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Introduction

Interferometric imaging by sparse inversion:

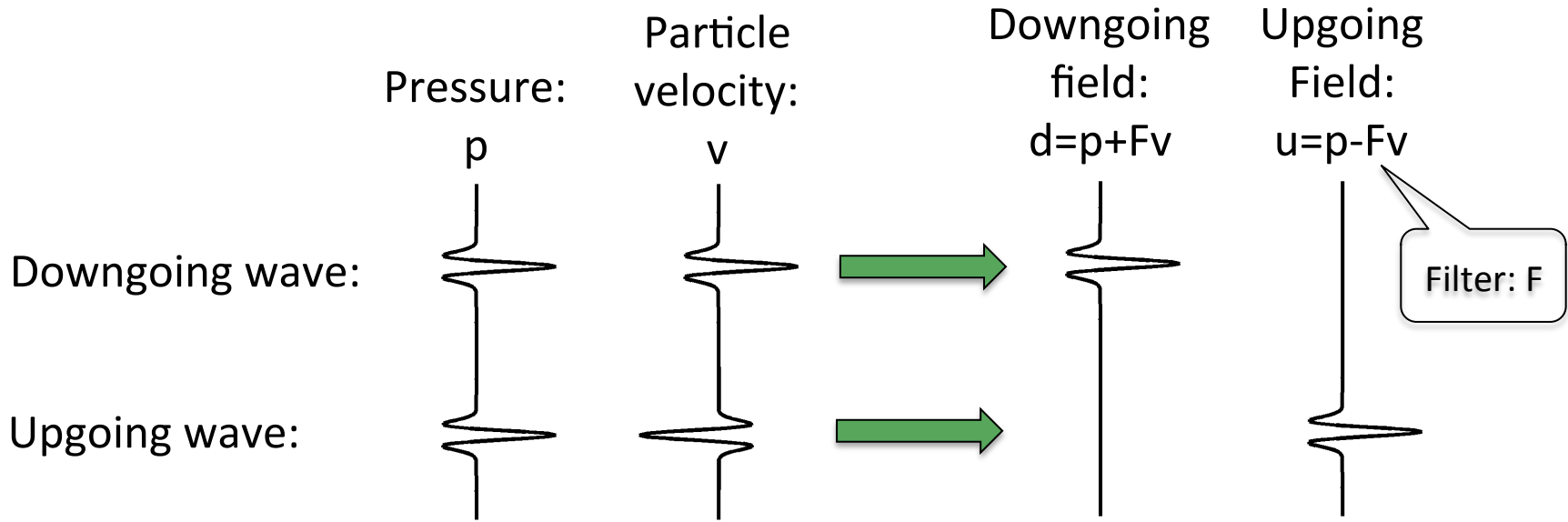
1. Up / down decomposition
2. Interferometric redatuming
3. Local imaging

Ongoing research

Conclusion

Introduction

Up / down separation



Composition:

(Claerbout, 1971; Wapenaar, 1998)

$$\begin{pmatrix} p \\ v \end{pmatrix} = L \begin{pmatrix} d \\ u \end{pmatrix}$$

Composition operator: L



Decomposition:

$$\begin{pmatrix} d \\ u \end{pmatrix} = L^{-1} \begin{pmatrix} p \\ v \end{pmatrix}$$

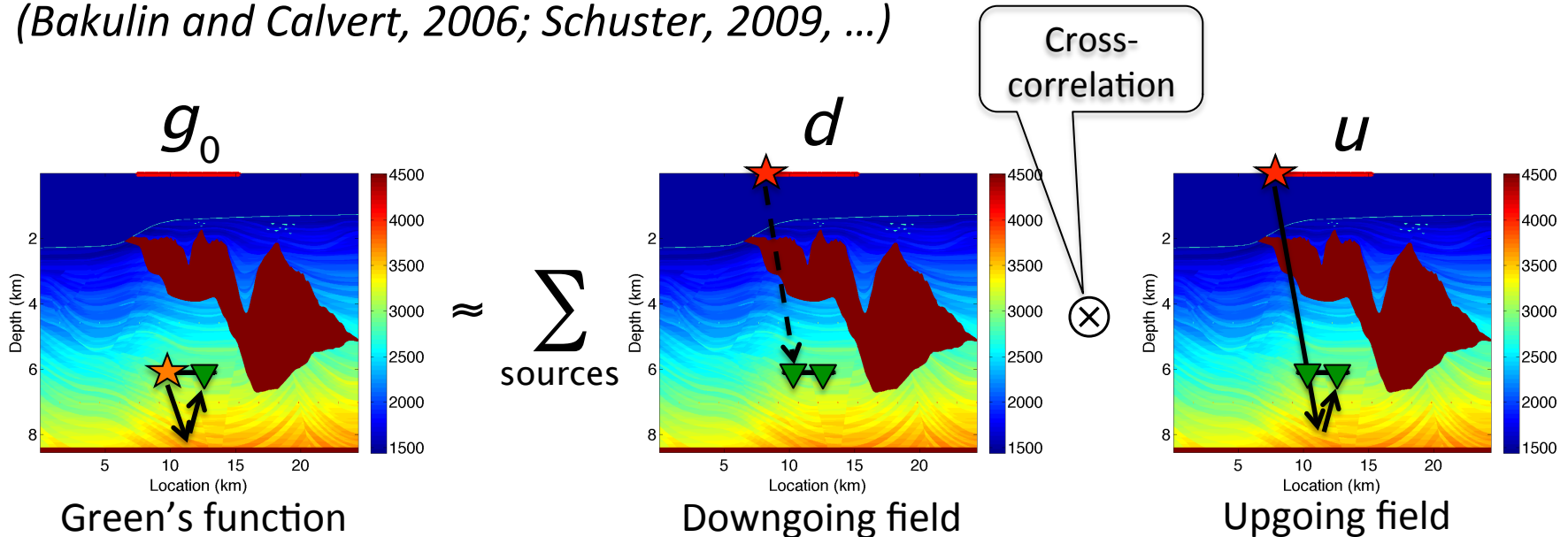
Decomposition operator: L⁻¹

Introduction

Interferometric redatuming by cross-correlation

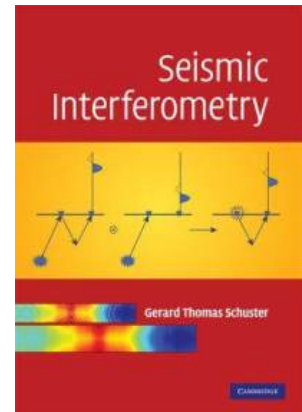
Interferometry by cross-correlation / virtual source method:

(Bakulin and Calvert, 2006; Schuster, 2009, ...)



Applications:

- Sub-salt imaging (Vasconcelos et al., 2008; this presentation)
- Monitoring below a complex overburden (Bakulin et al., 2007)
- Salt-flank imaging (Hornby et al., 2007)
- Various others (Schuster, 2009)

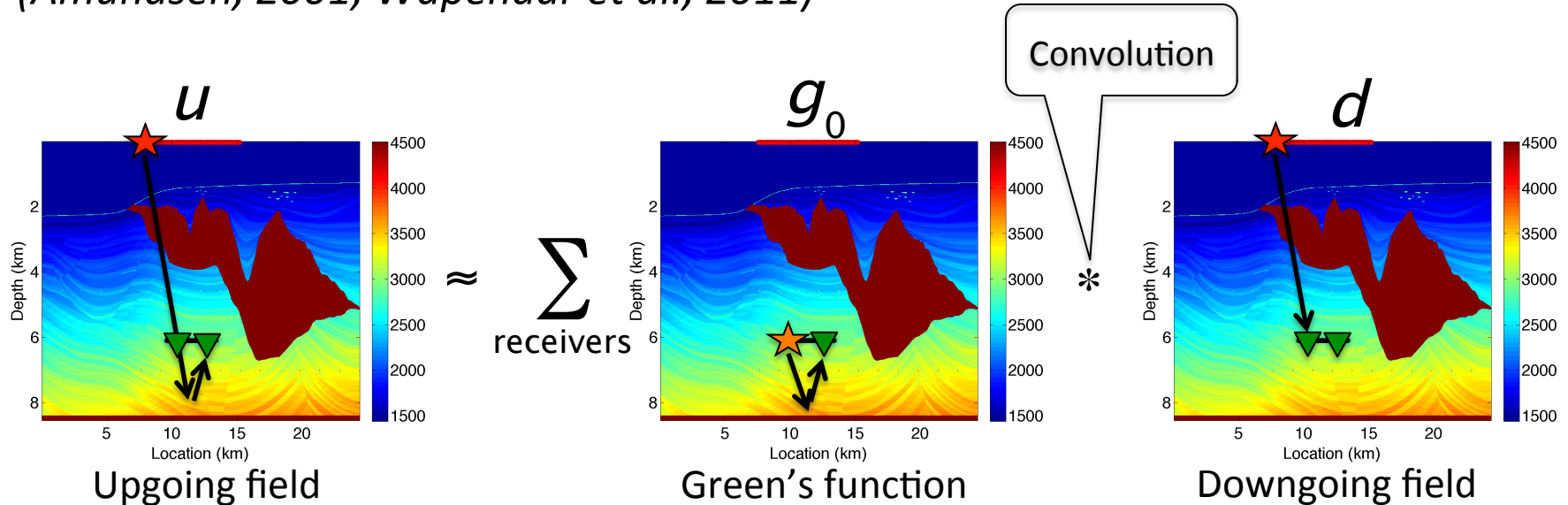


Introduction

Interferometric redatuming by multidimensional deconvolution

Interferometry by multidimensional deconvolution:

(Amundsen, 2001; Wapenaar et al., 2011)



Least-squares inversion:

$$\min_{g_0} \|u - Dg_0\|_2^2 + \lambda^2 \|g_0\|_2^2.$$

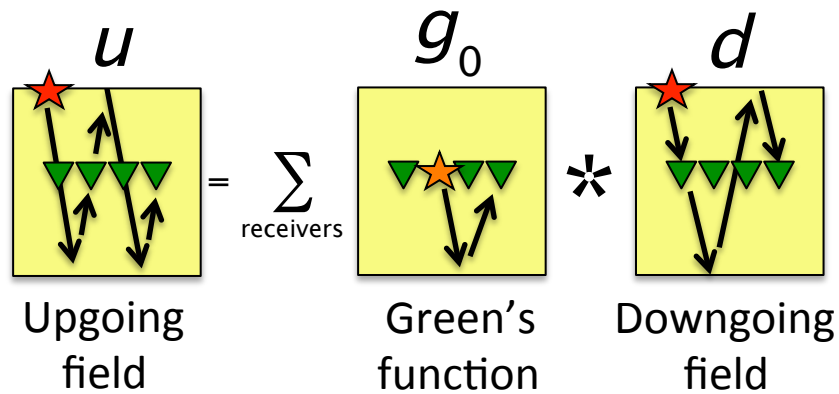
Operator: D

Regularization parameter: λ

Introduction

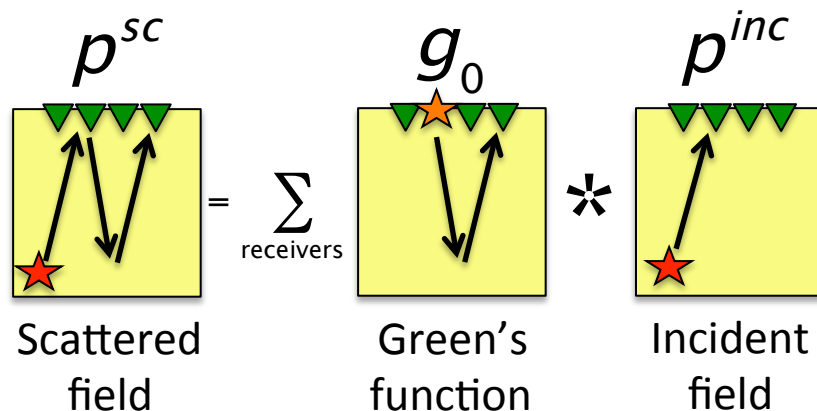
Connection with passive seismic interferometry

Interferometric redatuming problem:



Passive seismic interferometry problem:

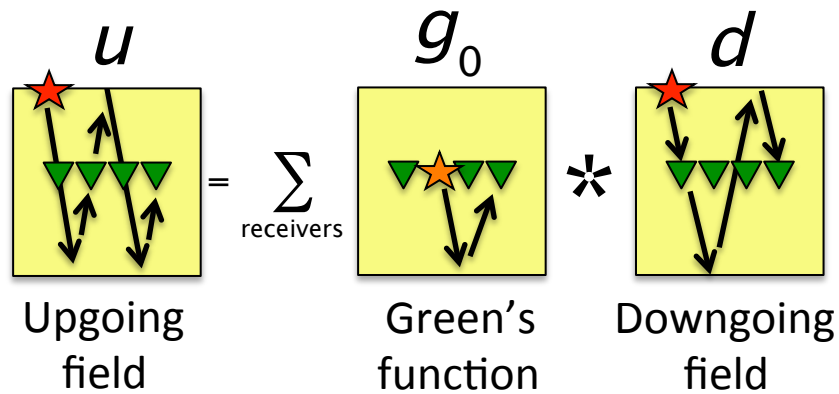
(Wapenaar et al., 2008)



