FWI from the West Coasts:
lessons learned from "Gulf of Mexico Imaging Challenges: What Can Full Waveform Inversion Achieve?"

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Dream team










Goals

- nonlinear Conjugate gradients
  - ***linear*** *anisotropic* smoothing of *gradients*
- *modified* Gauss-Newton
  - ***nonlinear*** "smoothing"/ *approximations* of *gradients* with *anisotropic* curvelets

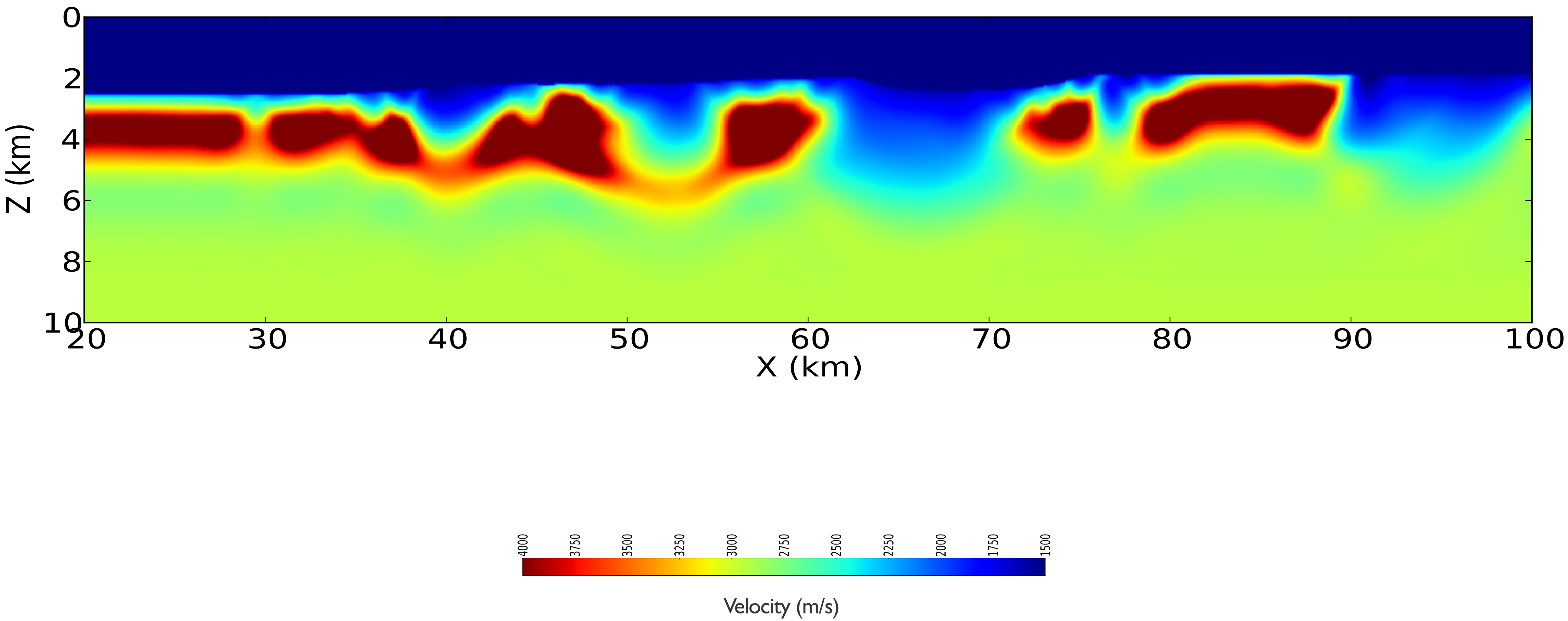
Acoustic FWI workflows with *minimal* parameters & user *intervention*... *Migration* was computationally *infeasible*.

Preprocessing

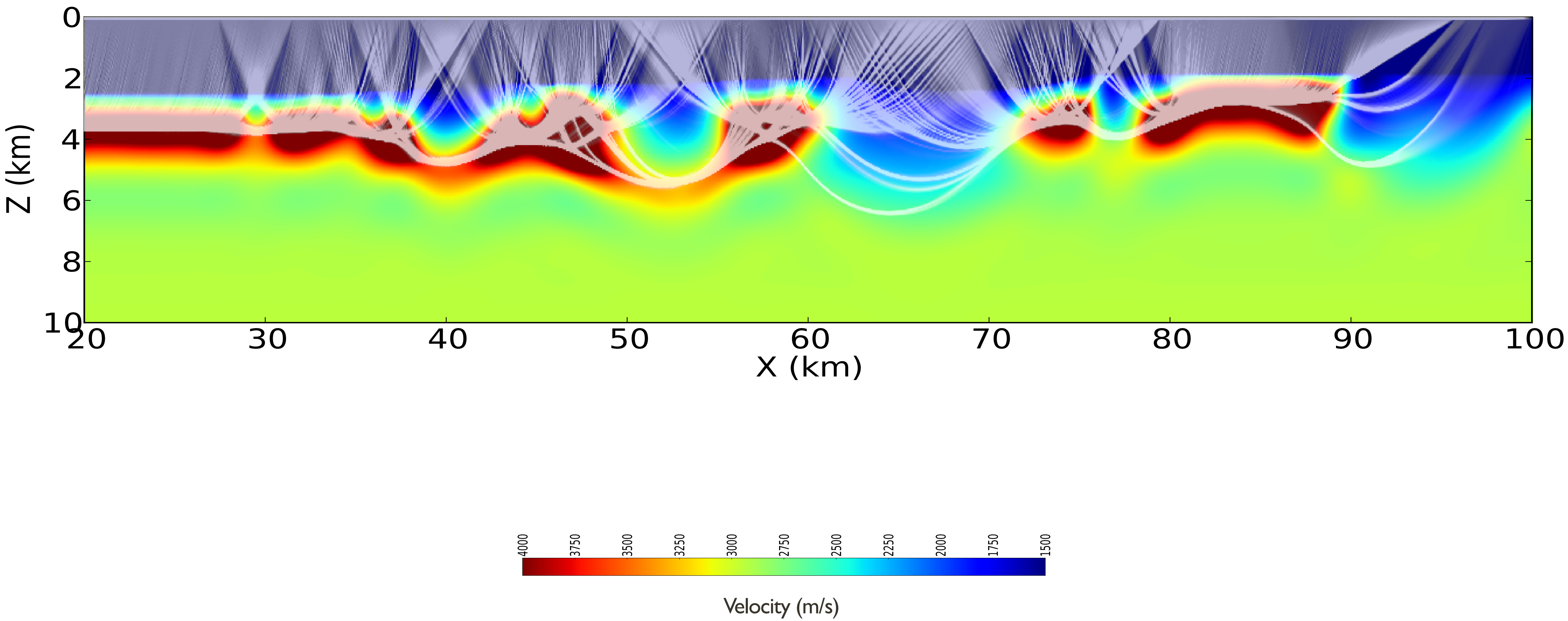
Travel-time *tomography* based on hand-picked *first* breaks ⟶ *initial velocity model*

Curvelet-denoising at selected *low*-frequencies ⟶ *improve SNR* for FWI

Initial model
[ray-based tomography]



Ray paths
[RMS traveltime misfit 11ms]



Denoising

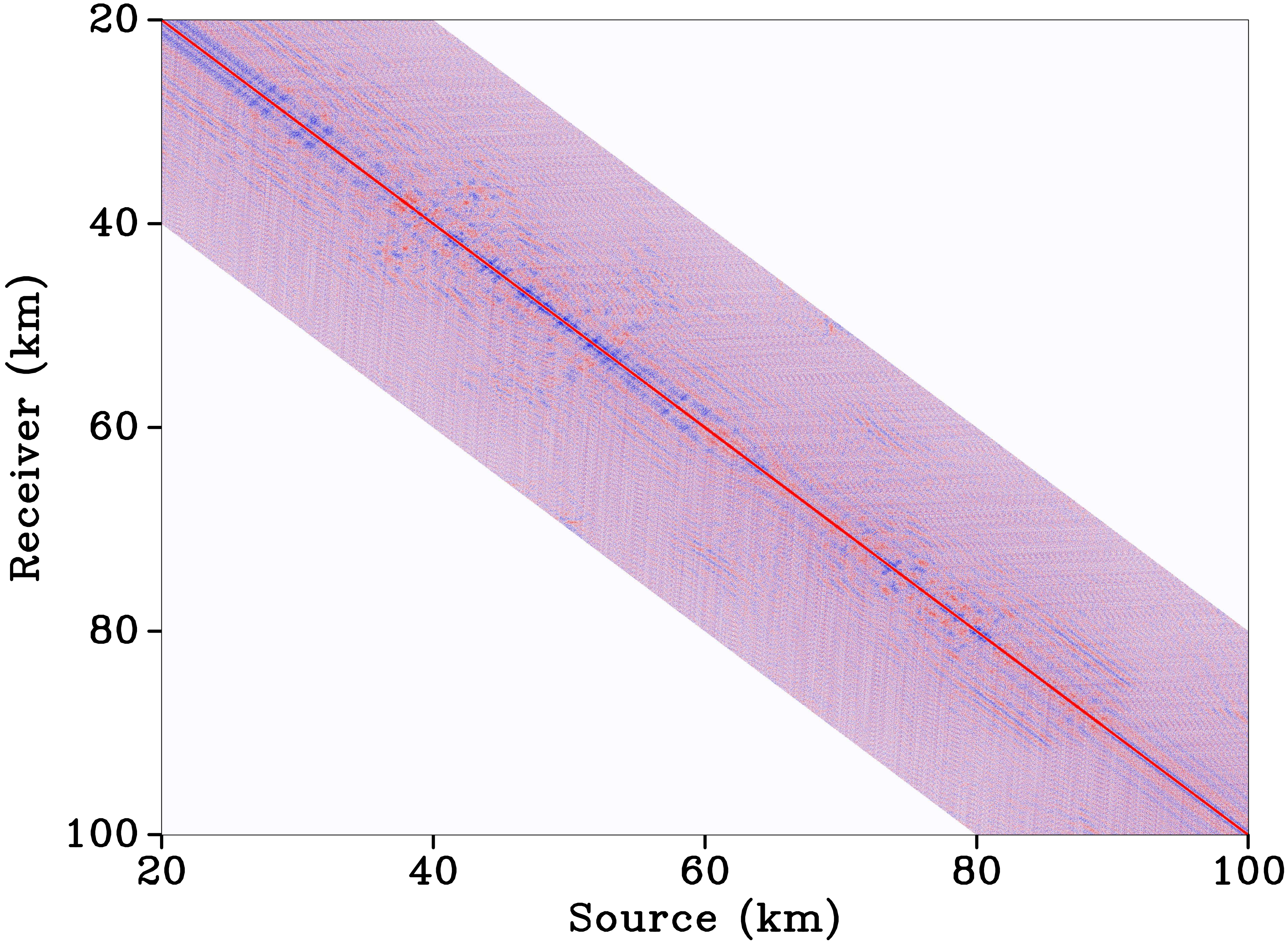
Work on "frequency slices" in the *source/receiver* plane

Find *support* by *hand*-selected *thresholding* of *syntheses* curvelet coefficients

Followed by *debiasing* to restore *amplitudes*...

Before denoising

[real part at 2.0 Hz]



