Hierarchical Tucker Tensor Optimization - Applications to 4D Seismic Data Interpolation

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High Dimensional Interpolation

Motivation:

- High dimensional data volumes have dependencies between dimensions that we can exploit in order to interpolate missing data
- Full source data is useful in the context of generating simultaneous sources for FWI, quality control

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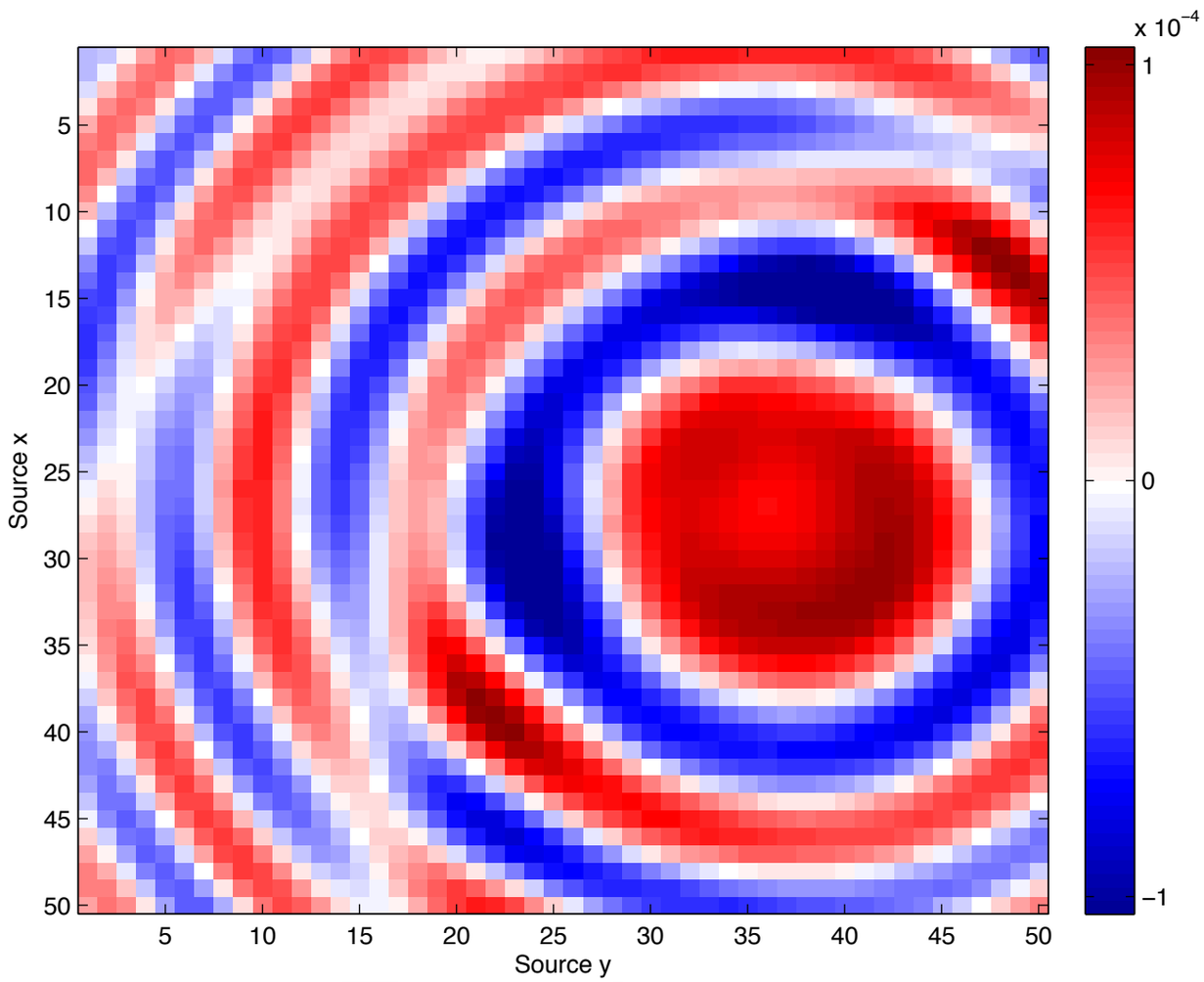
High Dimensional Interpolation

Goal:

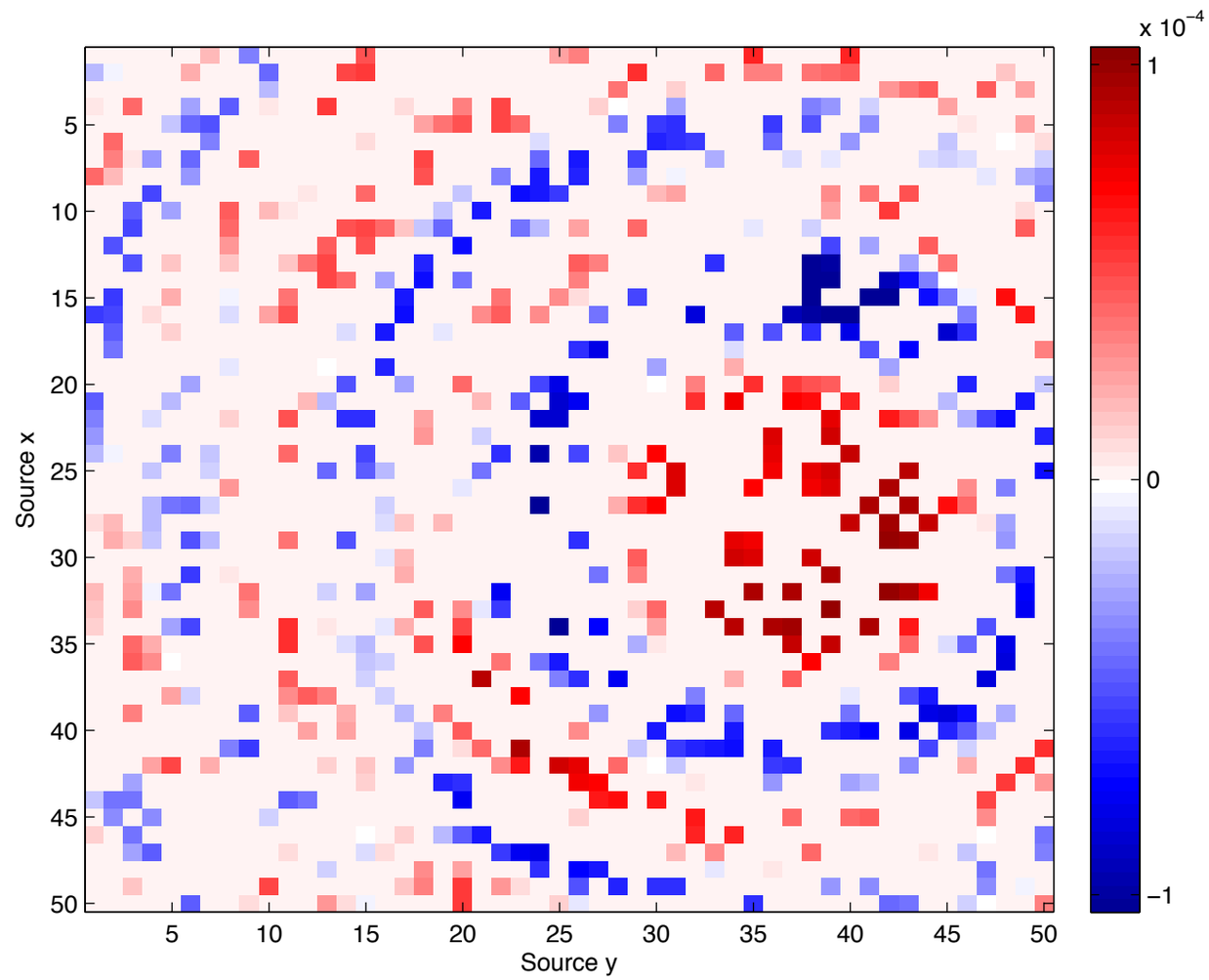
- Use our knowledge about favourable sampling conditions in 1D to design an optimization paradigm that favours missing data recovery
- Using randomized source acquisition to reduce costs of acquiring full source data

75% Missing Source Pairs



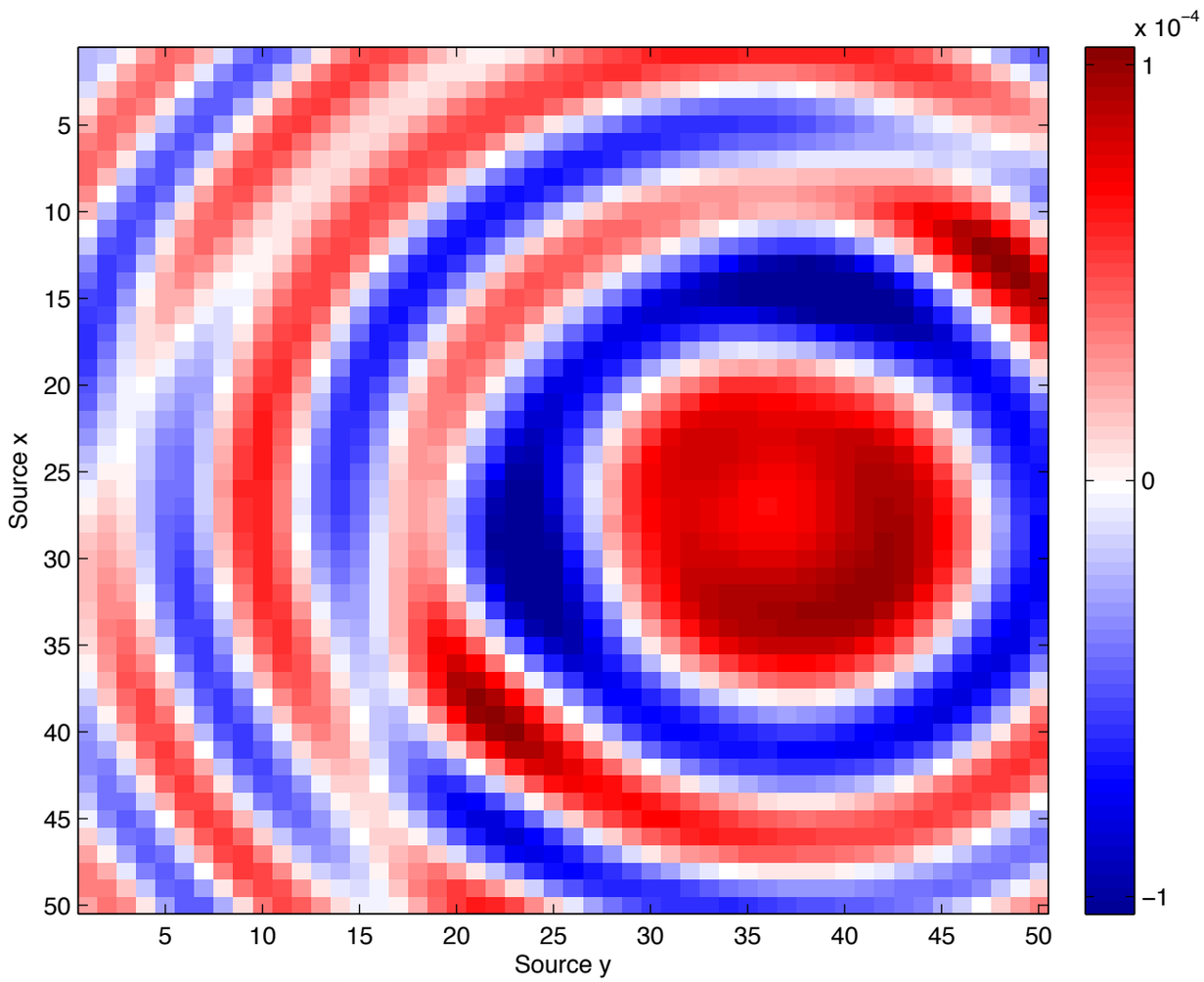
True data

(Rec x, Rec y) = (30,39)



Subsampled data

75% Missing Source Pairs



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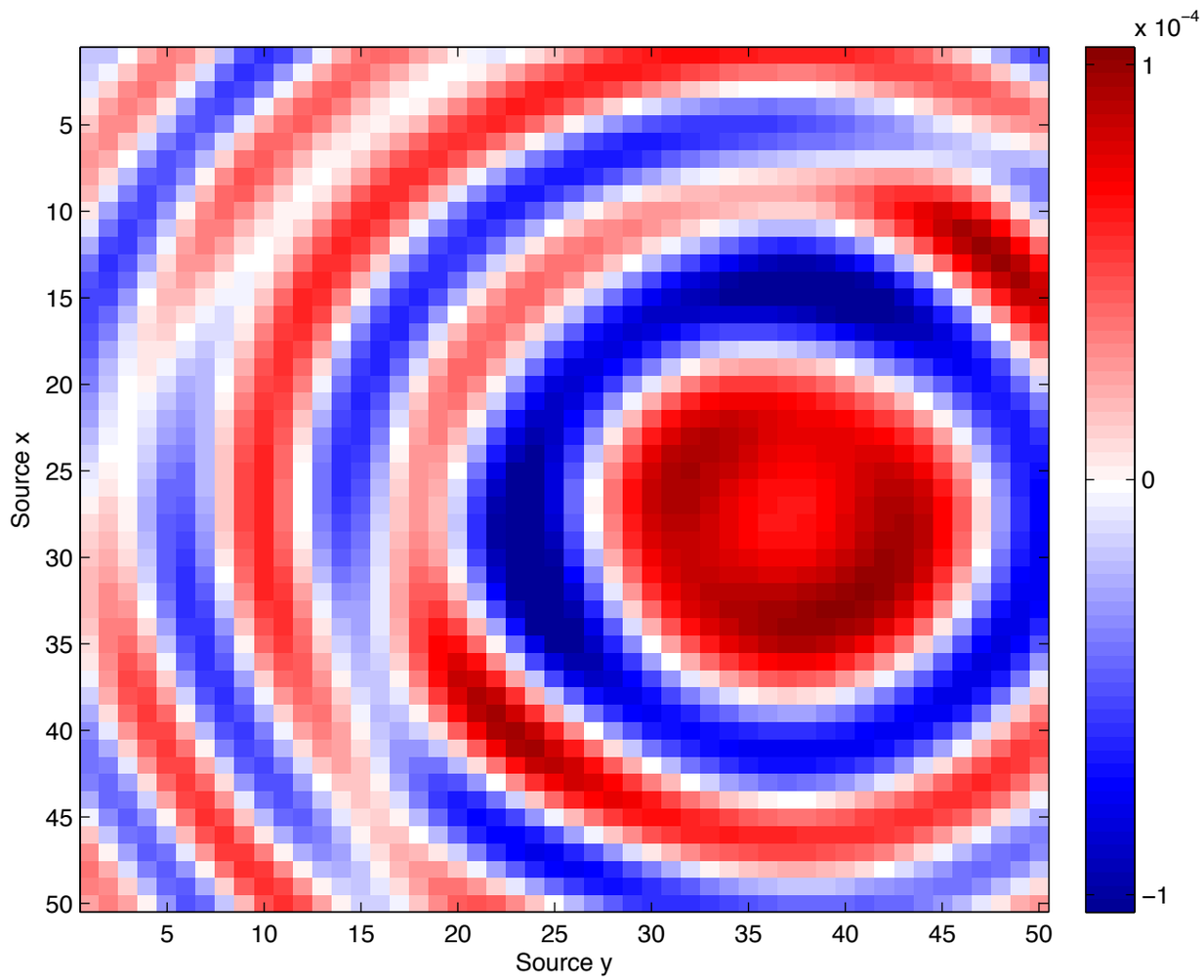

Recovered data

SNR - 18.7 dB

High Dimensional Interpolation

Goal:

- Generalize our notions of rank for matrices to higher-dimensional objects (tensors) and develop an efficient representation for these objects
- Avoid SVDs on full data

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Hierarchical Tucker Format

Matricization

- The *matricization* of a tensor X with dimensions $1, \ldots, d$ along the dimensions $t = (t_1, \ldots, t_r)$ is the matrix formed by placing the dimensions t along the rows and dimensions t^c along the columns
- Denoted $X^{(t)}$

Example in Matlab

```
n1 = 20; n2 = 20; n3 = 20; n4 = 20;
% Tensor
x = randn(n1,n2,n3,n4);
% Matricization along dimensions 1 and 2
x12 = reshape(x,n1 * n2, n3 * n4);
% Matricization along dimensions 3 and 4
y34 = permute(x,[3 4 1 2]);
x34 = reshape(x, n3 * n4, n1 * n2);
% Matricization along dimensions 1 and 3
y13 = permute(x,[1 3 2 4]);
x13 = reshape(x,n1 * n3, n2 * n4);
```

$X^{(1,2)}$

$X^{(3,4)}$

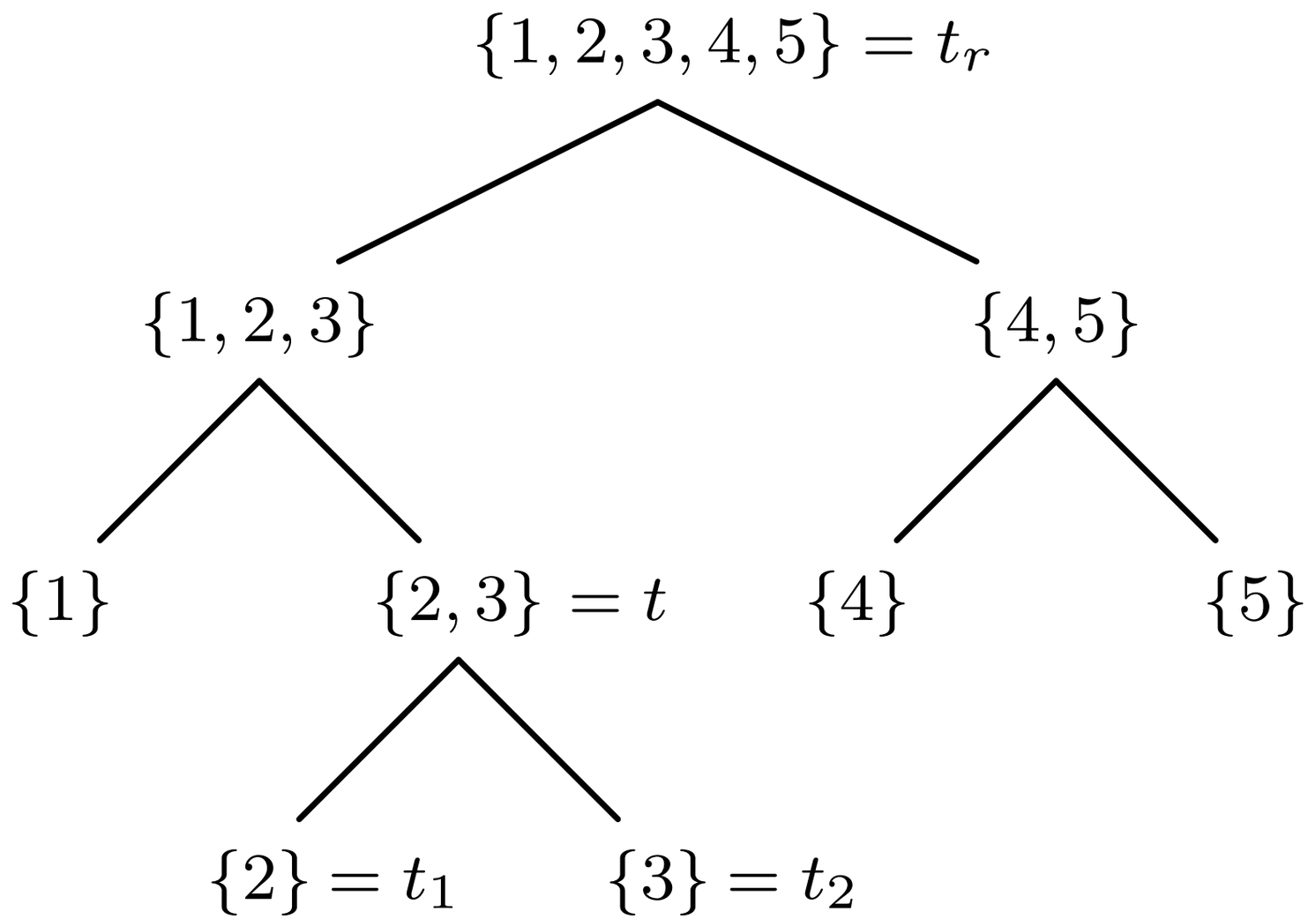
$X^{(1,3)}$

Hierarchical Tucker Format

- A *dimension tree* T for dimensions $\{1, \ldots, d\}$ is a non-trivial binary tree such that
  - the root, t_{root}, is has the label $\{1, \ldots, d\}$
  - each non-leaf node, t, can be written as $t = t_l \cup t_r$ where t_l is the left child of t, t_r is the right child of t $\quad t_l \cap t_r = \emptyset$

A. Uschmajew, B. Vandereycken. The geometry of algorithms using hierarchical tensors. 2012

Example



Hierarchical Tucker Format

- A tensor X can be written in the *Hierarchical Tucker format* corresponding to a dimension tree T and a vector of hierarchical ranks

$$(k_t)_{t \in T}, k_{\text{root}} = 1$$

if it can be written as

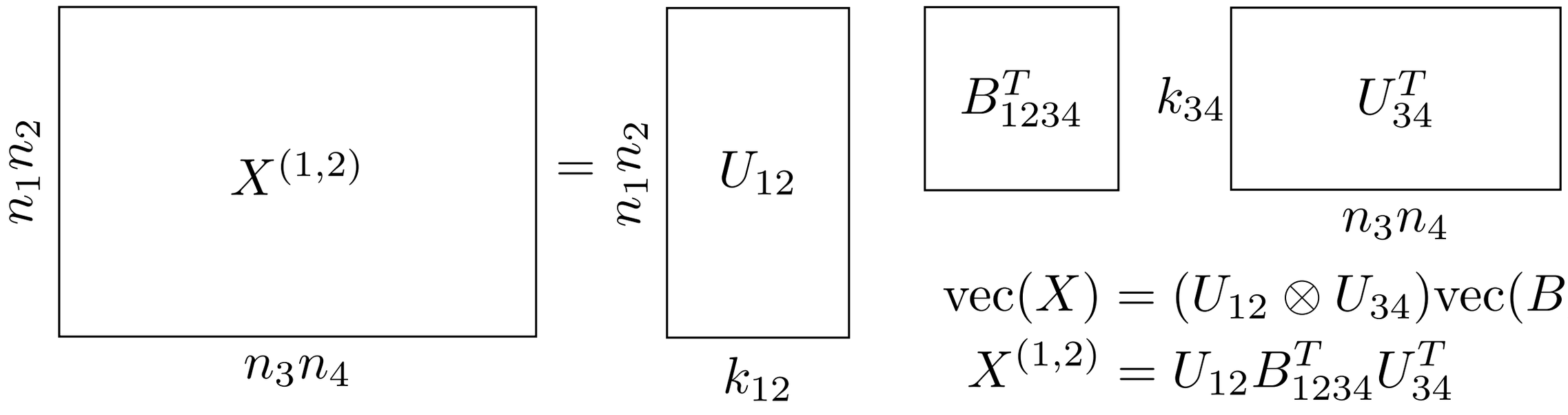
Hierarchical Tucker Format

$$\mathrm{vec}(X) = (U_{t_l} \otimes U_{t_r}) B_{t_{\mathrm{root}}}^{(1,2)} \quad t = t_{\mathrm{root}}$$

$$U_t = (U_{t_l} \otimes U_{t_r}) B_t^{(1,2)} \quad t \text{ not a leaf}$$

$$U_t \in \mathbb{C}^{n_t \times k_t} \qquad B_t \in \mathbb{C}^{k_l \times k_r \times k_t}$$

Example $X - n_1 \times n_2 \times n_3 \times n_4$ tensor

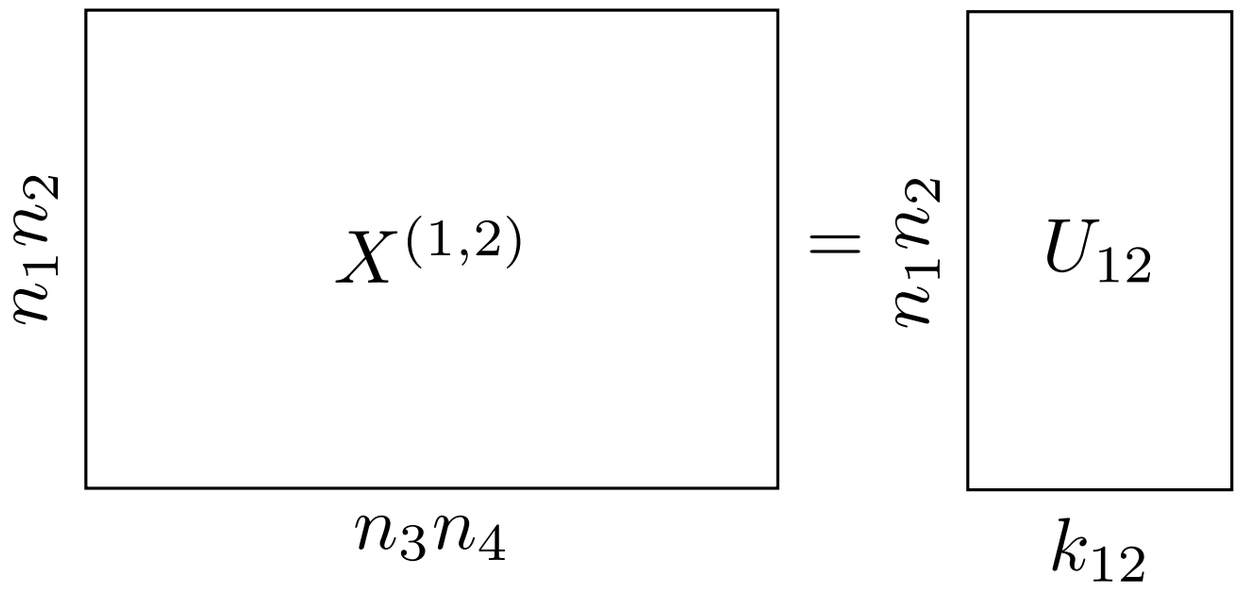


$$\text{vec}(X) = (U_{12} \otimes U_{34})\text{vec}(B_{1234})$$

$$X^{(1,2)} = U_{12} B^T_{1234} U^T_{34}$$

Example

$X - n_1 \times n_2 \times n_3 \times n_4$ tensor



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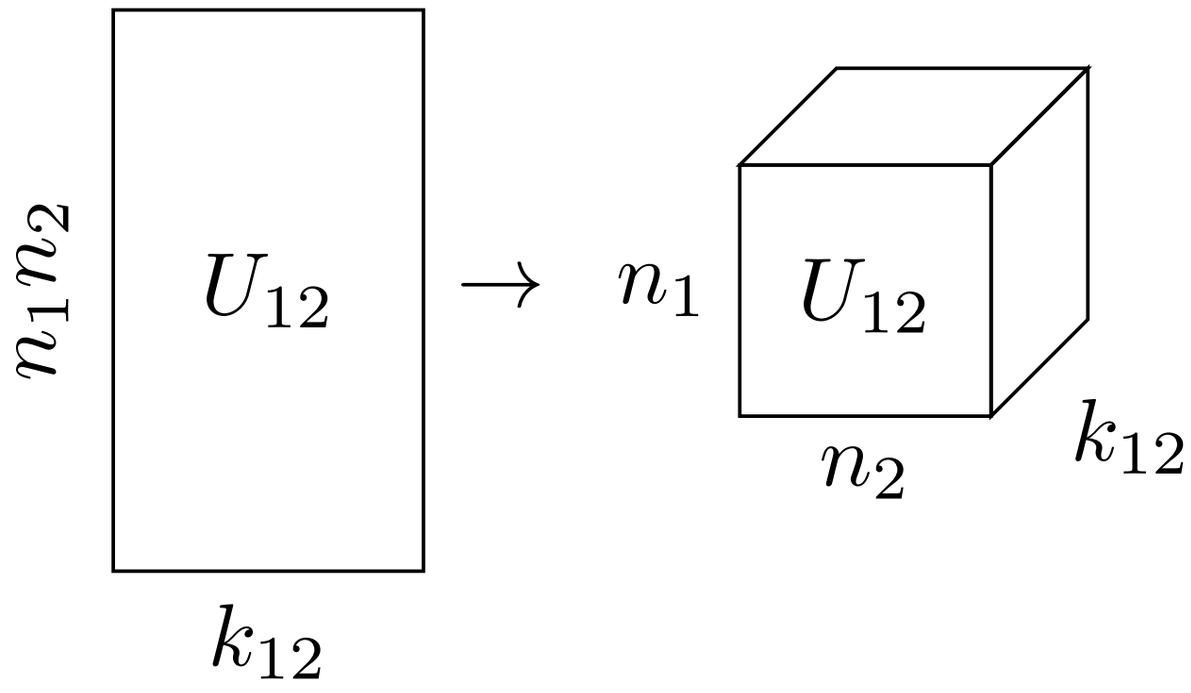

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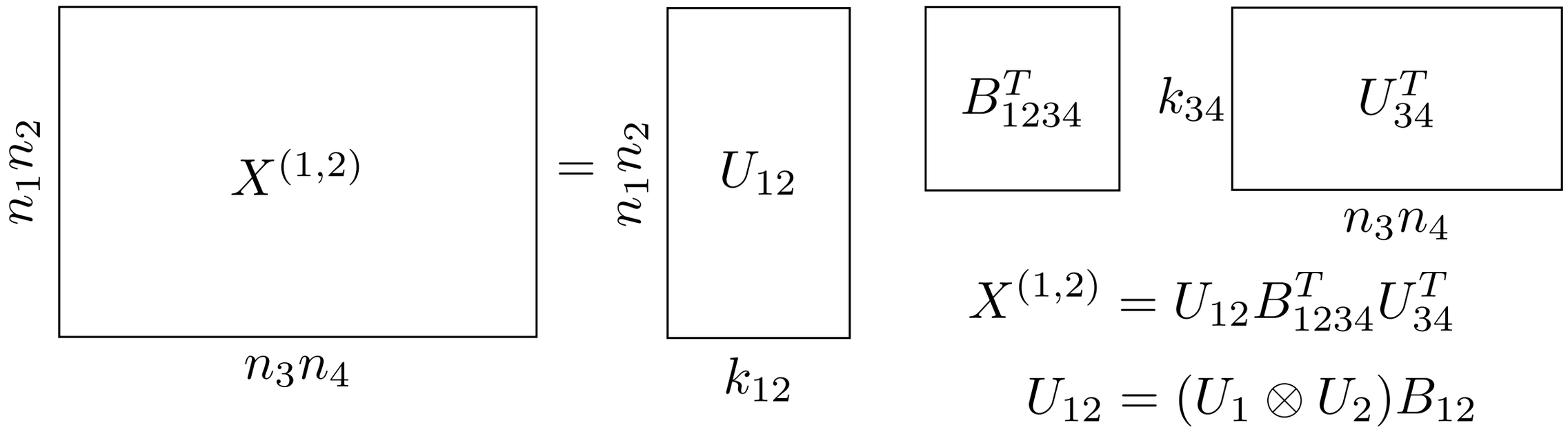

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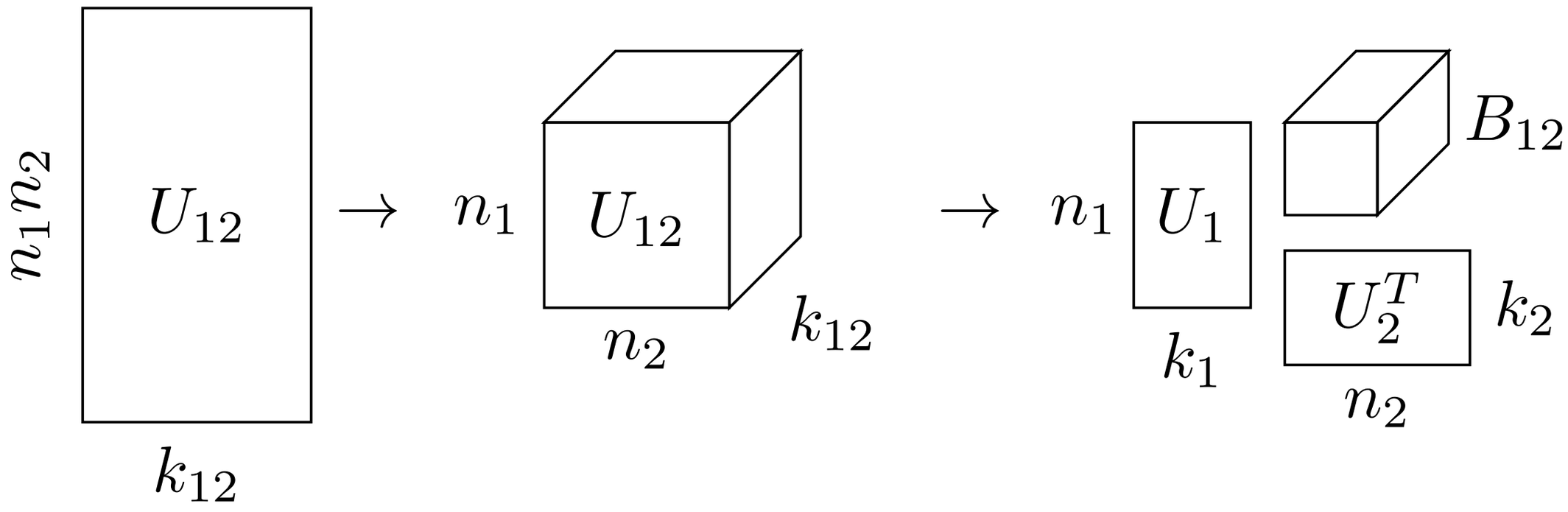
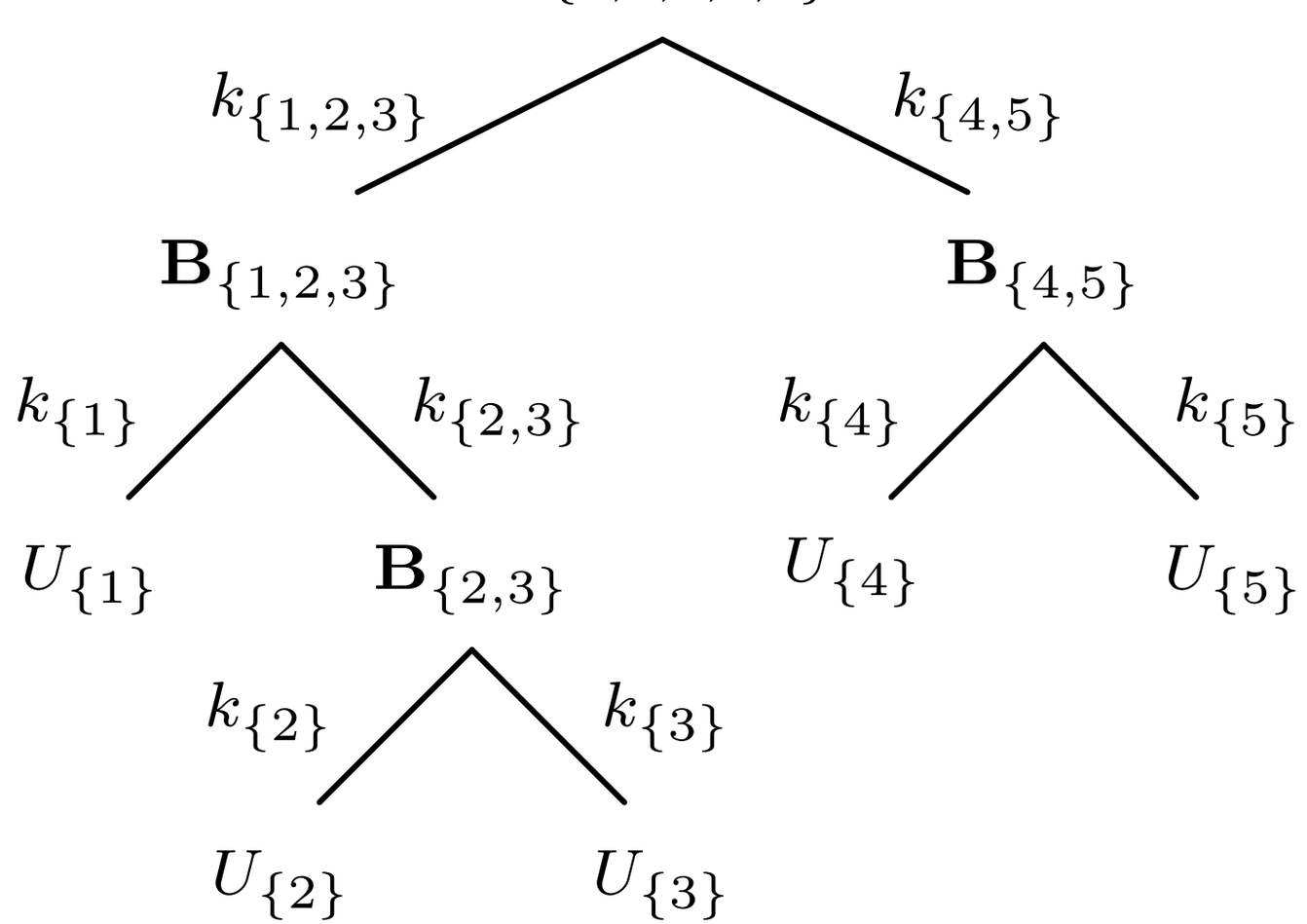


A. Uschmajew, B. Vandereycken. The geometry of algorithms using hierarchical tensors. 2012

Example

$$\mathrm{vec}(X) = (U_{\{1,2,3\}} \otimes U_{\{4,5\}})\mathbf{B}_{\{1,2,3,4,5\}}$$

$$k_{t_r} = 1$$



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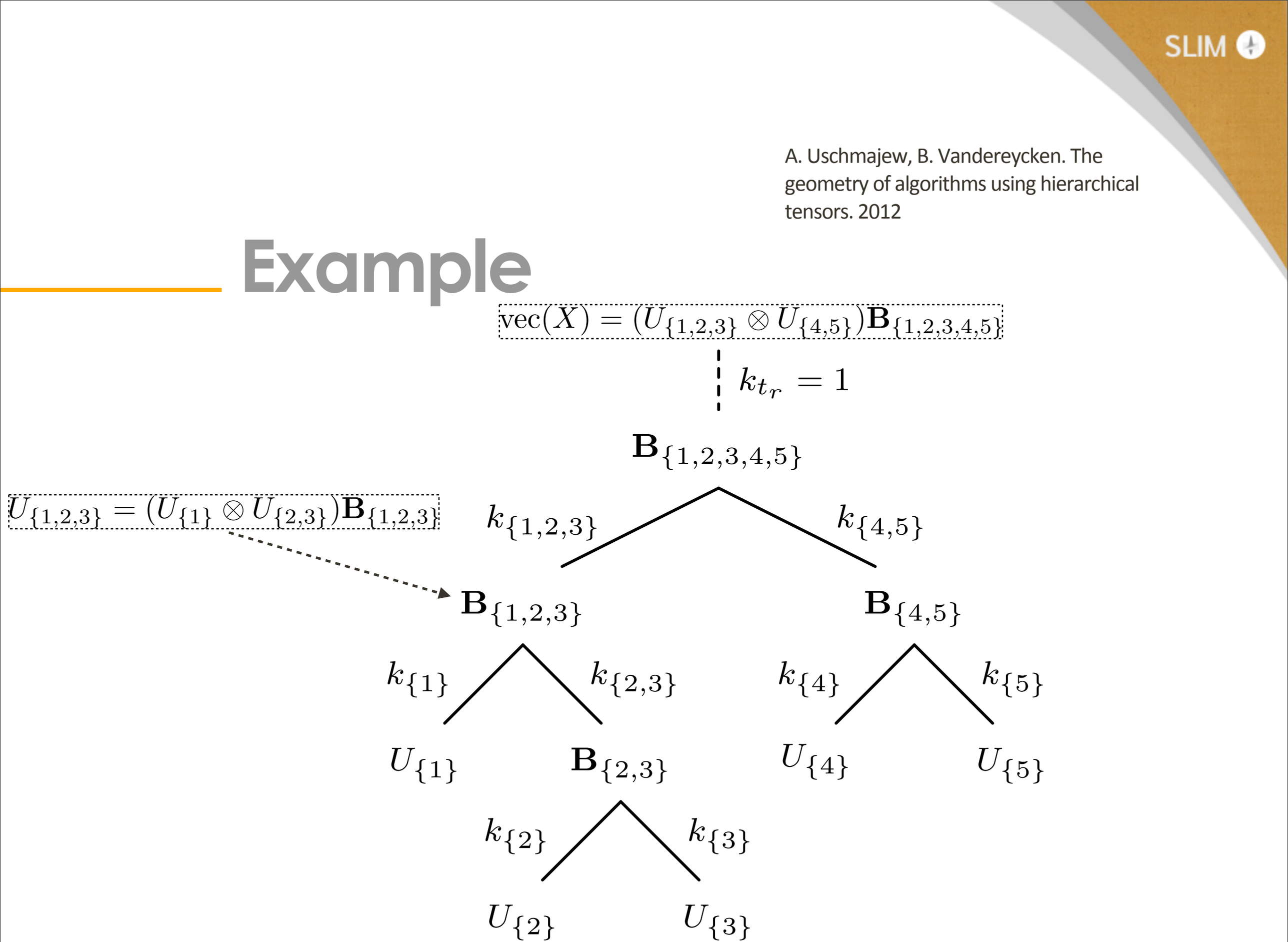

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Example

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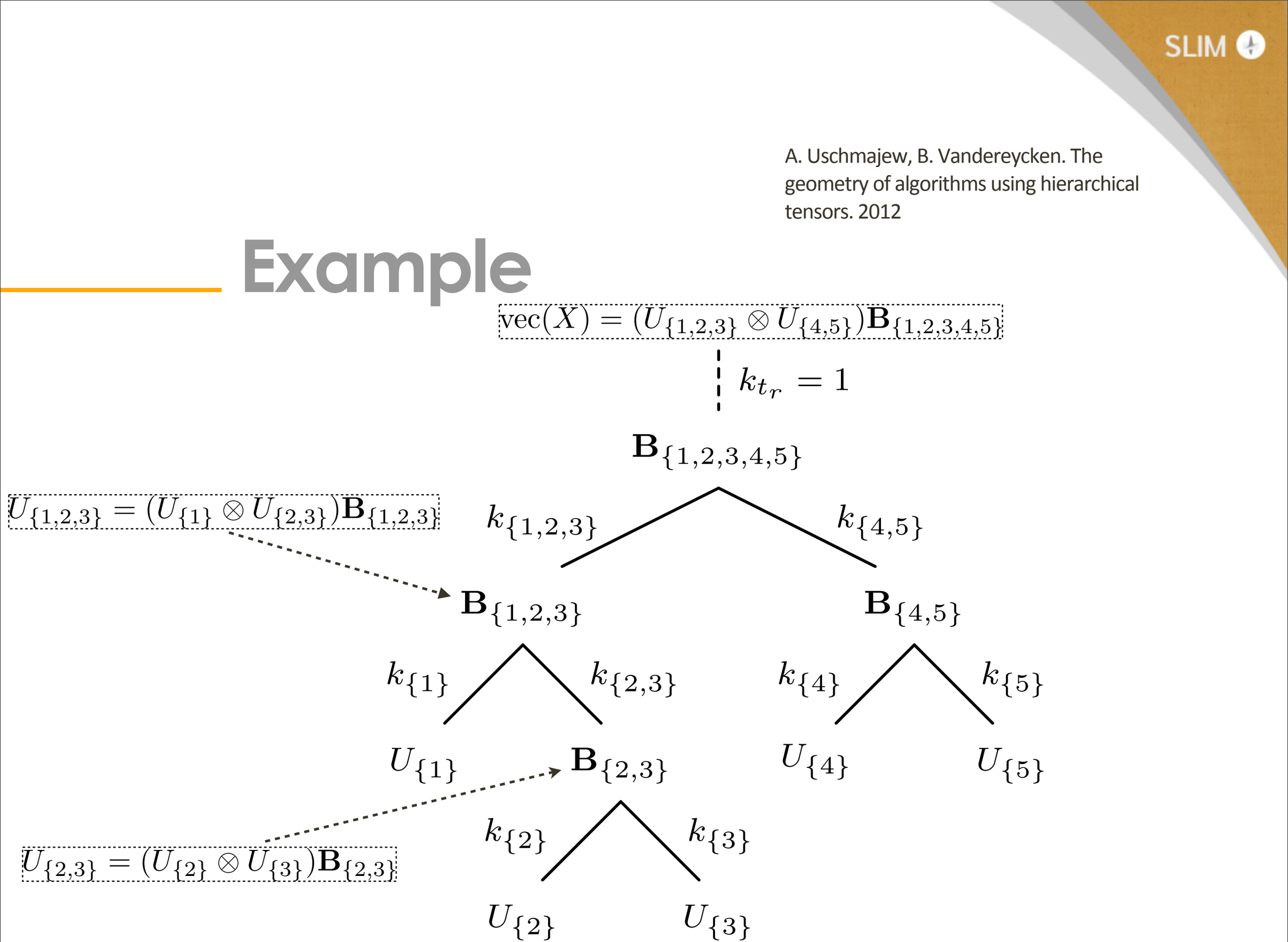
$$k_{t_r} = 1$$

$$\mathbf{B}_{\{1,2,3,4,5\}}$$

$$U_{\{1,2,3\}} = (U_{\{1\}} \otimes U_{\{2,3\}})\mathbf{B}_{\{1,2,3\}}$$



$$U_{\{2,3\}} = (U_{\{2\}} \otimes U_{\{3\}})\mathbf{B}_{\{2,3\}}$$

A. Uschmajew, B. Vandereycken. The geometry of algorithms using hierarchical tensors. 2012

Example

$$\mathrm{vec}(X) = (U_{\{1,2,3\}} \otimes U_{\{4,5\}})\mathbf{B}_{\{1,2,3,4,5\}}$$

$$k_{t_r} = 1$$

$$\mathbf{B}_{\{1,2,3,4,5\}} \qquad U_{\{4,5\}} = (U_{\{4\}} \otimes U_{\{5\}})\mathbf{B}_{\{4,5\}}$$

$$U_{\{1,2,3\}} = (U_{\{1\}} \otimes U_{\{2,3\}})\mathbf{B}_{\{1,2,3\}}$$

$k_{\{1,2,3\}}$ $k_{\{4,5\}}$

$\mathbf{B}_{\{1,2,3\}}$ $\mathbf{B}_{\{4,5\}}$

$k_{\{1\}}$ $k_{\{2,3\}}$ $k_{\{4\}}$ $k_{\{5\}}$

$U_{\{1\}}$ $\mathbf{B}_{\{2,3\}}$ $U_{\{4\}}$ $U_{\{5\}}$

$$U_{\{2,3\}} = (U_{\{2\}} \otimes U_{\{3\}})\mathbf{B}_{\{2,3\}}$$

$k_{\{2\}}$ $k_{\{3\}}$

$U_{\{2\}}$ $U_{\{3\}}$

Hierarchical Tucker Format

- We don't need to store the matrices U_t when t is not a leaf node
  - We only need to store U_t for the leaves, B_t for the internal nodes
  - We don't need to store the (full) tensor

Hierarchical Tucker Format

- Storage $\leq dNK + (d-2)K^3 + K^2$

  where $K = \max_{t \in T} k_t$, $N = \max_{i=1,\ldots,d} n_i$

- Compare to N^d storage for the full tensor
- Effectively breaking the curse of dimensionality when $K \ll N$

Hierarchical Tucker Format

- E.g. for a 100 x 100 x 100 x 100 cube with max rank 20, $N = 100 \quad d = 4 \quad k = 20$

  Naive storage: $N^d = 10^8$ parameters

  HTucker storage:

  $$dNK + (d-2)K^3 + K^2$$

  $$= 24400 \text{ parameters}$$

  Compression of a factor of 99.97%

www.sam.math.ethz.ch/NLAgroup/
htucker_toolbox.html

Hierarchical Tucker Format

- Existing tools for using this format

hTucker toolbox

- Matlab code for performing operations in this framework
- truncation, addition/subtraction, tensor-tensor contraction, etc.

Hierarchical Tucker Format

- Truncation of a full tensor X to a rank-k hTucker tensor requires knowledge of *all* of the entries of X
- Requires SVDs of underlying matricizations, which are expensive
- Problematic in seismic data cases, where not all of the data is available

Hierarchical Tucker Format

- We can use knowledge of the multidimensional structure of, say, a frequency slice of data with missing sources/receivers in order to recover the full data via solving an optimization problem
- Analogous to $l1$ regularized curvelet interpolation in the 2D seismic case

Optimization Format

- Given data D with missing sources and/or receivers, subsampling operator A, full tensor expansion operator

$$\phi : (U_t, B_t) \rightarrow \mathbb{C}^{n_1 \times \cdots \times n_d}$$

solve

$$\min_{(U_t, B_t)} \|A\phi(x) - D\|_2^2$$

Derivatives

- Derivatives of a particular node with respect to its children can be computed efficiently
- The chain rule gives the total derivative of the function ϕ

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Derivatives

- Messy painful derivation, available on request
- After much simplification, boils down to about 20 lines of matlab code, using the SPOT framework + code from the hTucker toolbox

Derivative code

```
function [dUdL,dUdR,dUdB] = dHTuck(Uleft, Uright, B)

    [kr,kl,k] = size(B);
    nl = size(Uleft,1); nr = size(Uright,1);
    Bhat = matricize(B,1,[2 3]);
    Bhat = dematricize(Uright * Bhat,[nr,kl,k],1,[2,3]);

    Bhat = matricize(Bhat,[1 3],2);
    dUdL = opFunction(nl*nr*k, nl*kl, @(x,mode) dUdL_func(x,mode, Bhat,nr,nl,k,kl));
    BhatR = dematricize( Uleft*matricize(permute(B,[2 1 3]),1, [2 3]), [nl,kr,k],1,[2 3]);
    dUdR = opKron(matricize(BhatR,[1 3],2),opDirac(nr));
    dUdB = opKron(opDirac(k),Uleft,Uright);

end

function y = dUdL_func(x,mode,Bhat,nr,nl,k,kl)
    if mode == 1
        %Forward mode
        v = reshape(x,nl,kl);
        q = Bhat * v';

        result = reshape(q,nr,k,nl);
        result = permute(result,[1 3 2]);
        y = vec(result);
    else
        %Adjoint mode
        v = reshape(x,nr,nl,k);
        q = Bhat' * matricize(v,[1 3],2);

        %Block transposition
        q = reshape(q,kl,nl);
        y = vec(q');
    end
end
```

Derivatives

- Only involve matrix-matrix multiplication of small matrices compared to the ambient, full-tensor space
- Immediately parallelizable
  - Replace “opKron” with “oppKron2Lo”

Differential Geometry

- The function

$$\phi : (U_t, B_t) \rightarrow \mathbb{C}^{n_1 \times \cdots \times n_d}$$

  is not injective
- There are sets of distinct matrices $(U_t, B_t), (V_t, C_t)$ where $\phi(U_t, B_t) = \phi(V_t, C_t)$

Differential Geometry

- This non-uniqueness (somewhat artificially) leads to *multiple minima* in the optimization program
- Can be resolved by characterizing the differential geometric structure of this format
- This gives us a *smooth manifold* (i.e. curved surface)

[1] A. Uschmajew, B. Vandereycken. The geometry of algorithms using hierarchical tensors. 2012

Differential Geometry

- The authors in [1] study the non-uniqueness of the general, non-orthogonalized hTucker format
- Characterization of the horizontal space (tangent vectors which change ϕ) and the vertical space (tangent vectors which do not change ϕ)

Differential Geometry

- We restrict ourselves to the *orthogonalized Hierarchical Tucker format*
- The matrices (U_t, B_t) not at the root are constrained to be orthogonal

Differential Geometry

- Different geometric structure, but still within a workable framework
- Projections become much better behaved with these constraints

P.A. Absil, R. Mahony, and R. Sepulchre. *Optimization algorithms on matrix manifolds*. Princeton Univ Press, 2008.

Differential Geometry

- With these orthogonality constraints, the standard matrix trace turns this format into a *Riemannian manifold*
- Large amount of existing research dealing with solving optimization problems on Riemannian manifolds
- Standard algorithms such as Steepest Descent translatable into this framework

Sampling

- With 2D volumes, we know several things
  - Low-rank structure can be recovered through solving a rank-minimizing optimization problem
  - Random removal of points increases the rank of the underlying matrix, which favours recovery by rank-minimization

Sampling

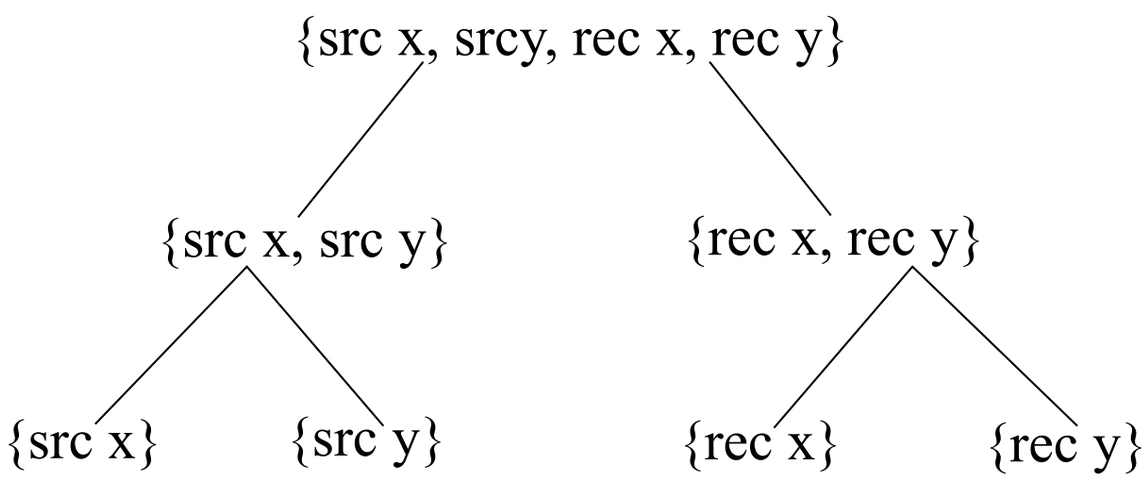
- In higher dimensions, there are multiple ways to ‘separate’ different dimensions
- No unique notion of rank, as in the matrix case

Sampling - Seismic Example

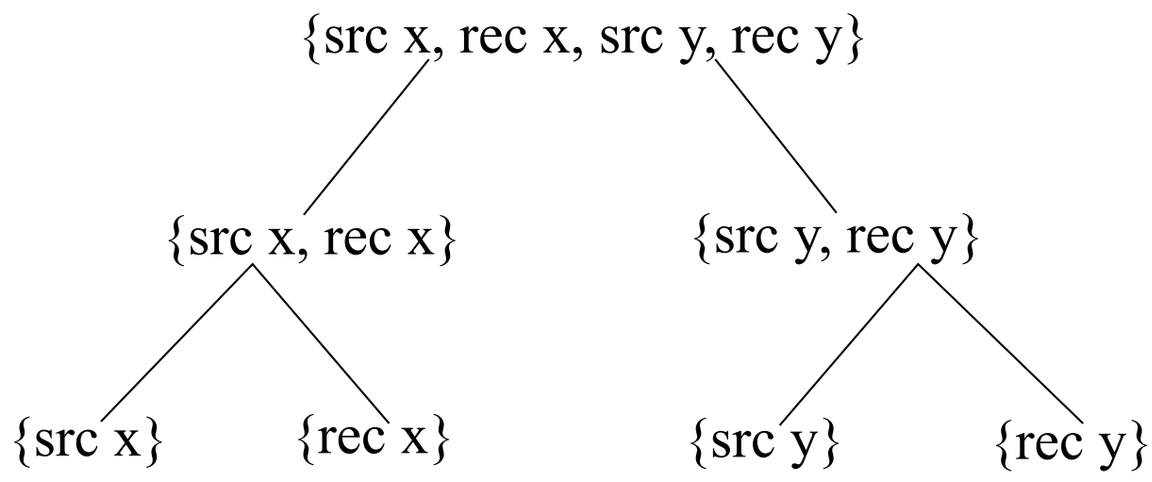
- We consider a 3D seismic survey with coordinates (src x, src y, rec x, rec y, time)
- We take a Fourier transform in time and restrict ourselves to a single frequency slice

Sampling

- For a frequency slice with coordinates (src x, src y, rec x, rec y), there are essentially two choices of dimension tree (by reciprocity)



