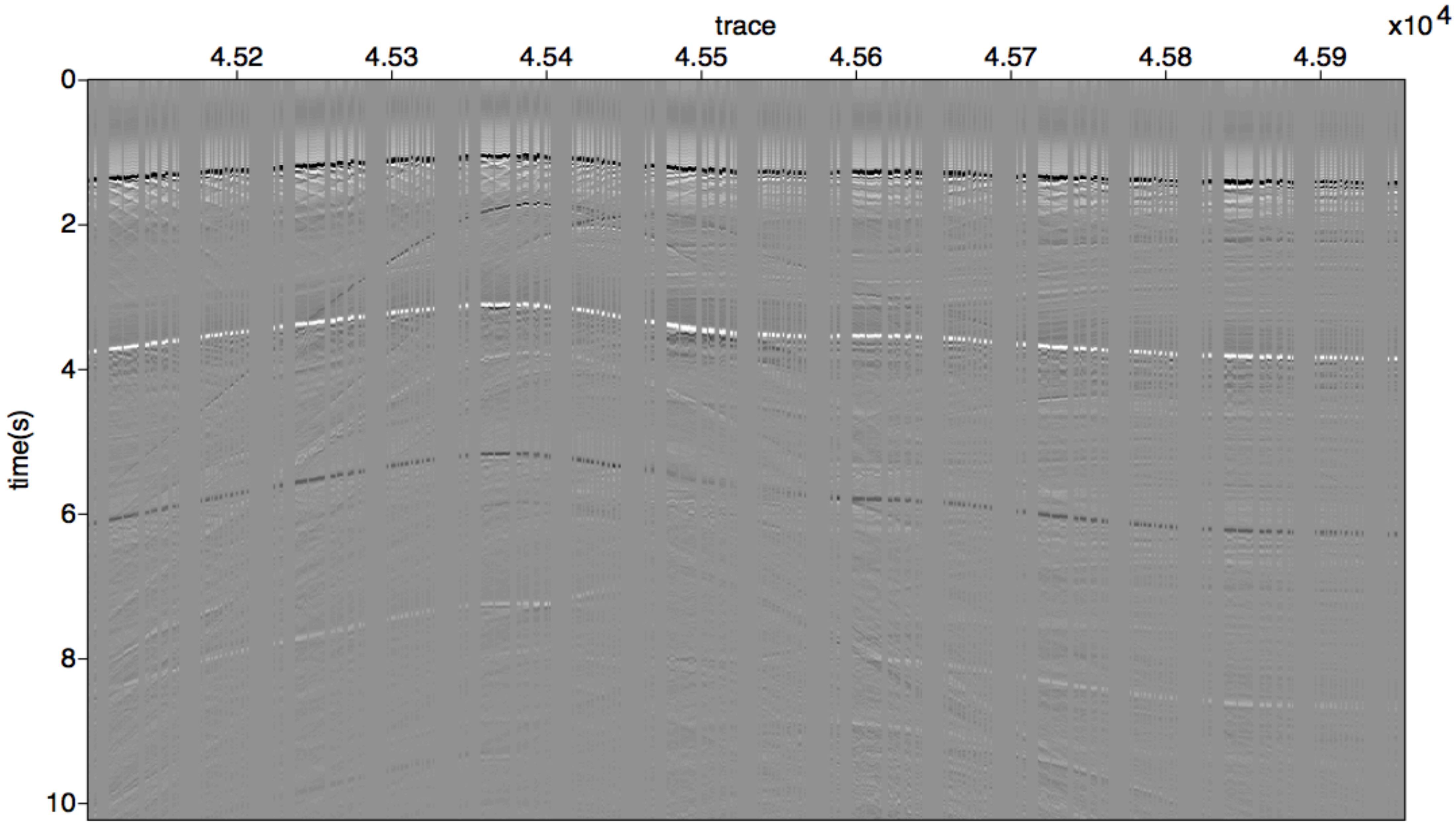
Seismic Data Interpolation using SVD-free Low Rank Optimization

Rajiv Kumar, Sasha Aravkin, Hassan Mansour, and Felix J. Herrmann

SLIM
University of British Columbia

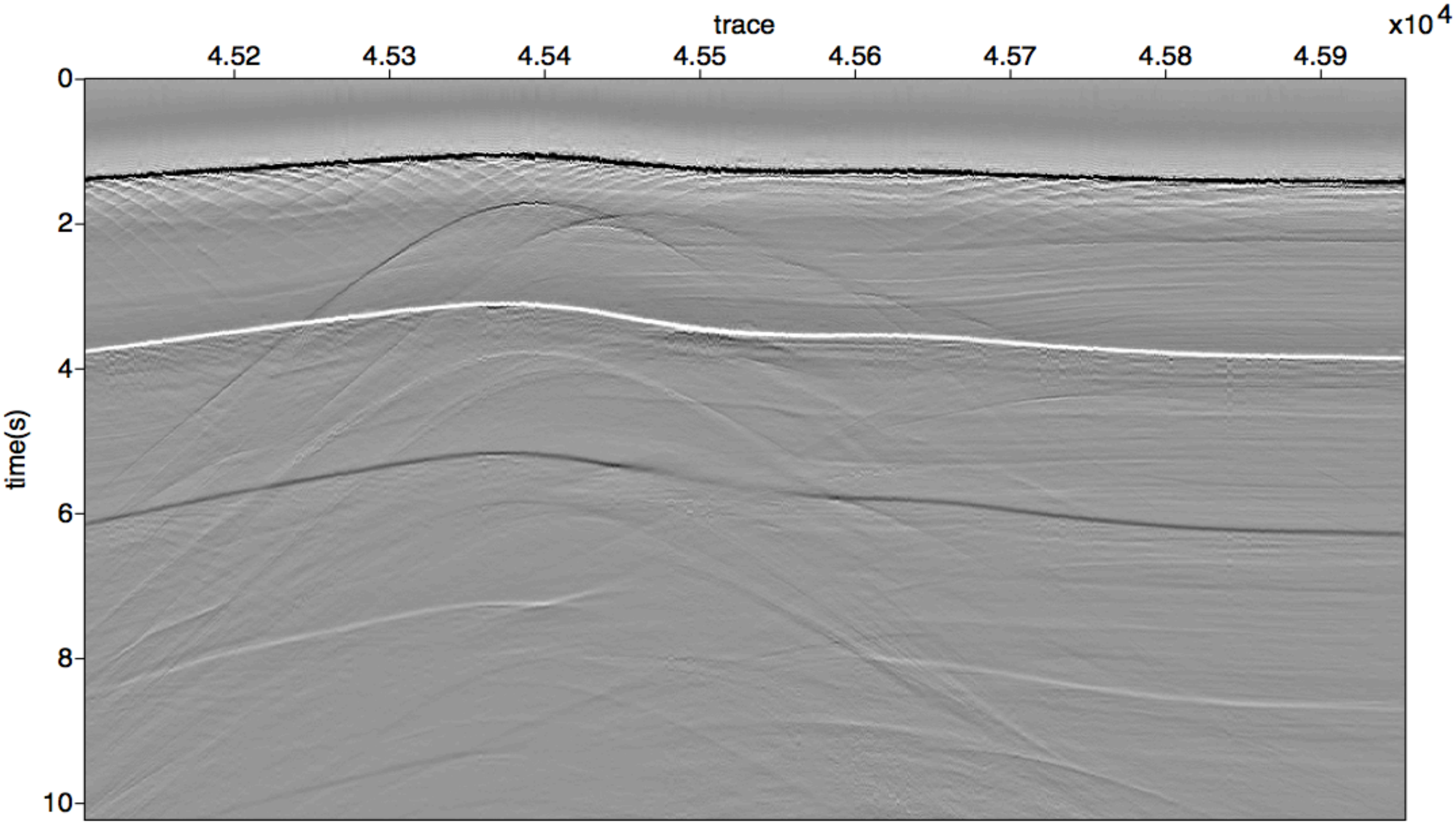
Motivation



Input data

SLIM

Motivation



Interpolation with 2D Curvelet

- ***is sparsity the only inherent structure in seismic??***

Outline

- rank structure in seismic
- matrix factorization
- regularized matrix factorization

Rank Structure in Seismic

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Setup

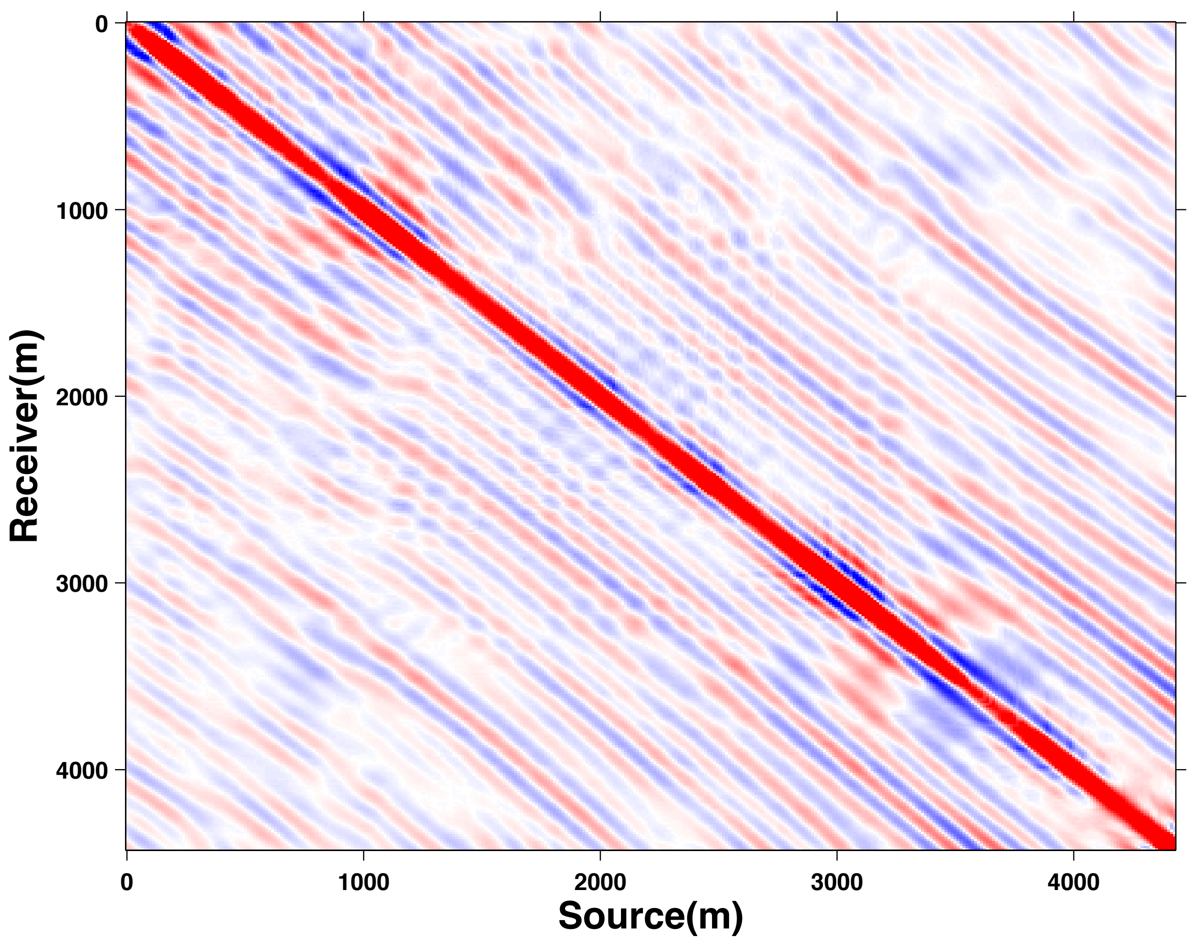
- let $\mathbf{D} \in \mathbb{R}^{n_r \times n_s}$ is spatial acquired data on regular grid
- monochromatic frequency

$$\mathbf{D}_\omega = \begin{pmatrix} D_{1,1} & D_{1,2} & \cdots & D_{1,n_s} \\ D_{2,1} & D_{2,2} & \cdots & D_{2,n_s} \\ \vdots & \vdots & \ddots & \vdots \\ D_{n_r,1} & D_{n_r,2} & \cdots & D_{n_r,n_s} \end{pmatrix}$$

- n_r, n_s represent receiver # and source #

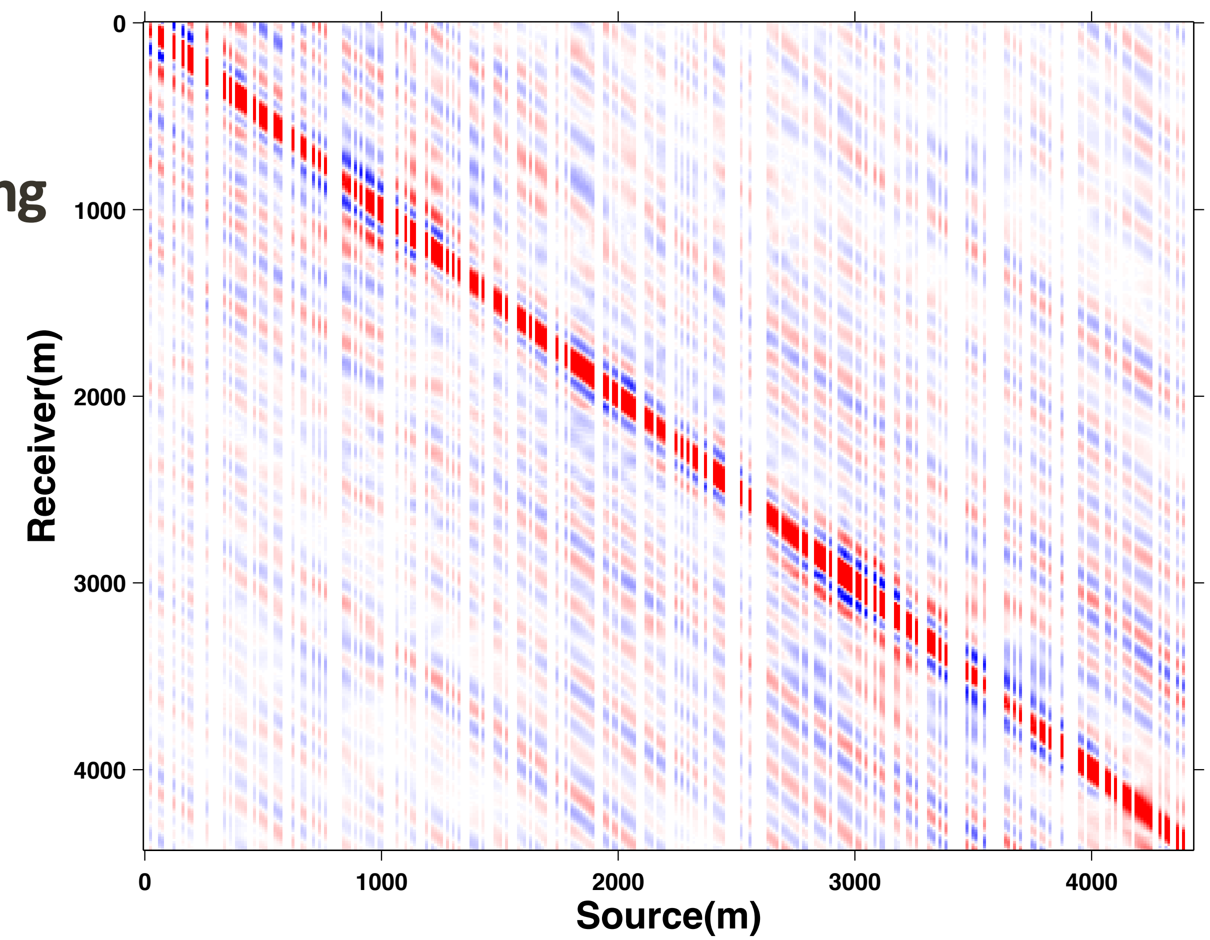
2D Acquisition

Acquisition impediments
[Frequency -12 Hz]



Regular Sampled data

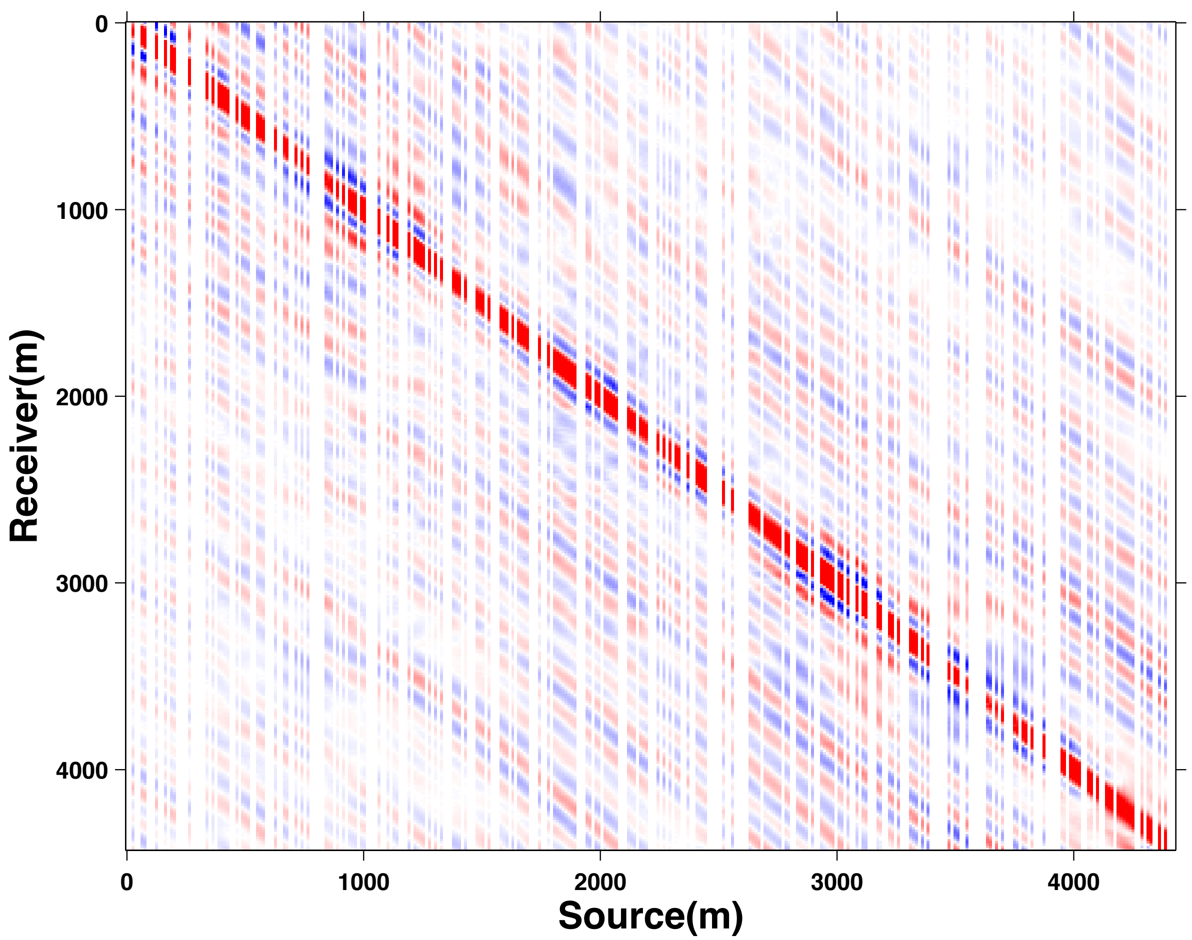
50% missing shots



Irregular Sampled data
[Missing columns do not increase rank]

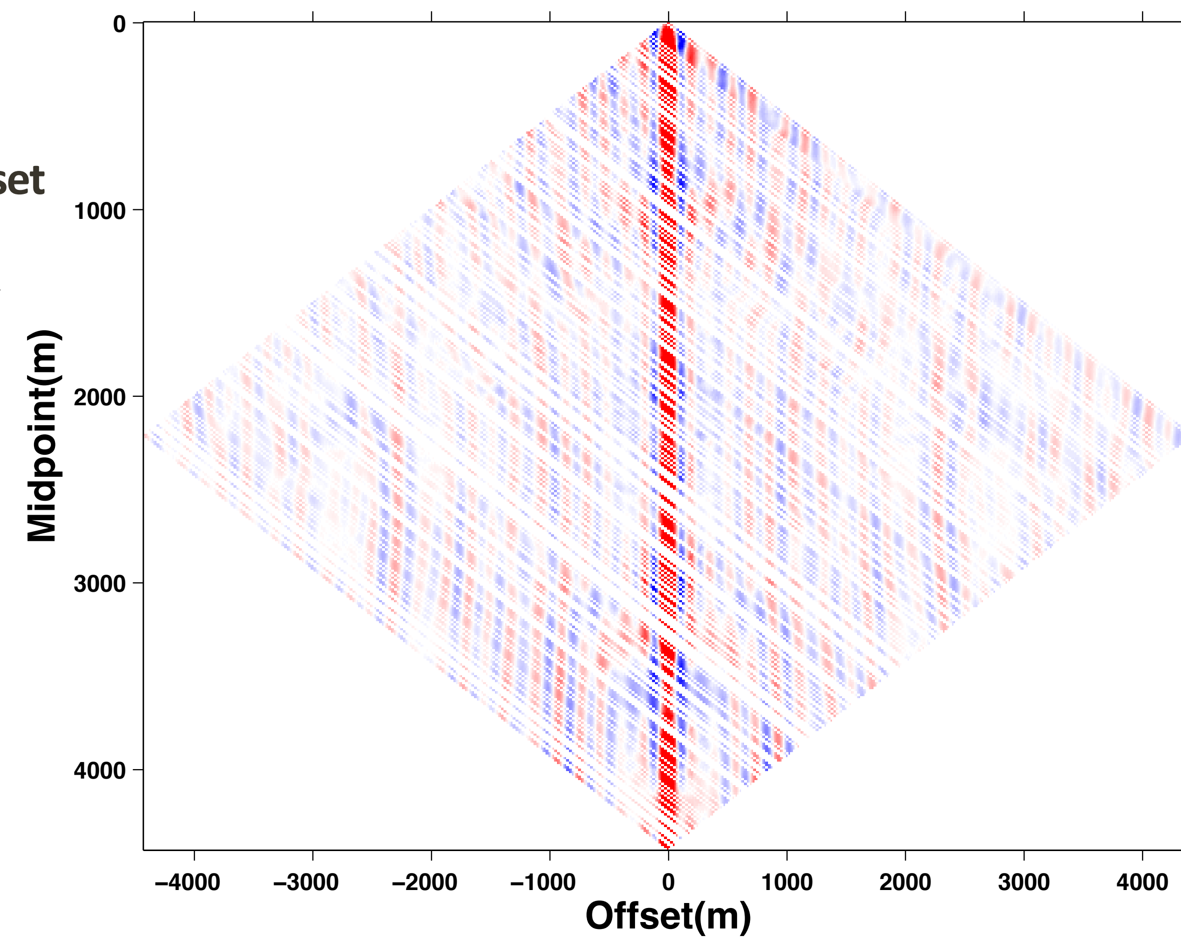
Missing data scenario
[Frequency -12 Hz]

