Recent developments on **Robust EPSI**

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EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

true primary wavefield SRME–produced primary

$$\mathbf{P_o} = \mathbf{P} - A(f)\mathbf{P_o}\mathbf{P}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{P_o}$ primary wavefield

$A(f)$ “matching” operator

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EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

true primary wavefield

SRME–produced primary

$$\mathbf{P_o} \approx \mathbf{P} - A(f)\mathbf{PP}$$

SRMP

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
$A(f)$	“matching” operator

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

adaptive subtraction

$$\min_A \sum_f \|\mathbf{P} - A(f)\mathbf{P}\mathbf{P}\|$$

SRMP

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
$A(f)$	"matching" operator

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

true primary wavefield SRME–produced primary

$$\mathbf{P_o} = \mathbf{P} - A(f)\mathbf{P_o}\mathbf{P}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{P_o}$ primary wavefield

$A(f)$ "matching" operator

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{P_o} + A(f)\mathbf{P_o}\mathbf{P}$$

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
$A(f)$	“matching” operator

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{P_o} + A(f)\mathbf{P_o}\mathbf{P}$$

$$\mathbf{P_o} = \mathbf{Q}\mathbf{G}$$
$$A(f) = -\mathbf{Q}^{-1}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{G}$ primary impulse response

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data predicted data from primary IR

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

$\mathbf{P}$	total up-going wavefield
$\mathbf{Q}$	down-going source signature
$\mathbf{G}$	primary impulse response

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data   predicted data from primary IR

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

EPSI Problem

In time domain (lower-case: whole dataset in time domain)

recorded data predicted data from primary IR

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathcal{M}(\mathbf{g}, \mathbf{q}) := \mathcal{F}_{\mathrm{t}}^{\dagger} \mathrm{BlockDiag}_{\omega_1 \cdots \omega_{nf}} [(q(\omega)\mathbf{I} - \mathbf{P})^{\dagger} \otimes \mathbf{I}] \mathcal{F}_{\mathrm{t}} \mathbf{g}$$

Inversion objective:

$$f(\mathbf{g}, \mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathcal{M}(\mathbf{g}, \mathbf{q})\|_2^2$$

EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

In fact it is bilinear:

$$\mathbf{M}_{\tilde{q}}\mathbf{g} = \mathcal{M}(\mathbf{g}, \tilde{\mathbf{q}}) \qquad \mathbf{M}_{\tilde{g}}\mathbf{q} = \mathcal{M}(\mathbf{q}, \tilde{\mathbf{g}})$$

EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}} \right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}} \right)_{\tilde{g}}$$

Associated objectives:

$$f_{\tilde{q}}(\mathbf{g}) = \frac{1}{2} \|\mathbf{p} - \mathbf{M}_{\tilde{q}} \mathbf{g}\|_2^2 \qquad f_{\tilde{g}}(\mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathbf{M}_{\tilde{g}} \mathbf{q}\|_2^2$$

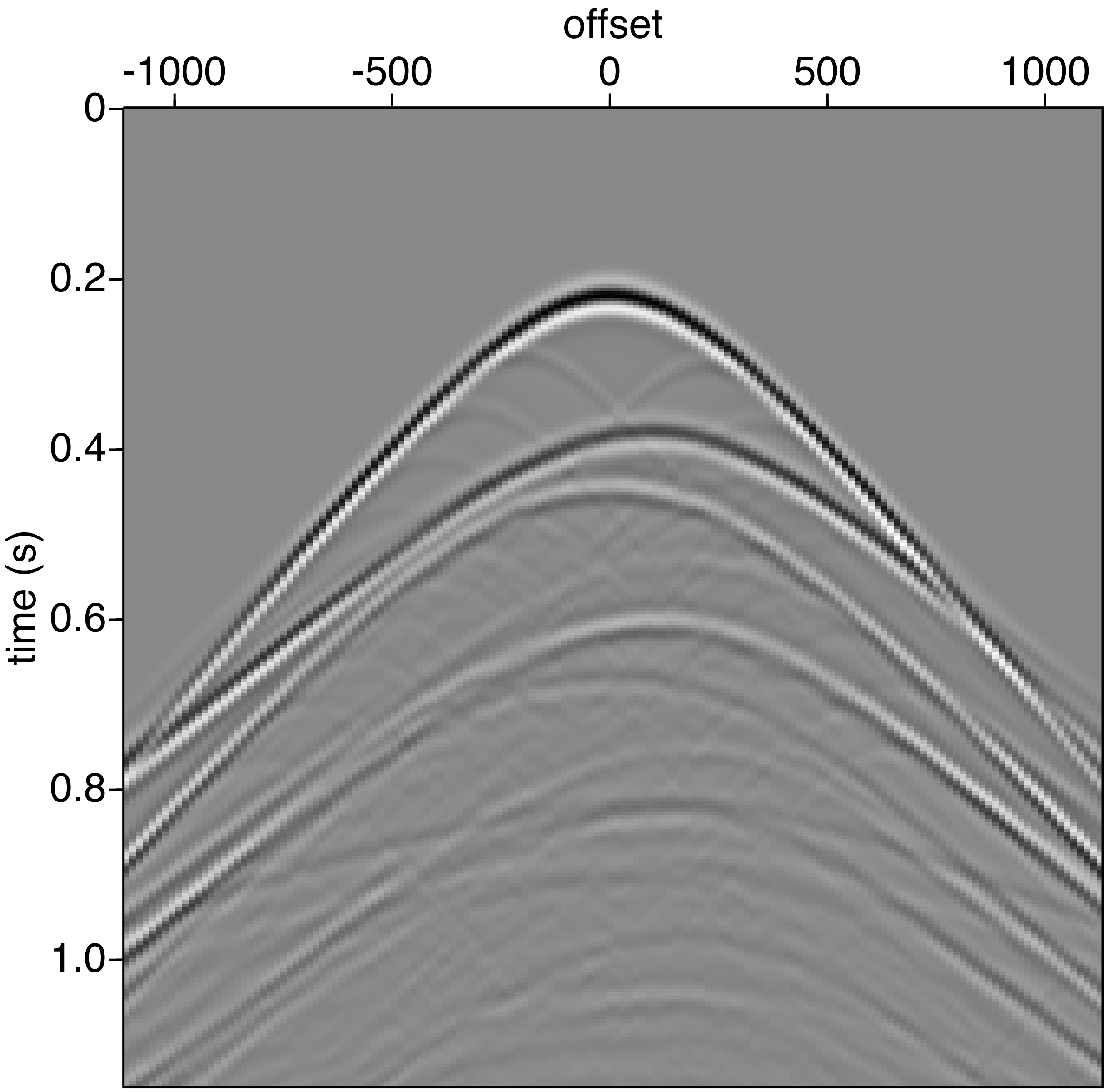
EPSI Procedure

Do:

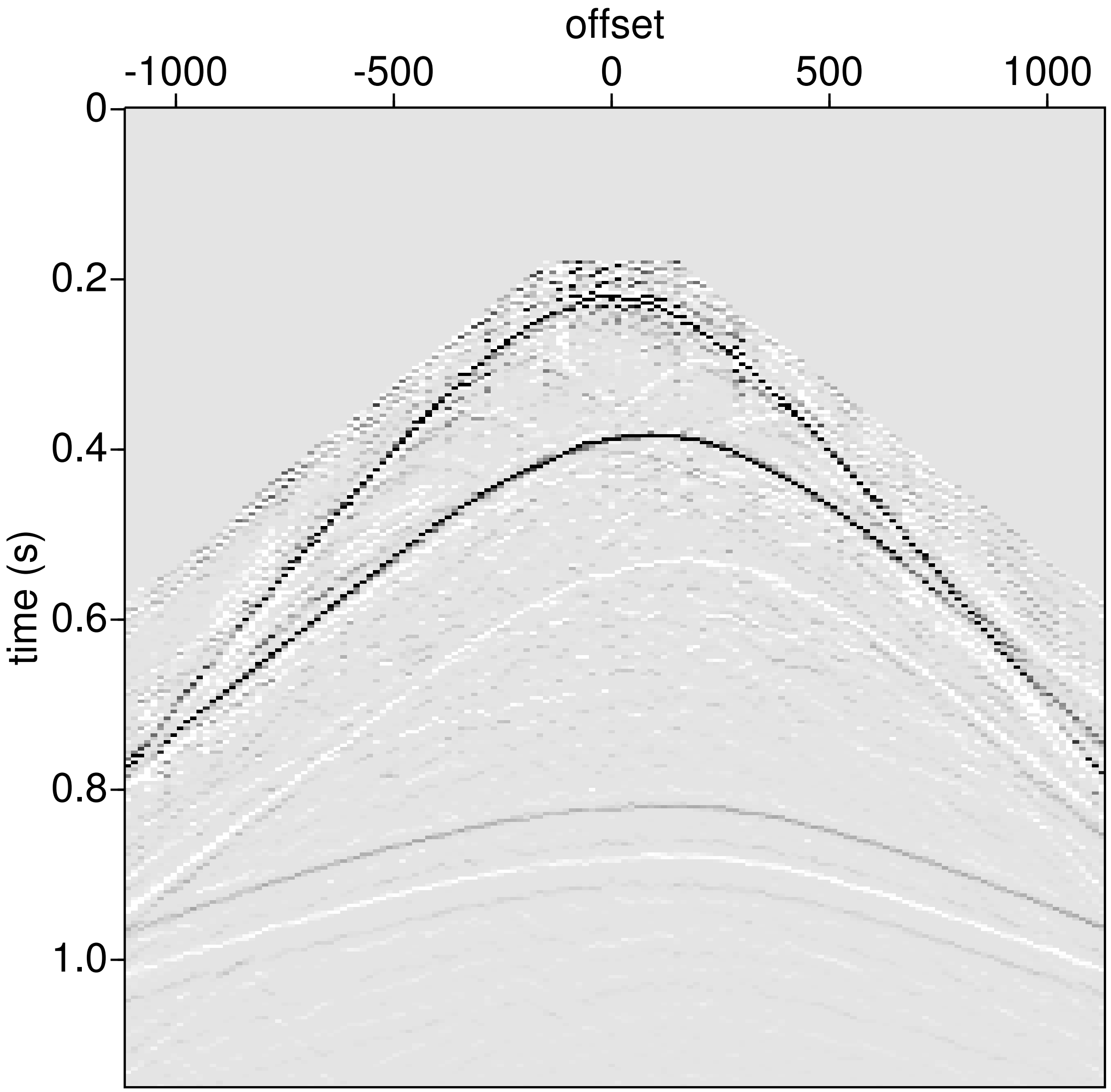
$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \mathcal{S}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Gradient sparsity

$\mathcal{S}$: pick largest ρ elements per trace


offset
-1000
-500
0
500
1000
time (s)
0
0.2
0.4
0.6
0.8
1.0

Data


offset
-1000
-500
0
500
1000
time (s)
0
0.2
0.4
0.6
0.8
1.0

EPSI IR

Robust EPSI procedure

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

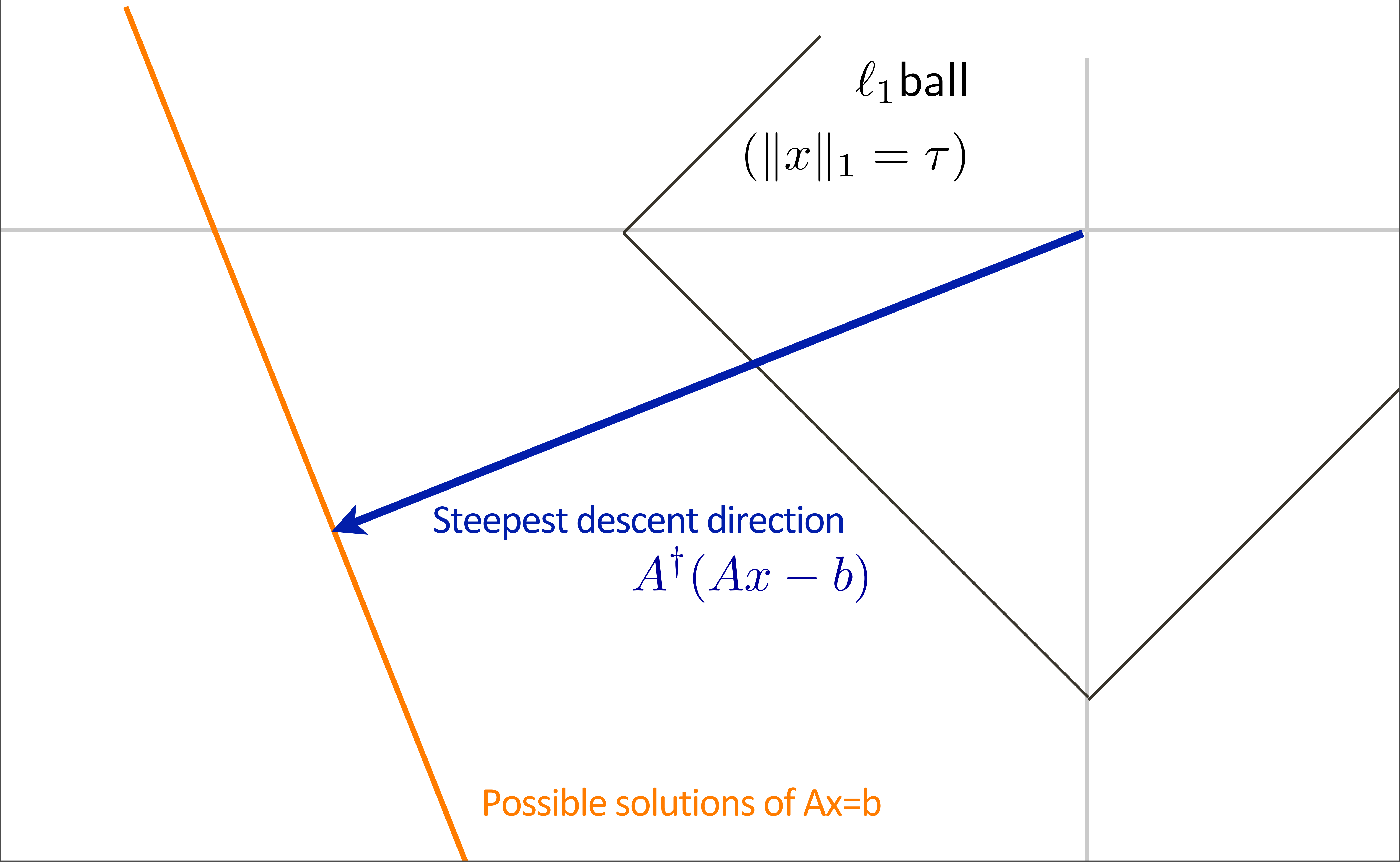
(Solve with SPG part of SPGL1 until Pareto curve reached)

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)

