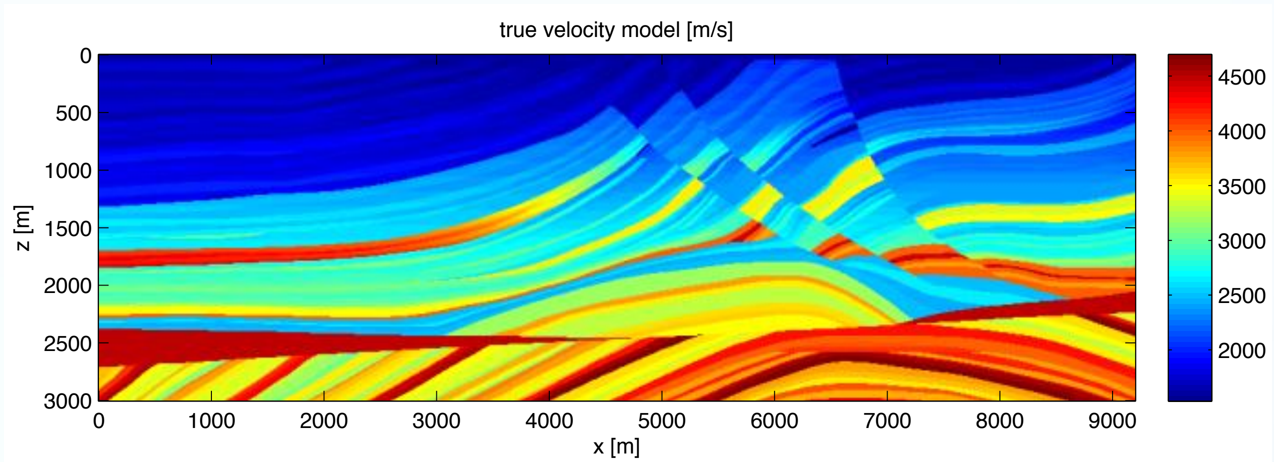
M	Number of frequencies
i	Indexes the frequencies
$\mathbf{N_i}$	Number of **measurements** per frequency ($\mathbf{N_i = n_{rec} \times n_{src}}$)
$\mathbf{d_i}$	Fourier transform of the recorded time series for frequency i (**data**)
$\mathbf{F_i}$	**Modeling operator** for frequency i ($\mathbf{F_i(x) = P\, A_i(x)^{-1}\, Q_i}$)
x	Vector of unknown velocity parameters

Experiments

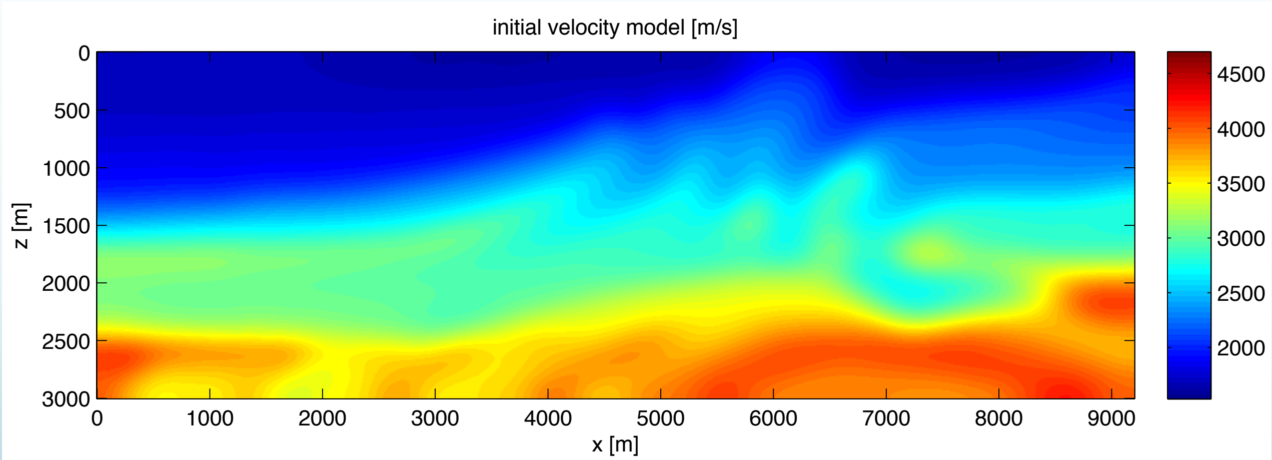
Synthetic model

- **Model**
  - Marmousi model
  - 301 x 921 grid with 10m spacing
  - 307 receivers
  - 185 sources

True velocity model (m/s)



Initial velocity model (m/s)



Experiment 1

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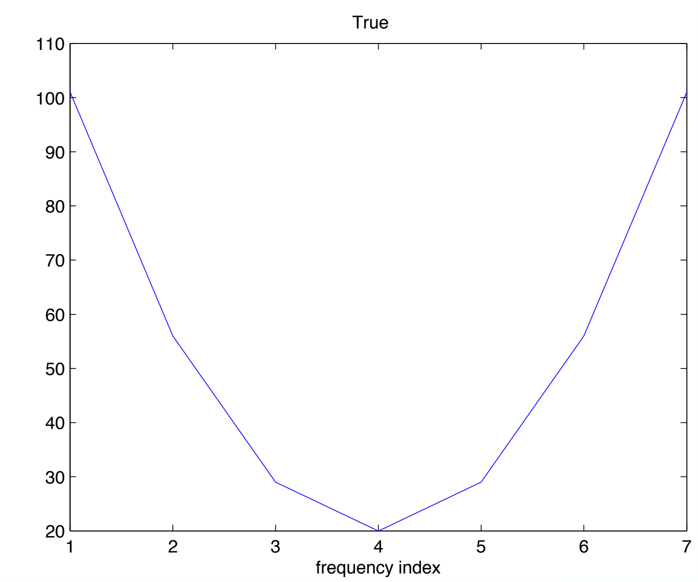
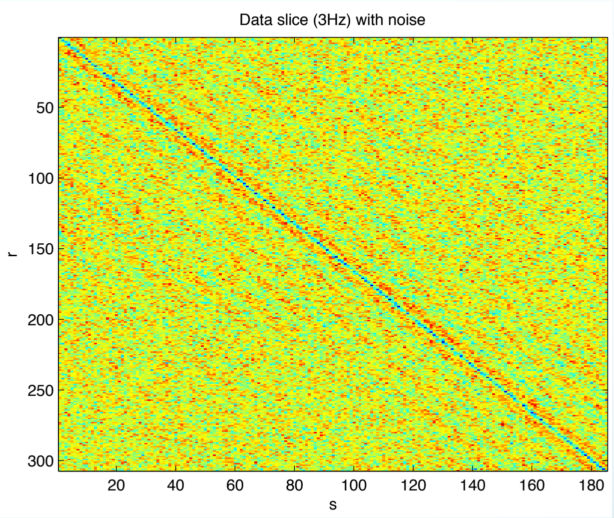


