Wavefield modelling and inversion in Matlab

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Algorithm design

Environment for developing FWI algorithms:

- Flexible
- Reasonably efficient and scalable
- Easy to maintain, test and debug

Algorithm design

- data cube and model `objects'

  `D, M`

- modelling operator

  `D = F(M)`

- Jacobian (linear operator)

  `D = J*M`

  `M = J'*D`

Algorithm design

Data/model objects:

- elementwise operations (`+,-,*`)
- reduction/transformation (`norm,fft`)
- may carry header information

Algorithm design

Linear operators

- forward/adjoint multiply (Ax, A^Tb)
- composition (`C = A*B`)
- coordinate free

Algorithm design

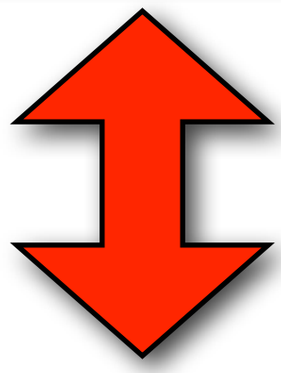
Solvers

- ask for operations on data
- ask for properties of data (e.g., norm)
- coordinate free

Algorithm design

1D linear operations can be combined via Kronecker products:

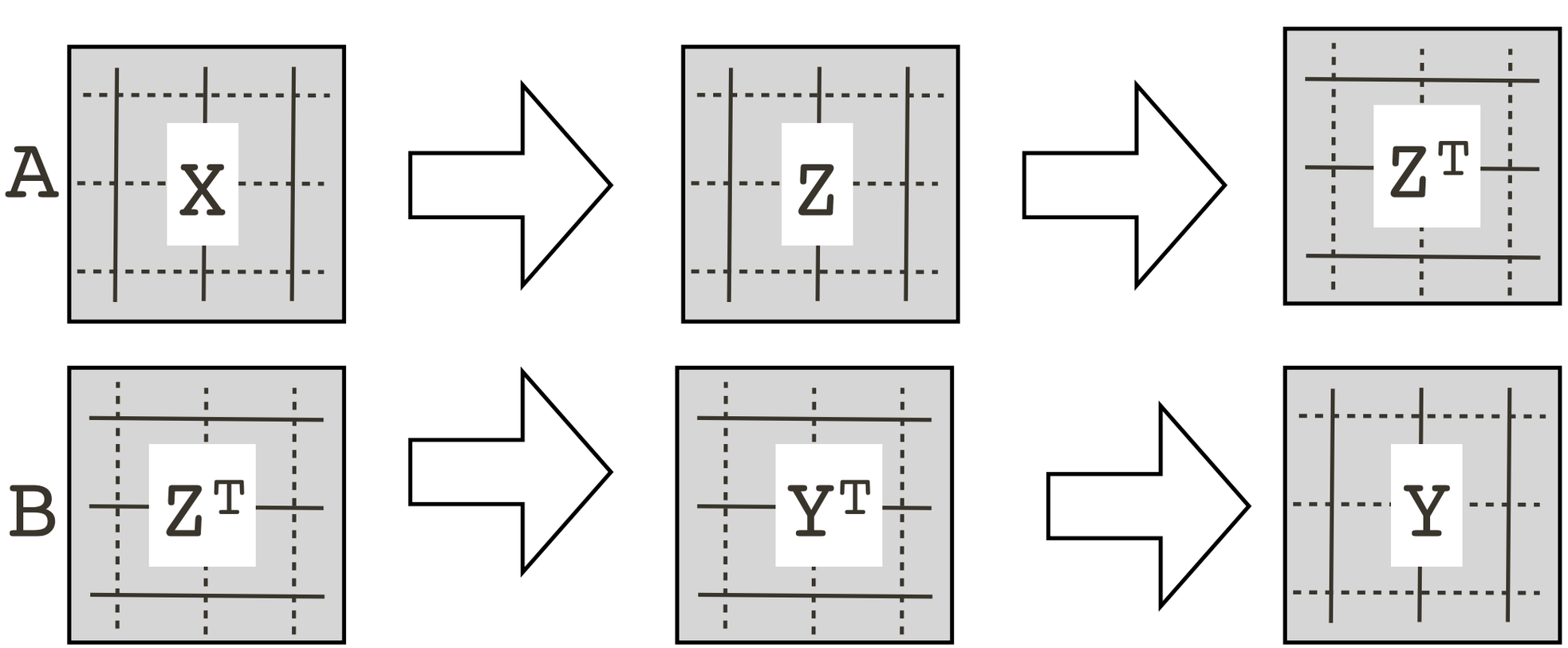
$$Y = AXB^T$$



$$\text{vec}(Y) = (B \otimes A)\text{vec}(X)$$

Algorithm design

Computations are easily parallelized over columns



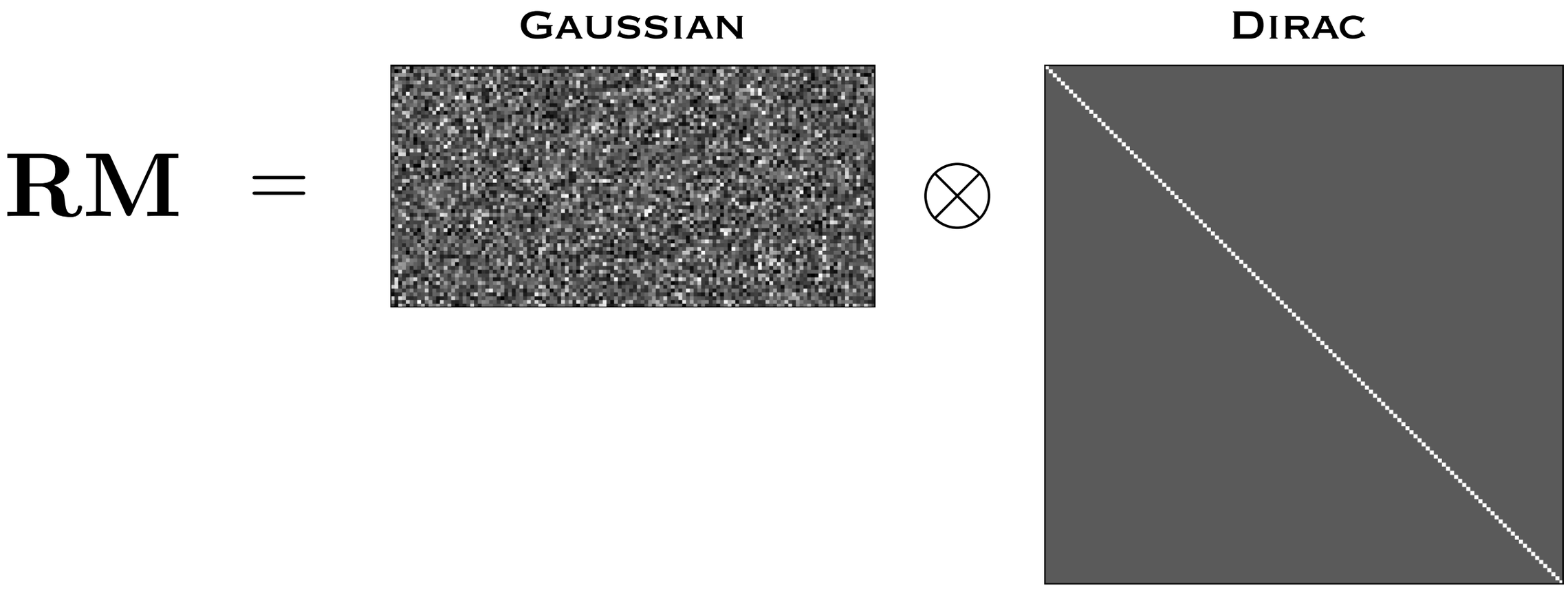
Sampling operator (RM)

```
% Subsampling ratio
p    = 1/2;

% Number of simultaneous-source experiments
nse  = round(p * dim(3));

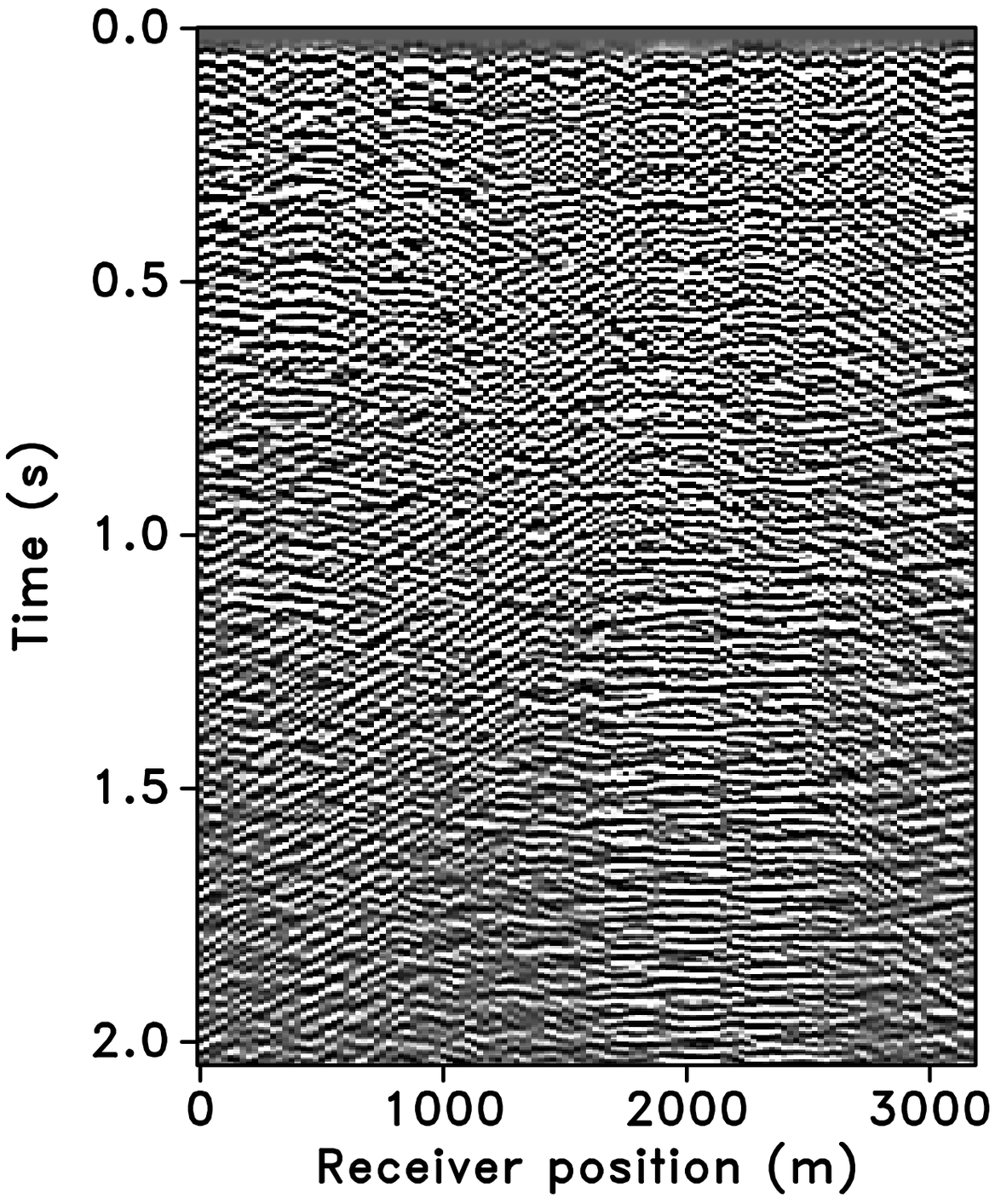
% Dimensions for simultaneous-source experiments
sdim = [dim(1) dim(2) nse];

% Construct operator (RM)
RM   = opKron(opGaussian(sdim(3),dim(3)),opDirac(sdim(1)*sdim(2)));
```