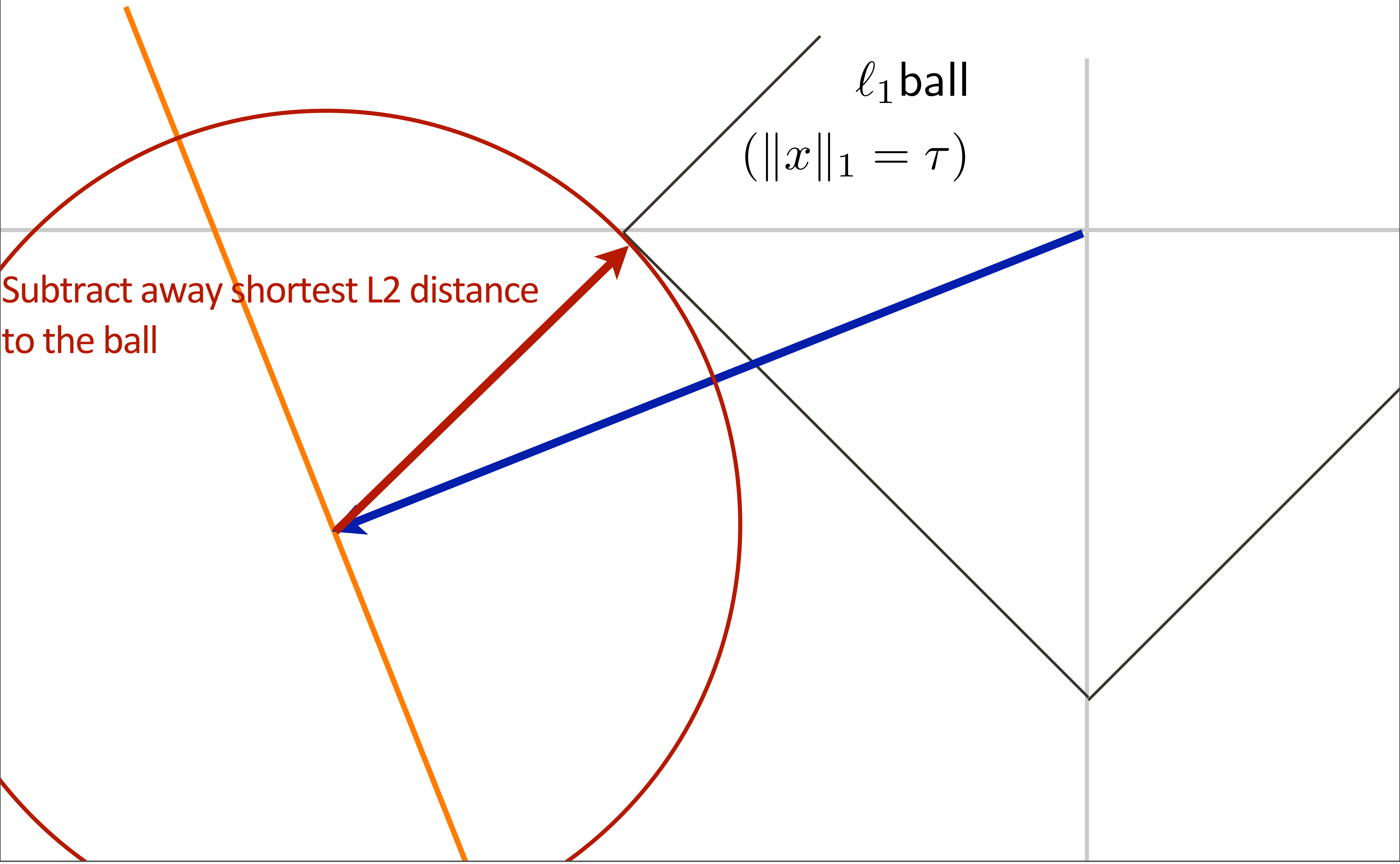
Solving with SPG

$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \le \tau \end{array}$$



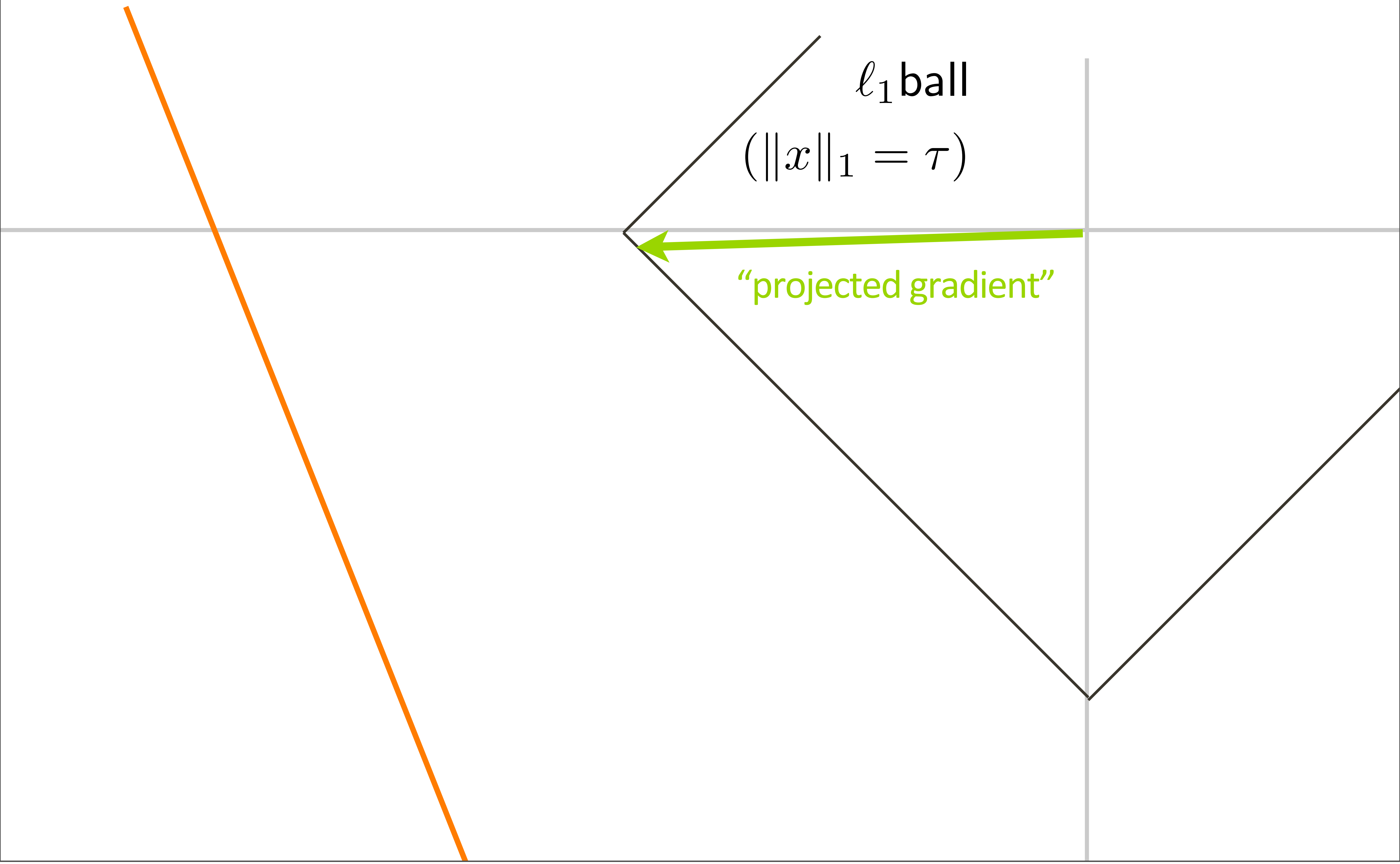
Solving with SPG

minimize $\|Ax - b\|_2$
subject to $\|x\|_1 \leq \tau$

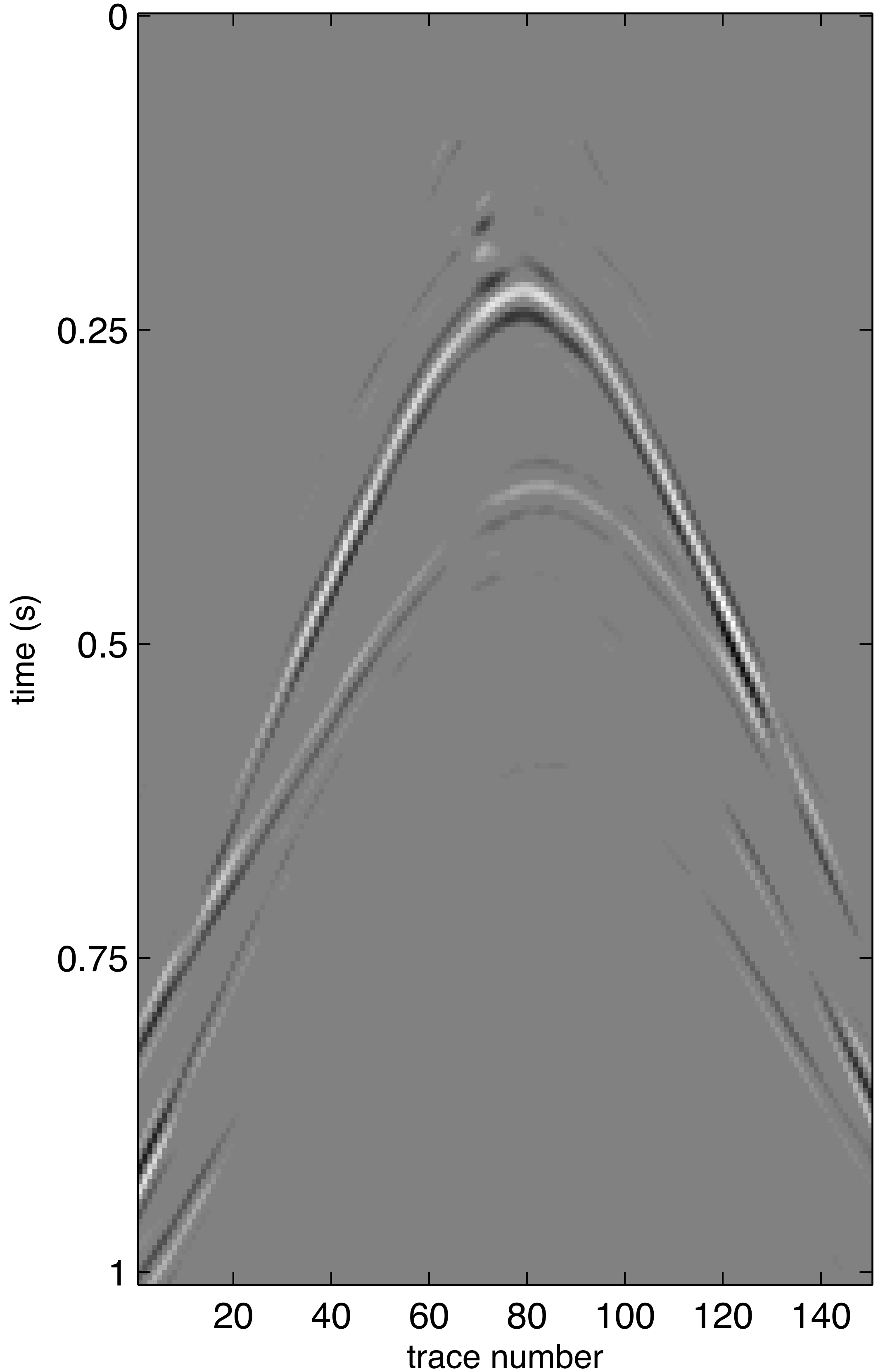


Solving with SPG

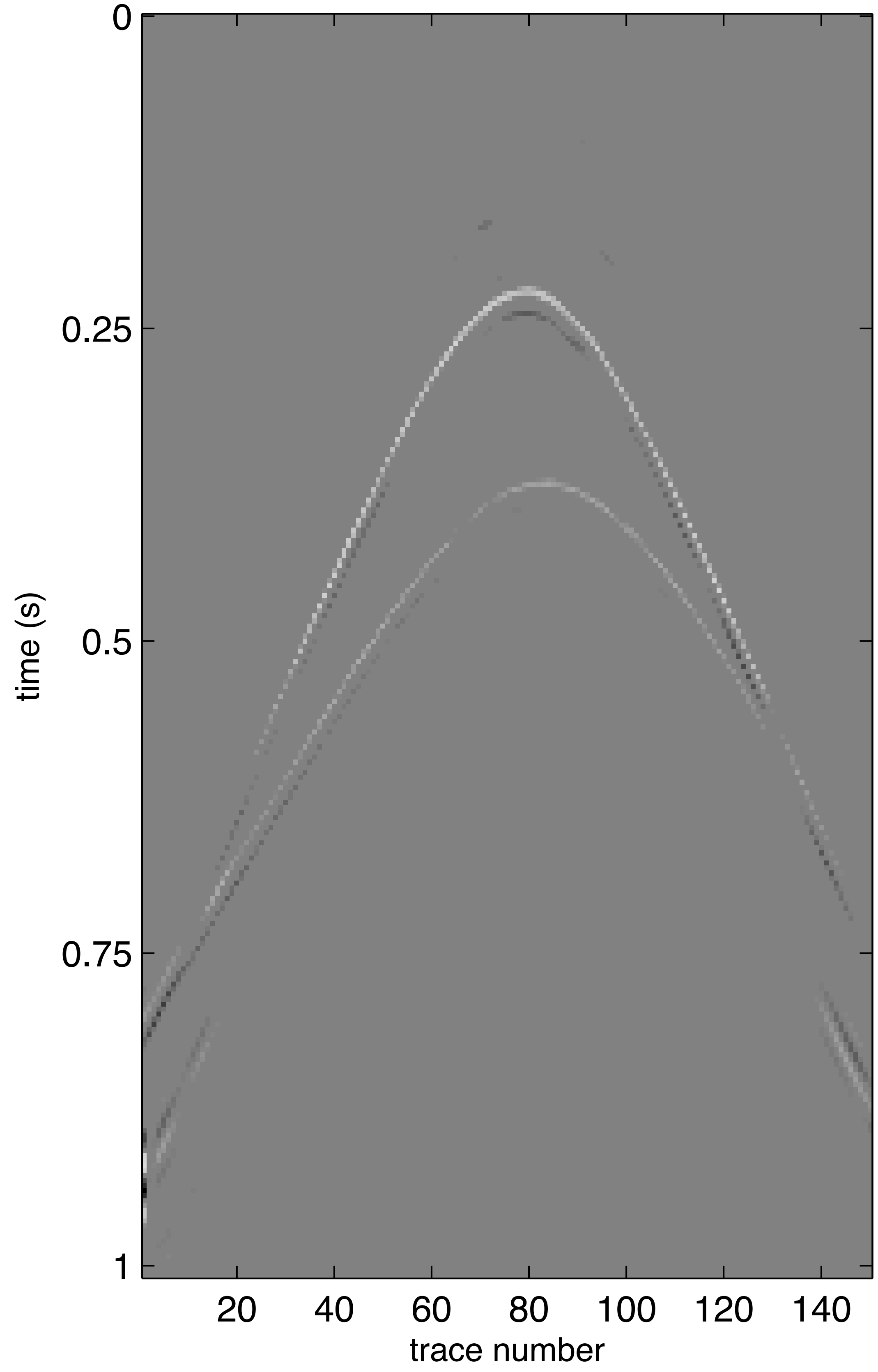
$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \le \tau \end{array}$$



SPG start



SPG at Pareto curve



Solving with SPG

minimize $\|Ax - b\|_2$
subject to $\|x\|_1 \leq \tau$

Projecting onto 1-ball

1) Find $\Delta\tau := \|\mathbf{x}\|_1 - \tau$

2) Subtract each element of $\mathbf{x}$ by $\dfrac{\Delta\tau}{N}$

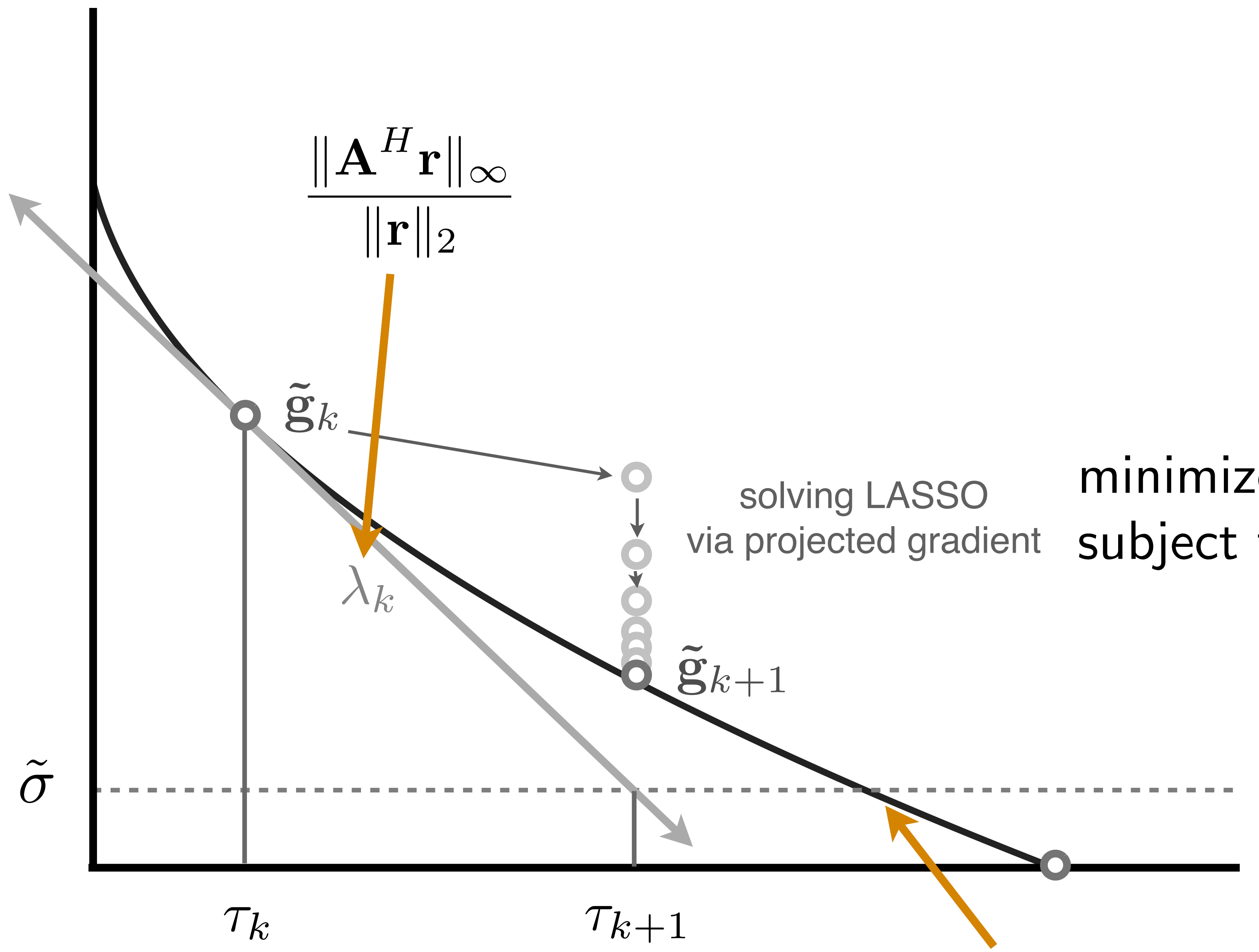
3) Repeat until $\|\mathbf{x}\|_1 \leq \tau$