Missing column do not increase rank



Midpoint-Offset transform

Missing column DOES increase rank

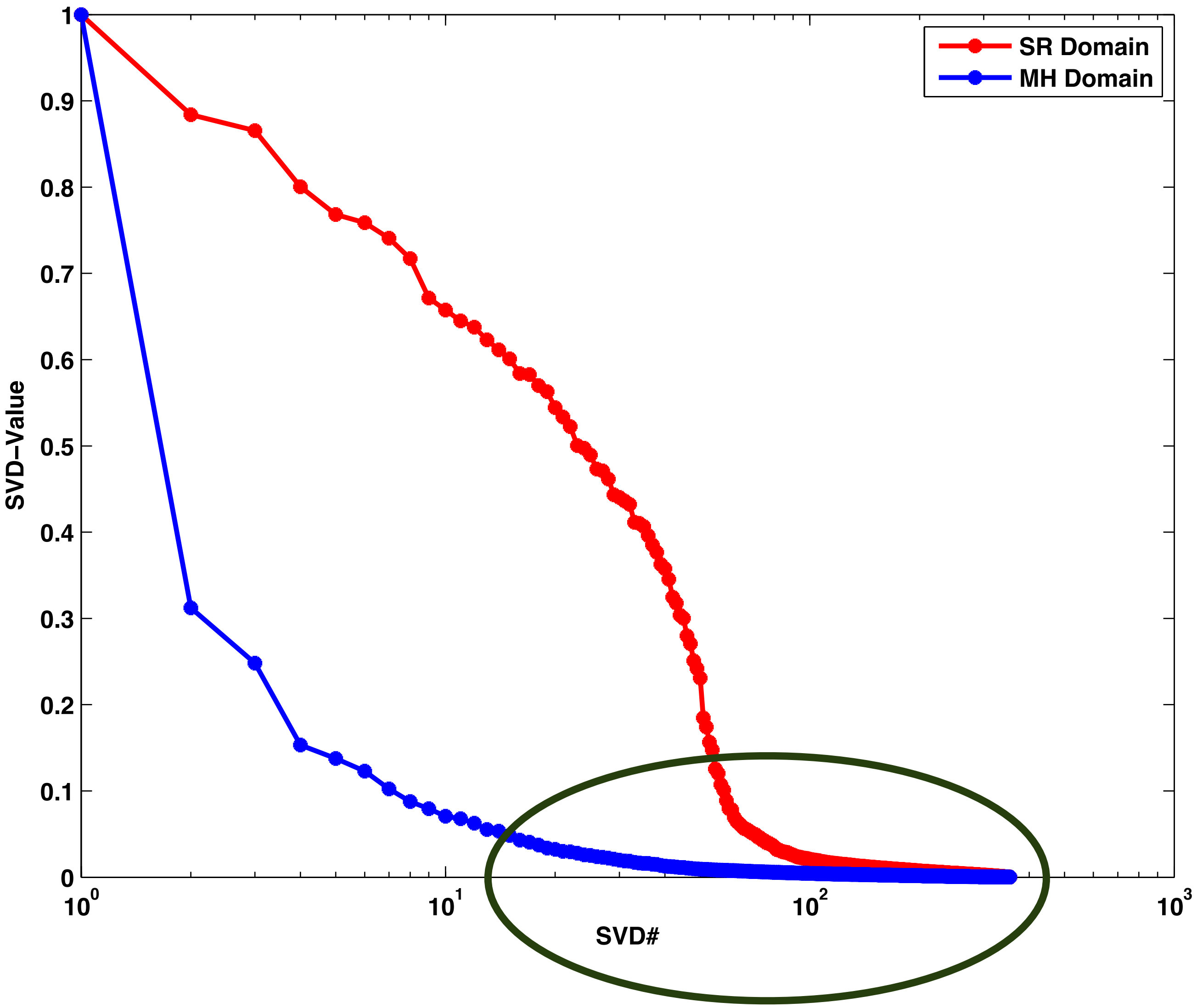


- **Midpoint -** $0.5(s+r)$
- **Offset -** $(s-r)$

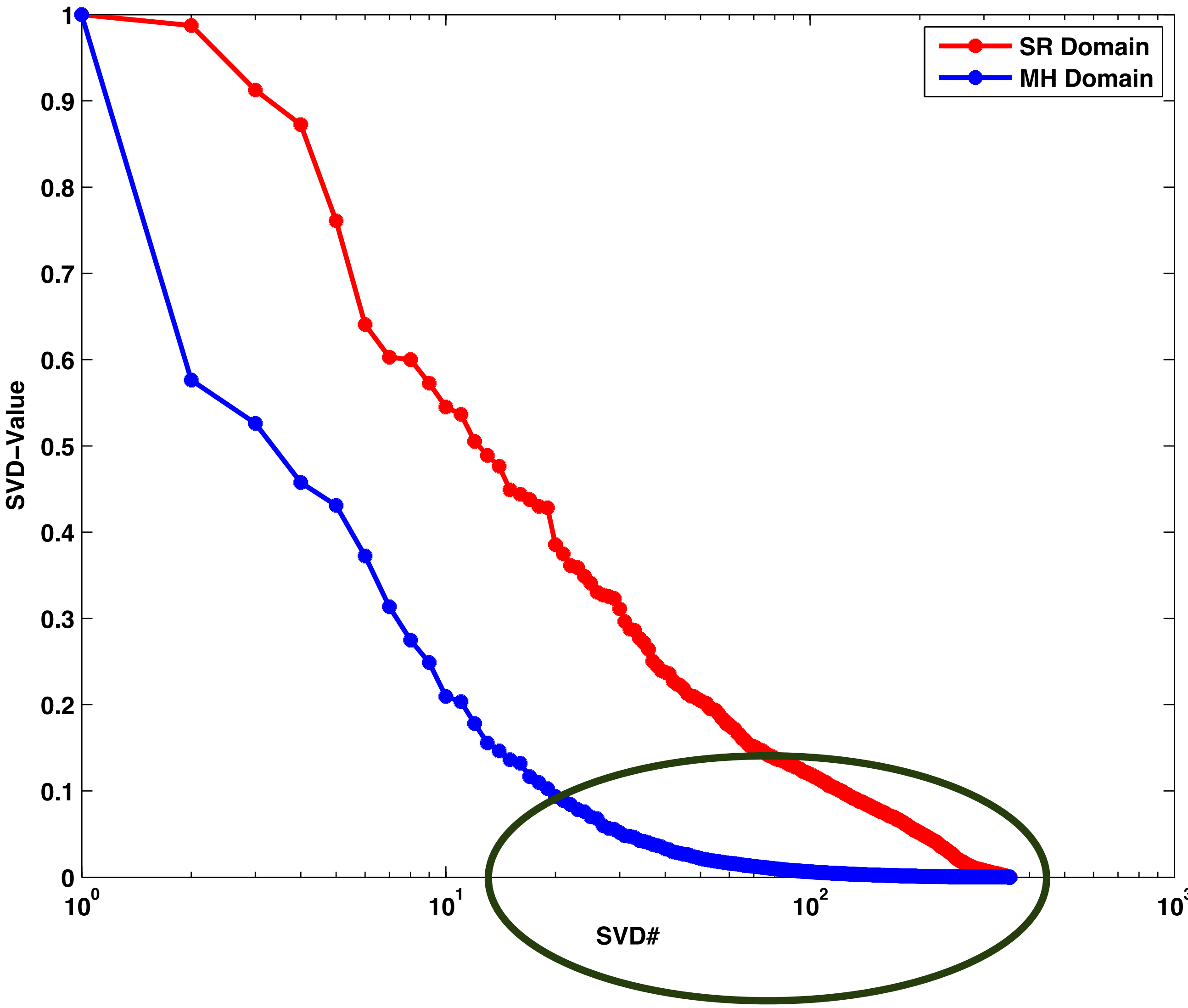
where s **is source position and** r **is receiver position**

Regular sampled data
[Singular value decay]

Freq : 12 Hz

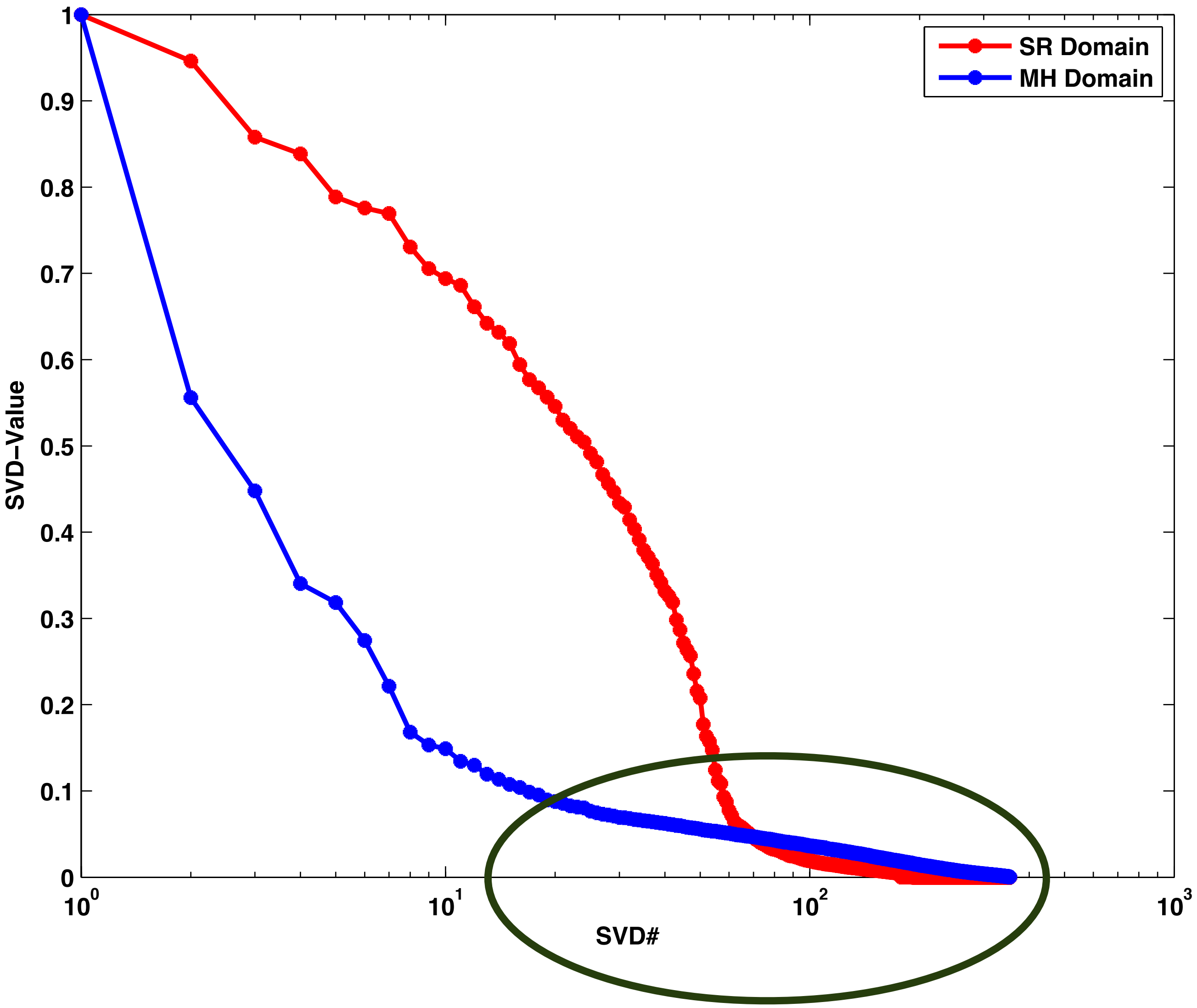


Freq : 60 Hz

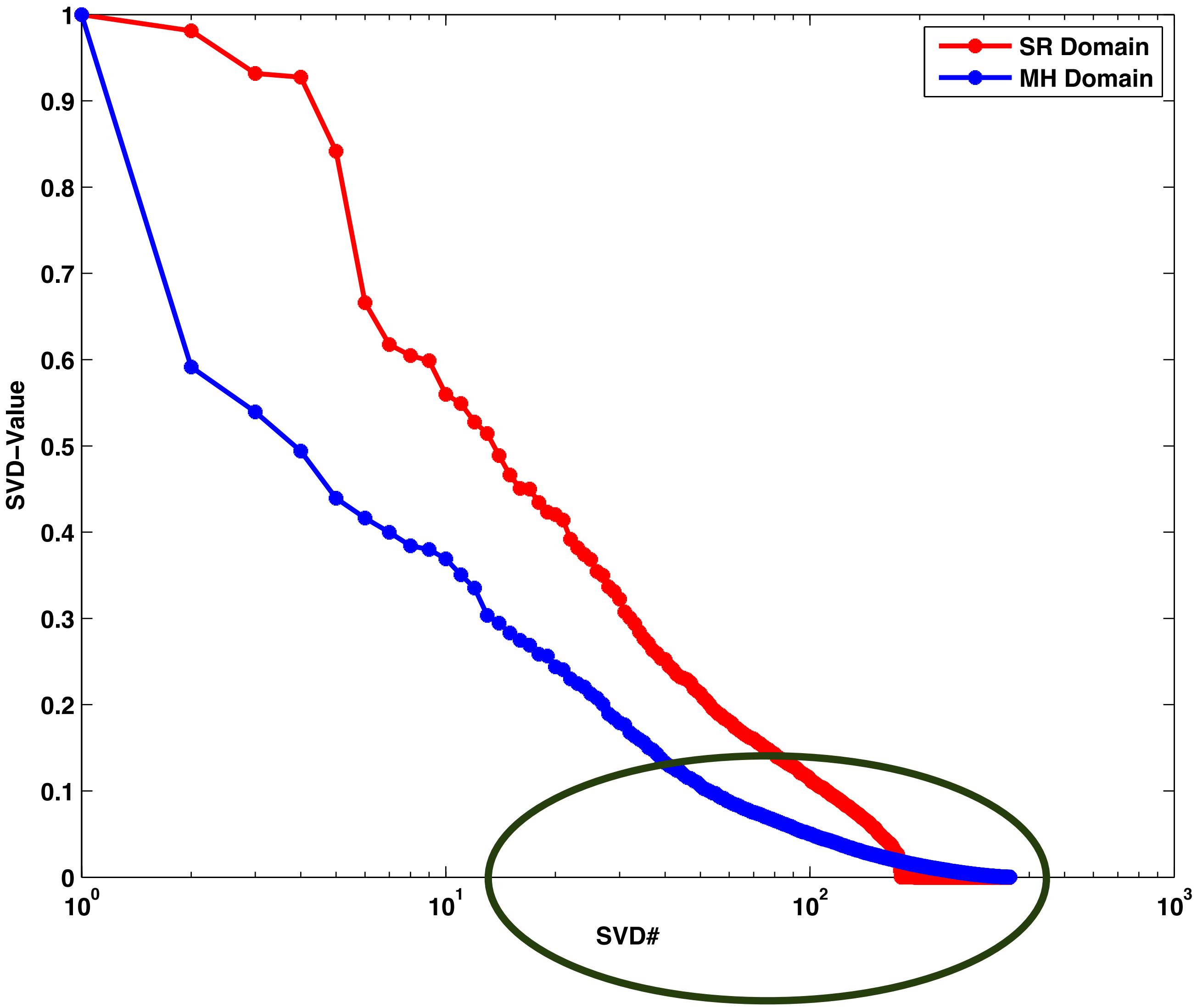


50 % Missing data
[Singular value decay]

Freq : 12 Hz

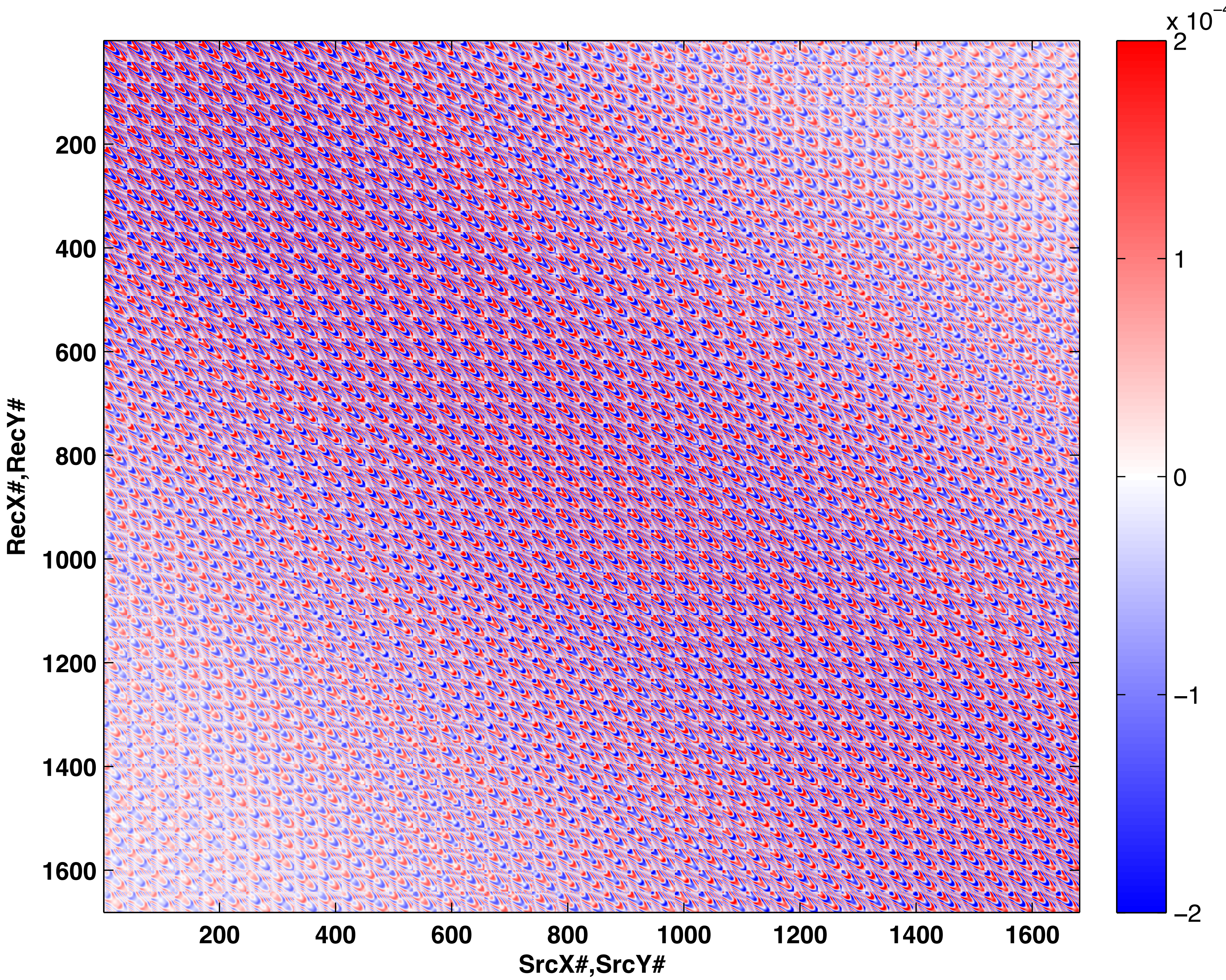


Freq : 60 Hz

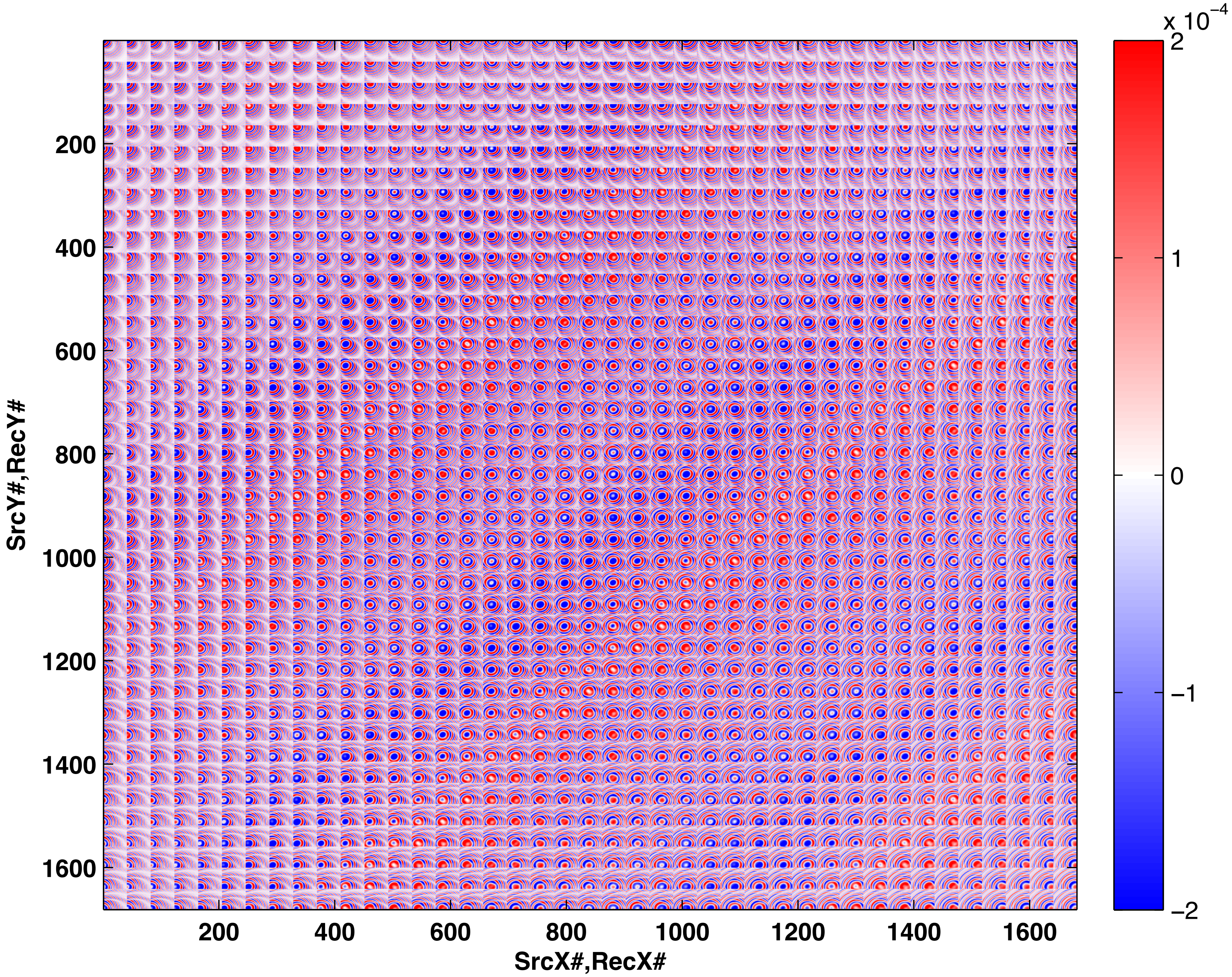


3D Acquisition

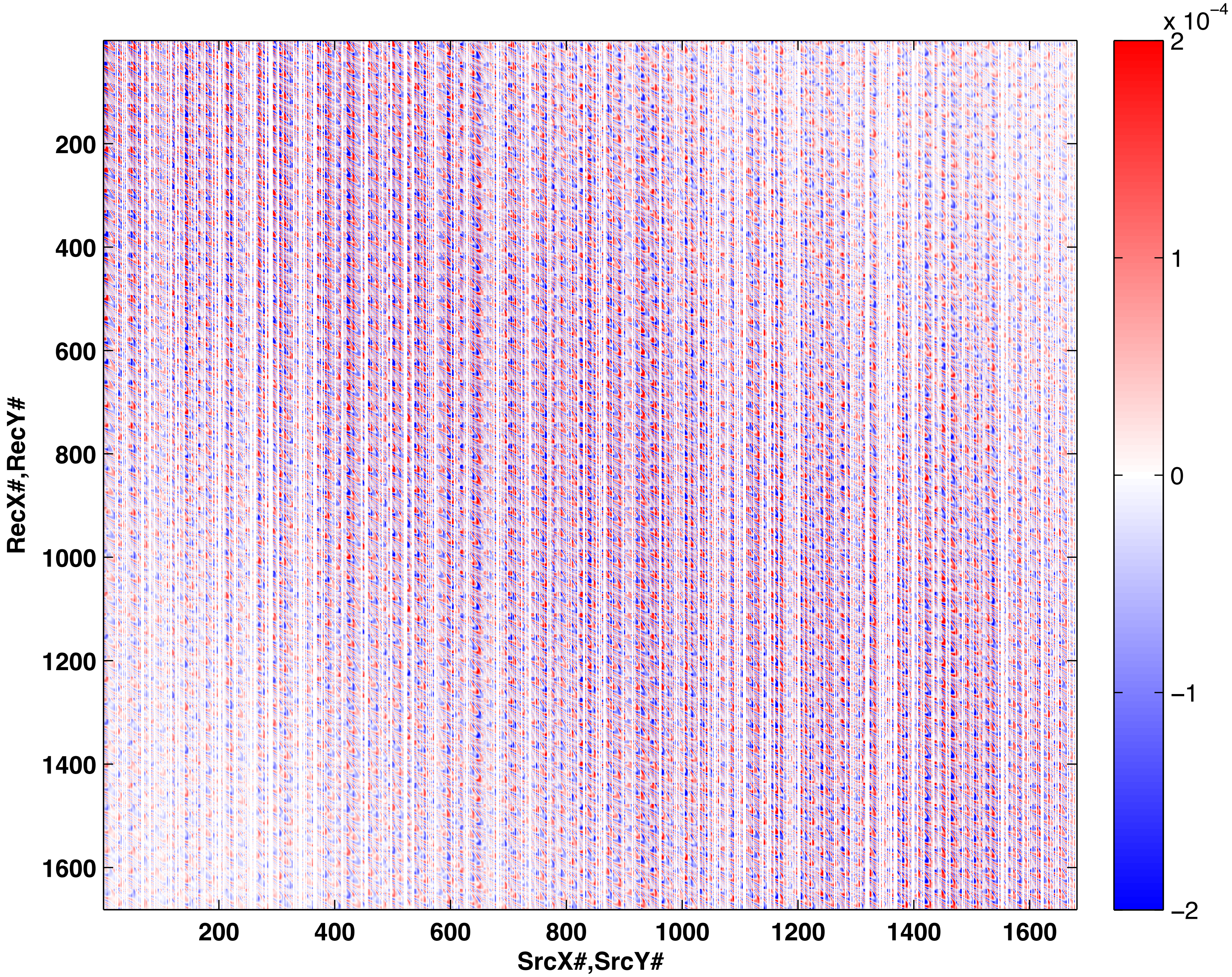
3D Acquisition
[Regular sampled data]



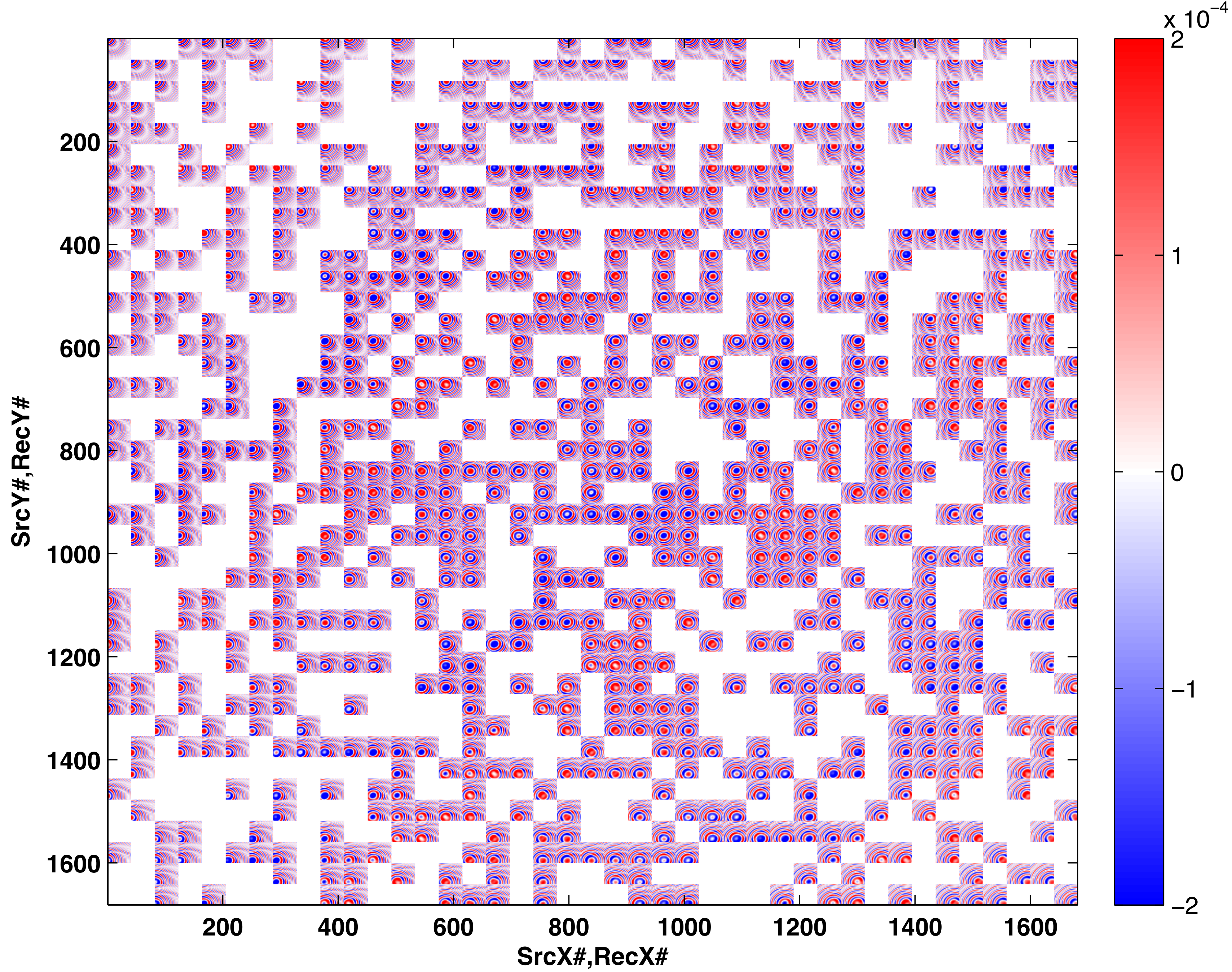
3D Acquisition
[Regular sampled data - “Transform” domain]