Canonical Decomposition

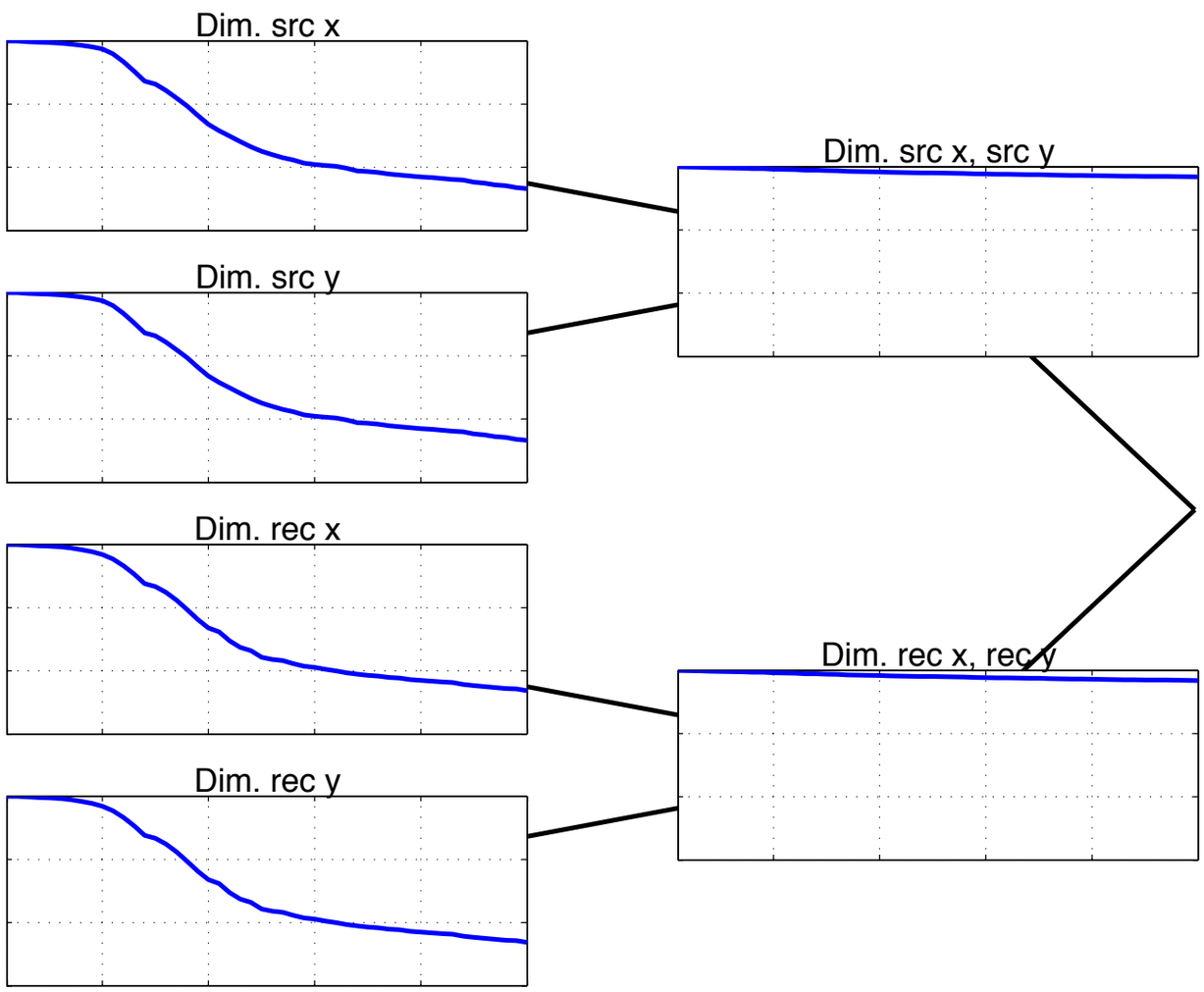


Non-canonical Decomposition

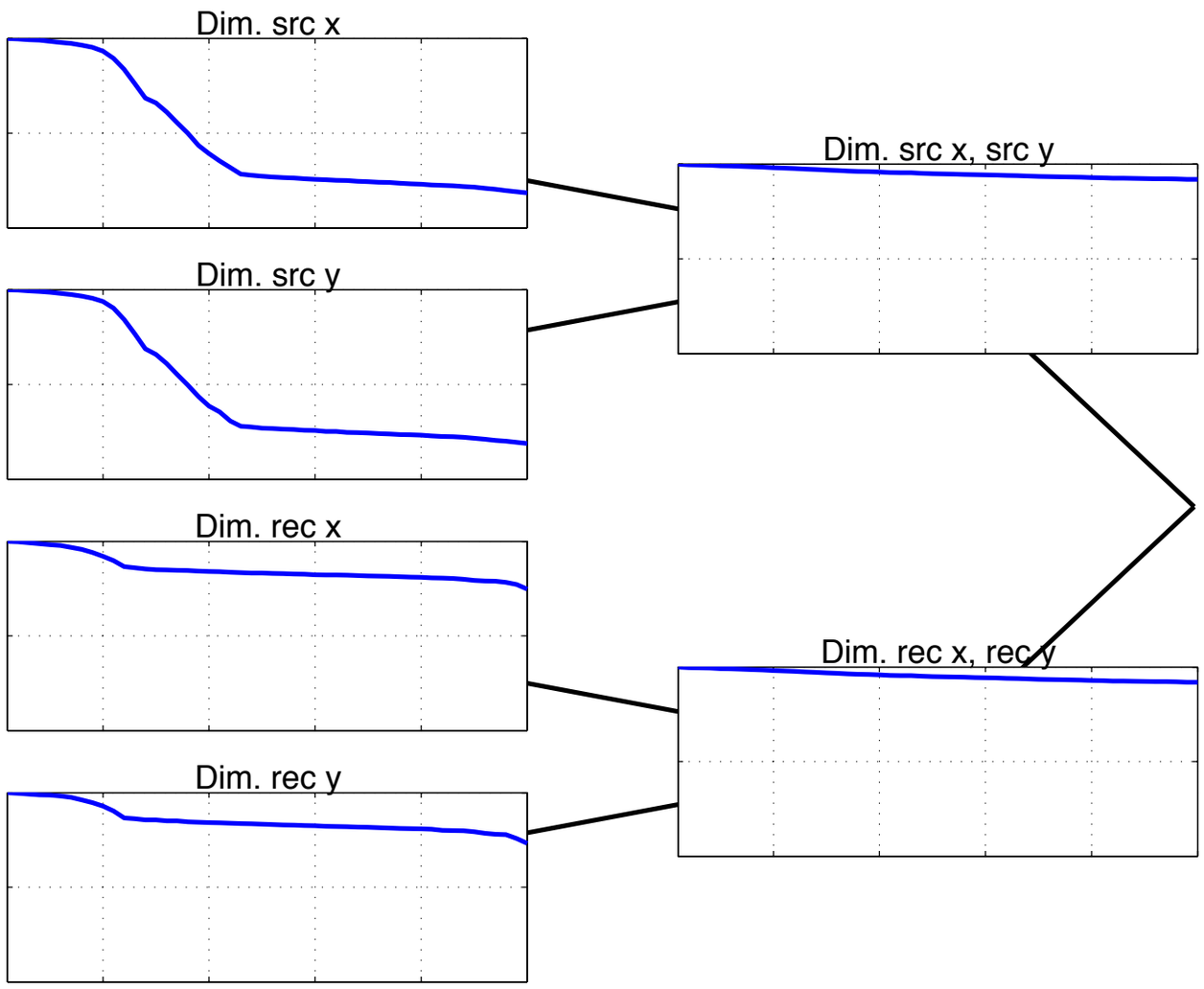
Effects of sampling

25% random receiver pairs removed

- (src x, src y), (rec x, rec y) pairing



No subsampling

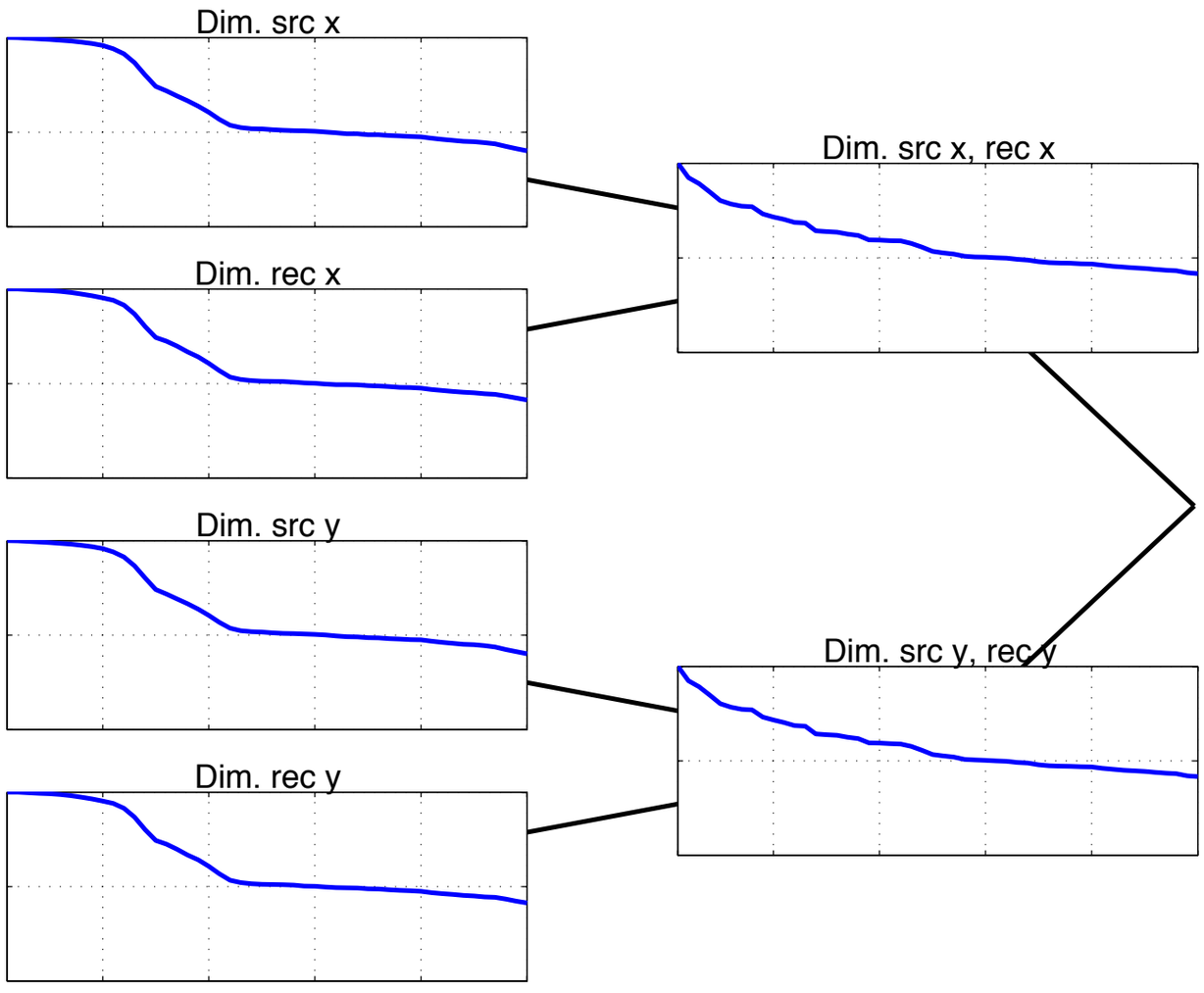


Rec subsampling

Effects of sampling

25% random receiver pairs removed

- (src x, rec x), (src y, rec y) pairing



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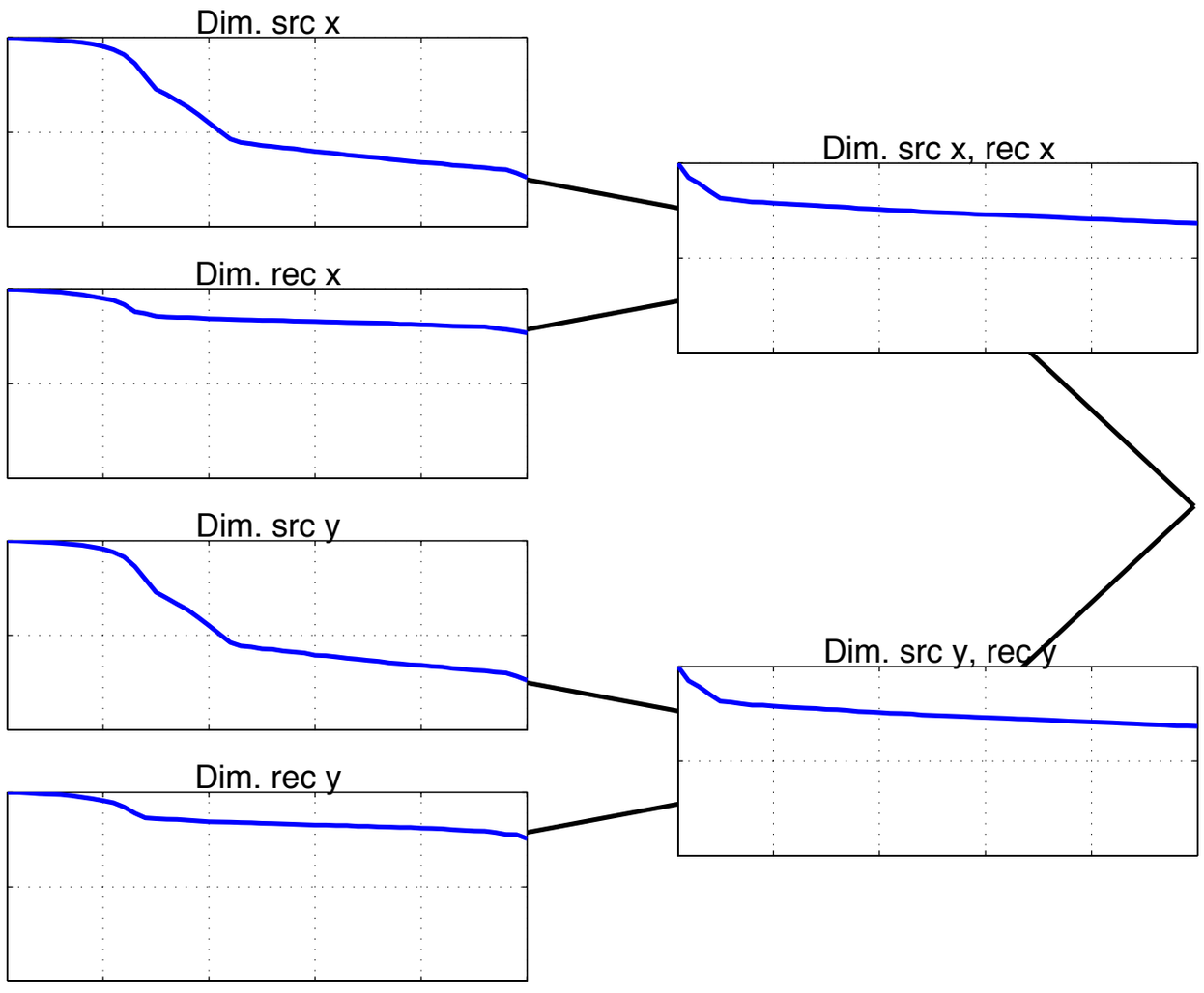
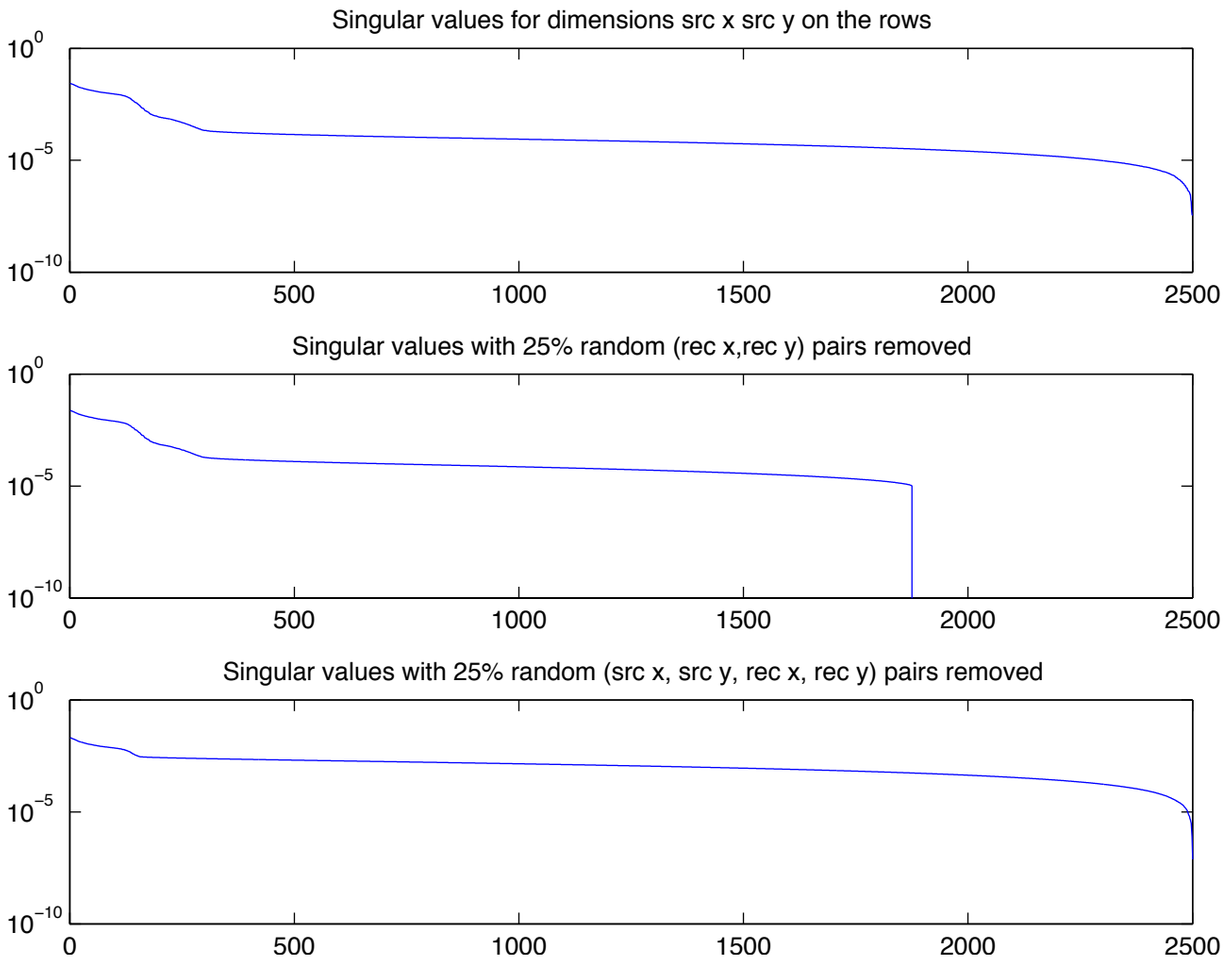


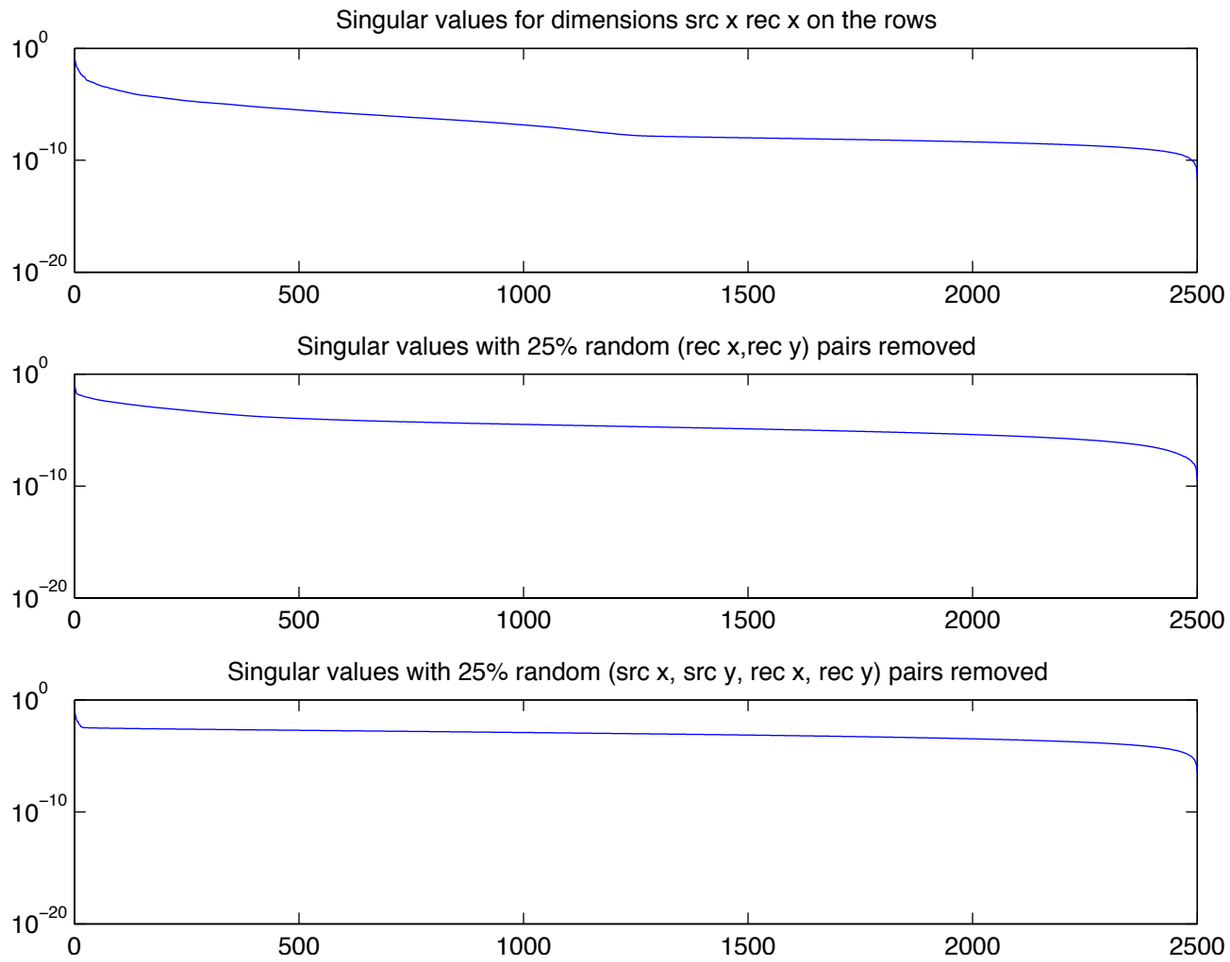
Rec subsampling

Effects of sampling

This phenomenon can also be understood in the context of different matricizations of the tensor

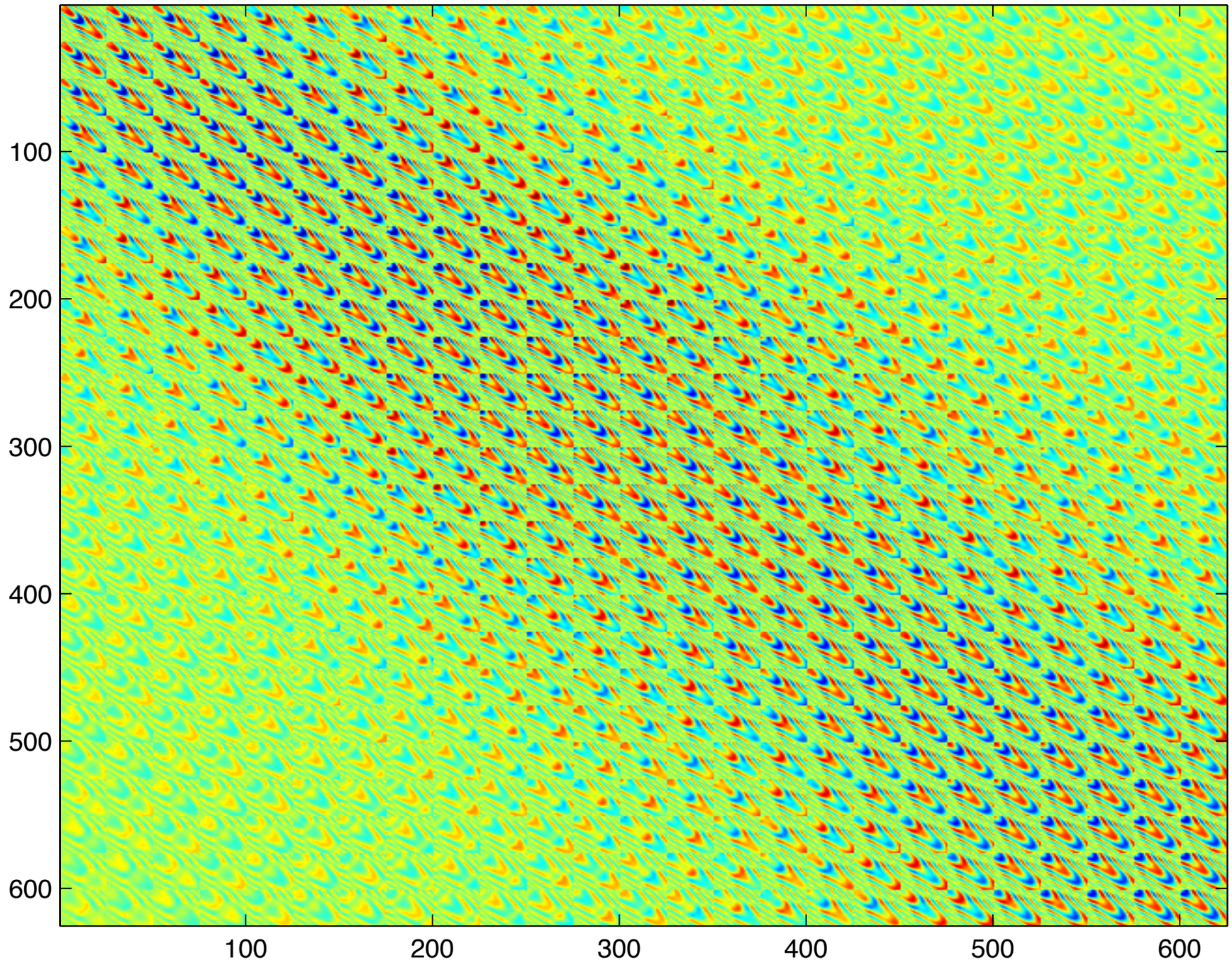


Canonical Grouping



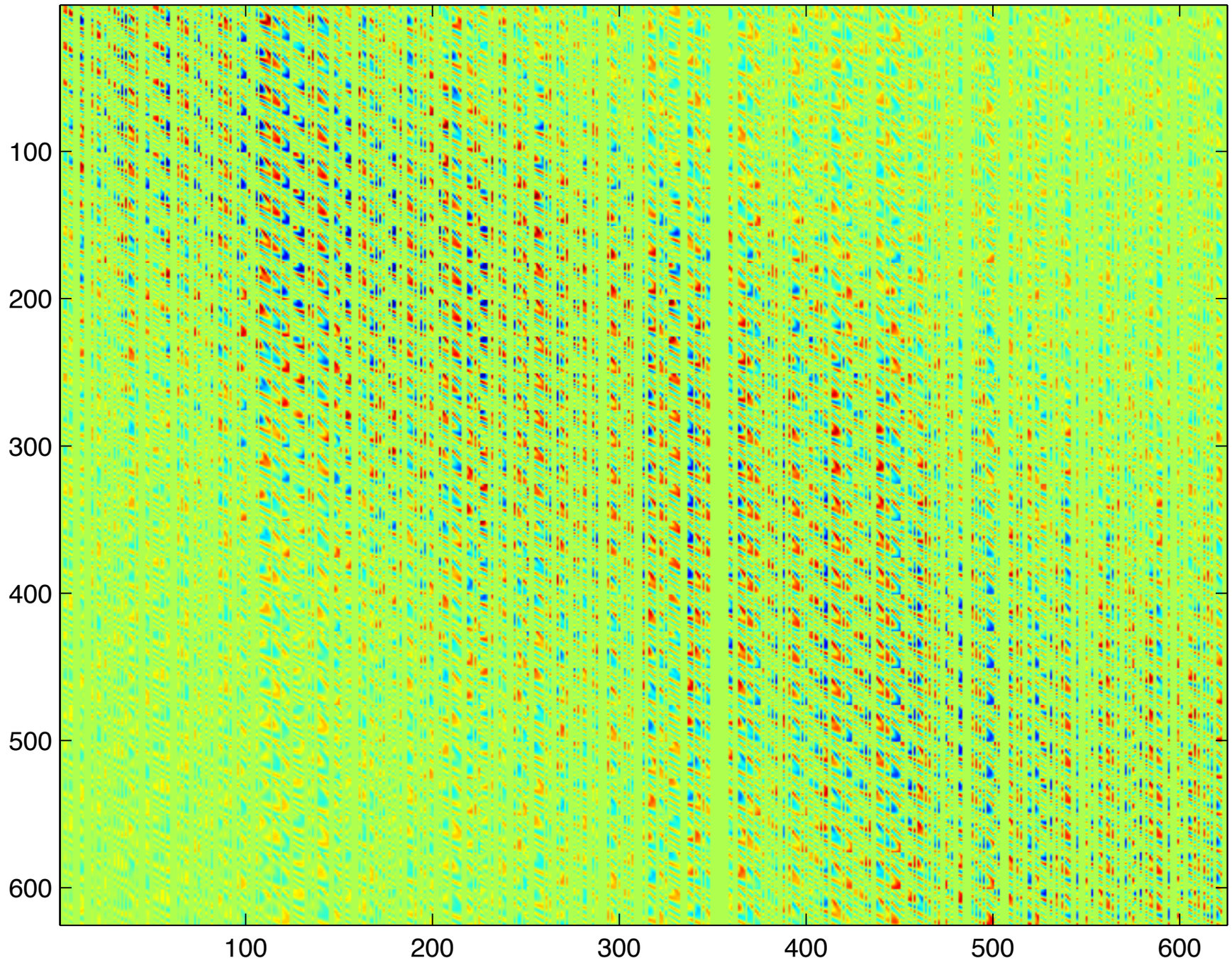
Non-canonical Grouping

Matricizations + Sampling



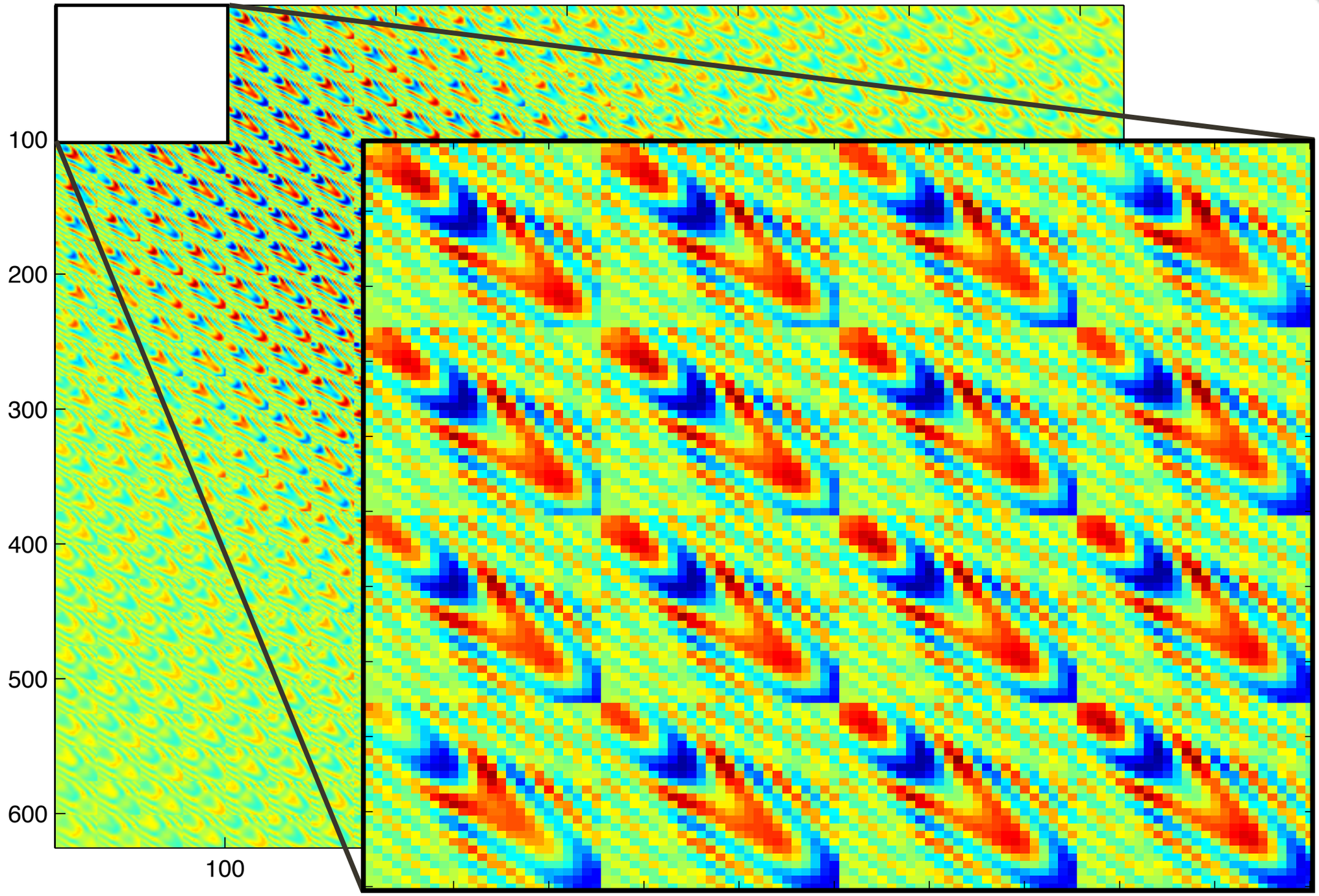
(Src x, Src y) matricization - full data

Matricizations + Sampling



(Src x, Src y) matricization - 50% missing receivers

Matricizations + Sampling



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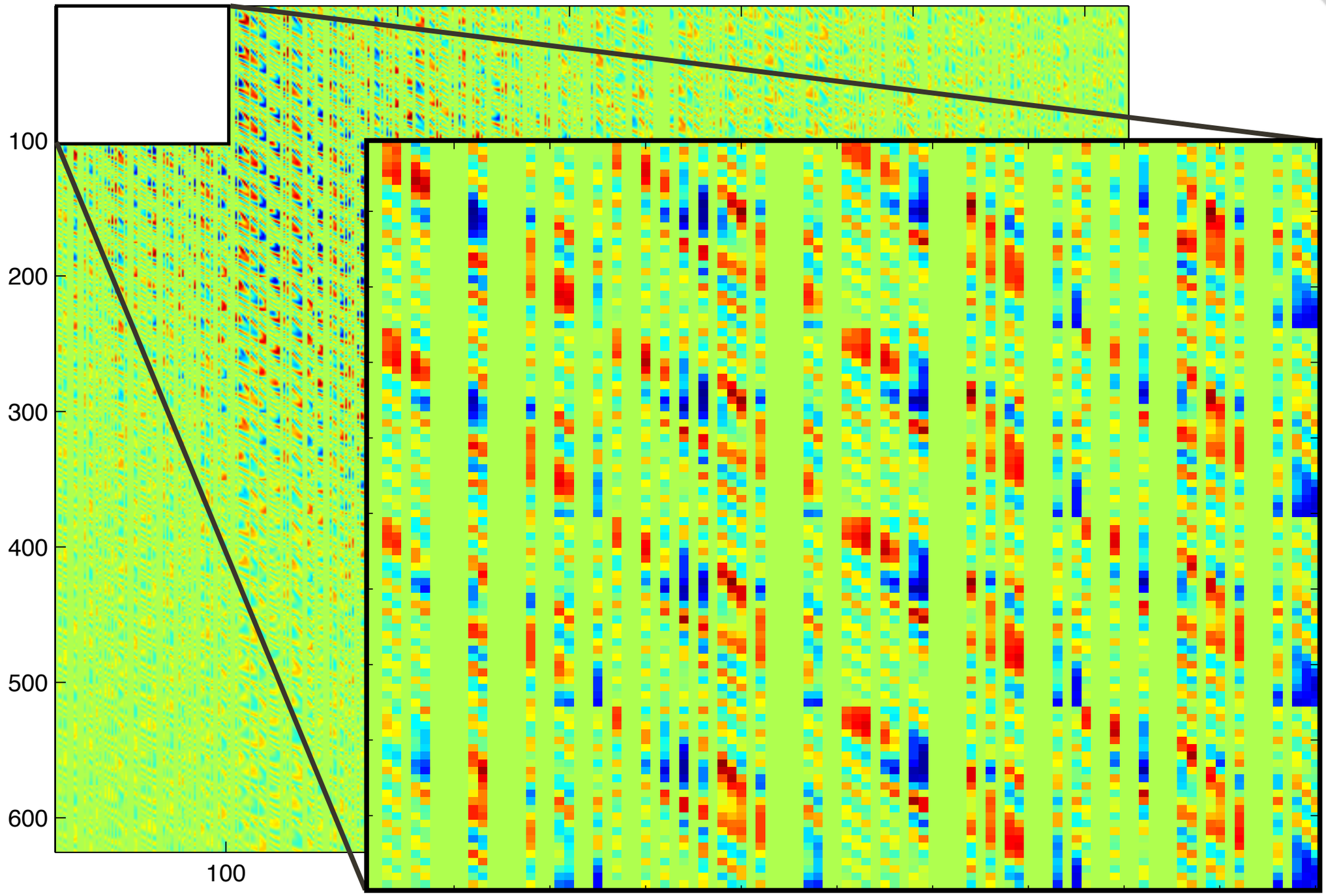

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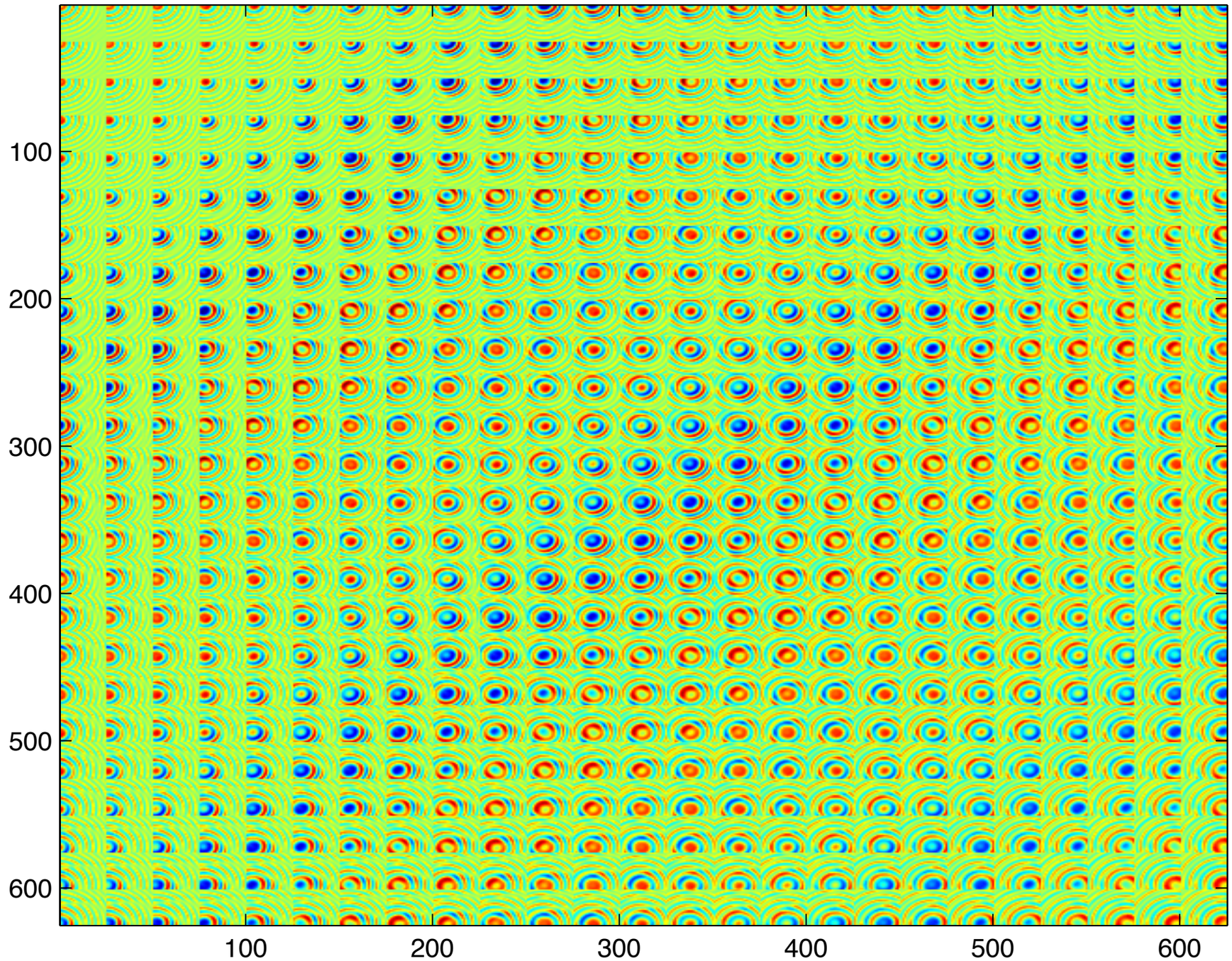

(Src x, Rec x) matricization - full data

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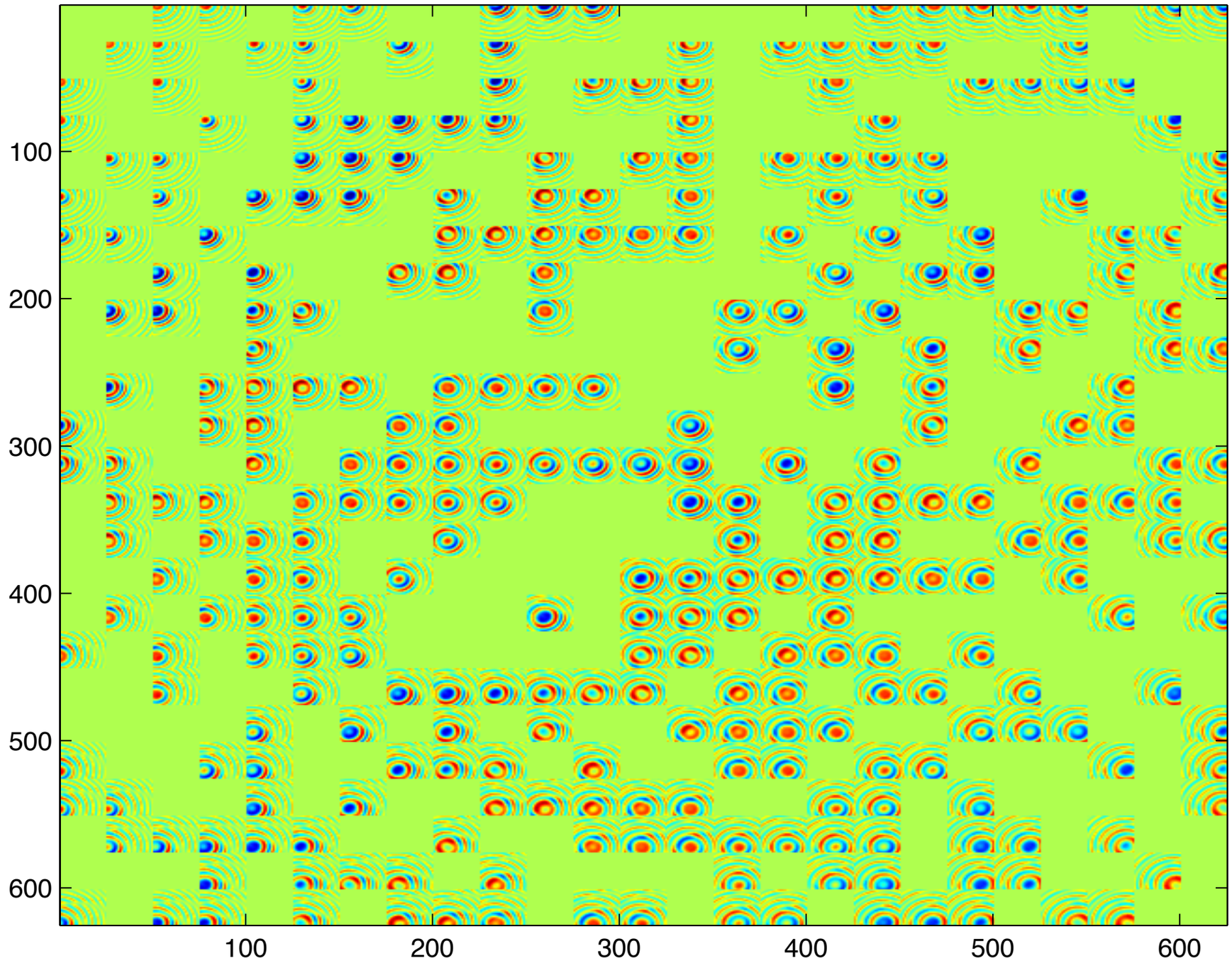

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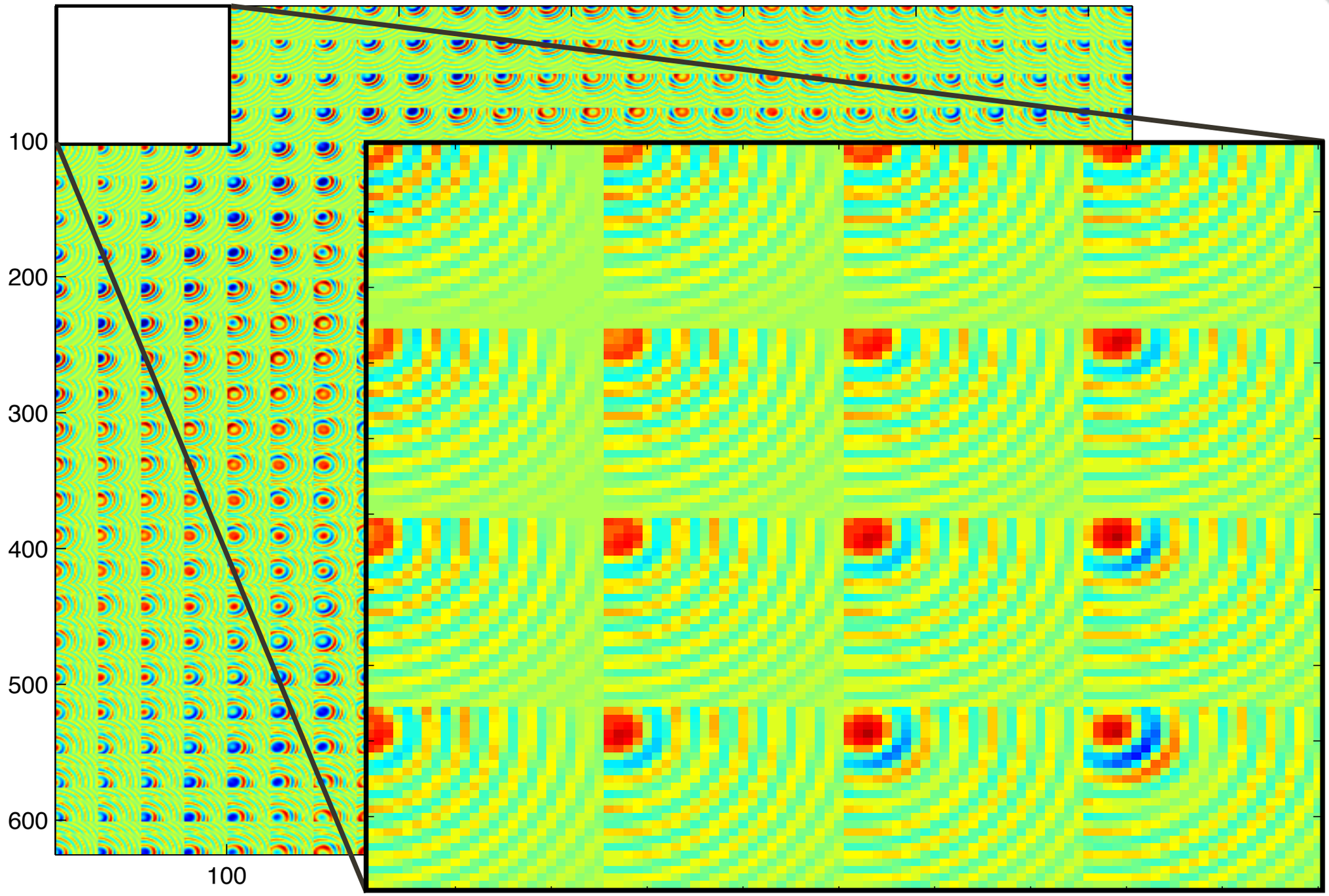

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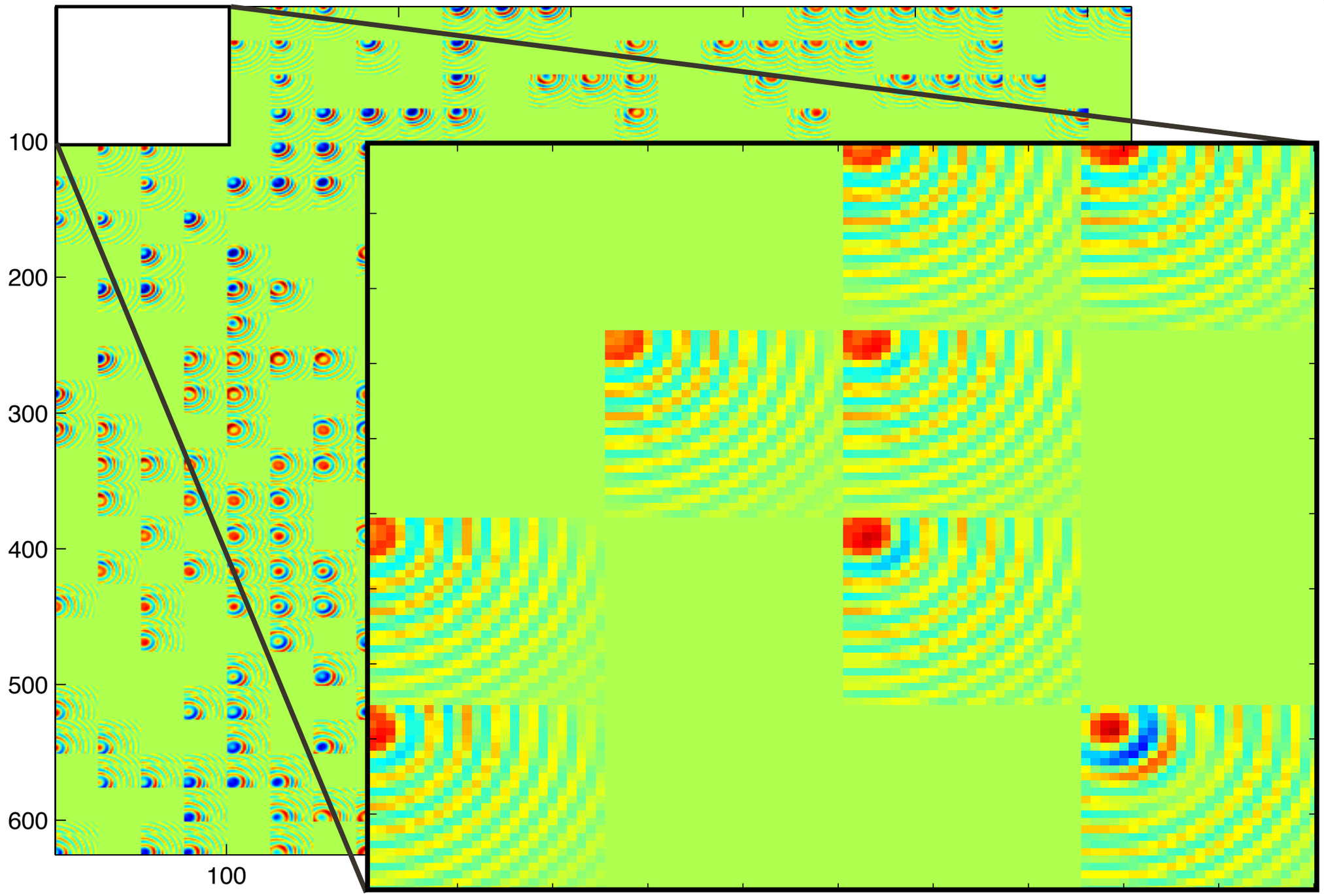

(Src x, Rec x) matricization - 50% missing receivers

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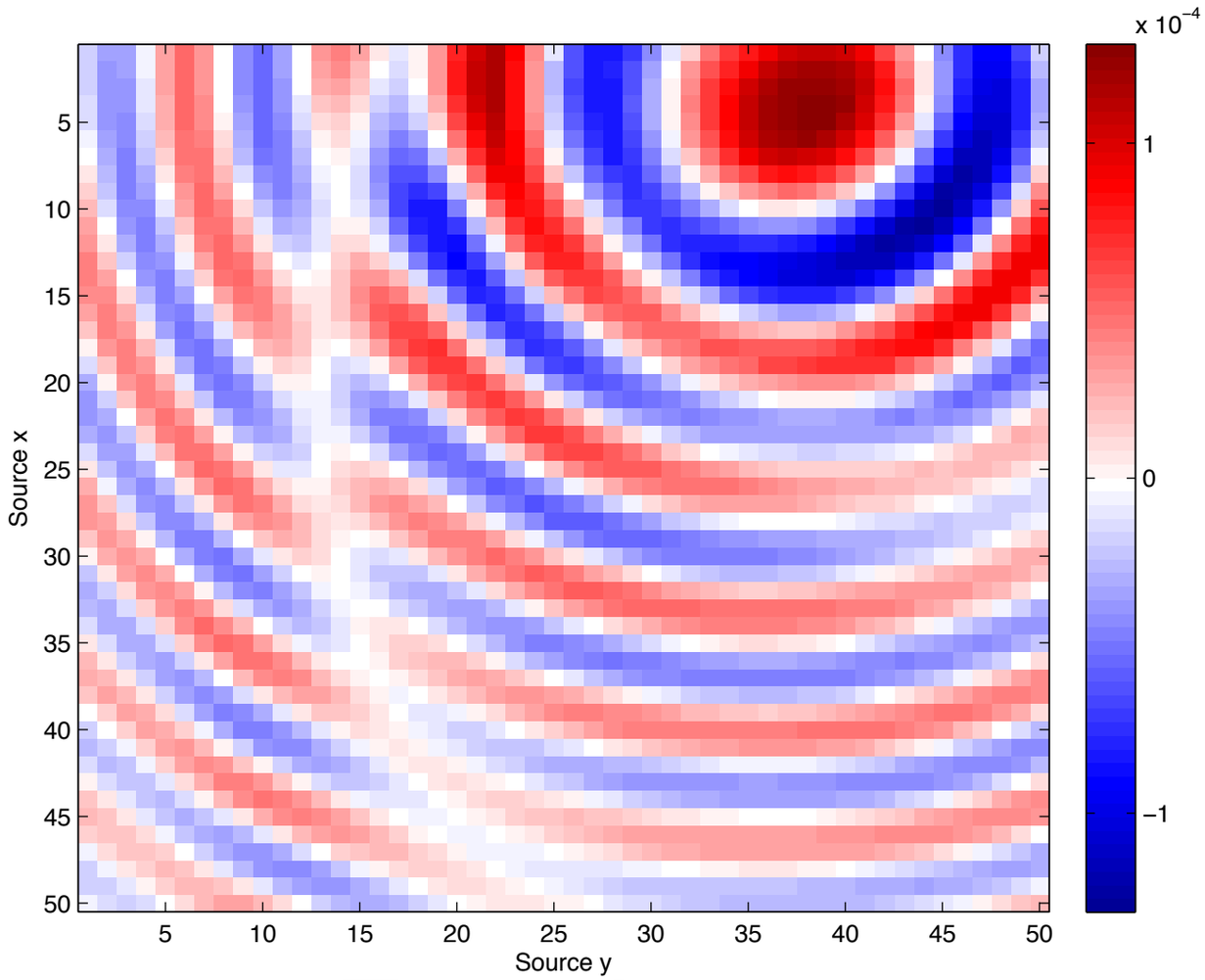
Results

Synthetic 4D Data

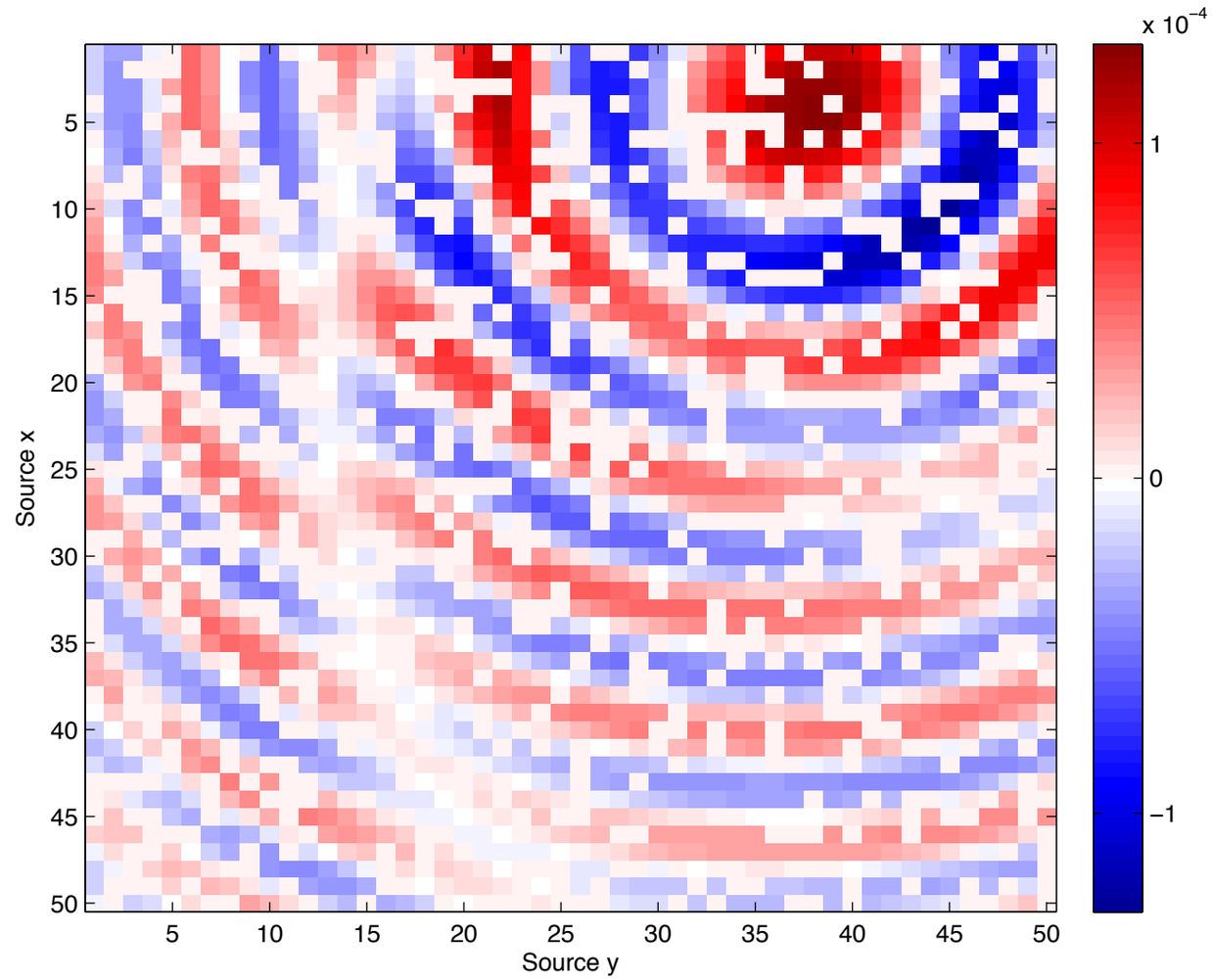
- Single reflector model
- Single frequency slice (real part)
- 50 x 50 sources
- 50 x 50 receivers
- Nonlinear CG in this format - 200 iterations
- Hierarchical rank 20 (internal + individual dimension rank)

25% Missing Source Pairs



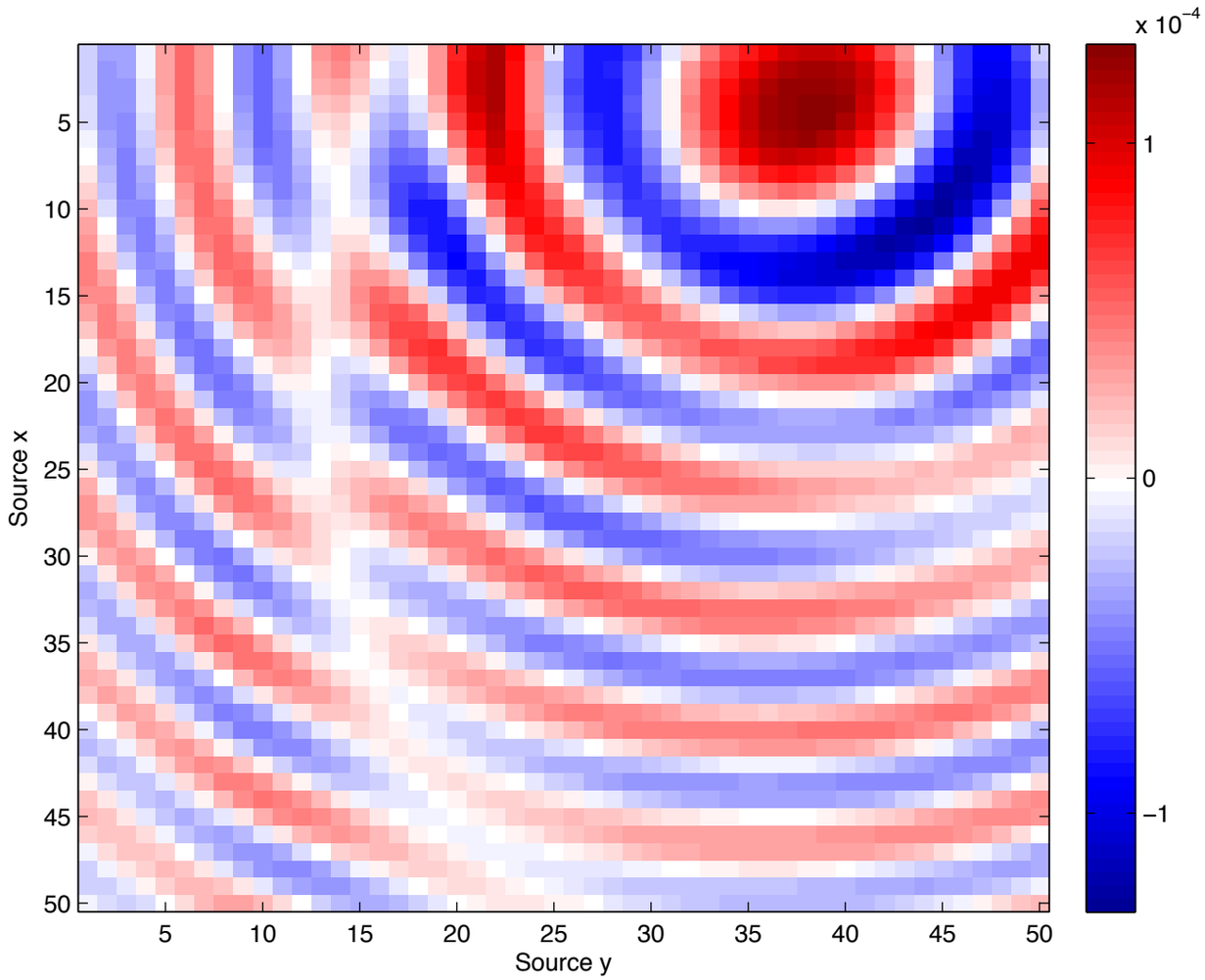
True data

(Rec x, Rec y) = (5,39)



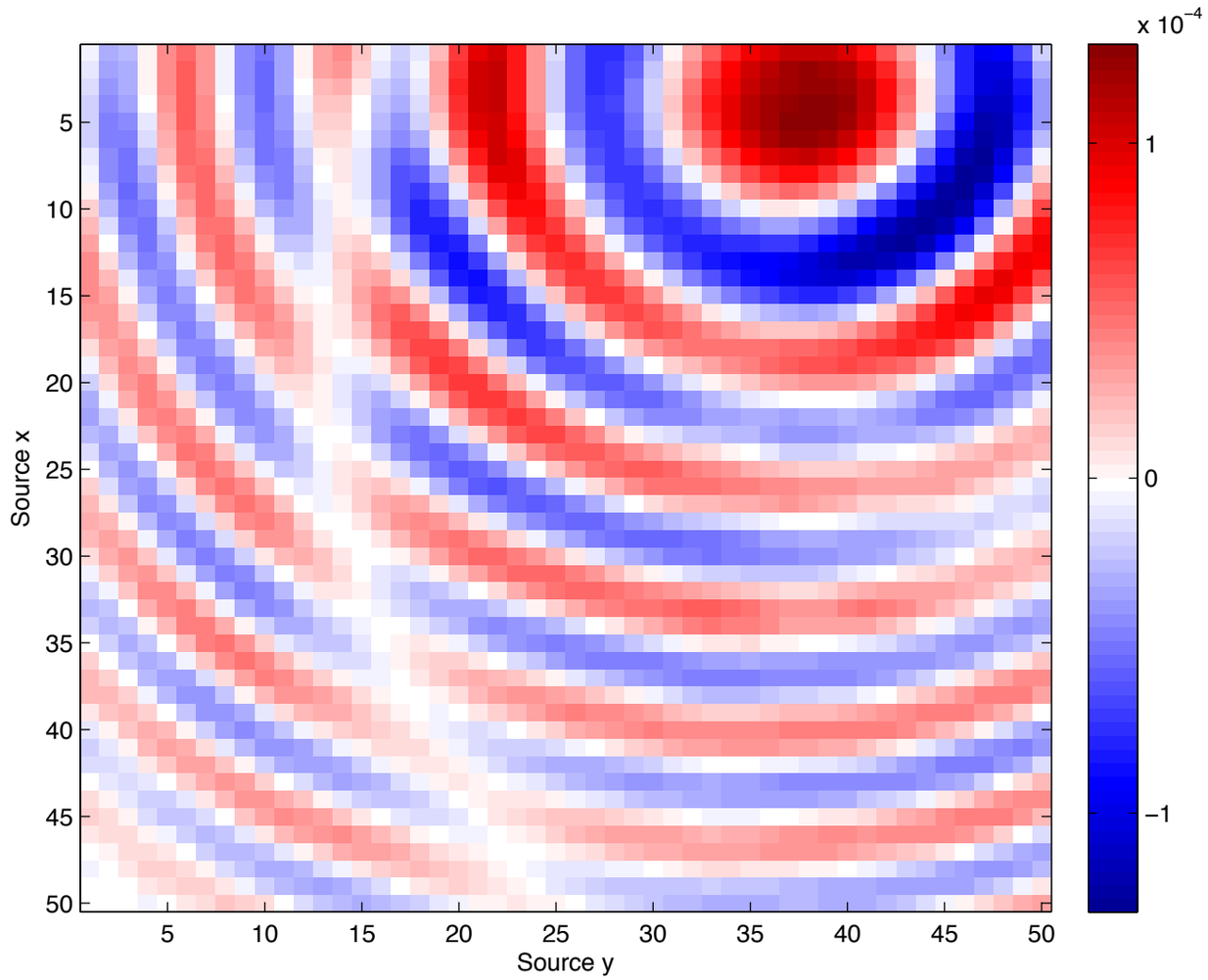
Subsampled data

25% Missing Source Pairs



True data

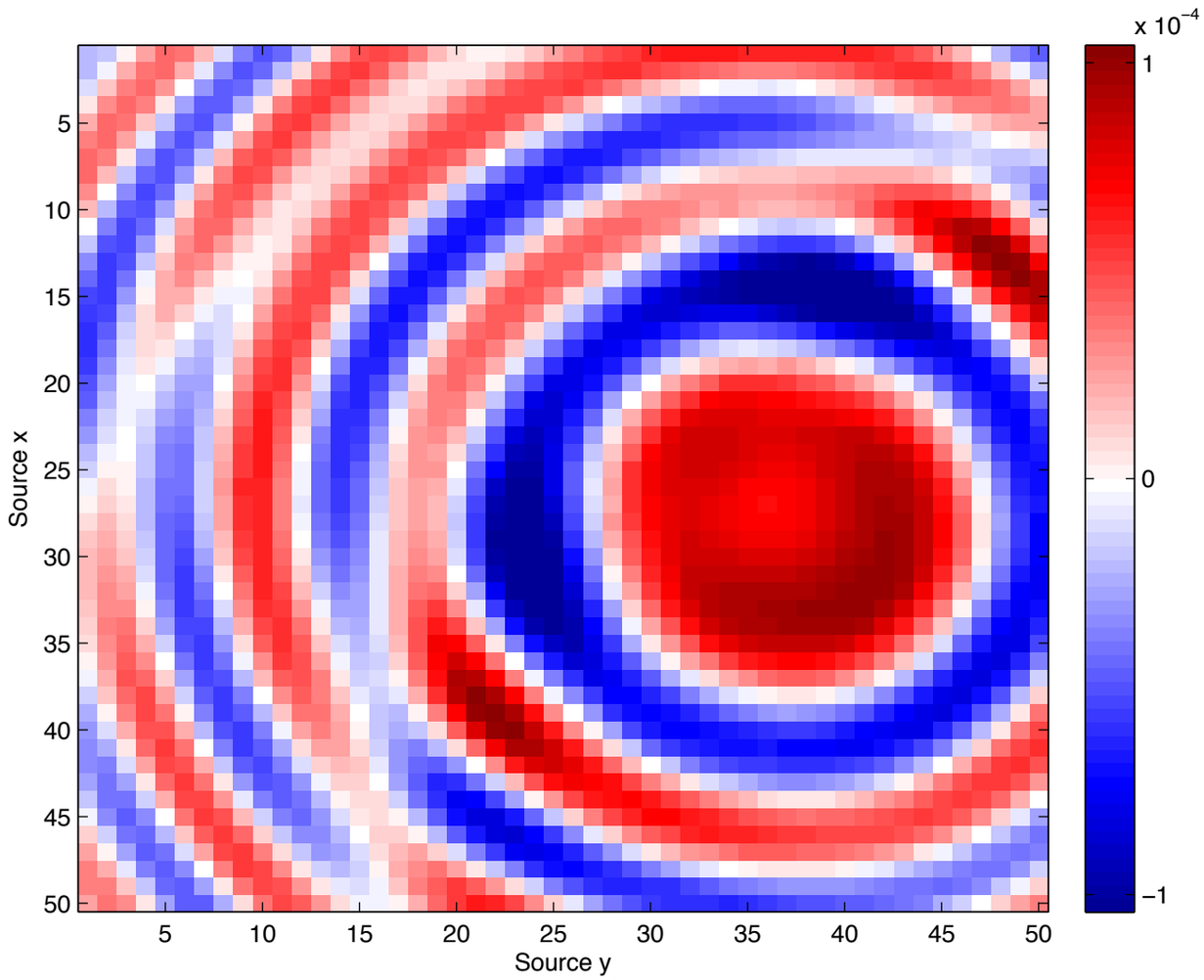
(Rec x, Rec y) = (5,39)