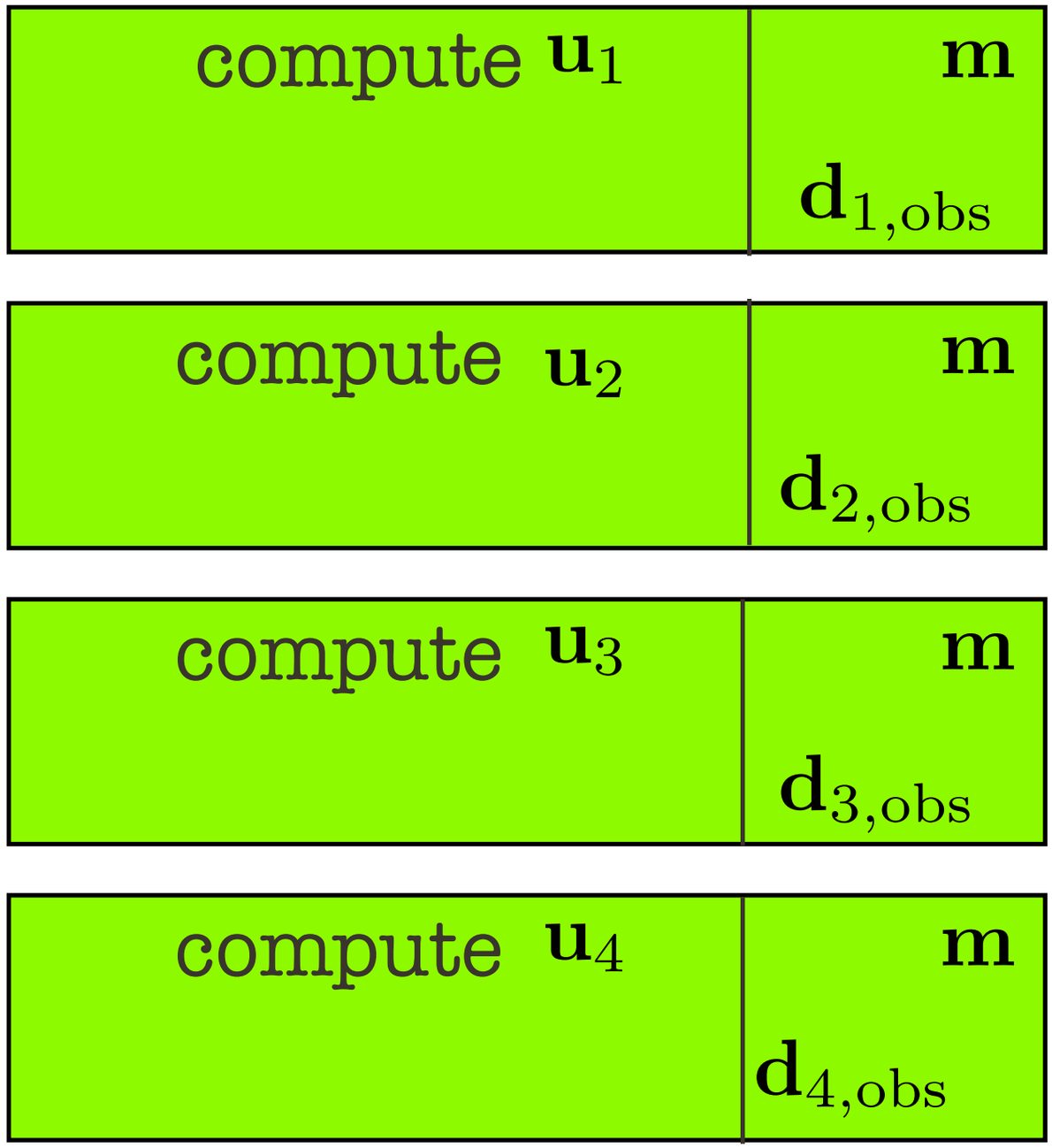
Simultaneous data

```
% Simultaneous measurements
b = RM*D(:);
```