Introduction

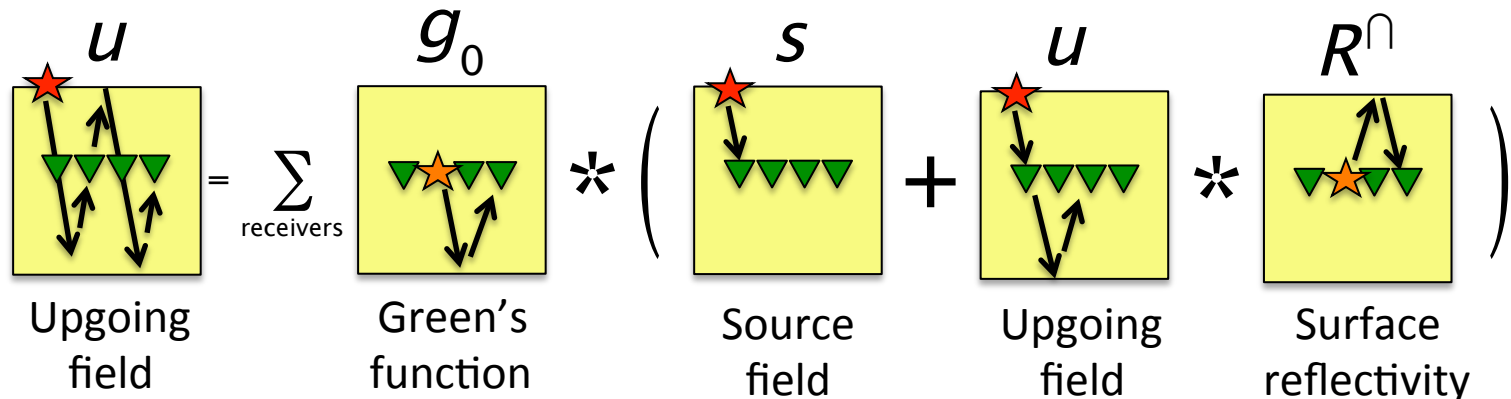
Connection with surface-related demultiple

Interferometric redatuming problem:



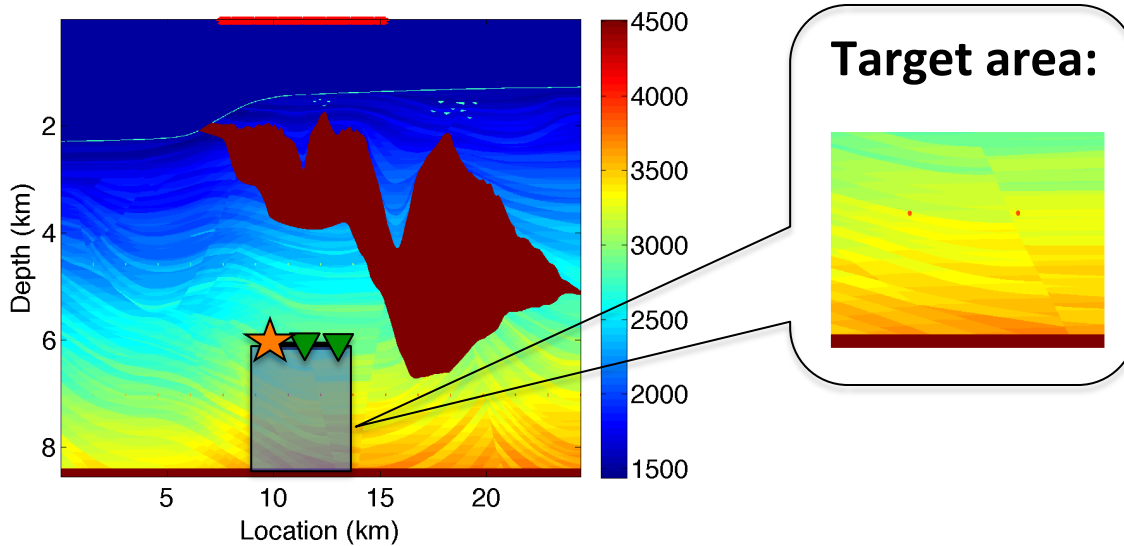
Surface-related demultiple problem:

(Amundsen, 2001; Berkhout and Verschuur, 1997; Frijlink et al., 2011)



Introduction

Local imaging

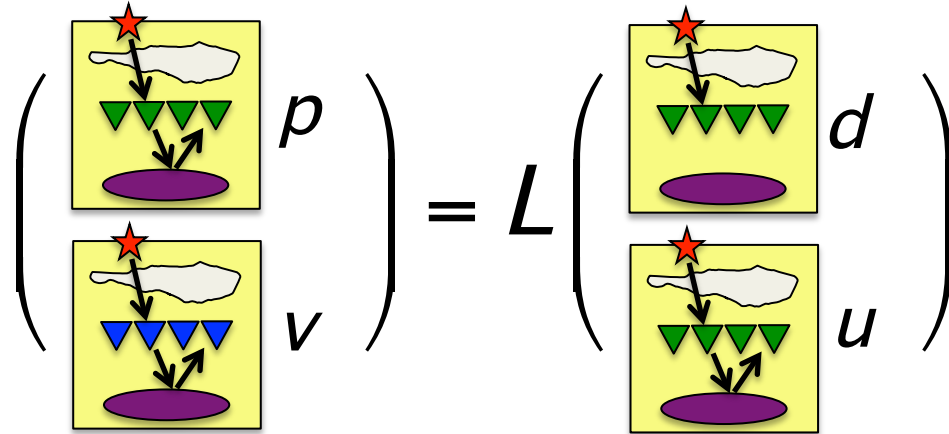


- Local imaging in target area.
- Velocity information in target area is required.
- No velocity information of the overburden / salt is required.
- Recent developments: least-squares imaging
(see presentations by Xiang Li and Tu Ning).

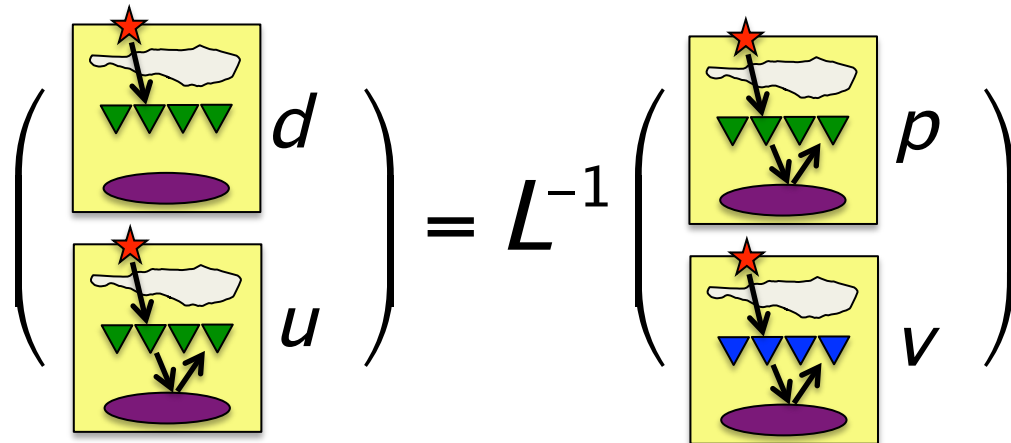
1. Up / down decomposition

Previous work

Forward problem:



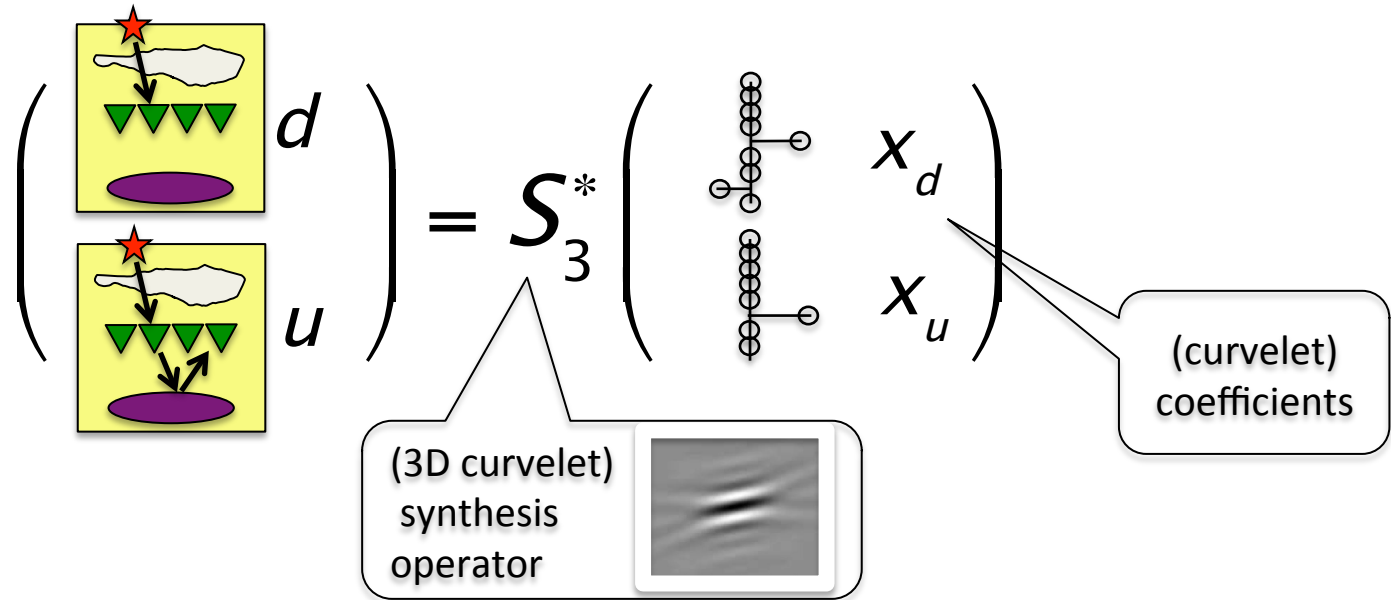
Analytic inversion:



1. Up / down decomposition

Sparse inversion

Synthesis:



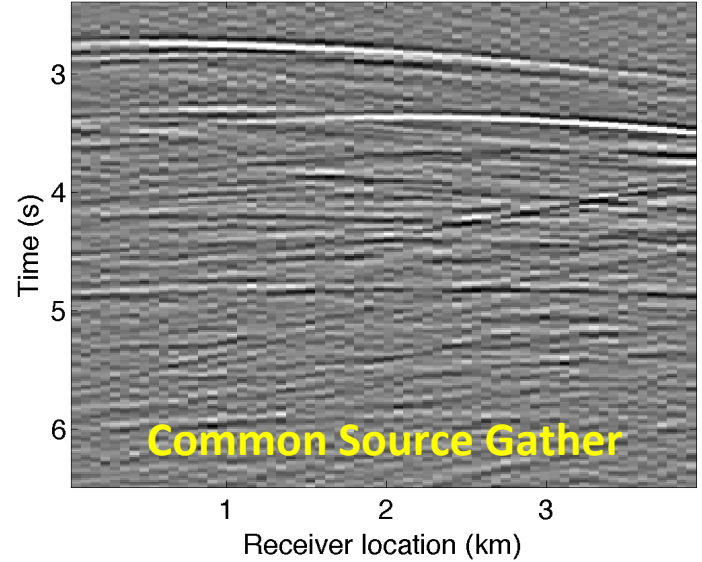
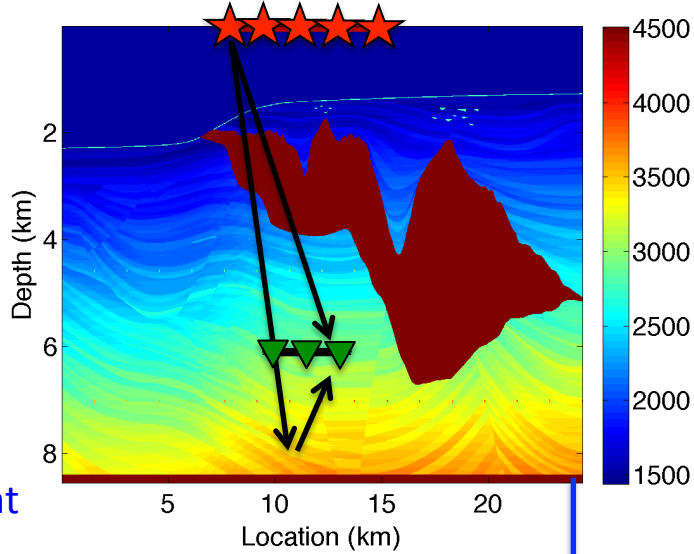
Sparse inversion:

$$\min_{X_d, X_u} \left\| \begin{pmatrix} X_d \\ X_u \end{pmatrix} \right\|_1 \text{ subject to } \left\| \begin{pmatrix} p \\ v \end{pmatrix} - LS_3^* \begin{pmatrix} X_d \\ X_u \end{pmatrix} \right\|_2 \leq \sigma$$