*** usually 3 to 5 iterations, easily parallelizeable**

Pareto curve

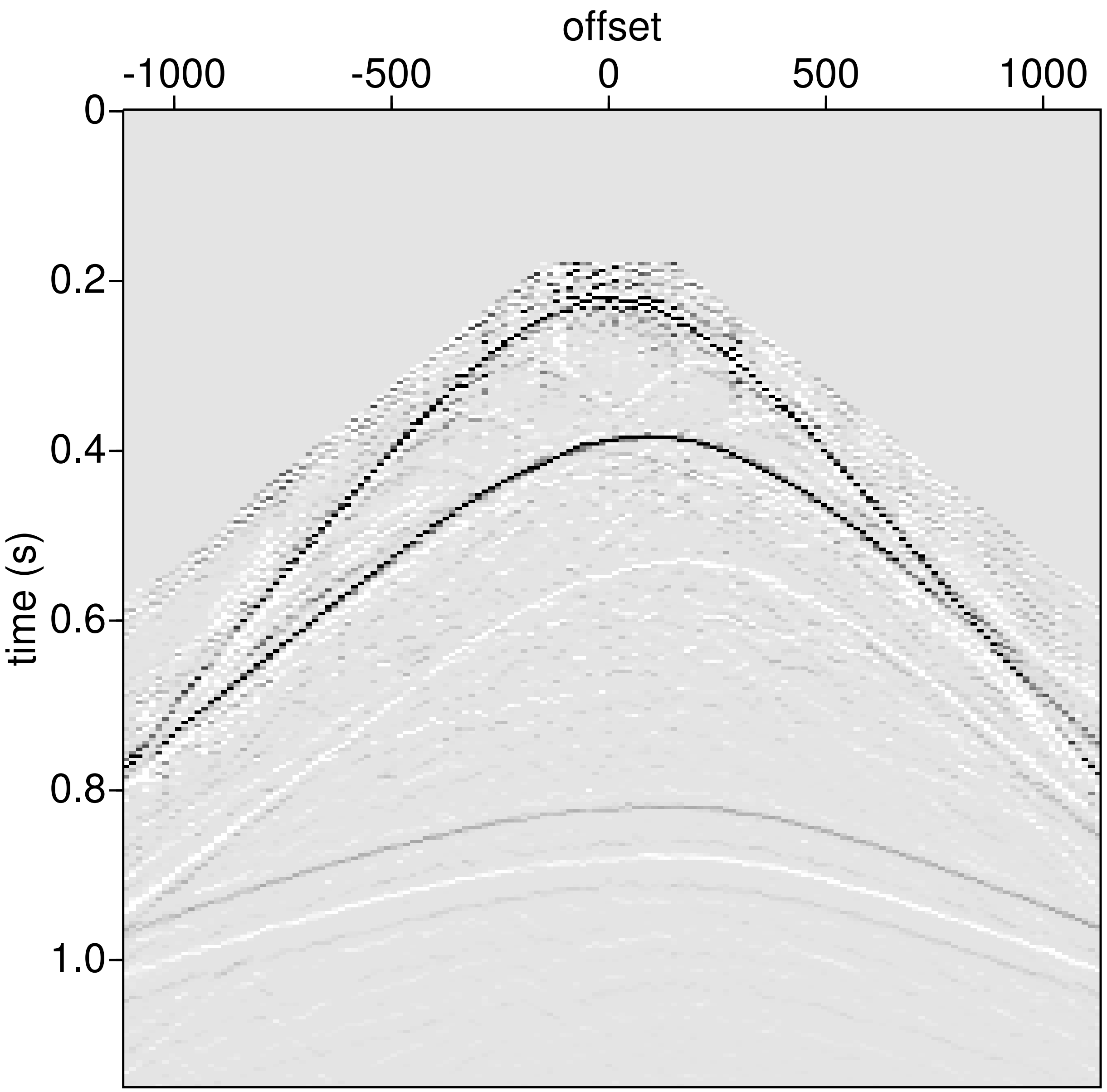
minimize $\|x\|_1$
subject to $\|Ax - b\|_2 \le \sigma$

Look at the solution space and the line of optimal solutions (Pareto curve)

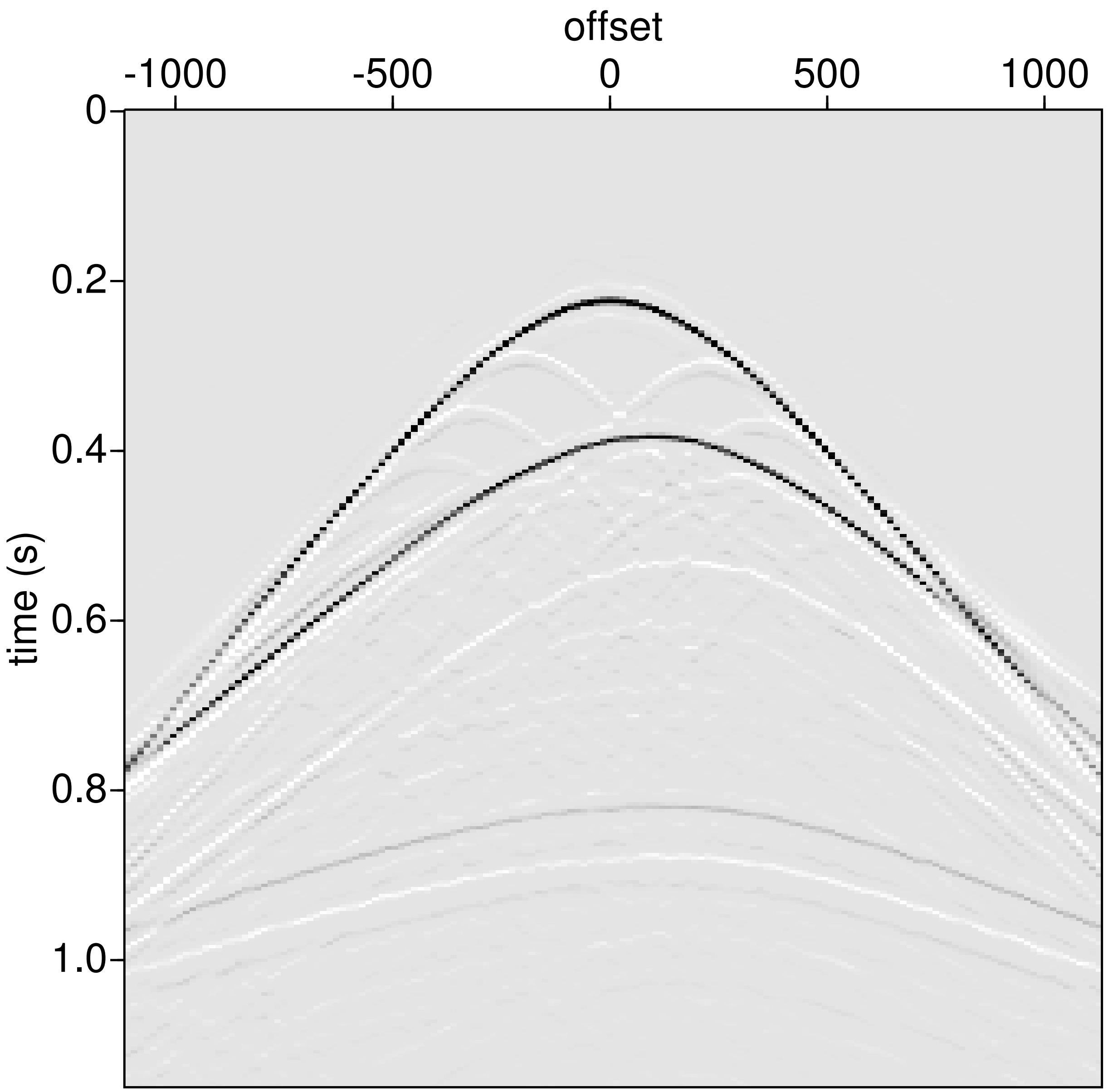


minimize $\|Ax - b\|_2$
subject to $\|x\|_1 \le \tau$

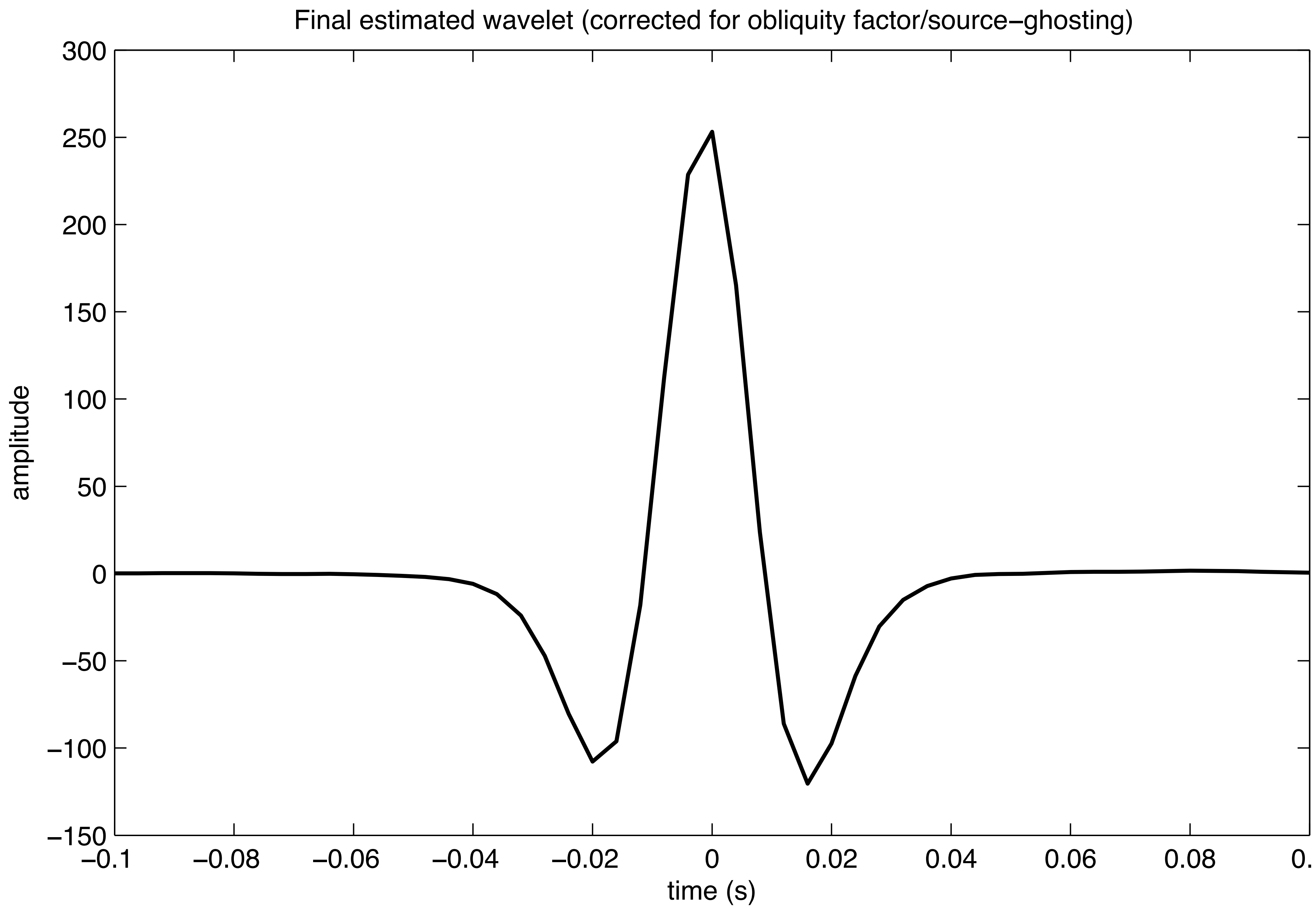
feasible solution with smallest $\|x\|_1$



EPSI IR



Robust EPSI IR



Final estimated wavelet (corrected for obliquity factor/source−ghosting)
300
250
200
150
100
50
0
−50
−100
−150
amplitude
−0.1
−0.08
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
0.08
0.1
time (s)

Convolution model

Convolution Model

$$\text{Up-going Primary} = \mathbf{GQ}$$

EPSI Model

$$\text{Up-going Primary} + \text{Multiples} = \mathbf{GQ} + \mathbf{GRP}$$

additional info on G

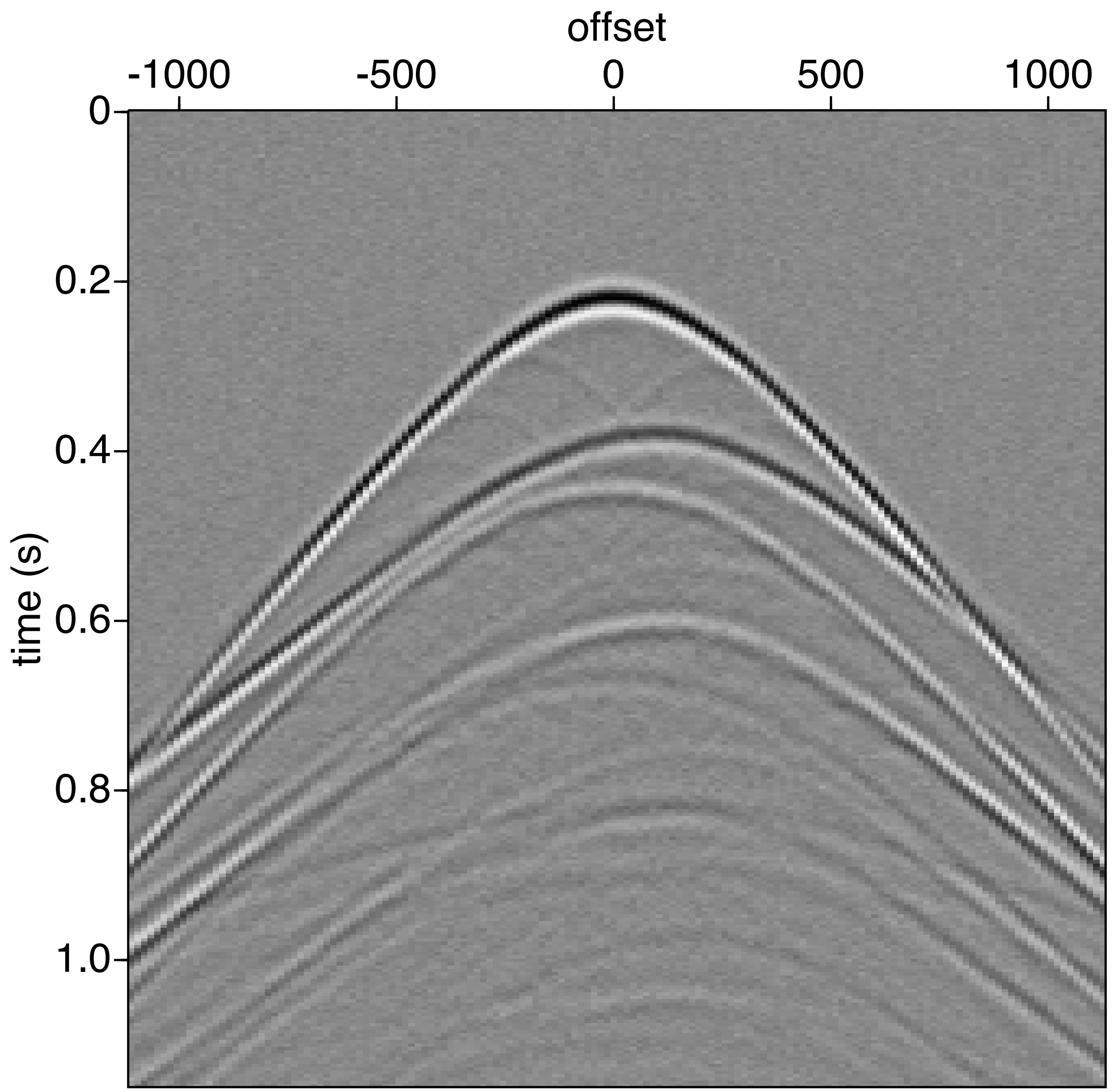
$\mathbf{P}$ total recorded up-going wavefield