- **Noise in the data**
  - Gaussian noise
  - Standard deviation $\sigma_i \sim (i-4)^2$

  ➡ **Higher variance** for the **low** and **high** frequencies



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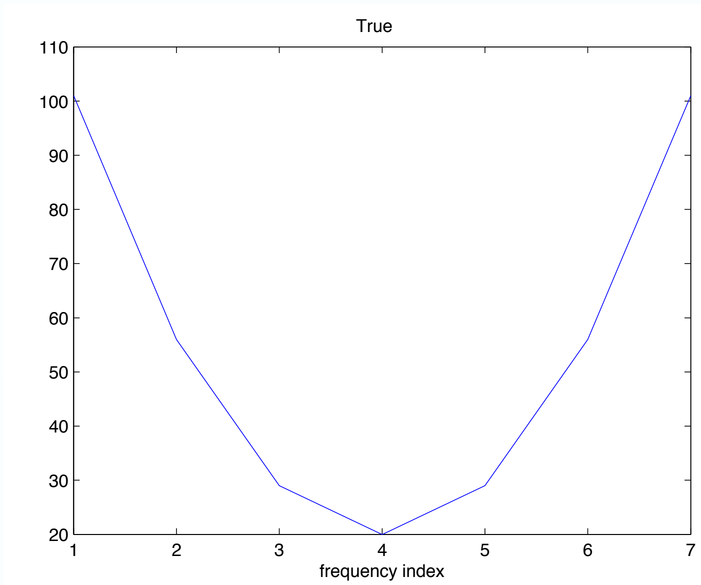

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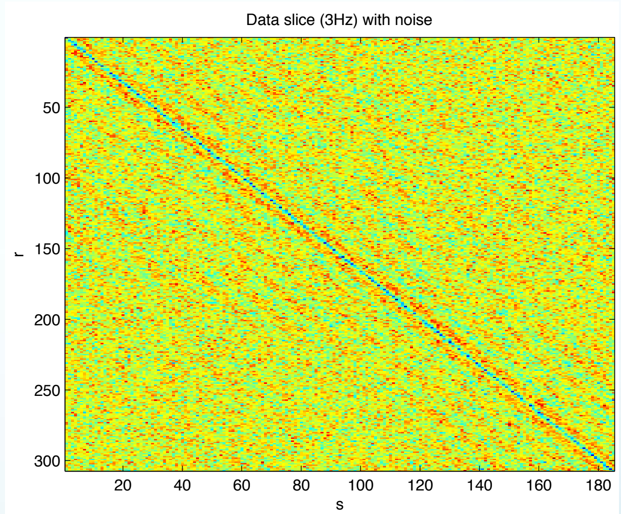
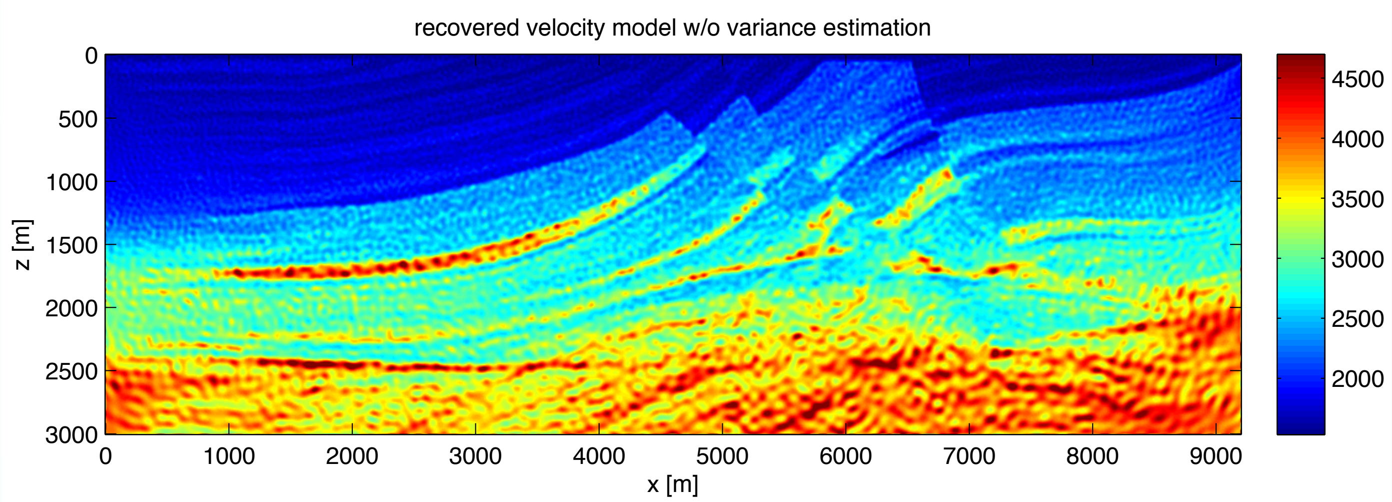


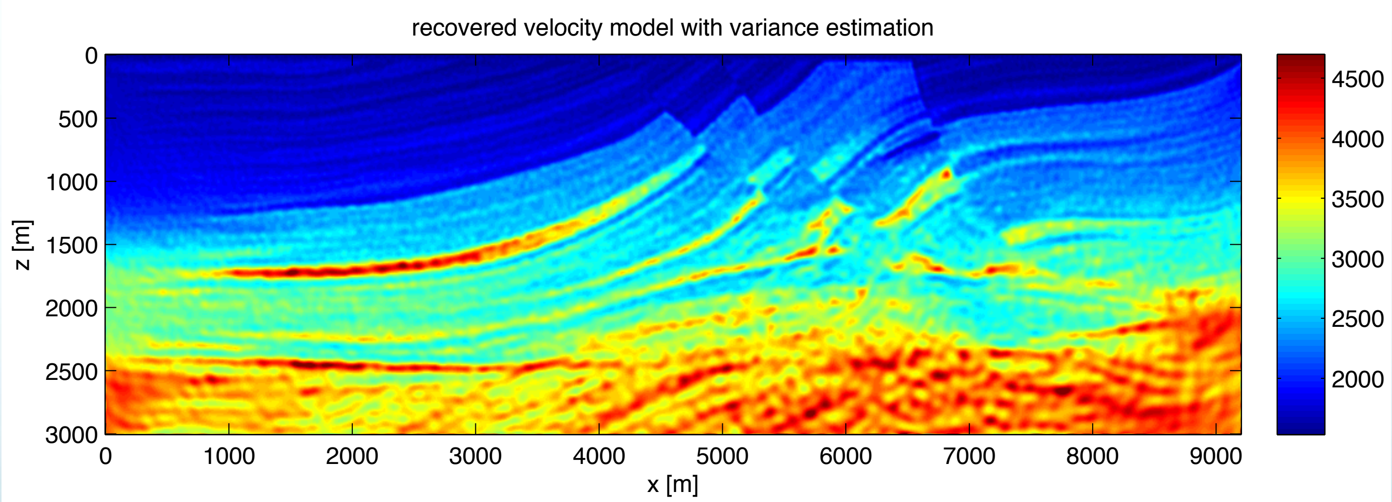
- **Processing**
  - 7 frequencies
  - 3 **overlapping** frequency bands of 3 frequencies each
  - Optimization problem solved using **L-BFGS**
  - **Without** variance estimation, problem solved for a fixed $\sigma_i = 1$ for all i