Recovered data

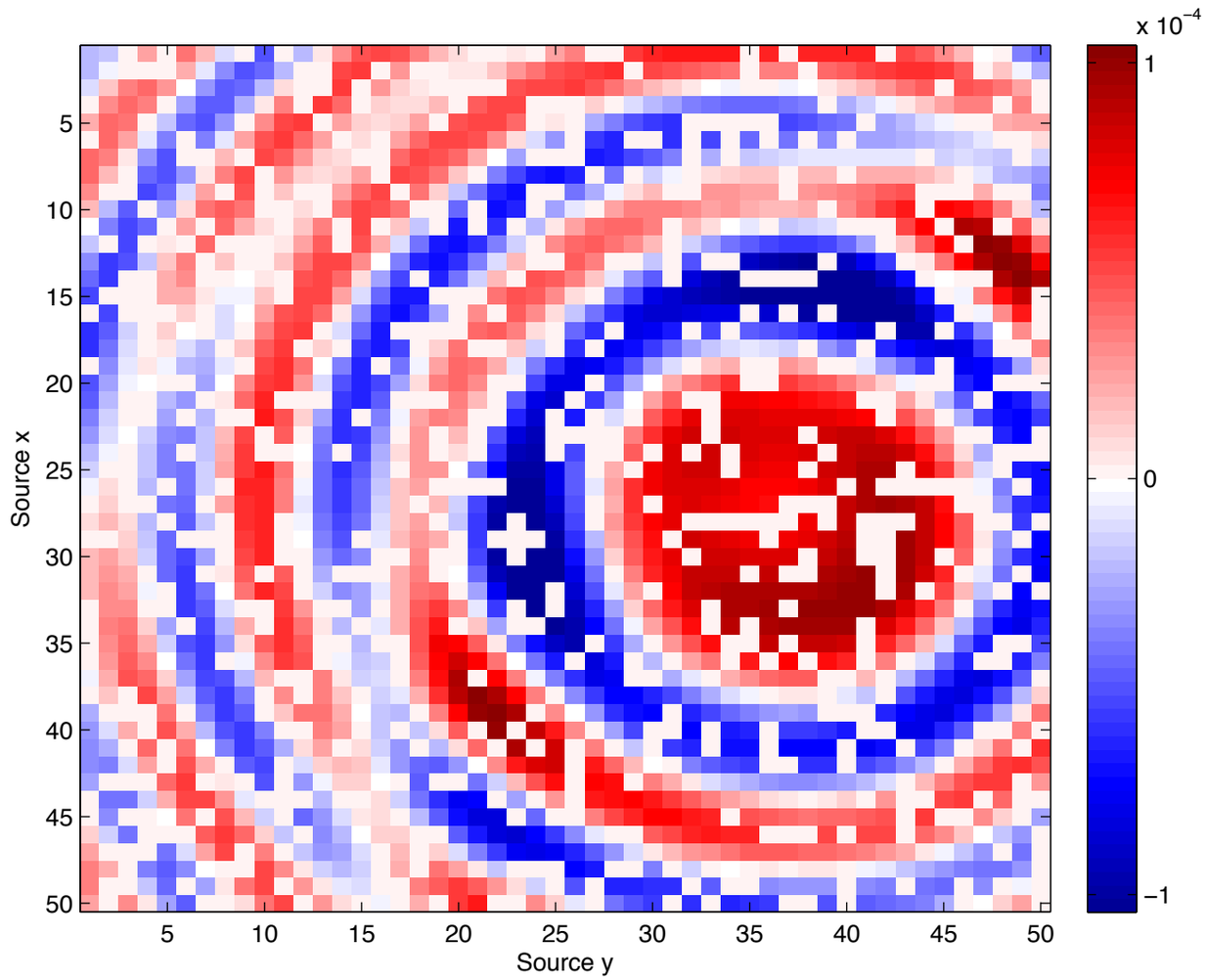
SNR - 16.8 dB

25% Missing Source Pairs



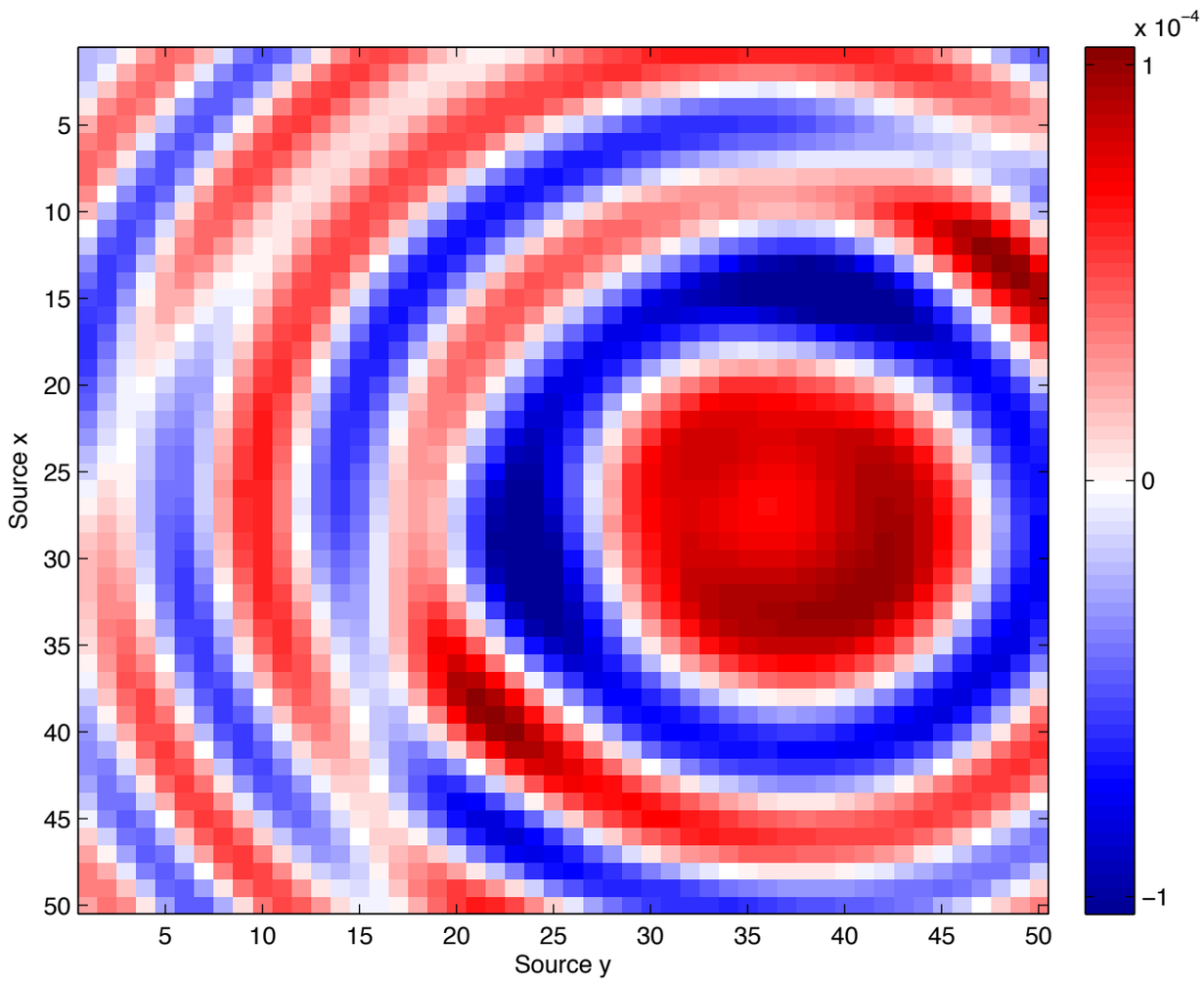
True data

(Rec x, Rec y) = (30,39)



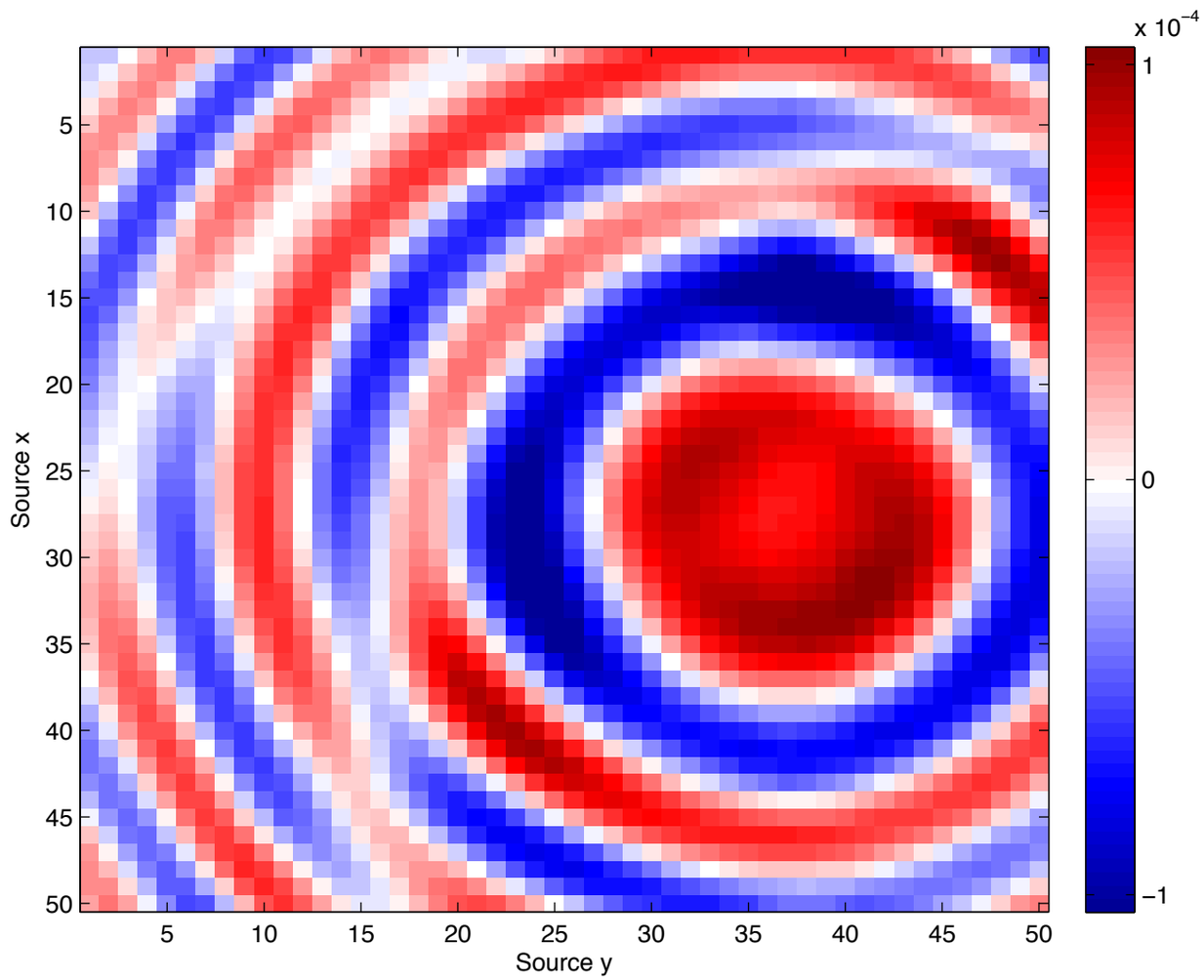
Subsampled data

25% Missing Source Pairs



True data

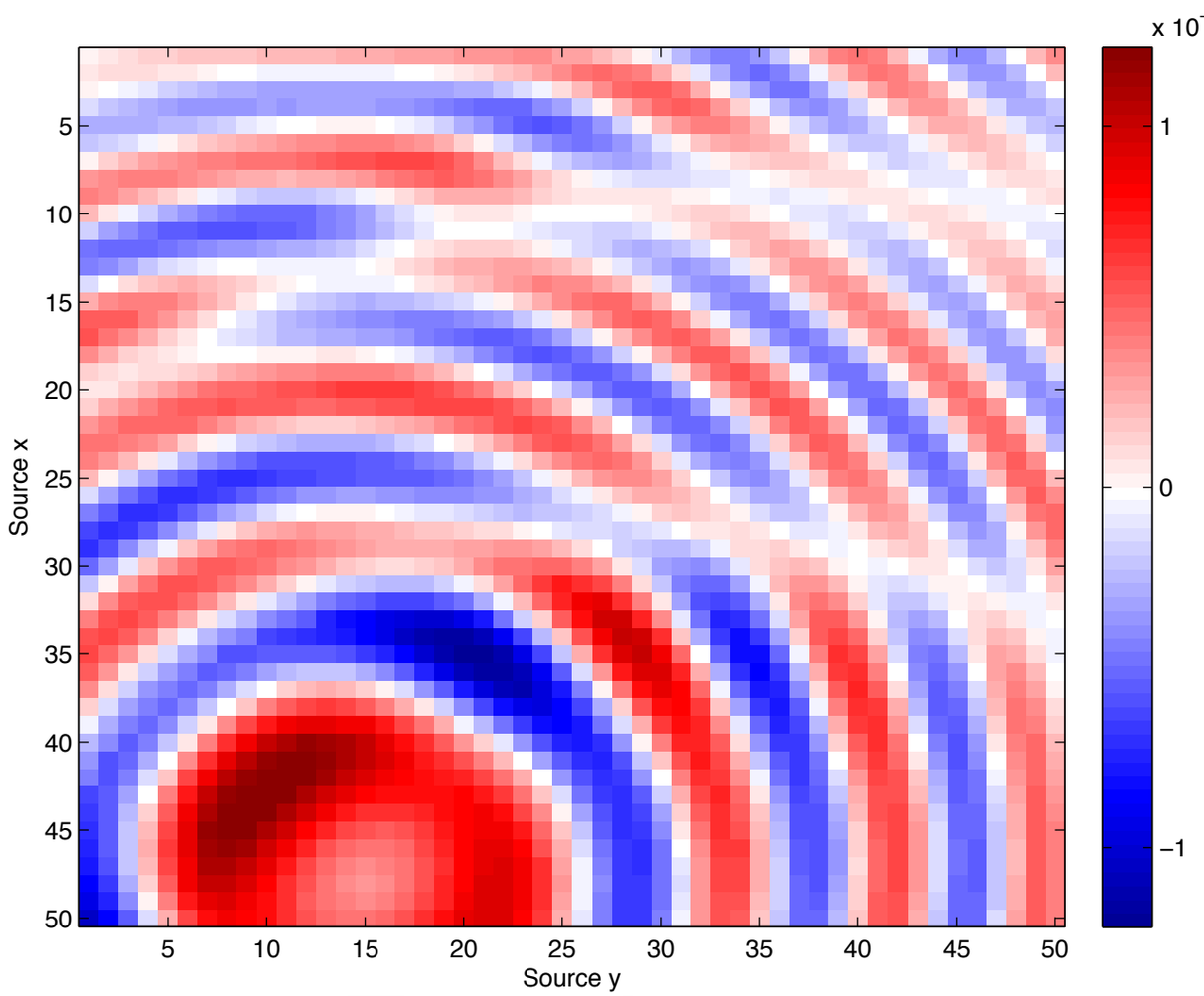
(Rec x, Rec y) = (30,39)



Recovered data

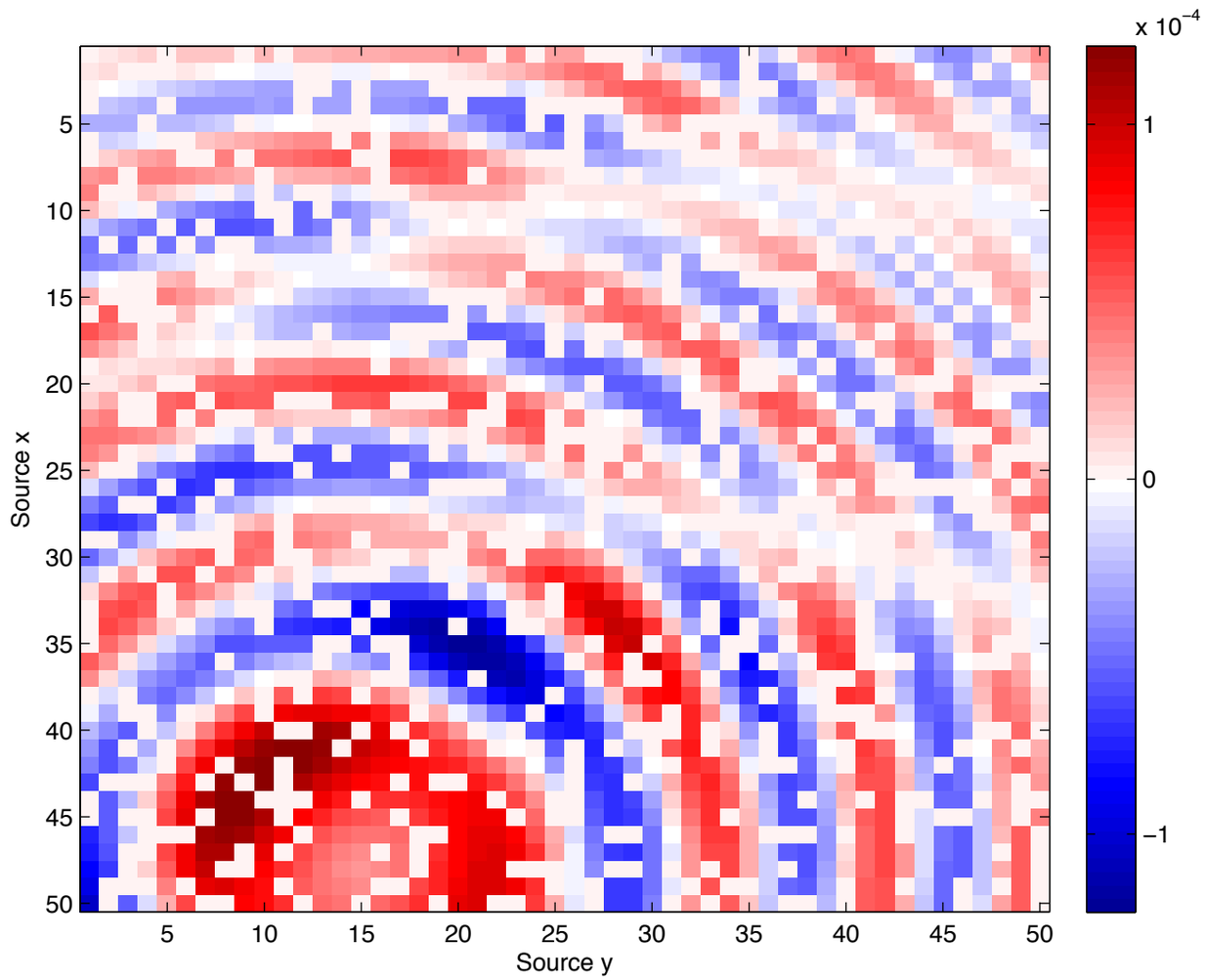
SNR - 18.7 dB

25% Missing Source Pairs



True data

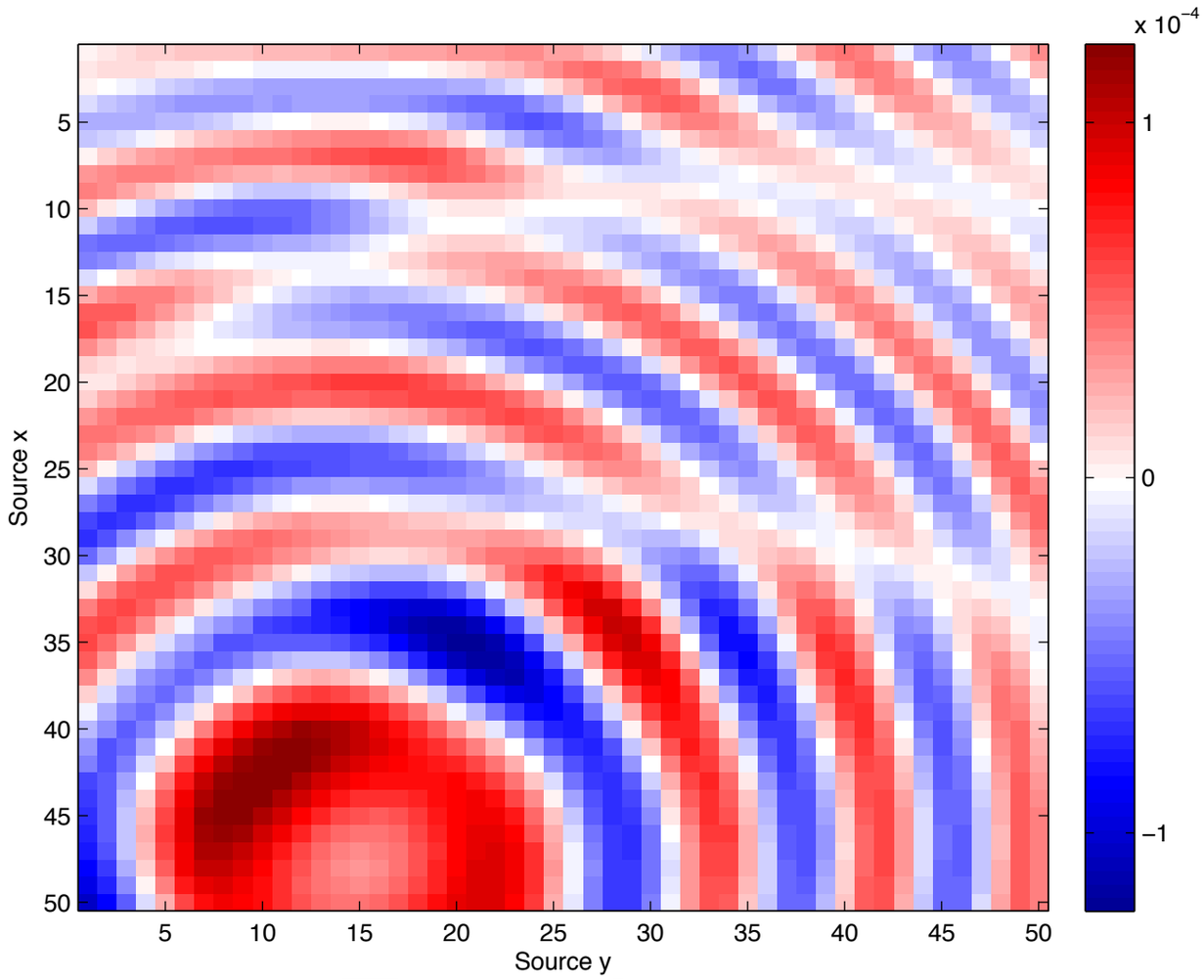
(Rec x, Rec y) = (50,17)



Subsampled data

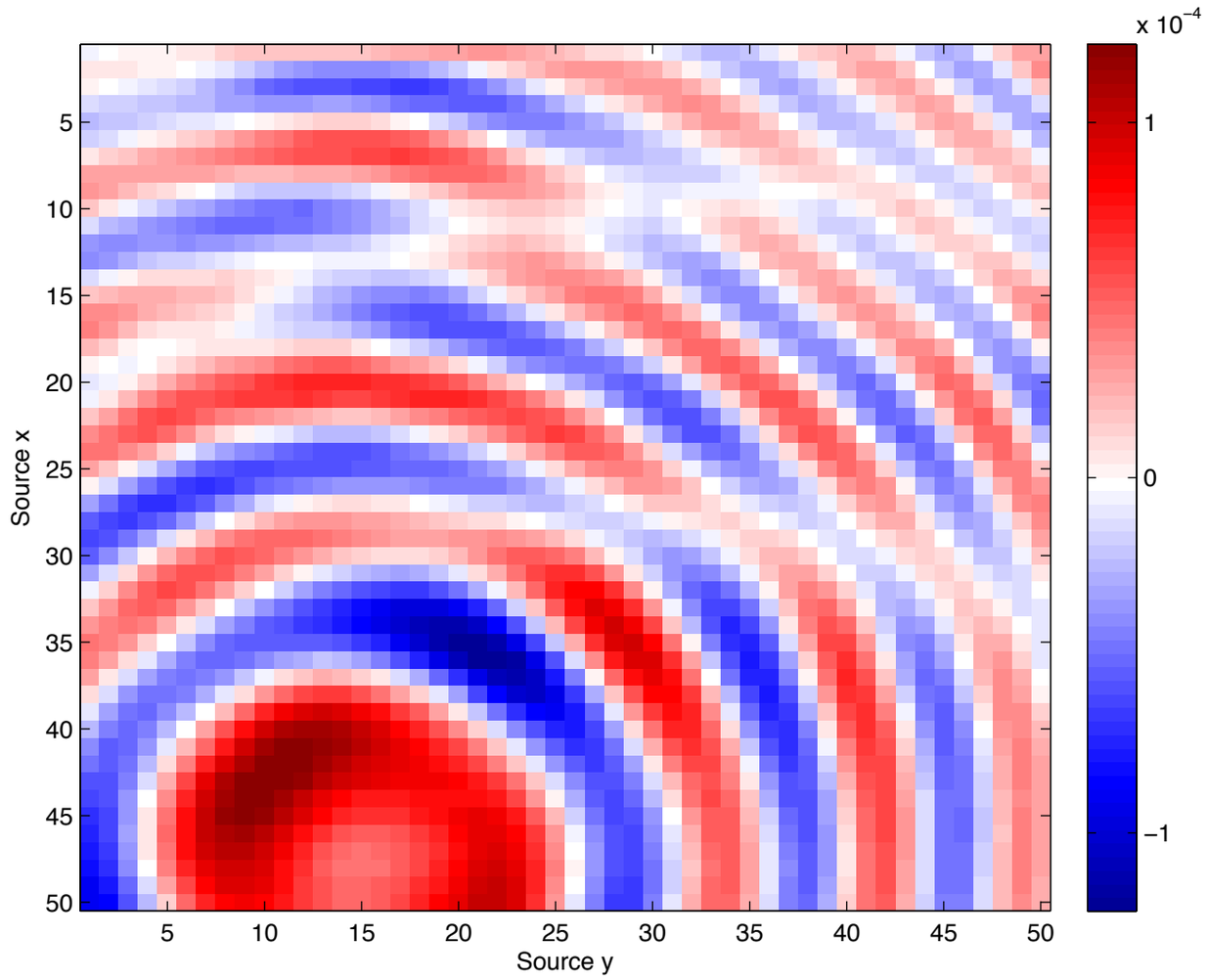
SLIM

25% Missing Source Pairs



True data

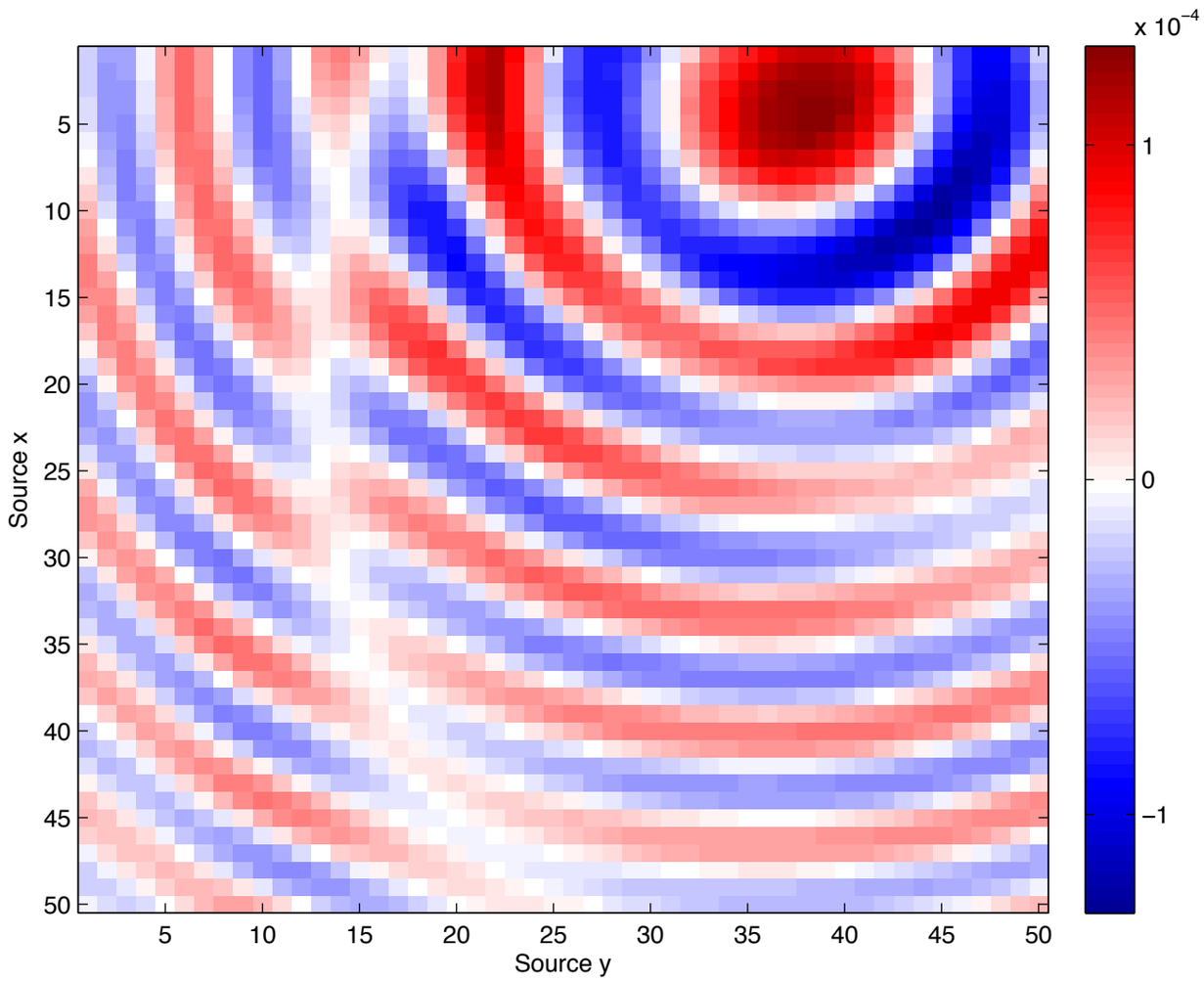
(Rec x, Rec y) = (50,17)



Recovered data

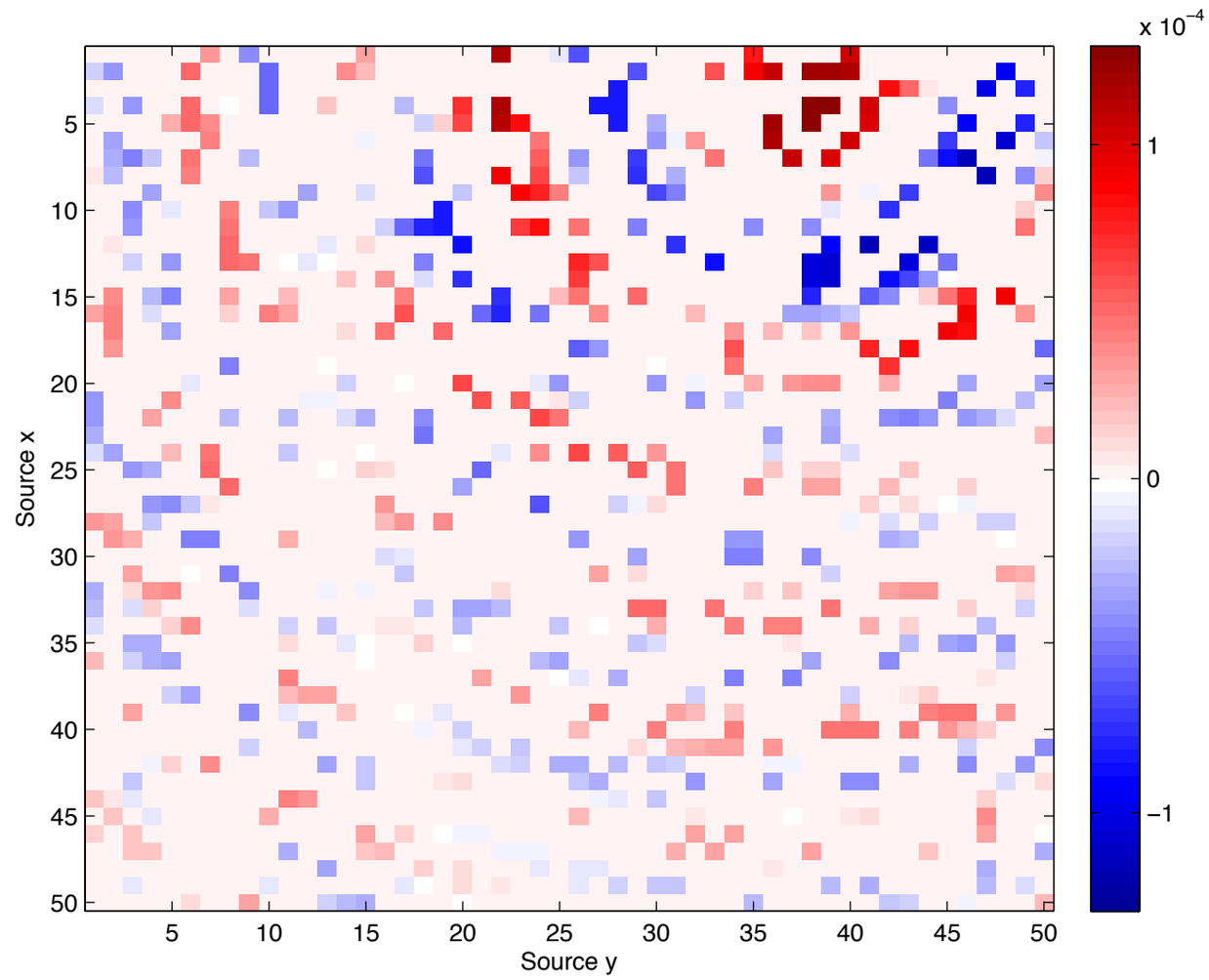
SNR - 14.8 dB

75% Missing Source Pairs



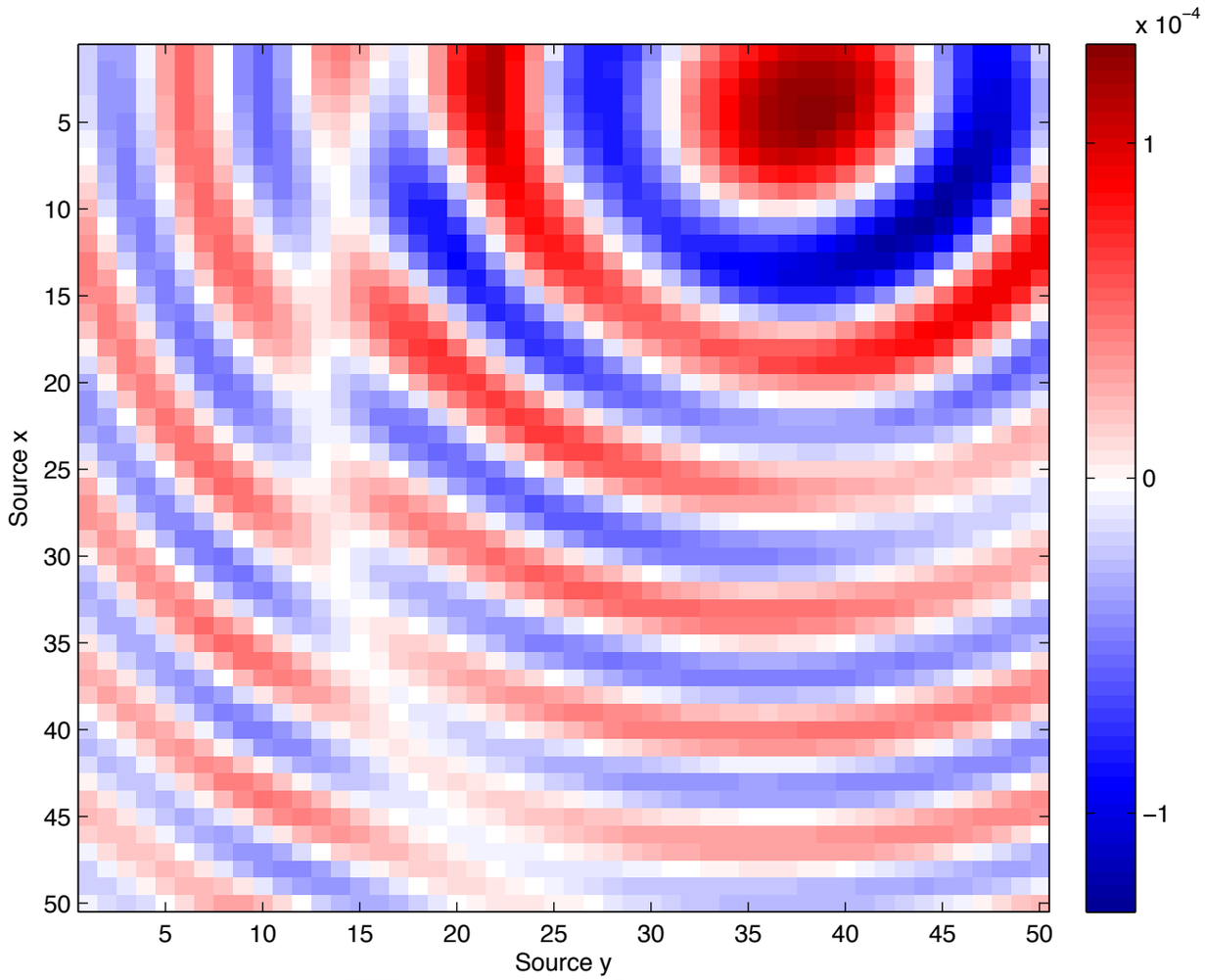
True data

(Rec x, Rec y) = (5,39)



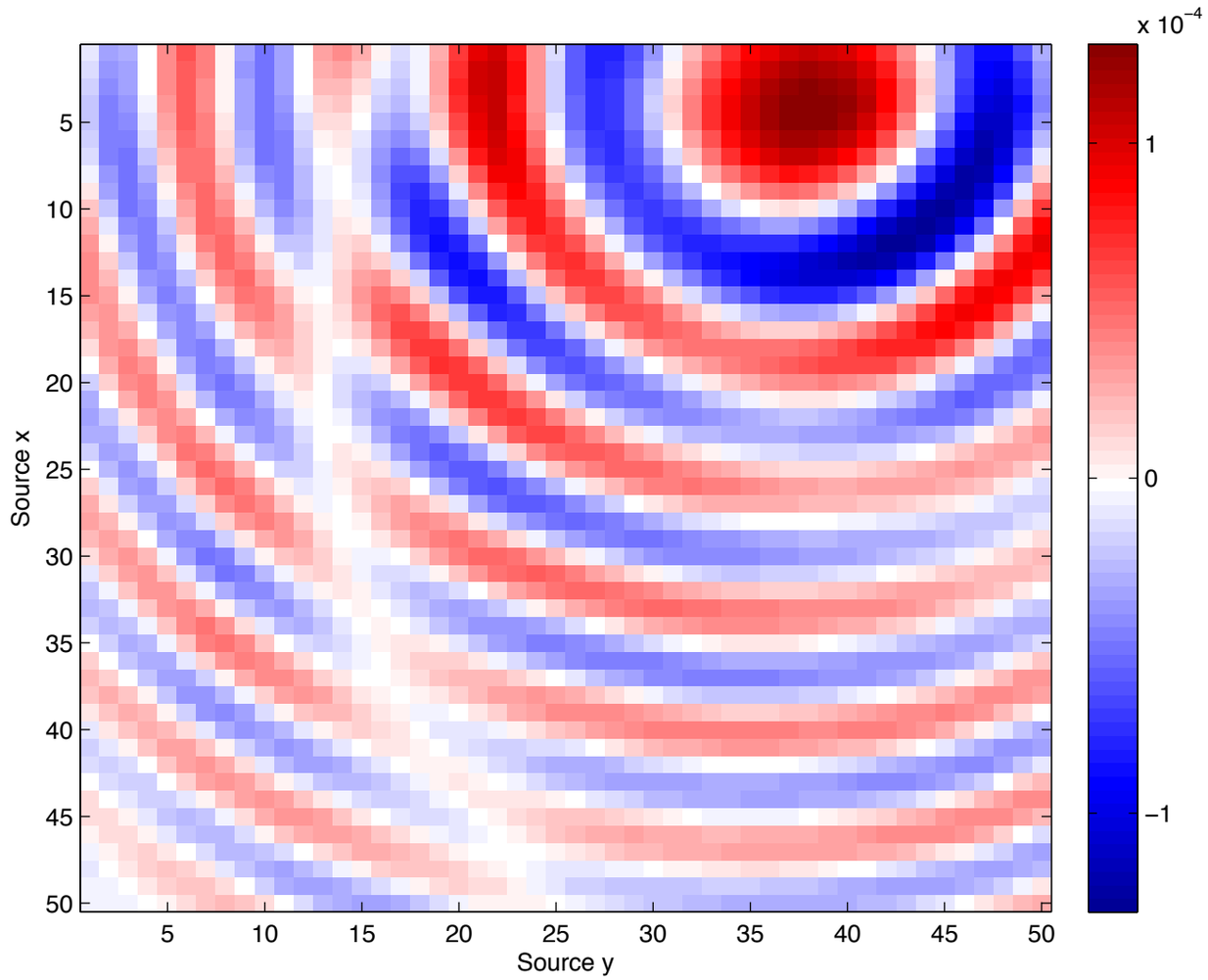
Subsampled data

75% Missing Source Pairs



True data

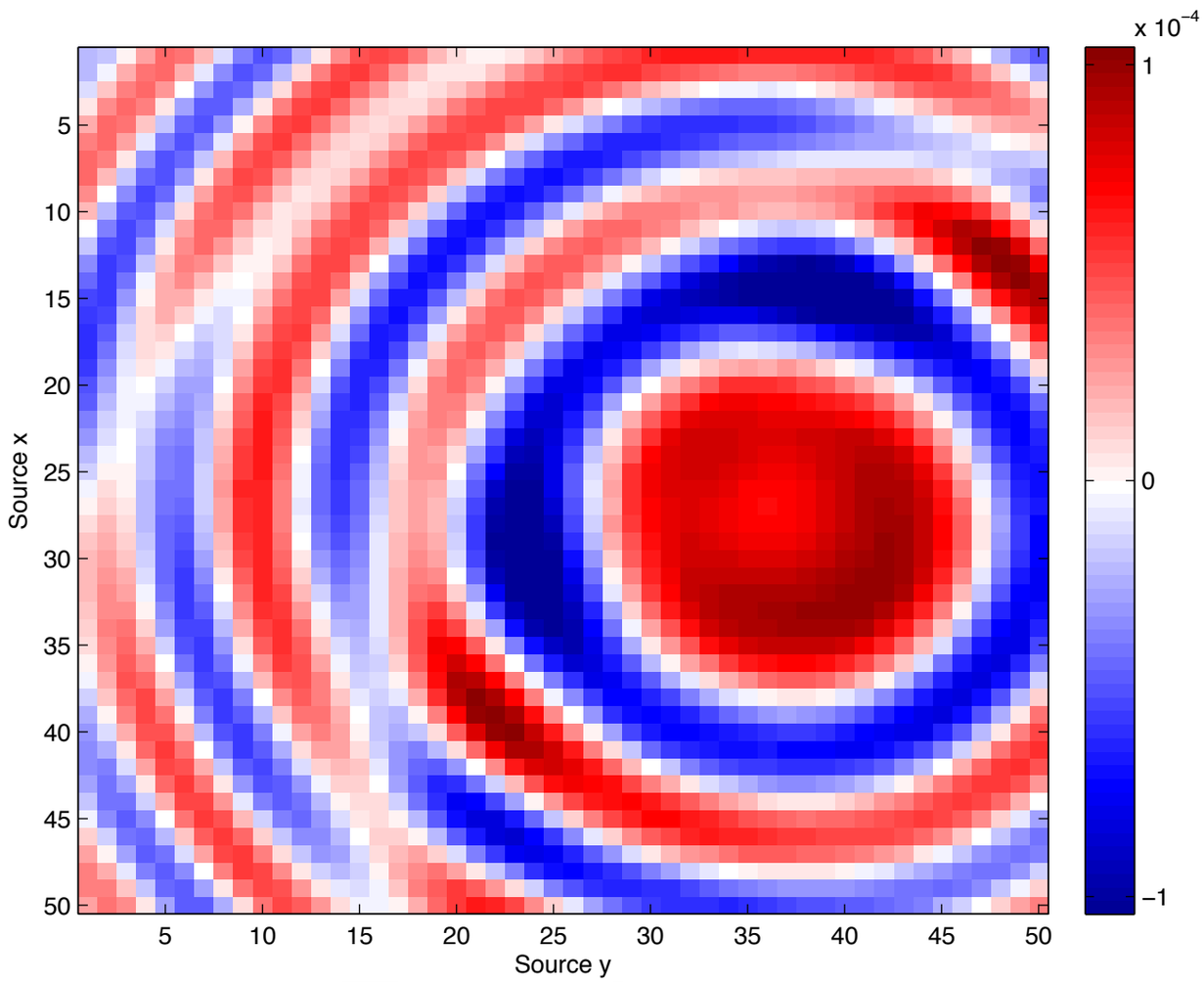
(Rec x, Rec y) = (5,39)



Recovered data

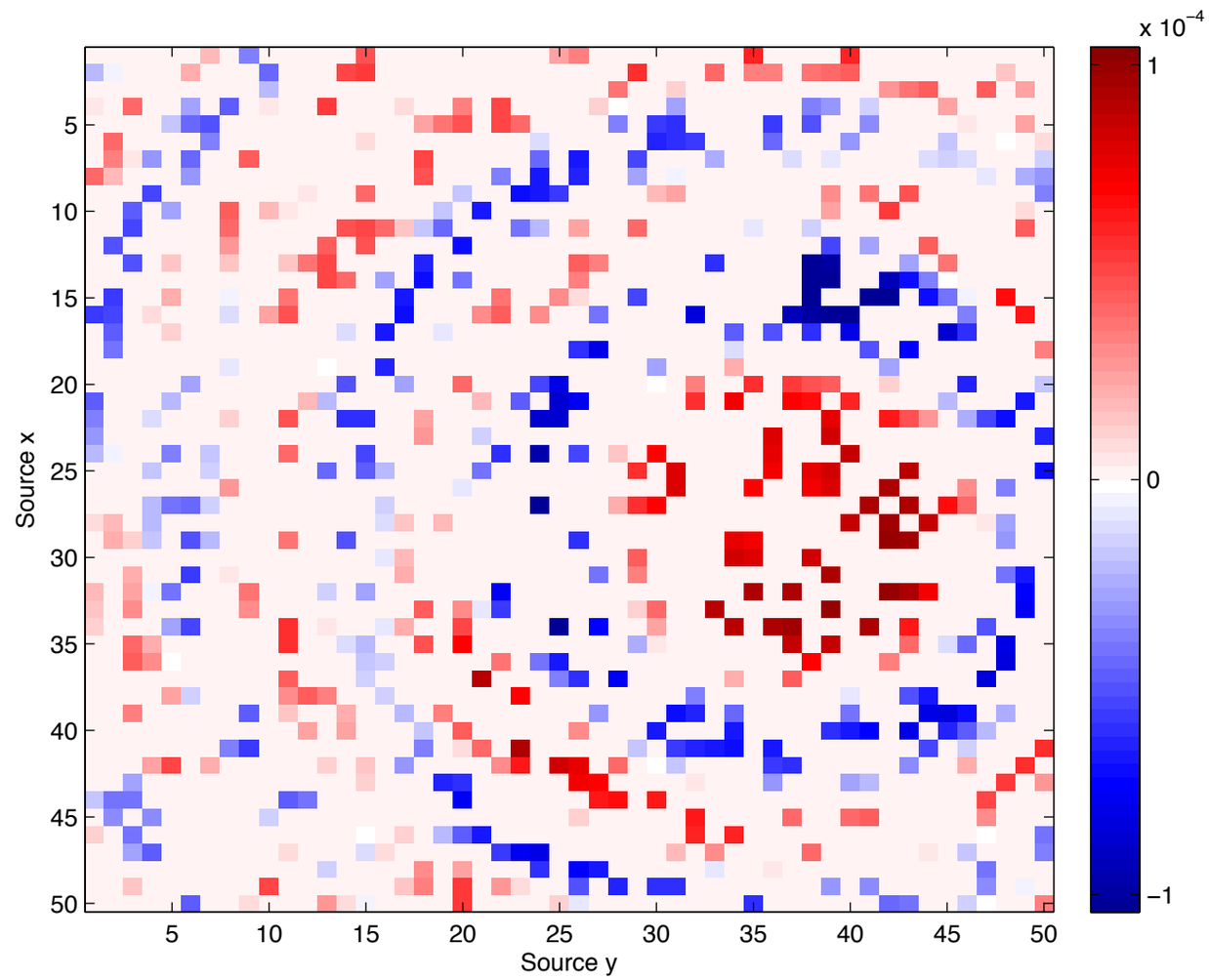
SNR - 18.5dB

75% Missing Source Pairs



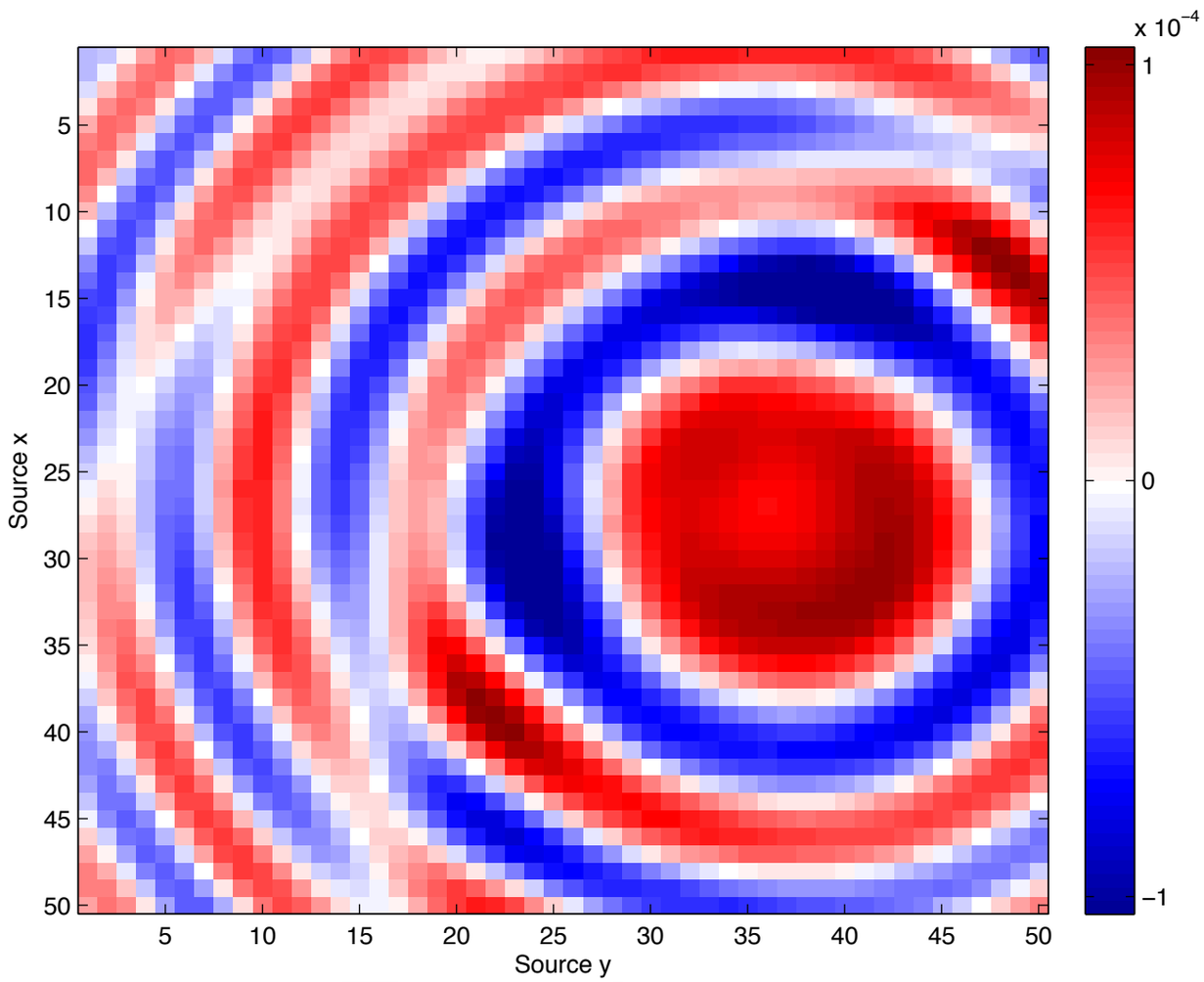
True data

(Rec x, Rec y) = (30,39)



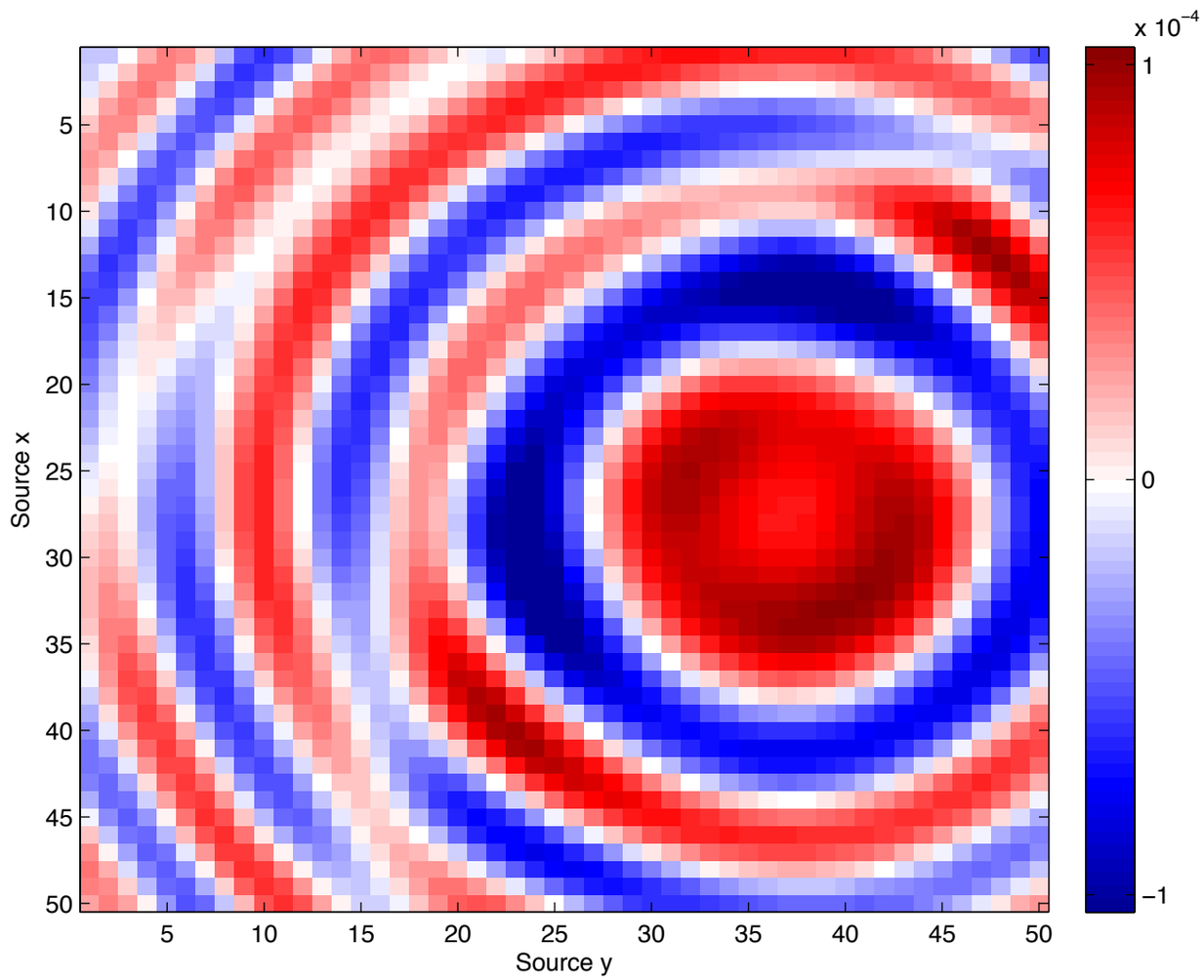
Subsampled data

75% Missing Source Pairs



True data

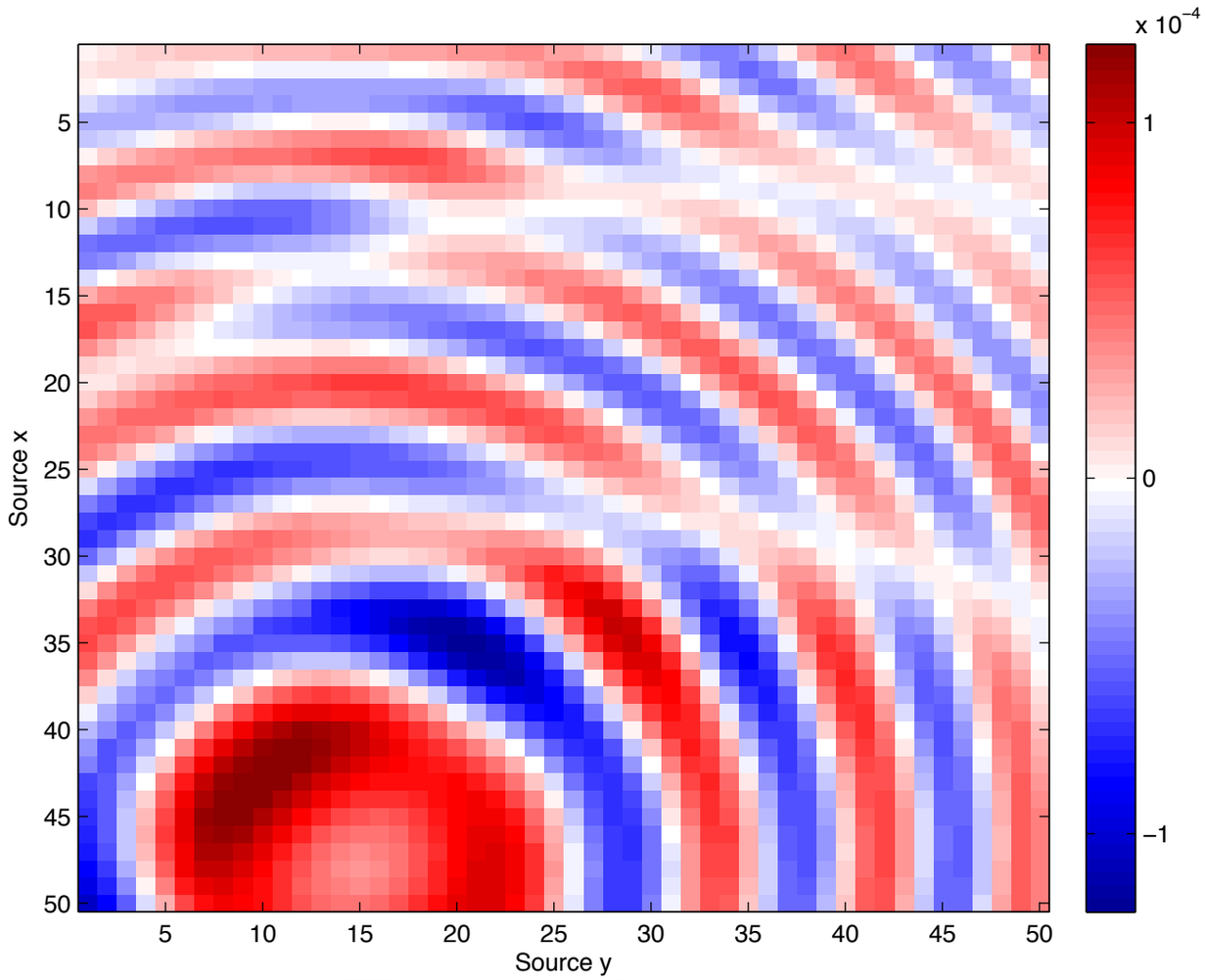
(Rec x, Rec y) = (30,39)



Recovered data

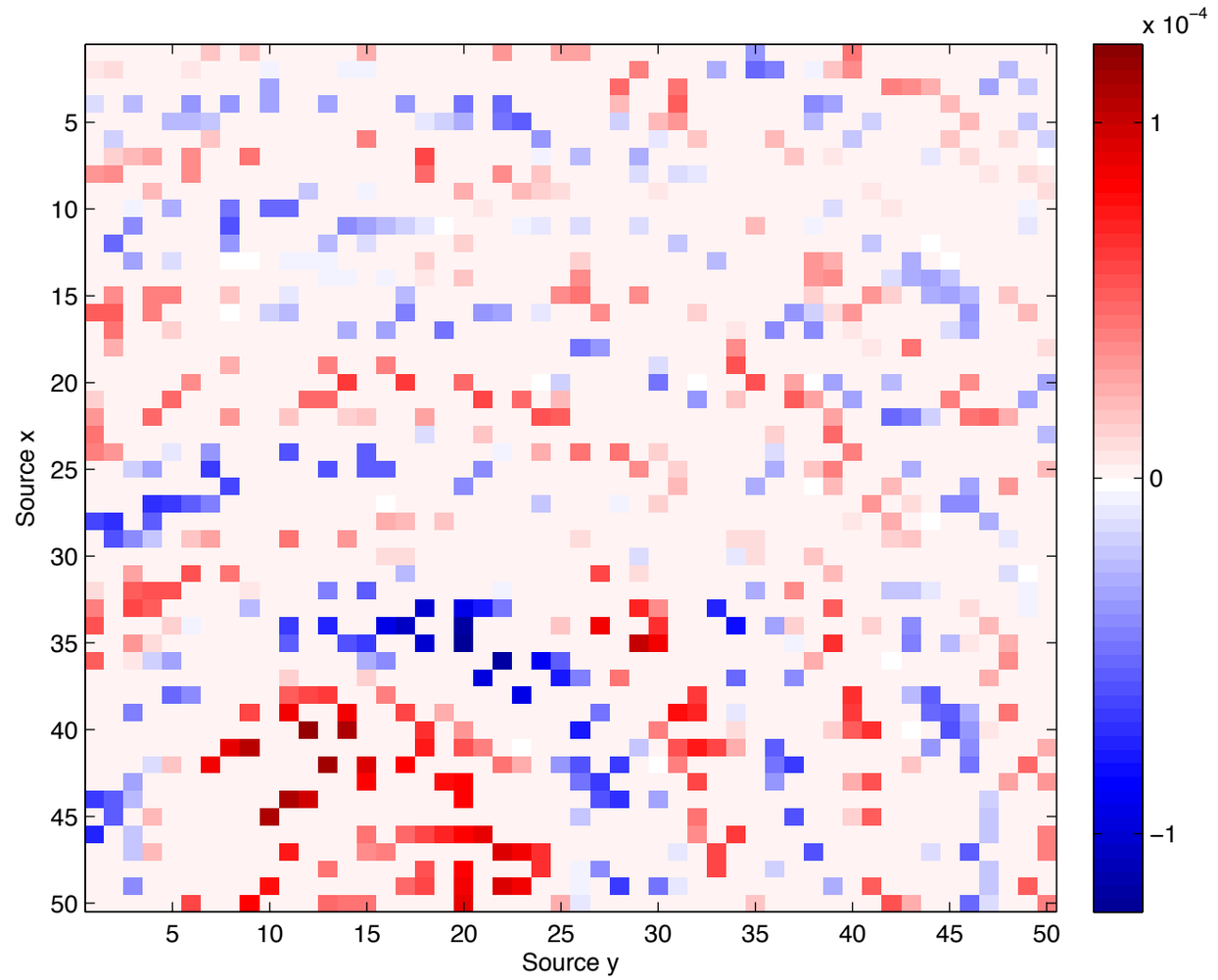
SNR - 18.7 dB

75% Missing Source Pairs



True data

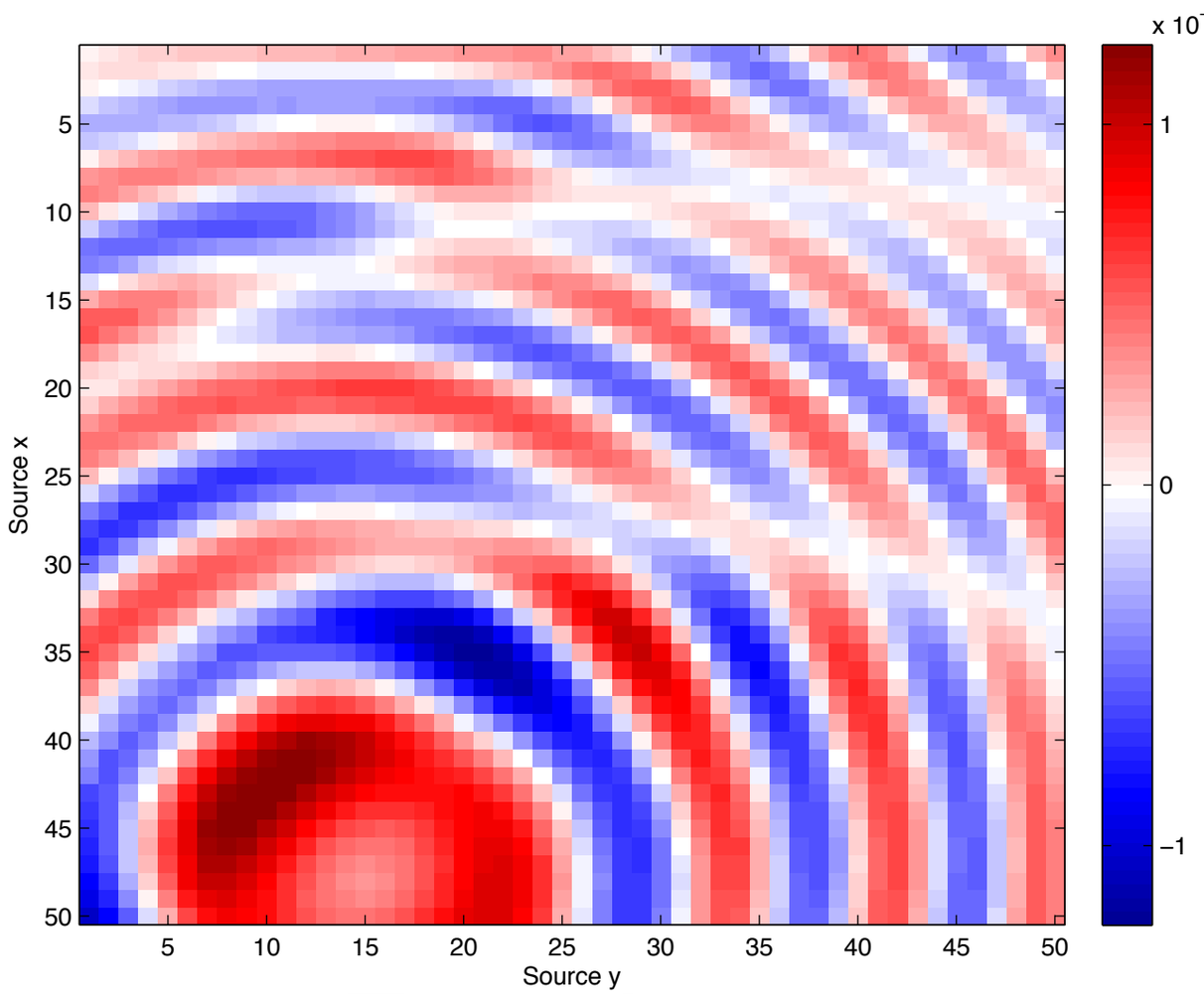
(Rec x, Rec y) = (50,17)



