Fast imaging with surface-related multiples

Ning Tu and Felix J. Herrmann

SLIM
University of British Columbia

Lin, Tu, and Herrmann, 2010; Verschuur and Berkhout, 2011; Whitmore et.al., 2010; Liu et.al., 2011

Motivation

- making use of primaries and multiples *simultaneously*
- *eliminating artifacts* from multiples
- looking for a computationally *efficient* approach

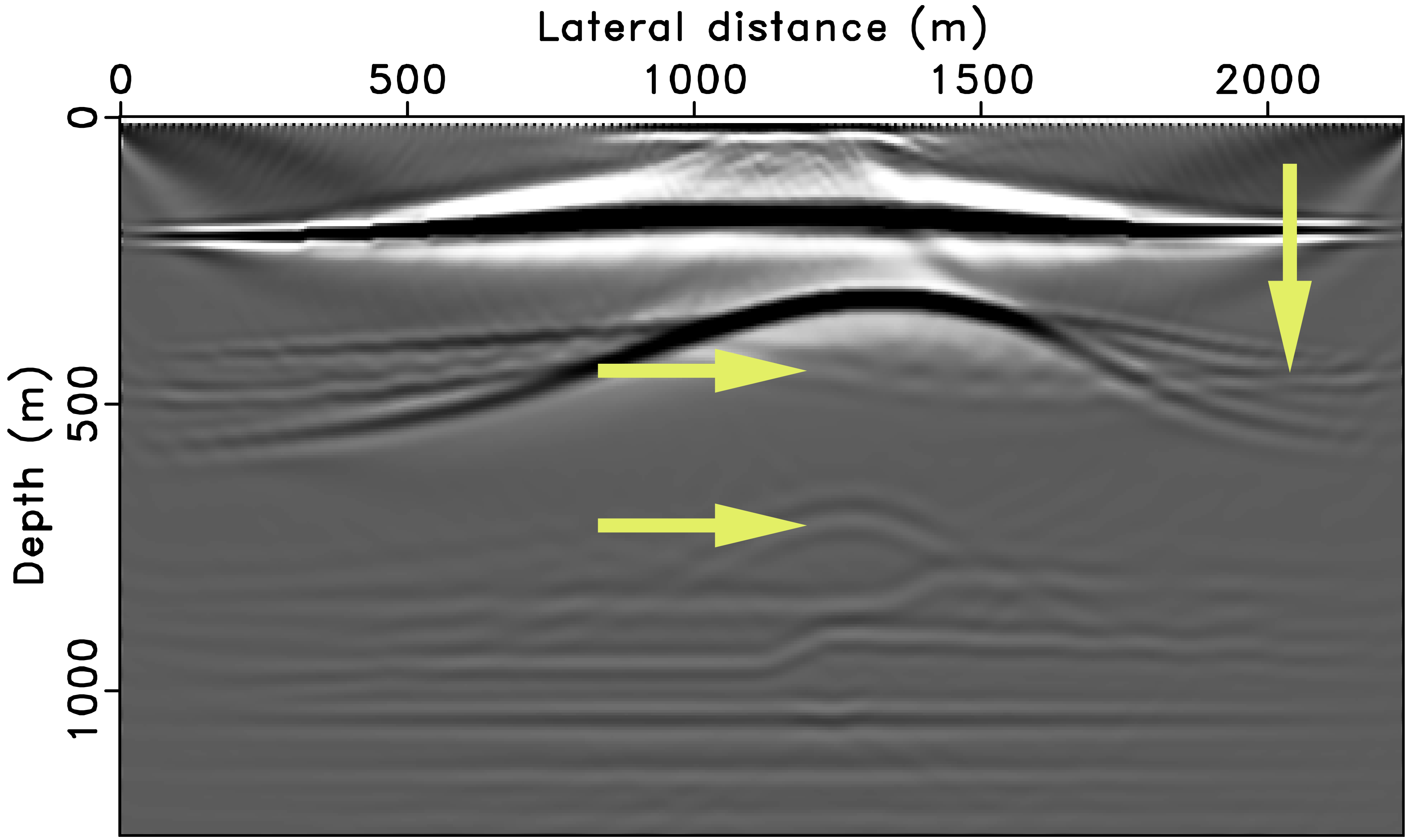
Tu and Herrmann, 2011; Verschuur and Berkhout, 2011; Lu et.al., 2011; Liu et.al., 2011

Primaries & multiples: not ‘or’ but ‘and’

- primaries have a higher signal-to-noise ratio
- multiples can be useful if used correctly
- separating them can be very expensive
- they are not always separable

Conventional RTM image

[use the primary imaging operator]

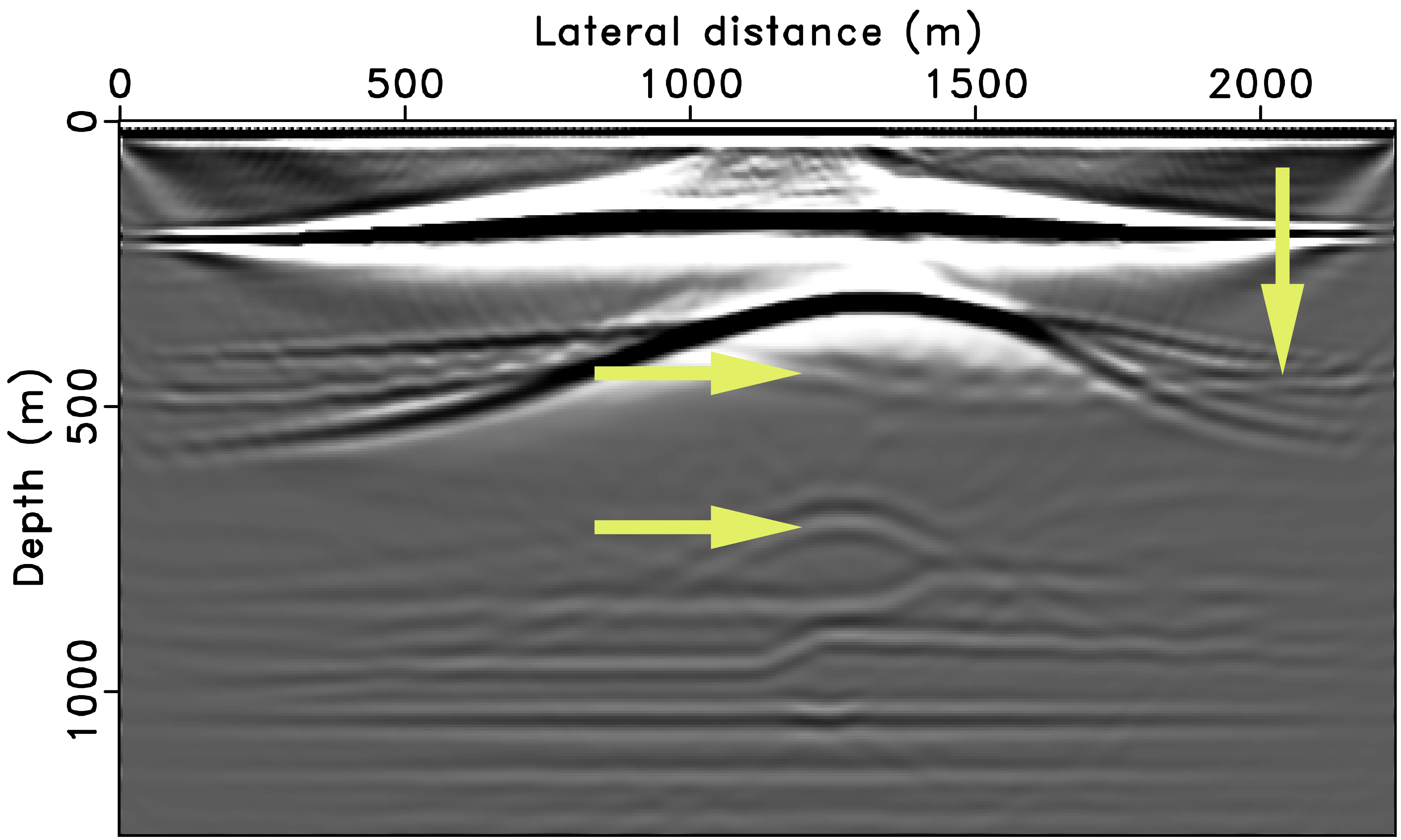


Reverse time migration of *primaries+multiples*, without accounting for multiples

Muijs et al., 2007; Whitmore et.al., 2010; Liu et.al., 2011

RTM of total data

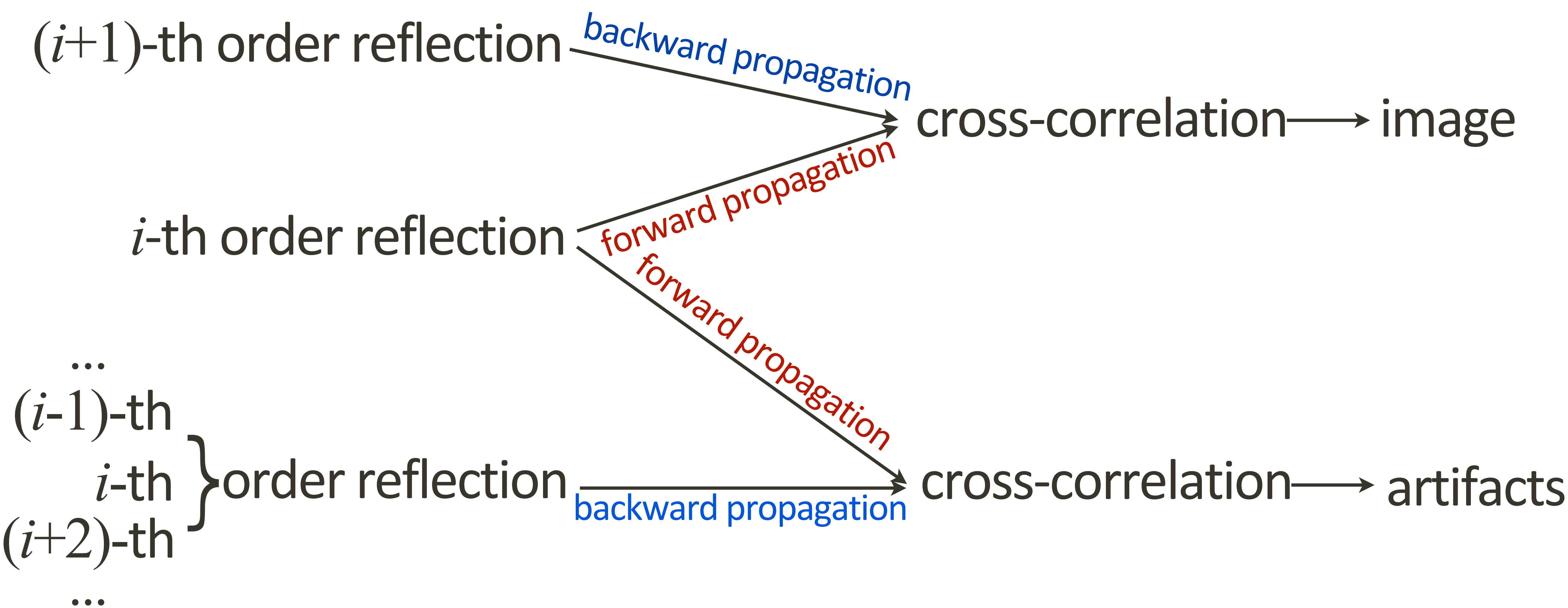
[the imaging operator includes the areal source to account for multiples]



Reverse time migration of *primaries+multiples*, accounting for multiples

Muijs et al., 2007; Liu et.al., 2011

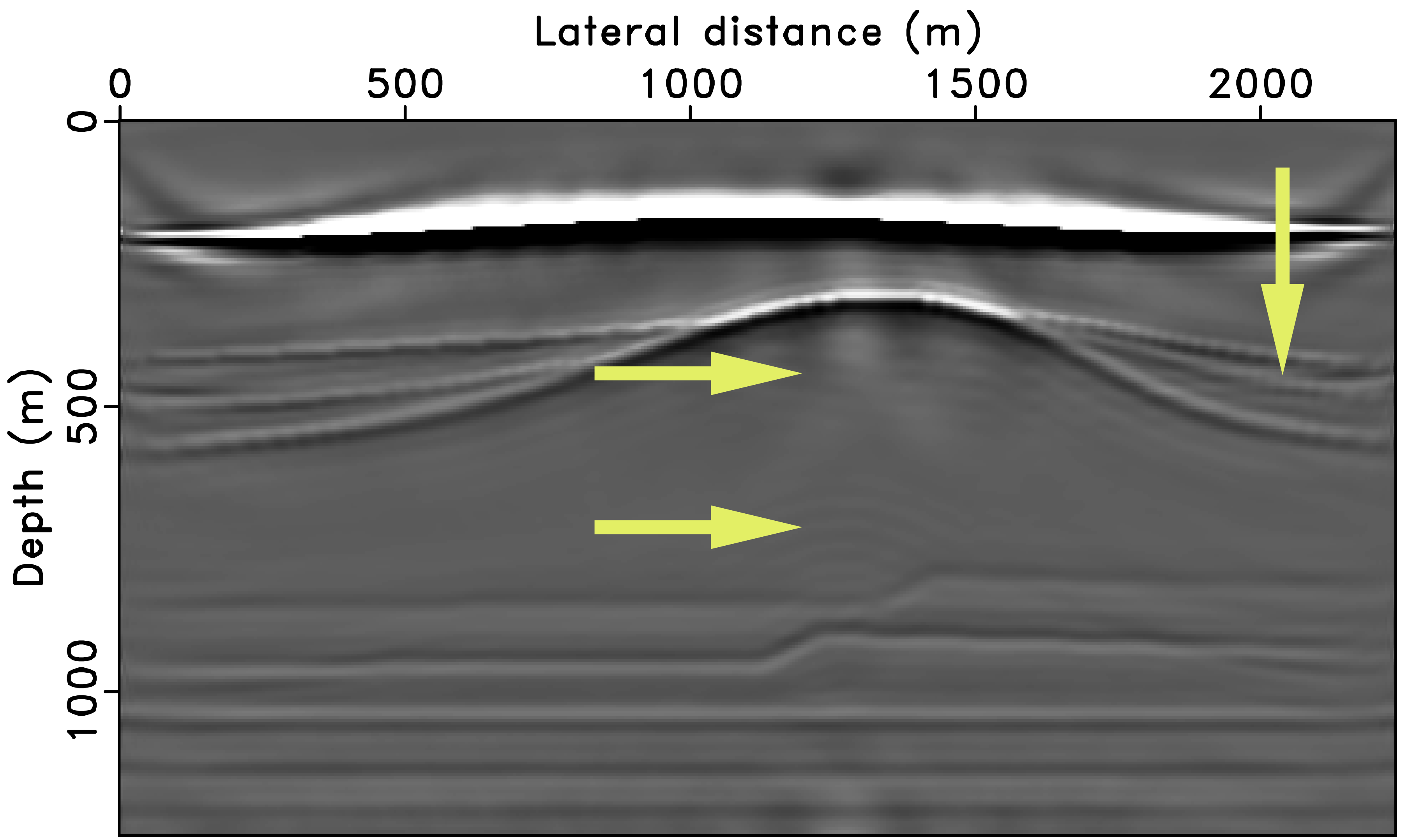
When a free-surface is present



Lin et. al., 2010; Tu and Herrmann, 2011a

SLIM

Artifacts-free image by inversion



Imaging of *primaries+multiples* by *inversion*

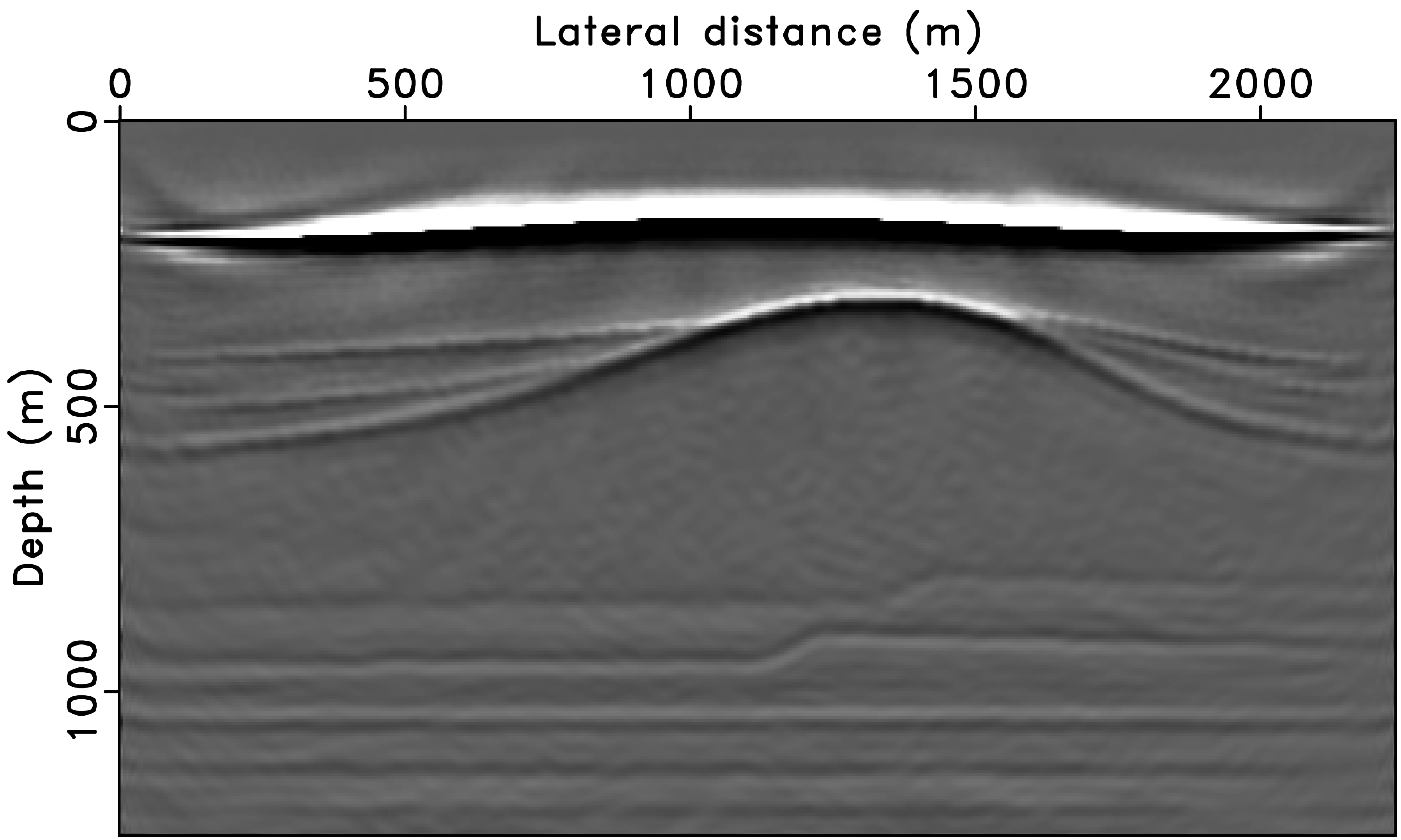
SLIM

Inversion? Sounds expensive...