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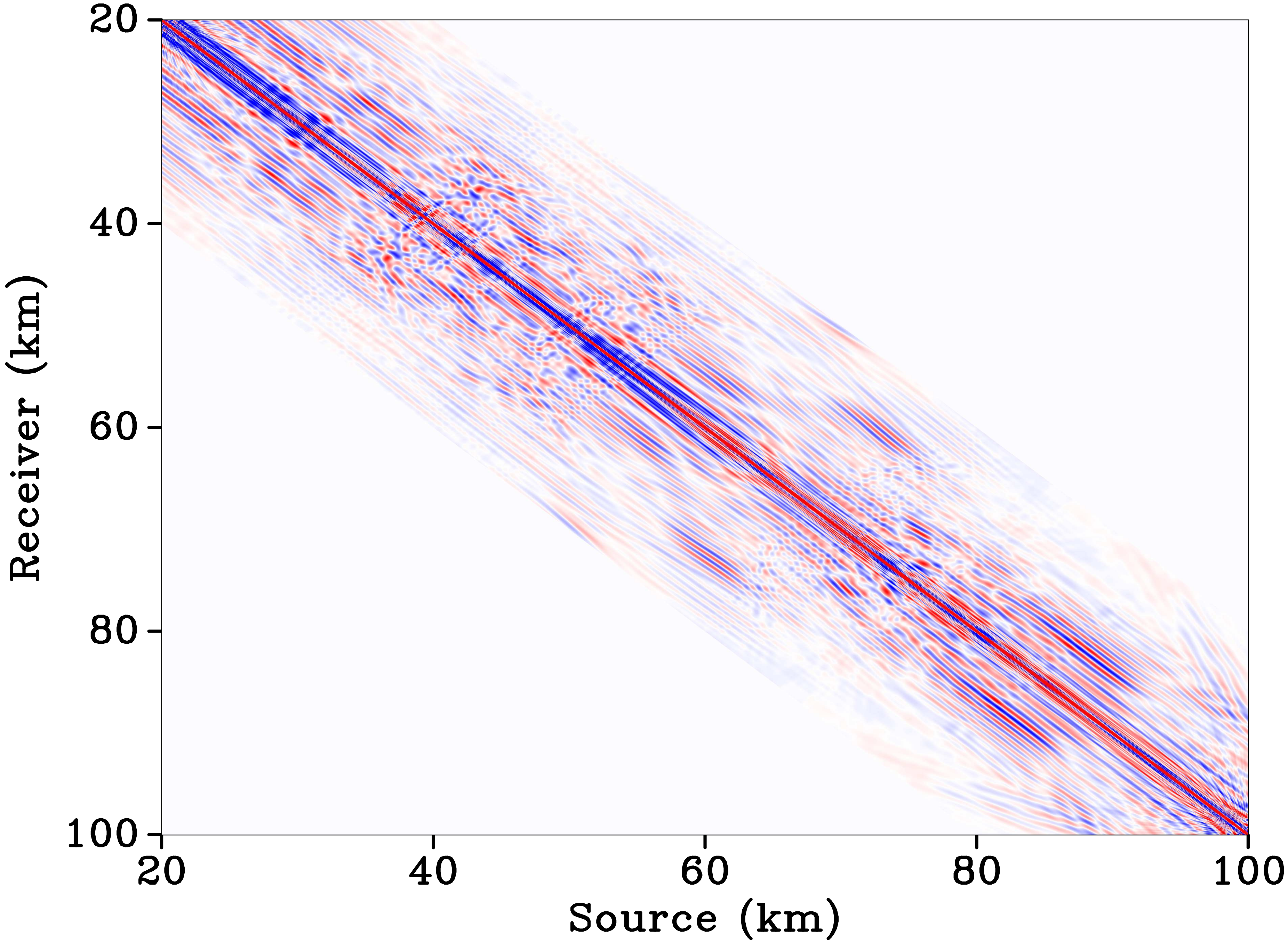
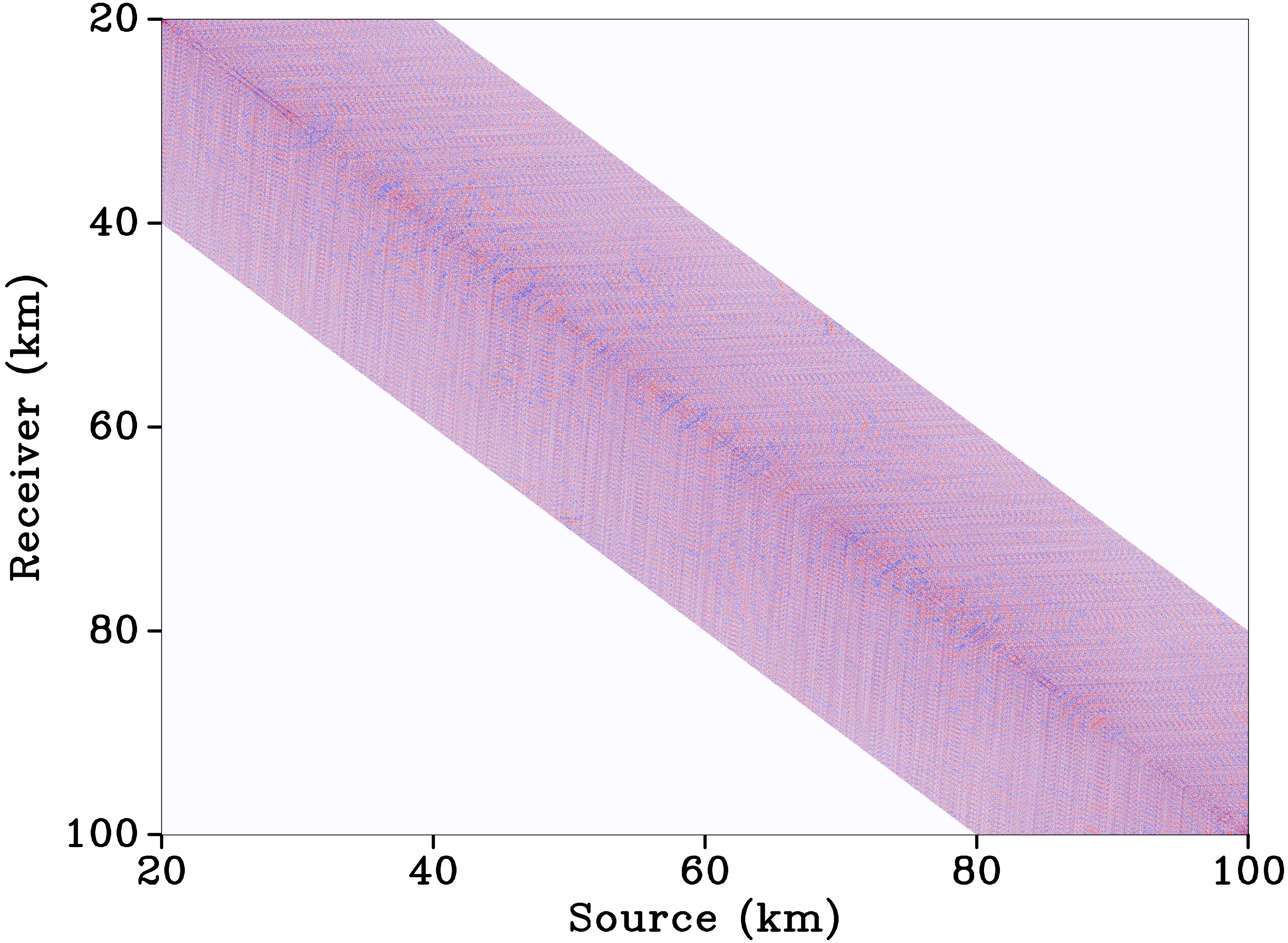
After denoising
[real part at 2.0 Hz]



Difference

[real part at 2.0 Hz]



Denoising

Removed mostly *incoherent* energy

Preserved amplitudes of the *coherent* events

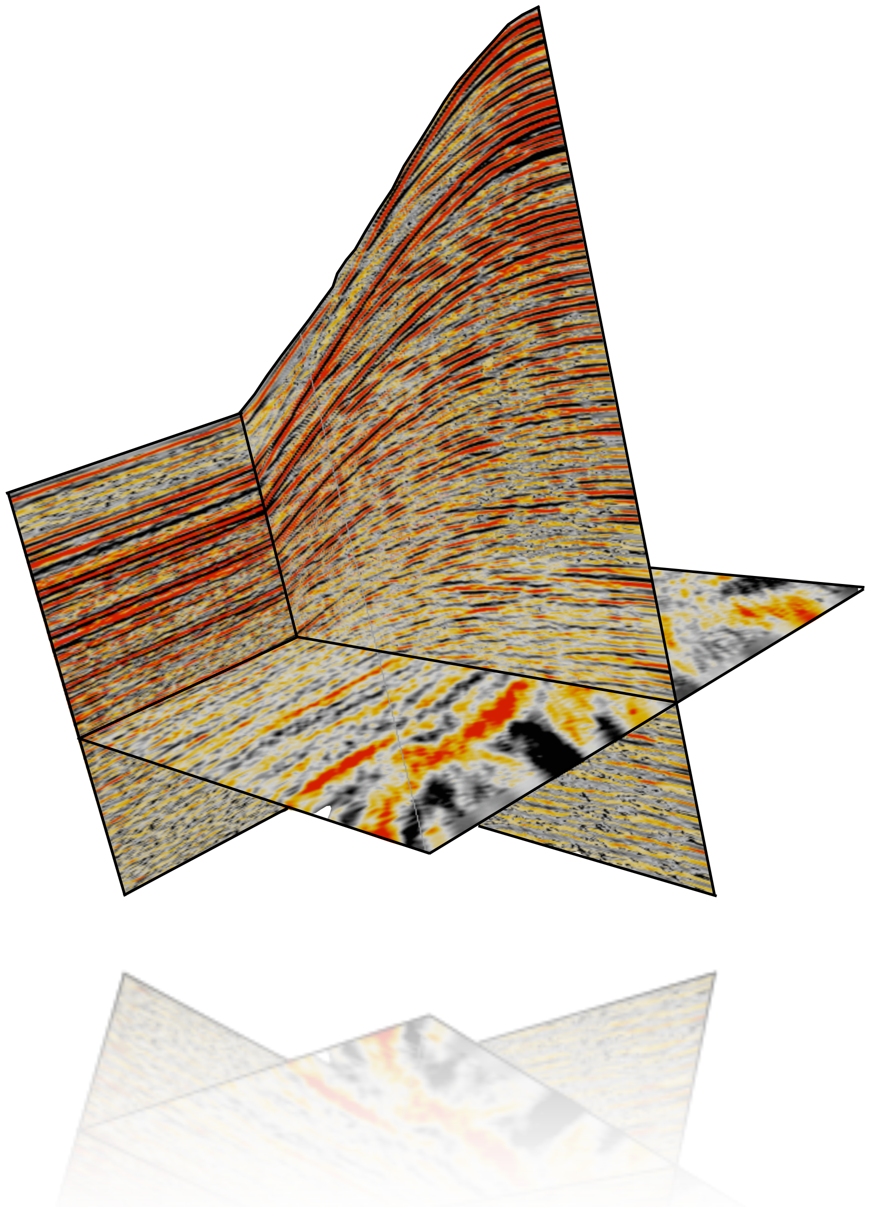
Improved data *quality* in the "eye ball" norm

Nonlinear Conjugate Gradients

Pratt, R.G., Shin, Changsoo and Hicks, G.J., 1998. Gauss-Newton and full Newton methods in frequency domain seismic waveform inversion. Geophysical Journal International, 133, 341-362.

Brenders, A. J. and Pratt, R. G., 2007. Full waveform tomography for lithospheric imaging: results from a blind test in a realistic crustal model, Geophysical Journal International, 168, 133-151

Takam Takougang, E. M. and Calvert, A. J., 2011. Application of waveform tomography to marine seismic reflection data from the Queen Charlotte Basin of western Canada, Geophysics,76, B1-B16



Methodology

Non-linear conjugate gradients (first order)

Scaling by *RMS* from (initial) *forward* model

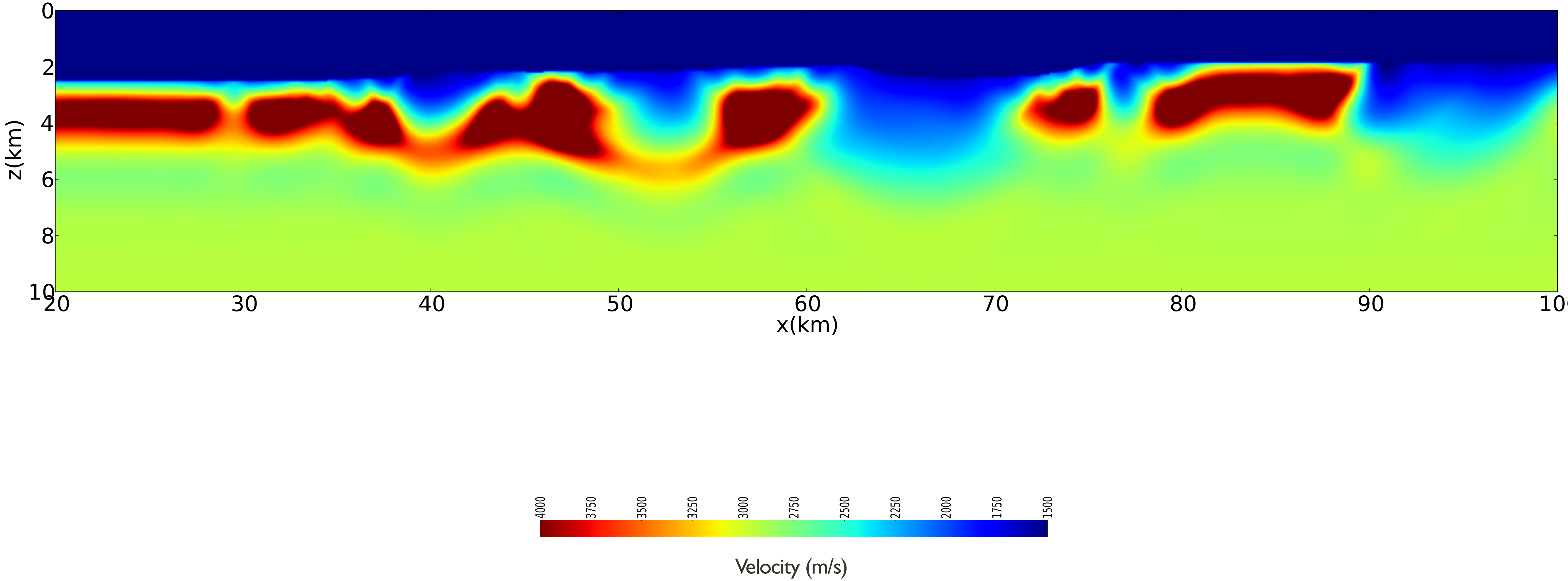
Layer stripping with gradient & offset weights

Two-staged inversion (40-80)% 3s window

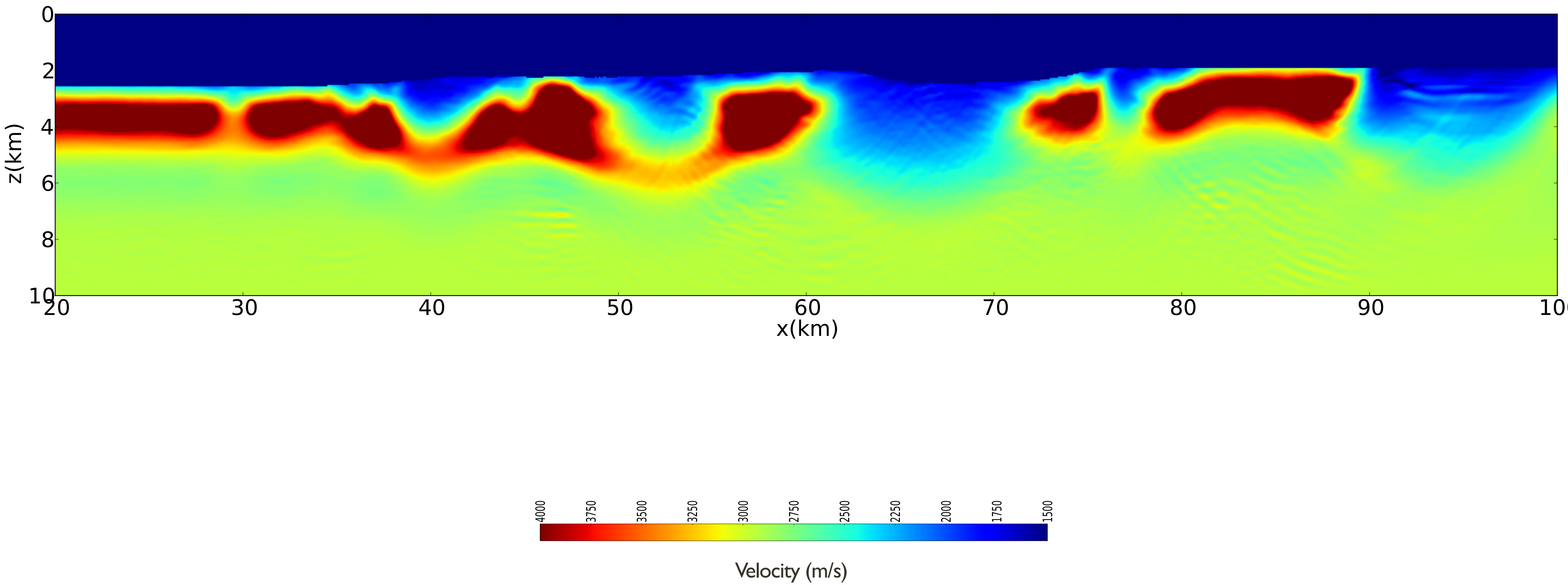
- *phase* only 2–7 Hz

Anisotropic *smoothing* on *gradients*...

