Probing the extended image volume

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Velocity analysis in complex areas:

- wave-equation image volumes
- automated focusing optimization (DSO)

Both computation and storage of these image volumes is expensive

Can we work with the image volume implicitly?

Overview

- Extended modelling
- Wave-equation MVA
- Probing the image volume
- Multiscale MVA
- Examples
- Conclusions

Extended modelling

- **Physical Helmholtz equation:**

$$\left[\omega^2 \mathrm{diag}(\mathbf{m}) + \nabla^2\right]\mathbf{u} = \mathbf{q}$$

- **Extension**

$$\left[\omega^2 M + \nabla^2\right]\mathbf{u} = \mathbf{q}$$

Non-stationary convolution, allows for action-at-a-distance

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Extended modelling

Correct model should be able to explain the data without violating physics:

minimize off-diagonal energy in M
and fit the data

Extended modelling

$$\min_{M} ||W \odot M||_F^2 \quad \text{s.t.} \quad \sum_{\omega} ||F[M]Q - D||_F^2 \leq \sigma$$

$Q = [\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_N]$ **sources**

$D = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_N]$ **monochromatic data matrix**

$F[M] = PH[M]^{-1}$ **modelling operator**

$w_{ij} \propto r_{ij}$ **penalty term**

Wave-equation MVA

Linearization: $M = \text{diag}(\mathbf{m}) + E$

$$\min_{\mathbf{m},E} ||W \odot E||_F^2 \quad \text{s.t.} \quad \sum_{\omega} ||DF[\mathbf{m}]E + F[\mathbf{m}]Q - D||_F^2 \leq \sigma$$

$$DF[\mathbf{m}, Q]E = PH[\mathbf{m}]^{-1}(\omega^2 EU)$$
$$U = H[\mathbf{m}]^{-1}Q$$

extended born modelling: allows non-local interaction between background wavefield and reflectivity

Wave-equation MVA

Approximate elimination of constraint leads to `conventional' MVA formulation:

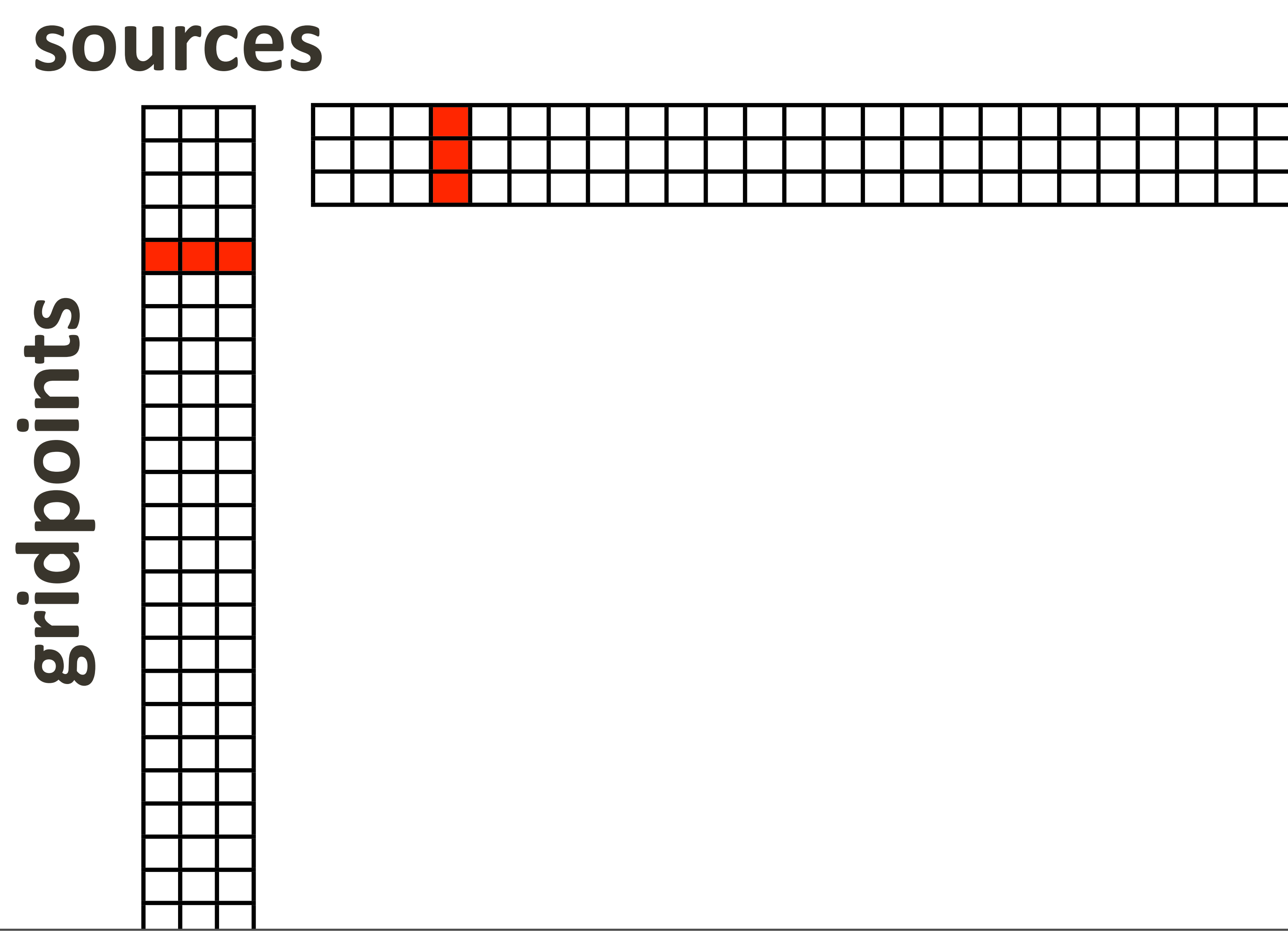
$$\min_{\mathbf{m}} ||W \odot E[\mathbf{m}]||_F^2$$

