Towards *extended* modelling for velocity *inversion*

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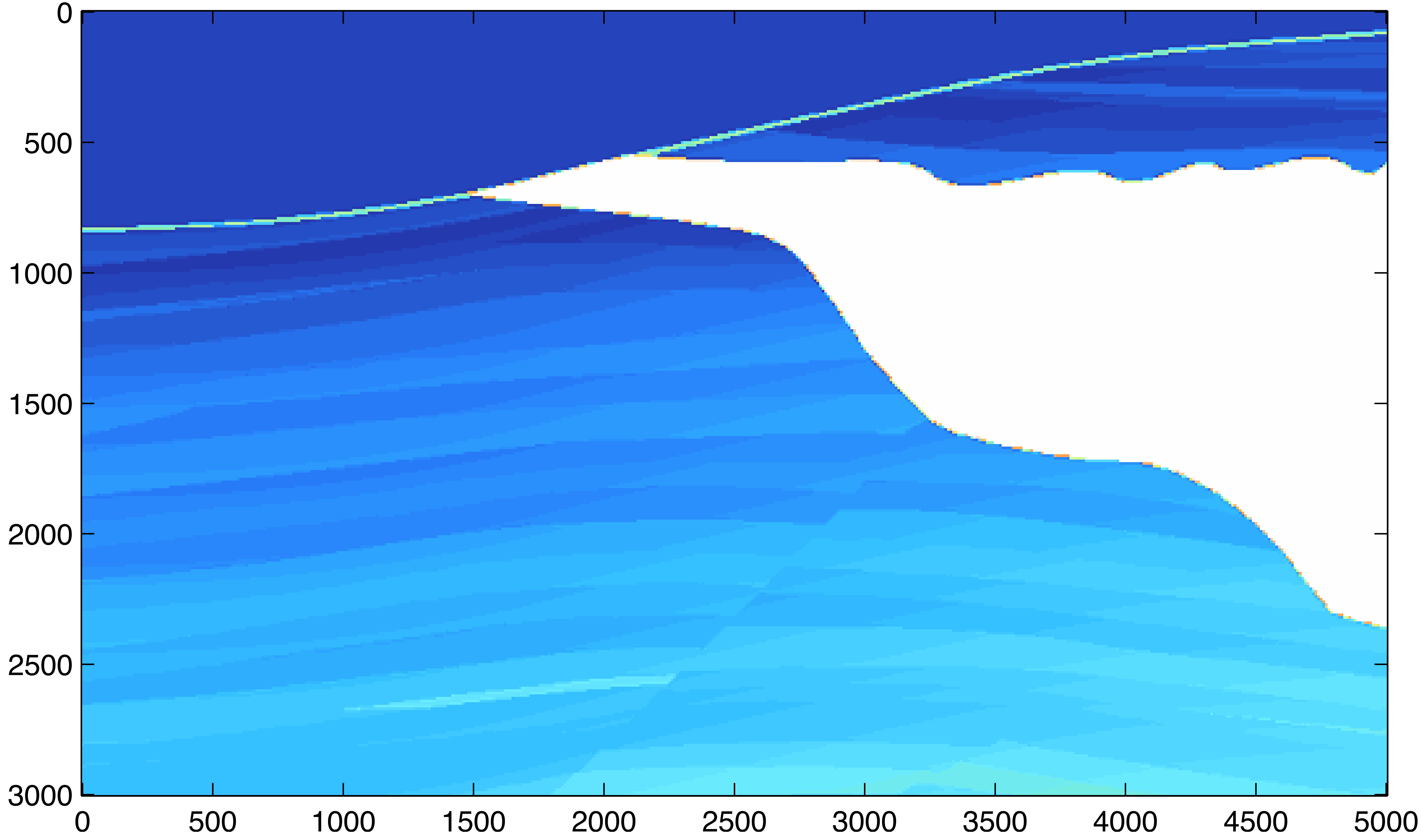

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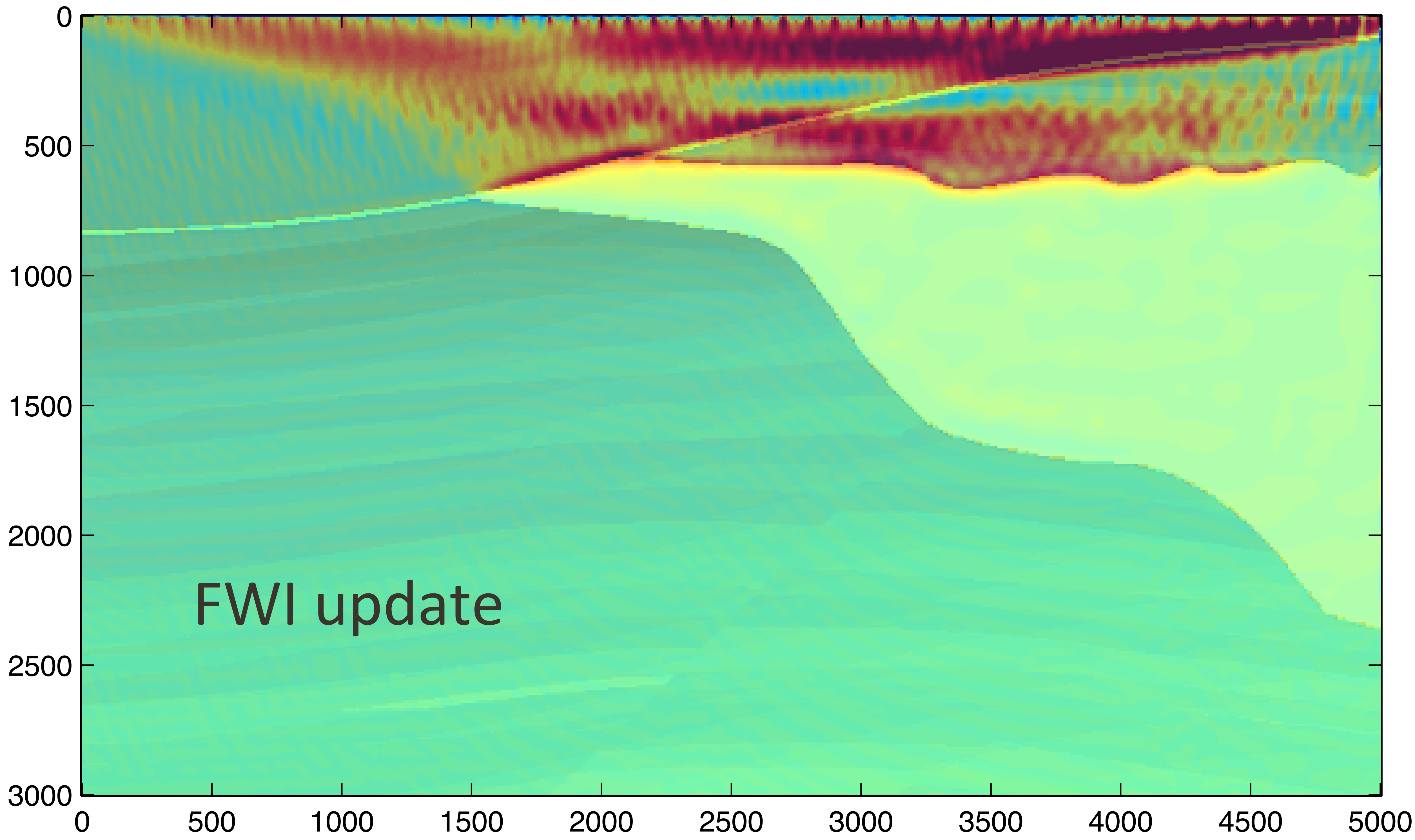
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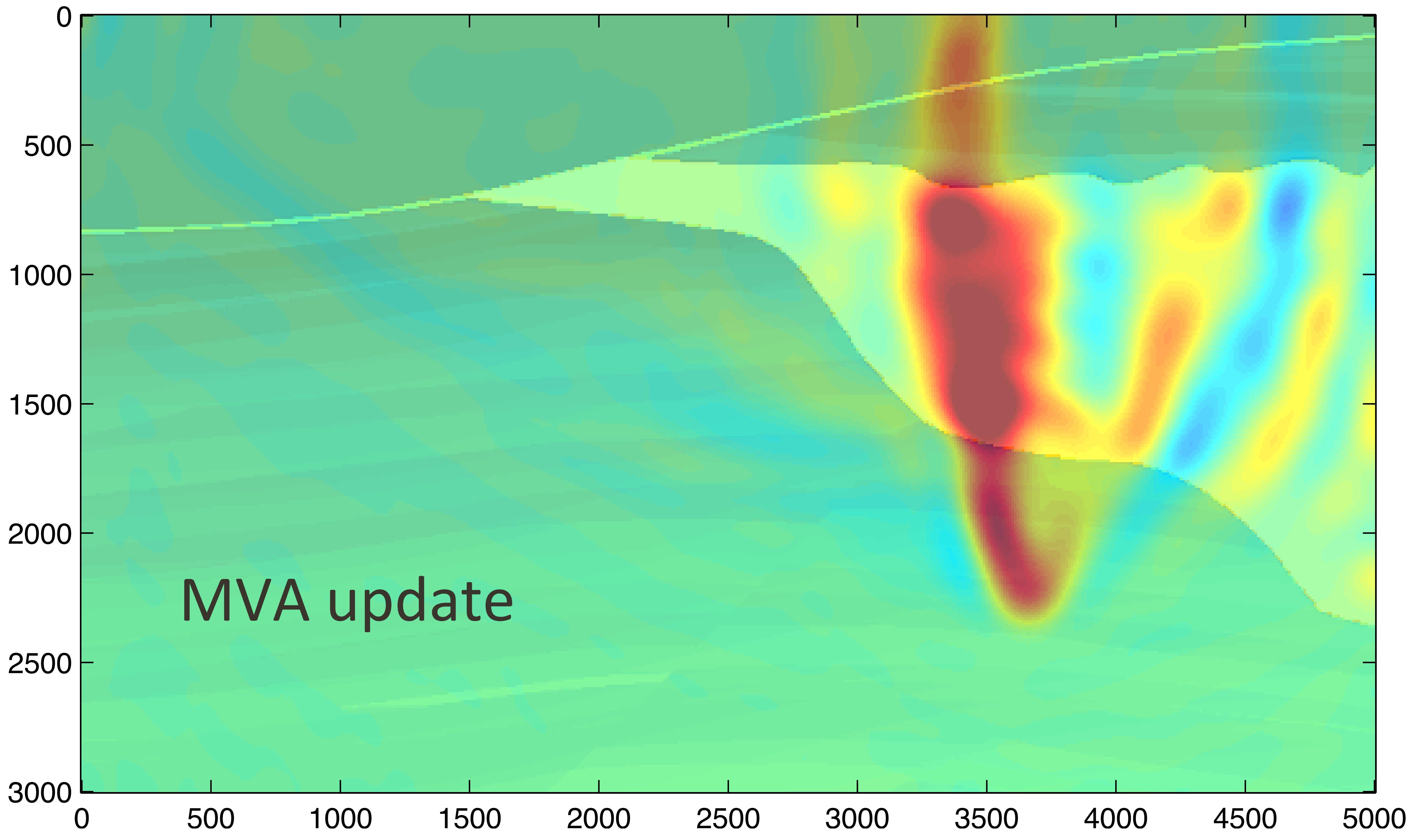
FWI update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MVA update

Overview

- Extended modelling
- Wave-equation MVA
- Probing the image volume
- Towards a non-linear formulation

Extended modeling

- Physical Helmholtz equation:

$$\left[\omega^2 \mathrm{diag}(\mathbf{m}) + \nabla^2\right] \mathbf{u} = \mathbf{q}$$

- Extension

$$\left[\omega^2 M + \nabla^2\right] \mathbf{u} = \mathbf{q}$$

Non-stationary convolution, allows for action-at-a-distance

[Symes '08]

Extended modeling

Correct model should be able to explain the data without violating physics:

minimize off-diagonal energy in M
and *fit the data*

Extended modelling

$$\min_{M} A[M] \quad \text{s.t.} \quad ||F[M]Q - D||_F^2 \leq \sigma$$

$Q = [\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_N]$ sources

$D = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_N]$ monochromatic data matrix

$F[M]$ extended modelling operator

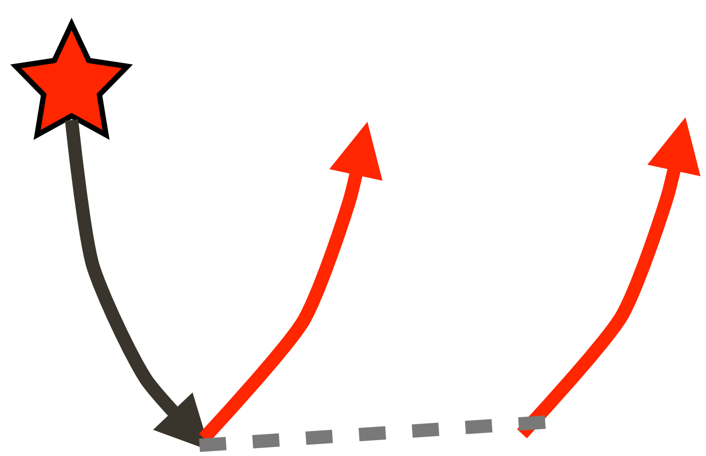
$A[M]$ focusing penalty

Wave-equation MVA

Linearization: $M = \text{diag}(\mathbf{m}) + E$

$$\min_{\mathbf{m},E} A[E] \quad \text{s.t.} \quad ||DF[\mathbf{m}]E + F[\mathbf{m}]Q - D||_F^2 \leq \sigma$$

extended born modelling allows *non-local* interaction



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Wave-equation MVA

Elimination of constraint leads to `conventional' MVA formulation:

$$\min_{\mathbf{m}} A[E]$$

where

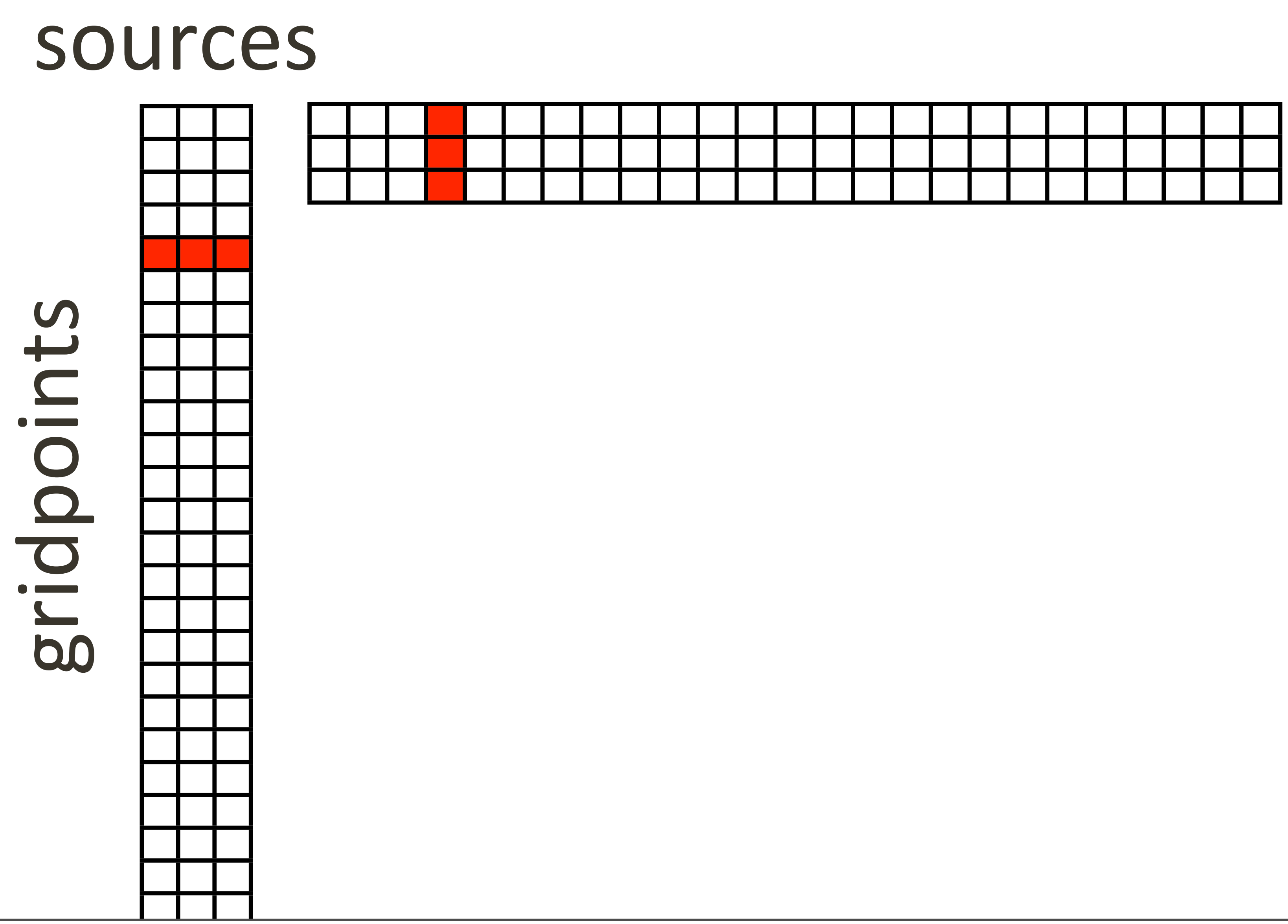
$$E[\mathbf{m}] = \sum_{\omega} \omega^2 V U^*$$