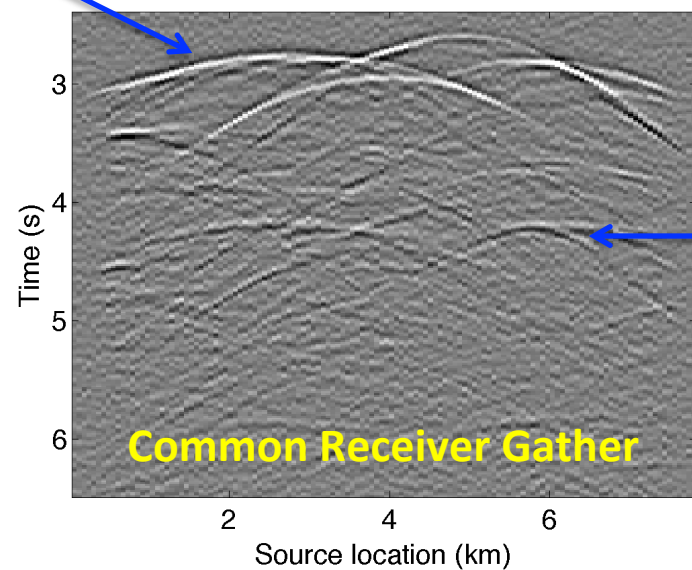
Noise level: σ

1. Up / down decomposition

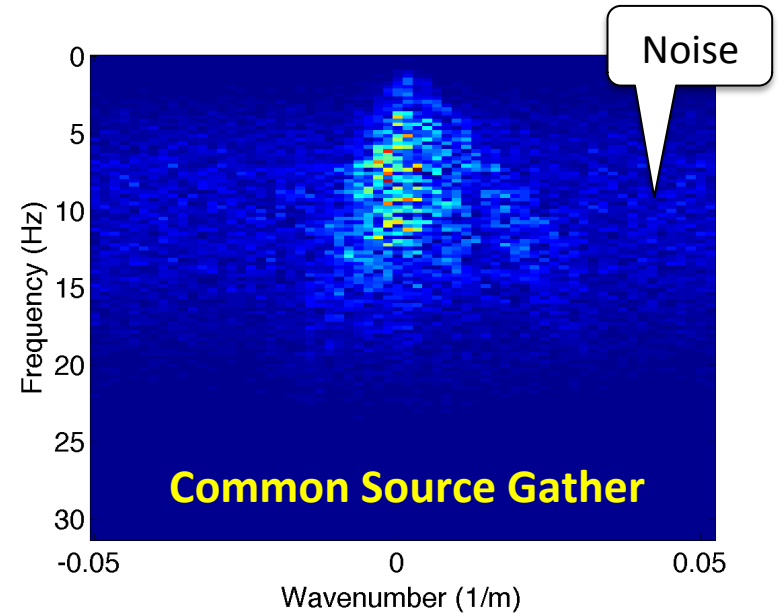
Downhole pressure field (noise added)



incident field

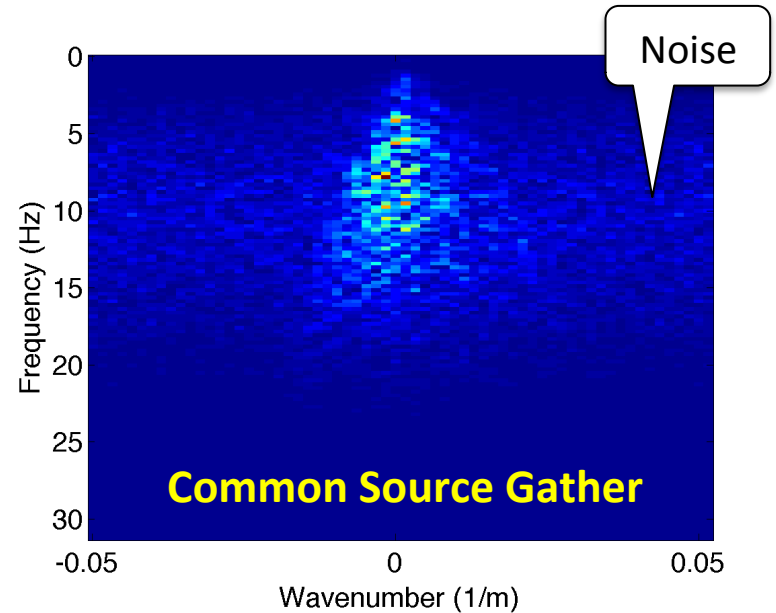
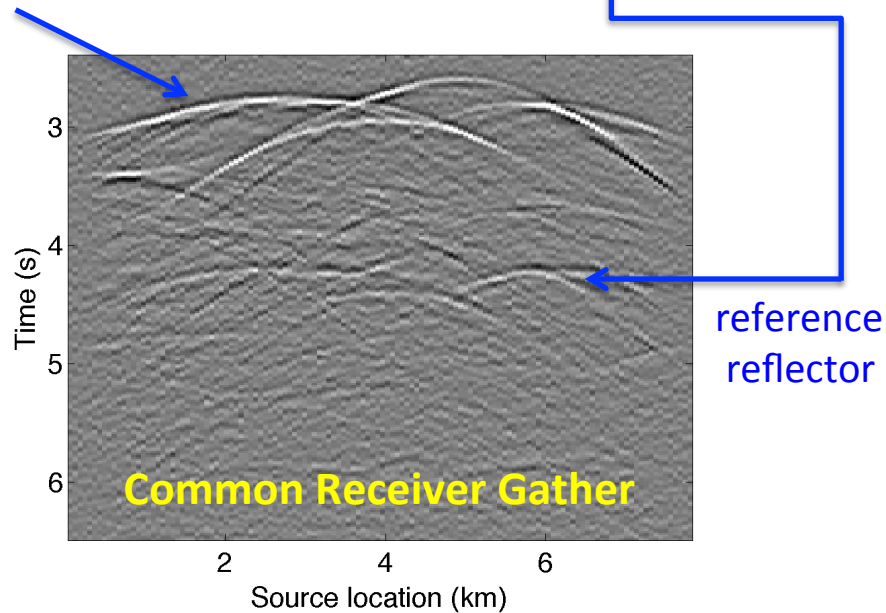
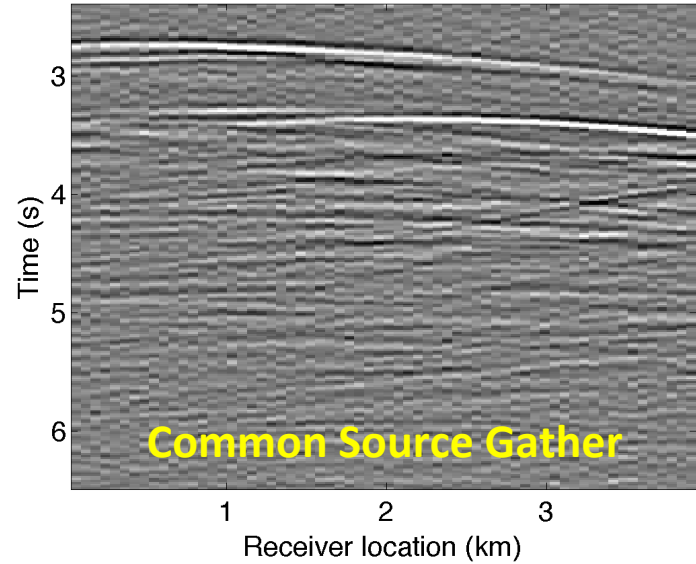
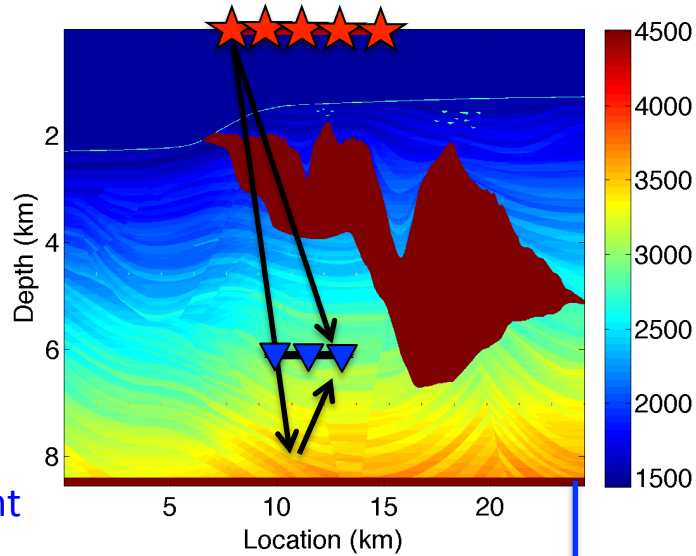


reference reflector



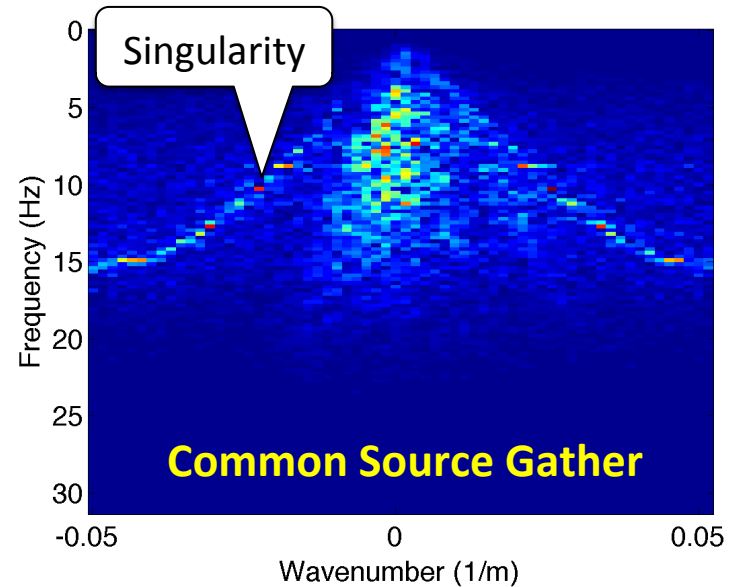
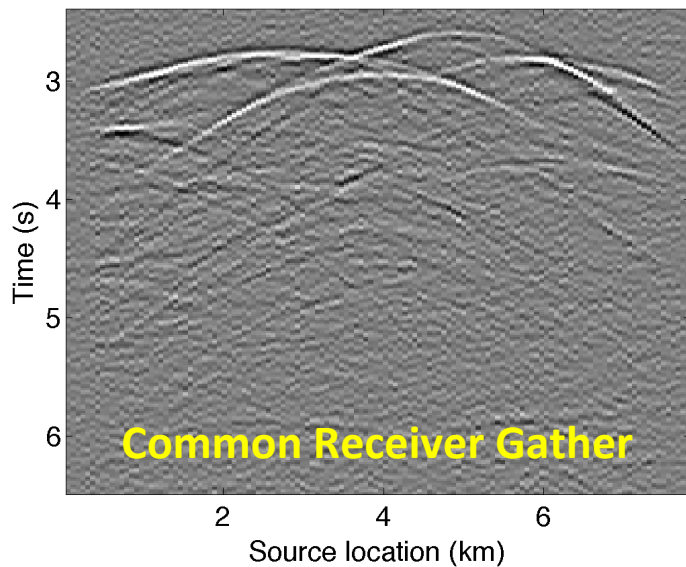
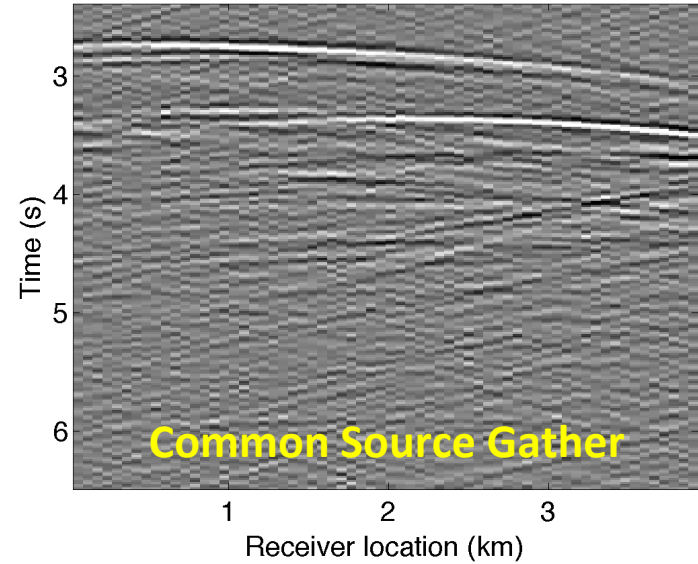
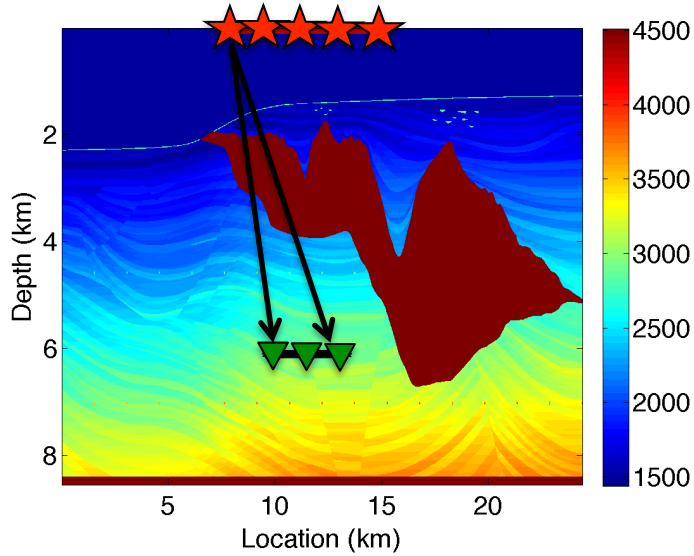
1. Up / down decomposition

Downhole particle velocity field (noise added)



