Vienna '11

Robust EPSI in a Curvelet-like Representation Domain

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SINBAD 2011 Spring

SLIM
University of British Columbia

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EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data predicted data from primary IR

$$\mathbf{P} = \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})$$

$\mathbf{P}$ **total up-going wavefield**

$\mathbf{Q}$ **down-going source signature**

$\mathbf{R}$ **reflectivity of free surface (assume -1)**

$\mathbf{G}$ **primary impulse response**

(all monochromatic data matrix, implicit ω **)**

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EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data   predicted data from primary IR

$$\mathbf{P} = \mathbf{G}(\mathbf{Q} + \mathbf{RP})$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - \mathbf{G}(\mathbf{Q} + \mathbf{RP})\|_2^2$$

EPSI Problem

In time domain **(lower-case: whole dataset in time domain)**

recorded data   predicted data from primary IR

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathcal{M}(\mathbf{g}, \mathbf{q}) := \mathcal{F}_{\mathrm{t}}^{\dagger} \mathrm{BlockDiag}_{\omega_1 \cdots \omega_{nf}} [(q(\omega)\mathbf{I} - \mathbf{P})^{\dagger} \otimes \mathbf{I}] \mathcal{F}_{\mathrm{t}} \mathbf{g}$$

Inversion objective:

$$f(\mathbf{g}, \mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathcal{M}(\mathbf{g}, \mathbf{q})\|_2^2$$

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EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

In fact it is bilinear:

$$\mathbf{M}_{\tilde{q}}\mathbf{g} = \mathcal{M}(\mathbf{g}, \tilde{\mathbf{q}}) \qquad \mathbf{M}_{\tilde{g}}\mathbf{q} = \mathcal{M}(\mathbf{q}, \tilde{\mathbf{g}})$$

EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

Associated objectives:

$$f_{\tilde{q}}(\mathbf{g}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2^2 \qquad f_{\tilde{g}}(\mathbf{q}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2^2$$

EPSI Procedure

Do:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \nabla f_{q_k}(\mathbf{g}_k)$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Alternating updates (Gauss-Seidel) to the linearized problem

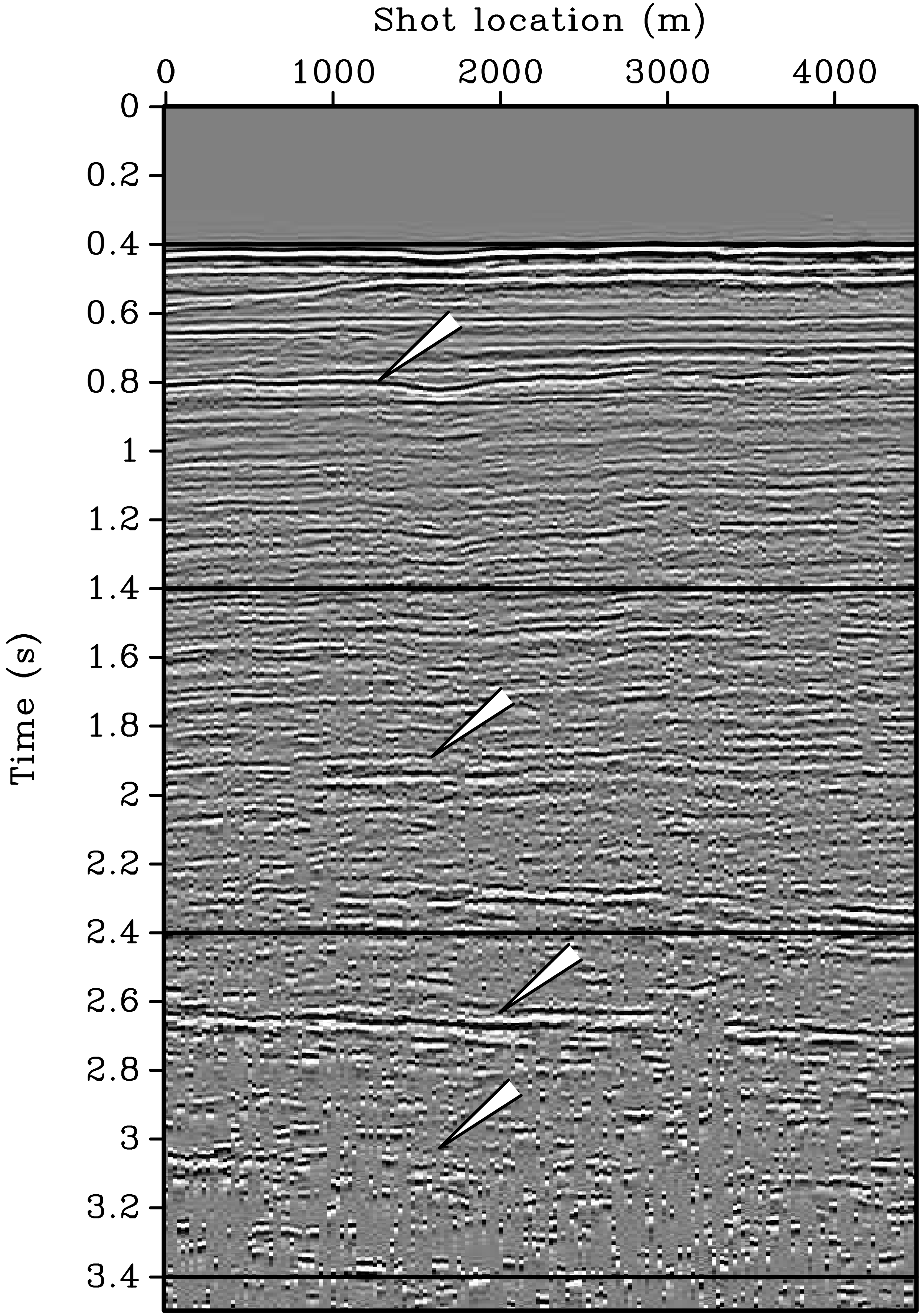
EPSI Procedure

Do:

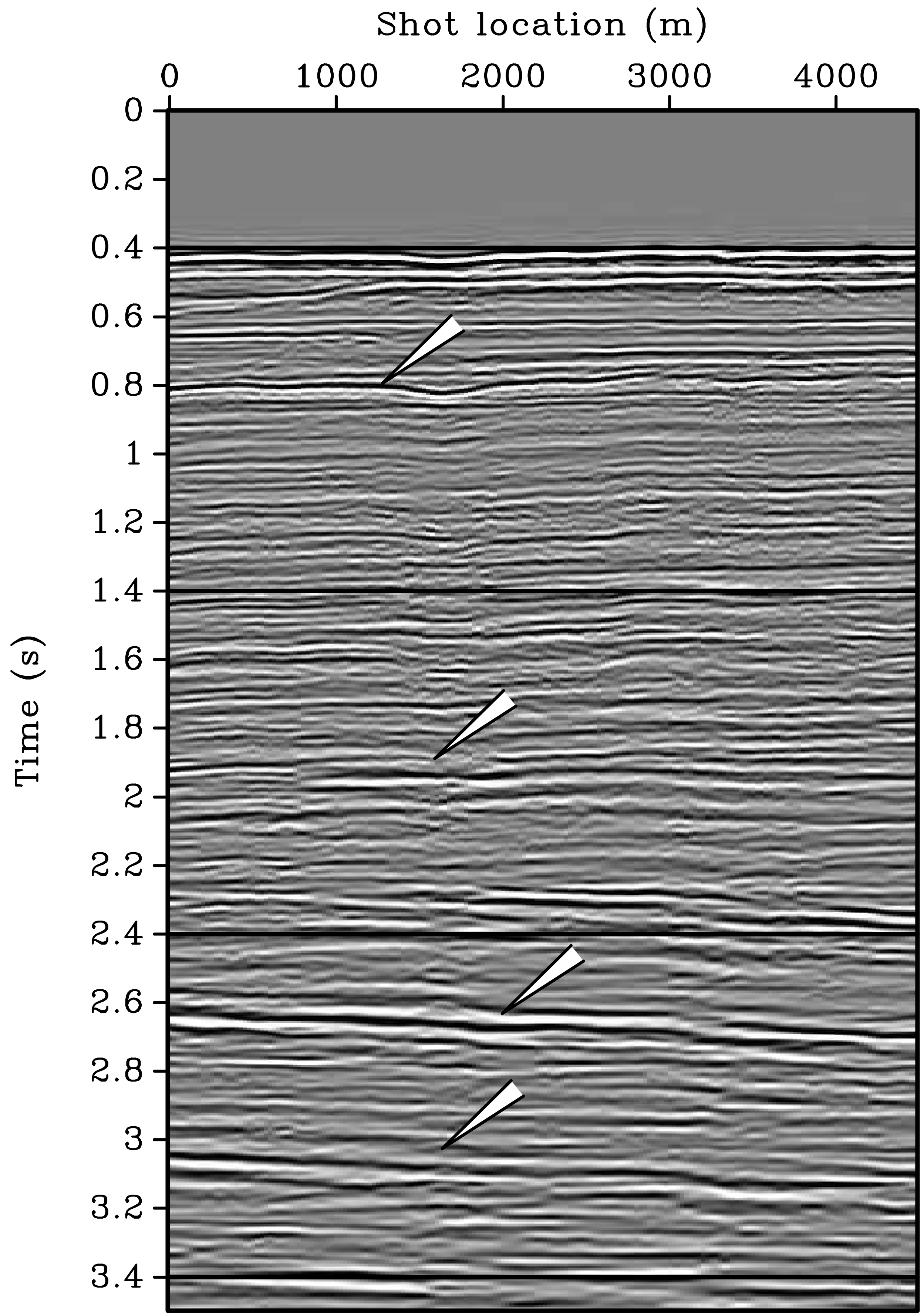
$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \mathcal{S}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Gradient sparsity

$\mathcal{S}$: pick largest ρ elements per trace



Shot location (m)
0
1000
2000
3000
4000
Time (s)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4


Shot location (m)
0
1000
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0
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0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4

EPSI Procedure

Related to two underlying sub-problems:

$$\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \quad \text{s.t.} \quad \text{nnz}(\mathbf{g}) \leq \rho$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Attempting to approximate:

$$\min_{\mathbf{g}} \text{nnz}(\mathbf{g}) \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

(notion of sparsest solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

EPSI Procedure

Can be made non-combinatorial (convex) by:

$$\min_{\mathbf{g}} \|\mathbf{g}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

(minimum L1 solution usually the sparsest solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Convex EPSI

Do:

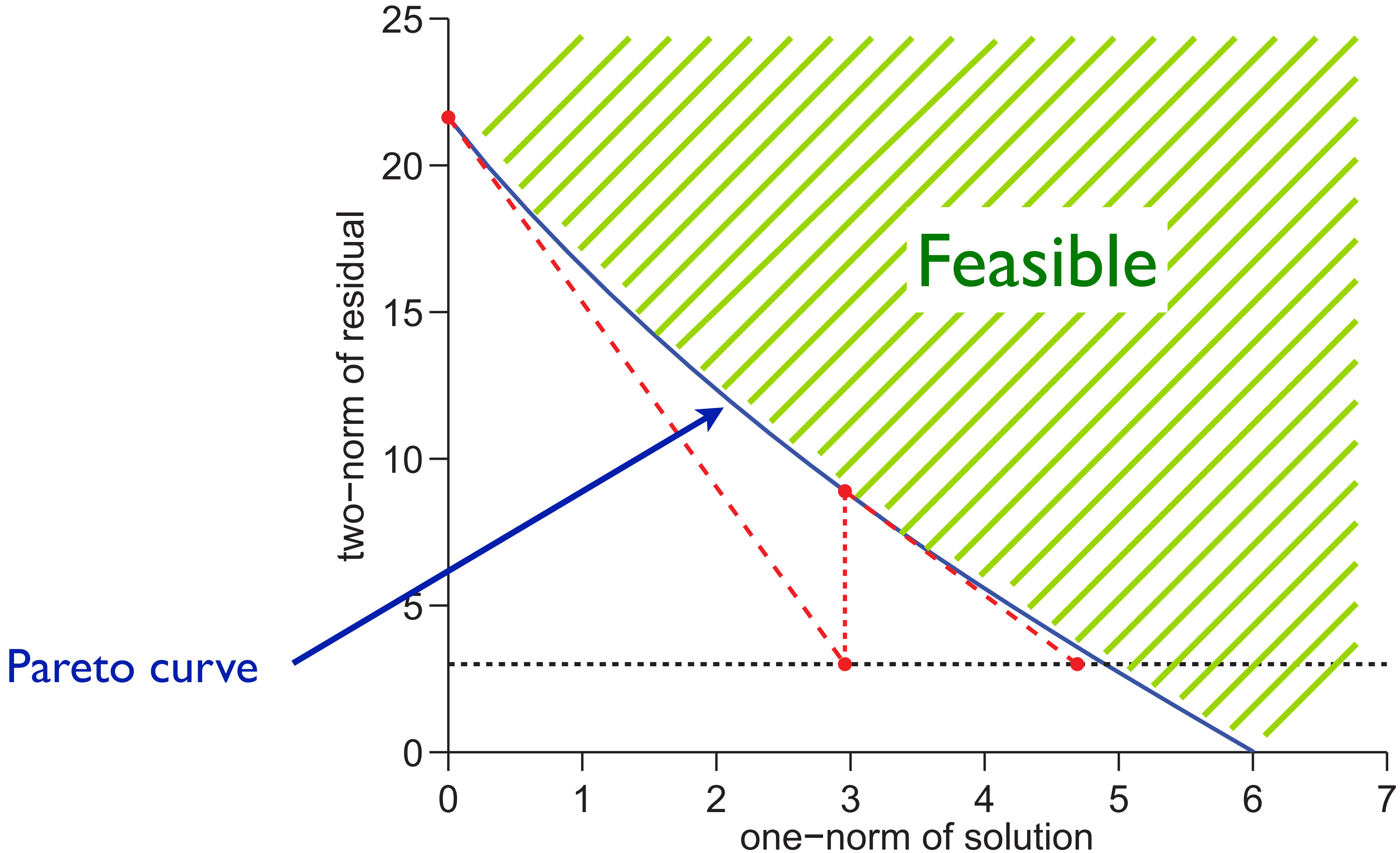
$$\mathbf{g}_{k+1} = \mathrm{SoftTh}_{\phi}(\mathbf{g}_k + \alpha \nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Soft-thresholding solves an L1 minimization problem, but how is ϕ determined?

Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax-b\|_2 \le \sigma \end{array}$$

Look at the solution space and the line of optimal solutions (Pareto curve)

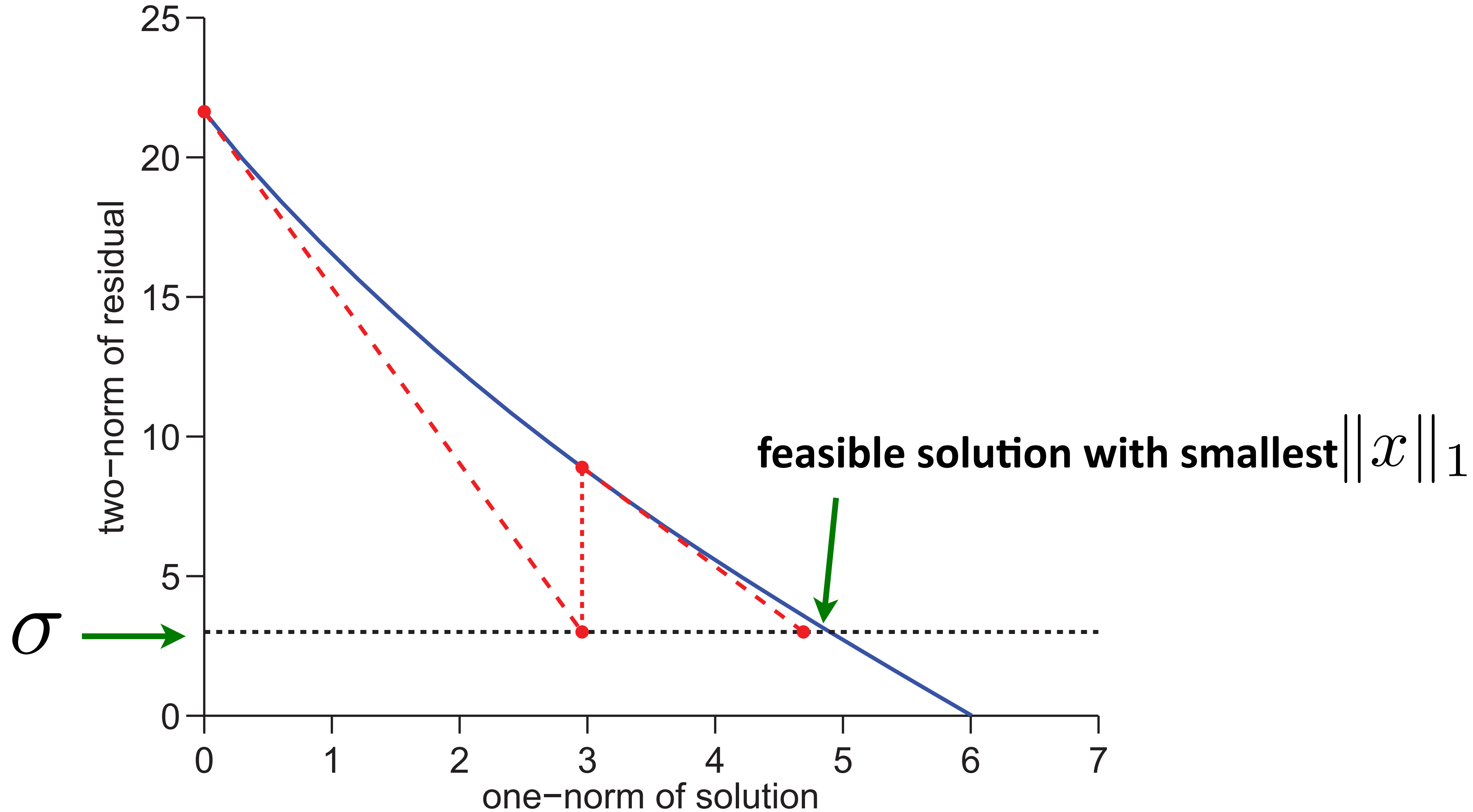


(van den Berg, Friedlander, 2008)