$\mathbf{Q}$ source signature (incl. src ghosts)

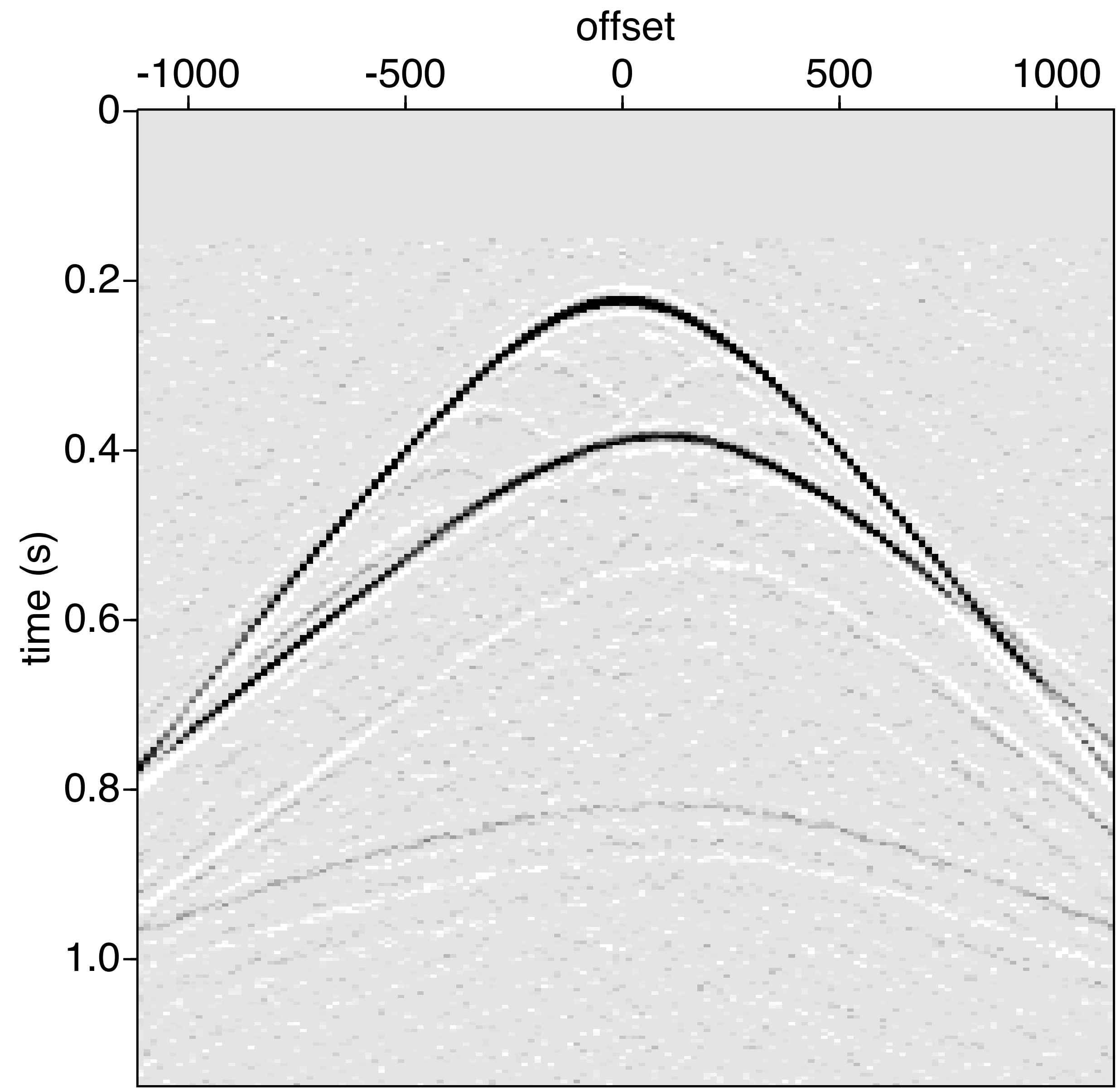
$\mathbf{R}$ reflectivity of free surface (assume -1)

$\mathbf{G}$ primary impulse response

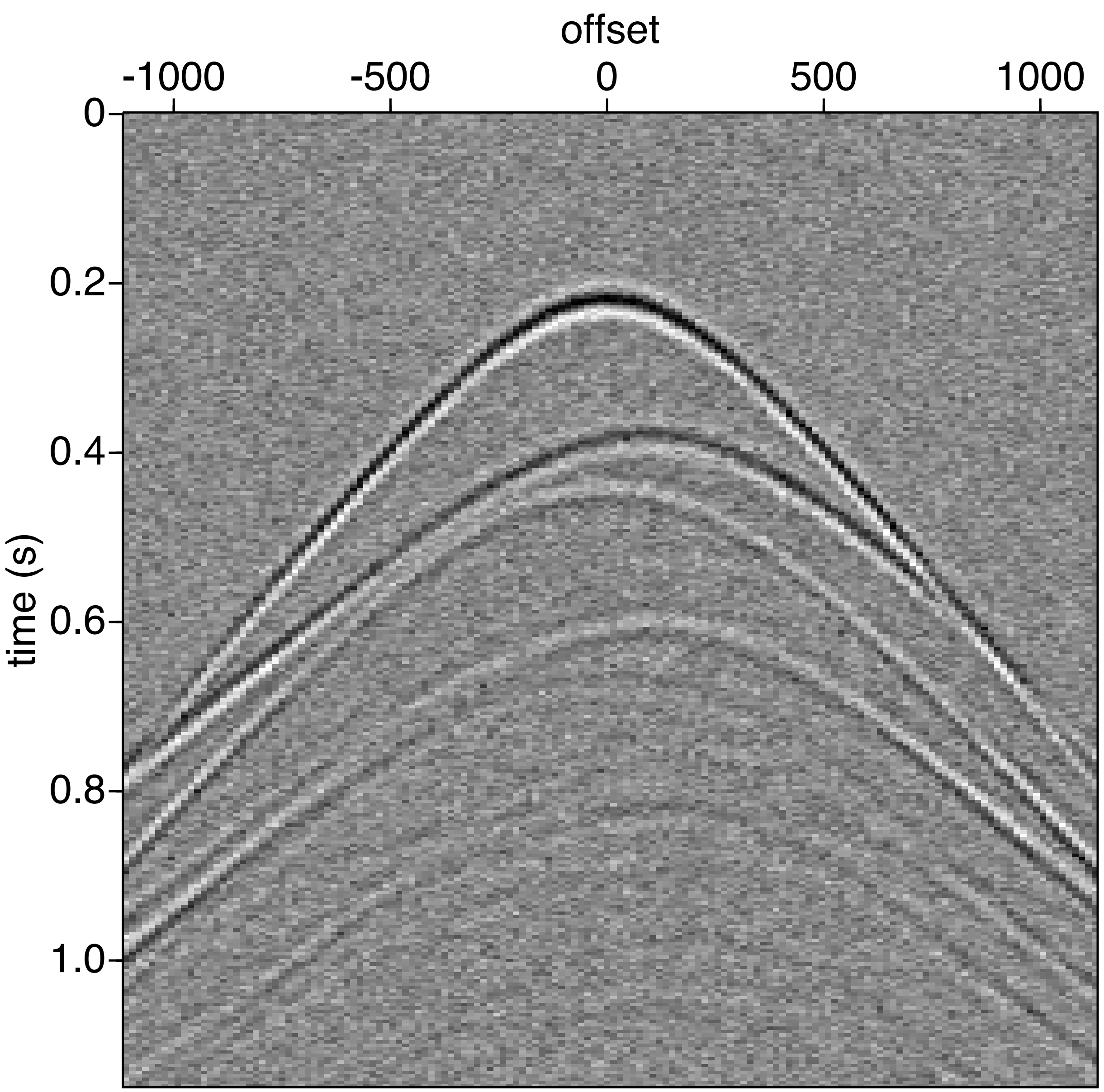
(all monochromatic data matrix, implicit ω)



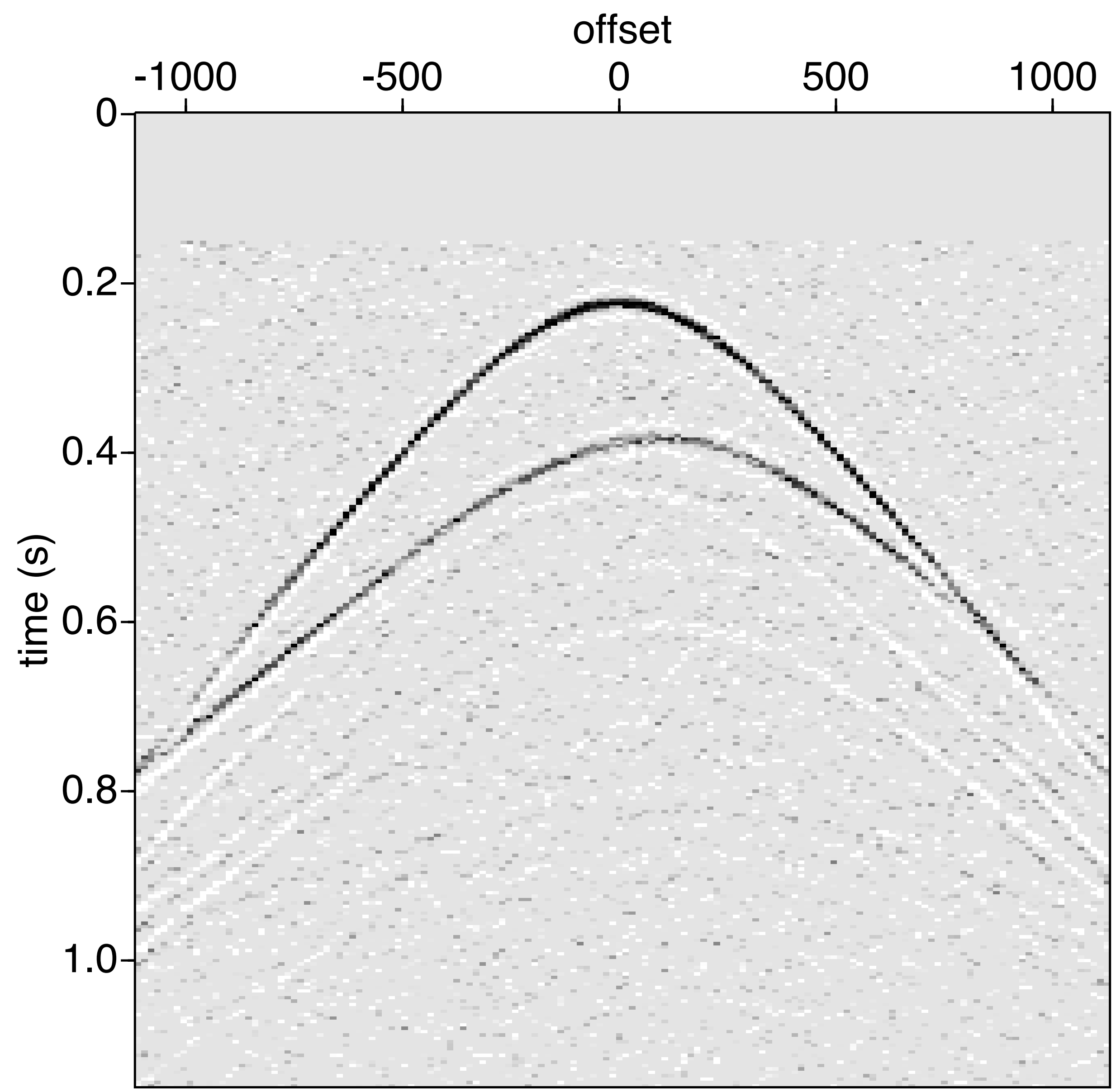
Data + 40% noise (SNR 7)



Robust EPSI IR



Data + 100% noise (SNR 0)



Robust EPSI IR

Enforcing reciprocity

recorded data

predicted data from primary IR

$$\mathbf{P} = \mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P}$$

P symmetric
(physical reciprocity)

Enforcing reciprocity

recorded data predicted data from primary IR

$$\mathbf{P} = \mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P}$$

P symmetric
(physical reciprocity)

Q symmetric
(by construction)

Enforcing reciprocity

recorded data　　predicted data from primary IR

$$\mathbf{P} = \mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P}$$

P symmetric
(physical reciprocity)

G *should* be symmetric

Q symmetric
(by construction)

Enforcing reciprocity

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{g}_{(t,x_s,x_r)} = \mathbf{g}_{(t,x_r,x_s)}$$

(Just add new set constraint to SPG problem)

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

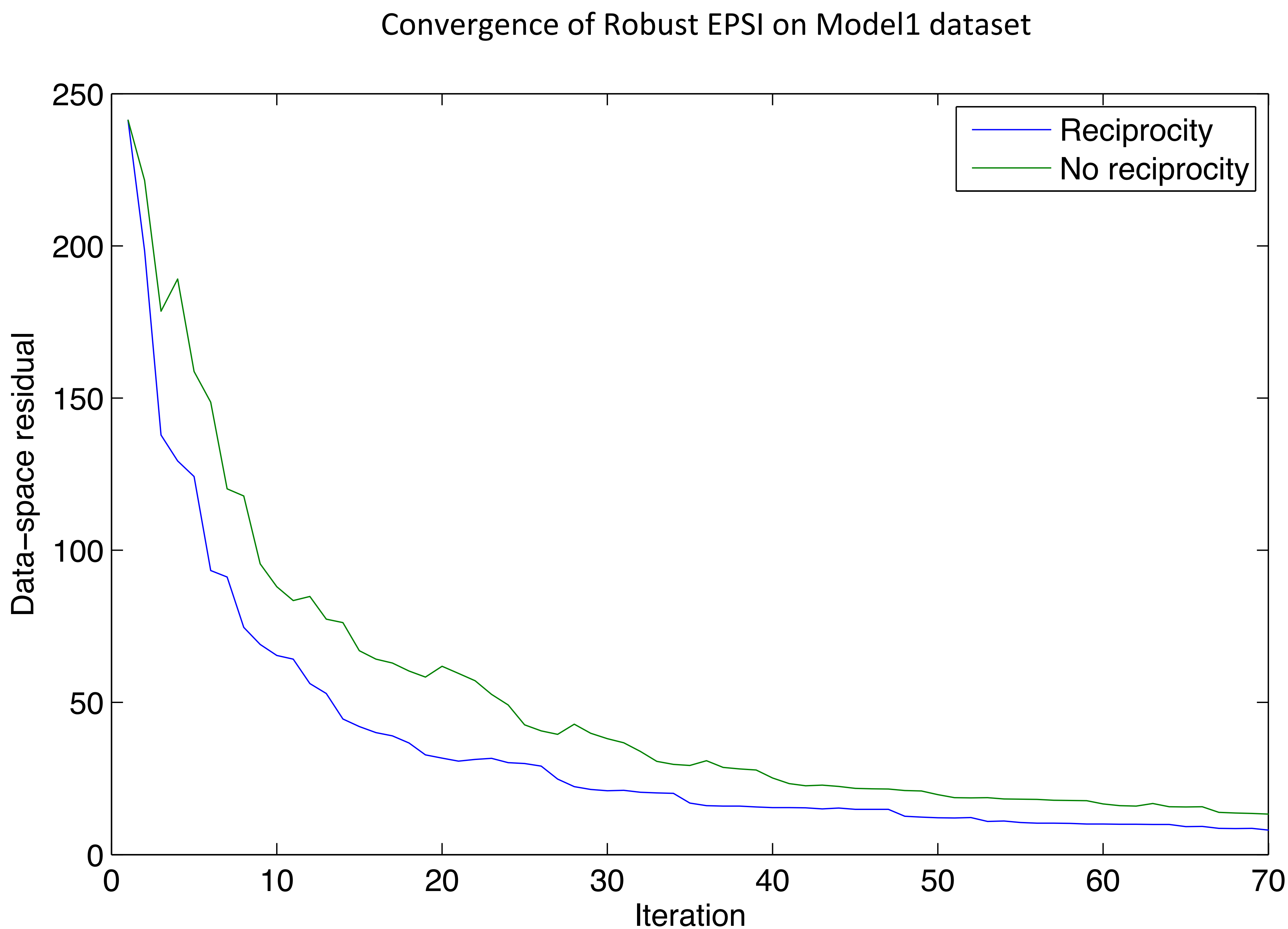
Enforcing reciprocity

- Set of all symmetric matrices is a convex set
- Projection onto set of symmetric matrices, followed by projection onto L1-ball, projects onto the intersection of both sets
- Projection onto symmetric matrix set is cheap:

$$\mathcal{P}_{\mathrm{symm}}(\mathbf{G}) = \frac{\mathbf{G} + \mathbf{G}^T}{2}$$

OPEN: Efficiently enforcing reciprocity of *forward modeled* wavefield (i.e., GP)?