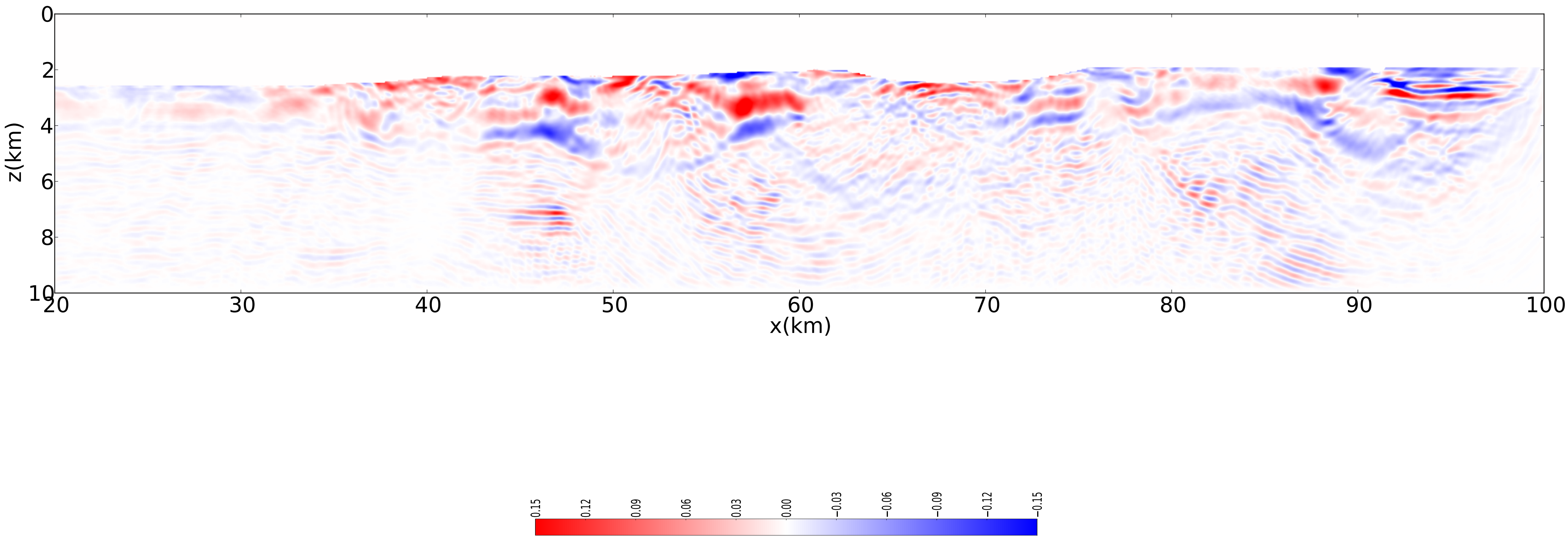
Initial model
[ray-based tomography]



Final result
[w/o denoising]



Relative update $\Delta(V)/V$



Batched FWI with nuisance parameter estimation

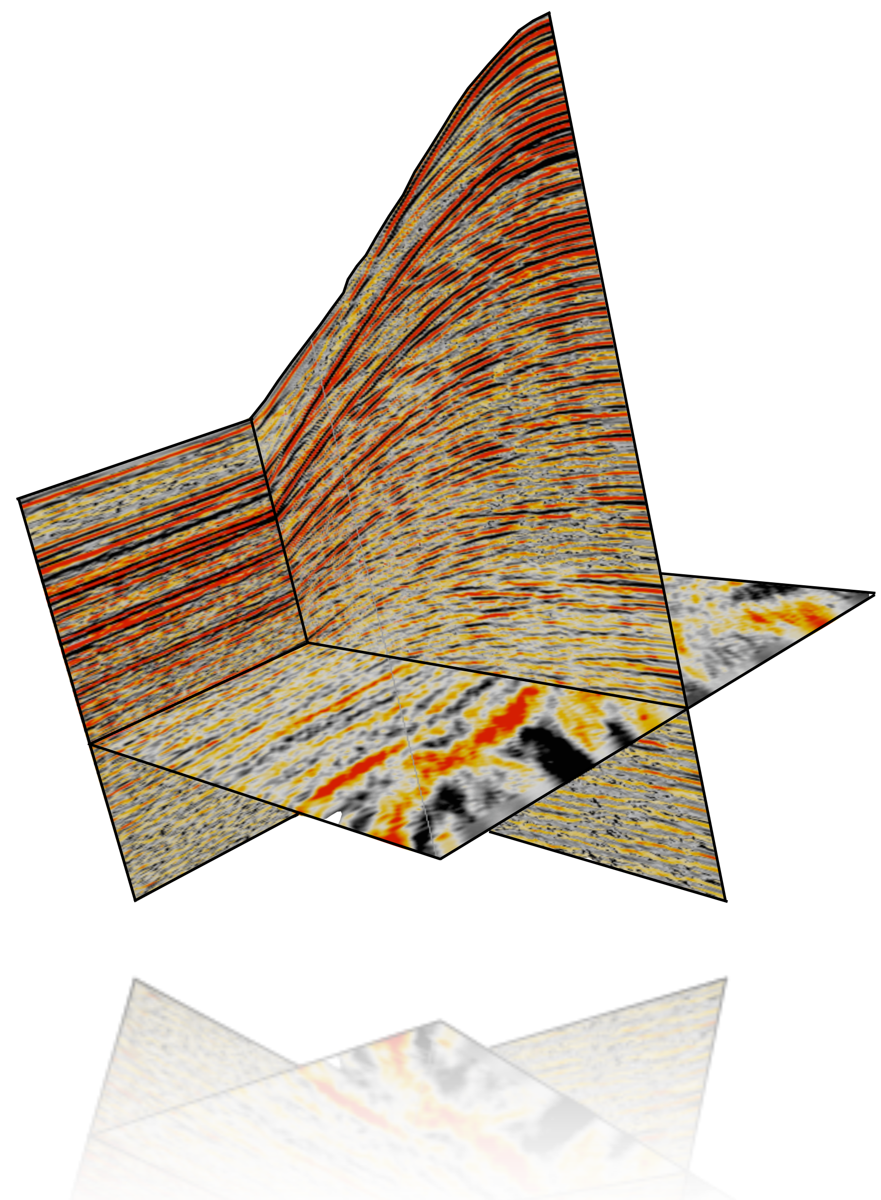
A.Y. Aravkin and T. van Leeuwen - Estimating nuisance parameters in inverse problems. Inverse Problems, 2012.

T. van Leeuwen and F.J. Herrmann - Fast waveform inversion without source encoding. Geophysical Prospecting, 2012

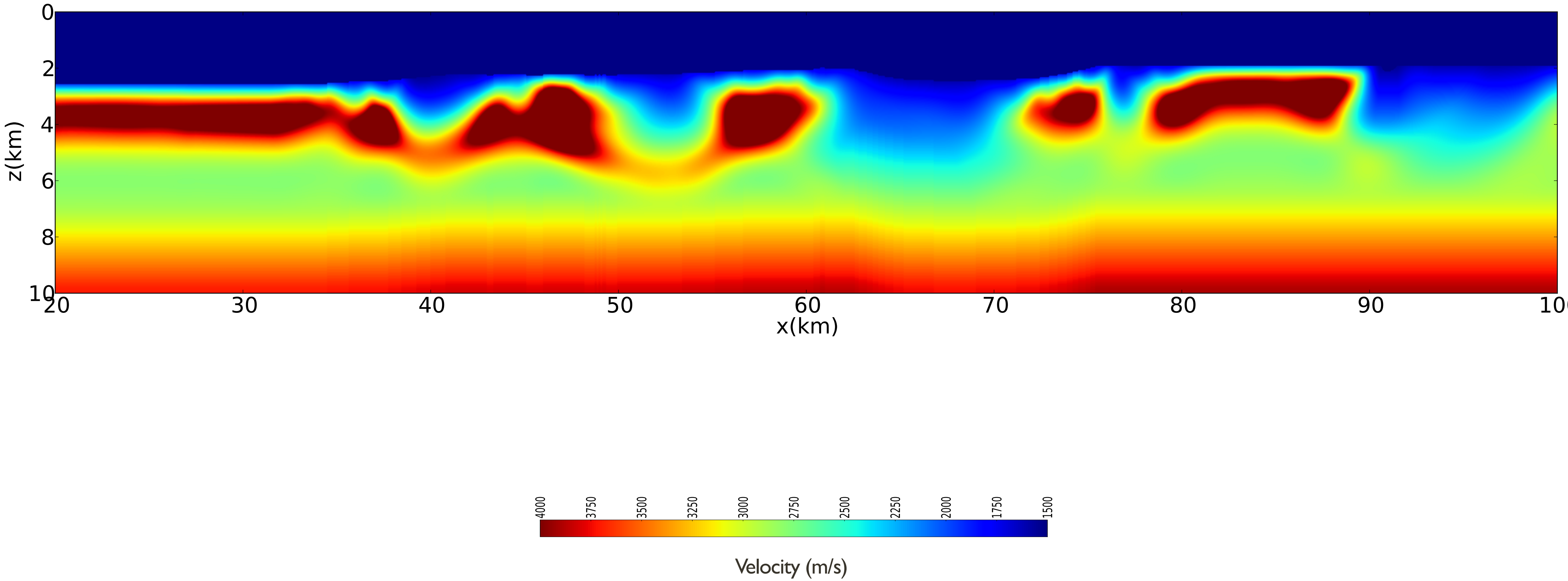
A. Aravkin, M.P. Friedlander, F.J. Herrmann and T. van Leeuwen - Robust inversion, dimensionality reduction and randomized sampling. Mathematical Programming, 2012.

Tristan van Leeuwen, “A parallel matrix-free framework for frequency-domain seismic modelling, imaging and inversion in Matlab”. 2012.

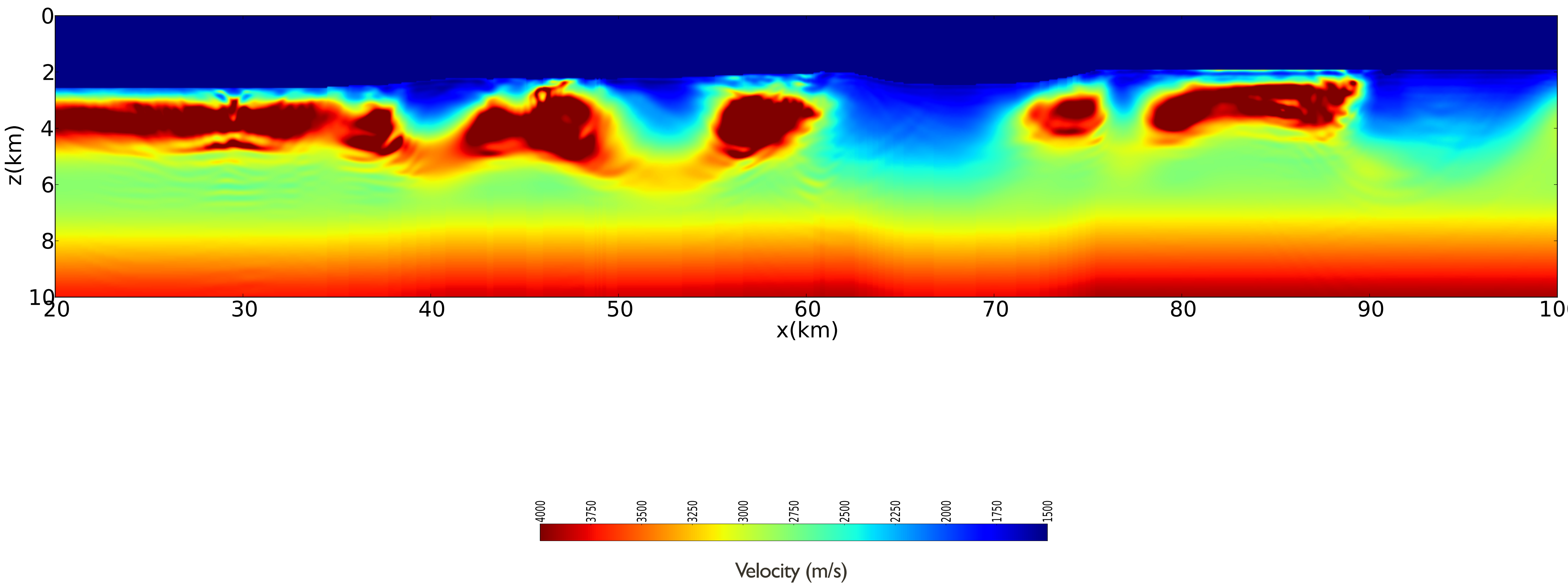
Tristan van Leeuwen and Felix J. Herrmann, “A parallel, object-oriented framework for frequency-domain wavefield imaging and inversion.”. 2012.