- repeated evaluations of the Born scattering operator and its adjoint
- each evaluation requires solving 4*(#source)*(#frequencies) PDEs

Sneak peek of our result

[with a 120X speed-up compared to the previous image]



Fast imaging of *primaries+multiples* by sparse inversion

Method

Physics of the free surface

Total data and the surface-free Green's function can be related by the SRME formulation:

$$\mathbf{P}_{\mathrm{i}} = \mathbf{G}_{\mathrm{i}}(\mathbf{Q}_{\mathrm{i}} + \mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}})$$

$\mathbf{P}$: total up-going wavefield
$\mathbf{G}$: surface-free Green's function
$\mathbf{Q}$: source wavelet
$\mathbf{R}$: surface reflectivity

Expressed in model space

$$\begin{aligned}\mathbf{G}_{\mathrm{i}} &= \mathbf{D}_{\mathrm{r}}\mathbf{H}_{\mathrm{i}}^{-\mathbf{1}}[\mathbf{m}](\mathbf{D}_{\mathrm{s}}^{*}\mathbf{I}) - \mathbf{D}_{\mathrm{r}}\mathbf{H}_{\mathrm{i}}^{-\mathbf{1}}[\overline{\mathbf{m}}](\mathbf{D}_{\mathrm{s}}^{*}\mathbf{I}) \\ &\doteq \mathrm{vec}^{-1}(\mathbf{F}_{\mathrm{i}}[\mathbf{m},\mathbf{I}]) - \mathrm{vec}^{-1}(\mathbf{F}_{\mathrm{i}}[\overline{\mathbf{m}},\mathbf{I}])\end{aligned}$$

$\mathbf{F}$: forward modelling operator

$\mathbf{m}$: true model

$\mathbf{I}$: impulsive source array

$\mathbf{D}_{\mathrm{r}}$,$\mathbf{D}_{\mathrm{s}}$: detection operator at receiver/source locations

$\mathbf{H}$: time-harmonic Helmholtz operator

$\overline{\mathbf{m}}$: homogeneous model

Expressed in model space

$$\mathbf{P}_\mathrm{i} = \mathrm{vec}^{-1}(\mathbf{F}_\mathrm{i}[\mathbf{m}, \mathbf{I}])(\mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) - \mathrm{vec}^{-1}(\mathbf{F}_\mathrm{i}[\overline{\mathbf{m}}, \mathbf{I}])(\mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i})$$

$$\begin{aligned} & \mathrm{vec}^{-1}(\mathbf{F}_\mathrm{i}[\mathbf{m}, \mathbf{I}])(\mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) \\ &= \mathbf{D}_\mathrm{r}\mathbf{H}_\mathrm{i}^{-1}[\mathbf{m}](\mathbf{D}_\mathrm{s}^{*}\mathbf{I})(\mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) \\ &= \mathbf{D}_\mathrm{r}\mathbf{H}_\mathrm{i}^{-1}[\mathbf{m}](\mathbf{D}_\mathrm{s}^{*}(\mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i})) \\ &\doteq \mathrm{vec}^{-1}(\mathbf{F}_\mathrm{i}[\mathbf{m}, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}]) \end{aligned}$$

$$\mathbf{P}_\mathrm{i} = \mathrm{vec}^{-1}(\mathbf{F}_\mathrm{i}[\mathbf{m}, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}]) - \mathrm{vec}^{-1}(\mathbf{F}_\mathrm{i}[\overline{\mathbf{m}}, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}])$$

Linearized forward modelling
[monochromatic]

$$\mathbf{p}_{\mathrm{i}} = \boxed{\nabla \mathbf{F}_{\mathrm{i}}[\mathbf{m}_0, \mathbf{Q}_{\mathrm{i}} + \mathbf{R}_{\mathrm{i}} \mathbf{P}_{\mathrm{i}}] \delta \mathbf{m}}$$ + higher order reflections

$\nabla \mathbf{F}$: Born scattering operator

$\mathbf{m_0}$: background model

$\delta \mathbf{m}$: model perturbation

$\mathbf{p}$: vectorized wavefield *difference*

Linearized forward modelling
[*all* frequencies]

$$\mathbf{p} \approx \begin{bmatrix} \nabla\mathbf{F}_1(\mathbf{m}_0, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) \\ \vdots \\ \nabla\mathbf{F}_\mathrm{nf}(\mathbf{m}_0, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) \end{bmatrix} \delta\mathbf{m}$$

$$\doteq \nabla\mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\delta\mathbf{m}$$

$$\Downarrow$$

$$\delta\mathbf{m} \approx \nabla\mathbf{F}^\dagger[\mathbf{m}_0, \mathbf{Q} + \mathbf{RP}]\mathbf{p}$$

Sparse inversion

We use a sparsity-promoting formulation:

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \operatorname*{argmin}_{\delta\mathbf{x}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\mathbf{p} - \nabla\mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

$\mathbf{C}$: curvelet transform

solver: $\mathrm{SPG}\ell_1$

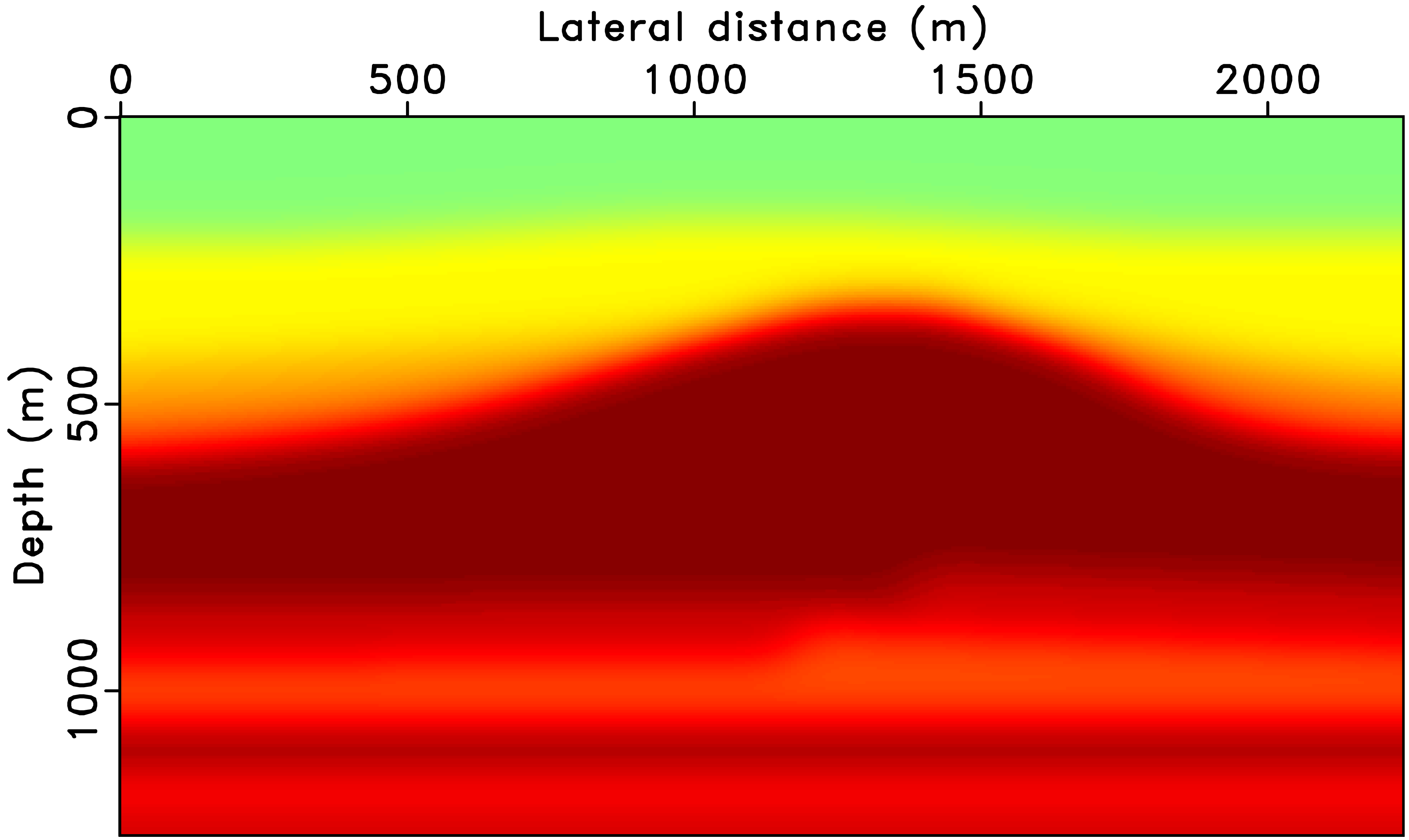
Example using a simple model

- model grid spacing: 5 meters
- using linearized data including surface related multiples:

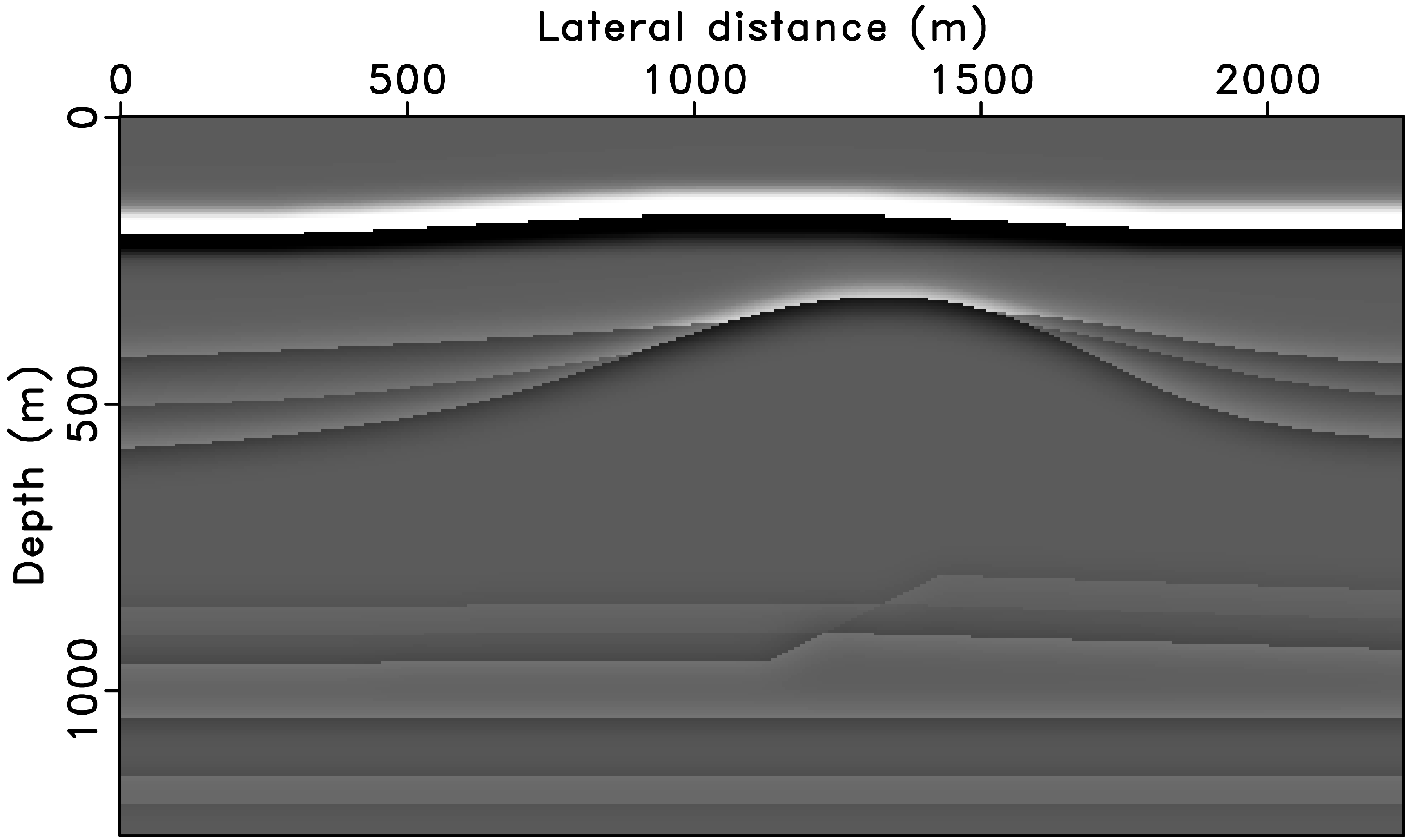
$$\nabla \mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\delta \mathbf{m}$$