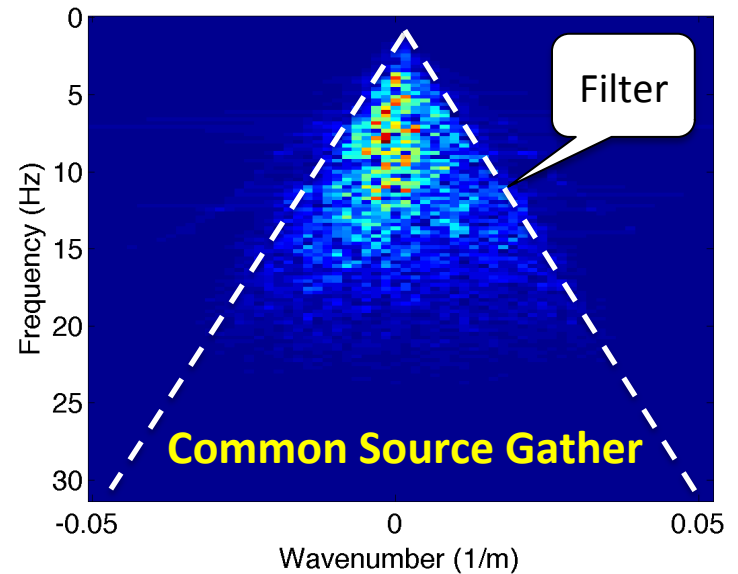
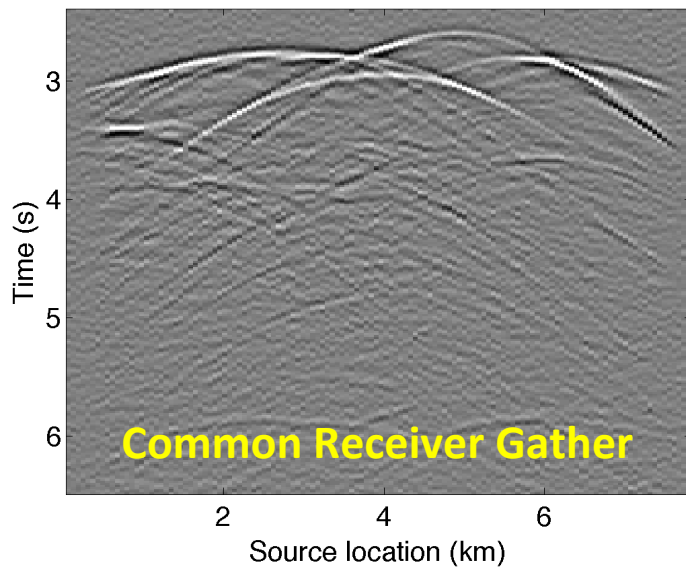
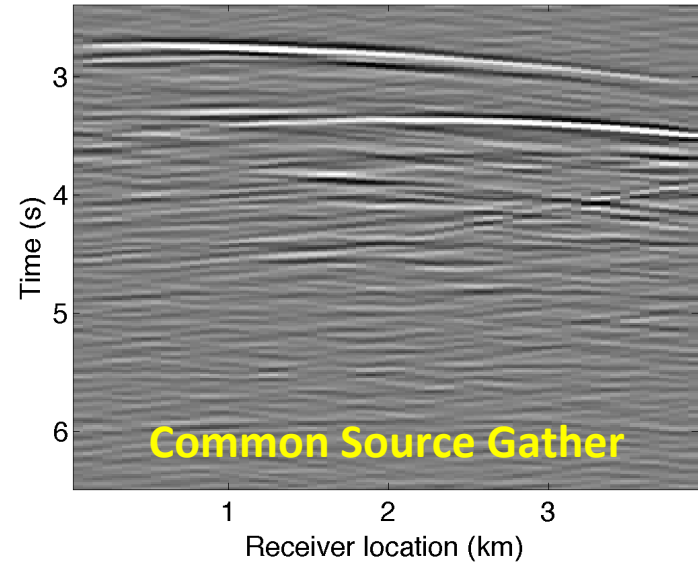
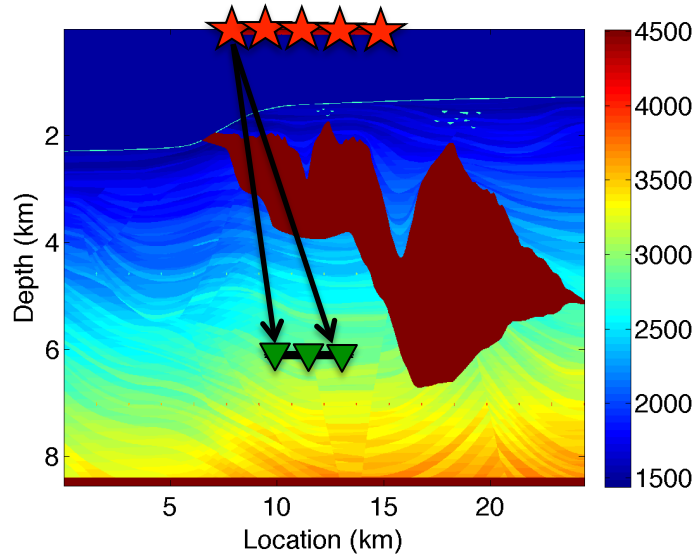
1. Up / down decomposition

Downgoing field retrieved by **analytic decomposition without FK-filter** (noise added)



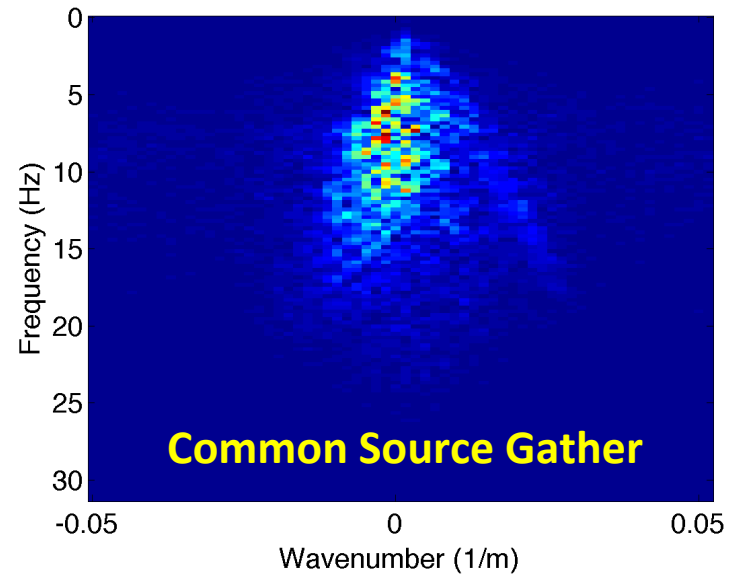
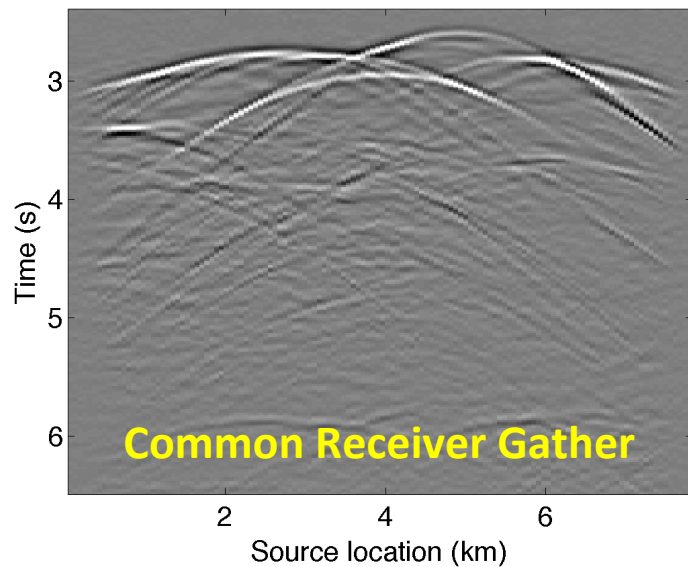
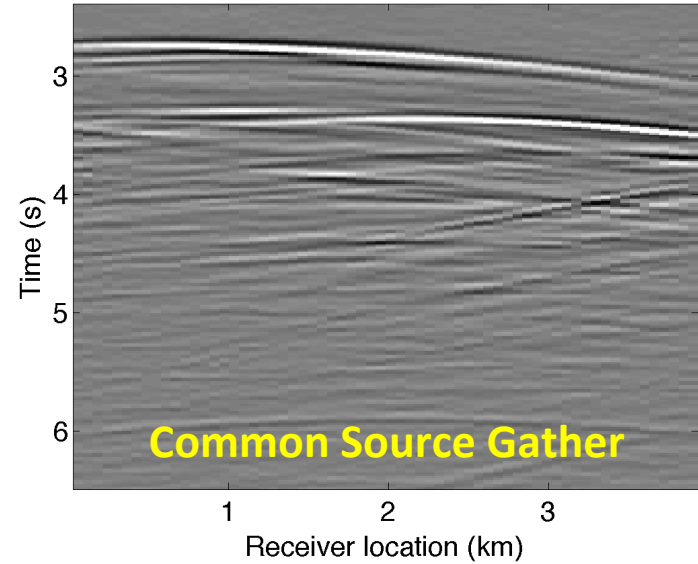
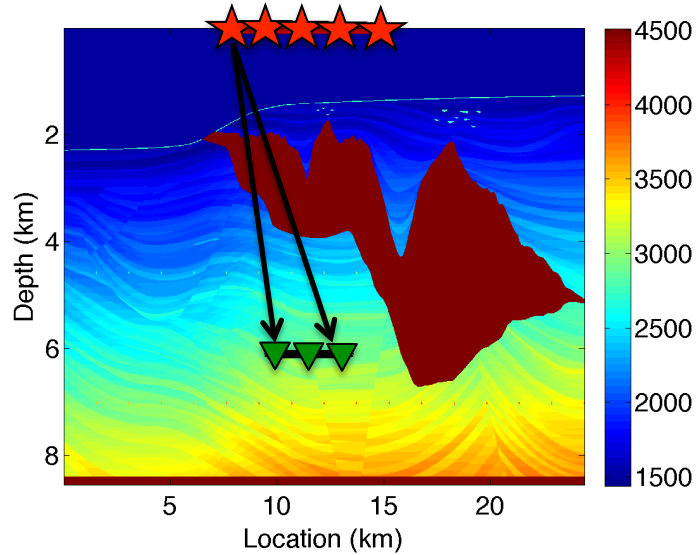
1. Up / down decomposition

Downgoing field retrieved by **analytic decomposition with FK-filter** (noise added)



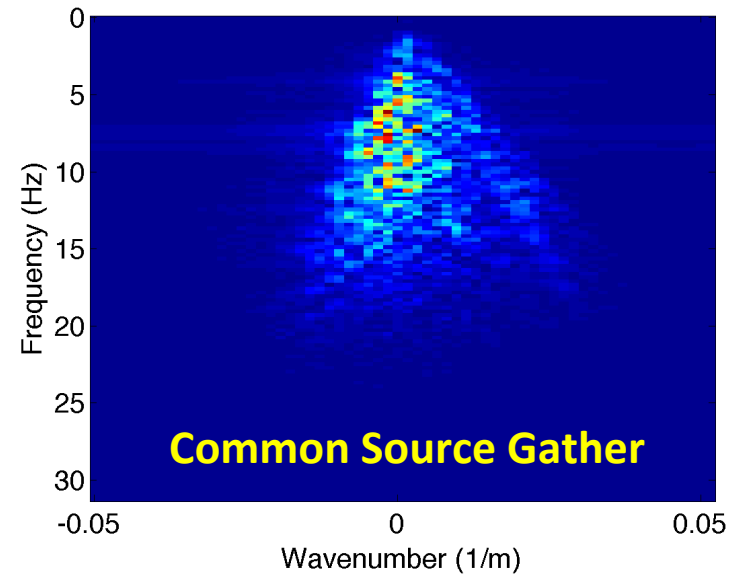
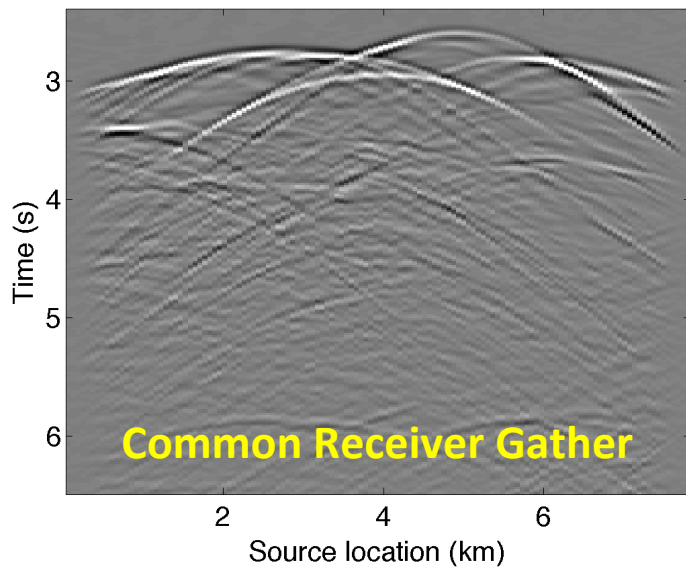
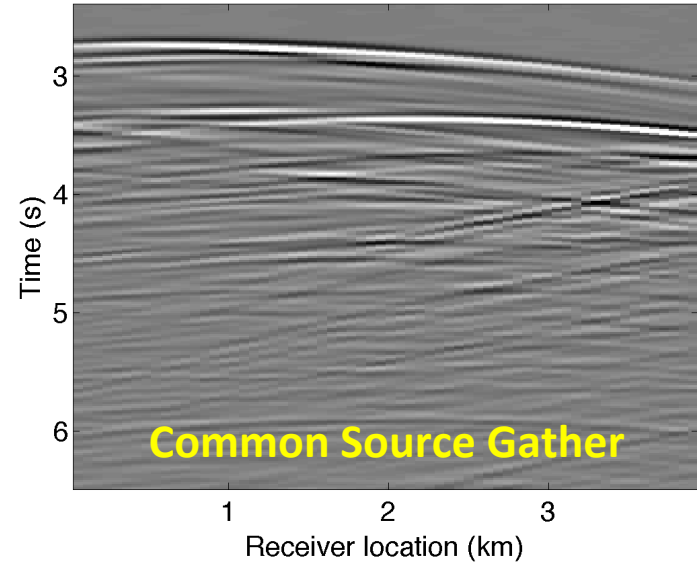
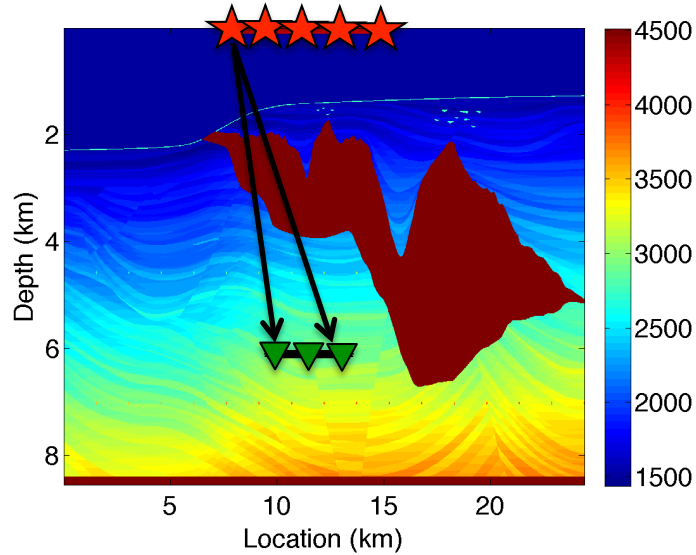
1. Up / down decomposition