RTM image is given by $\mathrm{diag}(E)$

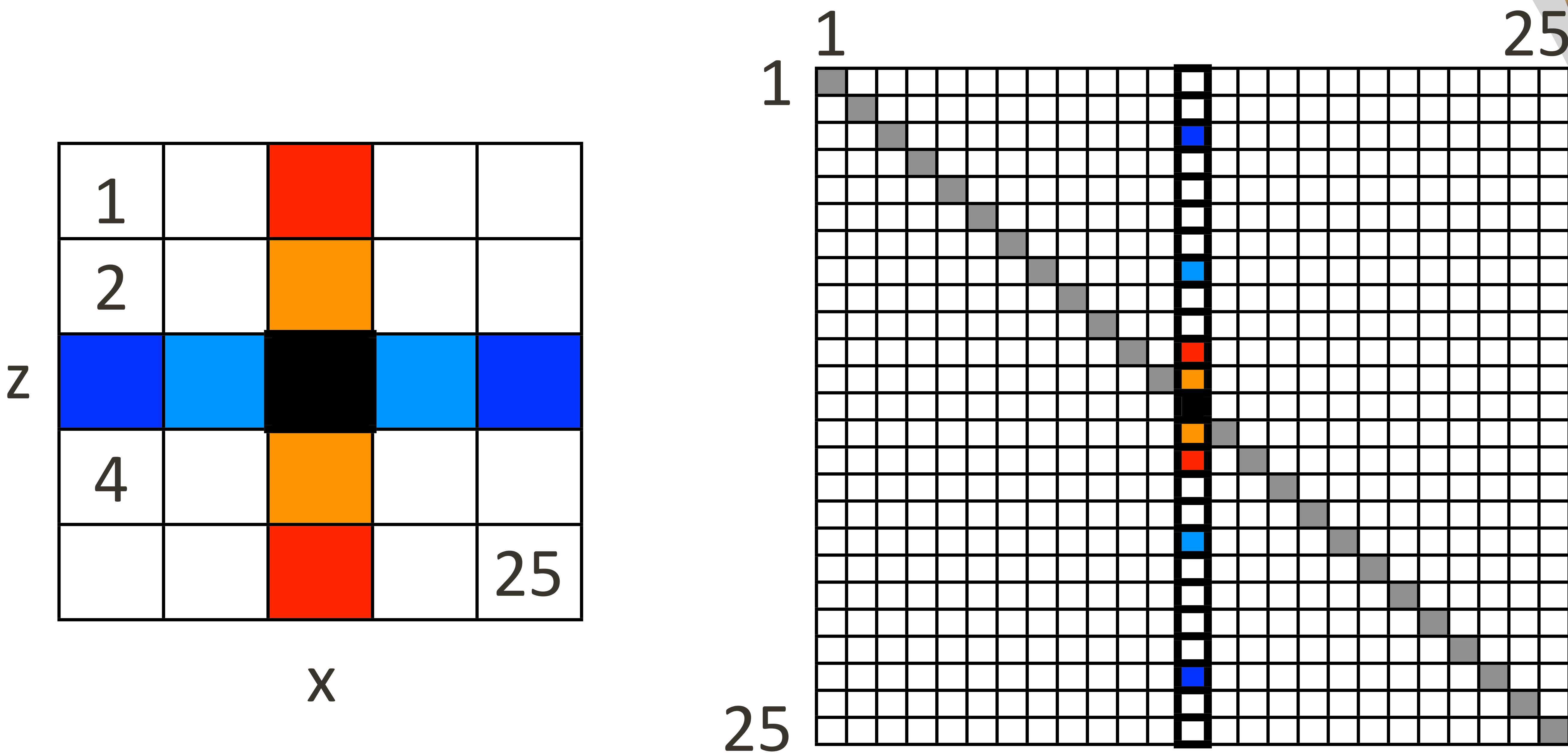
[Claerbout '74; Rickett '02 ;Shen '08]

Wave-equation MVA

$$E = \sum_{\omega} \omega^2 U V^* \qquad e_{i,j} = \sum_{\omega} \sum_{s} \omega^2 u_{i,s} v_{j,s}$$



Wave-equation MVA



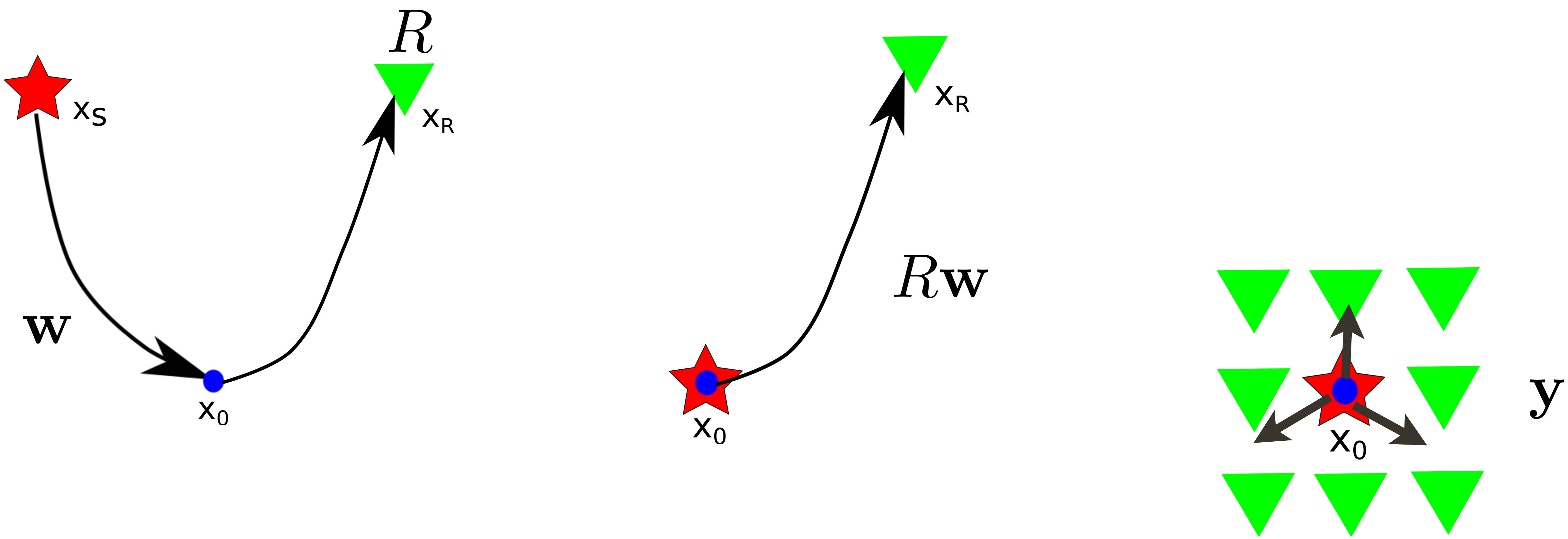
Probing the image volume

- ***prohibitively*** expensive to form *complete* image volume
- Cheap to calculate action on vector $\mathbf{y} = E\mathbf{x} = VU^*\mathbf{x}$

1. source wavefield $U = H[\mathbf{m}]^{-1}Q$
2. data residual $R = P^*(PU - D)$
3. adjoint source weights $\mathbf{w} = U^*\mathbf{x}$
4. Solve for *one* r.h.s. $\mathbf{y} = H[\mathbf{m}]^{-1}(R\mathbf{w})$

Probing the image volume

Interferometric interpretation: $\mathbf{x} = \delta_{ij}$



Greens function

Source redatuming

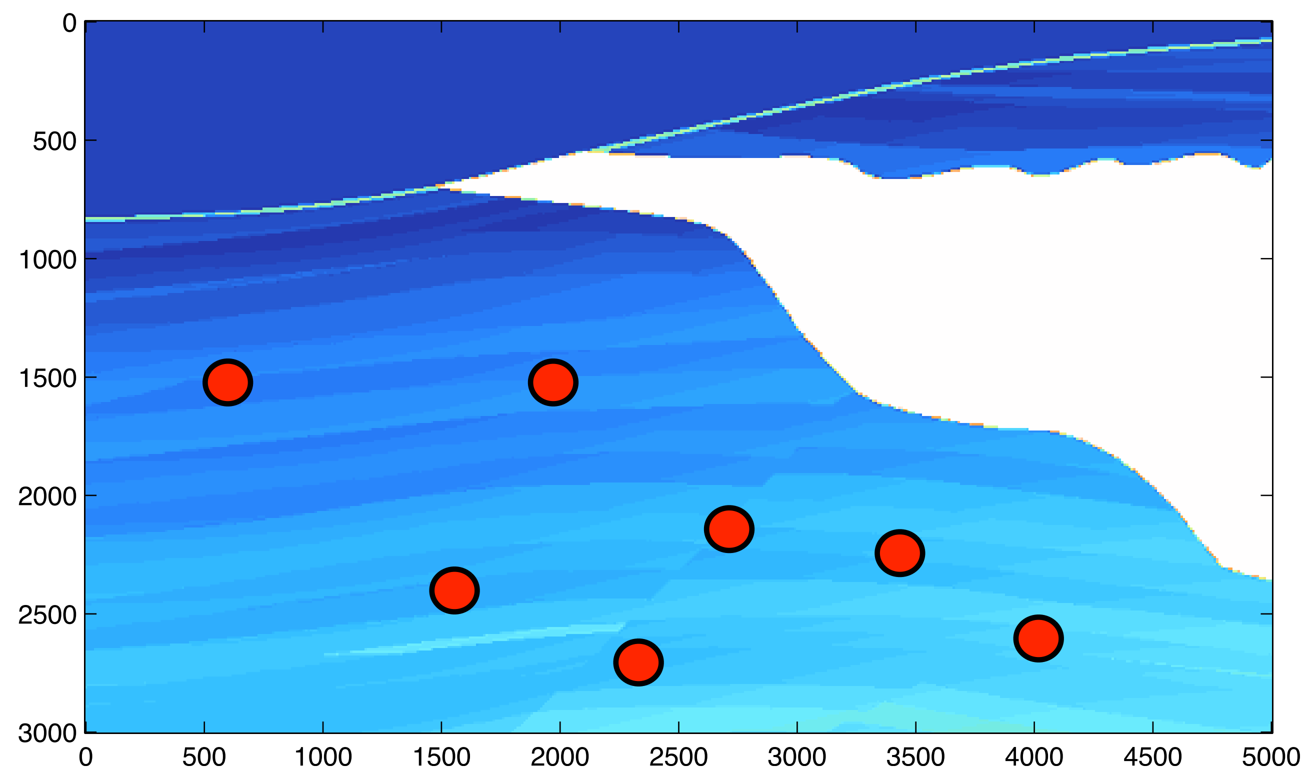
Receiver redatuming

[Vasconcelos ’10]

Probing the image volume

Calculate focusing penalty for *small* number of points

$$\min_{\mathbf{m}} \sum_{i \in I} A[E[\mathbf{m}]\mathbf{y}_i]$$



Gradient calculation

The sensitivity $\mathbf{g} = \left(\frac{\partial E\mathbf{y}}{\partial \mathbf{m}}\right)^{*}\mathbf{r}$
is given by

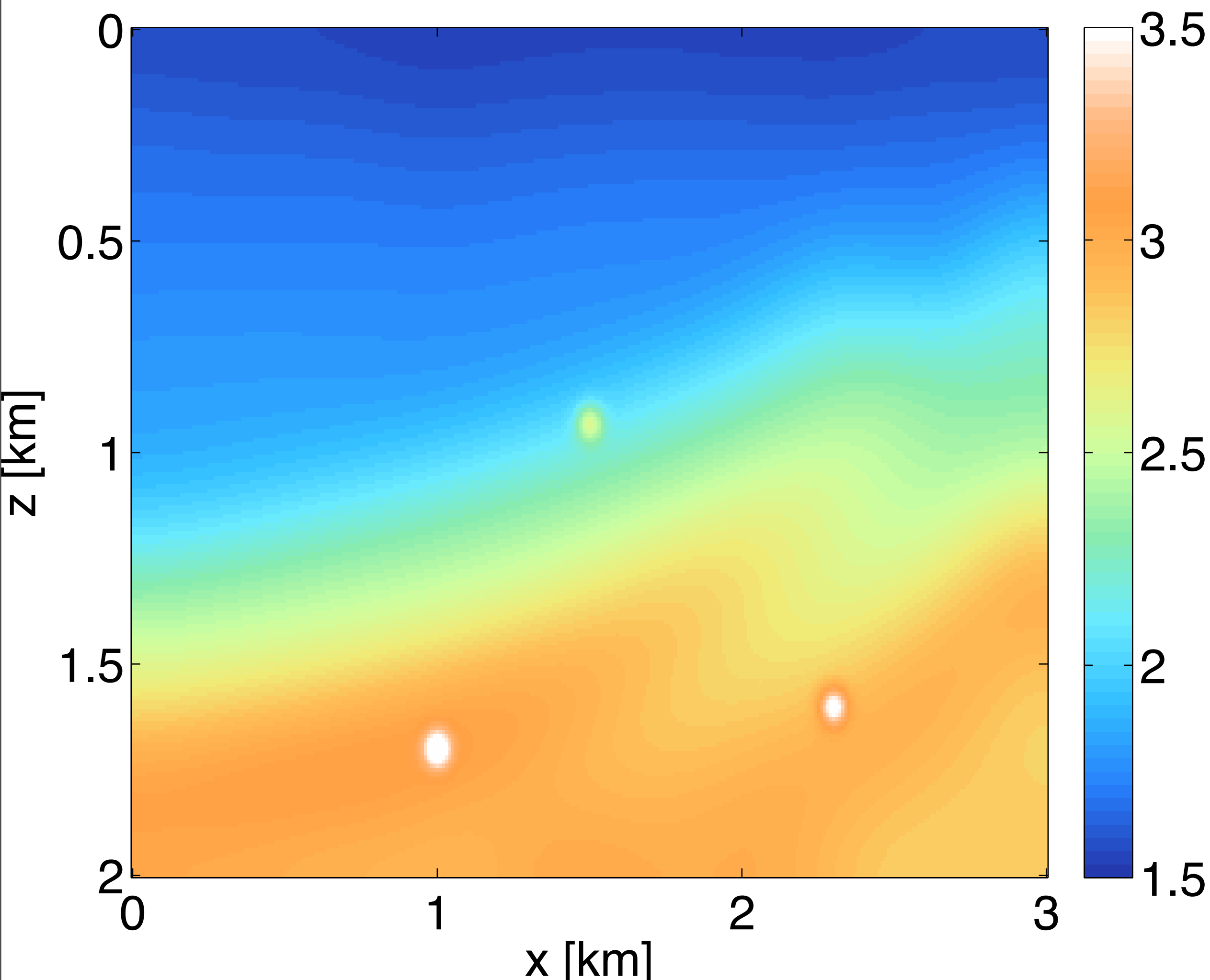
$$\mathrm{diag}\left(\left(H^{-1}\mathbf{r}\right)\left(V_0U_0^{*}\mathbf{y}\right)^{*}\right)+\mathrm{diag}\left(\left(H^{-*}\mathbf{y}\right)\left(U_0V_0^{*}\mathbf{r}\right)^{*}\right)$$

usual suspects

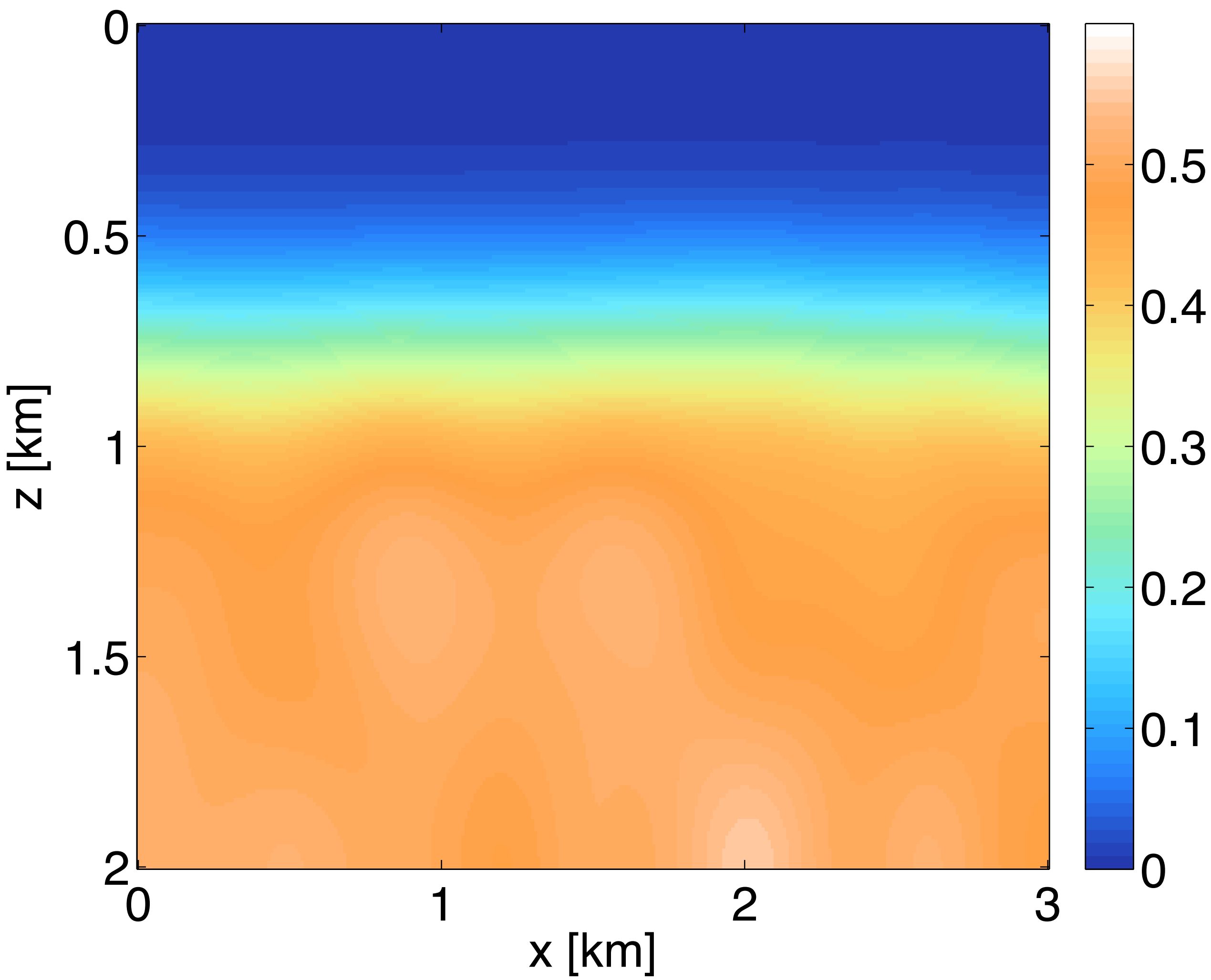
PDE solve for 1 r.h.s.

zero-lag correlation

Examples



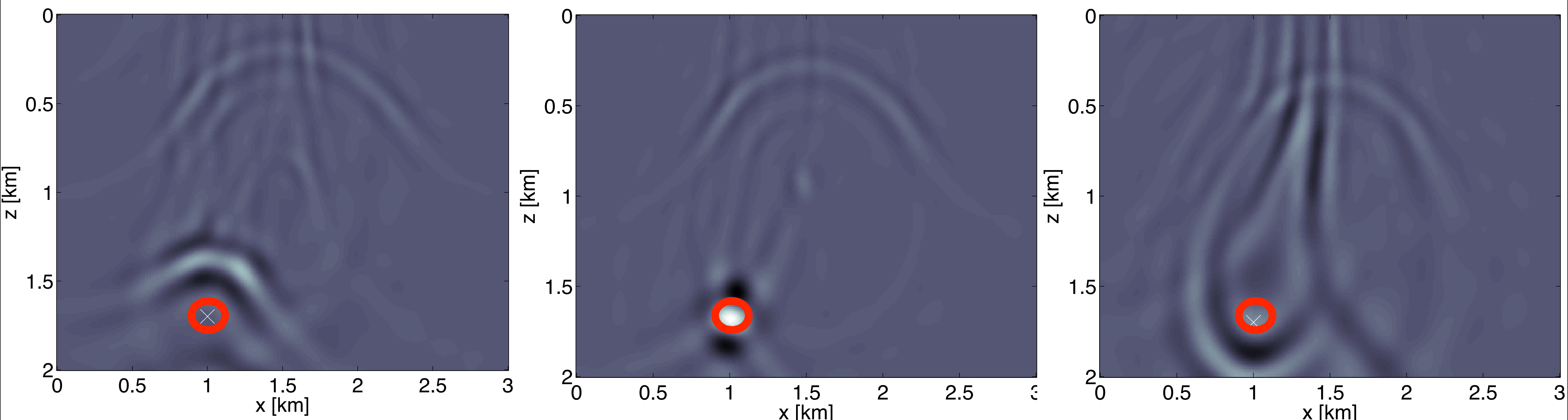
velocity [km/s]



perturbation [km/s]