Algorithm design

```
D=F(M)
R=D-Dobs
f=norm(R)
G=J*R
M=M-a*G
```



Algorithm design

```
D=F(M)
R=D-Dobs
f=norm(R)
G=J*R
M=M-a*G
```

$\mathbf{r}_1 = P\mathbf{u}_1 - \mathbf{d}_{1,\text{obs}}$	$\mathbf{u}_1\,\mathbf{m}$ $\mathbf{d}_{1,\text{obs}}$
$\mathbf{r}_2 = P\mathbf{u}_2 - \mathbf{d}_{2,\text{obs}}$	$\mathbf{u}_2\,\mathbf{m}$ $\mathbf{d}_{2,\text{obs}}$
$\mathbf{r}_3 = P\mathbf{u}_3 - \mathbf{d}_{3,\text{obs}}$	$\mathbf{u}_3\,\mathbf{m}$ $\mathbf{d}_{3,\text{obs}}$
$\mathbf{r}_4 = P\mathbf{u}_4 - \mathbf{d}_{4,\text{obs}}$	$\mathbf{u}_4\,\mathbf{m}$ $\mathbf{d}_{4,\text{obs}}$

Algorithm design

```
D=F(M)
R=D-Dobs
f=norm(R)
G=J*R
M=M-a*G
```

$$f = \sum_i f_i$$

$f_1 = \|\|\mathbf{r}_1\|\|_2^2$	$\mathbf{r}_1\ \mathbf{u}_1\ \mathbf{m}$ $\mathbf{d}_{1,\text{obs}}$
$f_2 = \|\|\mathbf{r}_2\|\|_2^2$	$\mathbf{r}_2\ \mathbf{u}_2\ \mathbf{m}$ $\mathbf{d}_{2,\text{obs}}$
$f_3 = \|\|\mathbf{r}_3\|\|_2^2$	$\mathbf{r}_3\ \mathbf{u}_3\ \mathbf{m}$ $\mathbf{d}_{3,\text{obs}}$
$f_4 = \|\|\mathbf{r}_4\|\|_2^2$	$\mathbf{r}_4\ \mathbf{u}_4\ \mathbf{m}$ $\mathbf{d}_{4,\text{obs}}$

Algorithm design

```
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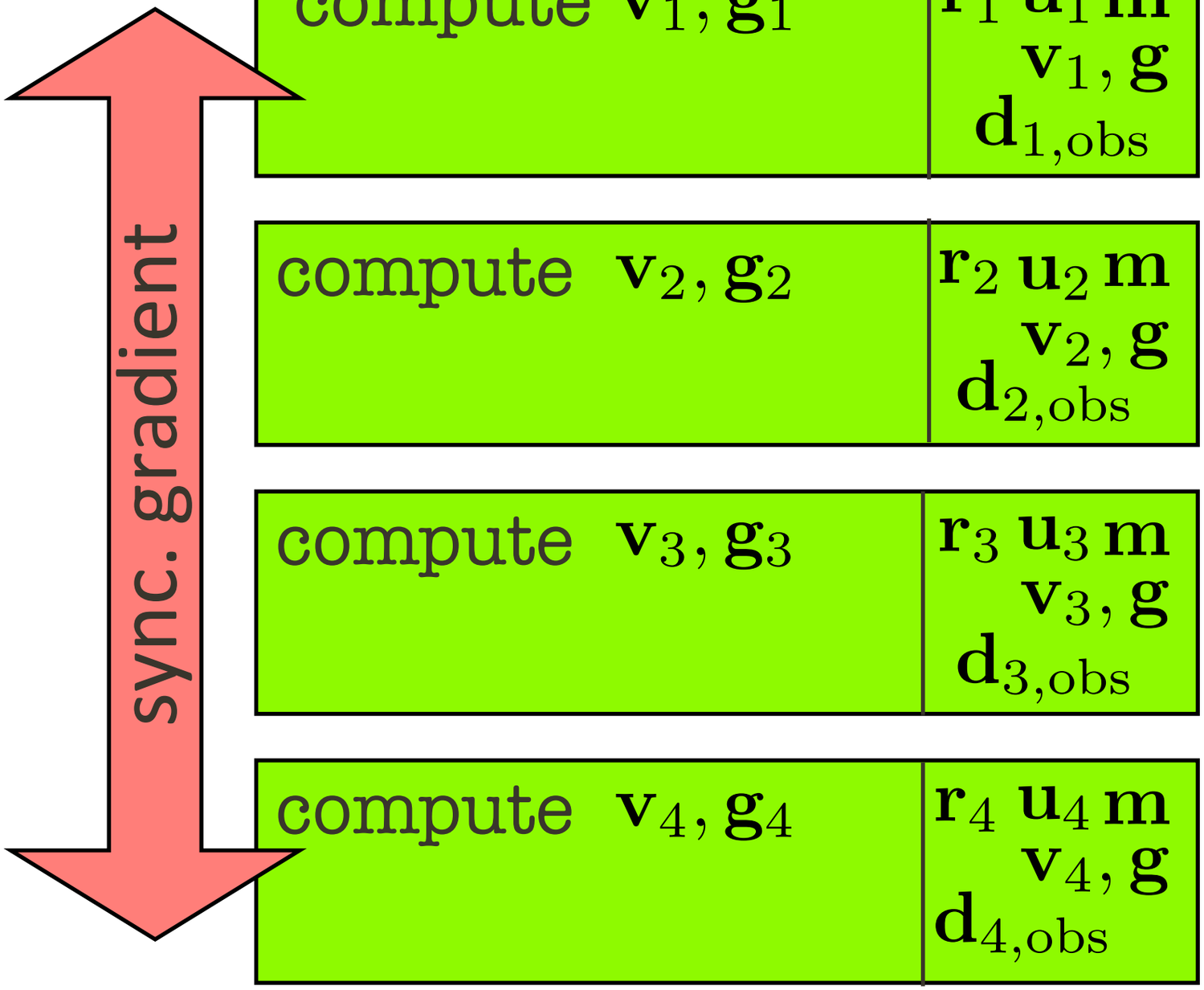


Algorithm design

