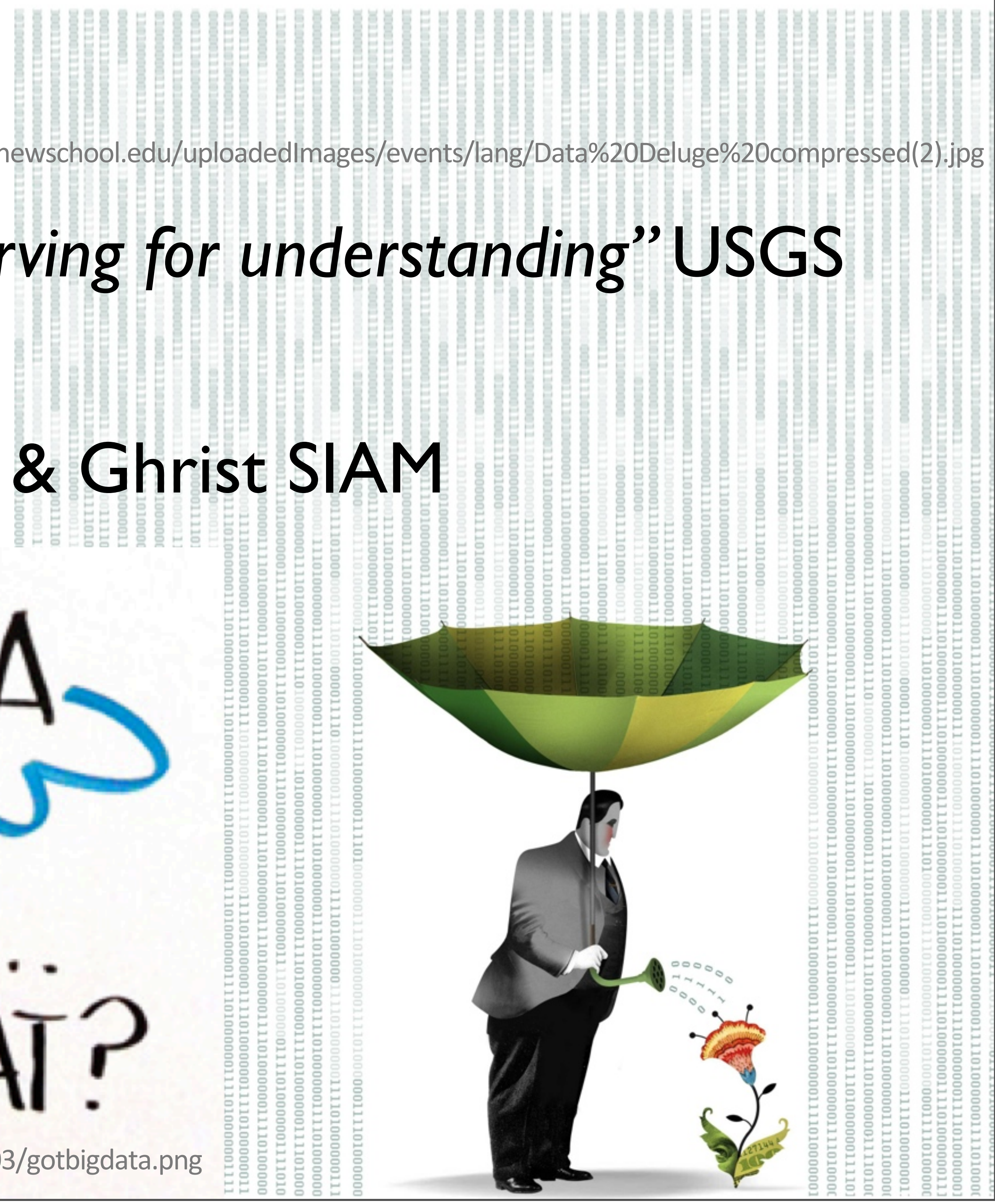
Fast sparsity-promoting imaging with *message* passing

Felix J. Herrmann

thanks to Xiang Li

SLIM
Seismic Laboratory for Imaging and Modeling
the University of British Columbia

Big data



http://www.newschool.edu/uploadedImages/events/lang/Data%20Deluge%20compressed(2).jpg

"We are drowning in data but starving for understanding" USGS director Marcia McNutt

"Got data now what" Carlsson & Ghrist SIAM

http://bigdatablog.emc.com/wp-content/uploads/2012/03/gotbigdata.png

Drivers

Recent technology push calls for collection

- high-quality *broad-band* data *volumes* (>100k channels)
- *larger* offsets & *full* azimuth

Exposes vulnerabilities in our *ability* to control

- *acquisition* costs / time / quality
- *processing* costs / time / quality

Drivers cont'd

Complexity of *inversion* algorithms *exposes* the "*curse of dimensionality*" in

- **sampling**: *exponential* growth of # *samples* for *high* dimensions
- **optimization**: *exponential* growth of # parameter *combinations* that need to be evaluated to minimize our *objective* functions

Credit to John McCaplin, University of Texas, HPC.

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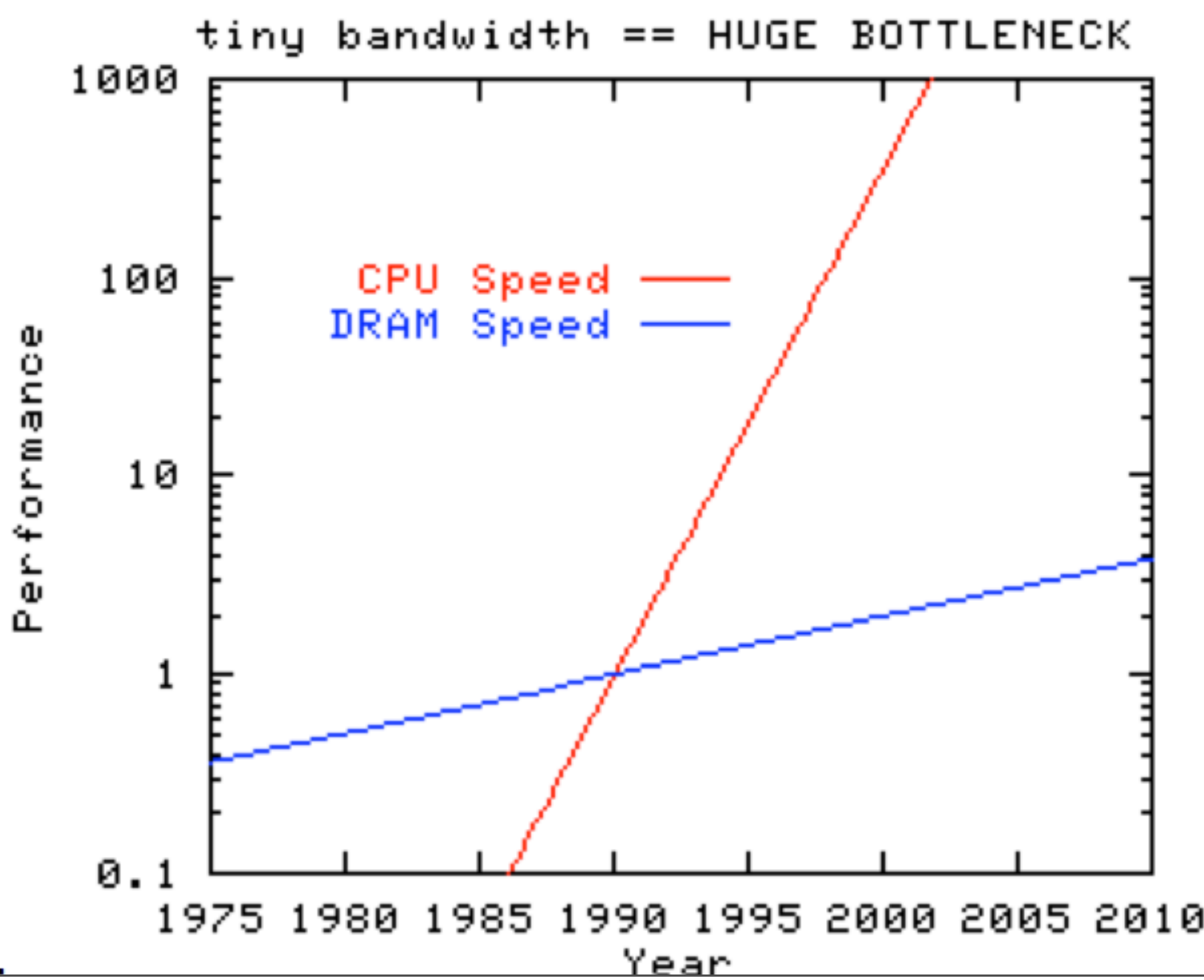
Drivers cont'd

Problems exacerbated by IO bottleneck:



Goals

Replace a '*sluggish*' inversion *paradigm* that

- relies on *touching* **all** *data* all the *time*

by an *agile* optimization *paradigm* that works on

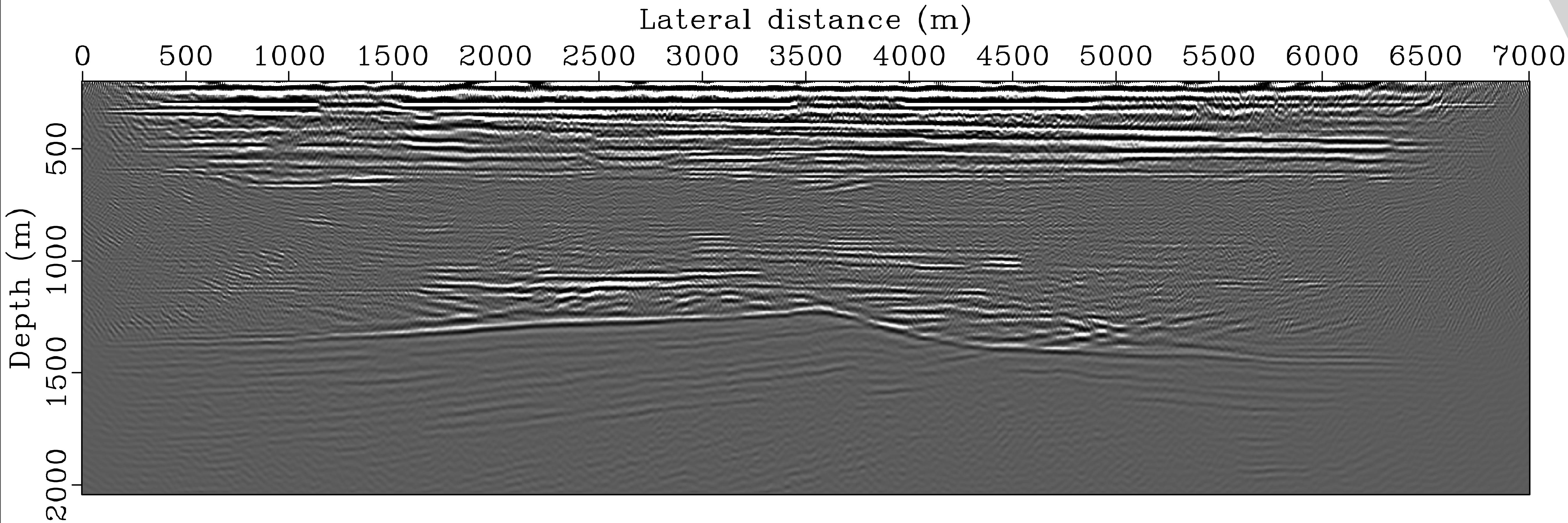
- **small** *randomized* subsets of data *iteratively*

Confront "*data explosion*" by

- *reducing* acquisition *costs*
- *removing* IO & PDEs-solve *bottlenecks*

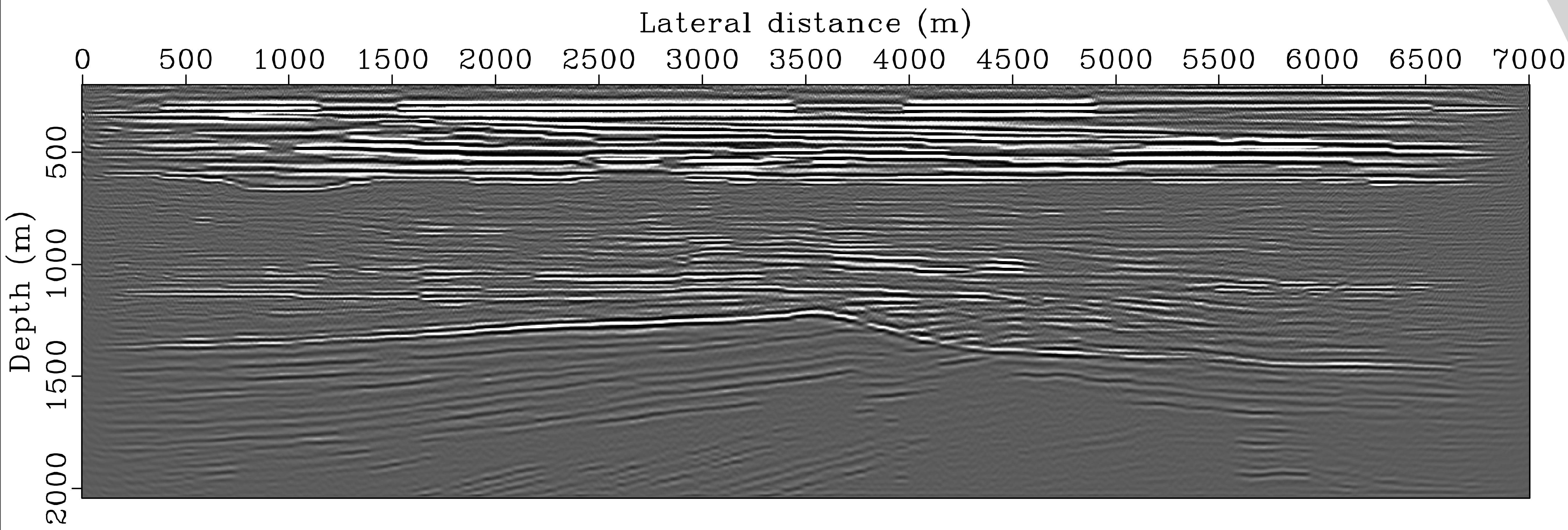
Imaging results

[migration w/ "*all*" data]



Imaging results

[*linearized* inversion w/ *small subsets*]



Key technologies

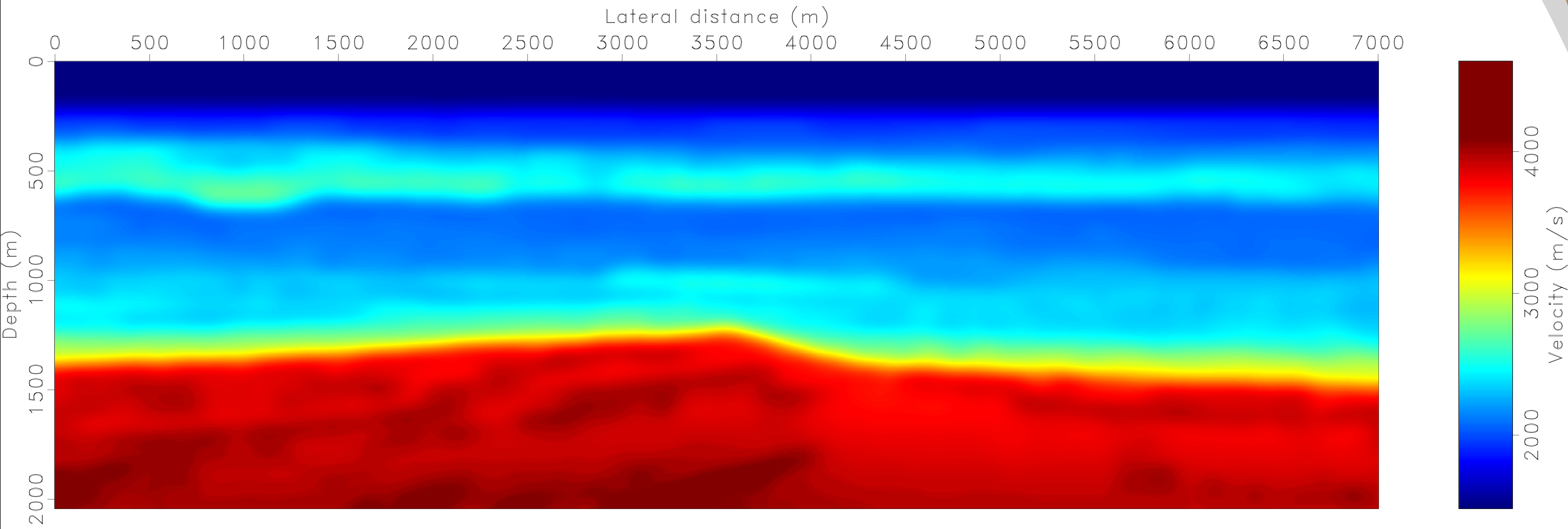
Fast imaging with Stochastic optimization / Compressive Sensing:

- *subsets* of *simultaneous* sources – *supershots* generated by *random* amplitude-weighted *superpositions*
- *random* subsets of *sequential* sources

Imaging via large-scale *curvelet*-domain *sparsity* promoting *convex* optimization with *cooling*