Enforcing reciprocity



Sampling Issues

- Near-offset traces unavailable
- Shot undersampling in inline and crossline direction

Inverting for unknown data

While $\|\mathbf{p}_k - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p}_k - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

(Solve with SPG part of SPGL1 until Pareto curve reached)

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha\Delta\mathbf{p}(\mathbf{g}_{k+1}, \mathbf{q}_{k+1}, \mathbf{p}_k)$$

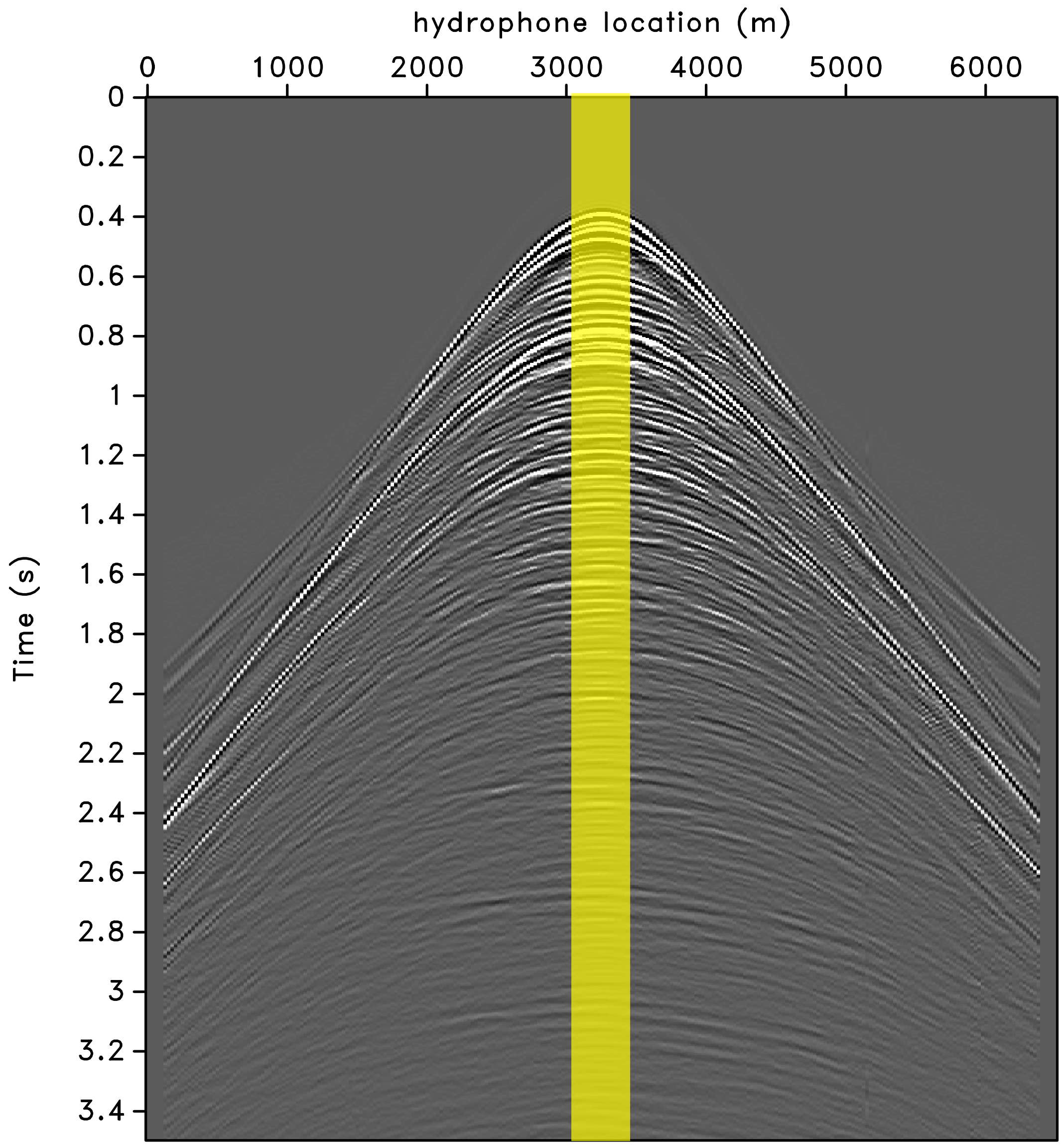
(Gradient update on data)

Inverting for unknown data

$$\Delta\mathbf{P}(\mathbf{G}_{k+1},\mathbf{Q}_{k+1},\mathbf{P}_k) := -(\mathbf{I}+\mathbf{G}_{k+1})^H(\mathbf{P}_k - \mathbf{Q}_{k+1}\mathbf{G}_{k+1} + \mathbf{G}_{k+1}\mathbf{P}_k)$$

$$\Delta\mathbf{P}(\mathbf{G}_{k+1},\mathbf{R}_{k+1}) := -(\mathbf{I}+\mathbf{G}_{k+1})^H(\mathbf{R}_{k+1})$$

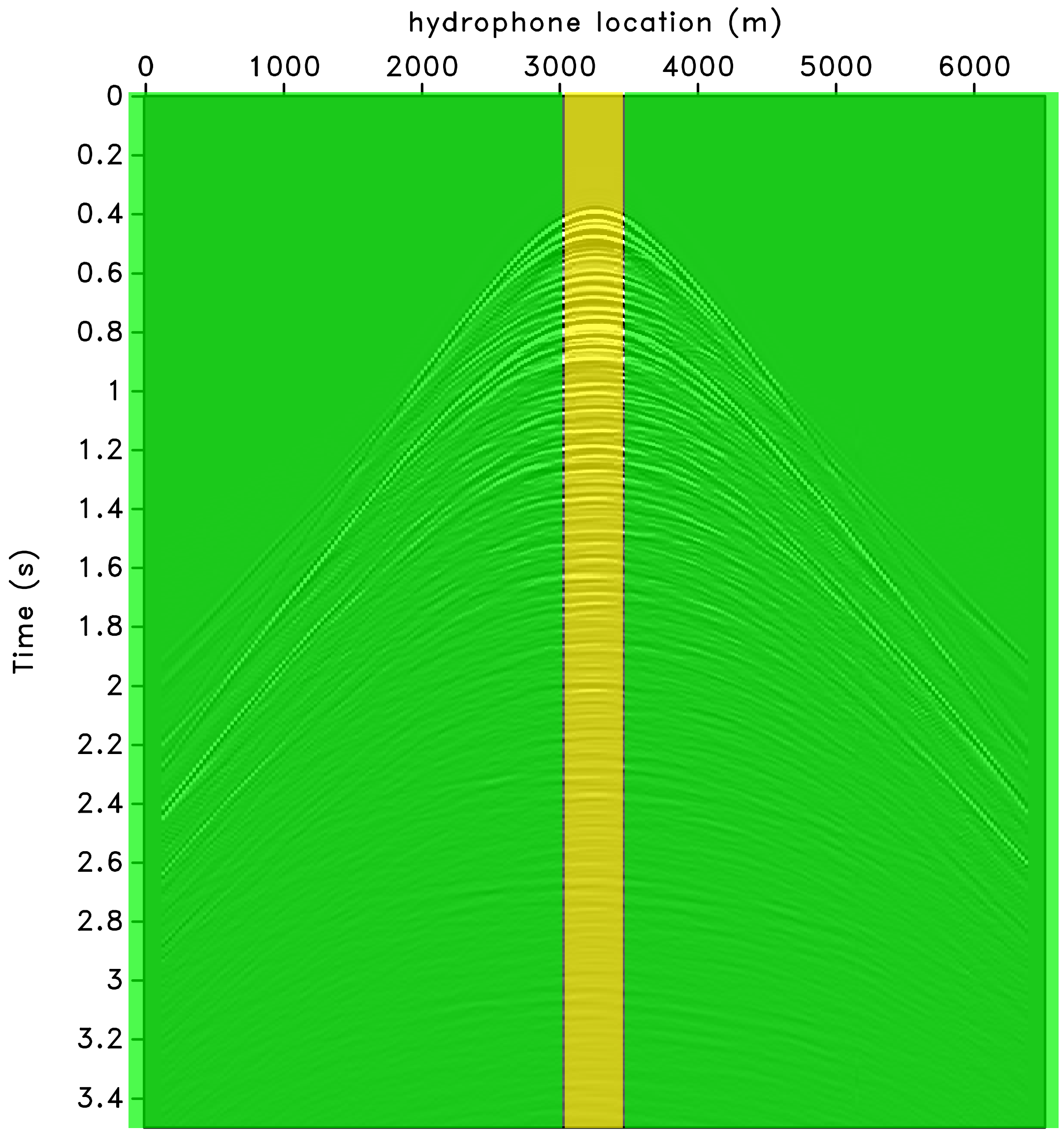
Inverting for unknown data



Restrict update $\Delta\mathbf{p}$ to unknown/ uncertain traces

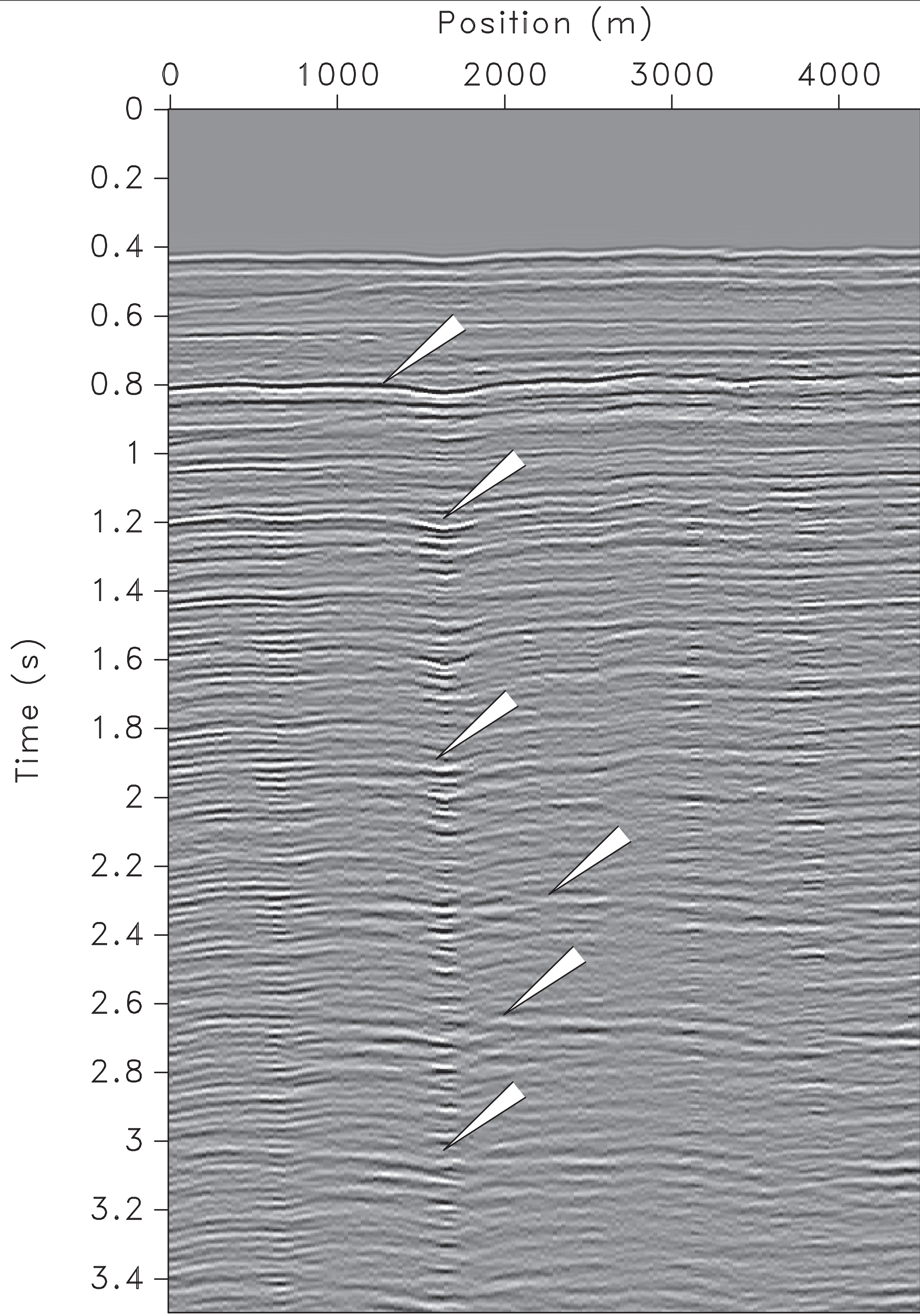
(near offset)

Inverting for unknown data



Restrict update $\Delta \mathbf{p}$ to unknown/ uncertain traces (near offset)