$$E[\mathbf{m}] = \sum_{\omega} \omega^2 V U^*$$

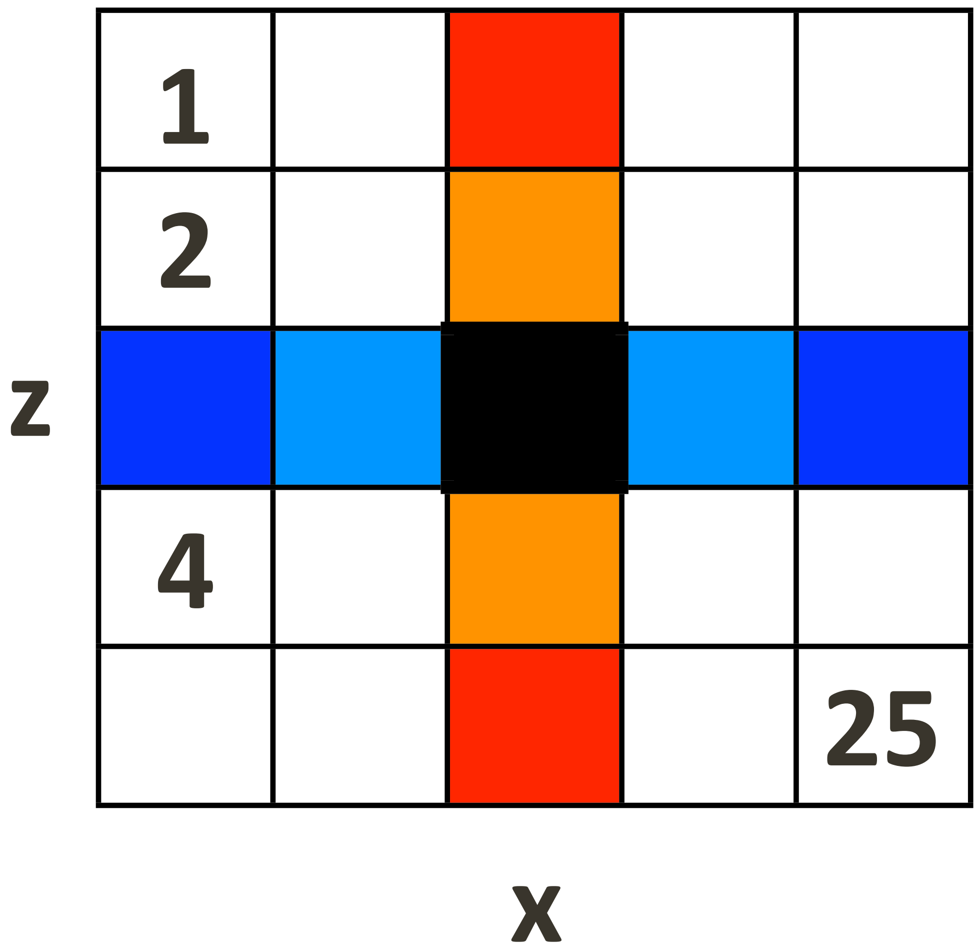
$$V = H[\mathbf{m}]^{-*} P^* (D - F[\mathbf{m}]Q)$$