Pareto curve

minimize $\|x\|_1$
subject to $\|Ax-b\|_2 \leq \sigma$

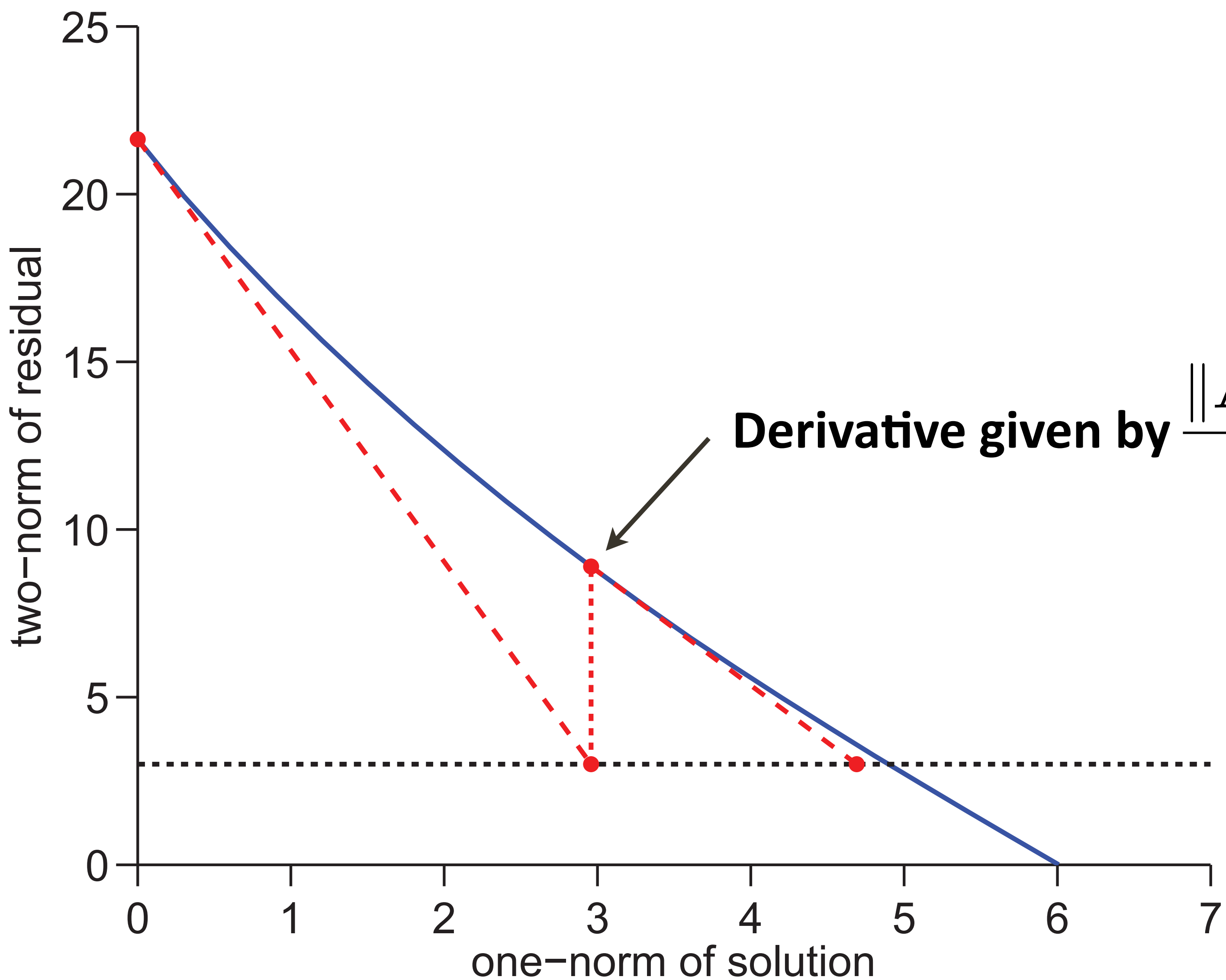
Look at the solution space and the line of optimal solutions (Pareto curve)



Pareto curve

minimize $\|x\|_1$
subject to $\|Ax - b\|_2 \leq \sigma$

Look at the solution space and the line of optimal solutions (Pareto curve)



Derivative given by $\dfrac{\|\mathbf{A}^H\mathbf{r}\|_\infty}{\|\mathbf{r}\|_2}$

Pareto curve

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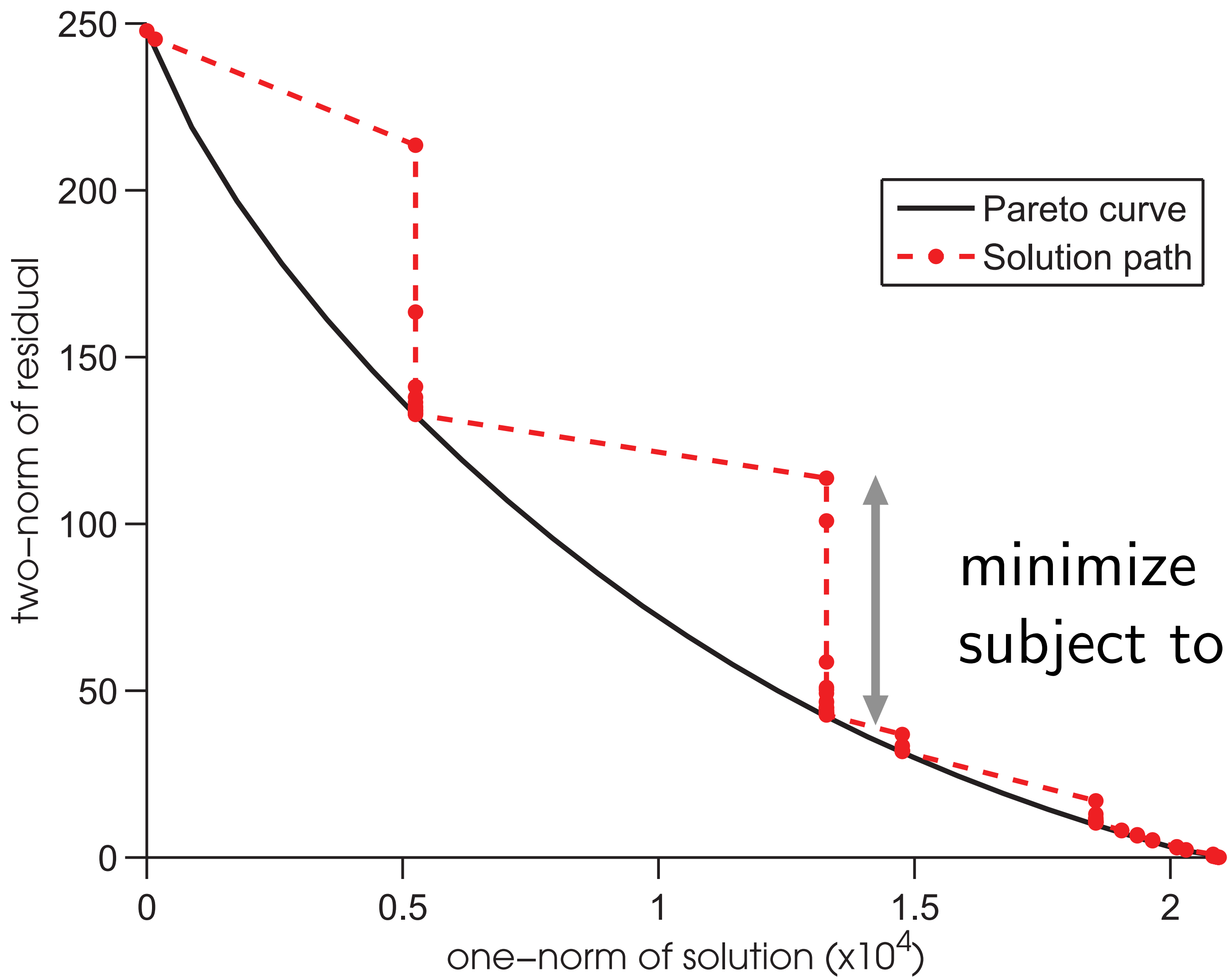
$$\begin{array}{ll}\text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax-b\|_2 \le \sigma\end{array}$$

Look at the solution space and the line of optimal solutions (Pareto curve)

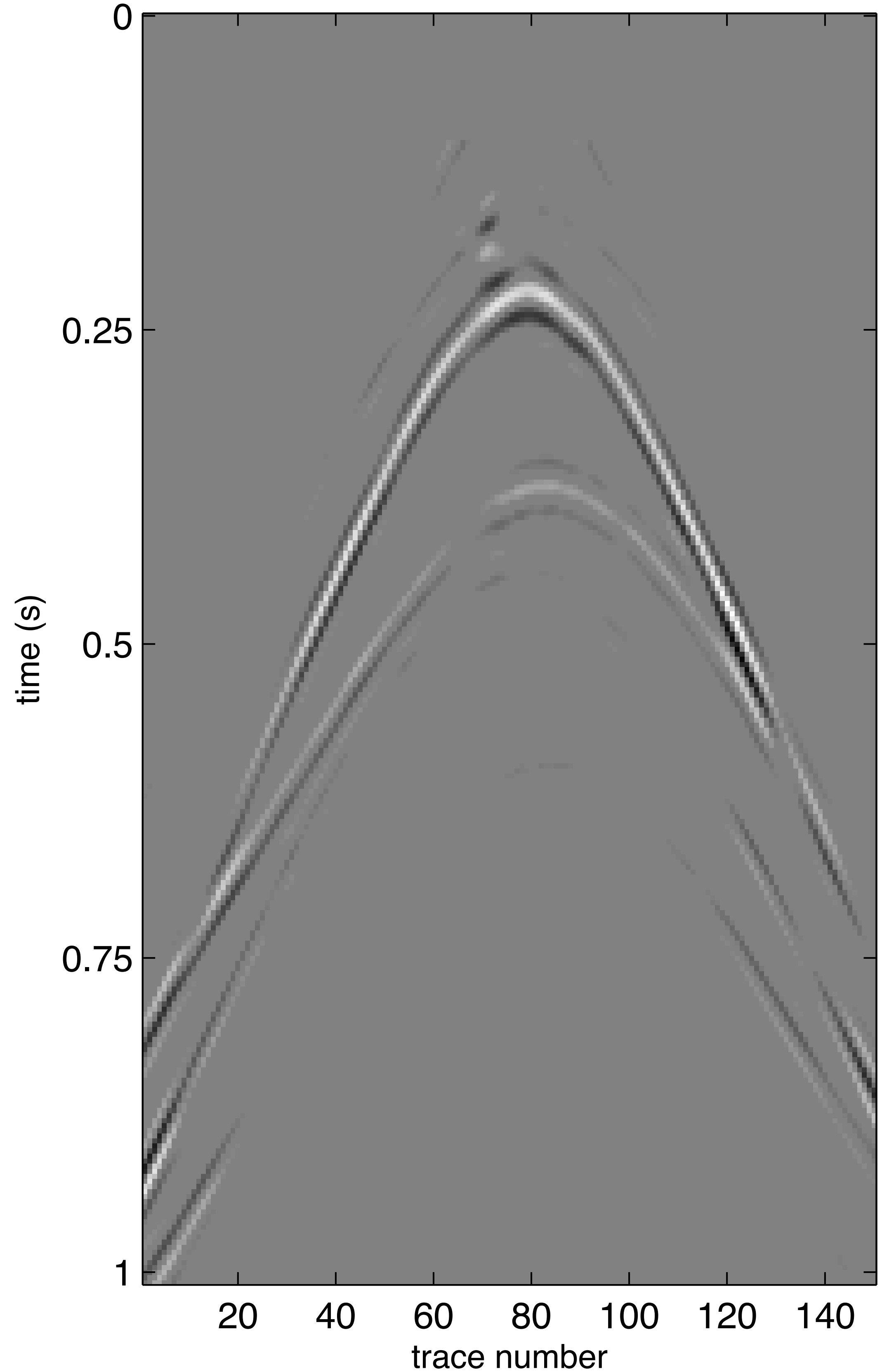


$$\begin{array}{ll}\text{minimize} & \|Ax-b\|_2 \\ \text{subject to} & \|x\|_1 \le \tau\end{array}$$

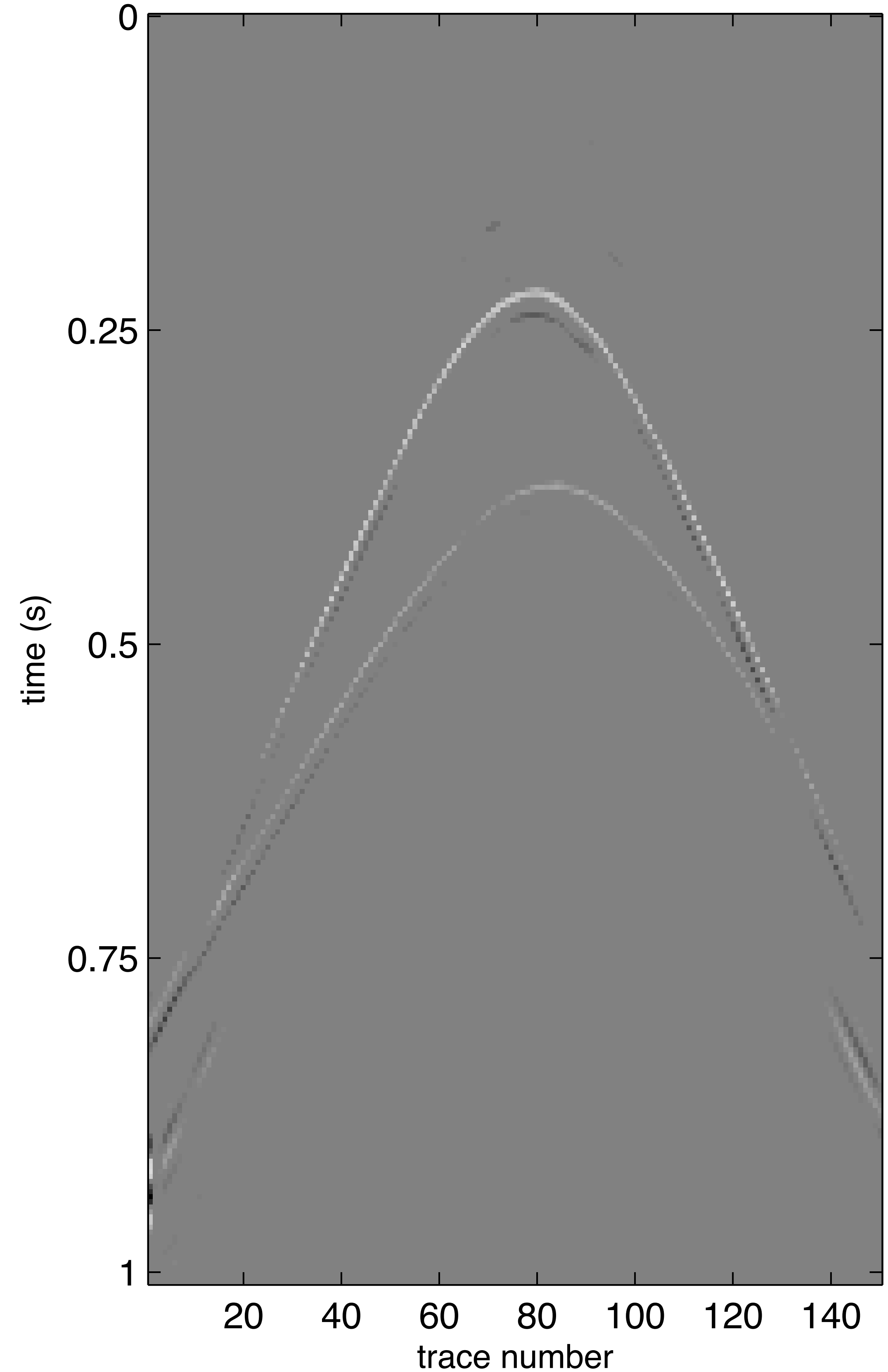
solve with SPG

(spectral projected gradients)