```
D=F(M)
R=D-Dobs
f=norm(R)
G=J*R
M=M-a*G
```

a

$\mathbf{m} = \mathbf{m} - \alpha \mathbf{g}$	$\mathbf{r}_1\ \mathbf{u}_1\ \mathbf{m}$ $\mathbf{v}_1, \mathbf{g}$ $\mathbf{d}_{1,\mathrm{obs}}$

$\mathbf{m} = \mathbf{m} - \alpha \mathbf{g}$	$\mathbf{r}_2\ \mathbf{u}_2\ \mathbf{m}$ $\mathbf{v}_2, \mathbf{g}$ $\mathbf{d}_{2,\mathrm{obs}}$

$\mathbf{m} = \mathbf{m} - \alpha \mathbf{g}$	$\mathbf{r}_3\ \mathbf{u}_3\ \mathbf{m}$ $\mathbf{v}_3, \mathbf{g}$ $\mathbf{d}_{3,\mathrm{obs}}$

$\mathbf{m} = \mathbf{m} - \alpha \mathbf{g}$	$\mathbf{r}_4\ \mathbf{u}_4\ \mathbf{m}$ $\mathbf{v}_4, \mathbf{g}$ $\mathbf{d}_{4,\mathrm{obs}}$

Parallel Matlab

- Supports *distributed* arrays
- Most basic linear algebra is *overloaded* (parallel matvec's)
- No need to code MPI
- Low-level MPI commands are available if needed

Matlab implementation

modelling:

```
[D,J] = F(m,Q,model)
```

`D` - data cube as Matlab distributed array

`J` - Jacobian as SPOT operator

`m` - model as Matlab array

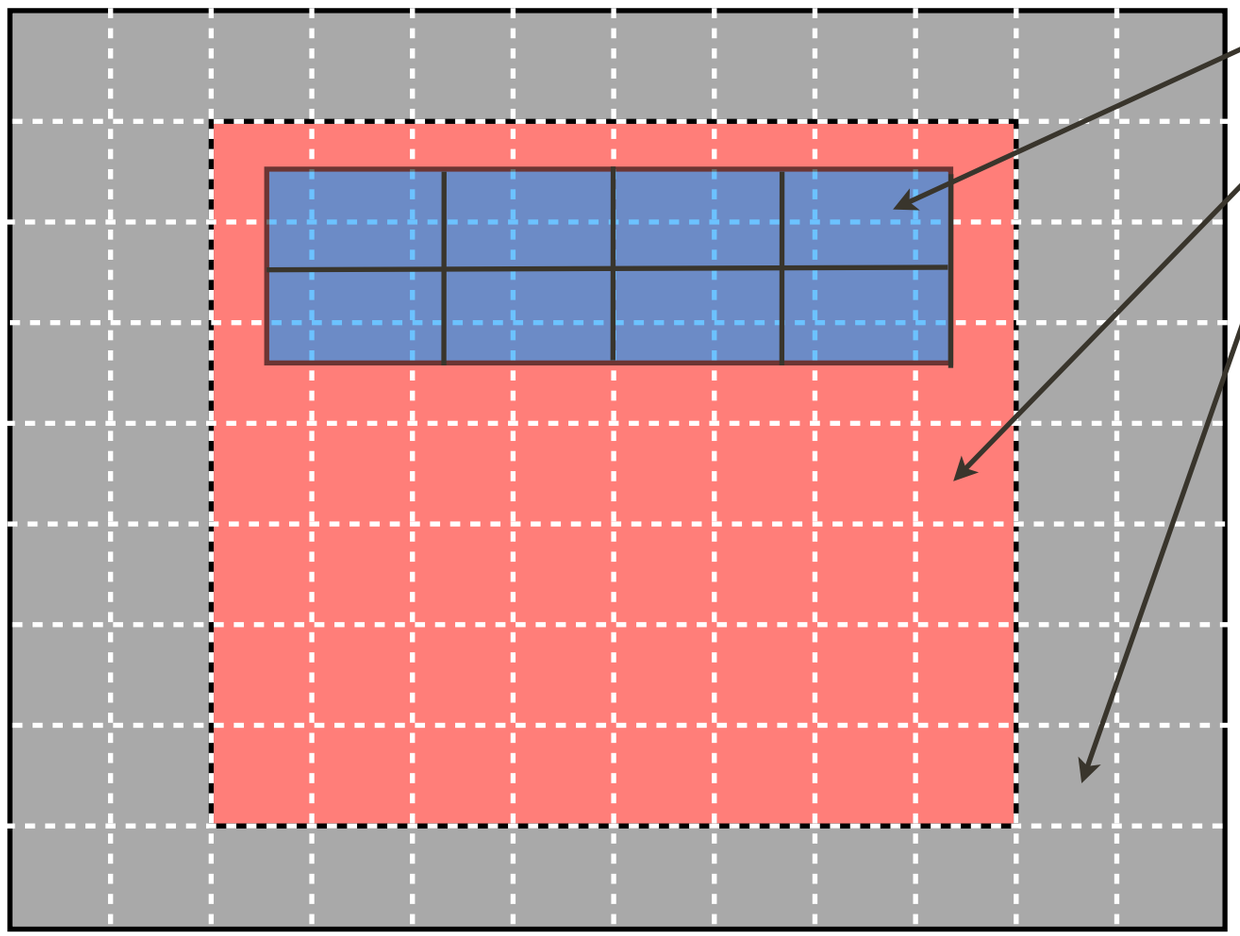
`Q` - source functions as matrix

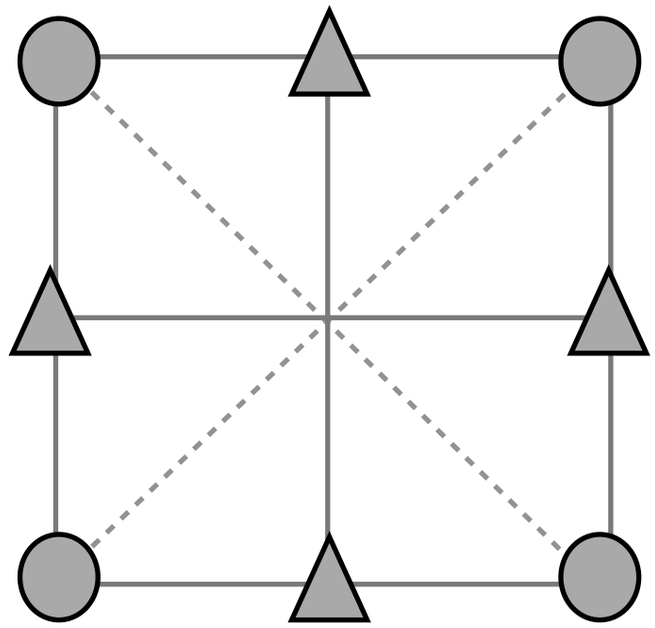
`model` - struct containing acquisition setup etc.

Matlab implementation

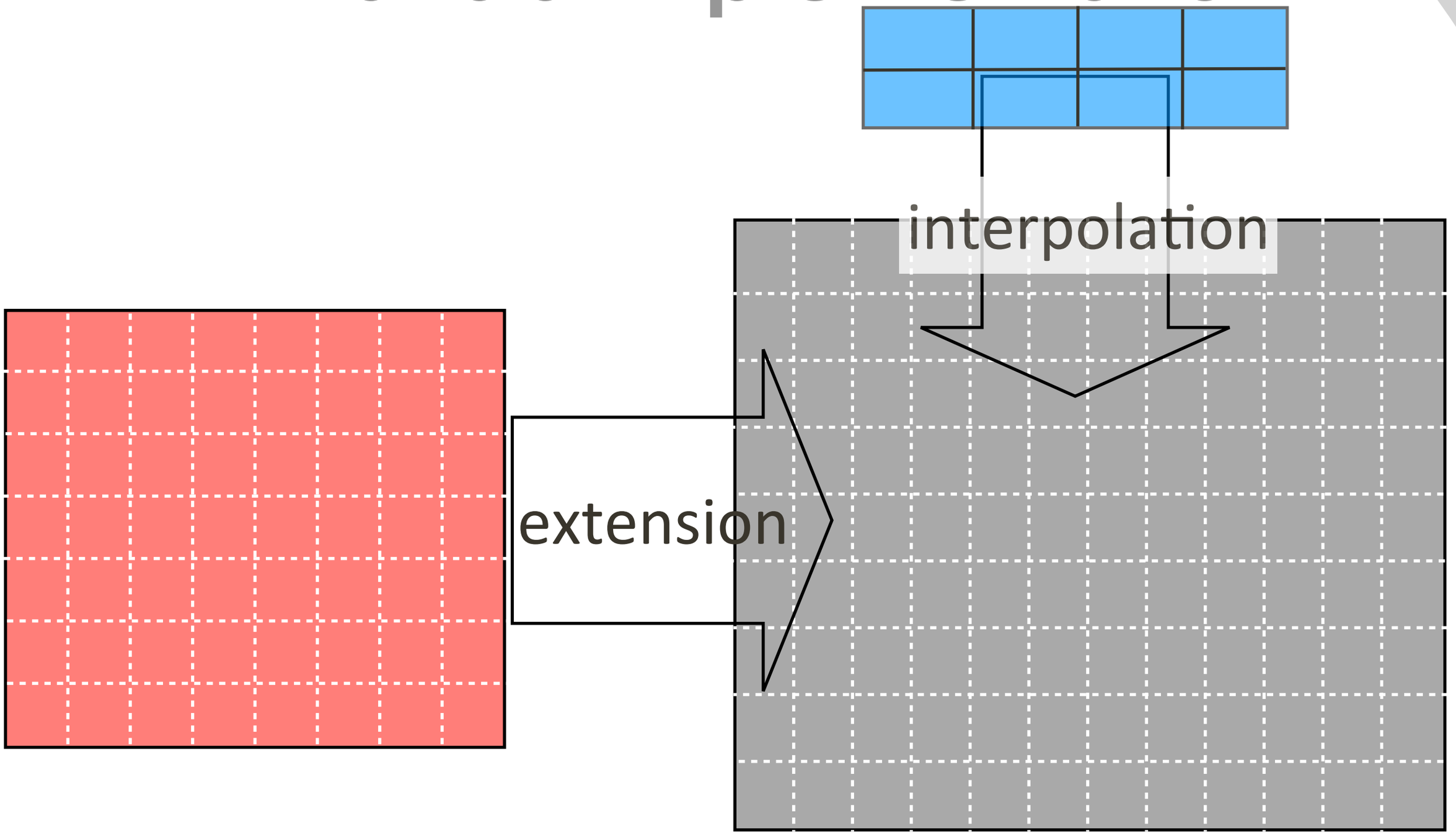
- frequency-domain FD modelling
- 9-point mixed-grid stencil
- `sponge' boundary
- source-injection and sampling via cubic interpolation
- parallel over frequencies
- Direct solver (Matlab->UMFPACK)

Matlab implementation





Matlab implementation



Matlab implementation

Born modelling and RTM