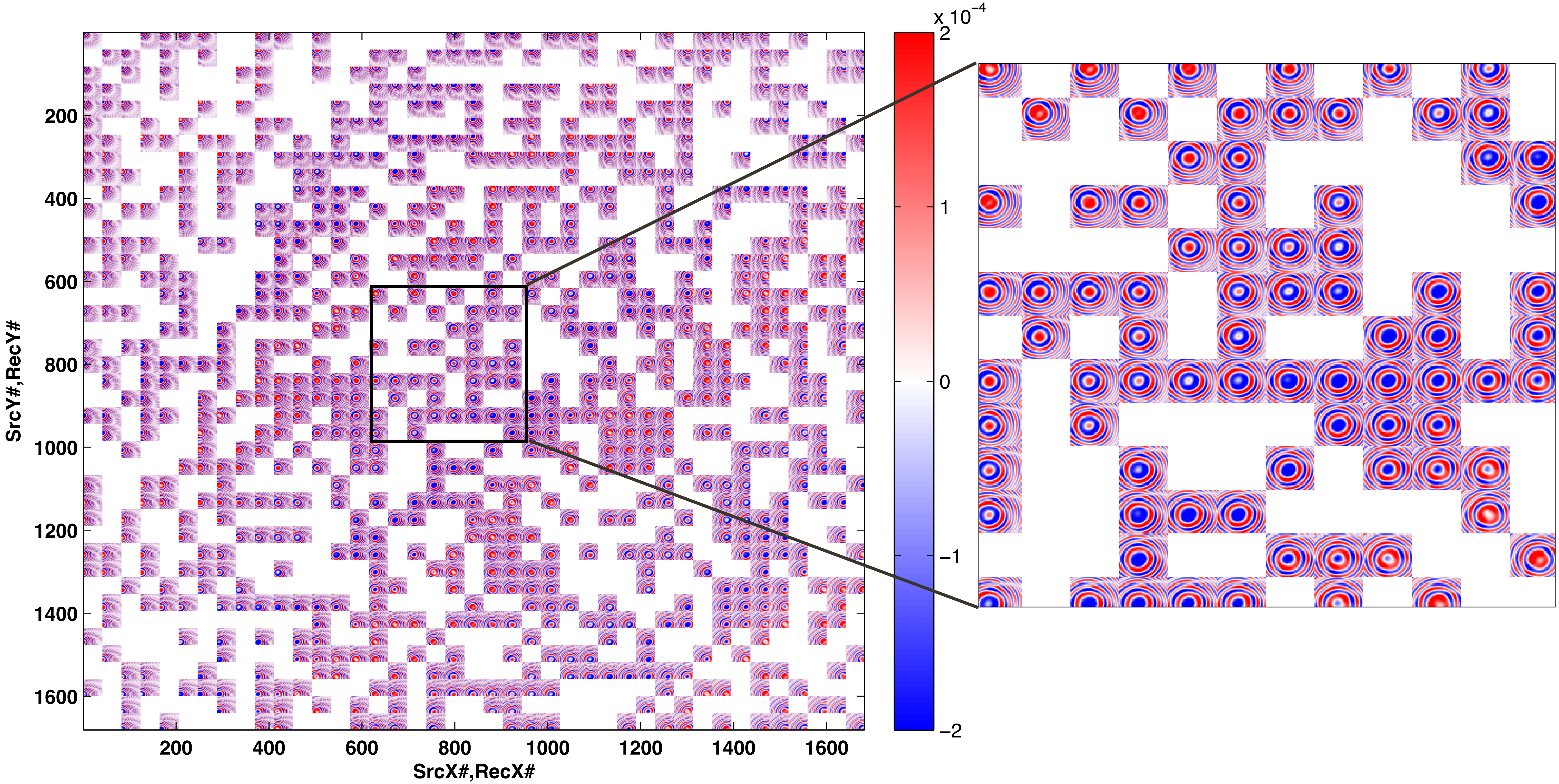
3D Acquisition
[Irregular sampled data]



3D Acquisition
[Irregular sampled data - "Transform" domain]

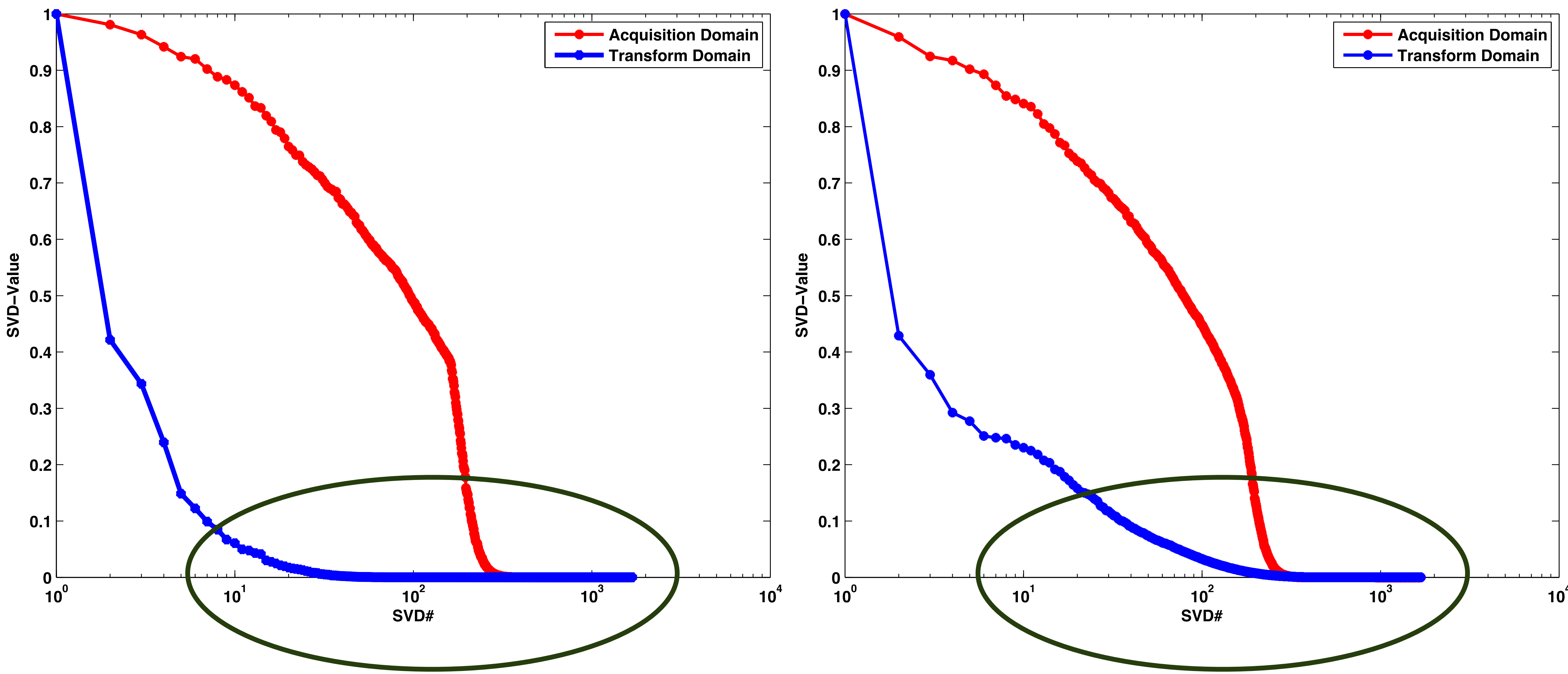


3D Acquisition
[Irregular sampled data - “Transform” domain]



3D Acquisition
[Singular value decay]

- **Acquisition domain - (SourceX, SourceY) & (ReceiverX, ReceiverY)**
- **Transform domain - (SourceX, ReceiverX) & (SourceY, ReceiverY)**



Original Data

Missing Data

Low Rank domain - Seismic

- 2D acquisition :
  - Midpoint-Offset
- 3D acquisition :
  - (SourceX,ReceiverX) and (SourceY,ReceiverY)
  - Midpoint-Offset-Azimuth

Sparsity vs Rank

Sparsity

- ***sparse/compressible***
- missing traces make signal ***less sparse*** in transform domain
- recovery using ***sparsity promoting*** scheme

Rank

- ***low rank/fast decay*** of singular values
- missing traces ***increase rank*** in "transform domain"
- recovery using ***rank penalization*** scheme

Observations

- Sampling become *incoherent* in “transform” domain
- *Slow decay* of singular values in “transform” domain

Low Rank Optimization

Methodology

- given data matrix $\mathbf{b}$ (in a "*transform* domain")

$$(BPDN_\sigma) \qquad \min_X ||X||_* \quad \text{s.t. } ||\mathcal{A}(X) - \mathbf{b}||_2^2 \leq \sigma$$

where $*$ is the *nuclear* norm given by :

$$\|X\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

where λ_i are the *singular* values

Methodology

- $\mathcal{A}$ is the sampling operator in the "*transform*" domain defined by :

$$\mathcal{A} = \mathbf{RM}$$

where

$\mathbf{R}$: restriction operator

$\mathbf{M}$: measurement operator

Impediments

- requires repeated application of *SVD* for projection
- *expensive* to compute for large system
- can we exploit *rank structure* *“**SVD free**”?*

Matrix Factorization

(Recht et al.)

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Methodology

- given data matrix $\mathbf{b}$ (in a "*transform* domain")
- parameterize $X \in \mathbb{R}^{n \times m}$ using factors:

$$X = LR'$$

where $L \in \mathbb{R}^{n \times k}$ and $R \in \mathbb{R}^{m \times k}$

- note that rank $k << max(m, n)$ is enforced through parametrization

Methodology

- solve the least-squares problem :

$$\min_{L,R} \|\mathcal{A}(LR') - \mathbf{b}\|_2^2$$

- implemented using *L-BFGS* with 300 iterations

Experiment & Results

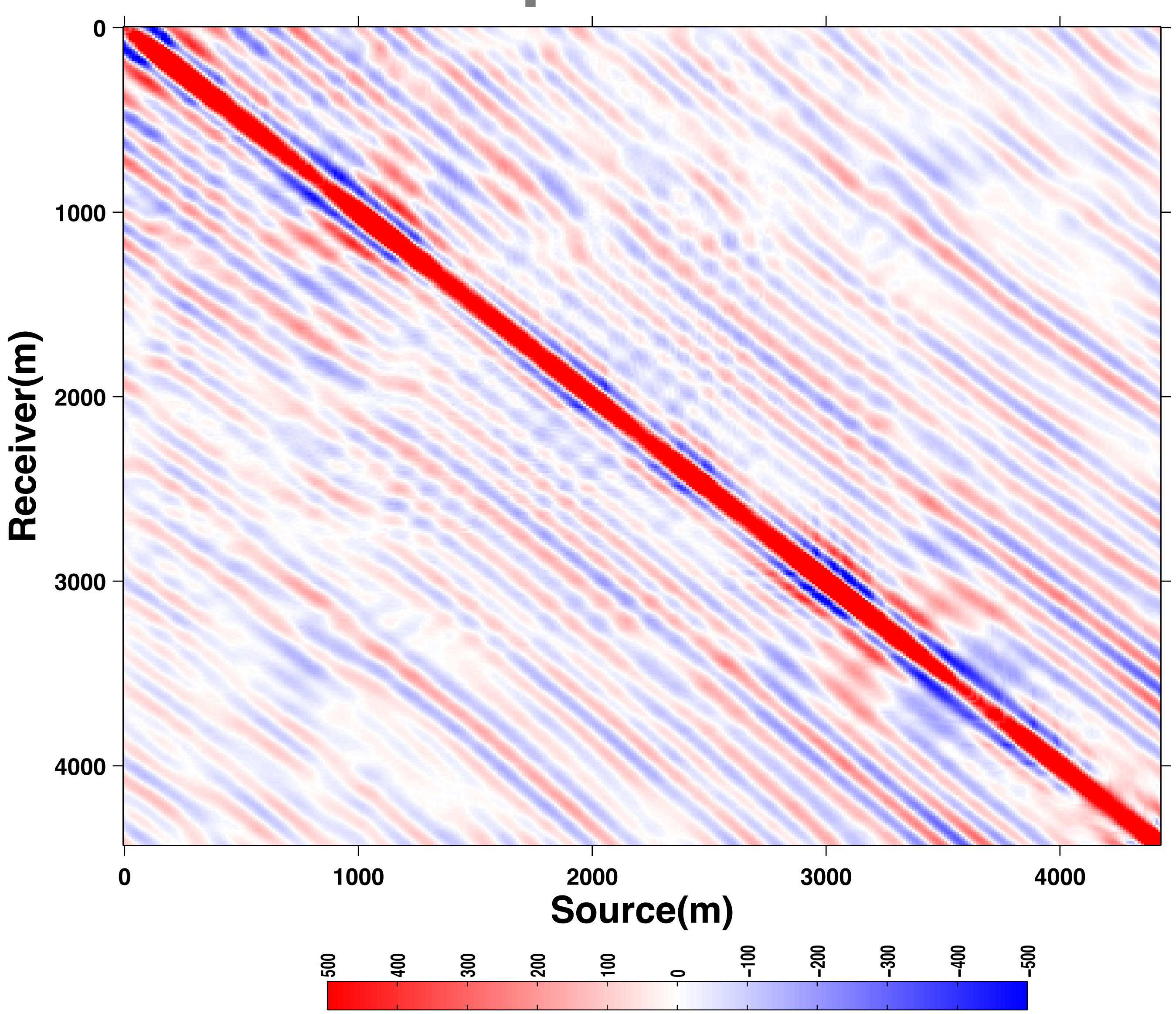
2D Interpolation

2D Acquisition

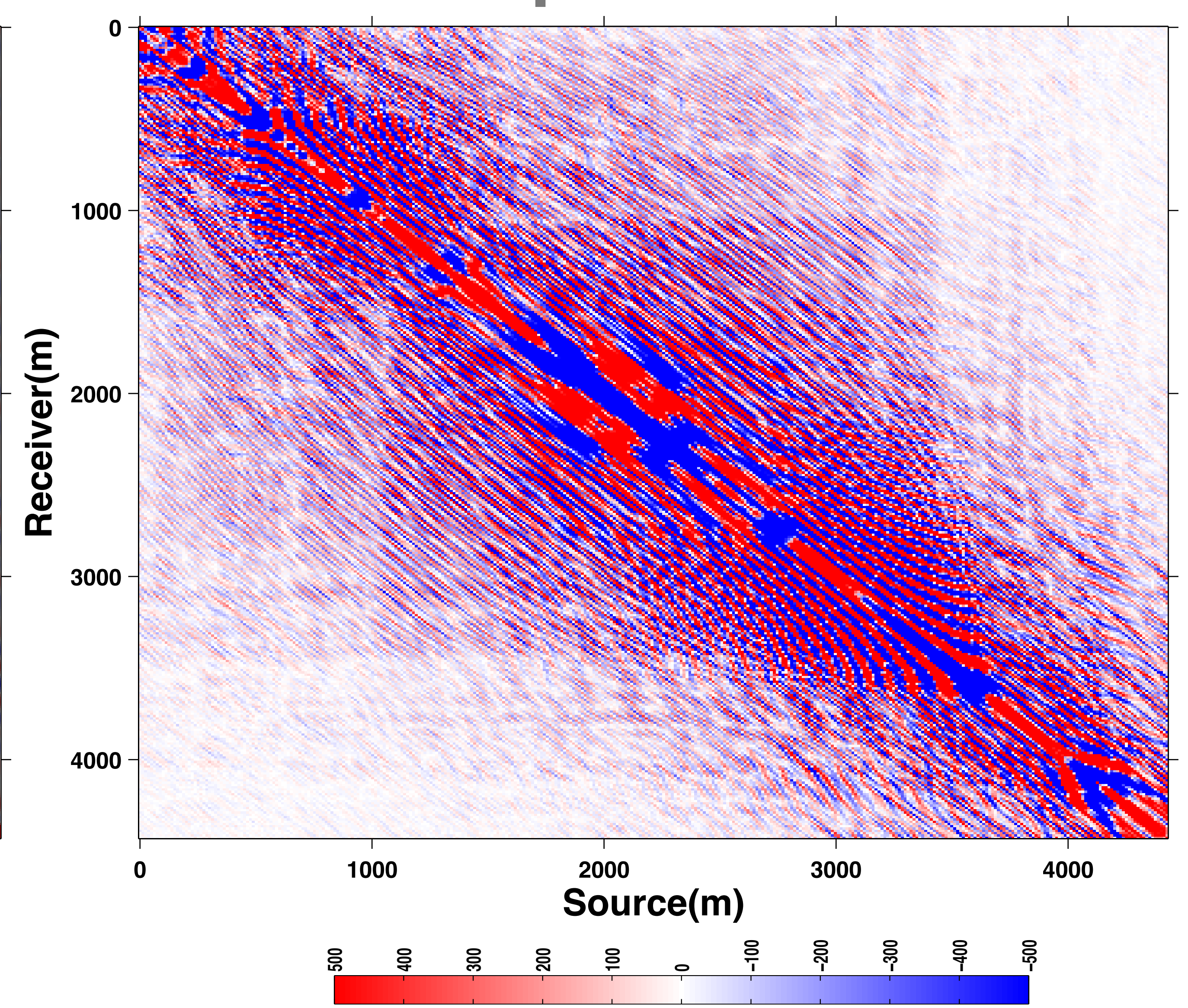
- Gulf of Suez data set
- 354 receivers, 354 sources, 1024 time samples
- monochromatic frequency slice
- rank adjusted according to frequencies