Calculate step-length α by its influence elsewhere

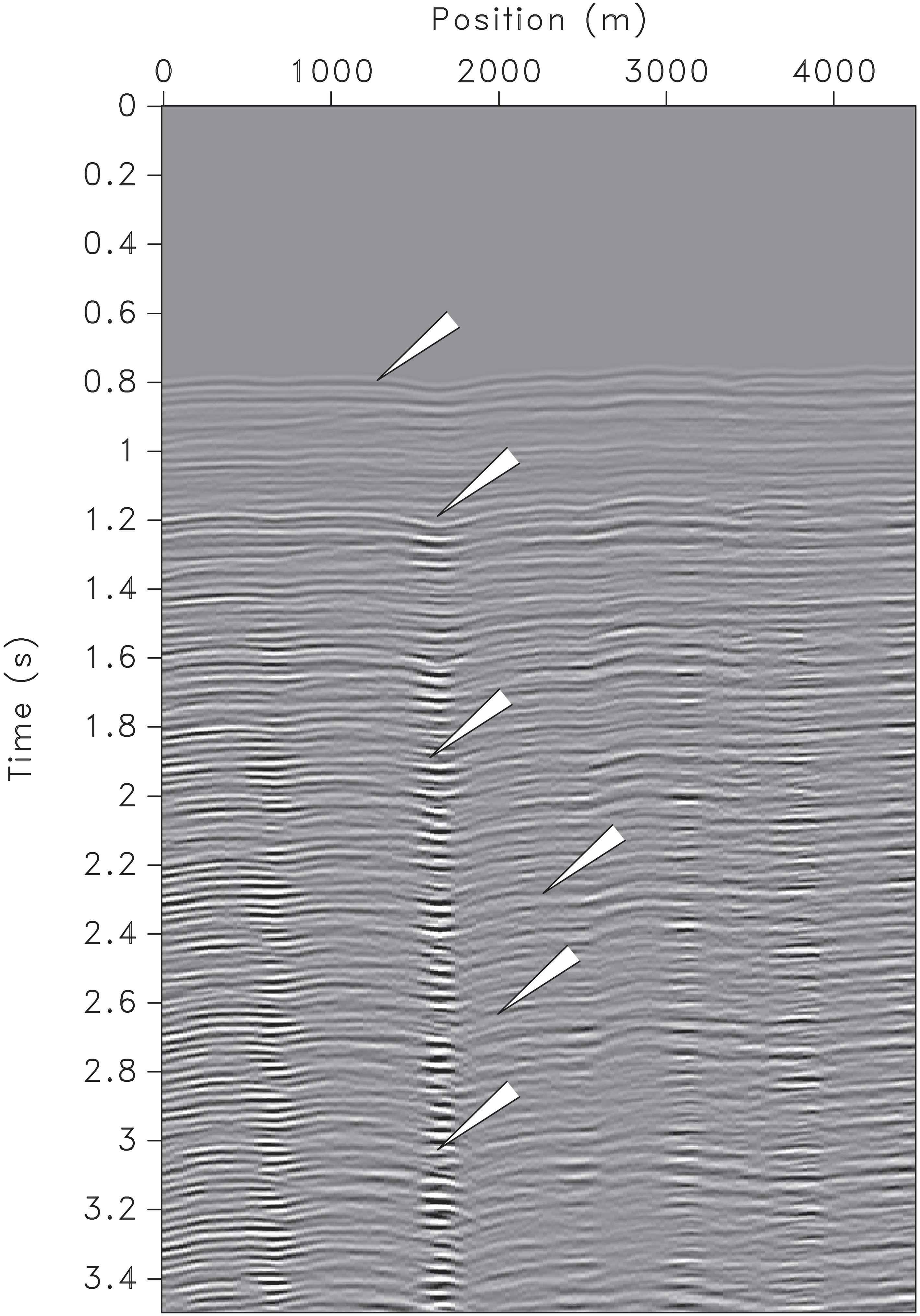


North Sea

data

offset gather 200m

time-gain panel

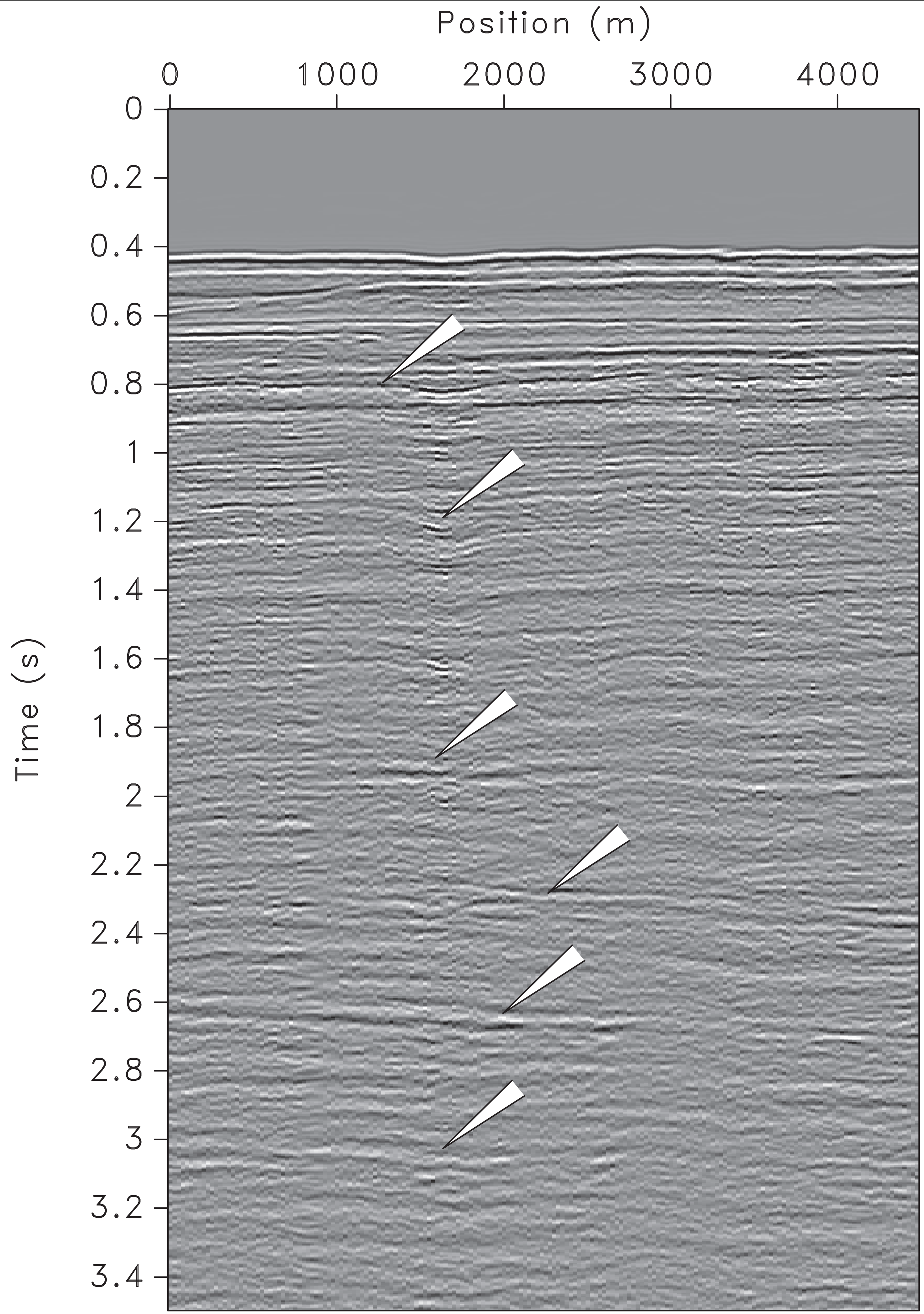


North Sea

SRMP multiple

offset gather 200m

time-gain panel



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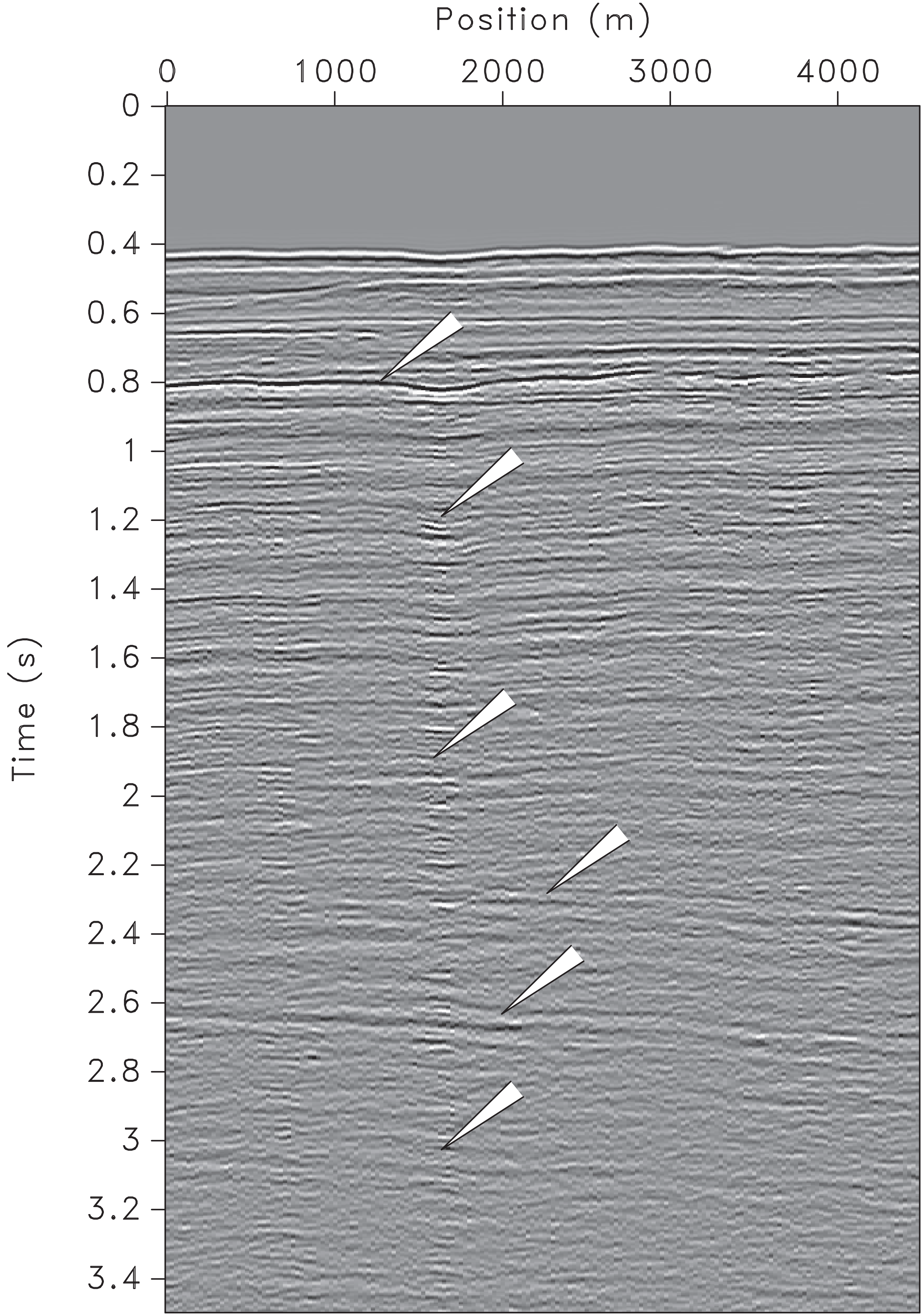
North Sea