Subsampled data

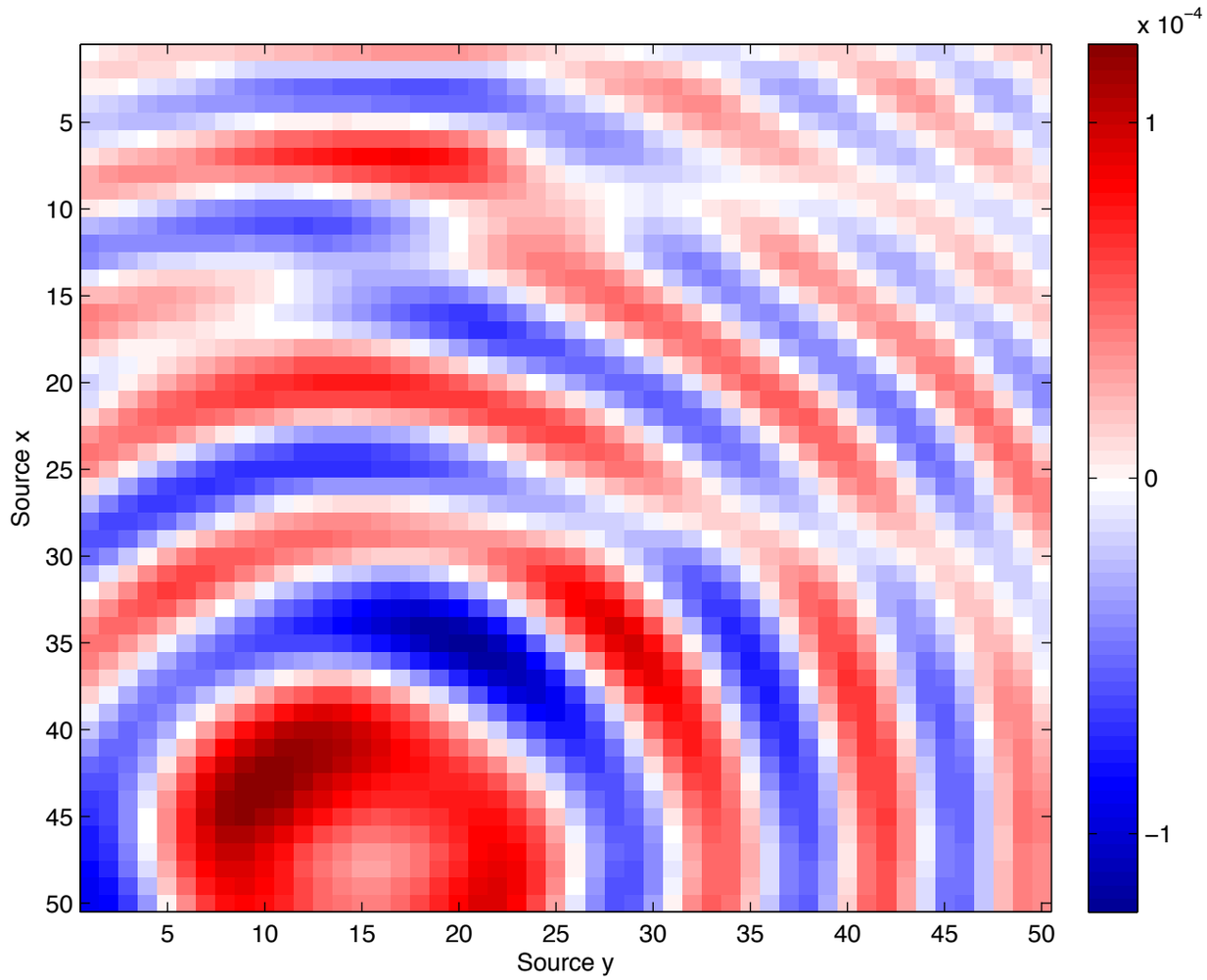
SLIM

75% Missing Source Pairs



True data

(Rec x, Rec y) = (50,17)



Recovered data

SNR - 13 dB

Synthetic BG Data

- Unknown model, unknown full data
- 200 shots (subsampled srcs) x 401 x 401 receivers data at 7.34 Hz
- Receivers subsampled to 101 x 101, fourier interpolated back to 401 x 401 to produce figures (due to low frequency content of slice)

Synthetic BG Data

- Single frequency slice (real part) - normalized
- 80 internal rank, 80 receiver rank, various source ranks

Synthetic BG Data

- minConf_PQN - optimization code by M. Schmidt, M. Friedlander, and K. Murphy (2009)
- L-BFGS code for optimization problems with costly objectives, cheap constraint projection
  - Exactly our regime

Synthetic BG Data

- Interpolating from 200 irregularly sampled shots -> 100 x 100 grid (10000 shots) - 98% missing data
- No free lunch here, illustrates the effects of overly aggressive upsampling
- Better results on a less-greedy grid
  - Can’t coarsen grid too much

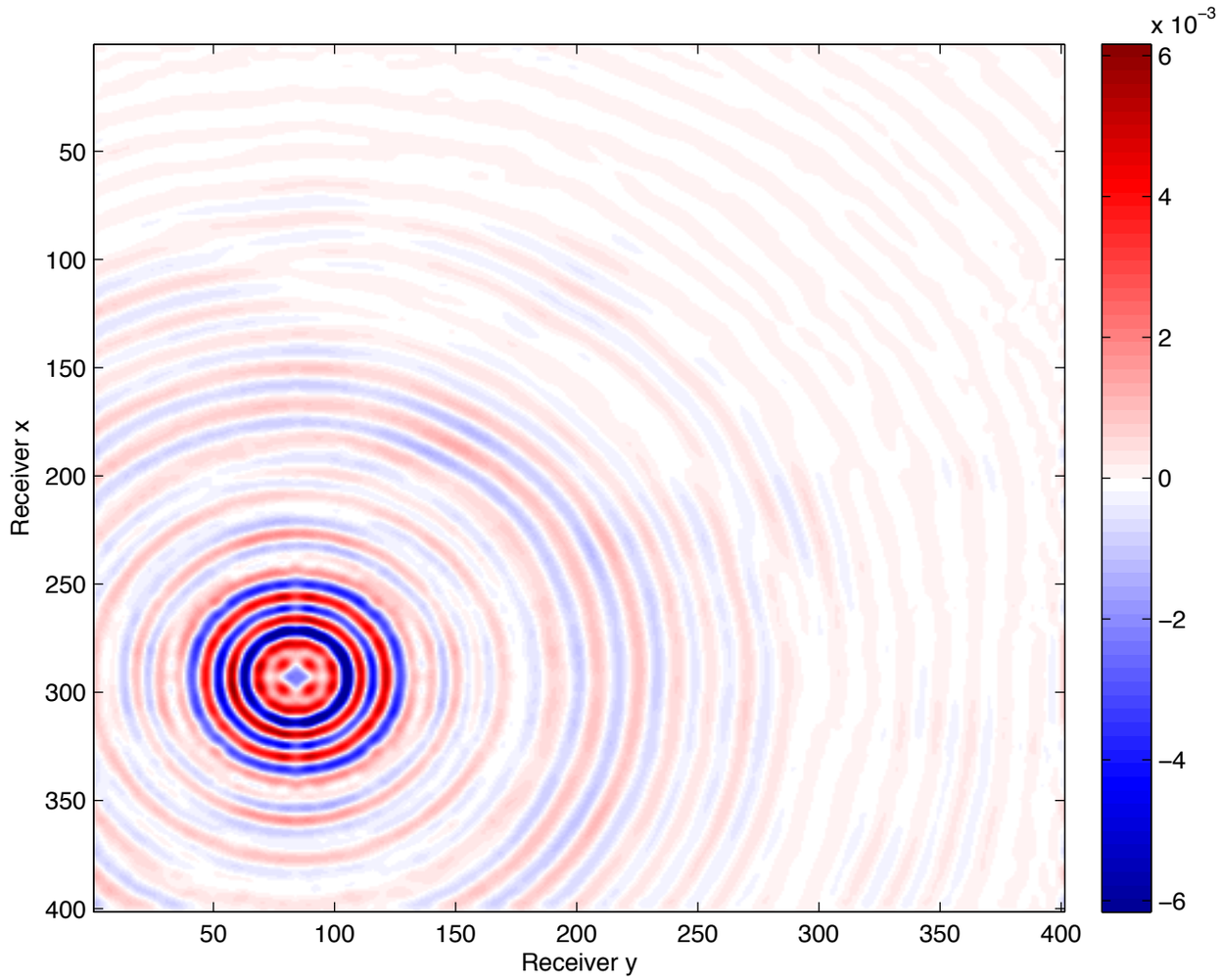
Synthetic BG Data

- Source rank 20
- Locations of poor results aren't structured
  - Doesn't really matter if we weren't going to obtain anything at those points anyways

Synthetic BG Data

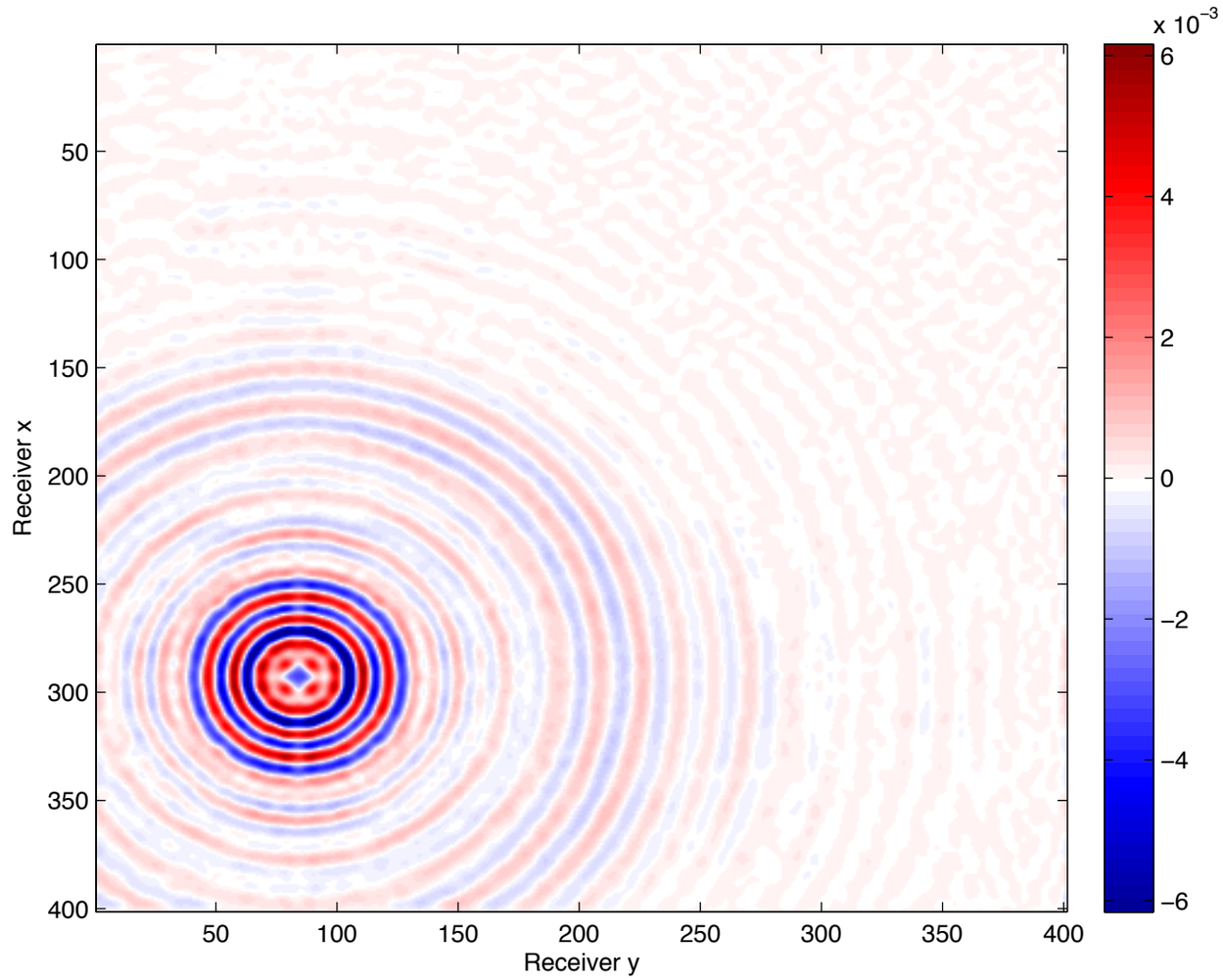
- Source rank 20 - good tradeoff between interpolation quality + SNR
- Number of parameters: 282560
- Compare to 102010000 for the full 100 x 100 x 101 x 101 cube
- Compression of a factor of 99.7%

Source rank 20 - 500 iterations



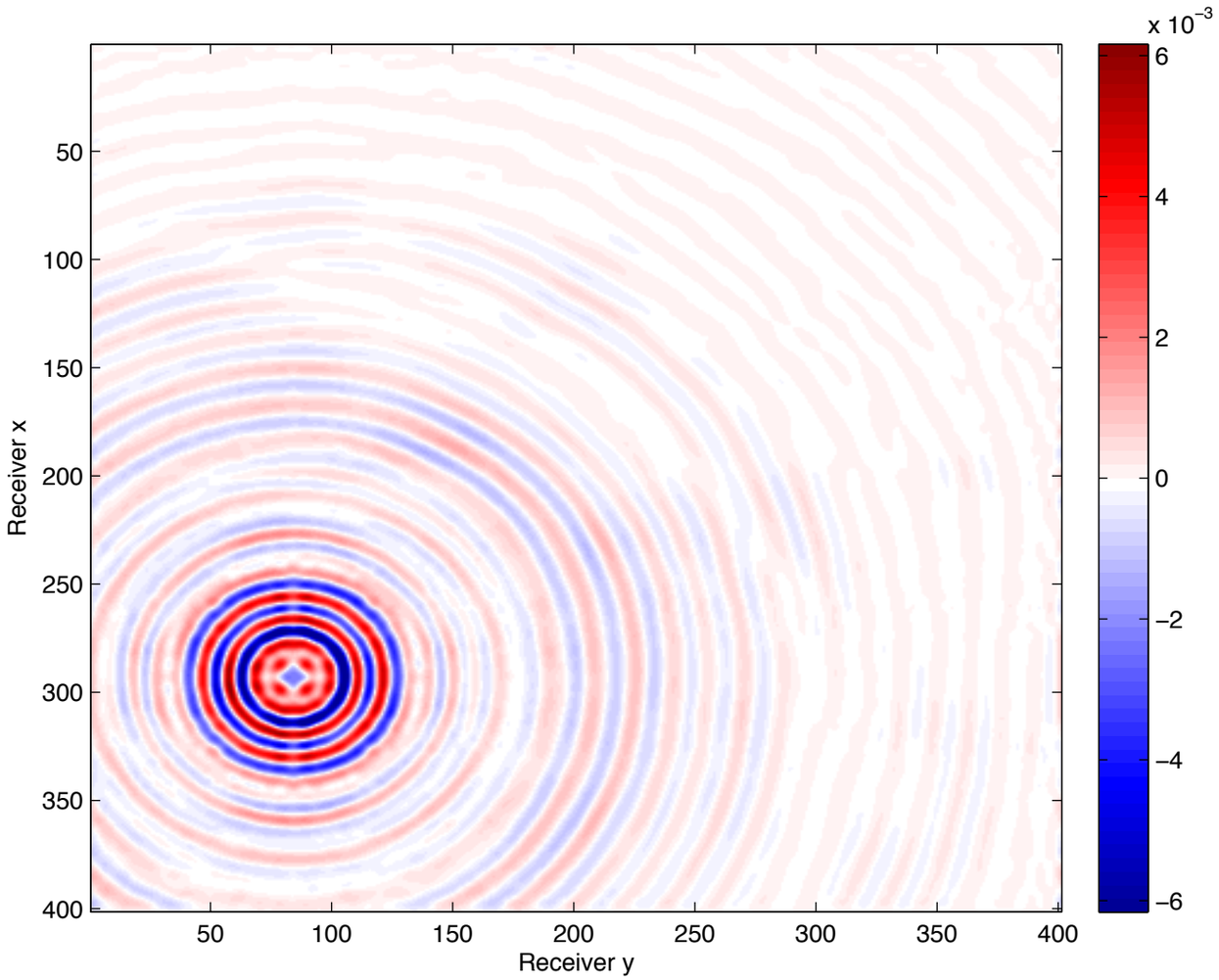
Known data

(Src x, Src y) = (50,15)



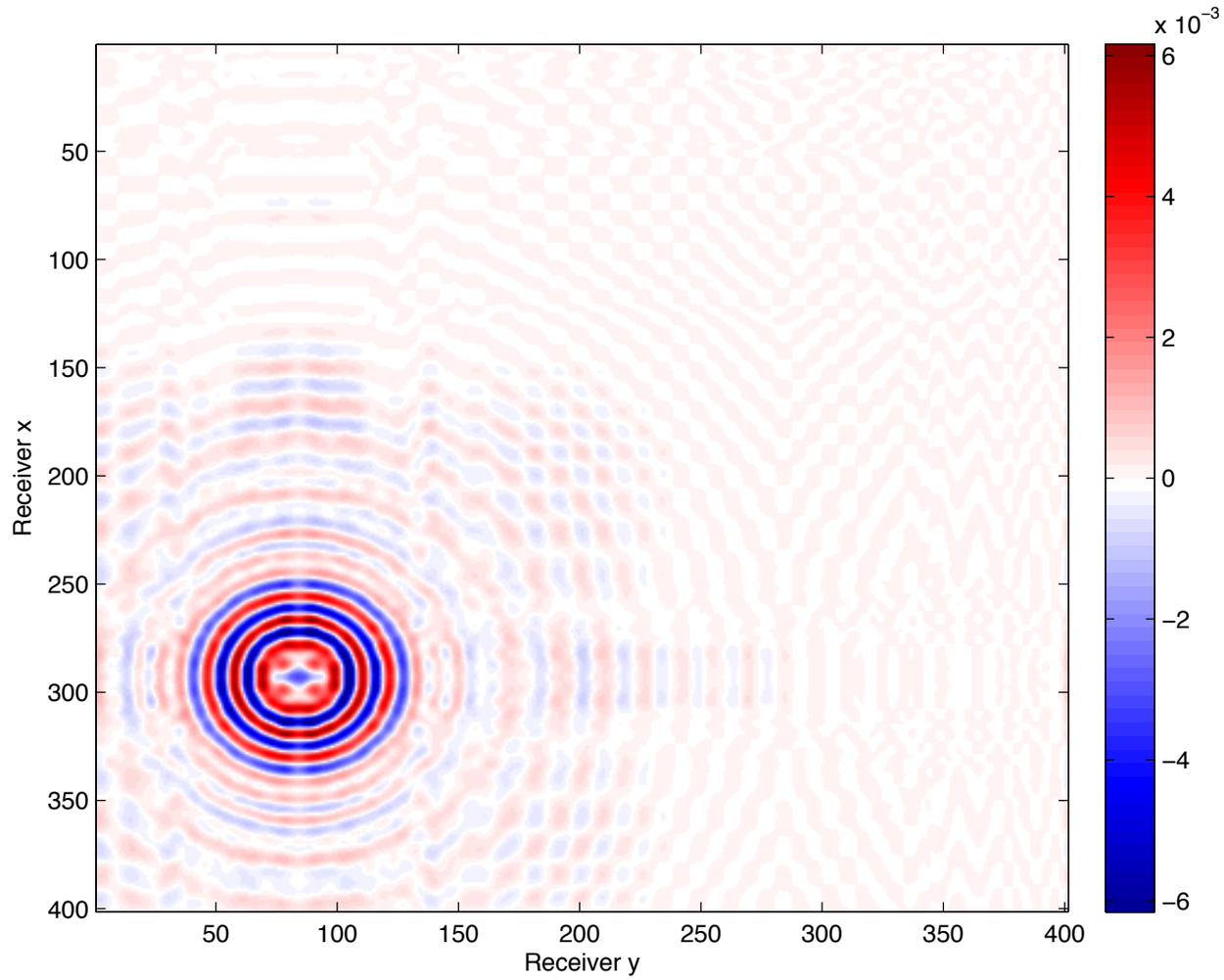
Interpolated data - SNR 13.6 dB

LR Matrix Completion



Known data

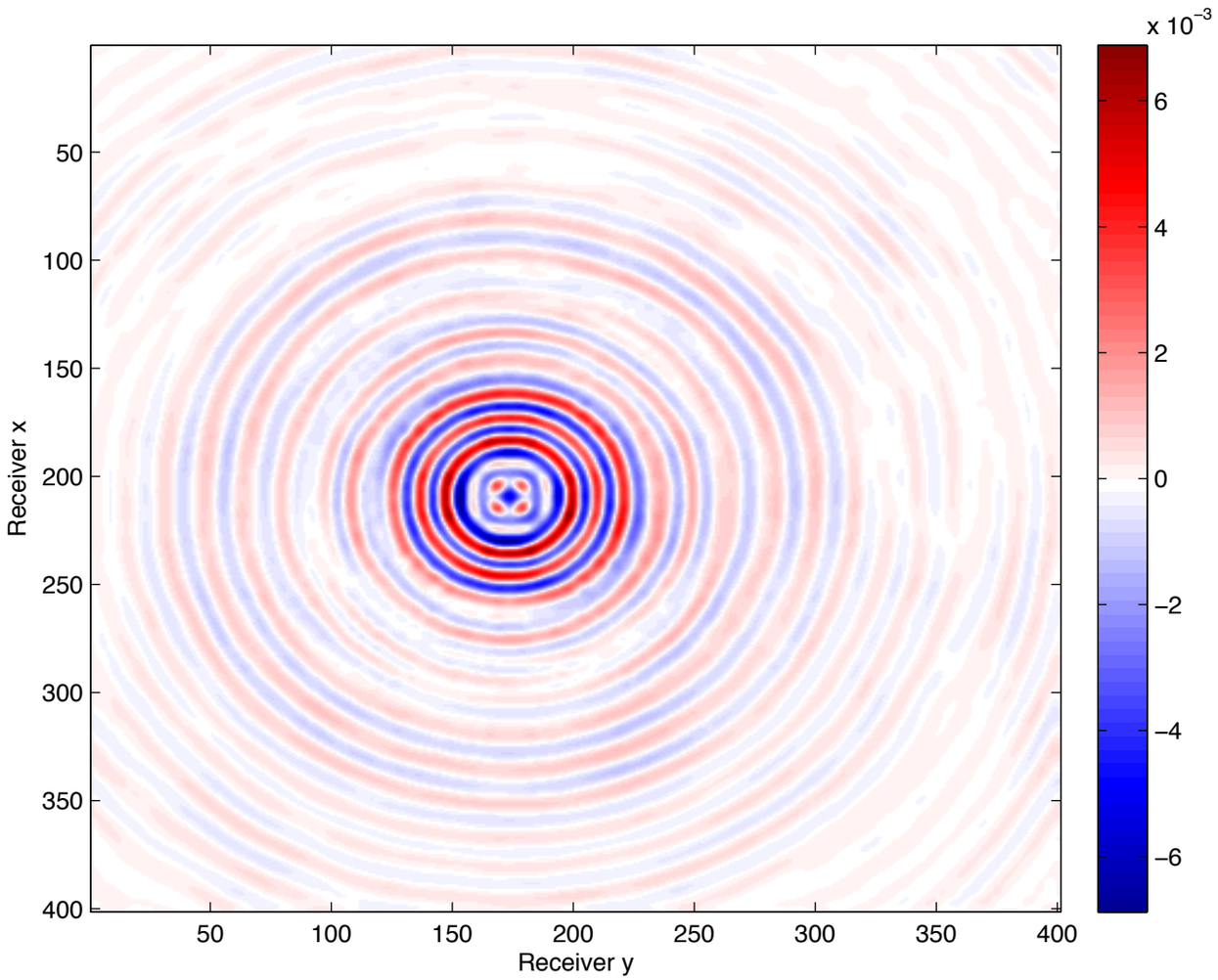
(Src x, Src y) = (50,15)



Interpolated data -

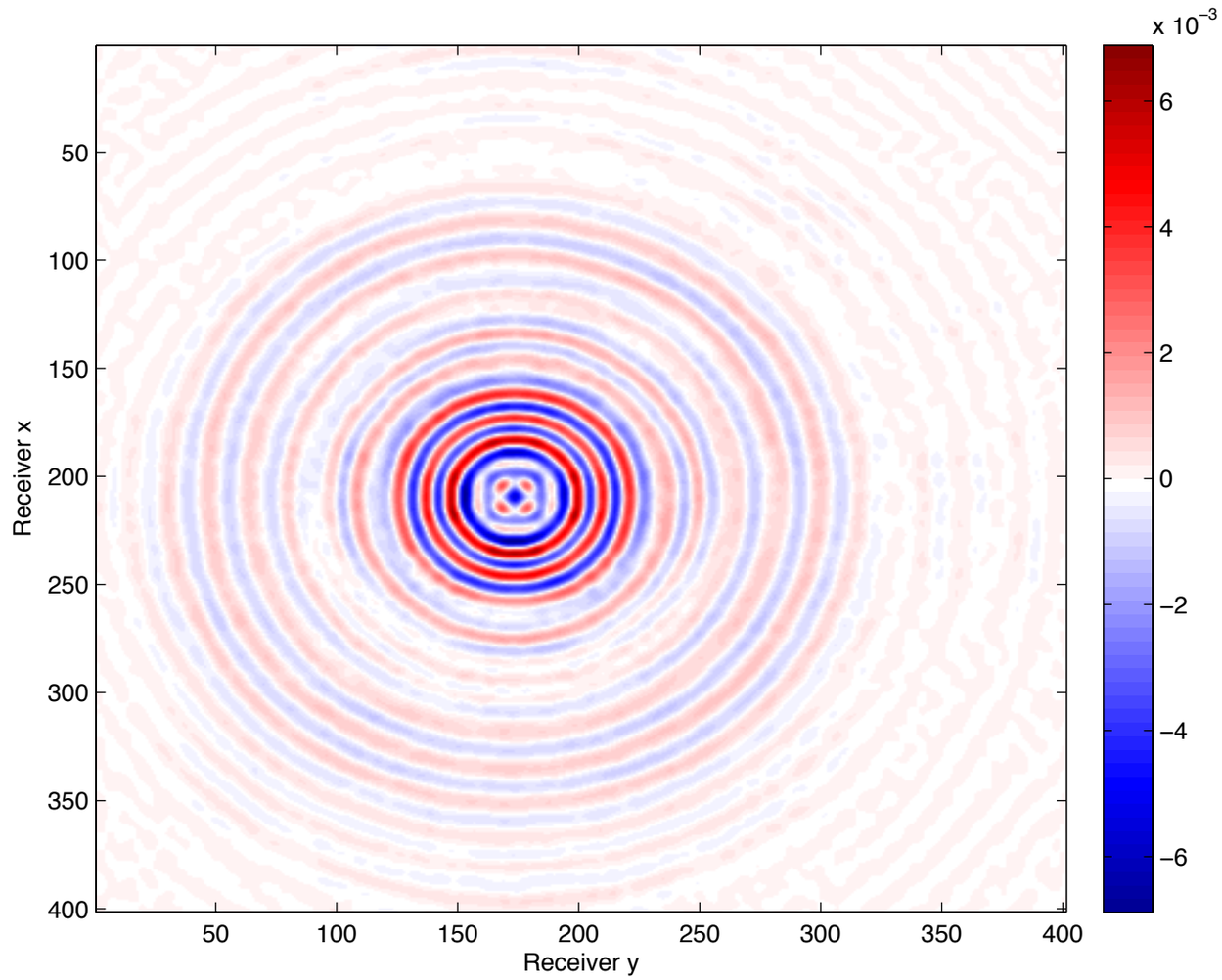
SNR 8.14 dB

Source rank 20 - 500 iterations



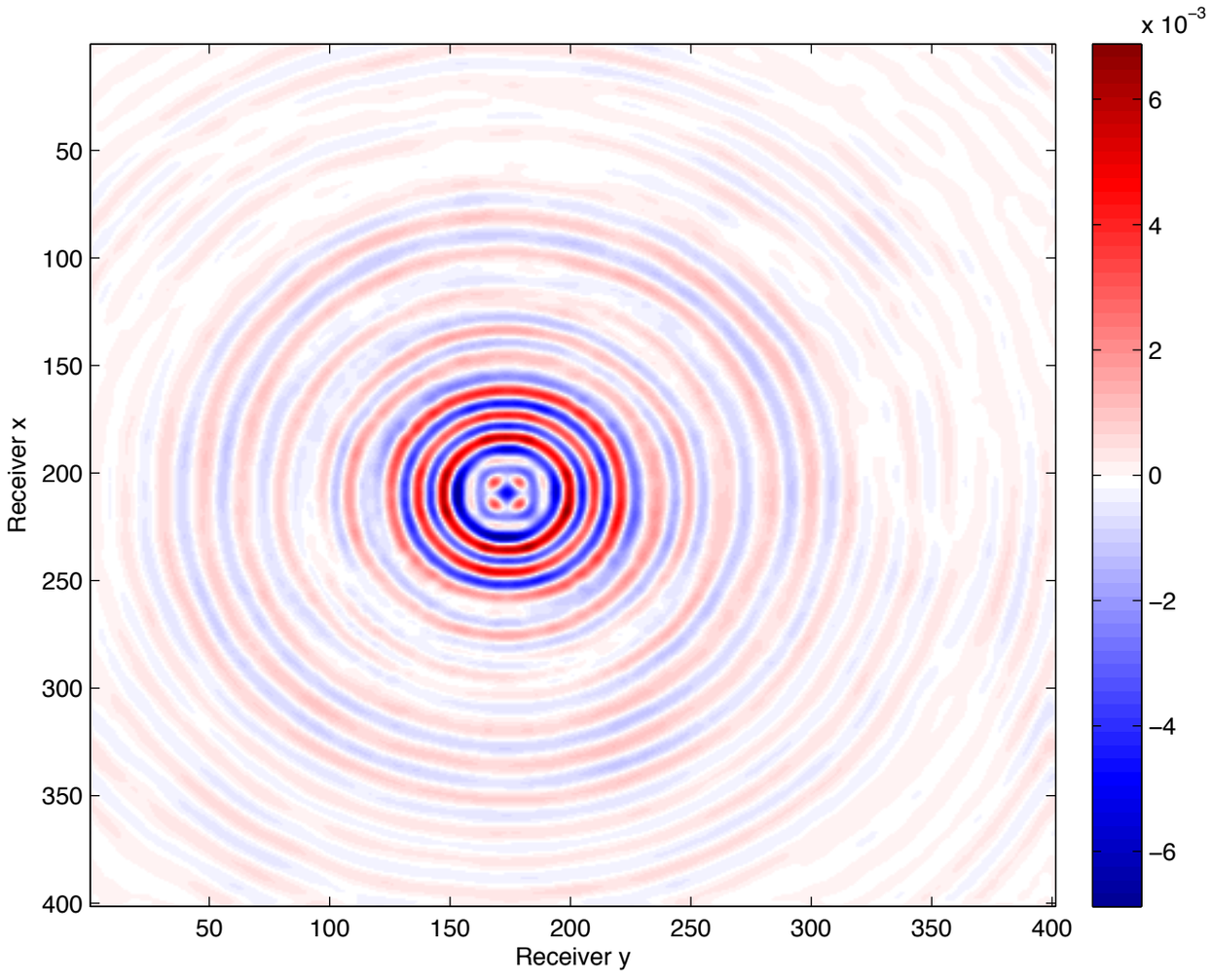
Known data

(Src x, Src y) = (36,30)



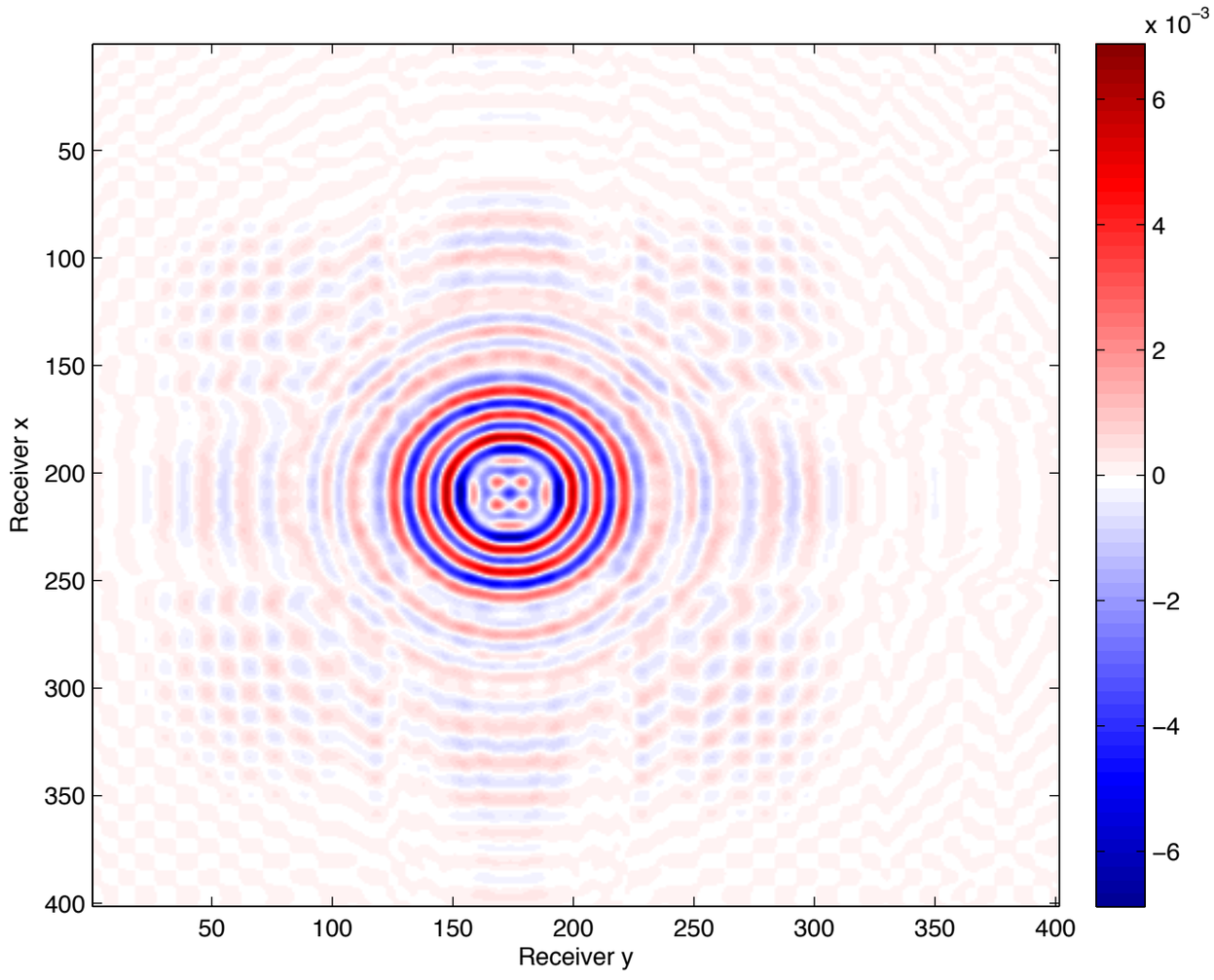
Interpolated data - SNR 12.6 dB

LR Matrix Completion



Known data

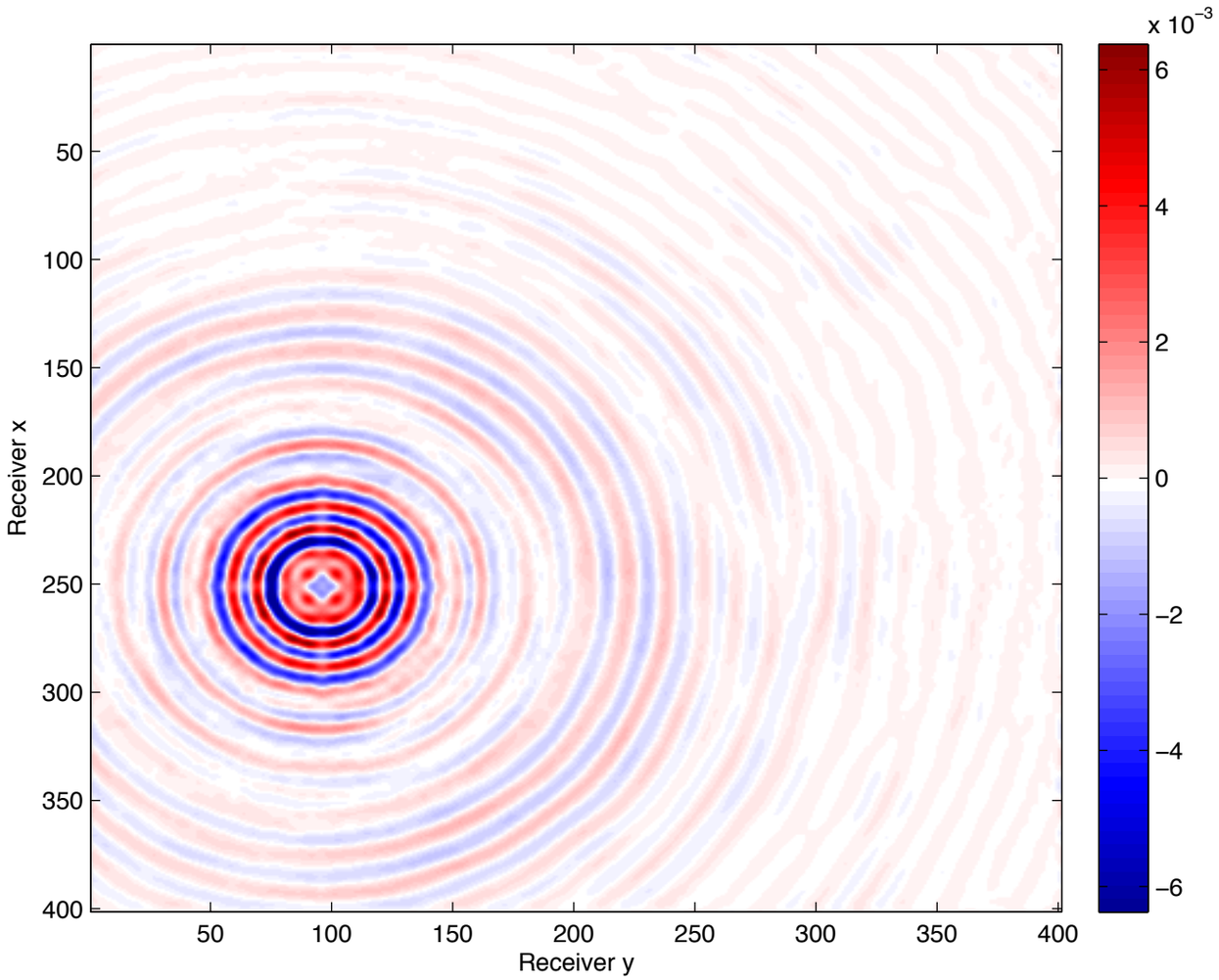
(Src x, Src y) = (36,30)



Interpolated data -

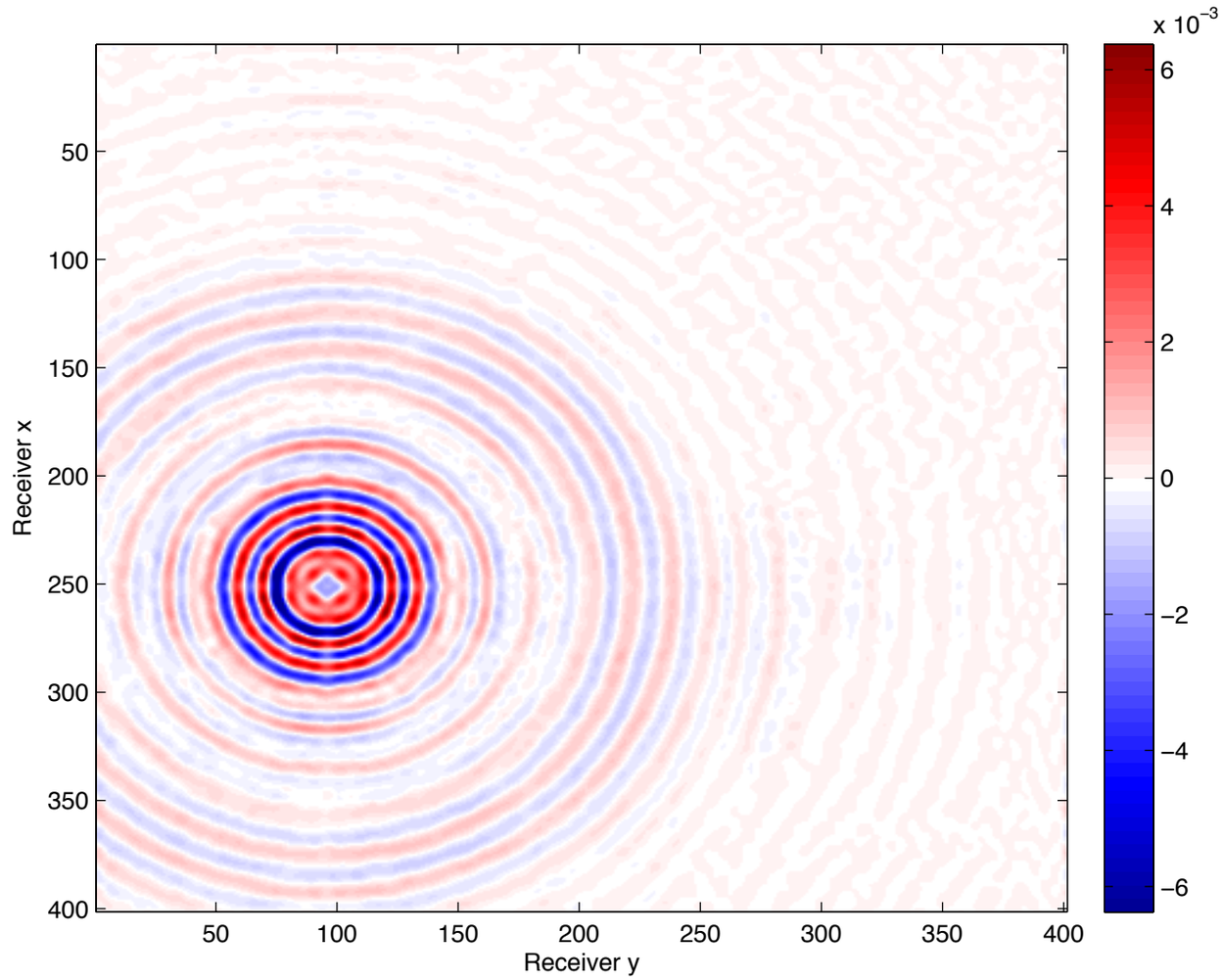
SNR 8.49 dB

Source rank 20 - 500 iterations



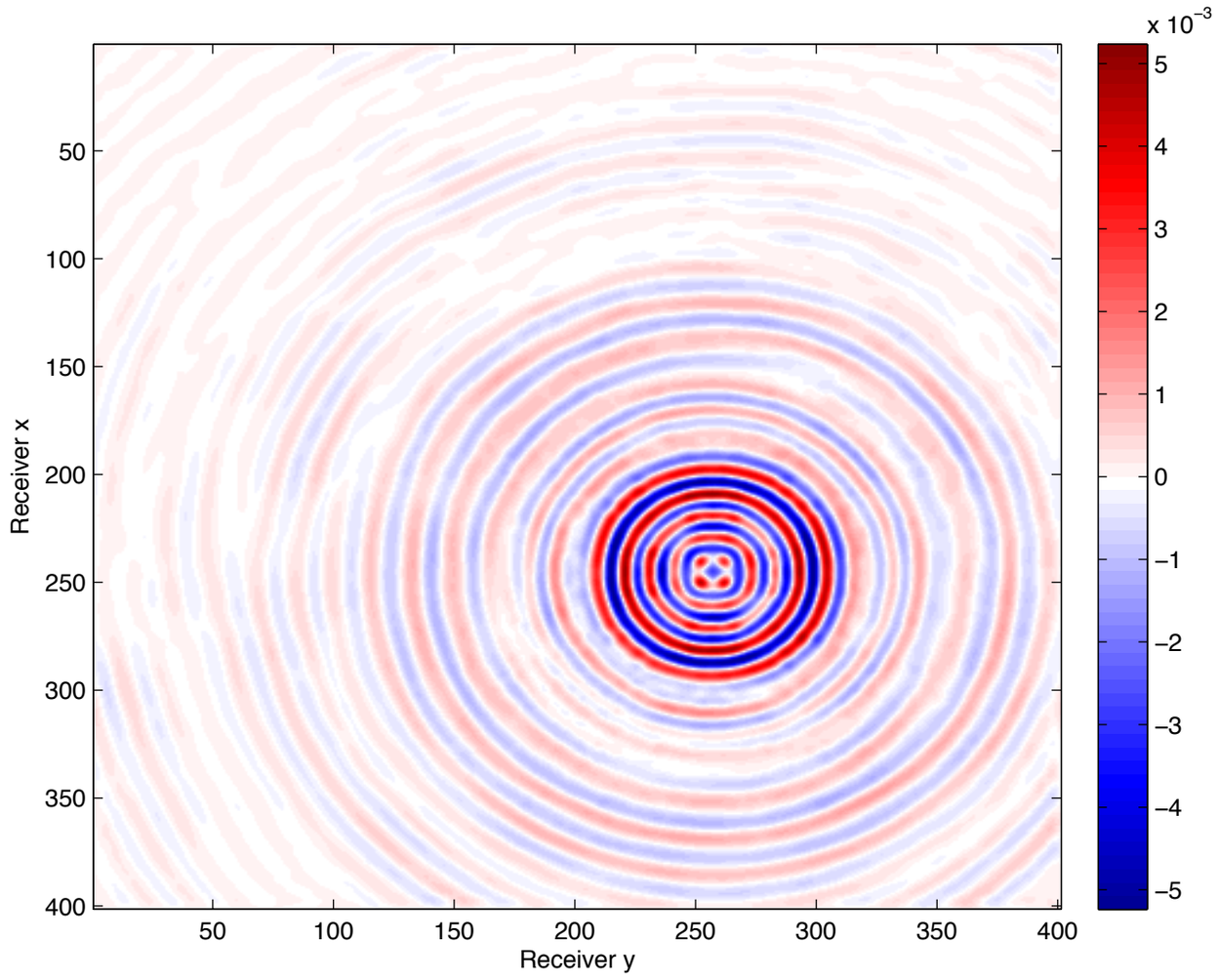
Known data

(Src x, Src y) = (43,17)



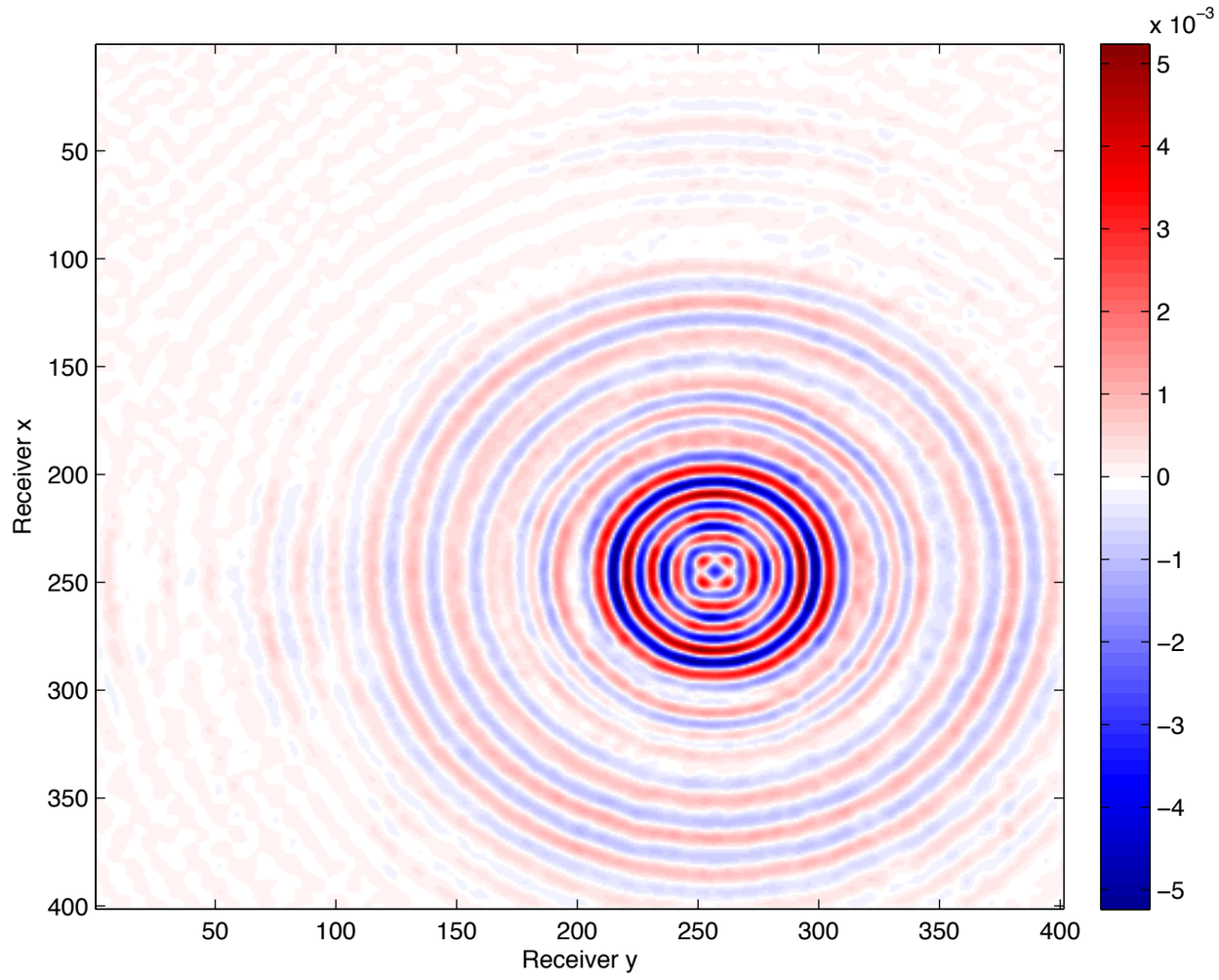
Interpolated data - SNR 13.9 dB

Source rank 20 - 500 iterations



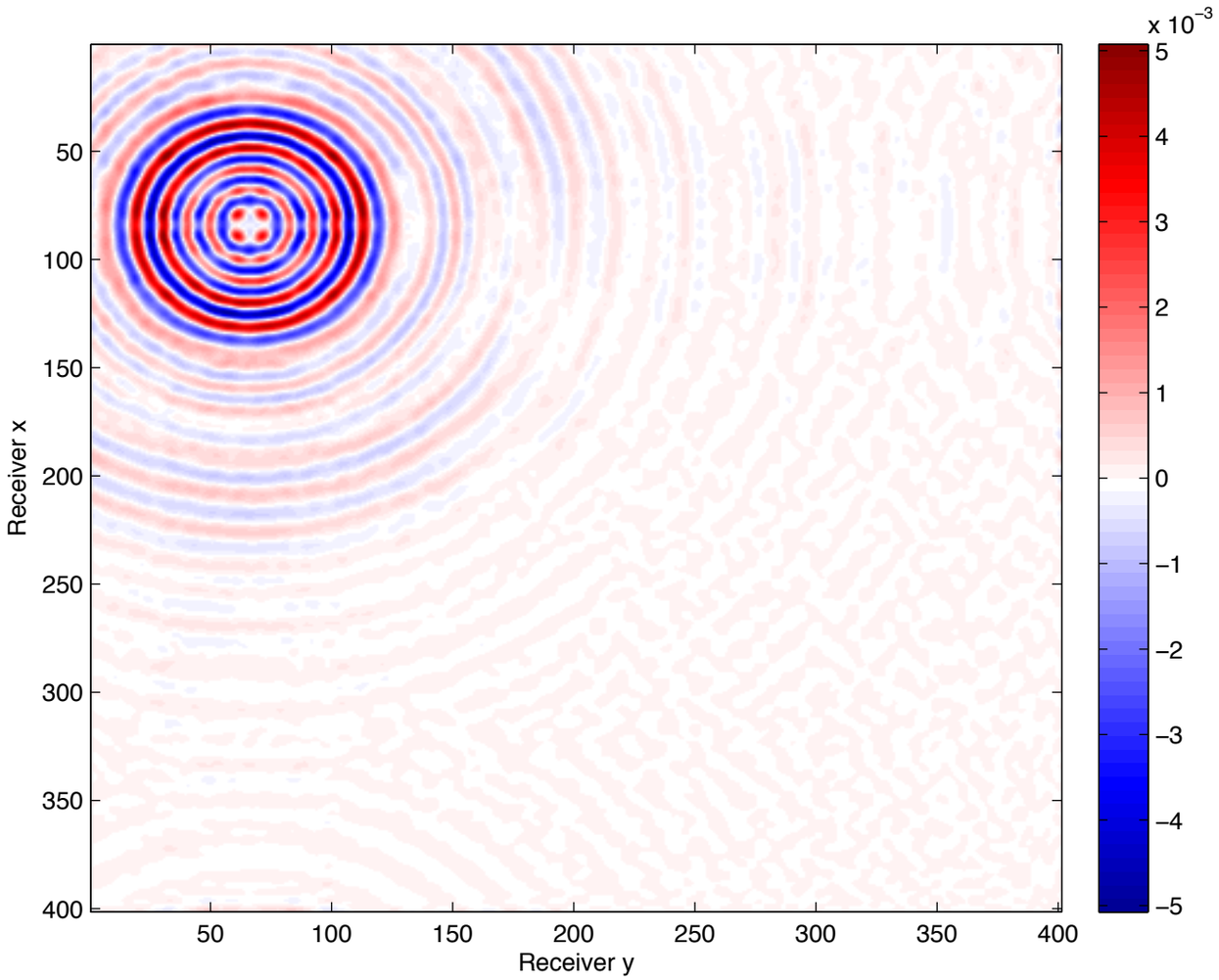
Known data

(Src x, Src y) = (42,44)



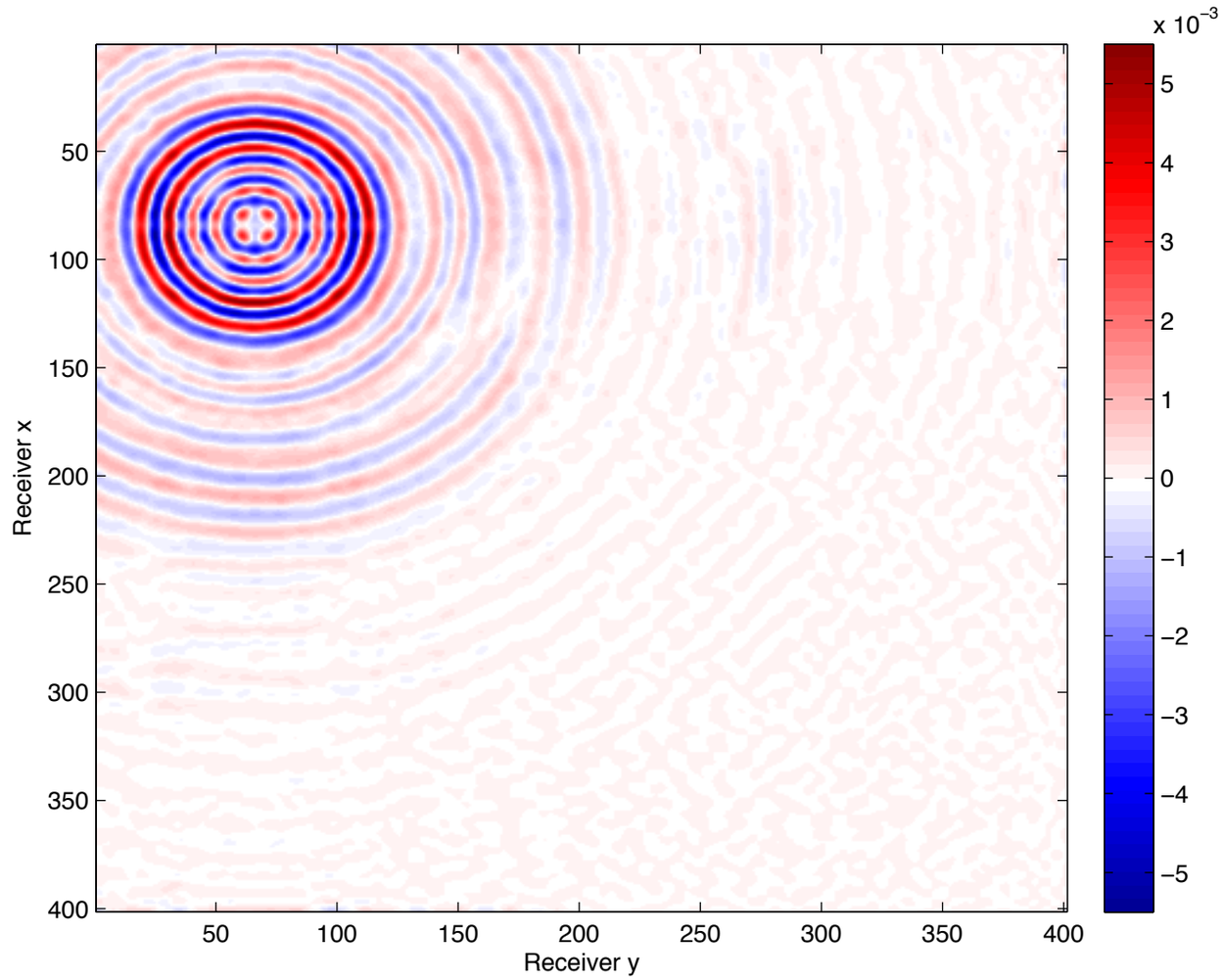
Interpolated data - SNR 13.2 dB

Source rank 5 vs source rank 20



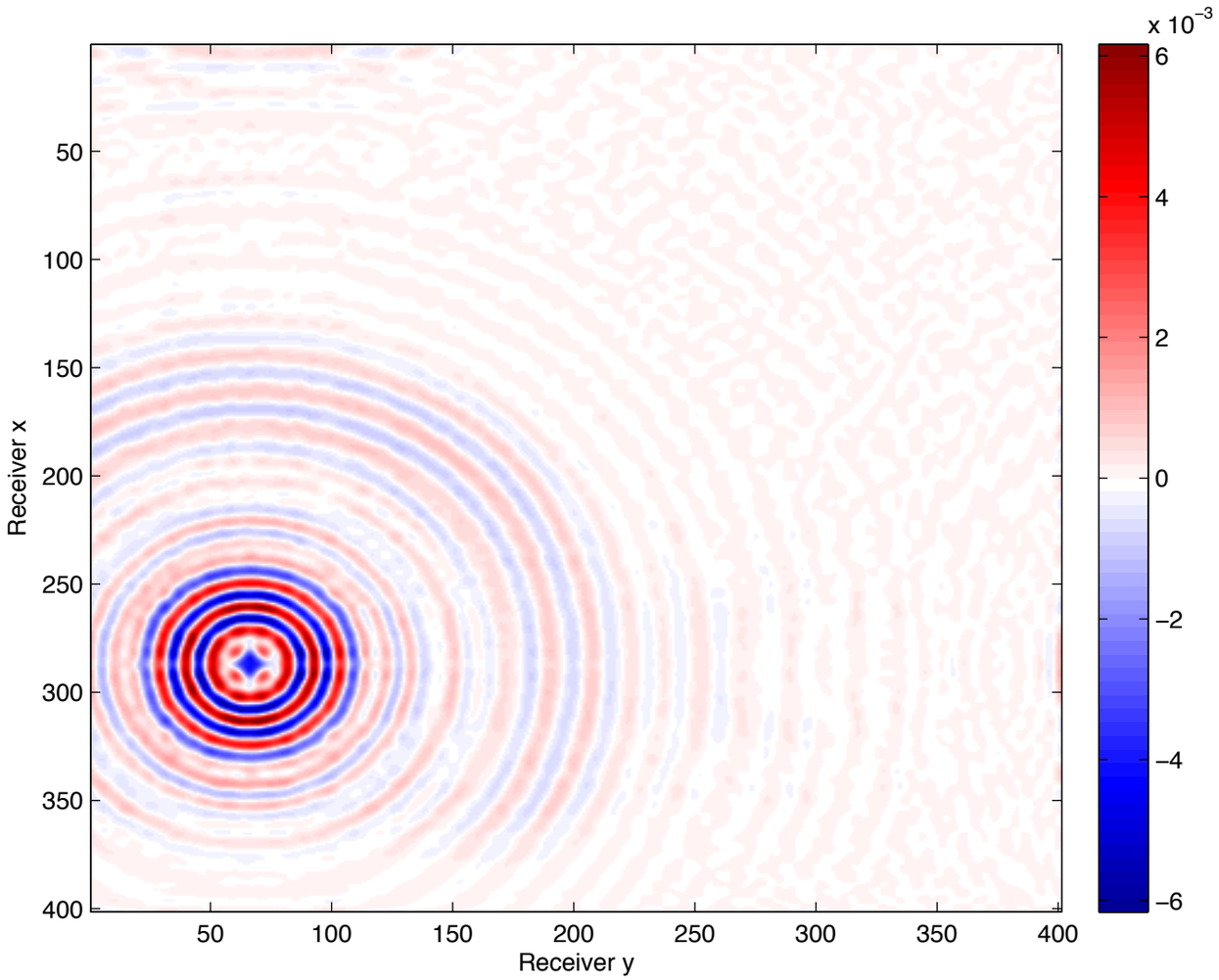
Rank 5

(Src x, Src y) = (15,12)