Examples

all sequential sources

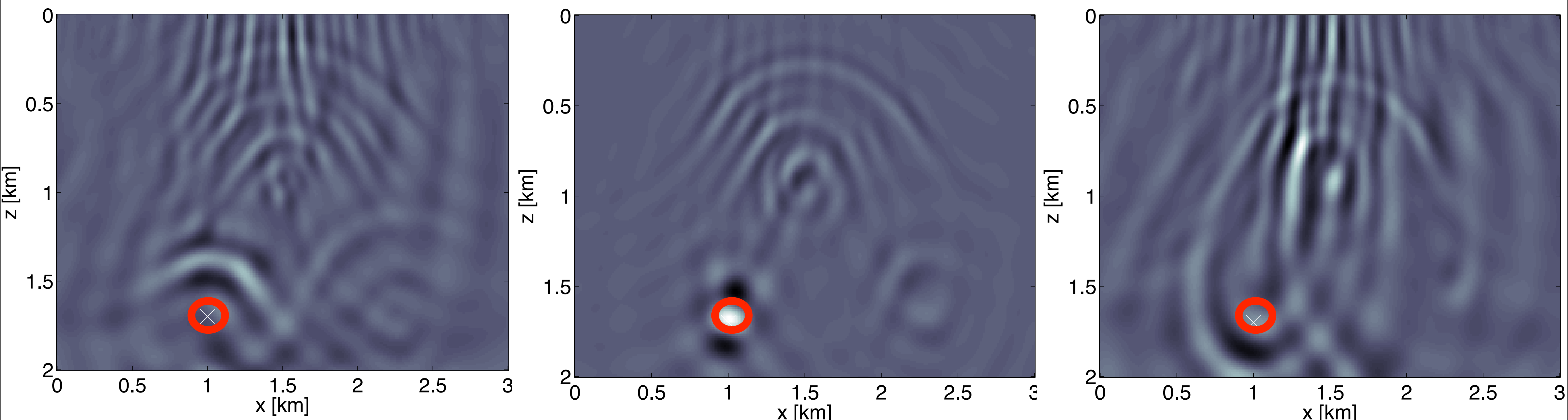


low

high

Examples

10 simultaneous sources



low

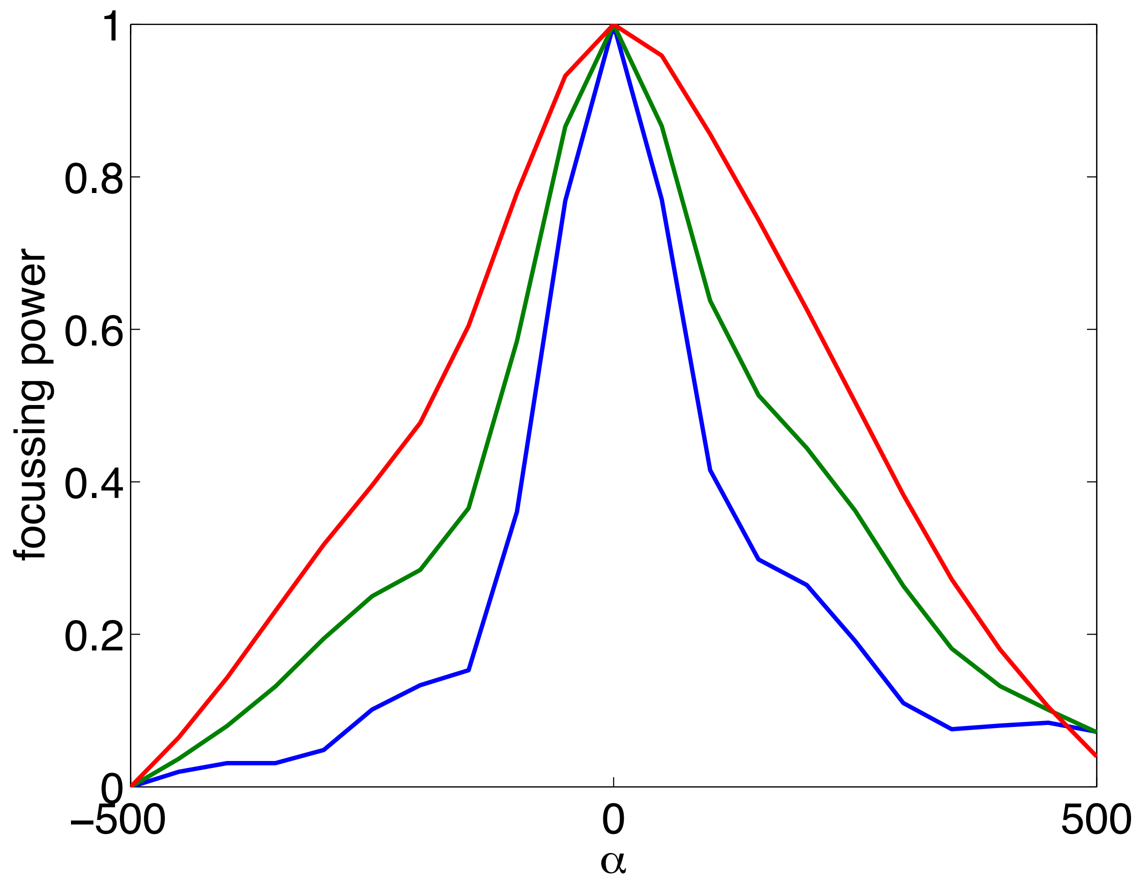
high

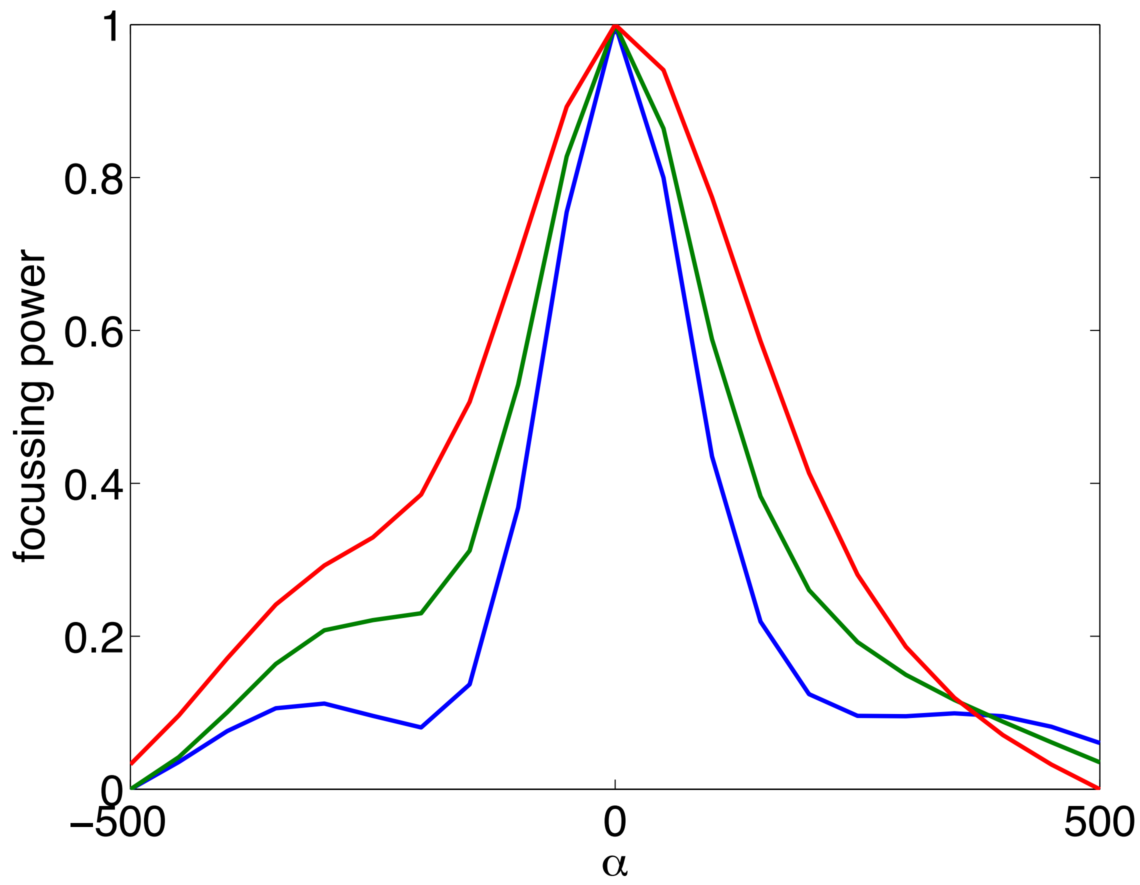
Focussing power

- maximize energy around focusing point by using Gaussian penalty
- width of Gaussian determines max. distance of interaction
- behaves like *stackpower* for small width