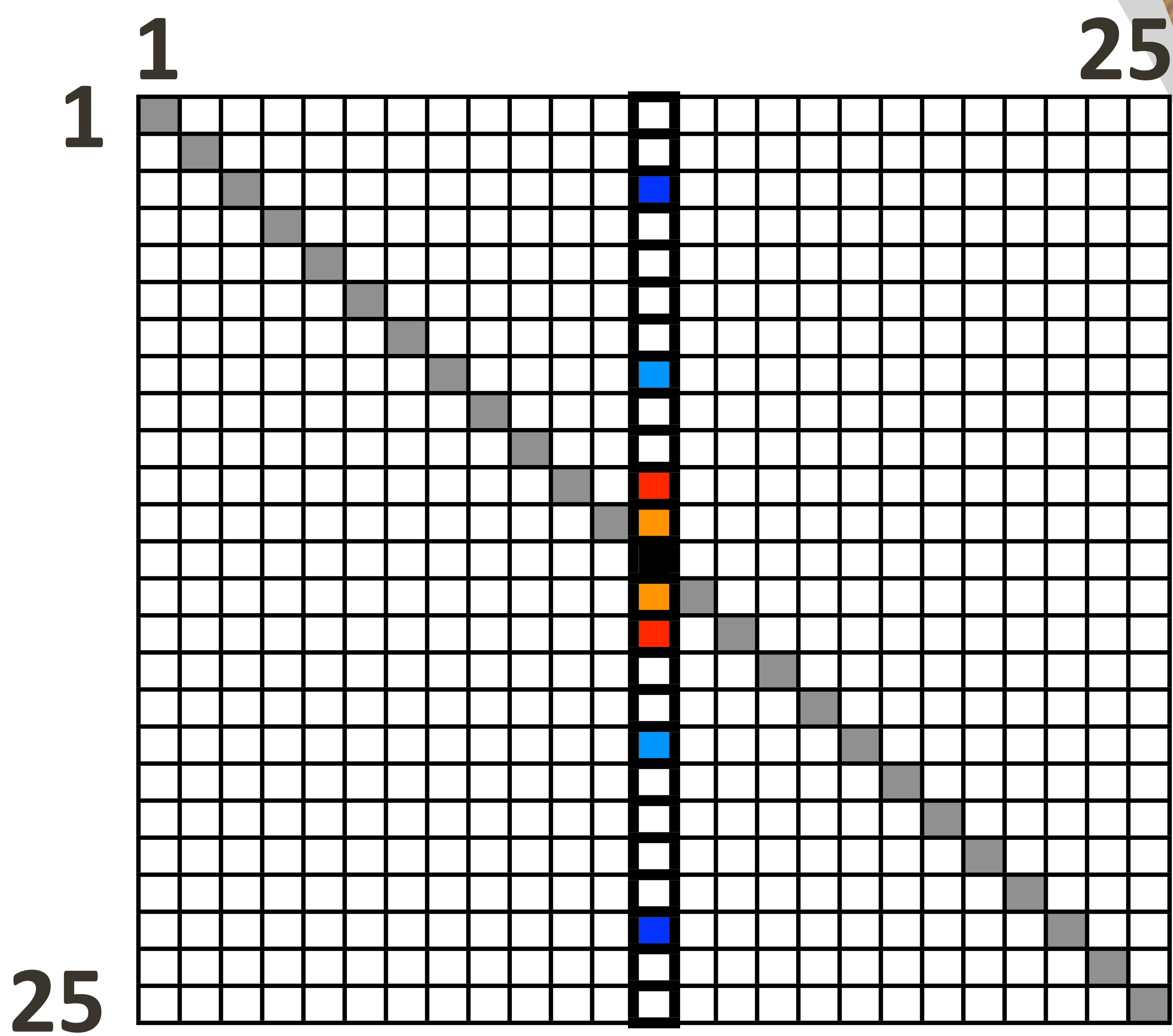
Wave-equation MVA

$$E = \sum_{\omega} \omega^2 U V^* \qquad e_{i,j} = \sum_{\omega} \sum_{s} \omega^2 u_{i,s} v_{j,s}$$



Wave-equation MVA





Probing the image volume

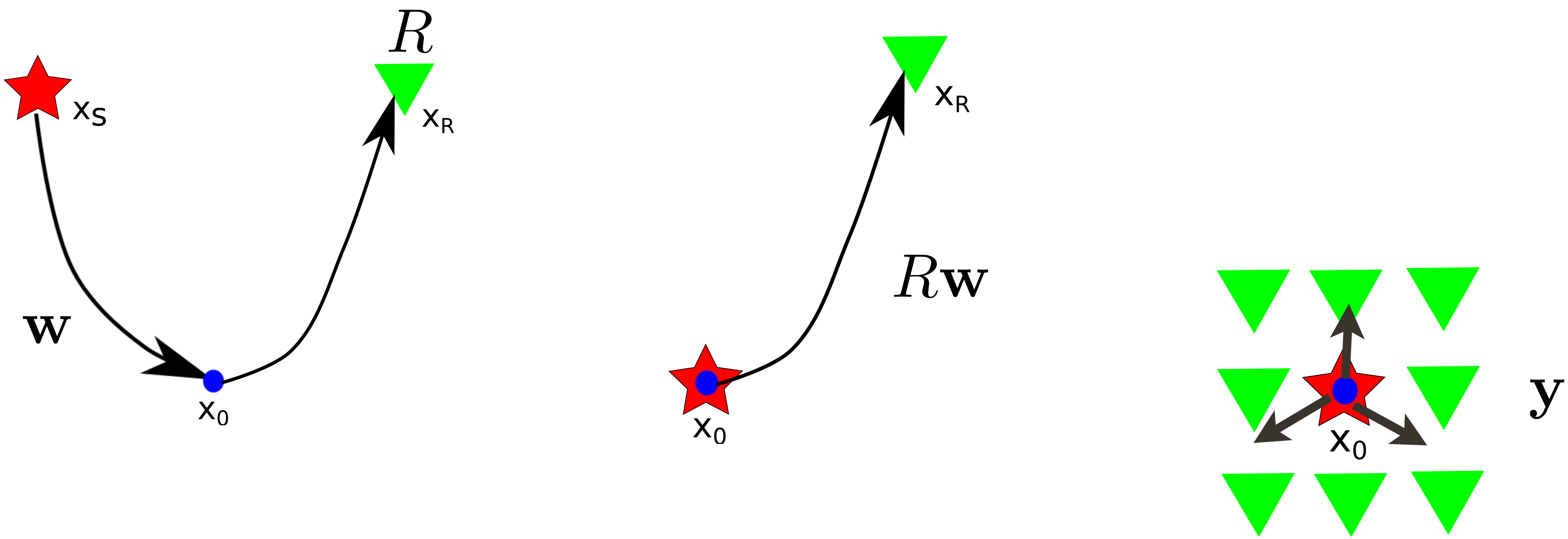
- VERY expensive to form complete image volume
- Cheap to calculate action on vector