- 150 collocated sources/receivers
- 122 frequencies in 0-60Hz range

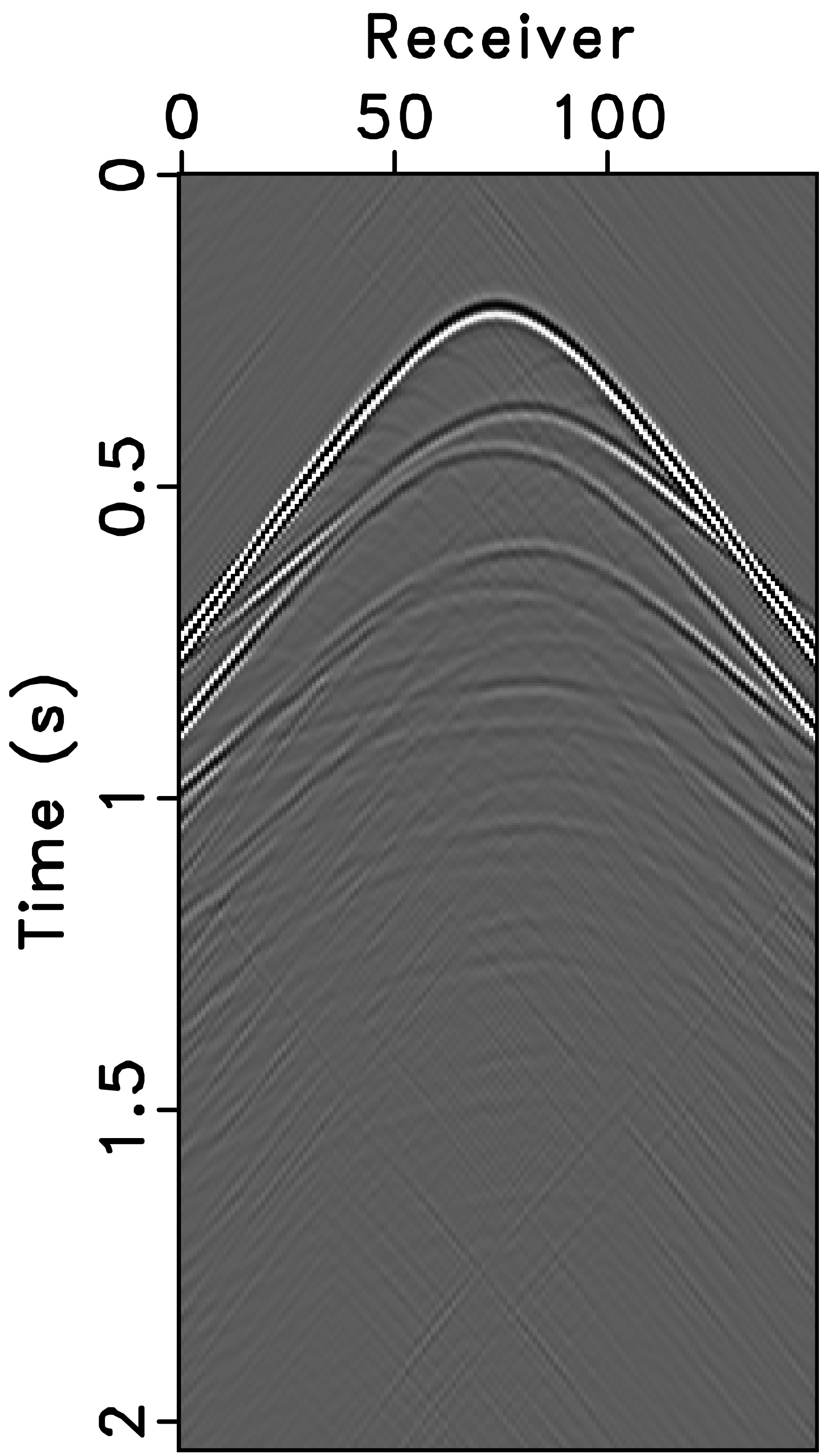
Background velocity model



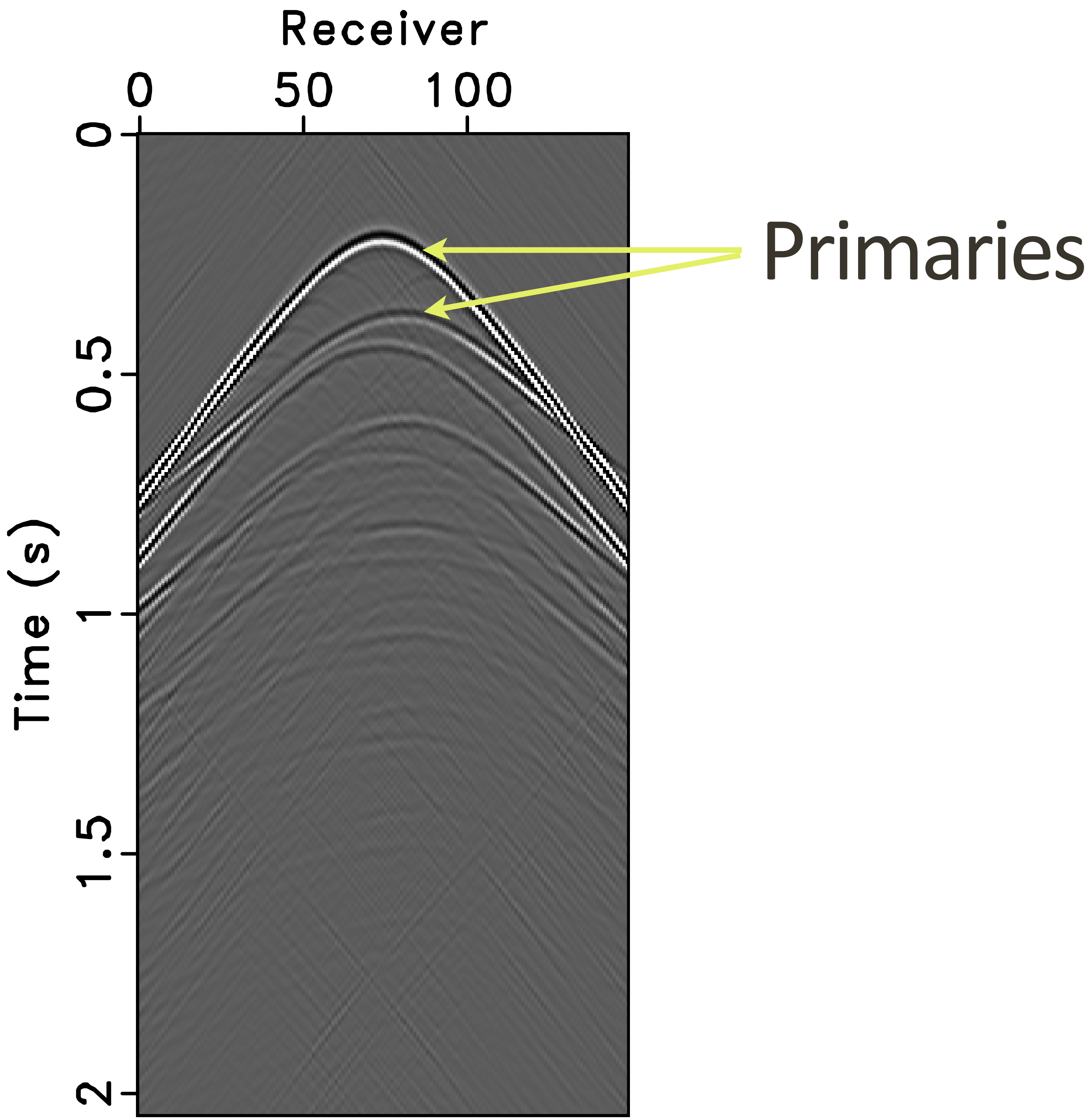
True perturbation



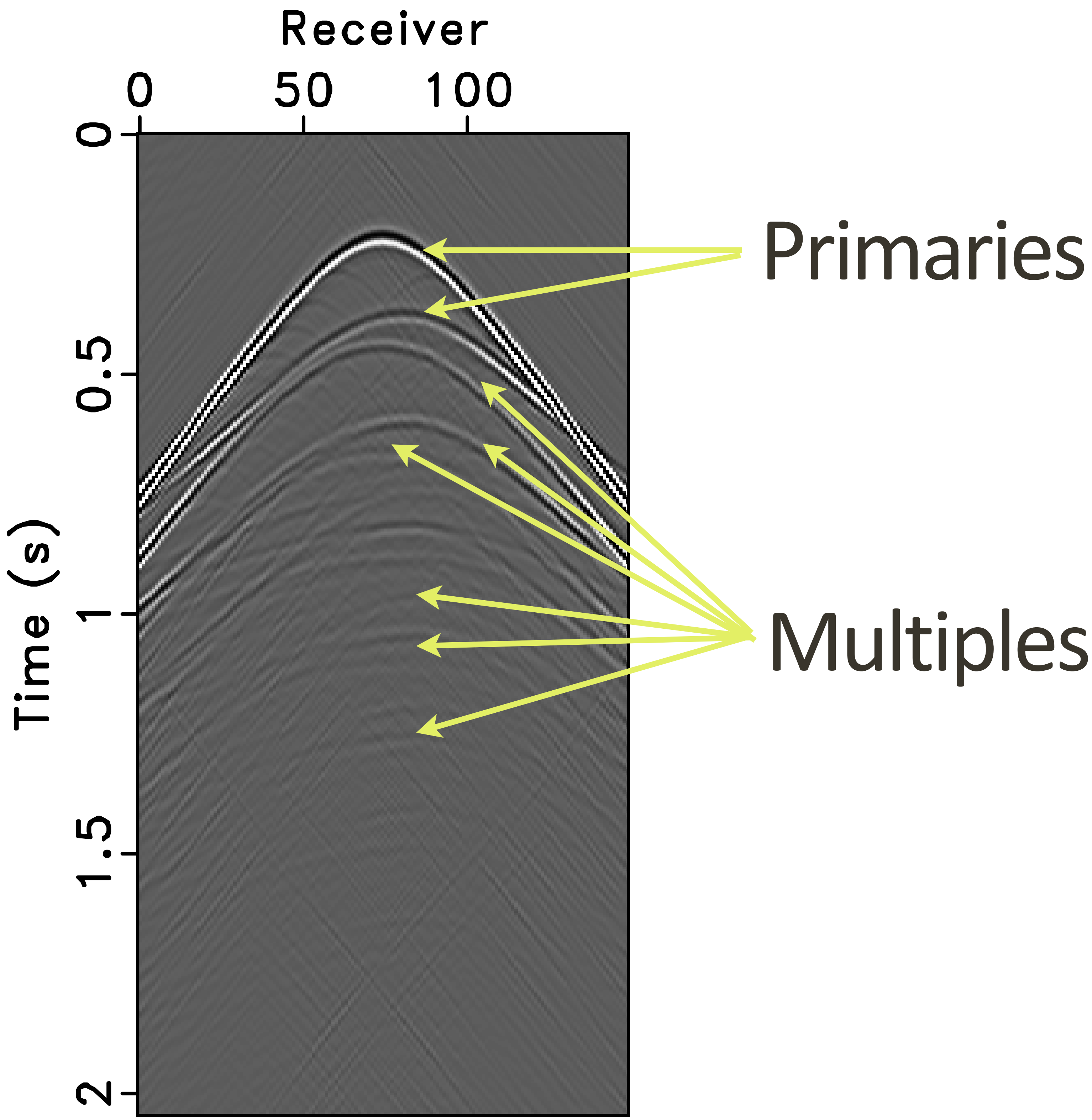
Linearized total data



Linearized total data

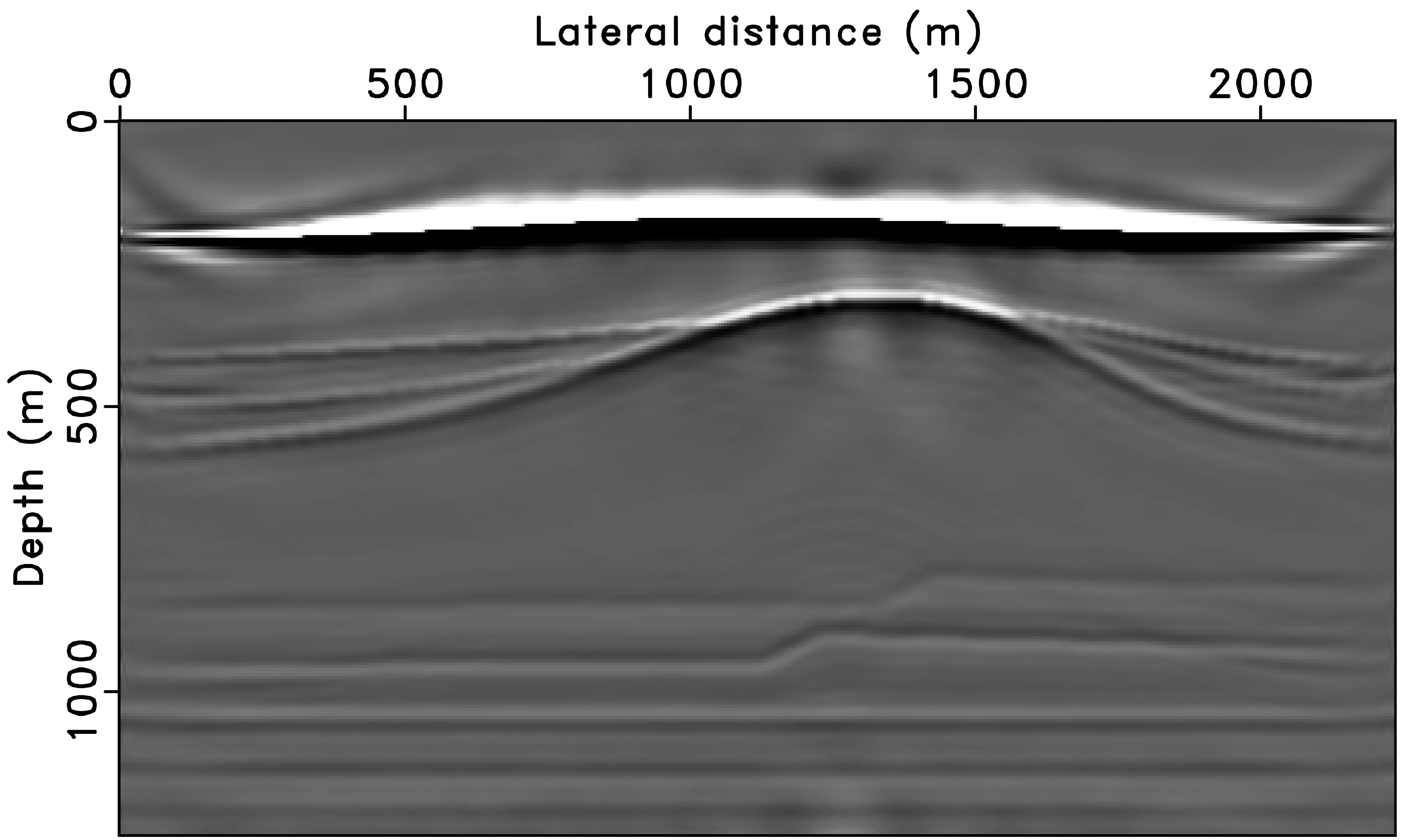


Linearized total data



Inversion of total data

[by computing the inverse of the Born scattering operator]



Inversion of the total up-going wavefield using all sequential sources and all frequencies
number of PDE solves: ~4.4 million (by calculation)

Speed up inversion

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1$$

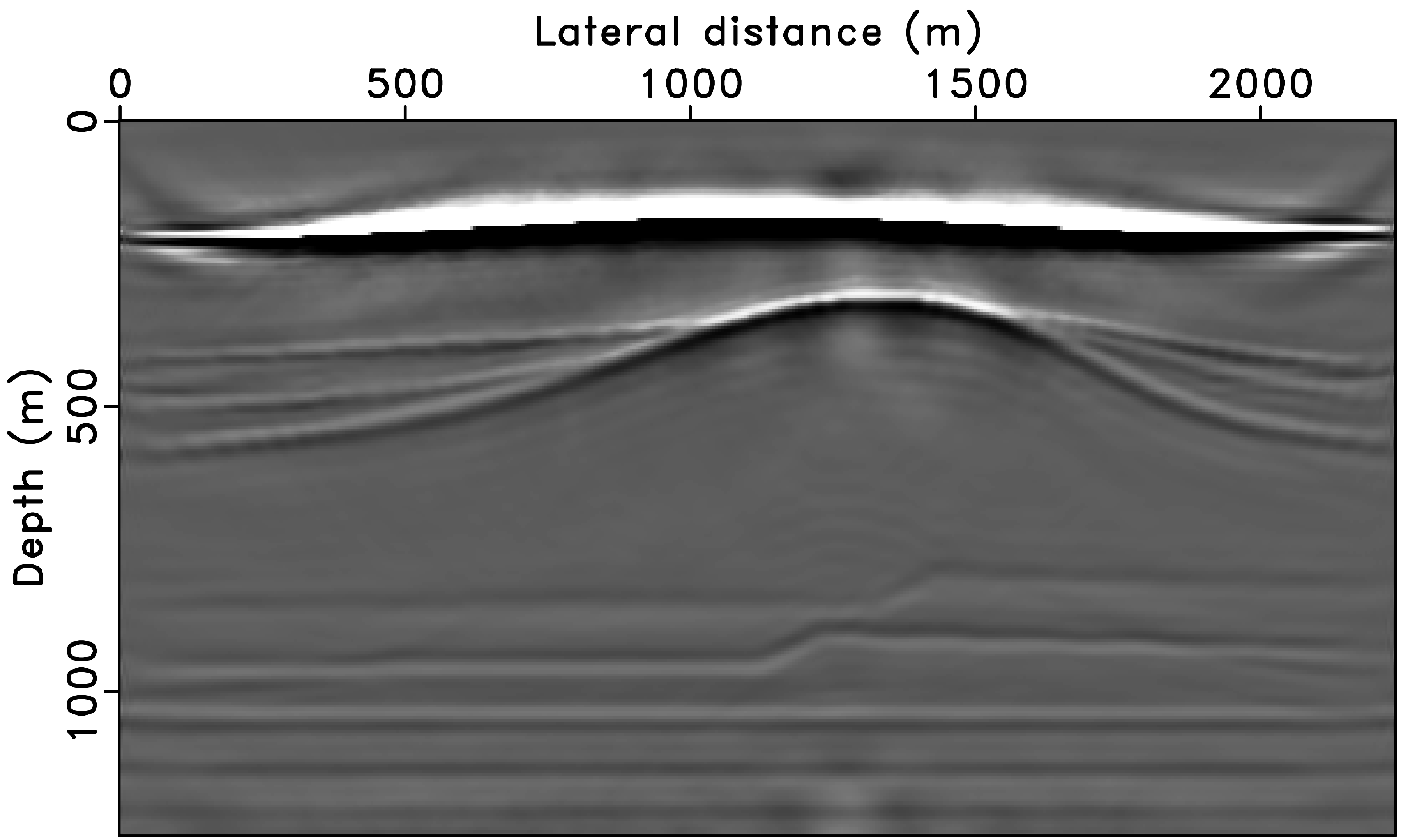
$$\text{subject to } ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

source: combine all sequential sources into a few simultaneous sources

frequency: randomly choose a subset from all of them

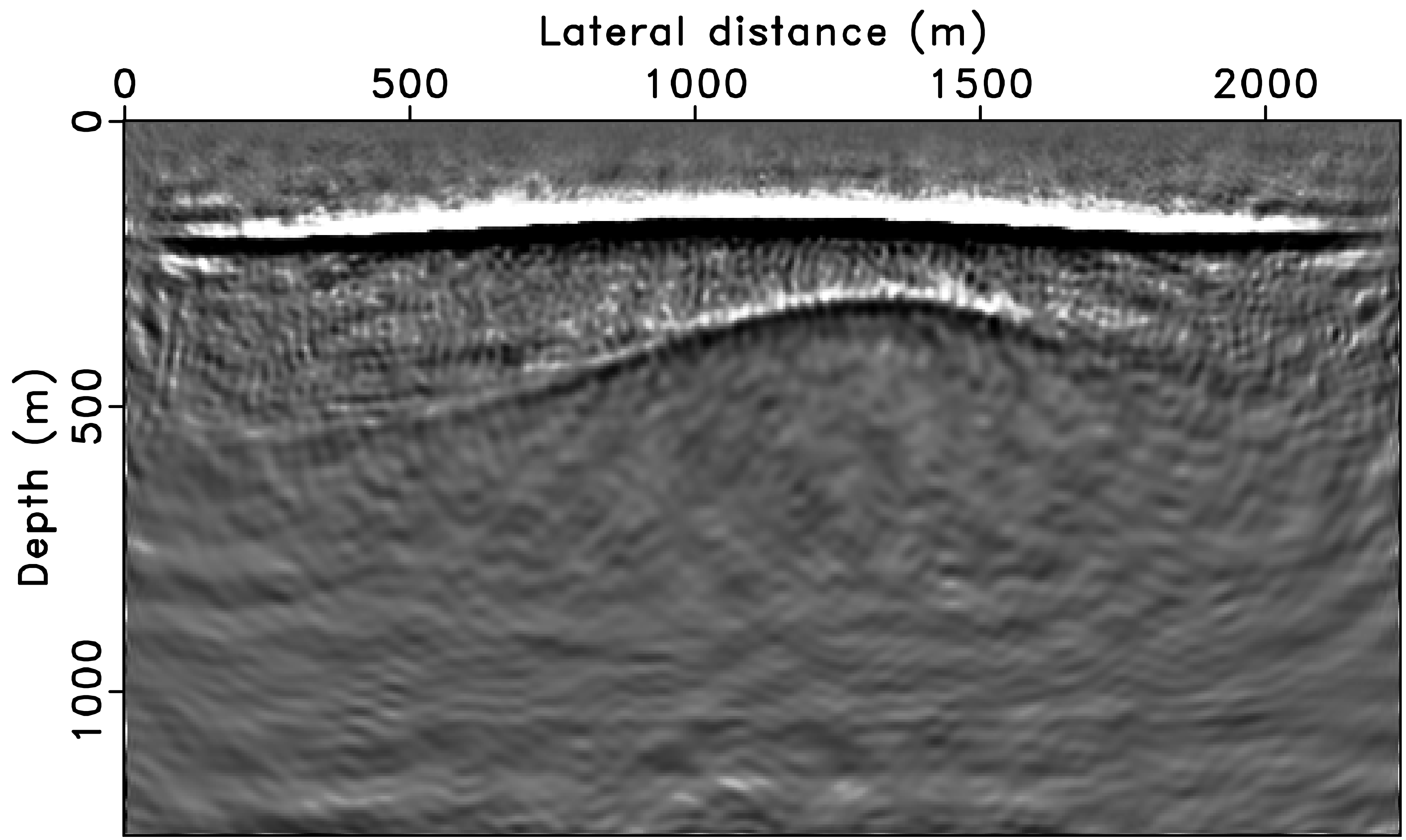
SLIM

Result with 15x speed-up



Inversion of the total up-going wavefield using 10 simultaneous sources and all frequencies
number of PDE solves: ~0.3 million[15X speed-up]

Too much subsampling brings artifacts [120X speed-up]



Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand [120X speed-up]

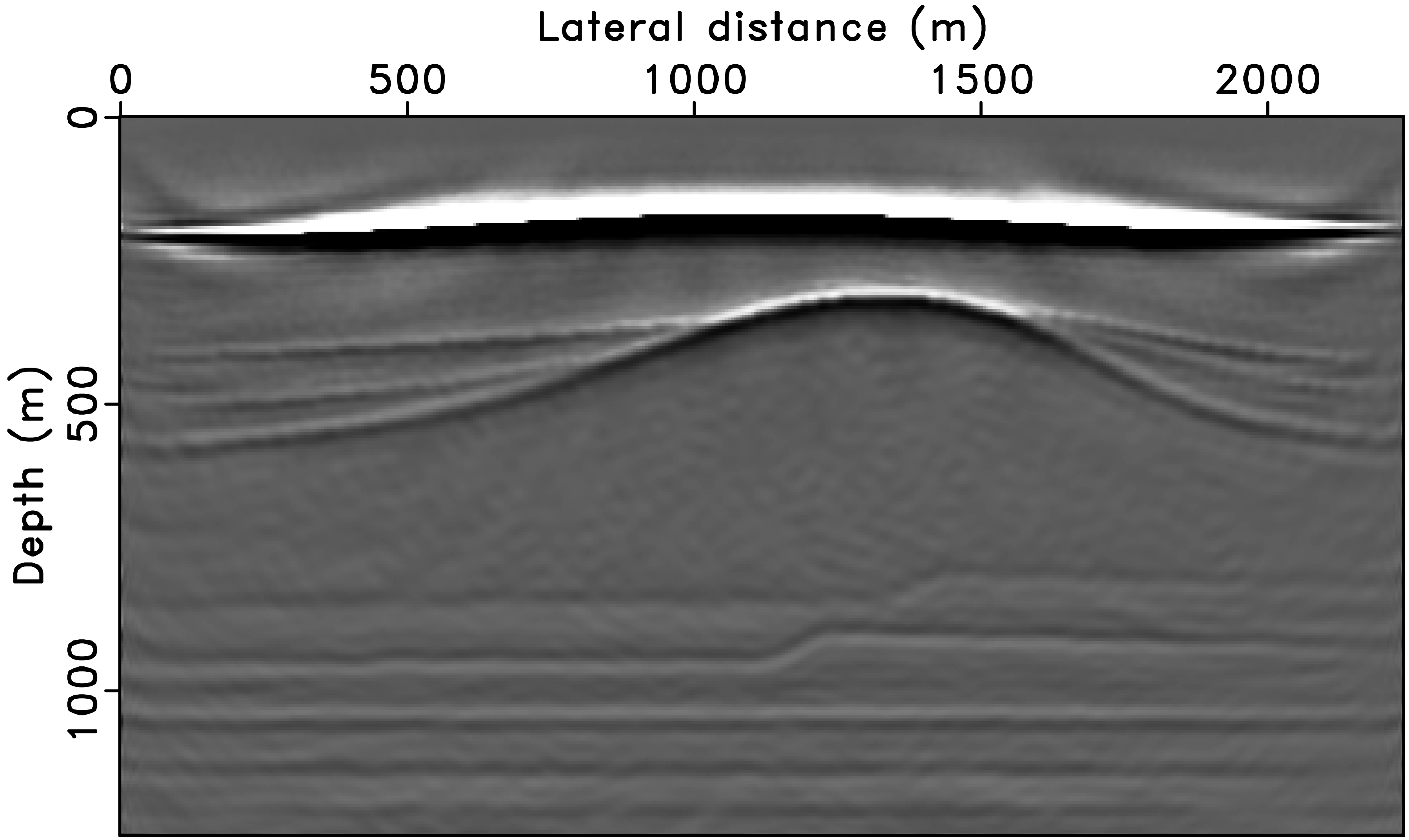
Rerandomization

- $\mathrm{SPG}\ell_1$ solves a series of subproblems:

$$\underset{\delta\mathbf{x}}{\mathrm{argmin}} \, ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2$$
$$\text{subject to } ||\delta\mathbf{x}||_1 \leq \tau$$