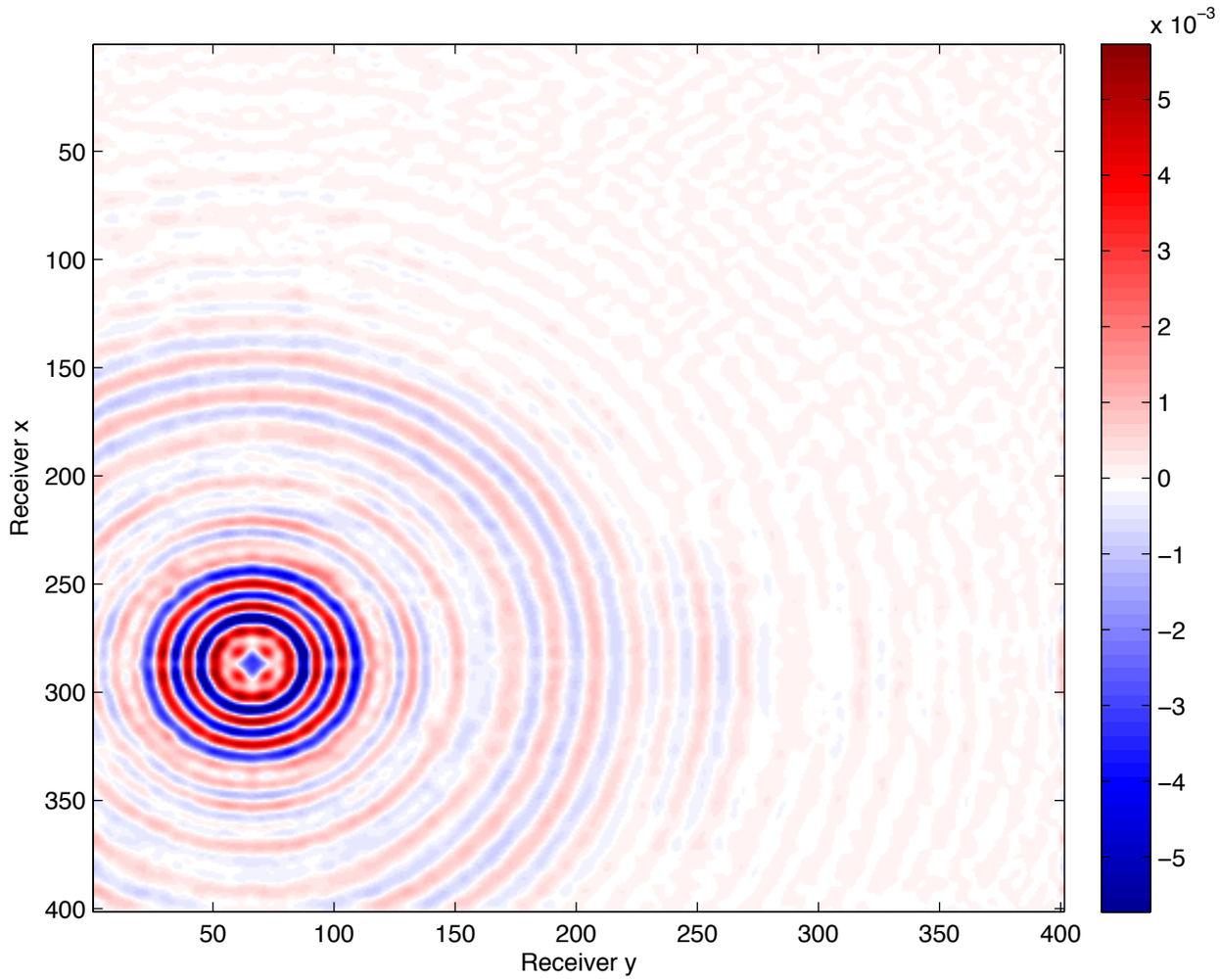
Rank 20

Source rank 5 vs source rank 20



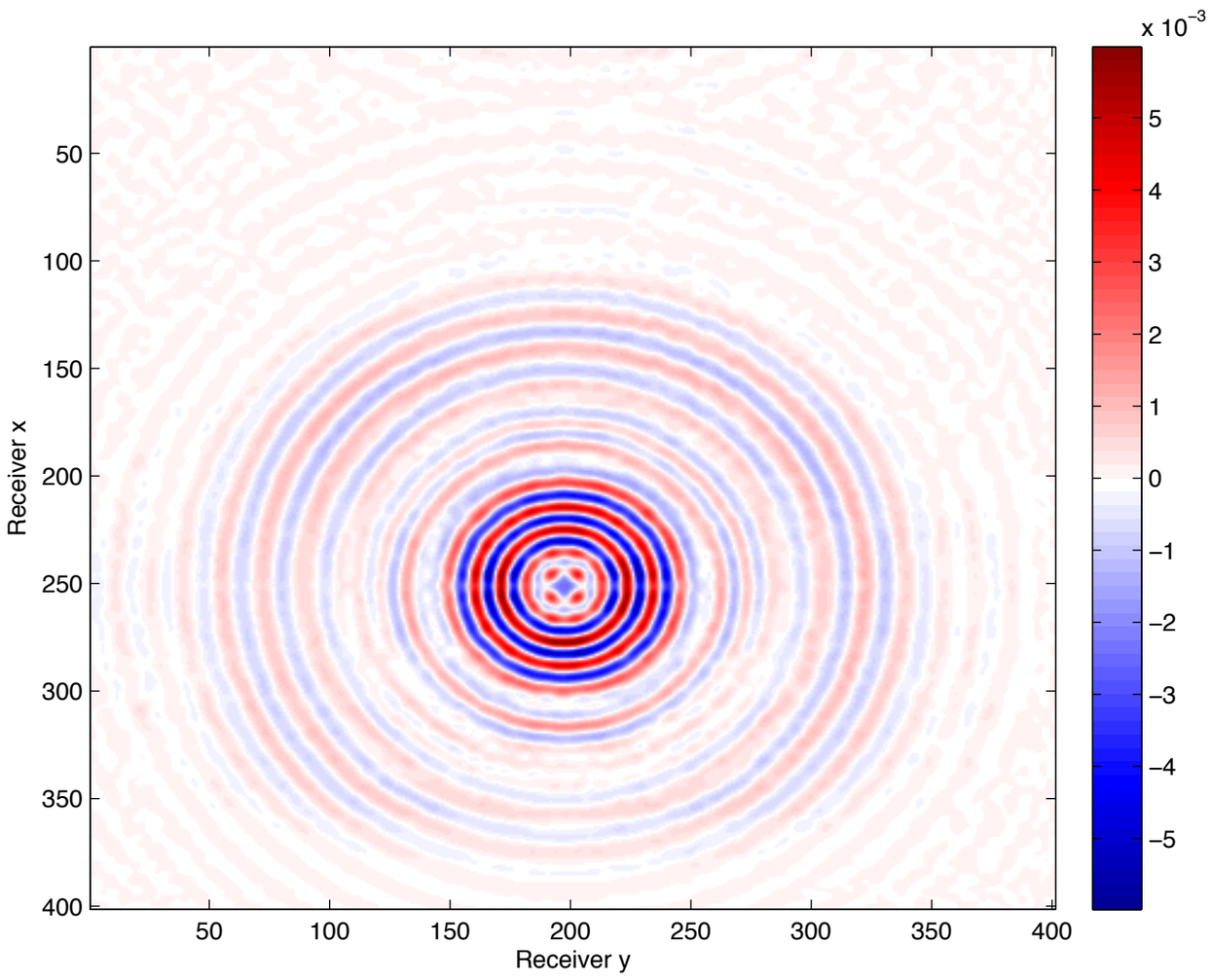
Rank 5

(Src x, Src y) = (49,12)



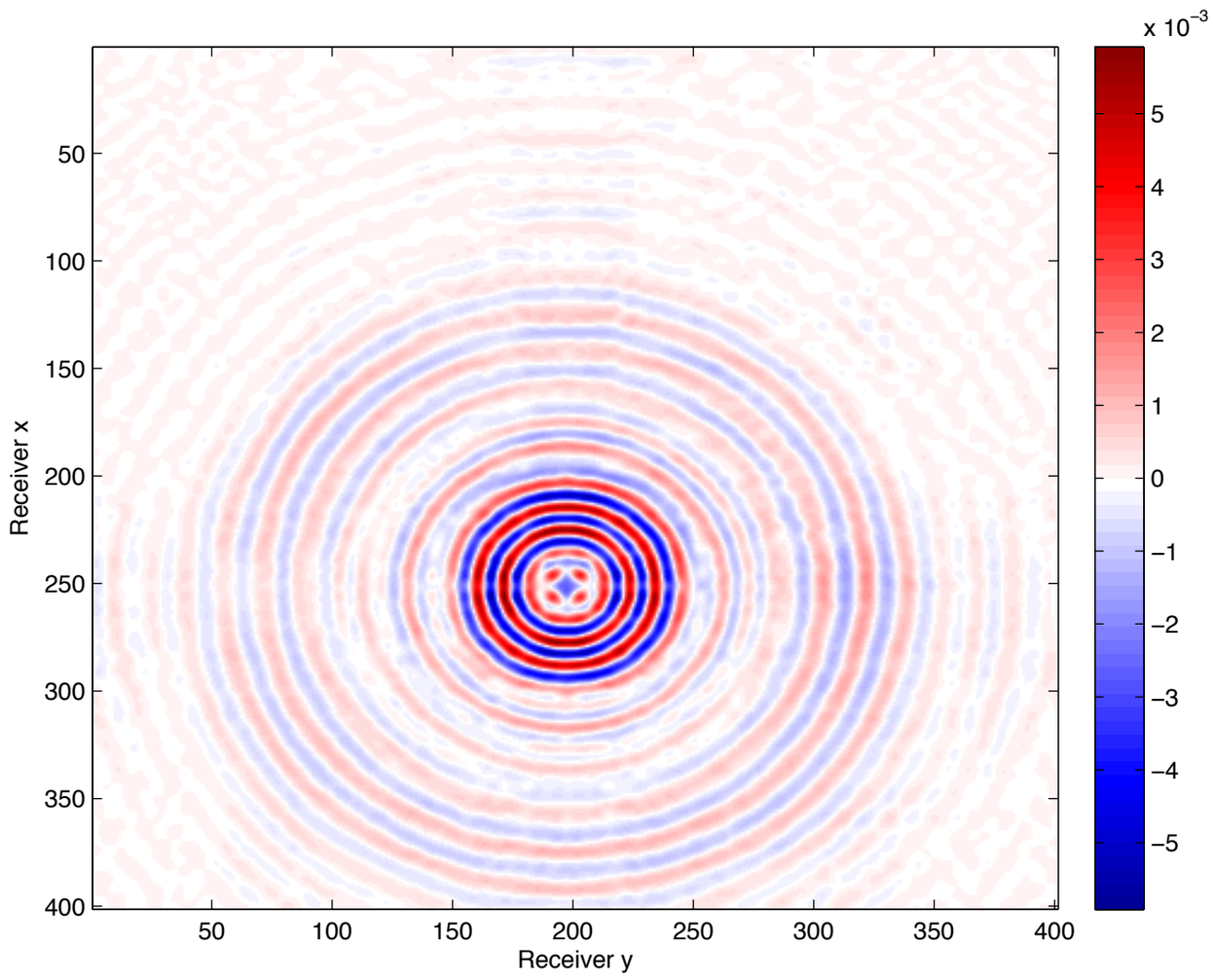
Rank 20

Source rank 5 vs source rank 20



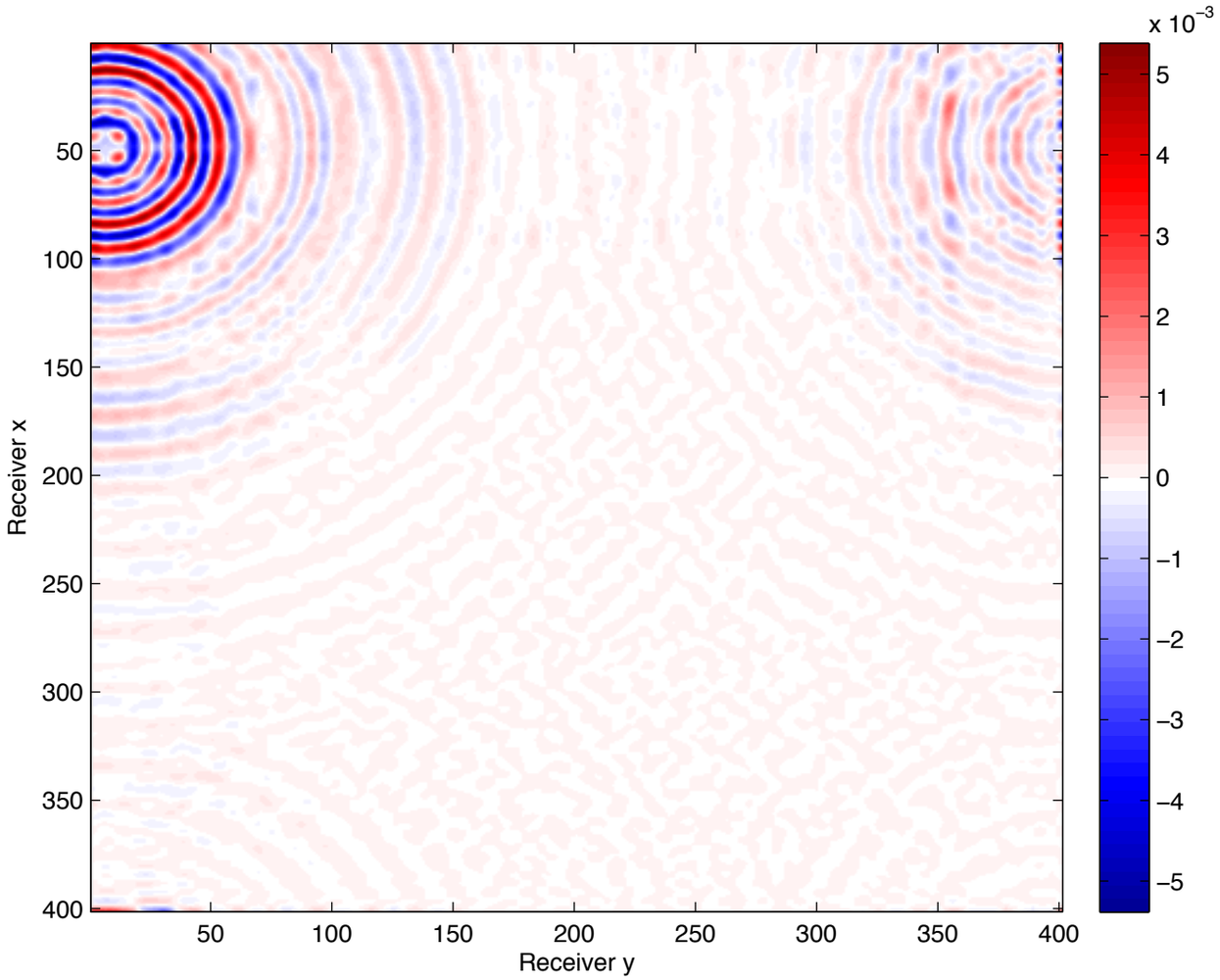
Rank 5

(Src x, Src y) = (43,34)



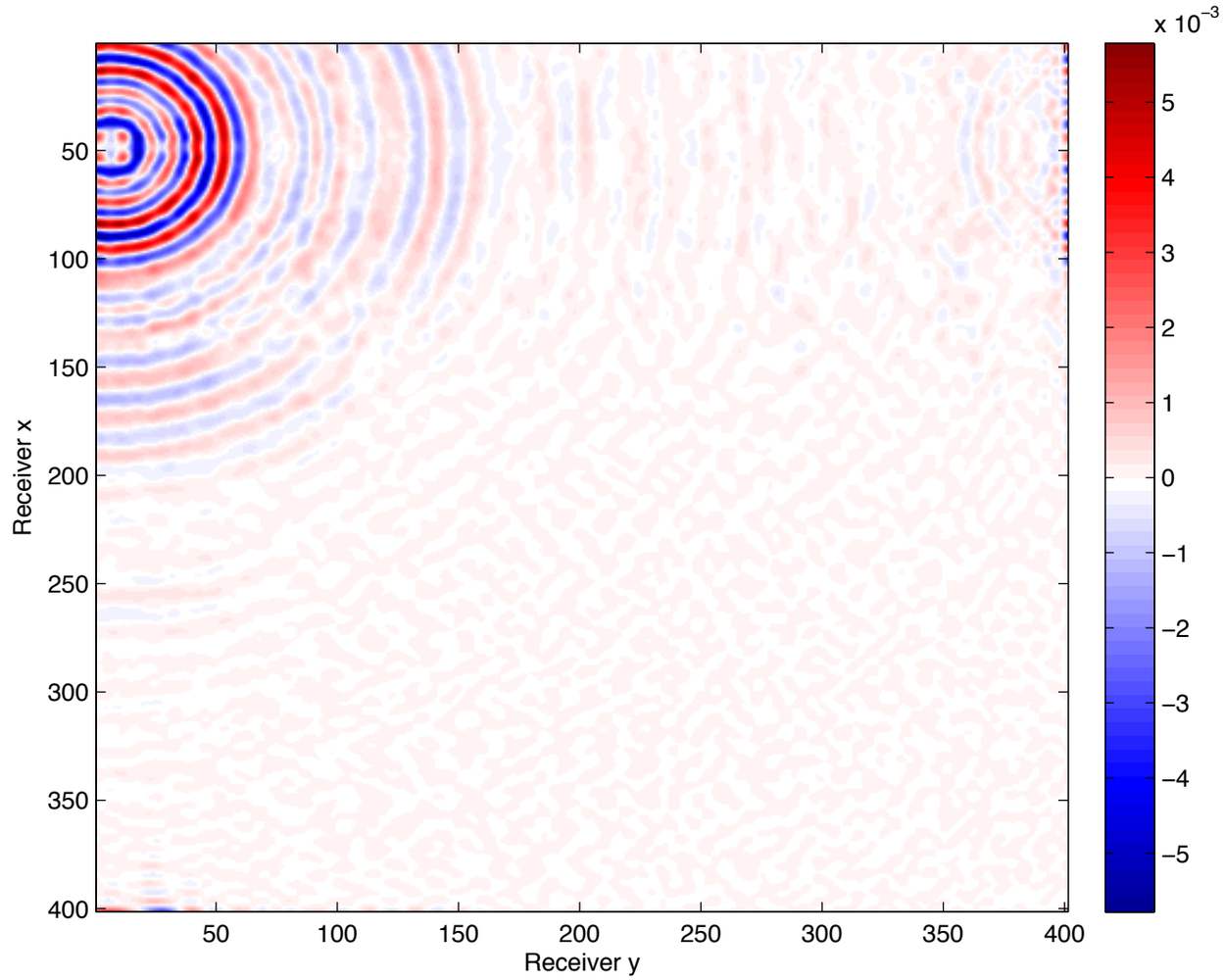
Rank 20

Source rank 5 vs source rank 20



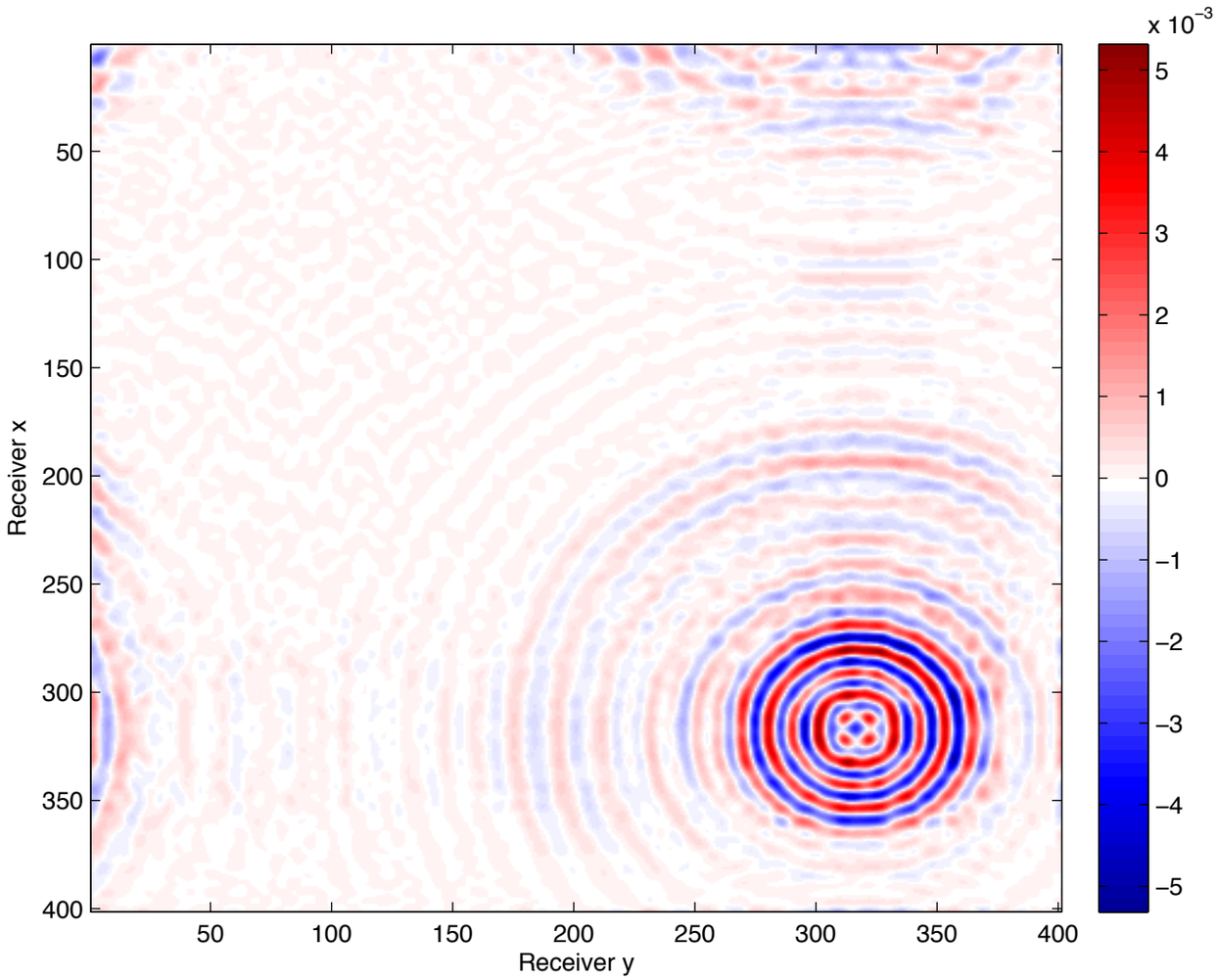
Rank 5

(Src x, Src y) = (9,2)



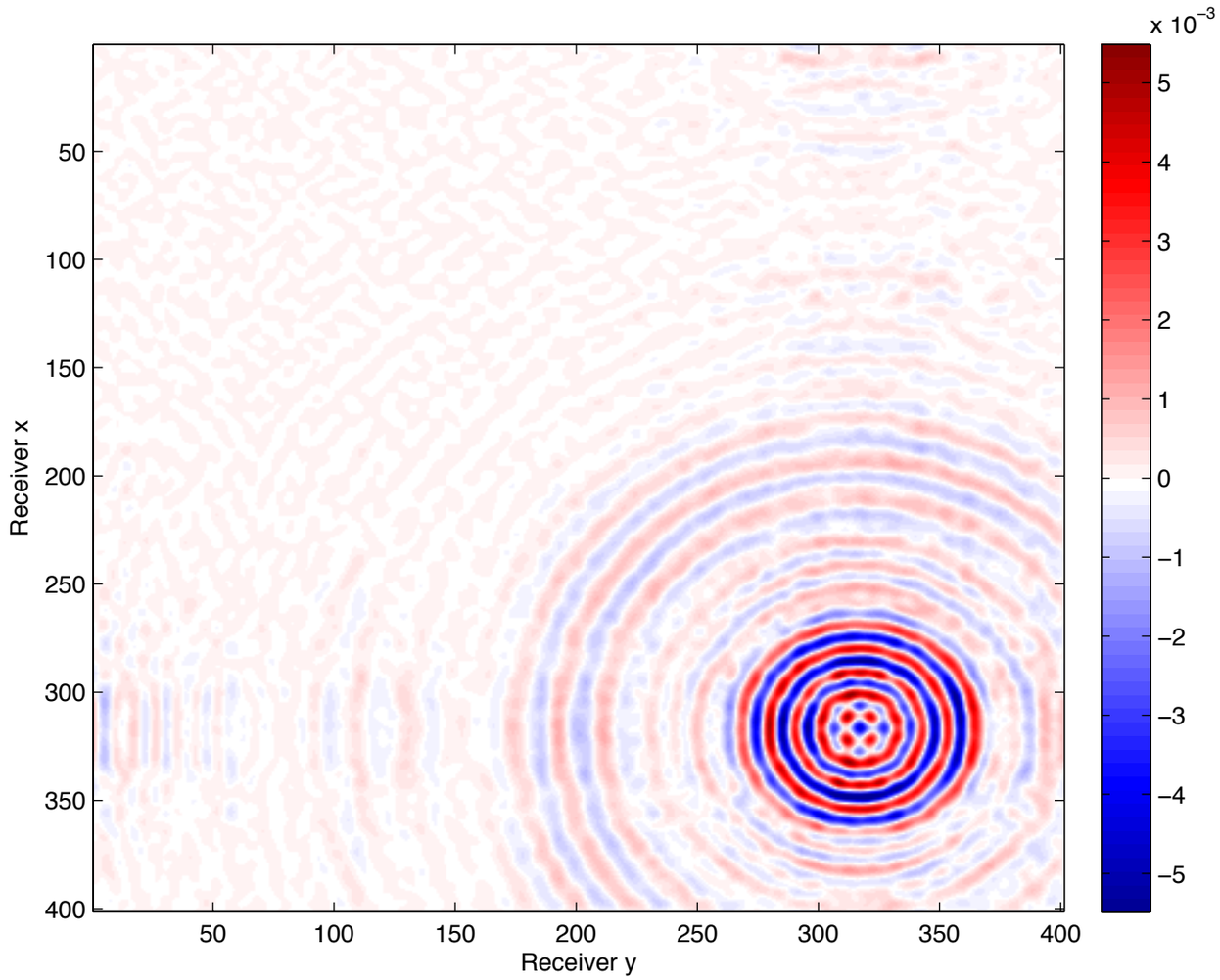
Rank 20

Source rank 5 vs source rank 20



Rank 5

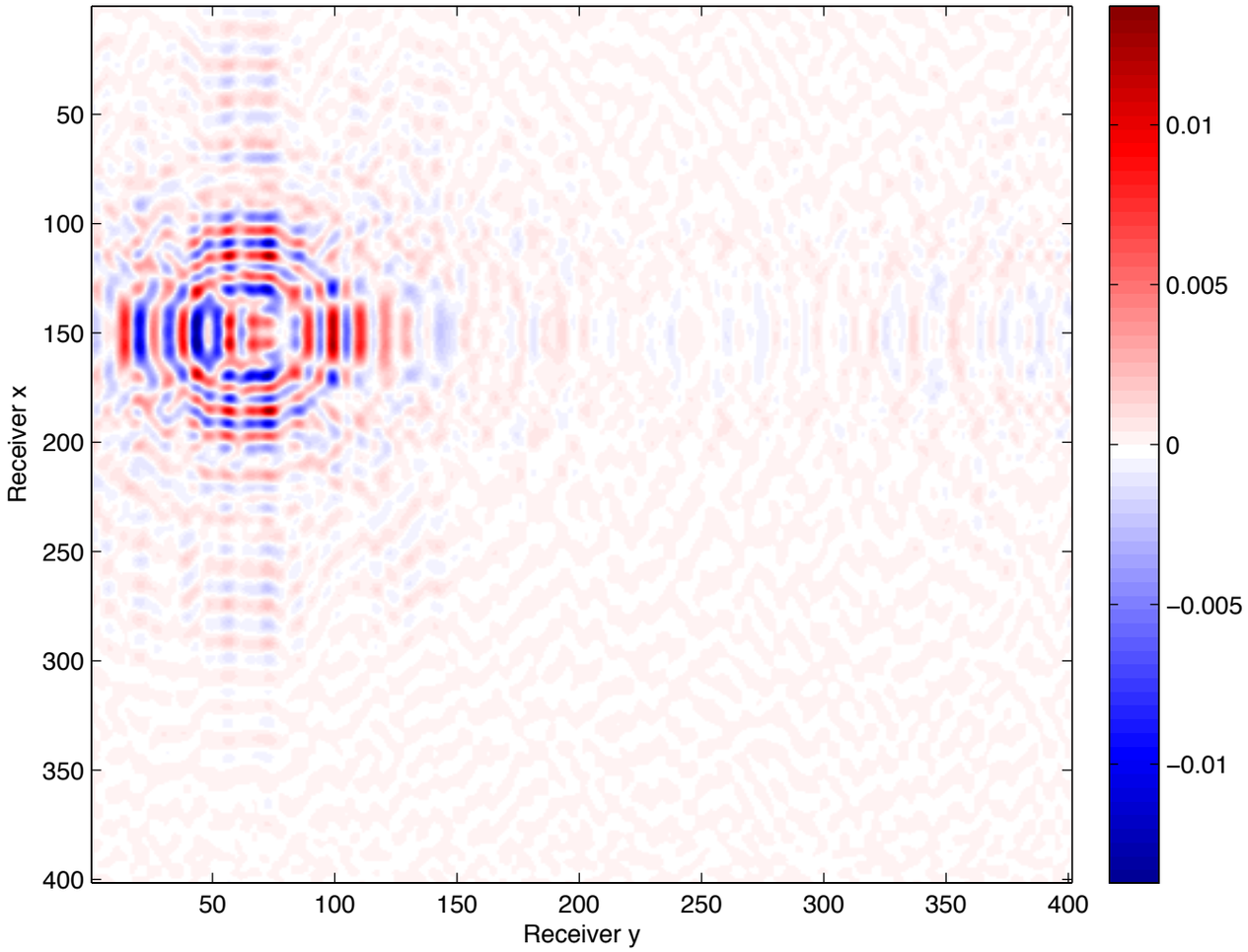
(Src x, Src y) = (54,54)



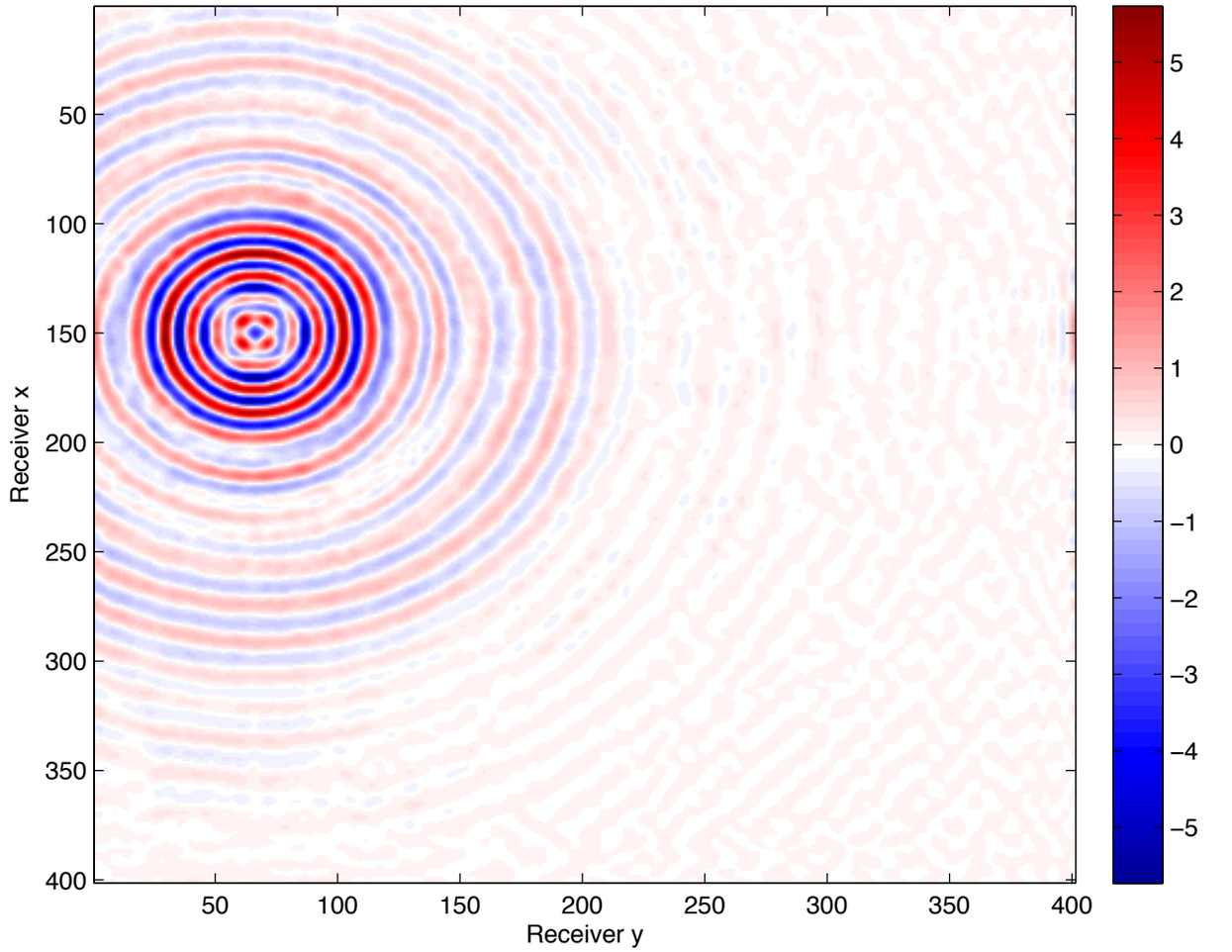
Rank 20

Source rank 20 - 500 iterations

Missing sufficient data spread



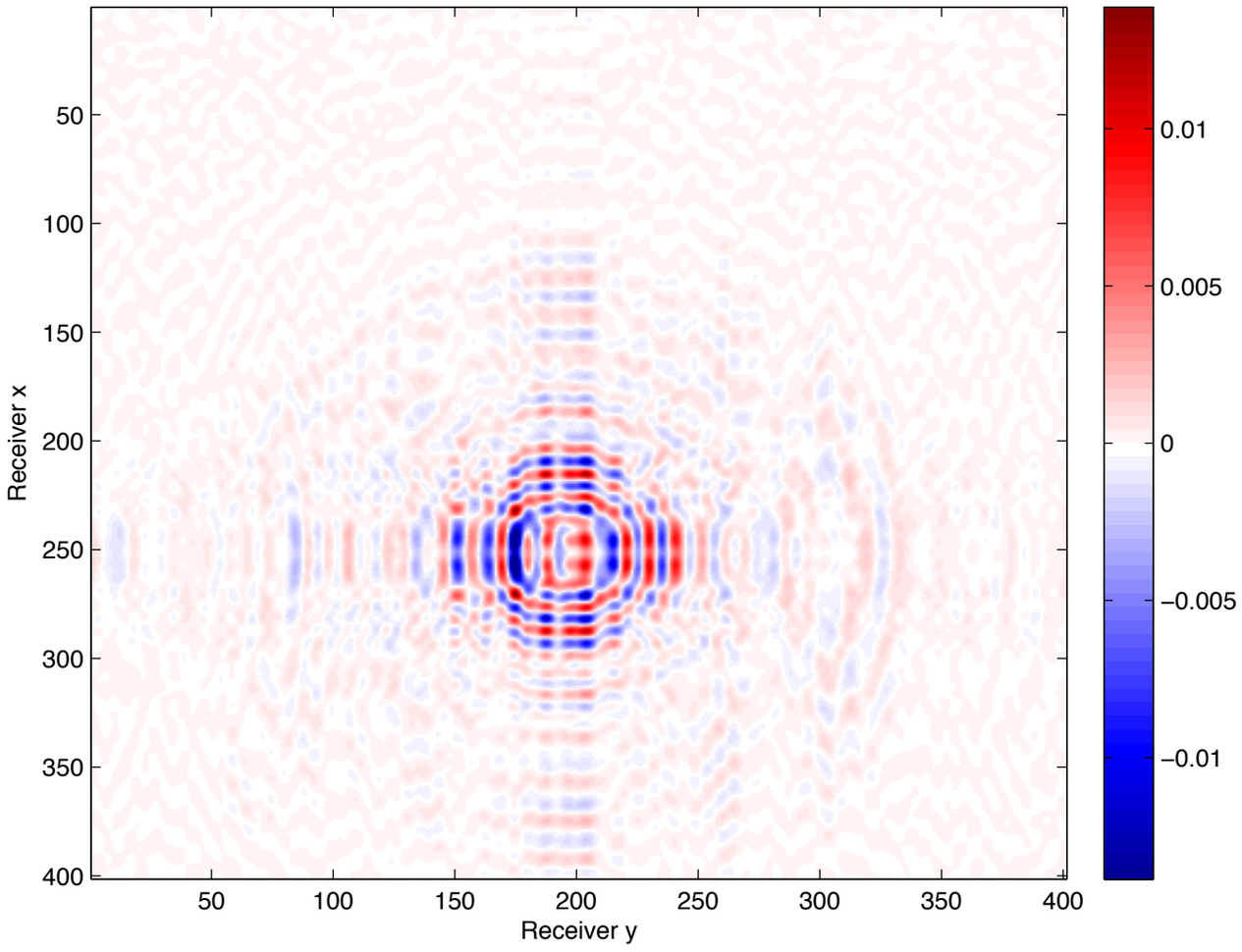
Interpolated data
(Src x, Src y) = (26,11)



Interpolated data
(Src x, Src y) = (26,12)

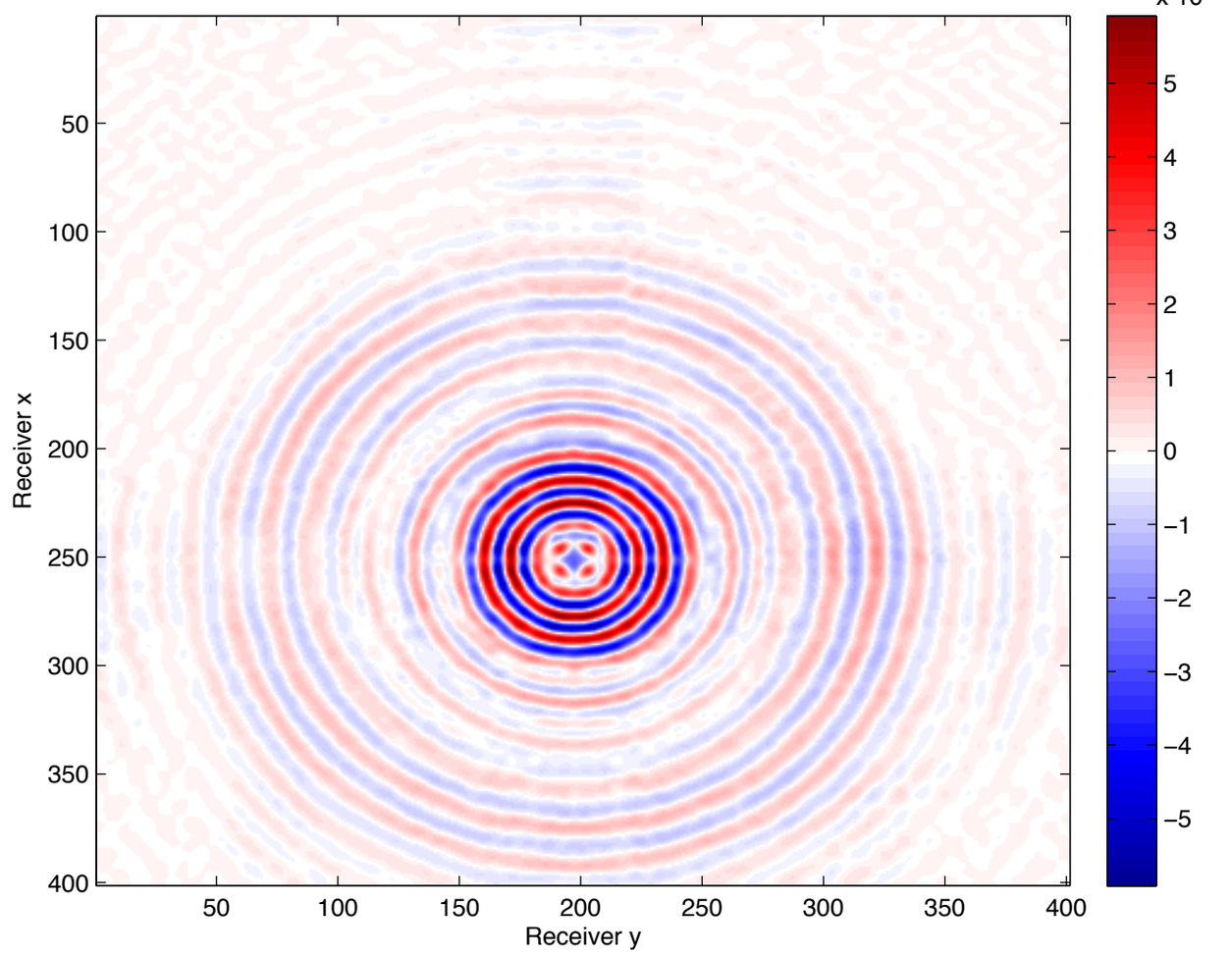
Source rank 20 - 500 iterations

Missing sufficient data spread



Interpolated data

(Src x, Src y) = (43,33)



Interpolated data

(Src x, Src y) = (43,34)

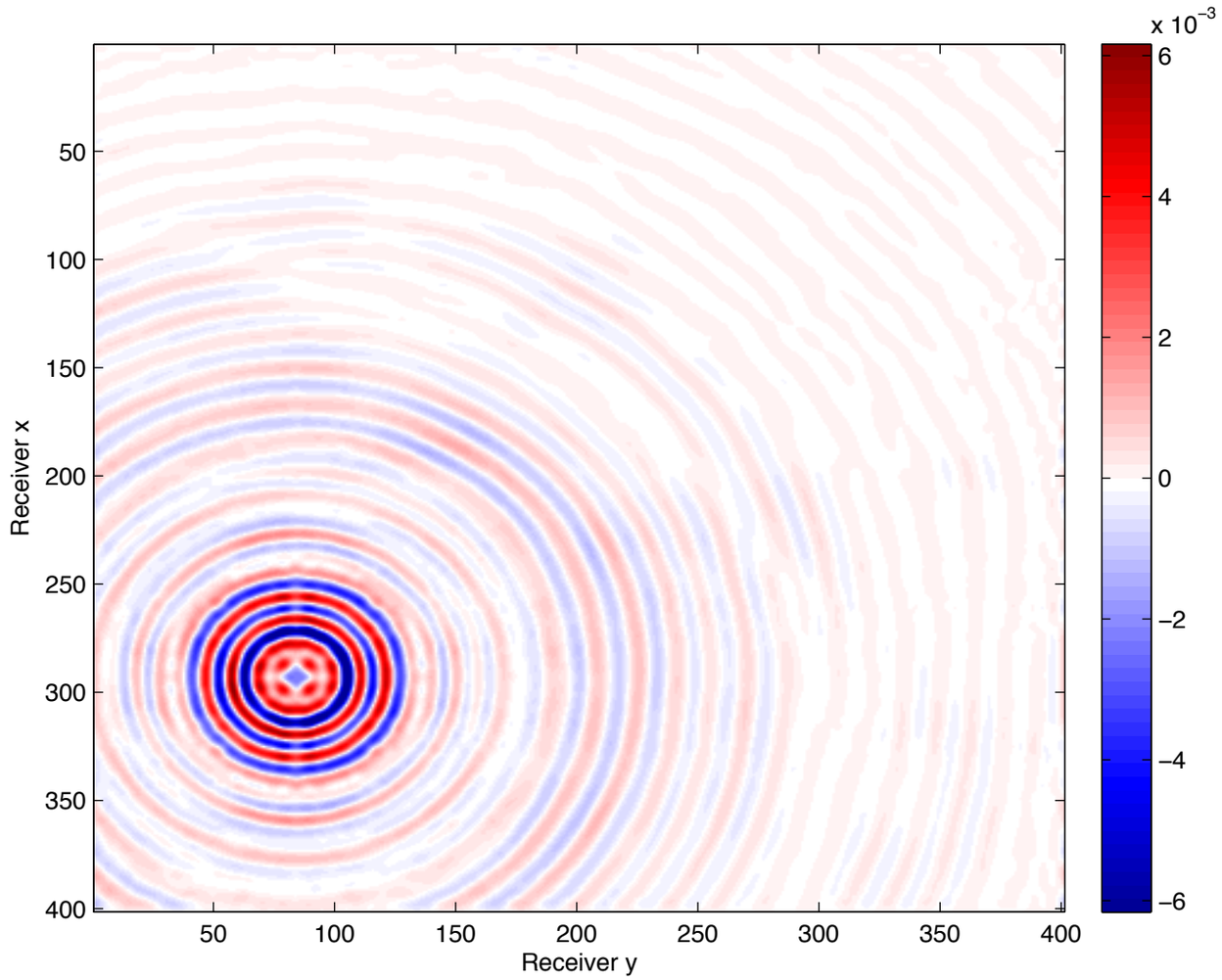
Synthetic BG Data

- Currently we are only trying to minimize the energy misfit between our model + the data we have (as well as choosing a rank for our underlying model)
- Our assumption is that the underlying tensor has *quickly*-decaying singular values in different matricizations

Synthetic BG Data

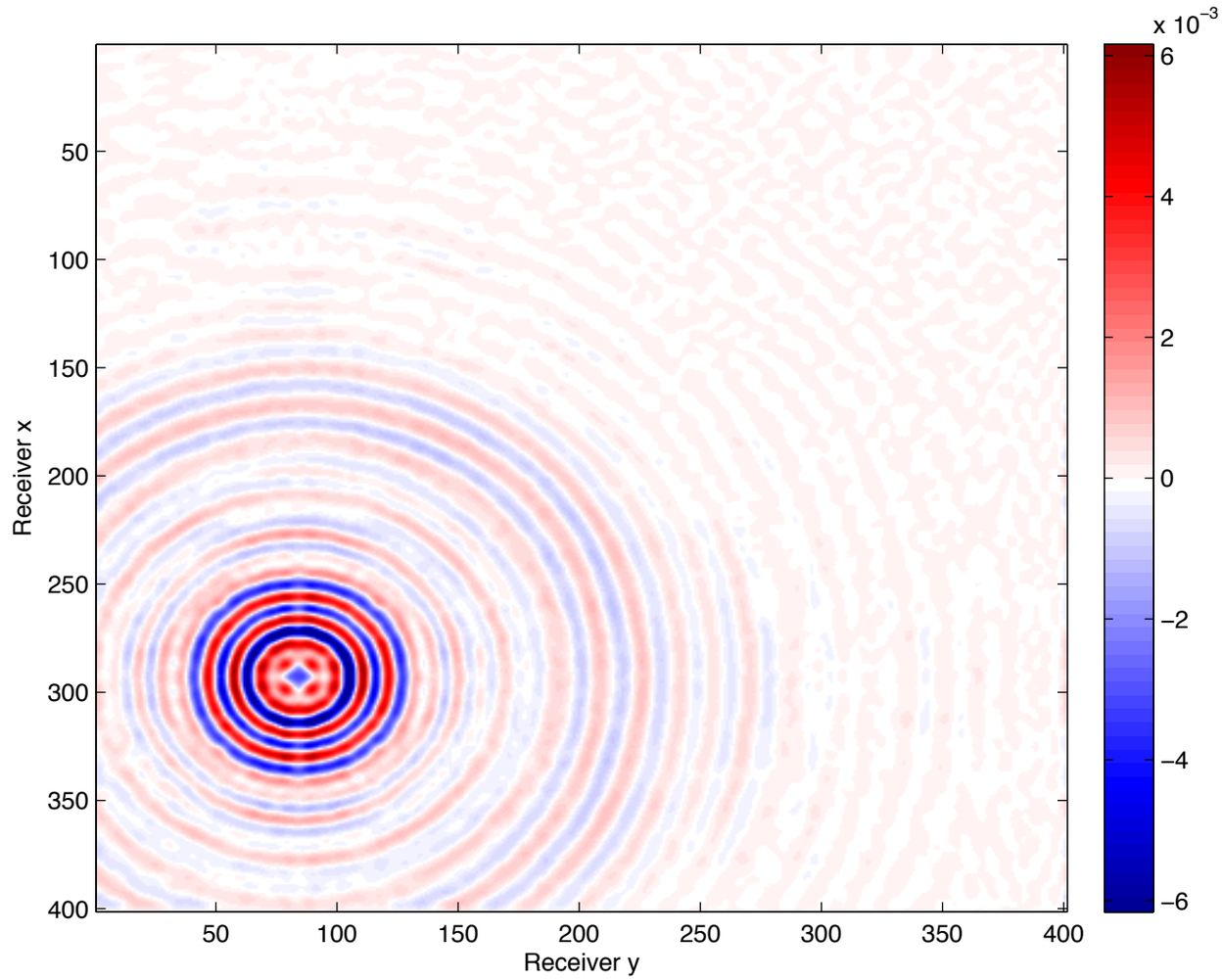
- We can impose a penalty on the decay of the singular values of these matricizations
  - This talk is far too long already, so we'll skip the details here
  - Suffice to say, it can be done, and inexpensively at that (full tensor not used, only interior parameter matrices)

Source rank 20 - 500 iterations



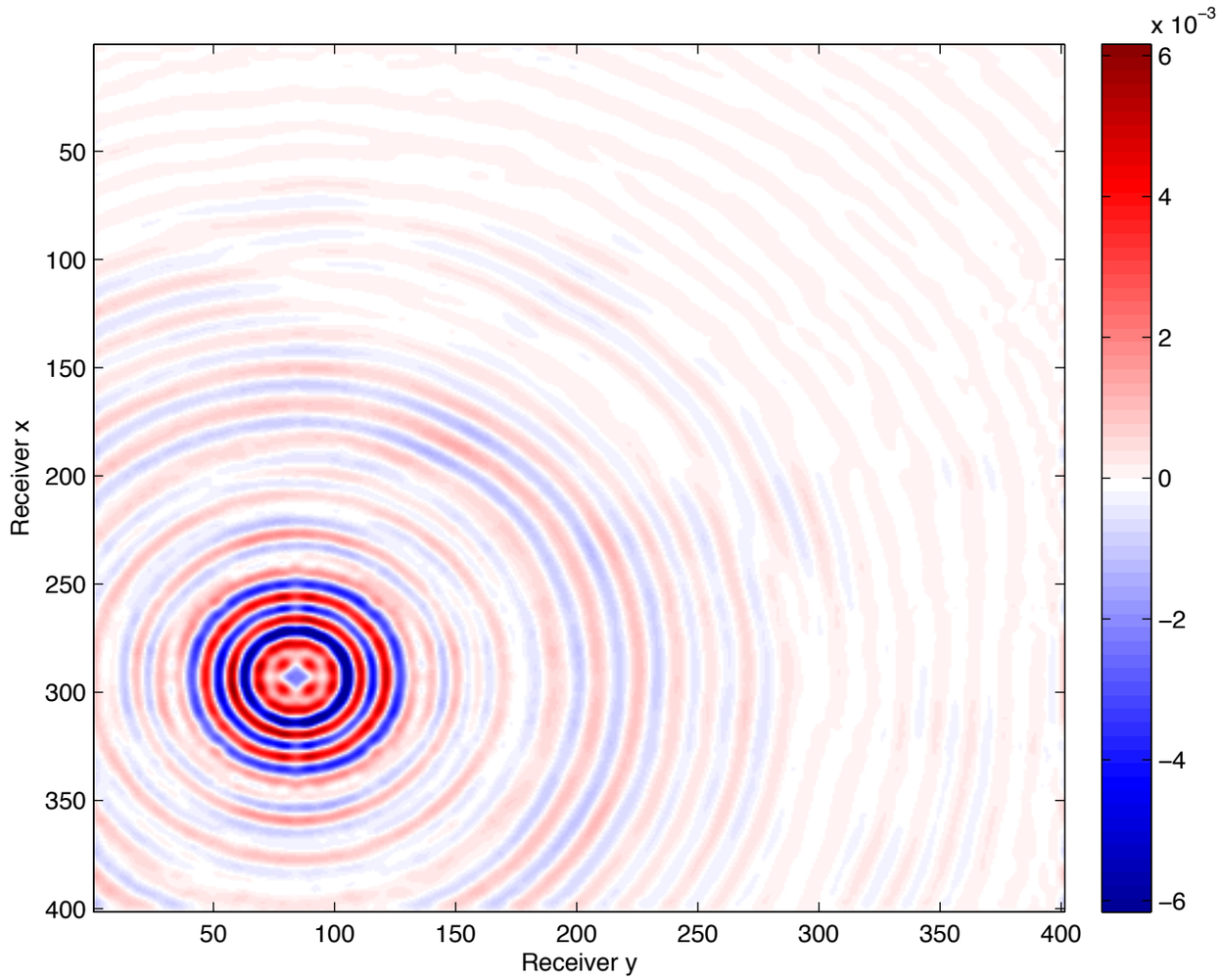
Known data

(Src x, Src y) = (50,15)



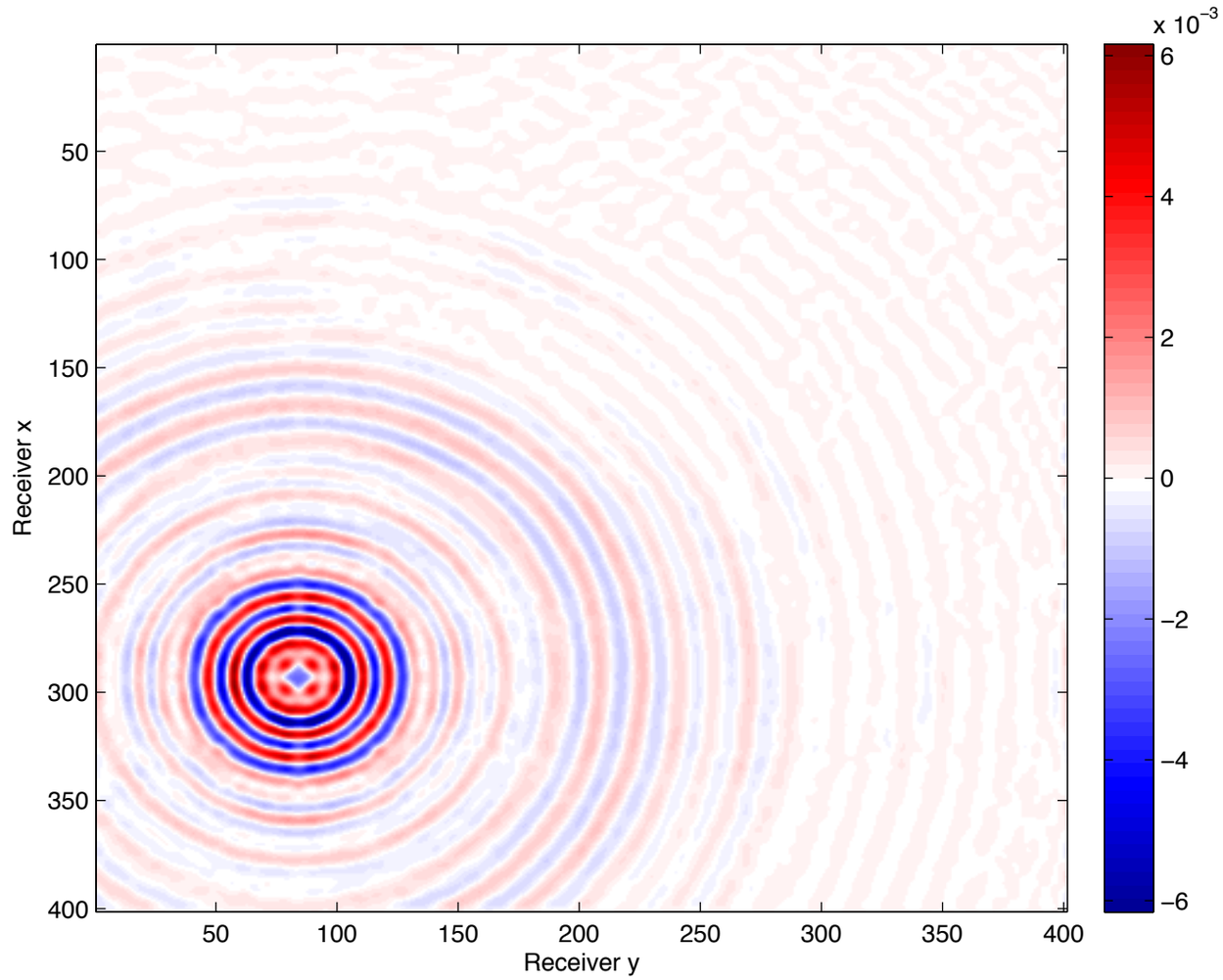
Interpolated data - SNR 13.6 dB

Source rank 20 - 500 iterations



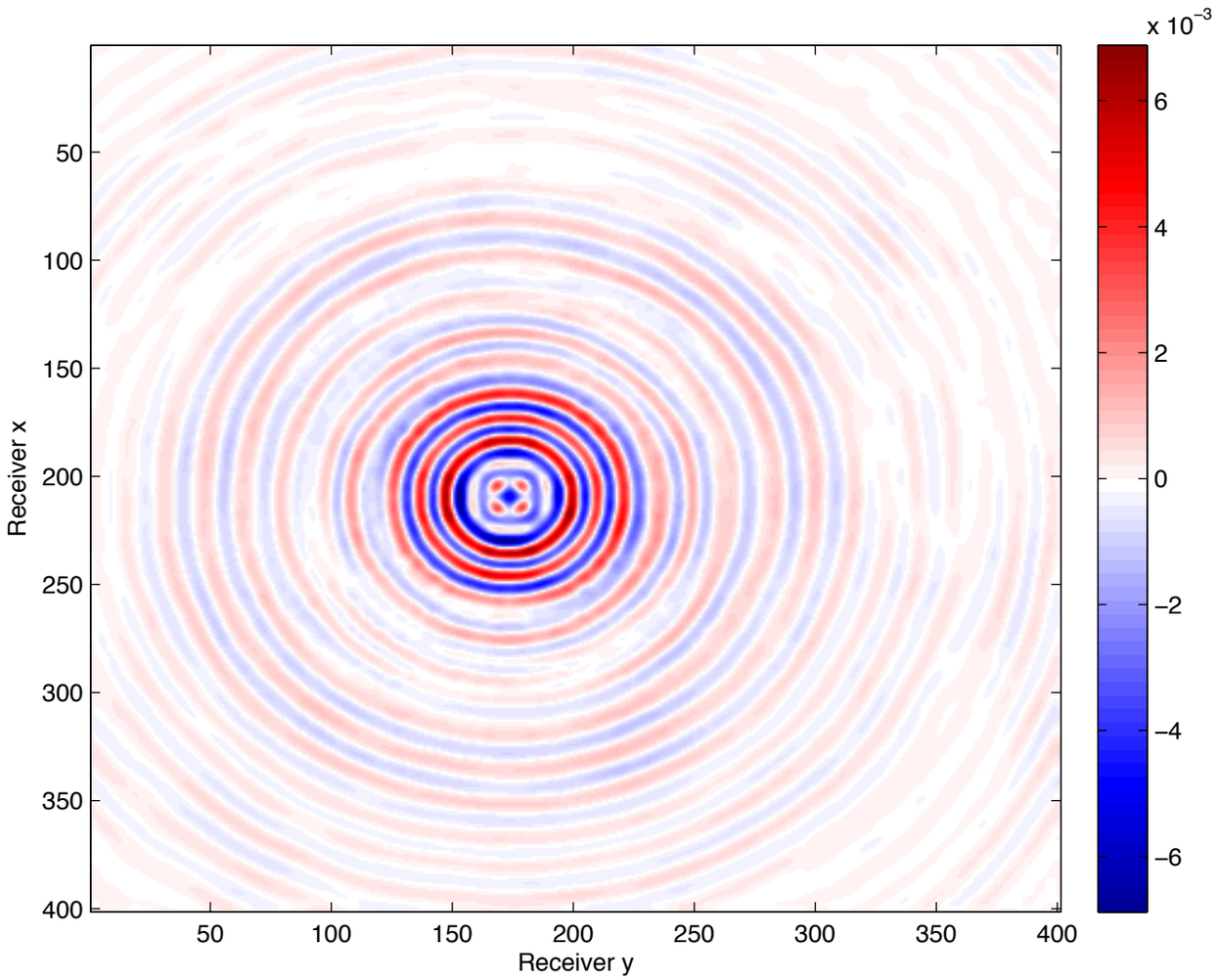
Known data

(Src x, Src y) = (50,15)



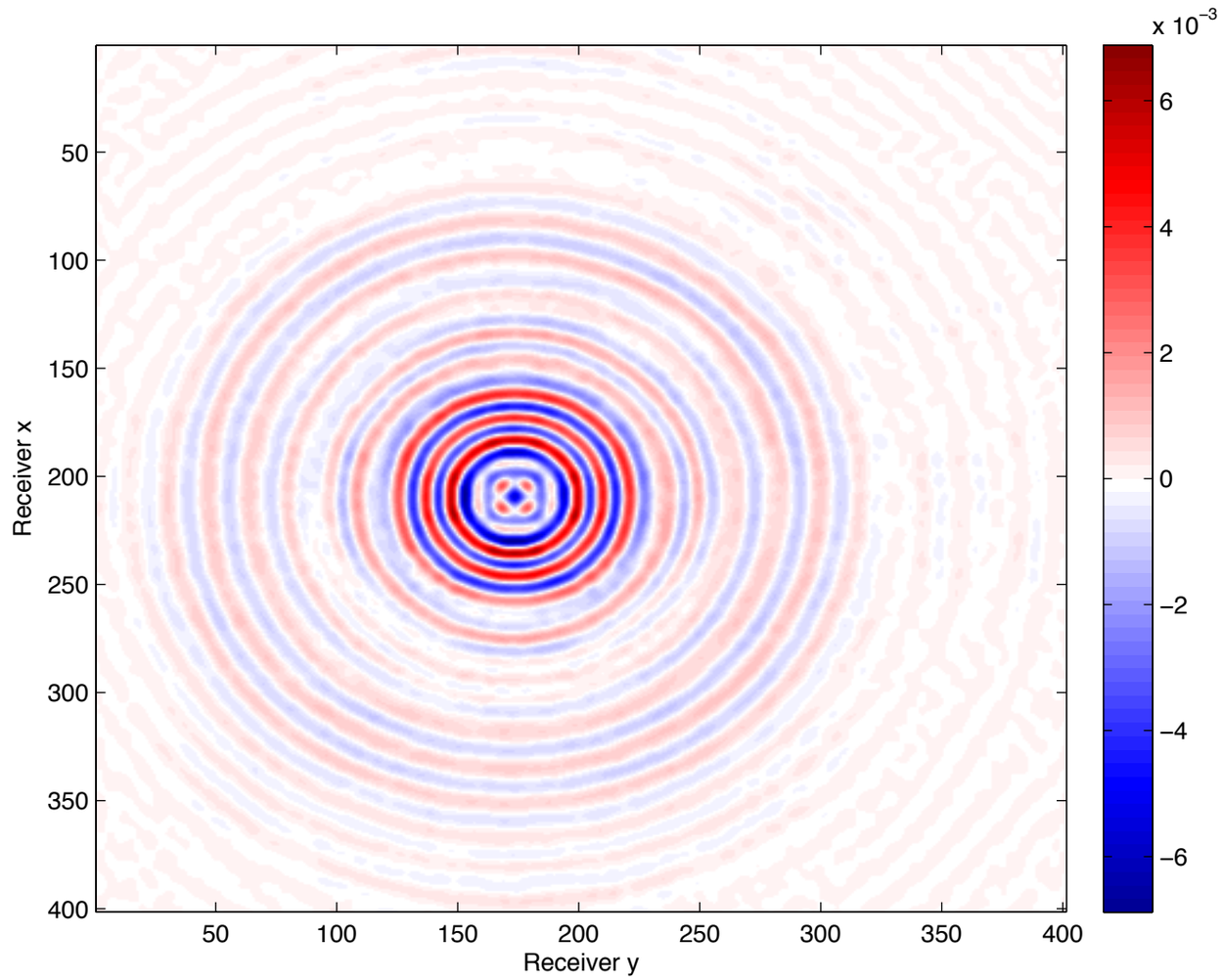
Interpolated data - SNR 15.2 dB

Source rank 20 - 500 iterations



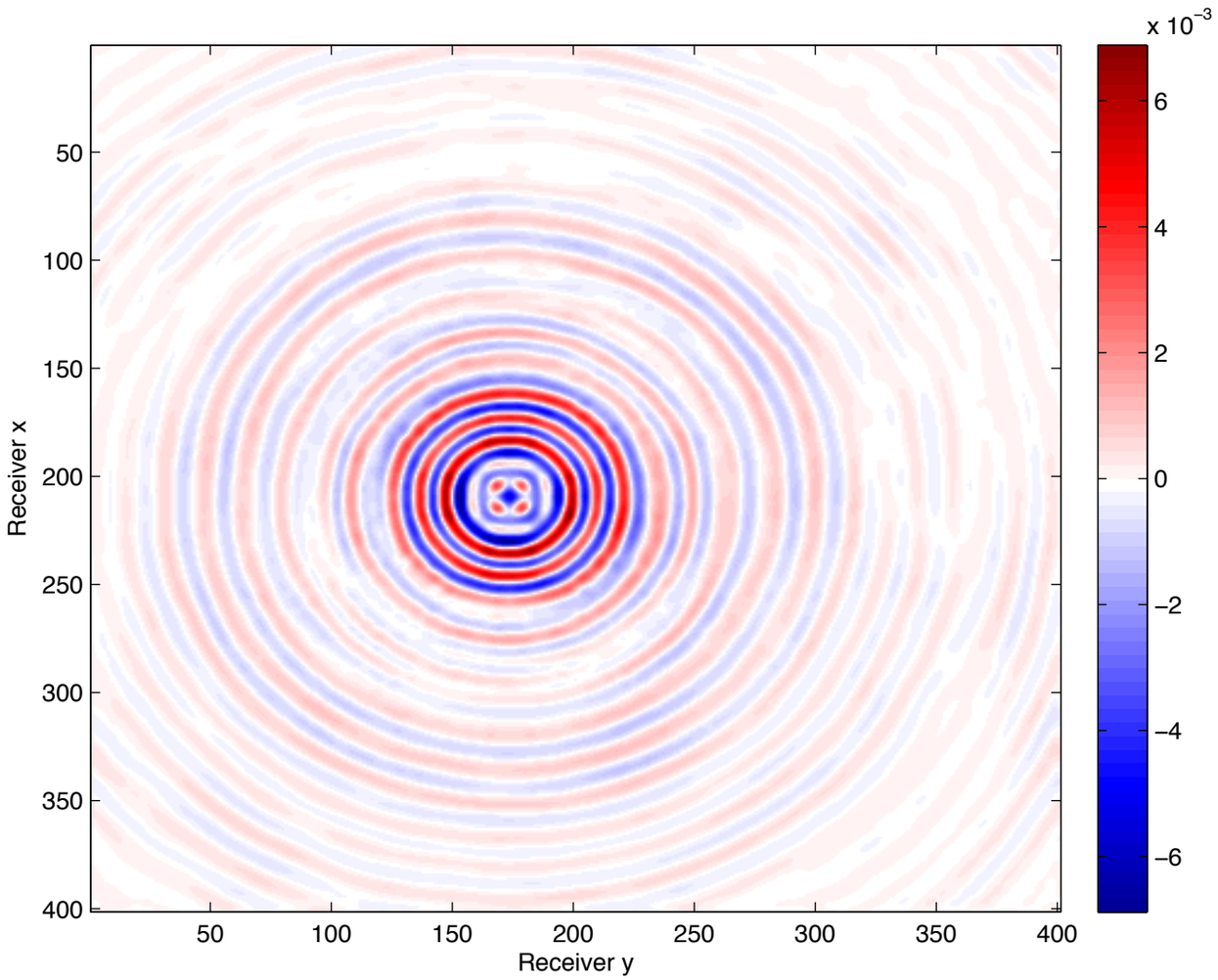
Known data

(Src x, Src y) = (36,30)