SPG start



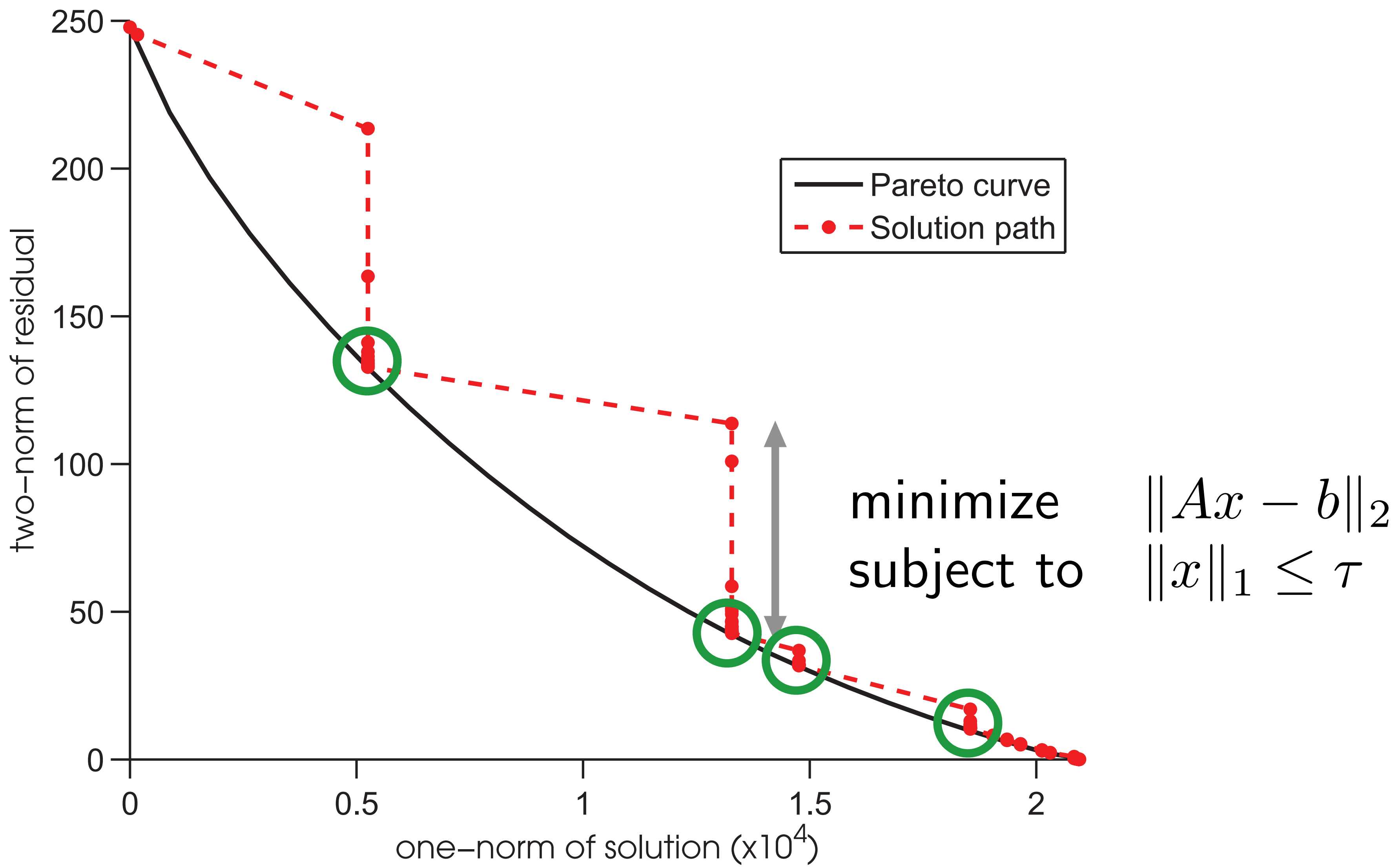
SPG at Pareto curve



Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax - b\|_2 \leq \sigma \end{array}$$

Only solve least-squares matching for q when solution reaches Pareto curve



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Robust EPSI procedure

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

(Solve with SPGL1 until Pareto curve reached)

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)

REPSI in transform domain

Modify just the problem for g:

$$\min_{\mathbf{g}} \|\mathbf{g}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

REPSI in transform domain

Modify just the problem for g:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{S}^\dagger\mathbf{x}\|_2 \leq \sigma, \quad \mathbf{g} = \mathbf{S}^\dagger\mathbf{x}$$

(basis pursuit + denoise)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

$\mathbf{S}$: sparsifying representation for seismic signals

- **Should have spatially localized support**
- **ex: nd-Wavelets, Curvelets, etc...**

$\mathbf{S}^\dagger$: synthesis operator for $\mathbf{S}$

REPSI in transform domain

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

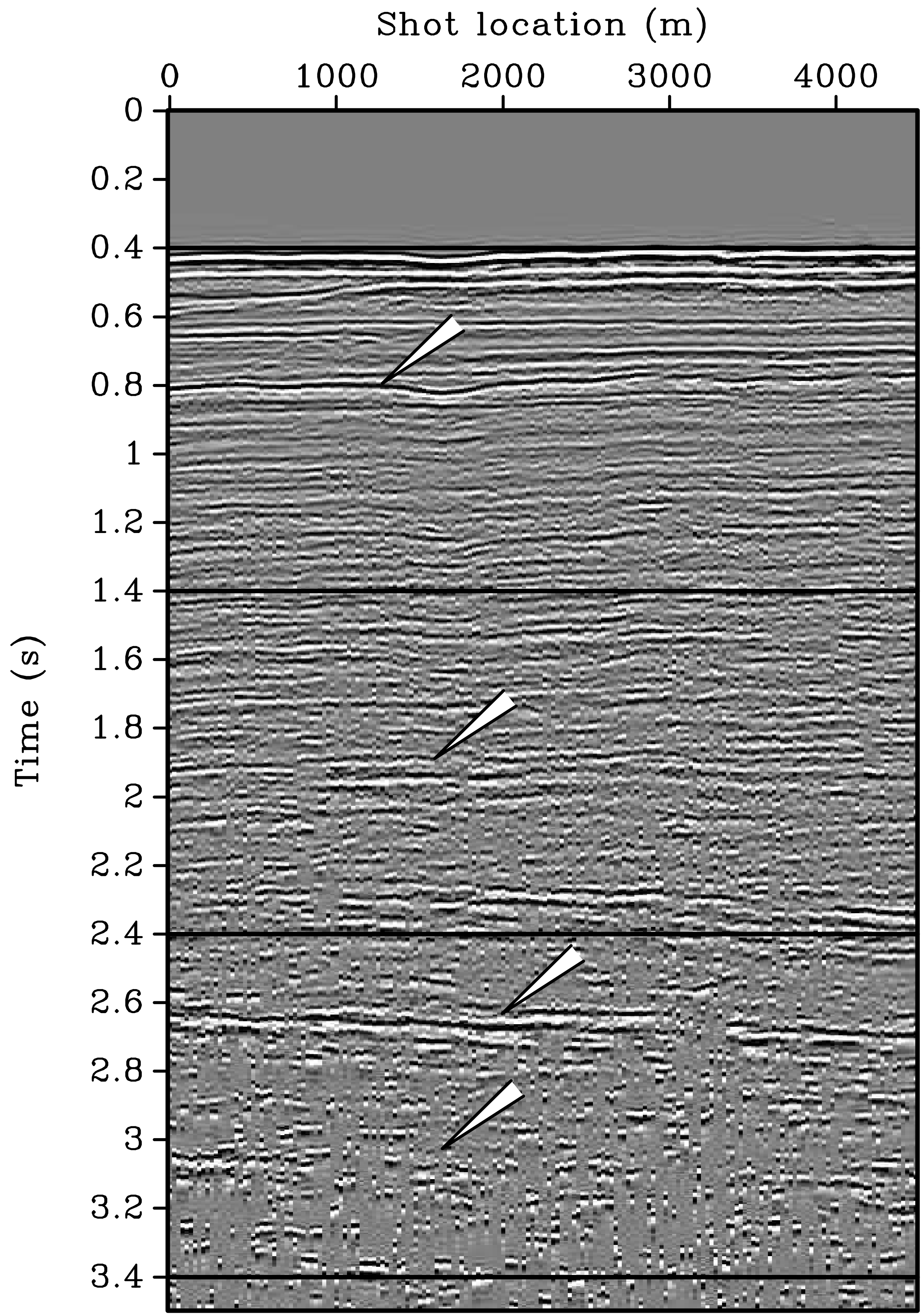
$$\mathbf{x}_{k+1} = \arg\min_{\mathbf{x}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{S}^\dagger\mathbf{x}\|_2 \text{ s.t. } \|\mathbf{x}\|_1 \leq \tau_k$$

(Solve with SPGL1 until Pareto curve reached)

$$\mathbf{g}_{k+1} = \mathbf{S}^\dagger\mathbf{x}_{k+1}$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)