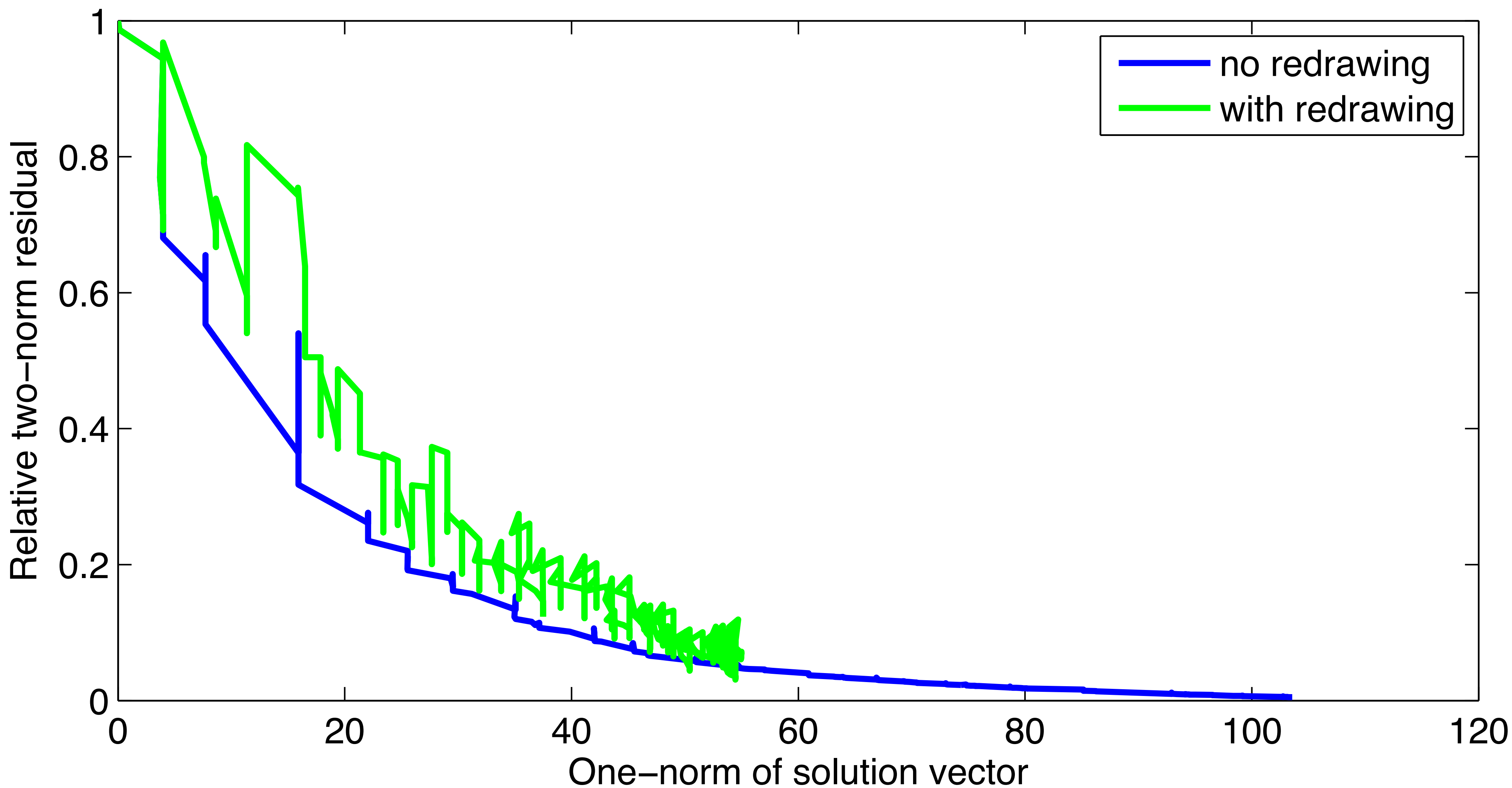
- redraw subsampling operator for each new subproblem
- motivated by insights from approximate message passing

Redraw sim. sources and frequencies [120X speed-up]

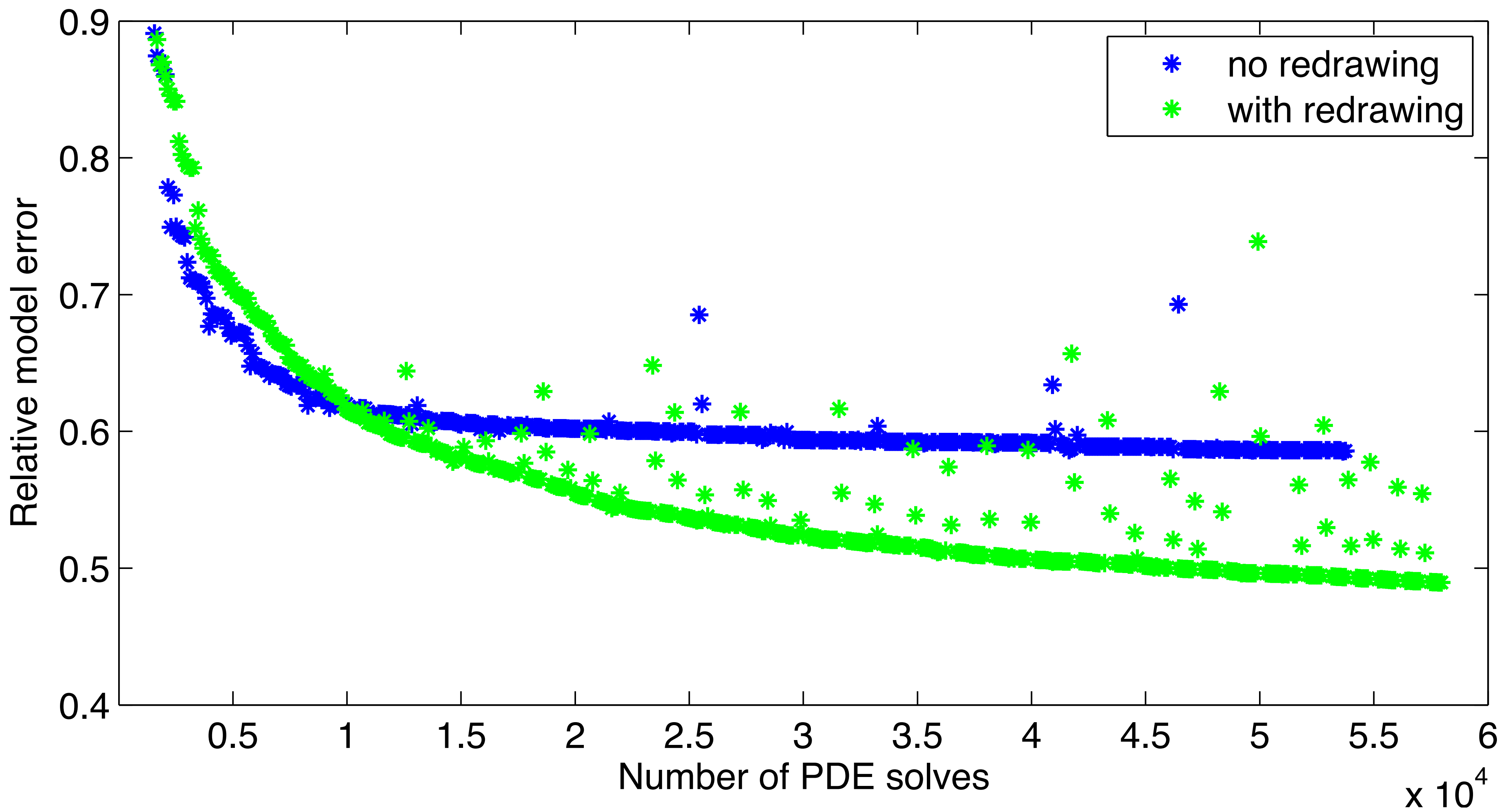


Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand (by calculation)

Solution path



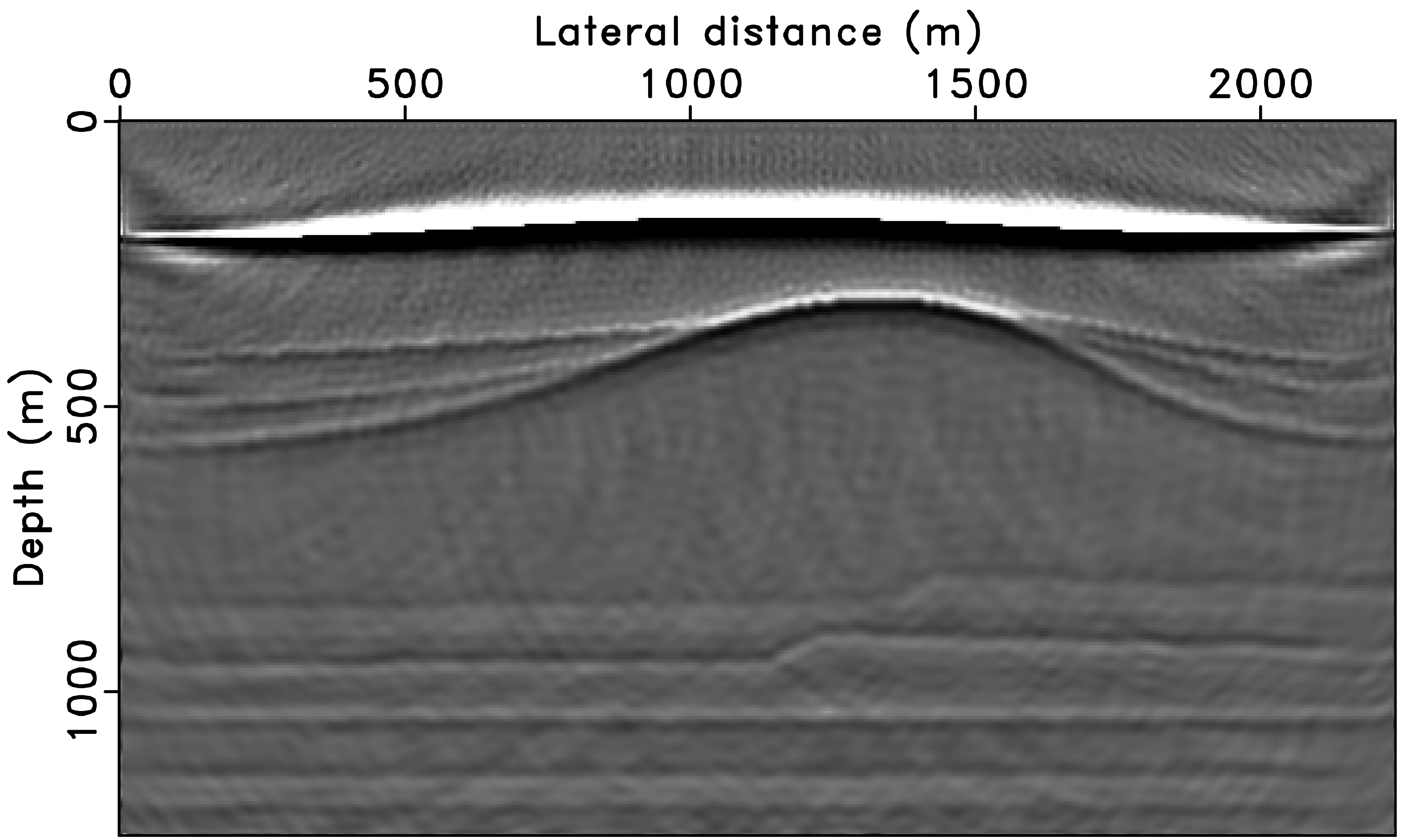
Model error decrease



Note: outliers are intermediate line-search results, not a concern; number of PDE solves in practice has ~50% overhead due to line search, etc.

L1 vs. L2 minimization

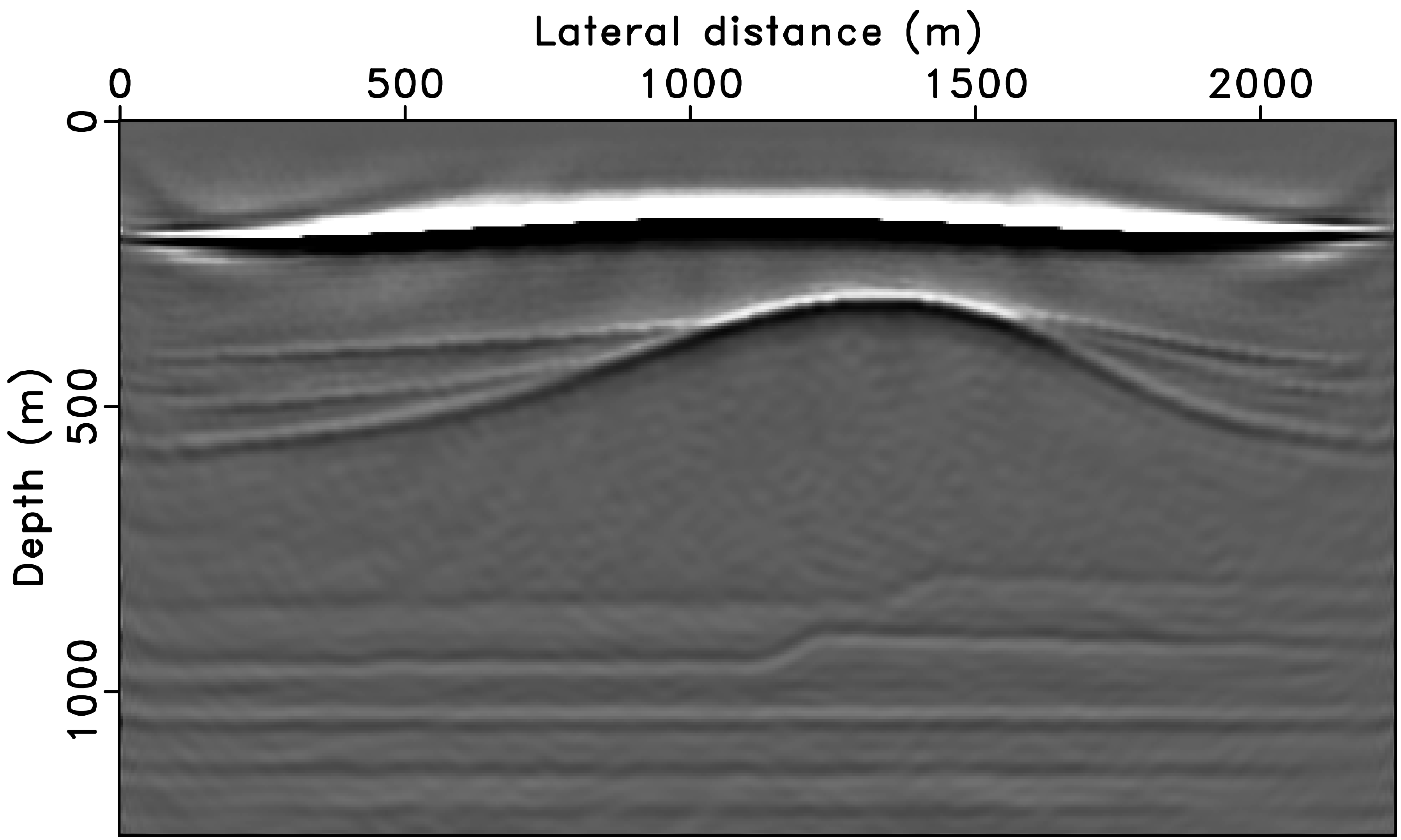
[with rerandomization]



By **L2** minimization: $\delta\tilde{\mathbf{m}} = \mathbf{C}^H \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_2$

$$\text{subject to } ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

L1 vs. L2 minimization
[both with rerandomization]



By **L1** minimization: $\delta\tilde{\mathbf{m}} = \mathbf{C}^H \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1$

$$\text{subject to } ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

Choosing number of sim. sources and frequencies
[fixed number of PDE solves]

batch size (#src*#freq.)	15	30	60	120	240	480
#iteration	610	305	152	76	39	20
operator aspect ratio	0.02	0.04	0.08	0.16	0.32	0.64
Result SNR (dB)	6.5	6.2	5.8	5.1	4.2	3.4

Choosing number of sim. sources and frequencies
[fixed number of PDE solves]

batch size (#src*#freq.)	15	30	60	120	240	480
#iteration	610	305	152	76	39	20
operator aspect ratio	0.02	0.04	0.08	0.16	0.32	0.64
Result SNR (dB)	6.5	6.2	5.8	5.1	4.2	3.4

Cheaper and more iterations win!

Choosing number of sim. sources and frequencies
[fixed number of PDE solves]

#sim. sources	2	3	6	10	15
#frequencies	15	10	5	3	2
Result SNR (dB)	6.2	6.2	6.2	6.3	6.7

Choosing number of sim. sources and frequencies

[fixed number of PDE solves]

#sim. sources	2	3	6	10	15
#frequencies	15	10	5	3	2
Result SNR (dB)	6.2	6.2	6.2	6.3	6.7

You have the freedom to choose #src & # freq once you choose a batch size.