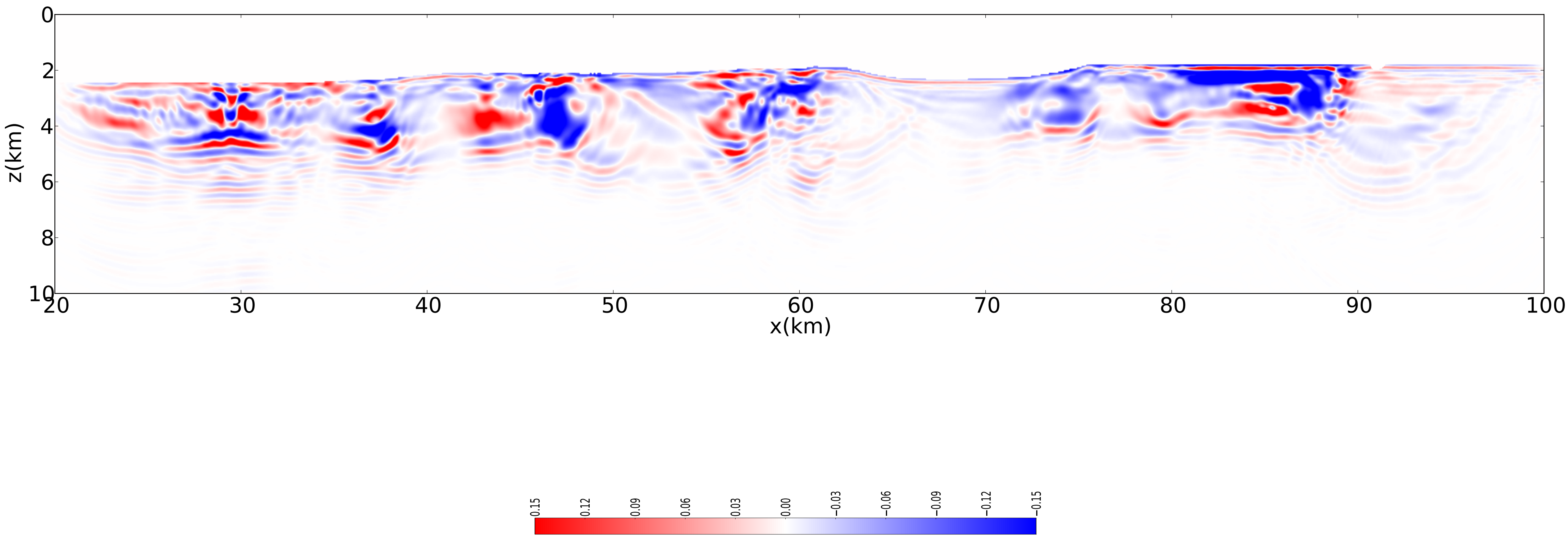
Input Model
[ray-based tomography + NMO]



Final result
[w/o denoising]



Relative update $\Delta(V)/V$

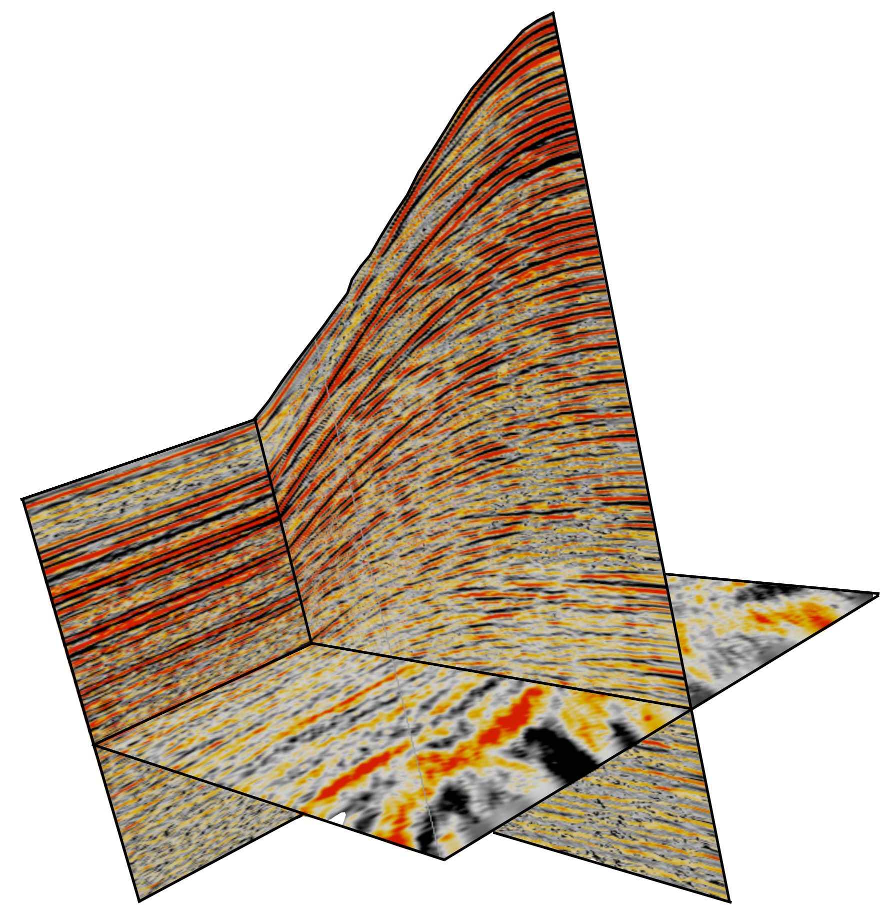


Modified Gauss-Newton

F. J. Herrmann, X. Li, A. Y. Aravkin, and T. van Leeuwen, “A modified, sparsity promoting, Gauss-Newton algorithm for seismic waveform inversion”, in Proc. SPIE, 2011, vol. 2011.

X. Li, F.J. Herrmann, A.Y. Aravkin and T. van Leeuwen - Fast randomized full waveform inversion with compressed sensing. Geophysics 77 (A13), 2012.

F.J. Herrmann, X. Li, A.Y. Aravkin, and T. van Leeuwen - A modified, sparsity-promoting, Gauss-Newton algorithm for seismic waveform inversion. Proc. SPIE 8138, 81380V (2011



Methodology

Modified Gauss-Newton:

- frequency continuation (7 bands 4 freqs, 2-5Hz)
- *rerandomized subsets* with *only* 600 shots
- 6 GN *iterations* per frequency *band*
- *preconditioning* of *Jacobian* by depth *weighting*
- *projection* of water *layer*

One-norm curvelet *regularized* gradients...

Convex composite structure [Burke & Ferris, '95.]

FWI:

$$\min_{\mathbf{m}} \quad \phi(\mathbf{m}) := \underbrace{\frac{1}{2}\|\overbrace{\mathbf{D} - \mathcal{F}[\mathbf{m};\mathbf{Q}]}^{\text{smooth}}\|_F^2}_{\text{convex}}$$

- exploit *convexity* by linearizing *within*

$$\min_{\mathbf{m}} \quad \phi(\mathbf{m}) := \frac{1}{2}\|\mathbf{D} - \mathcal{F}[\mathbf{m};\mathbf{Q}] - \nabla\mathcal{F}[\mathbf{m};\mathbf{Q}]]\boldsymbol{\delta}\mathbf{m})\|_F^2$$

- control the norm of the updates to *guarantee* convergence

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Gauss-Newton

Objective : $\phi(\mathbf{m}) = \|\mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}]\|_F^2$

Method: $\mathbf{m}^{k+1} = \mathbf{m}^k + \gamma_k \boldsymbol{\delta}\mathbf{m}^k$

GN Subproblem: $\boldsymbol{\delta}\mathbf{m}^k = \arg\min_{\boldsymbol{\delta}\mathbf{m}} \|\boldsymbol{\delta}\mathbf{D}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}]\boldsymbol{\delta}\mathbf{m}\|_F^2$

$$\boldsymbol{\delta}\mathbf{D}^k = \mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}]$$

Convergence : (1) $\phi'(\mathbf{m}^k;\boldsymbol{\delta}\mathbf{m}^k) < 0$
($\boldsymbol{\delta}\mathbf{m}^k$ is a descent direction)

(2) $\{\|\boldsymbol{\delta}\mathbf{m}^k\|\}$ diverges to infinity OR
$\mathbf{m}^k \rightarrow \bar{\mathbf{m}}$, which is stationary.

Modified Gauss-Newton

[Burke, personal com.]

Objective: $\boldsymbol{\phi}(\mathbf{m}) = \|\mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}]\|_F^2$

Method: $\mathbf{m}^{k+1} = \mathbf{m}^k + \gamma_k \boldsymbol{\delta}\mathbf{m}^k$

Subproblem: $\boldsymbol{\delta}\mathbf{m}^k = \underset{\boldsymbol{\delta}\mathbf{m}}{\arg\min} \left\{ \begin{array}{l} \|\boldsymbol{\delta}\mathbf{D}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}]\boldsymbol{\delta}\mathbf{m}\|_F^2 \\ \text{s.t. } \|\boldsymbol{\delta}\mathbf{m}\| \leq \tau^k \end{array} \right\}$

Convergence: (1) $\phi'(\mathbf{m}^k;\boldsymbol{\delta}\mathbf{m}^k) \leq \|\boldsymbol{\delta}\mathbf{D}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}]\boldsymbol{\delta}\mathbf{m}^k\|_F^2 - \phi(\mathbf{m}^k) < 0$

(2) $\mathbf{m}^k \rightarrow \bar{\mathbf{m}}$ as long as $\|\tau^k\|$ are bounded.

Modified Gauss-Newton
[with SAA]

Objective: $\underline{\phi}(\mathbf{m}) = \|\underline{\mathbf{D}} - \boldsymbol{\mathcal{F}}[\mathbf{m};\underline{\mathbf{Q}}]\|_F^2$

Method: $\mathbf{m}^{k+1} = \mathbf{m}^k + \gamma_k \boldsymbol{\delta}\mathbf{m}^k$

Subproblem: $\boldsymbol{\delta}\mathbf{m}^k = \arg\min_{\boldsymbol{\delta}\mathbf{m}} \left\{ \begin{array}{l} \|\underline{\boldsymbol{\delta}\mathbf{D}}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\underline{\mathbf{Q}}]\boldsymbol{\delta}\mathbf{m}\|_F^2 \\ \text{s.t. } \|\boldsymbol{\delta}\mathbf{m}\| \leq \tau^k \end{array} \right\}$

Convergence: (1) $\underline{\phi}'(\mathbf{m}^k;\boldsymbol{\delta}\mathbf{m}^k) \leq \|\underline{\boldsymbol{\delta}\mathbf{D}}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\underline{\mathbf{Q}}]\boldsymbol{\delta}\mathbf{m}^k\|_F^2 - \phi(\mathbf{m}^k) < 0$

(2) $\mathbf{m}^k \rightarrow \bar{\mathbf{m}}$ as long as $\|\tau^k\|$ are bounded.

Algorithm

Result: Output estimate for the model $\mathbf{m}$

$k \longleftarrow 0$; $\mathbf{m}^k \longleftarrow \mathbf{m}_0$

while not converged **do**

$\{\underline{\mathbf{D}}^k, \underline{\mathbf{Q}}^k\} \longleftarrow \{\mathbf{D}\mathbf{W}^k, \mathbf{Q}\mathbf{W}^k\}$ with $\mathbf{W}^k \subset [\mathbf{e}_1, \cdots, \mathbf{e}_{n_s}]$

$\underline{\delta\mathbf{D}}^k \longleftarrow \underline{\mathbf{D}}^k - \boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]$ $\tau^k \longleftarrow \|\underline{\delta\mathbf{D}}^k\|_F \,/\, \|\mathbf{C}_2 \nabla\boldsymbol{\mathcal{F}}^*[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\underline{\delta\mathbf{D}}^k\|_\infty$

$\delta\mathbf{x} \leftarrow \arg\min_{||\mathbf{x}||_1 \leq \tau_k} \|\underline{\delta\mathbf{D}}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\mathbf{C}_2^H \mathbf{x}\|_F^2$

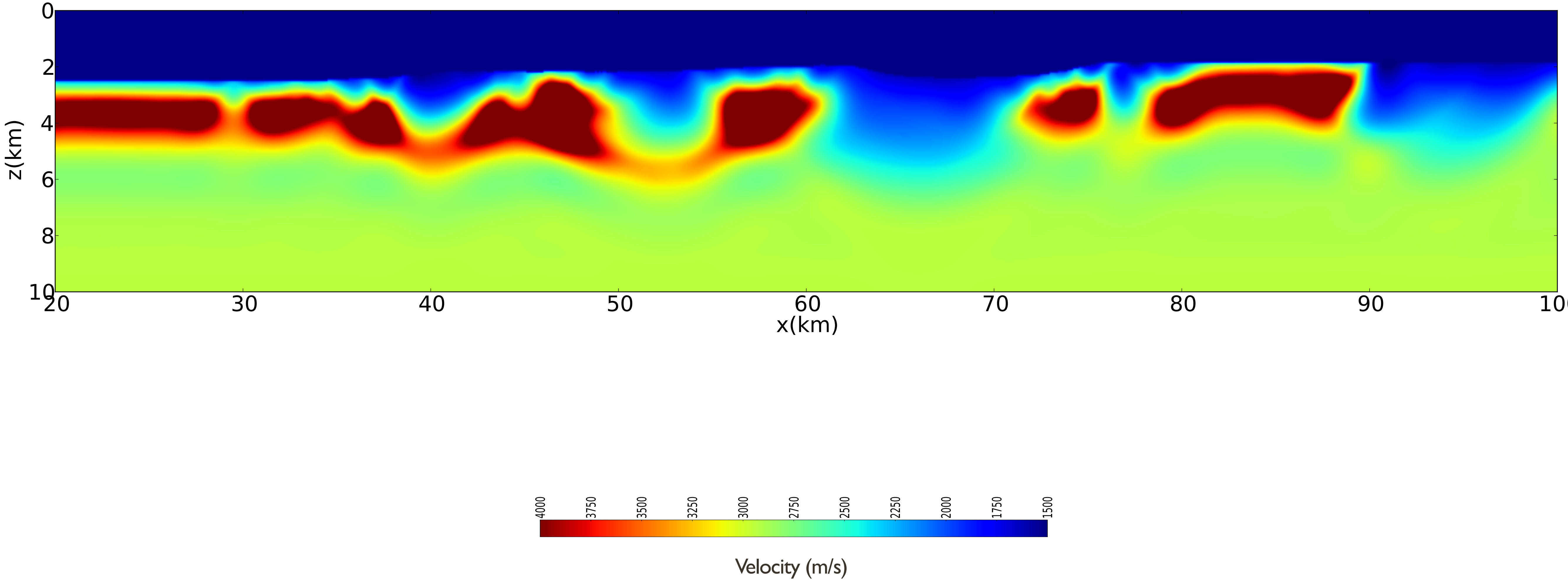
$\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k \mathbf{C}_2^H \delta\mathbf{x}$