Regular sampled data

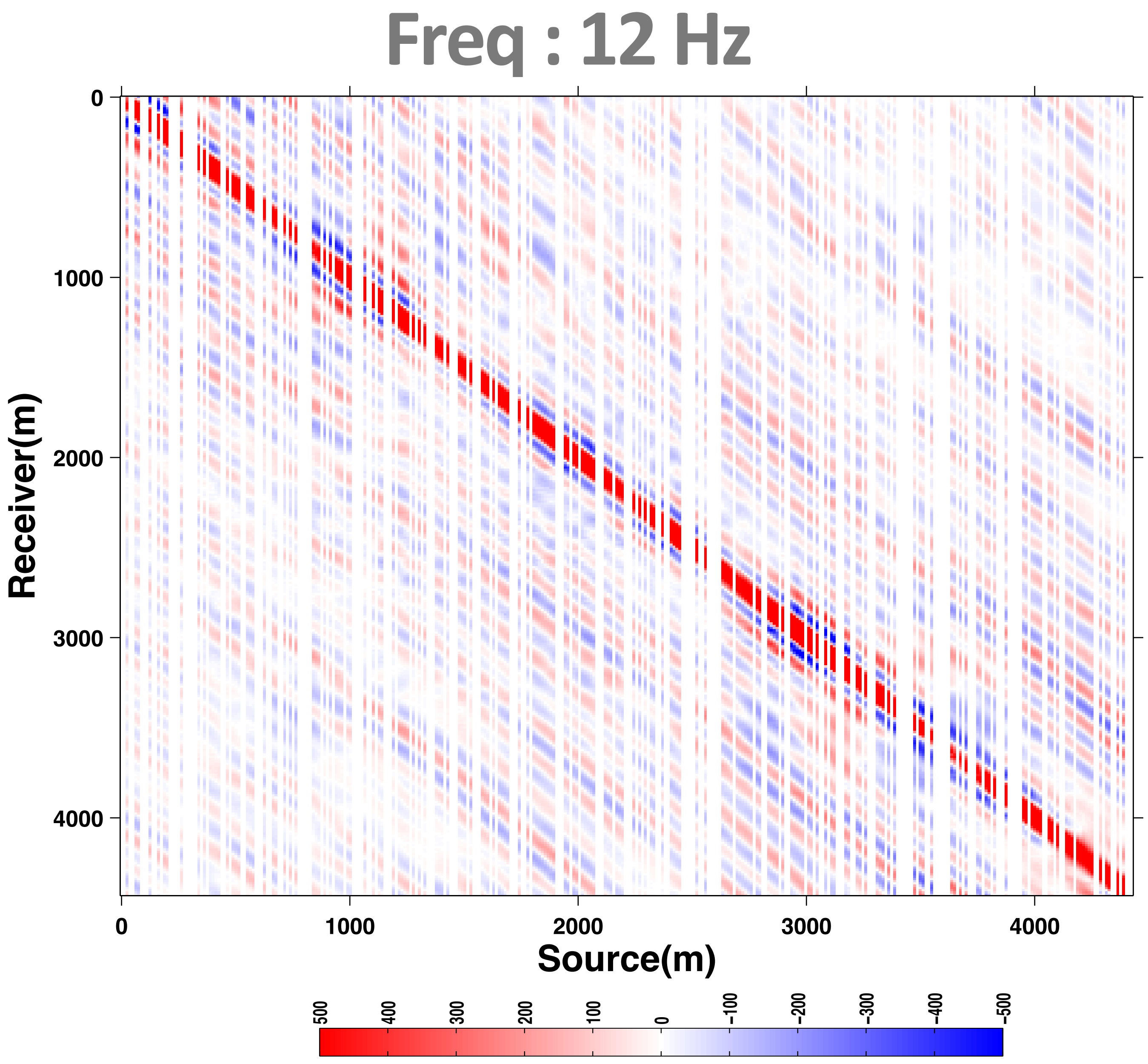
Freq : 12 Hz

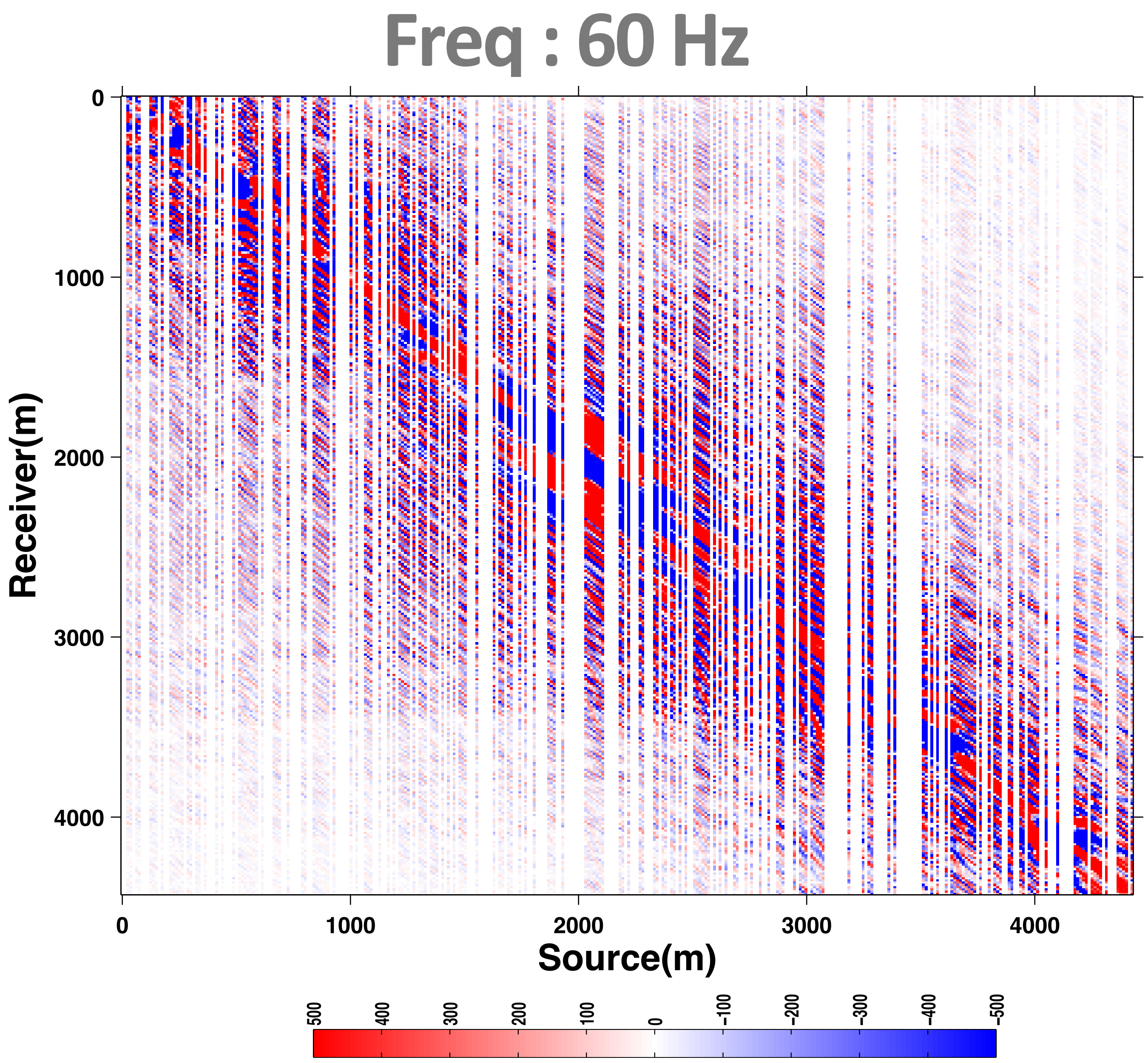


Freq : 60 Hz

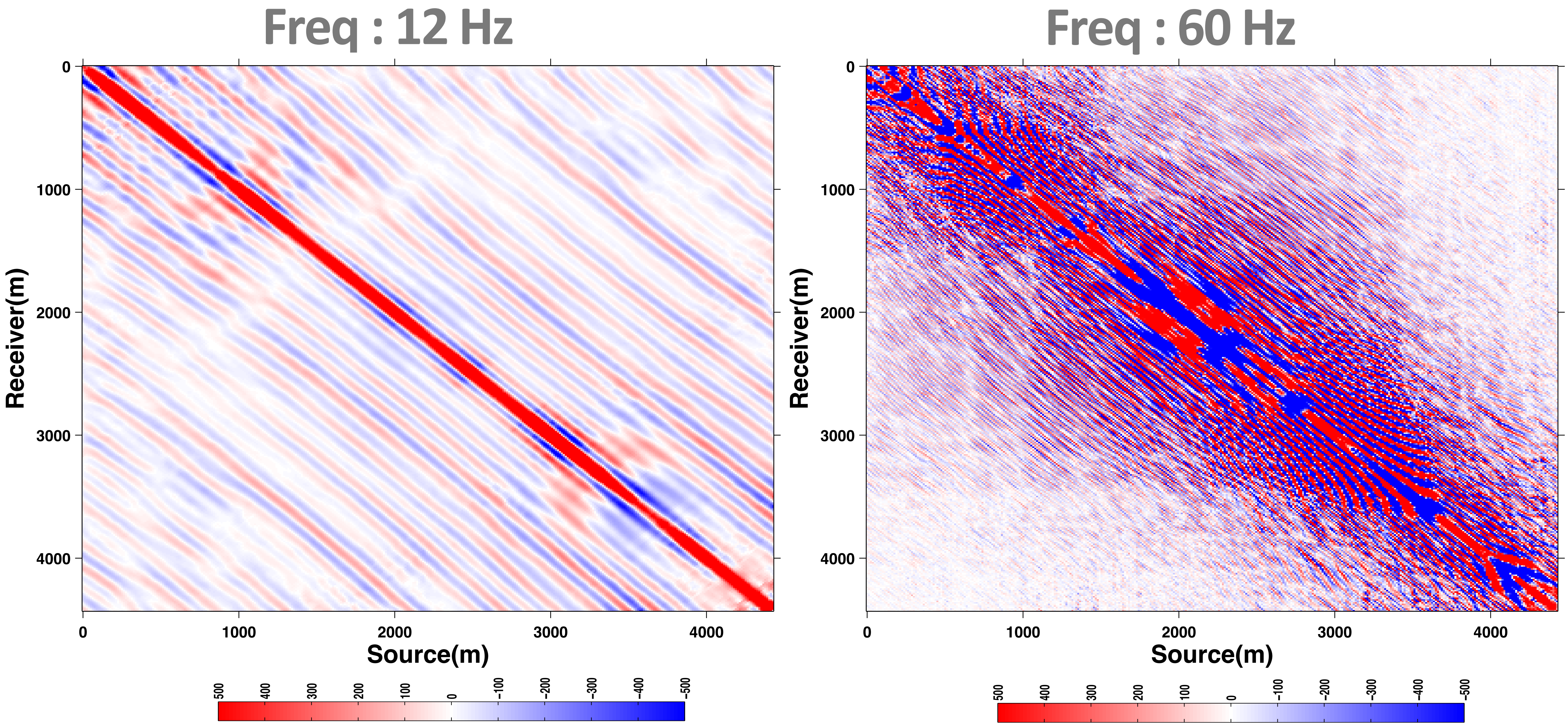


50% Missing Shots





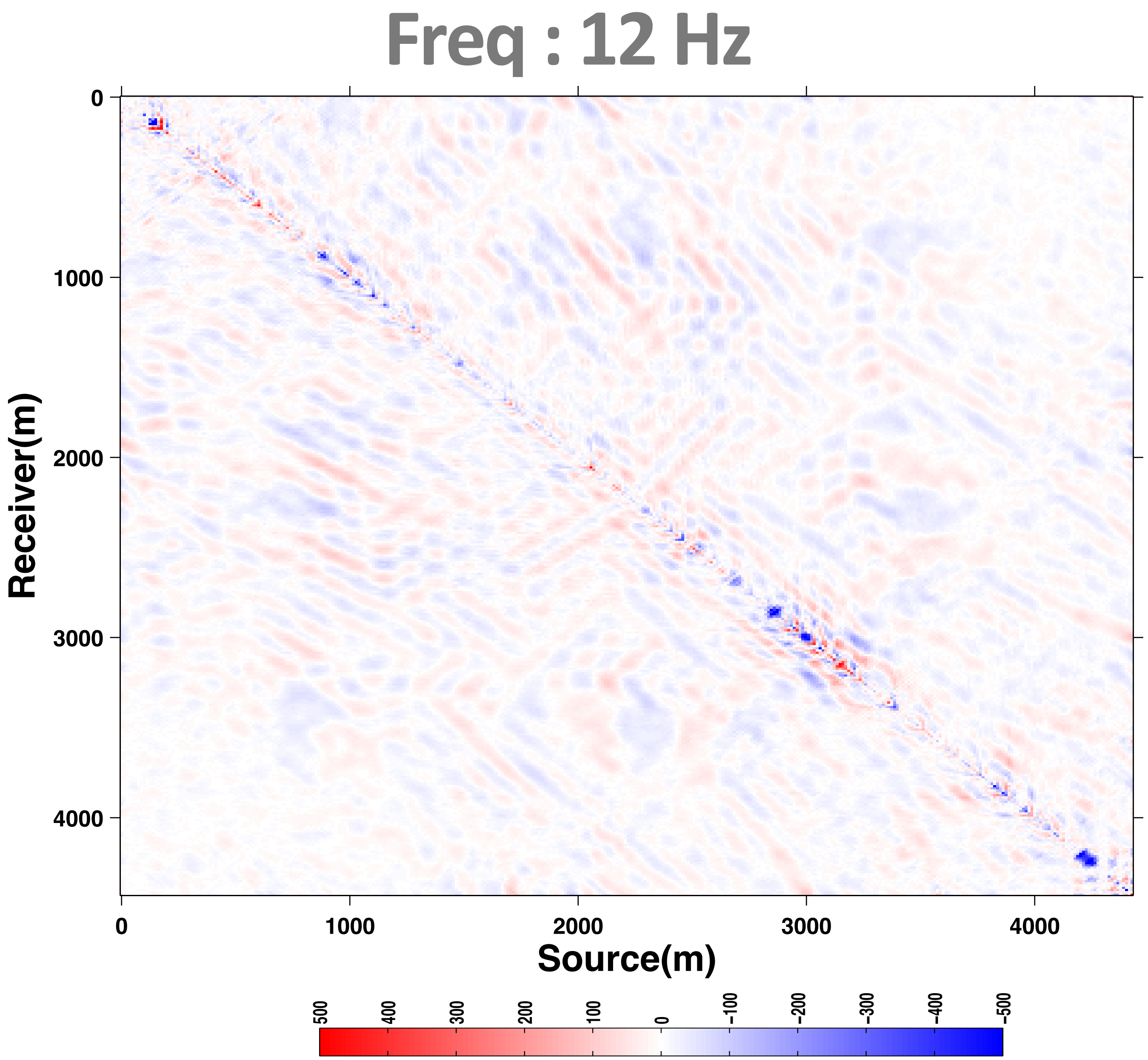
Recovery using MH formulation

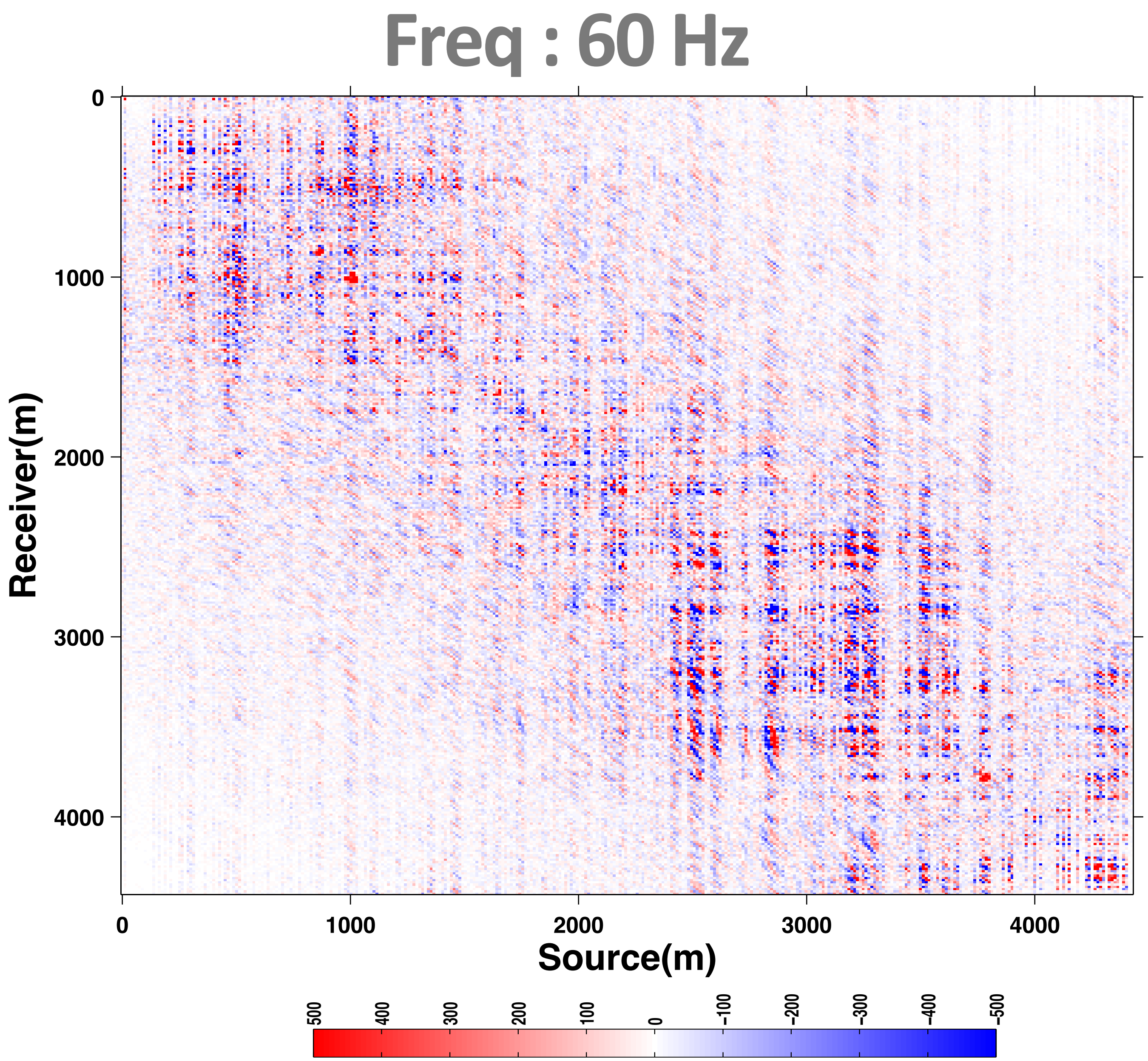


SNR = 13.8 db, Rank=5

SNR = 16.1 db, Rank=30

Residual


Freq : 12 Hz
Receiver(m)
Source(m)
0
1000
2000
3000
4000
500
400
300
200
100
0
-100
-200
-300
-400
-500


Freq : 60 Hz
Receiver(m)
Source(m)
0
1000
2000
3000
4000
500
400
300
200
100
0
-100
-200
-300
-400
-500

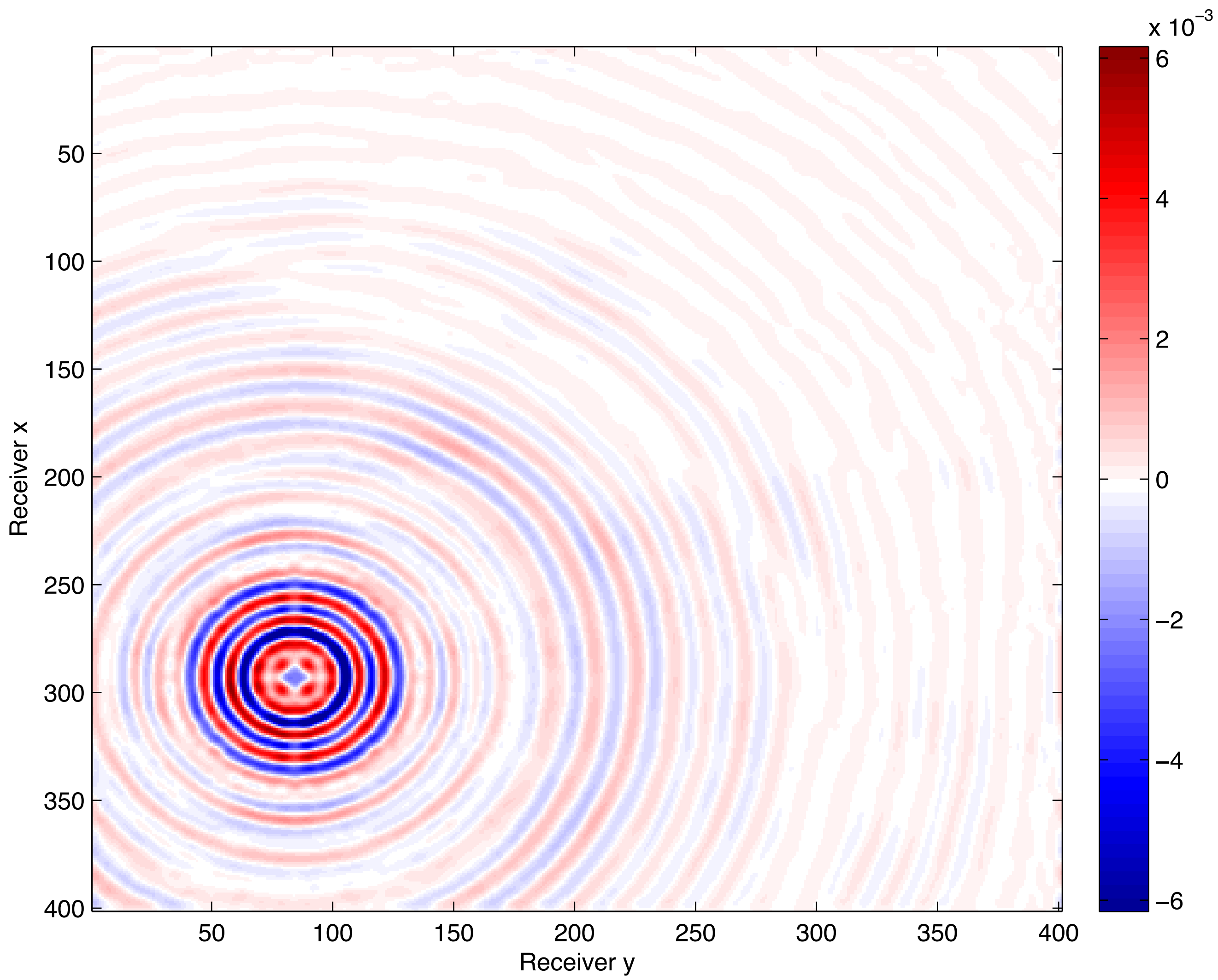
3D Interpolation

3D Acquisition

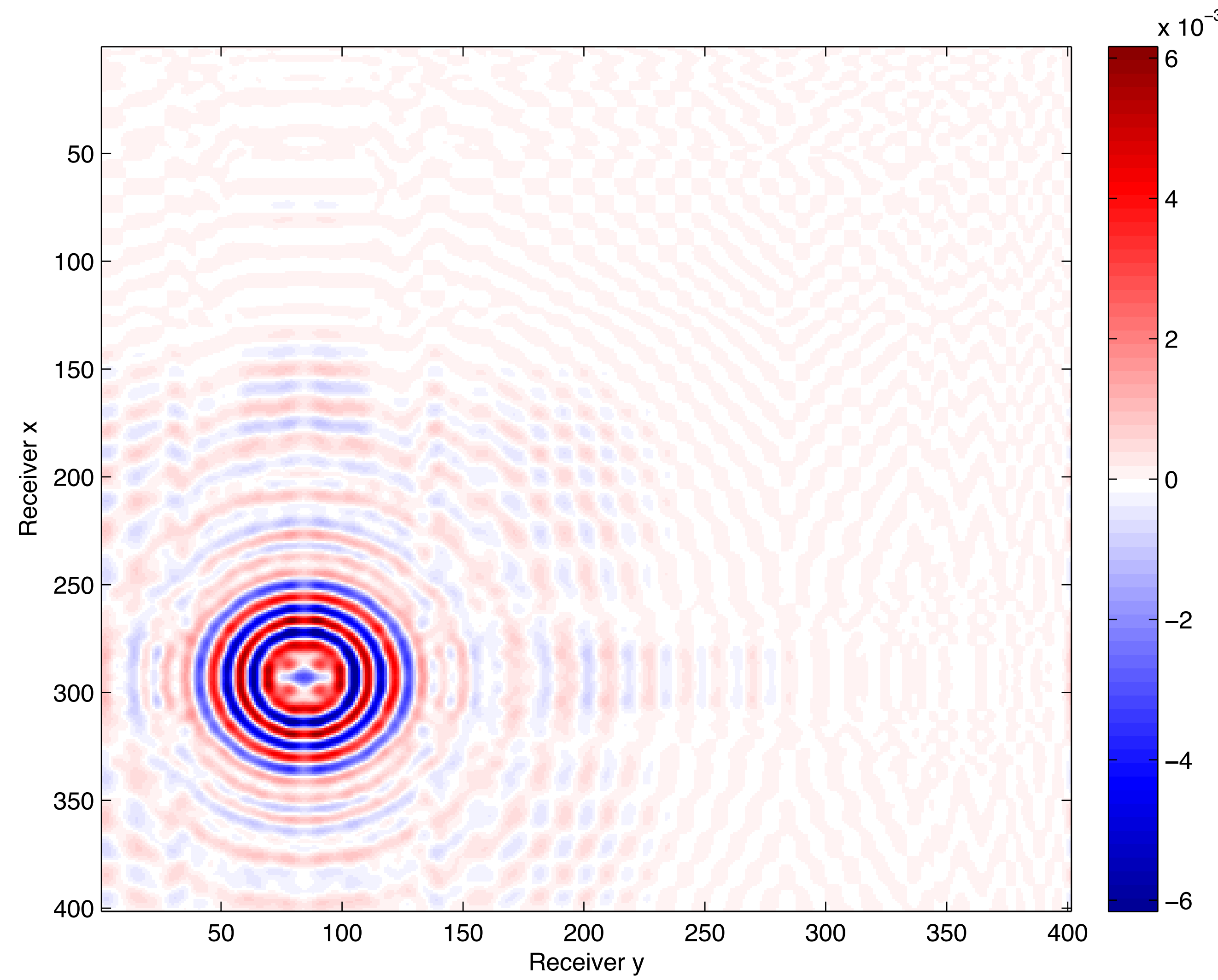
- Synthetic BG data set
- 200 *randomly subsampled* shots out of 4624 i.e 4% of shots
- 1201 x 1201 receivers
- Implementation of formulation on smaller grid with 401 x 401 receivers
- monochromatic frequency slice at 7.34 Hz
- recovery for rank 10

Recovery - Shot Gather
[Rank=10, 500 Iterations]

True data
(SrcX,SrcY = 50,15)

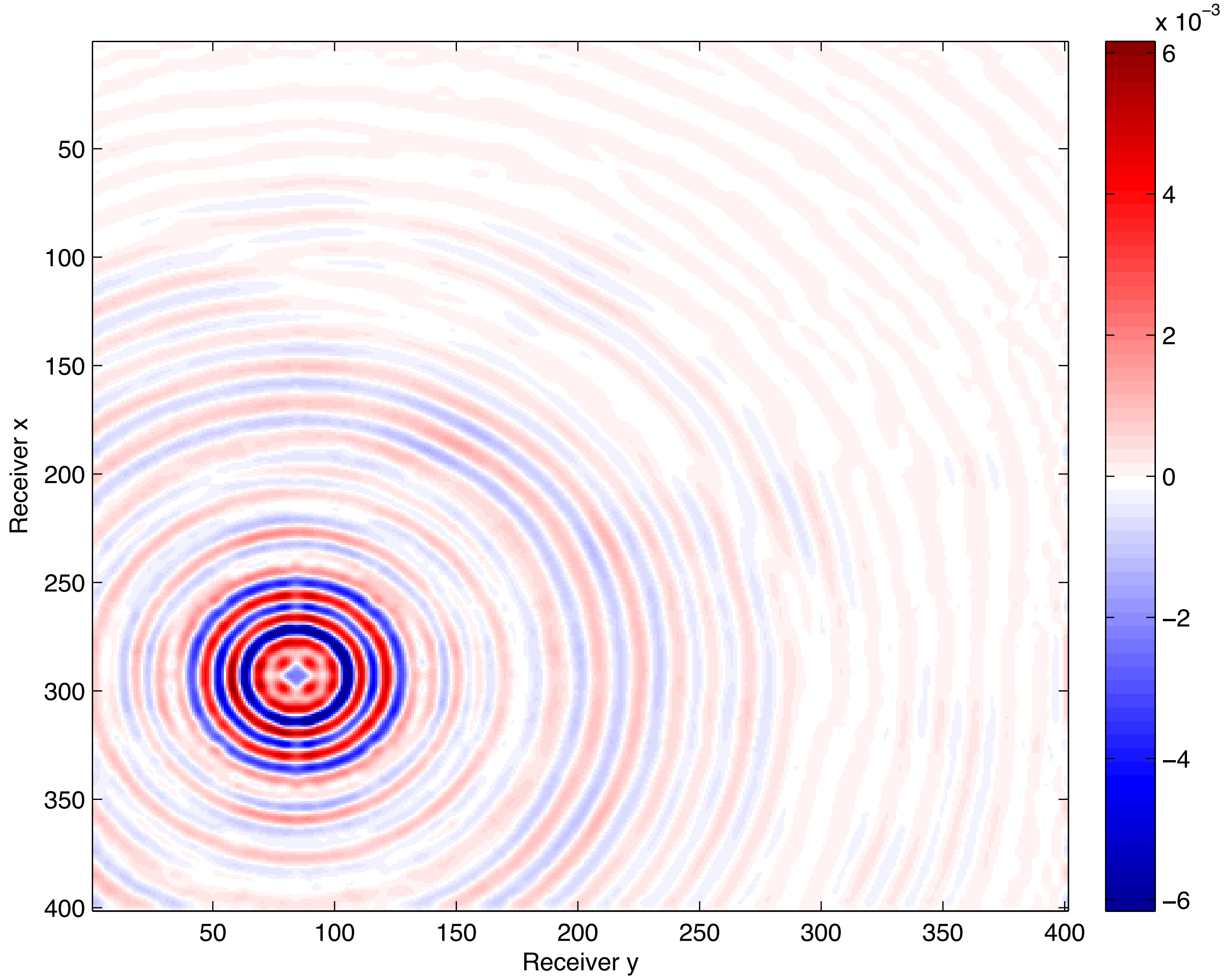


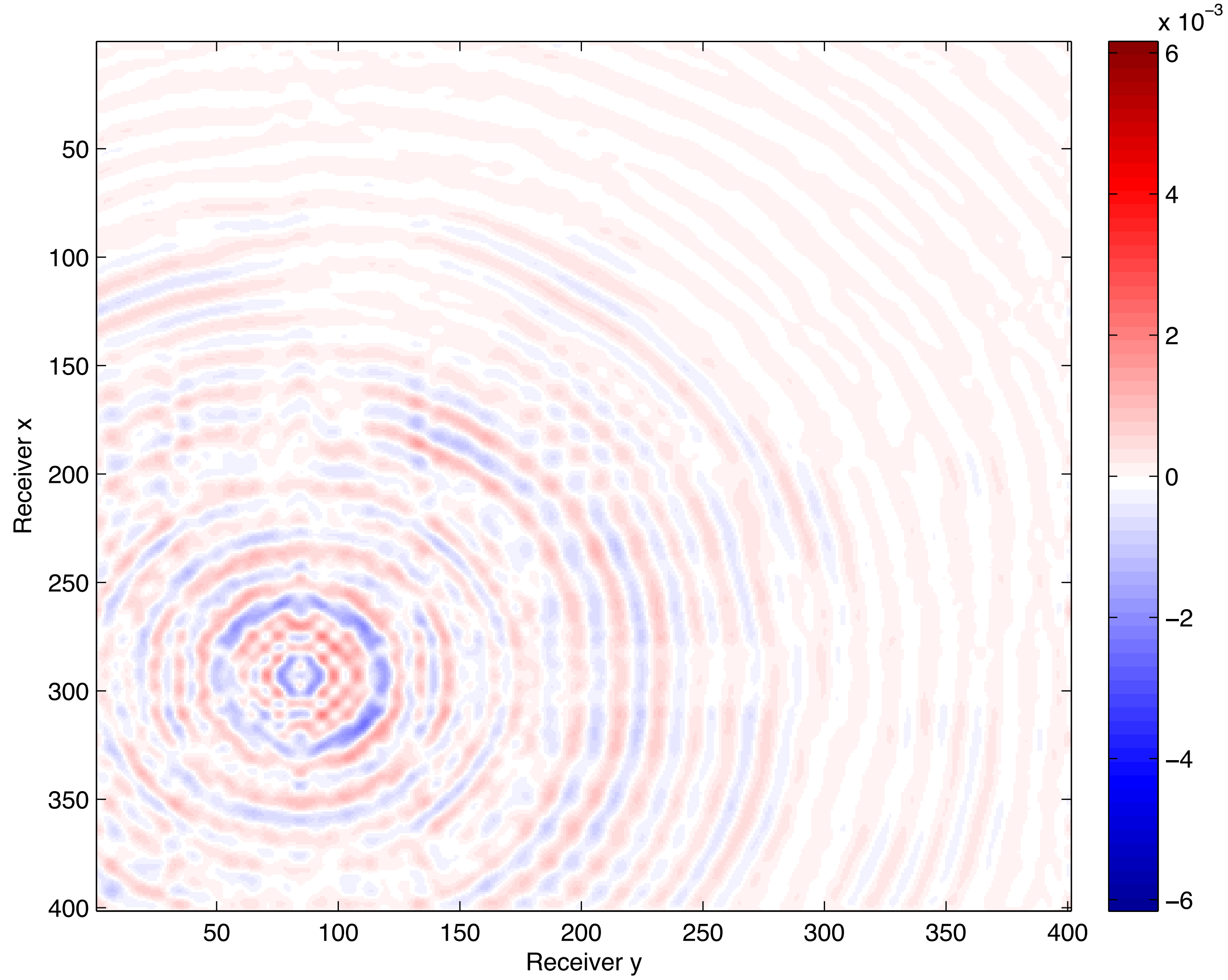
Approximated data
SNR=8.14 db



Residual - Shot Gather
[Rank=10, 500 Iterations]