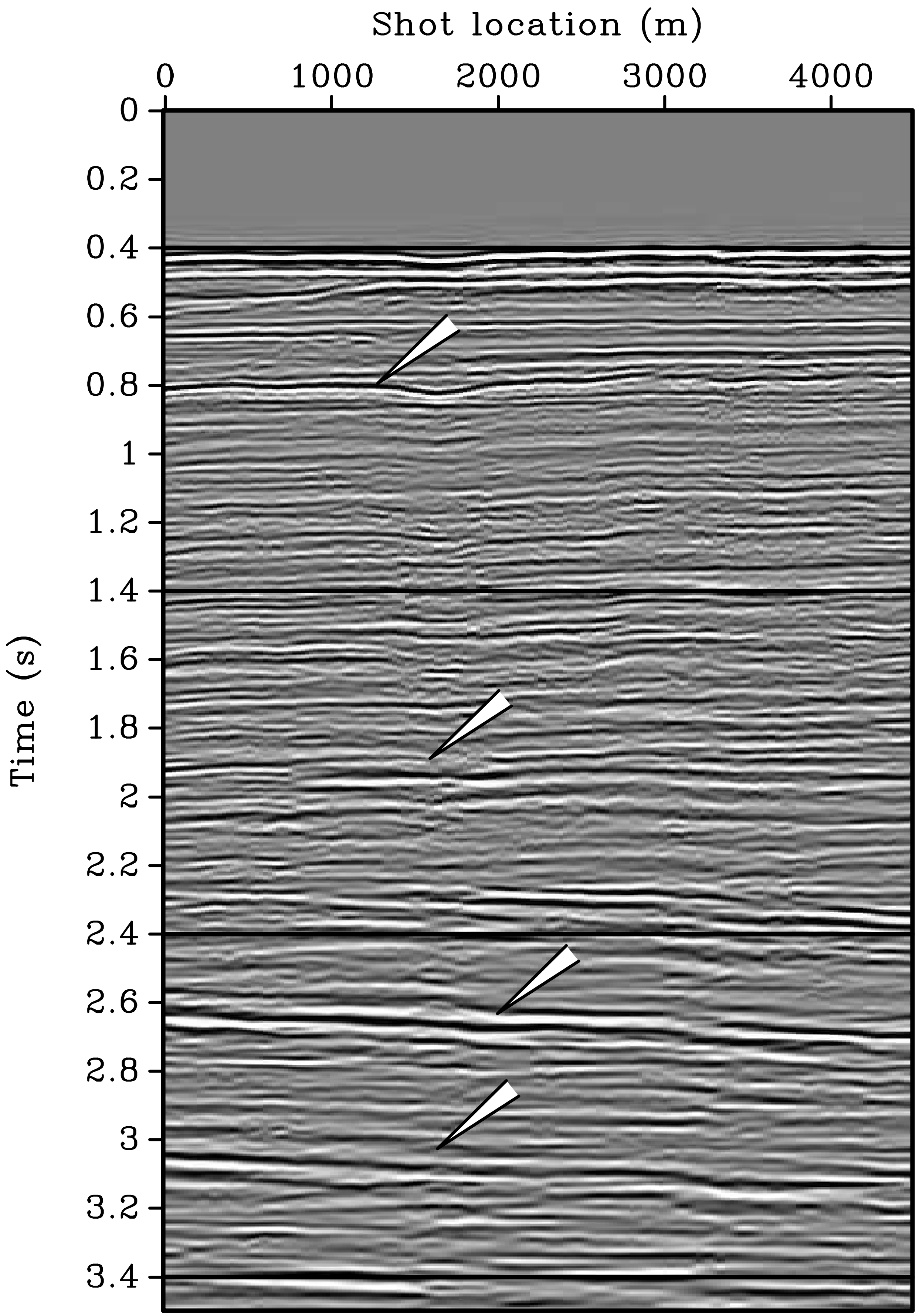
North Sea

REPSI

offset gather 200m

AGC display panels

75 SPG gradient iterations



North Sea

REPSI + Transform

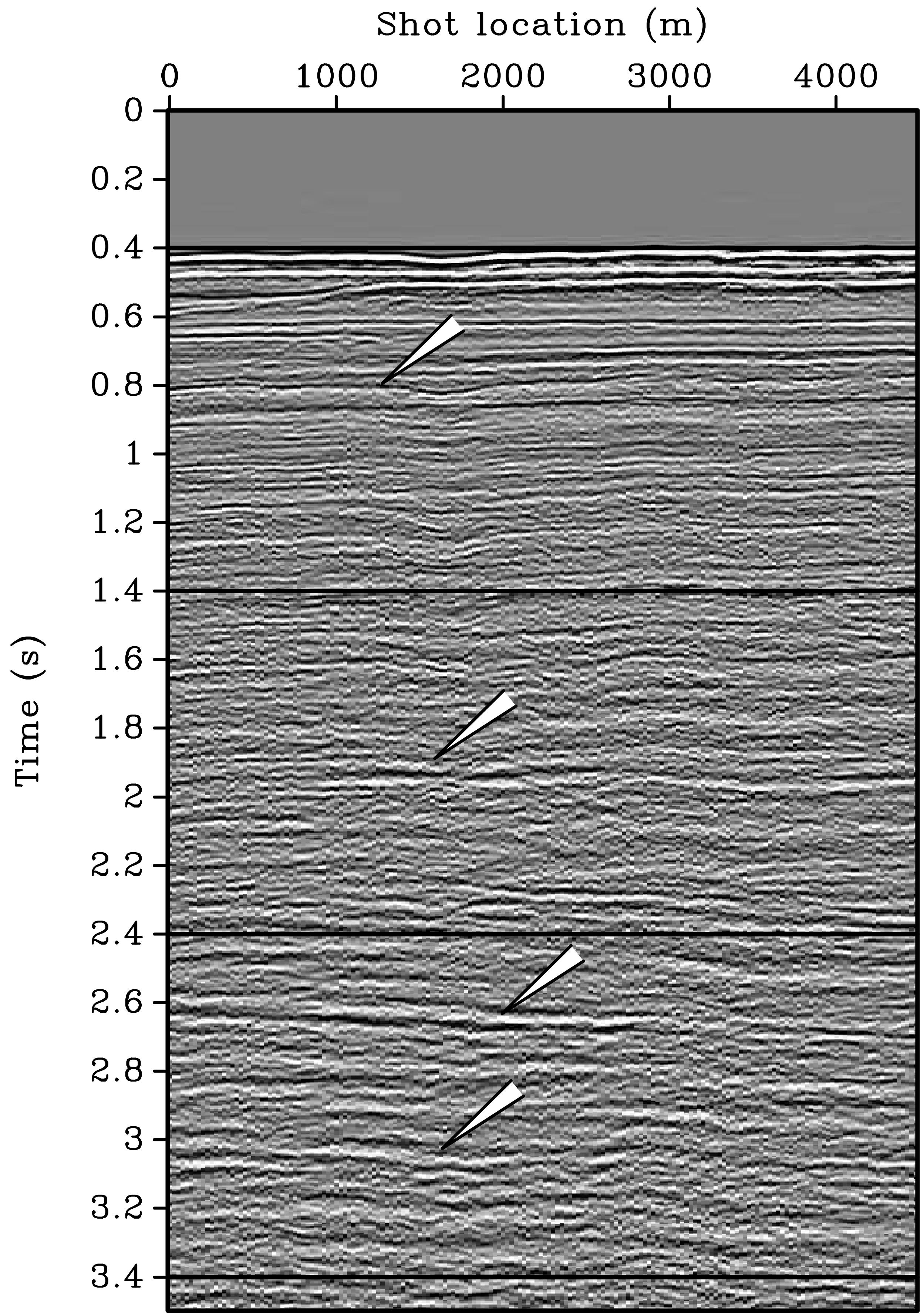
offset gather 200m

AGC display panels

2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

75 SPG gradient iterations



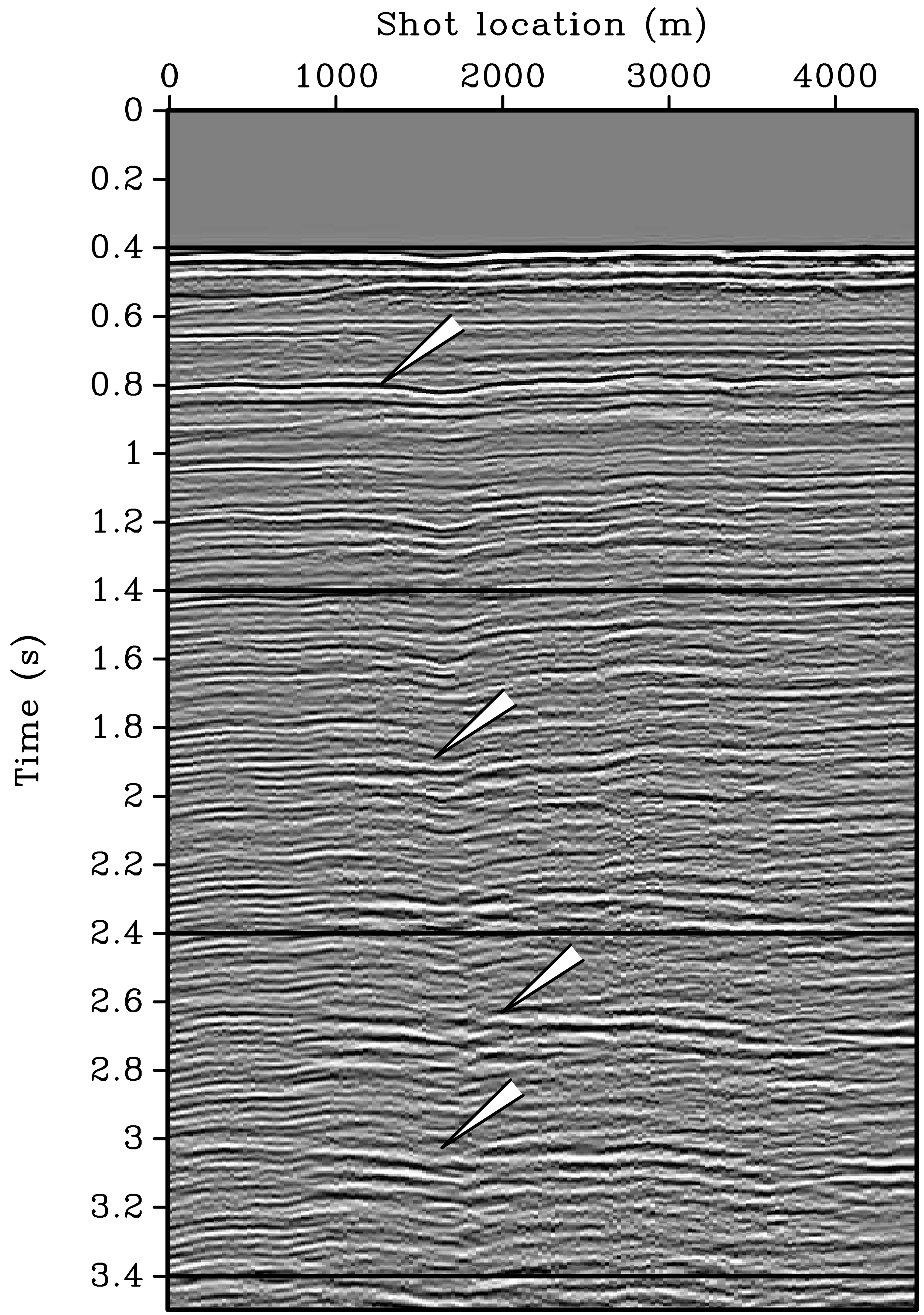
North Sea

SRME + LS subtraction

offset gather 200m

AGC display panels

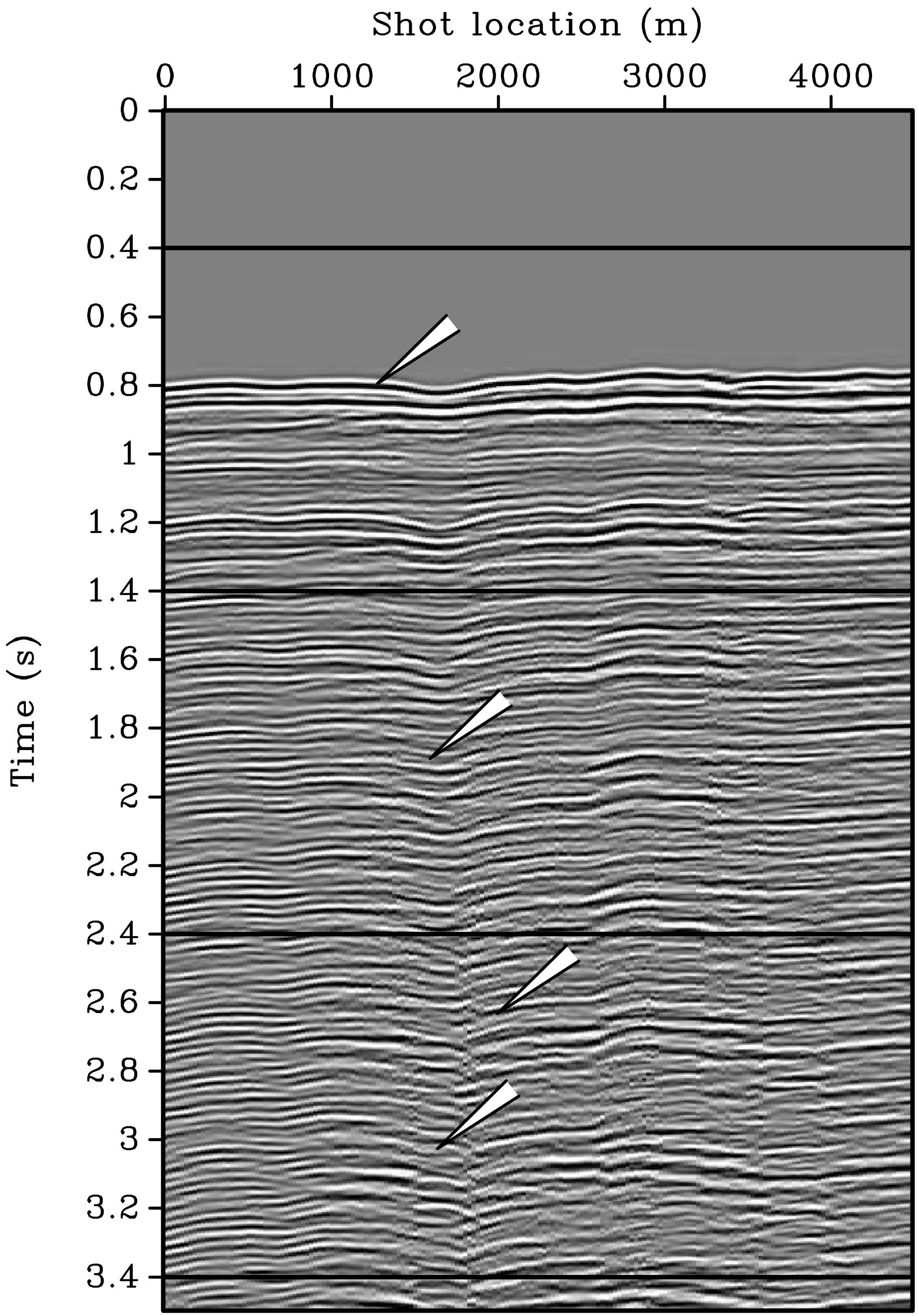
SLIM



North Sea data

offset gather 200m

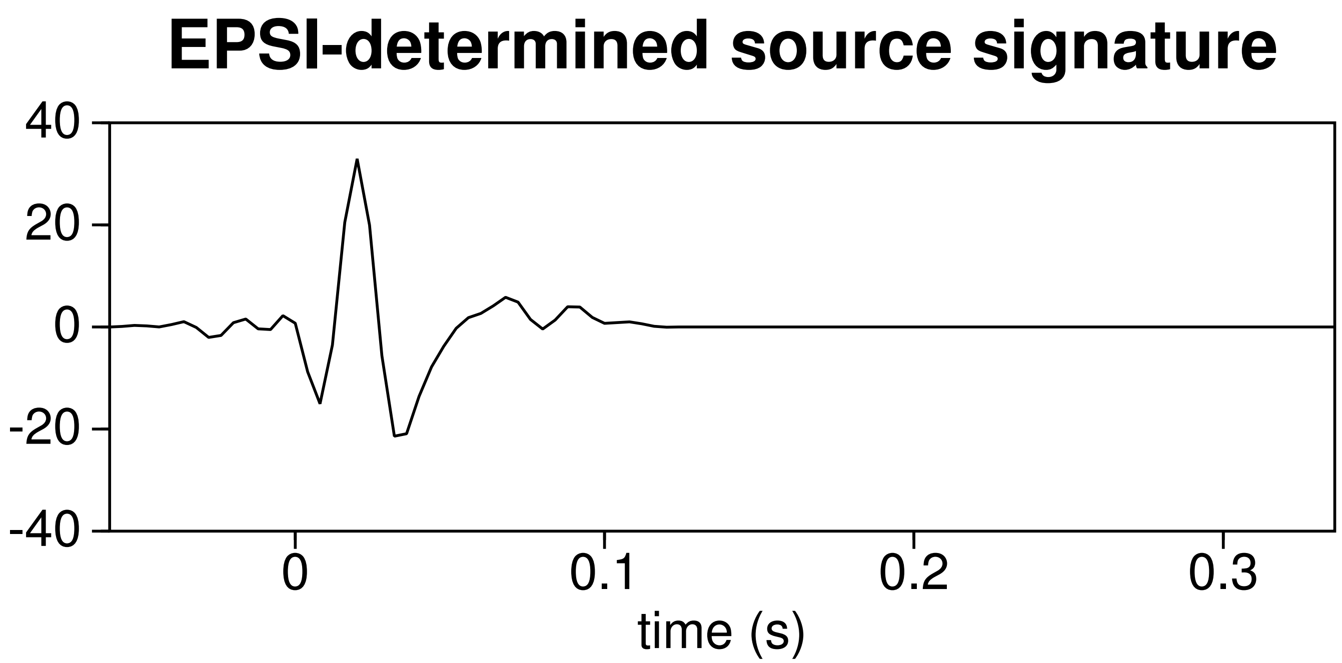
AGC display panels



North Sea
predicted multiples

offset gather 200m

AGC display panels

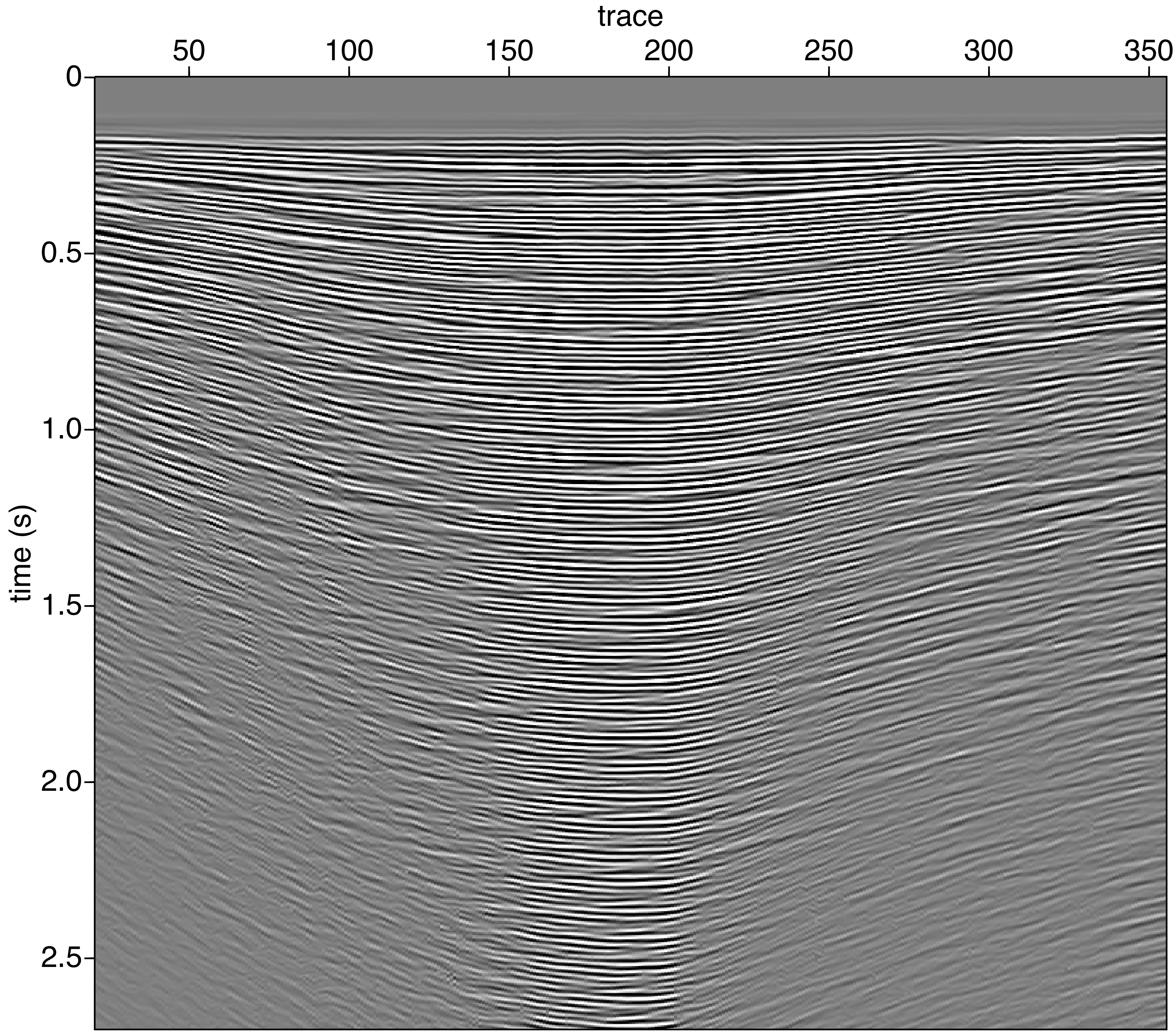