Experiment 1

Recovered Velocity Model



Without variance estimation

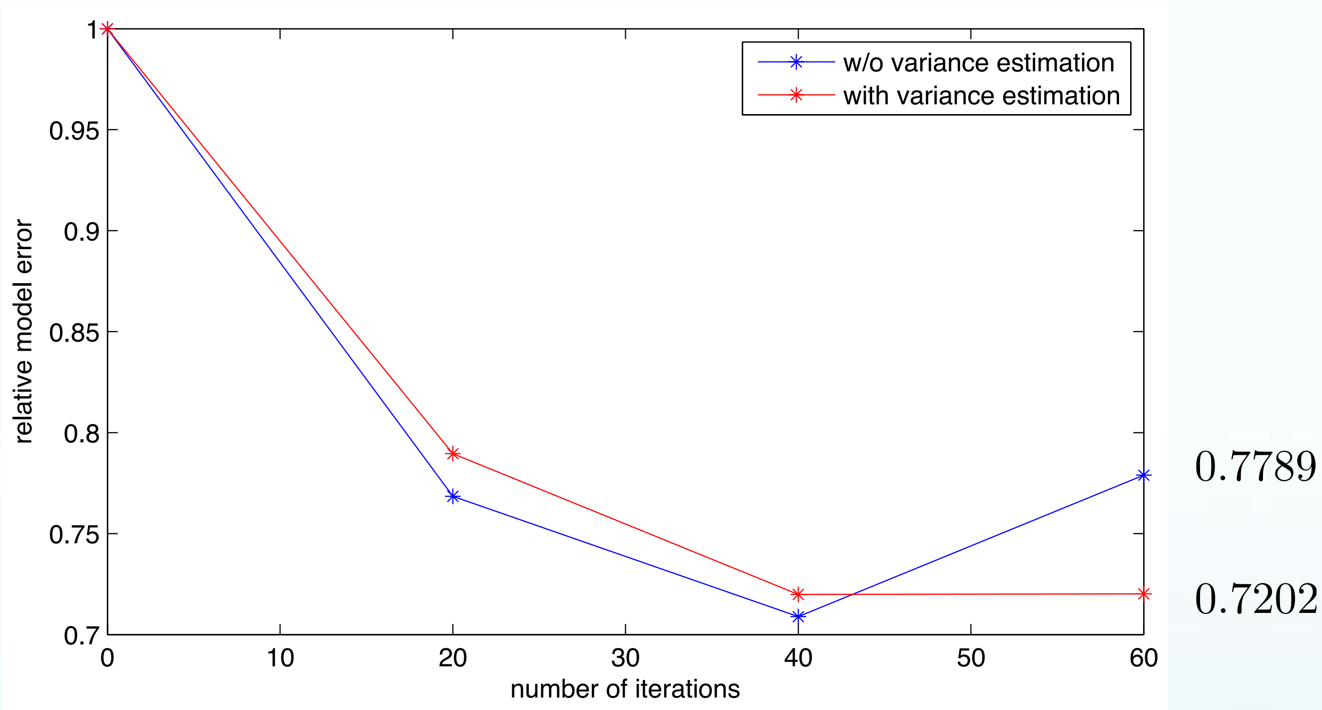


With variance estimation

Experiment 1

Relative Model Error

$$\frac{||\mathbf{m}_{true} - \hat{\mathbf{m}}||_2}{||\mathbf{m}_{true} - \mathbf{m}_0||_2}$$



Relative model error

Experiment 1

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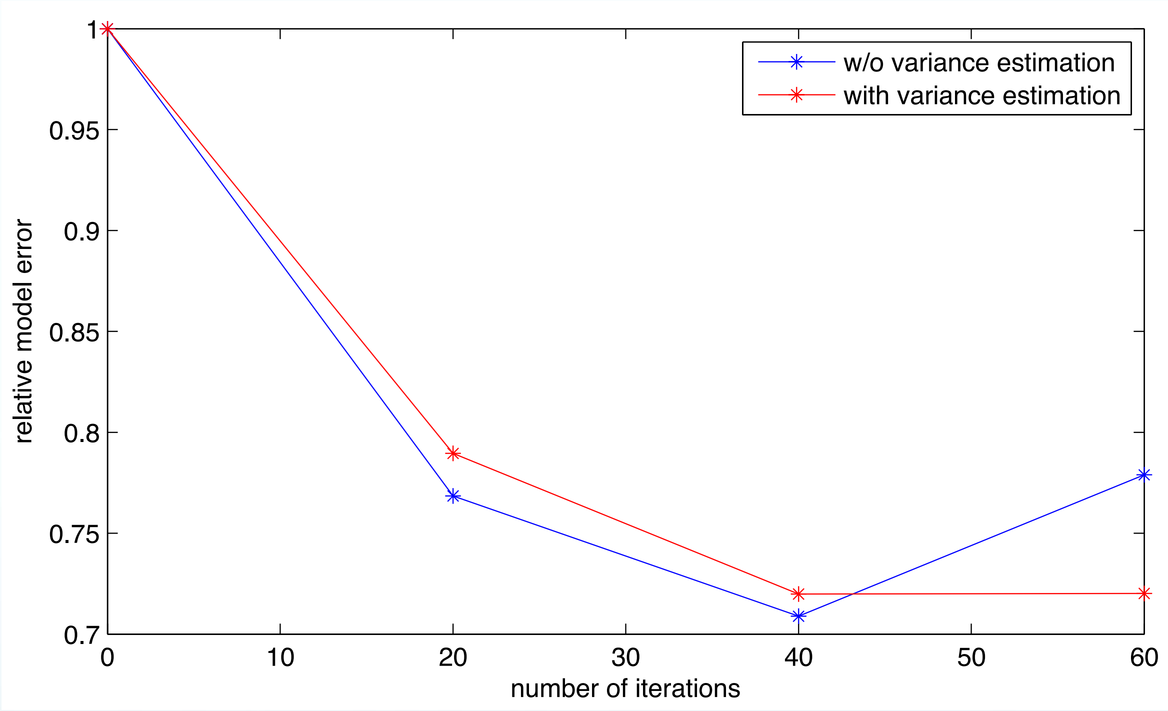