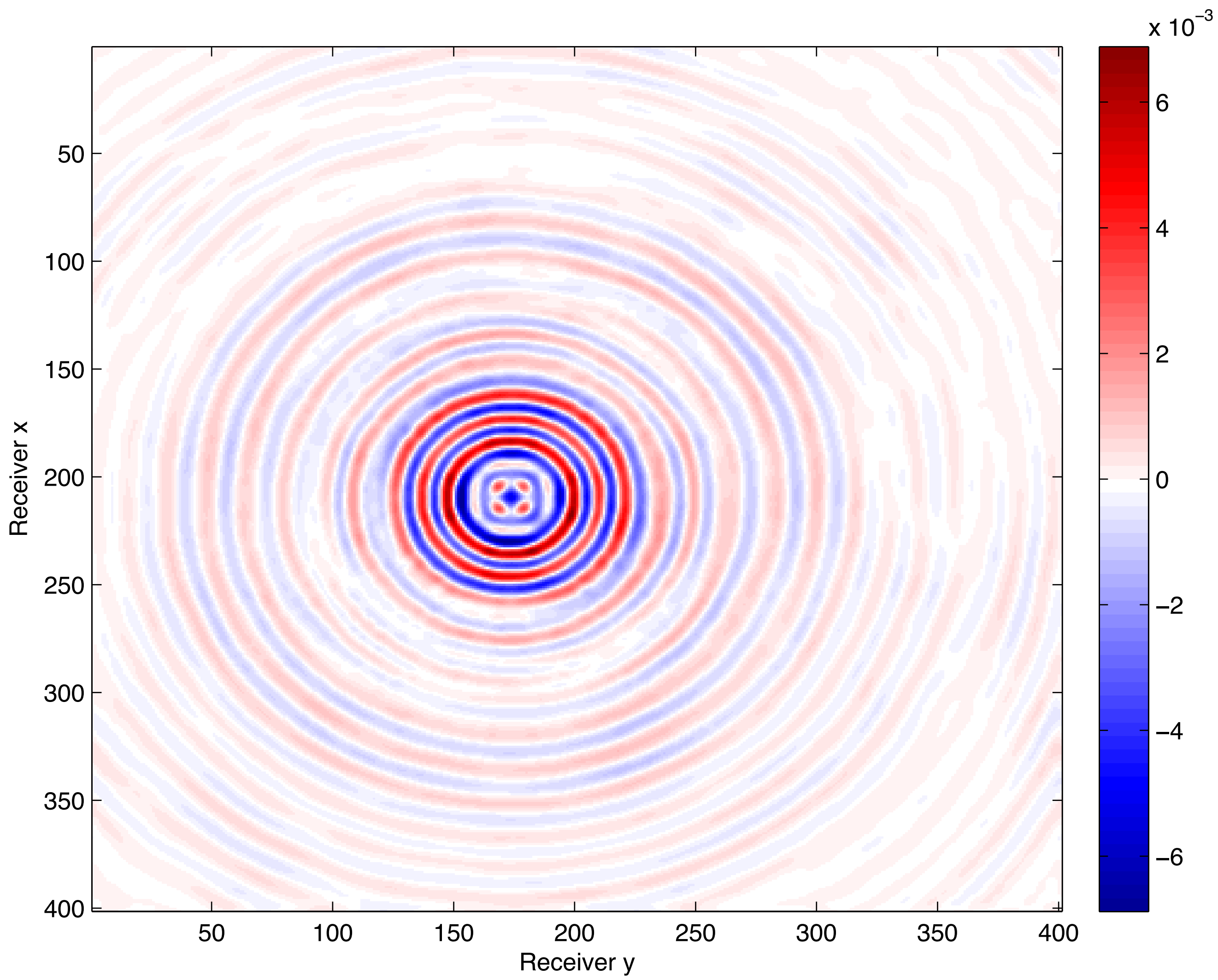
True data
(SrcX,SrcY = 50,15)



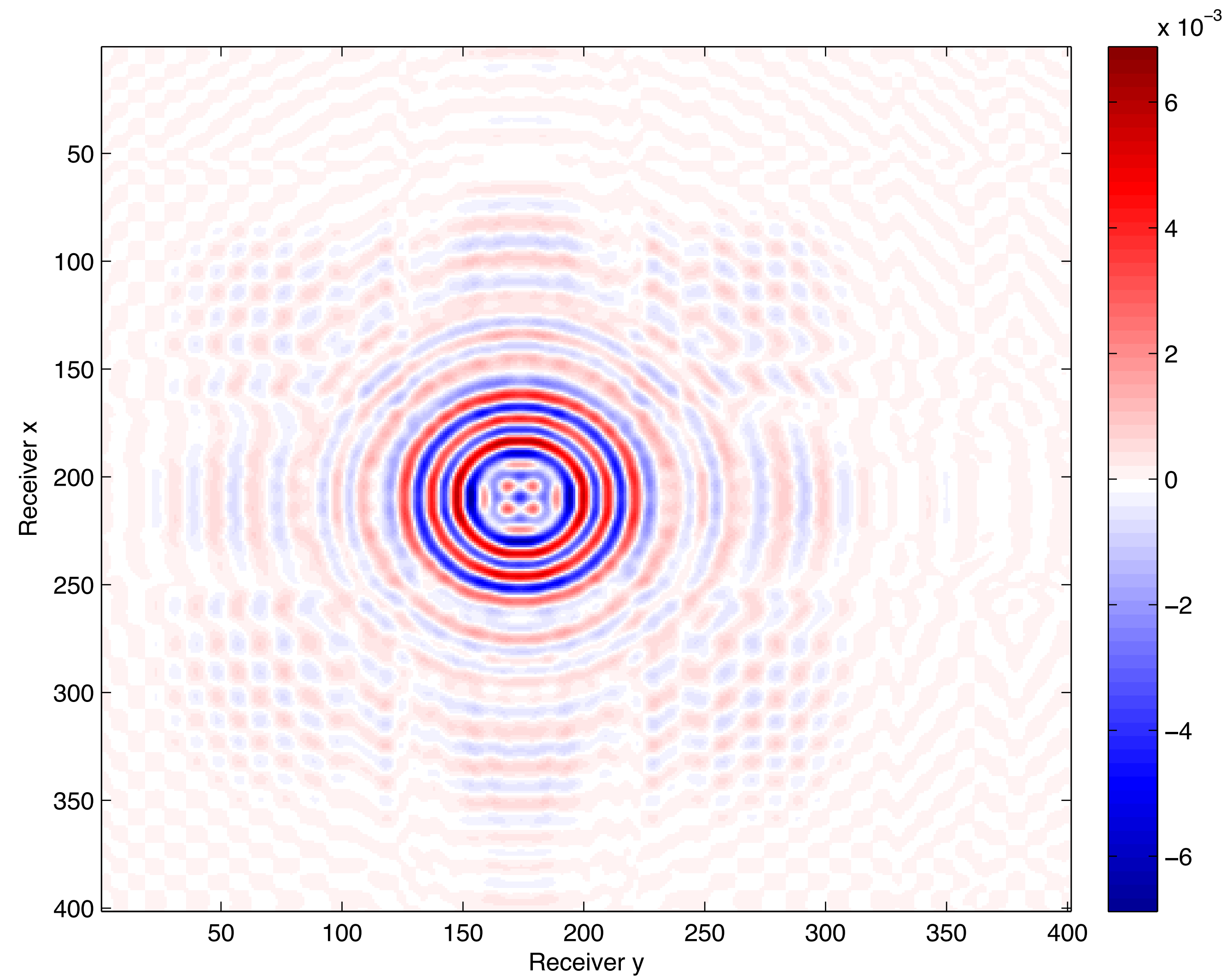


Recovery - Shot Gather
[Rank=10, 500 Iterations]

True data (SrcX,SrcY = 36,30)



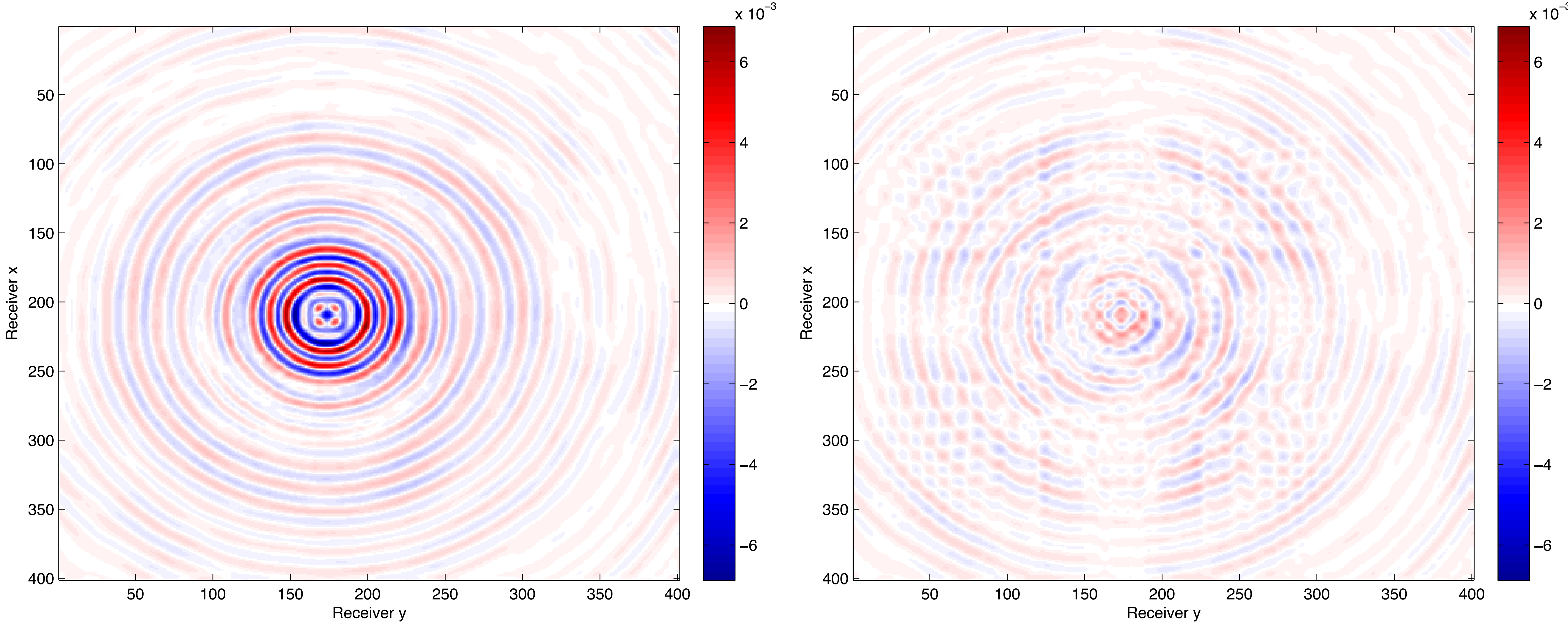
Approximated data SNR=8.49 db



Residual - Shot Gather
[Rank=10, 500 Iterations]

True data

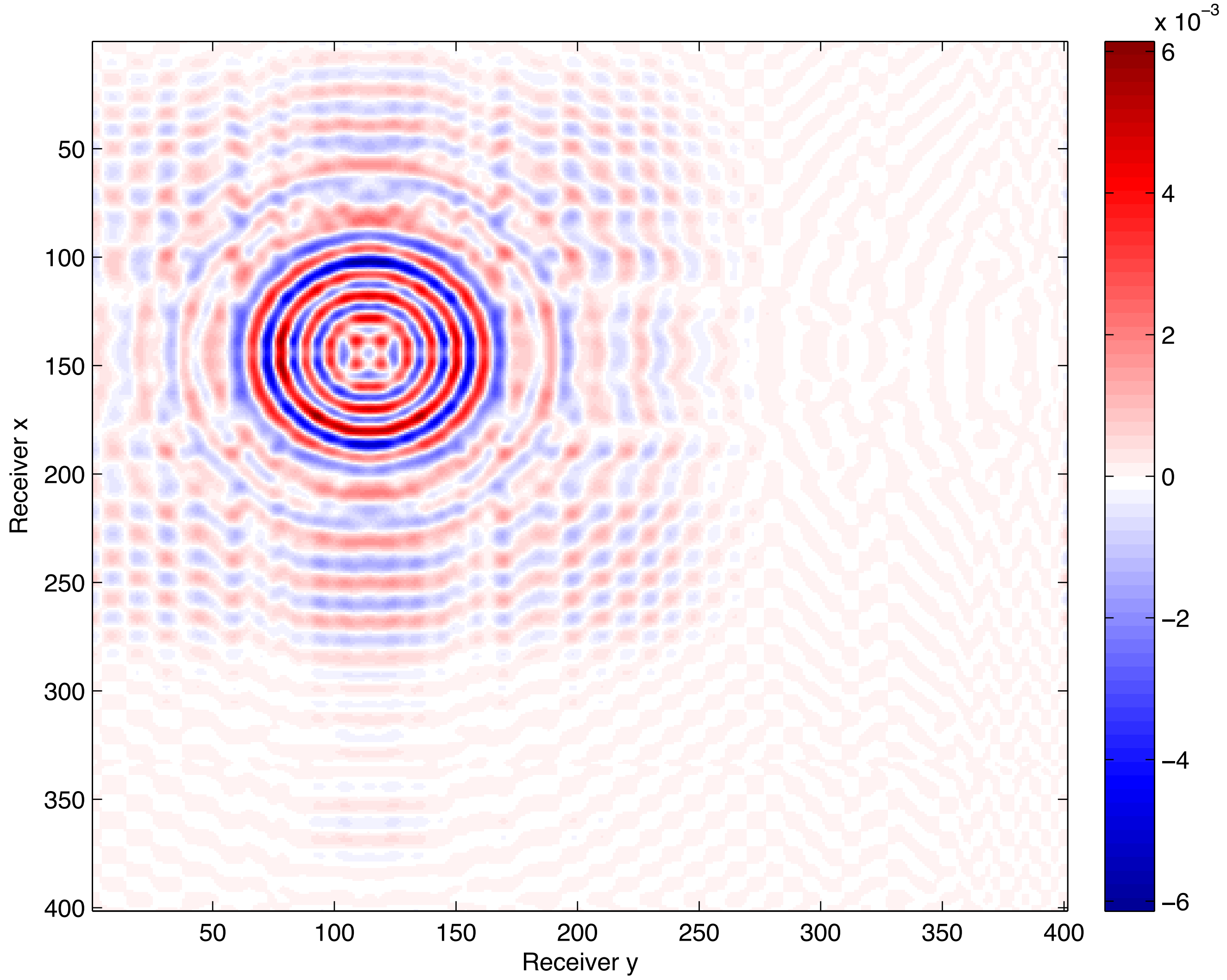
(SrcX,SrcY = 36,30)



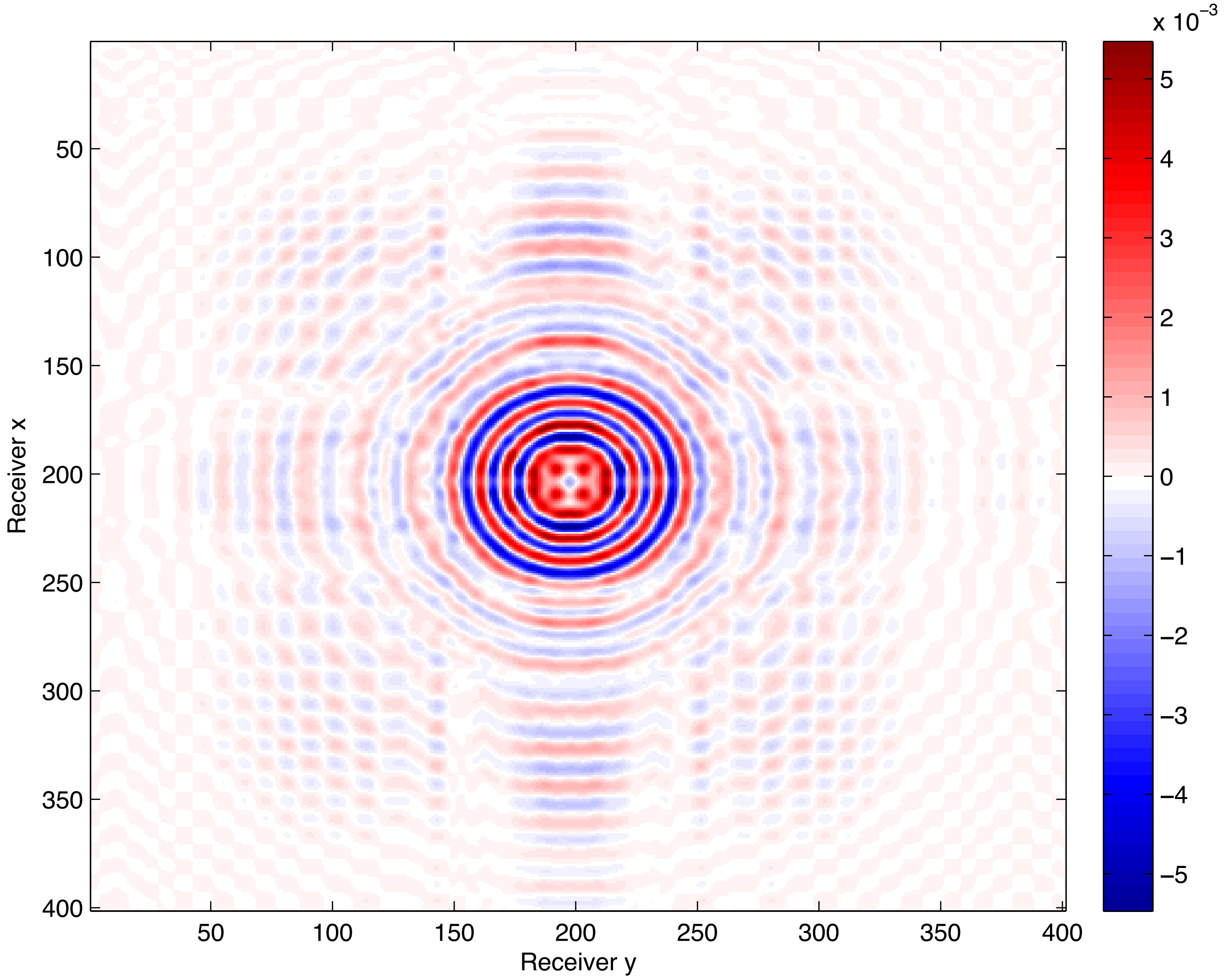
Monday, 3 December, 12

Recovery - Shot Gather
[Rank=10, 500 Iterations]

Interpolated data (SrcX,SrcY = 25,20)



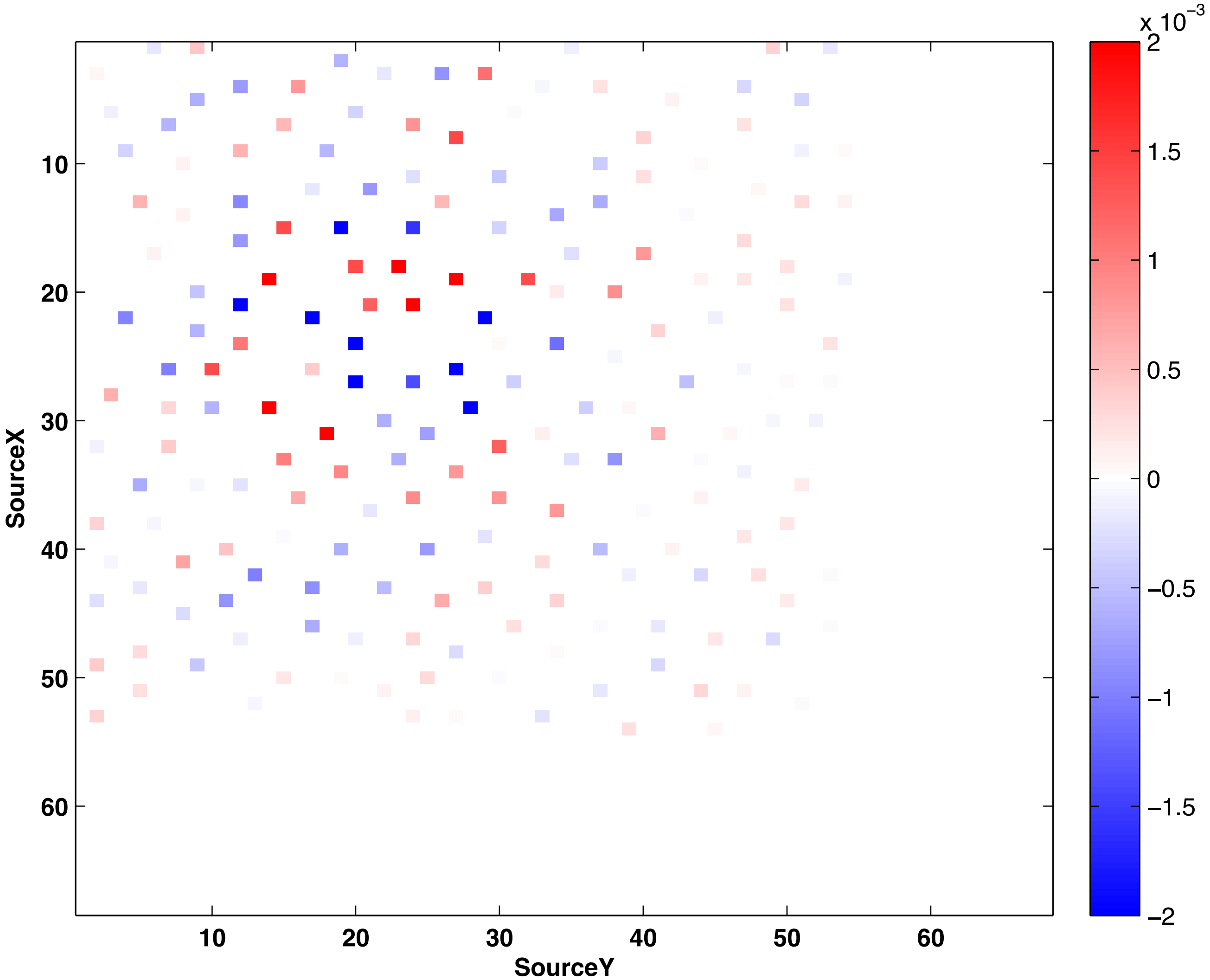
Interpolated data (SrcX,SrcY = 35,34)



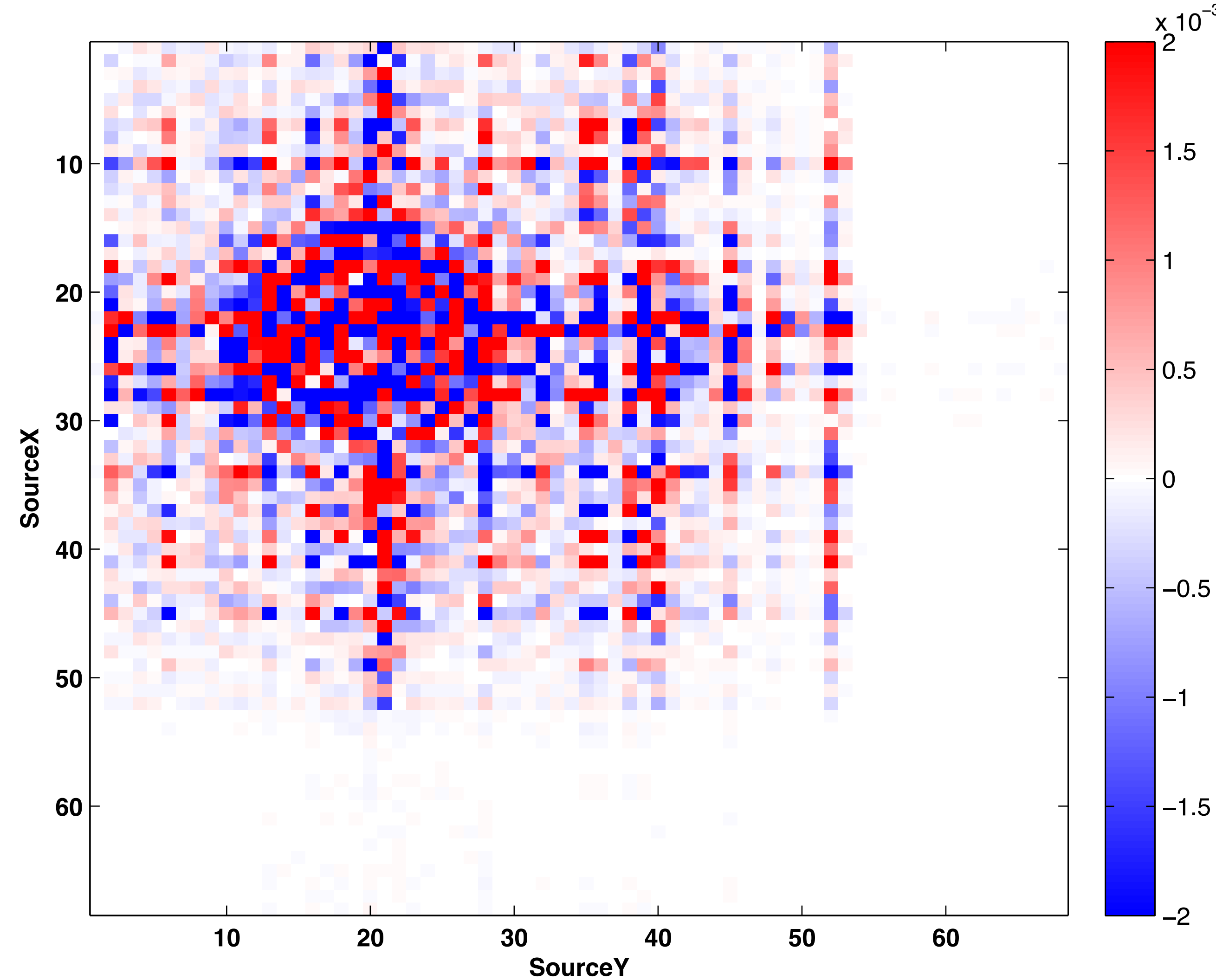
Recovery - Receiver Gather

[Rank=10, 500 Iterations]

Acquired data (RecX,RecY = 35,30)



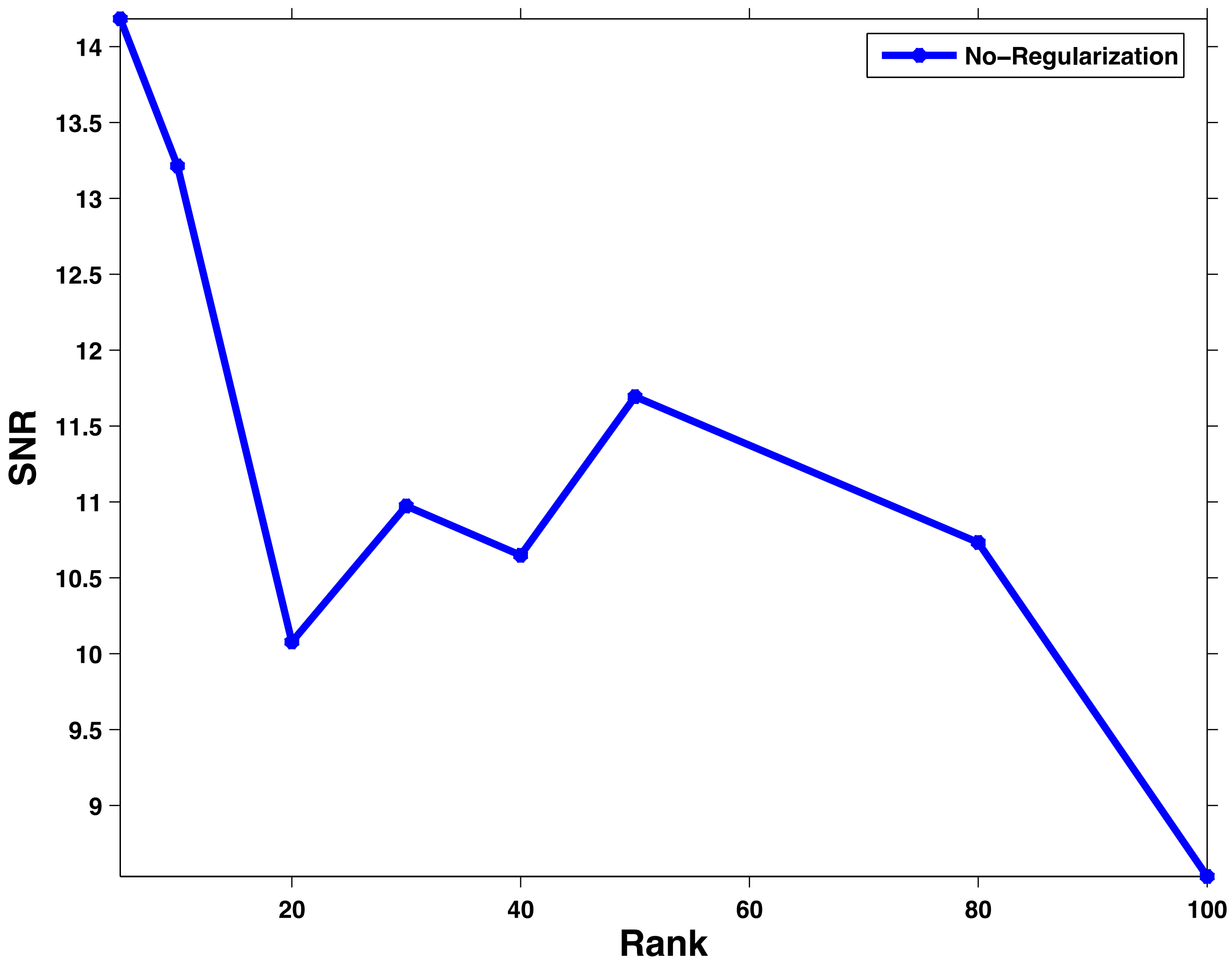
Interpolated data (RecX,RecY = 35,30)