Synthetic case study

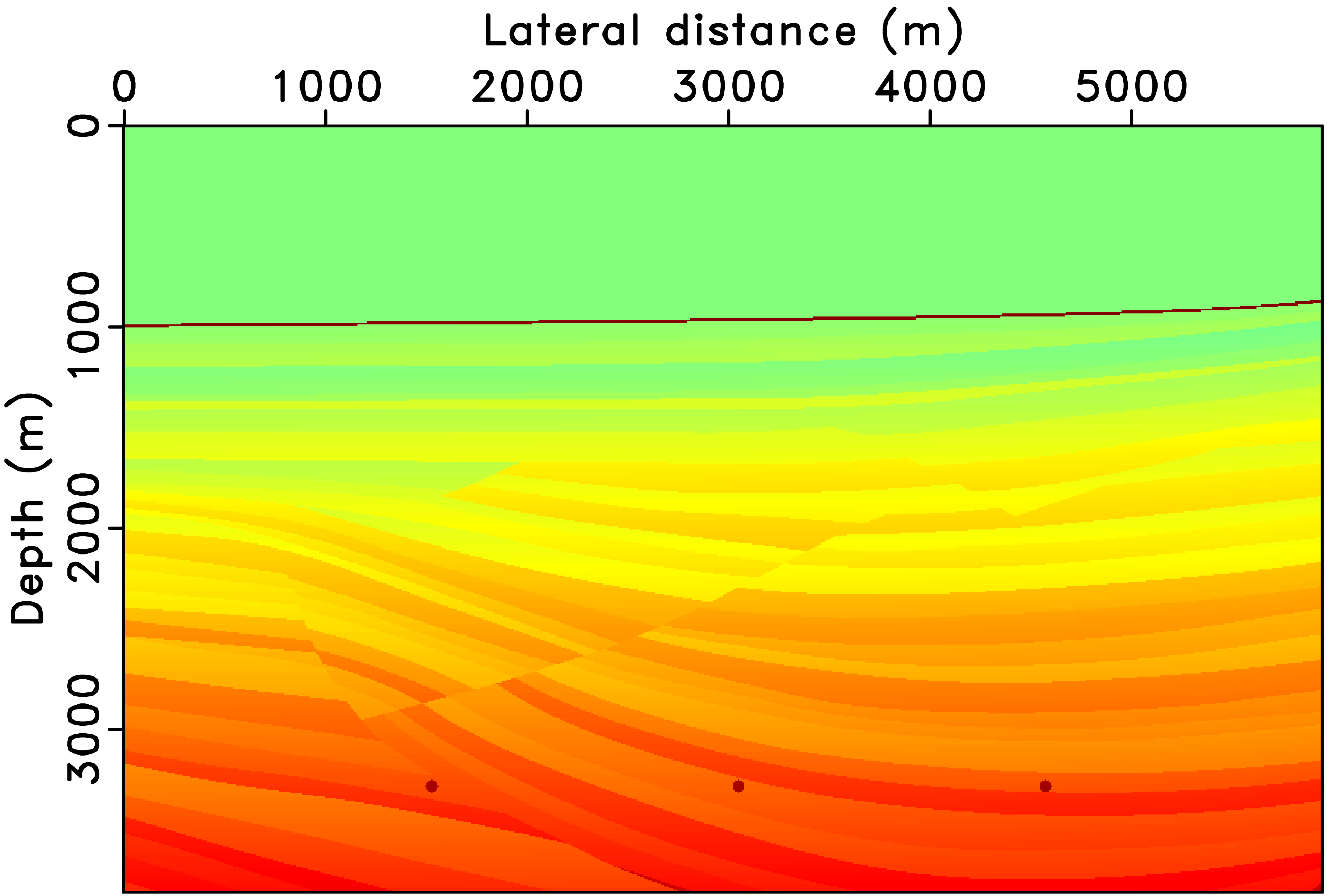
SLIM

Using a complex model

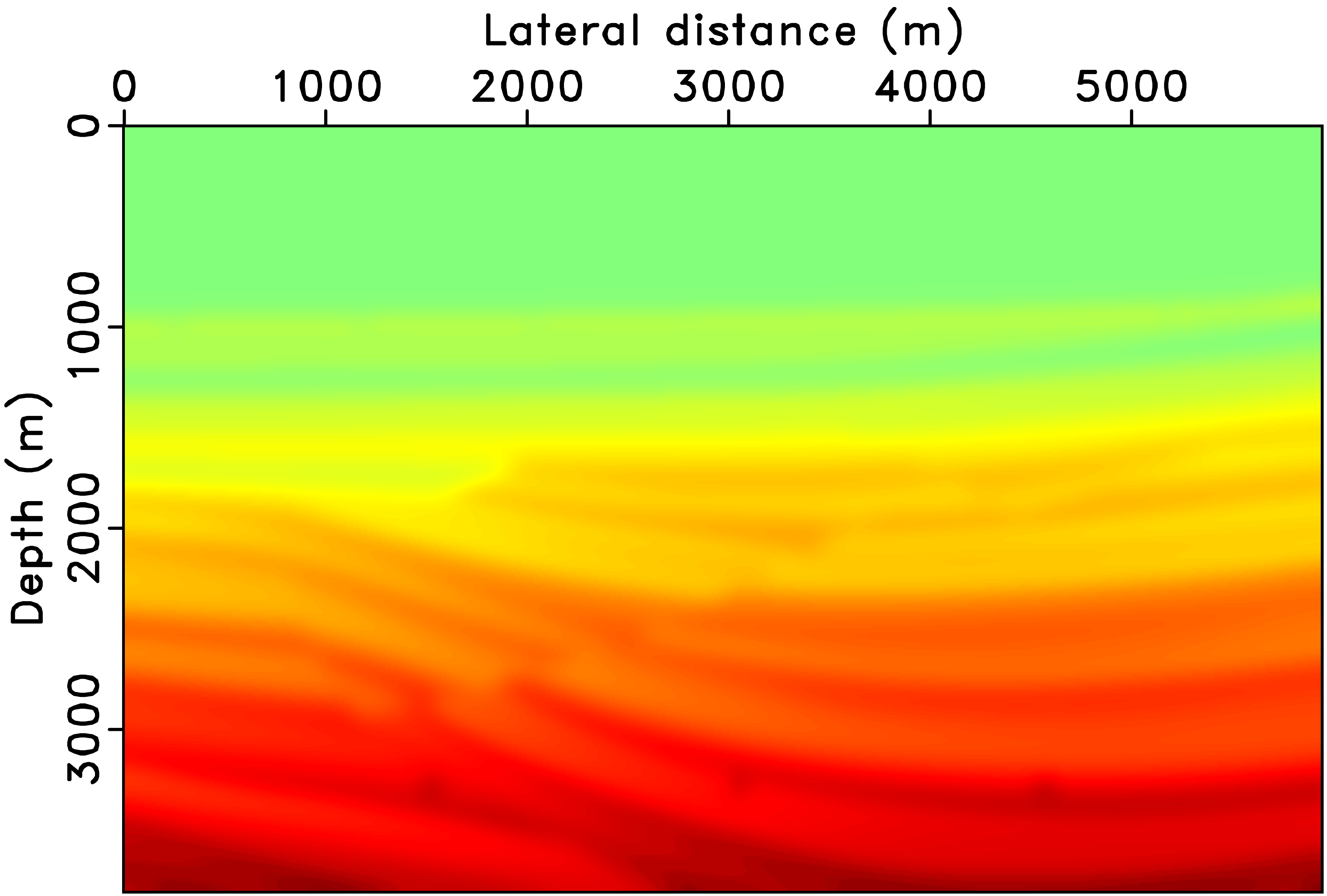
[cropped from the Sigsbee 2B model]

- model grid spacing: 7.62m
- using linearized data
- 261 sequential sources
- ~8s recording time, 278 frequencies in 0-34Hz range
- using 8 simultaneous sources and 15 frequencies with rerandomization