Examples

focussing power for small, medium and large width

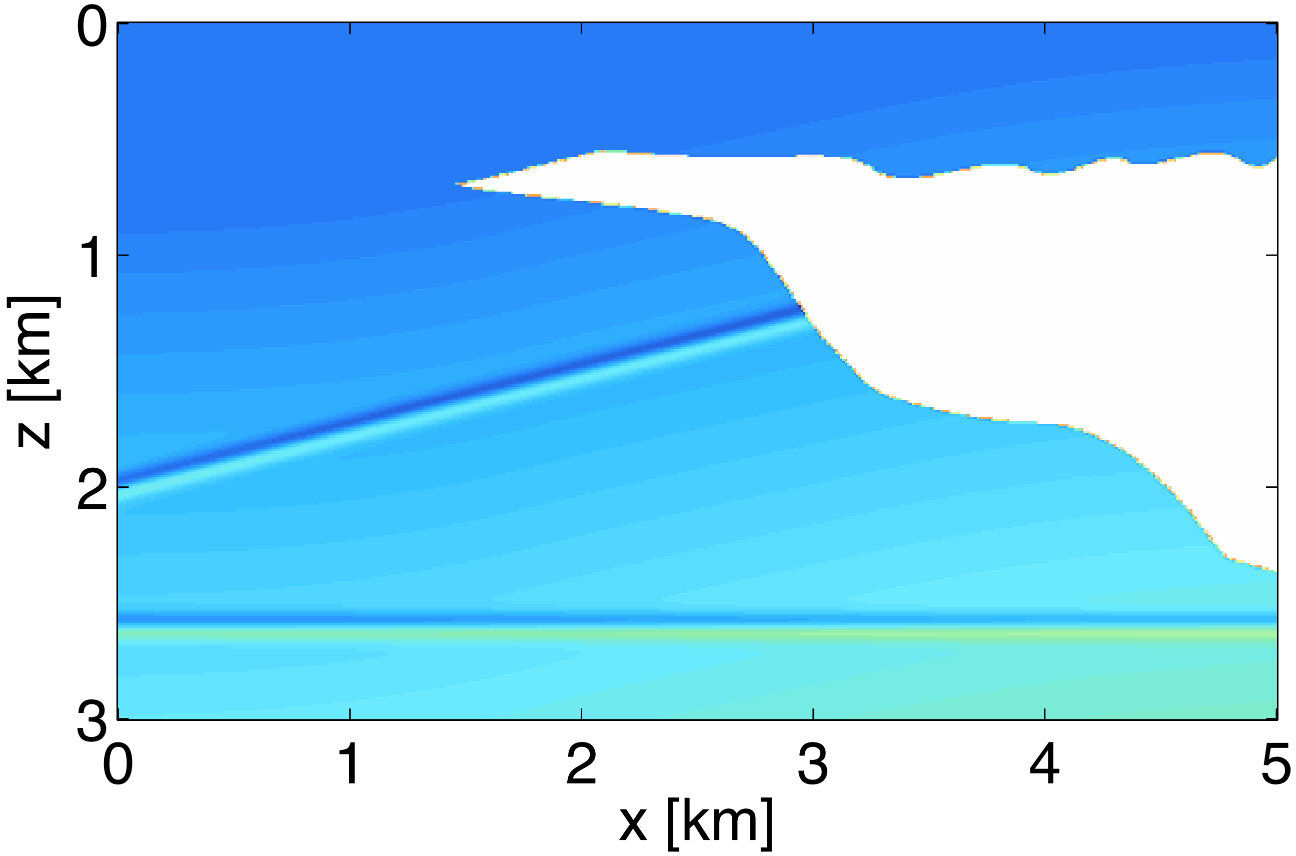


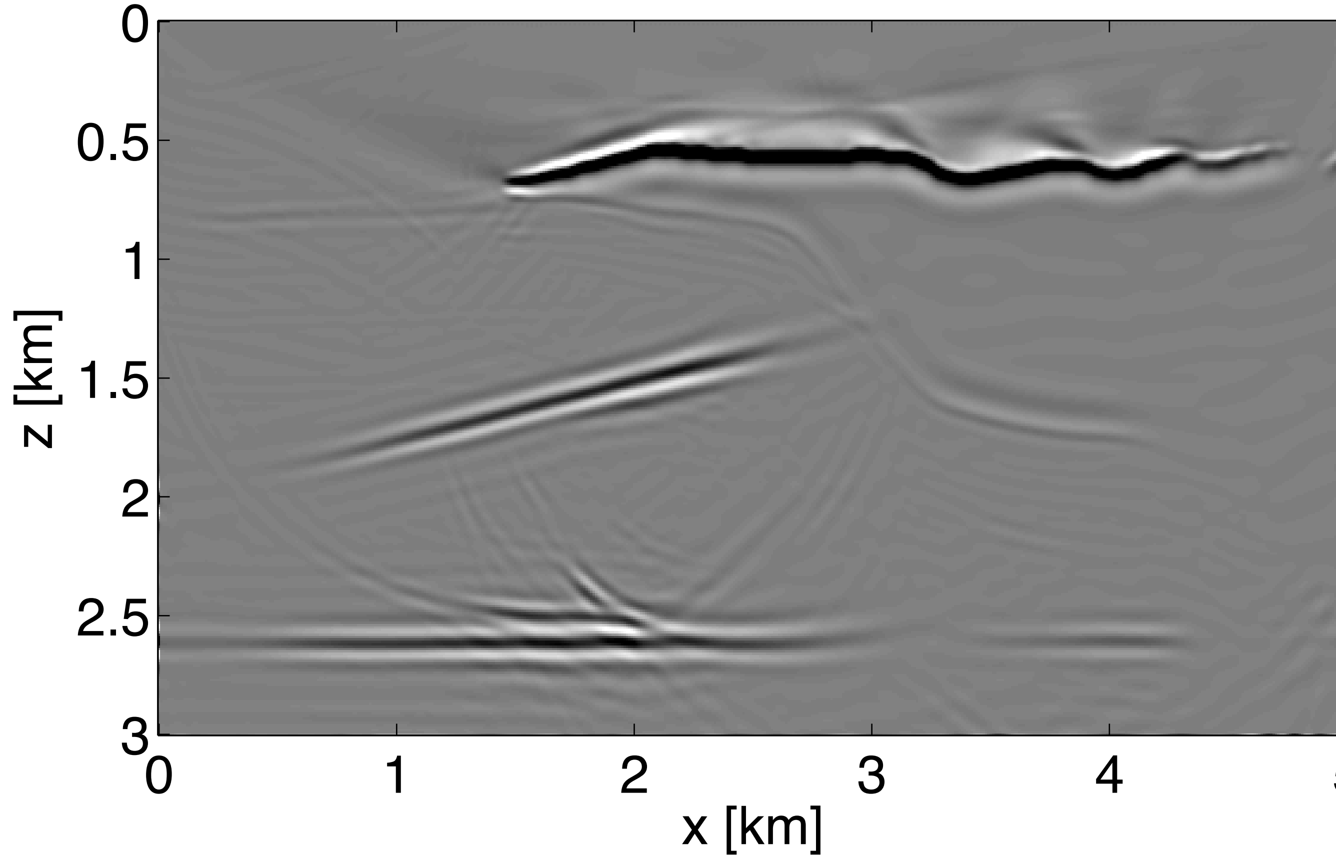


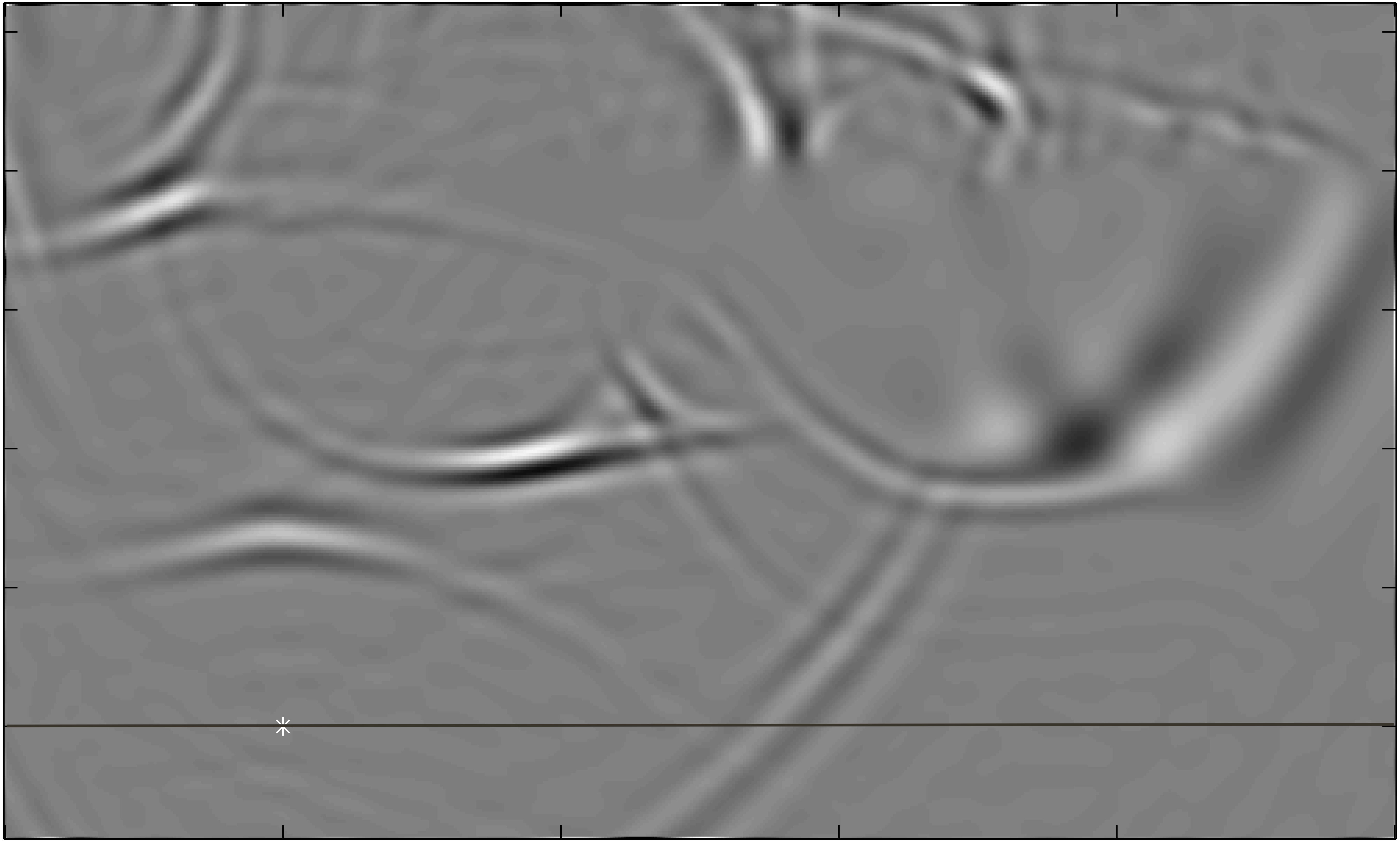
sequential sources

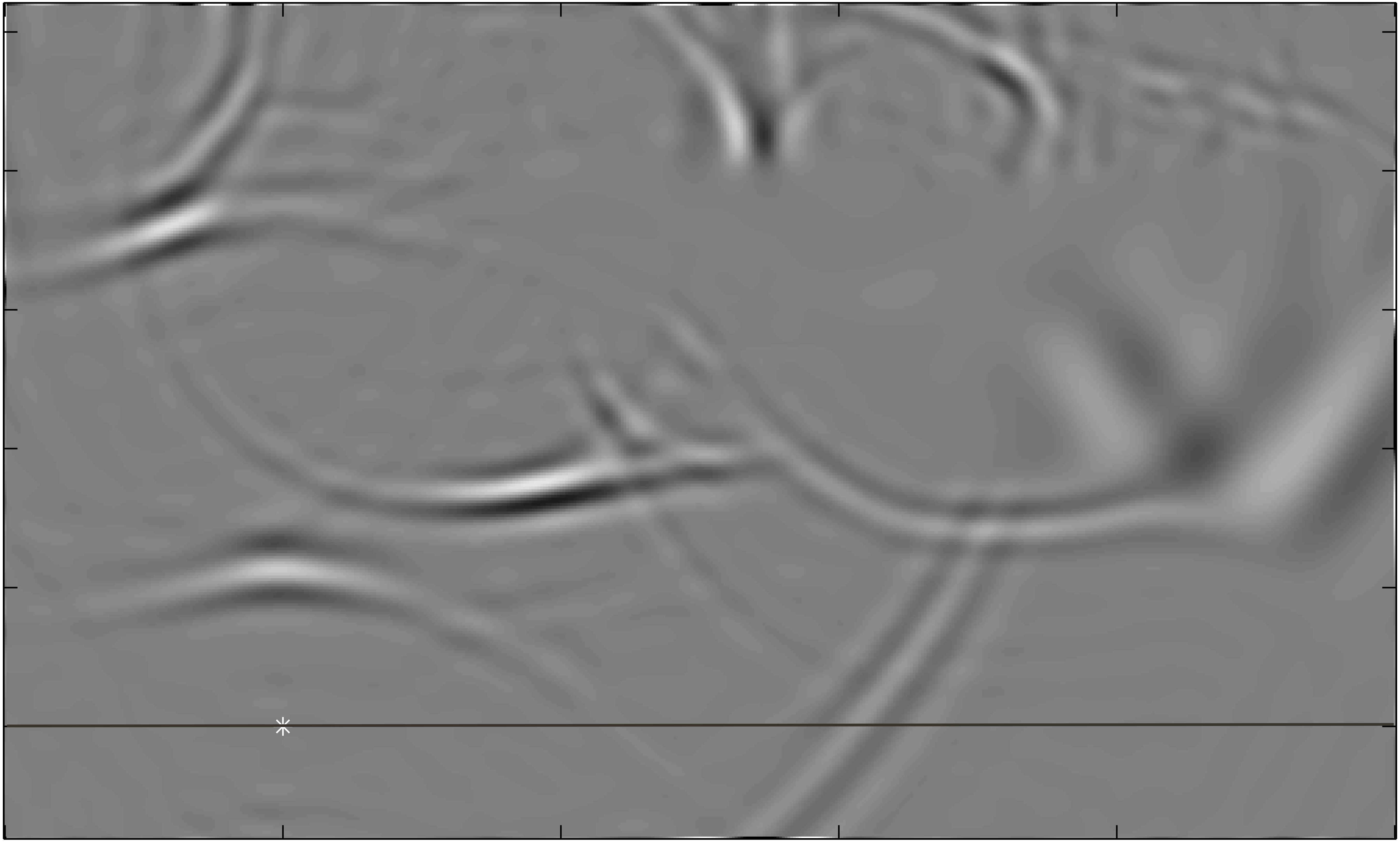
10 simultaneous sources

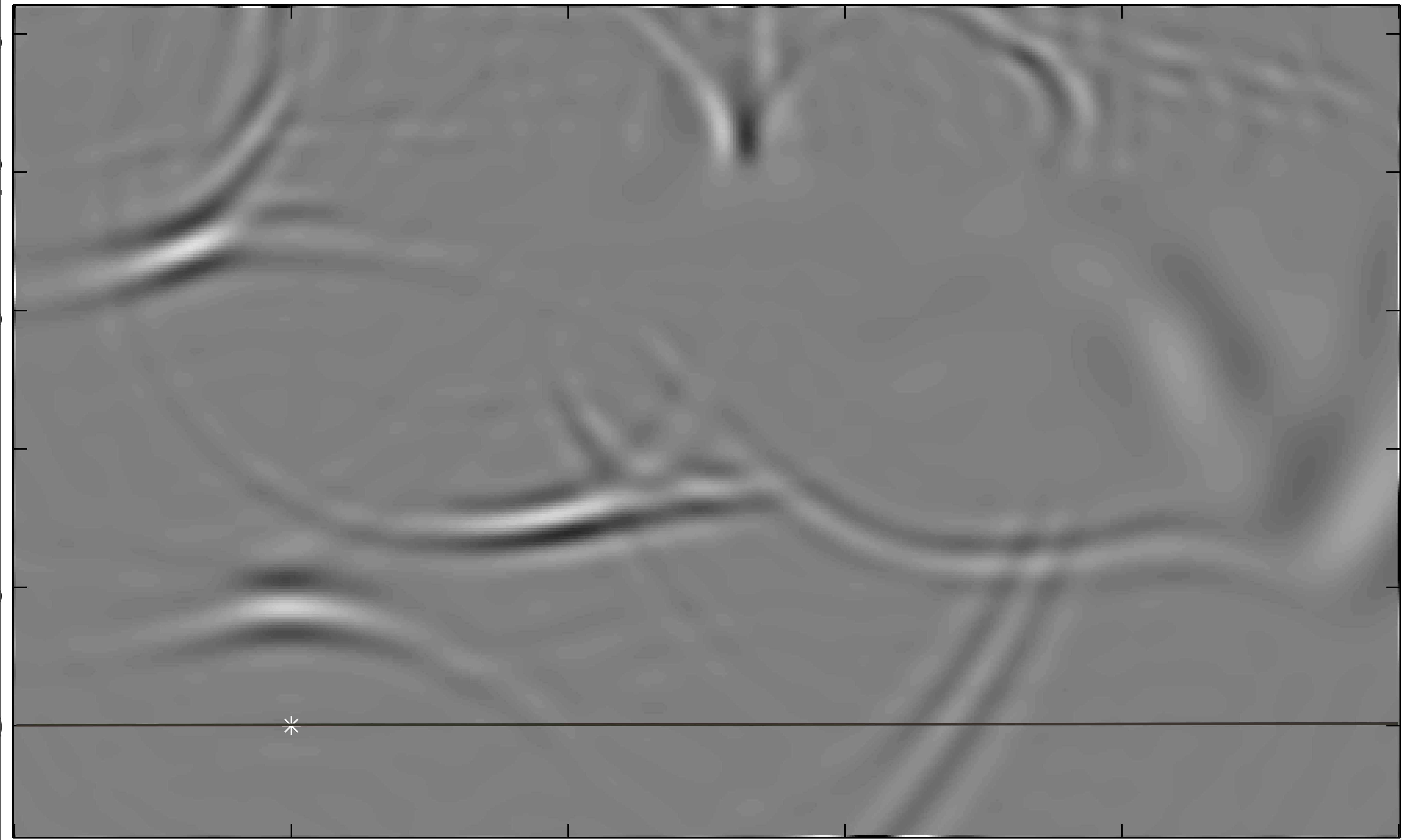
Examples

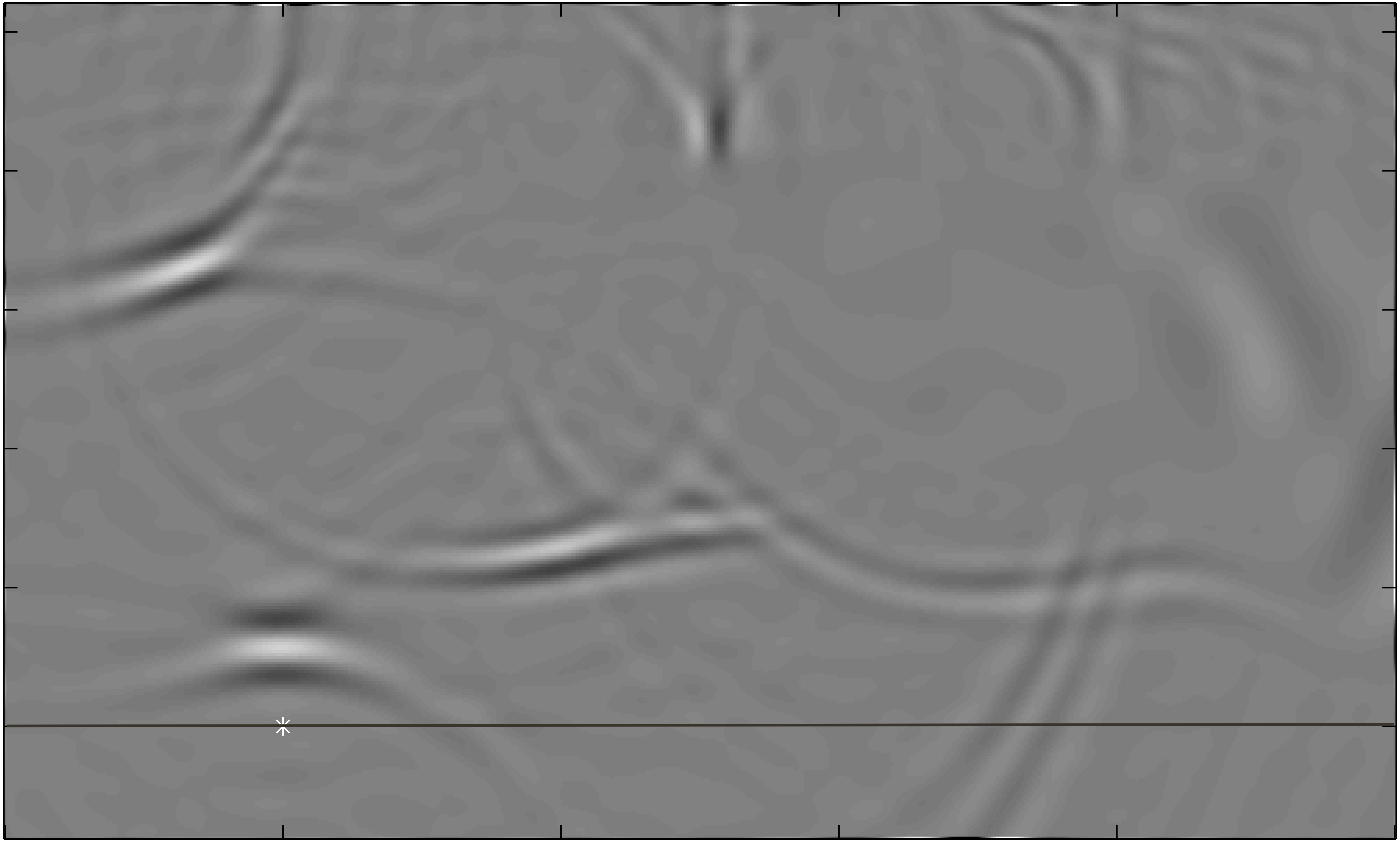


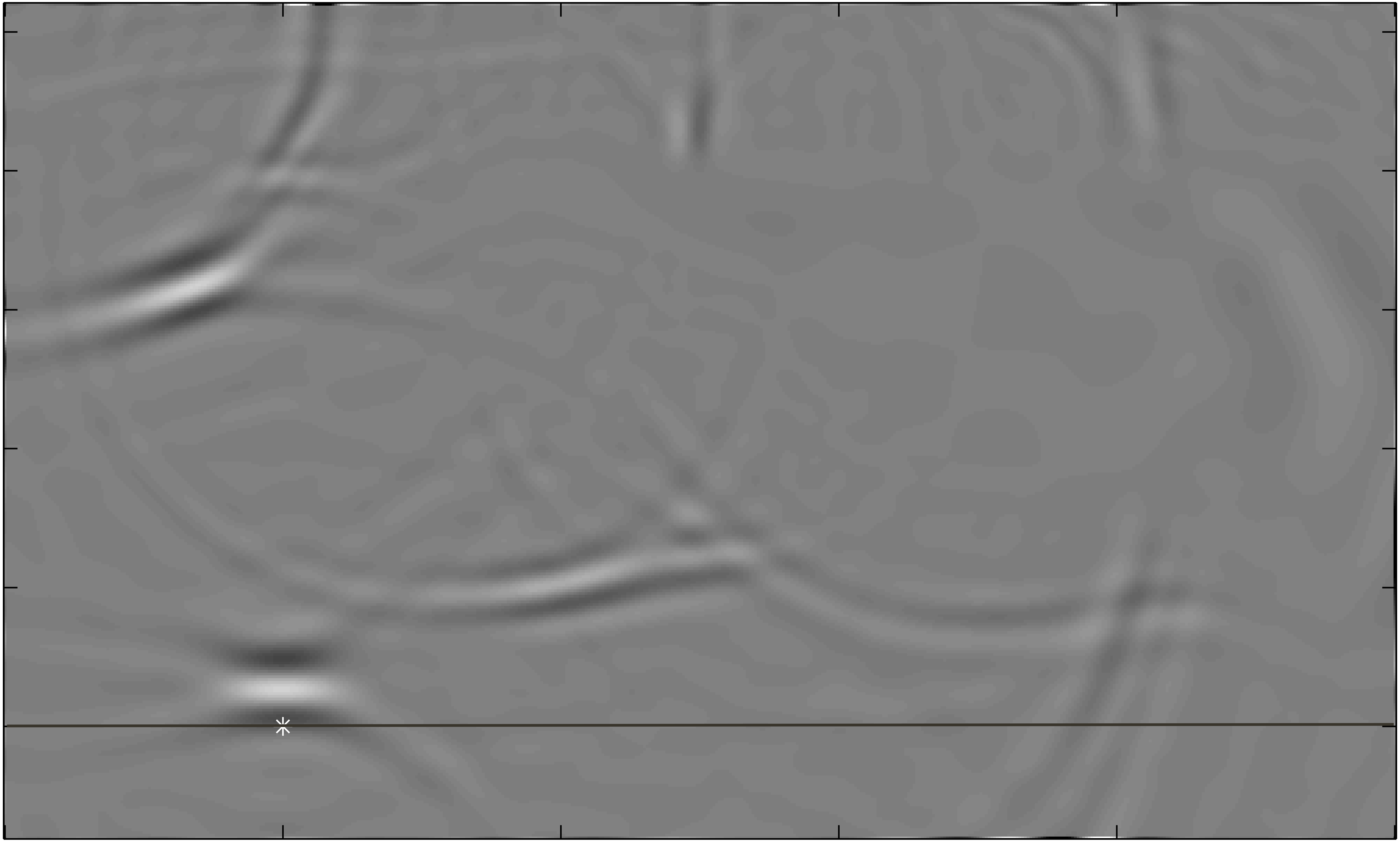


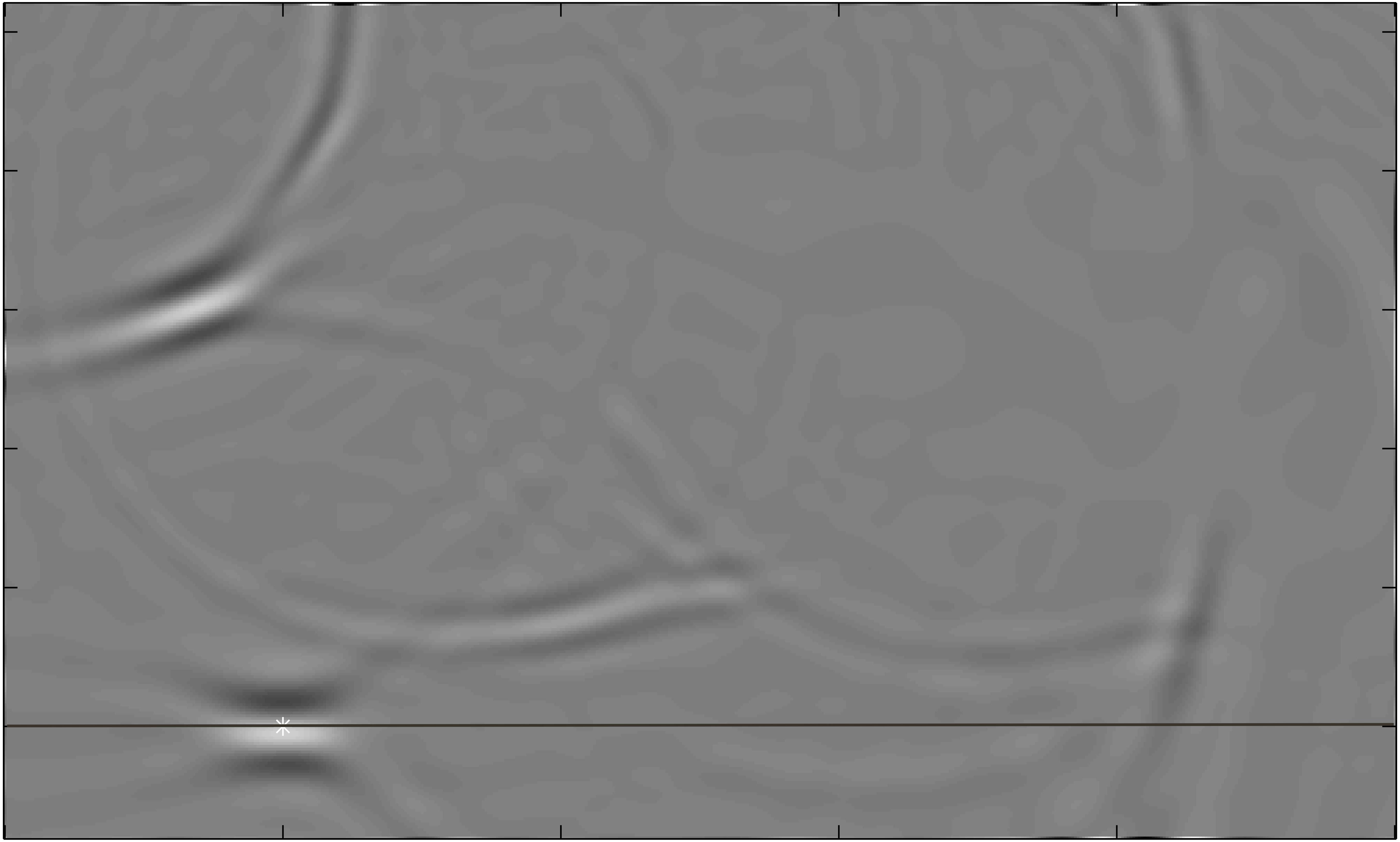


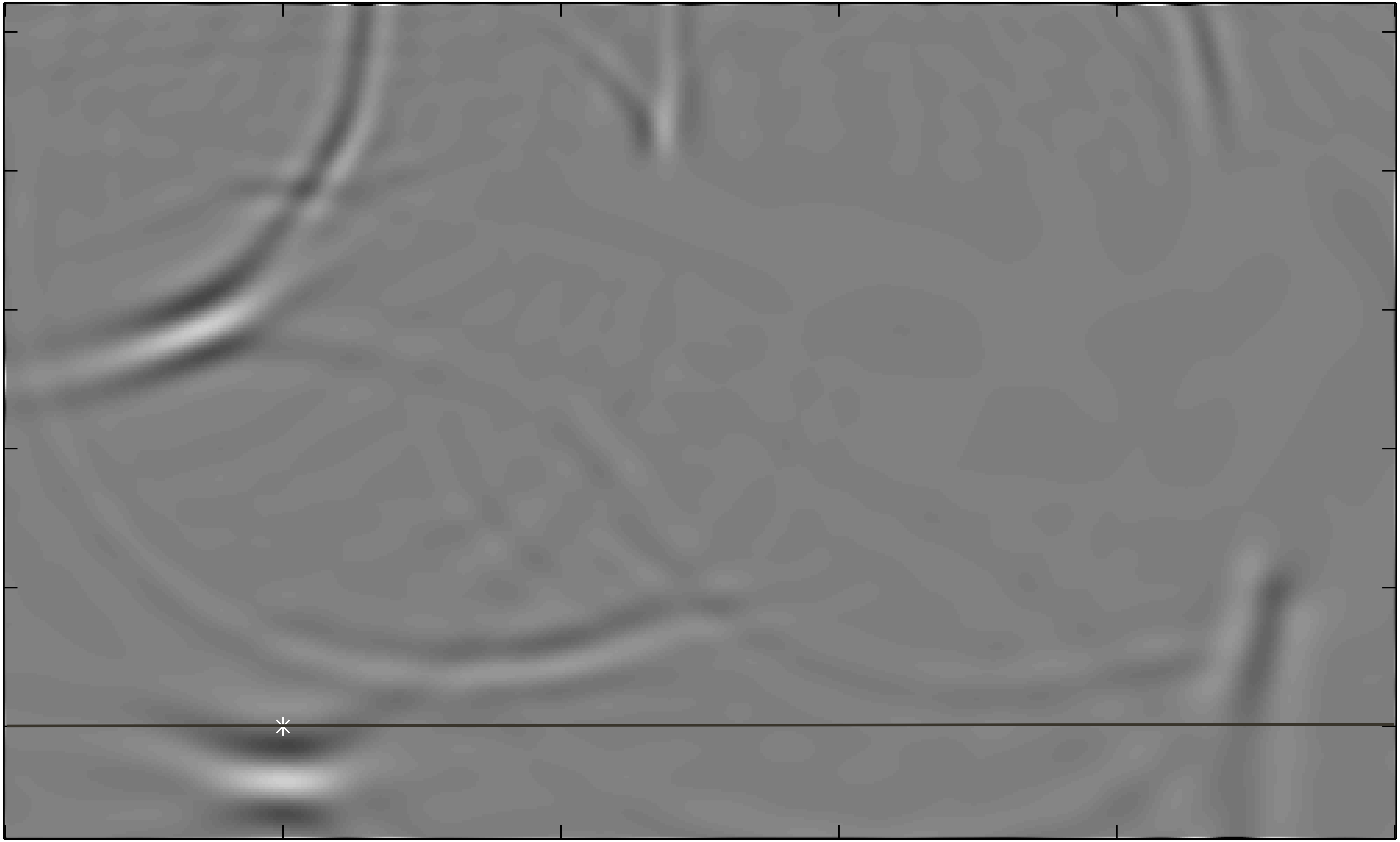


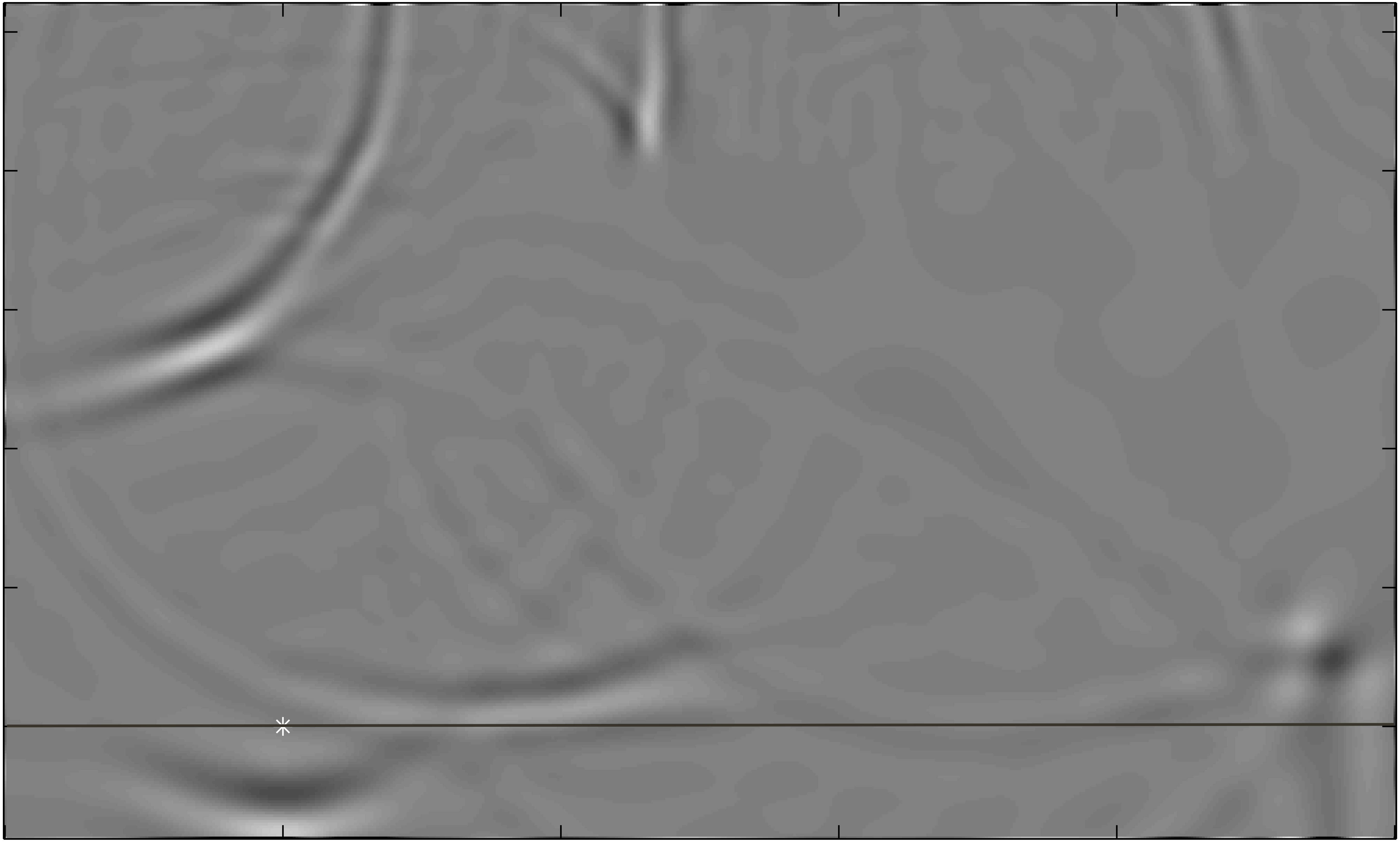


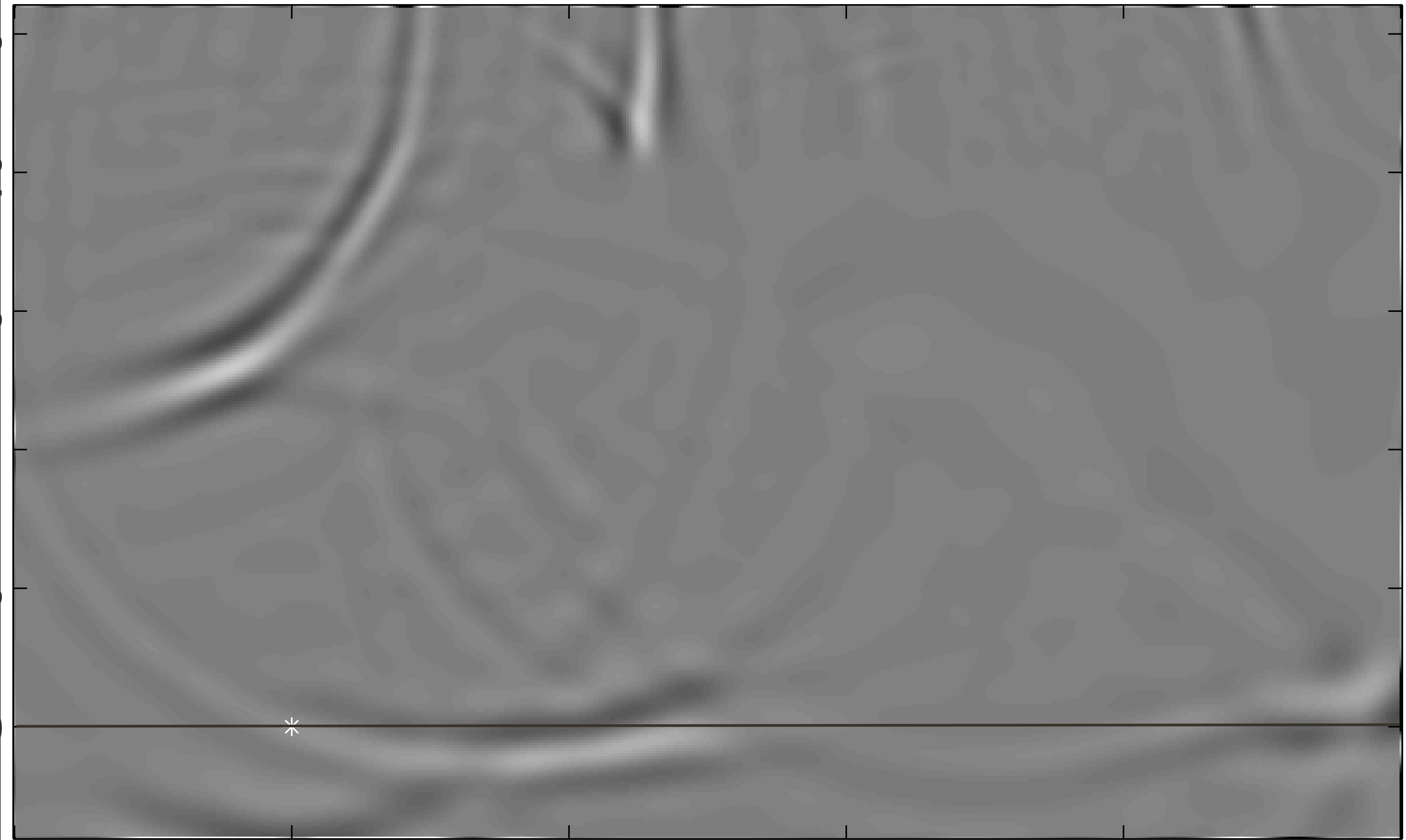