North Sea
REPSI estimated wavelet

offset gather 200m

AGC display panels

75 SPG gradient iterations

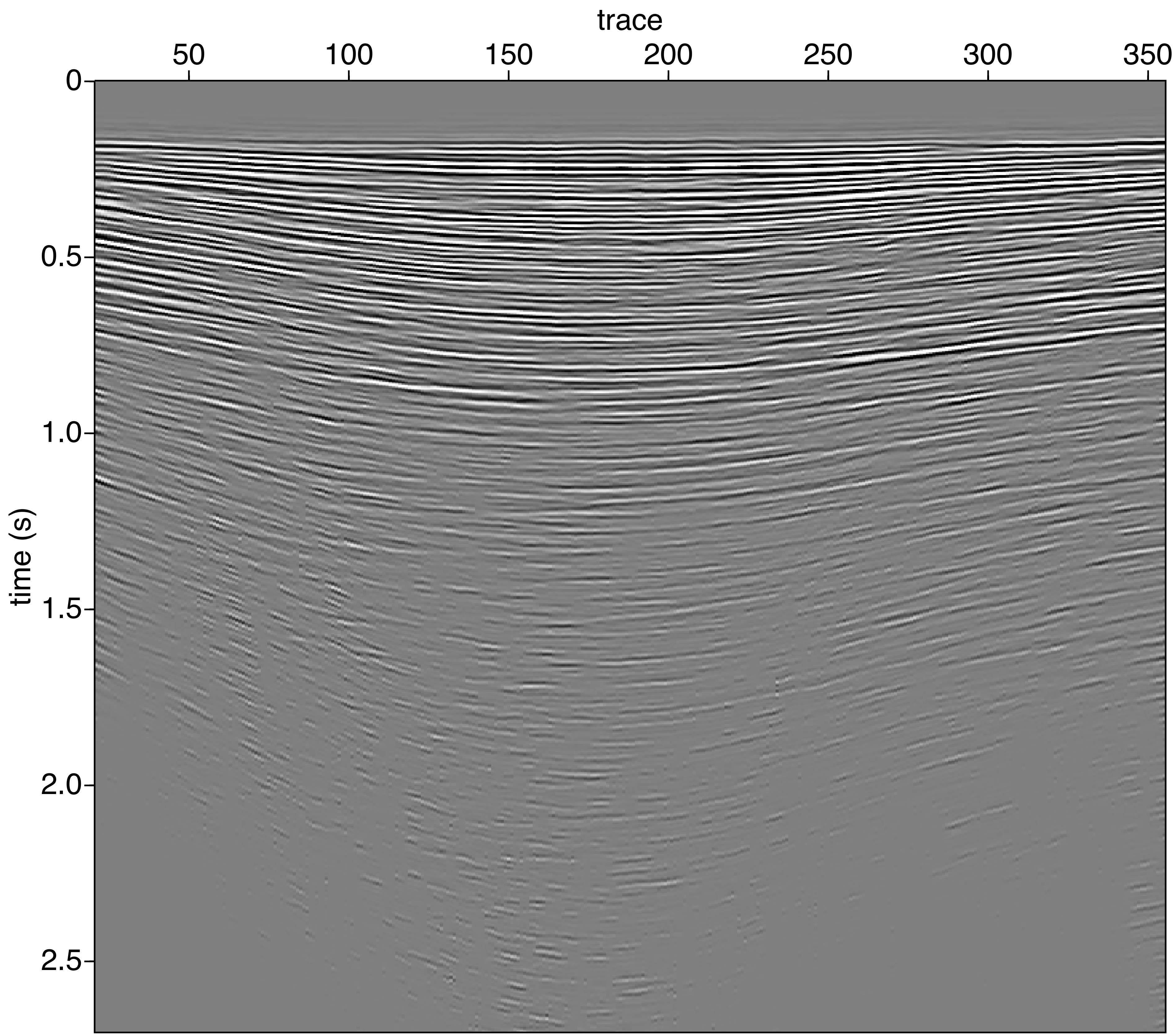


Input data

Gulf of Suez data

offset gather 250m

t-gain display panels



REPSI primary wavefield

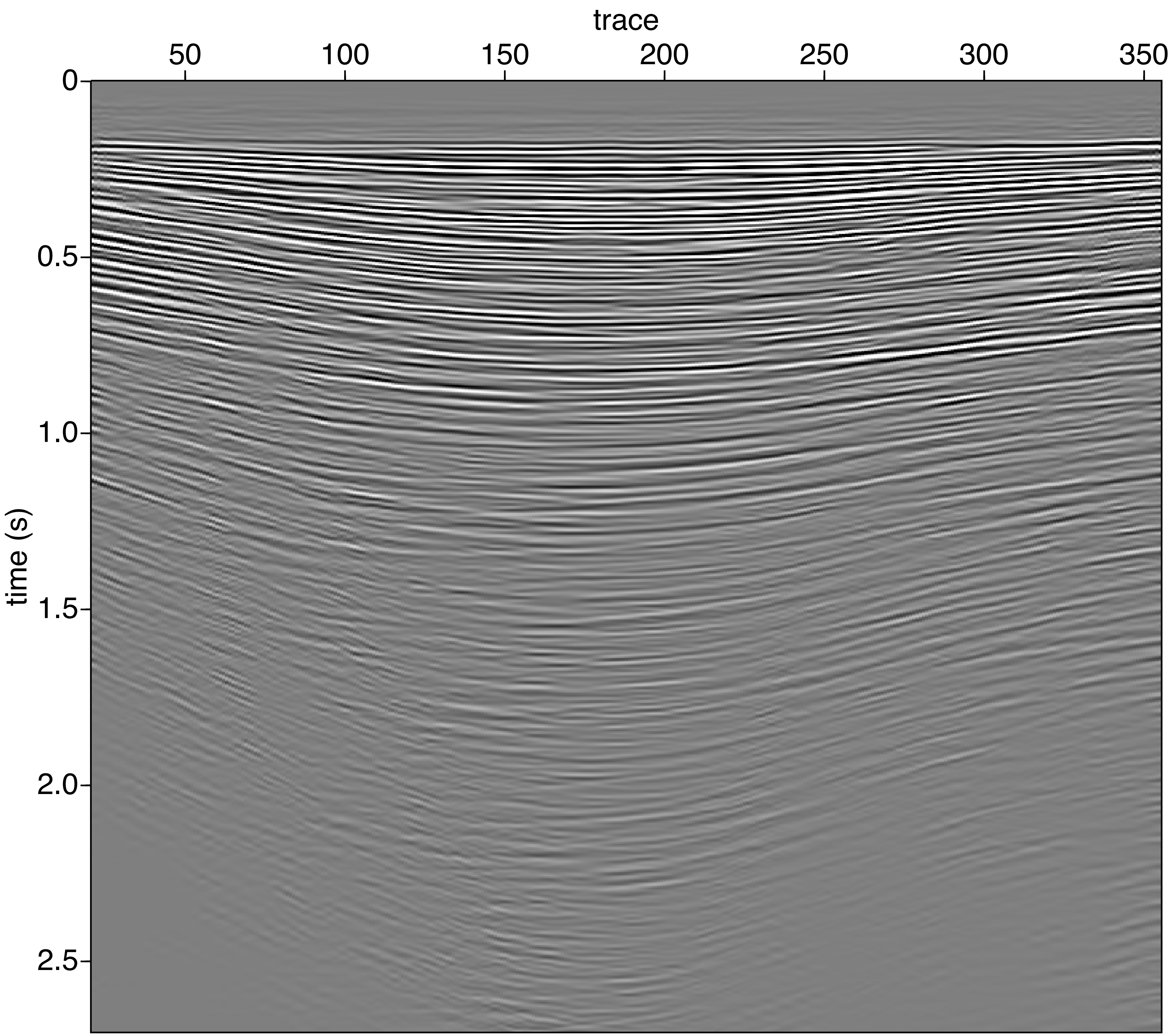
Gulf of Suez

REPSI

offset gather 250m

t-gain display panels

90 SPG grad. iterations



REPSI + transform domain primary wavefield

Gulf of Suez

REPSI + Transform

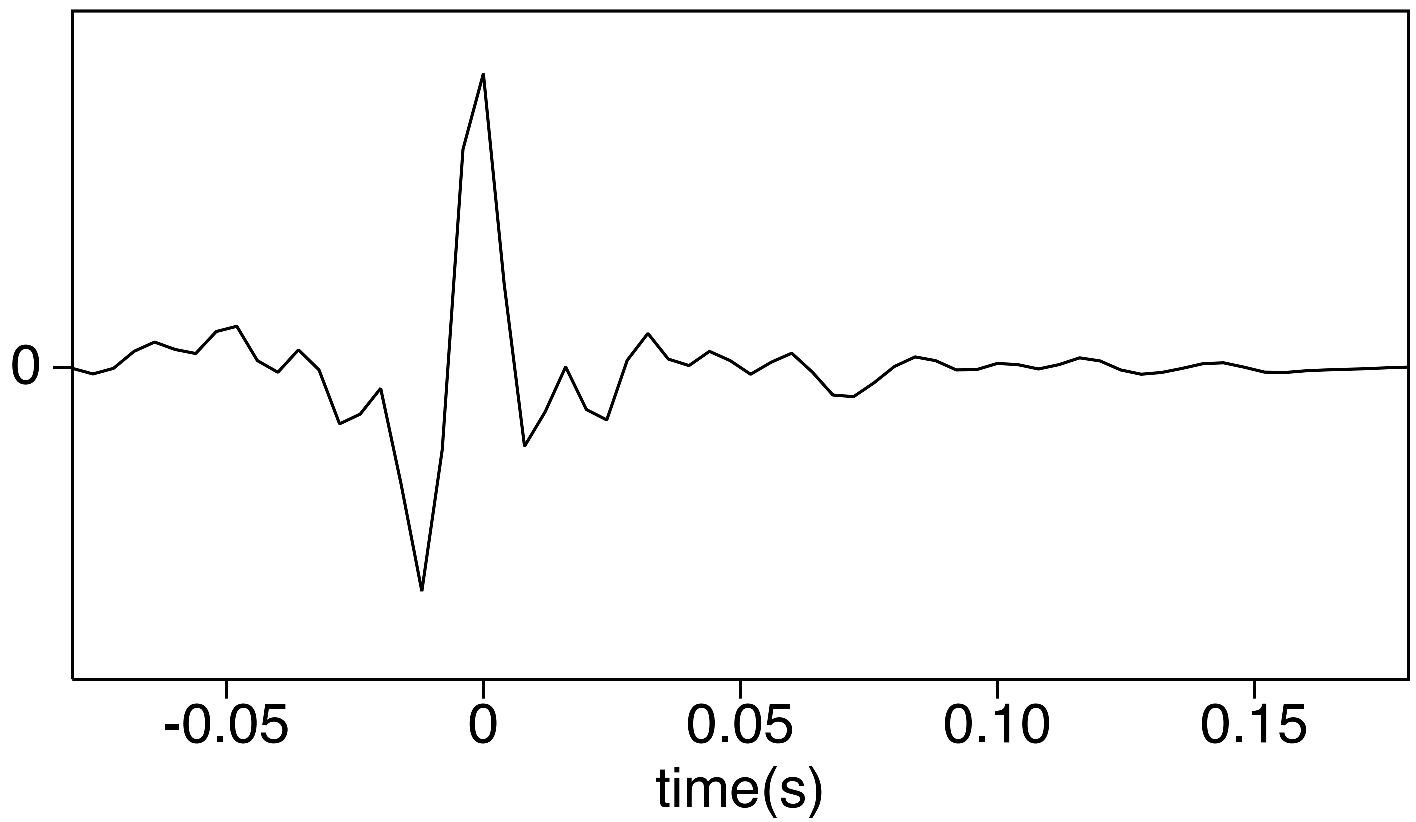
offset gather 250m

t-gain display panels

2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

90 SPG grad. iterations



Gulf of Suez

REPSI estimated wavelet