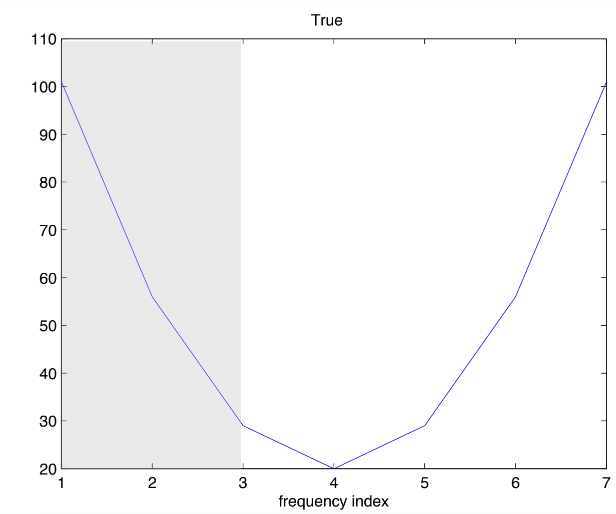
0.7789

0.7202

Relative model error

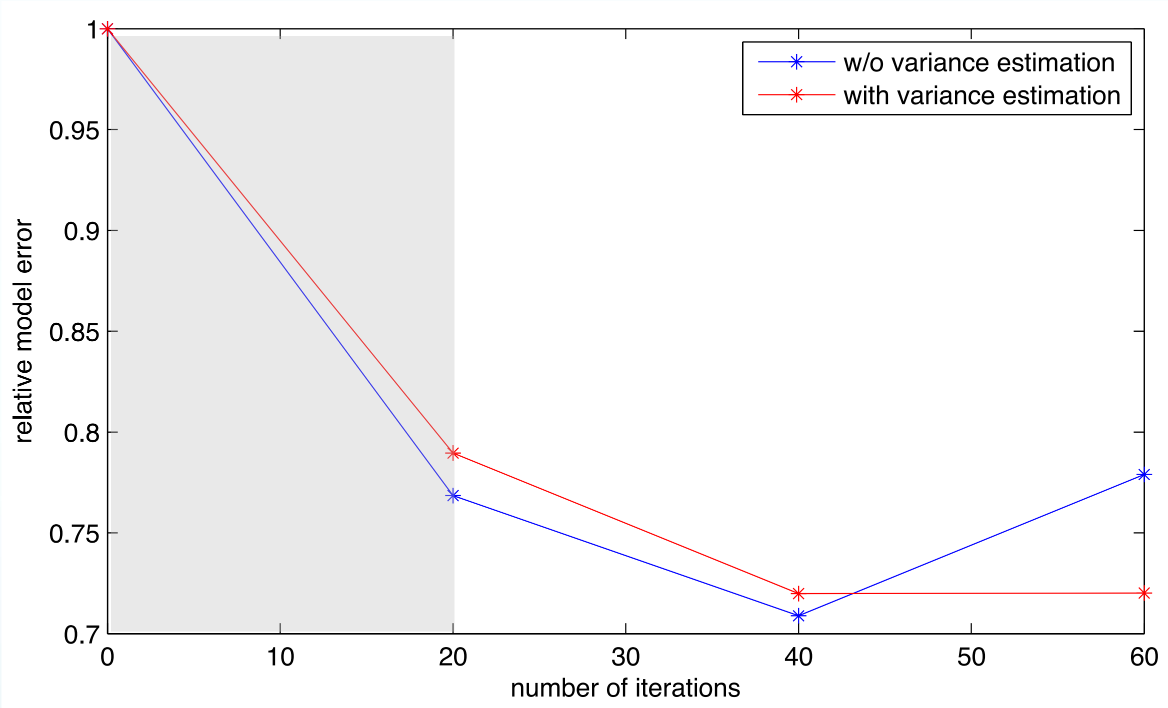
➡ Result **improved** with variance estimation