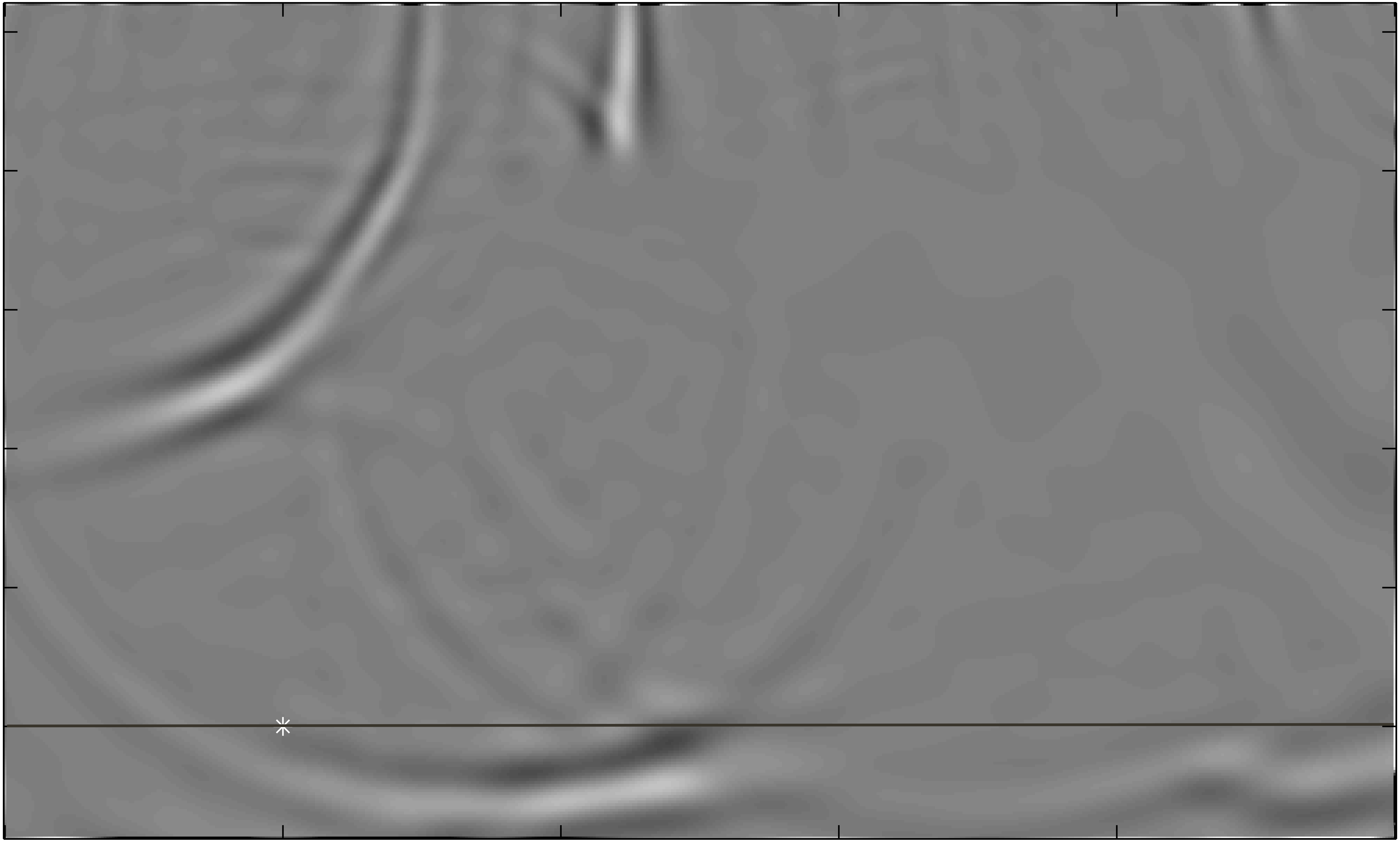


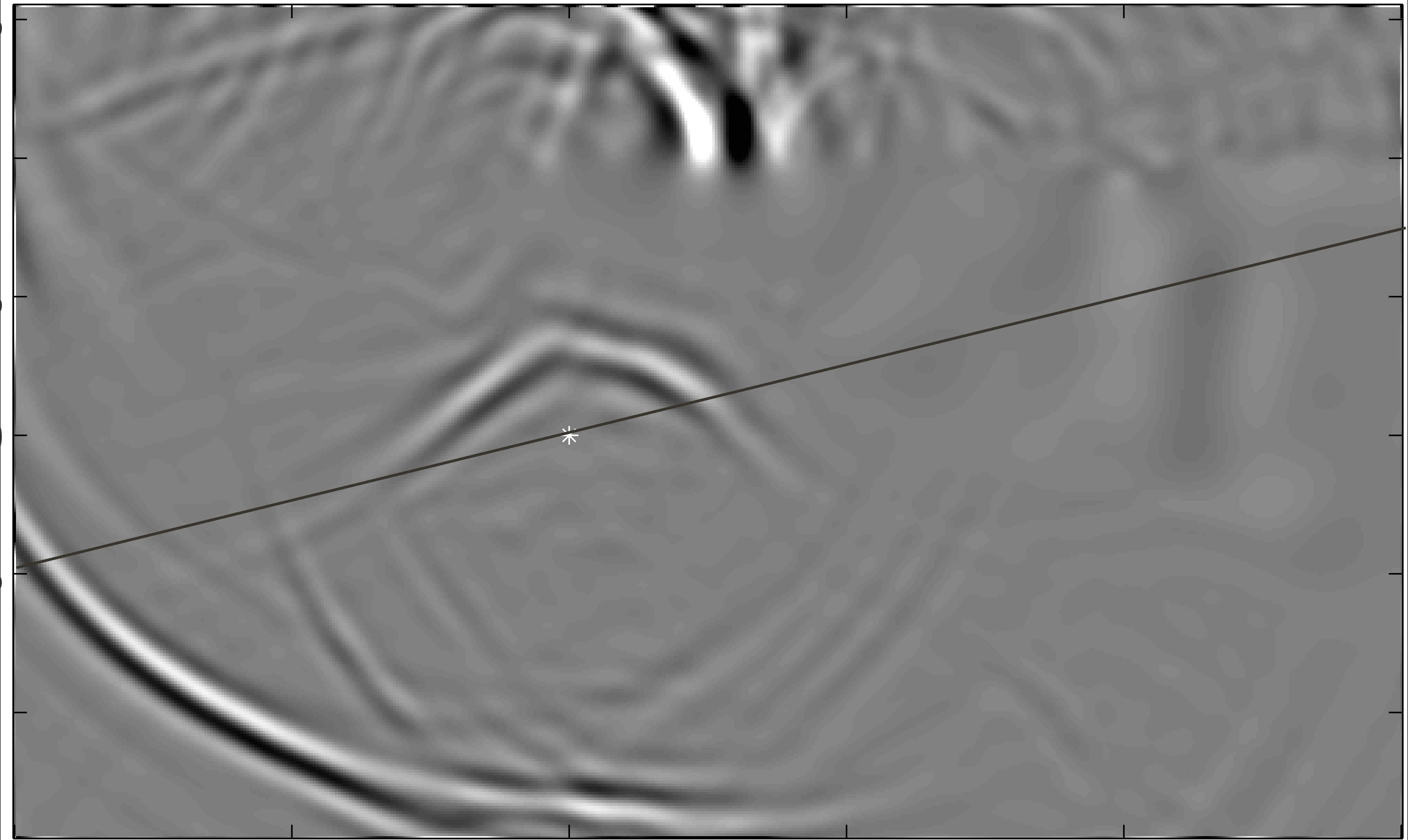


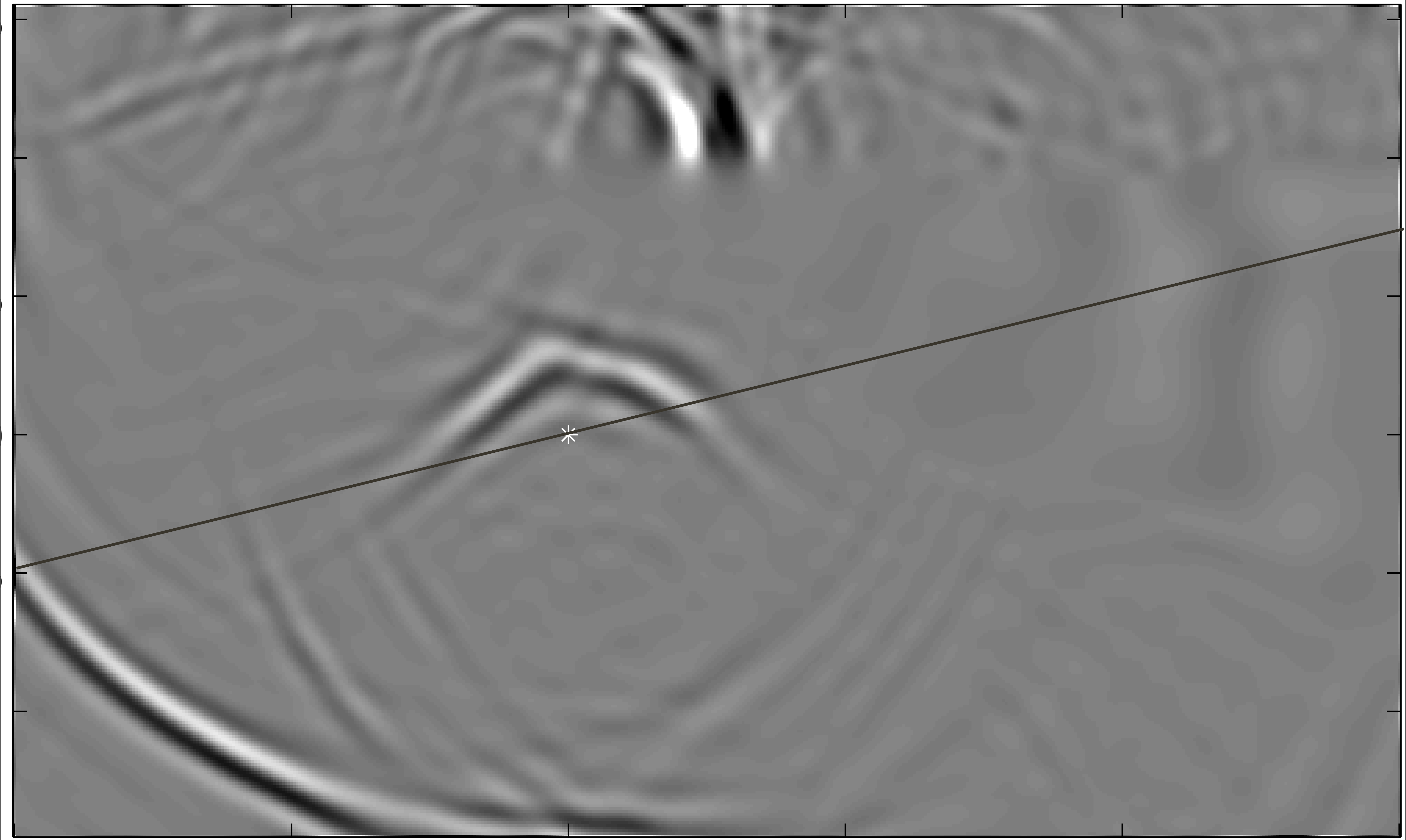


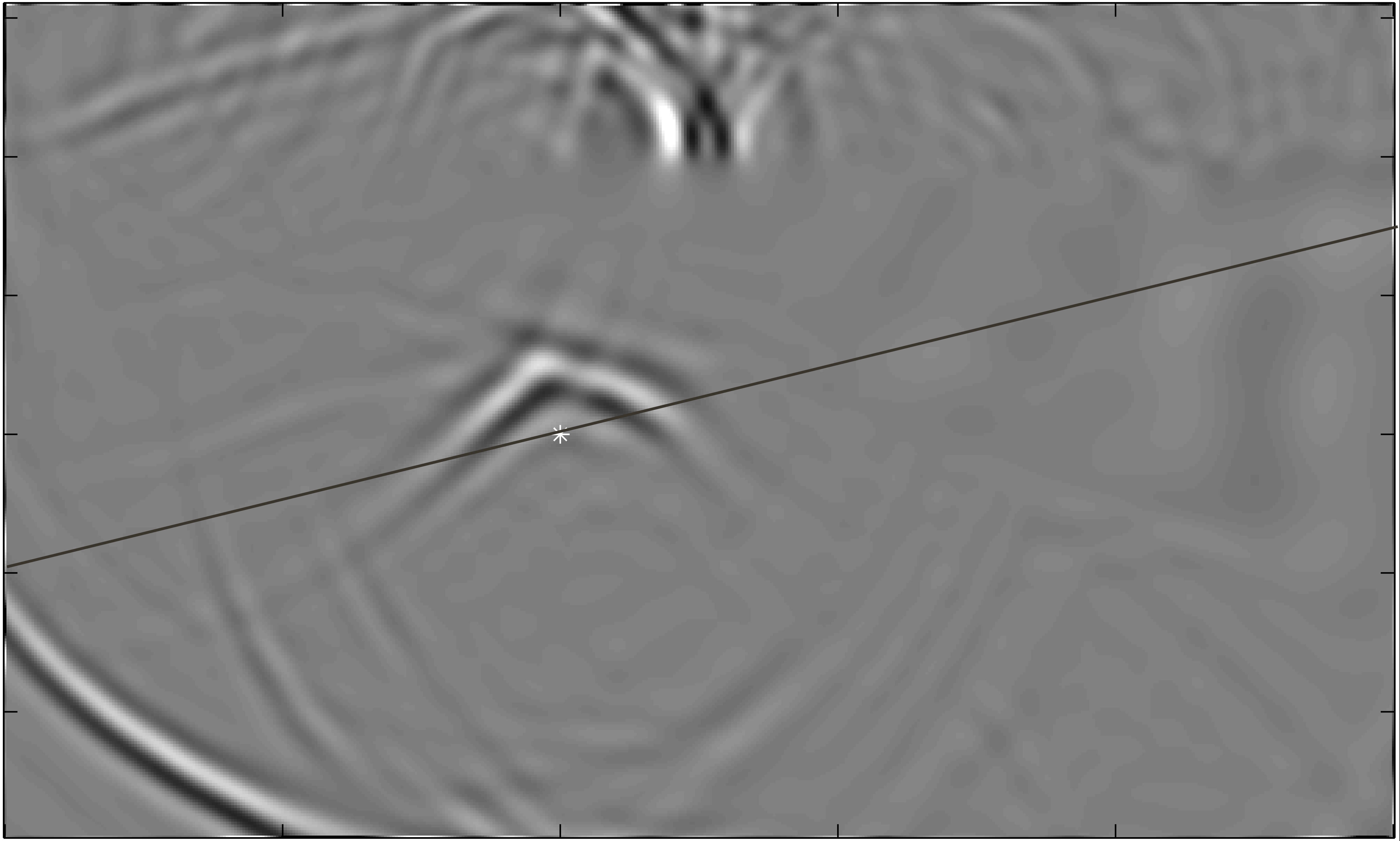


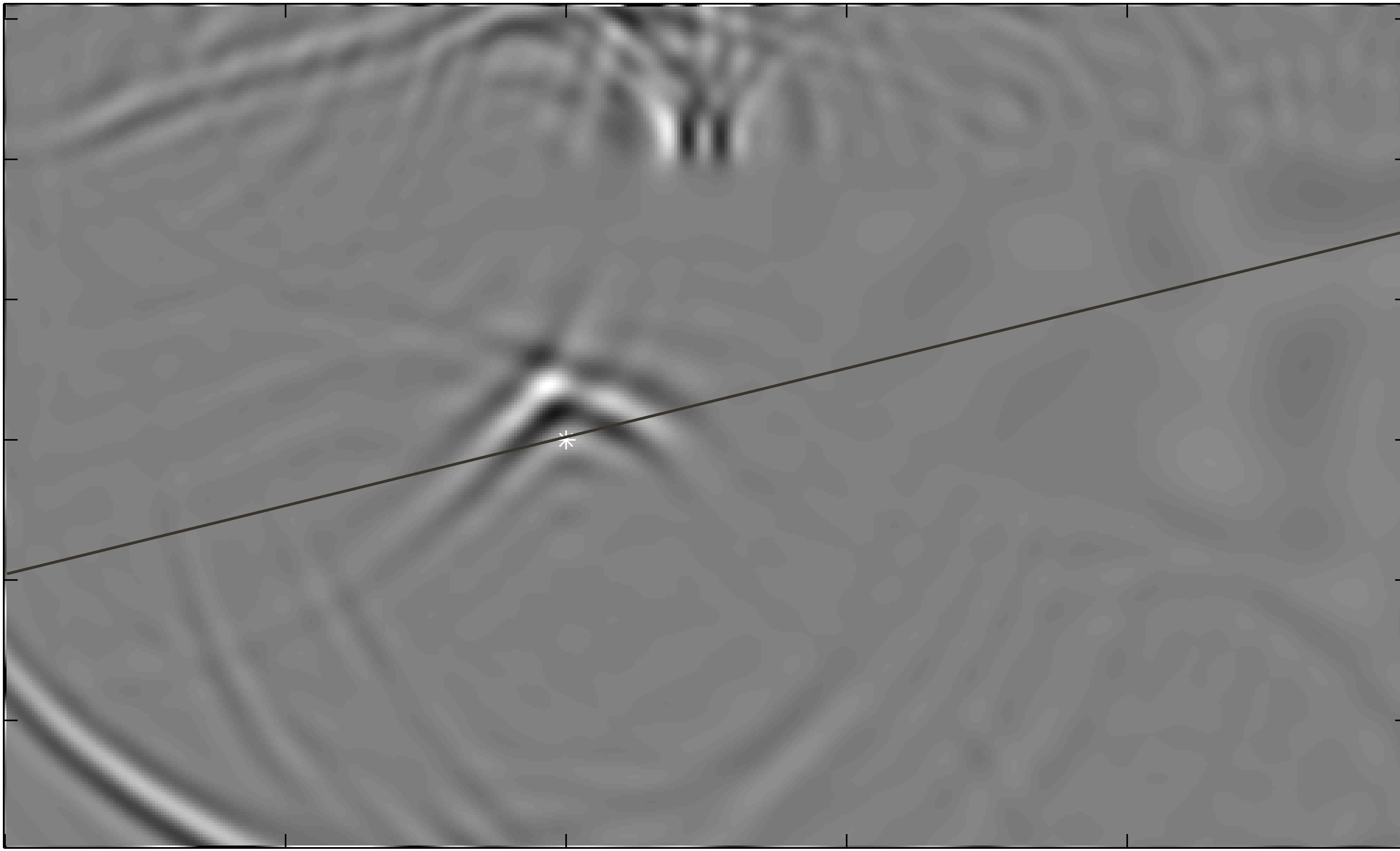


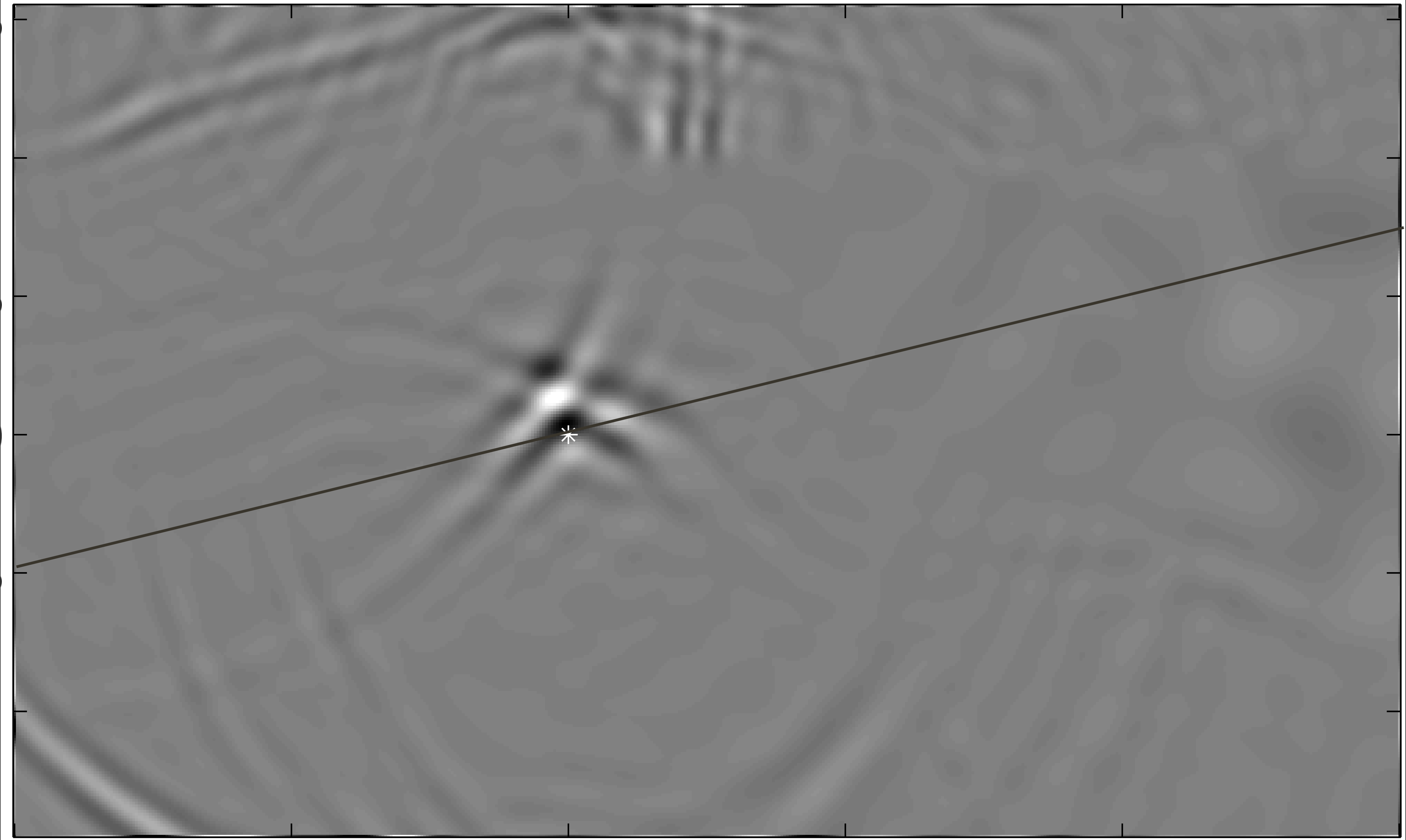


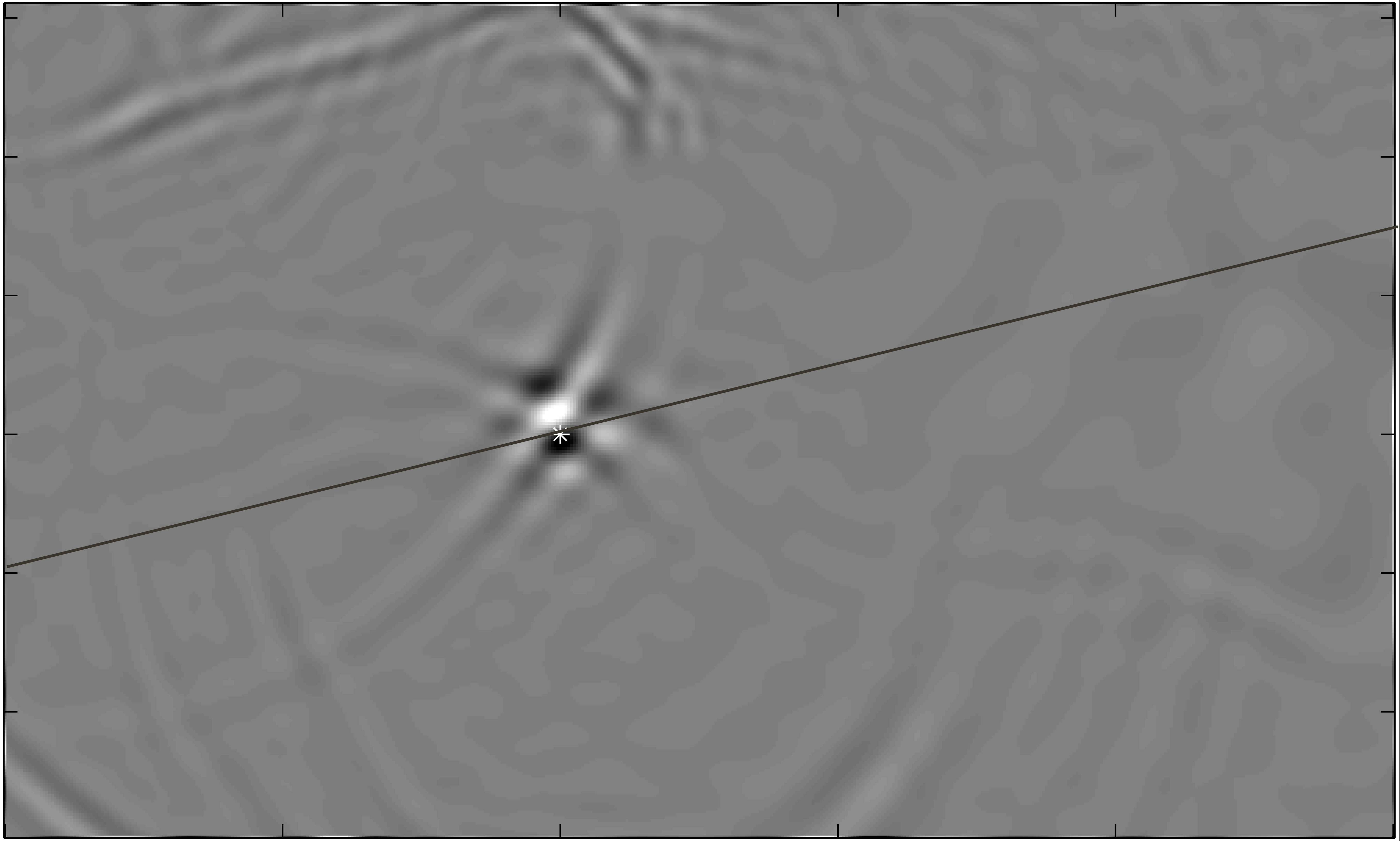


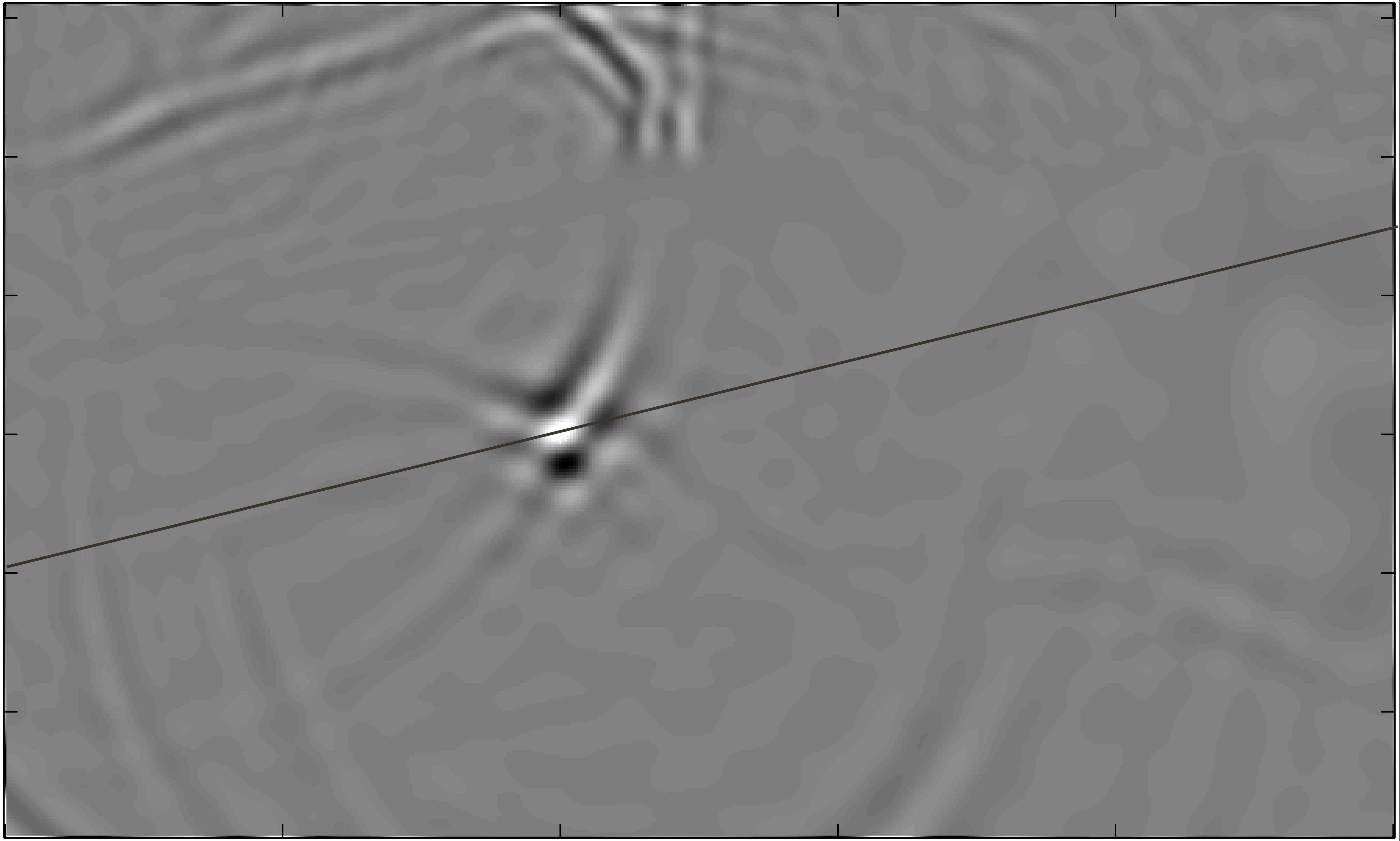


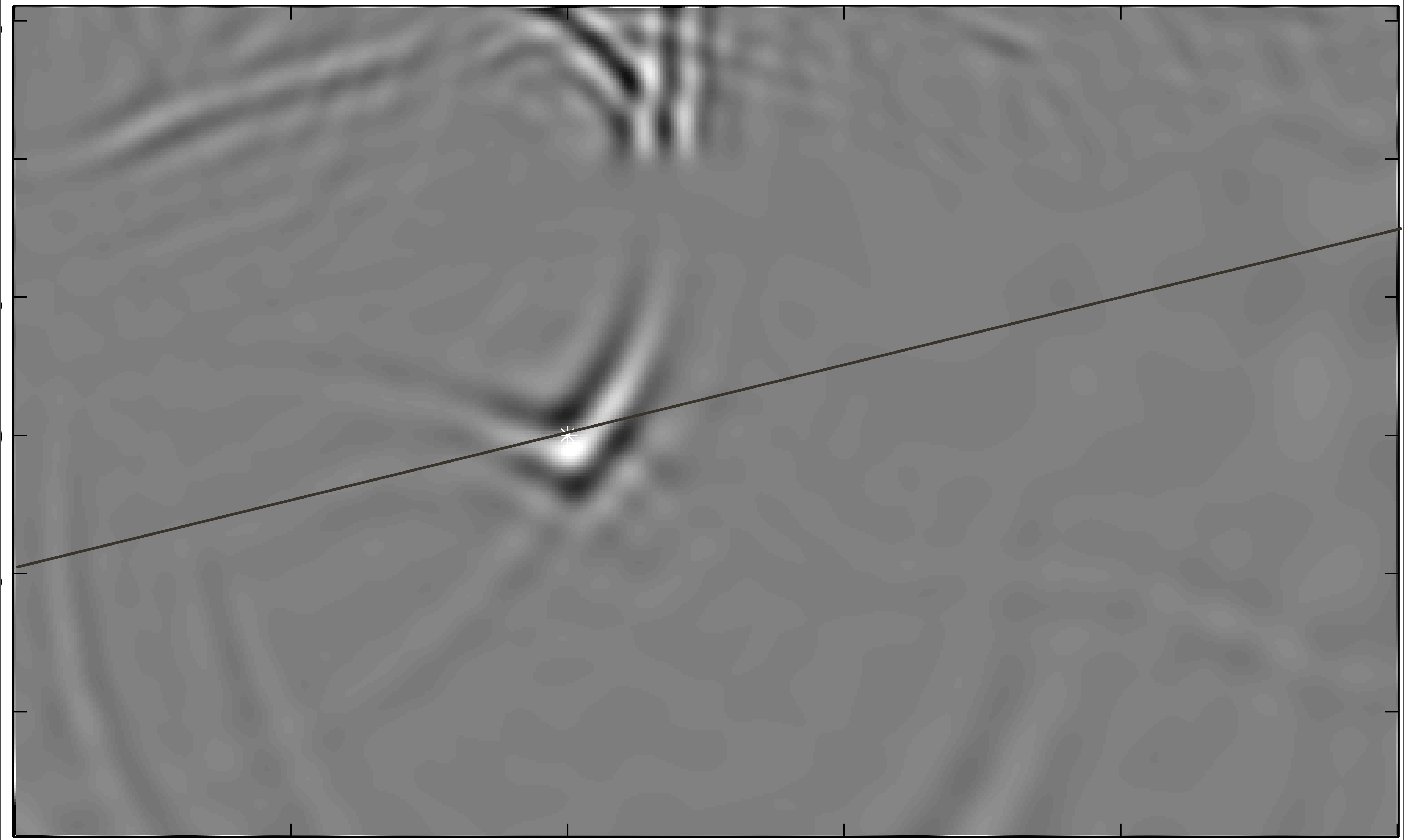


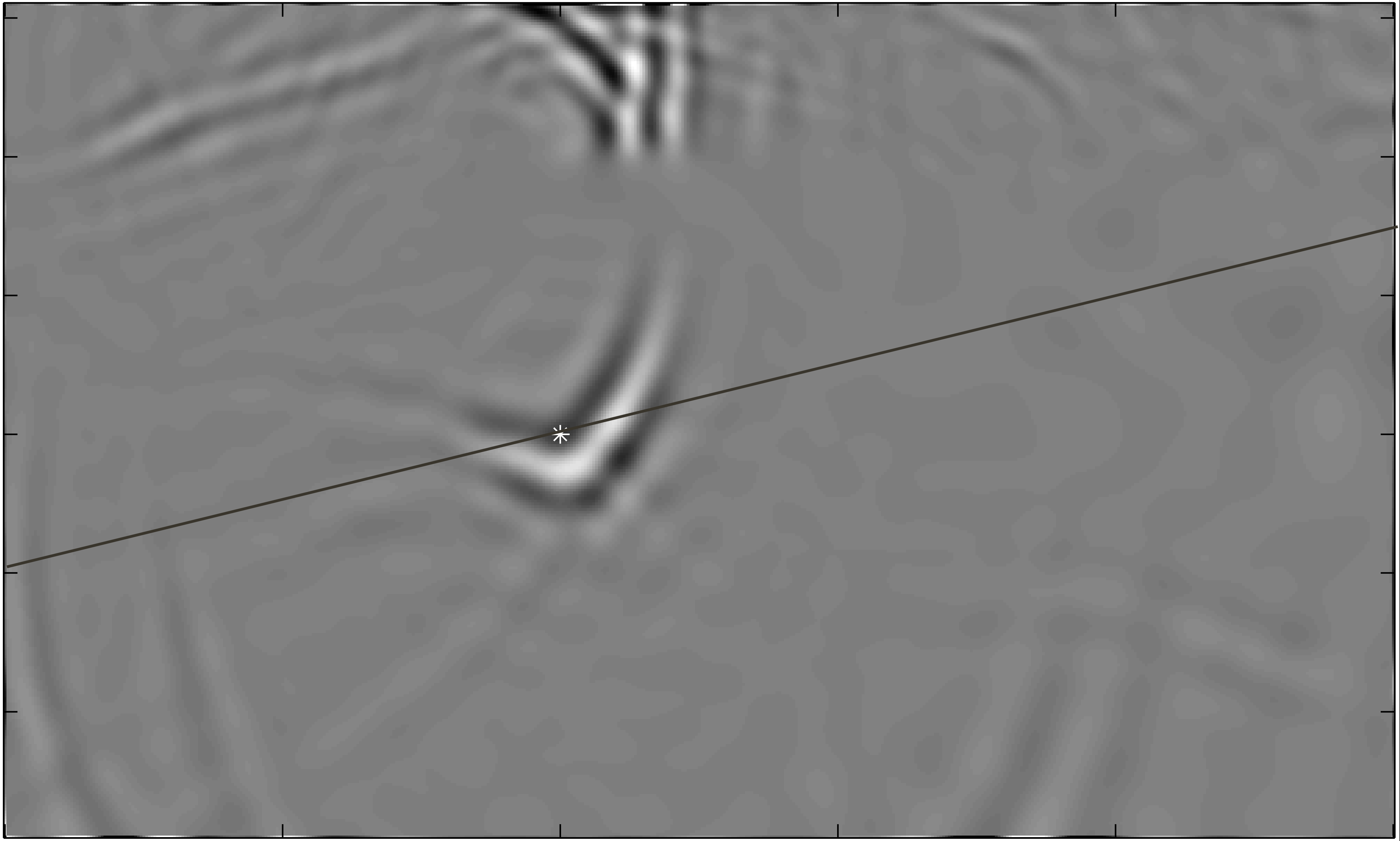


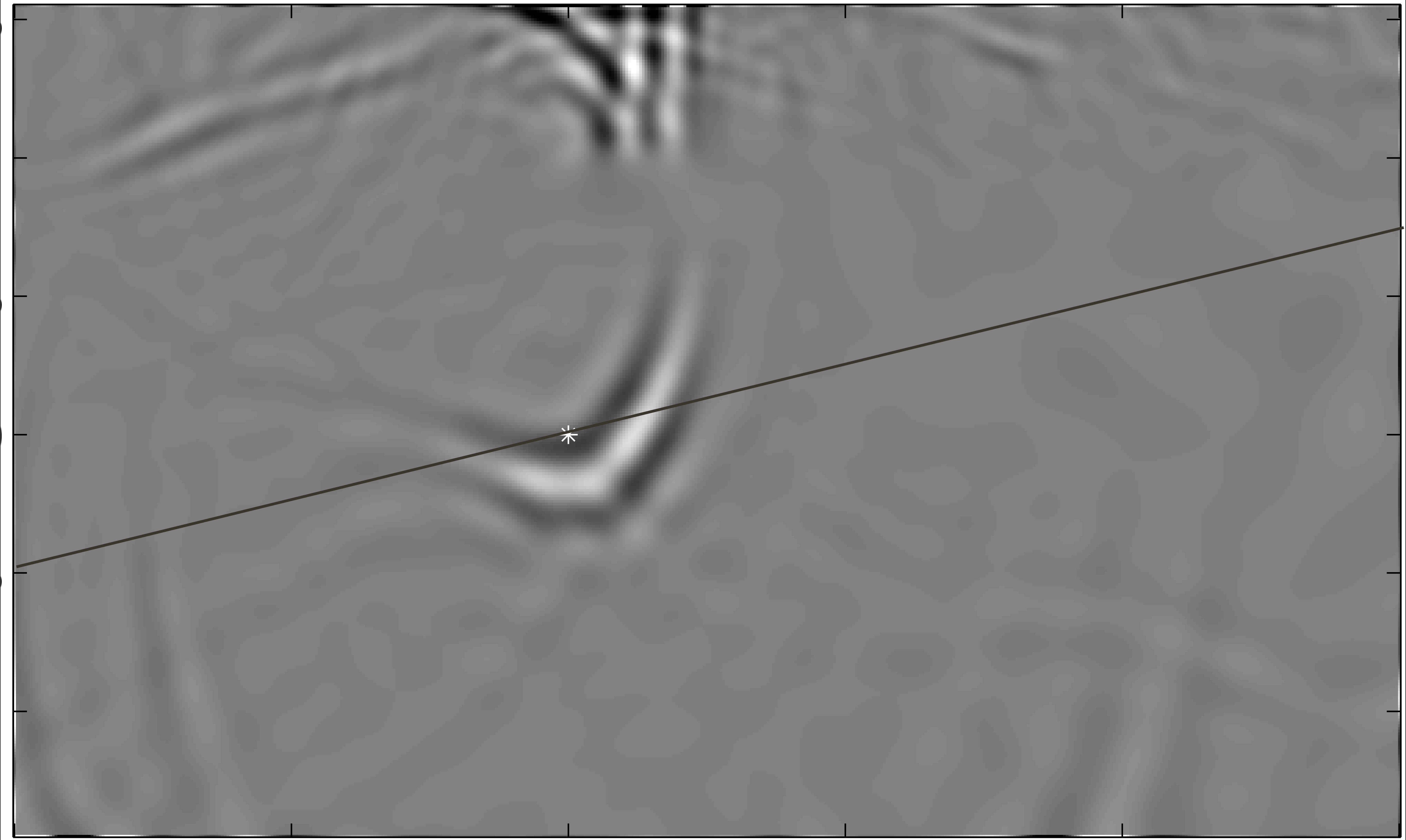


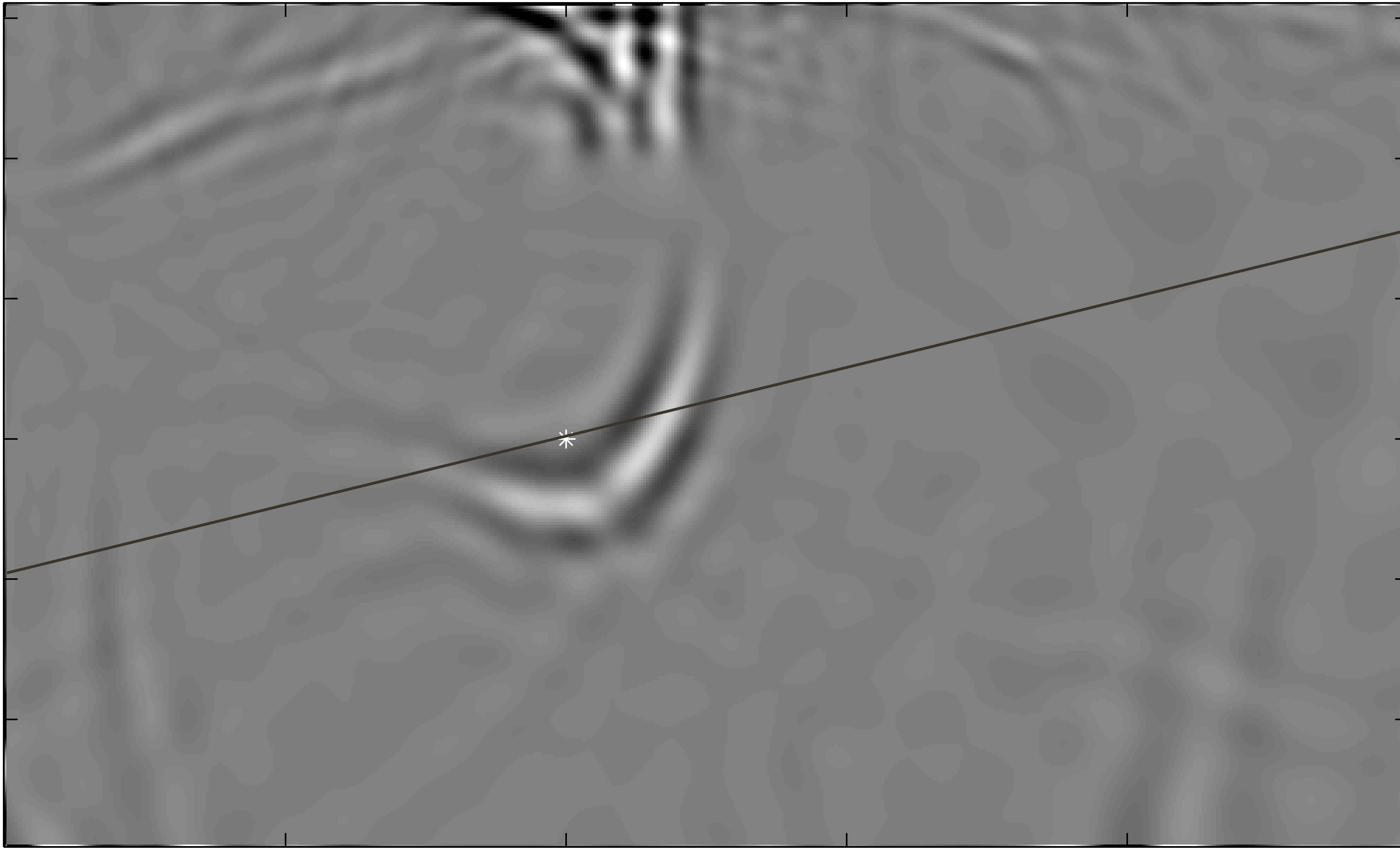


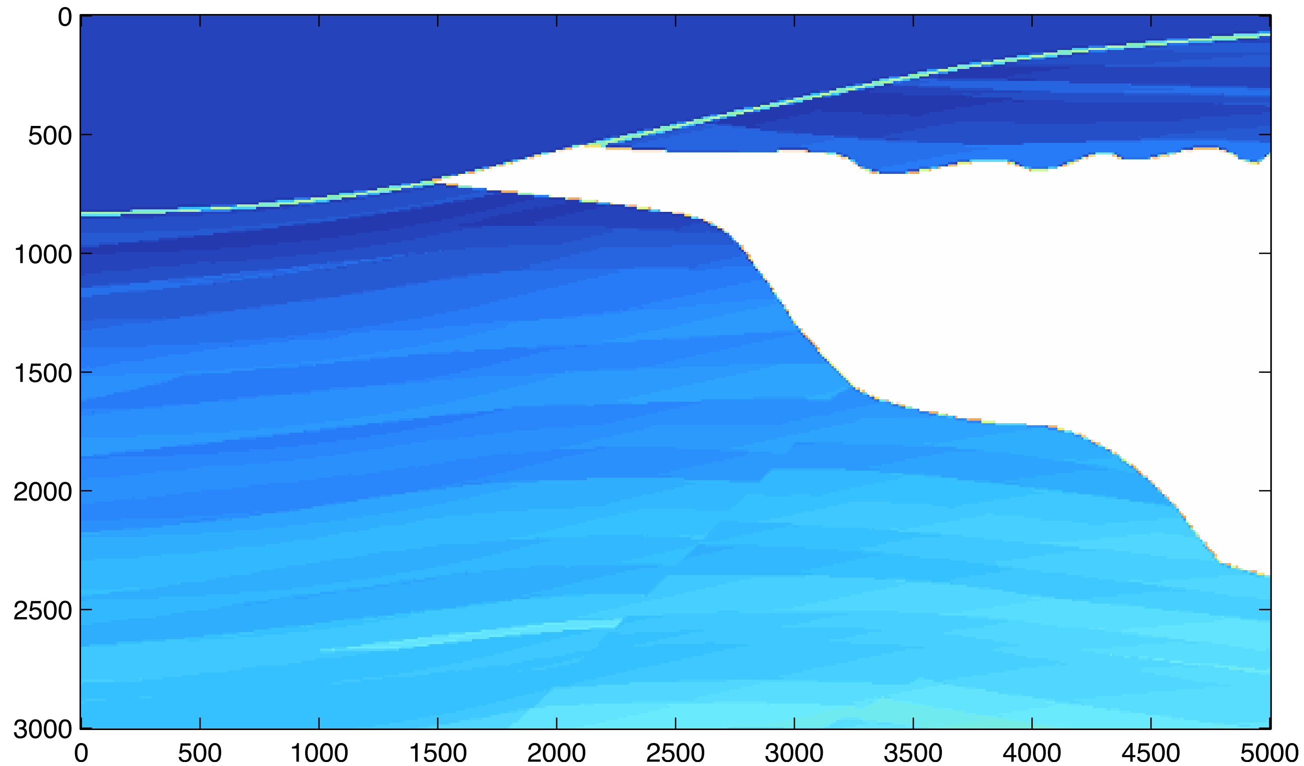


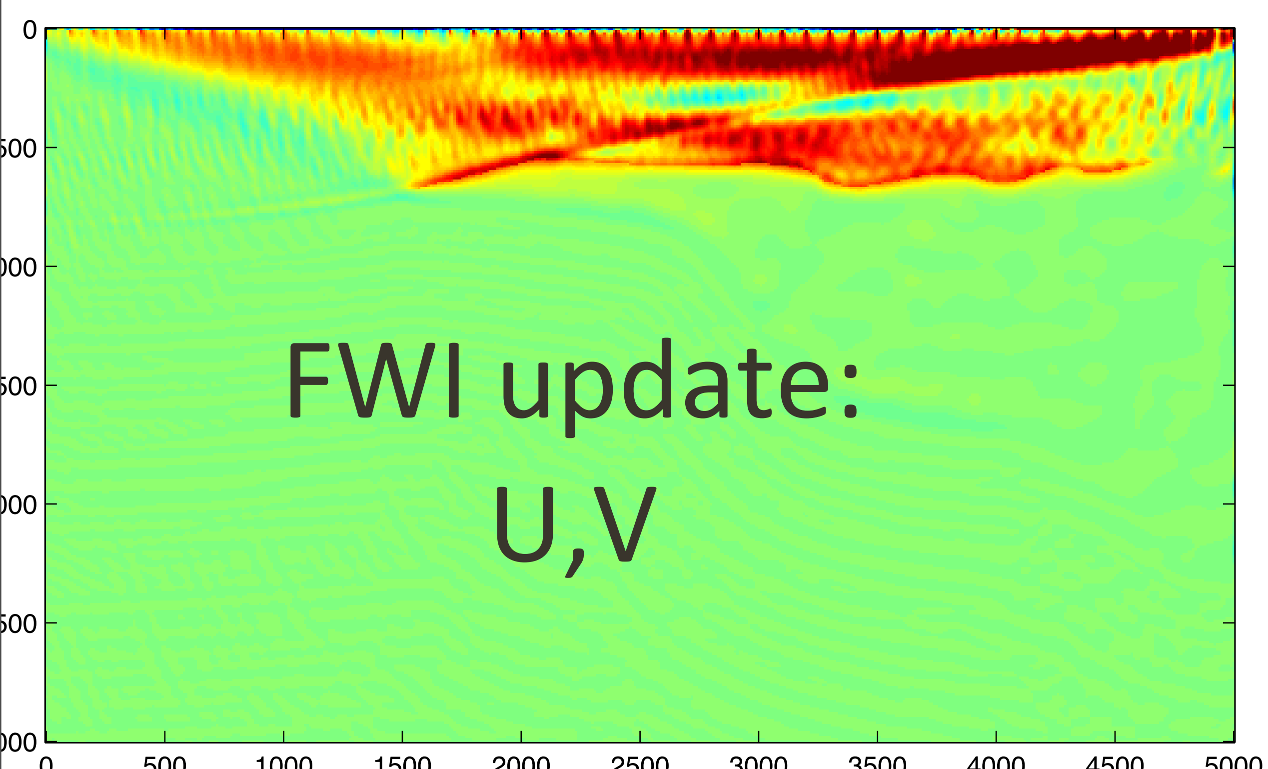