```
D    = J*dm;% calls J.multiply(dm,1)
dmt = J'*D;% calls J.multiply(D,-1)
```

remember, J is not a Matlab matrix but just an object.

Matlab implementation

- Jacobian based on linearization of the discretized system
- careful implementation of the adjoint
- recomputes wavefields

Matlab implementation

Testing

```
%test Jacobian
[D0,J0] = F(m0,Q,model);
D1      = F(m0+dm,Q,model);
dD      = J0*dm;
norm(D1 - D0 - dD)

%dottest: (Ax)'*b = x'*(A'*b)
(J*dm)'*dD
dm'(J'*dD)
```

Matlab implementation

least-squares migration

```
[D0,J0] = F(m0,Q,model);

%LSQR
m1 = LSQR(J0,D-D0);

%SPGL1
C  = opCurvelet(...);
x  = spgl1(J0*C',D-D0);
m2 = C'*x;
```

Matlab implementation

FWI

```
[f,g] = Jls(m,Q,Dobs,model)
%least-squares misfit

[D,J] = F(m,Q,model);
f     = .5*norm(D-Dobs).^2;
g     = J'*(D-Dobs);
```

Matlab implementation

FWI with source estimation

```
[f,g] = JlsSrc(m,Q,Dobs,model)
%least-squares misfit with source estimation

[D,J] = F(m,Q,model);
W     = SrcEst(D,Dobs);
f     = .5*norm(W*D-Dobs).^2;
g     = J'*(W'*df);
```

Matlab implementation

FWI with Students t

```
[f,g] = JStudentsT(m,Q,Dobs,model)
%Students T misfit

[D,J]   = F(m,Q,model);
[f,df] = StudentsT(D-Dobs);
g       = J'*df;
```

Matlab implementation

FWI with model parametrization

```
[f,g] = Jls(m,Q,Dobs,model)
%least-squares misfit

B      = opSpline(...);
[D,J] = F(B*m,Q,model);
f      = .5*norm(D-Dobs).^2;
g      = B'*J'*(D-Dobs);
```

Matlab implementation

Now, we can use *any* optimization code that uses only function values and gradients.

```
% function handle
fh = @(x)Jls(x,Q,D,model);

% optimization
mn = lbfgs(fh,m0);
```

Matlab implementation

```
m = GN(fh,m0,D)
% Gauss-Newton

m = m0;
for k = 1:maxit
  [Dk,Jk] = fh(m);
  dm      = LSQR(J,Dk - D);
  m       = m + dm;
end
```

Matlab implementation

Now, we can use *any* optimization code that uses only function values and gradients.

```
% function handle
fh = @(x)F(x,Q,model);

% optimization
mn = GN(fh,m0,D);
```

Future developments

The computational complexity of most of these algorithms lies in modelling, which we can do externally.

The memory complexity lies in the data volumes, which we do ***not*** have to import into Matlab

Future developments

- Interface to external modelling codes
- DataContainer object to access files as matlab arrays
- Interface to JavaSeis

Datacontainer

Matlab object that carries data and head

Command Window

```
>> help iCon.disp
 disp Display function for all dataContainers.
    disp(X) displays the header of our dataContainer and not the data
    itself. Displaying a dataContainer with default header values for double
    real serial array looks like:

    iCon dataContainer

    Variable Name:     unknown
    Variable Units:    unknown
    Dims:              n
    Size:              [ S1 S2 ... Sn]
    Origin:            [ O1 O2 ... On]
    Delta:             [ D1 D2 ... Dn]
    Label:             [ L1 L2 ... Ln]
    Unit:              [ U1 U2 ... Un]
    Precision:         double
    Complex:           No
    Distributed:       No

 Help for iCon/disp is inherited from superclass dataContainer

fx >>
```

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Datacontainer

Load data and define sampling intervals

Command Window

```
>> D = ReadSuFast('GulfOfSuez178.su');
>> whos D
  Name          Size                     Bytes  Class     Attributes

  D          1024x31684               259555328  double
```

Command Window

```
>> % Number of data samples
nt = 1024;          % Time
nr = 178;           % Receiver
ns = 178;           % Source


% Sampling intervals
dt = 0.004;         % Time
dr = 25;            % Receiver
ds = 25;            % Source
fx >>
```

Datacontainer

Construct data container

Command Window

```
>> D = iCon(reshape(D,nt,nr,ns), 'varName', 'Amplitude', 'varUnits', 'number','delta', [dt dr ds], ...
            'label', {'Time', 'Receiver position', 'Shot position'}, 'unit', {'s', 'm', 'm'});
>>
>> whos D
  Name         Size                     Bytes  Class    Attributes