SRME w/LS subtraction

offset gather 200m

time-gain panel



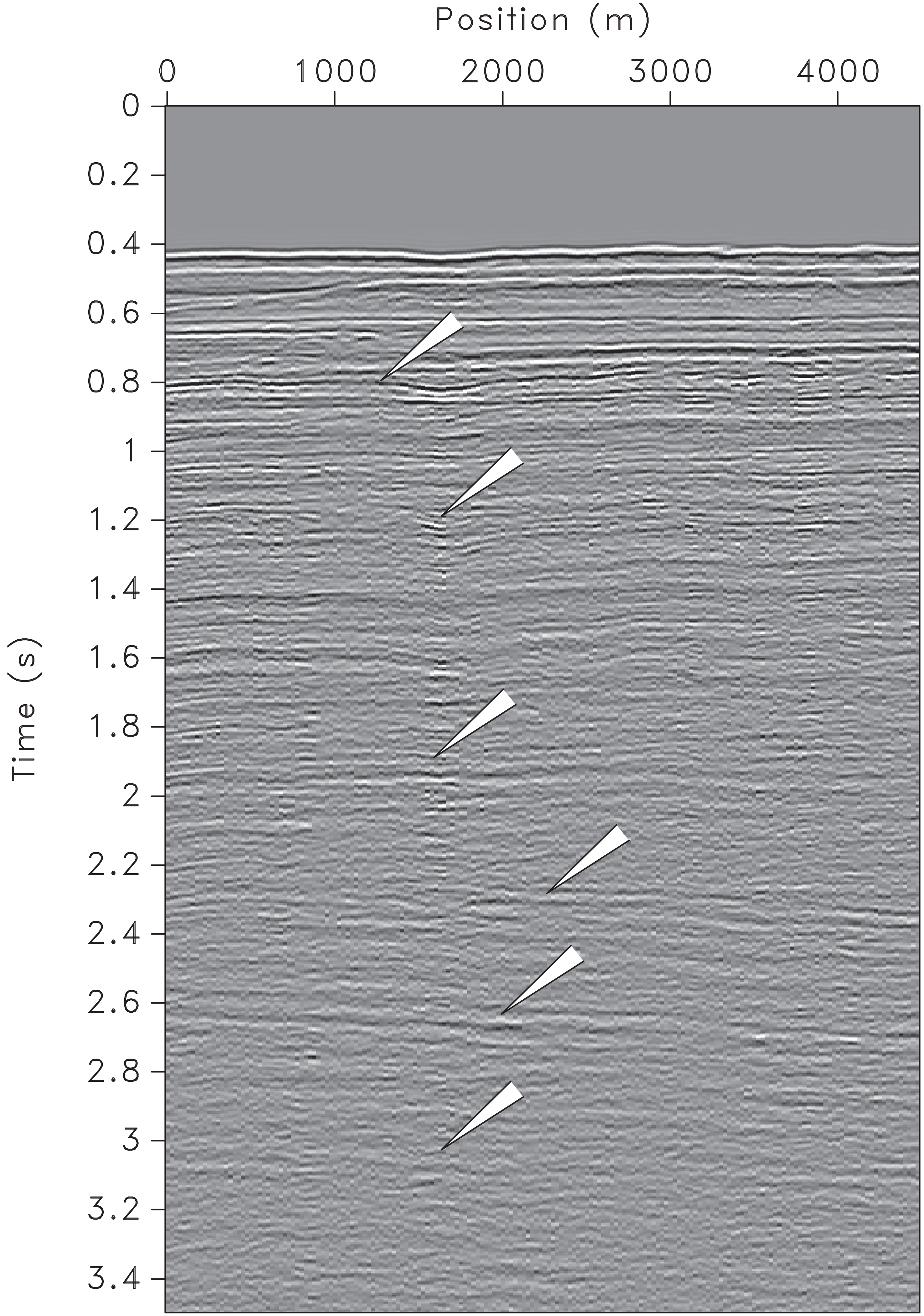
North Sea

REPSI

offset gather 200m

time-gain panel

75 iterations



SLIM

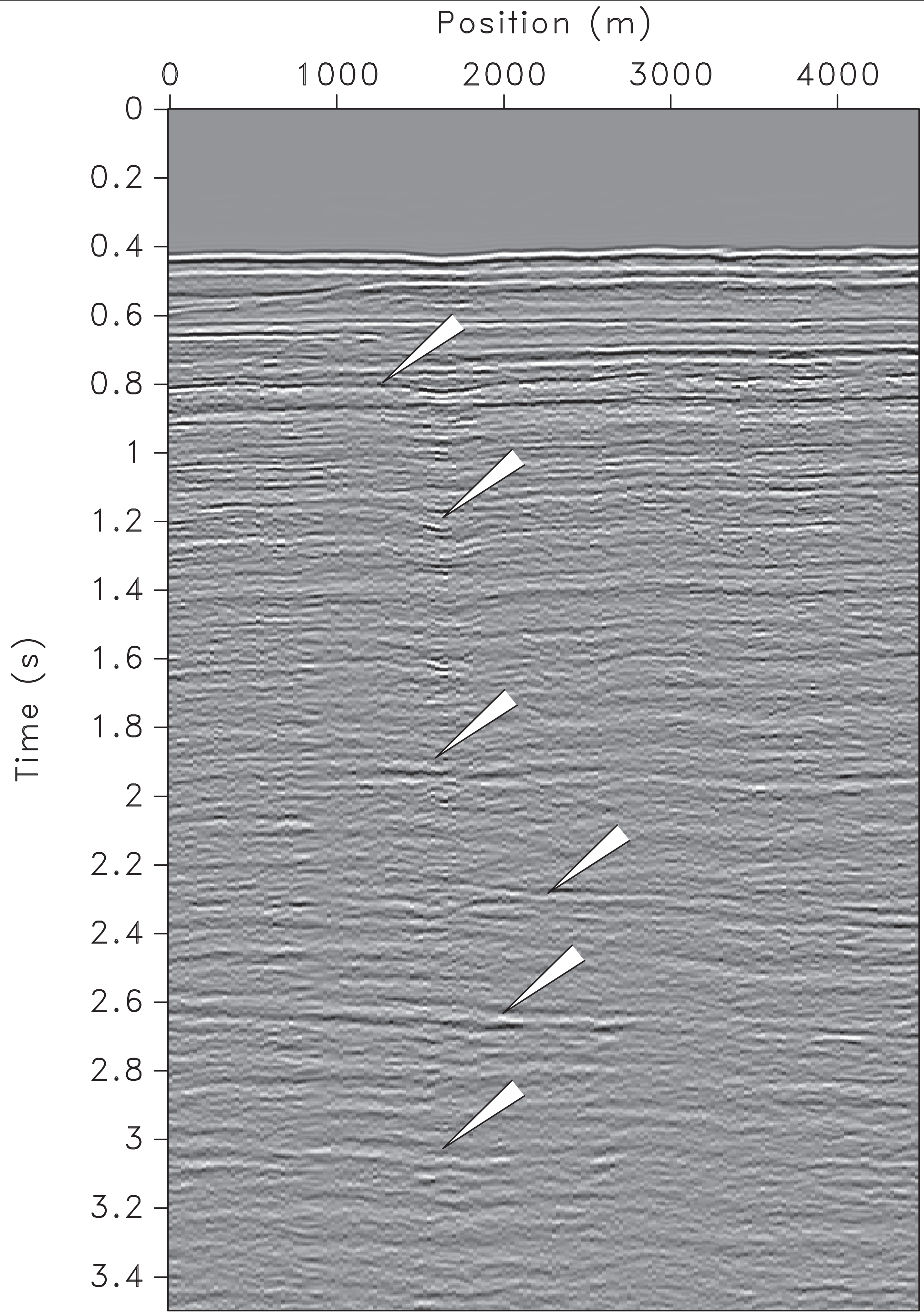
North Sea

REPSI + data update

offset gather 200m

time-gain panel

85 iterations

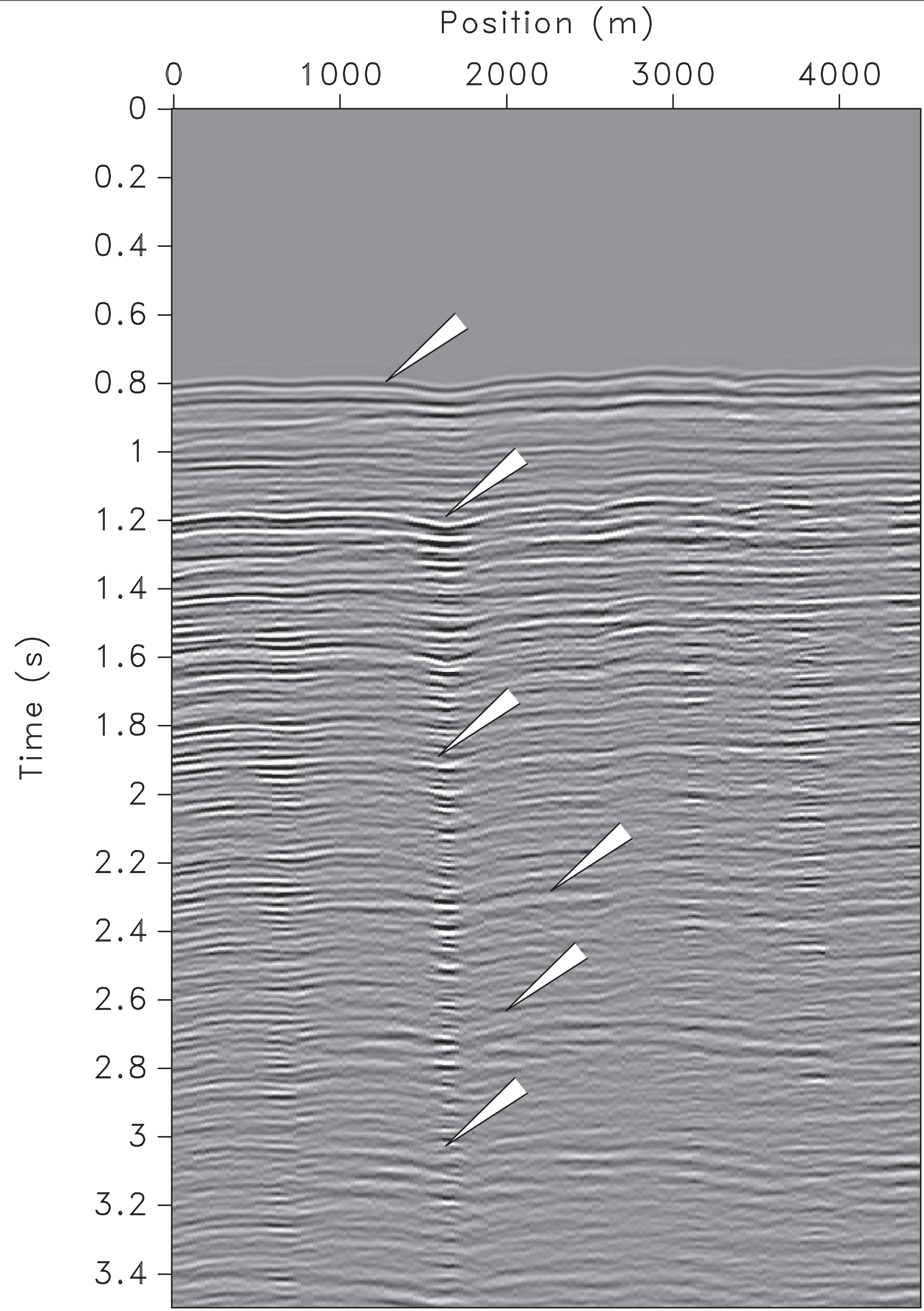


North Sea

SRME w/LS subtraction

offset gather 200m

time-gain panel



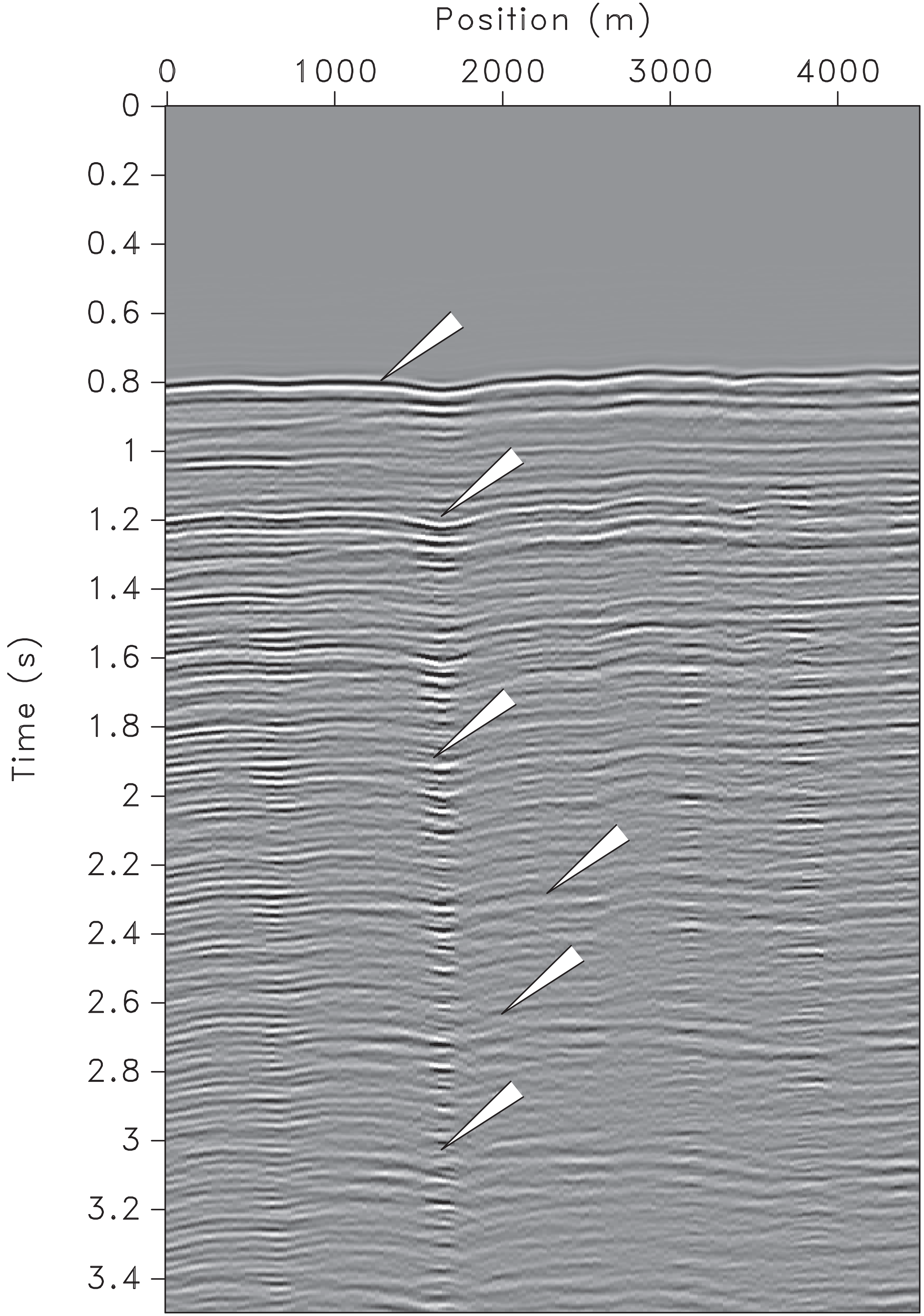
SLIM

North Sea

REPSI predicted mult

offset gather 200m

time-gain panel



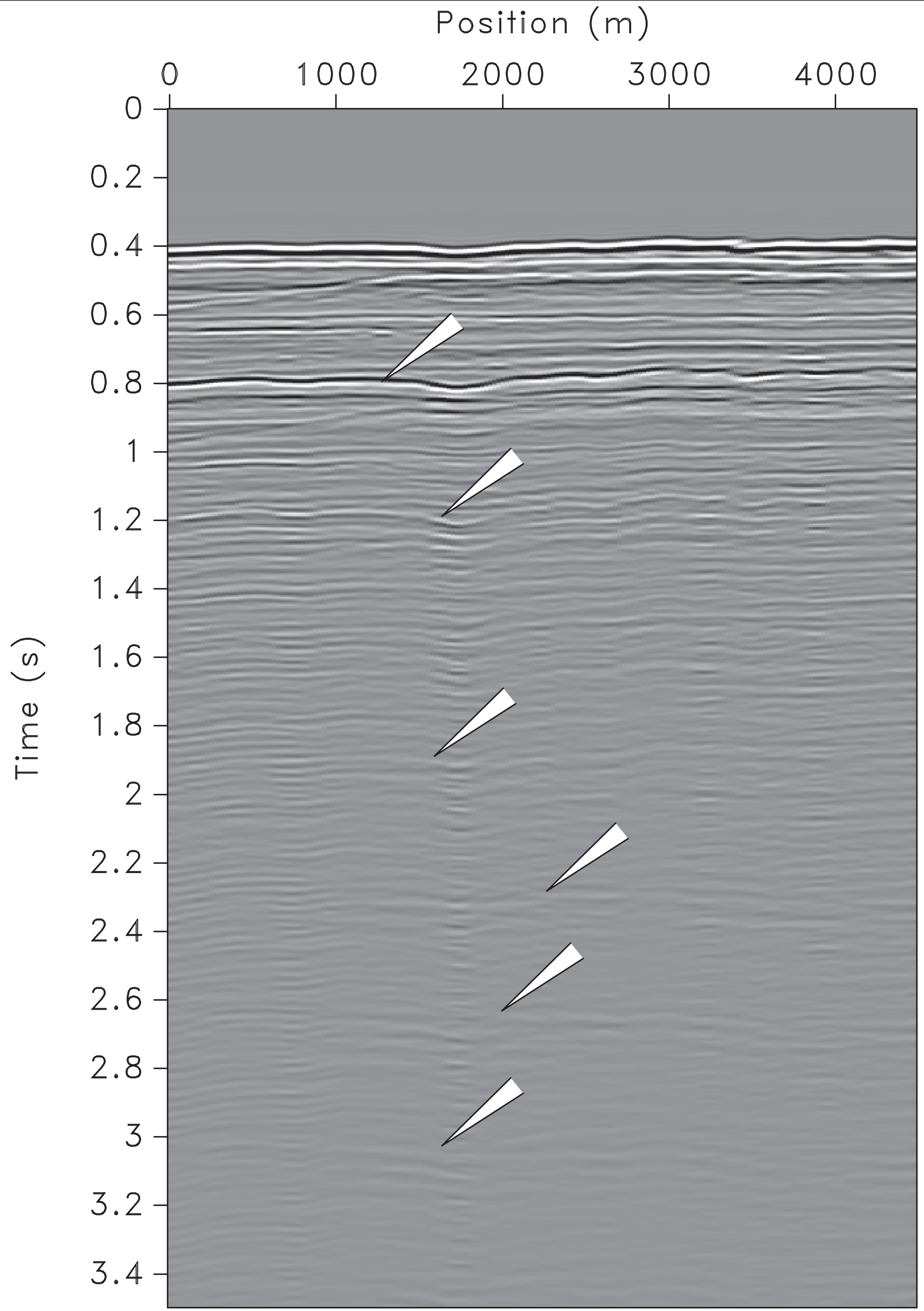
SLIM

North Sea

REPSI + data update predicted mult

offset gather 200m

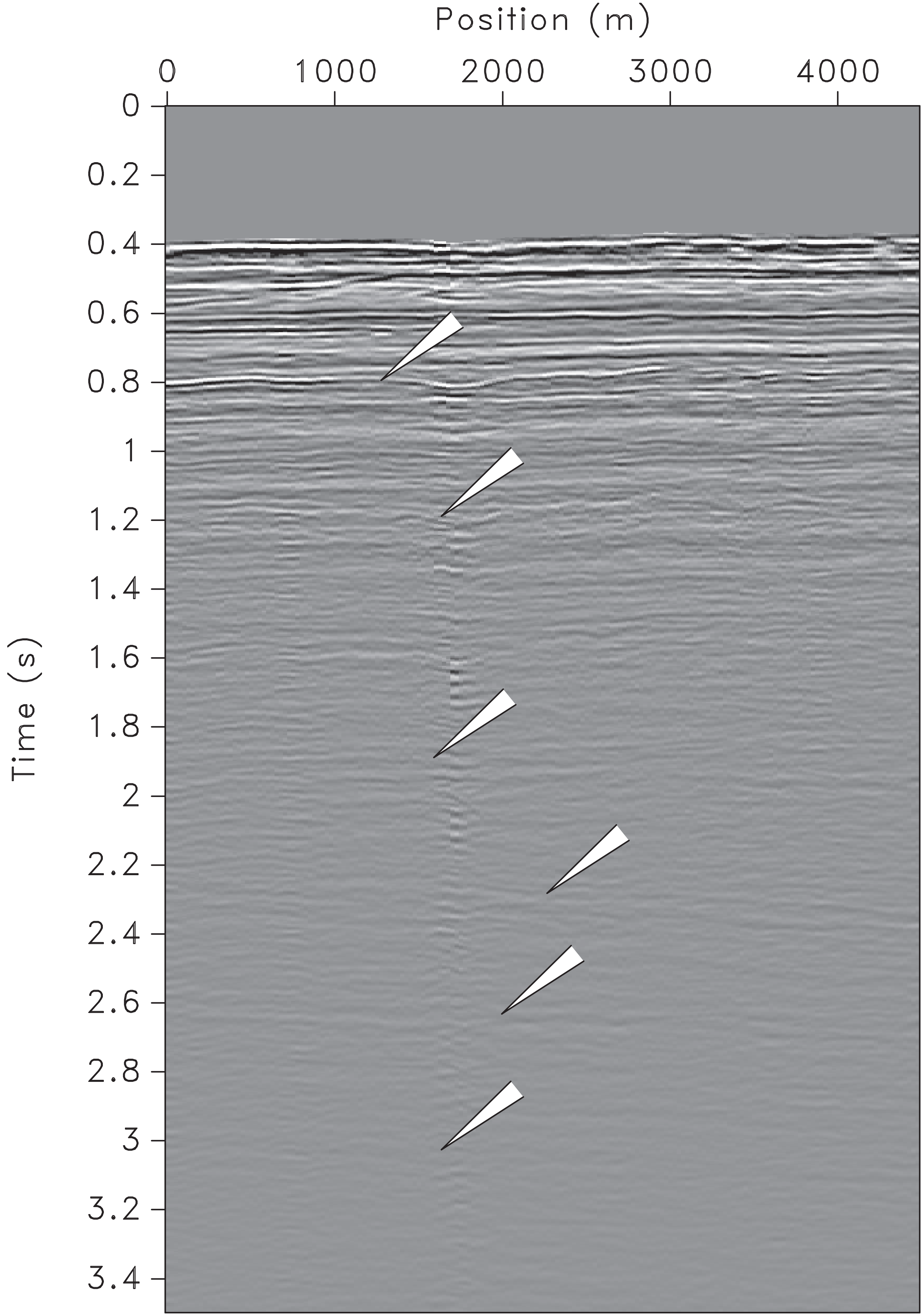
time-gain panel



SLIM

North Sea
data

zero-offset gather



SLIM

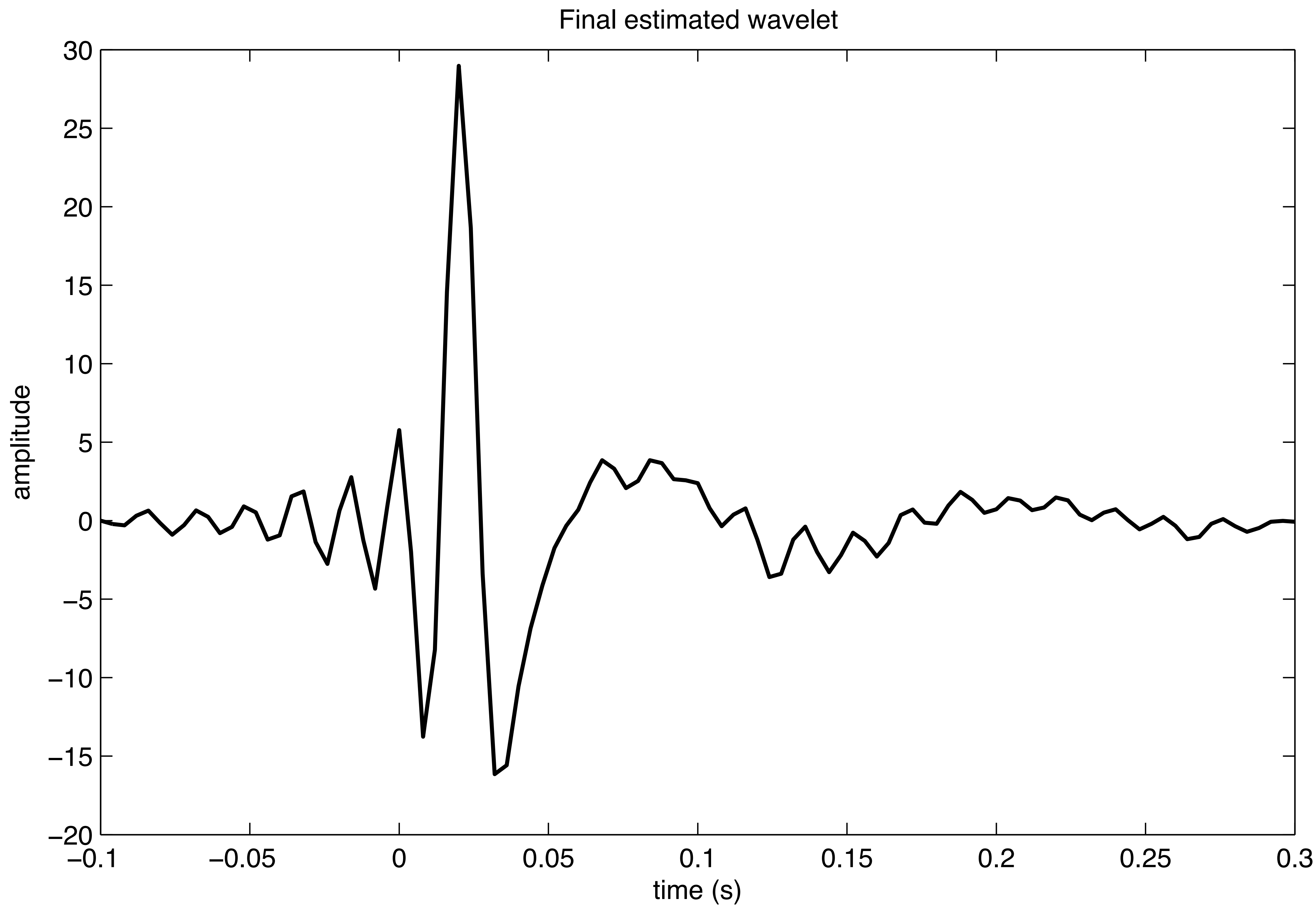
North Sea

data update from REPSI

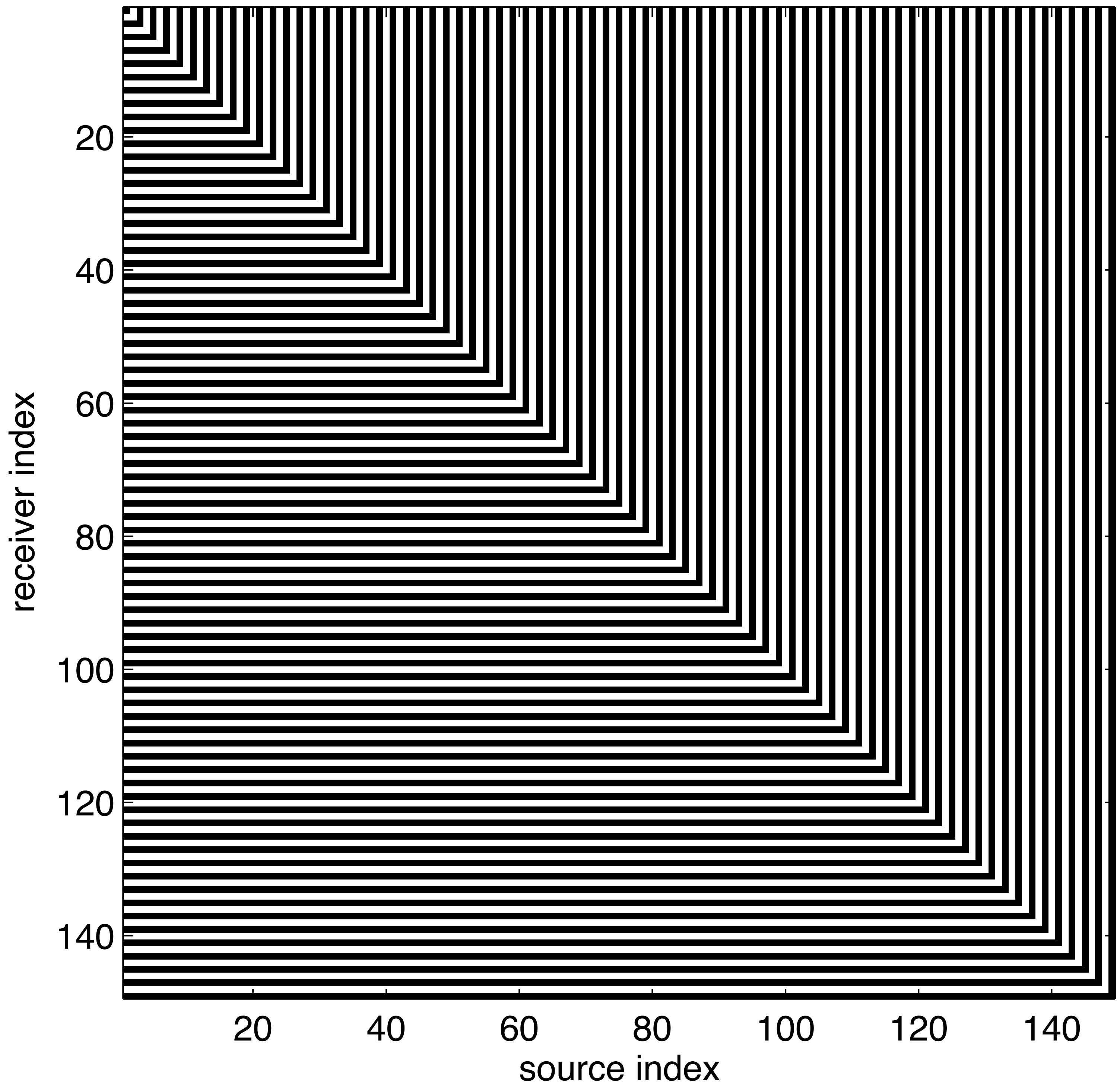
zero-offset gather

data updated 9 times

North Sea
REPSI estimated source wavelet

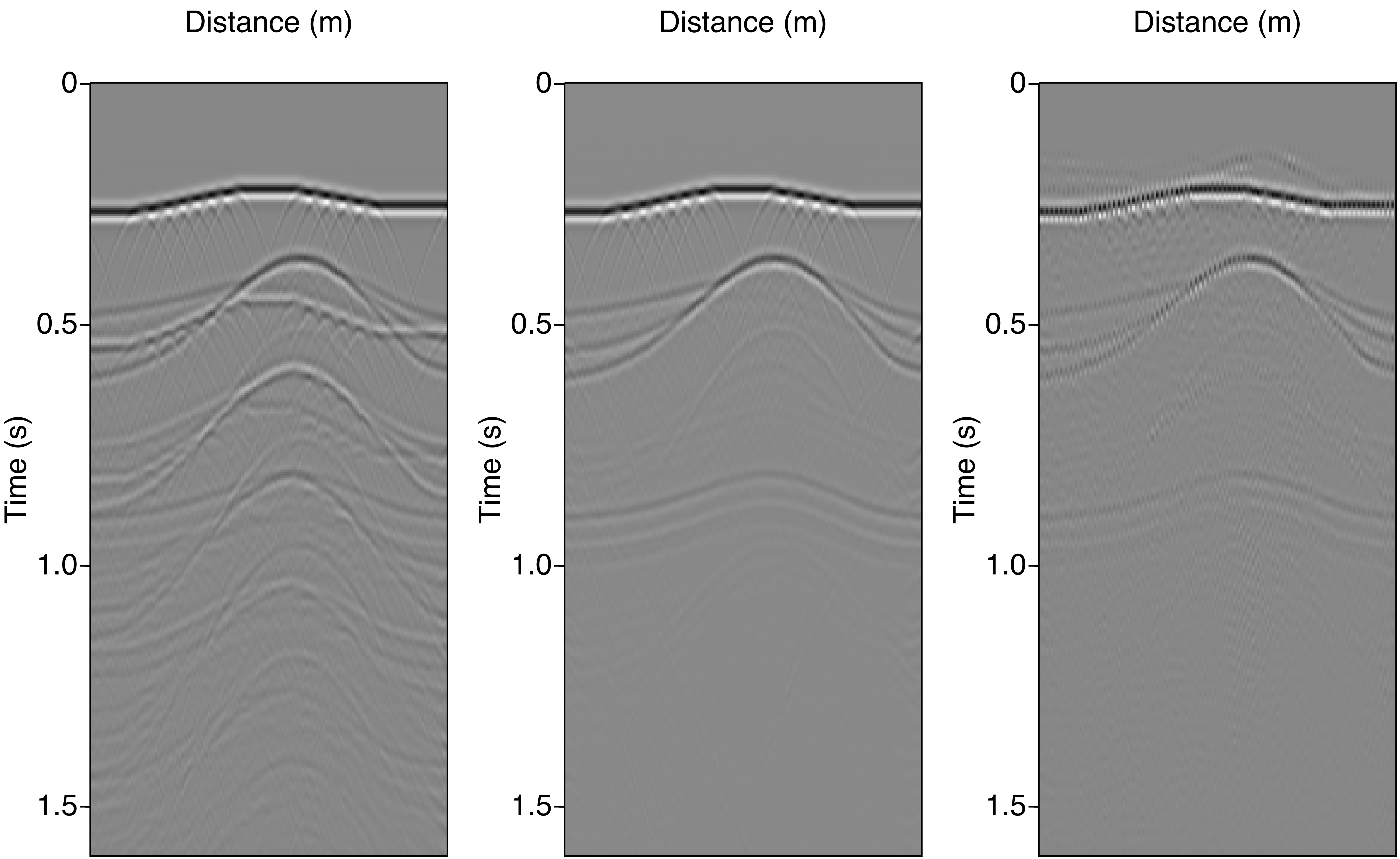


Shot-undersampling mask