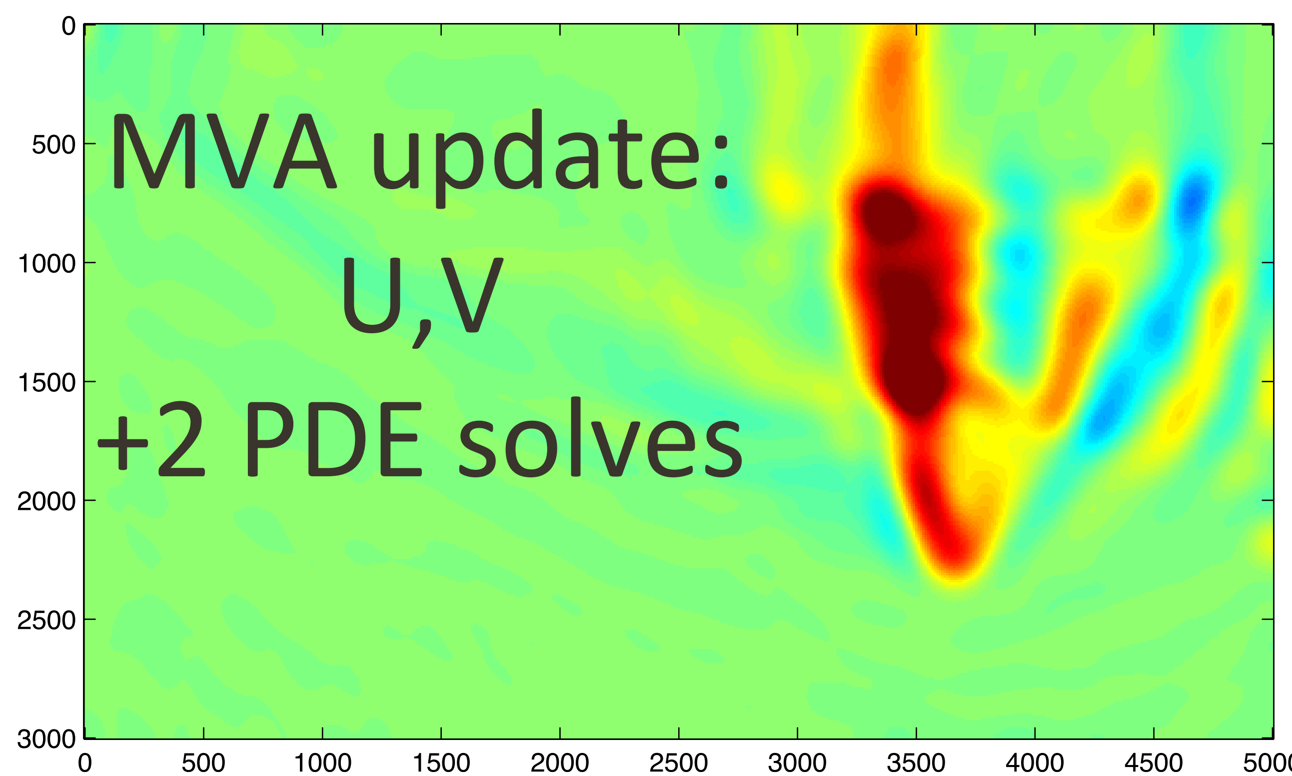









FWI update:
U,V


MVA update:
U,V
+2 PDE solves

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Conclusions

- We can *efficiently* probe the *extended* image volume
- No need to estimate dip because of using subsurface offset in *all* directions
- Sensitivity kernel is computed at *little* extra cost compared to FWI update

Future work

- Calculate penalty using matvec's with image volume
- Extract *conventional* image gathers from *image* volume
- Velocity *continuation*
- *Nonlinear* MVA

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Non-linear formulation