$$\mathbf{y} = E\mathbf{x} = VU^*\mathbf{x}$$

1. source wavefield $U = H[\mathbf{m}]^{-1}Q$
2. data residual $R = P^*(PU - D)$
3. adjoint source weights $\mathbf{w} = U^*\mathbf{x}$
4. Solve for one r.h.s. $\mathbf{y} = H[\mathbf{m}]^{-*}(R\mathbf{w})$

Probing the image volume

Interferometric interpretation: $\mathbf{x} = \delta_{ij}$



Greens function | **Source redatuming** | **Receiver redatuming**

Probing the image volume

- sparsely subsample the image

$$\min_{\mathbf{m}} \sum_{i \in \mathcal{I}} ||W_i \odot E[\mathbf{m}]\delta_i||_2^2$$

- can we randomly combine the subsurface sources?
- also allows for target-oriented approach

Multiscale MVA

- **Instead of penalizing off-diagonal energy, we reward near-diagonal energy** $w_{ij} \propto \exp[-(r_{ij}/\sigma)^2]$

- **When** $\sigma \downarrow 0$ **we measure only energy on the diagonal, i.e. stackpower which is equivalent to FWI!**

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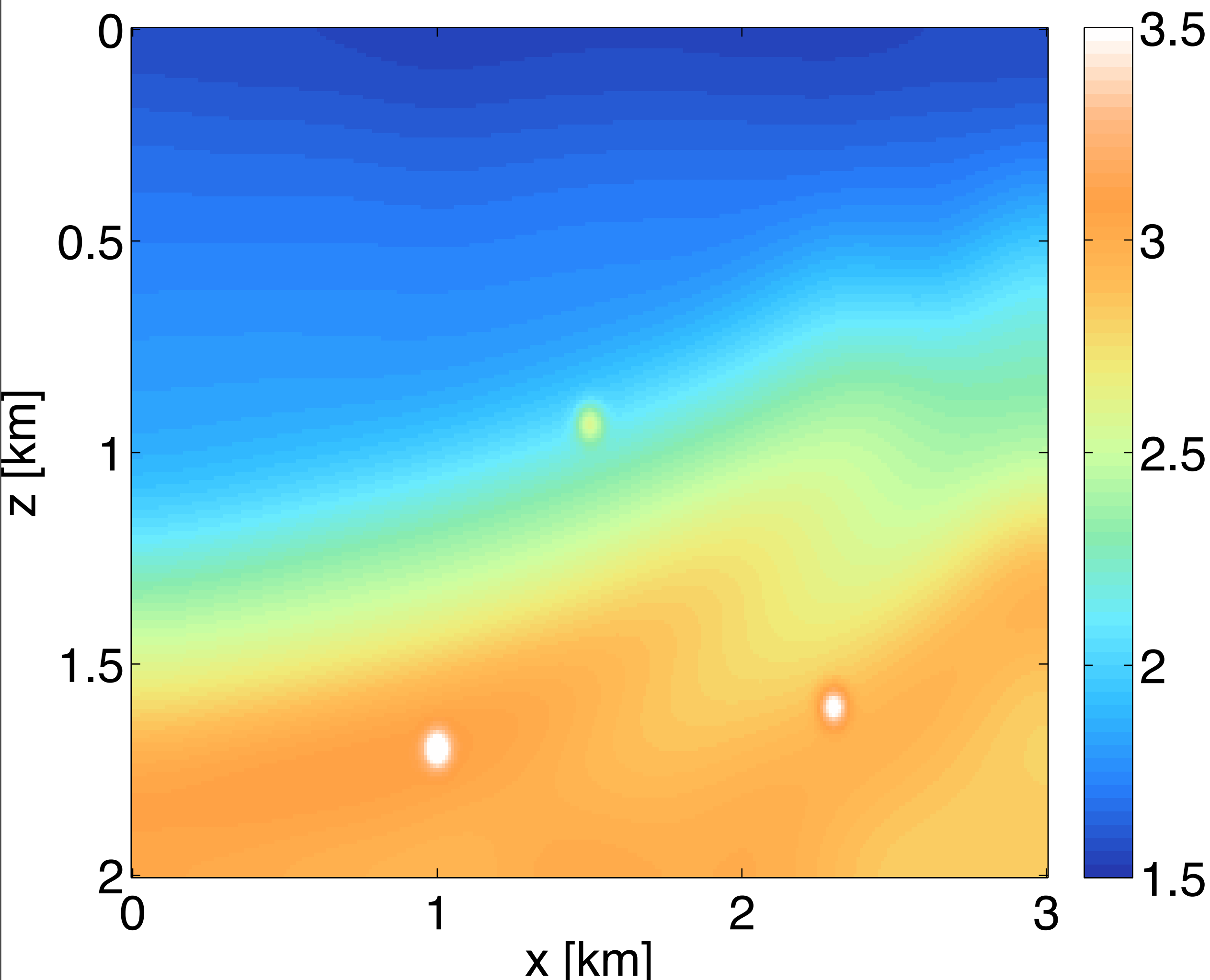
Multiscale MVA