Downgoing field retrieved by **sparse decomposition without FK-filter** (noise added)



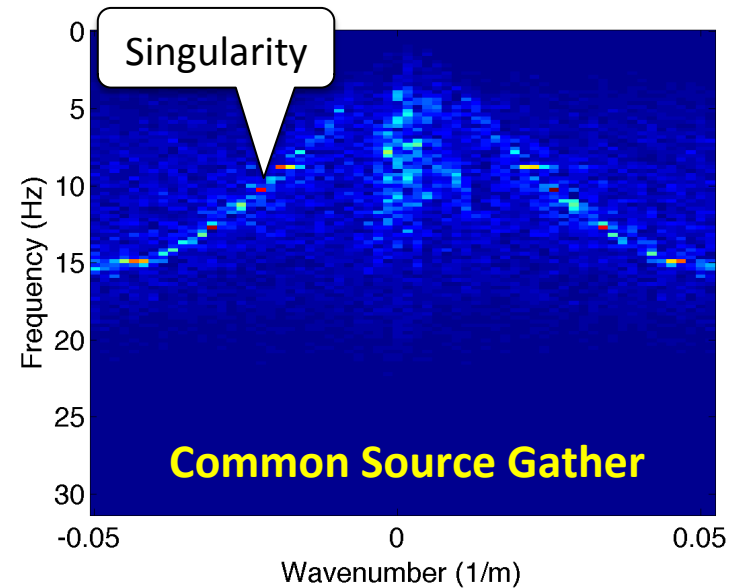
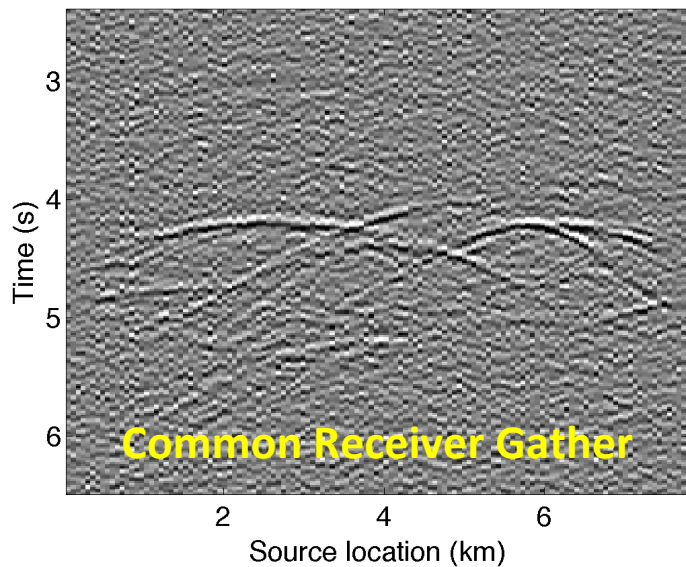
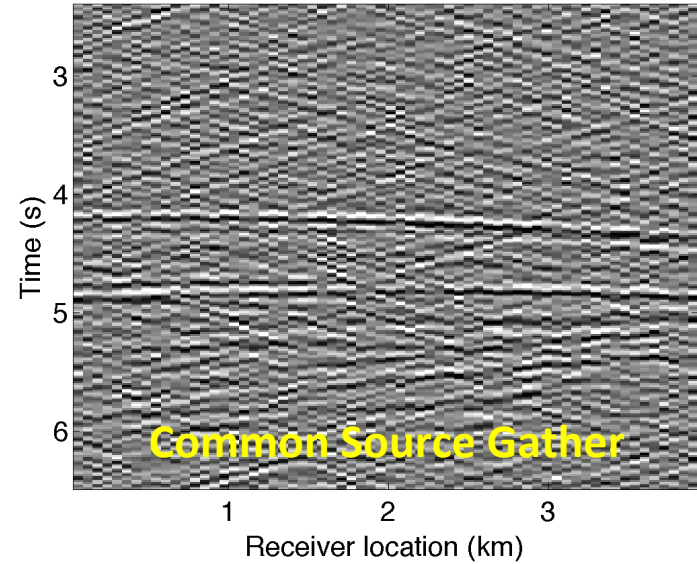
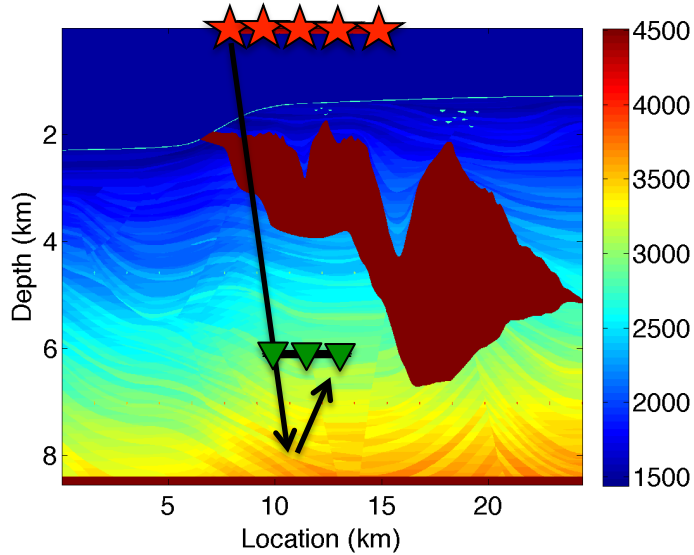
1. Up / down decomposition

Downgoing field retrieved by **sparse decomposition without FK-filter** (no noise added)



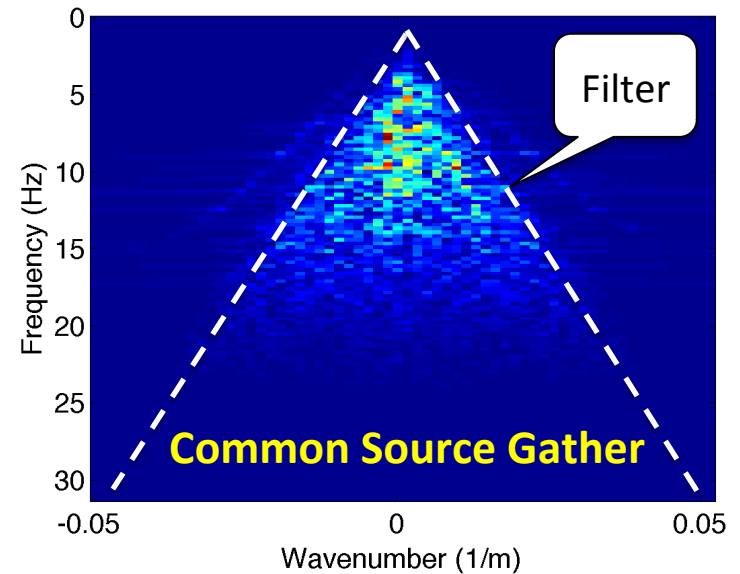
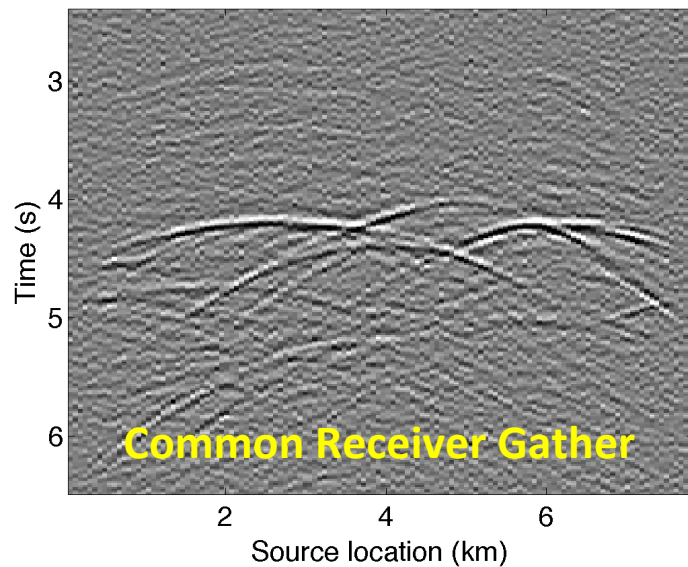
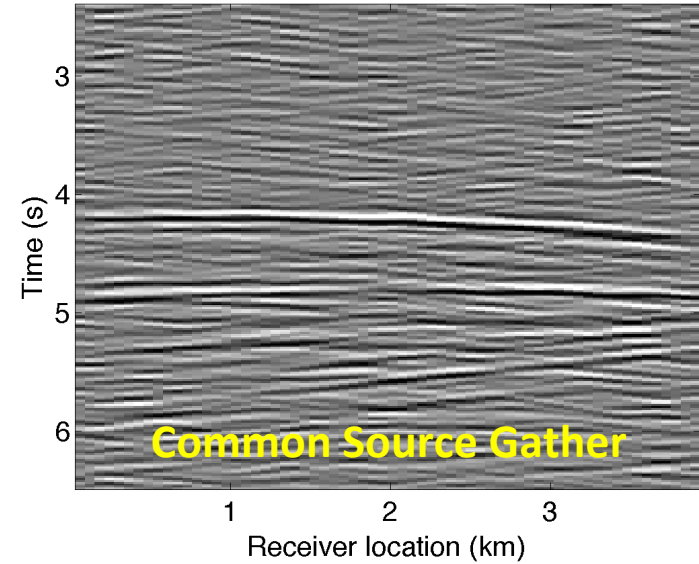
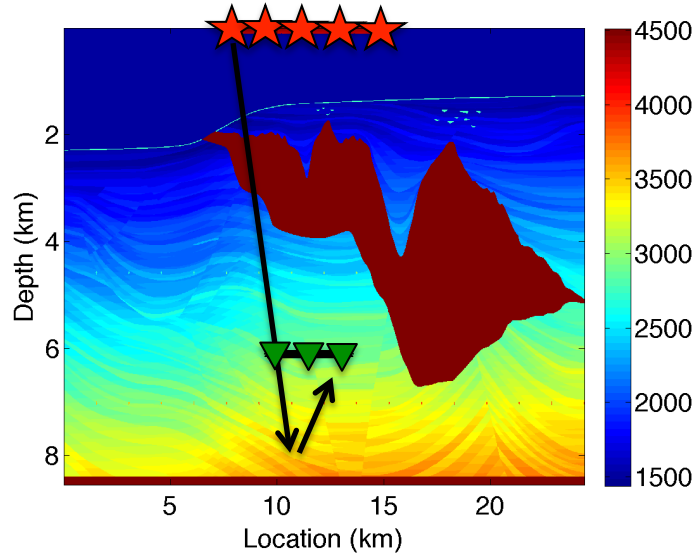
1. Up / down decomposition

Upgoing field retrieved by **analytic decomposition without FK-filter** (noise added)



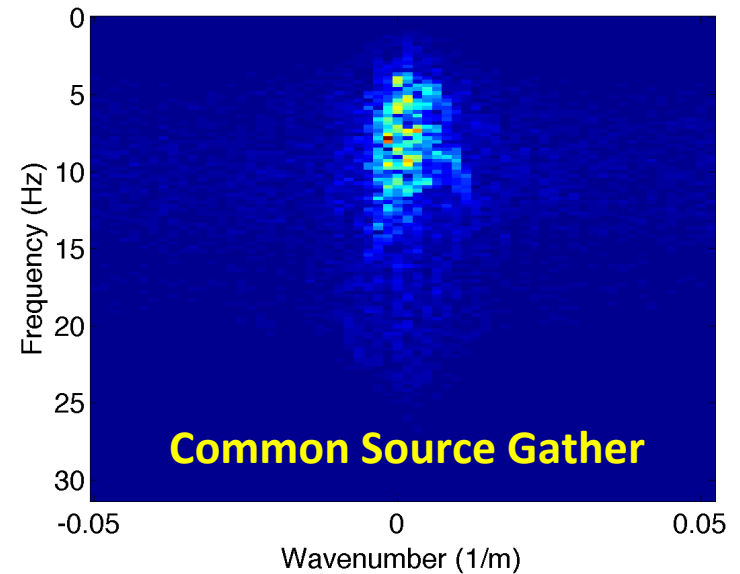
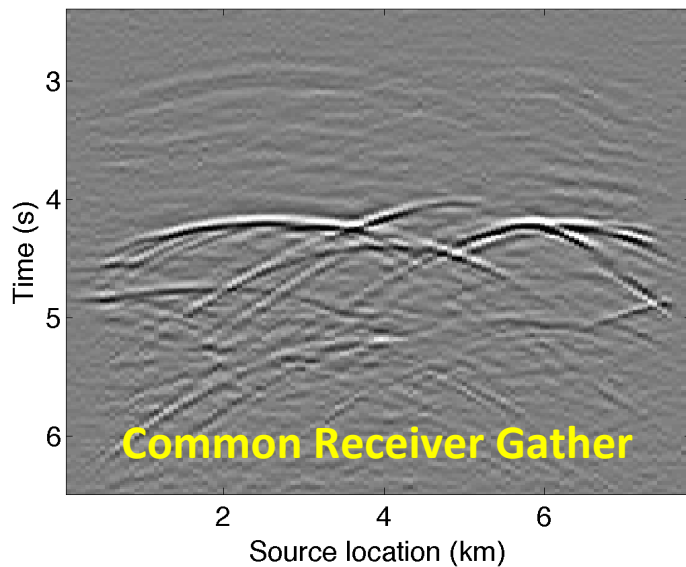
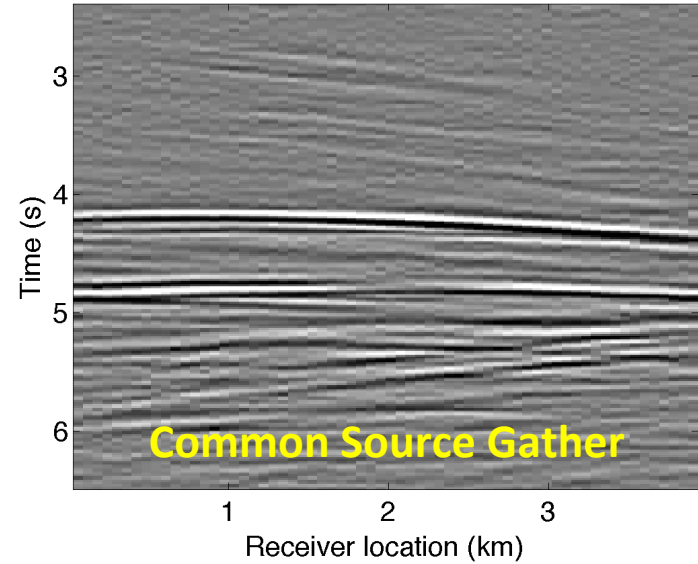
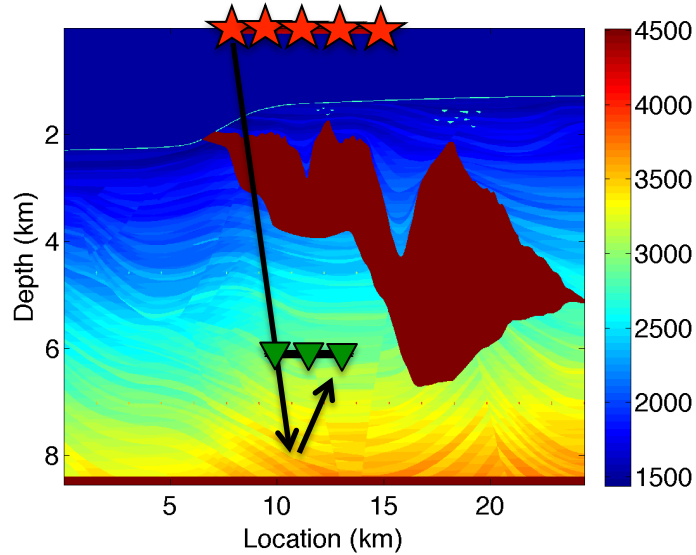
1. Up / down decomposition