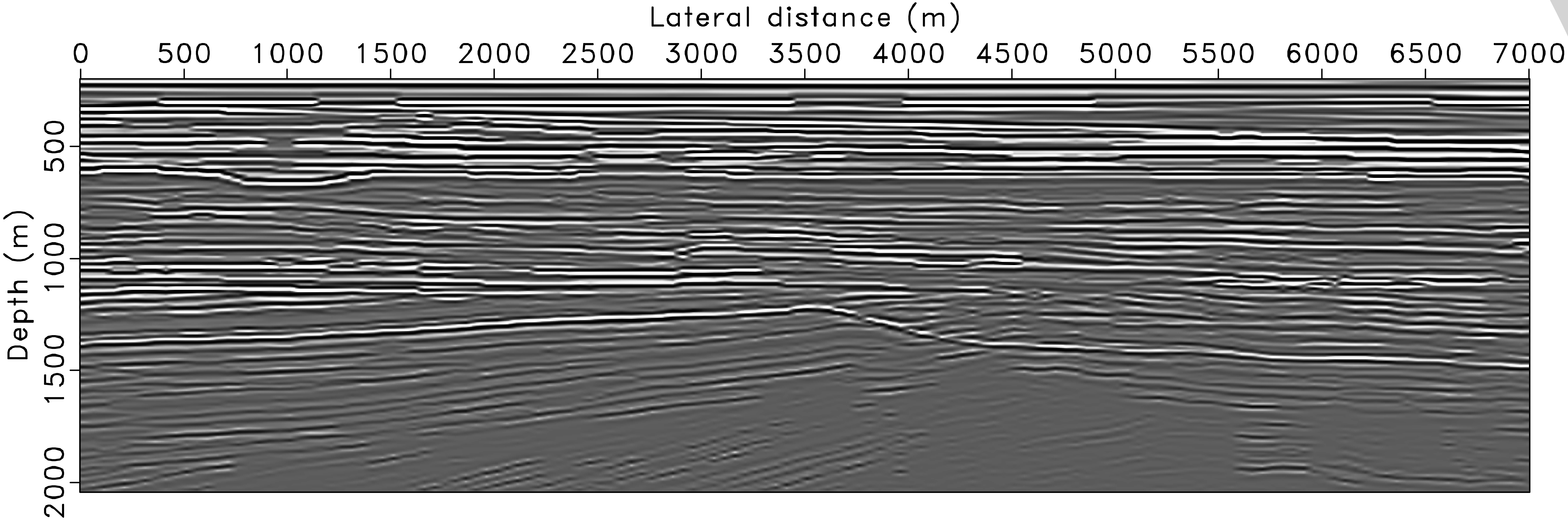
Acceleration with "*approximate* message passing"

Imaging
[background model]



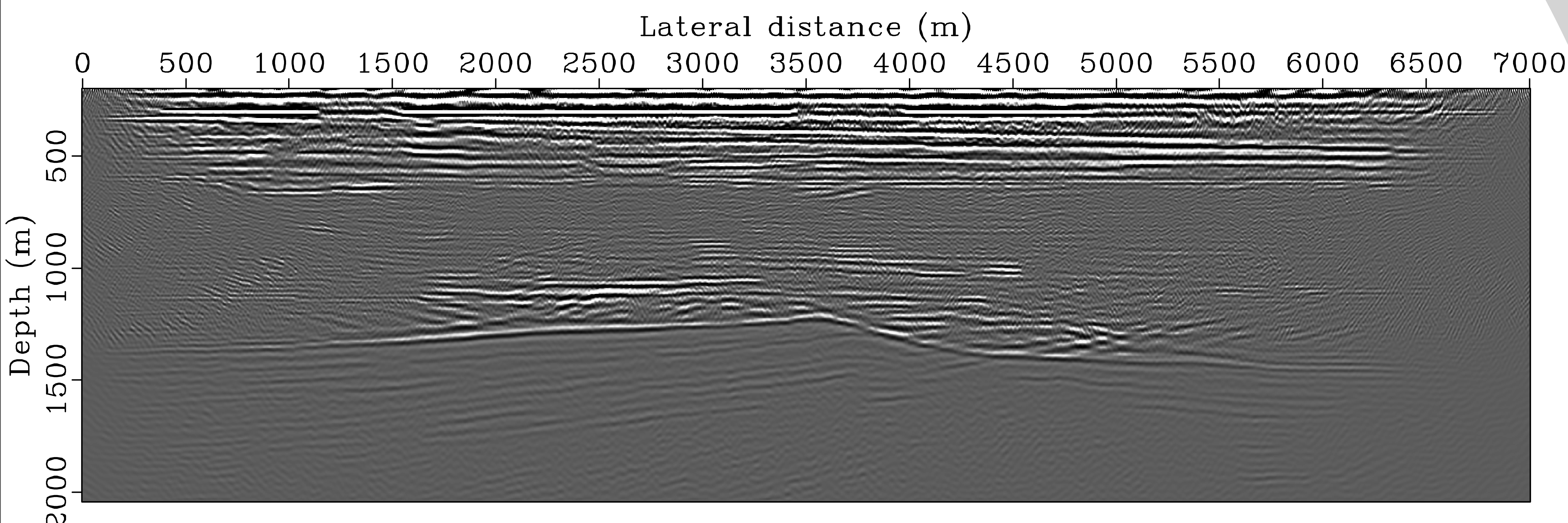
Imaging

[*true* perturbation]



Migration

[single migration with "*all*" data]



Too expensive to *invert* with "*all*" data...

Fast imaging

[*via* stochastic optimization]

*R*erandomized sampling

- *linear* speed up by *reducing* # PDE solves
- *increases* convergence but may *fail* to *converge*

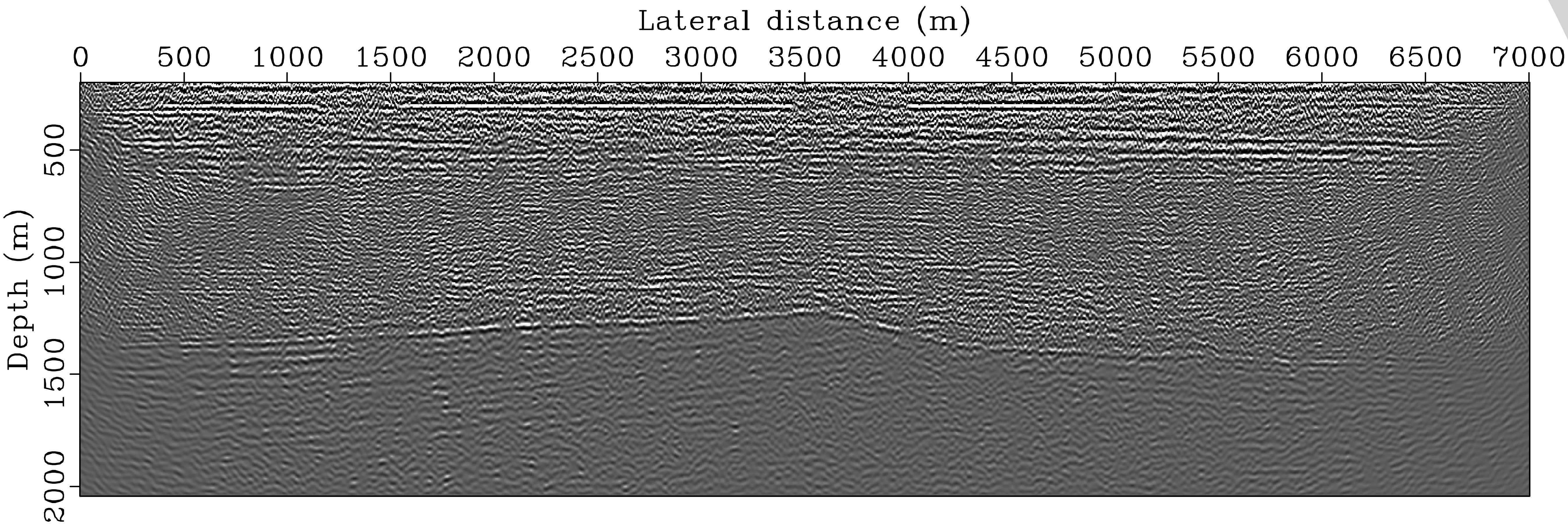
Exploits multi-experiment *redundancy* of seismic data volumes

- *regularly* draw *independent* subsets of shots
- cancels *crosstalk* by *rerandomization*

Heuristic of *current* phase-encoding migration/FWI methods

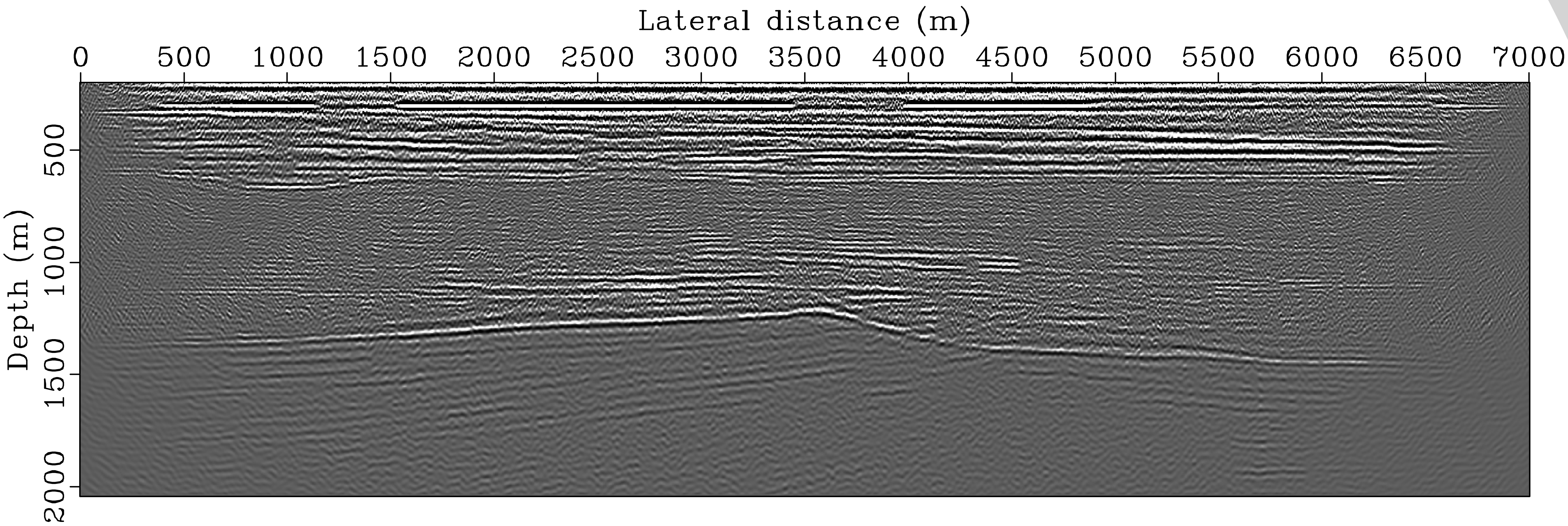
Linearized inversion

[ℓ_2 w/o rerandomization 3 *super* shots]



Linearized inversion

[ℓ_2 *w/* rerandomization 3 *super* shots]



Fast imaging
[*via* compressive sensing]

Incoherent randomized sampling

- *linear* speed up by *reducing* # PDE solves
- *coherent* source crosstalk turns into ***non-sparse*** *incoherent* noise

Exploits *structure* exhibited by migrated *images*

- leverages *curvelet*-domain ***sparsity*** promotion
- *maps* “noisy” crosstalk to *coherent* reflectors

Convex optimization

[p=2 or p=1]

Linearized inversion with *randomized supershots:*

$$\delta\widetilde{\mathbf{m}} = \mathbf{S}^* \arg\min_{\delta\mathbf{x}} \|\delta\mathbf{x}\|_{\ell_p} \quad \text{subject to} \quad \|\delta\underline{\mathbf{d}} - \overbrace{\nabla\mathcal{F}[\mathbf{m}_0;\underline{\mathbf{Q}}]}^{\text{demigration}}\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma$$

$\delta\mathbf{x}$ = Sparse curvelet-coefficient vector

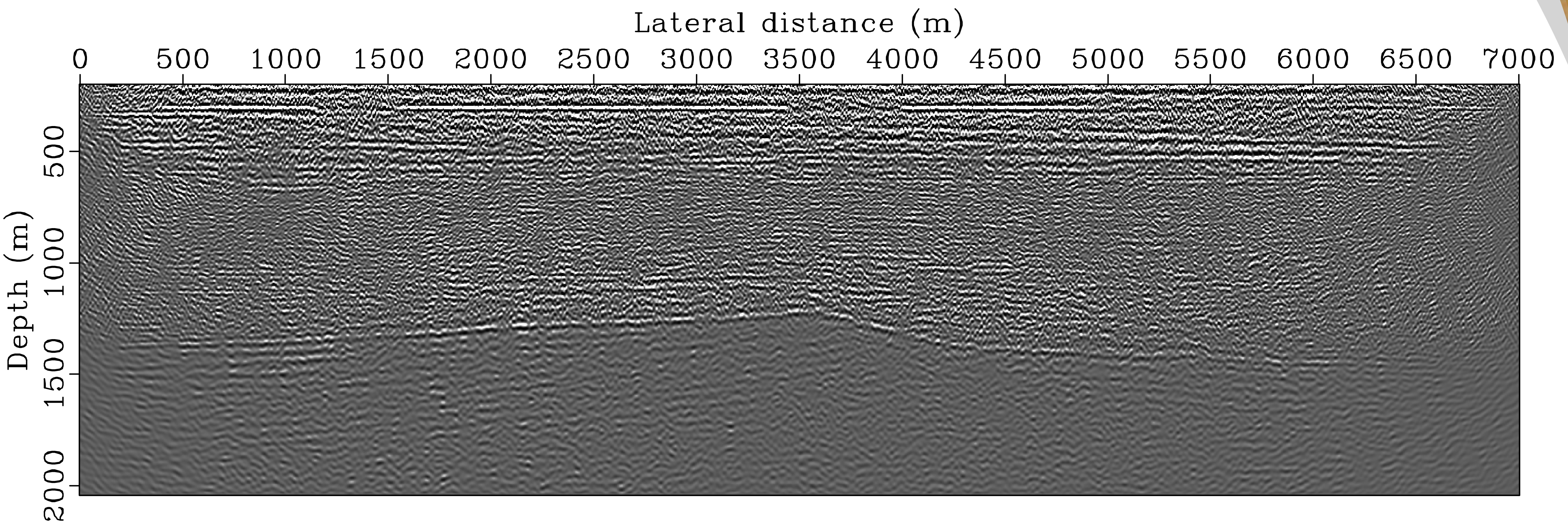
$\mathbf{S}^*$ = Curvelet synthesis

$\underline{\mathbf{Q}}$ = Simultaneous sources

$\delta\underline{\mathbf{d}}$ = Super shots

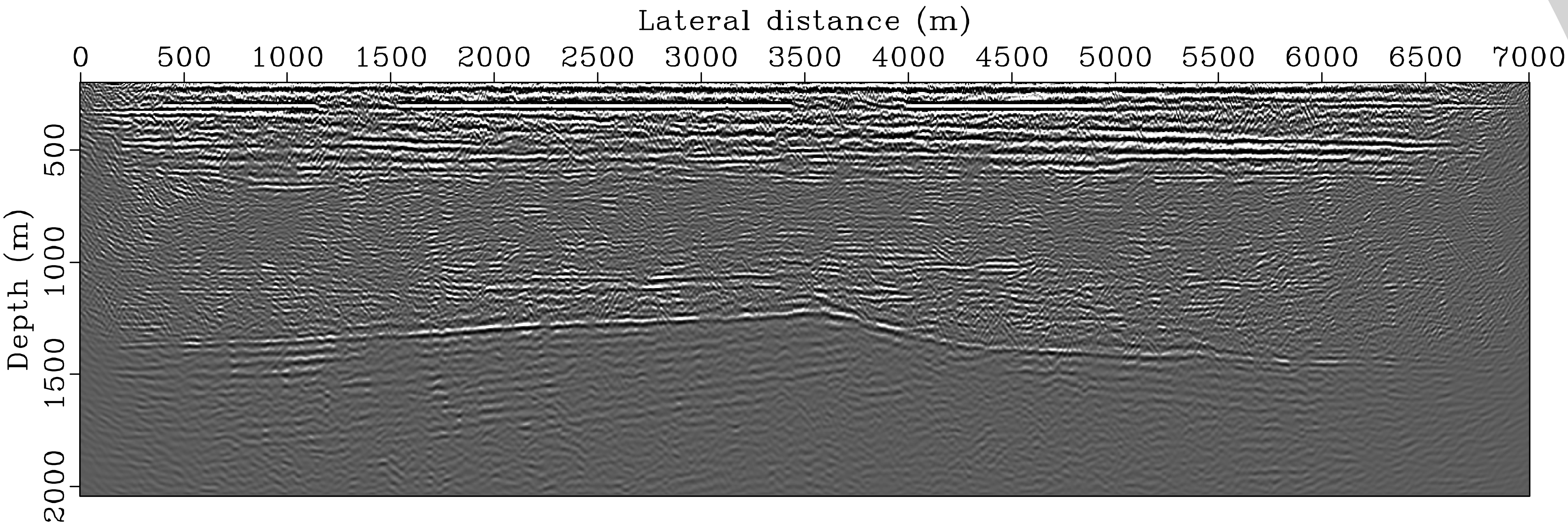
Linearized inversion

[ℓ_2 3 *super* shots]



Linearized inversion

[ℓ_1 3 *super* shots]



Observations
[reasonable PDE solve budget]

*R*erandomization and *curvelet*-domain *sparsity* promotion:

- *partly* eliminate "*noisy*" *crosstalk*
- *fail* to *remove* "small" incoherent crosstalk

Can we somehow combine these two methods?