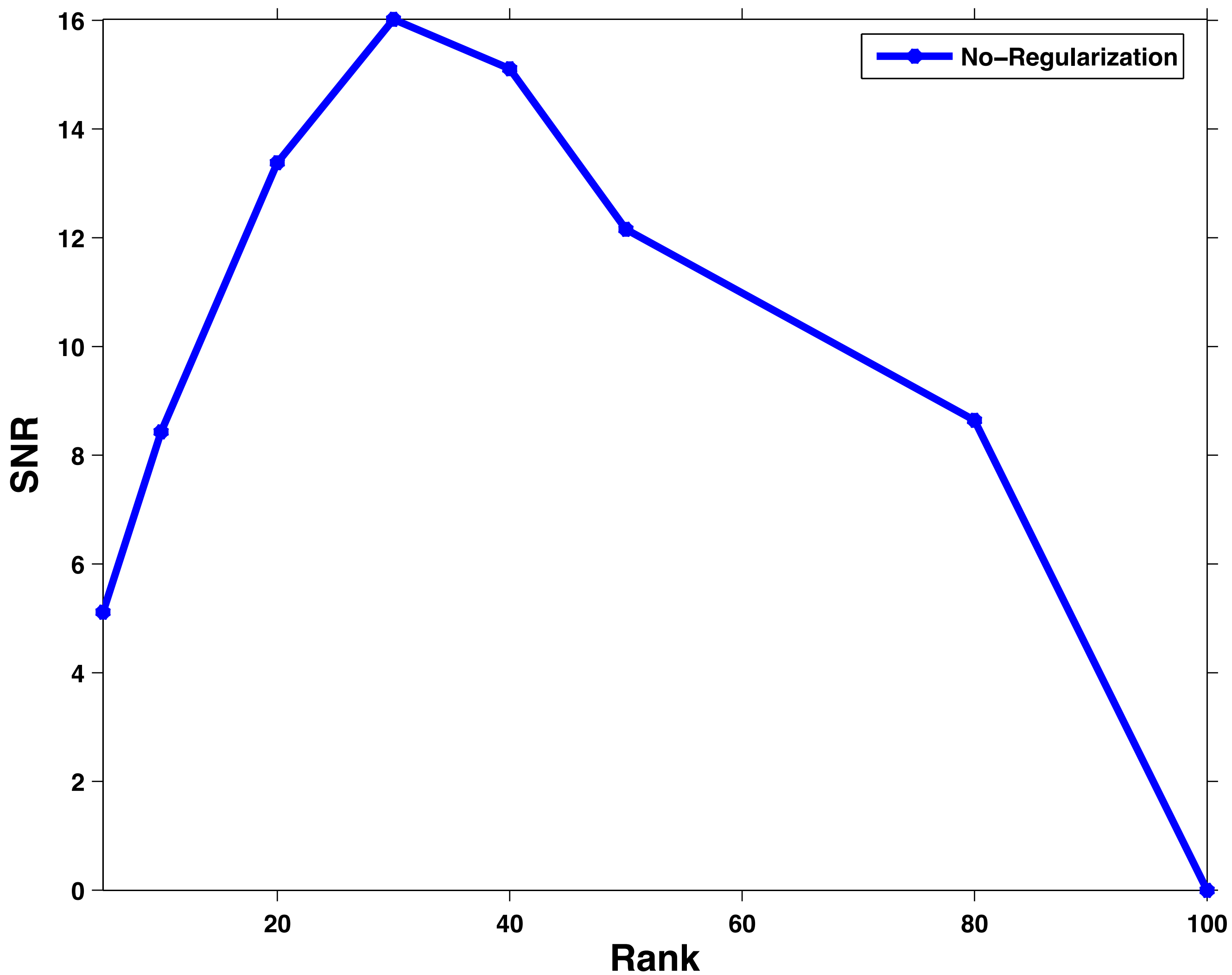
Recovery from 96% missing data

Limitation
[SNR vs Rank - 2D Interpolation]

Better SNR can achieve by increasing k, but unconstrained formulation starts over-fitting the data



Freq : 12 Hz



Freq : 60 Hz

Regularized Matrix Factorization

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Methodology

- given data matrix $\mathbf{b}$ (in a "*transform* domain")

- solve the following formulation :

$$(LASSO_\tau) \qquad \min_X ||\mathcal{A}(X) - \mathbf{b}||_2^2 \quad \text{s.t.} \quad ||X||_* \leq \tau$$

where τ is a rank regularization parameter.

(Recht et al.)

SLIM

Methodology

- Parametrize $X \in \mathbb{R}^{n\times m}$ using factors:

$$X = LR'$$

where $L \in \mathbb{R}^{n\times k}$ and $R \in \mathbb{R}^{m\times k}$

- Solve the following rank penalized problem:

$$\min_{L,R} \|\mathcal{A}(LR') - \mathbf{b}\|_2^2 \quad \text{s.t.} \; \|LR'\|_* \leq \tau$$

(Recht et al.)

Methodology

- nuclear norm is given by :

$$\|X\|_* = \inf_{X=LR'} \frac{1}{2} \left\| \begin{bmatrix} L \\ R \end{bmatrix} \right\|_F^2$$

where $\|.\|_F^2$ is sum of squares of all entries

- Therefore, $\|LR'\|_* \leq \frac{1}{2} \left\| \begin{bmatrix} L \\ R \end{bmatrix} \right\|_F^2$

  and we can avoid SVDs for projection.

Methodology

- We solve the BPDN problem

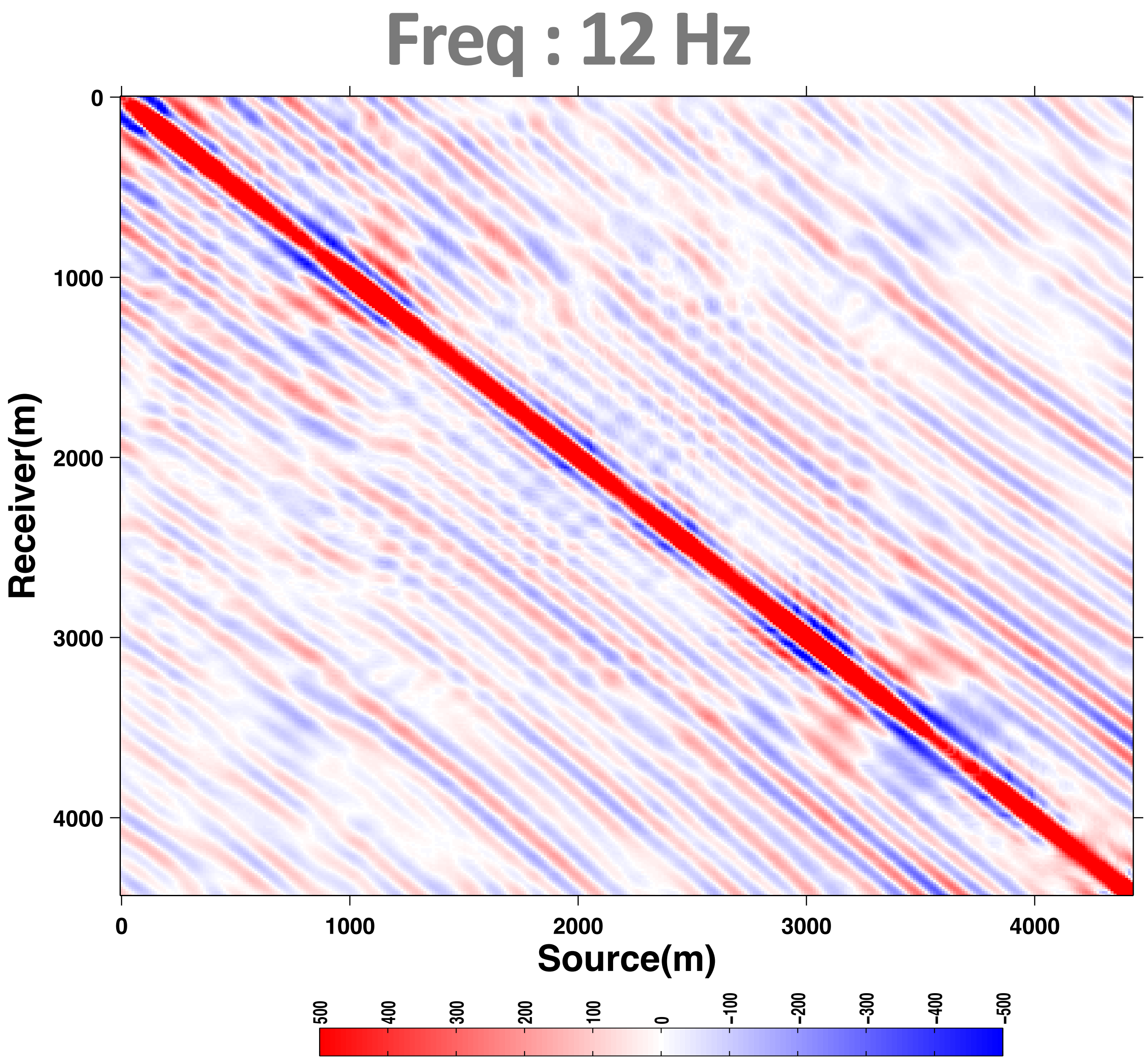
$$\min_X ||X||_* \quad \text{s.t. } ||\mathcal{A}(X) - \mathbf{b}||_2^2 \leq \sigma$$

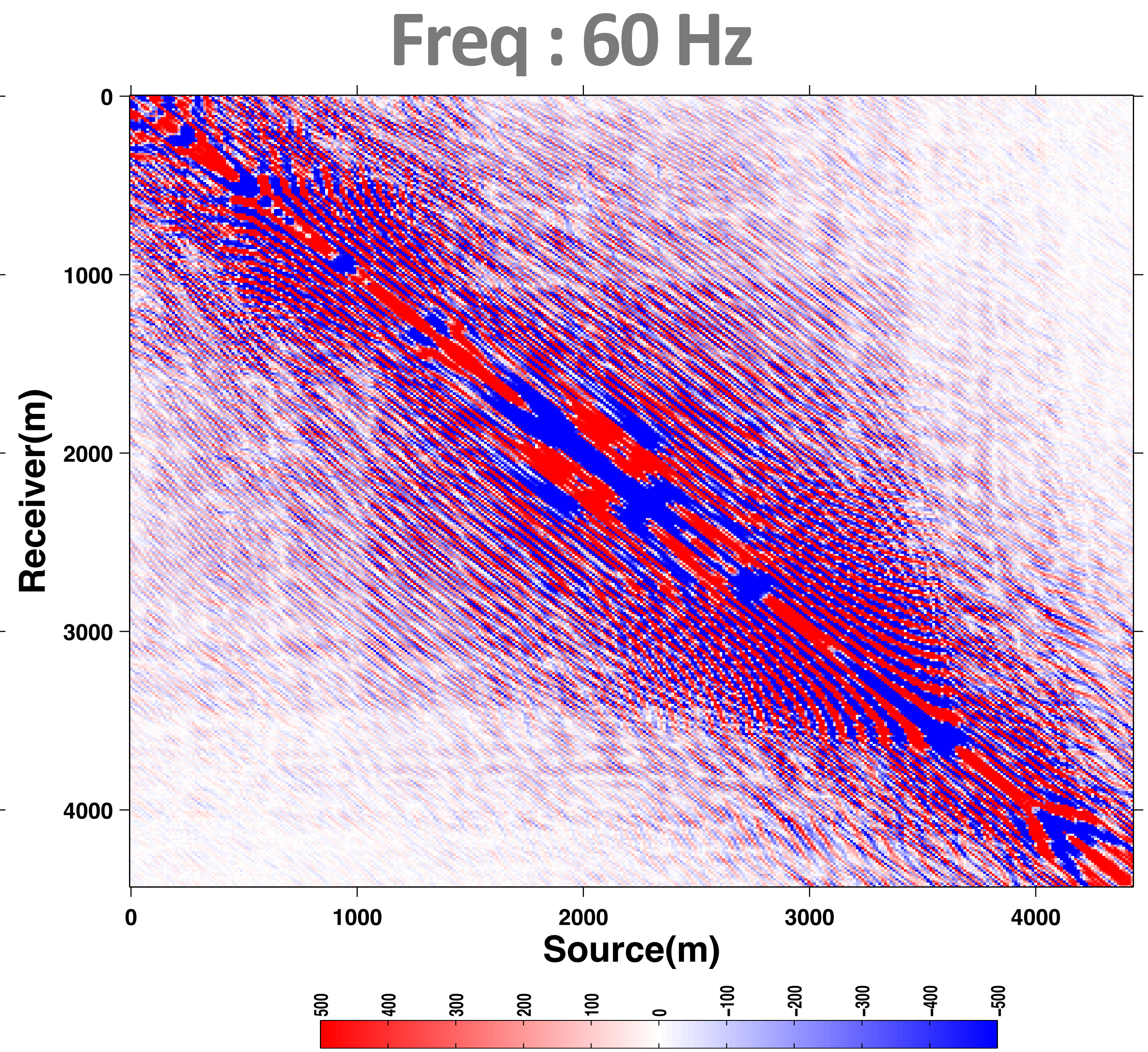
- We use 300 iterations of modified $\mathrm{SPG}\ell 1$ algorithm for the factorized formulation.

- Pareto curve uses the true dual of the $\|\cdot\|_*$ norm, but projection is done using factors.

Experiment & Results

Regular sampled data

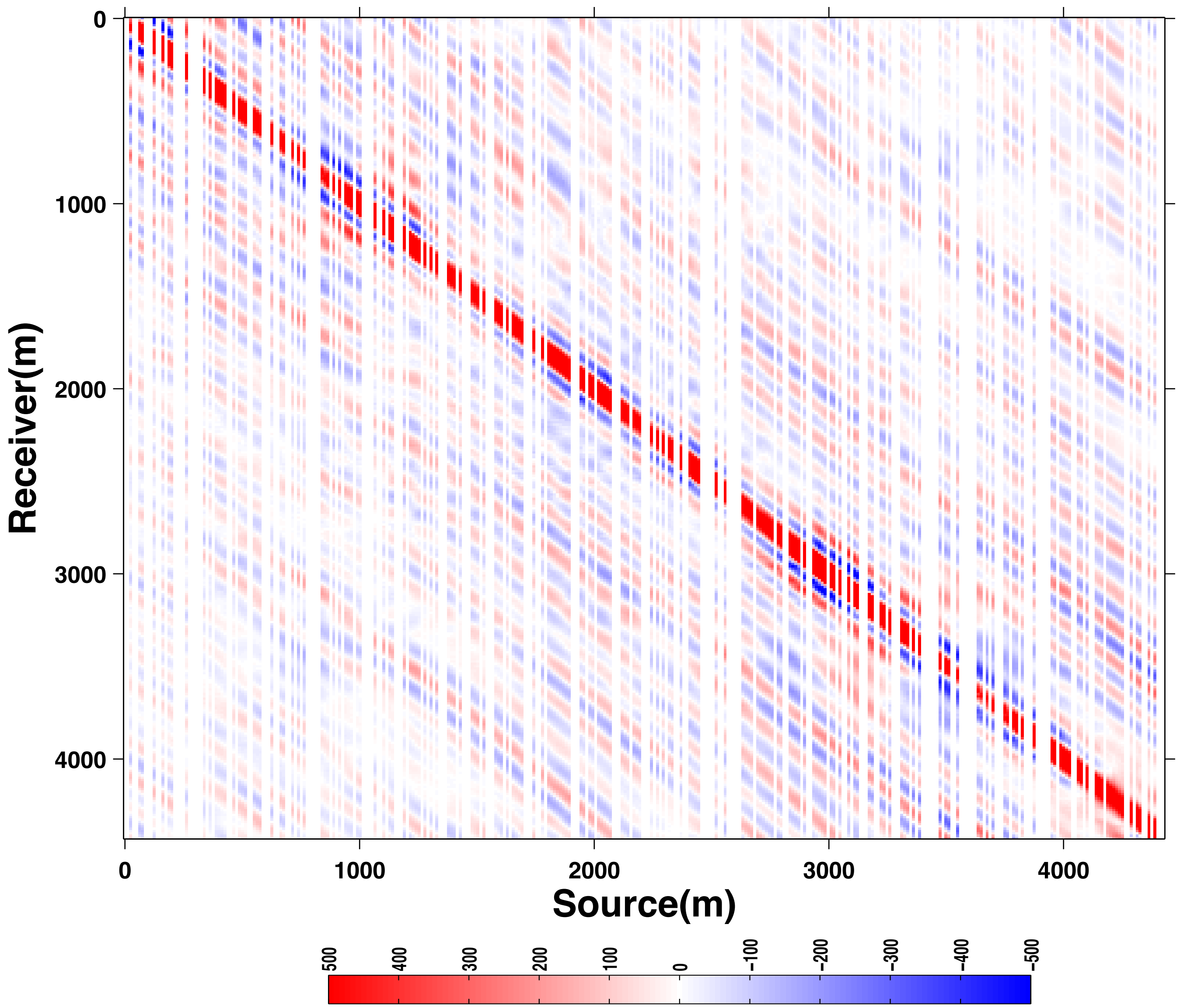




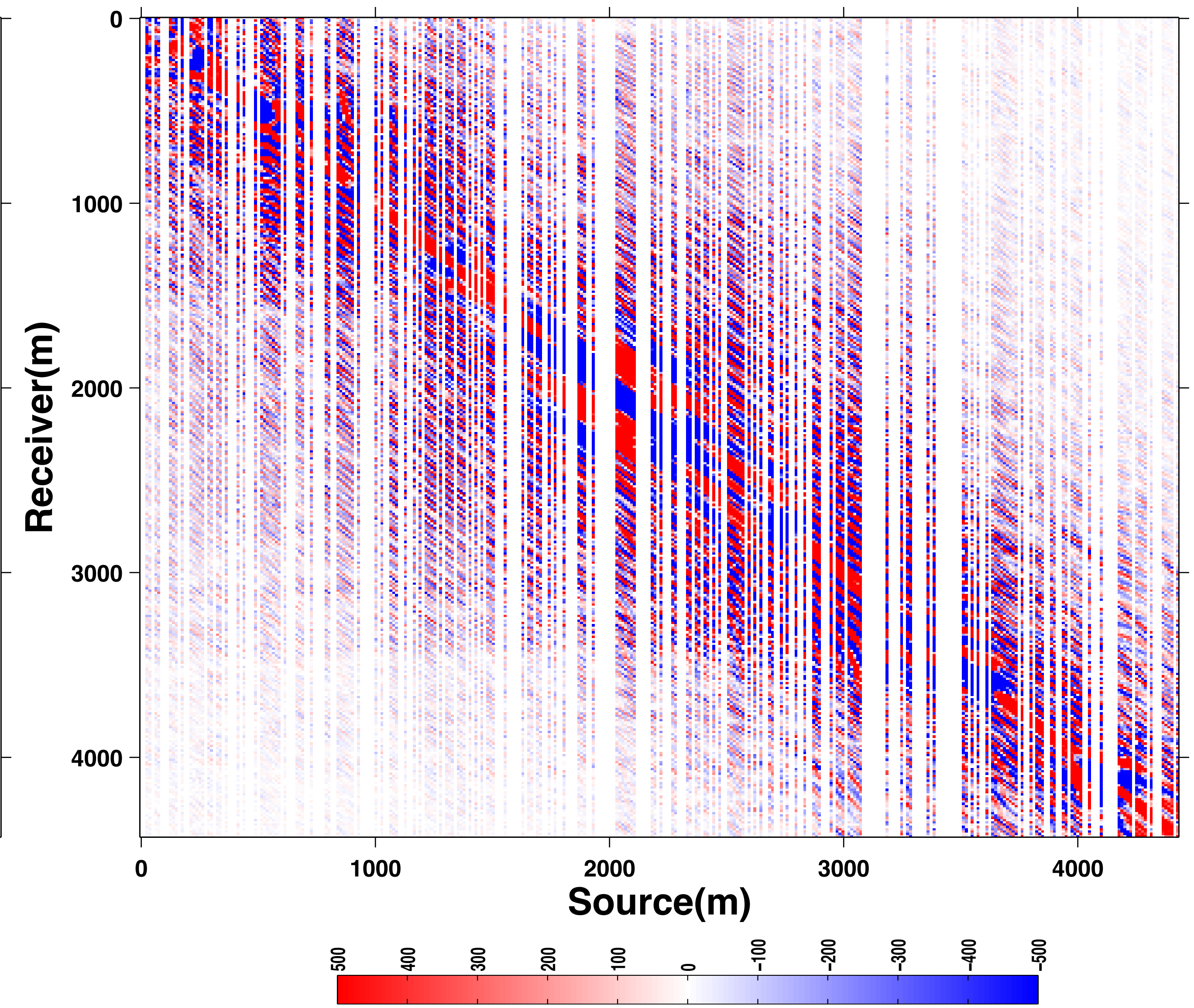
50% Missing Shots

SLIM

Freq : 12 Hz



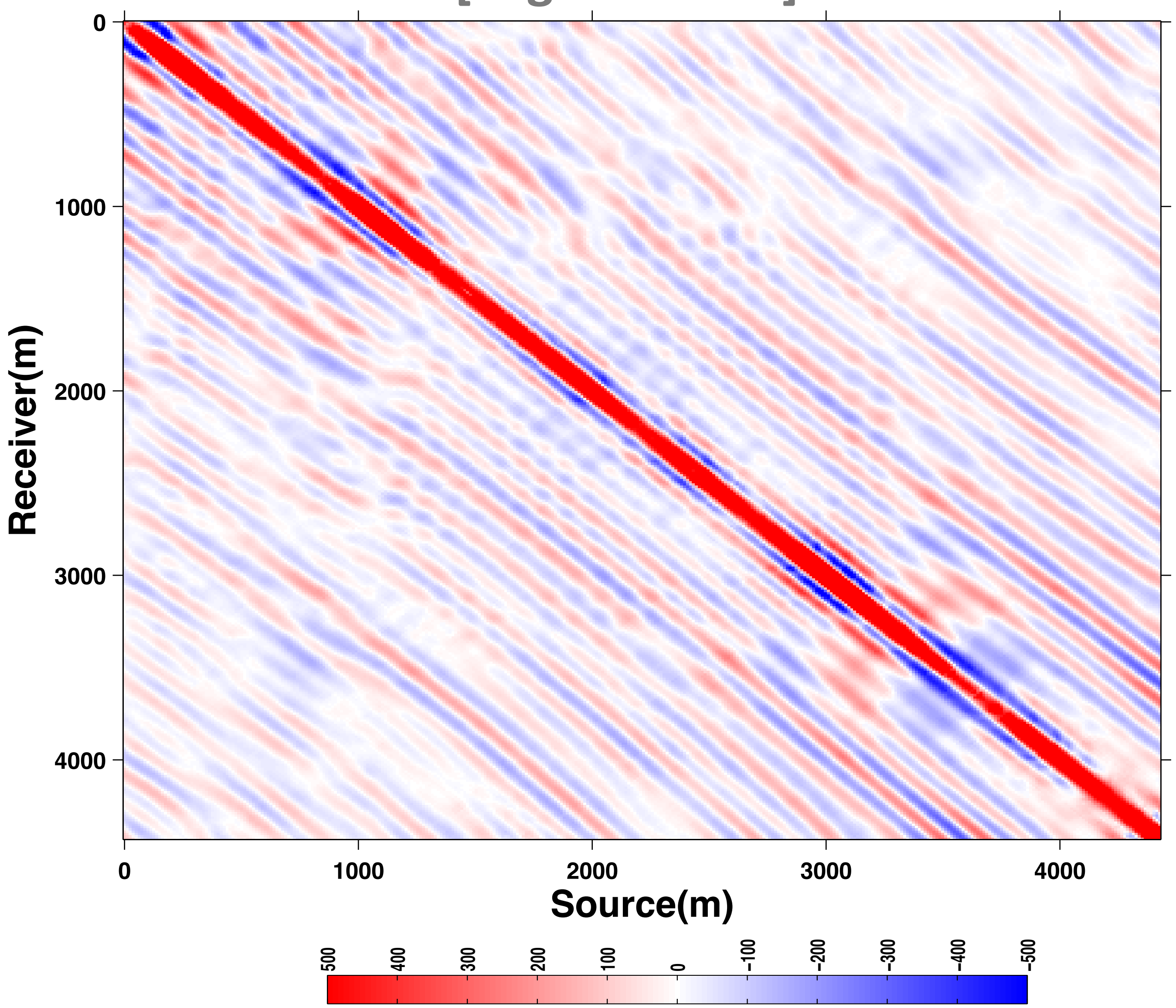
Freq : 60 Hz



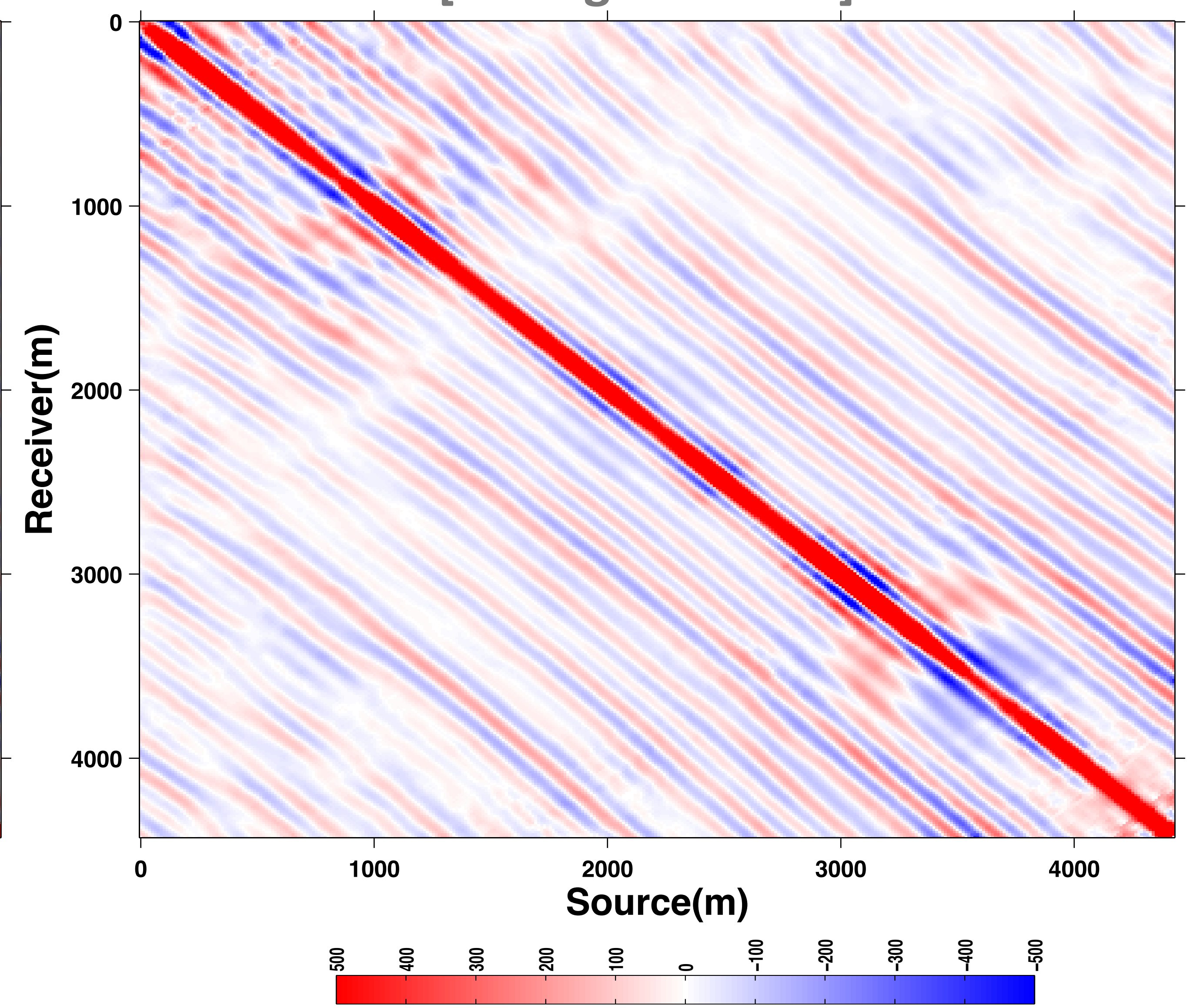
Recovery using MH formulation
[12 Hz]