  D         1024x178x178             259558978  iCon

>> D

D =

iCon dataContainer

Variable Name:     Amplitude
Variable Units:    number
Dims:              3
Size:              [ 1024 178 178 ]
Origin:            [ 0 0 0 ]
Delta:             [ 4.000000e-03 25 25 ]
Label:             [ Time Receiver position Shot position ]
Unit:              [ s m m ]
Precision:         double
Complex:           No
Distributed:       No
fx >>
```

Datacontainer

Permute data

```
Command Window
>> D = permute(D,[3,2,1]);
>> whos D
  Name        Size                      Bytes  Class    Attributes

  D         178x178x1024             259558978  iCon

>> D

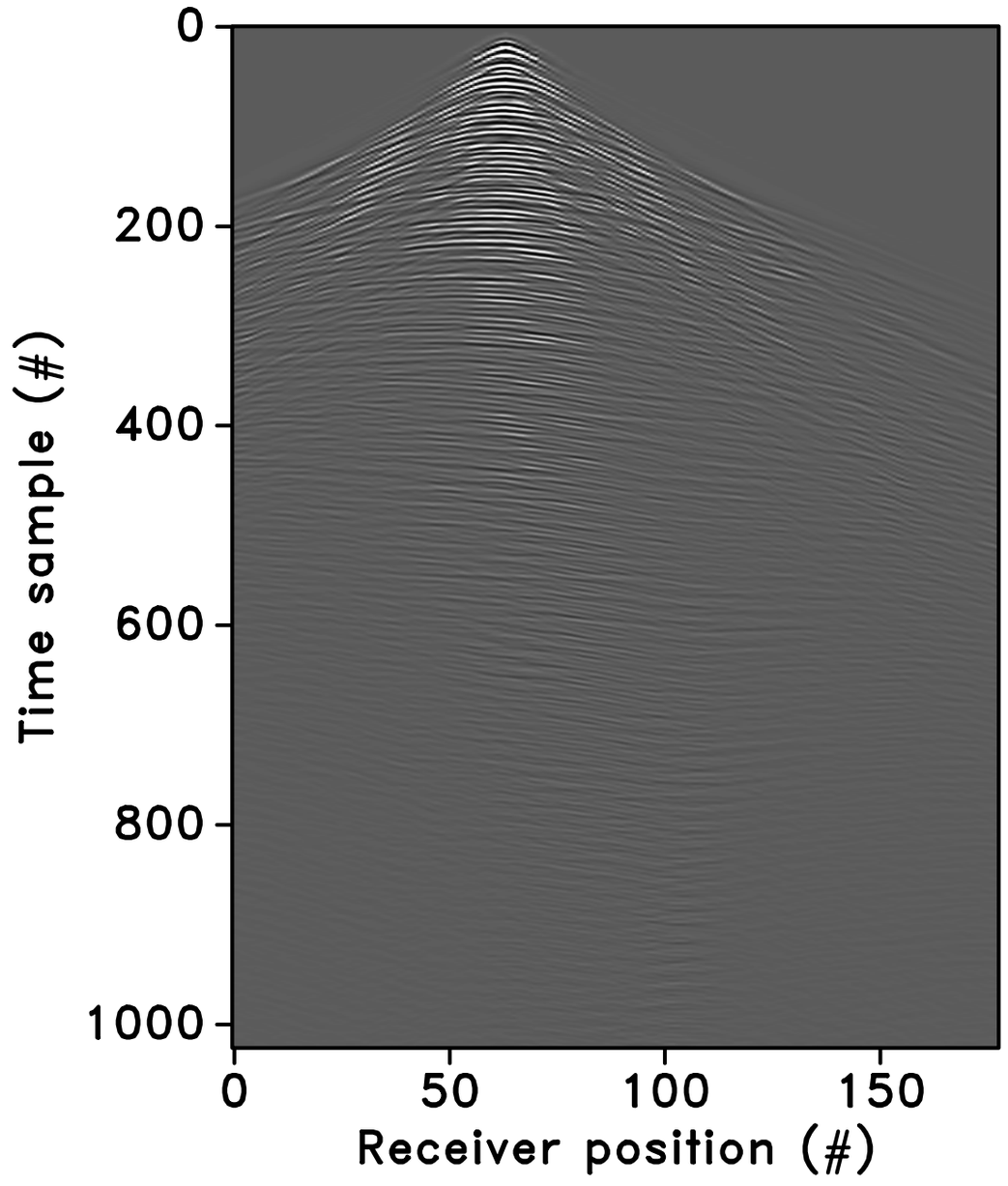
D =

iCon dataContainer

Variable Name:     Amplitude
Variable Units:    number
Dims:              3
Size:              [ 178 178 1024 ]
Origin:            [ 0 0 0 ]
Delta:             [ 25 25 4.000000e-03 ]
Label:             [ Shot position Receiver position Time ]
Unit:              [ m m s ]
Precision:         double
Complex:           No
Distributed:       No
fx >>
```

Datacontainer

```
Command Window
>> imagesc(D(:,:,64));
>>
```



Conclusions

- Use Matlab as `scripting language'
- allows us to quickly prototype and benefit from algorithms developed by `experts'
- codebase is modular and easy to maintain

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Conclusions

- Overloading allows us to call external modelling code, and access data from external sources (disk, memory)
- no need to explicitly import data into Matlab
- Switch easily between matlab-arrays/data-containers and different modelling kernels

Coming soon...

Software release

Acknowledgements

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