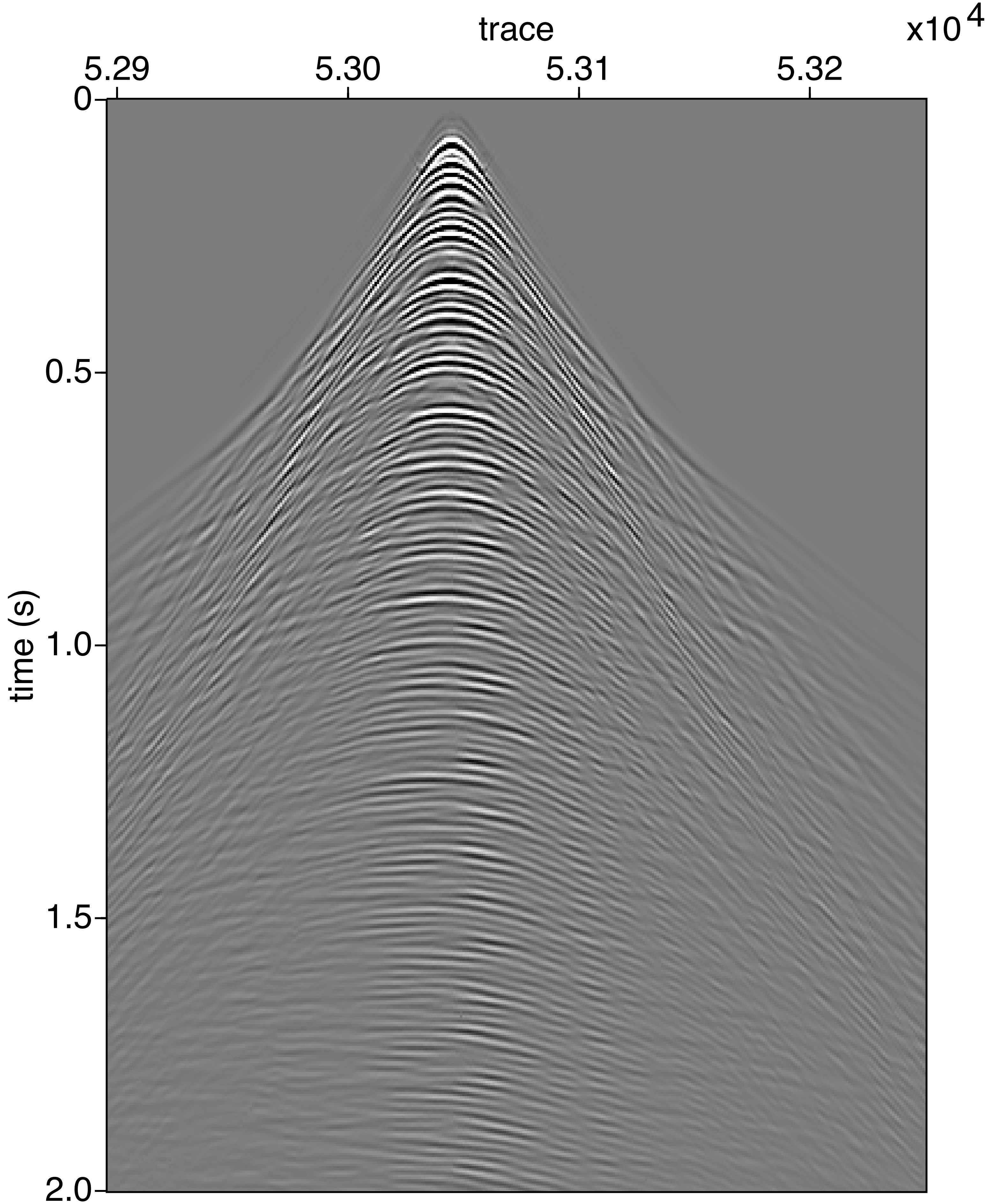
offset gather 250m

t-gain display panels

2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

90 SPG grad. iterations



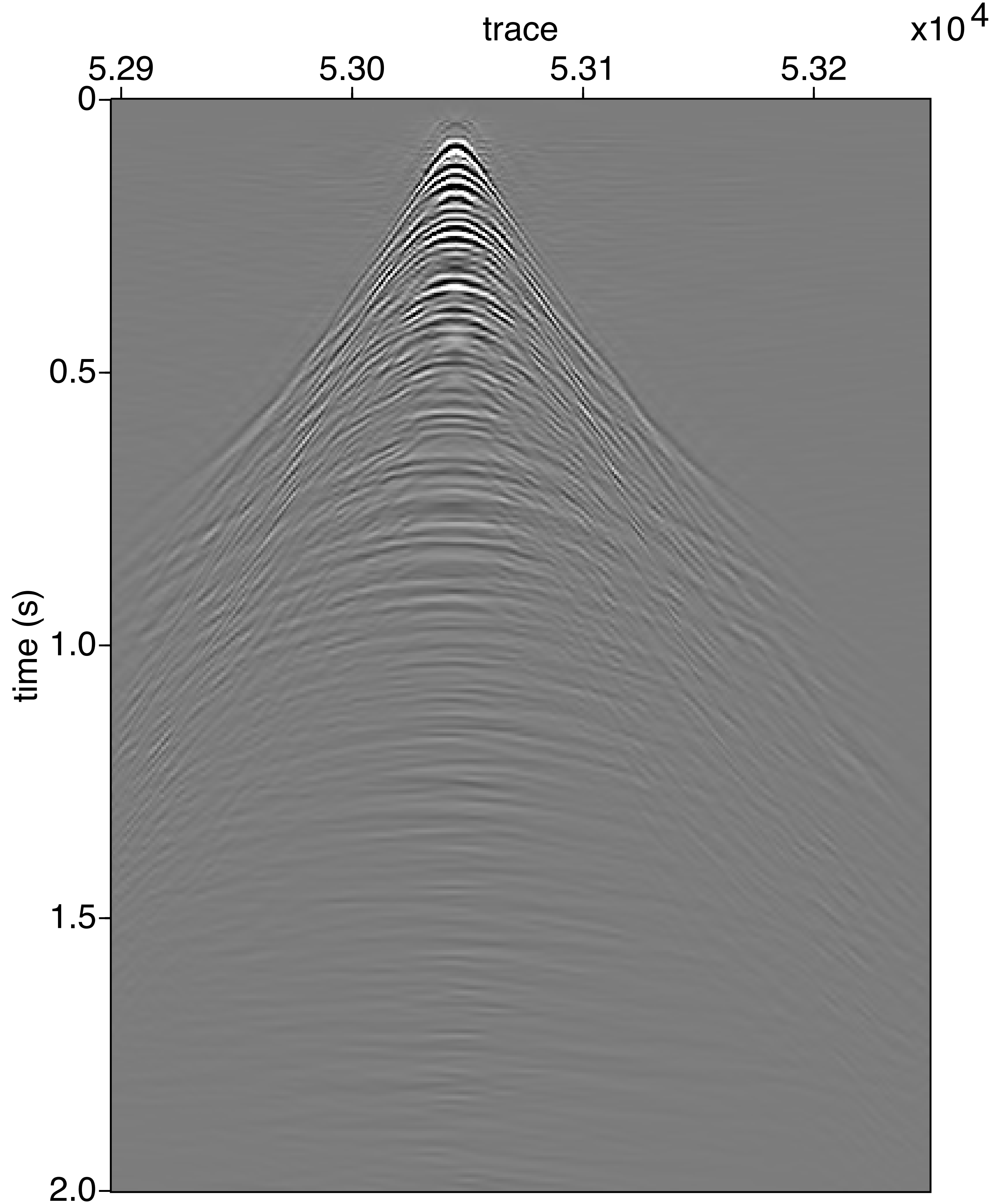
Gulf of Suez
data

shot gather

interpolated, muted

reciprocity

no deghosting



Gulf of Suez

REPSI + Transform

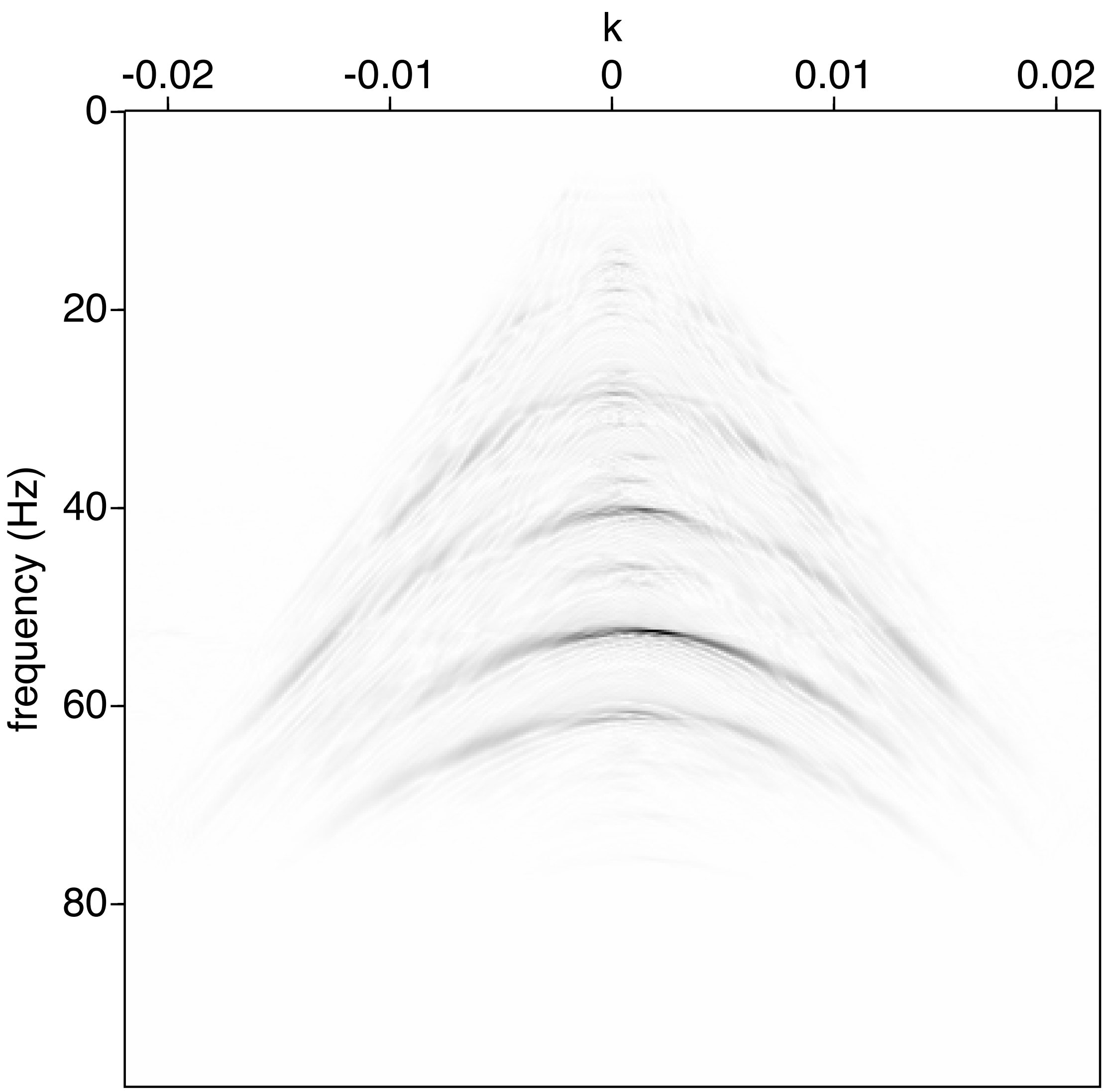
Primary IR (G)

shot gather

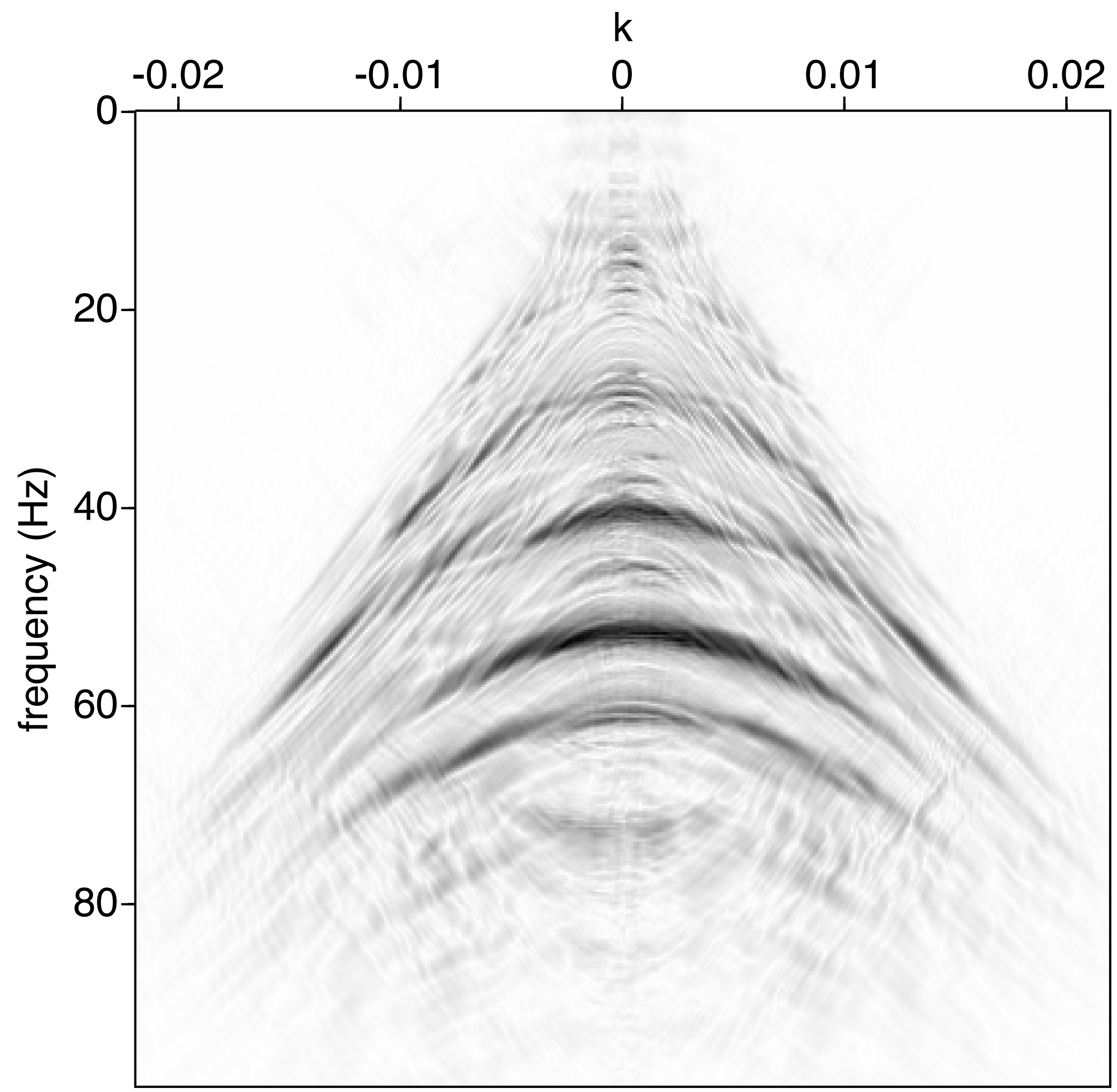
2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

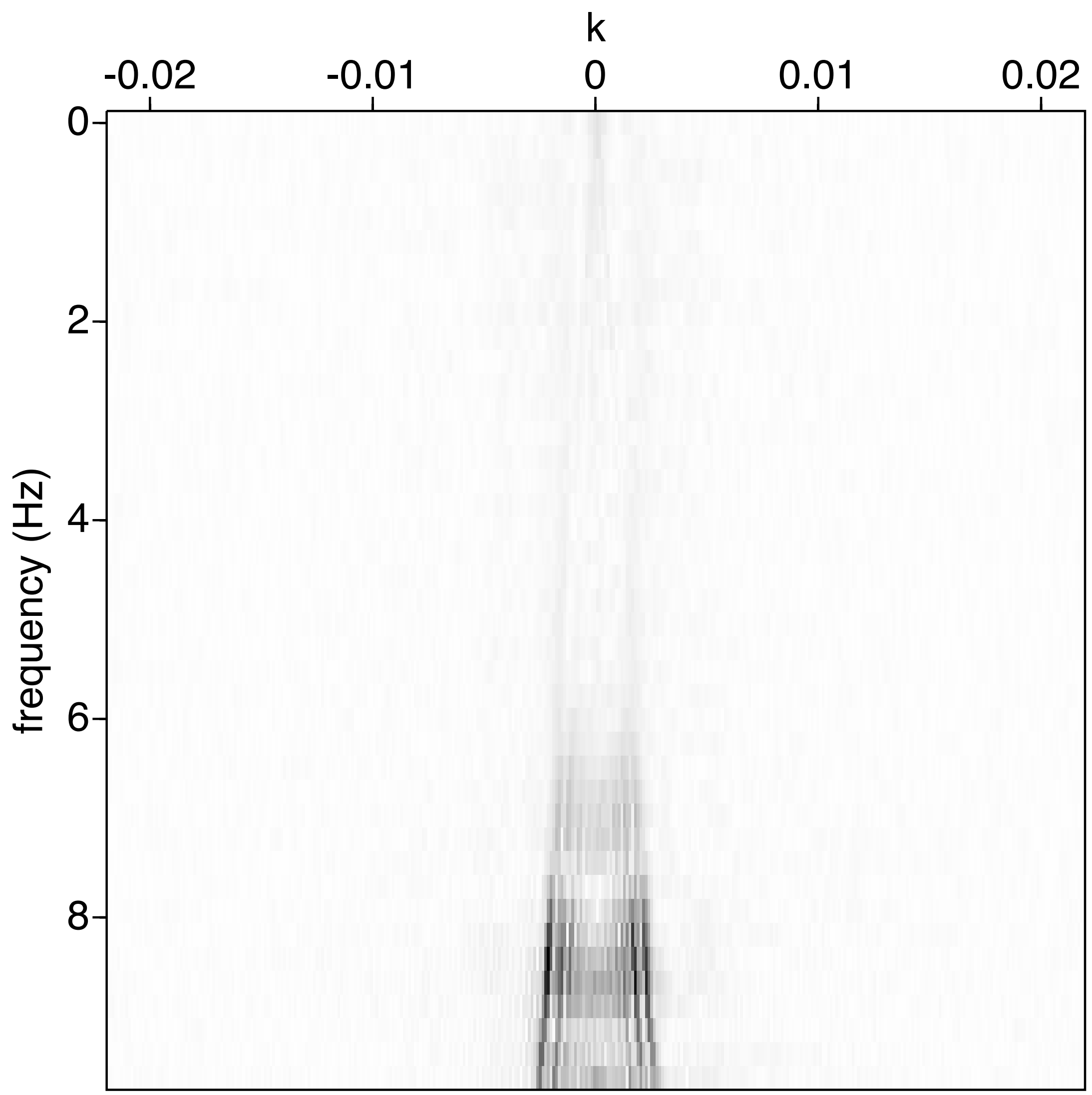
90 SPG grad. iterations



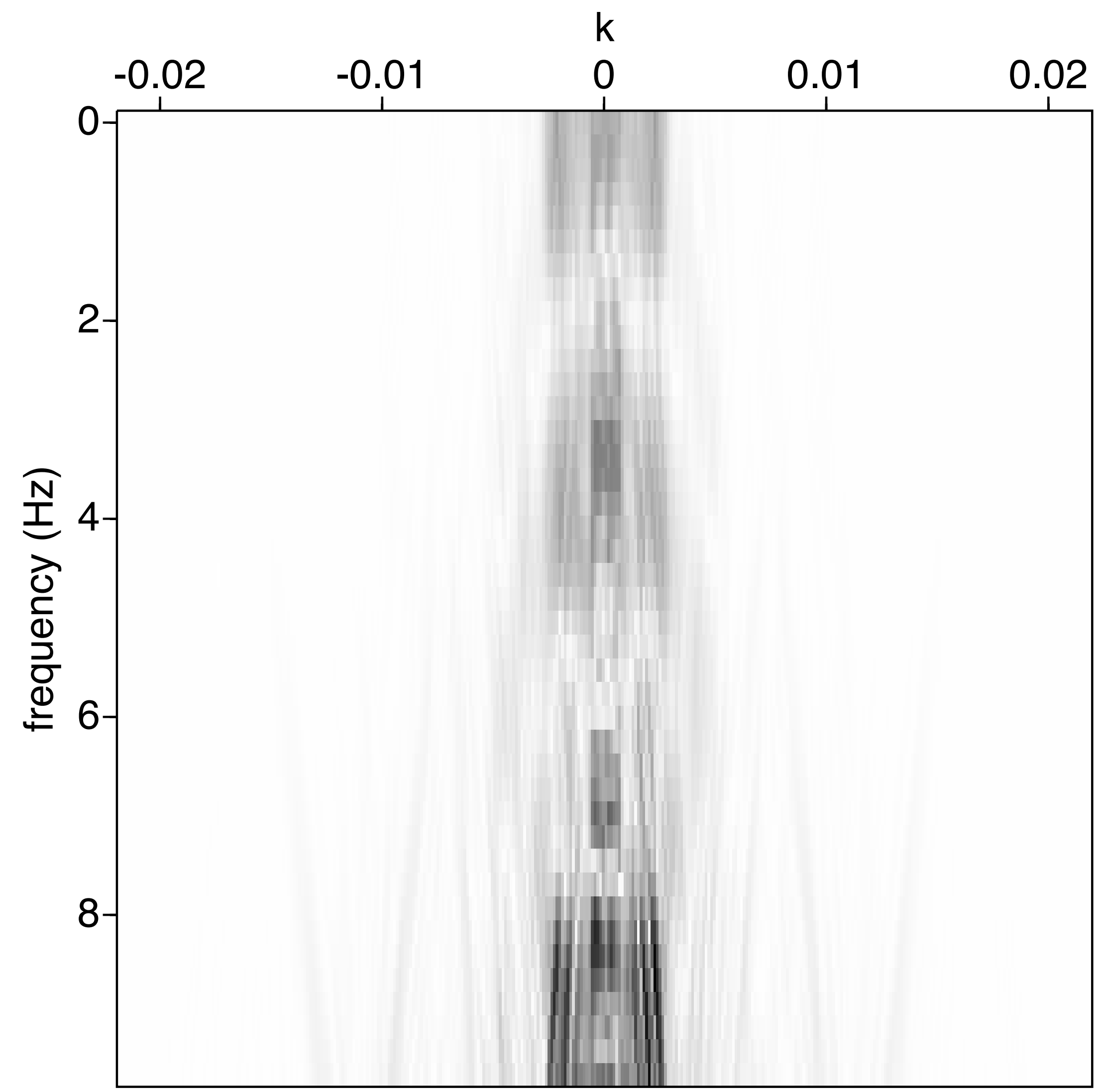
F-K Spectrum of data



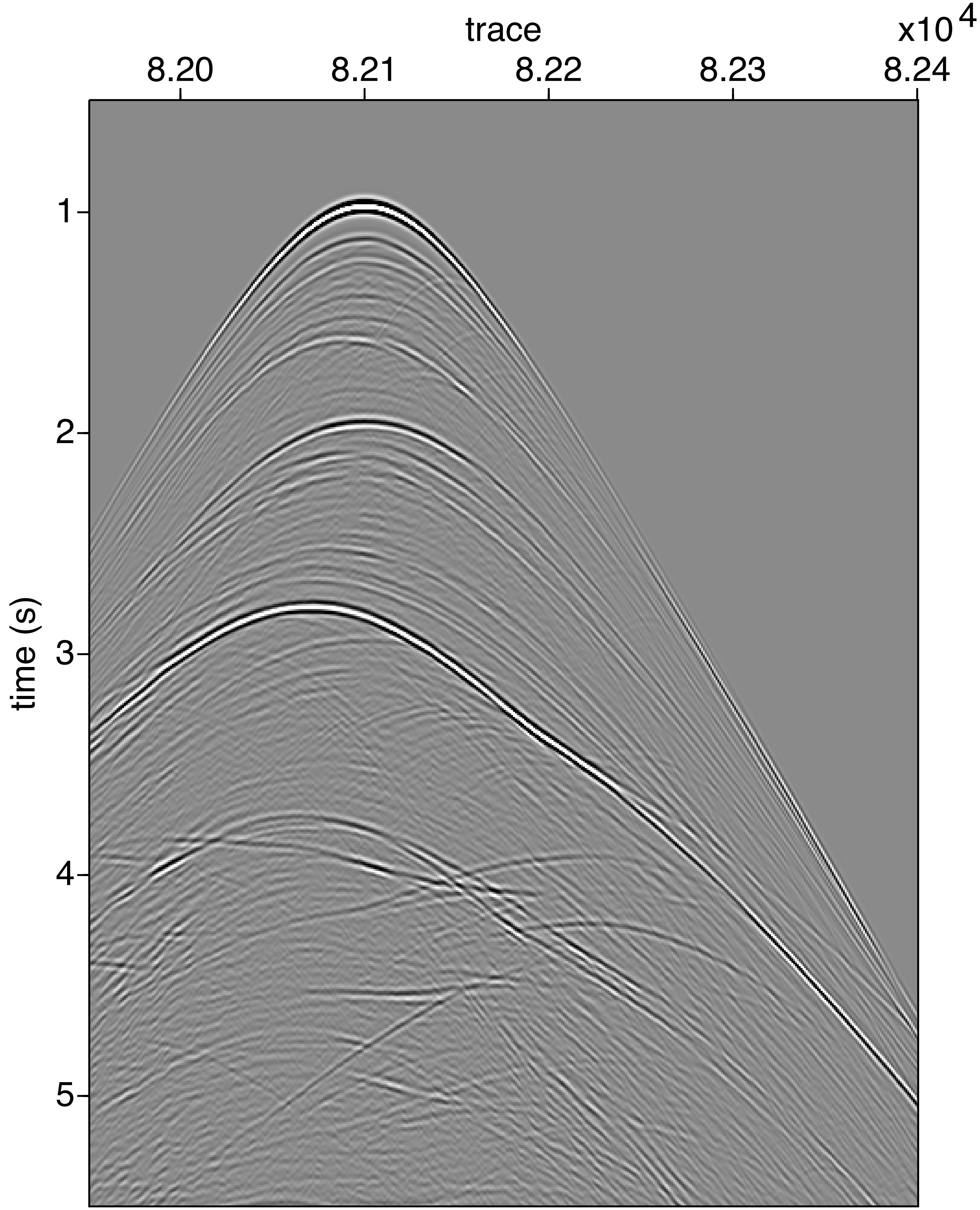
F-K Spectrum of REPSI+Transform Primary IR