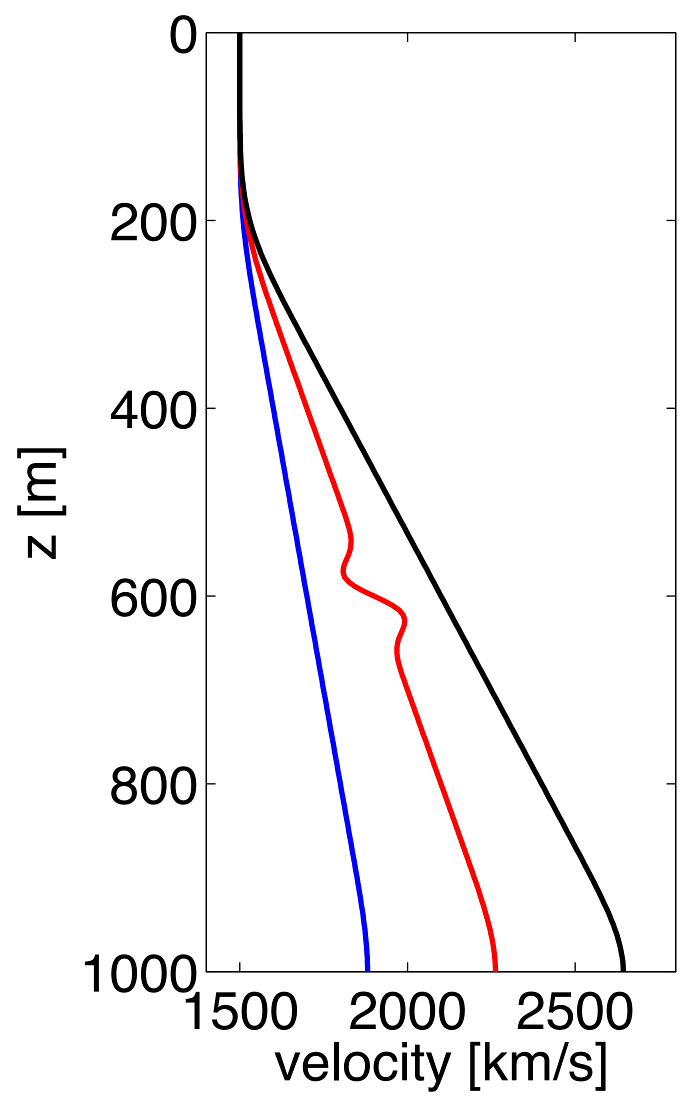
For layered models we have only lateral interaction:

$$Mu(z,x) = \int \mathrm{d}x' \, m(z, x - x') u(z,x)$$

which *diagonalizes* under the Fourier transform.

The resulting model $\hat{M}$ should be laterally invariant.

Non-linear formulation

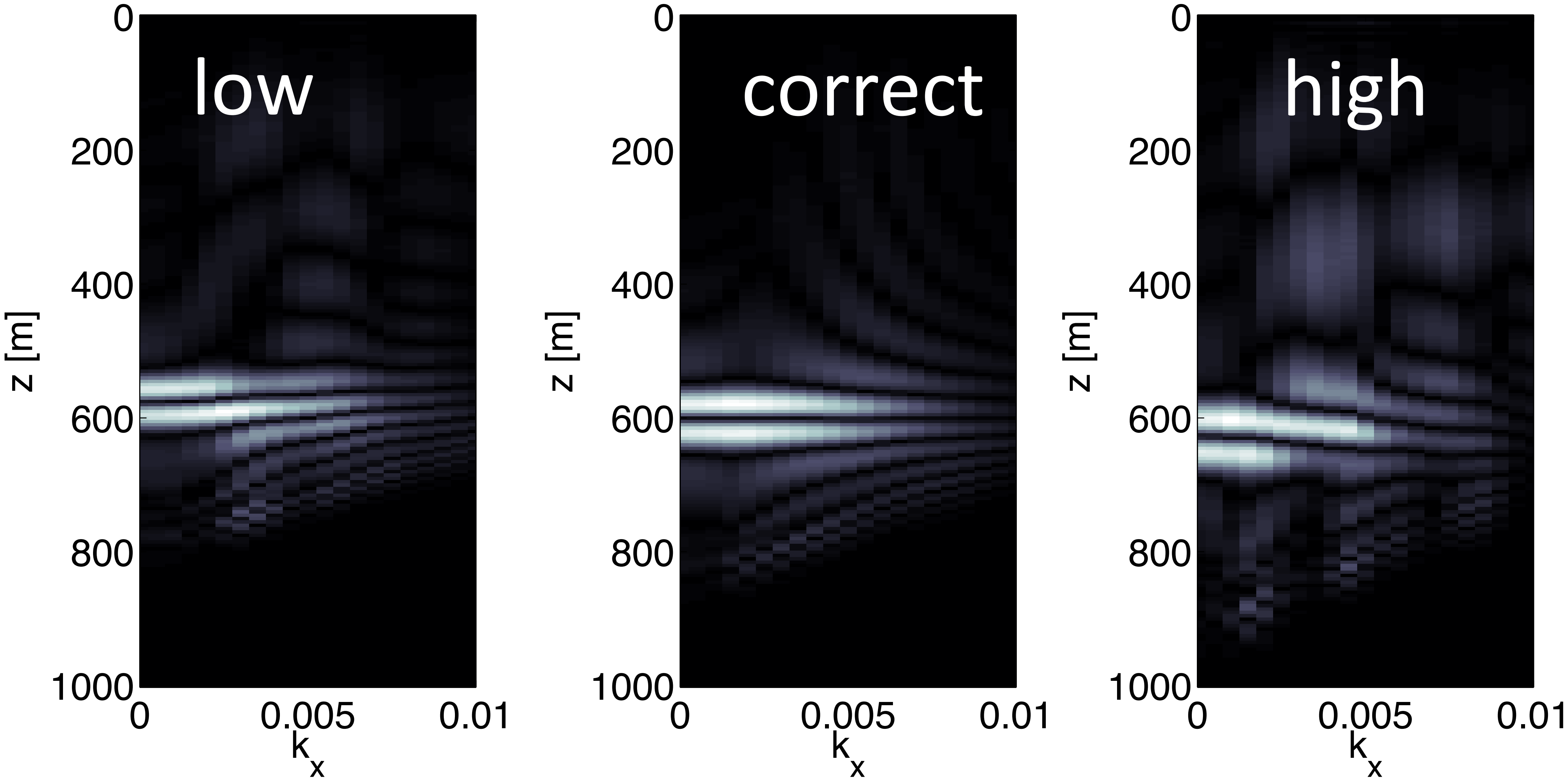


Reconstruct *extended* model for various reference models up to same data-misfit

$$\min_{\hat{M}} ||F[\hat{M}] - \mathbf{d}||_2^2$$

Non-linear formulation

reconstructed perturbations



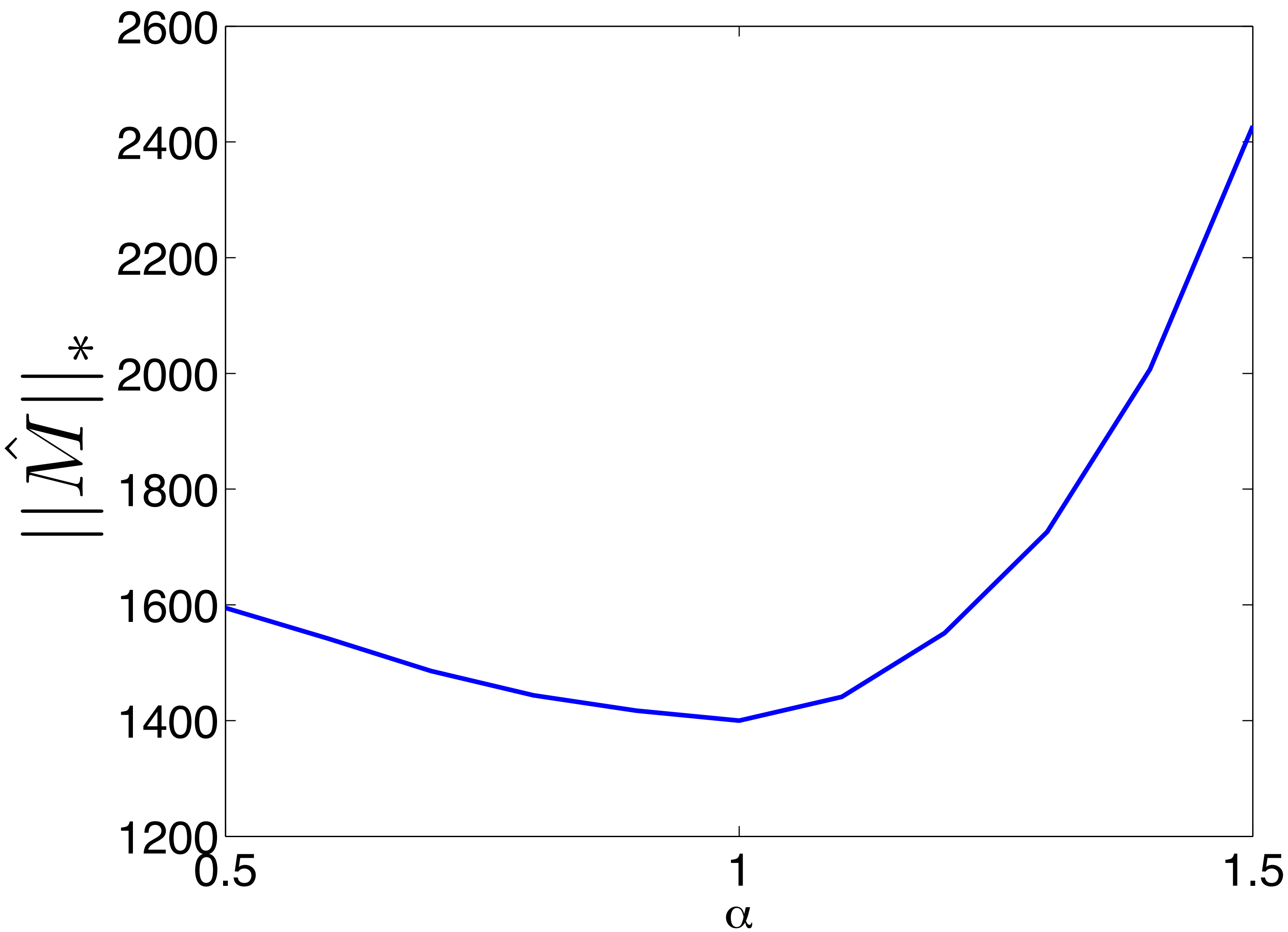
Non-linear formulation

We may now formulate the problem as

$$\min_{\hat{M}} ||\hat{M}||_* \quad \text{s.t.} \quad ||F[\hat{M}] - \mathbf{d}||_2 \leq \sigma$$

where $||\cdot||_*$ denotes the *nuclear norm* which is the sum of the singular values.

Non-linear formulation



The road ahead

Recent developments:

- The *max-norm*: proxy for the nuclear norm that is cheap to compute
- Keep factored form of model

$$\hat{M} = LR^T$$

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Acknowledgements

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BP, Chevron, ConocoPhillips, Petrobras, Total SA, BGP, PGS and WesternGeco.