Experiment 1



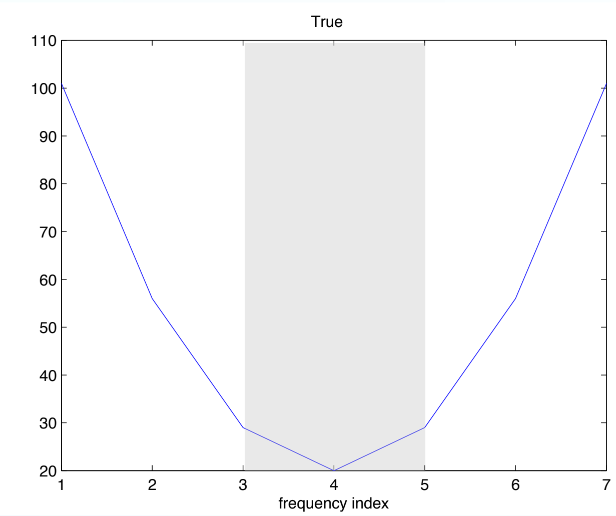
Relative Model Error

First frequency band



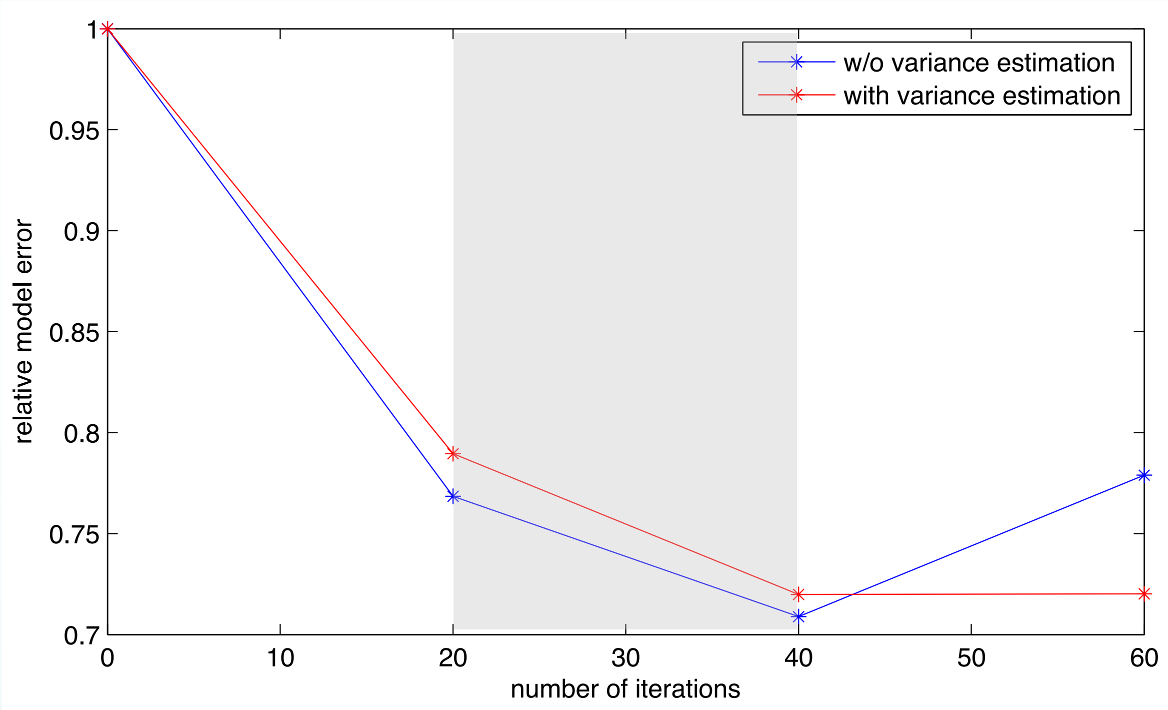
Relative model error

Experiment 1



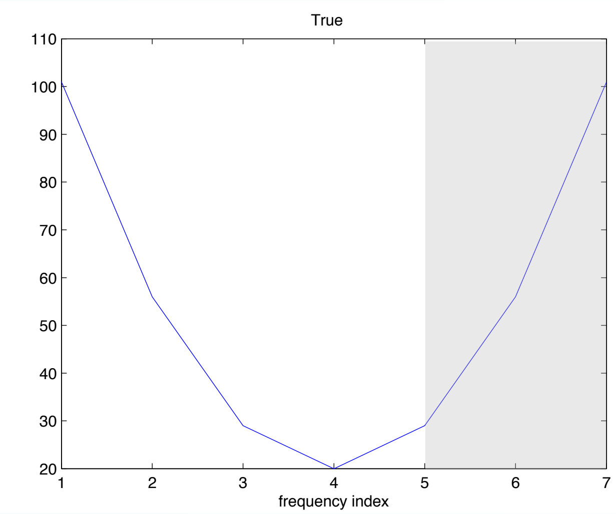
Relative Model Error

Second frequency band



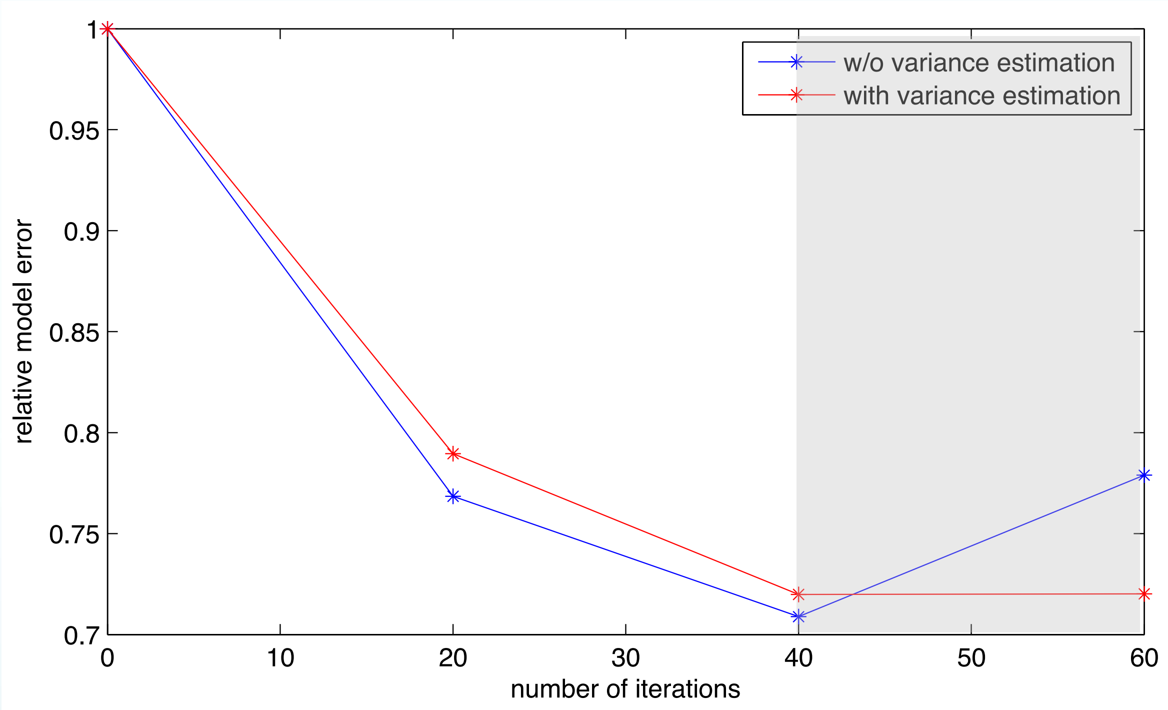
Relative model error

Experiment 1



Relative Model Error

Third frequency band

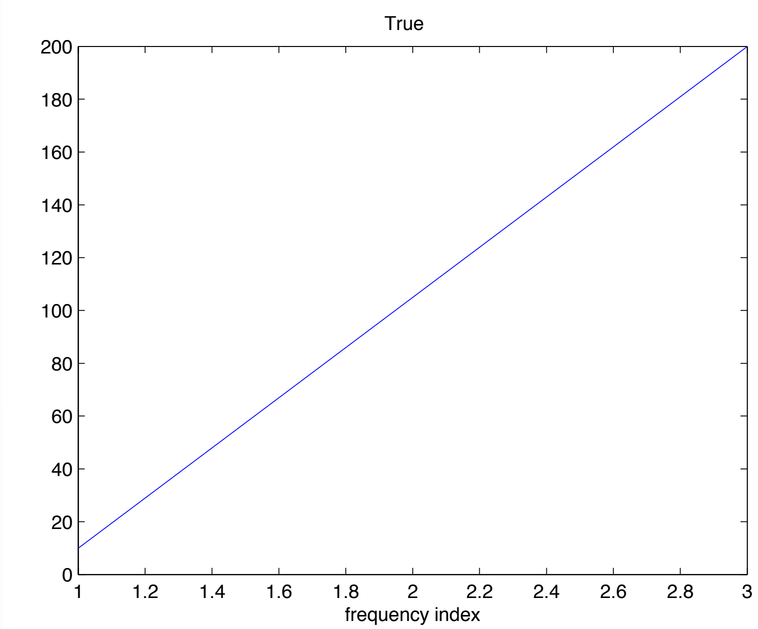


Relative model error

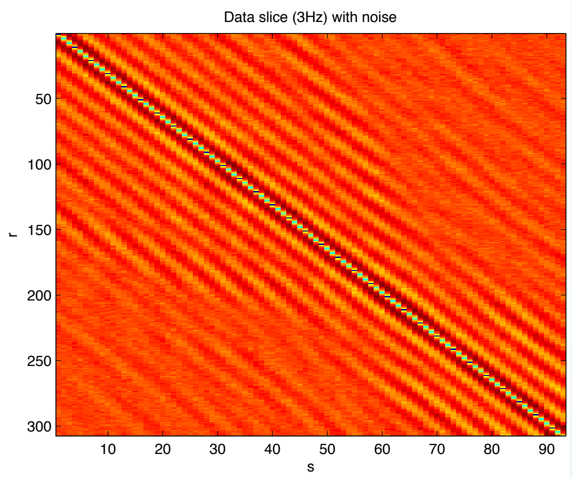
⇒ Variance estimation useful when the variance of the noise **increases** with the frequency