F-K Spectrum of data



F-K Spectrum of REPSI+Transform Primary IR



SLIM

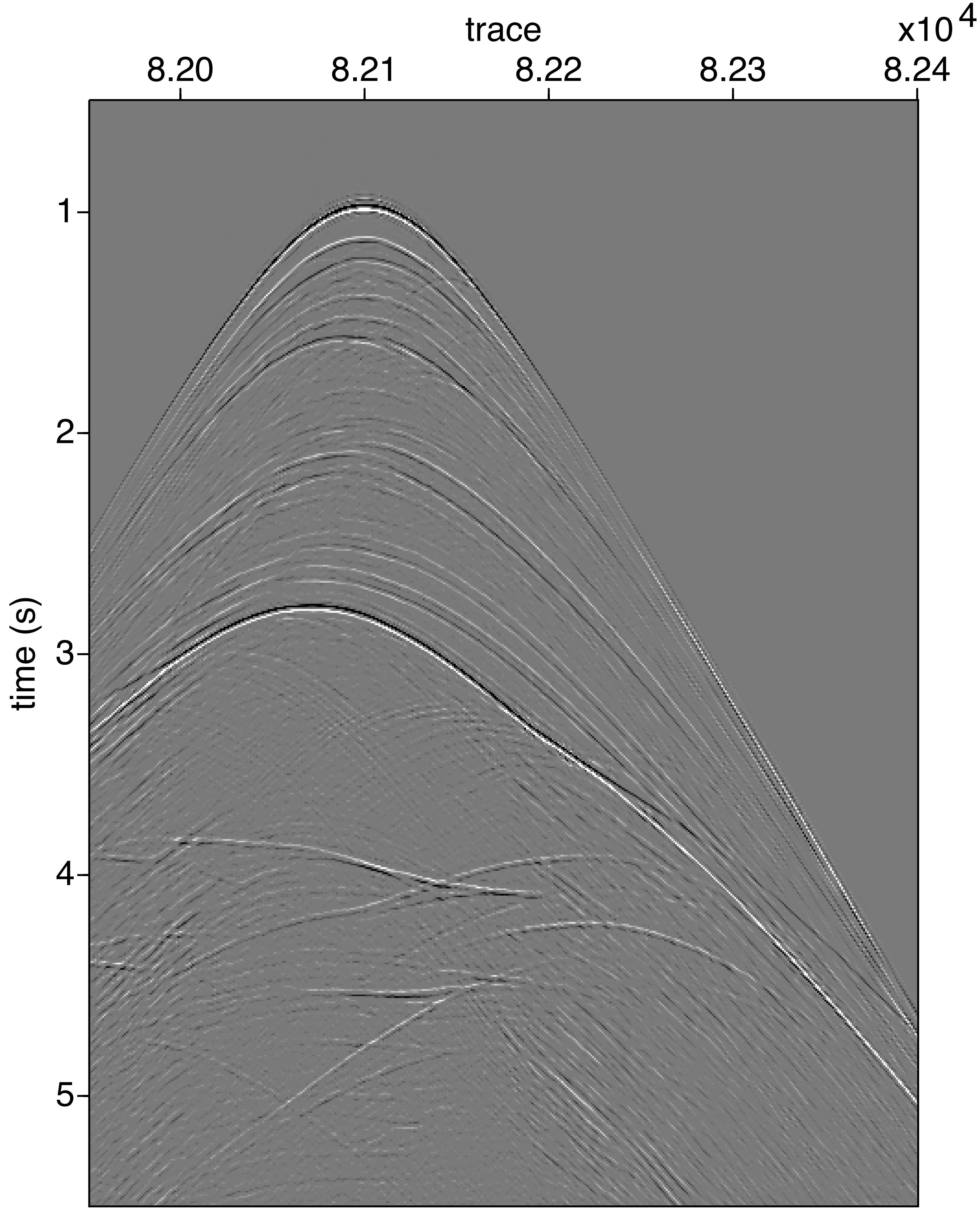
Pluto15

data

Elastic FD Modeling

muted

no deghosting



SLIM

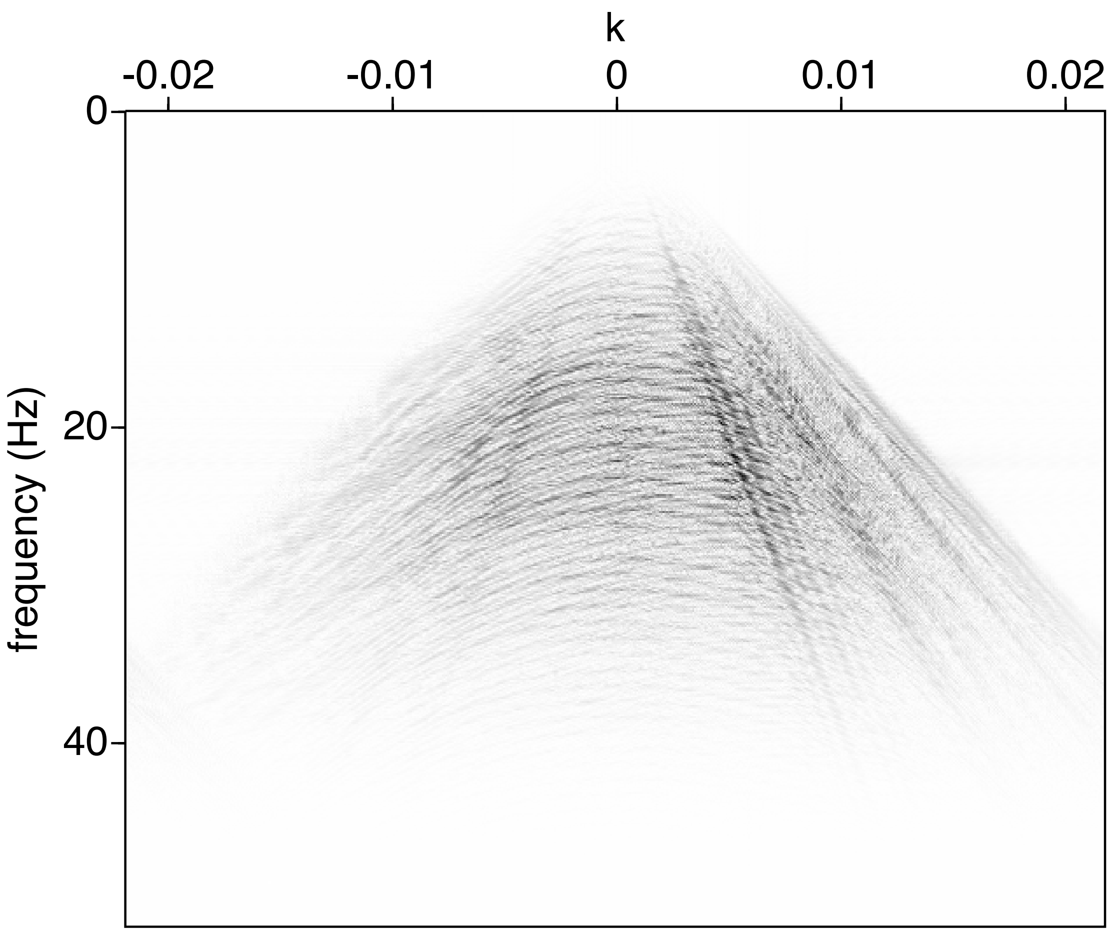
Pluto15

REPSI

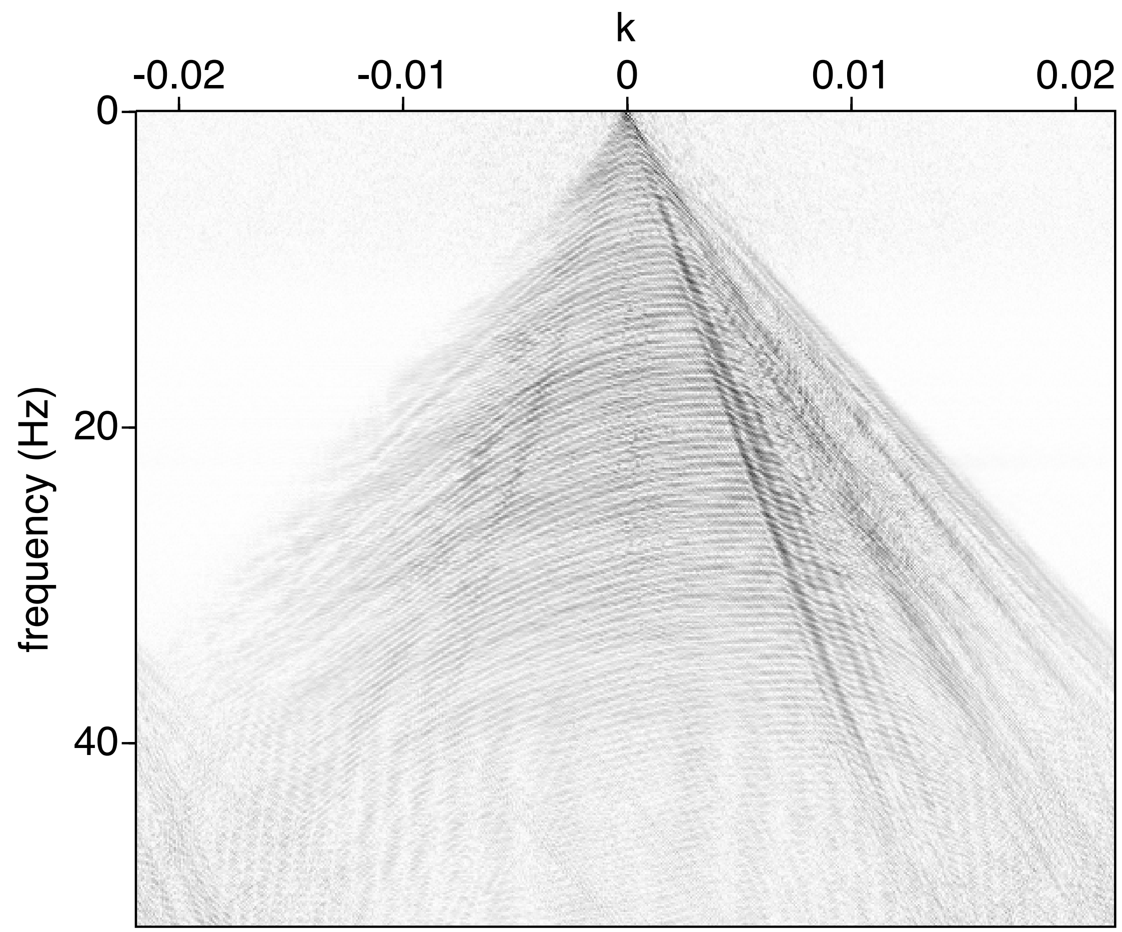
Primary IR (G)