$k \longleftarrow k + 1$;

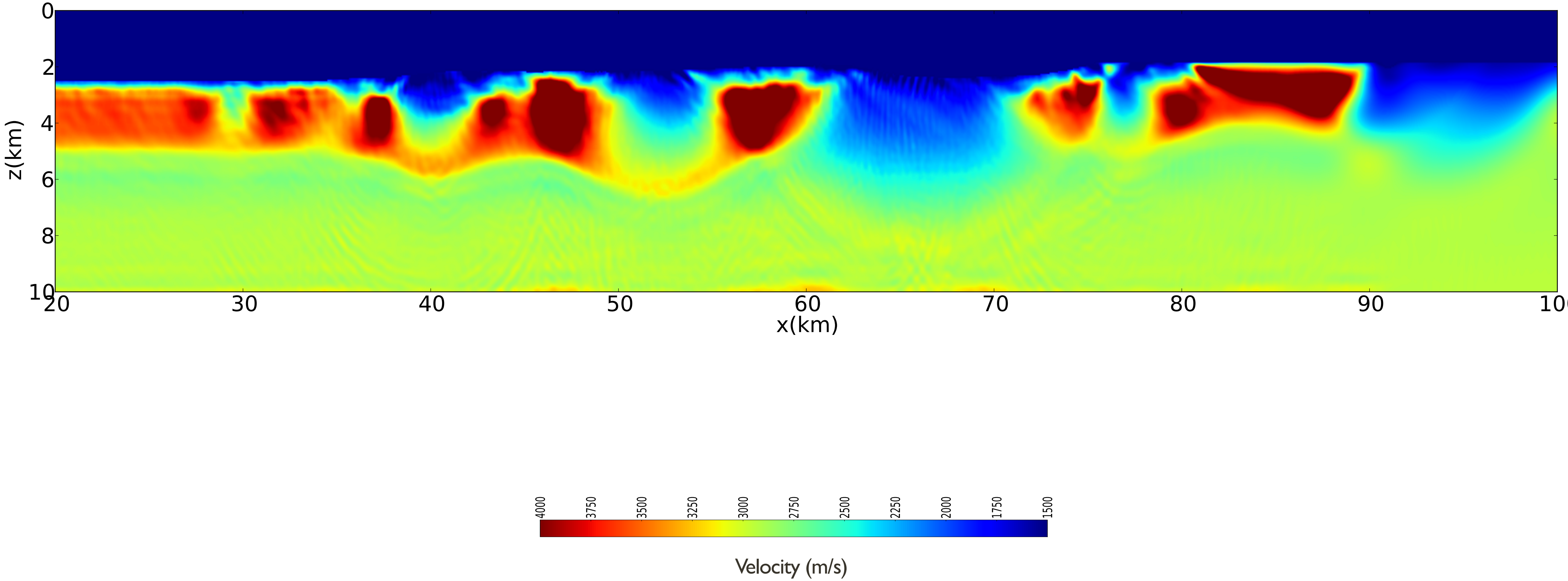
end

Algorithm 1: modified Gauss Newton with sparsity promotion

Initial model
[ray-based tomography]

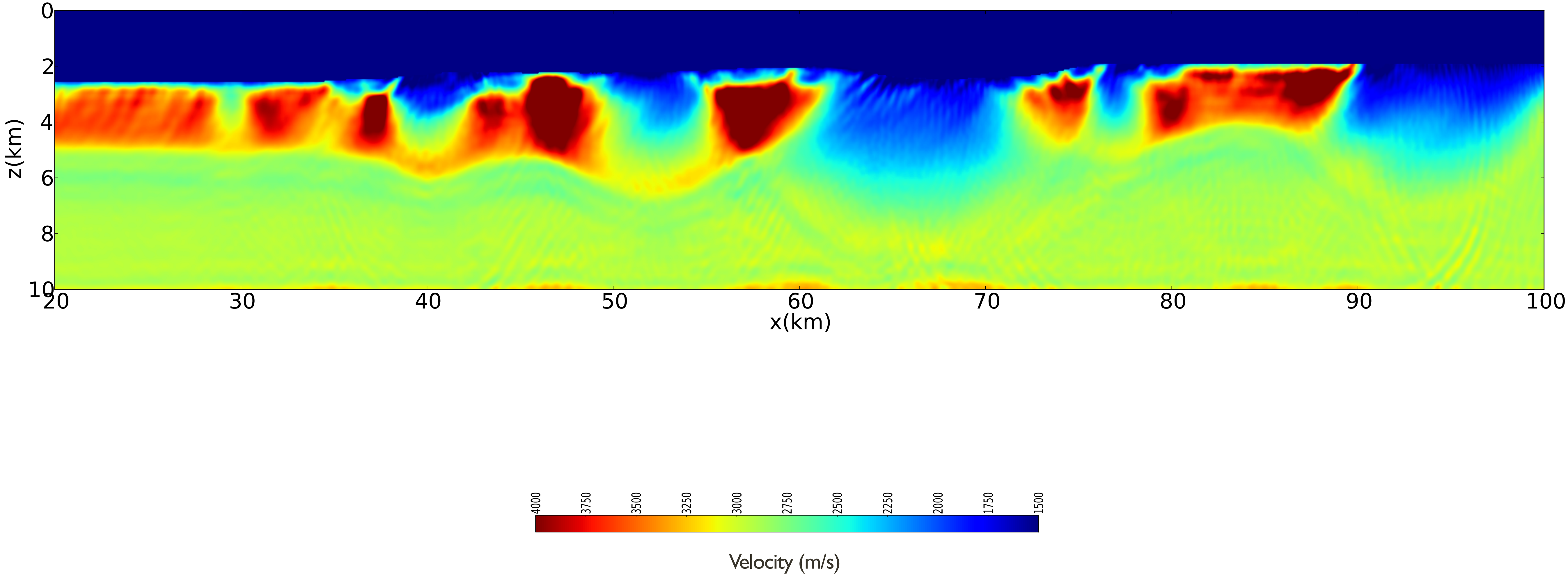


Final result submitted to workshop
[w/o denoising]

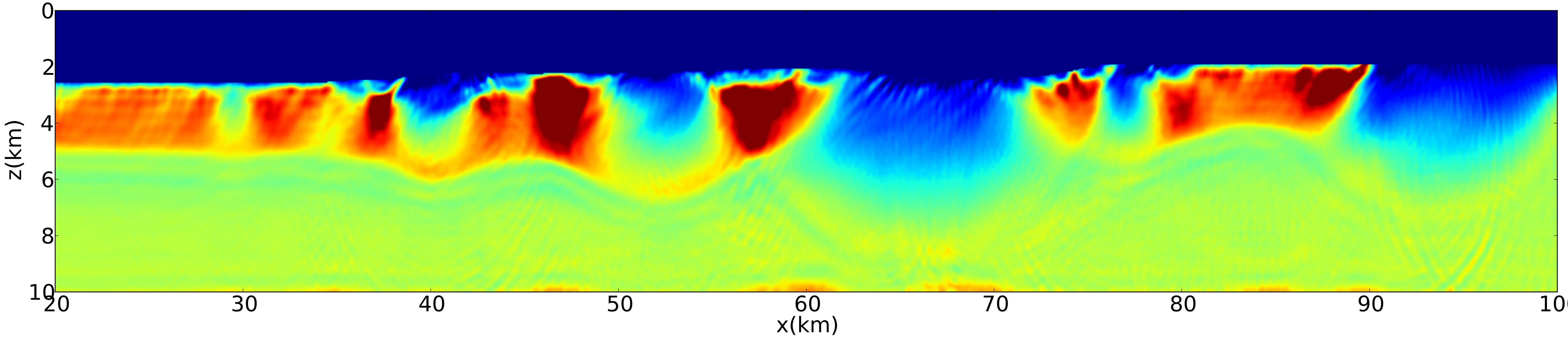


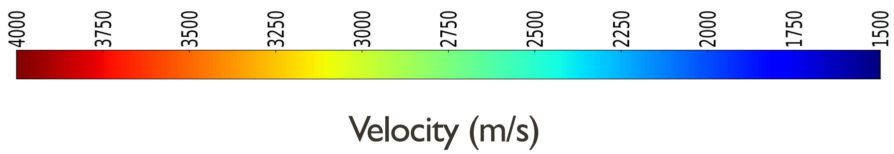
“Latest” final result

[w/o denoising]

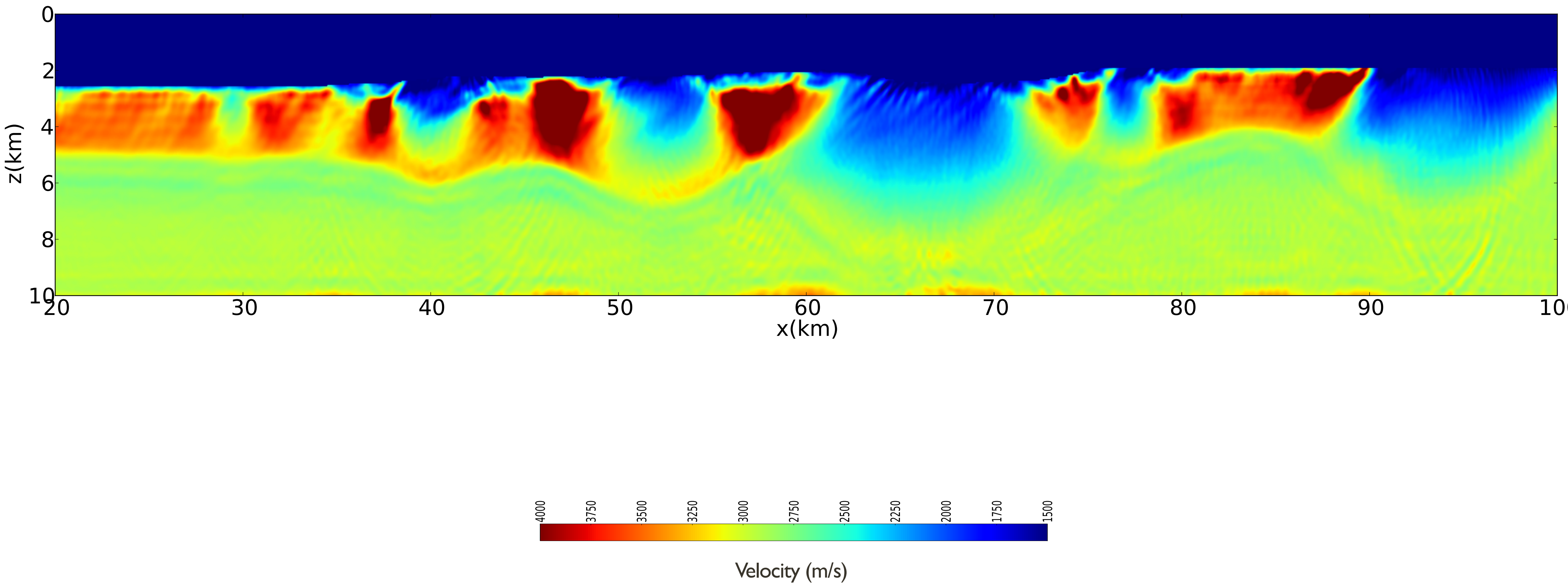


Final result
[w/ denoising]



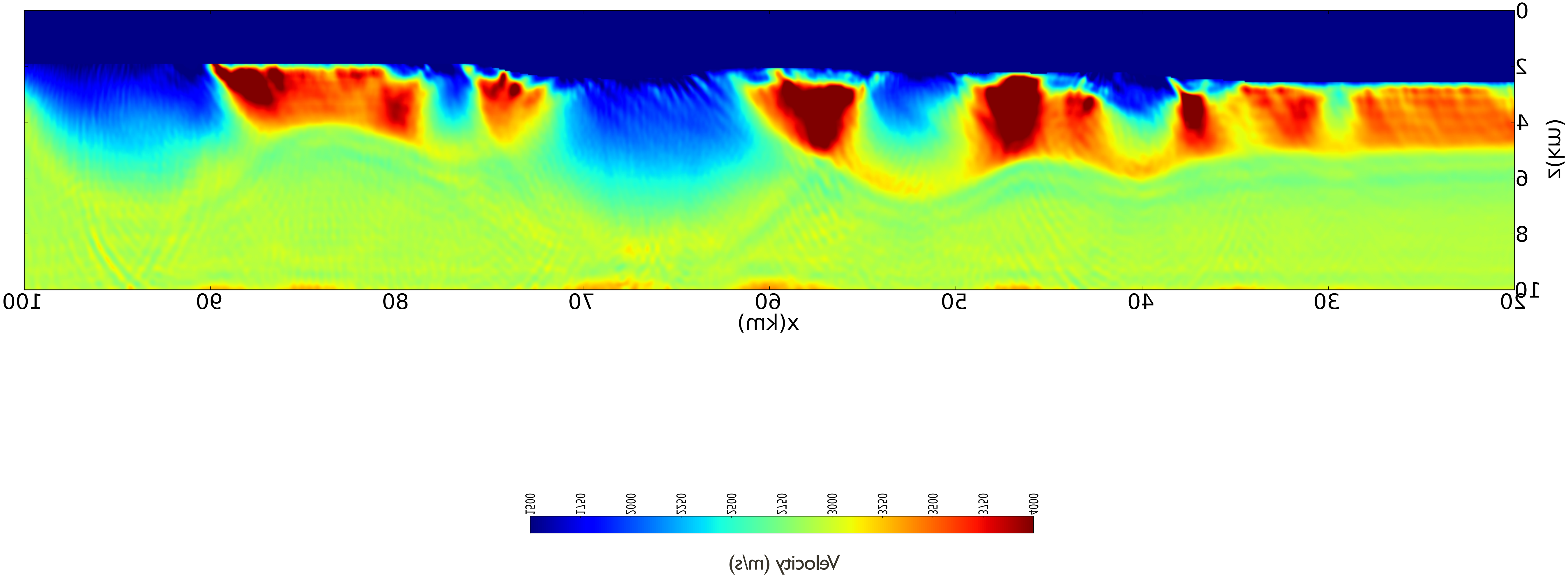


Final result
[w/ denoising]