Experiment 2

Experiment 2



- **Noise in the data**
  - Gaussian noise
  - **Variance increasing** with the frequency



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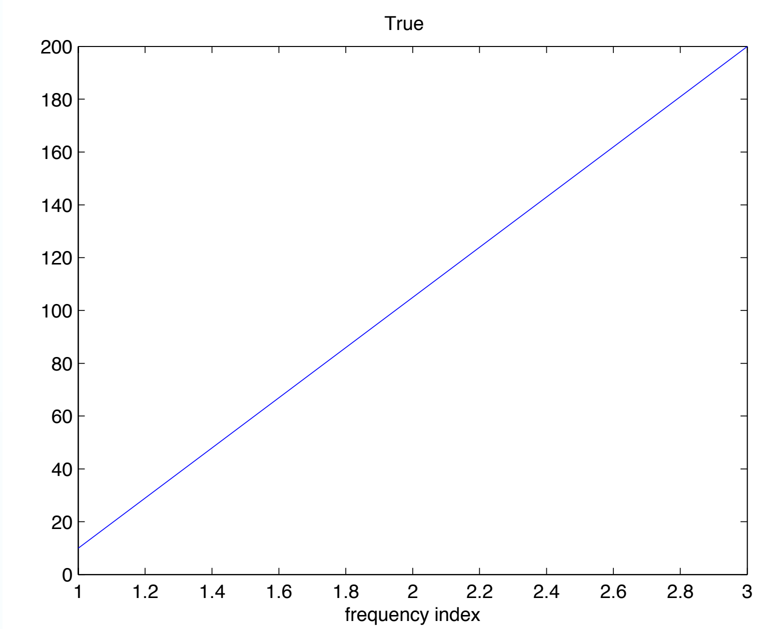

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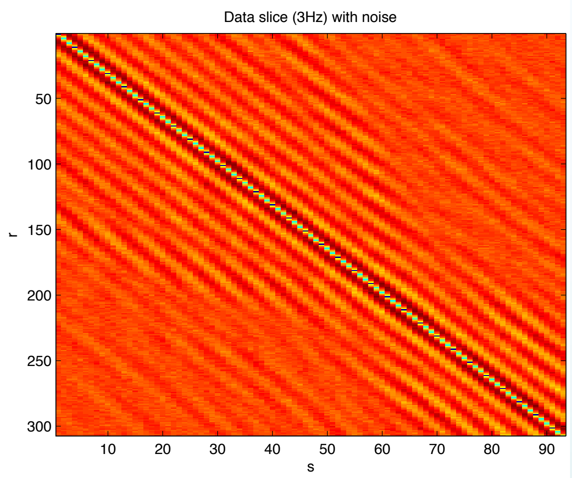

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  - **1 frequency band** of 3 frequencies
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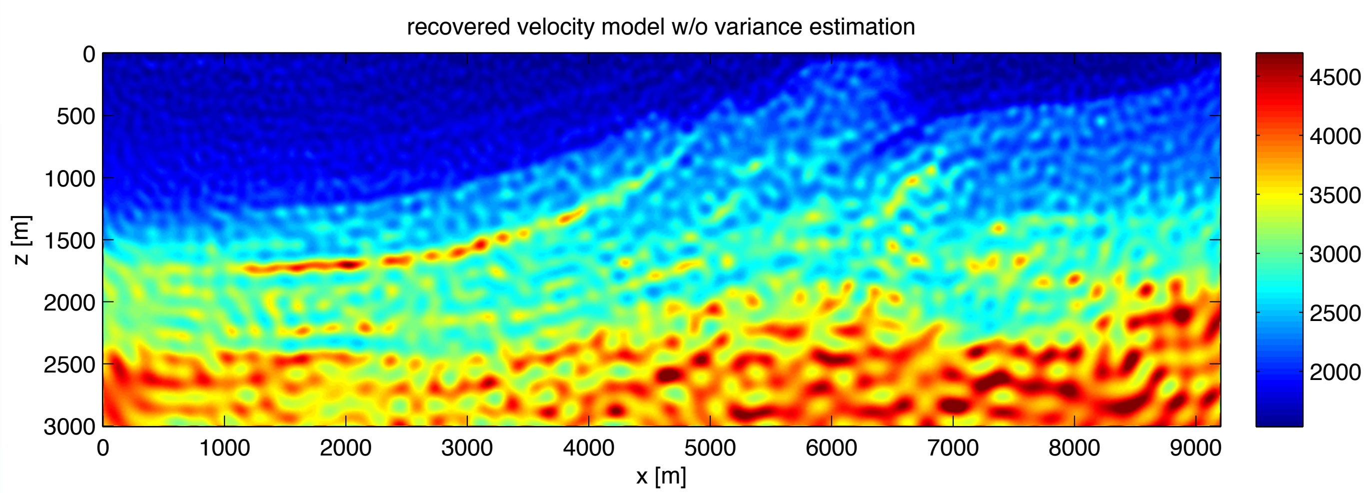
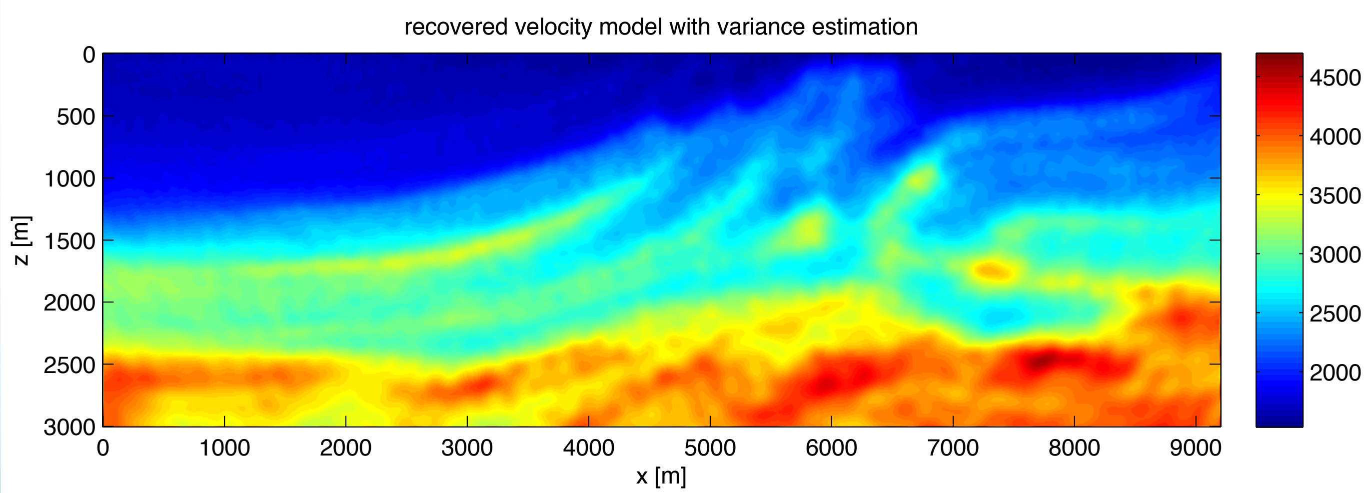
Experiment 2