SNR improvement by 6db

SNR = 19.3 db, Rank=20
[Regularization]



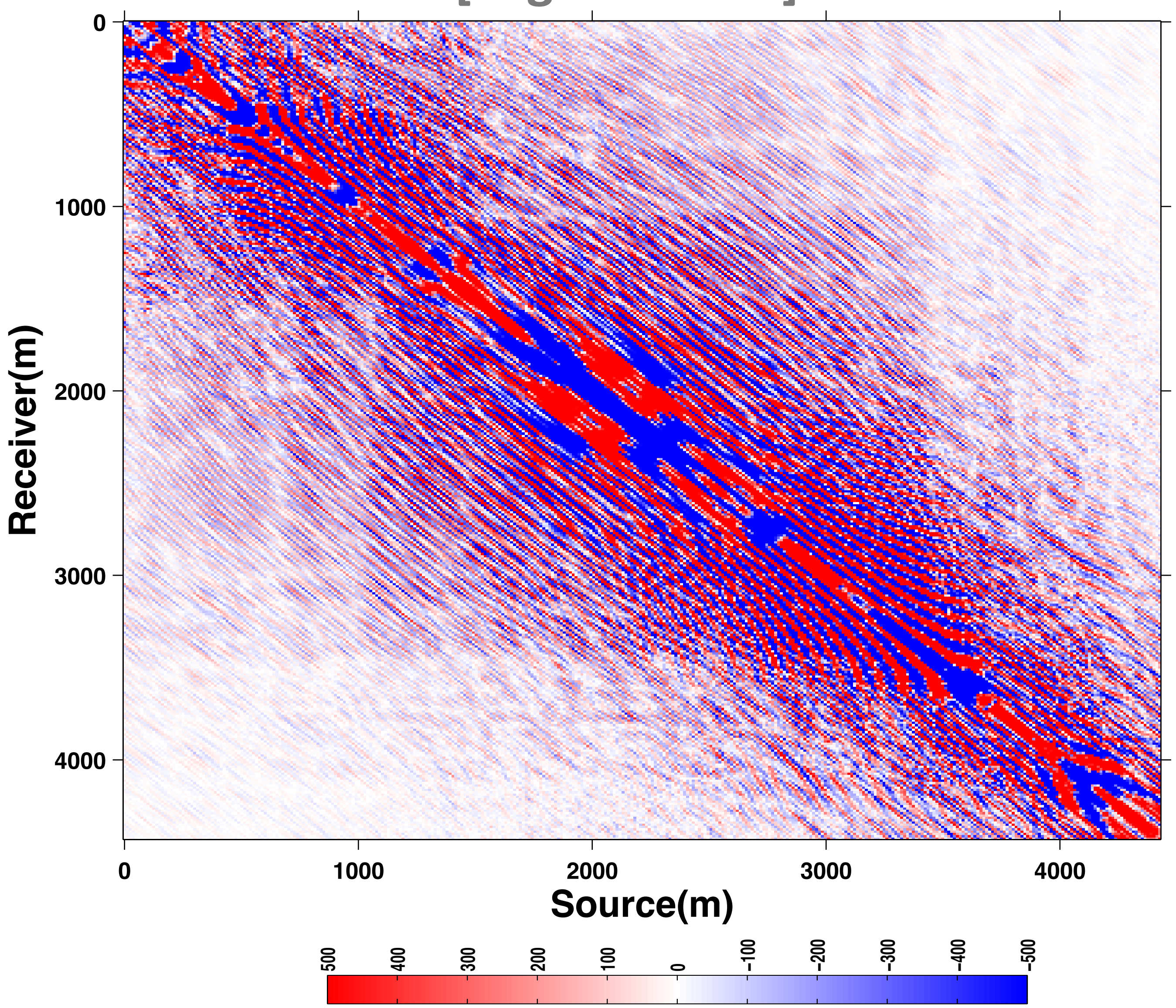
SNR = 13.8 db, Rank=10
[No Regularization]



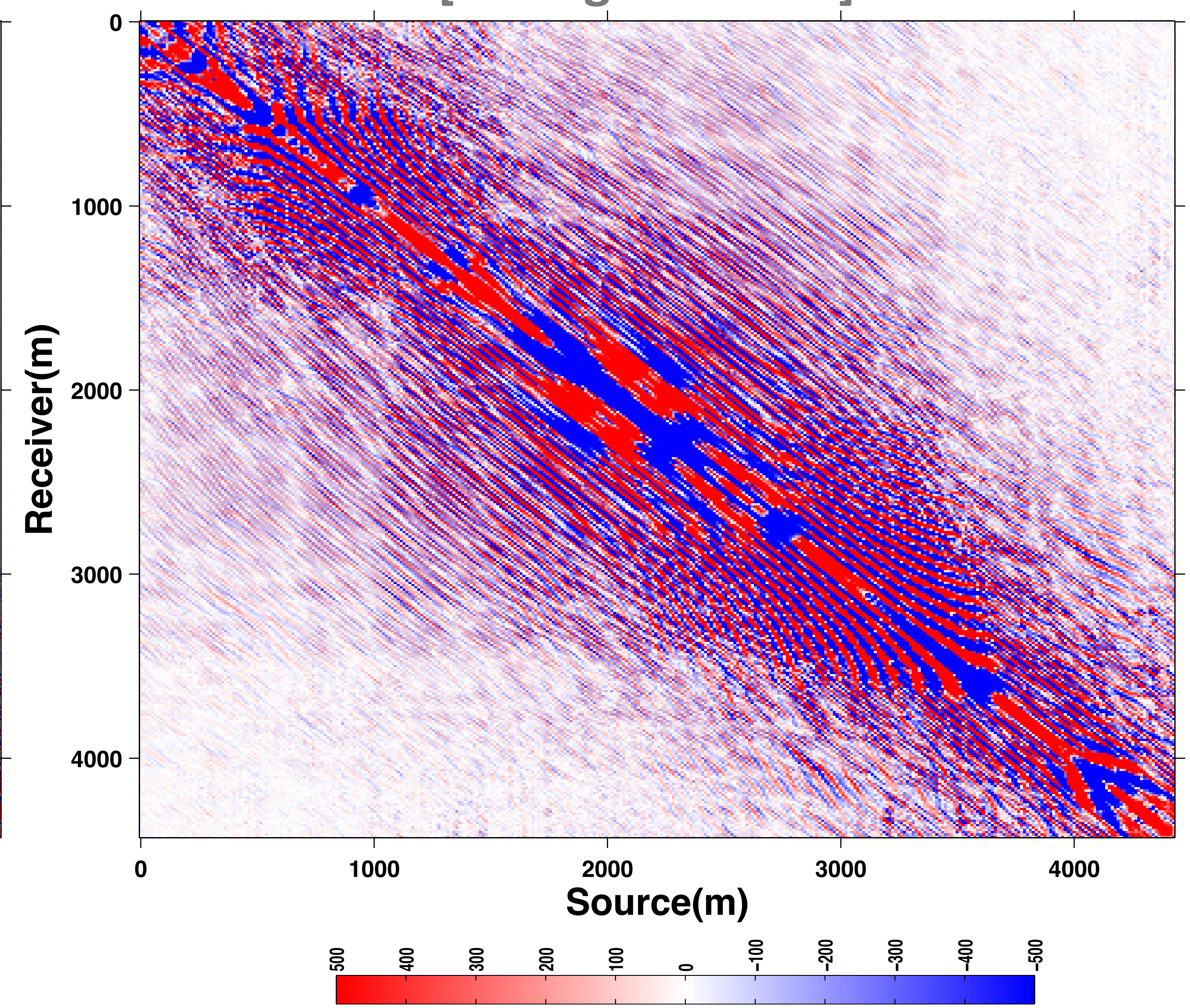
Recovery using MH formulation
[60 Hz]

SNR improvement by 1db

SNR = 17.5 db, Rank=40
[Regularization]

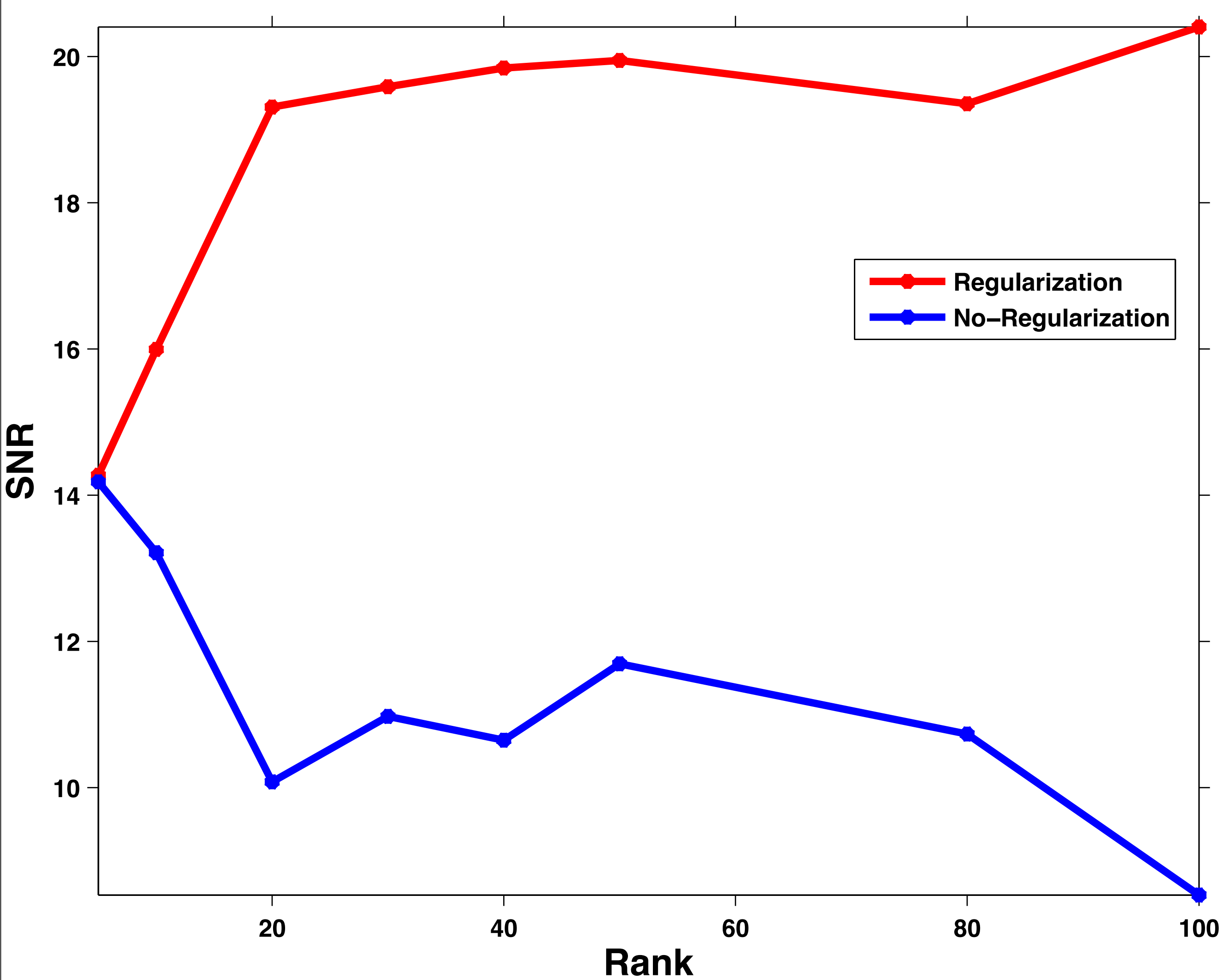


SNR = 16.1 db, Rank=30
[No Regularization]

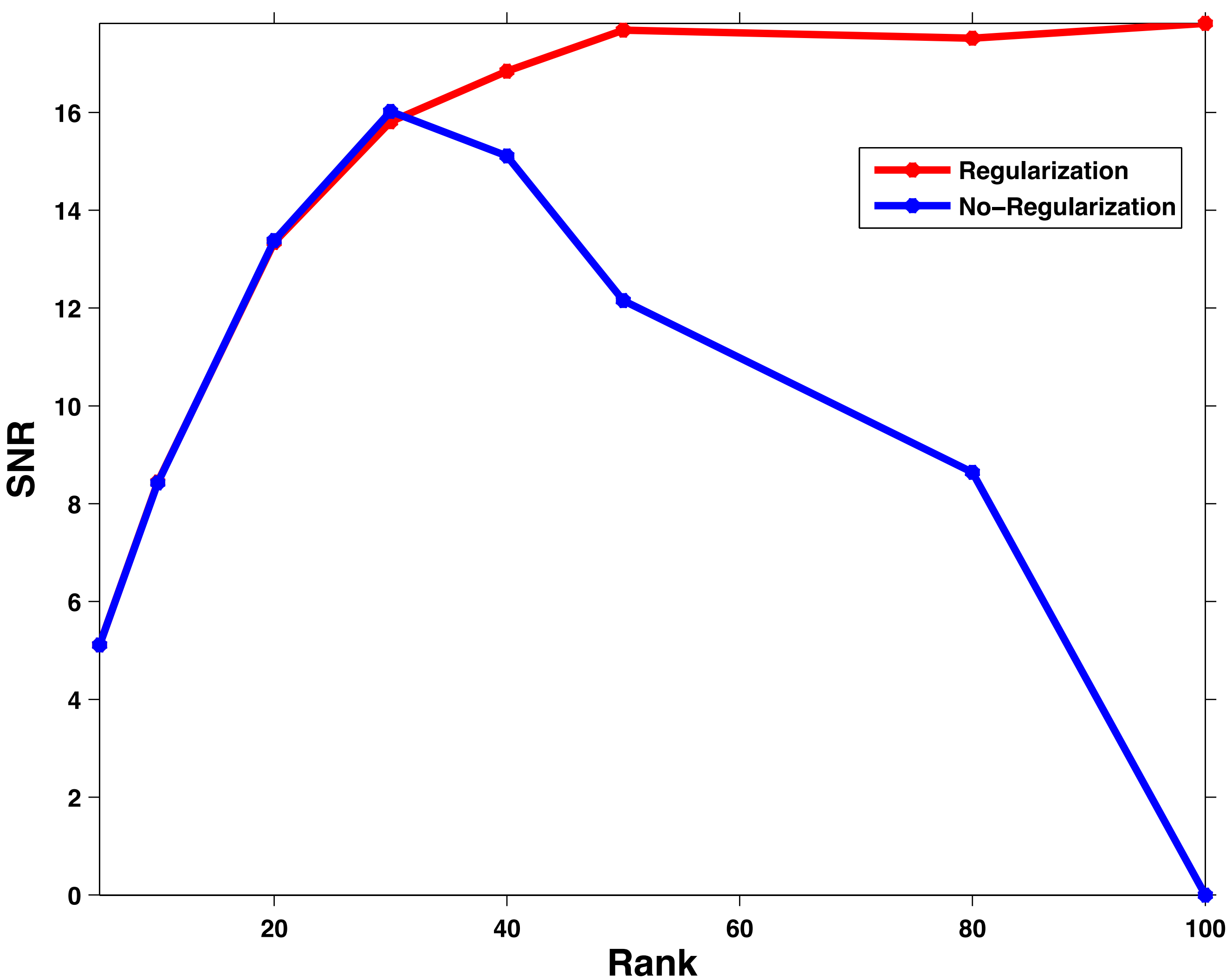


2D Interpolation
[SNR vs Rank]

Higher SNR can be achieved by increasing k



Freq : 12 Hz

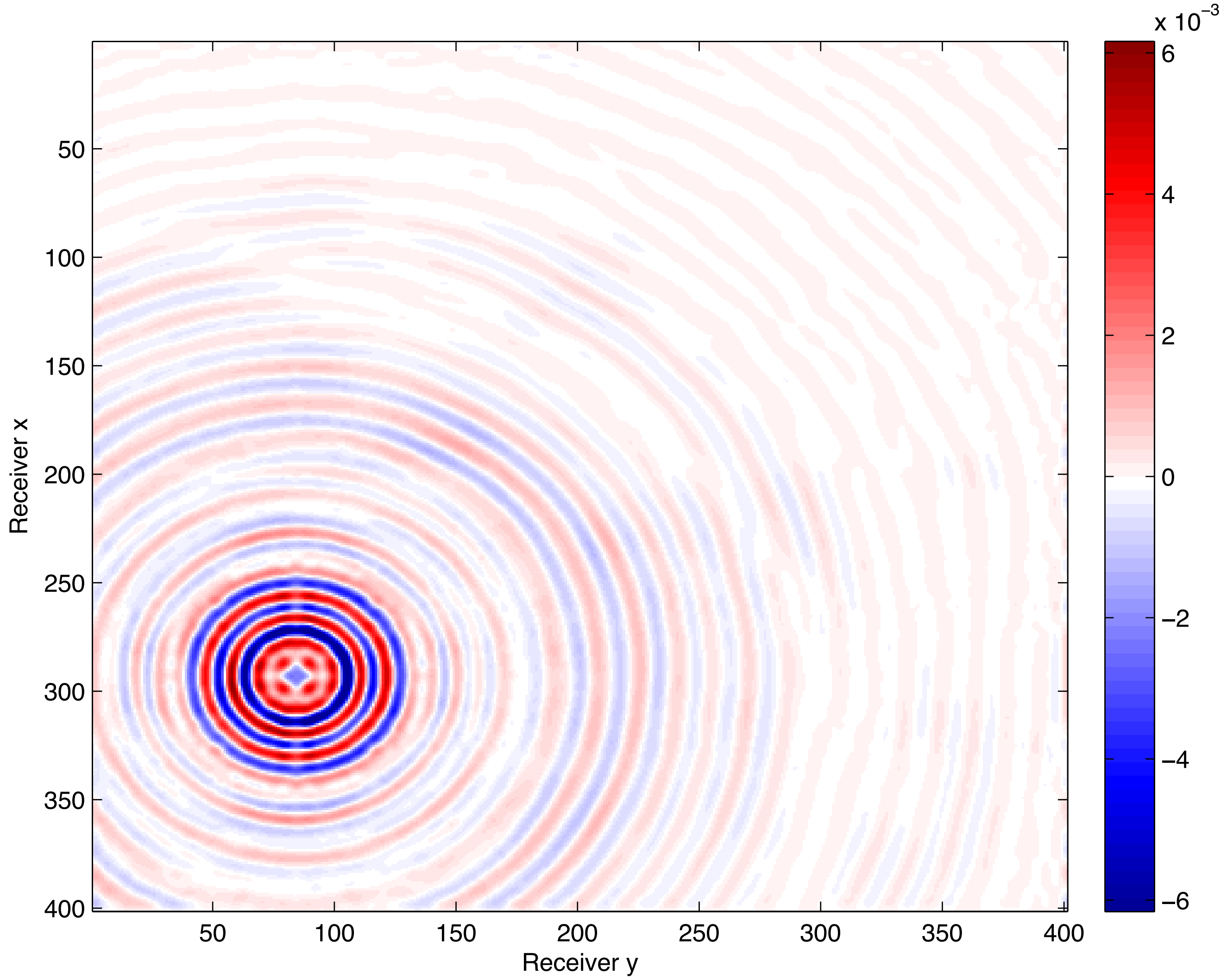


Freq : 60 Hz

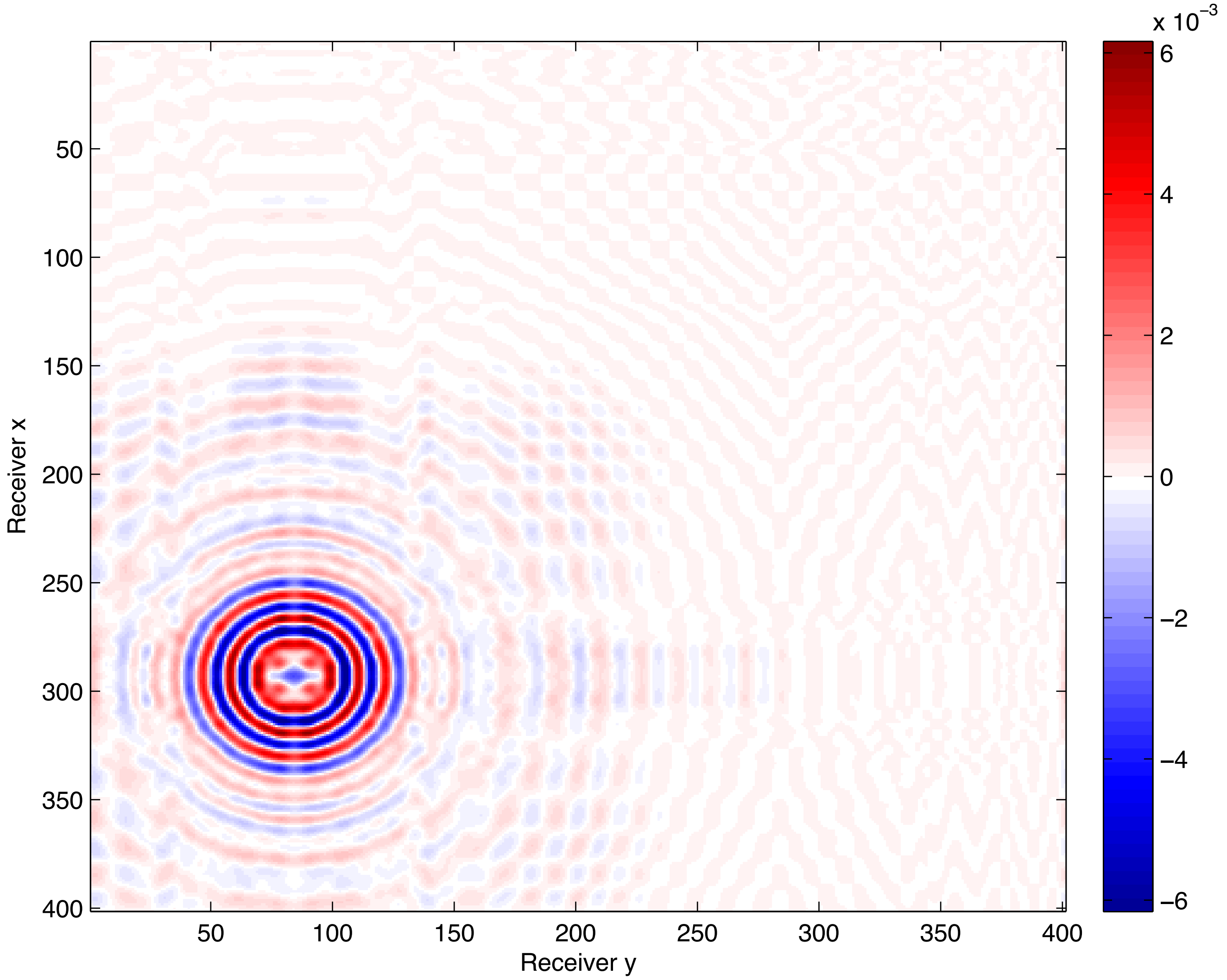
Recovery - Shot Gather

[Rank=10, 500 Iterations]

True data
(SrcX,SrcY = 50,15)