no transform used

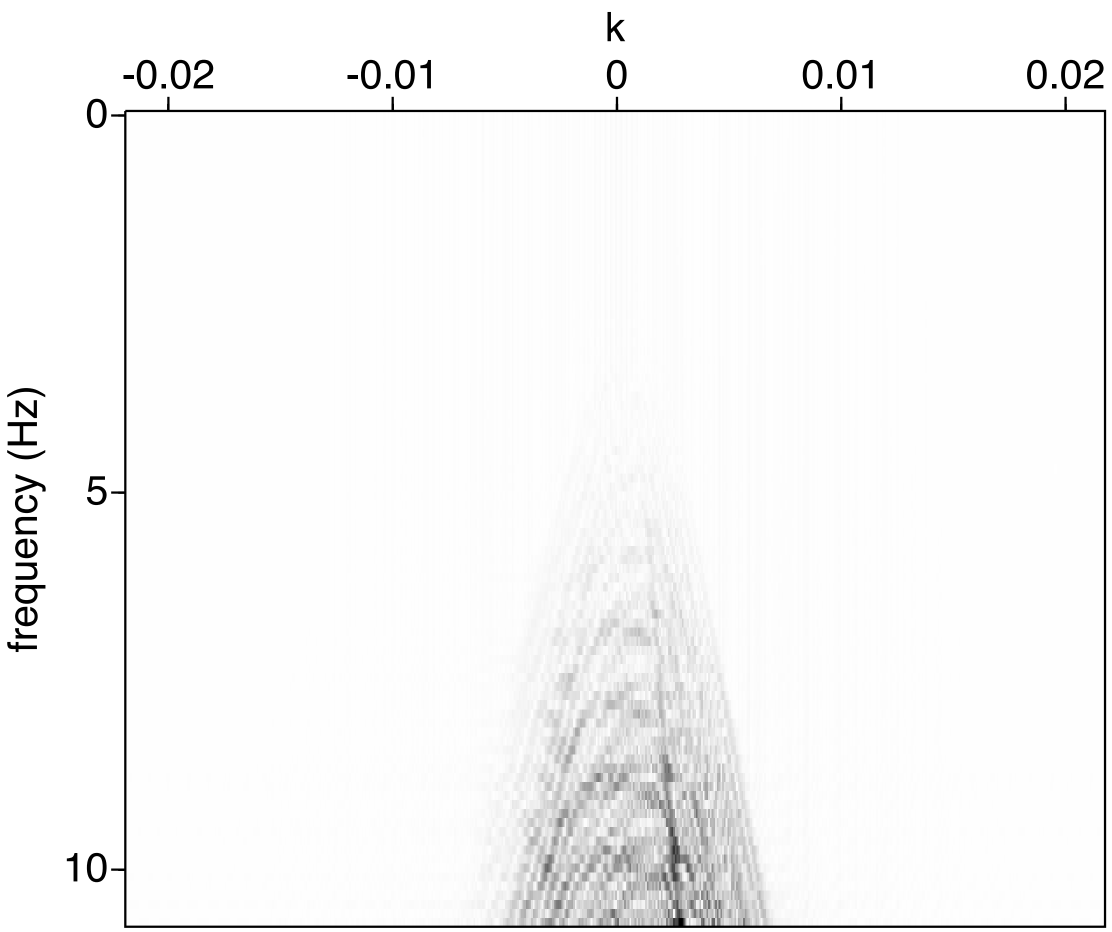
80 iters



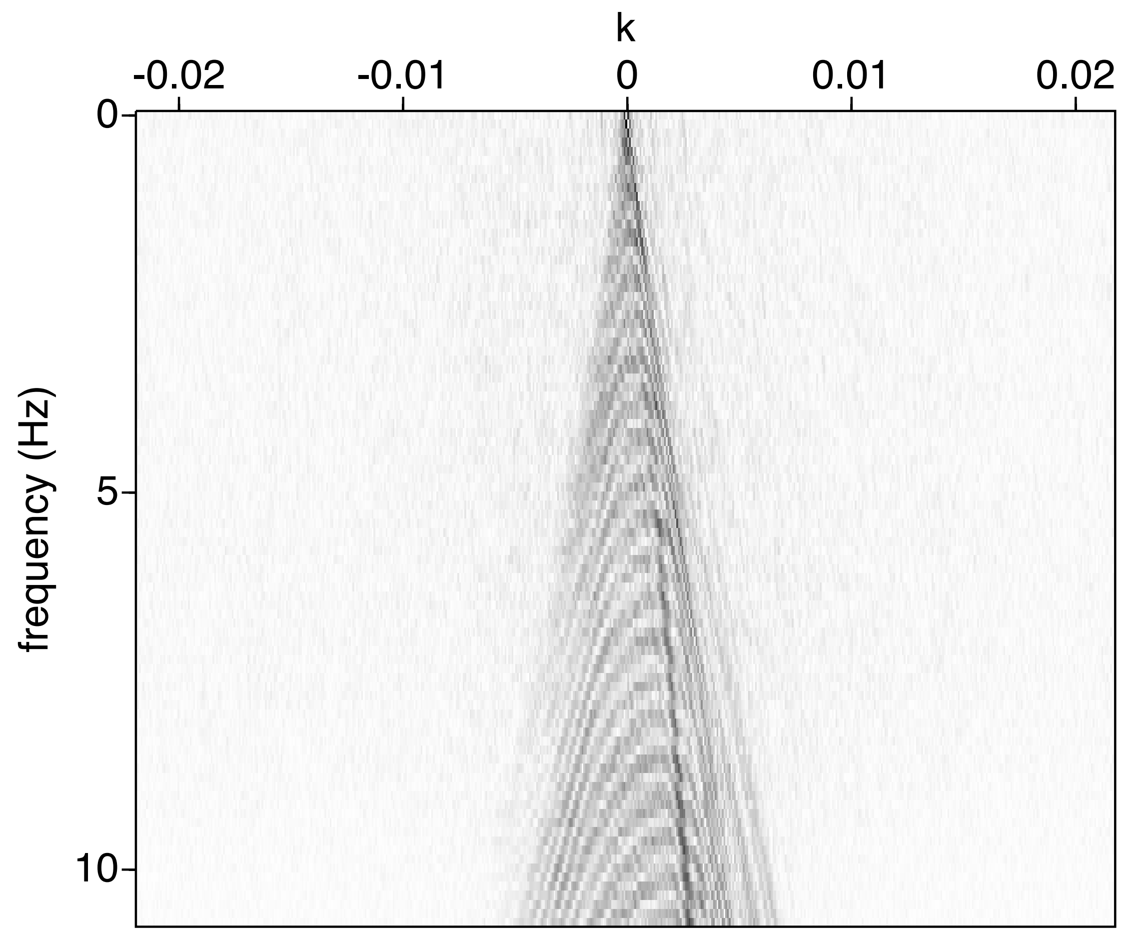
F-K Spectrum of data



F-K Spectrum of REPSI Primary IR



F-K Spectrum of data



F-K Spectrum of REPSI Primary IR

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summary

- L1-convexification better behaved than sparse gradients and has few free parameters
- Follows the Pareto curve into a series of projected gradient problems
- Easily incorporates seeking the solution in a transform domain that promotes continuity
- Transform domain REPSI acts as an effective reflection physics-based denoising
- Promising applications to decon

Acknowledgements

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M. Friedlander and E. van den Berg

SINBAD

NSERC CRSNG

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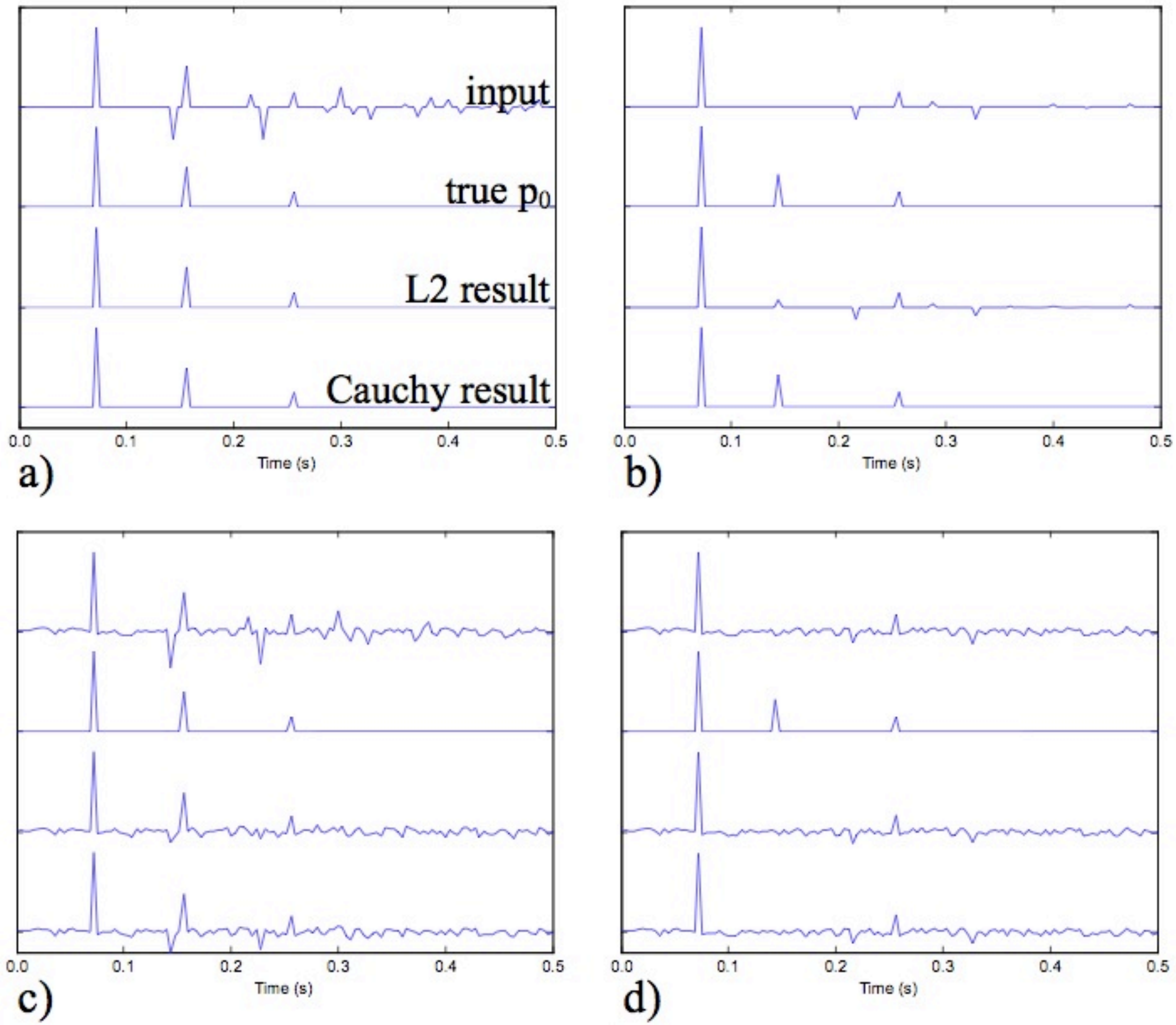
Primary estimation from single trace of plane wave data

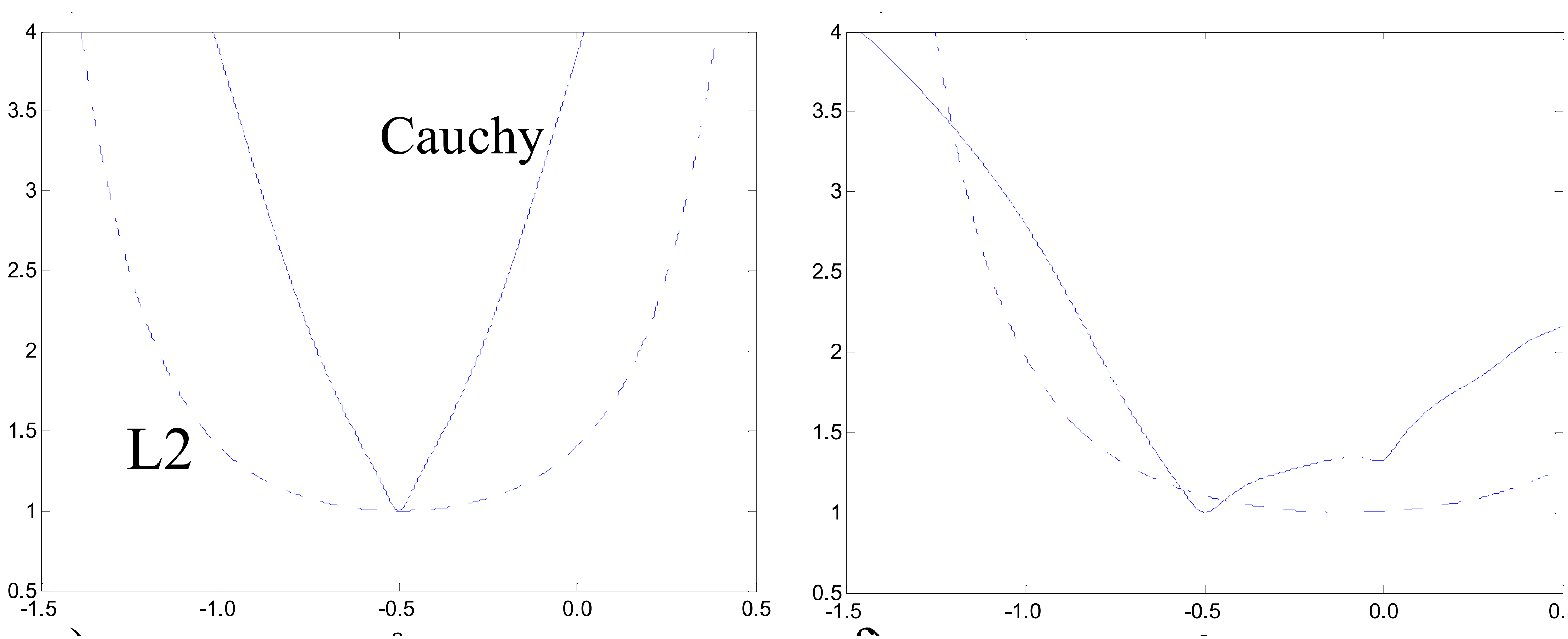


Cauchy “norm”:

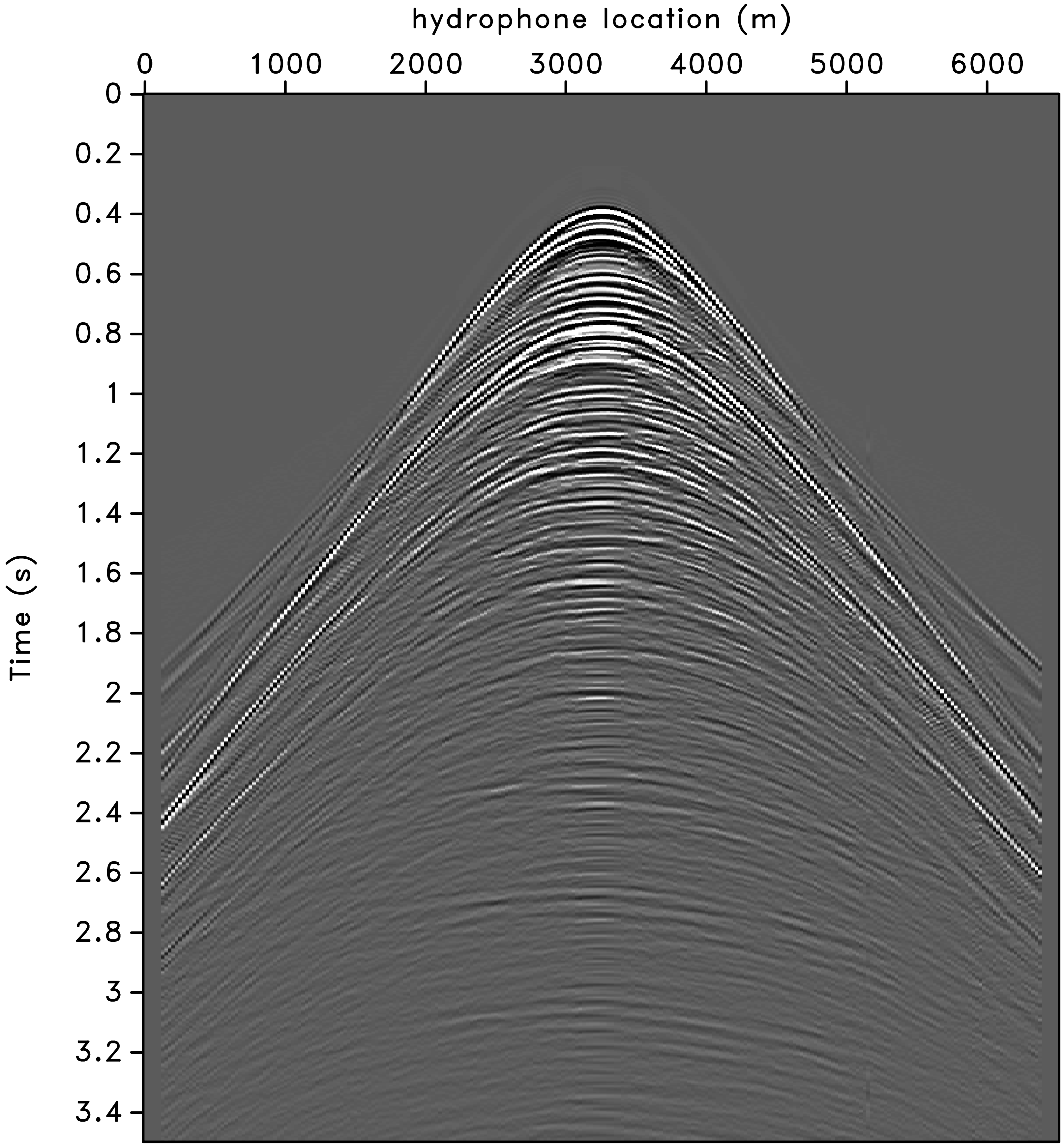
$$\sum_t \log(1 + \mathbf{p_o}(t)^2/\beta^2)$$

(van Groenestijn and Verschuur 09)

Primary estimation from single trace of plane wave data



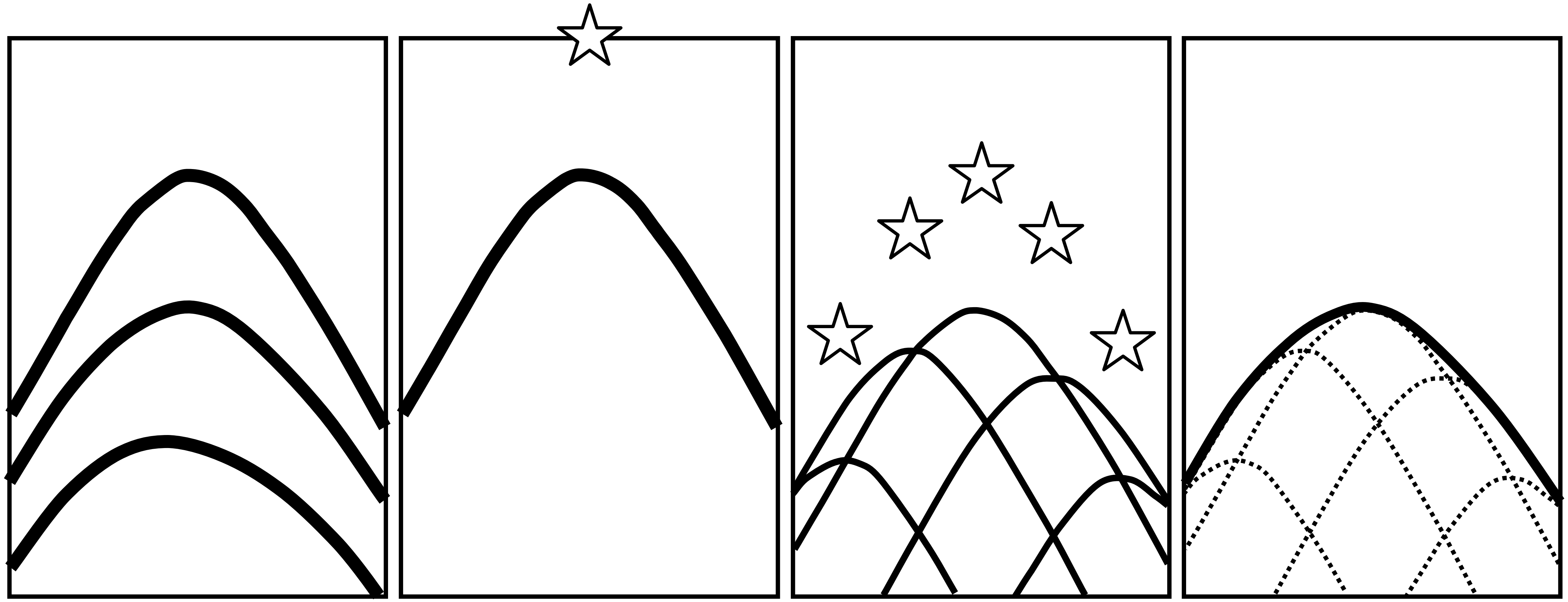
(van Groenestijn and Verschuur 09)



North Sea
data

shot gather 3250m

reciprocity + Radon interp



(van Groenestijn and Verschuur 08)