Recovered Velocity Model



Without variance estimation



With variance estimation

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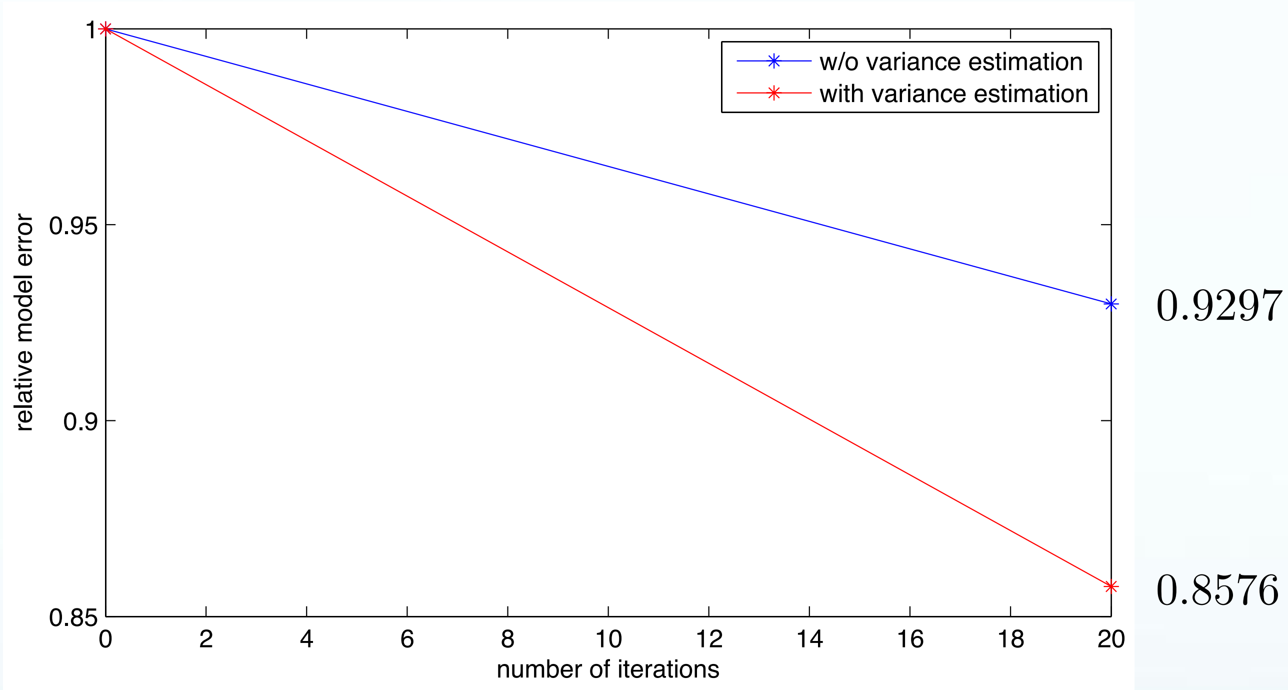