Upgoing field retrieved by **analytic decomposition with FK-filter** (noise added)



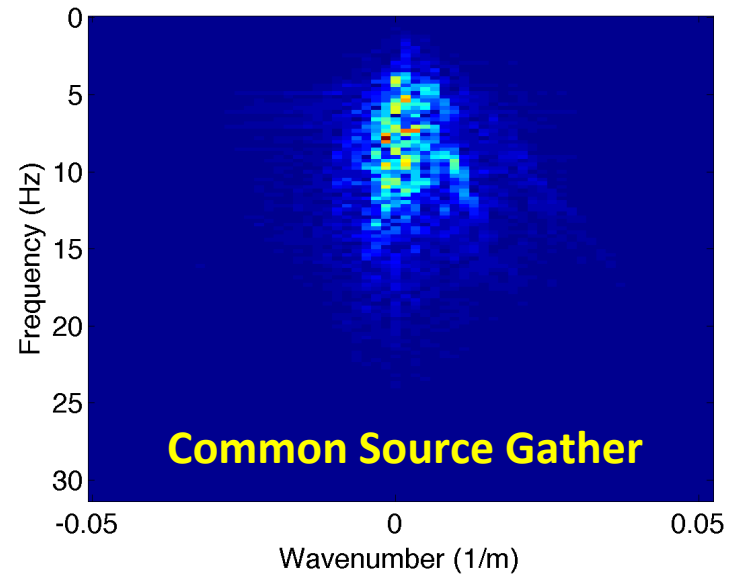
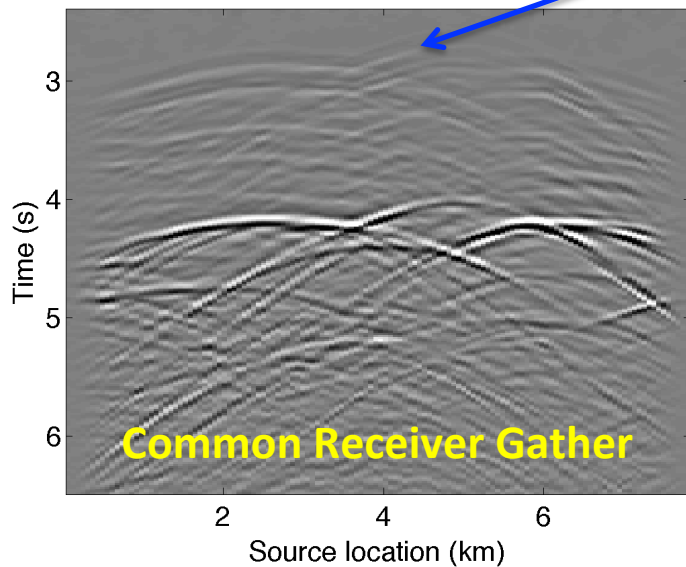
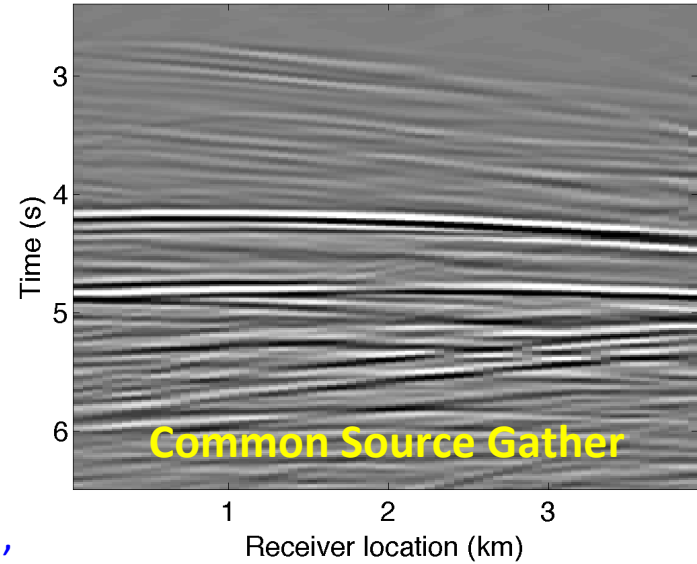
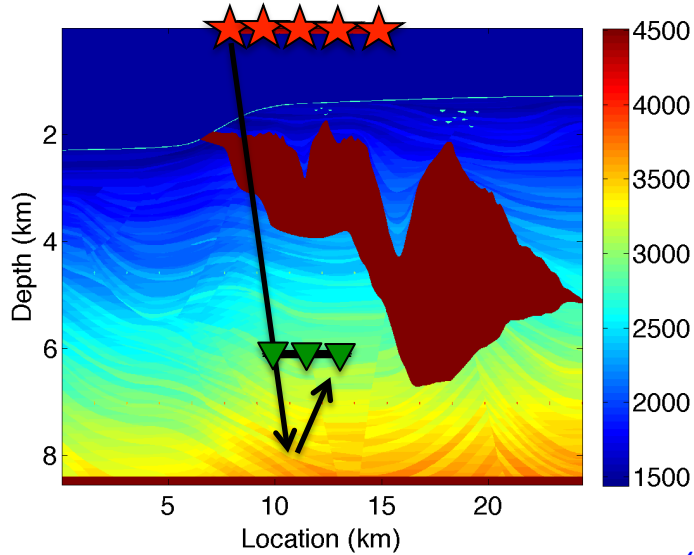
1. Up / down decomposition

Upgoing field retrieved by **sparse decomposition without FK-filter** (noise added)



1. Up / down decomposition

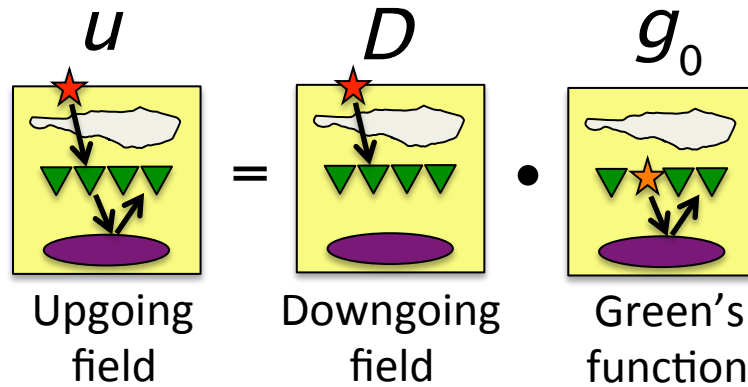
Upgoing field retrieved by **sparse decomposition without FK-filter** (no noise added)



2. Interferometric redatuming

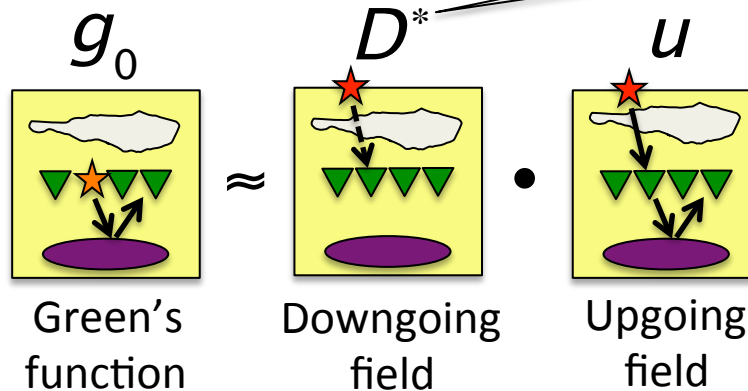
Previous work

Forward problem:



Adjoint

Cross-correlation approach:



Regularization parameter: λ

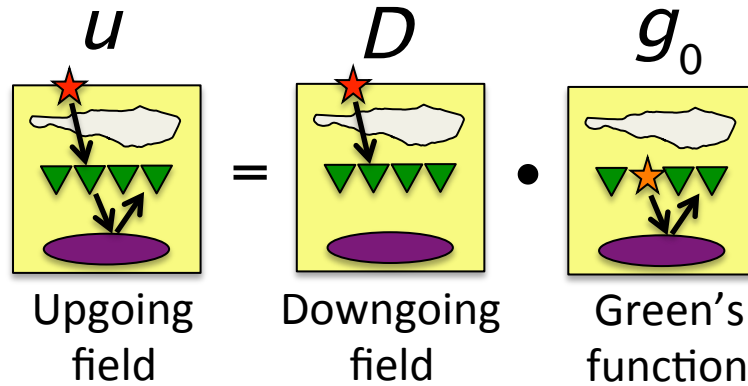
Multidimensional deconvolution approach:

$$\min_{g_0} \|u - Dg_0\|_2^2 + \lambda^2 \|g_0\|_2^2.$$

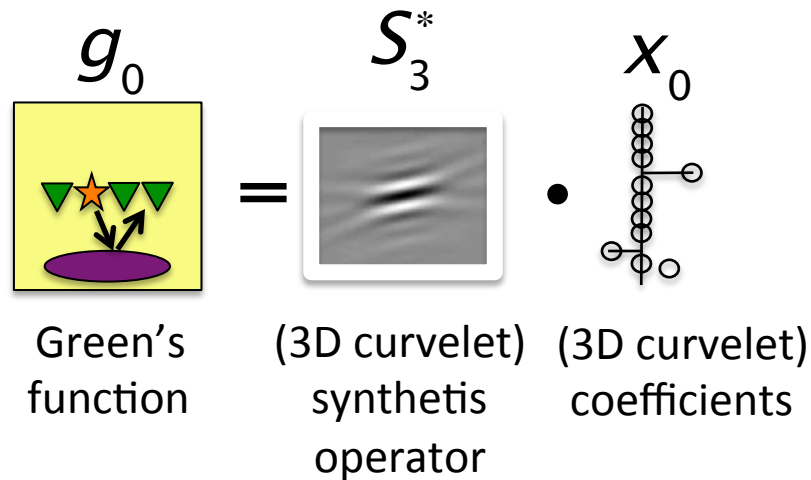
2. Interferometric redatuming

Sparse inversion

Forward problem:



Synthesis:



Sparse inversion:

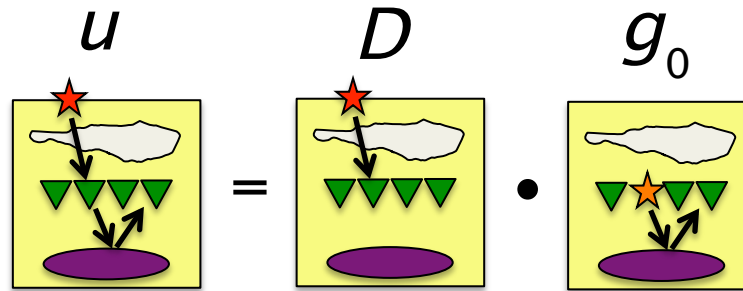
$$\min_{x_0} \|x_0\|_1 \text{ subject to } \|u - DS_3^*x_0\|_2 \leq \sigma$$

Noise level: σ

2. Interferometric redatuming

Which upgoing and downgoing fields to use?