- *continuation* method for large-scale *convex* optimization
- use *insights* from *approximate* message passing

[Daubechies et. al, '04; Hennenfent et. al.,'08, Mallat, '09, Donoho et. al, '09]

[Montanari, '12]

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Convex optimization

Involves *iterations* of the type

soft threshold ↓

$$\mathbf{x}^{t+1} = \eta_t \left(\mathbf{A}^* \mathbf{r}^t + \mathbf{x}^t \right)$$

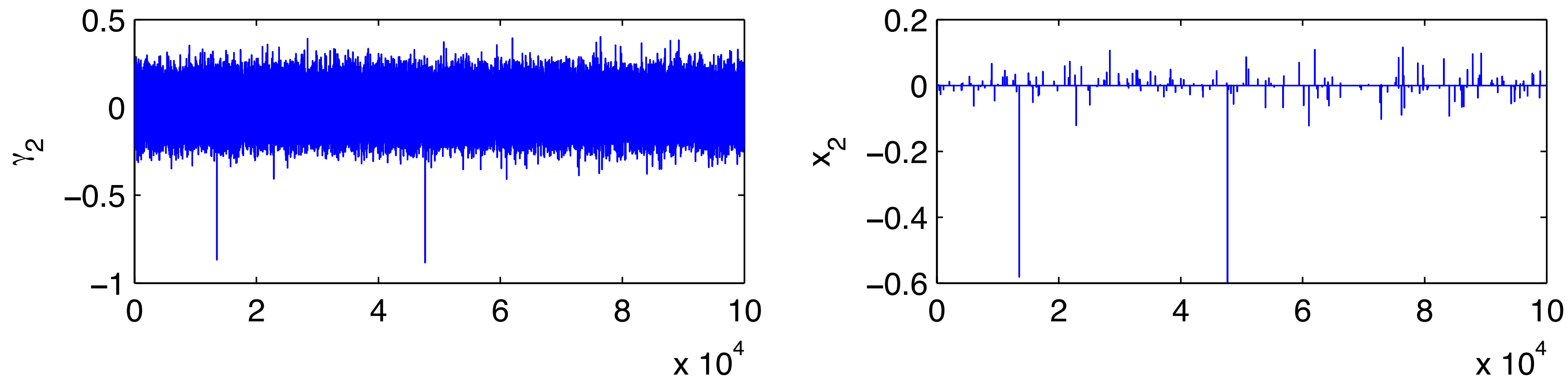
$$\mathbf{r}^t = \mathbf{b} - \mathbf{A}\mathbf{x}^t$$

Corresponds to *vanilla* denoising if **A** is a *Gaussian* matrix.

But does the *same* hold for later (t>1) *iterations*...?

Iteration t=1

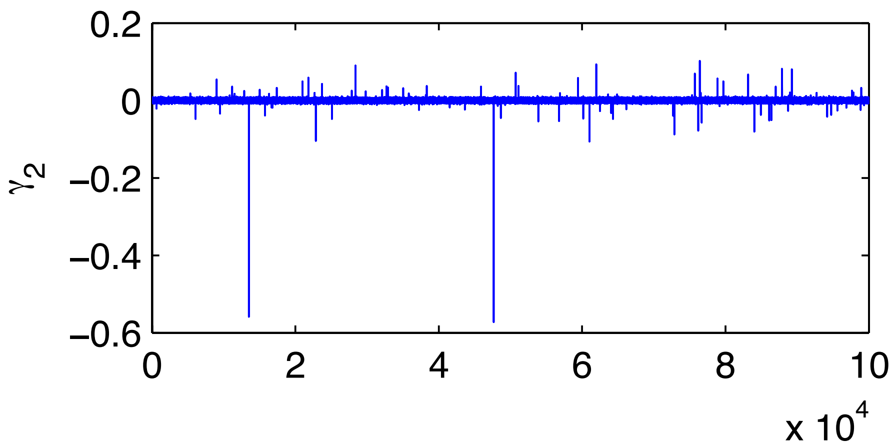
$$\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t \qquad \eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$



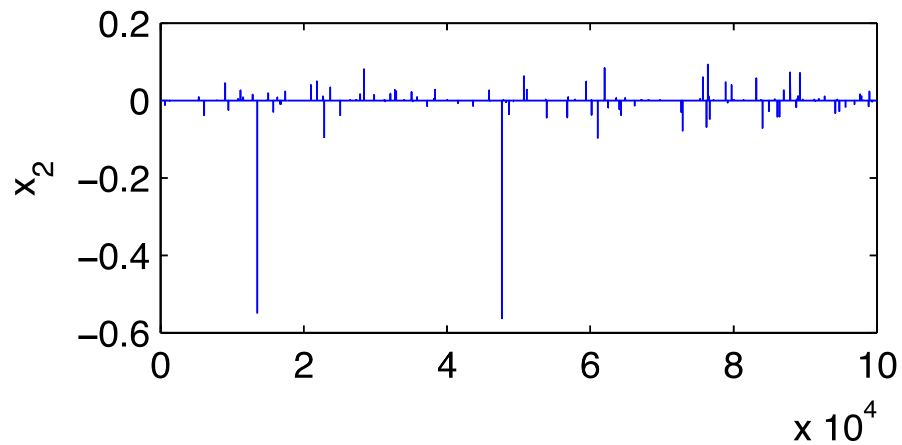
$$\mathbf{A}^*\mathbf{A}\mathbf{x}_0 \qquad \mathbf{x_0} \text{ is } k = 2\text{-sparse and } N = 10^4$$

Iteration t=2

$$\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t$$

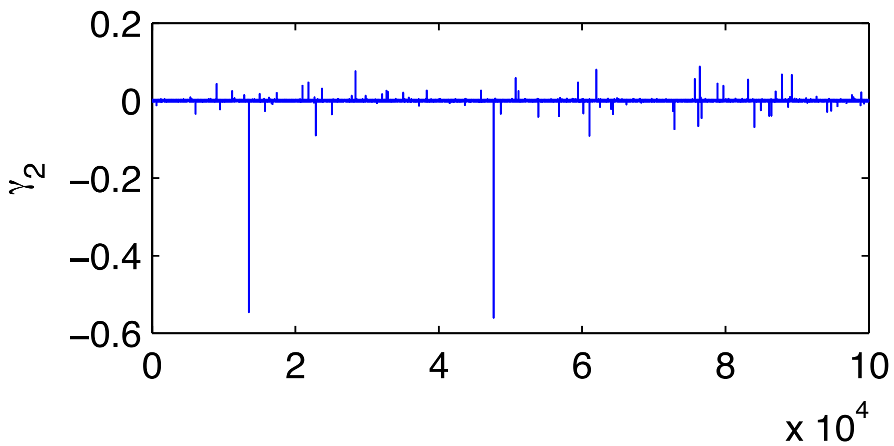


$$\eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$

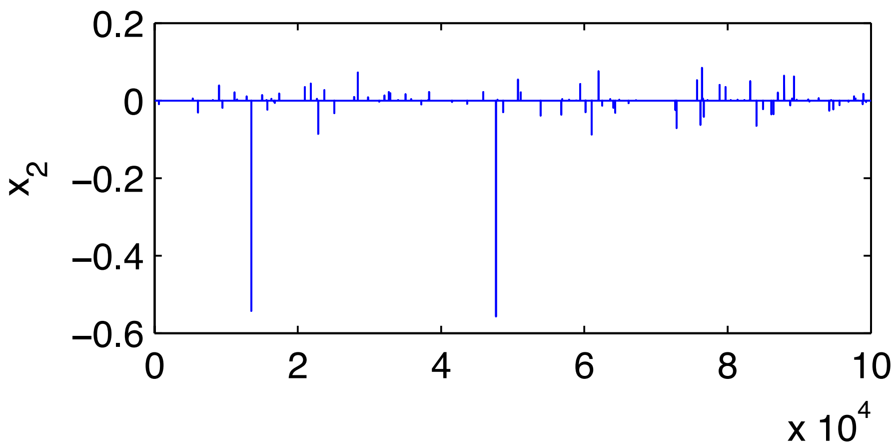


Iteration t=3

$$\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t$$

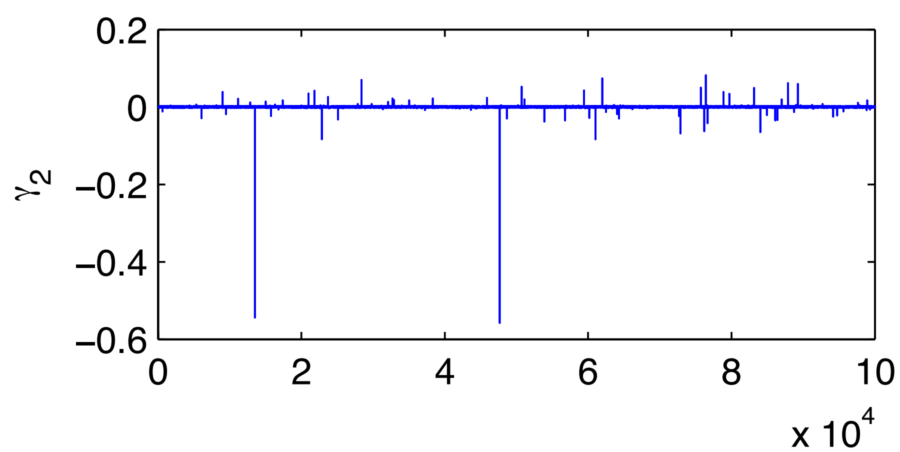


$$\eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$

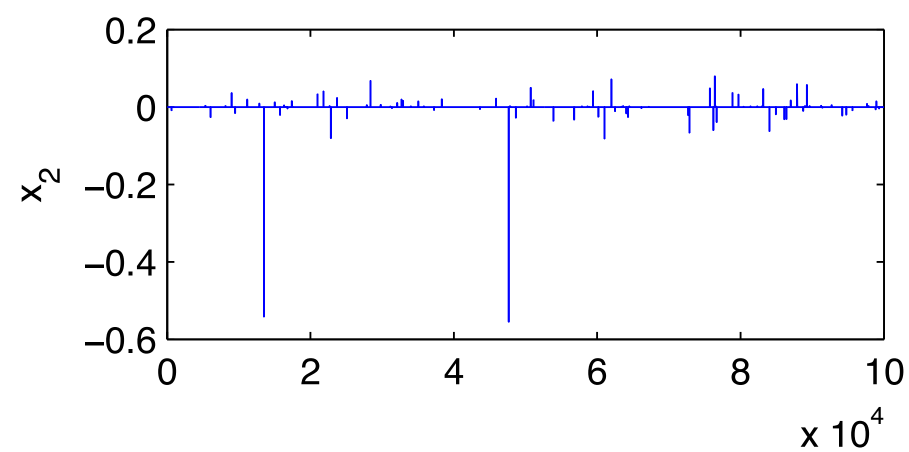


Iteration t=4

$$\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t$$



$$\eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$



Problem

After *first* iteration the *interferences* become 'spiky' because of *correlations* between model *iterate* $\mathbf{x}^t$ & the *matrix* $\mathbf{A}$

- *assumption* spiky vs Gaussian noise *no* longer *holds*
- renders soft *thresholding* less *effective*

Leads to *stalling* of sparsity-promoting *algorithms*...

Approximate message passing

Add a *term* to *iterative* soft *thresholding*, i.e.,

$$\mathbf{x}^{t+1} = \eta_t \left(\mathbf{A}^* \mathbf{r}^t + \mathbf{x}^t \right)$$

$$\mathbf{r}^t = \mathbf{b} - \mathbf{A}\mathbf{x}^t + \frac{\|\mathbf{x}^{t+1}\|_0}{n} \mathbf{r}^{t-1}$$ ← "message term"

Holds for

- *normalized* Gaussian matrices $\mathbf{A}_{ij} \in n^{-1/2} N(0,1)$
- large-scale limit and for specific thresholding *strategy*

[Montanari, '12]

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Approximate message passing

Statistically equivalent to

$$\mathbf{x}^{t+1} = \eta_t\left(\mathbf{A}_t^*\mathbf{r}^t + \mathbf{x}^t\right)$$
$$\mathbf{r}^t = \mathbf{b}_t - \mathbf{A}_t\mathbf{x}^t$$

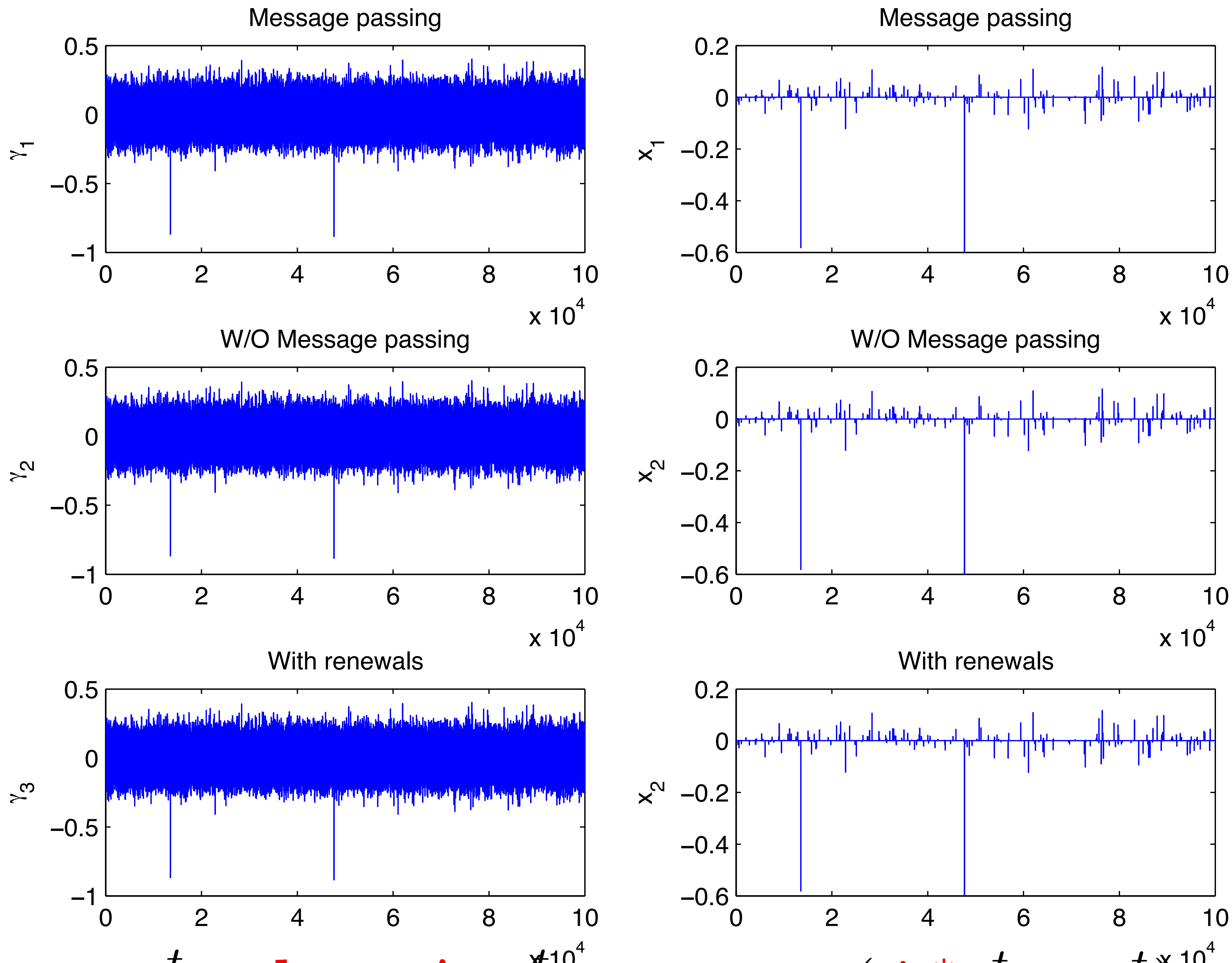
by drawing *new independent* pairs $\{\mathbf{b}_t, \mathbf{A}_t\}$ for *each* iteration

Changes the story *completely*

- breaks *correlation* buildup
- *faster* convergence

Iteration t=1

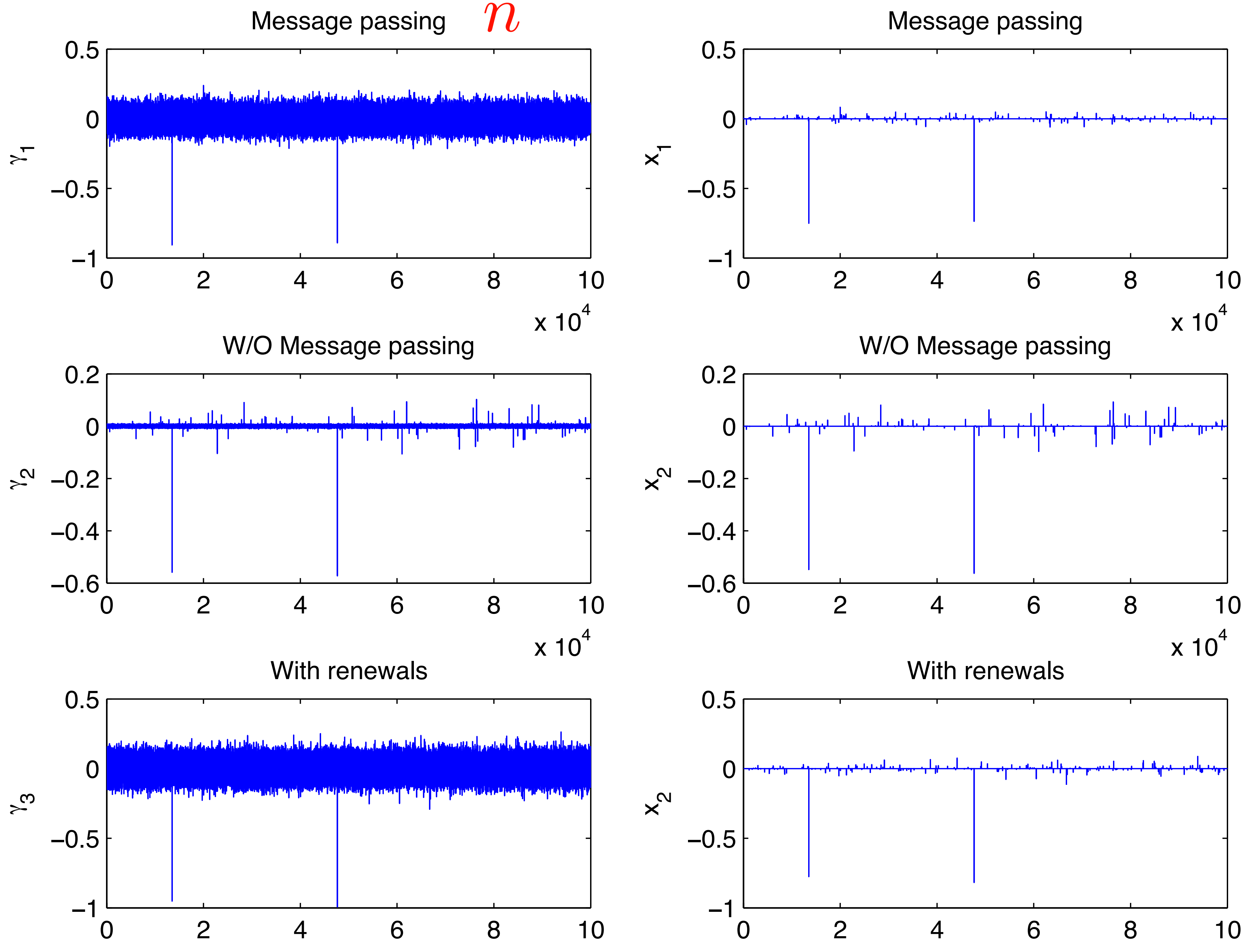
$$\mathbf{r}^t = \mathbf{b} - \mathbf{A}\mathbf{x}^t + \frac{\|\mathbf{x}^{t+1}\|_0}{}\mathbf{r}^{t-1} \qquad \eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$



$$\mathbf{r}^t = \mathbf{b}_t - \mathbf{A}_t\mathbf{x}^t \qquad \eta_t(\mathbf{A}_t^*\mathbf{r}^t + \mathbf{x}^t)$$