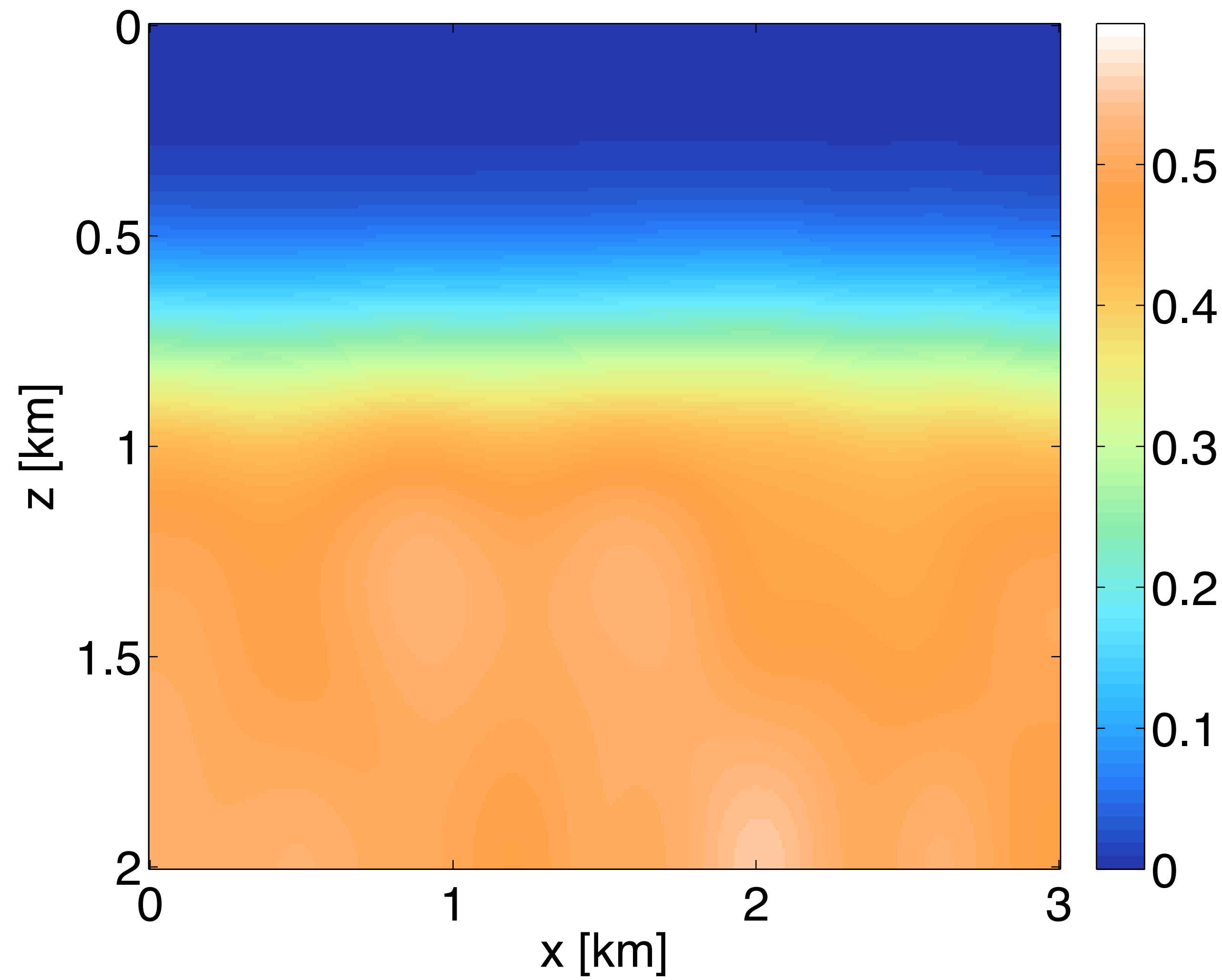
- Start with very sparse sampling and large width
- Gradually move to finer sampling and smaller widths
- Finally, compute only diagonal of extended image and move to FWI