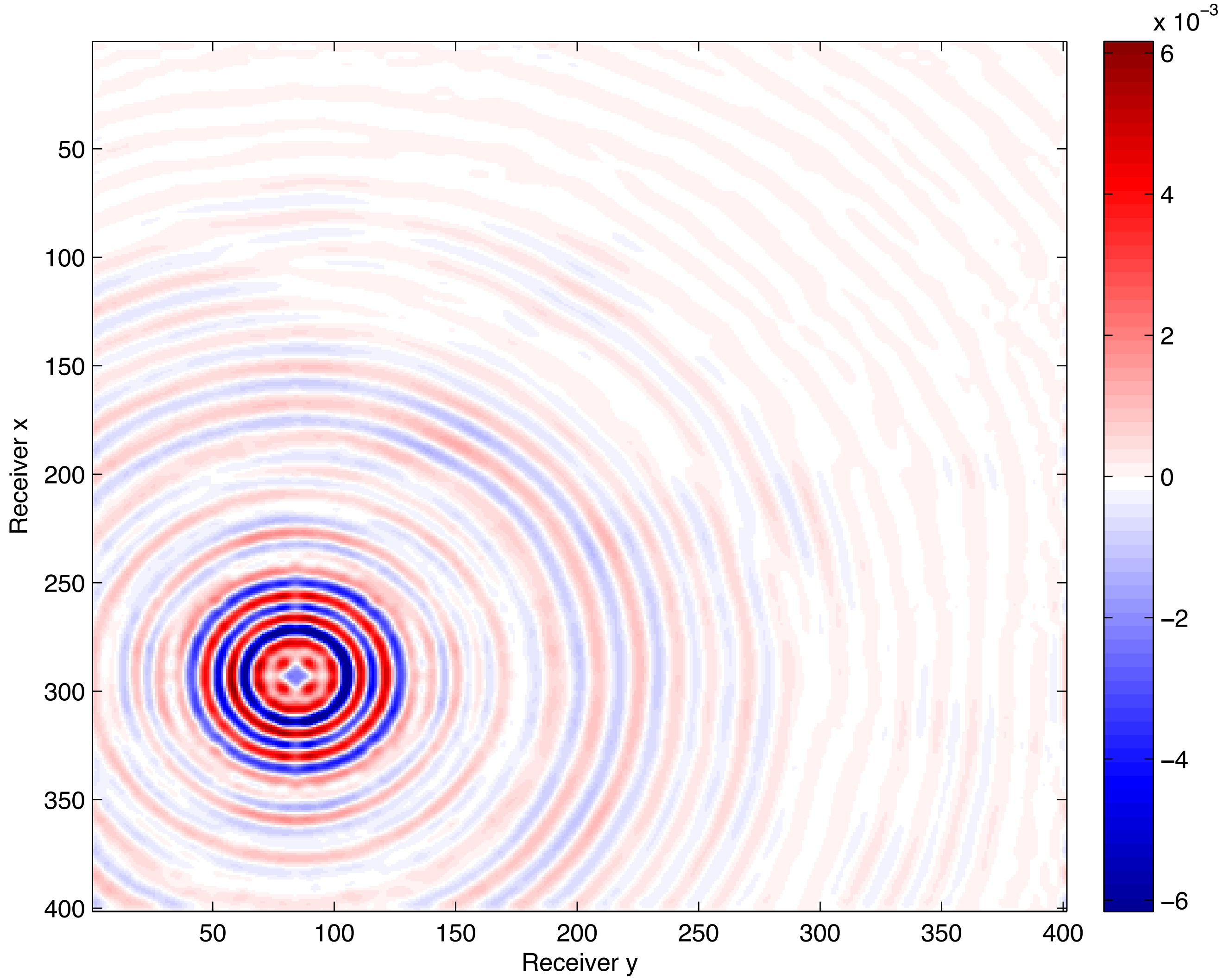


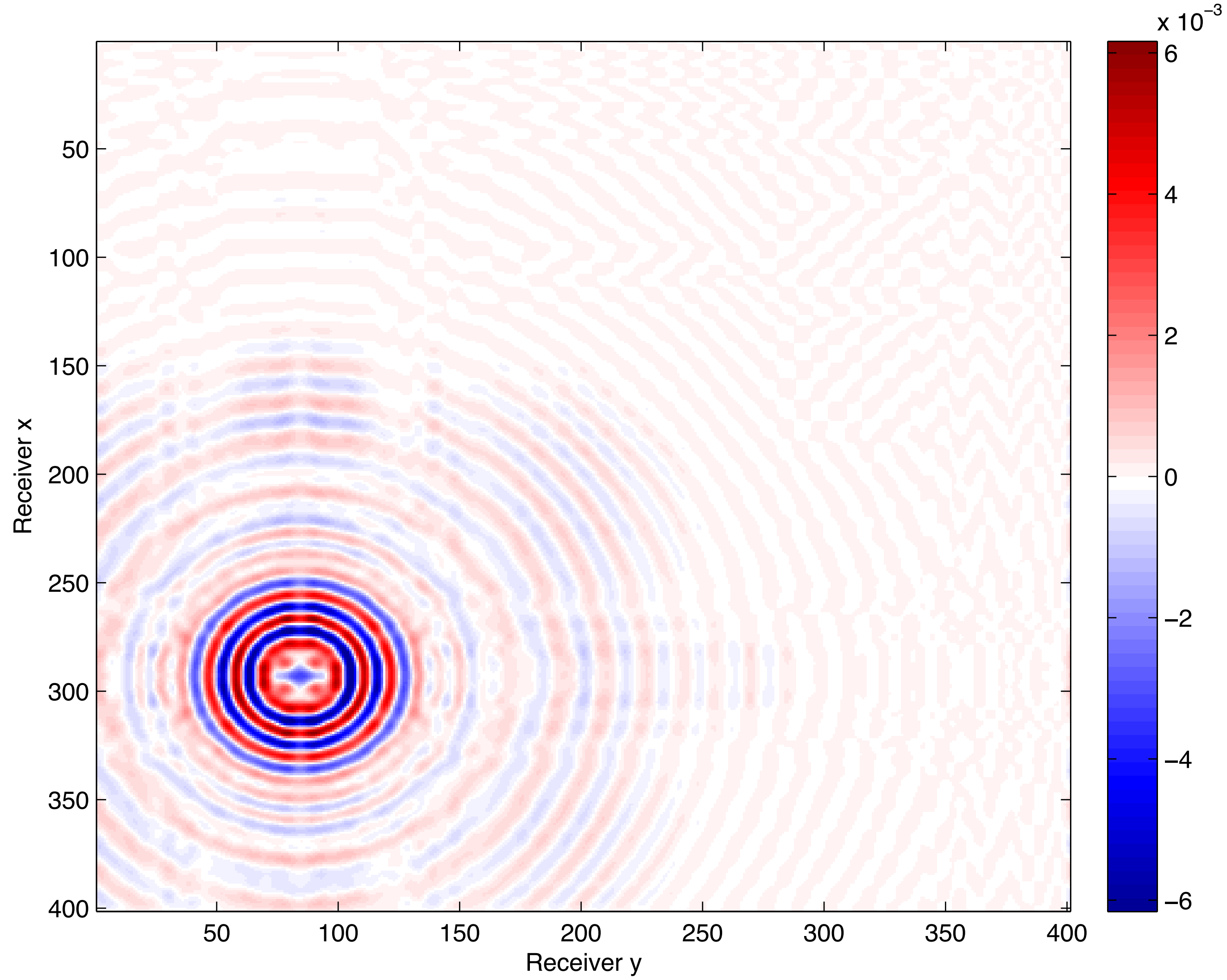
Approximated data
SNR=8.14 db



No-Regularization

Recovery - Shot Gather
[Rank=10, 500 Iterations]

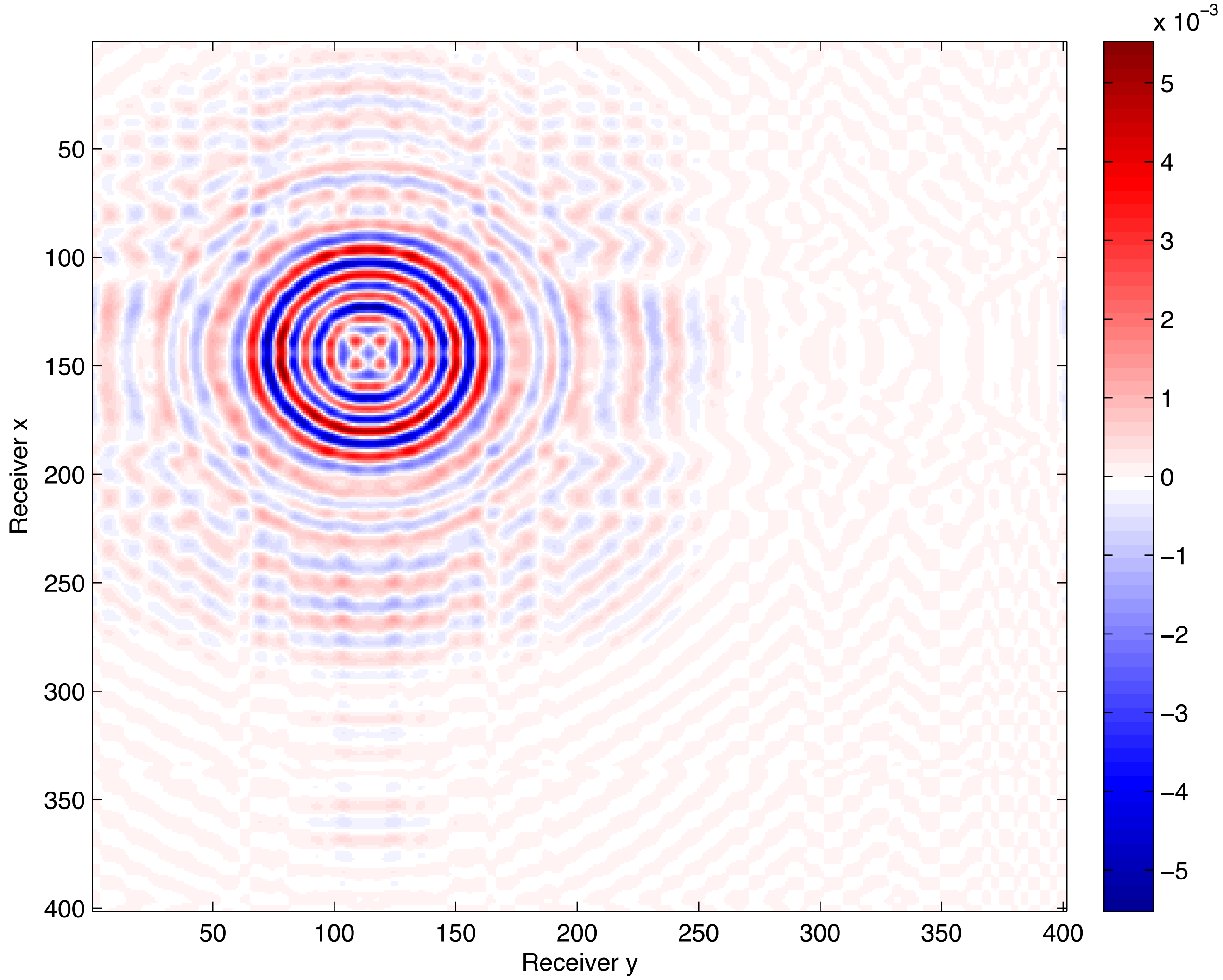




Regularization

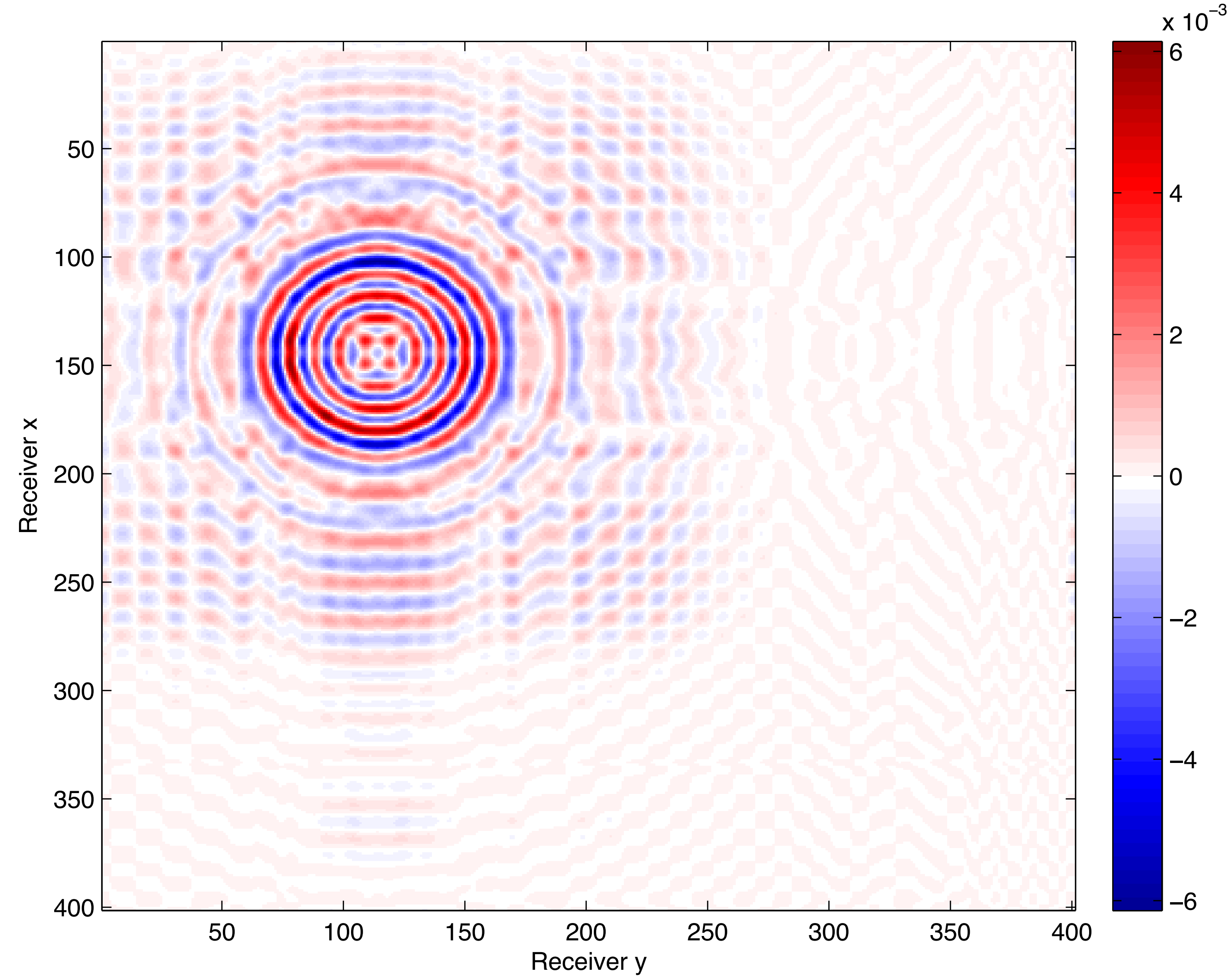
Recovery - Shot Gather
[Rank=10, 500 Iterations]

Interpolated data (SrcX,SrcY = 25,20)



Regularization

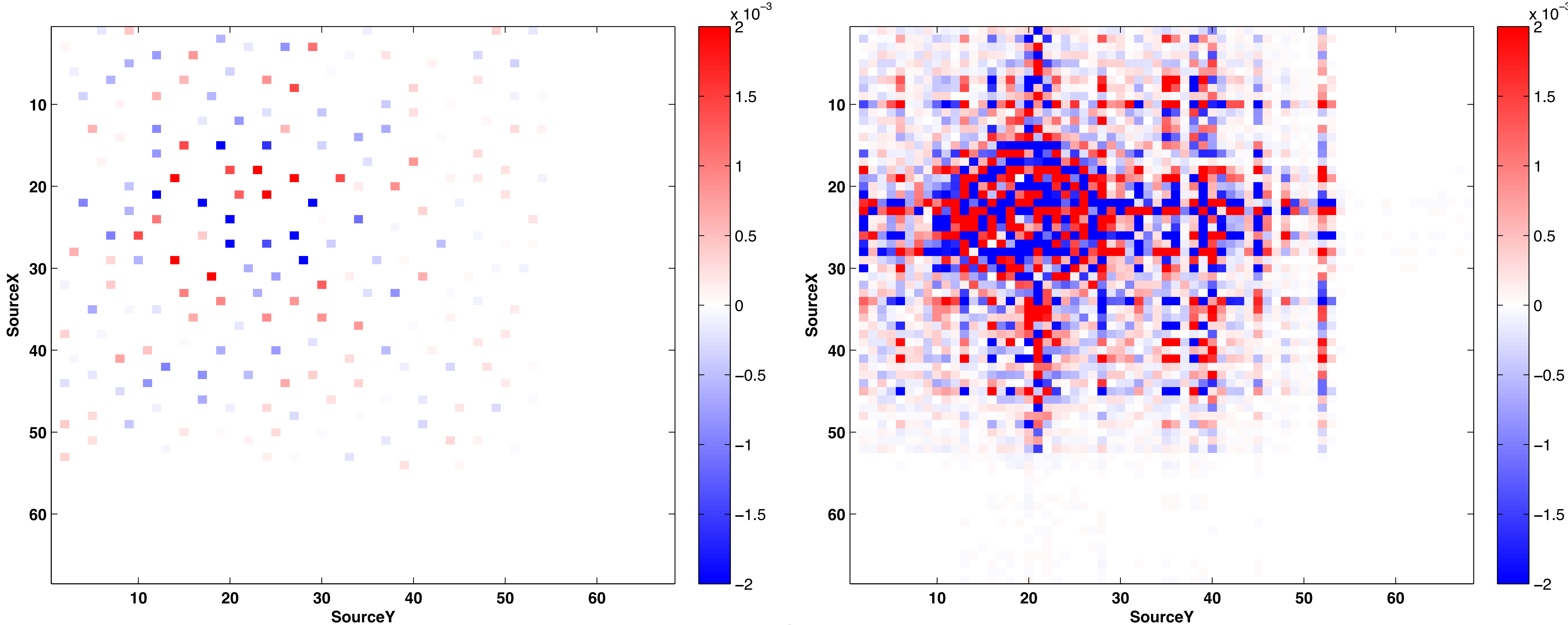
Interpolated data (SrcX,SrcY = 25,20)



No-Regularization

Recovery - Receiver Gather
[Rank=10, 500 Iterations]

Acquired data (RecX,RecY = 35,30) — **Interpolated data (RecX,RecY = 35,30)**



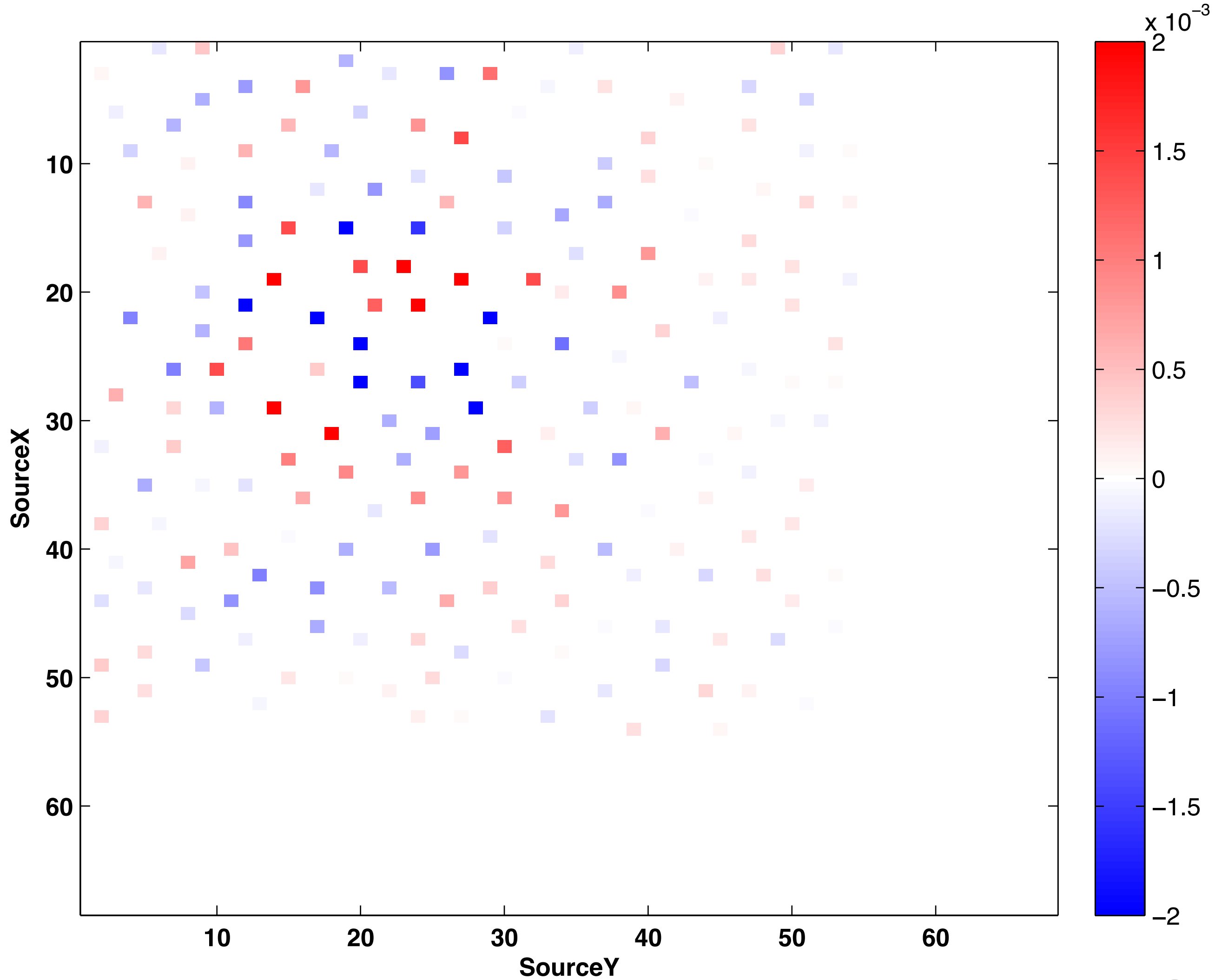
No-Regularization

[98 % missing data]

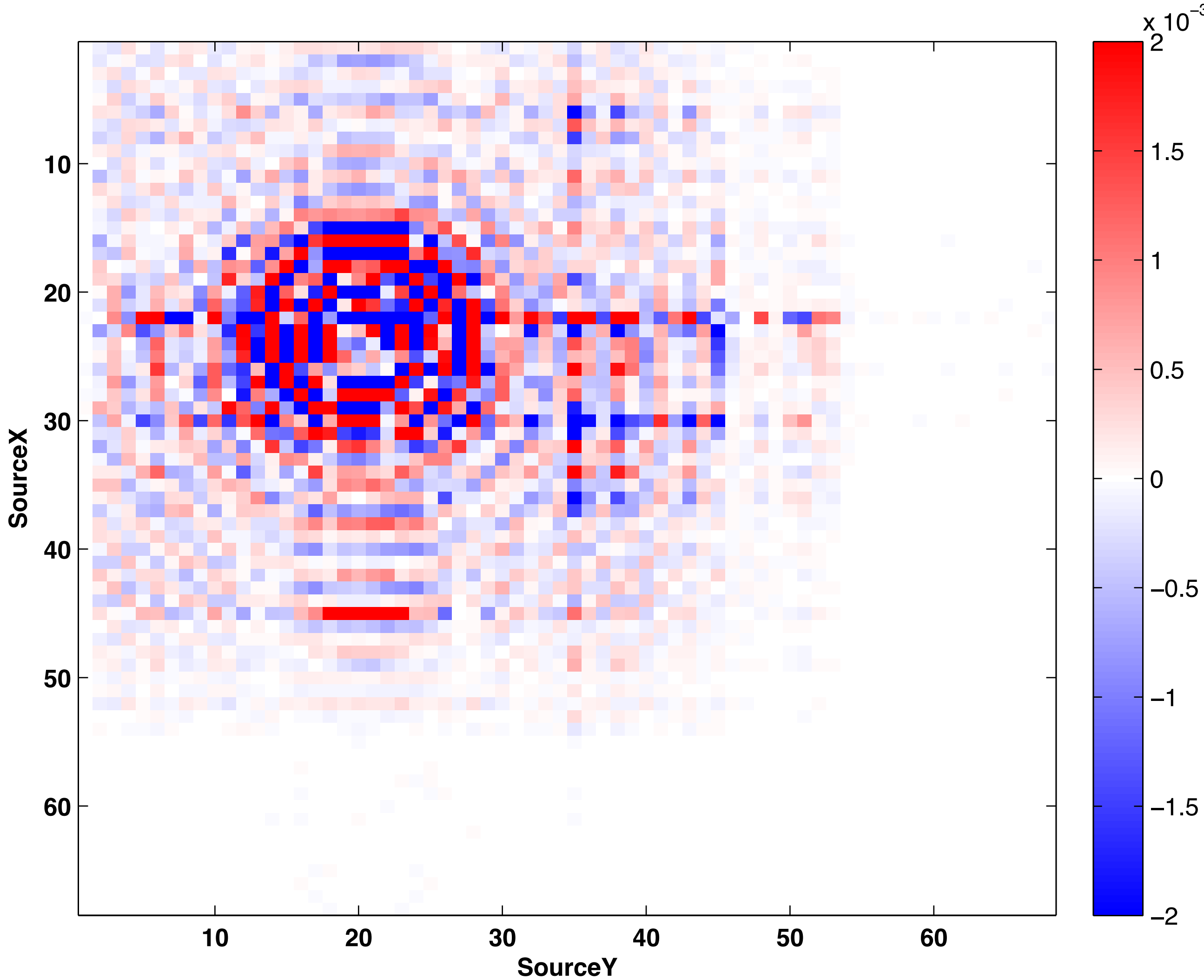
Recovery - Receiver Gather

[Rank=10, 500 Iterations]

Acquired data
(RecX,RecY = 35,30)



Interpolated data
(RecX,RecY = 35,30)



Regularization
[98 % missing data]

Conclusion

- matrix factorization allows *SVD-free* rank penalty methods that work fast on large data
- low-rank structure can be exploited *implicitly* (through factors) and *explicitly* (BPDN)
- low-rank structure holds promise for data recovery and more compact representation.

Future Work

- *weighted* regularized formulation
- Use factorization in *processing*
- implementation of *HSS* structure
- implementation of 3D interpolation in *Midpoint-Offset-Azimuth* domain

References

- **Kumar, R., A. Y. Aravkin, F. J. Herrmann, 2012,** Fast Methods for Rank Minimization with Applications to Seismic-Data Interpolation : TR-2012-04

- **Lee, J., B. Recht, R. Salakhutdinov, N. Srebro, and J. Tropp, 2010**, Practical large- scale optimization for max-norm regularization: Analysis, 23, 1–9.

- **van den Berg, E., and M. P. Friedlander, 2008**, Probing the pareto frontier for basis pursuit solutions: SIAM Journal on Scientific Computing, 31, 890–912.

- **Herrmann, F. J., 2008**, Compressive sampling: a new paradigm for seismic data acquistion and processing?: Presented at the ION.

- **Herrmann, F. J., and G. Hennenfent, 2008**, Non-parametric seismic data recovery with curvelet frames: Geophysical Journal International, 173, 233–248.

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