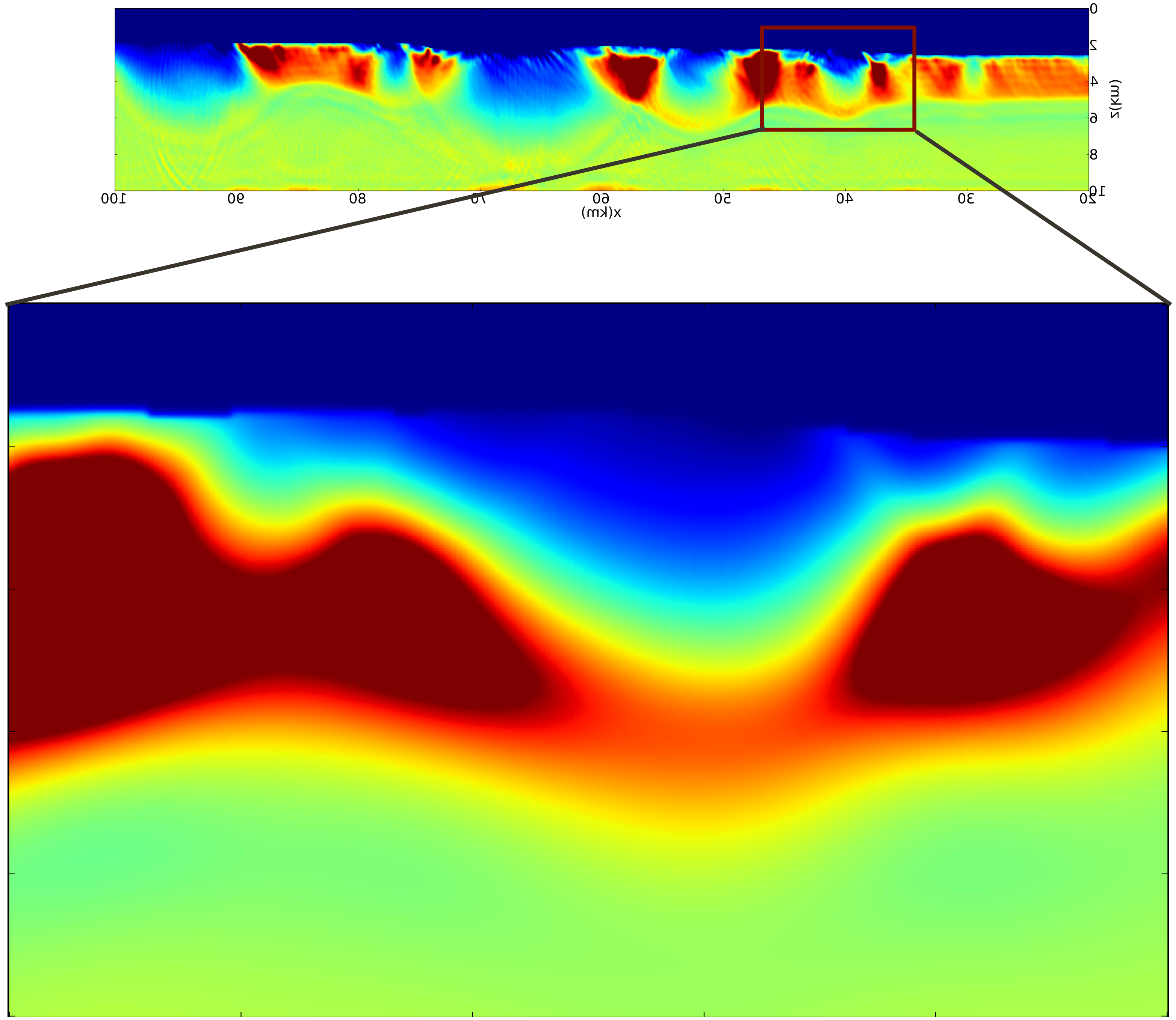


Final result
[w/ denoising]

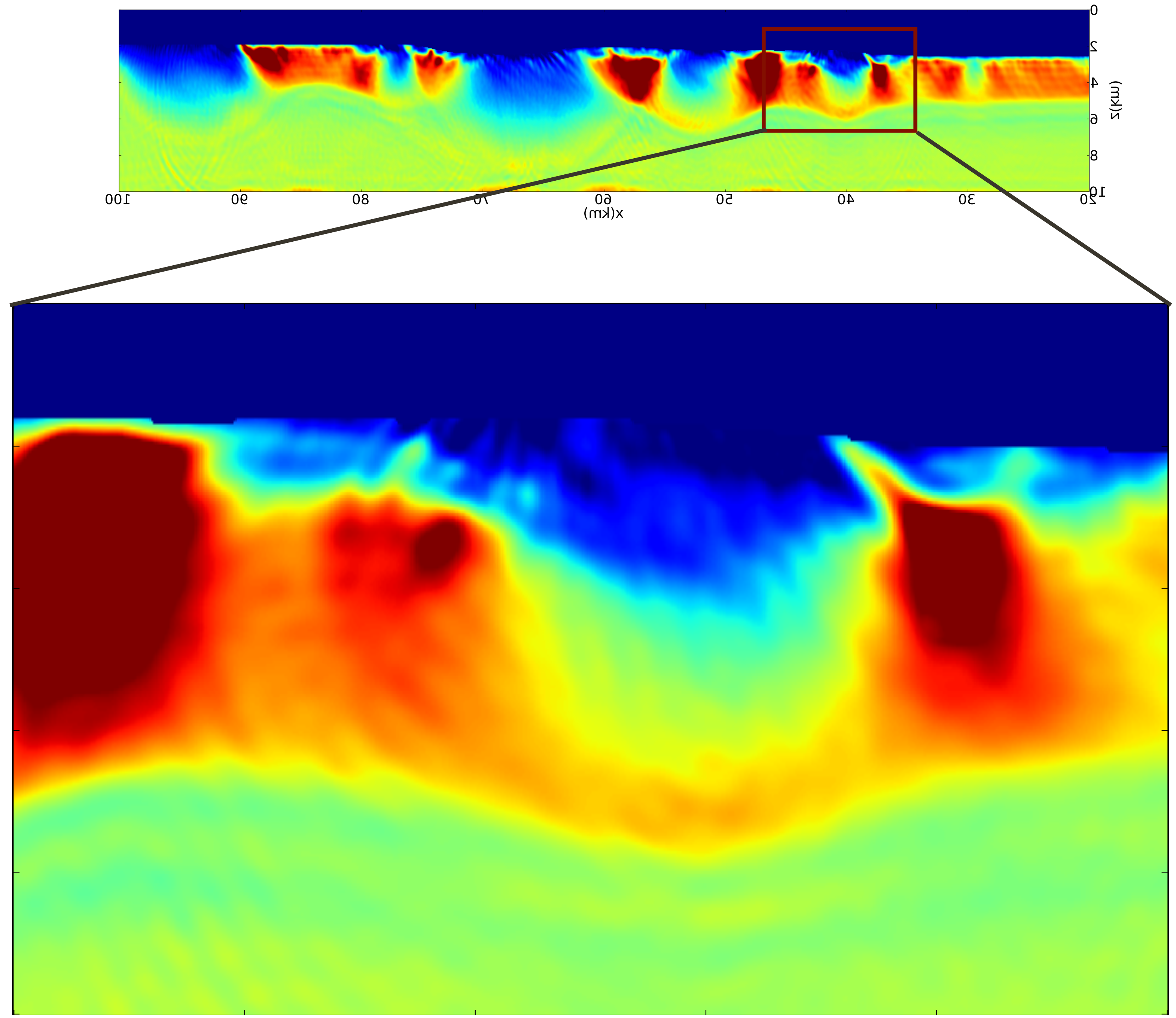
Final result
[w/ denoising]



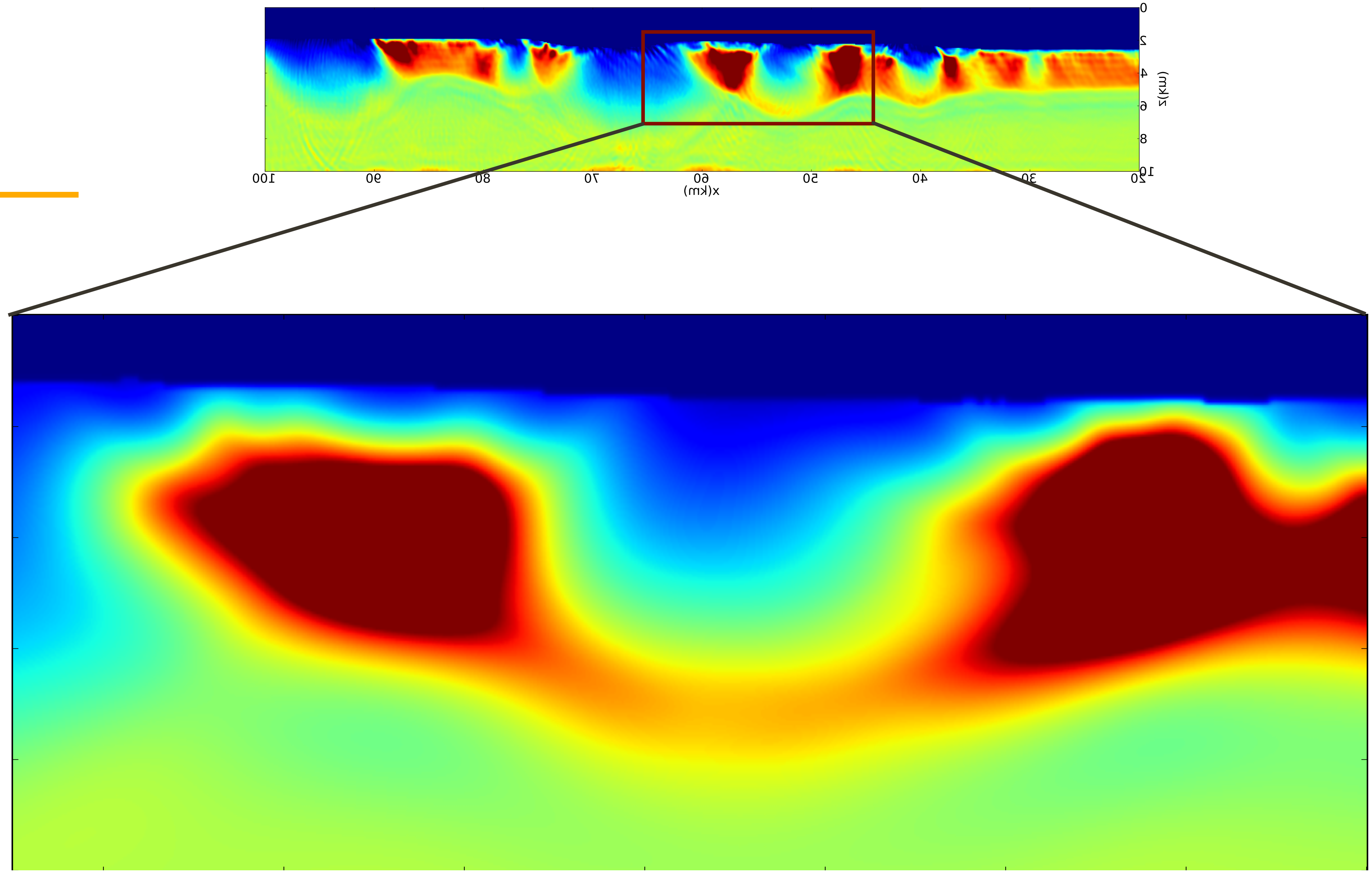
Final result
[w/ denoising]