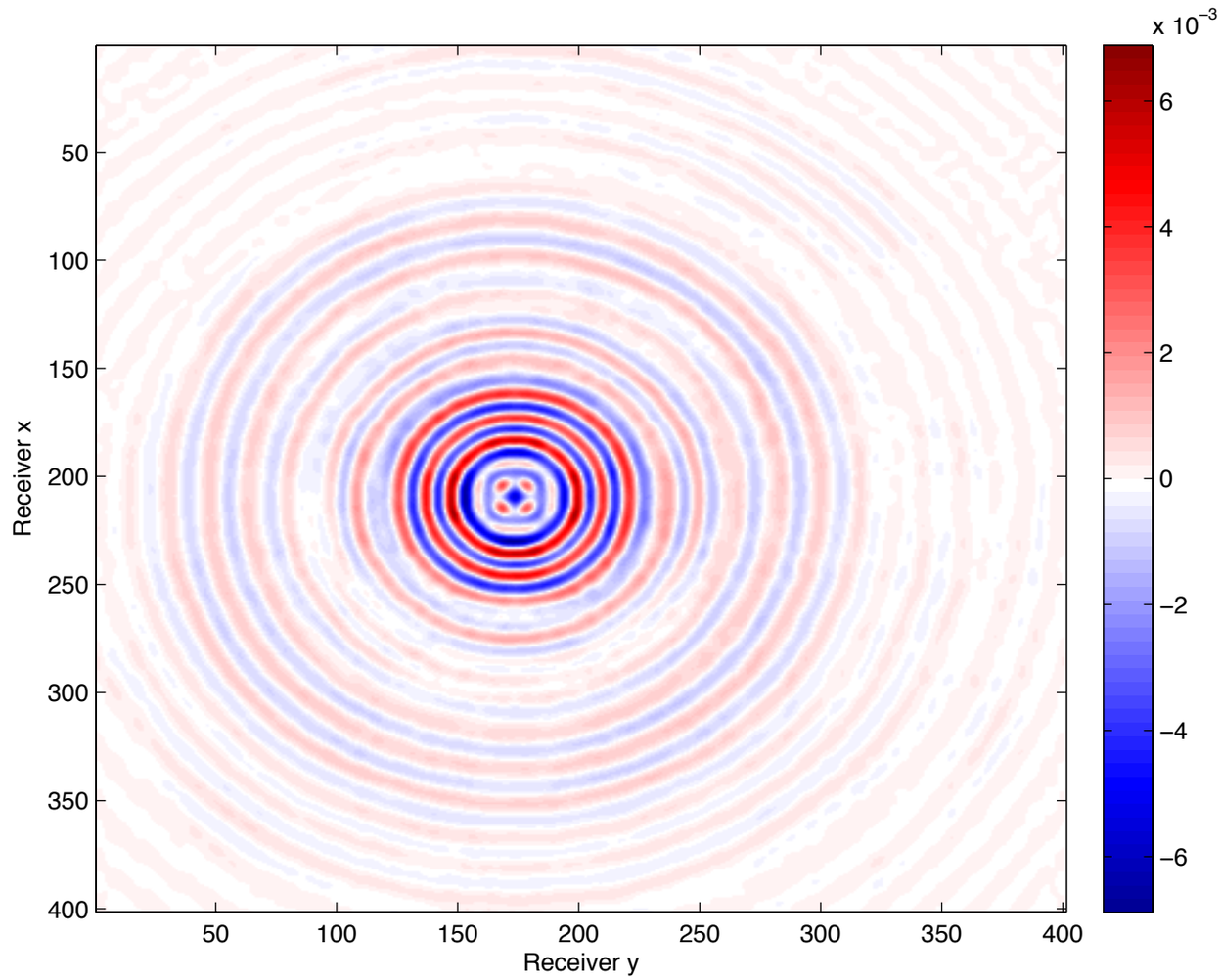
Interpolated data - SNR 12.6 dB

Source rank 20 - Regularization



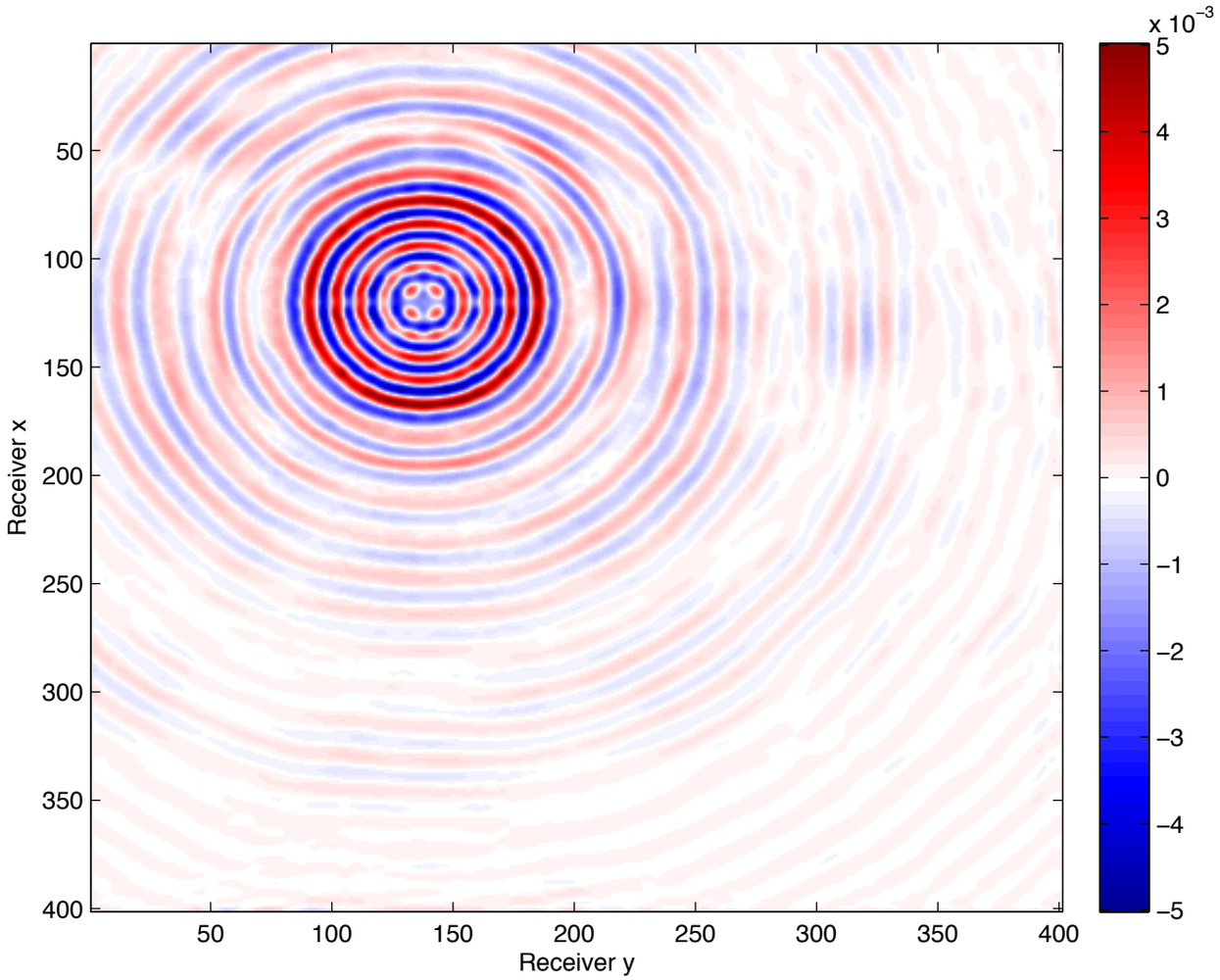
Known data

(Src x, Src y) = (36,30)



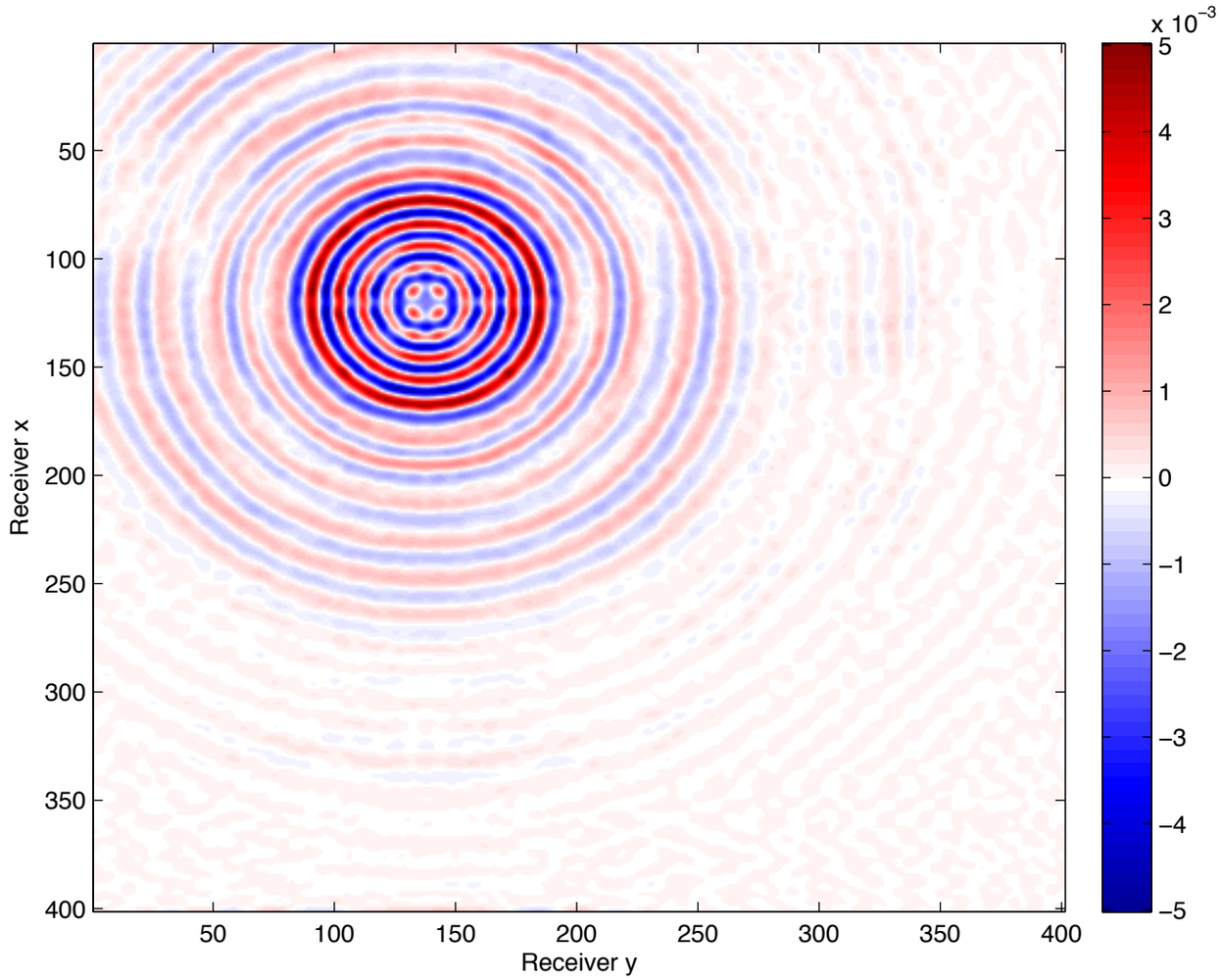
Interpolated data - SNR 14.2 dB

Source rank 20 - 500 iterations



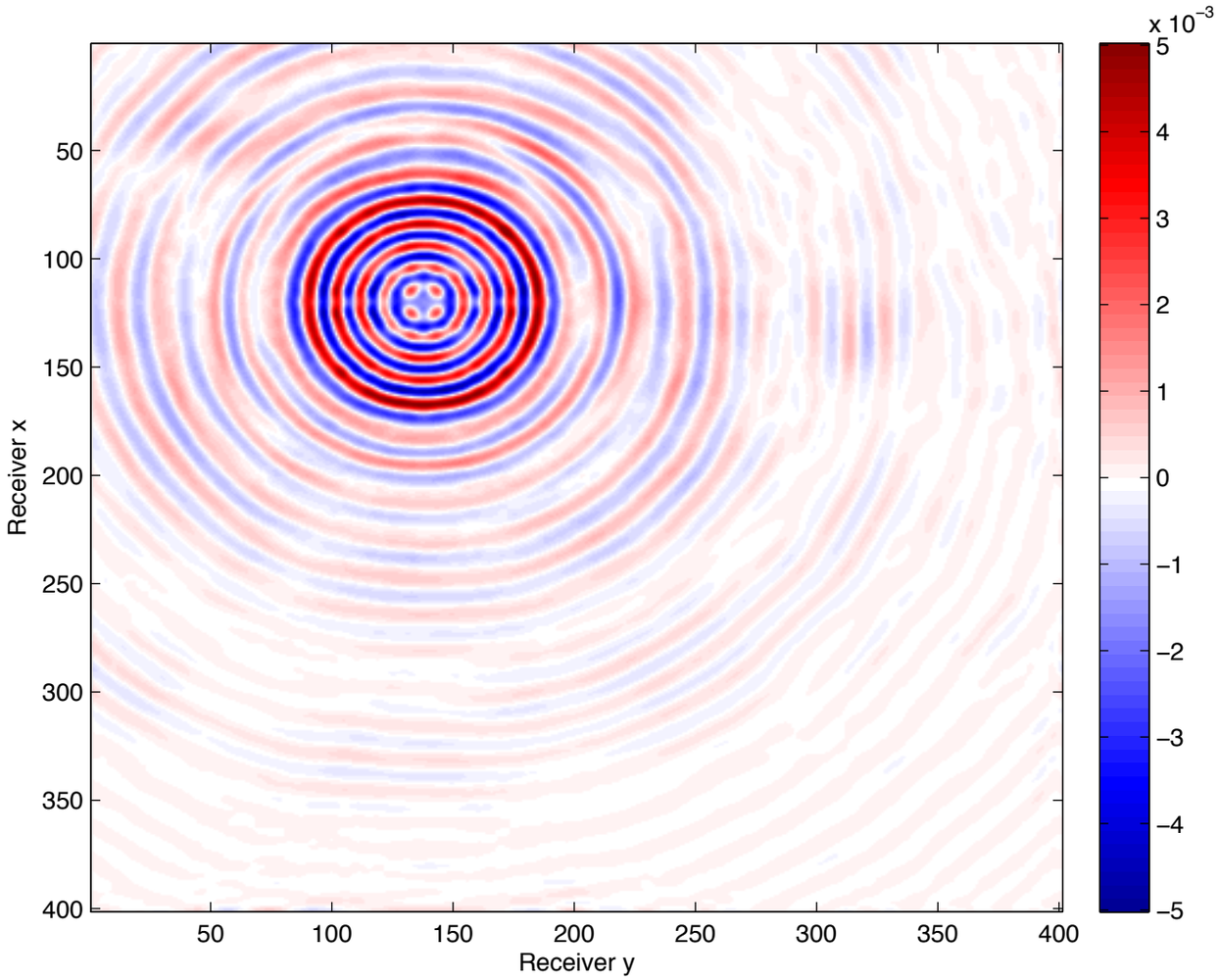
Known data

(Src x, Src y) = (21,24)



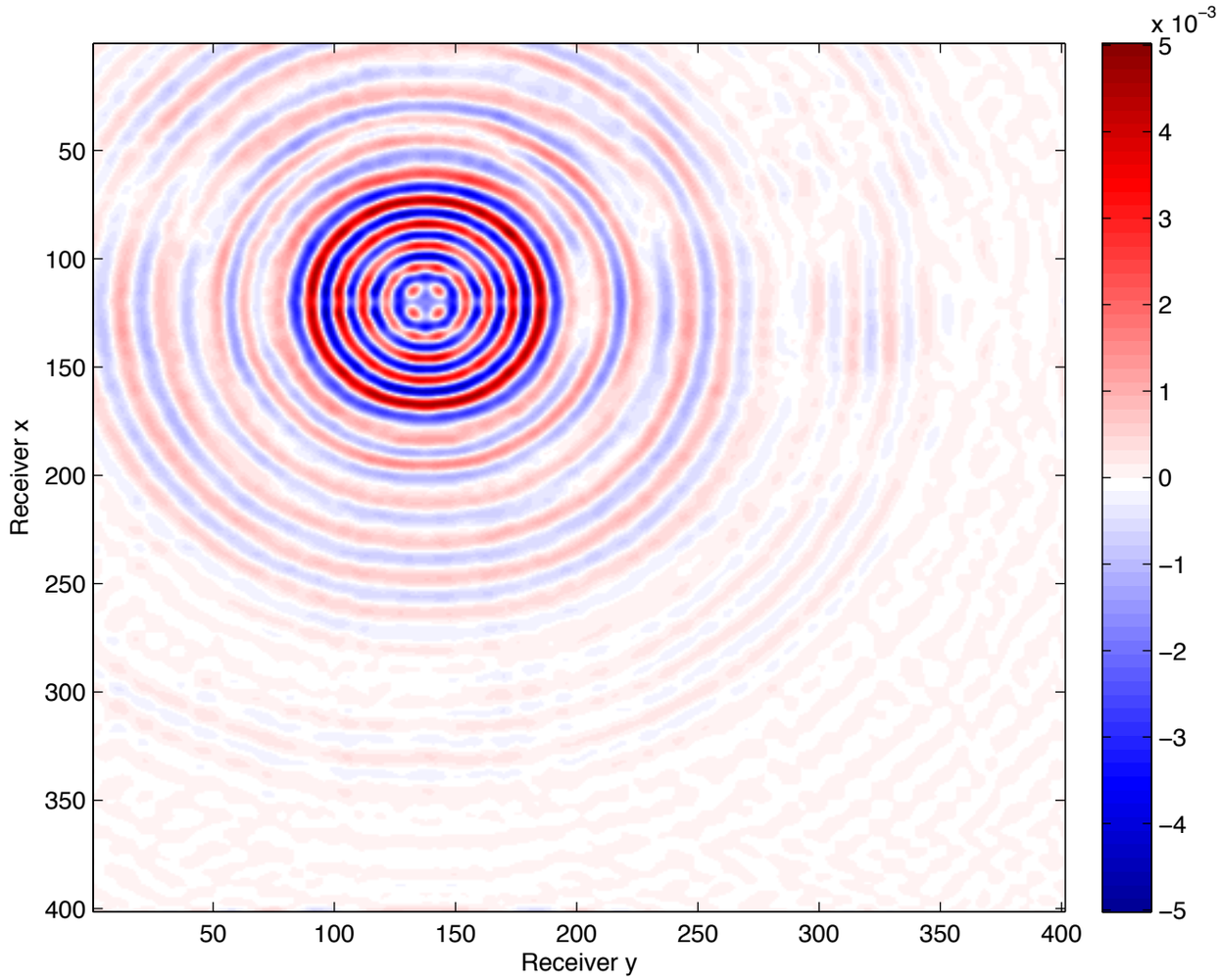
Interpolated data - SNR 12.5 dB

Source rank 20 - Regularization



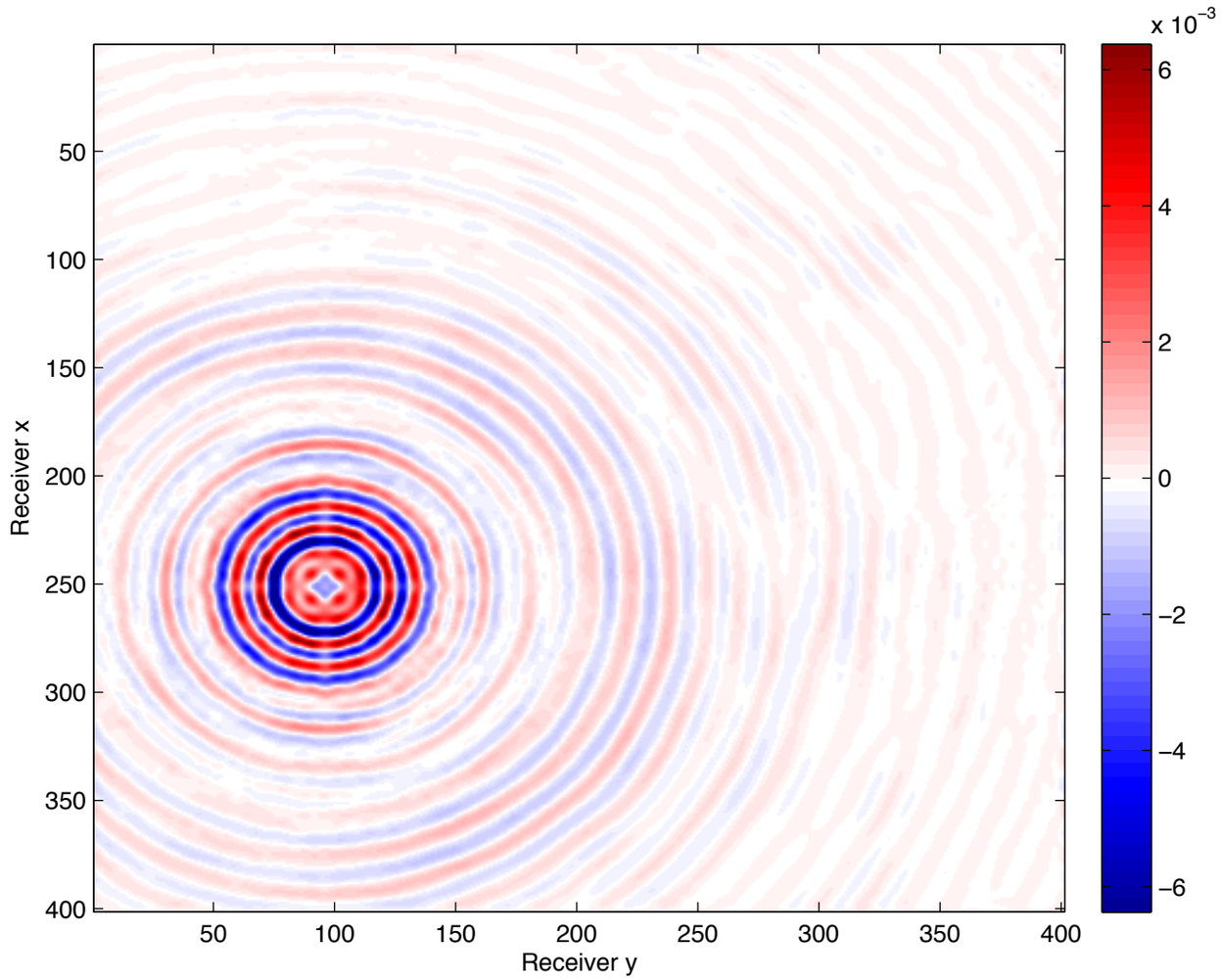
Known data

(Src x, Src y) = (21,24)



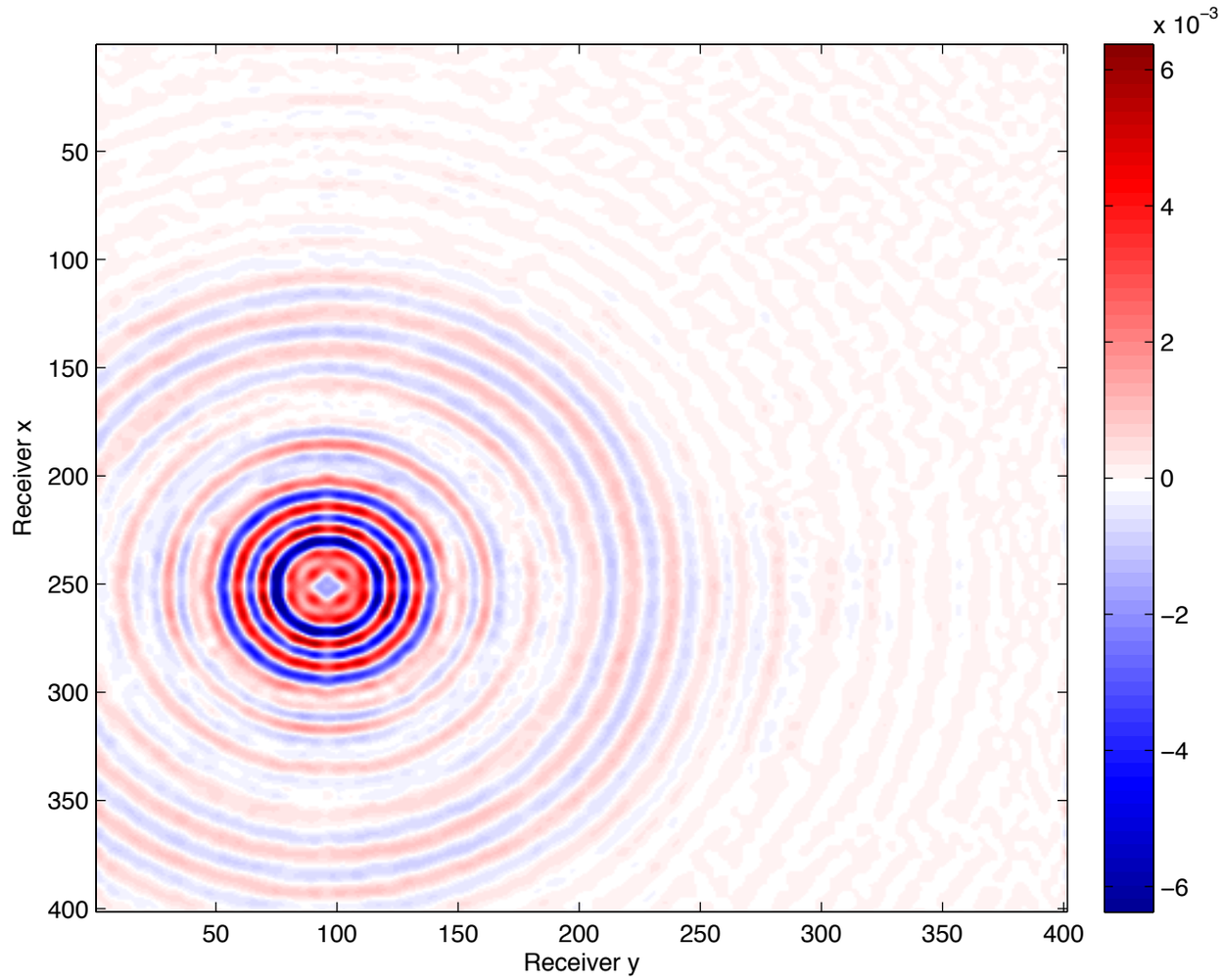
Interpolated data - SNR 14.2 dB

Source rank 20 - No Regularization



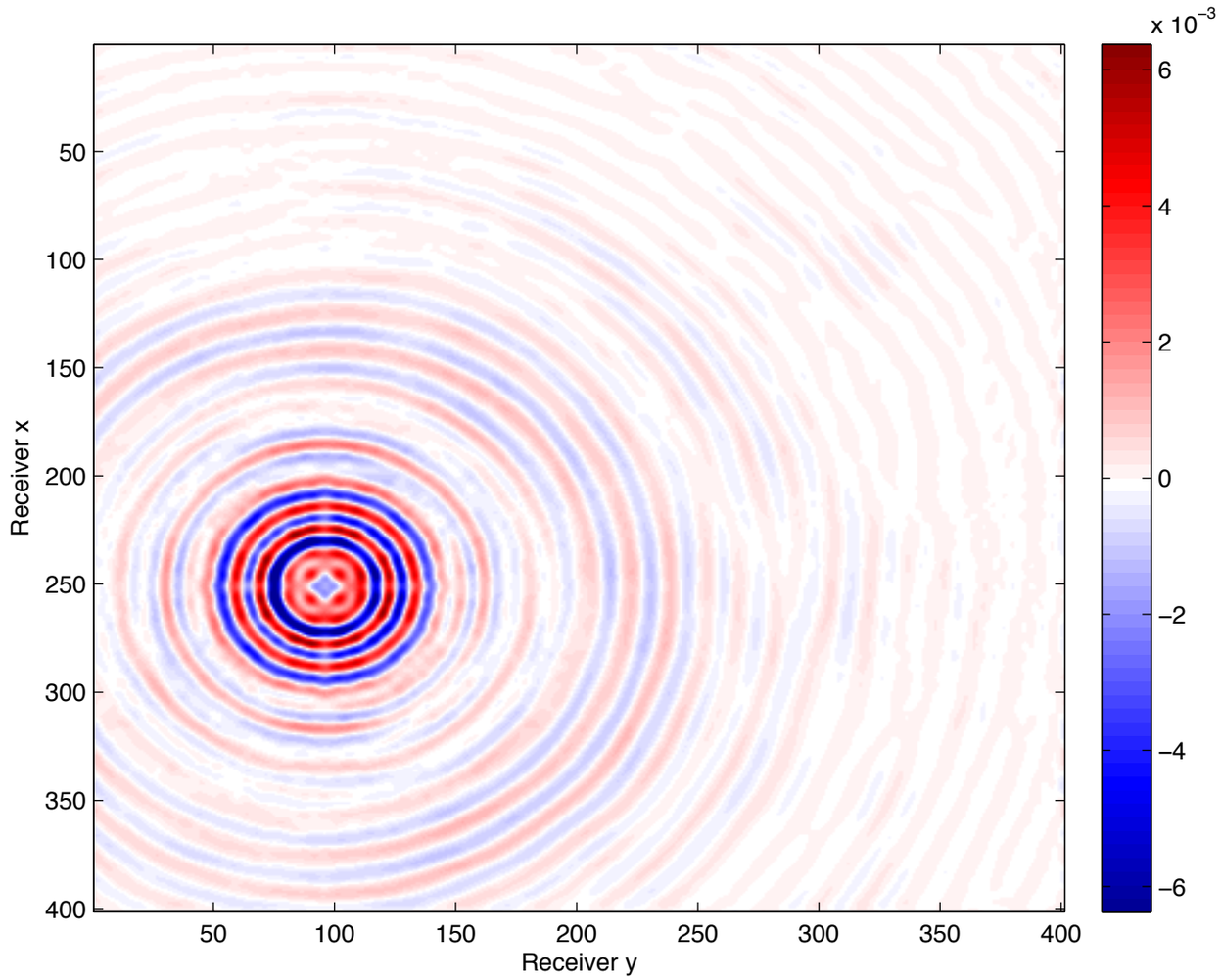
Known data

(Src x, Src y) = (64,25)



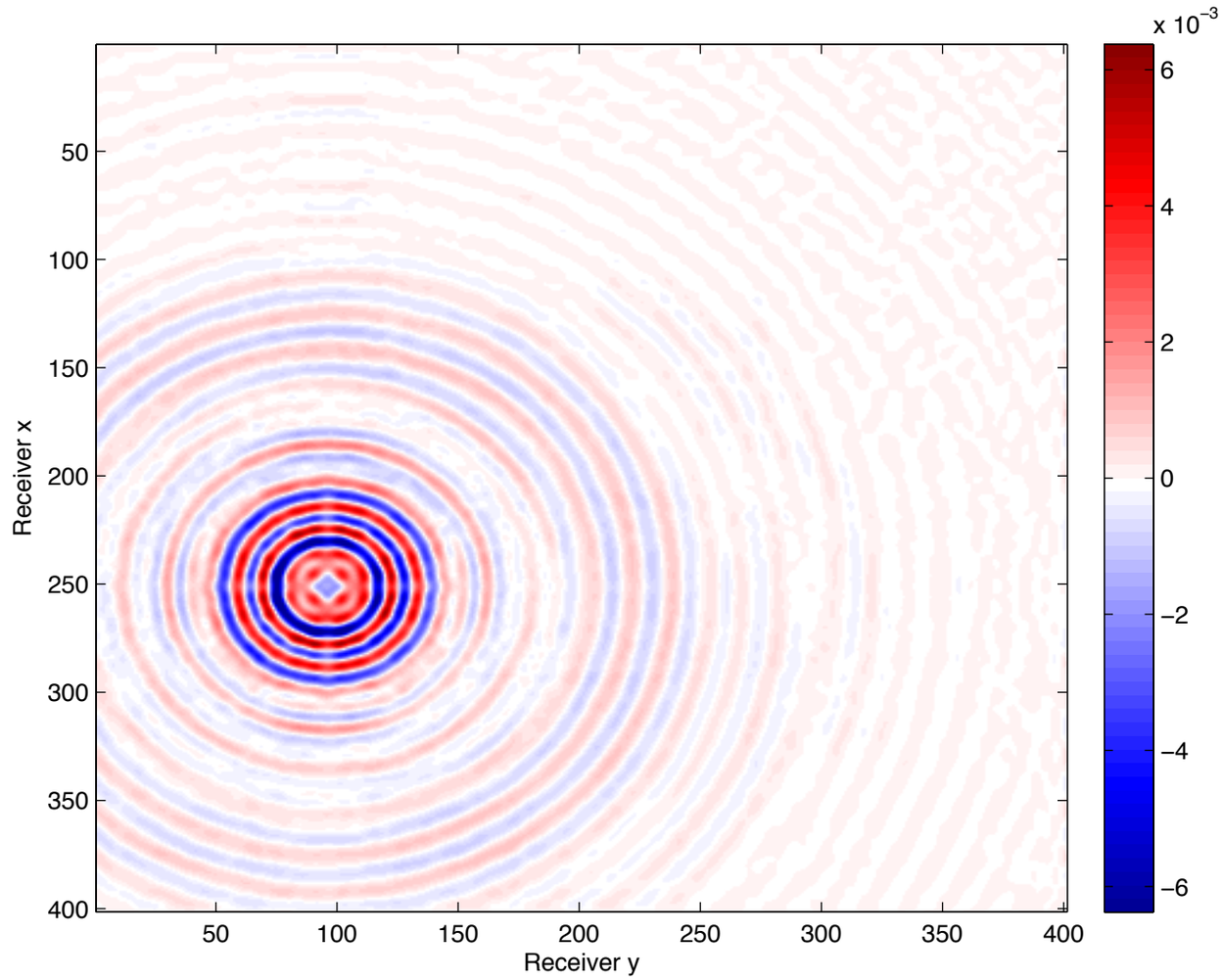
Interpolated data - SNR 13.9 dB

Source rank 20 - Regularization



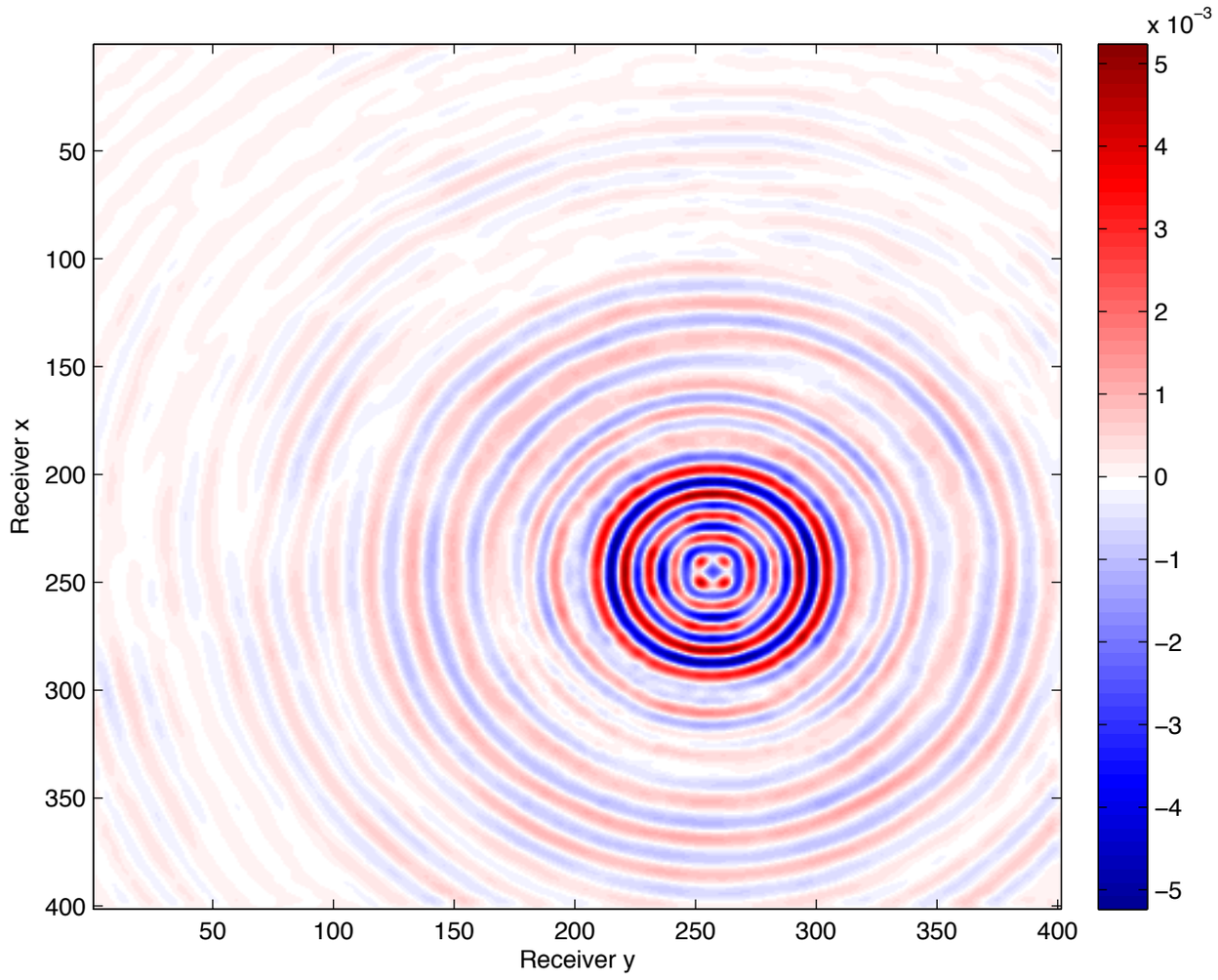
Known data

(Src x, Src y) = (64,25)



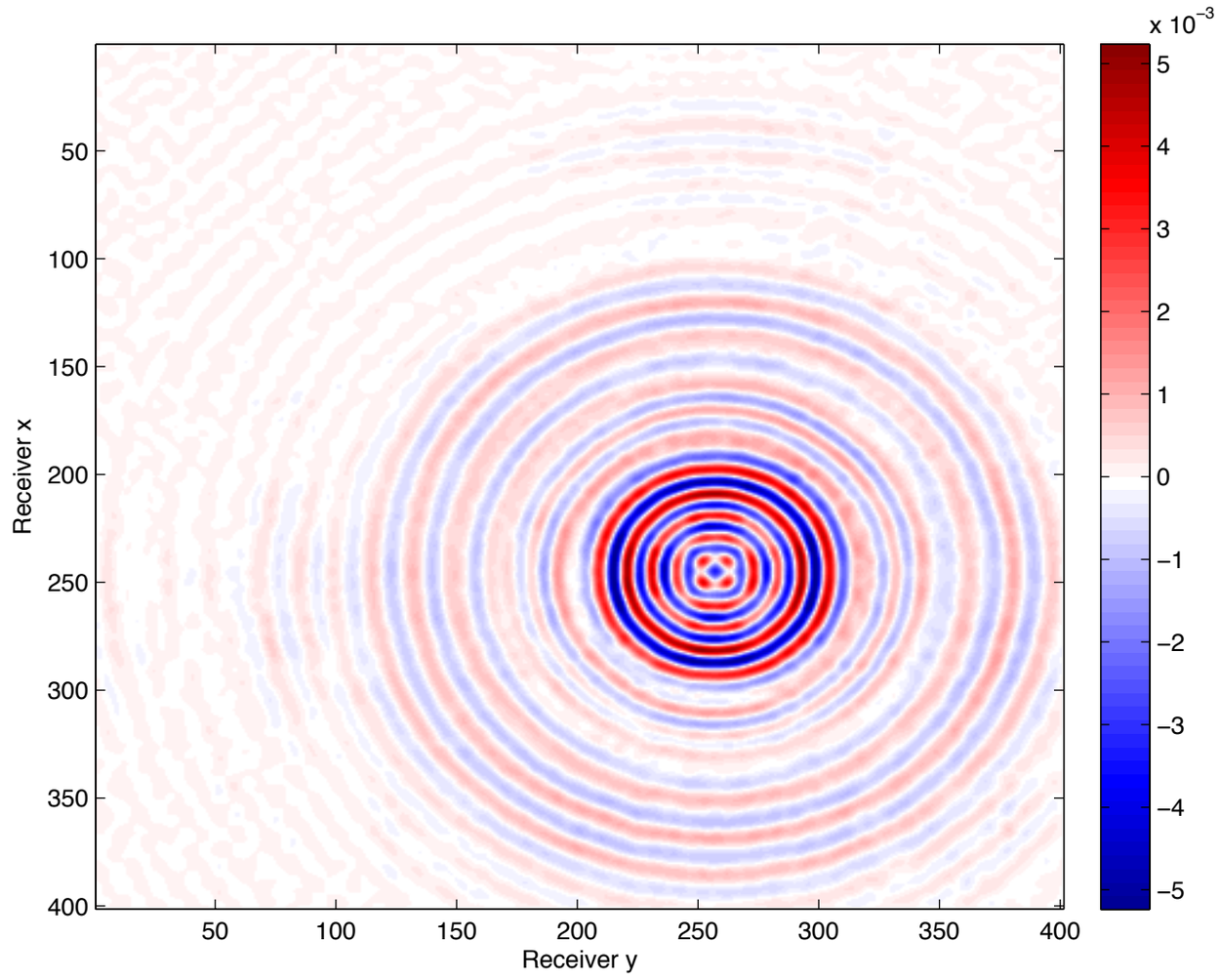
Interpolated data - SNR 15.4 dB

Source rank 20 - No Regularization



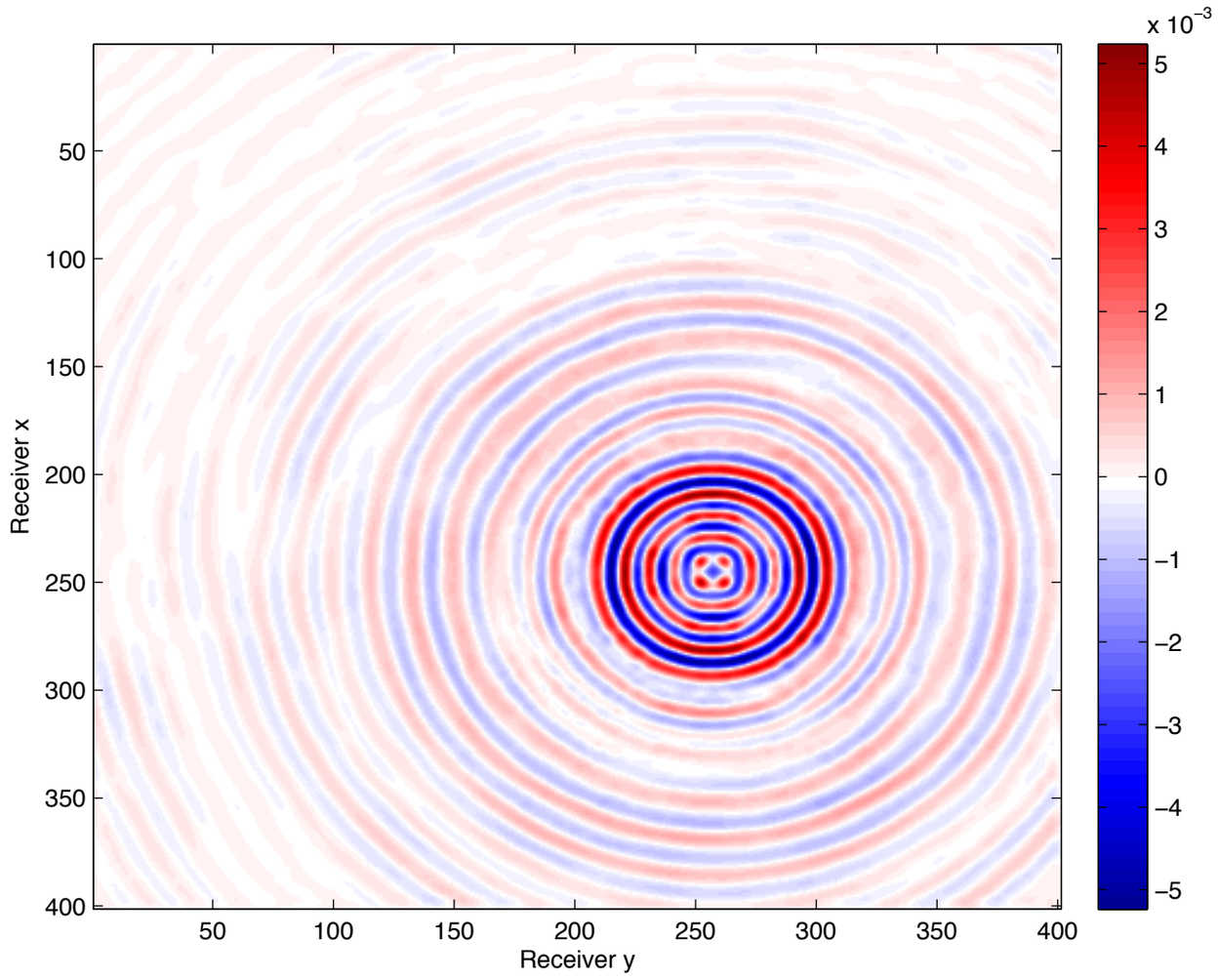
Known data

(Src x, Src y) = (63,66)



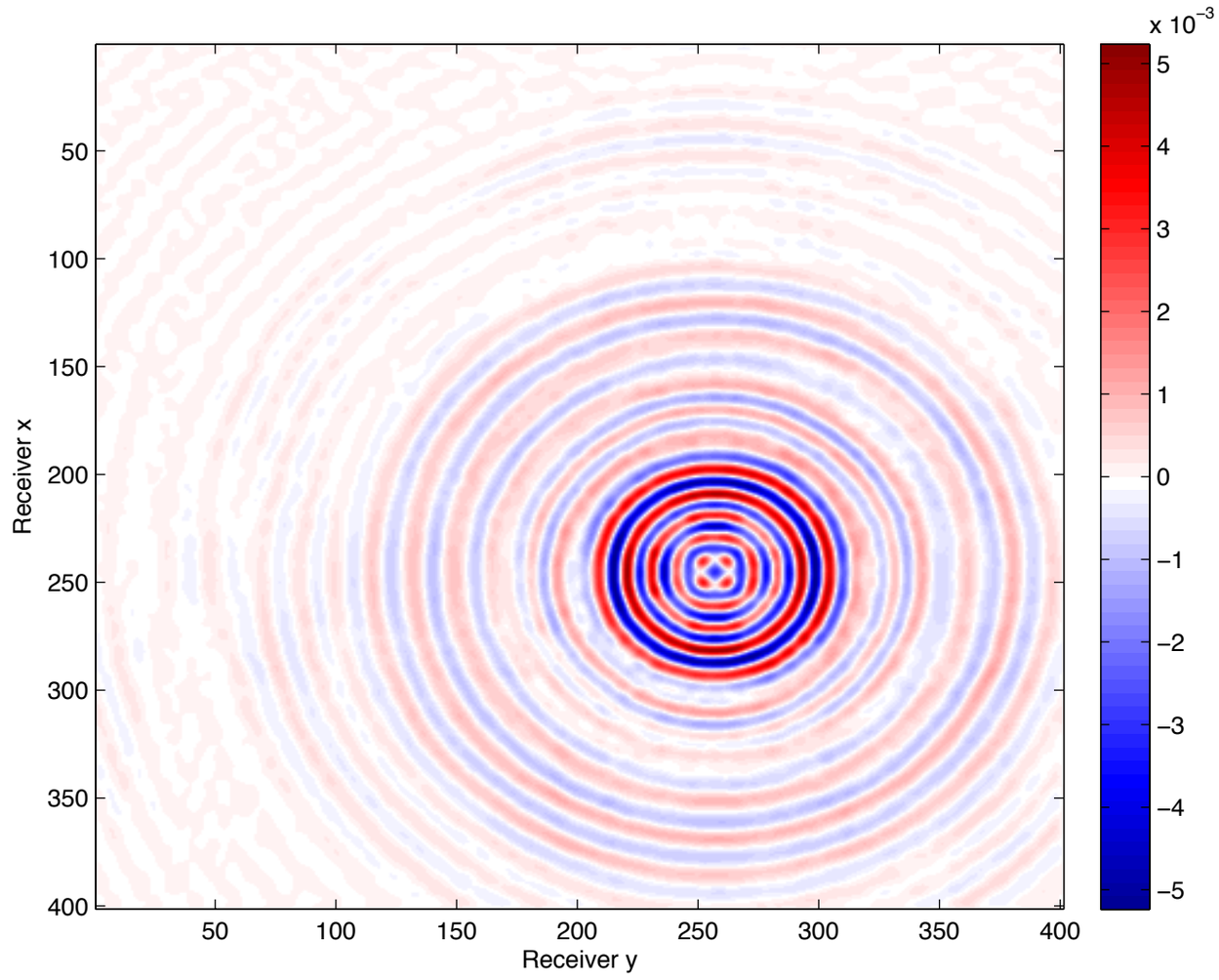
Interpolated data - SNR 13.2 dB

Source rank 20 - Regularization



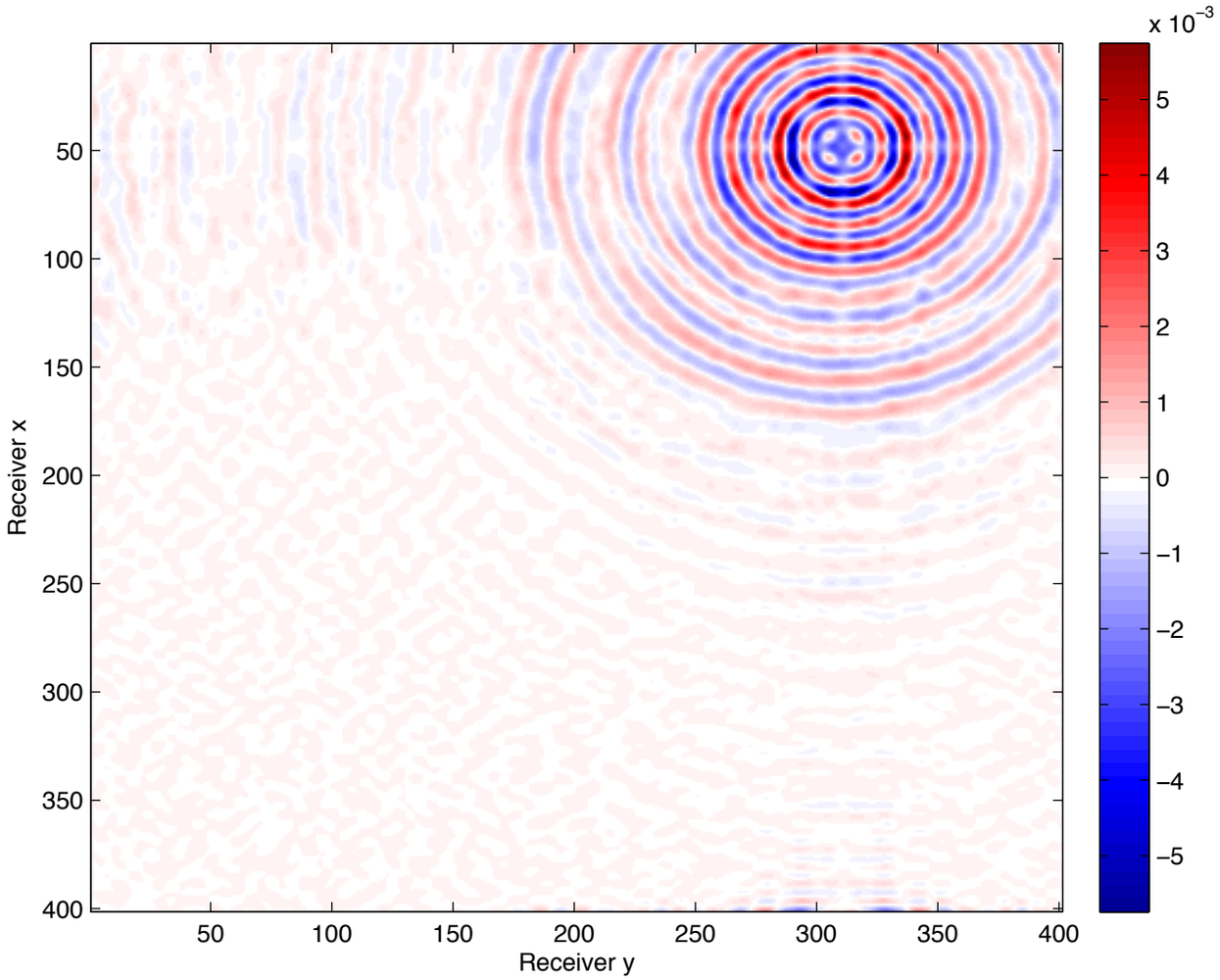
Known data

(Src x, Src y) = (63,66)



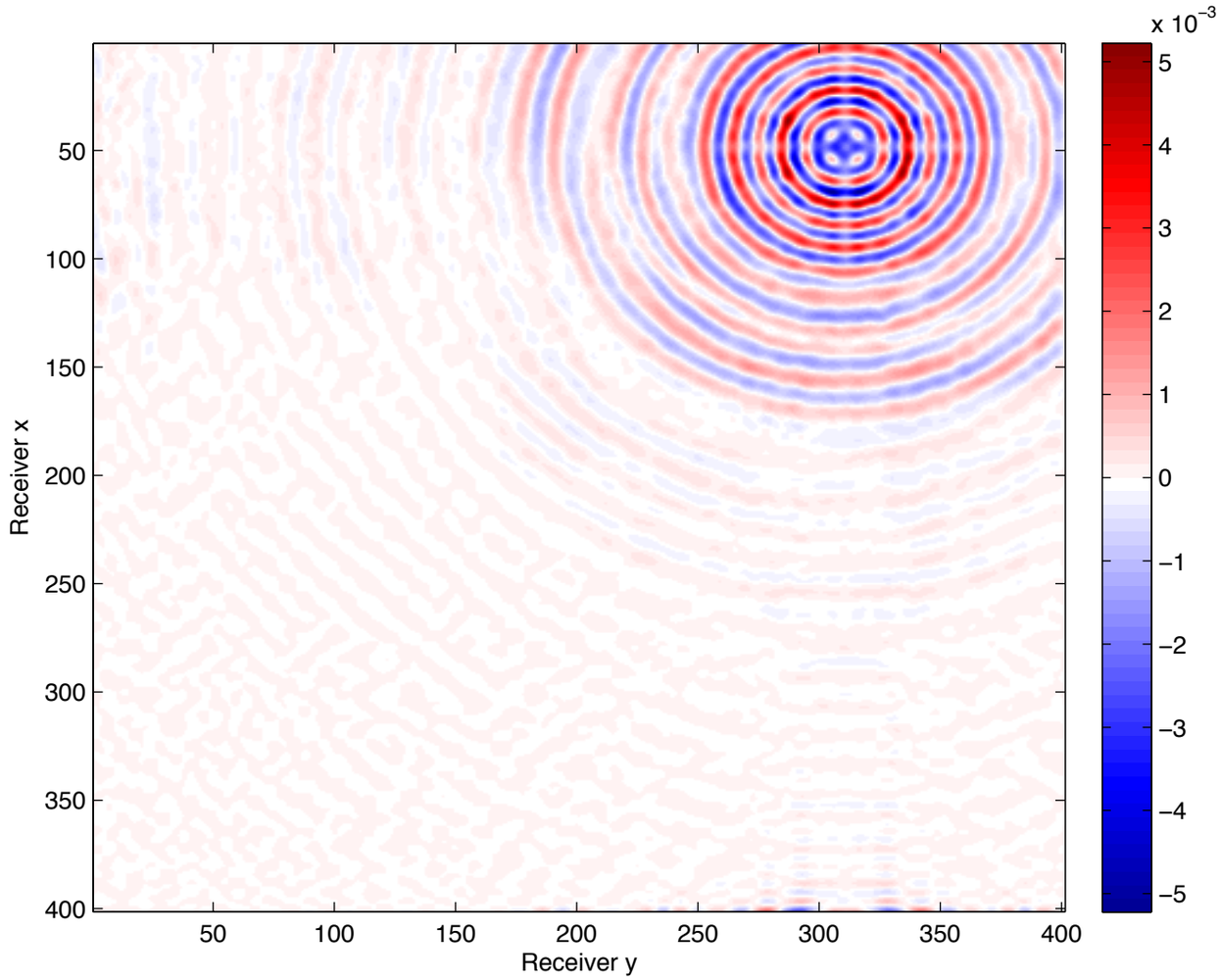
Interpolated data - SNR 15 dB

Interpolated Data



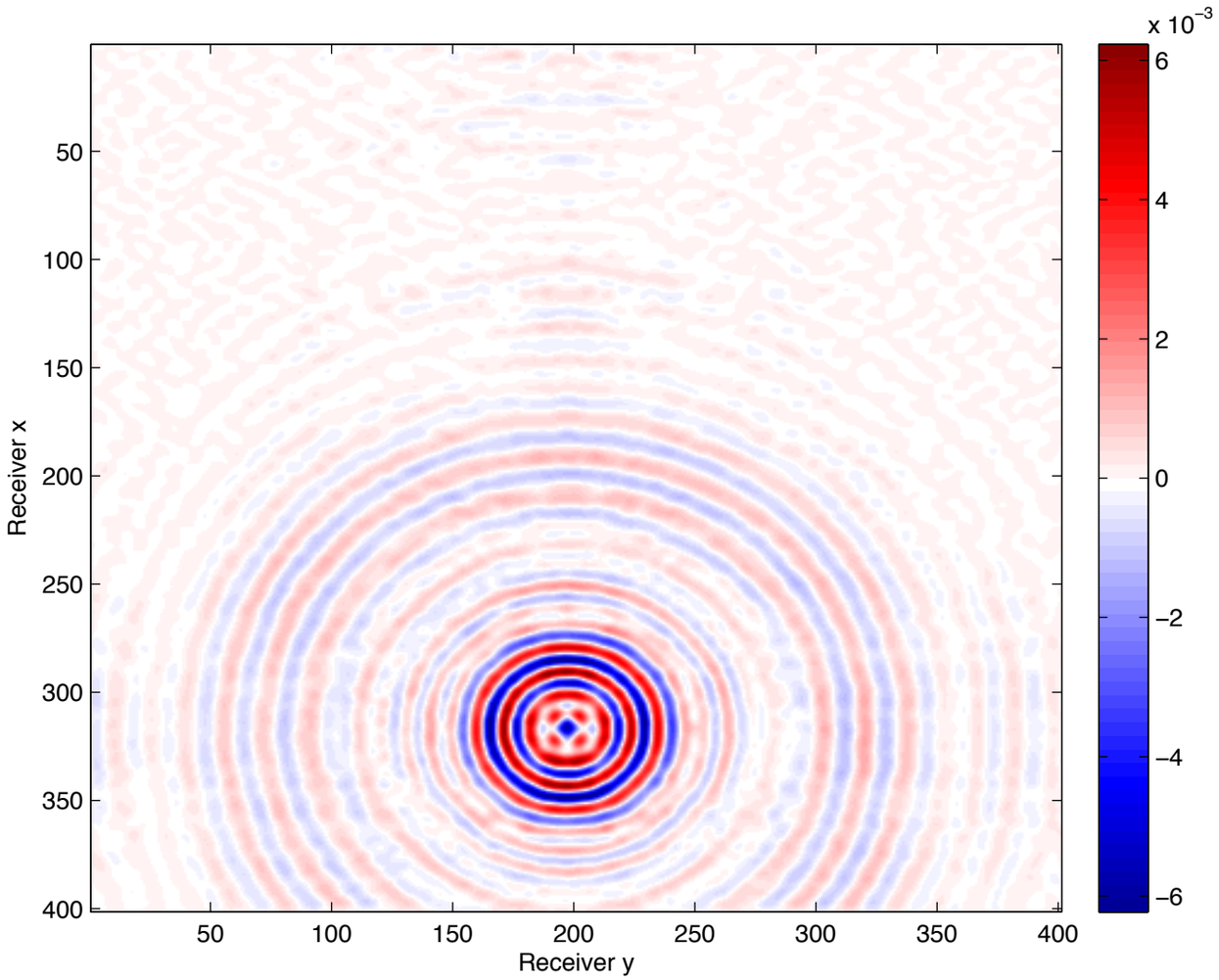
No Regularization

(Src x, Src y) = (9,53)