Examples

61 sources, 301 receivers, [3:0.5:25] Hz



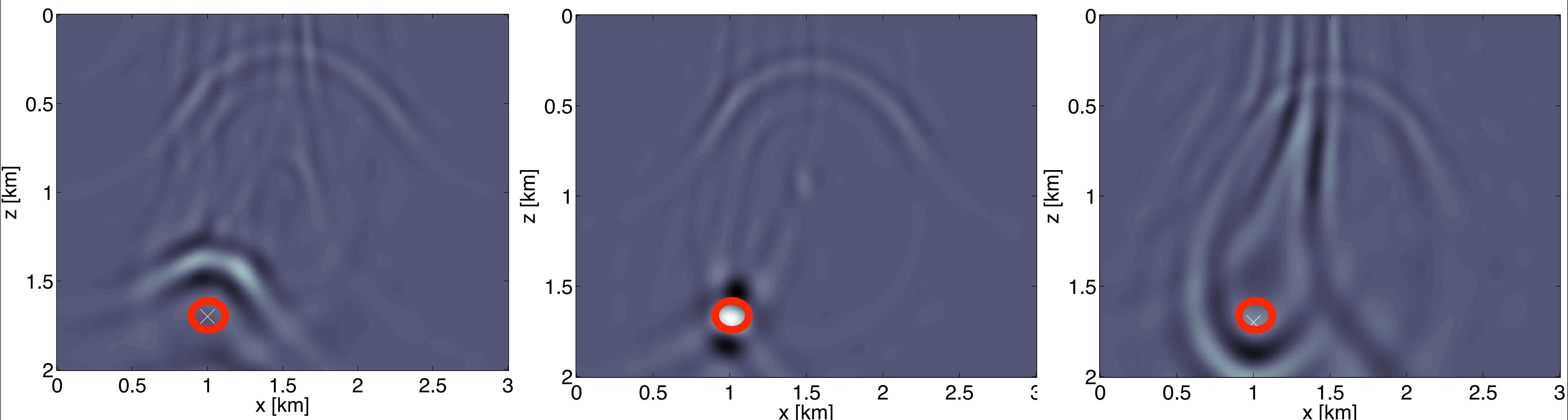
velocity [km/s]



perturbation [km/s]