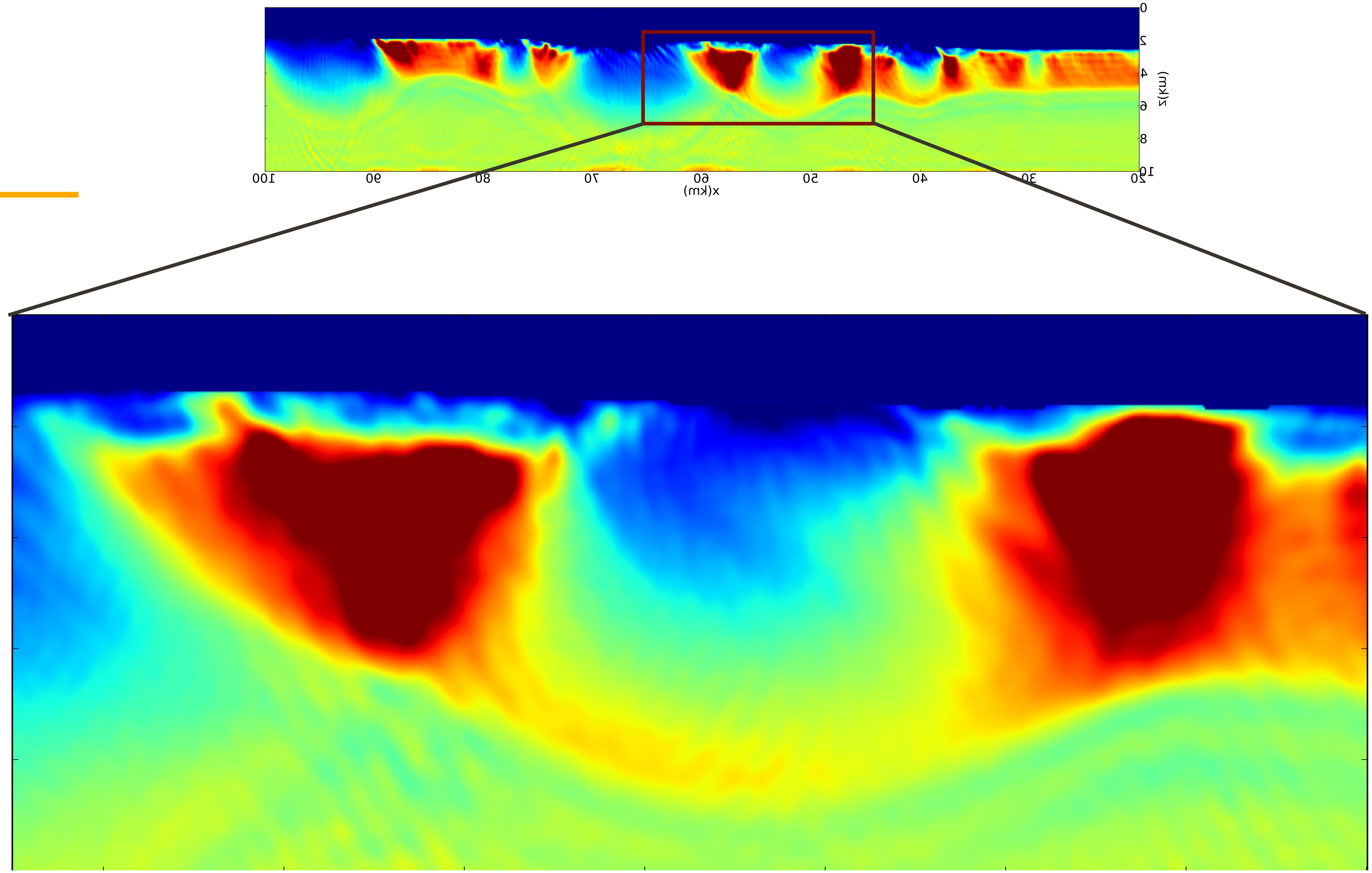
Final result
[w/ denoising]



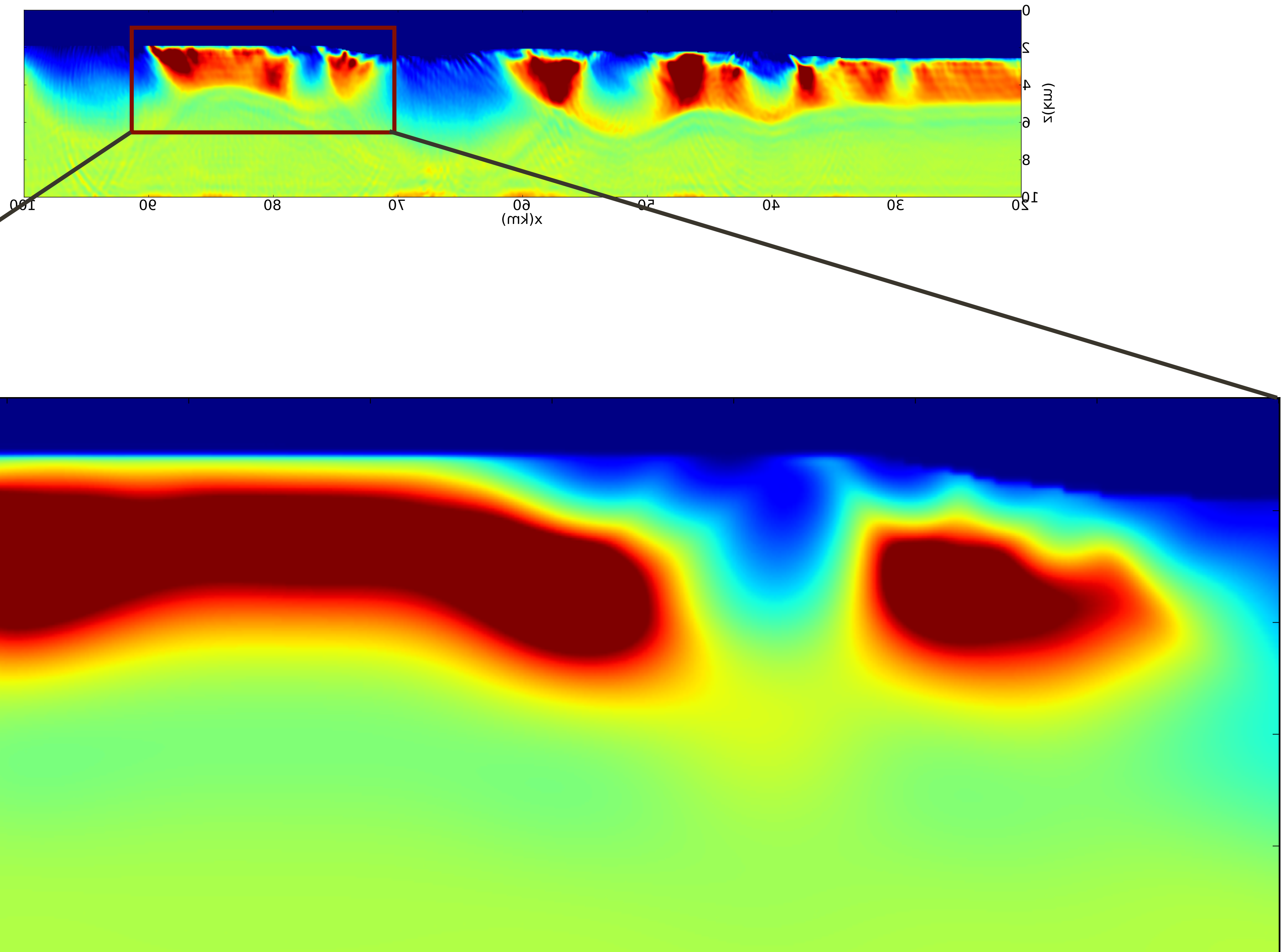
Final result
[w/ denoising]



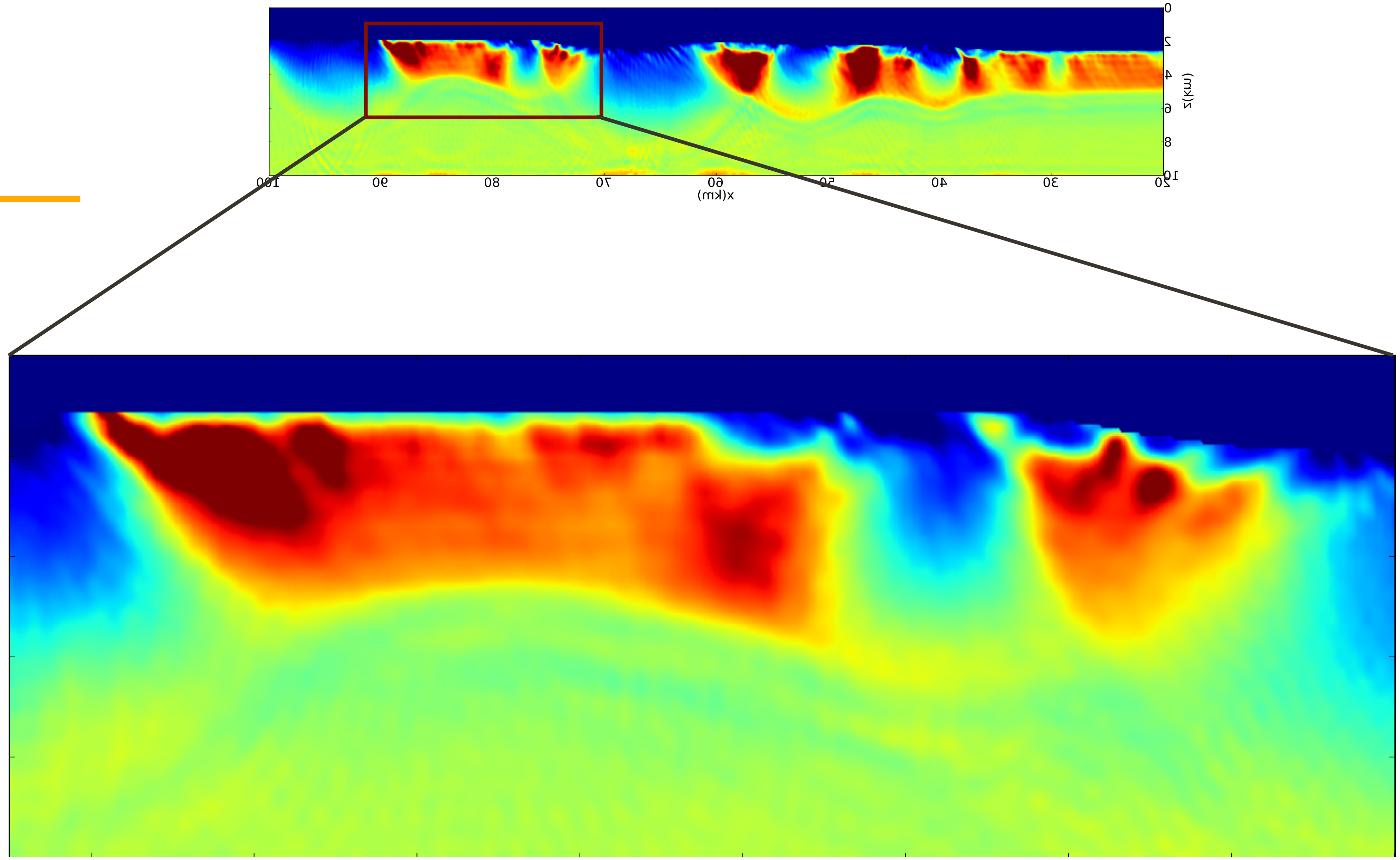
Final result
[w/ denoising]



Final result
[w/ denoising]

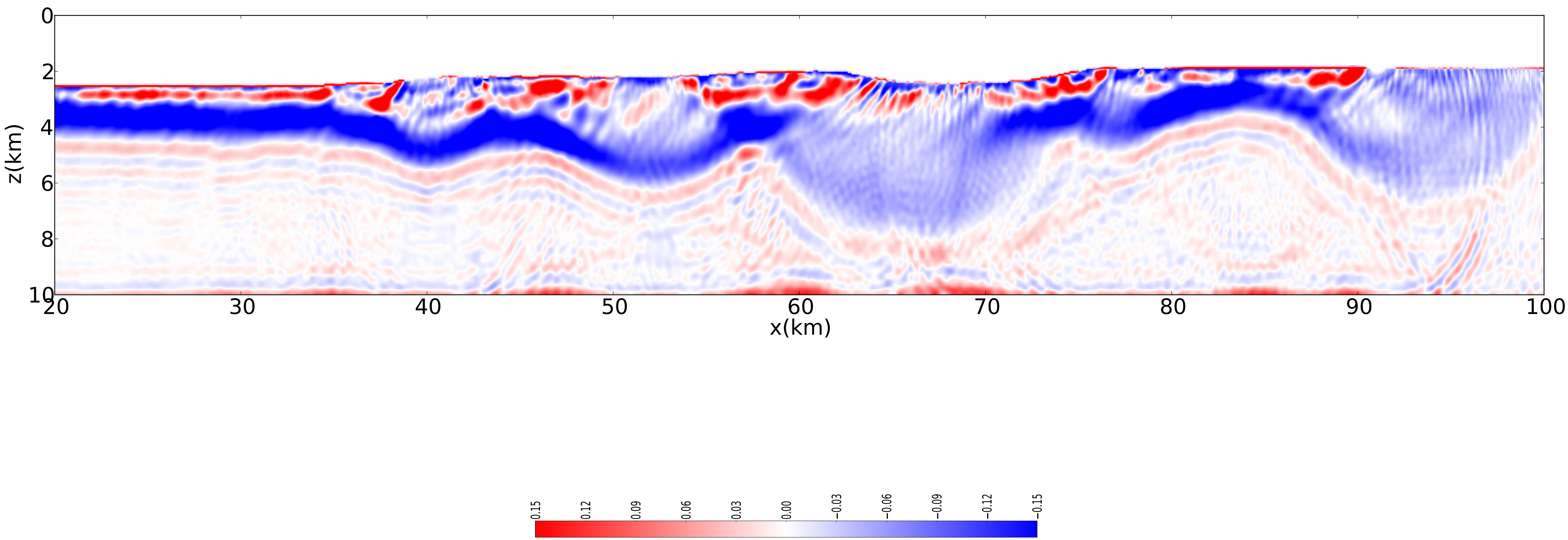


Final result
[w/ denoising]