Iteration t=2

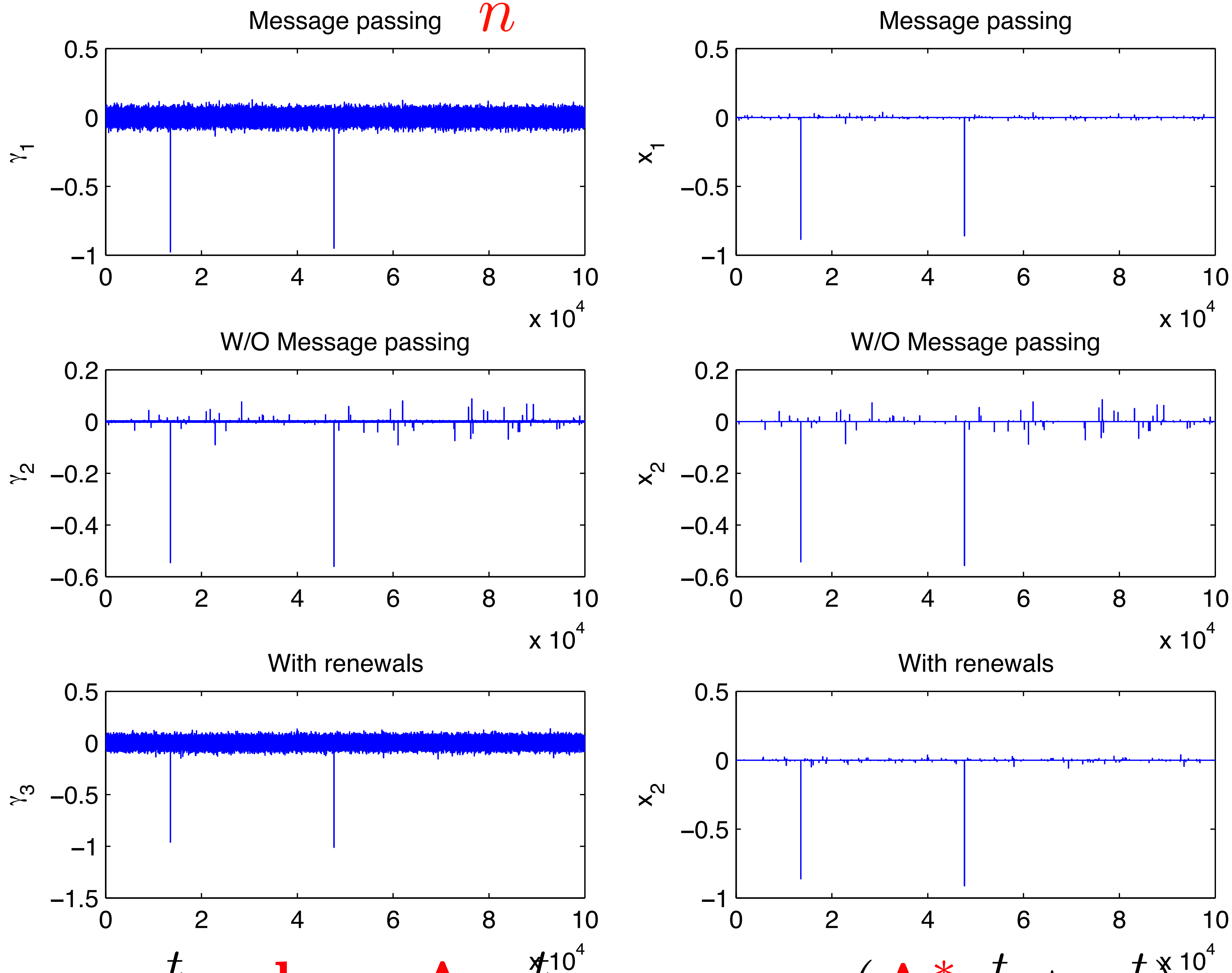
$$\mathbf{r}^t = \mathbf{b} - \mathbf{A}\mathbf{x}^t + \frac{\|\mathbf{x}^{t+1}\|_0}{n}\mathbf{r}^{t-1} \qquad \eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$



$$\mathbf{r}^t = \mathbf{b}_t - \mathbf{A}_t\mathbf{x}^t \qquad \eta_t(\mathbf{A}_t^*\mathbf{r}^t + \mathbf{x}^t)$$

Iteration t=3

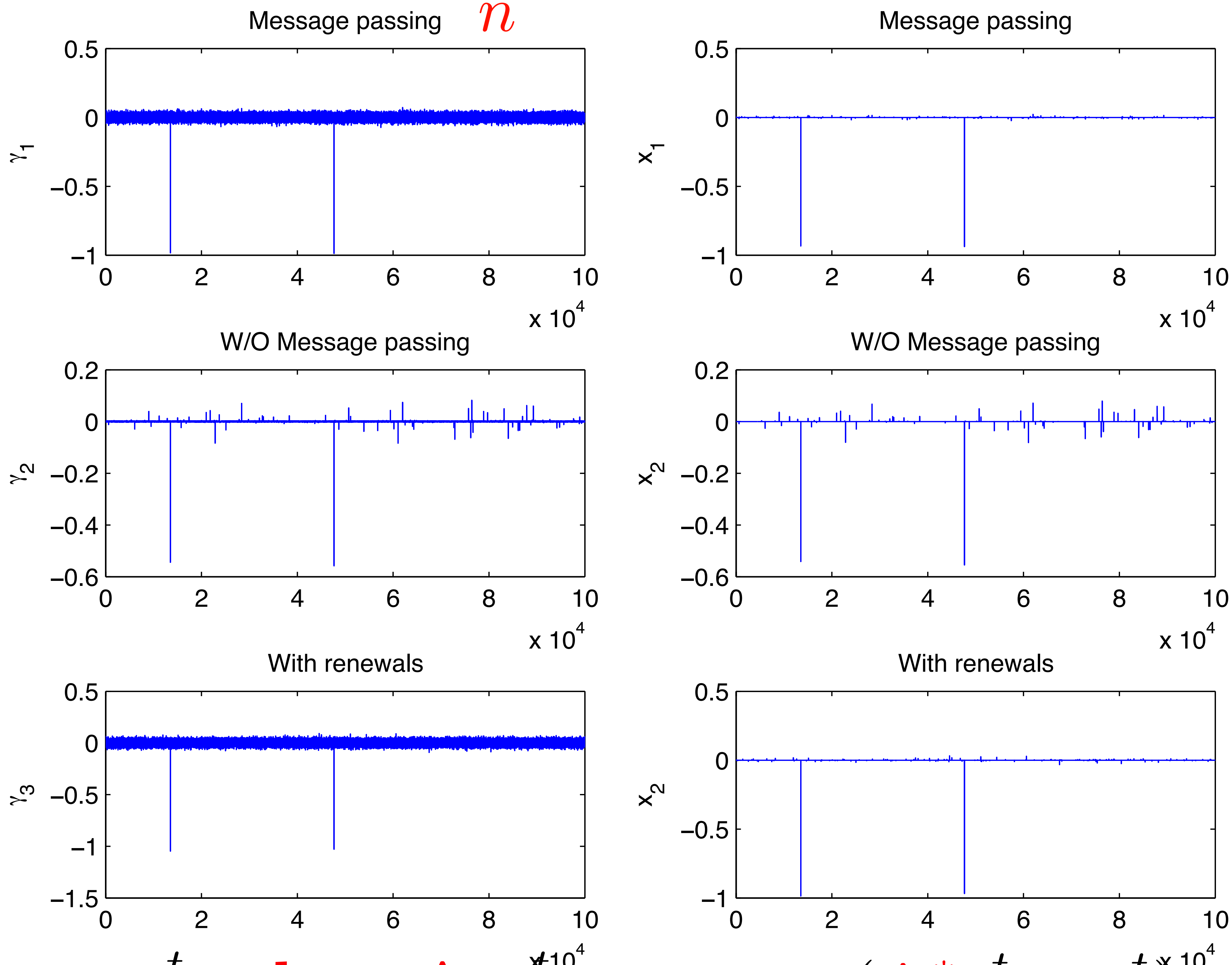
$$\mathbf{r}^t = \mathbf{b} - \mathbf{A}\mathbf{x}^t + \frac{\|\mathbf{x}^{t+1}\|_0}{n}\mathbf{r}^{t-1} \qquad \eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$



$$\mathbf{r}^t = \mathbf{b}_t - \mathbf{A}_t\mathbf{x}^t \qquad \eta_t(\mathbf{A}_t^*\mathbf{r}^t + \mathbf{x}^t)$$

Iteration t=4

$$\mathbf{r}^t = \mathbf{b} - \mathbf{A}\mathbf{x}^t + \frac{\|\mathbf{x}^{t+1}\|_0}{n}\mathbf{r}^{t-1} \qquad \eta_t(\mathbf{A}^*\mathbf{r}^t + \mathbf{x}^t)$$



$$\mathbf{r}^t = \mathbf{b}_t - \mathbf{A}_t\mathbf{x}^t \qquad \eta_t(\mathbf{A}_t^*\mathbf{r}^t + \mathbf{x}^t)$$

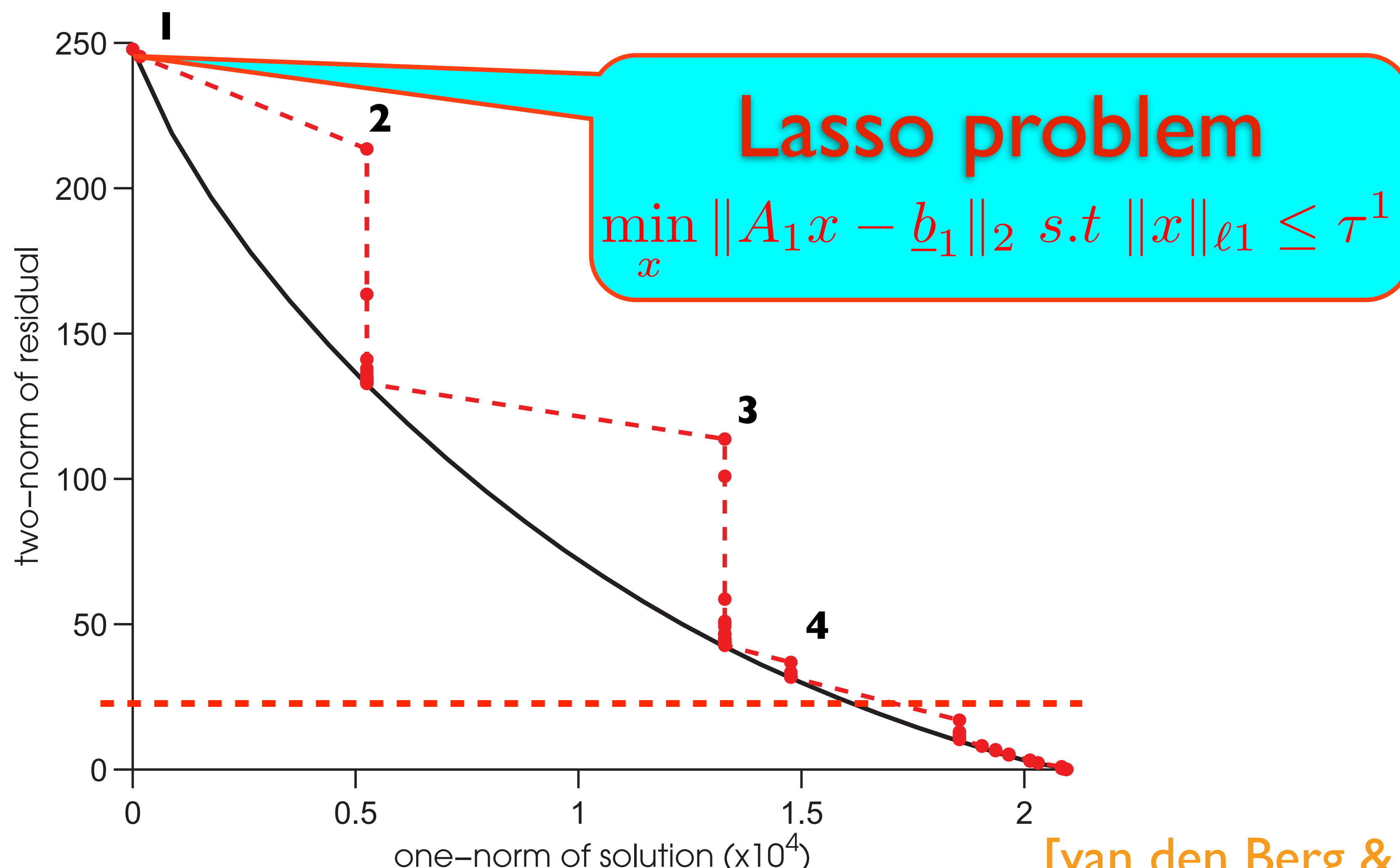
Supercooling

Break *correlations* between the model *iterate* and matrix $\mathbf{A}$ by *rerandomization*

- draw new *independent* $\{\mathbf{b}_t, \mathbf{A}_t\}$ after each subproblem is solved
- brings in *"extra"* information *without* growing the *system*
- ***minimal*** *extra* computational & memory cost

Supercooled

spectral-projected gradients



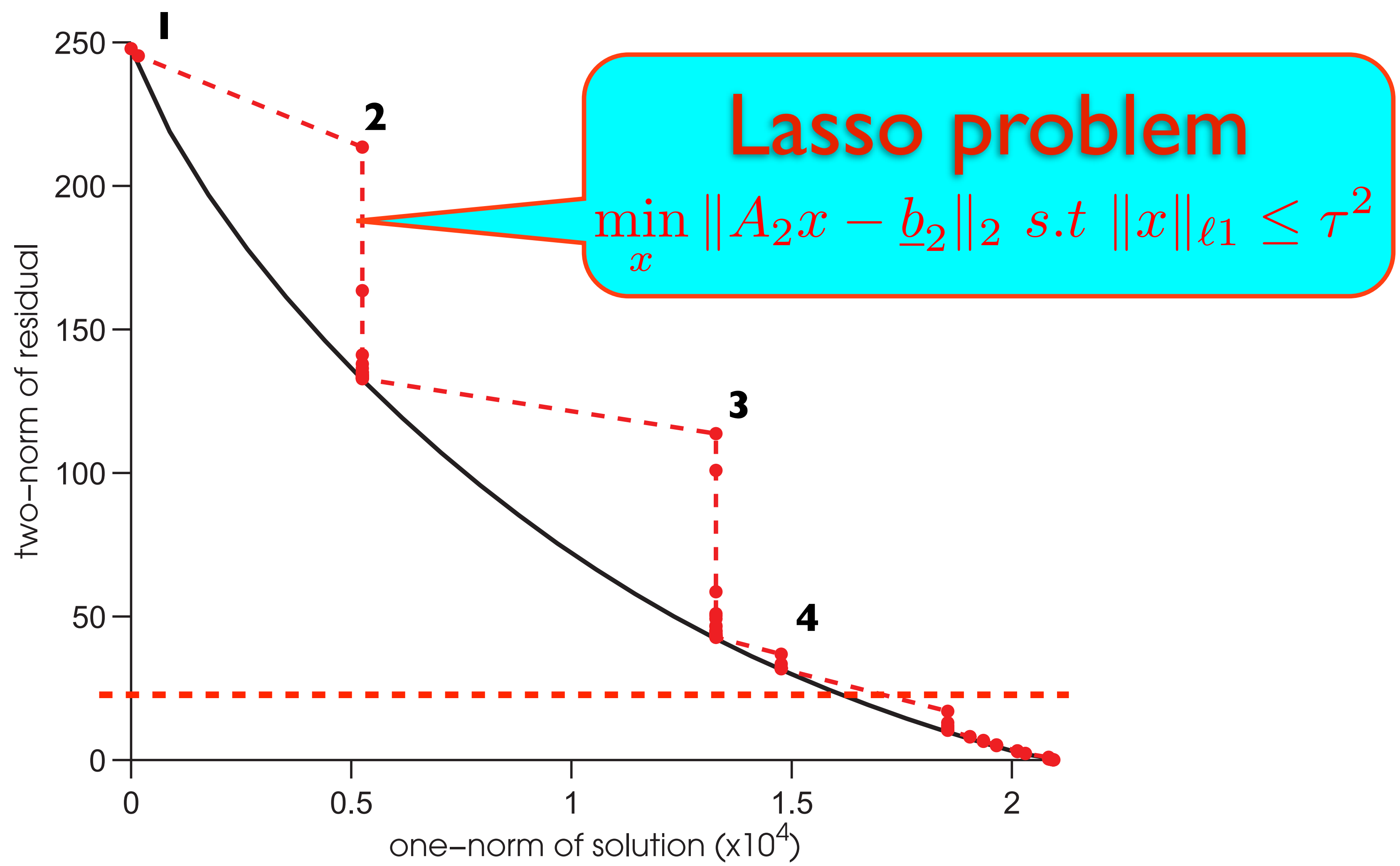
[van den Berg & Friedlander, ’08]