Relative model error

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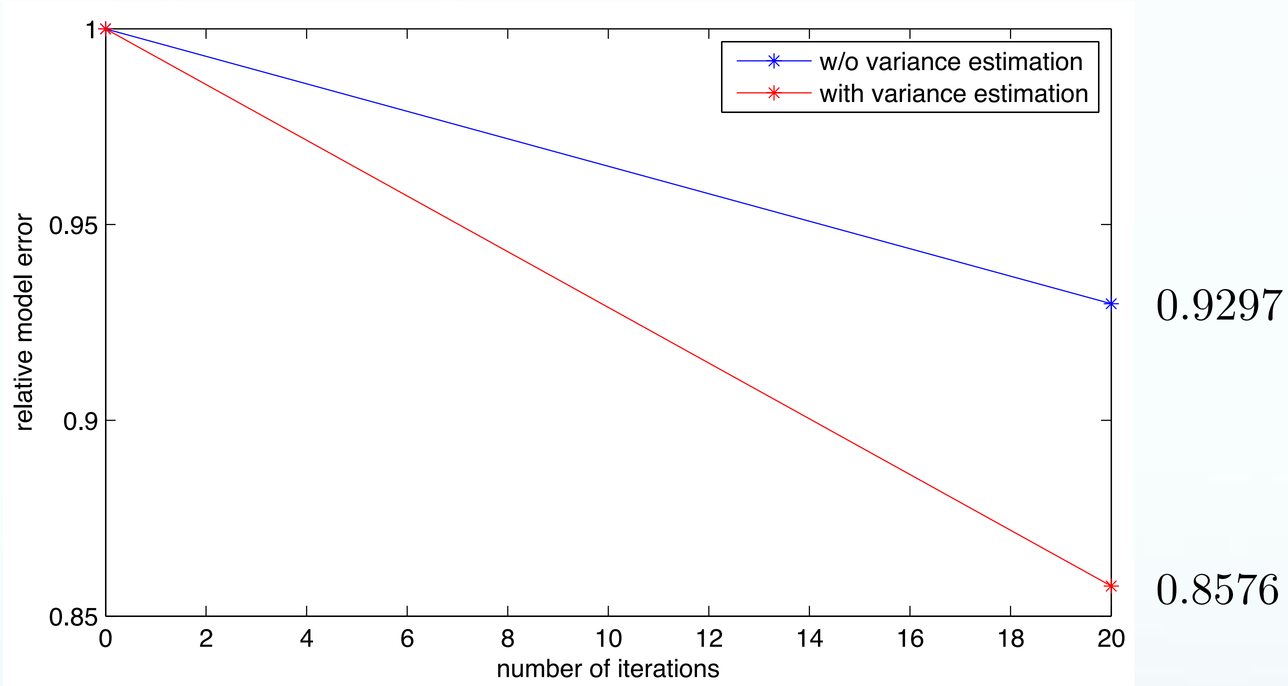
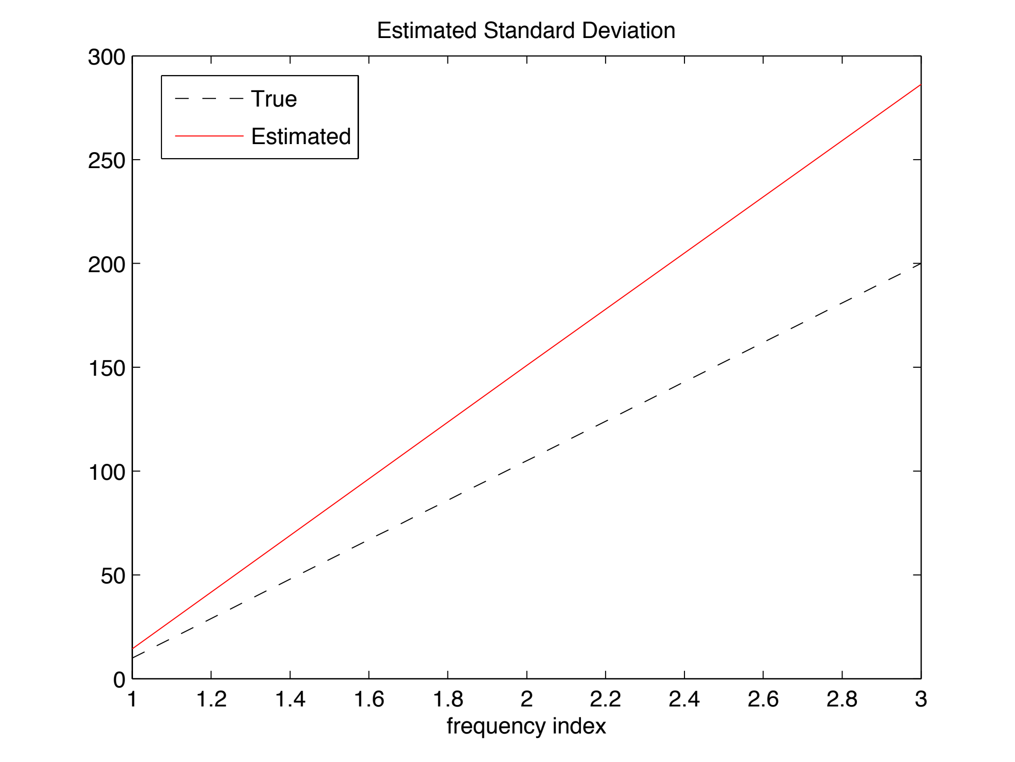


Relative model error

➡ Result improved with variance estimation when the variance **increases** with the frequency

Experiment 2

Estimated Standard Deviation



Estimated Standard Deviation

Experiment 2

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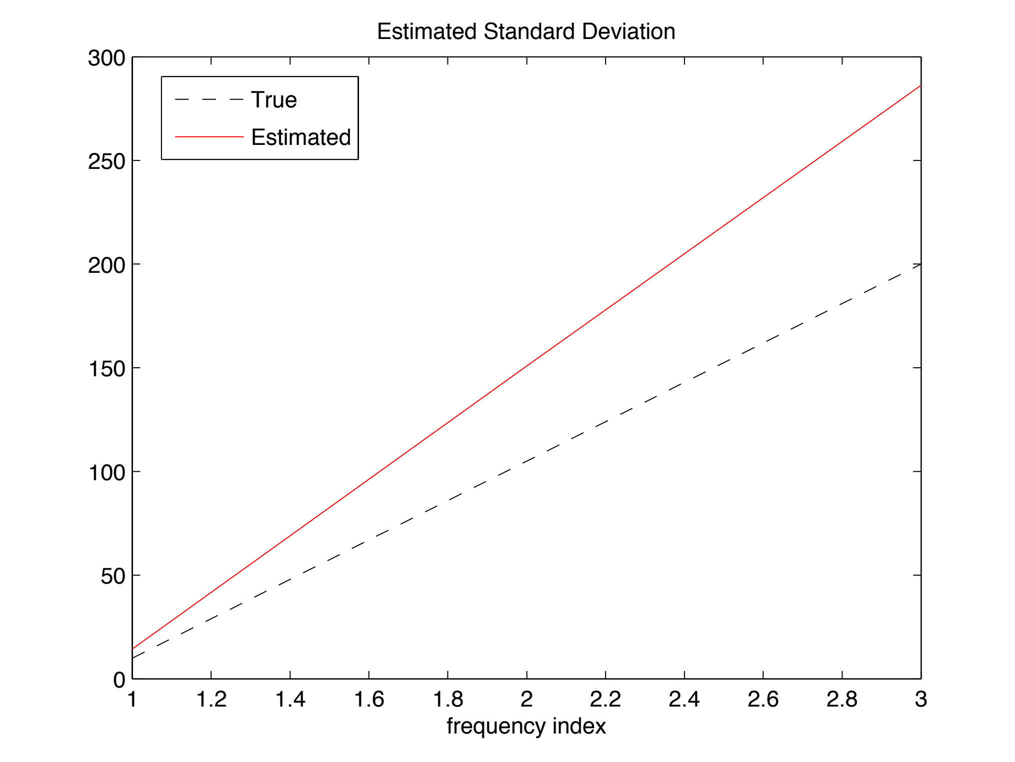

Estimated Standard Deviation

➡ Thanks to variance estimation, we are able to know the **nature** of the noise

Conclusion

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- **Estimation** of the variance **on the fly**, while solving the overall inverse problem

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- Ability to **easily modify algorithms** solving the inverse problem and **add variance estimation**

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- Ability to know the **nature** of the noise

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- **Estimation** of the variance **on the fly**, while solving the overall inverse problem

- Ability to **easily modify algorithms** solving the inverse problem and **add variance estimation**

- Ability to know the **nature** of the noise

- Application to FWI

→ Results **improved** with variance estimation when the variance increases with the frequency in the frequency band

Thank you for your attention!