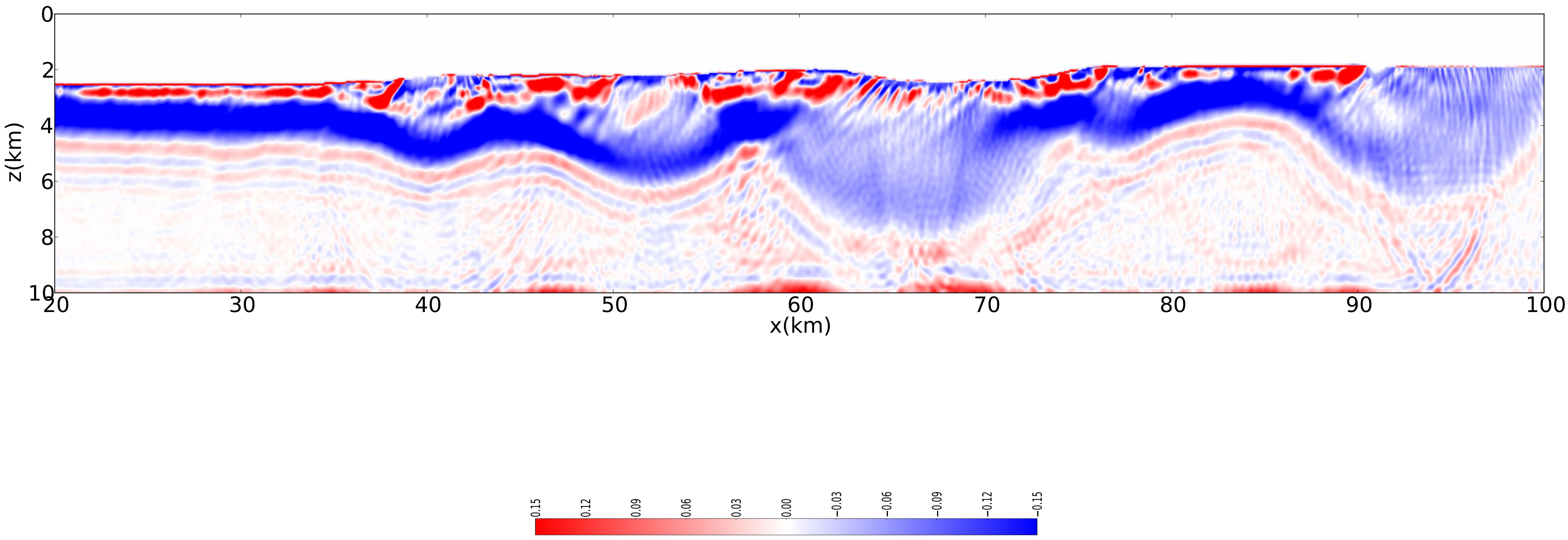
Relative update
[w/o denoising]

$$\Delta(V)/V$$

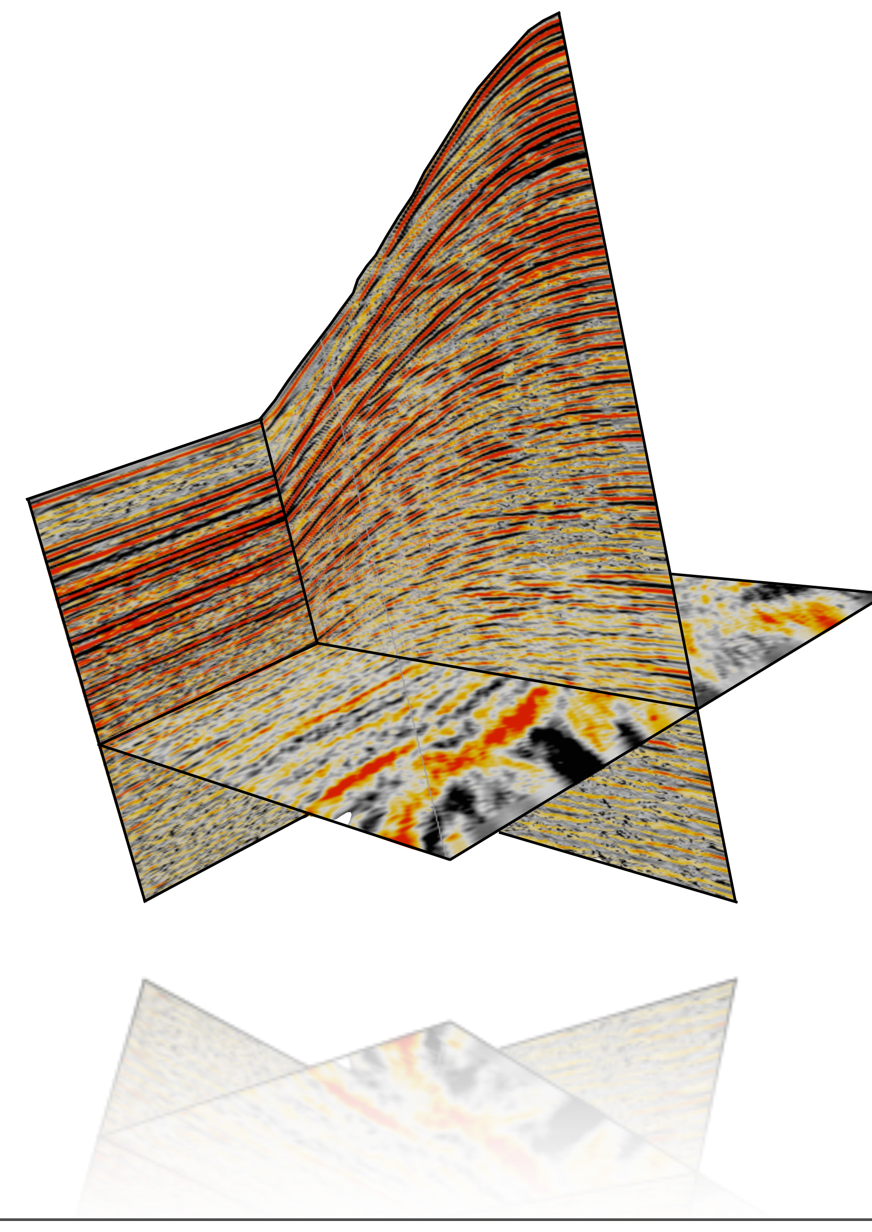


Relative update
[w/ denoising]

$$\Delta(V)/V$$

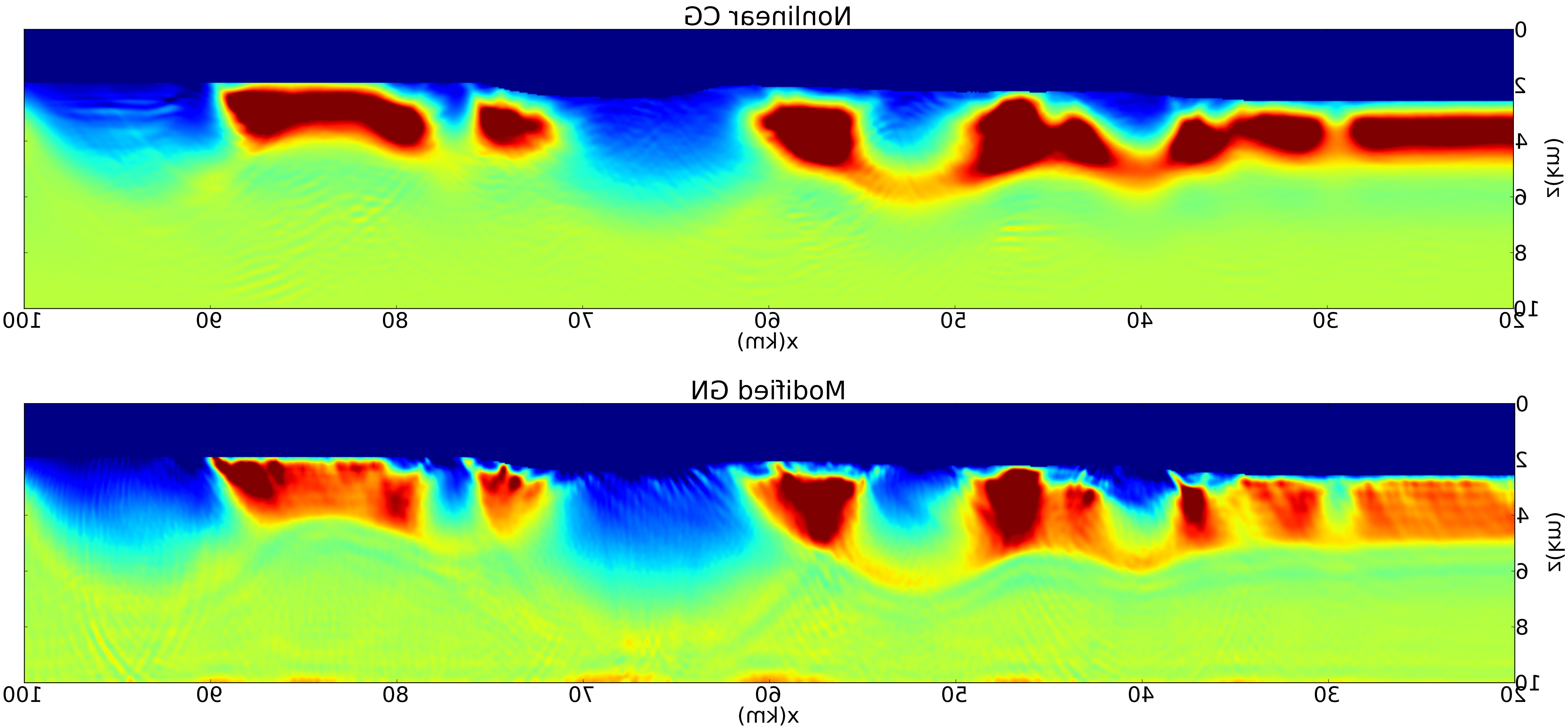


Lessons learned & road ahead



Comparison

[nonlinear CG versus modified GN]



SLIM

Verdict

Modified Gauss-Newton is the winner:

- “clearly defined” topology @ top salt
- sharper edges @ the base of the salt
- little parameter settings and intervention (no layer stripping & offset continuation)

Good *starting* model for *elastic* inversion.

Why does this work?

Continuation method in

- frequency
- complexity (sparse to less sparse)

Sparse *nonlinear* approximations of updates

- contain "skeleton" of the singularities
- "concentrates on acoustic phases"

Observations

	Nonlin. CG [phase]	Modified GN [amplitude]
Acoustic	mixed results	encouraging
Noise	?	important
Conditioning	involved	“simple”
Quality	smooth	sharp

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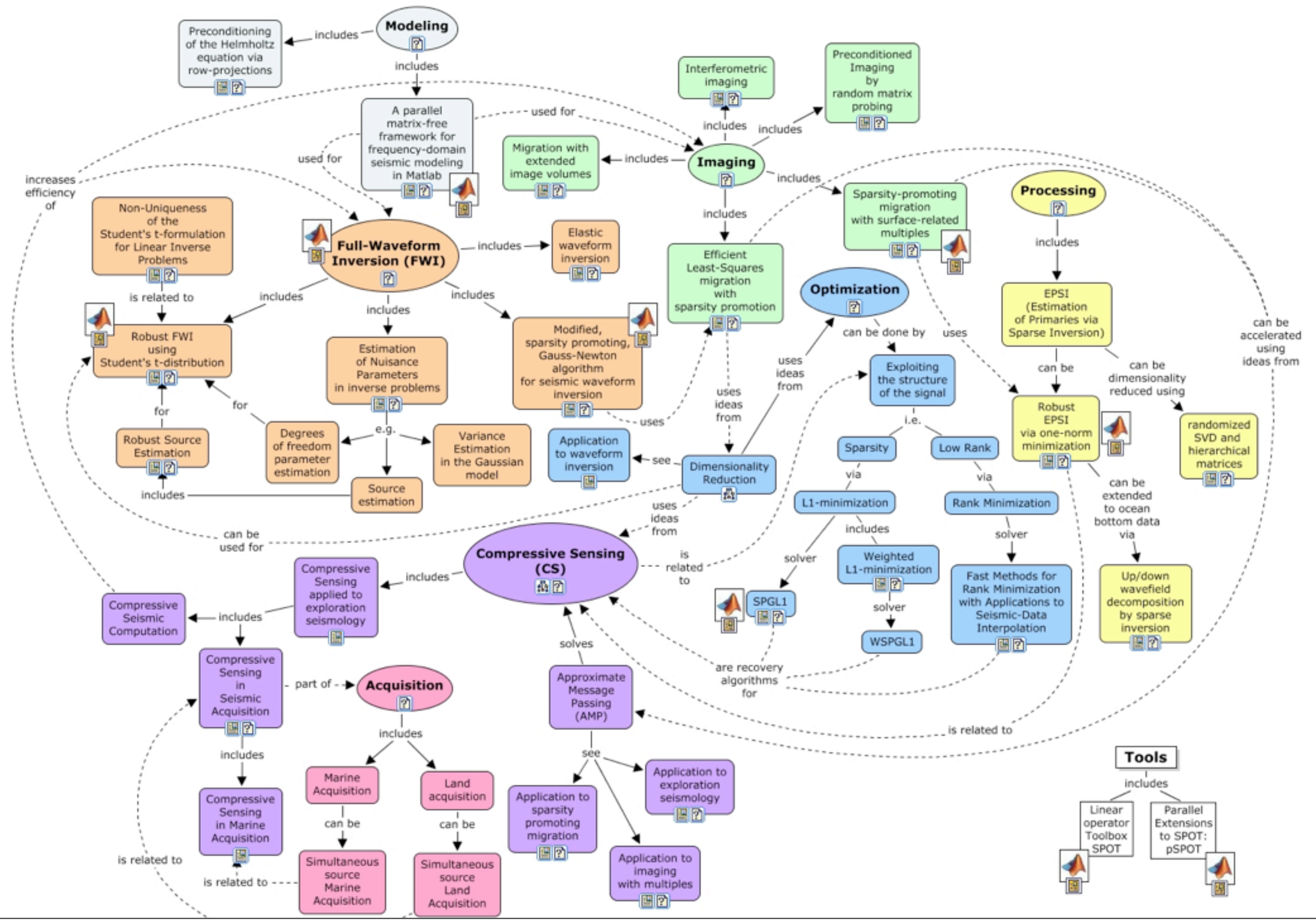

Wednesday, 5 December, 12

Acknowledgments

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NSERC
CRSNG

Thank you

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