[Hennefent et. al., ’08]

[Lin & FJH, ’09-]

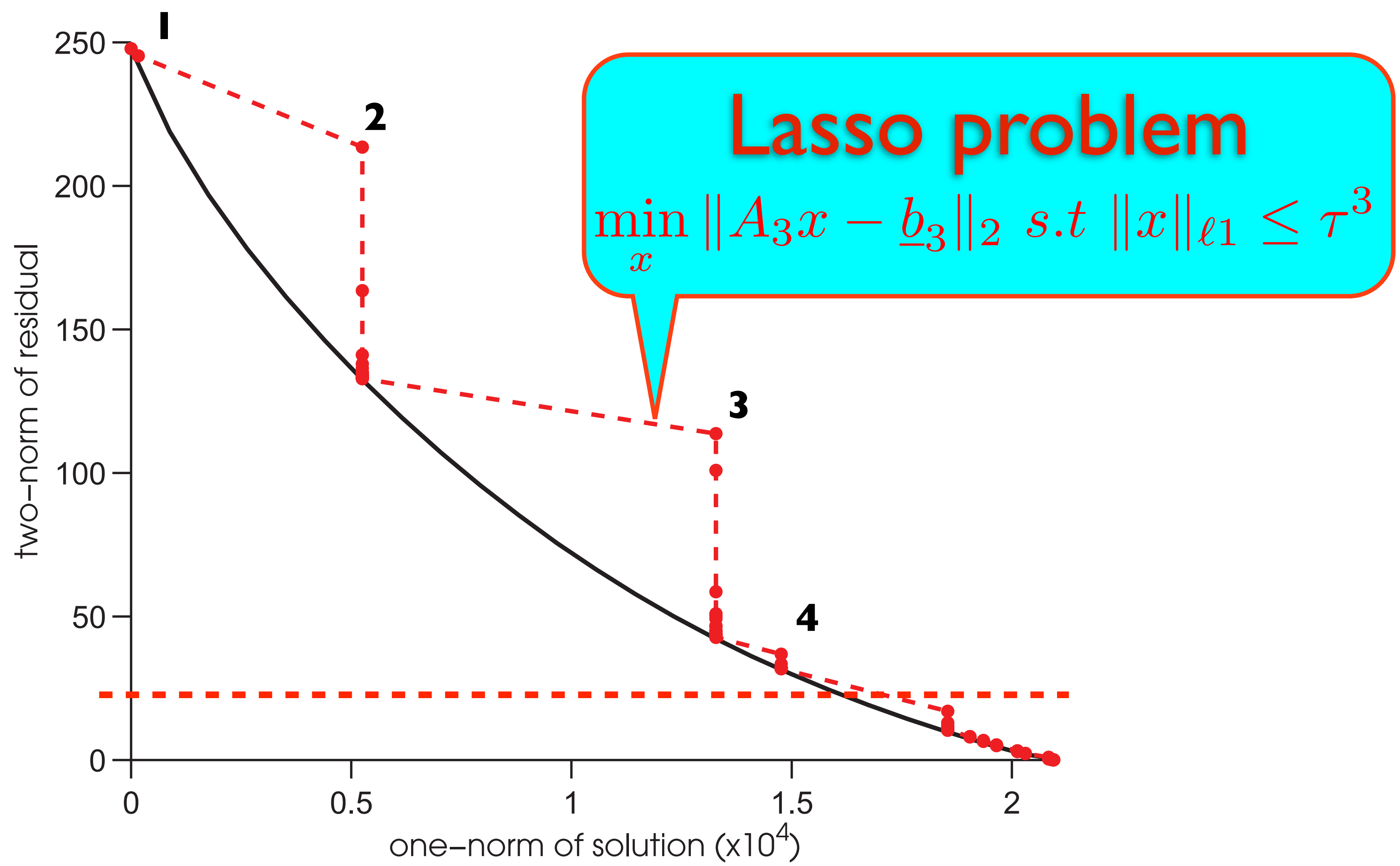
Supercooled

spectral-projected gradients



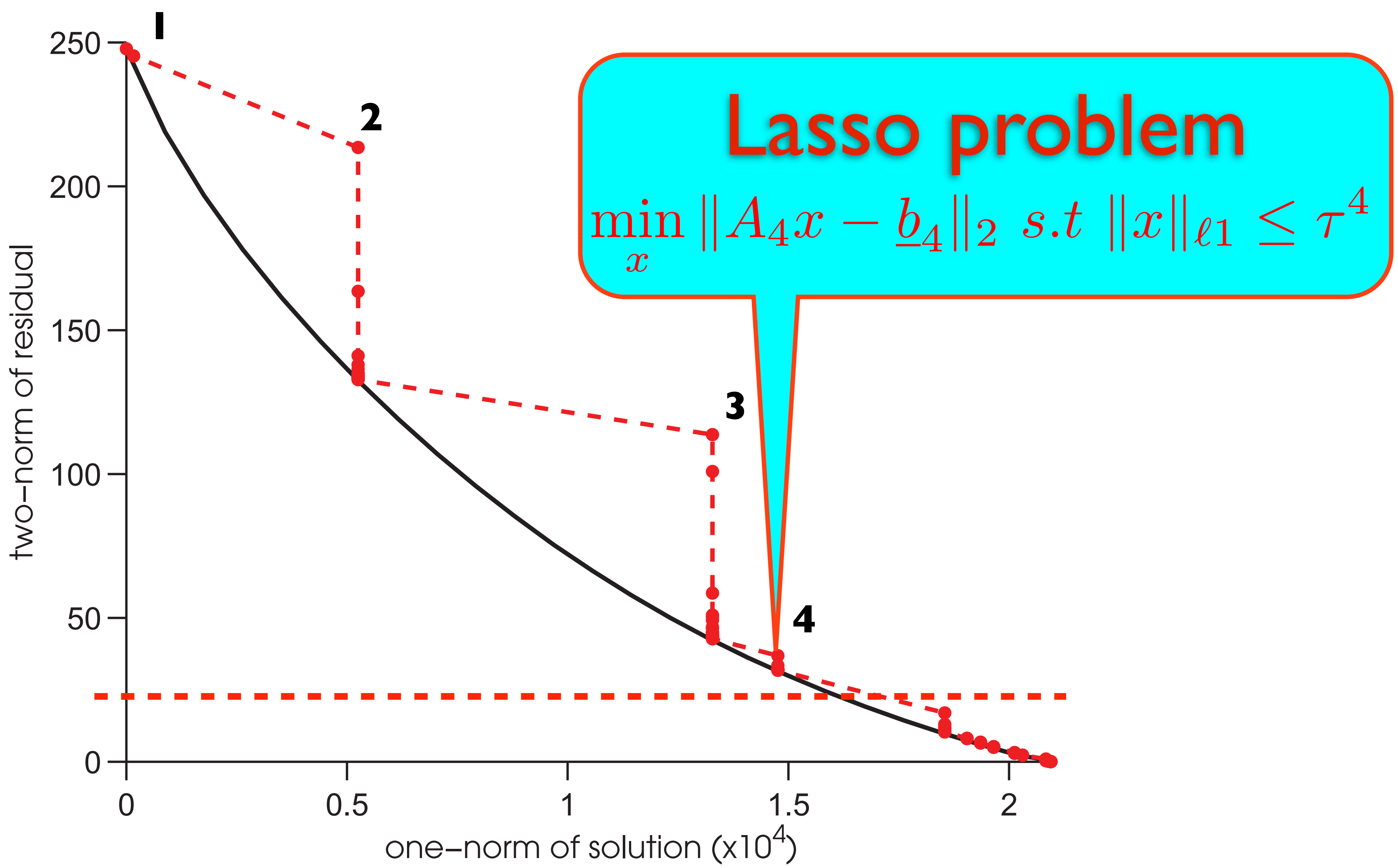
Supercooled

spectral-projected gradients



Supercooled

spectral-projected gradients



Supercooled

spectral-projected gradients

Algorithm 1: Modified SPGℓ_1 with message passing.

Result: Estimate for the model $\mathbf{x}^{t+1}$

$\mathbf{x}^0, \widetilde{\mathbf{x}} \longleftarrow \mathbf{0}$ and $t, \tau^0 \longleftarrow 0$; `// Initialize`

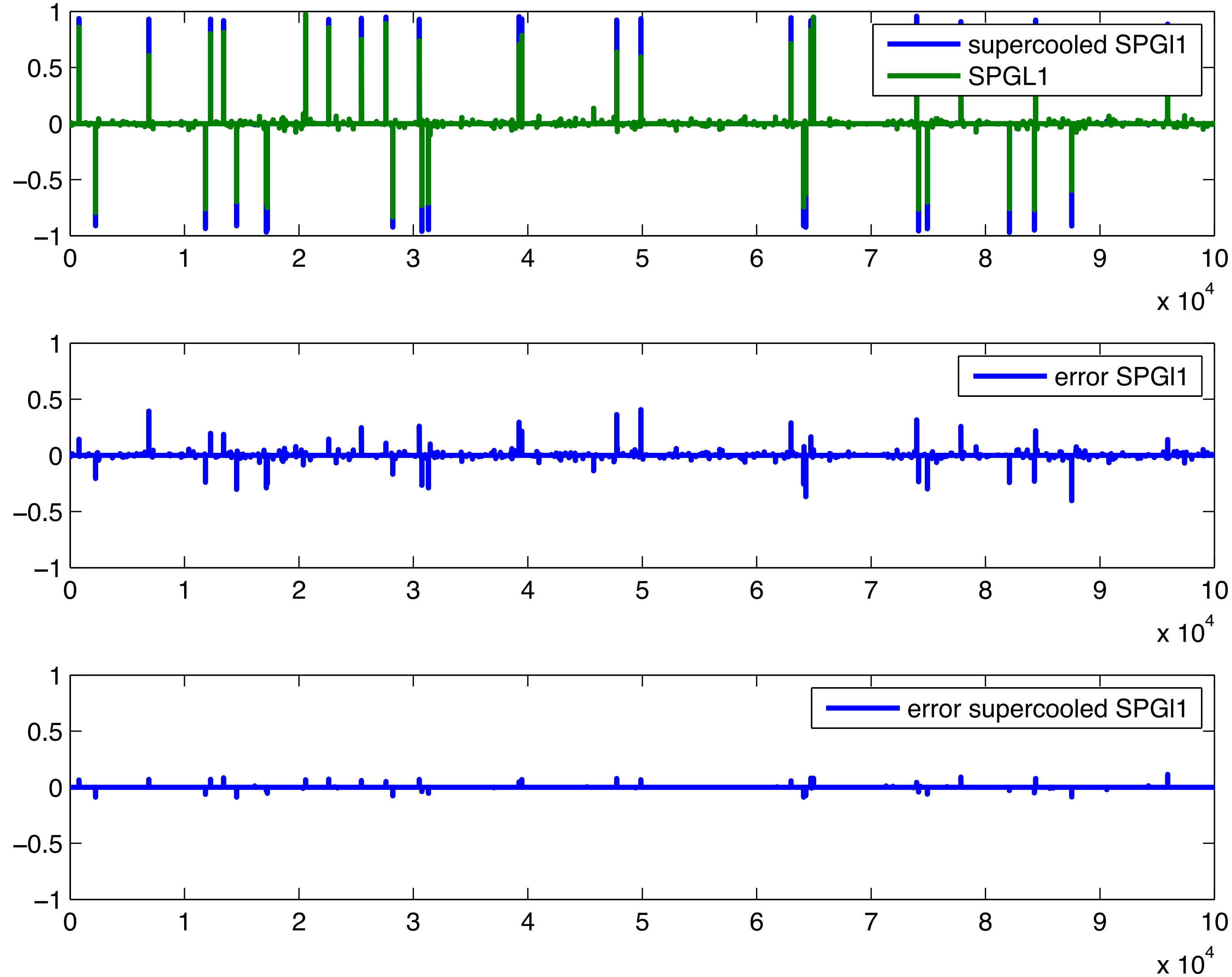
while $t \leq T$ **do**

- $\mathbf{A} \longleftarrow \mathbf{A} \sim P(\mathbf{A})$; `// Draw new sensing matrix`
- $\mathbf{b} \longleftarrow \mathbf{A}\mathbf{x}$; `// Collect new data`
- $\mathbf{x}^{t+1} \longleftarrow \mathrm{spgl1}(\mathbf{A}, \mathbf{b}, \tau^t, \sigma = 0, \mathbf{x}^t)$; `// Reach Pareto`
- $\tau^t \longleftarrow \|\mathbf{x}^{t+1}\|_1$; `// New initial τ value`
- $t \longleftarrow t + \Delta T$; ; `// Add # of iterations of spgl1`

end

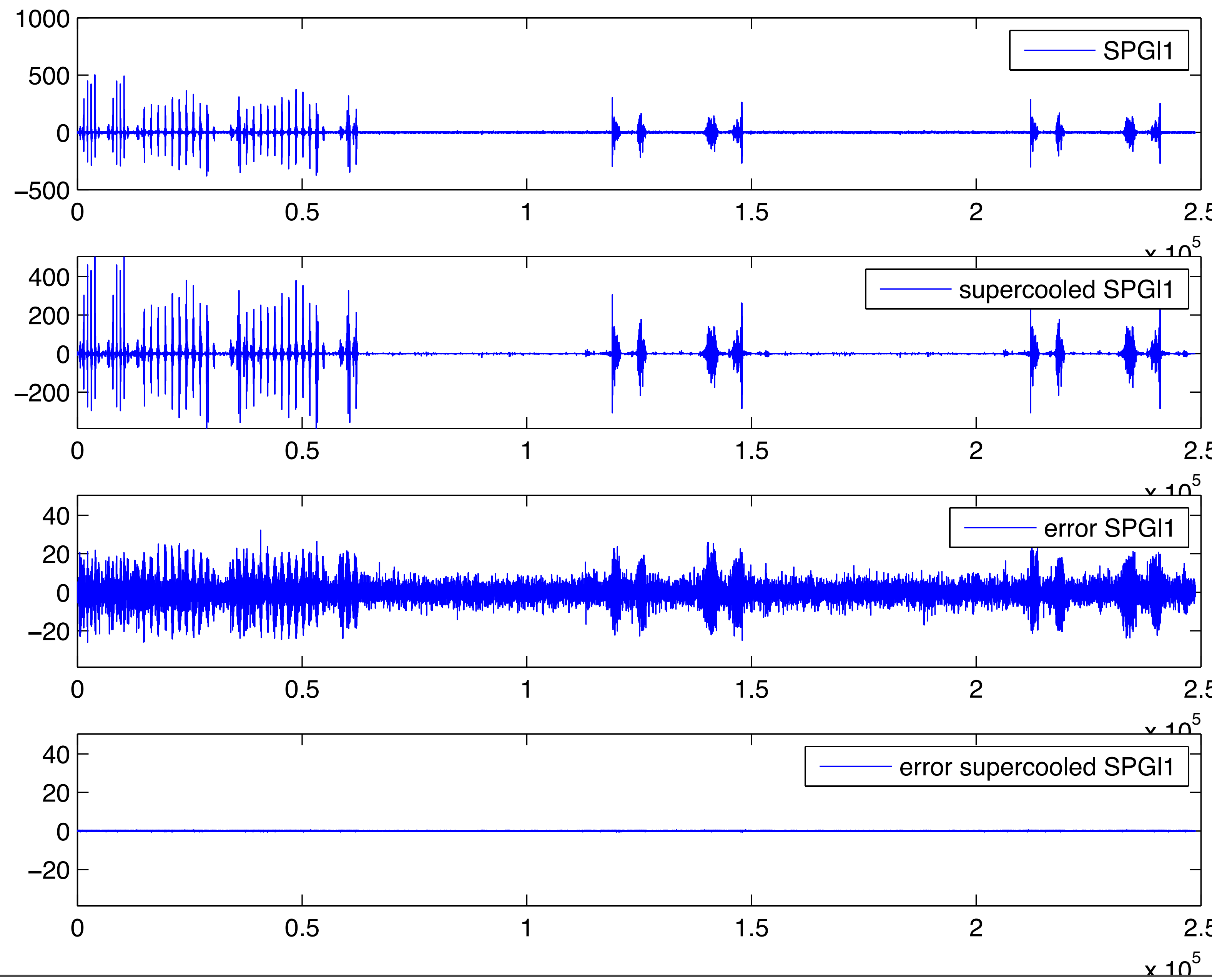
Sparse example

[n=500; N=10000; k=35; T=50]



Ideal ‘Seismic’ example

[n/N=0.13;N=248759;T=500]