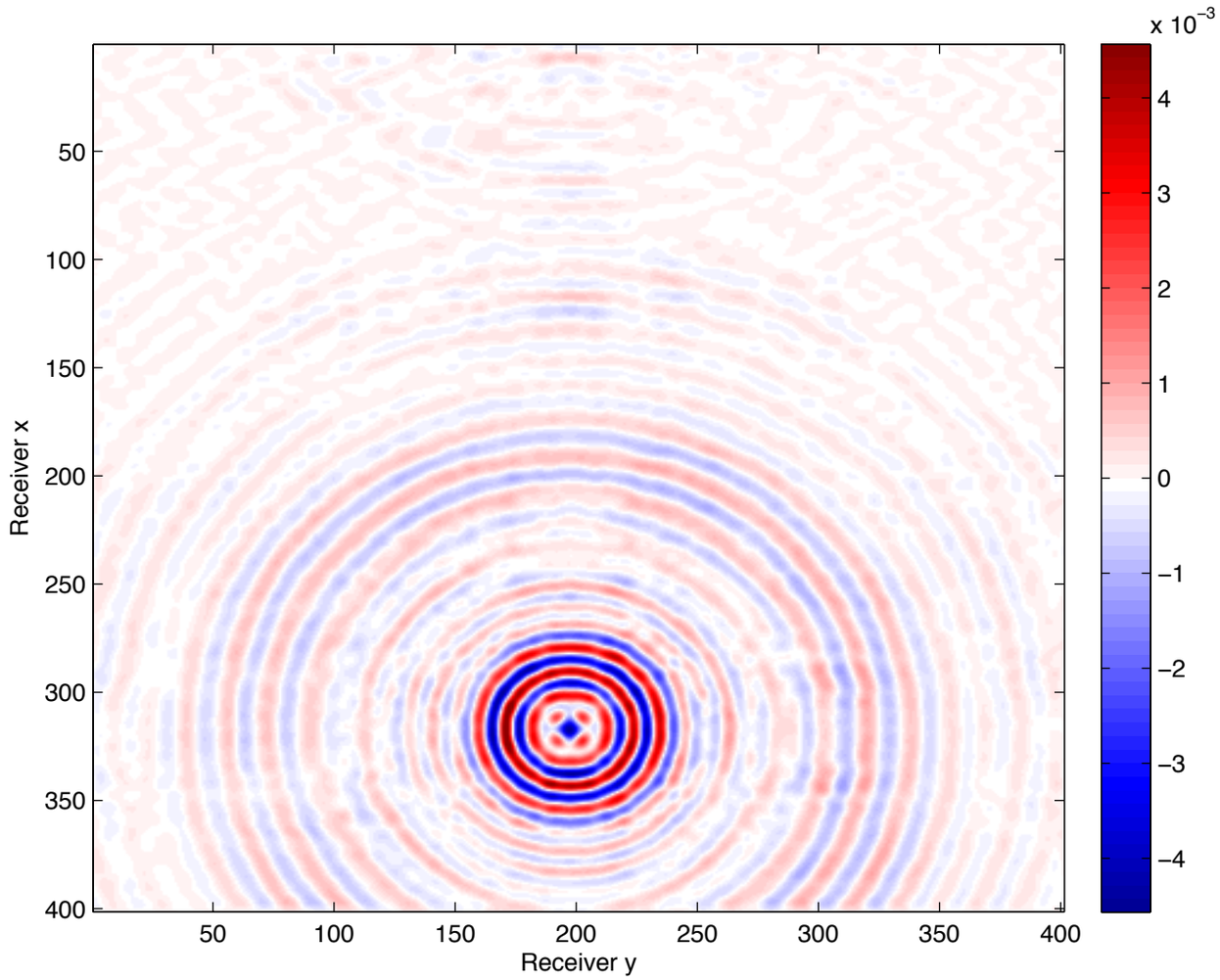
Regularization

Interpolated Data



No Regularization

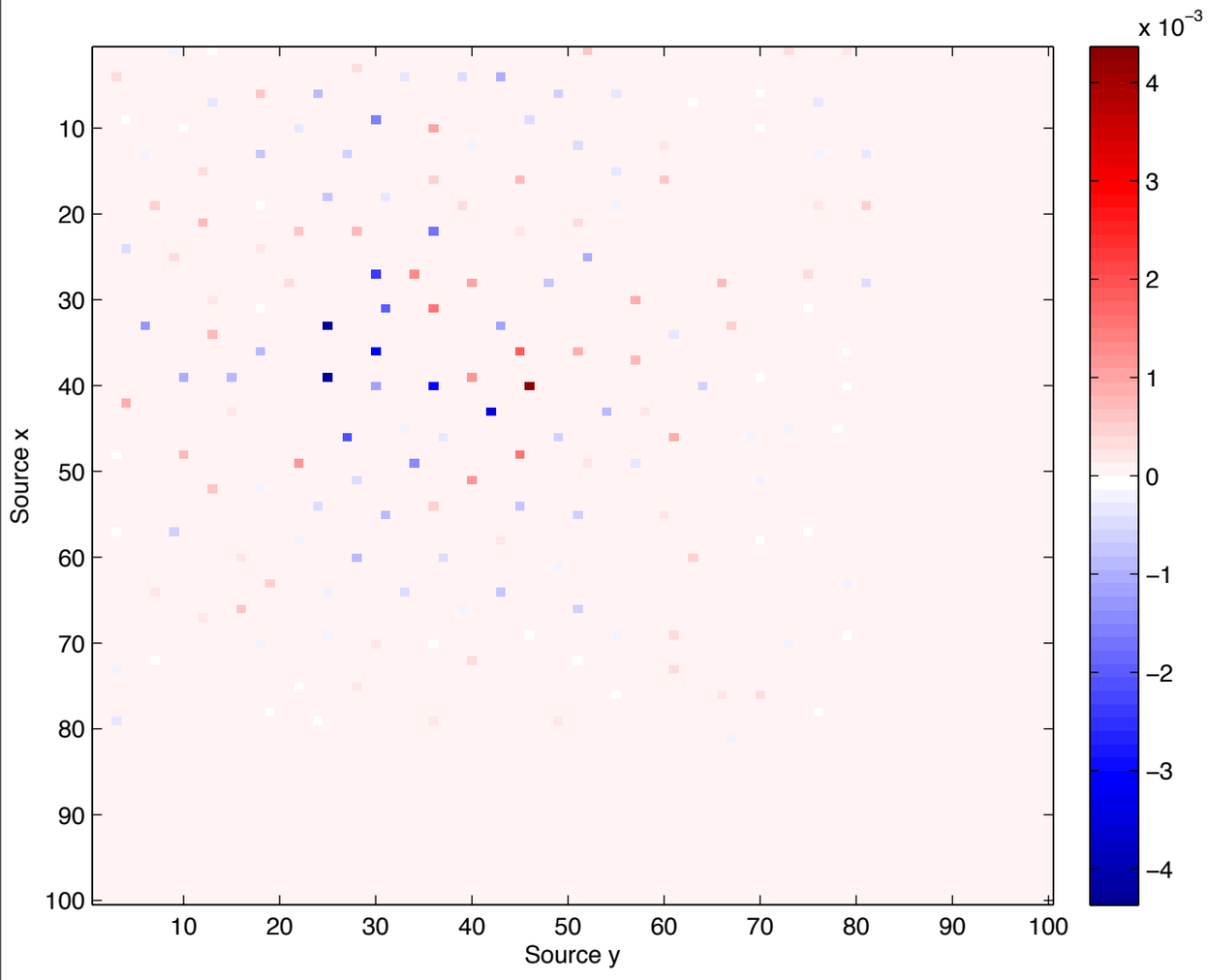
(Src x, Src y) = (54,34)