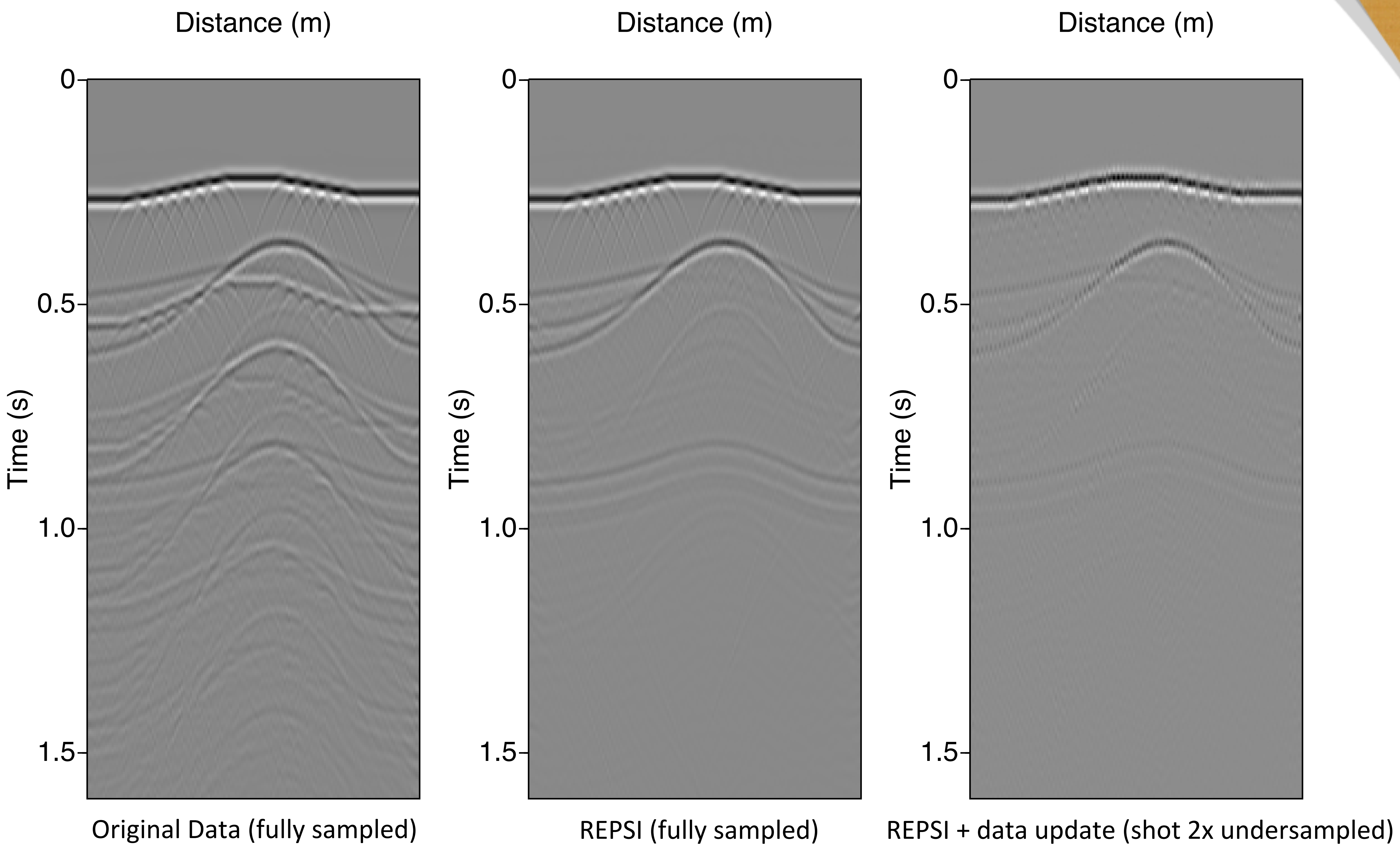
■ : missing data

Shot-undersampling (2x)

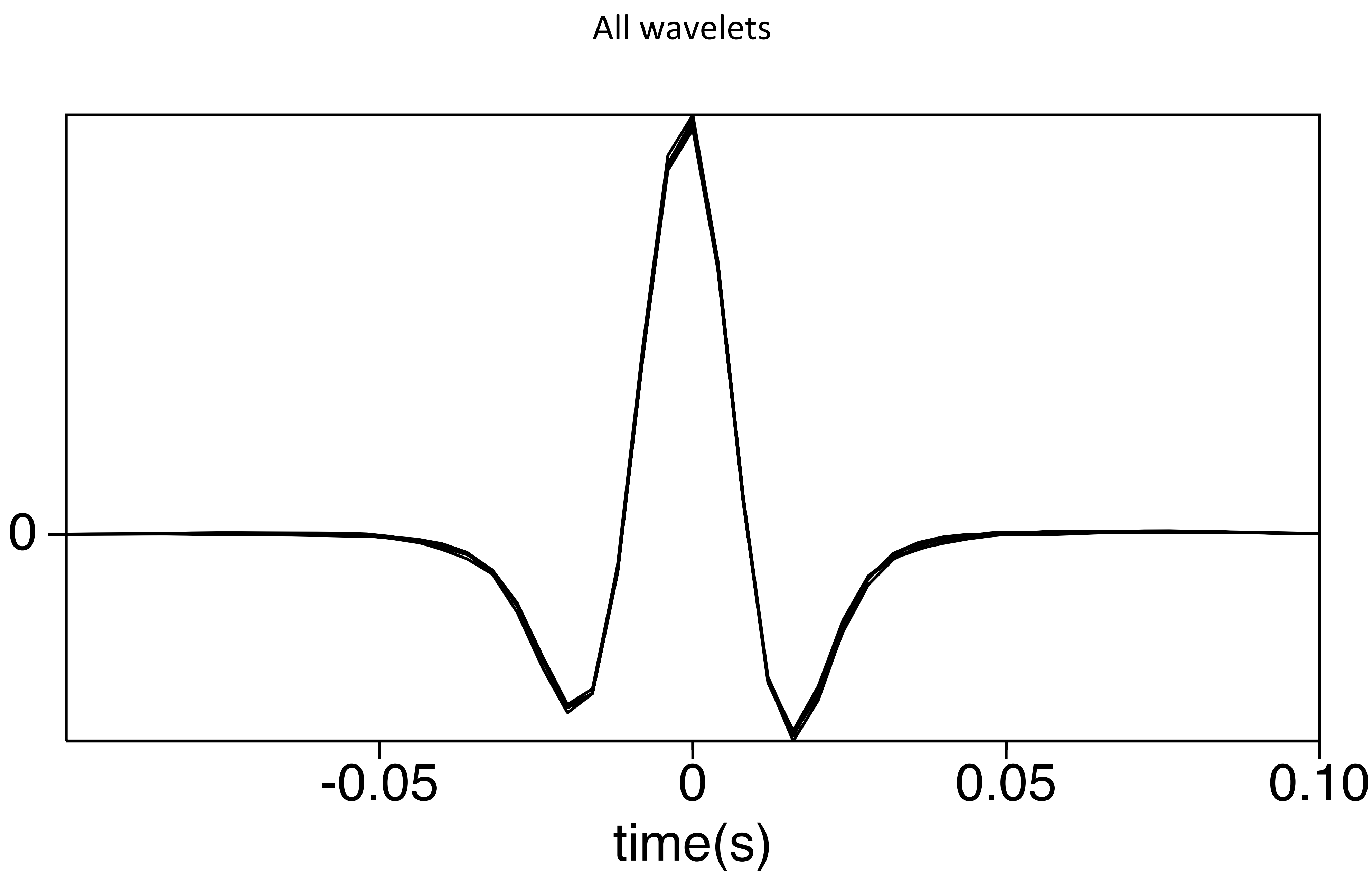


Original Data (fully sampled) | REPSI (fully sampled) | REPSI (shot 2x undersampled)

Shot-undersampling (2x)



Shot-undersampling (2x)



Pathway to 3D

- Continue investigation into sampling issue
- Reduce computation by randomized subsampling or low-rank tensor format

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Role in seismic workflow

- Automatic process, no hand-tuning of adaptive subtraction *(trade computer time for human time)*
- Gives phase-correct, amplitude correct prediction of multiples
- Gives Green's function and source wavelet in a stable manner (better than statistics-based blind deconvolution)
- Can correct interpolated data within the inversion process

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Acknowledgements

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SINBAD

NSERC CRSNG

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