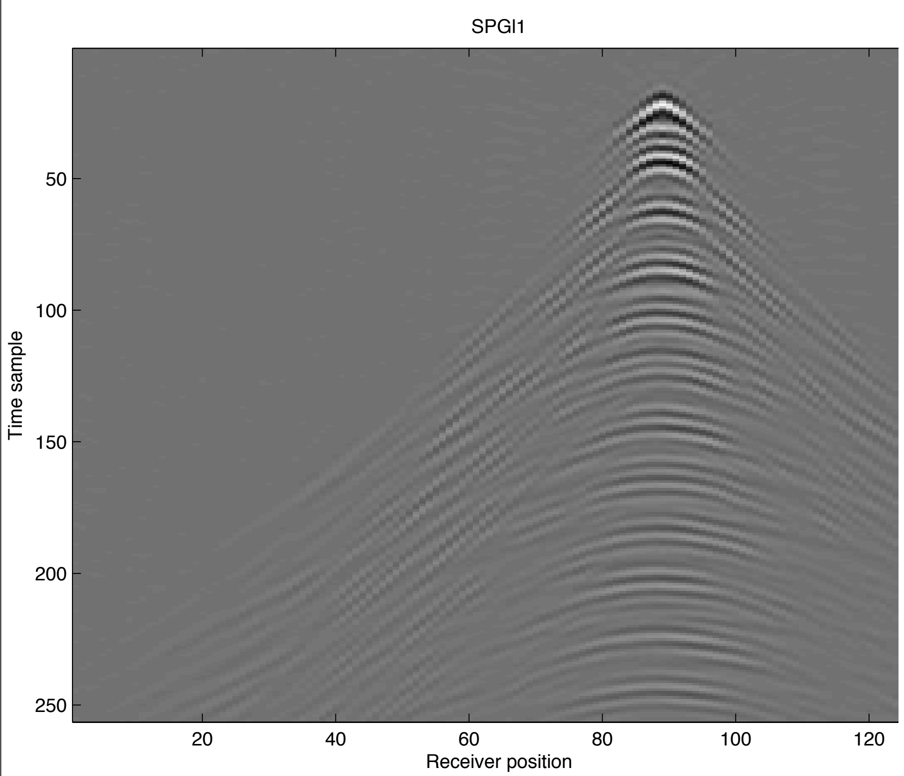
10 X

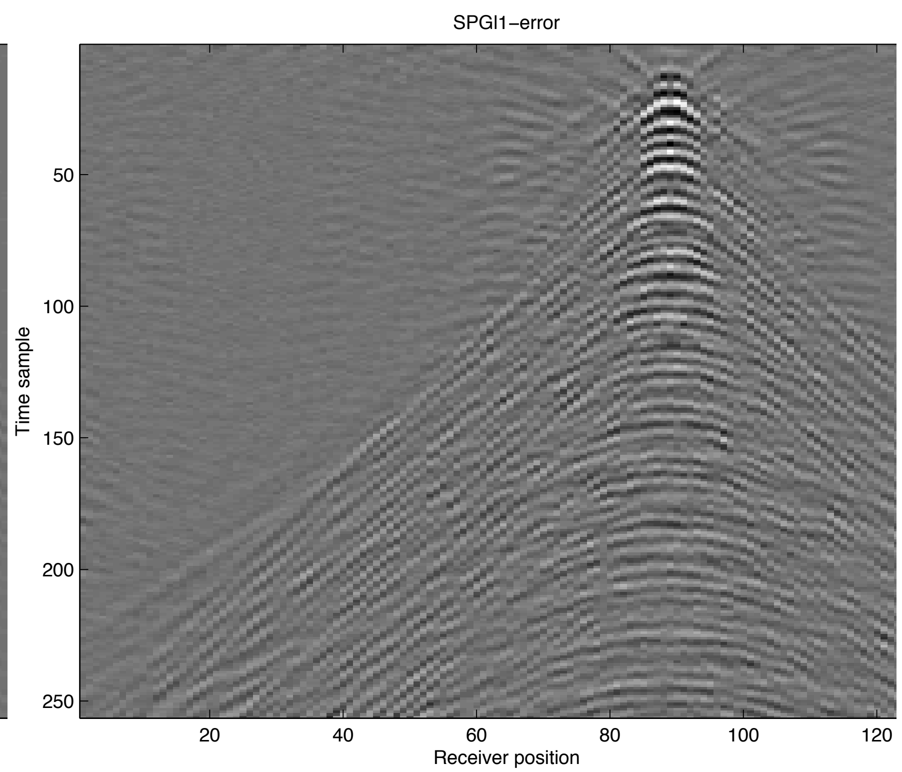
10 X

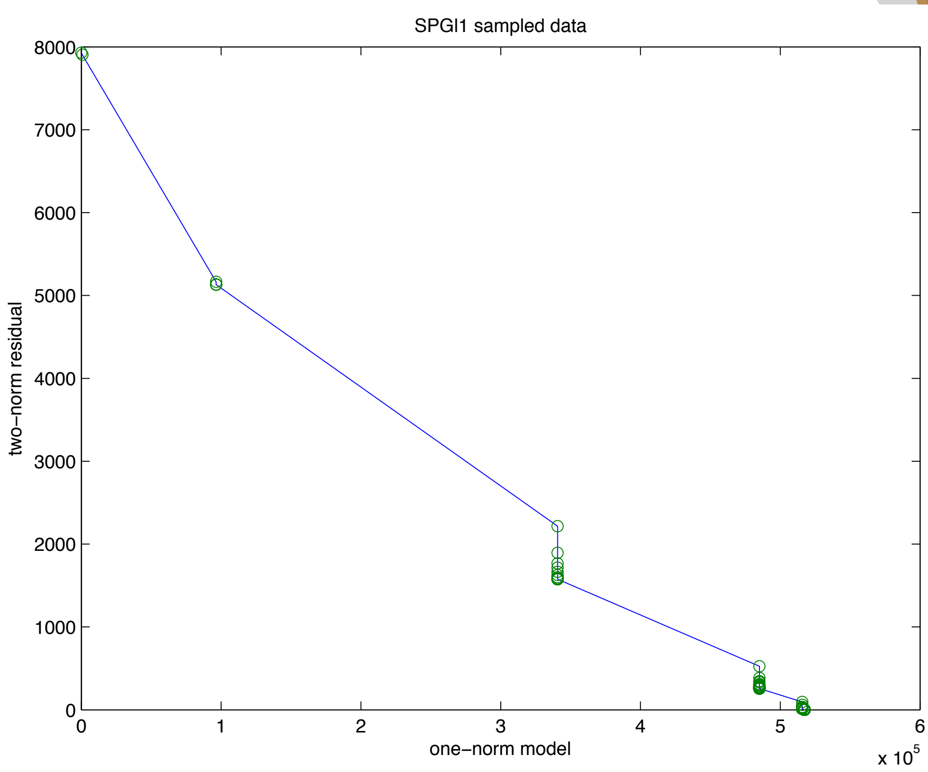
Ideal ‘Seismic’ example

[n/N=0.13;N=248759;T=500]

10 X





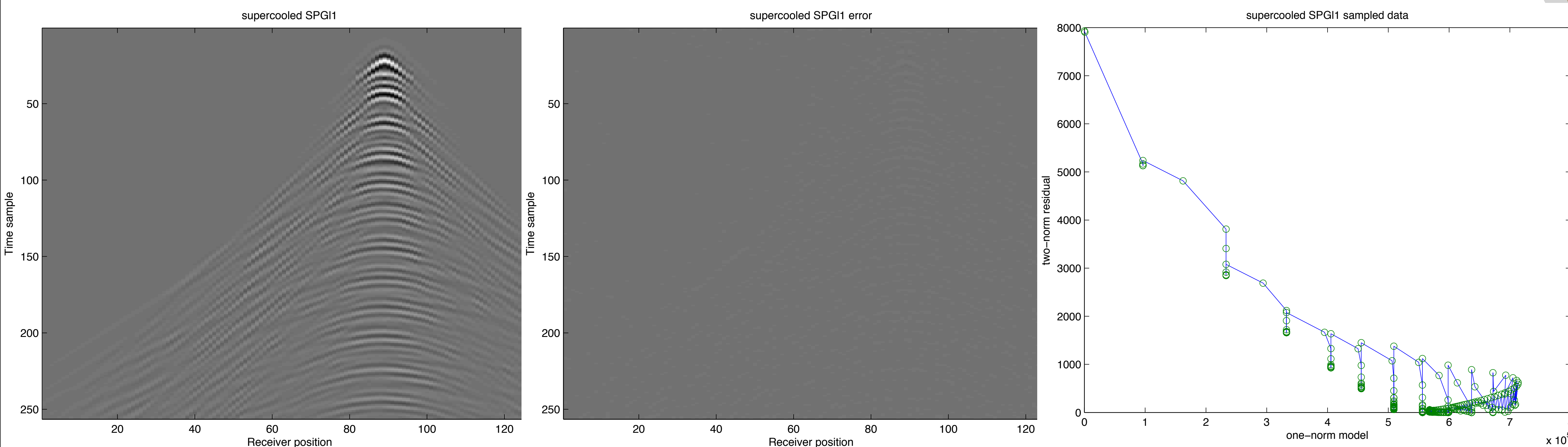


recovery *error* solution *path*

Cooled

Ideal ‘Seismic’ example
[n/N=0.13;N=248759;T=500]

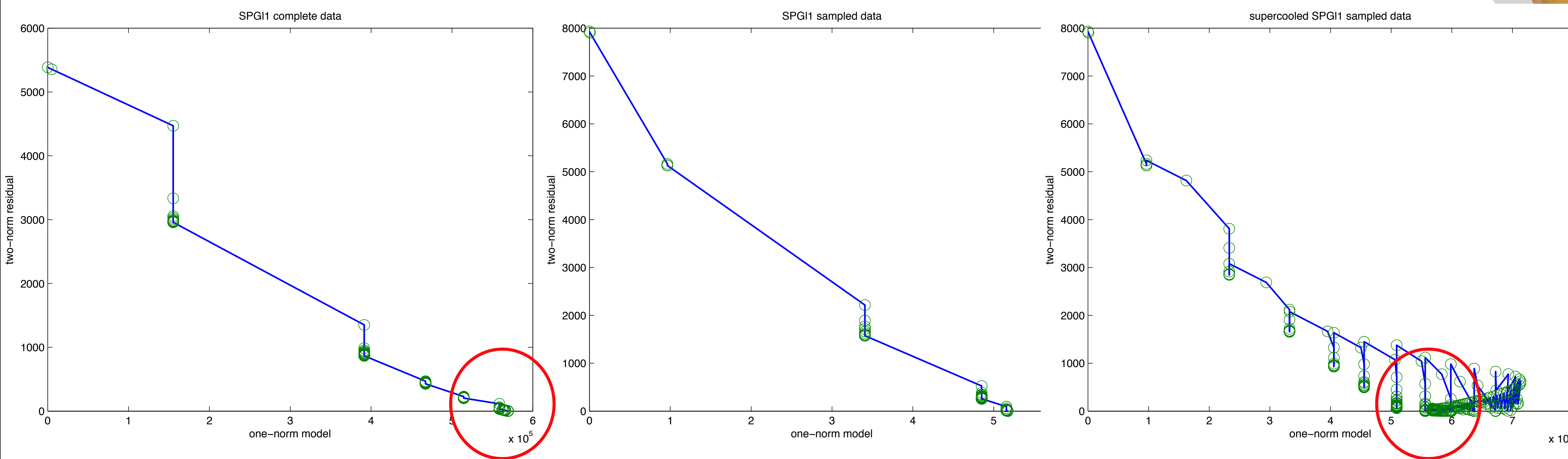
10 X



recovery | *error* | solution *path*

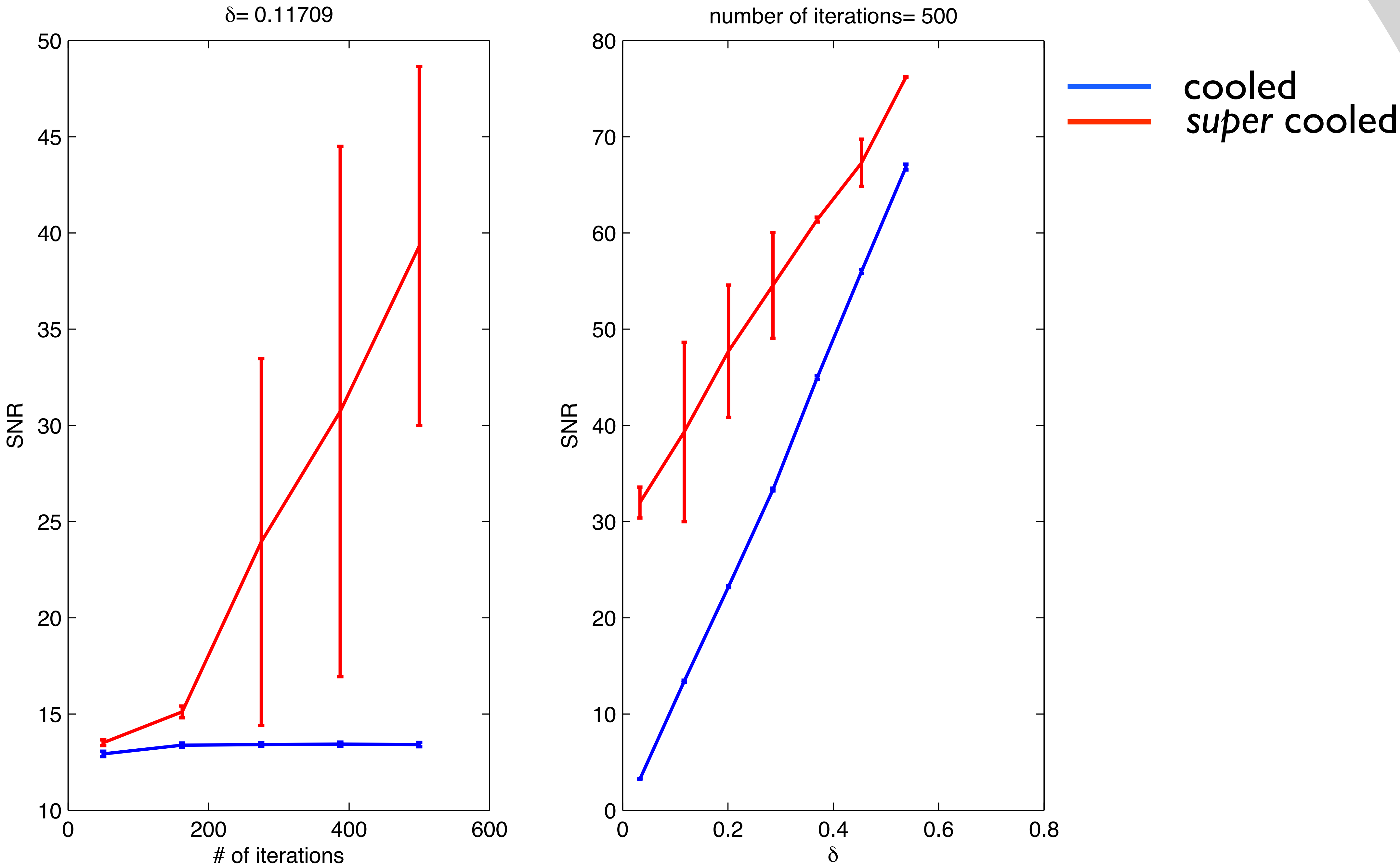
Supercooled

Solution paths



Independent redraws of $\{\mathbf{b}_t, \mathbf{A}_t\}$ lead to improved *recovery*

MCC experiments
[# of iterations & subsampling]



Linearized inversion

Split-spread surface-free 'land' acquisition:

- 350 sources with sampling interval 20m
- 701 receivers with sampling interval 10m
- maximal offset 7km (3.5 X depth of model)
- Ricker wavelet with central frequency of 30Hz
- recording time for each shot is 3.6s

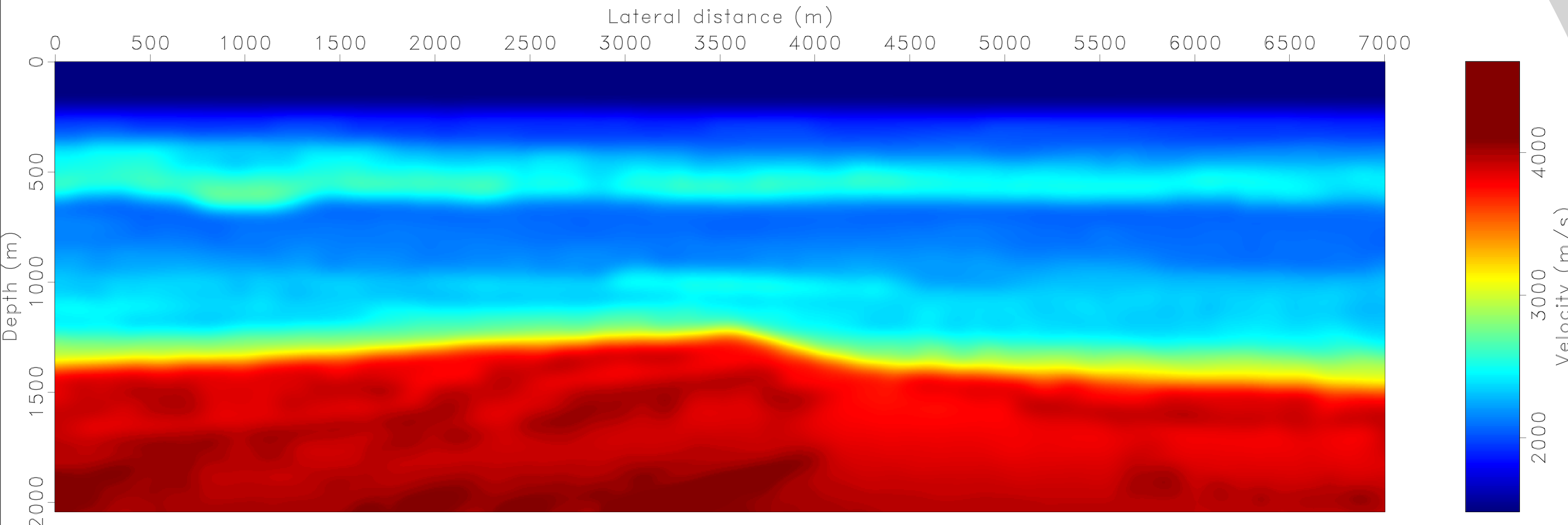
Linearized inversion

Time-harmonic Helmholtz:

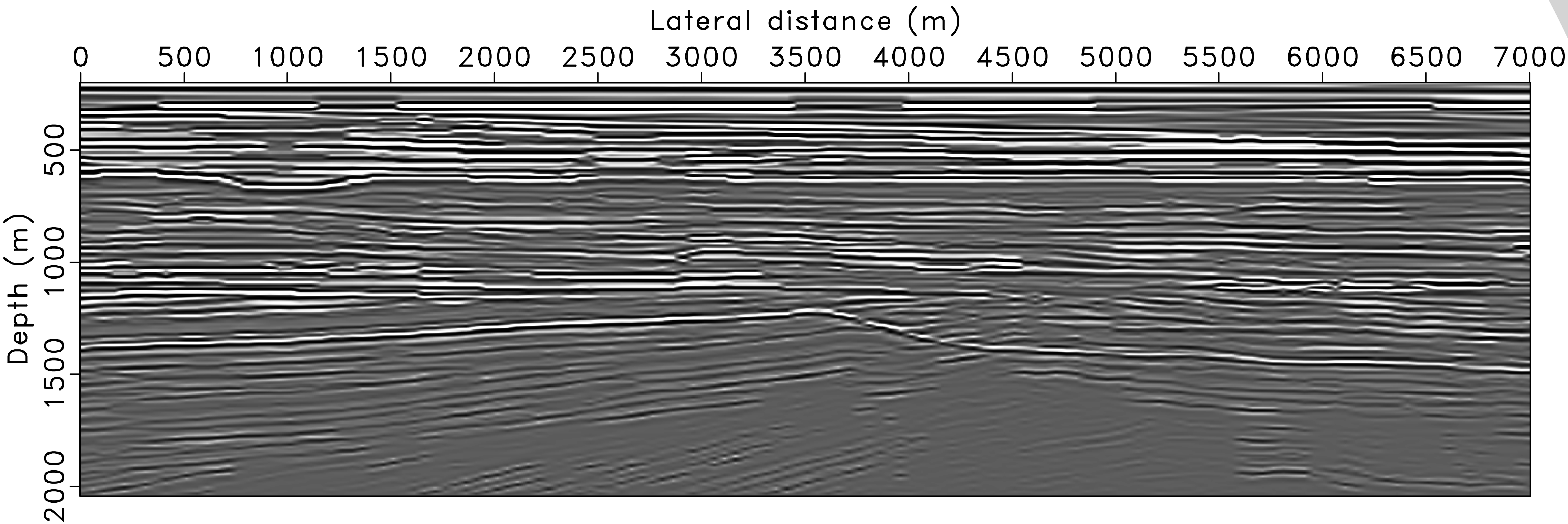
- 409 X 1401 with mesh size of 5m
- 9 point stencil [C. Jo et. al., '96]
- absorbing boundary condition with damping layer with thickness proportional to wavelength
- solve wavefields on the fly with direct solver

Linearized inversion

[background model]



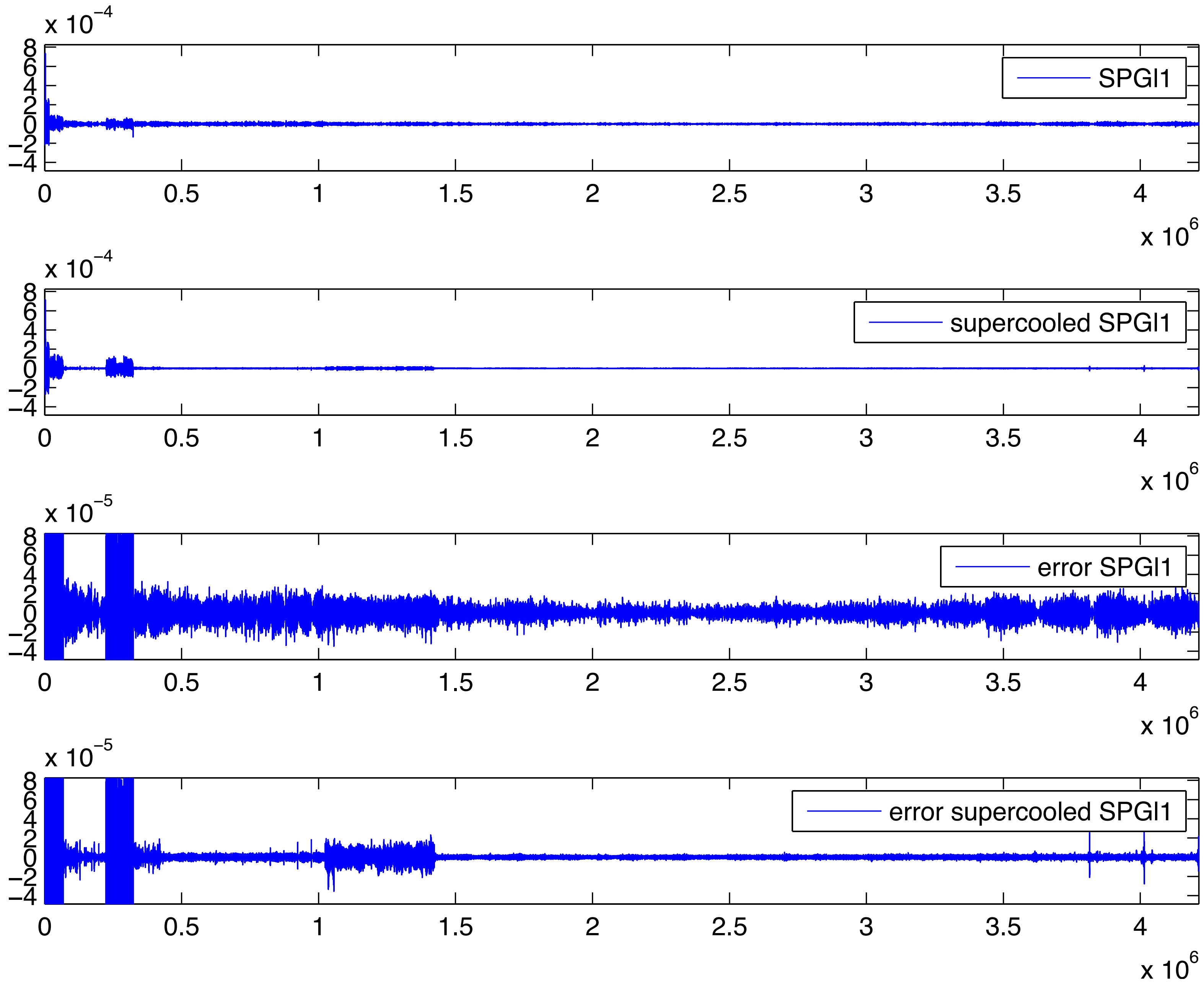
Linearized inversion
[*true* perturbation]



Linearized inversion
[estimated coefficients]

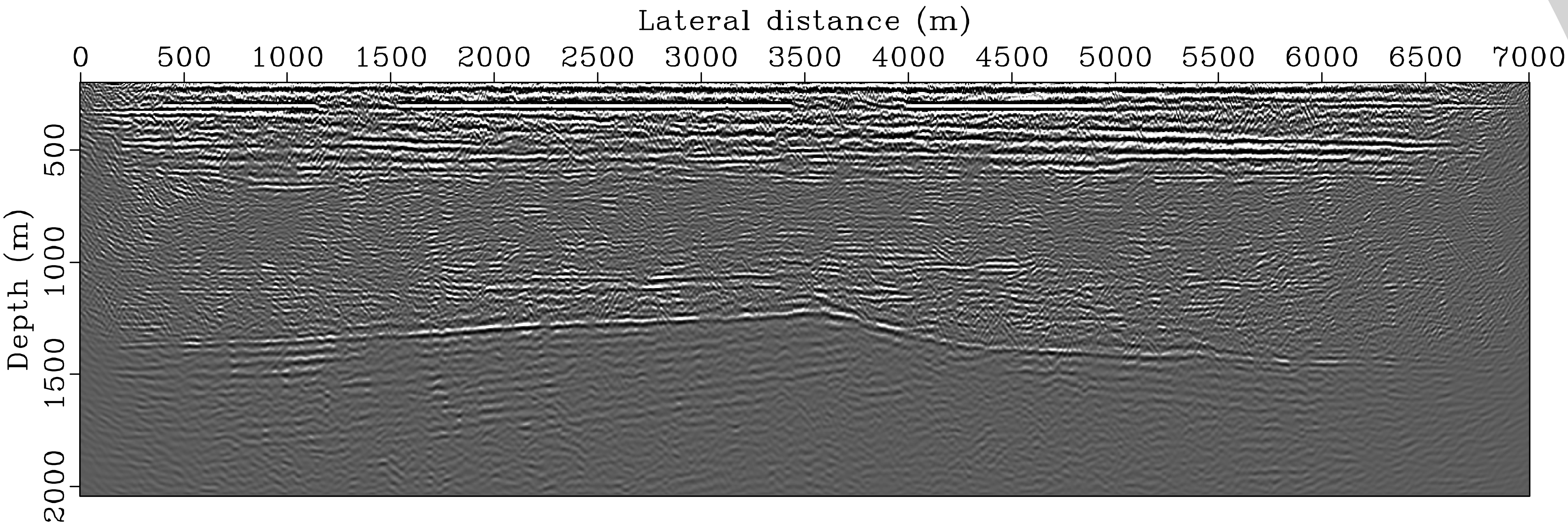
10 X

10 X



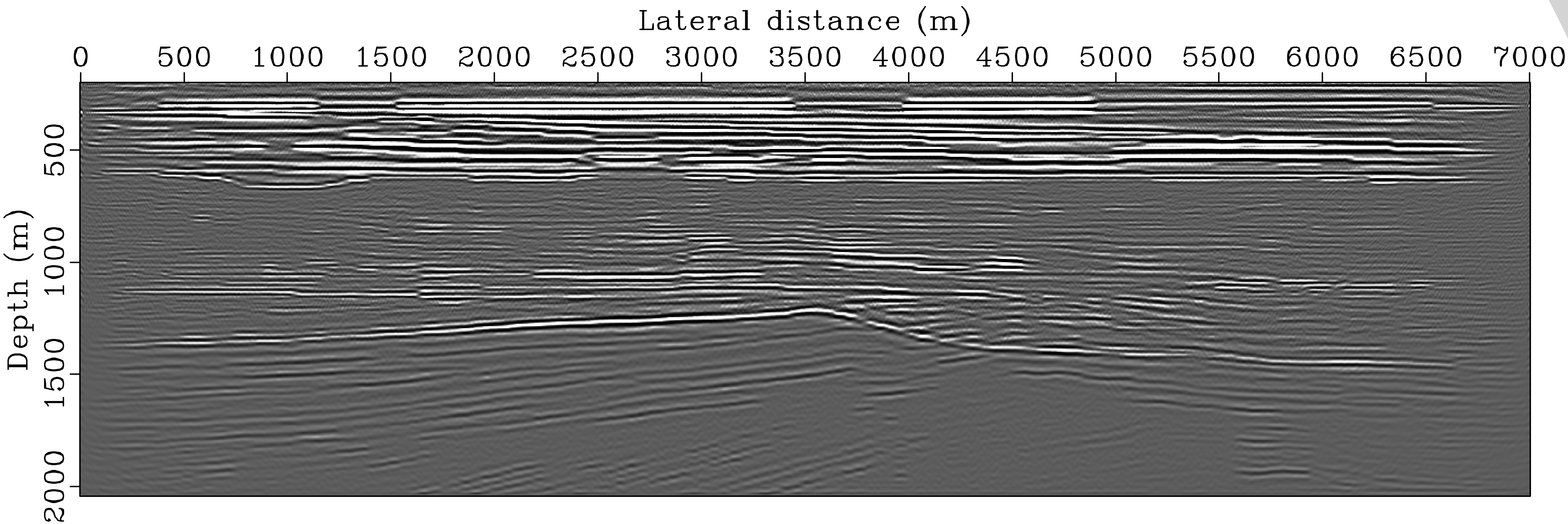
Linearized inversion

[ℓ_1 *w/o rerandomization 3 super shots*]



Linearized inversion

[ℓ_1 *w/* rerandomization 3 *super* shots]



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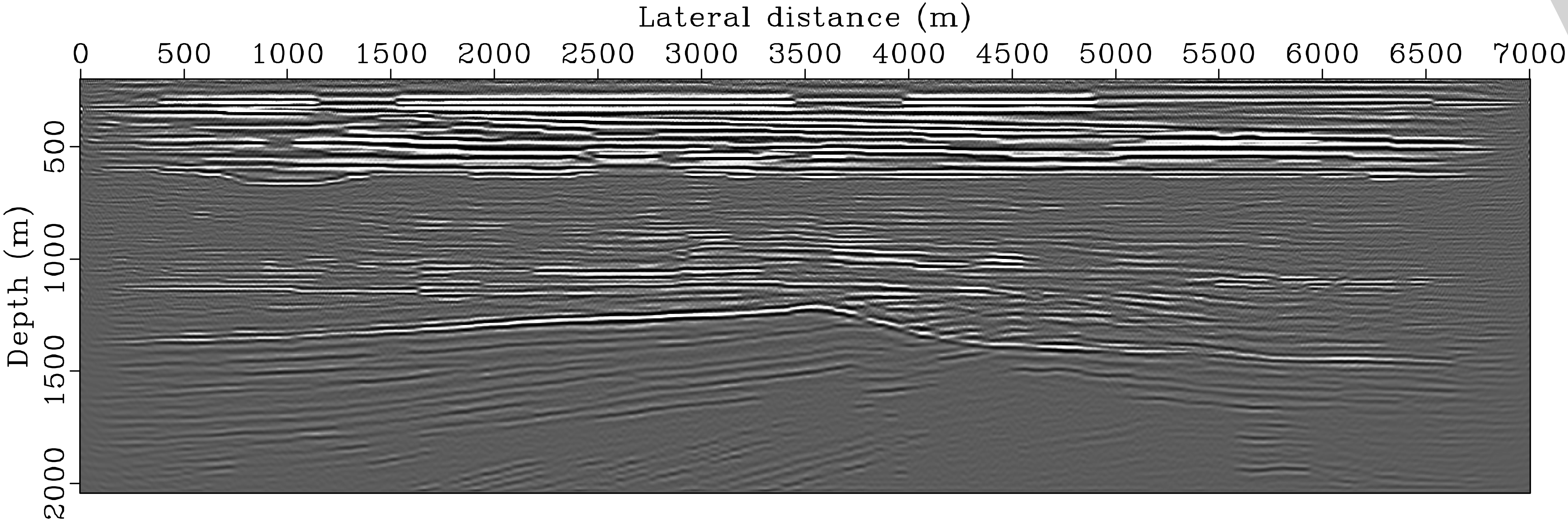
Linearized inversion

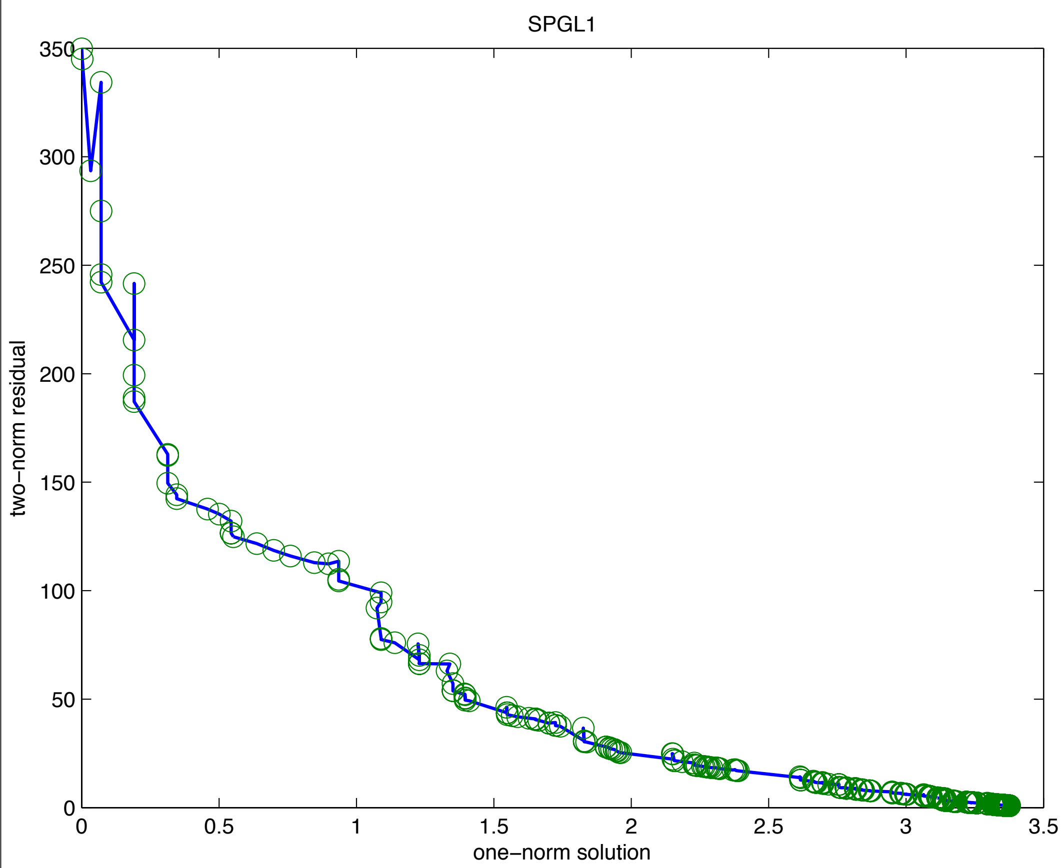
[ℓ_1 *w/* rerandomization 3 *super* shots]