Examples

all sequential sources

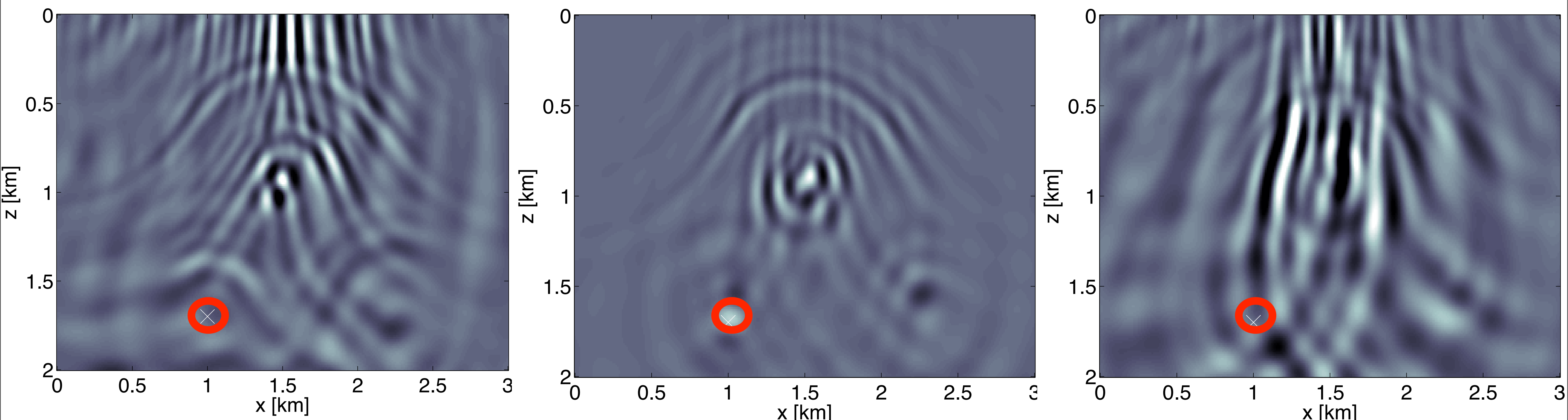


low **high**

Examples

1 simultaneous source

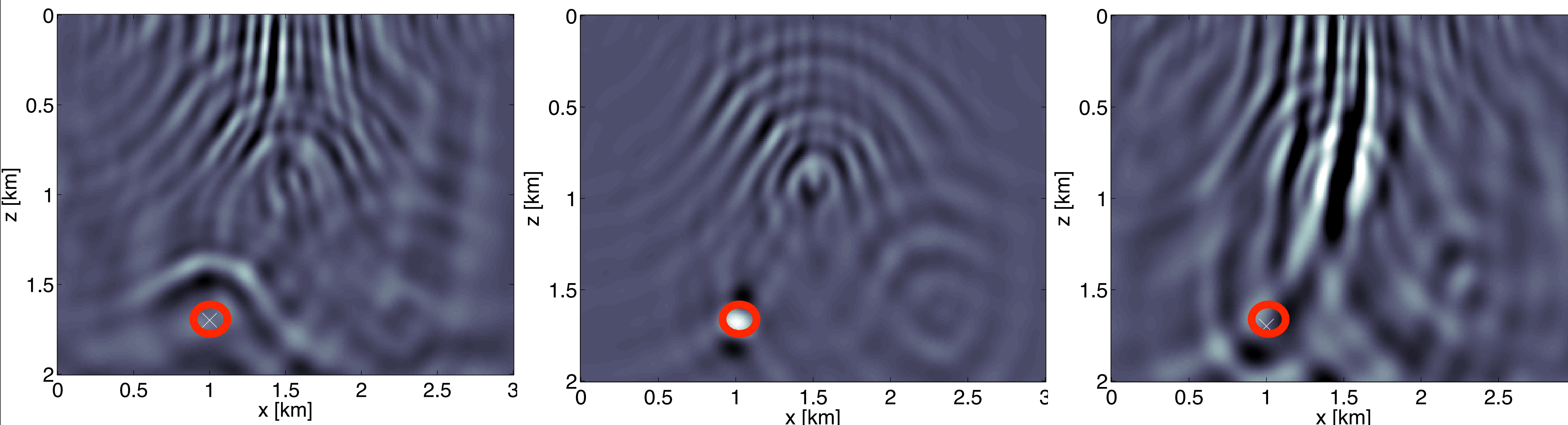
$$E = \sum_{\omega} \omega^2 UW(VW)^*$$



low **high**

Examples

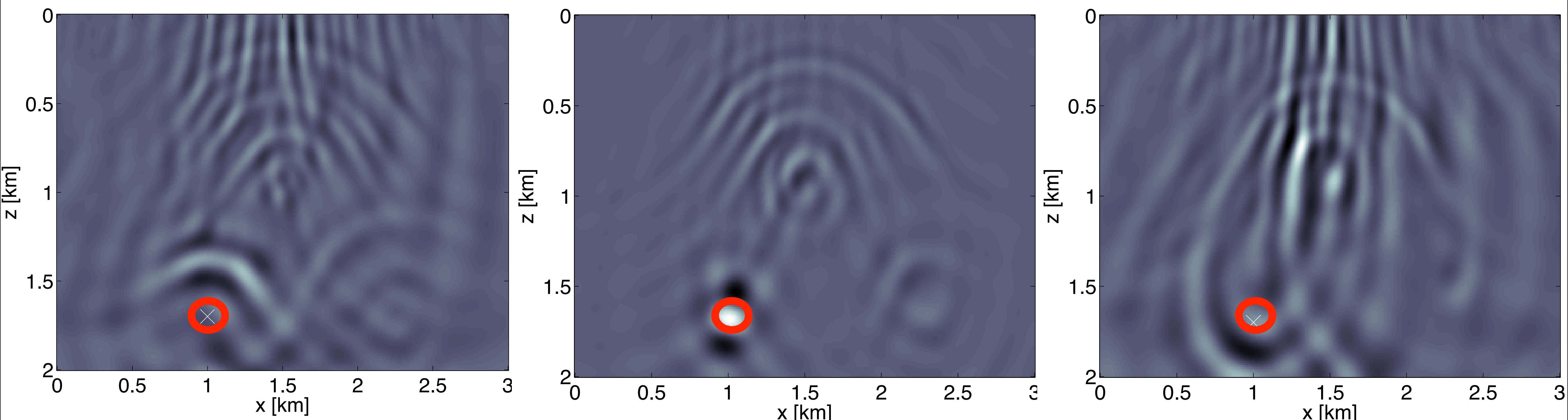
5 simultaneous sources $E = \sum_{\omega} \omega^2 UW(VW)^*$



low **high**

Examples

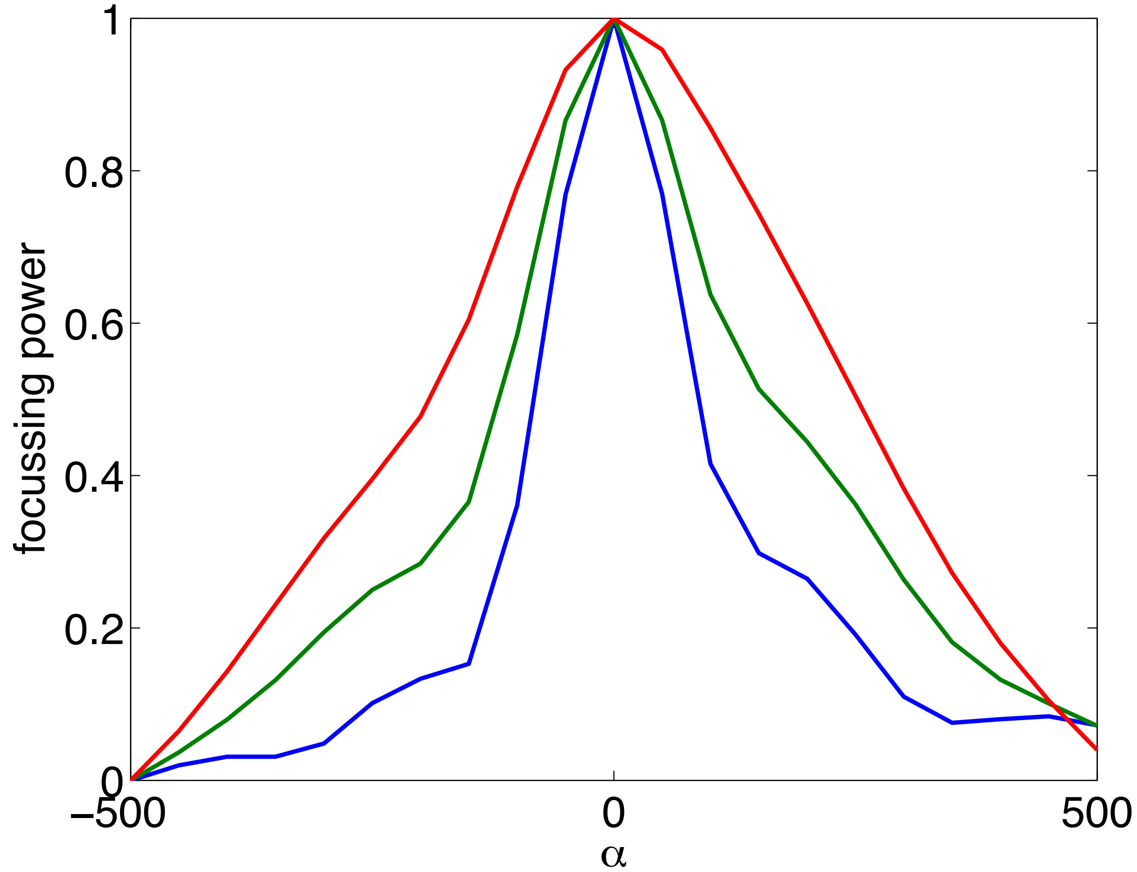
10 simultaneous sources $E = \sum_{\omega} \omega^2 UW(VW)^*$