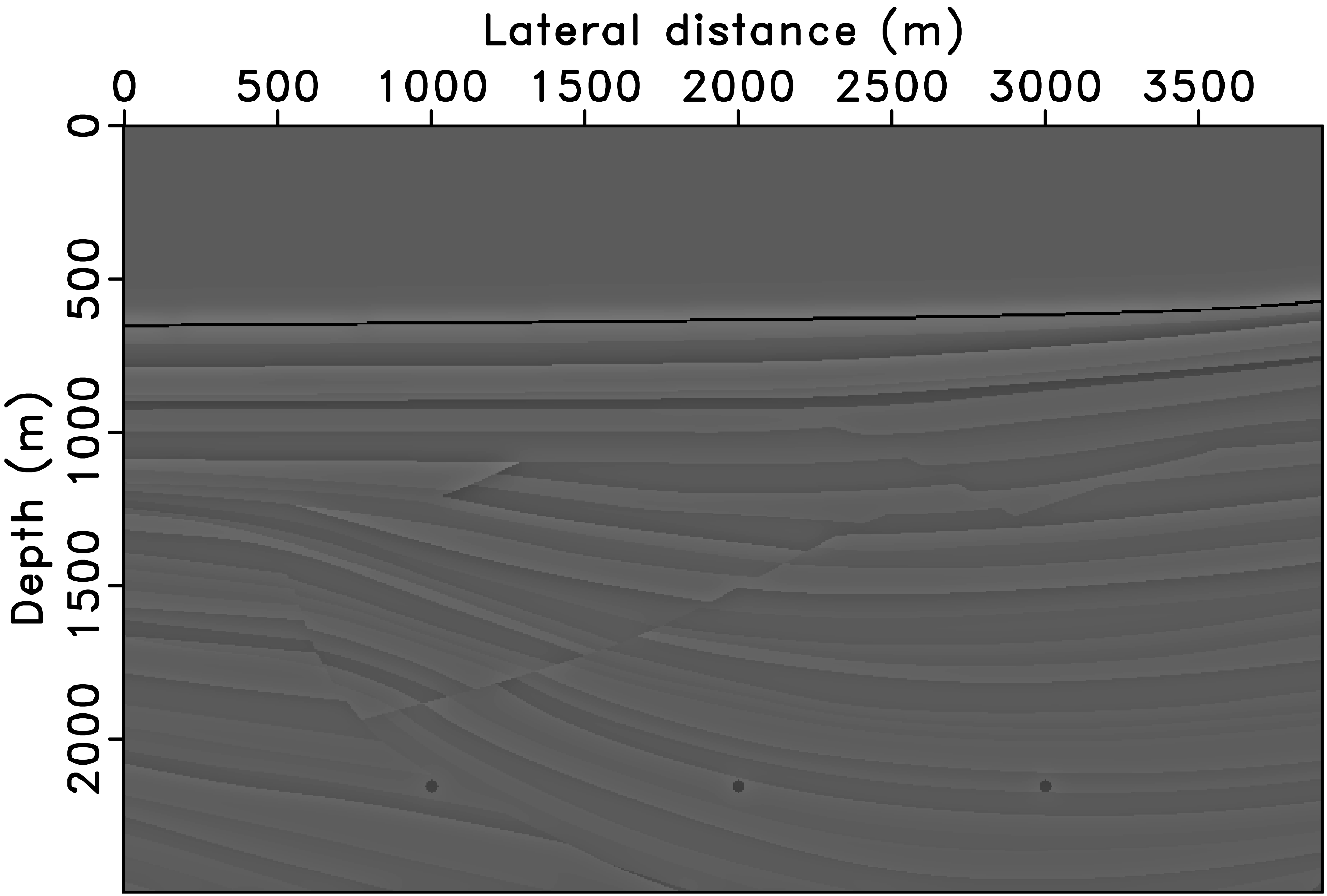
The true velocity model



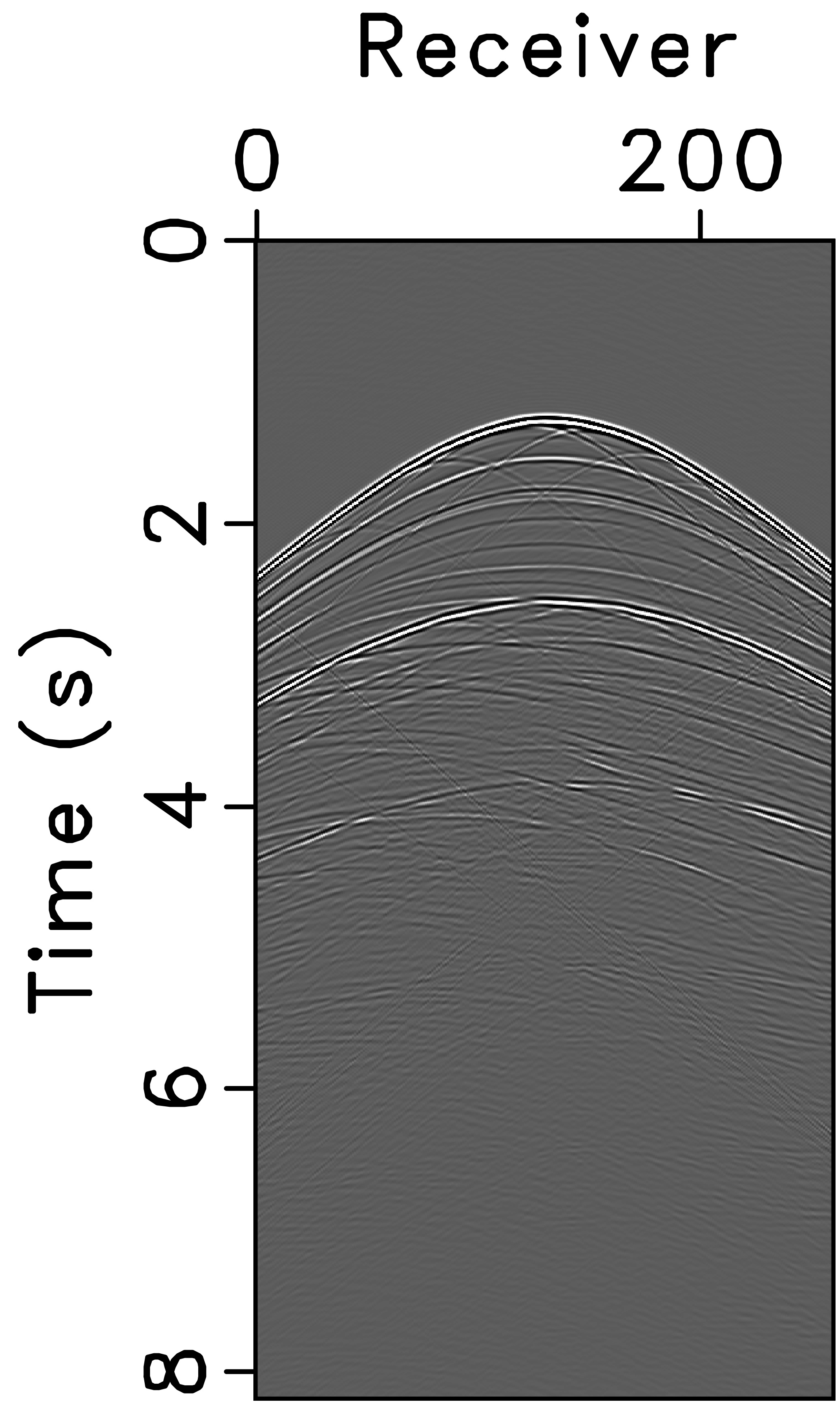
Background velocity model



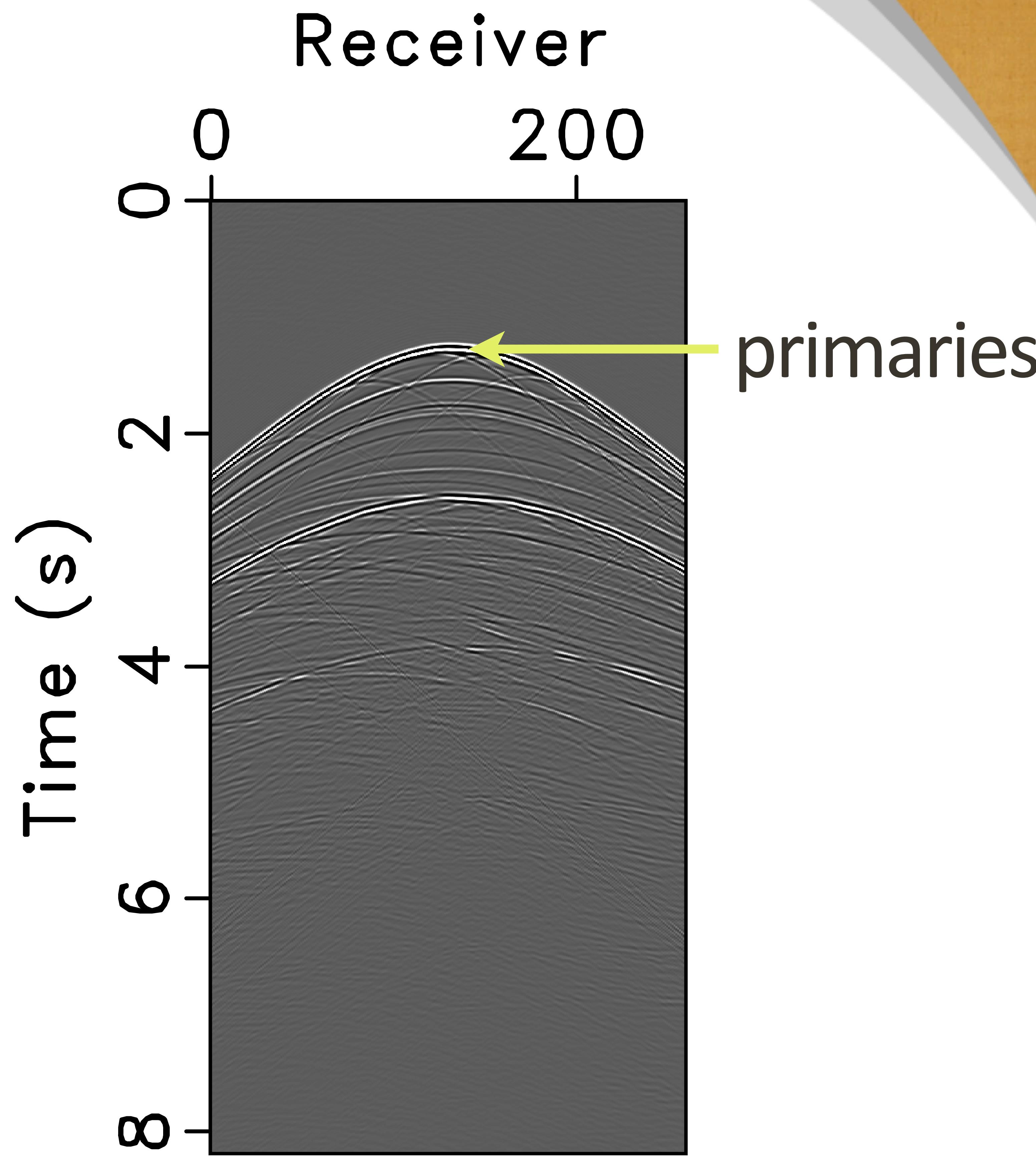
True perturbation



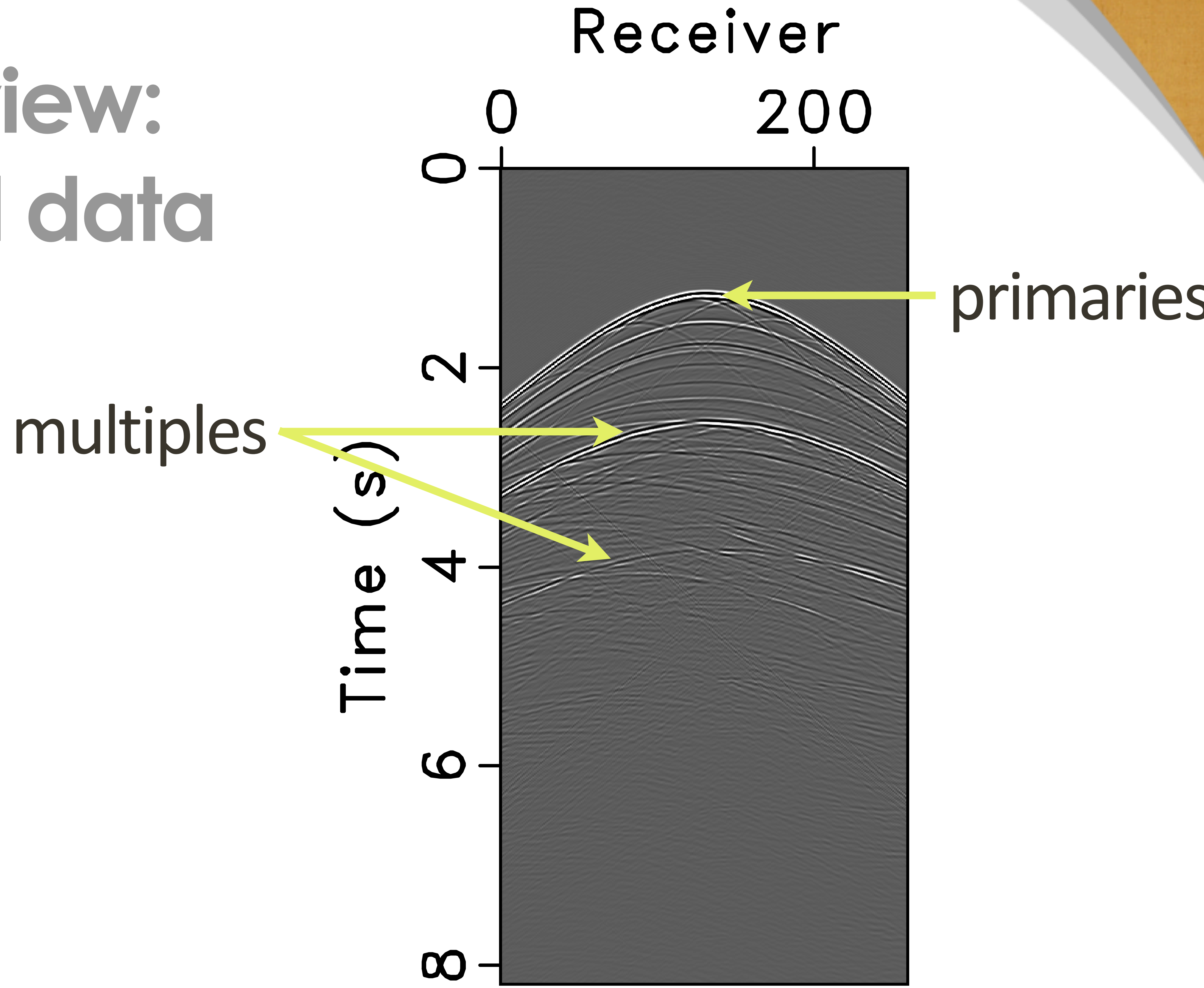
Preview: total data



Preview: total data

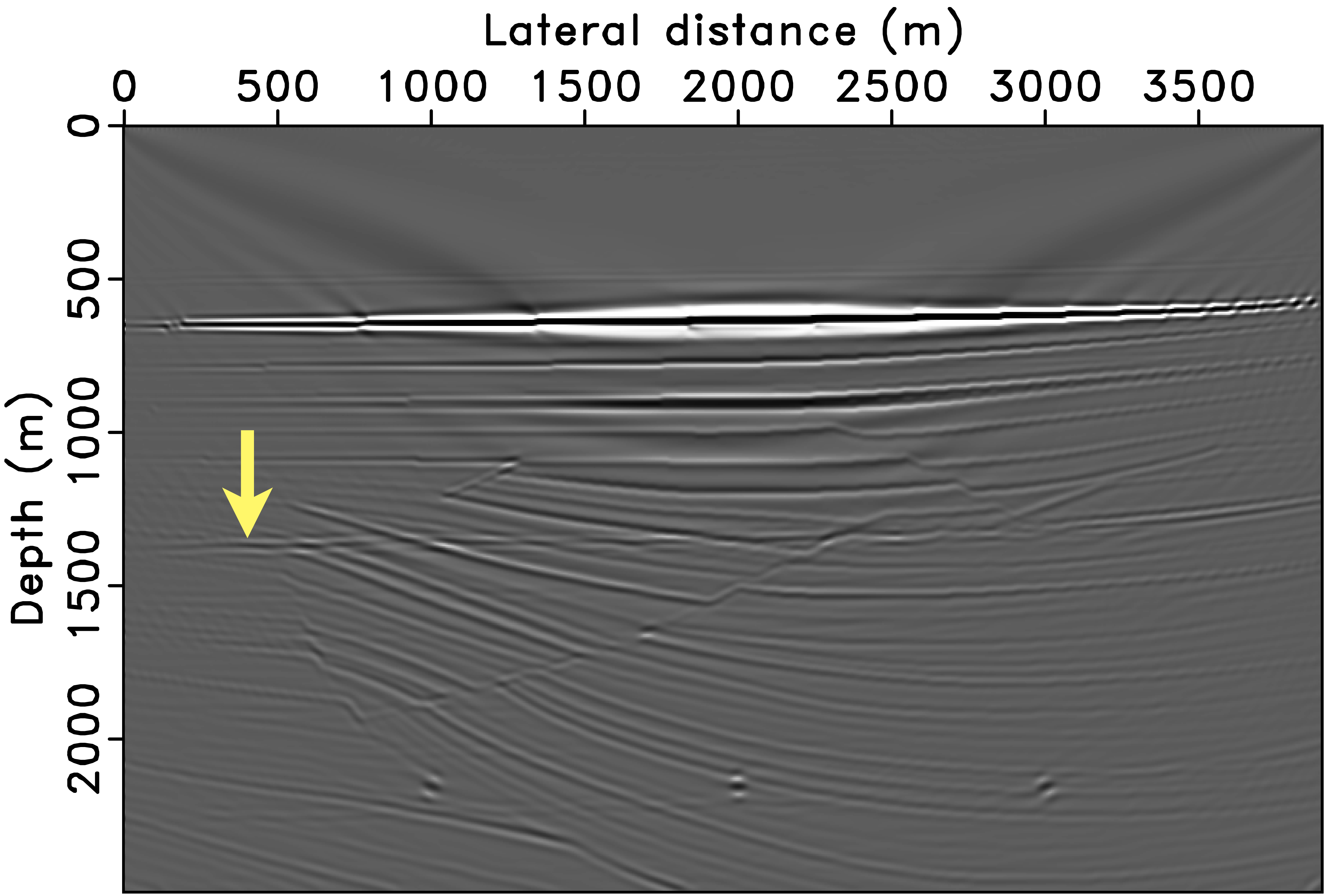


Preview: total data



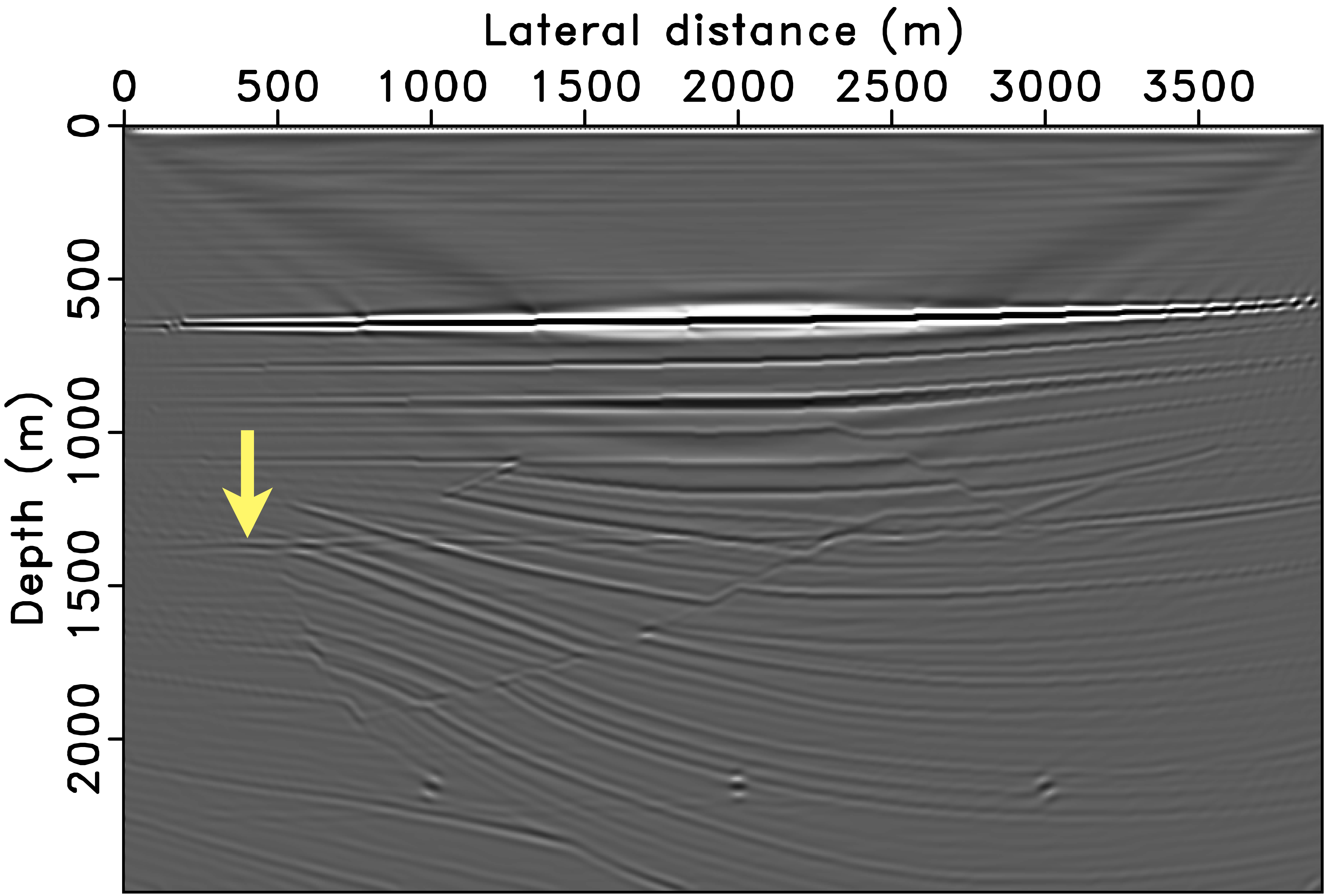
Conventional RTM image

[using the primary imaging operator]



RTM of total data

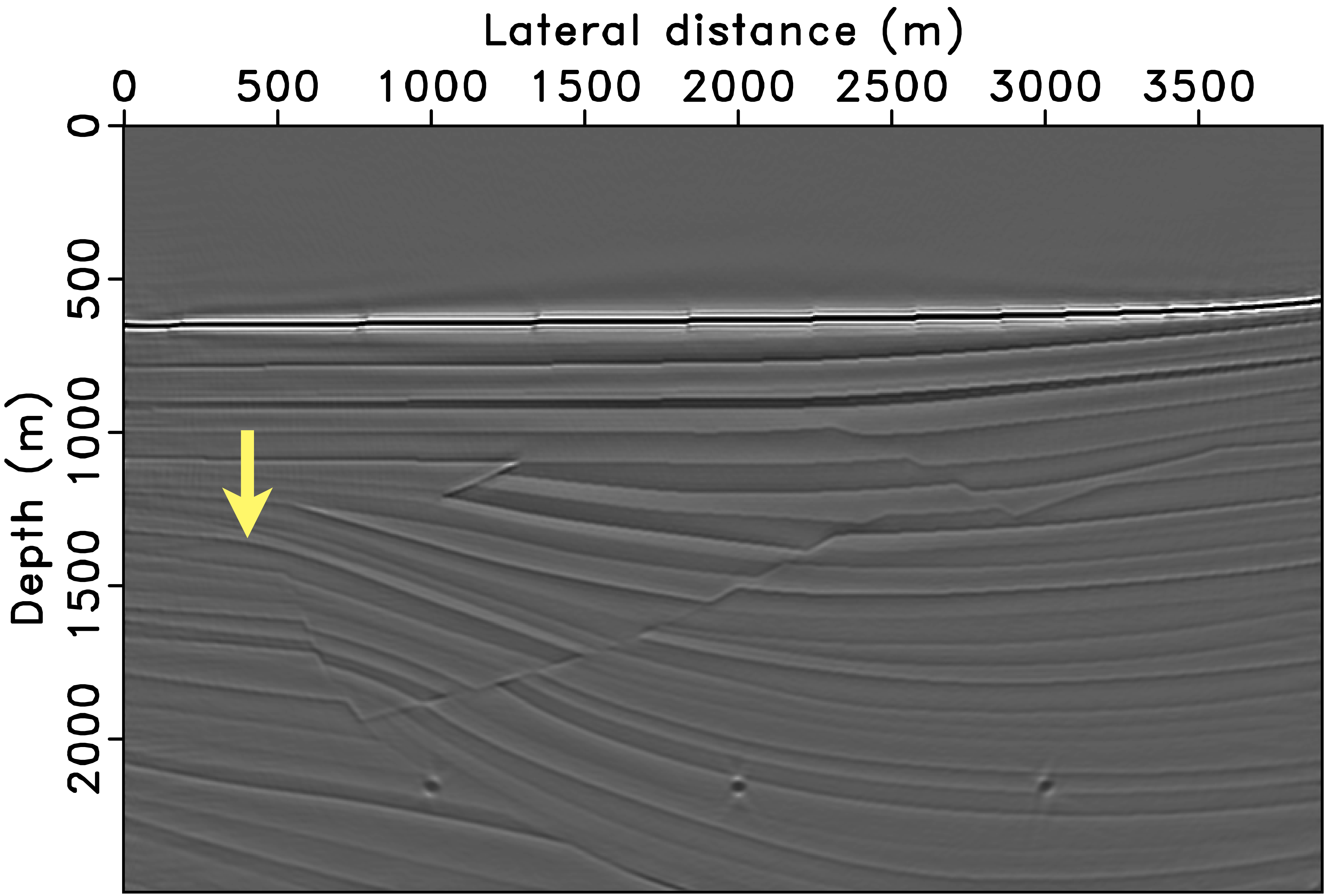
[imaging operator includes areal source for multiples]



SLIM

Fast inversion of total data

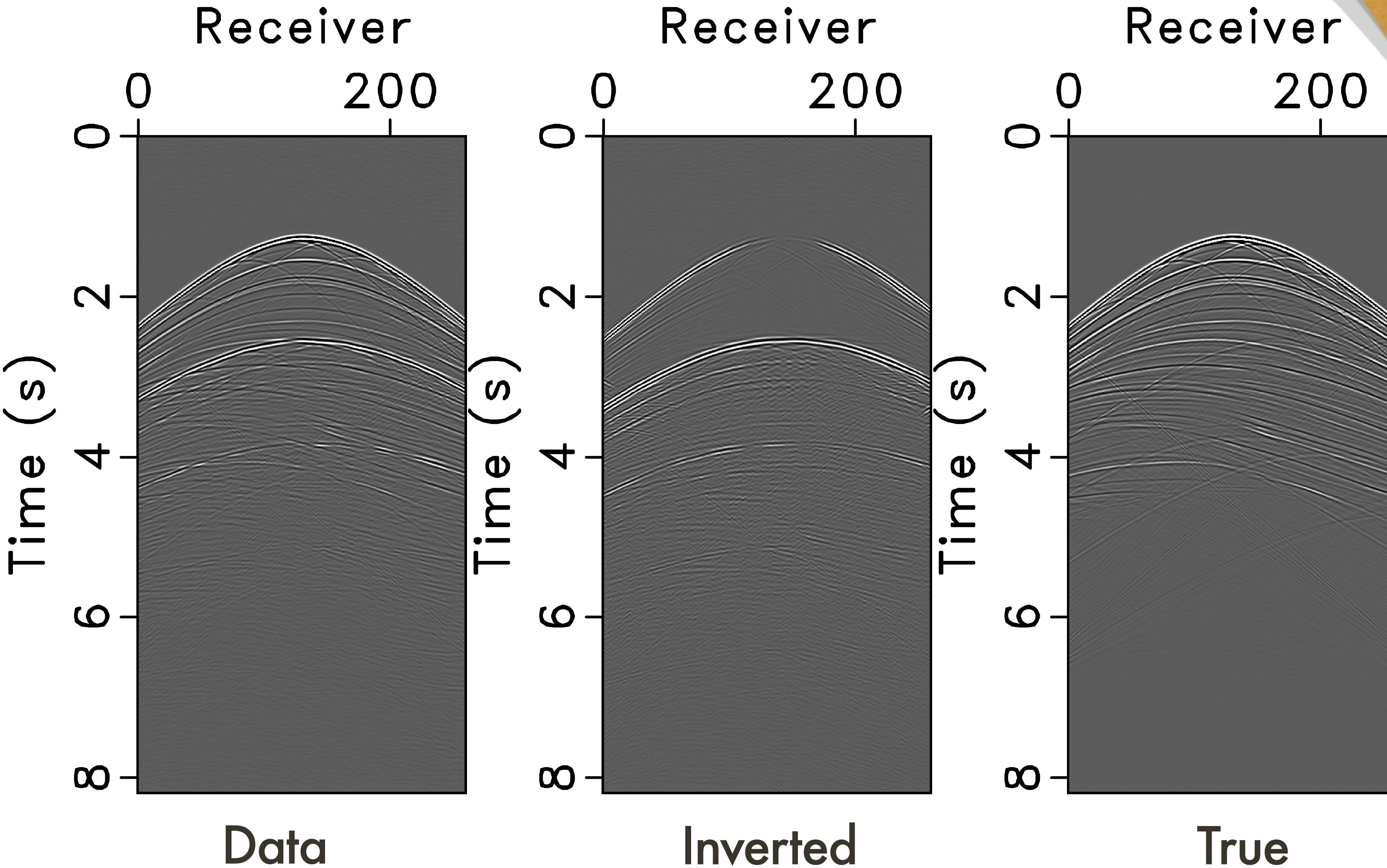
[with the **same** computational budget **as the previous two images**]



Example with coarse source sampling

- suppose only 21 shots regularly sampled from all 261 shots are available
- an analogue of limited number of ocean bottom nodes by source-receiver reciprocity
- SRME and EPSI have difficulty to predict or invert multiples
- we directly image from the total data using the proposed method