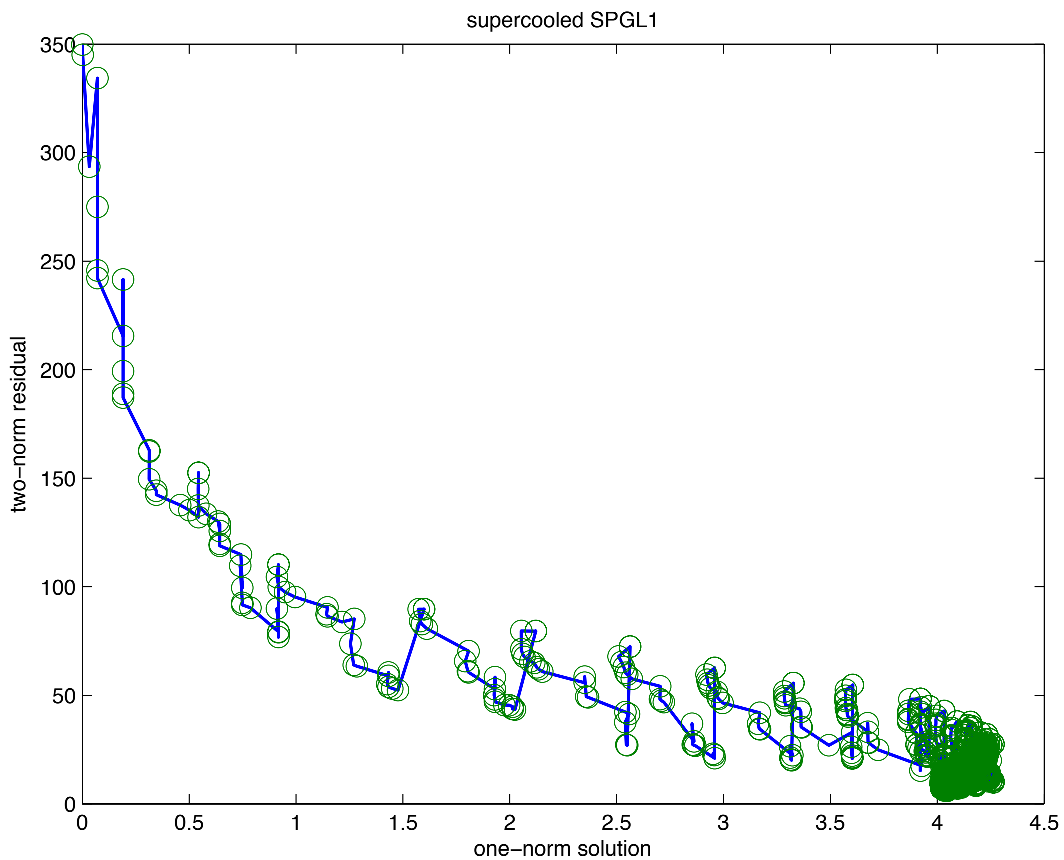
cost of 1/2 RTM w/ *all* data

Linearized inversion

[solution paths ℓ_1]



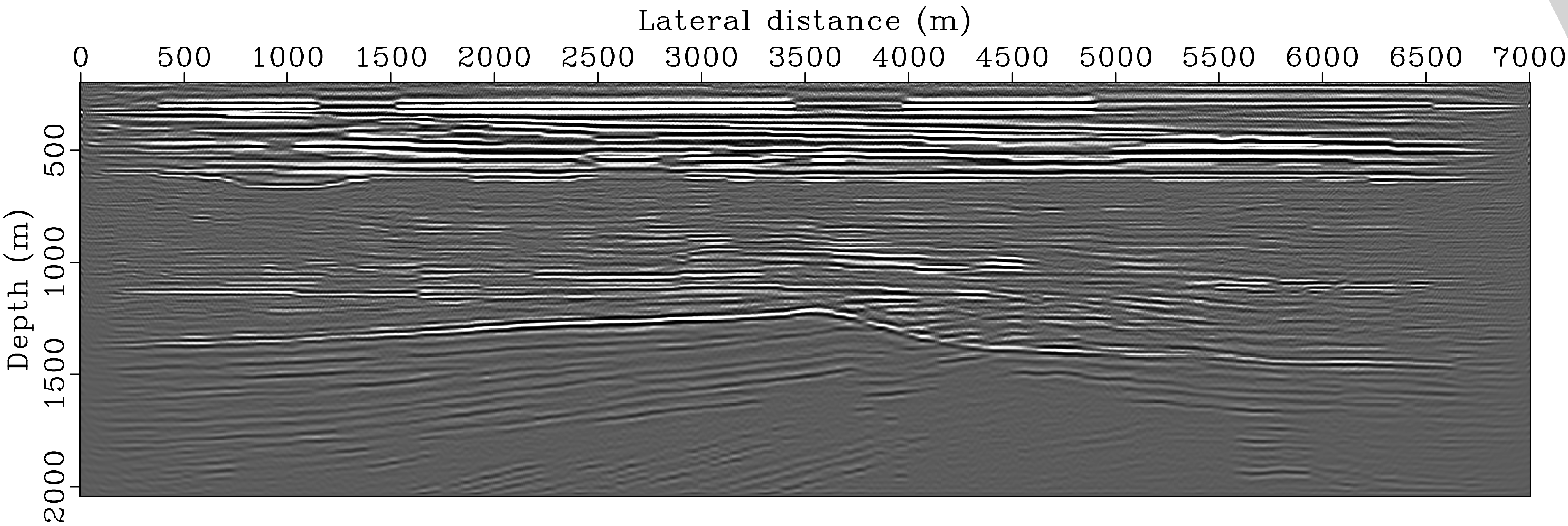
without rerandomization



with rerandomization

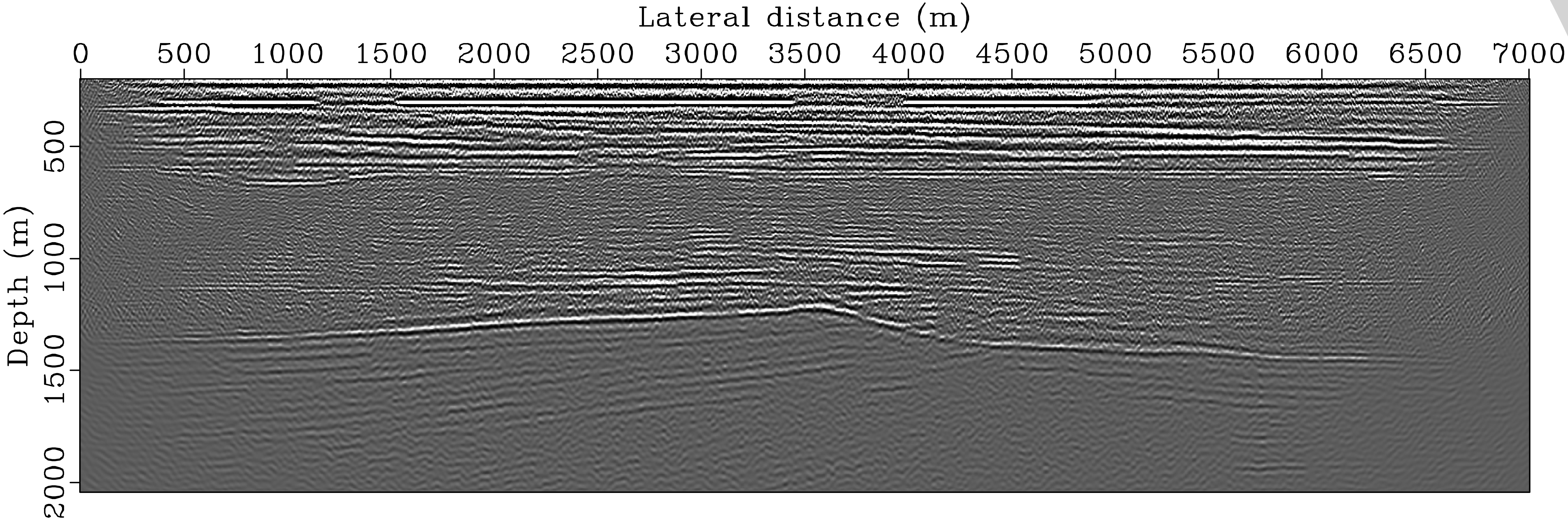
Linearized inversion

[ℓ_1 *w/* rerandomization 3 *super* shots]



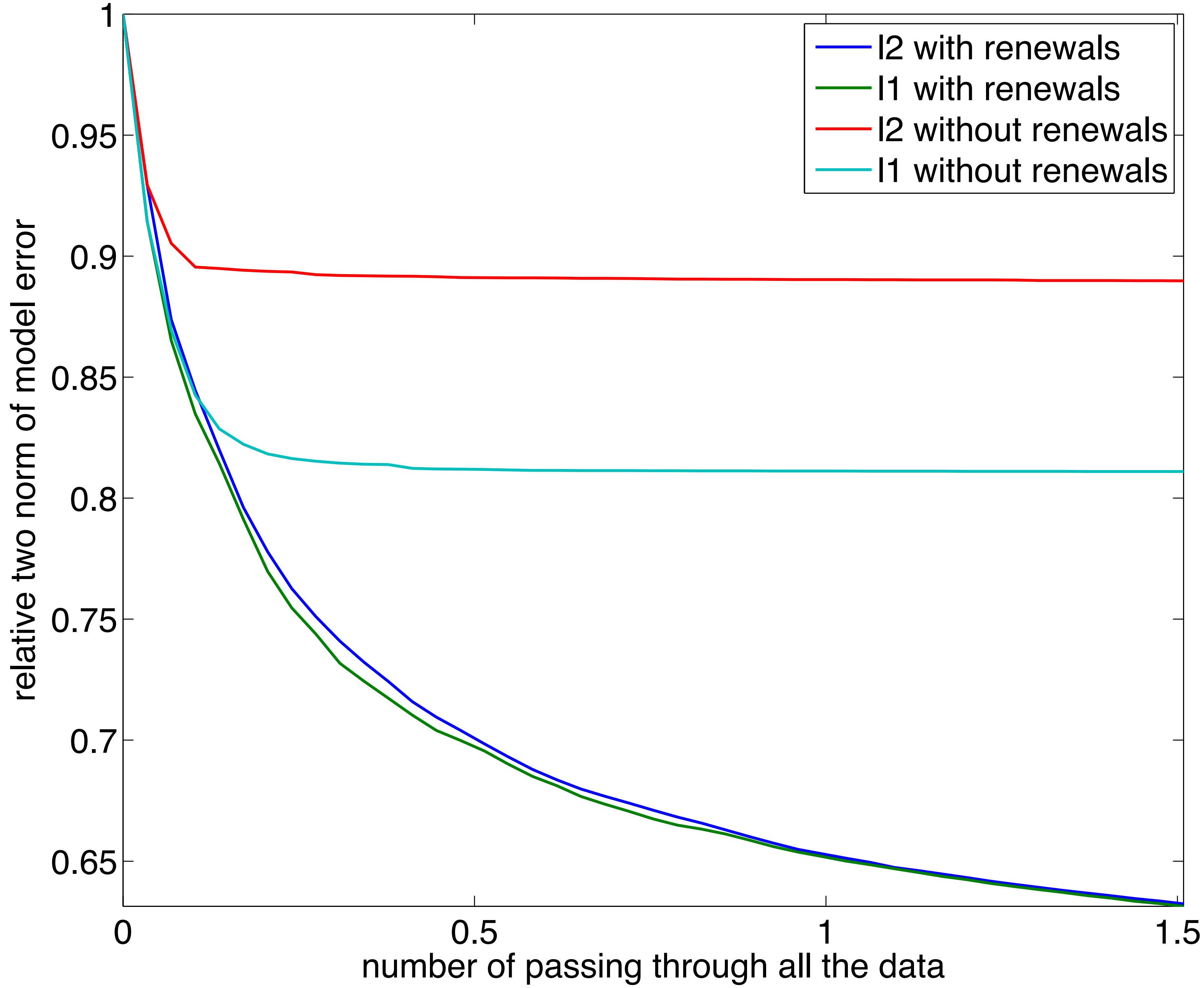
Linearized inversion

[ℓ_2 *w/* rerandomization 3 *super* shots]



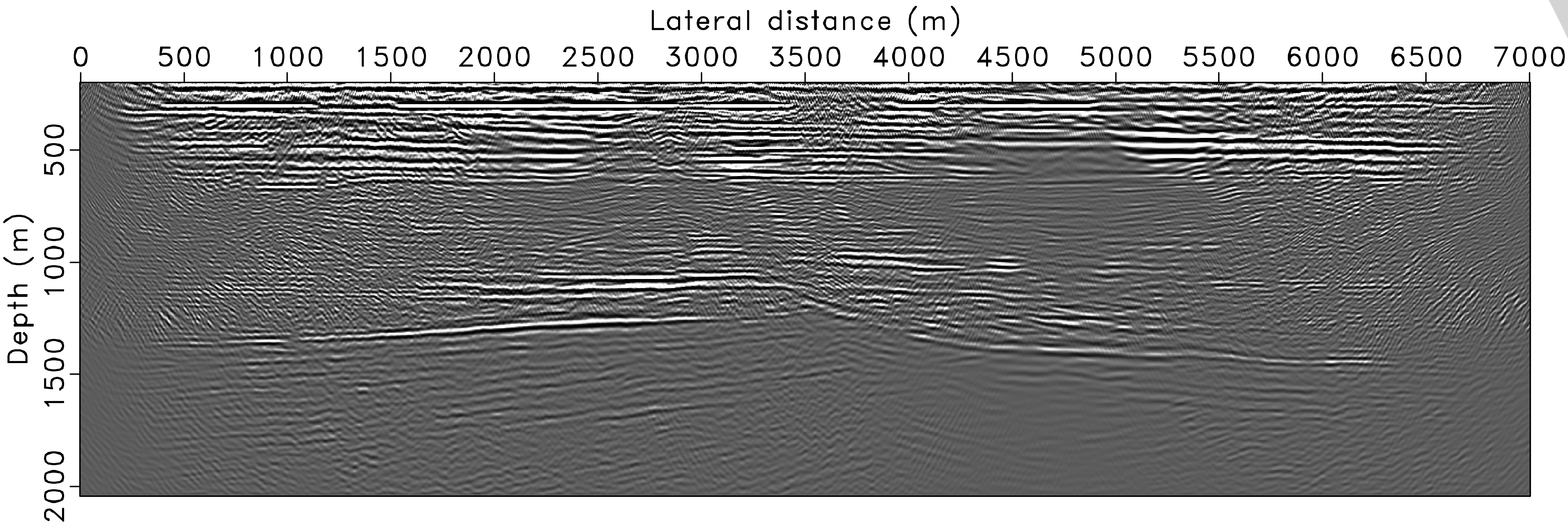
Linearized inversion

[model errors]



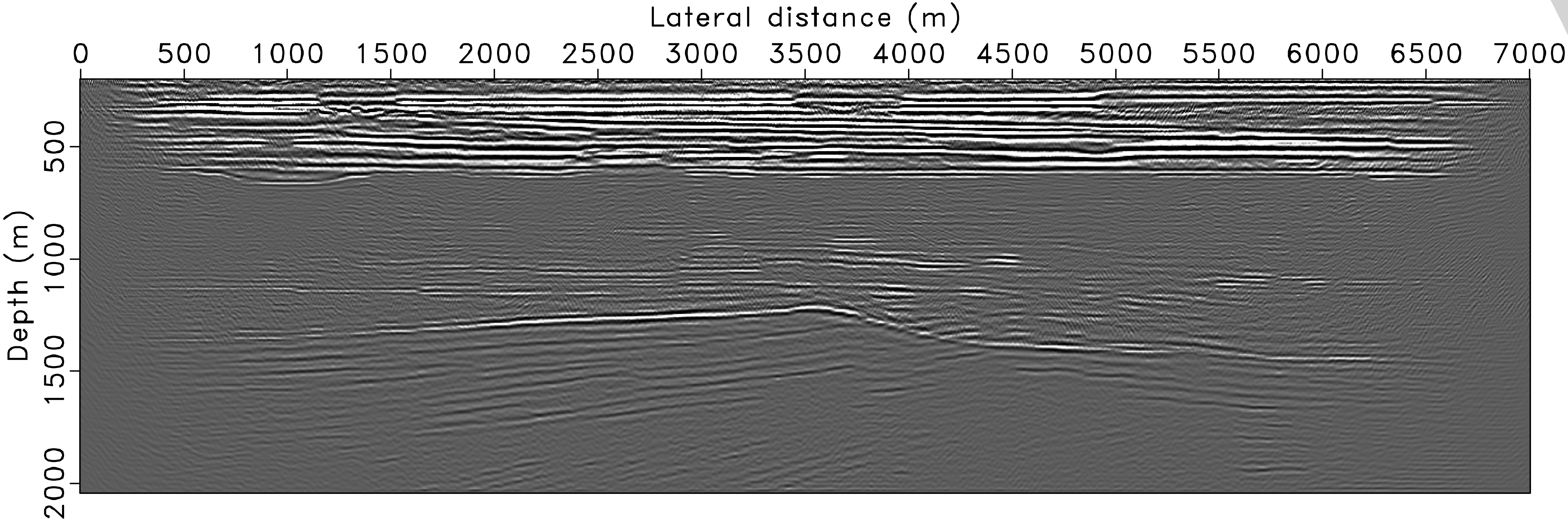
Marine linearized inversion

[ℓ_1 w/o rerandomization 17 shots]



Marine linearized inversion

[ℓ_1 *w/* rerandomization 17 shots]



Conclusions

Message passing improves *image* quality

- computationally *feasible* one-norm *regularization*

Message *passing* via *rerandomization*

- *small* system *size* with *small* IO and memory *imprints*

Possibility to exploit *new* computer *architectures* that employ *model* space *parallelism* to *speed up* wavefield *simulations*...

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Thank you

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