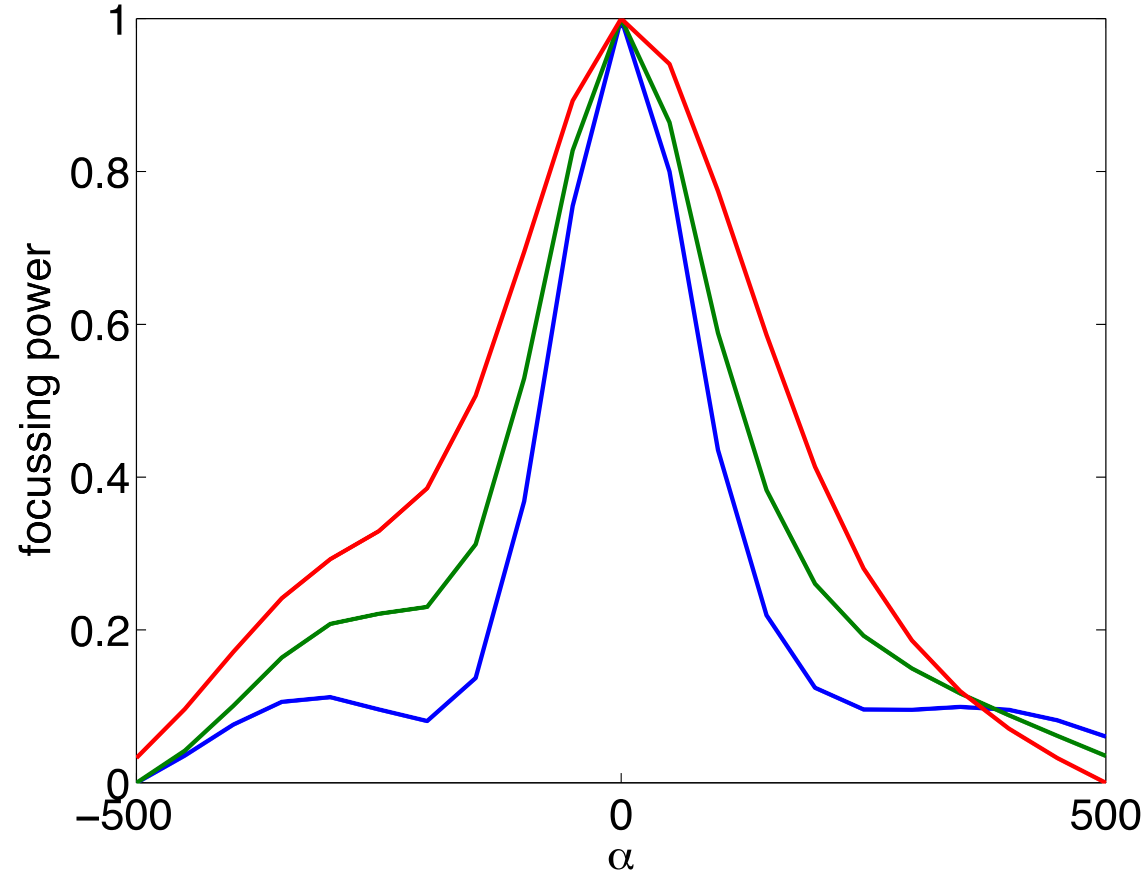
low **high**

Examples

focussing power for small, medium and large scale



sequential sources



10 simultaneous sources

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Conclusions

- We can efficiently probe the extended image volume
- No need to estimate local dip because of using subsurface offset in all directions
- Use sim. source ideas for both surface and subsurface sources
- Multiscale focusing criterion