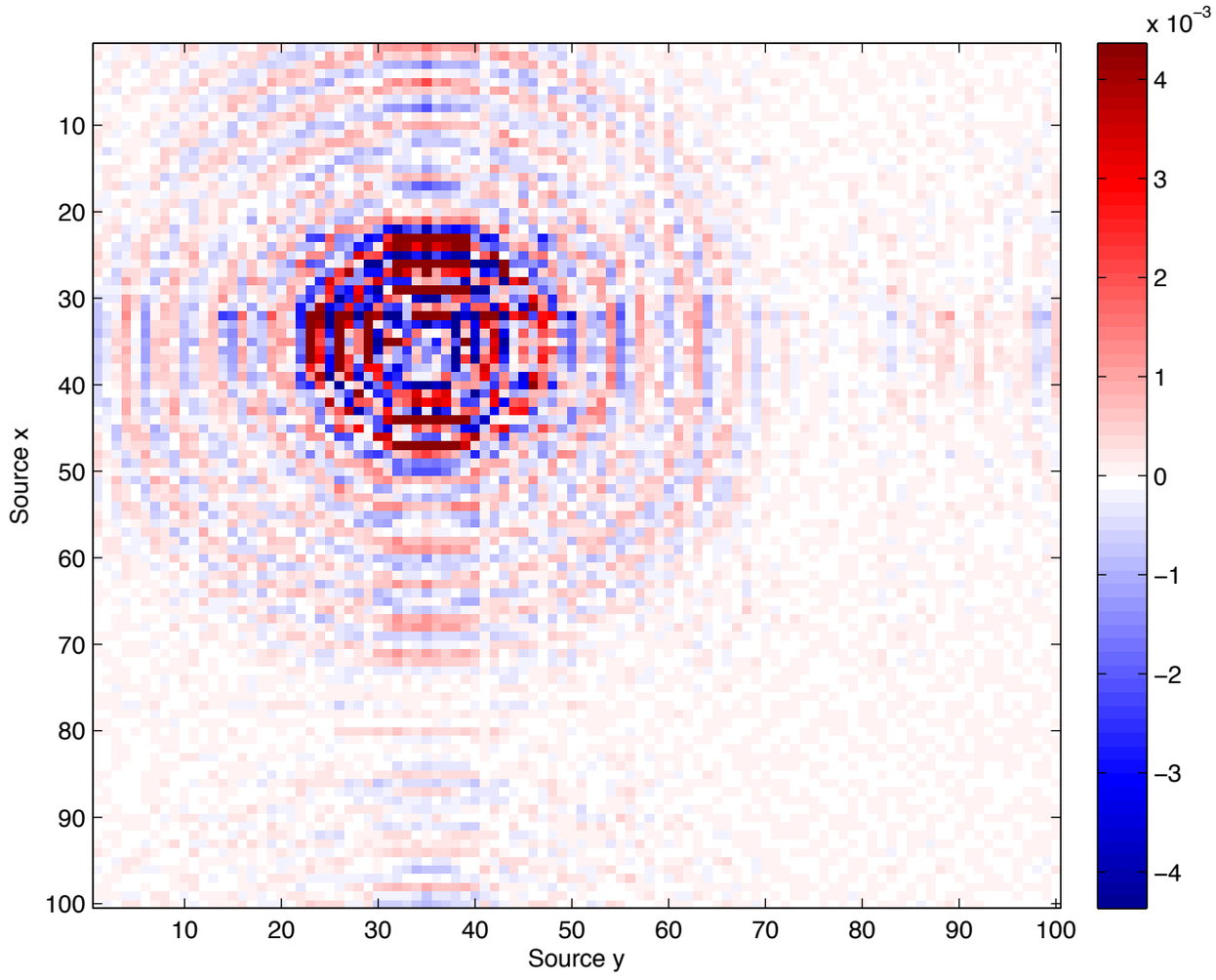
Regularization

Common Receiver Gathers

Rec x = 35, Rec y = 35



Known data

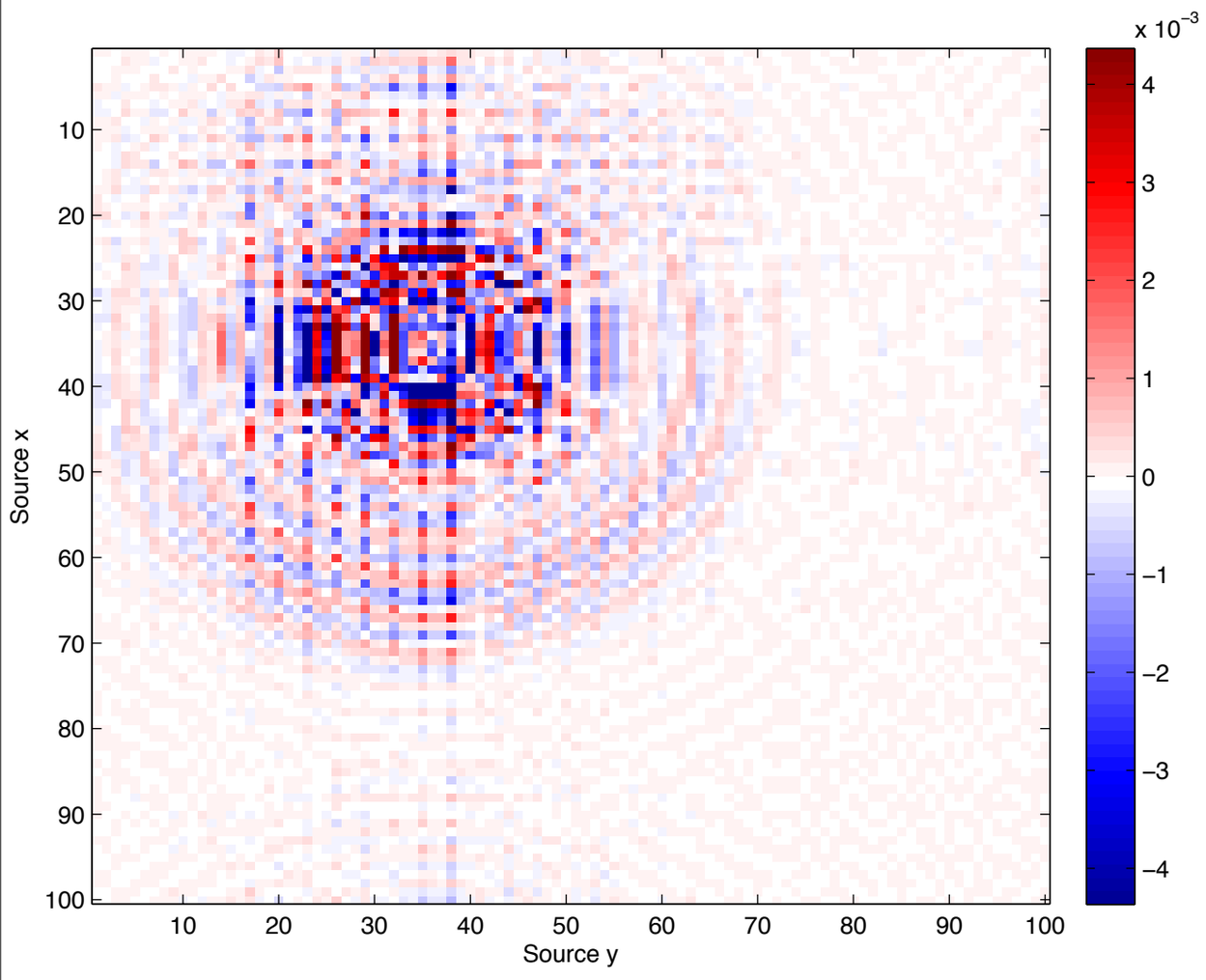


Interpolated data

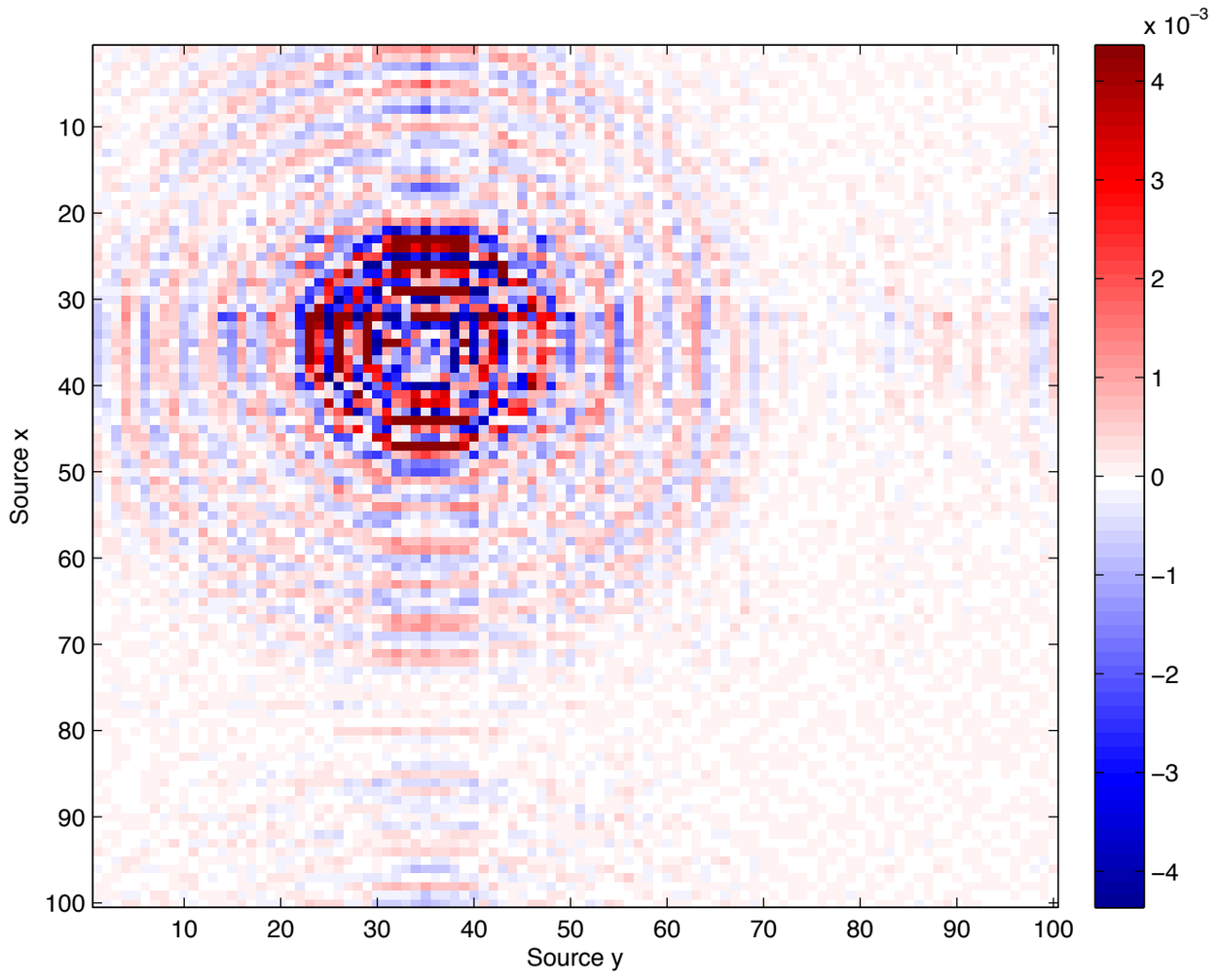
SLIM

Common Receiver Gathers

Rec x = 35, Rec y = 35



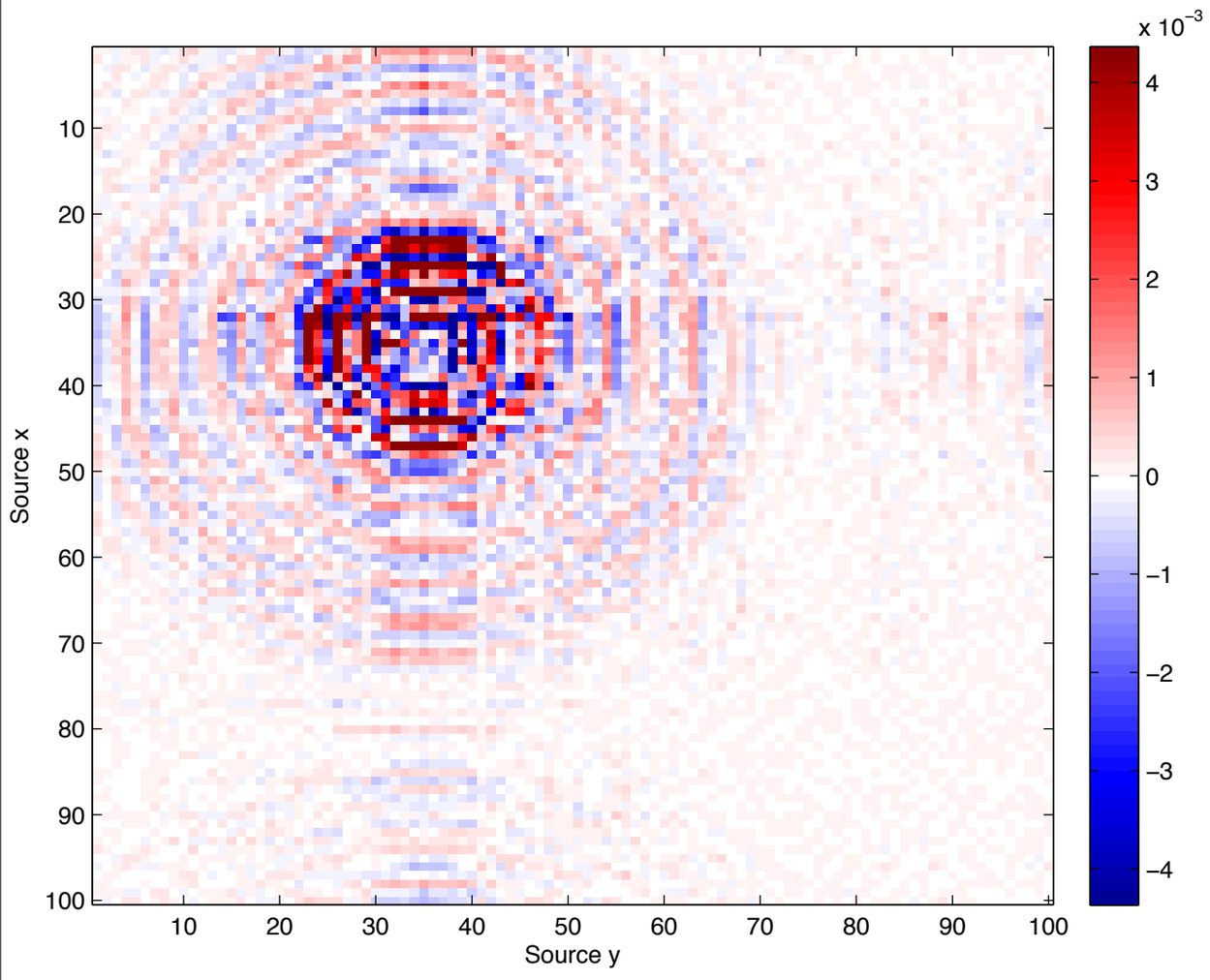
Source Rank 5



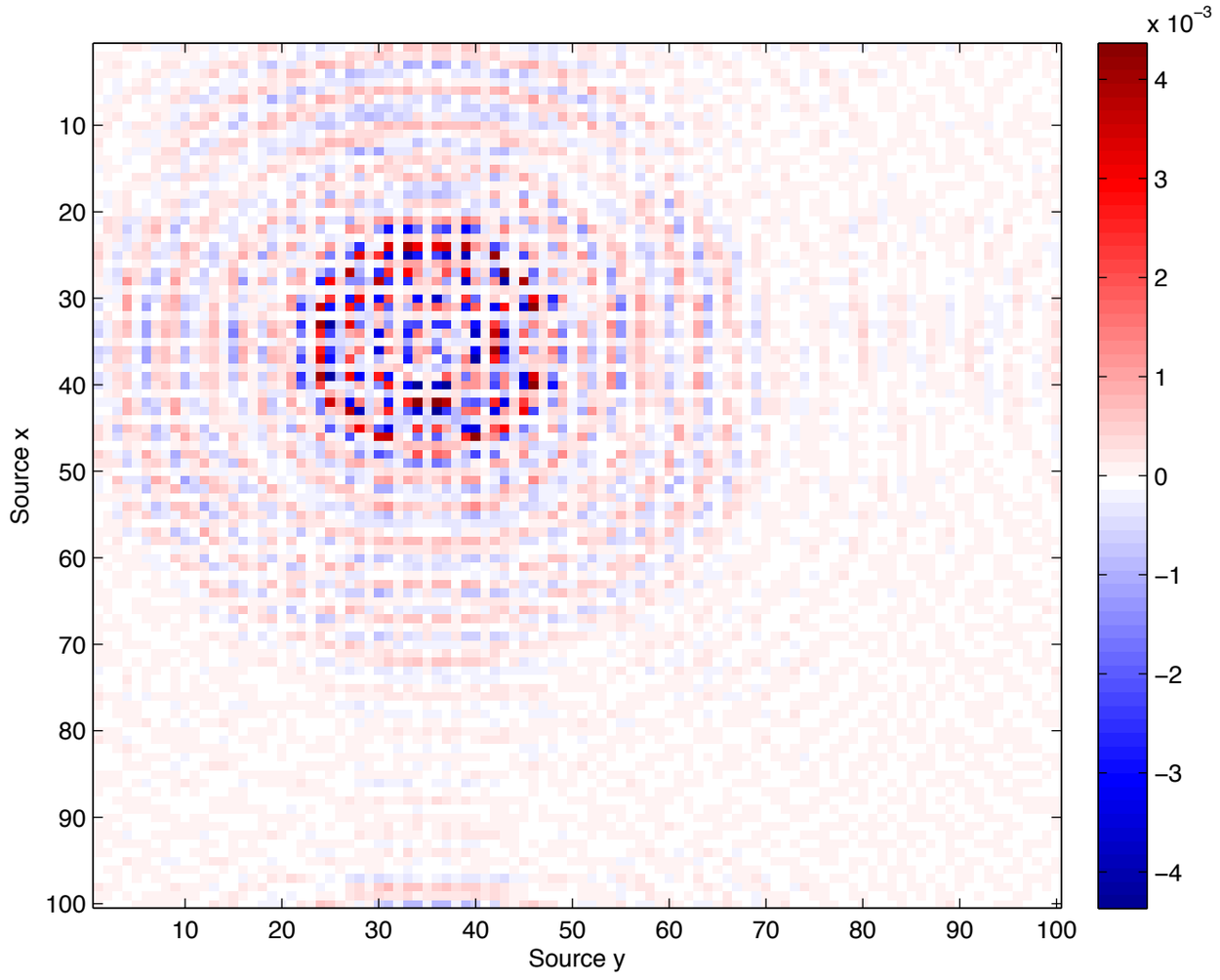
Source Rank 20

Common Receiver Gathers

Rec x = 35, Rec y = 35



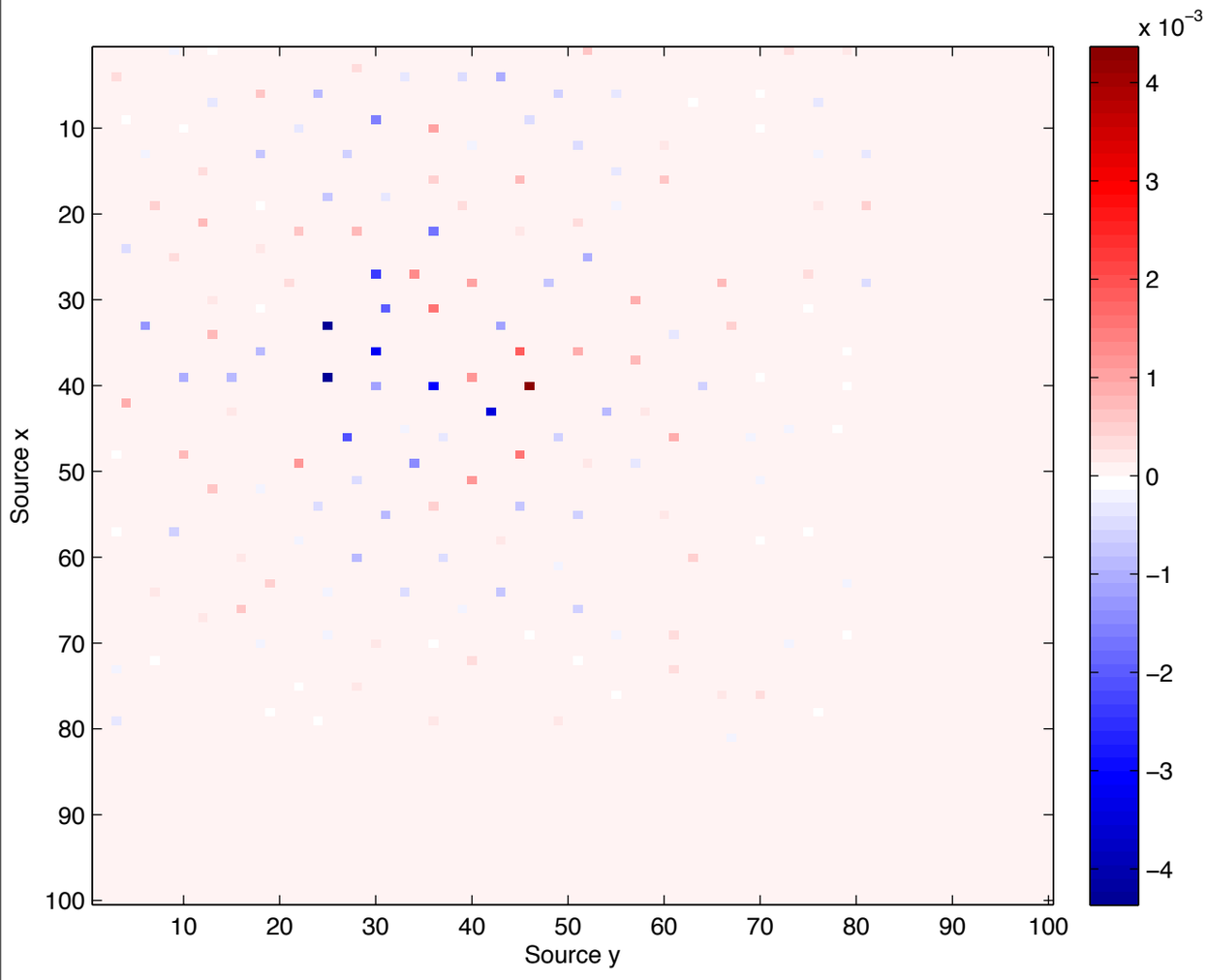
No Regularization



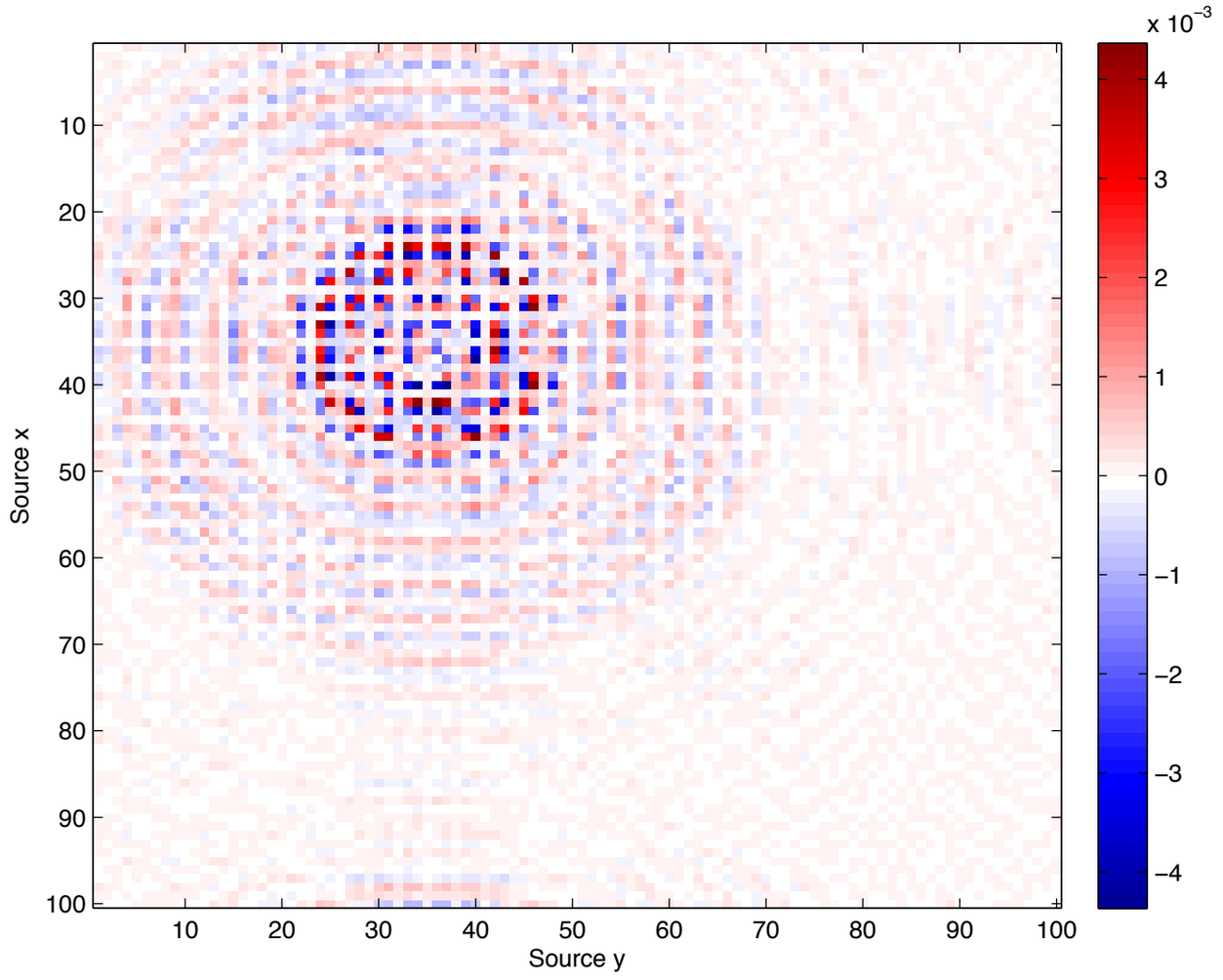
Regularization

Common Receiver Gathers

Rec x = 35, Rec y = 35



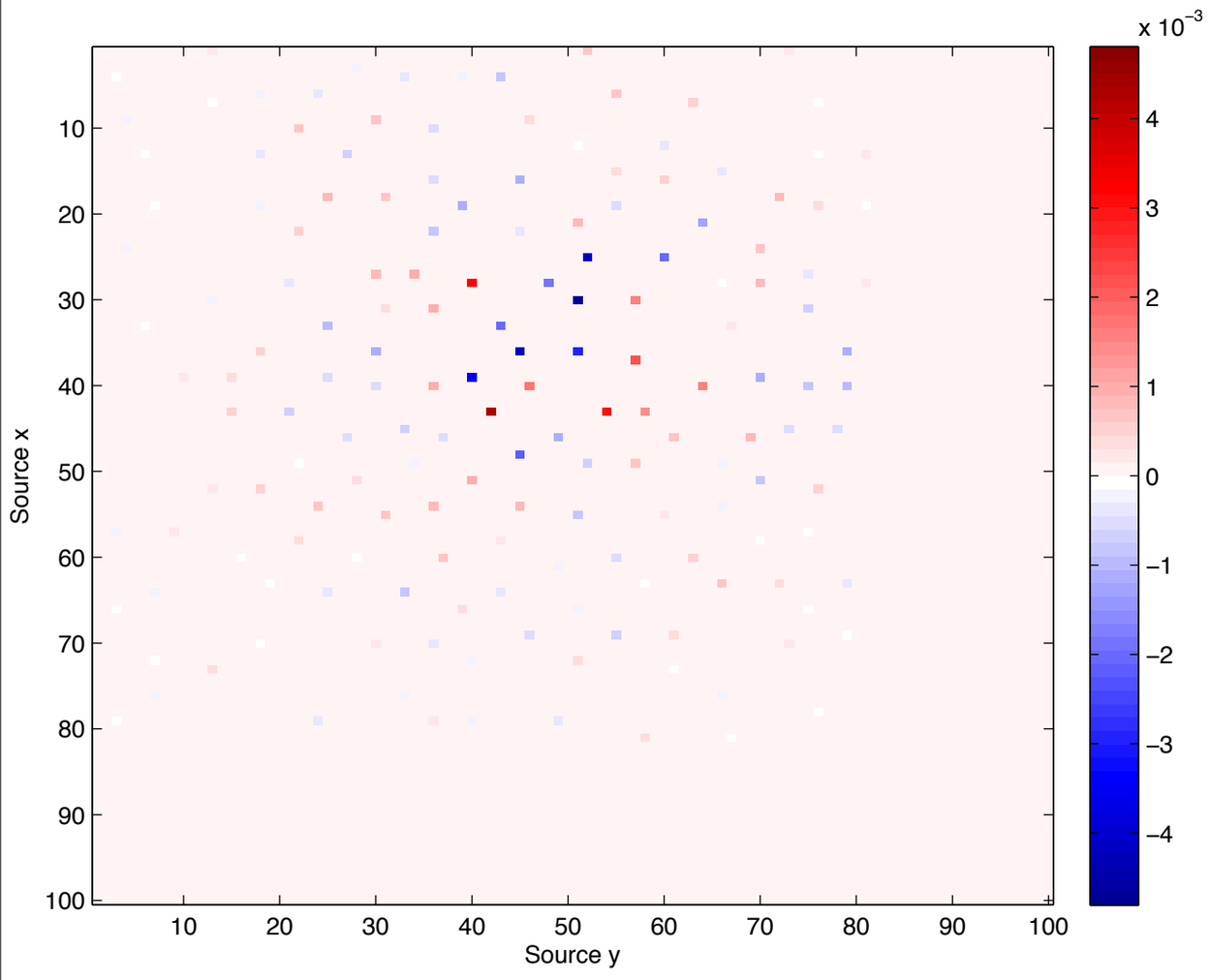
Known data



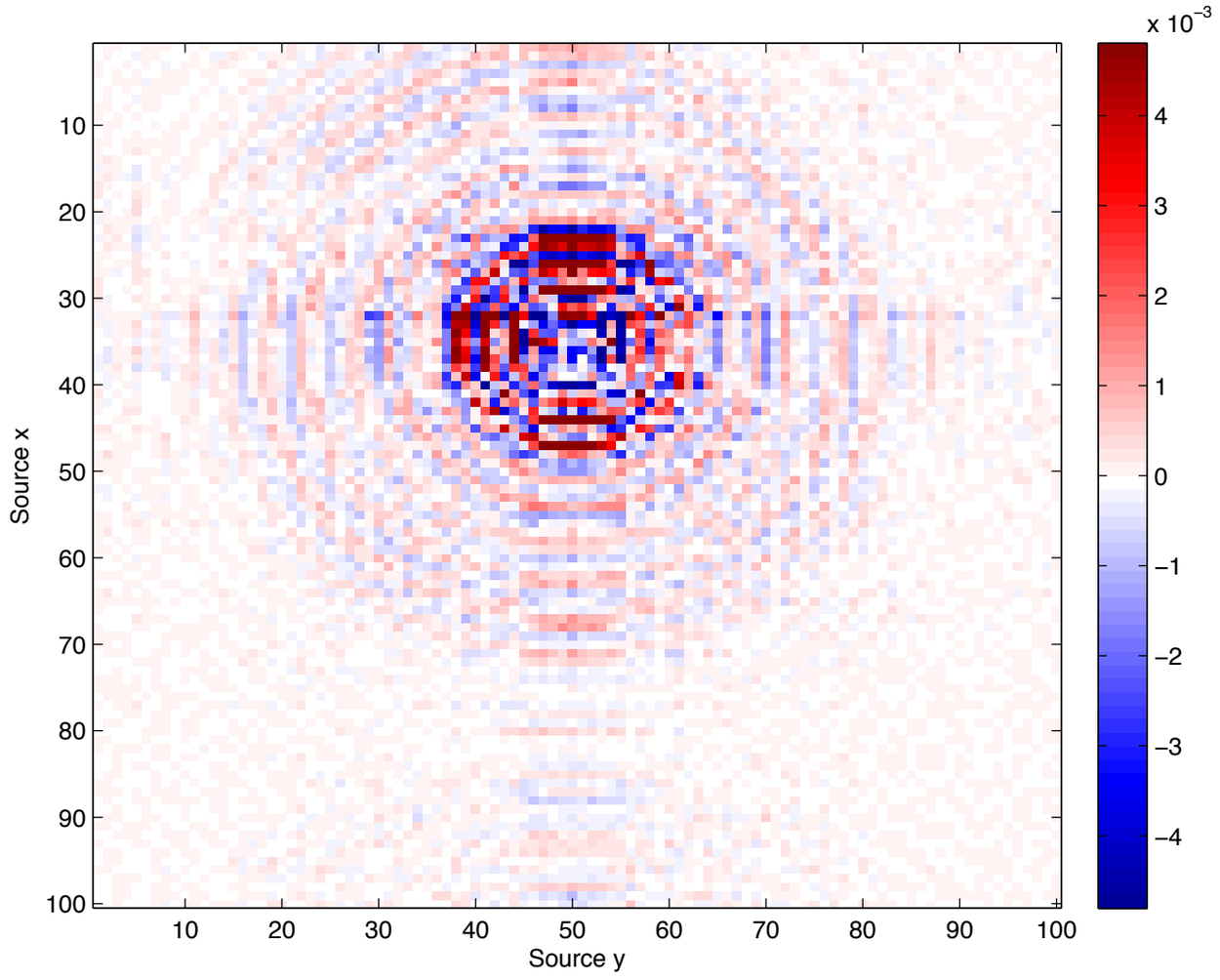
Regularized final result

Common Receiver Gathers

Rec x = 35, Rec y = 50



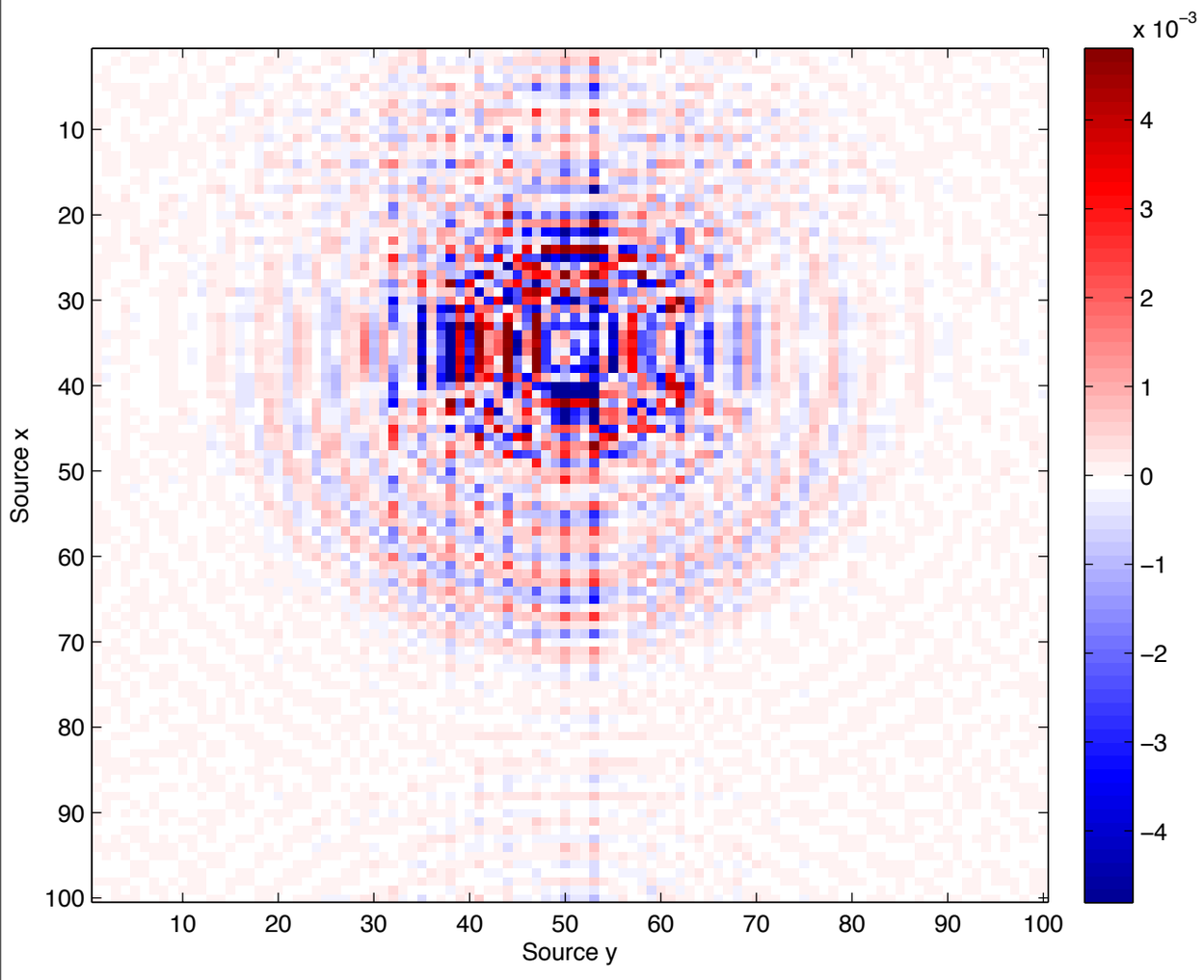
Known data



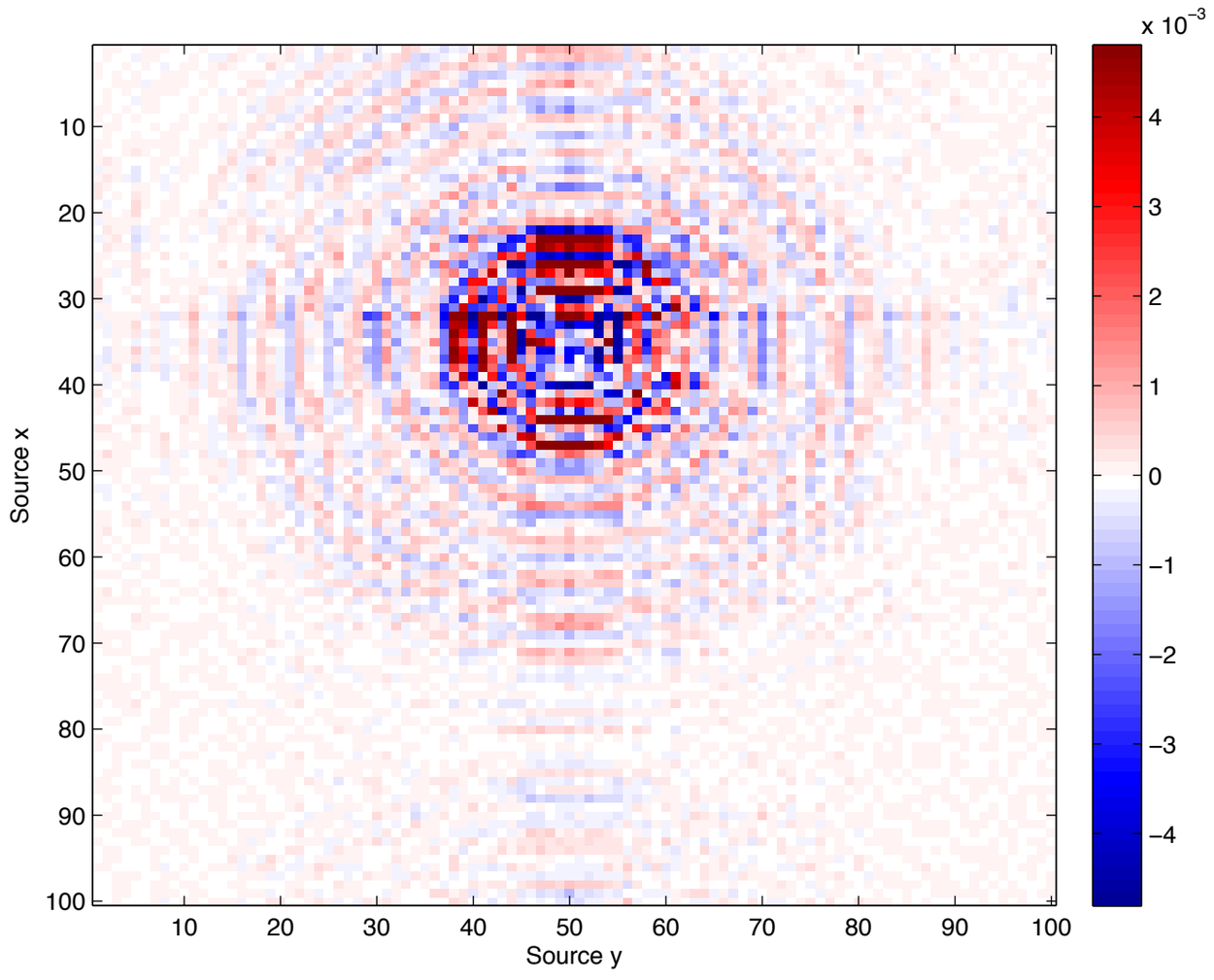
Interpolated data

Common Receiver Gathers

Rec x = 35, Rec y = 50



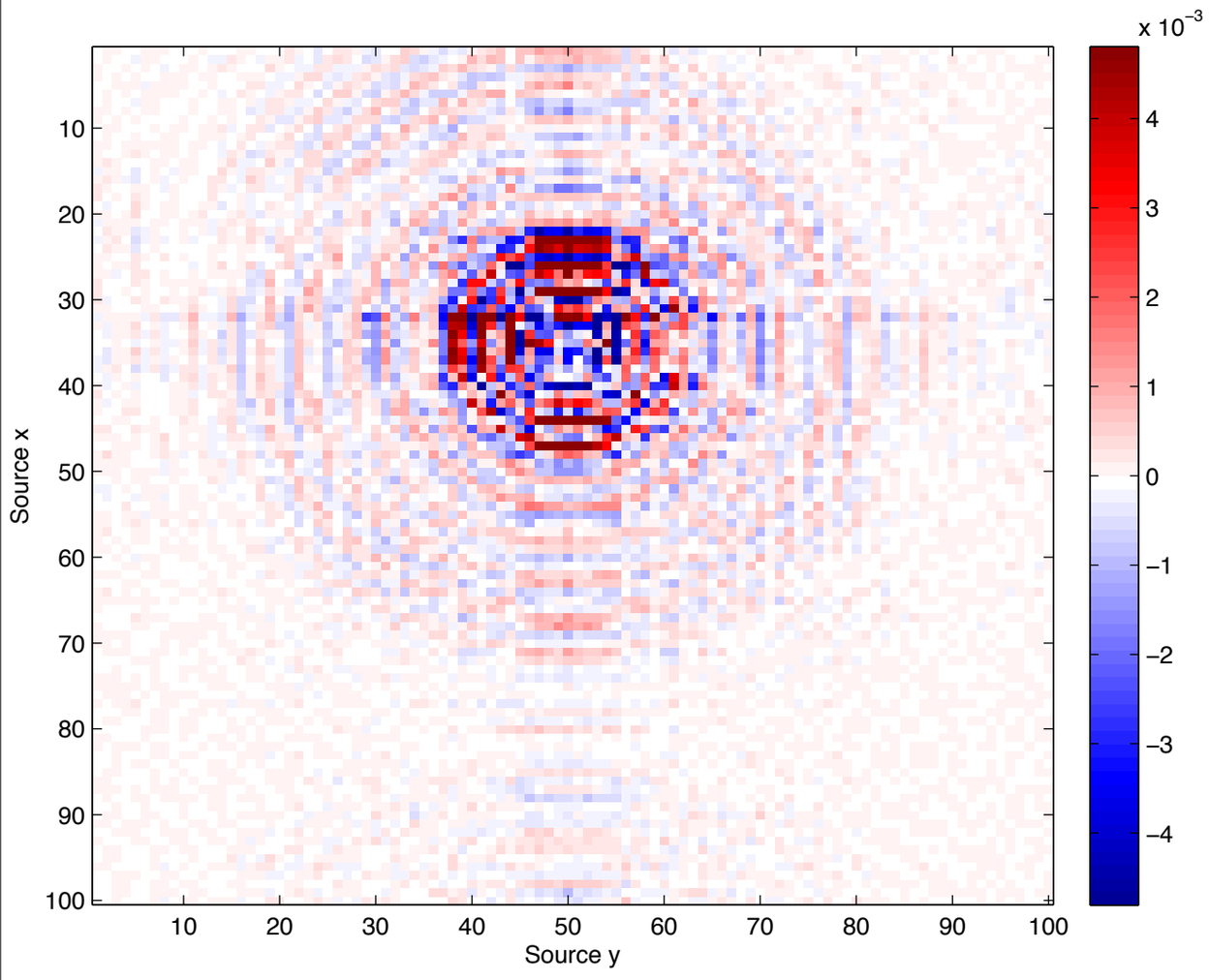
Source Rank 5



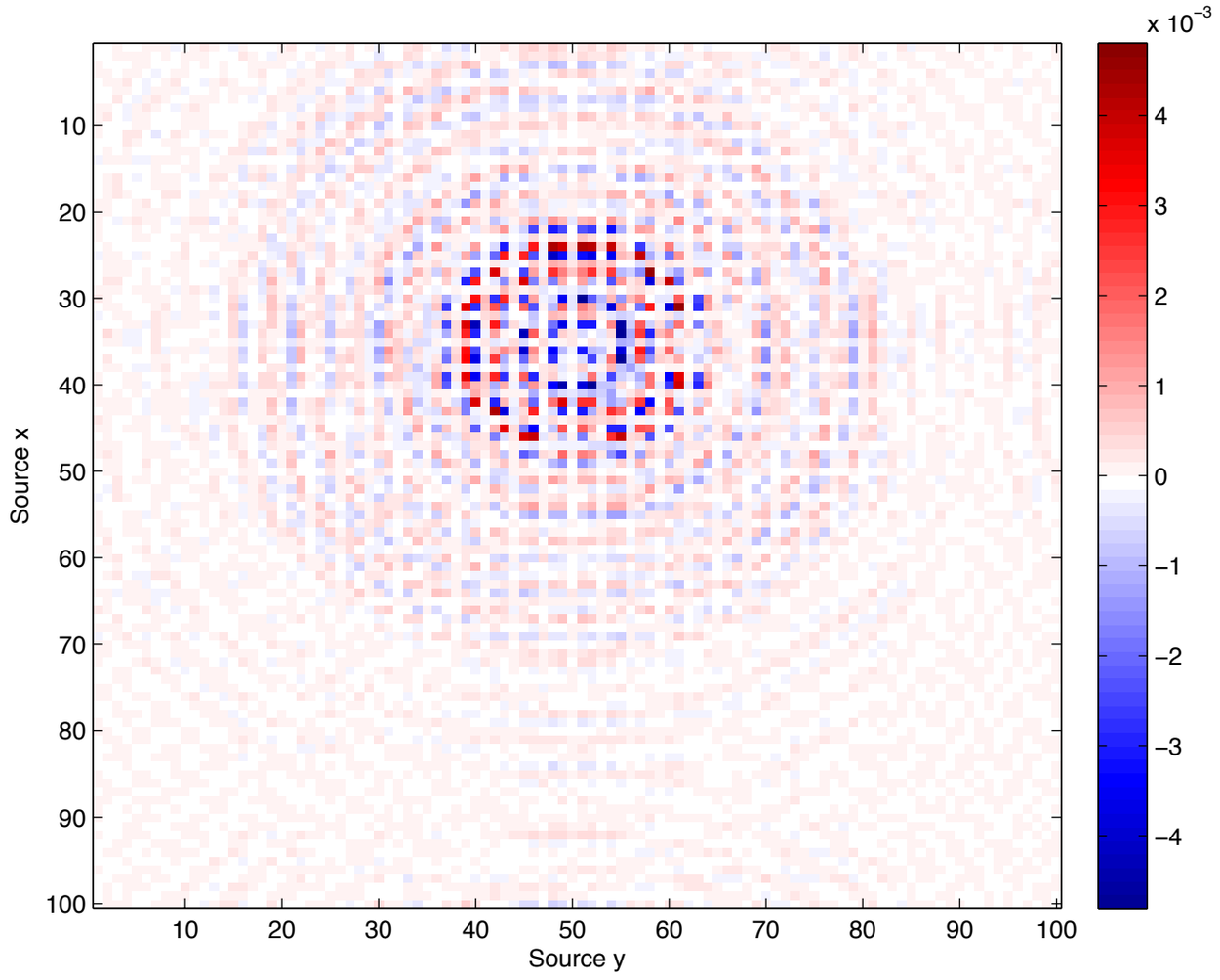
Source Rank 20

Common Receiver Gathers

Rec x = 35, Rec y = 50



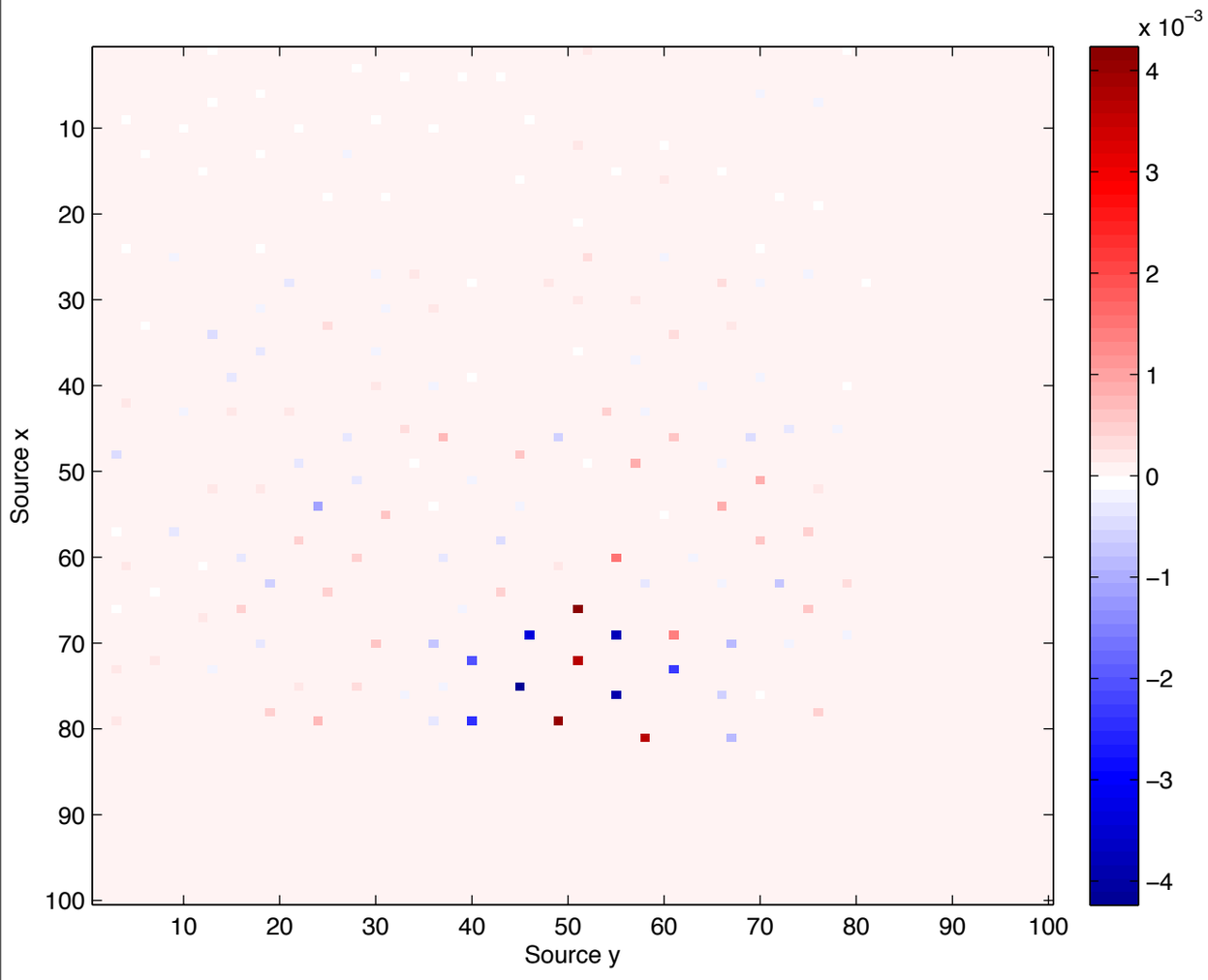
No Regularization



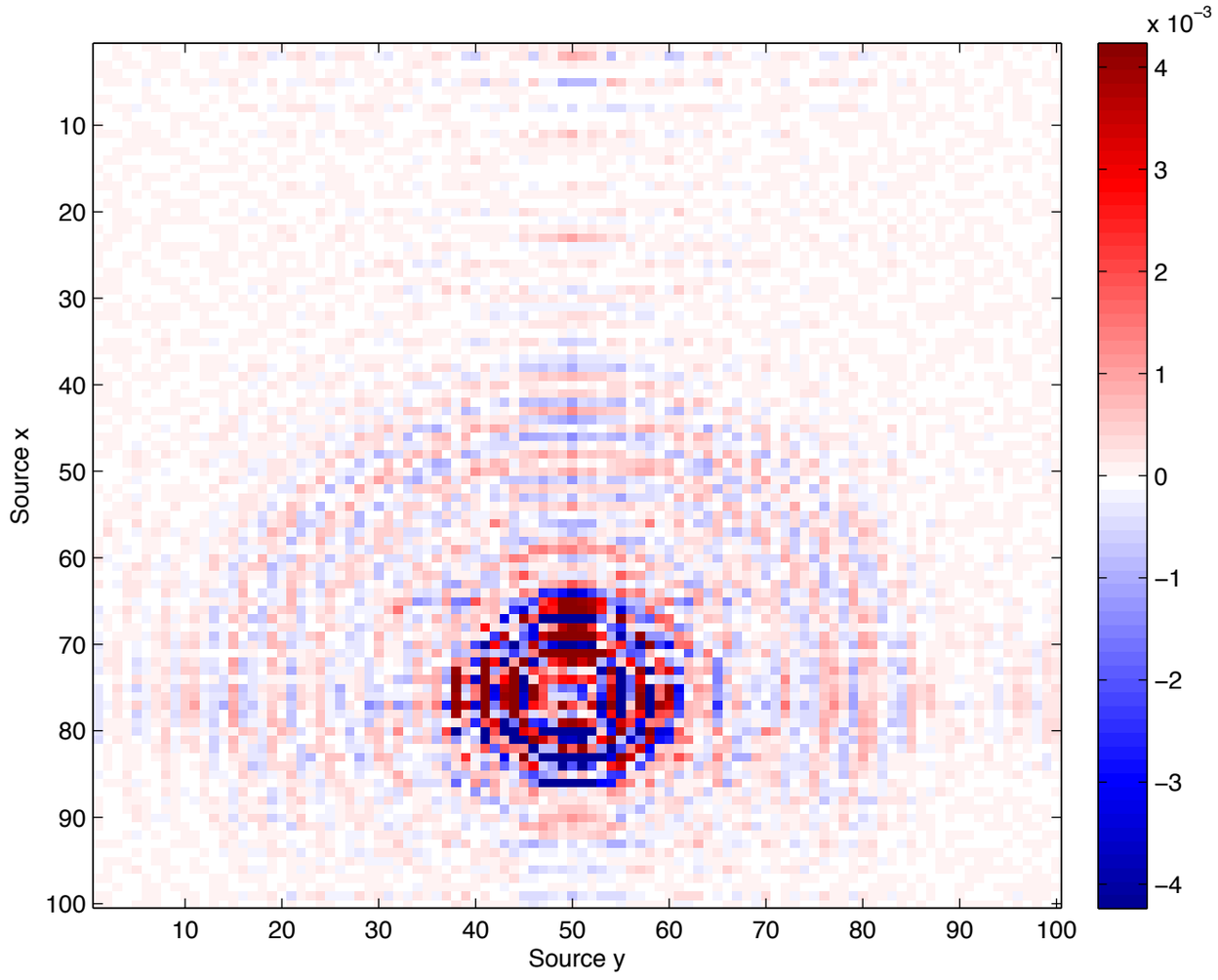
Regularization

Common Receiver Gathers

Rec x = 75, Rec y = 50



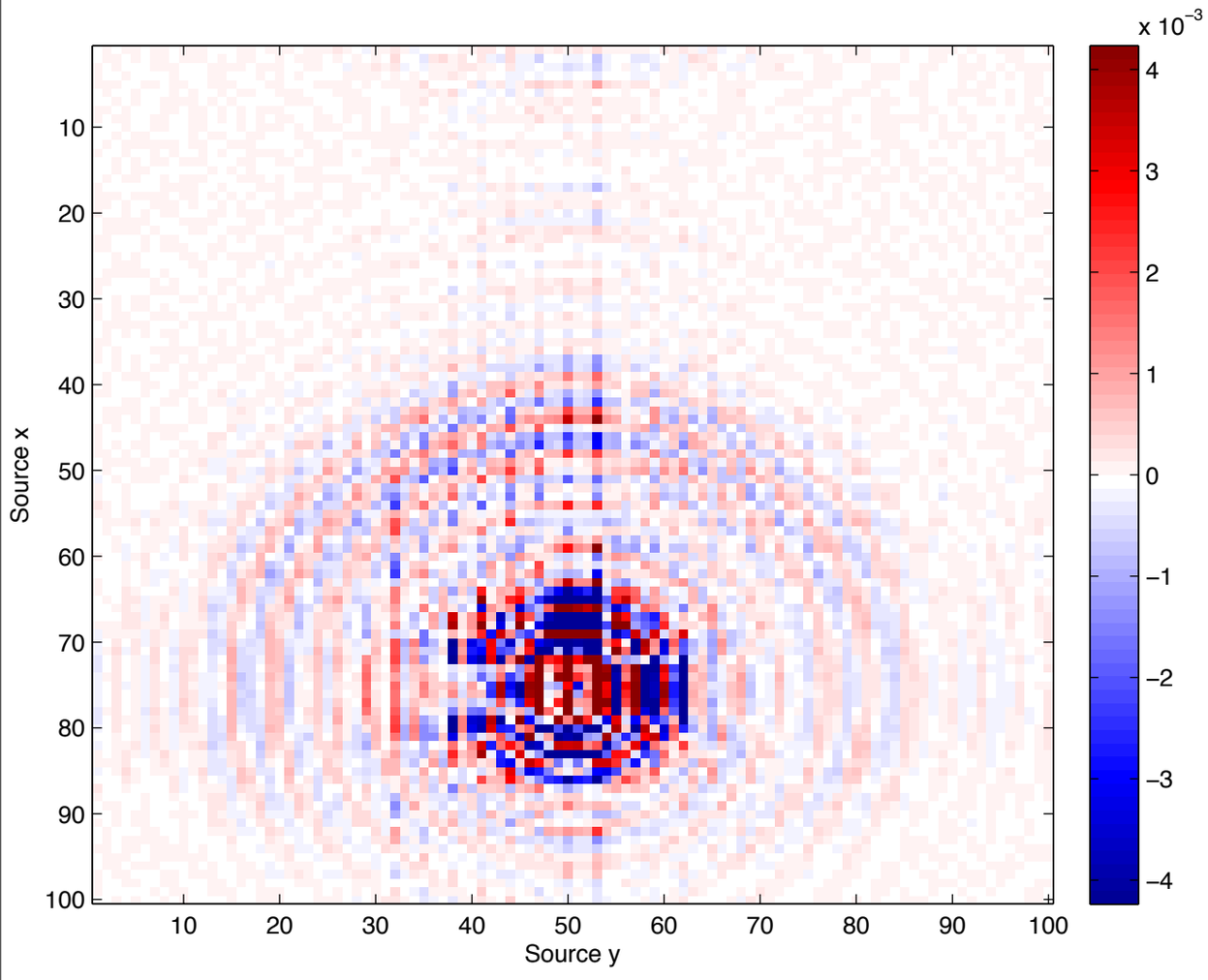
Known data



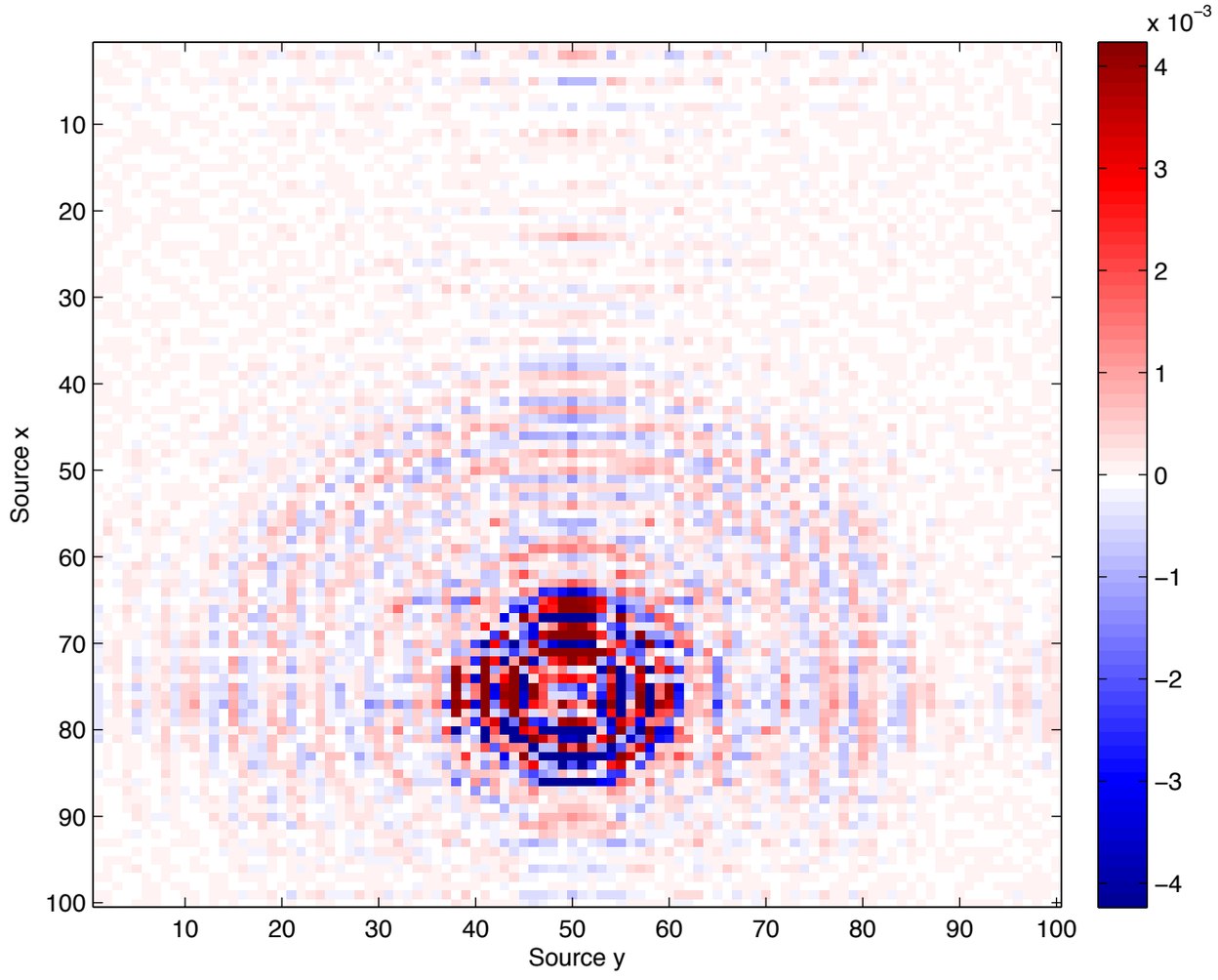
Interpolated data

Common Receiver Gathers

Rec x = 75, Rec y = 50



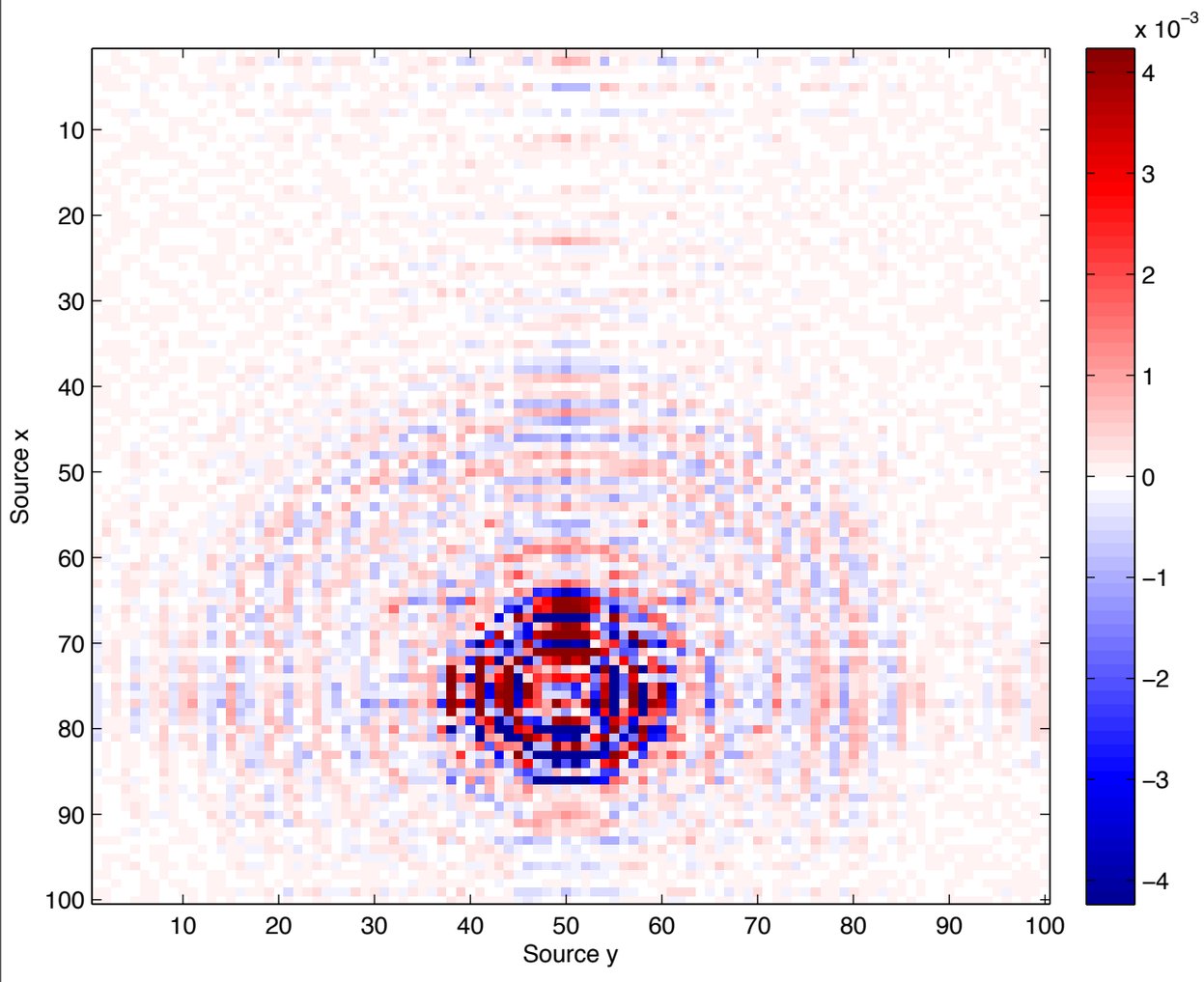
Source Rank 5



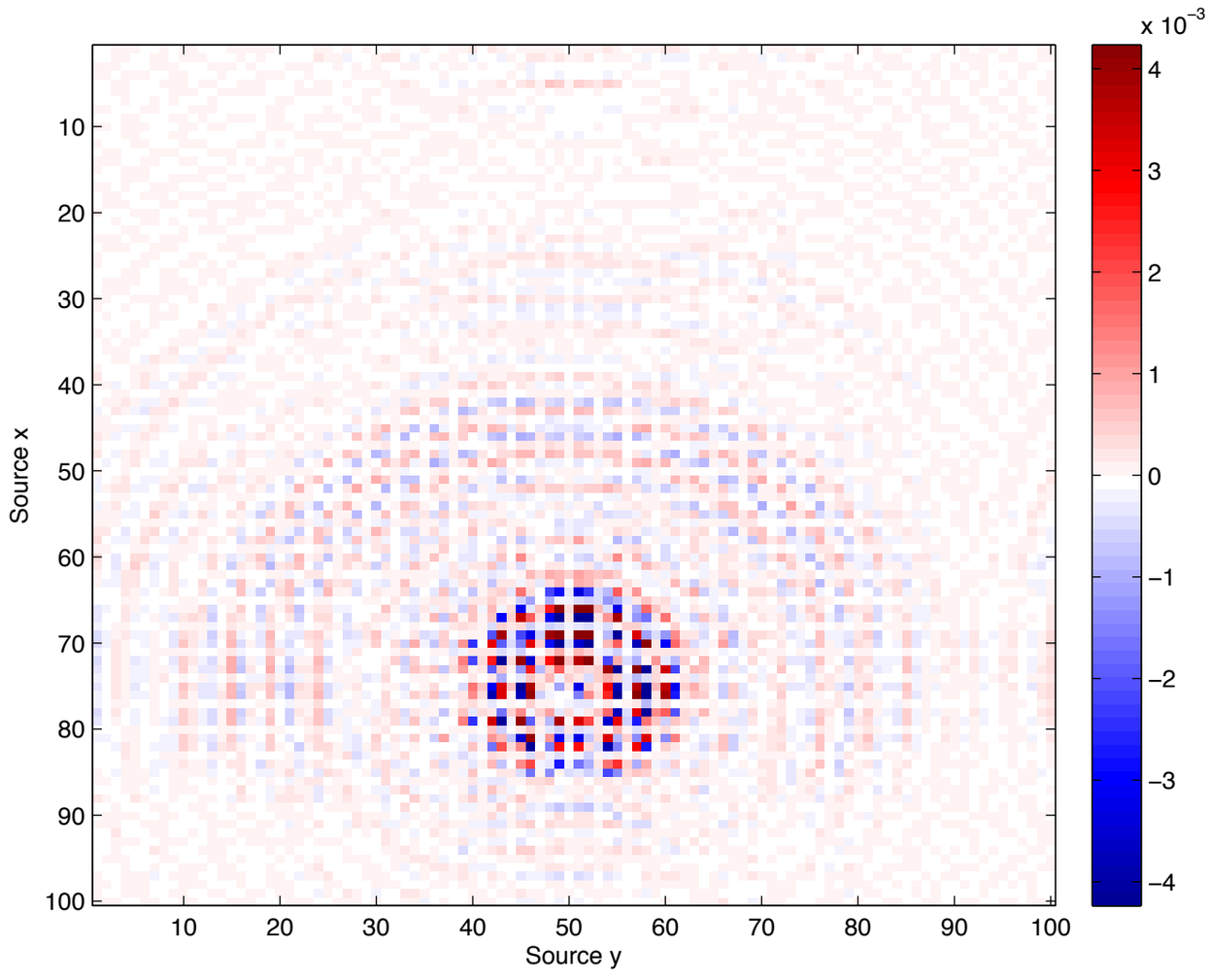
Source Rank 20

Common Receiver Gathers

Rec x = 75, Rec y = 50



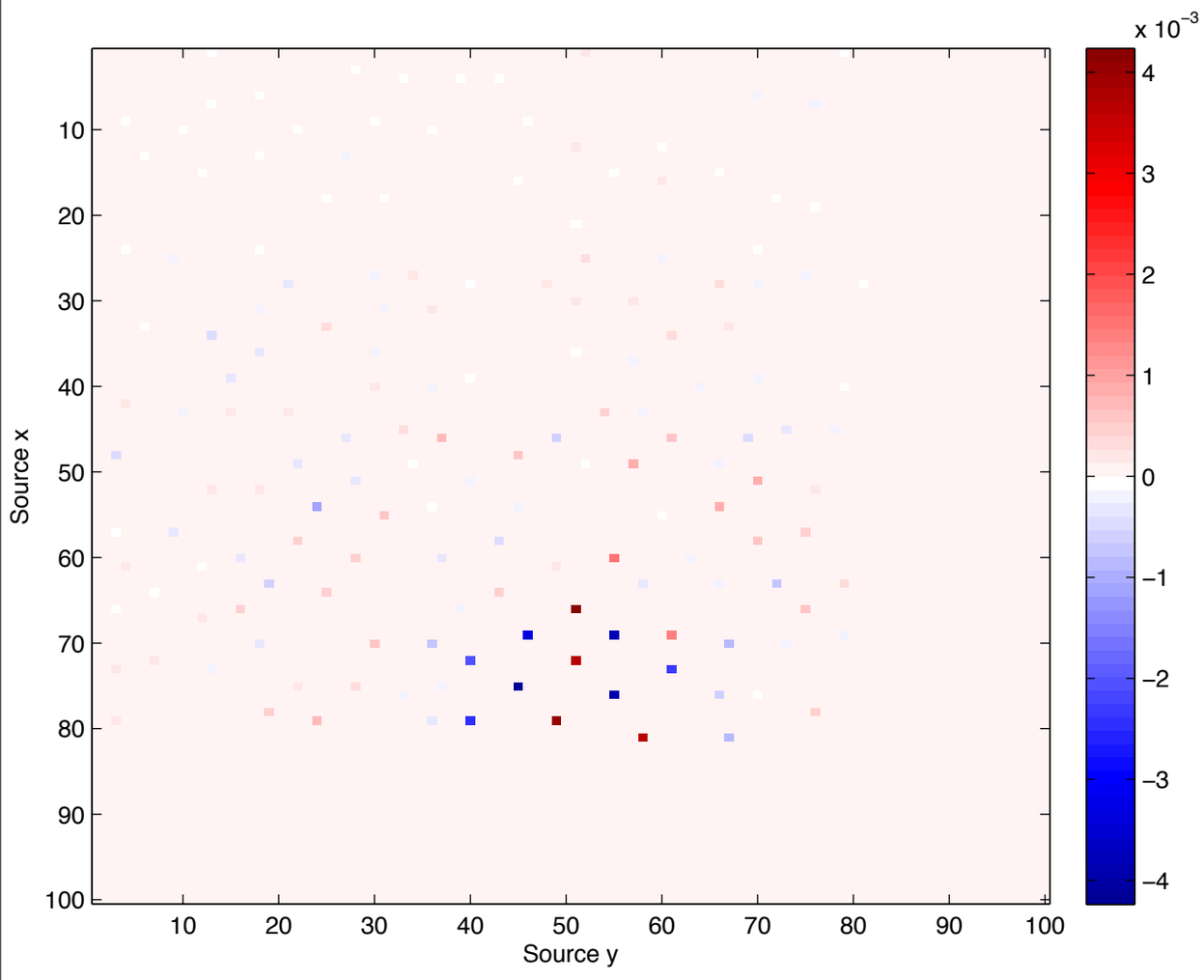
No Regularization



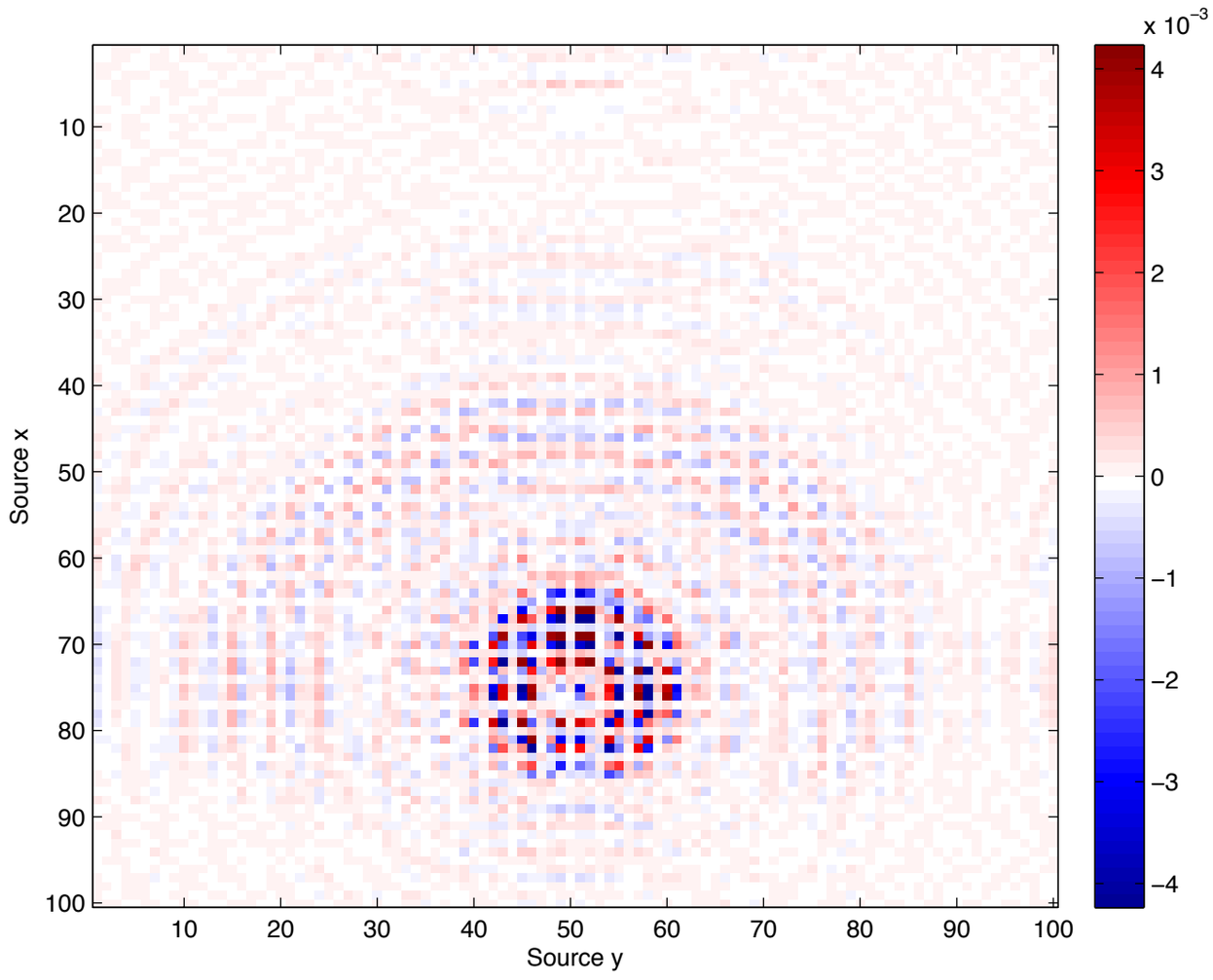
Regularization

Common Receiver Gathers

Rec x = 75, Rec y = 50



Known data

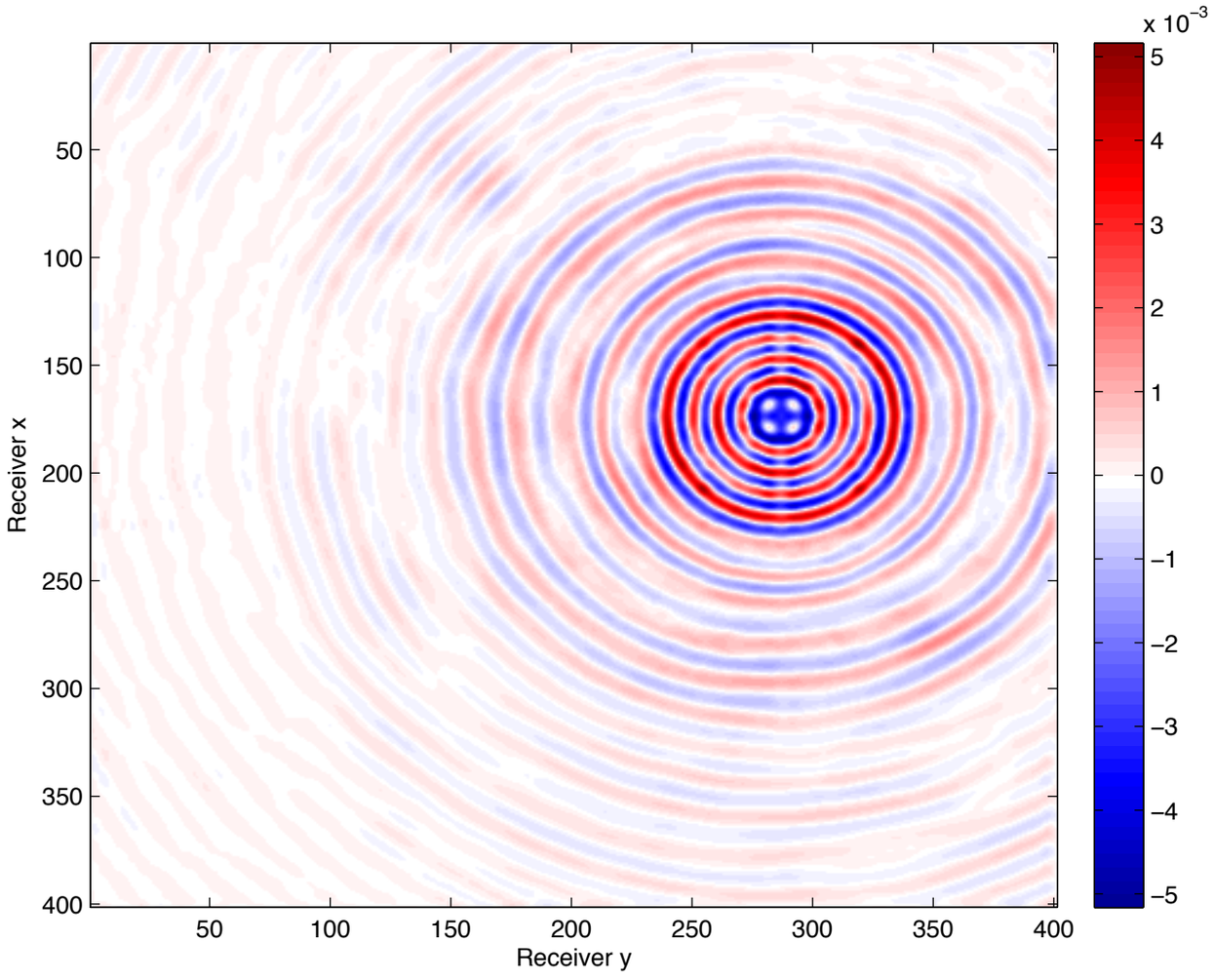


Regularized final result

Shot Reconstruction

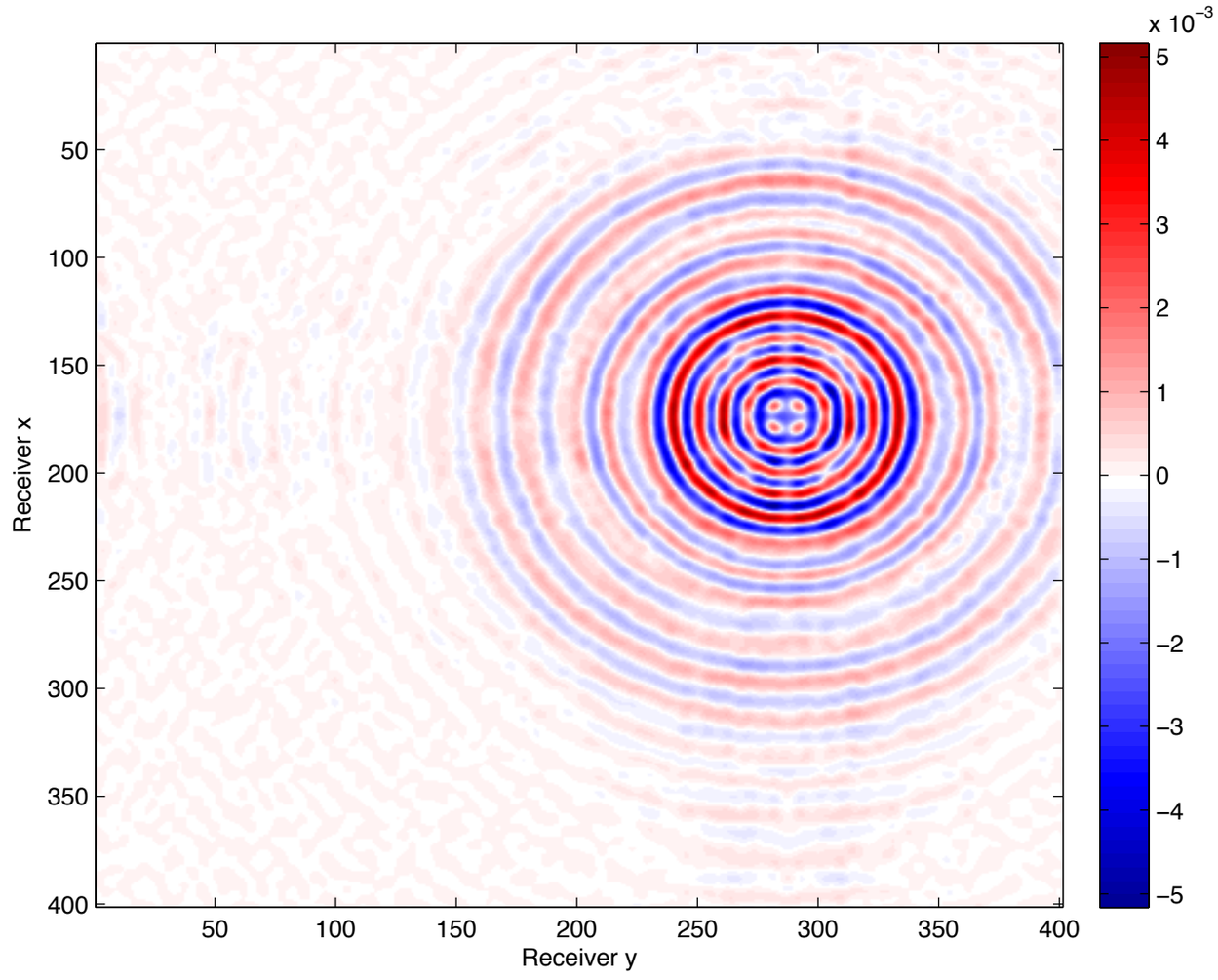
- We consider the reconstruction error of 3 shots on a reduced data set (using the remaining 197 shots)
- We obtain still reasonable recovery results

Shot Reconstruction



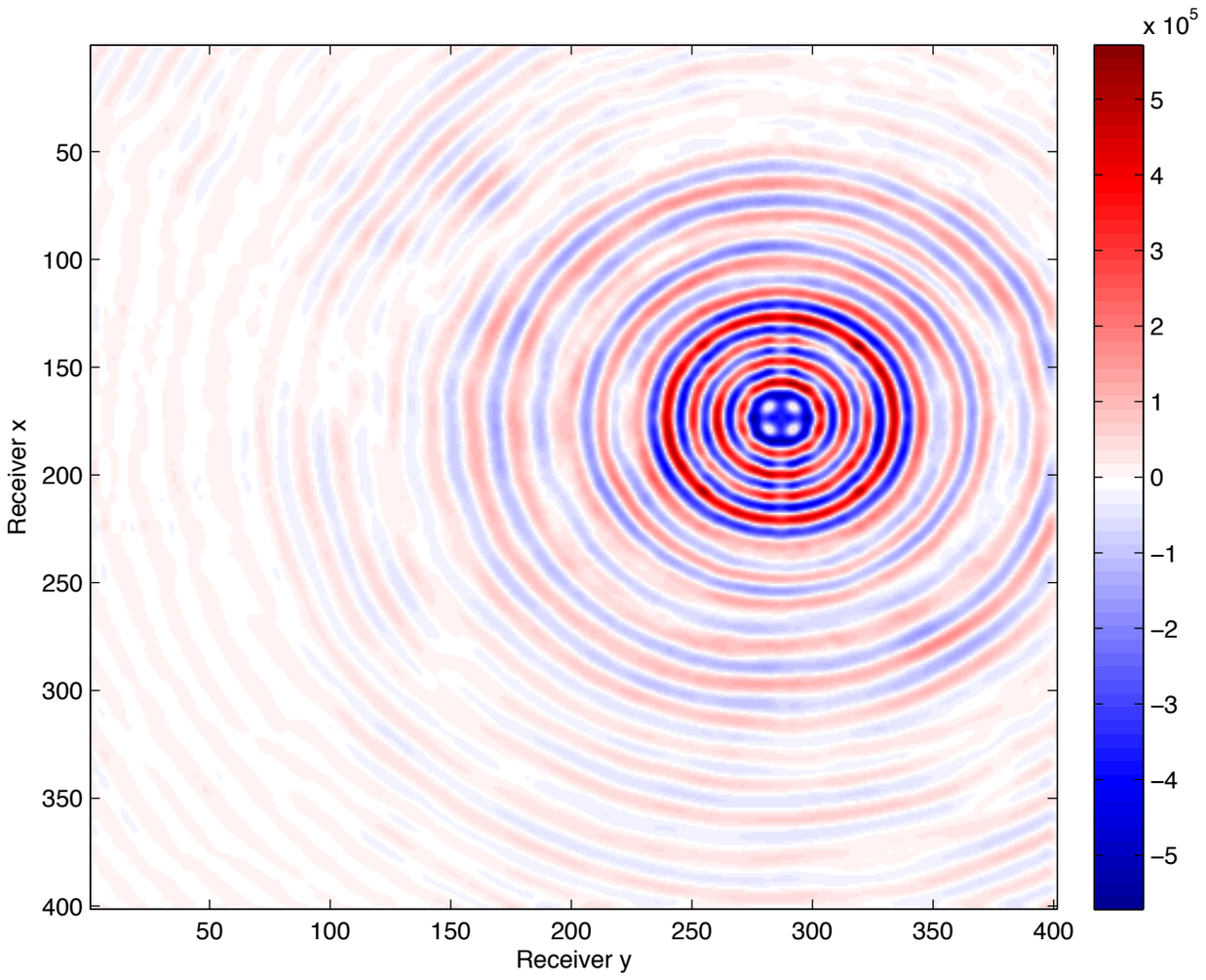
True data

(Src x, Src y) = (45,73)



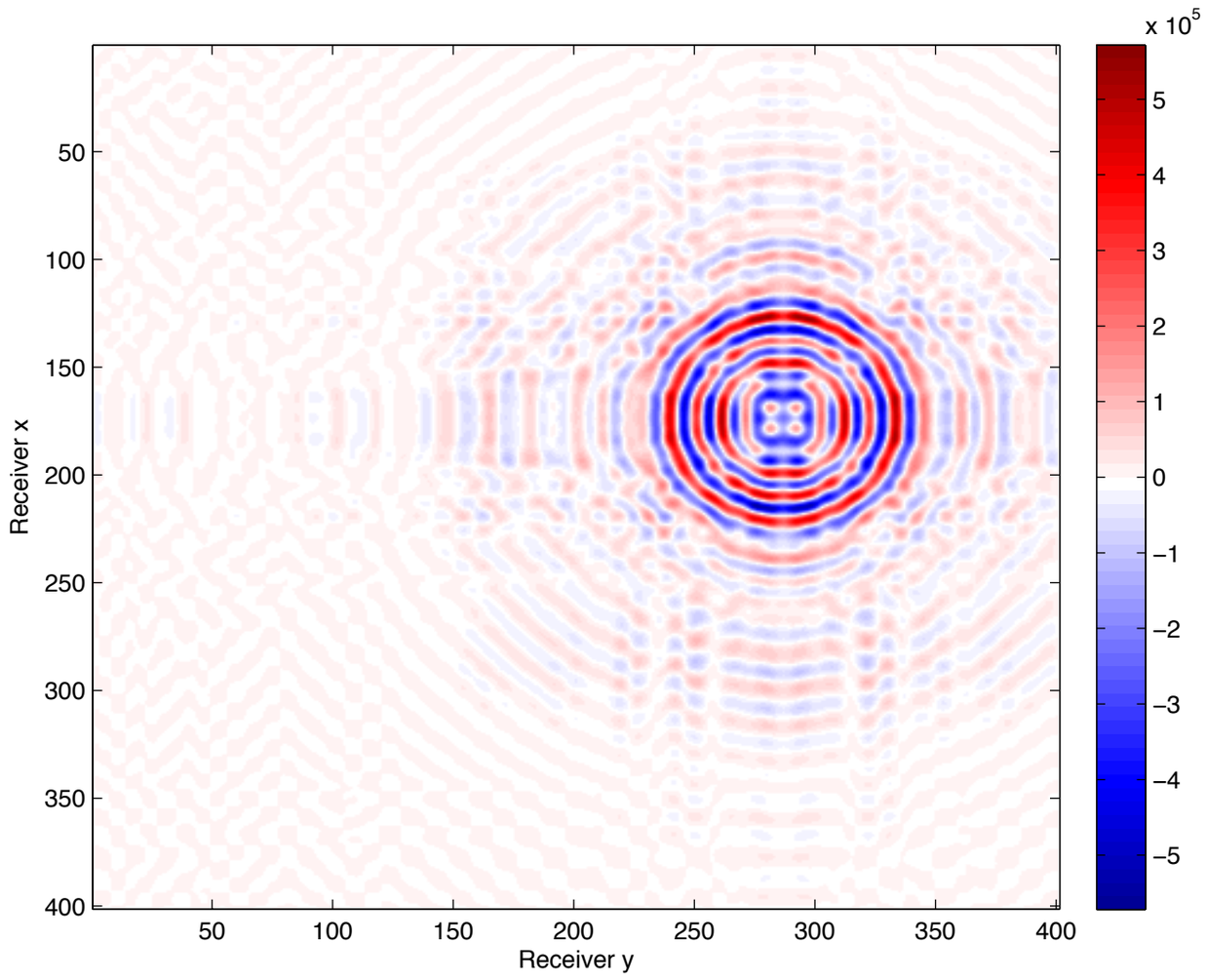
Interpolated data - SNR 8.46 dB

Shot Reconstruction - Jellyfish



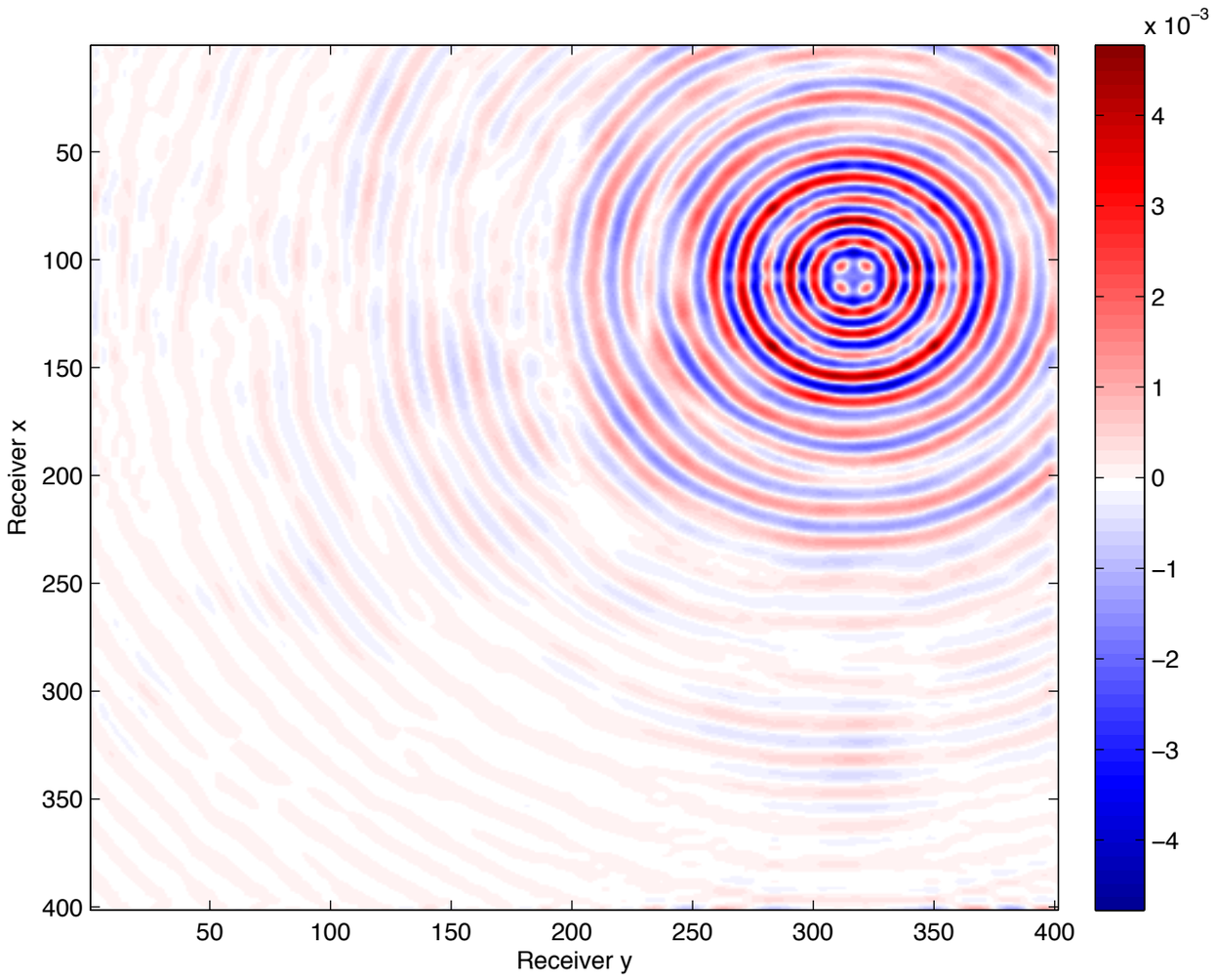
True data

(Src x, Src y) = (45,73)



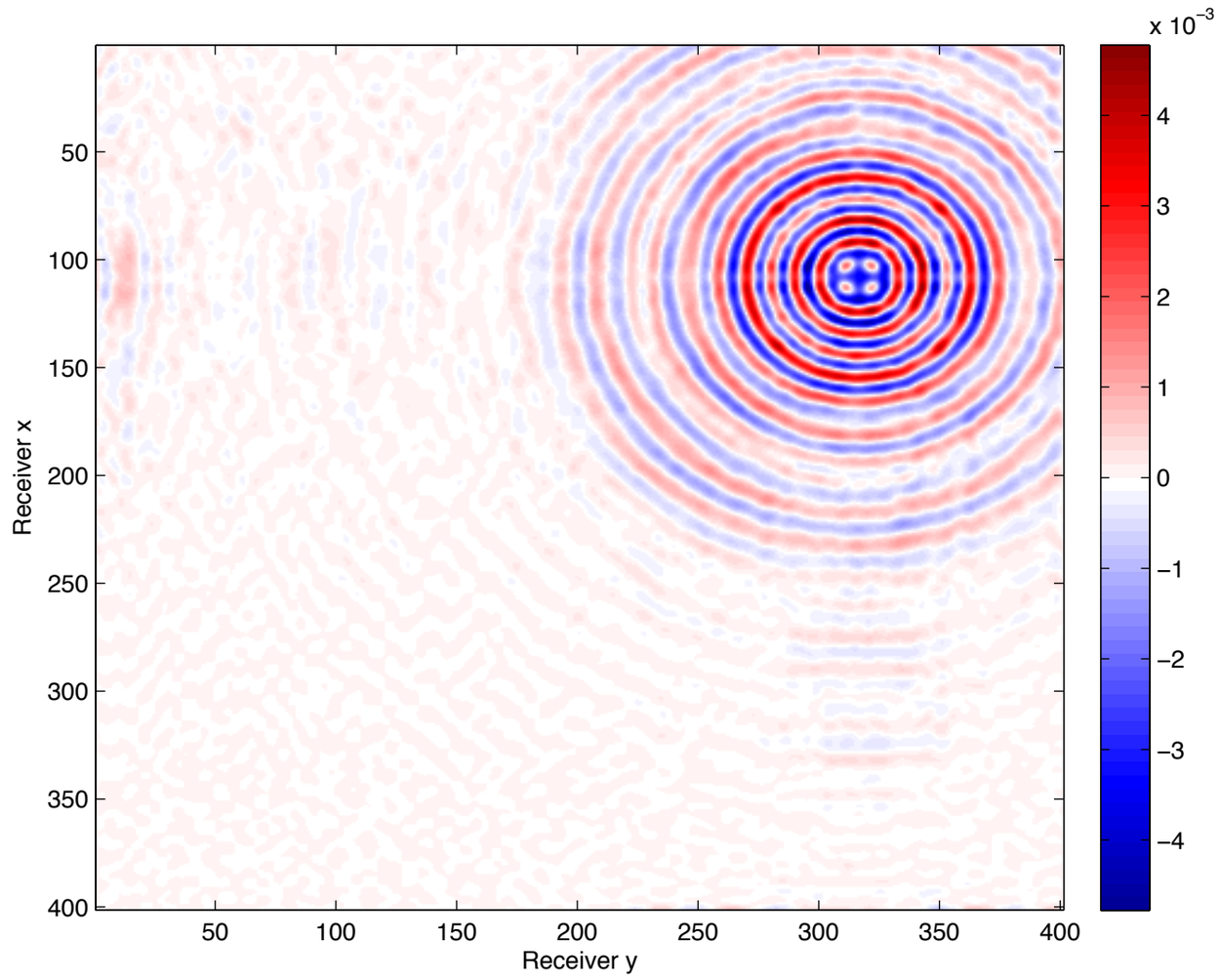
Interpolated data - SNR 3.86 dB

Shot Reconstruction



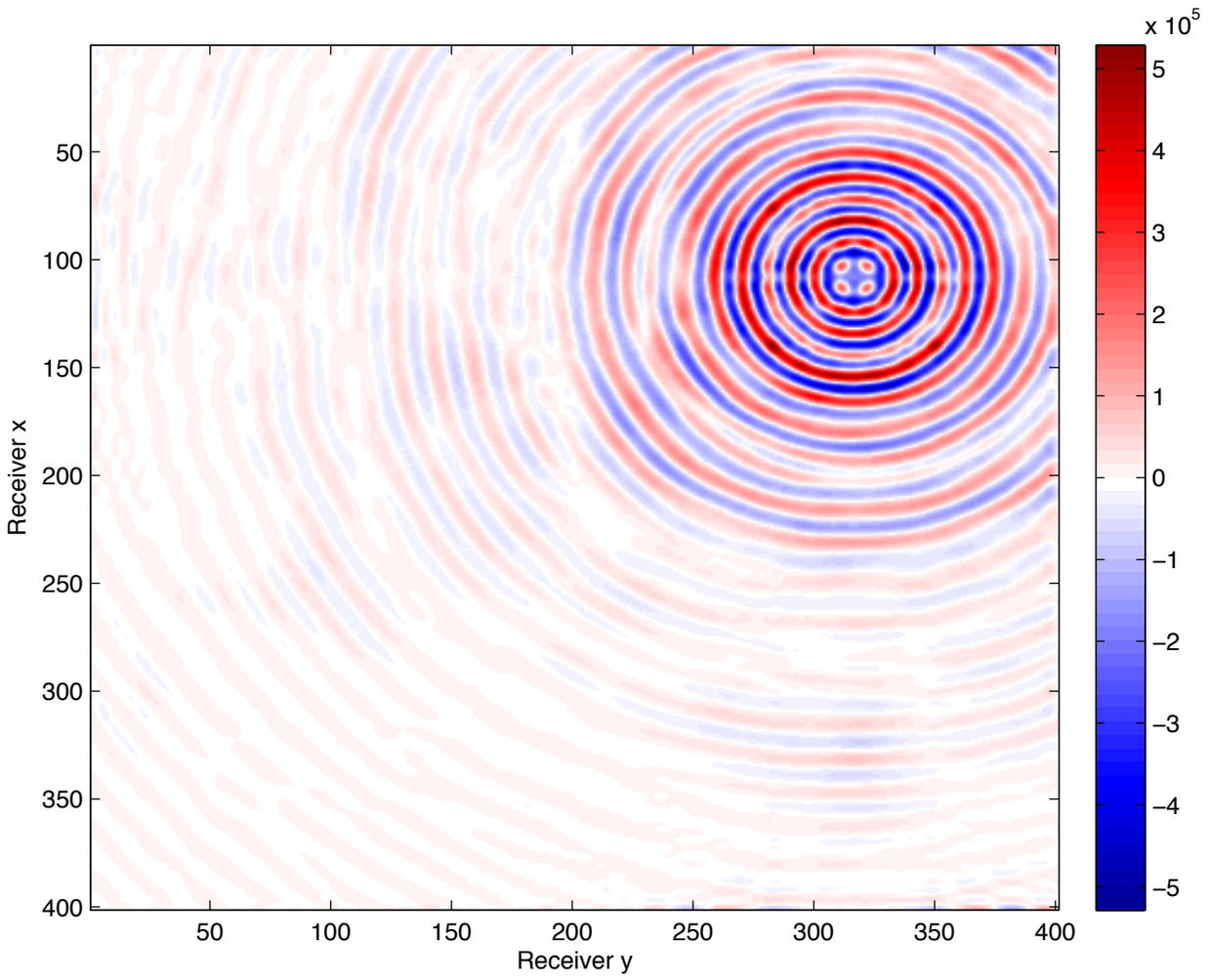
True data

(Src x, Src y) = (28,81)



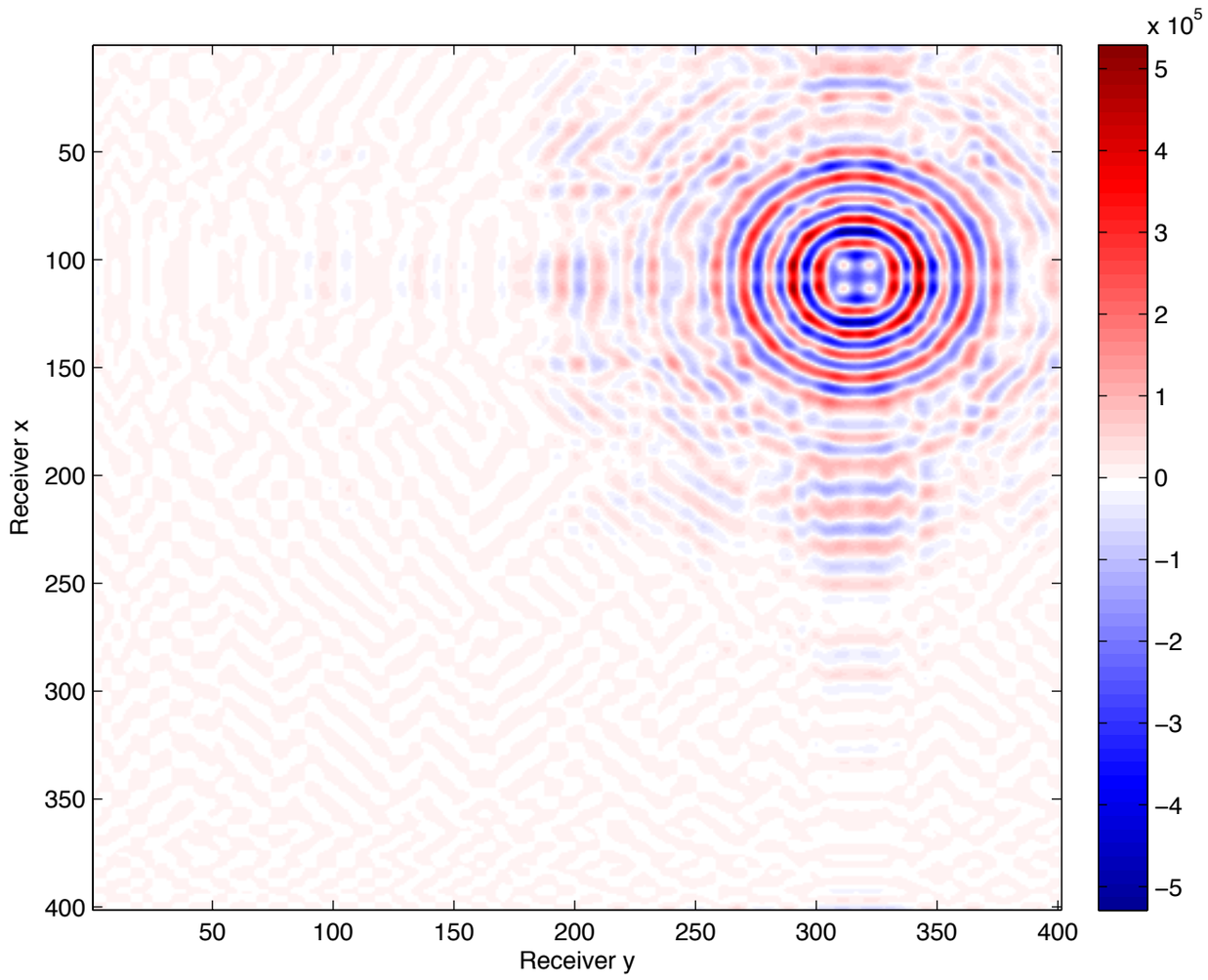
Interpolated data - SNR 7.98 dB

Shot Reconstruction - Jellyfish



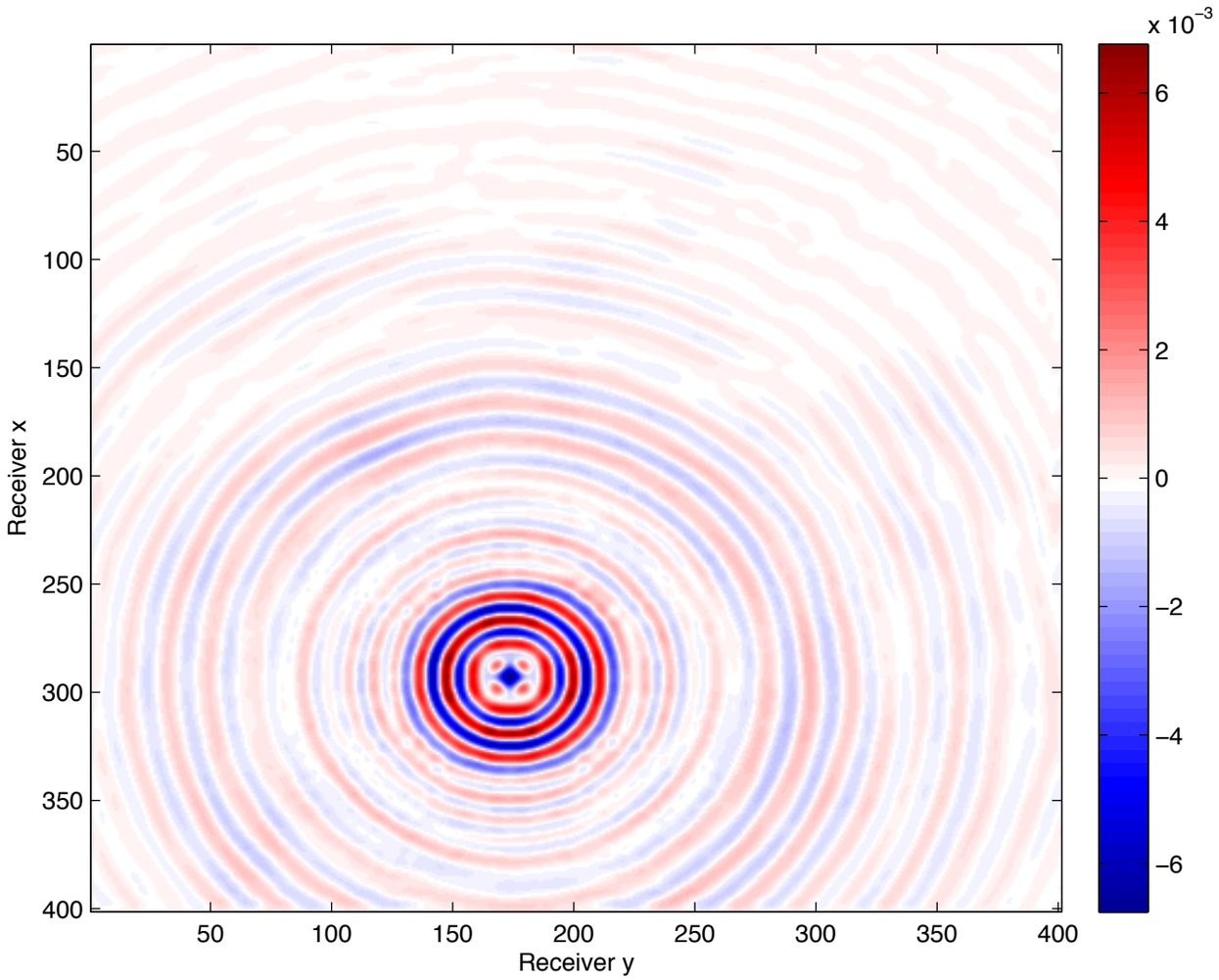
True data

(Src x, Src y) = (28,81)



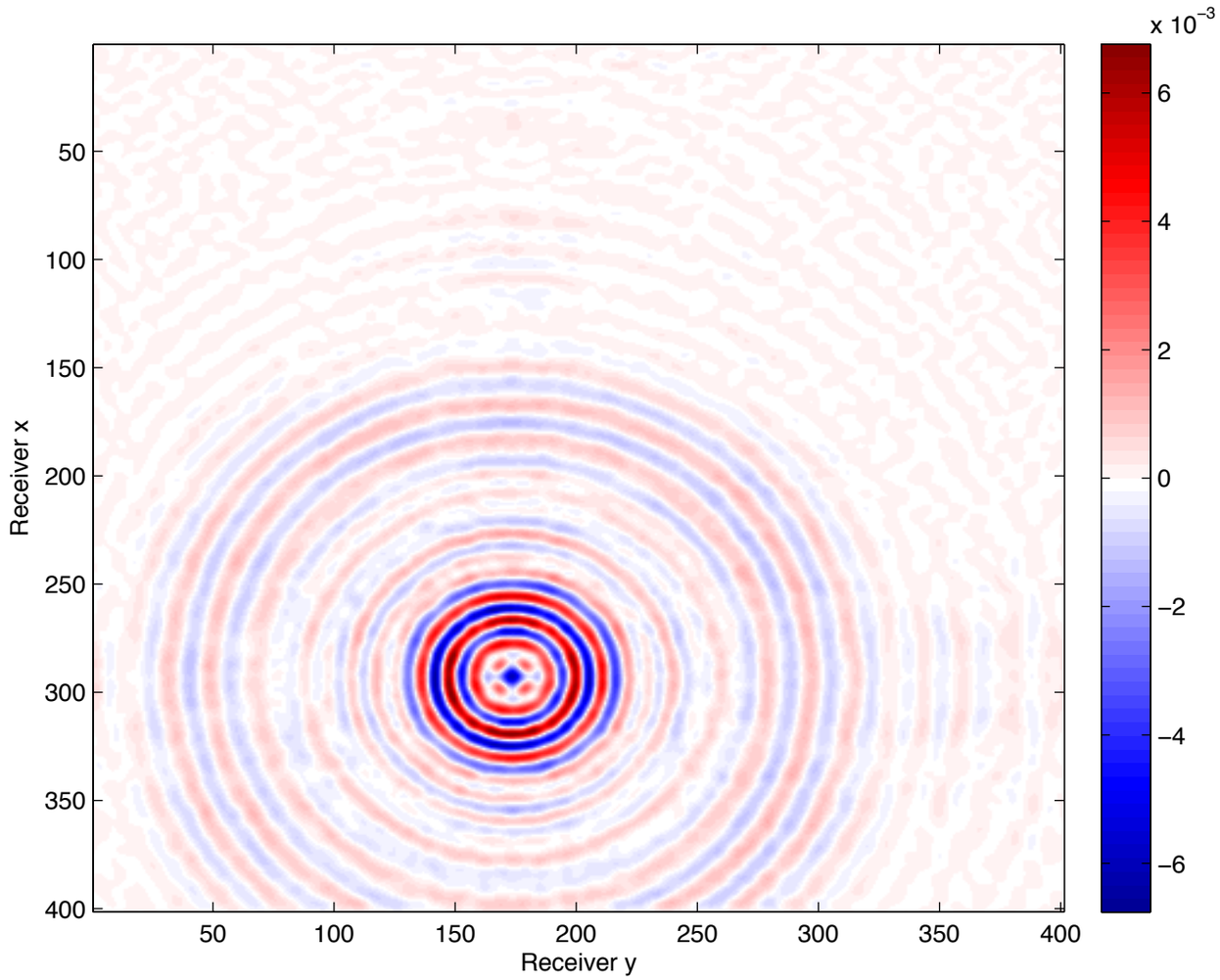
Interpolated data - SNR 3.89 dB

Shot Reconstruction



True data

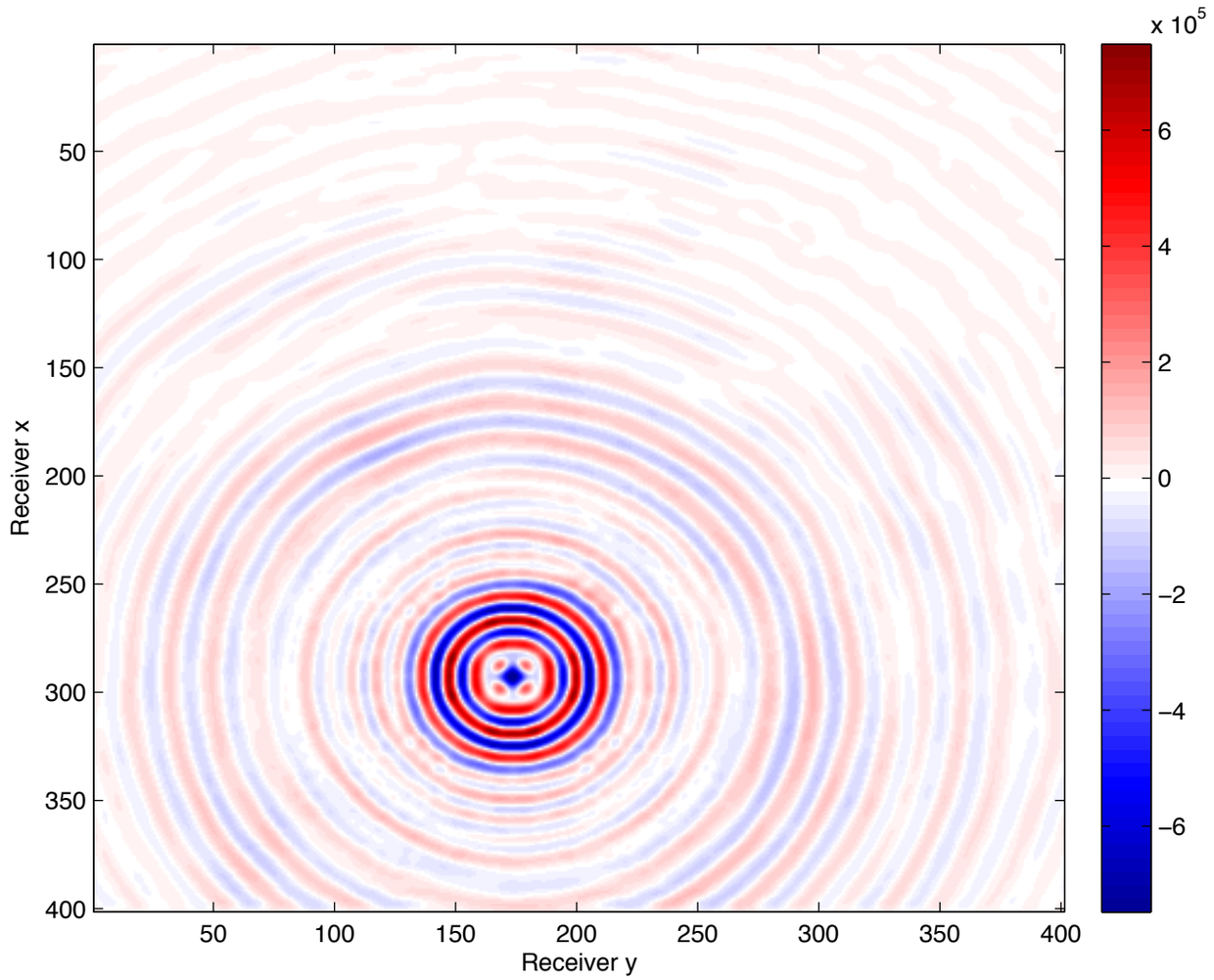
(Src x, Src y) = (75,45)



Interpolated data -

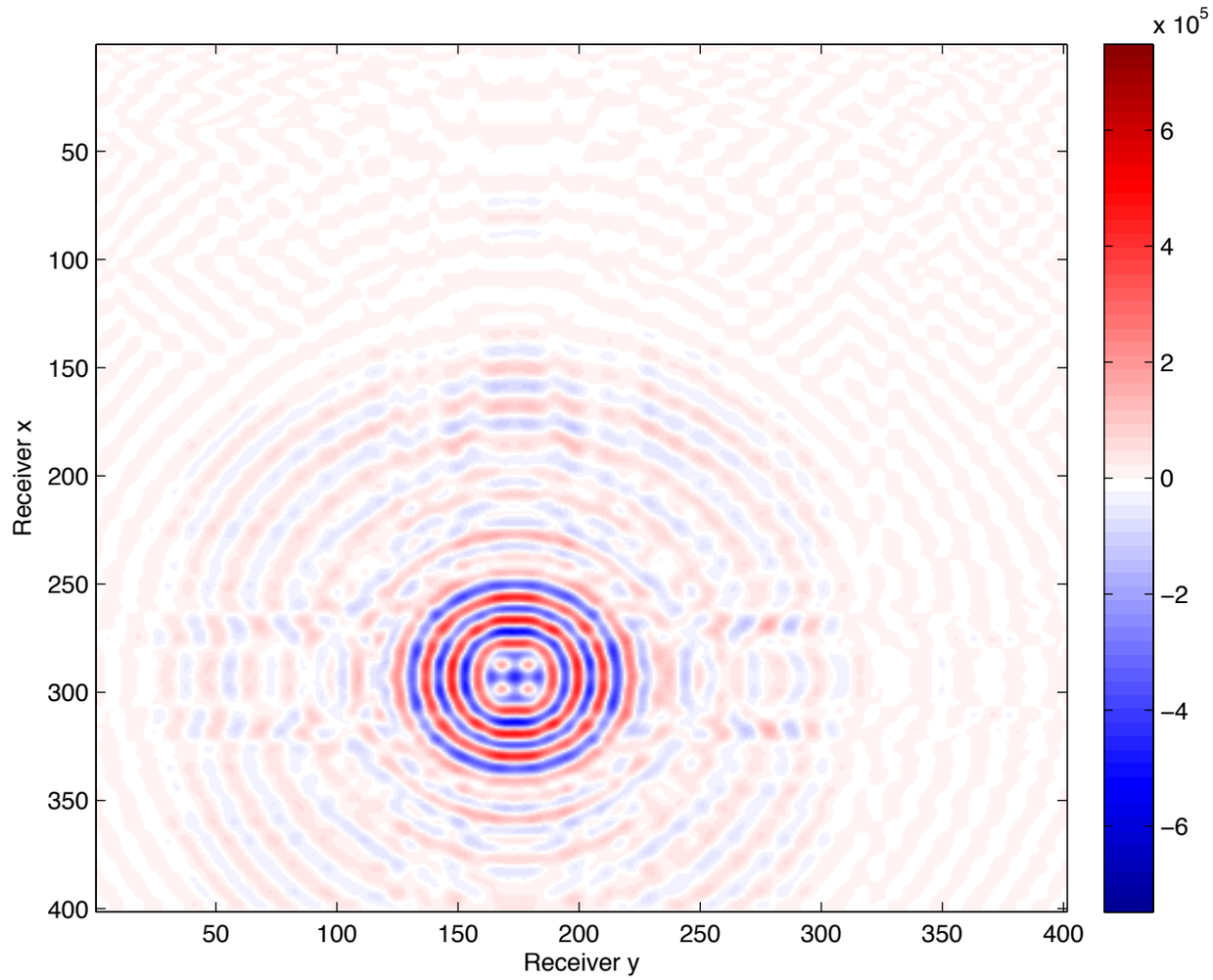
SNR 10.6 dB

Shot Reconstruction - Jellyfish



True data

(Src x, Src y) = (75,45)



Interpolated data - SNR 4.62 dB

Conclusion

- 3D seismic data has an underlying structure that we can exploit for interpolation (hierarchical tucker format)
- The choice of organization of this data has important implications in the success of the recovery

Conclusion

- We can impose low-rank regularization of the underlying matricizations of the tensor *without* having to form them explicitly/ compute SVDs
- Achieve reasonable results from hardly any data at all (2% of sources)

Future Work

- Using the Hierarchical Tucker format (or other tensor formats) to speed up multidimensional convolution
- Automatic parameter selection for regularization
- Adapt this approach to higher frequency data, which tend to be higher rank

Future Work

- Impose regularization based on an understanding the physics underlying these data sets
- E.g. use knowledge of the behaviour of wave-propagation in 3D to prevent spurious artifacts

Future Work

- Apply the Hierarchical Tucker format to high-dimensional image volumes for computing outer products efficiently
- Image focusing

Acknowledgements

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