Inverted primaries by EPSI



Inverted primaries by EPSI

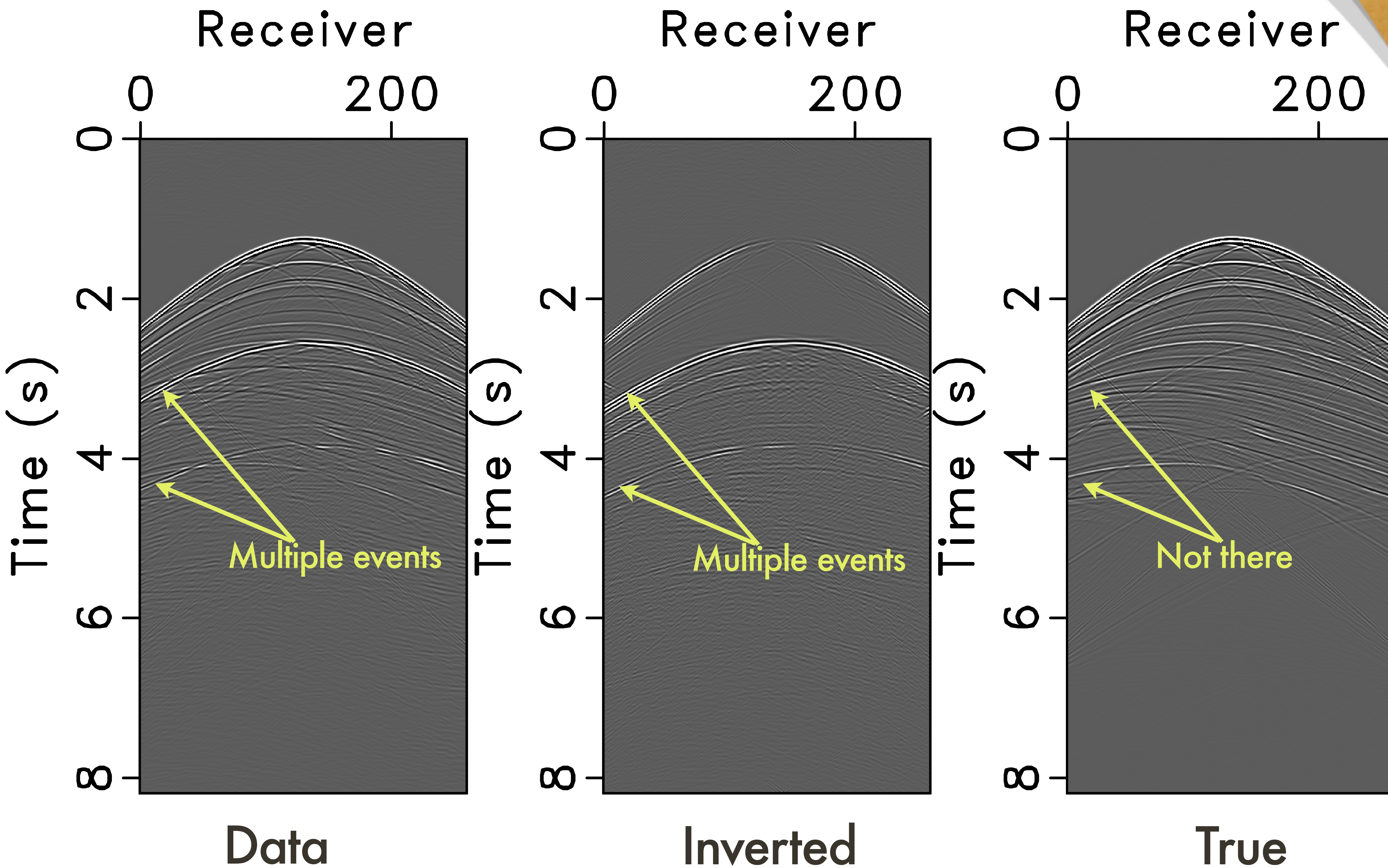
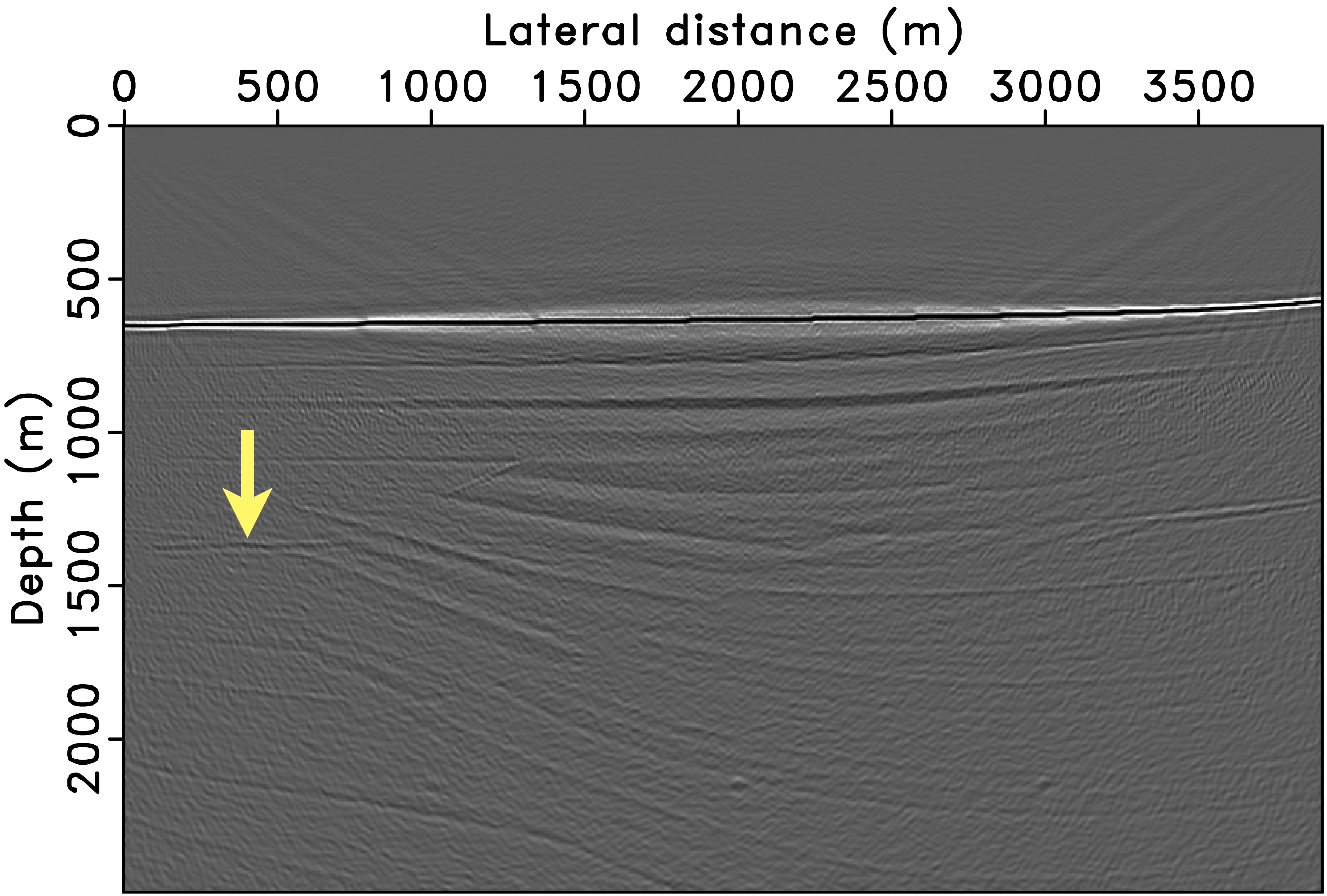
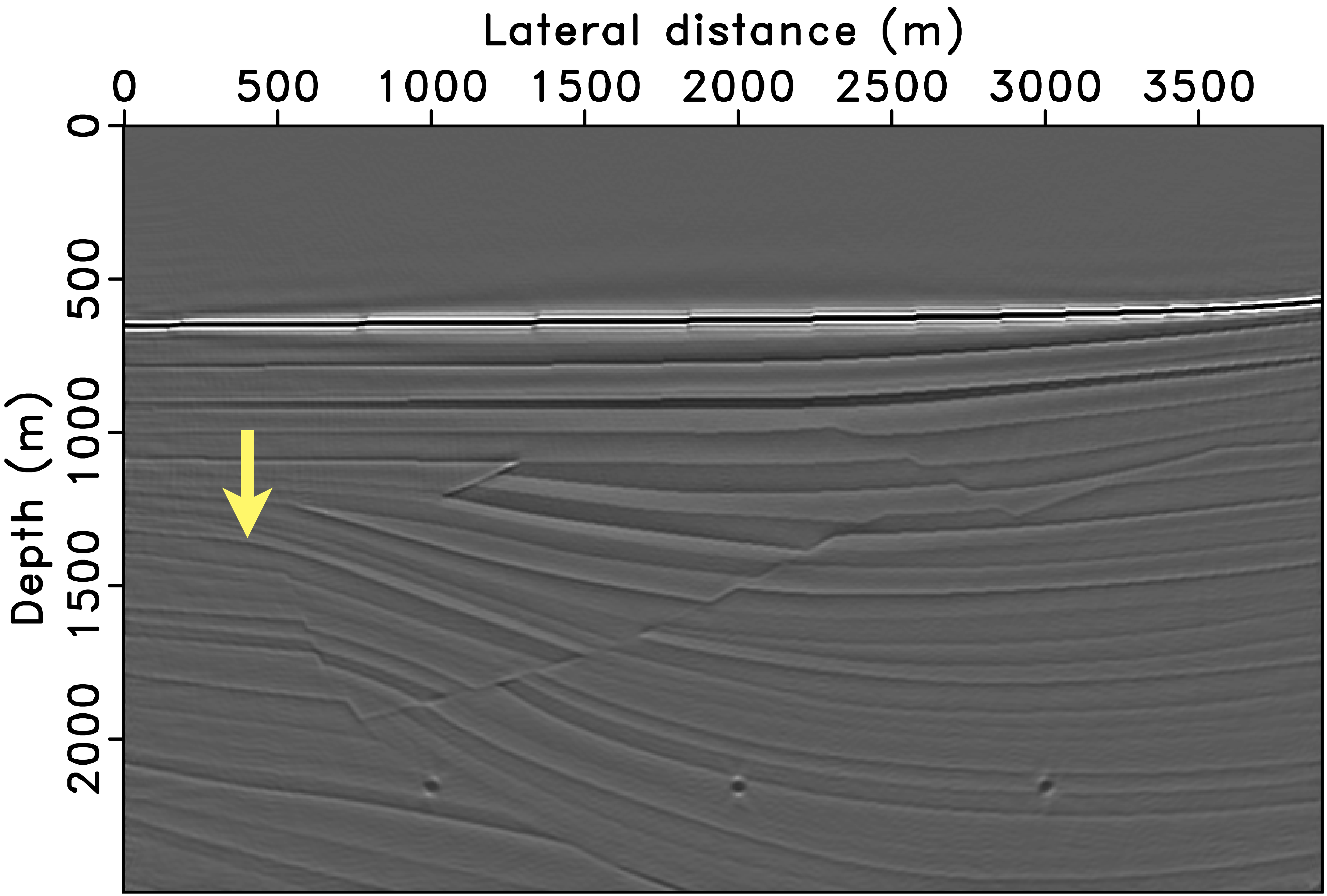


Image from EPSI inverted data

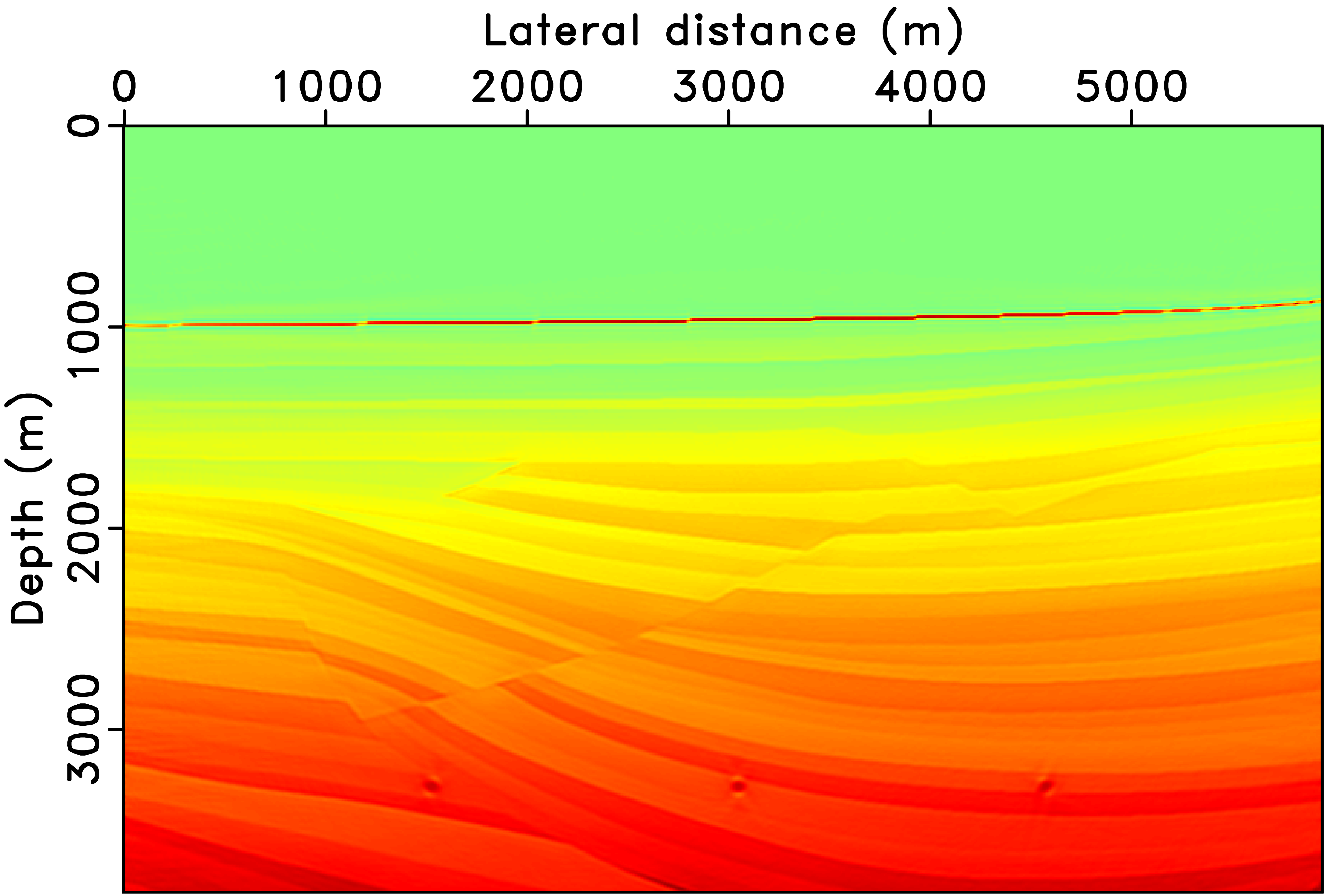


Our result


Lateral distance (m)
0
500
1000
1500
2000
2500
3000
3500
Depth (m)
0
500
1000
1500
2000

Our method retains amplitude

[adding inverted medium perturbation back to background model]



SLIM

Conclusions

- It is plausible to image with multiples without the artifacts from them.
- Non-causal cross correlations caused by multiples are eliminated by inversion.
- Multi-dimensional convolution in multiple prediction can be implicitly carried out by the wave-equation solver.

Conclusions *[cont.]*

- We gain significant speed-up in sparsity-promoting RTM by subsampling over sources/frequencies and rerandomization.
- Our method can handle data with very large source (or receiver by reciprocity) gaps by optimizing in the image space instead of data space (e.g., EPSI).

Future work

- Source estimation via variable projection
- Compare with methods that use deconvolutional imaging condition
- Compare with methods that include the free-surface in the background model
- Extend the idea to more generic interferometric imaging
- Extend the idea to velocity model building

Aravkin and van Leeuwen, 2012

Source estimation via variable projection [preliminary results]

$$\min_{x \in \mathcal{X}, \alpha} g(x, \alpha)$$

Two variables in one optimization problem, e.g., image and wavelet

↓

$$\bar{\alpha}(x) = \operatorname*{argmin}_{\alpha} g(x, \alpha)$$

Variable projection, e.g., fix the image, we can estimate the wavelet

↓

$$\tilde{g}(x) = g(x, \bar{\alpha}(x))$$

Modified objective function with only one variable

Example: a 1D EPSI analogue

Formulation:

$$\mathbf{d} = \mathbf{x} * \mathbf{w} - \mathbf{x} * \mathbf{d}$$

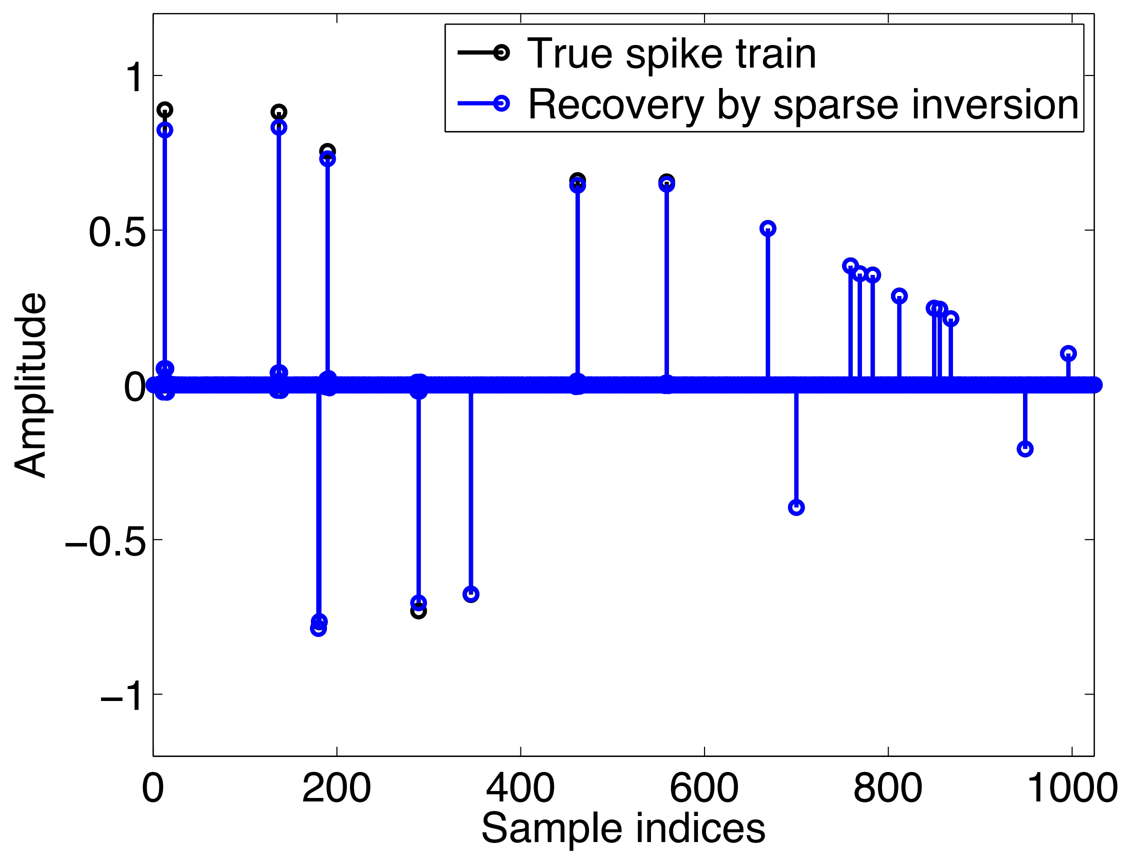
$\mathbf{d}$ data

$\mathbf{x}$ unknown spike train

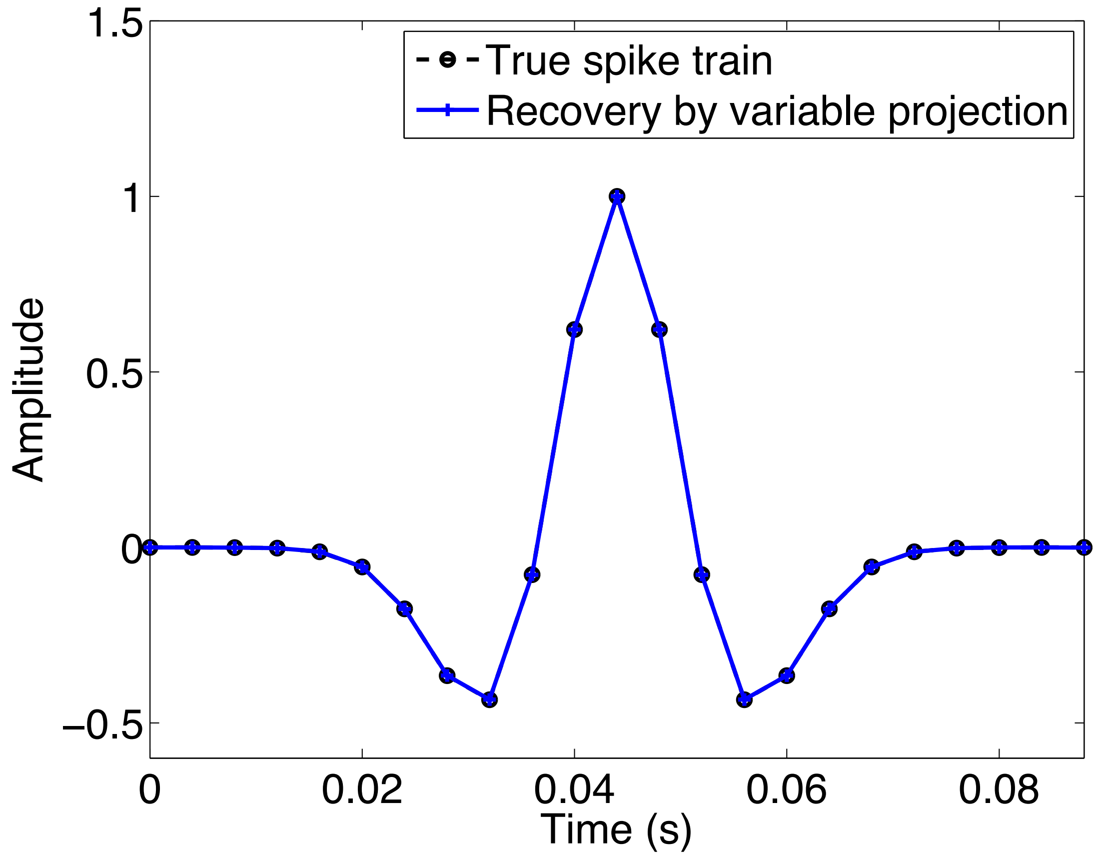
$\mathbf{w}$ unknown wavelet

$*$ convolution operator

Inversion result



Recovered spike train



Recovered wavelet

Example: sparsity promoting migration

Formulation:

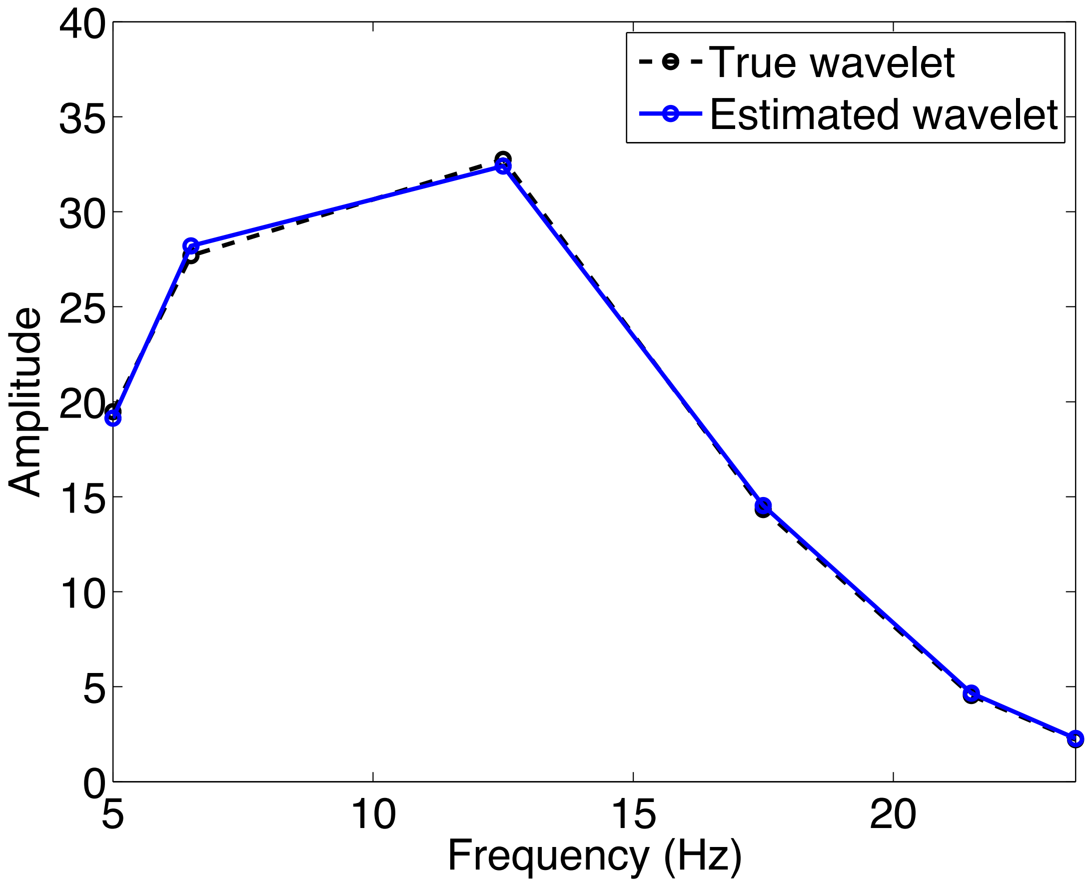
$$\min_{\mathbf{x},\alpha} \|\mathbf{x}\|_1 \quad \text{s.t} \quad \sum_{\mathrm{i}} \|\mathbf{d}_{\mathrm{i}} - \alpha_{\mathrm{i}} \nabla \mathbf{F}_{\mathrm{i}} \mathbf{C} \mathbf{x}\|_2 \leq \sigma.$$

- i frequency index
- $\mathbf{x}$ curvelet coefficients of model perturbation
- α_{i} wavelet spectrum at the i-th frequency
- $\mathbf{d}_{\mathrm{i}}$ data of the i-th frequency

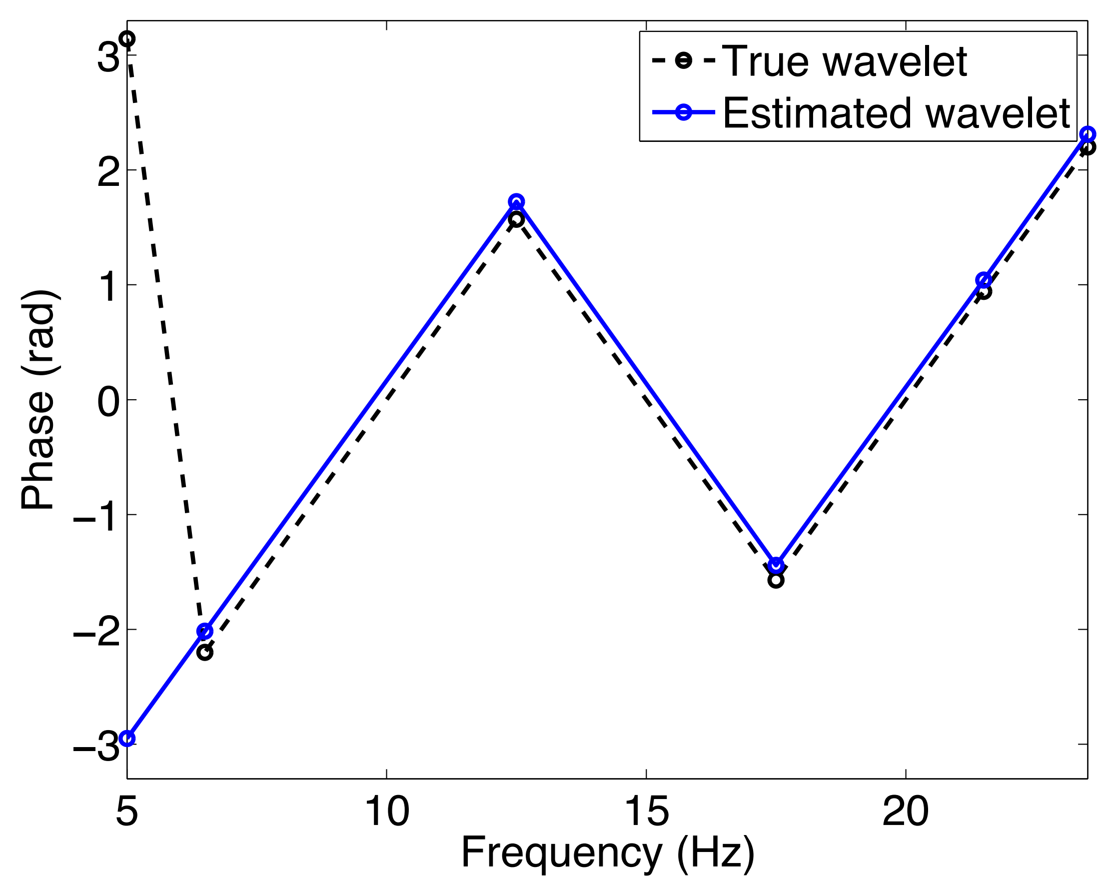
Example: sparsity promoting migration

- Three scenarios:
  - assuming the true wavelet is known
  - using a wrong wavelet
  - starting with the same wrong wavelet as above, but update it by variable projection
- Use 15 sim. sources and 6 frequencies, model dimensions: 201*301 with 10m spacing.

Estimated wavelet

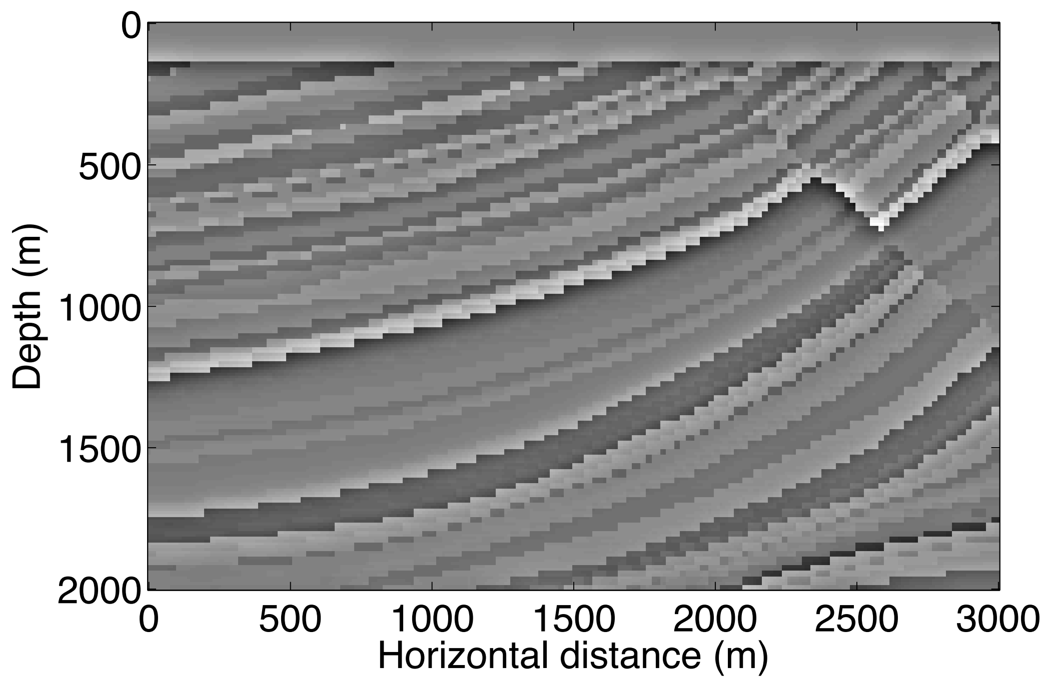


Amplitude spectrum

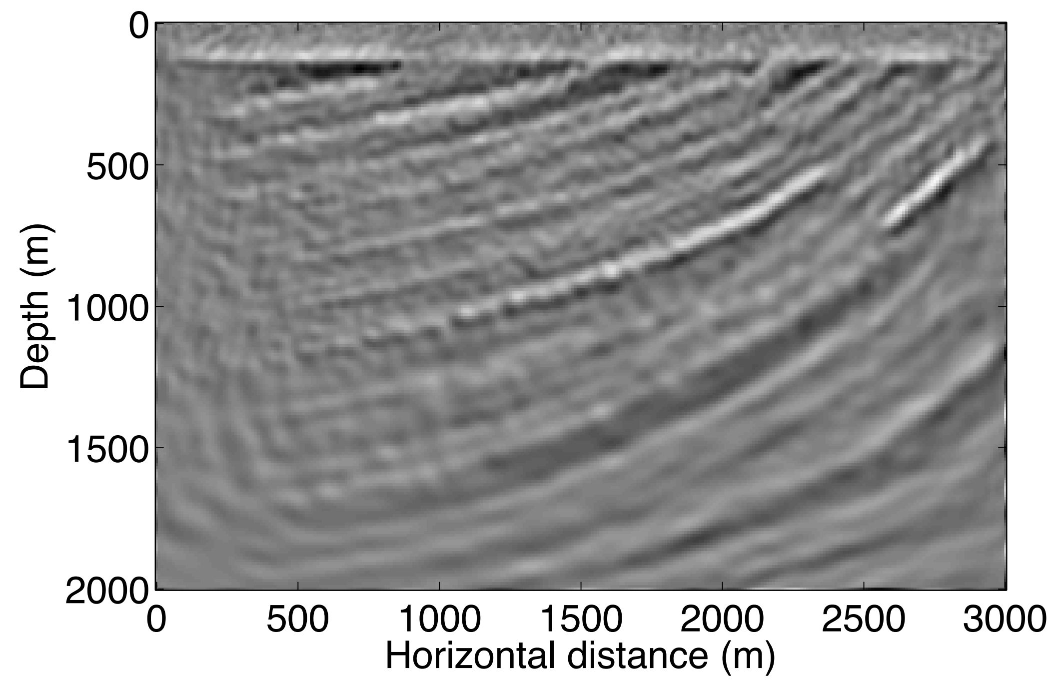


Phase spectrum (note -pi=pi)

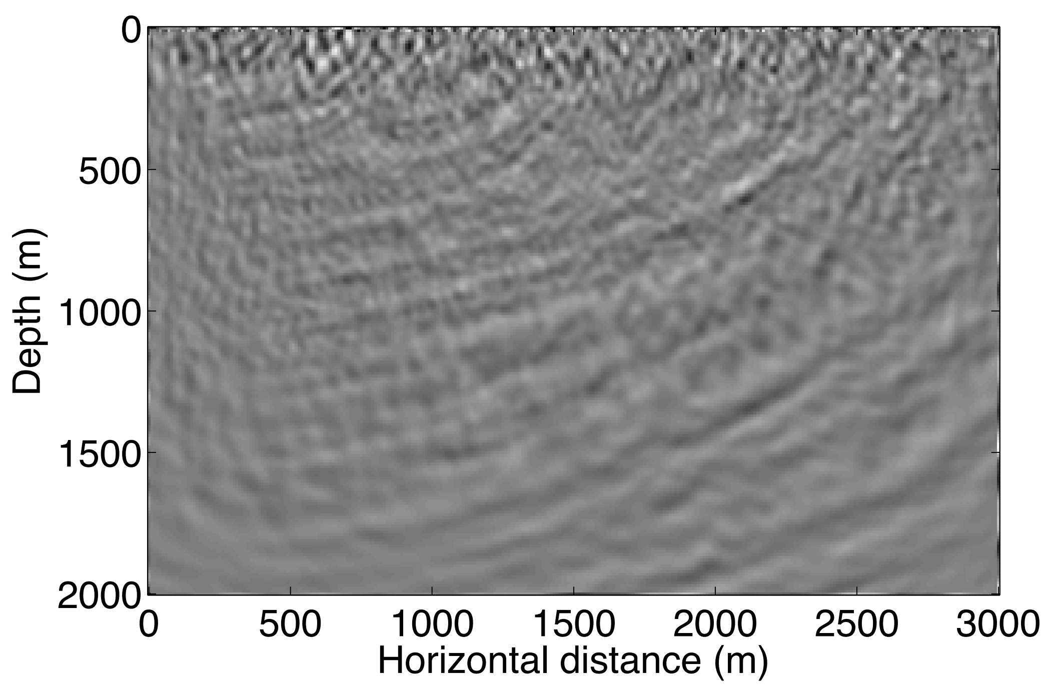
Inverted images



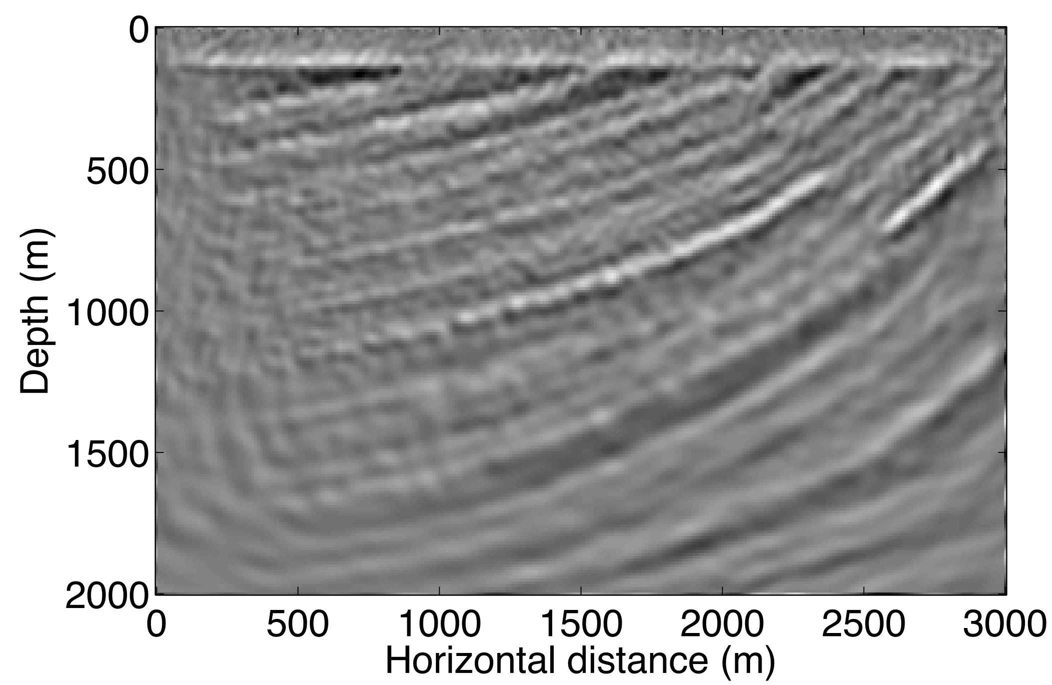
True perturbation



Using the true wavelet



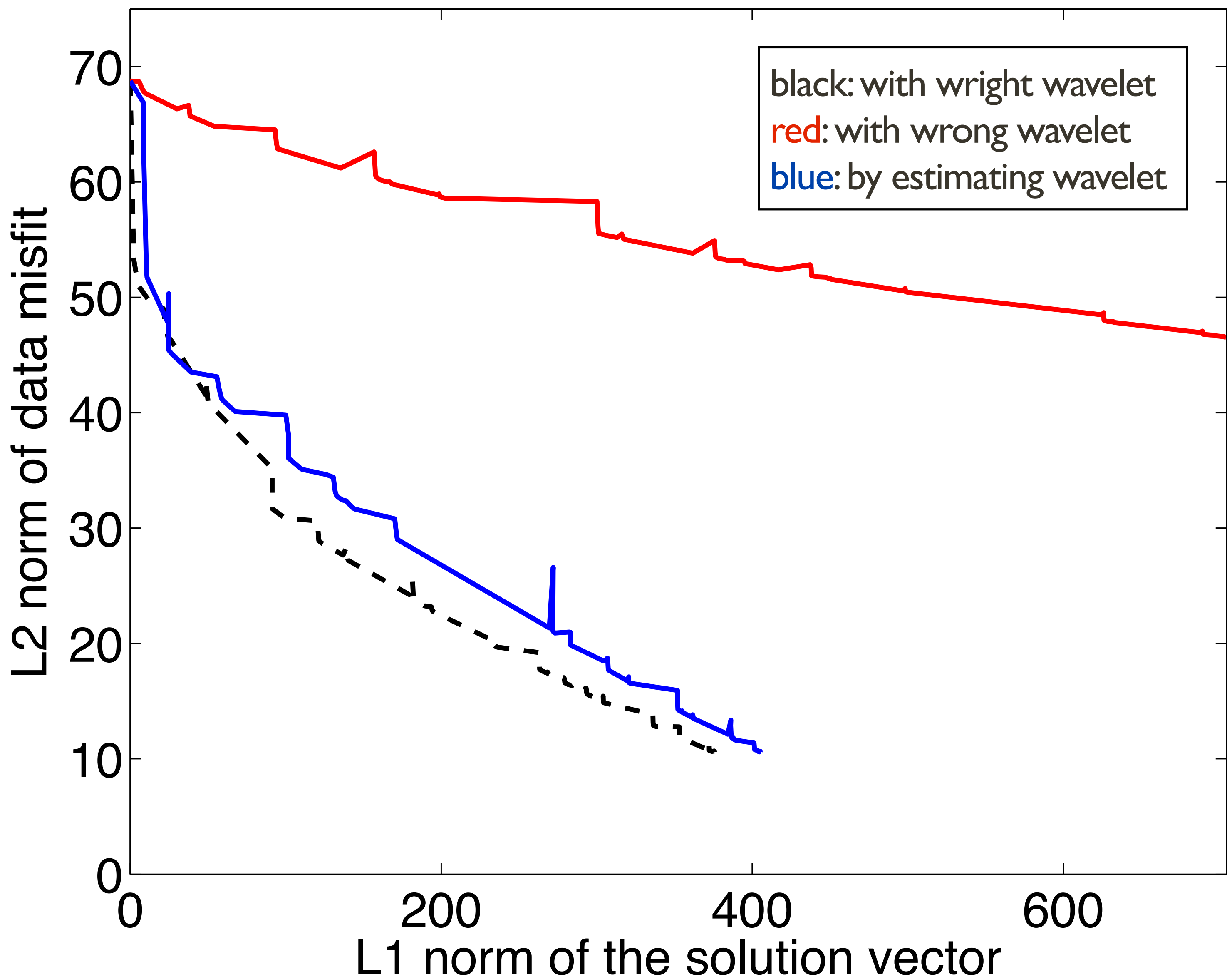
Using a wrong wavelet



Estimating the wavelet

Insights from the solution paths

[solve up to 100 iterations]



Acknowledgements

My colleagues for fruitful discussions.

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, and WesternGeco.