Forward problem:



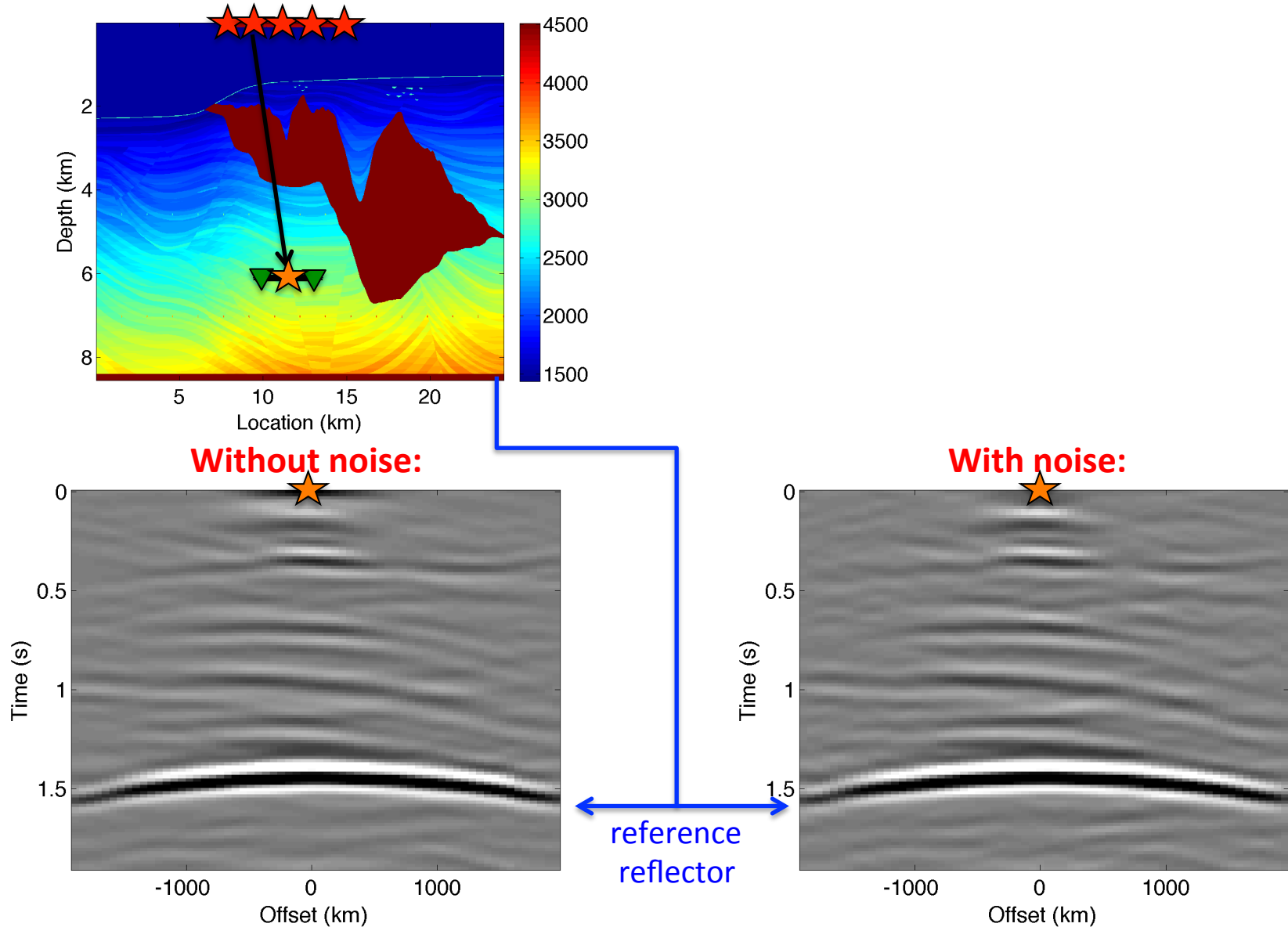
	Downgoing field	Upgoing field
Sensitivity to noise	High	Low
Sensitivity to missing data	Low	High

Suggestion:

Downgoing field ("operator" for redatuming)	Upgoing field ("data" for redatuming)
Sparse inversion	Analytic inversion

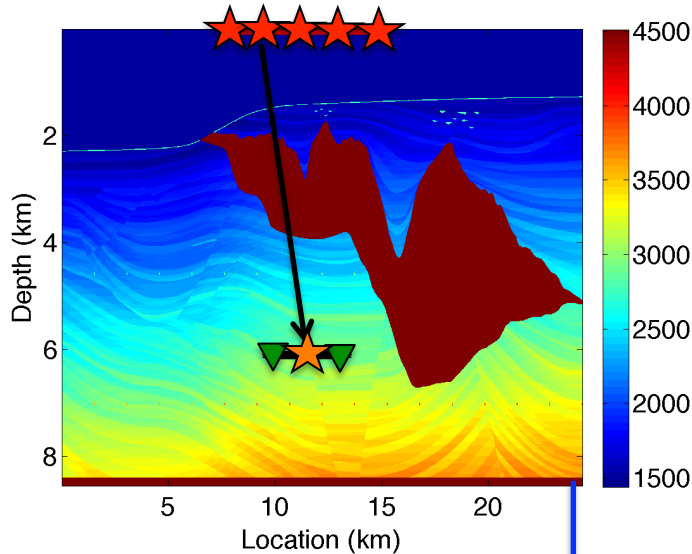
2. Interferometric redatuming

Virtual shot record retrieved with redatuming by **cross-correlation**



2. Interferometric redatuming

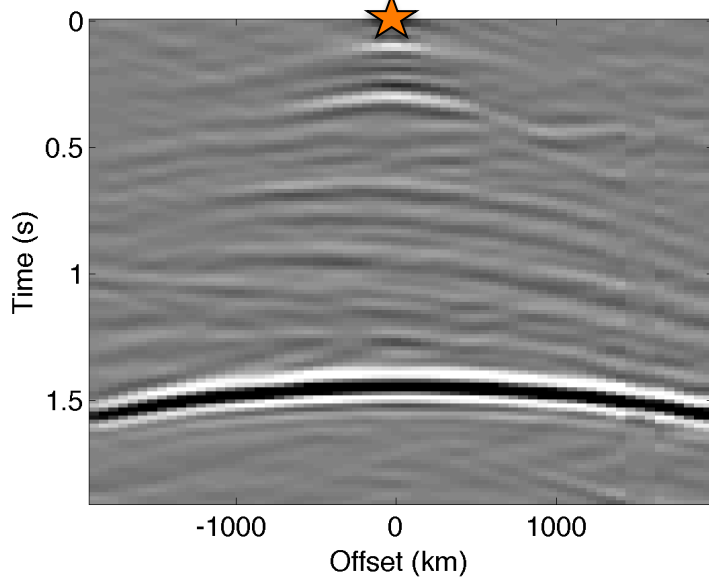
Virtual shot record retrieved with redatuming by **least-squares inversion**



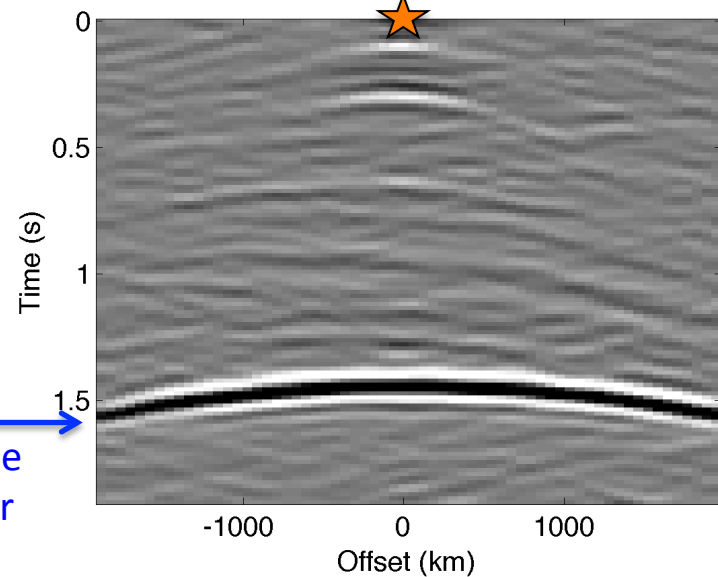
Observations:

- Improved focusing / broadening of frequency spectrum
- Results depend on regularization parameter λ / number of iterations

Without noise:



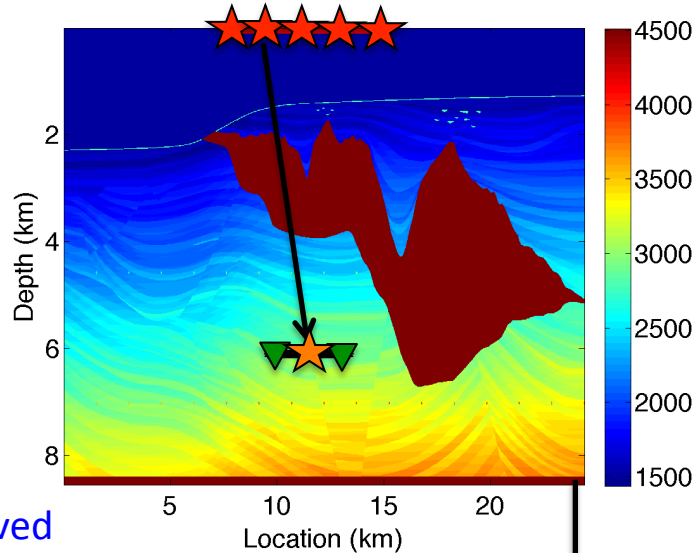
With noise:



reference
reflector

2. Interferometric redatuming

Virtual shot record retrieved with redatuming by **sparse inversion**

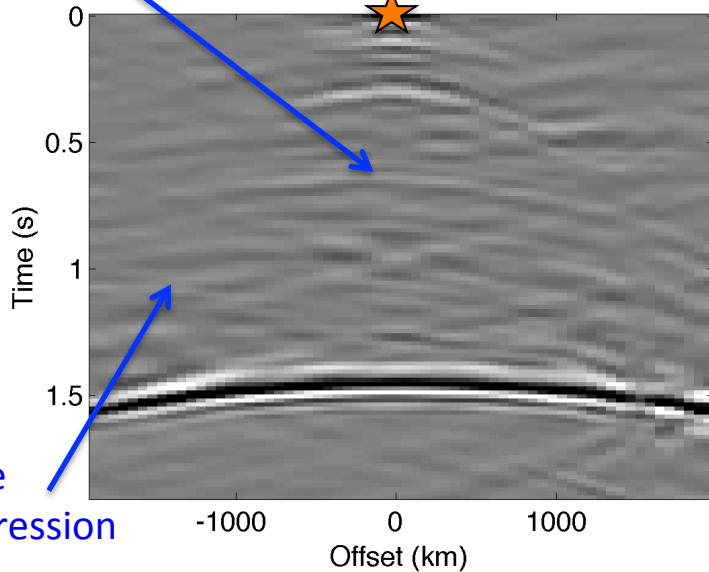


Observations:

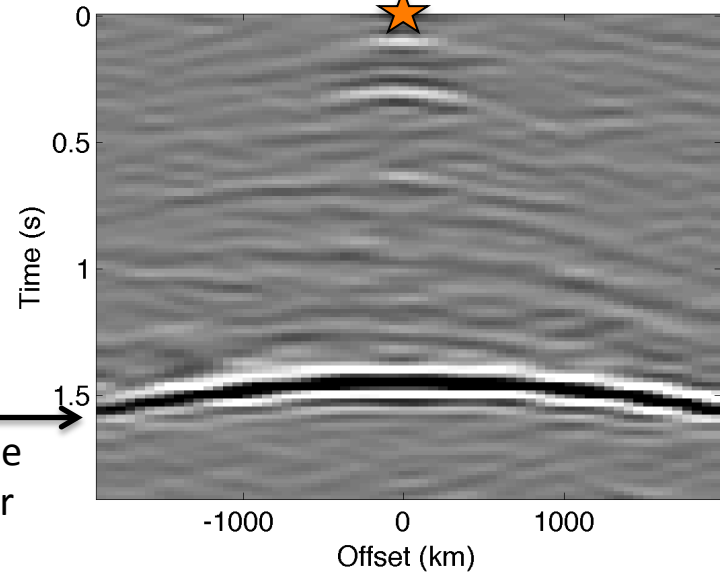
- Improved focusing / broadening of frequency spectrum
- Noise suppression
- Results depend on noise level σ / number of iterations

Improved focusing

Without noise:



With noise:

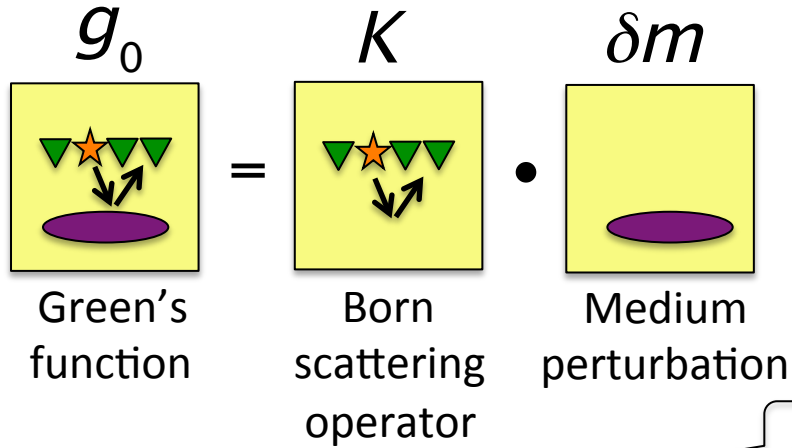


reference reflector

3. Local imaging

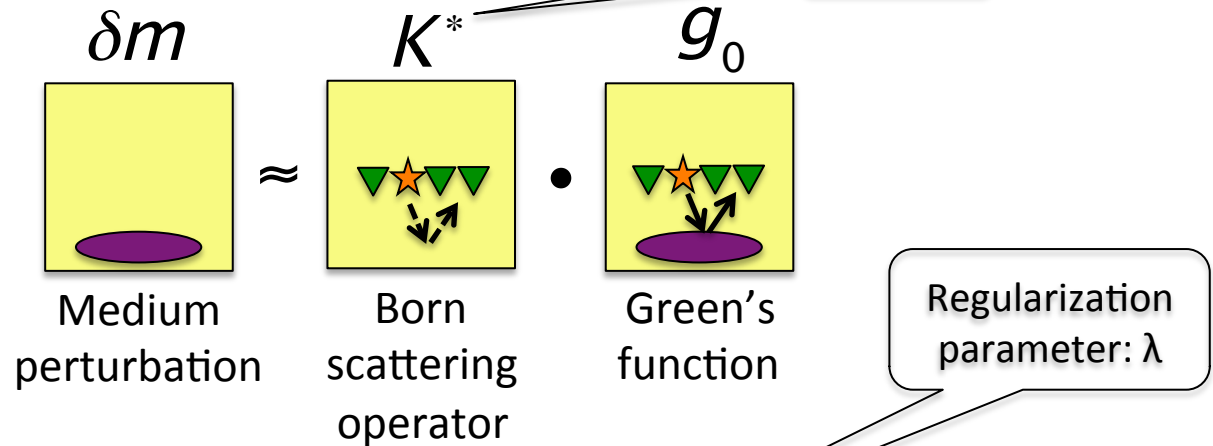
Previous work

Forward problem:



Reverse Time

Migration (RTM):



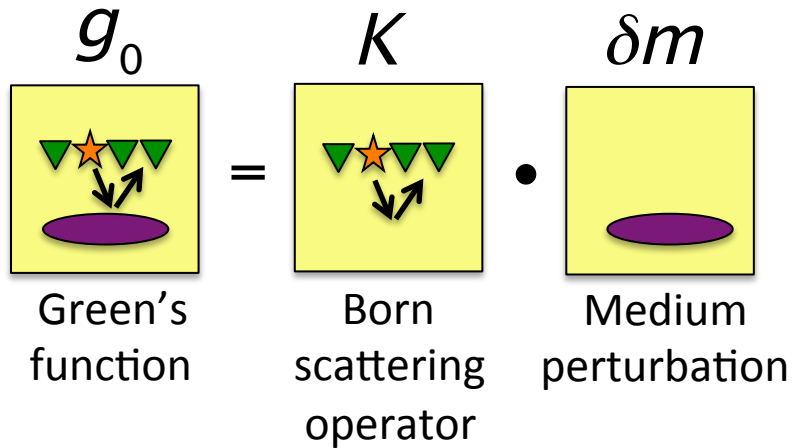
Least-squares imaging:

$$\min_{\delta m} \left\| g_0 - K \delta m \right\|_2^2 + \lambda^2 \left\| \delta m \right\|_2^2$$

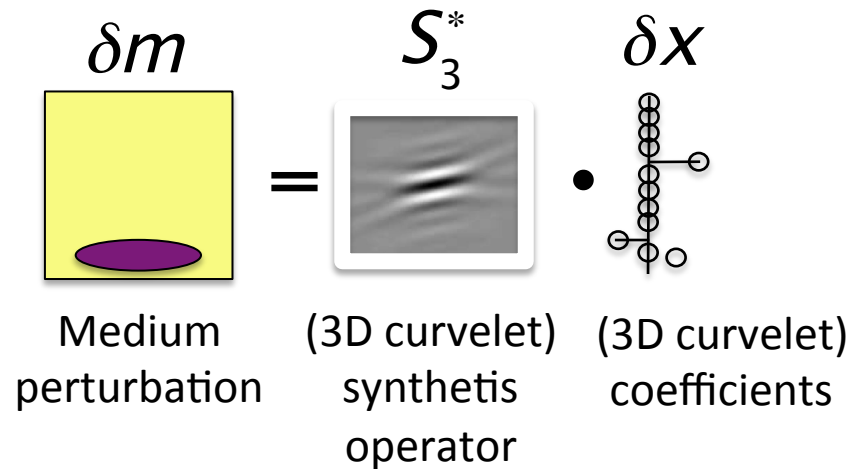
2. Interferometric redatuming

Sparse redatuming

Forward problem:



Synthesis:



Sparse inversion:

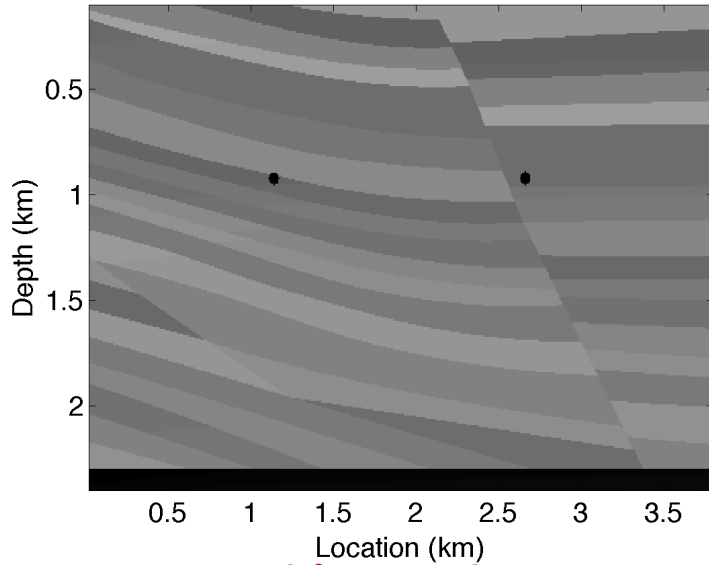
$$\min_{\delta x} \|\delta x\|_1 \text{ subject to } \|g_0 - KS_3^* \delta x\|_2 \leq \sigma$$

Noise level: σ

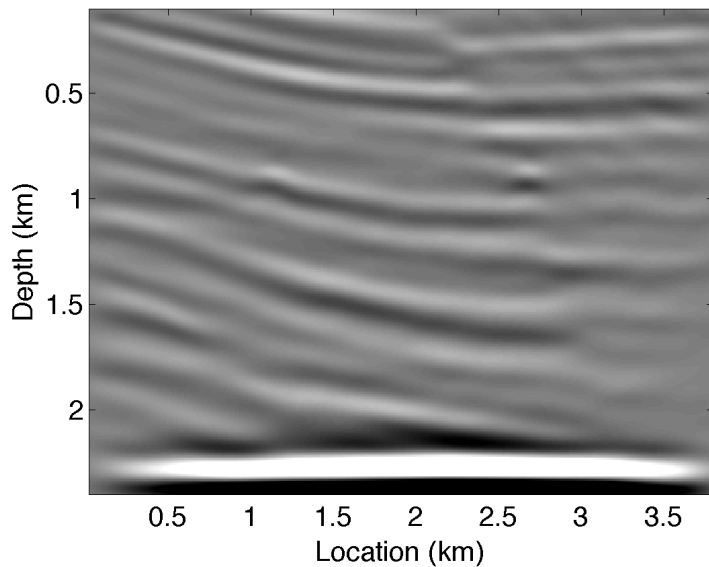
3. Local imaging

RTM image from data retrieved with redatuming by **cross-correlation**

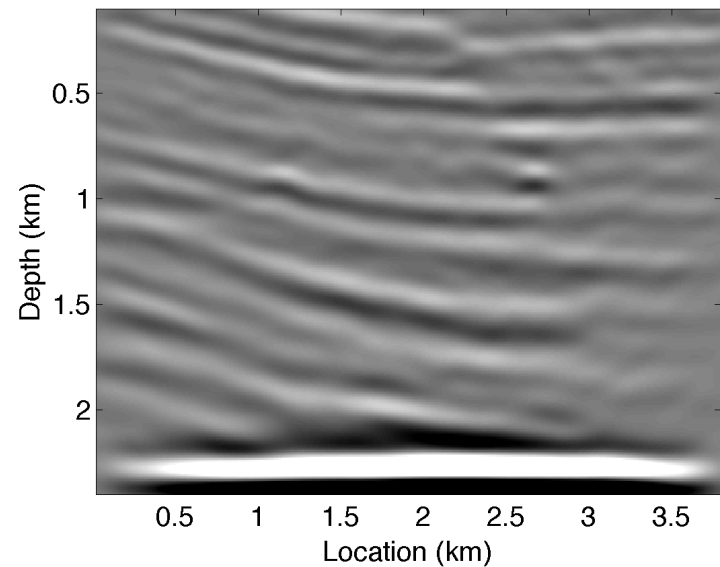
True medium perturbation:



Without noise:



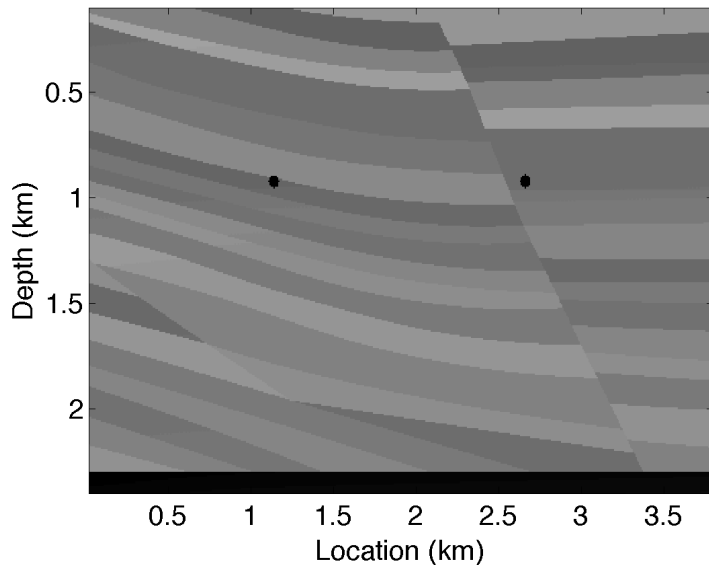
With noise:



3. Local imaging

RTM image from data retrieved with redatuming by **least-squares inversion**

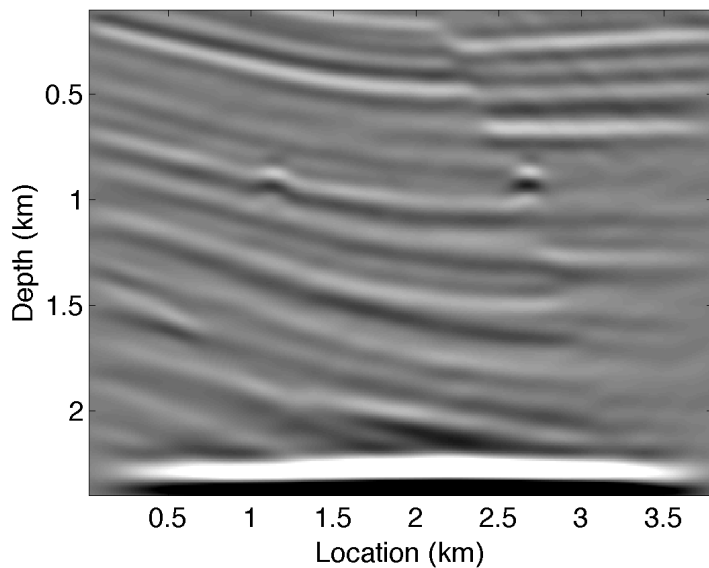
True medium perturbation:



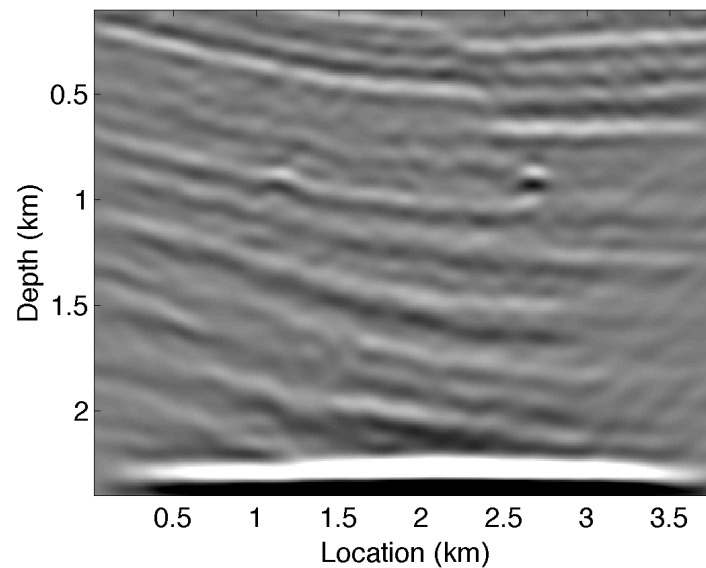
Observations:

- Improved focusing / broadening frequency spectrum

Without noise:



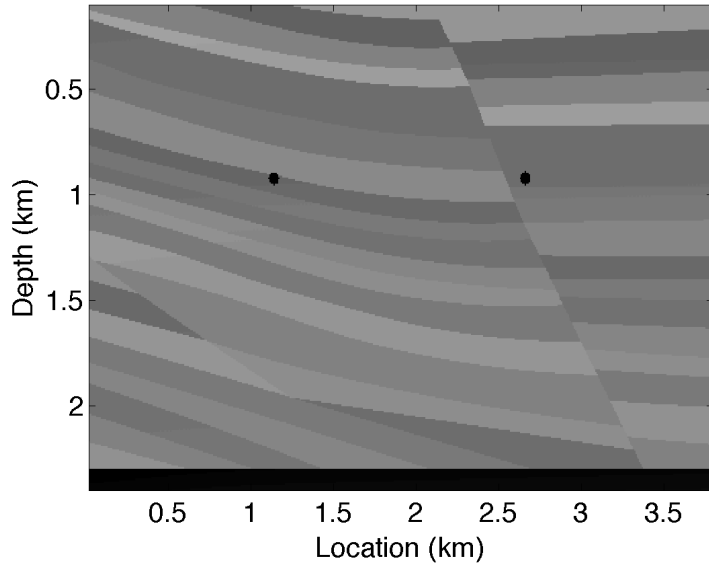
With noise:



3. Local imaging

RTM image from data retrieved with redatuming by **sparse inversion**

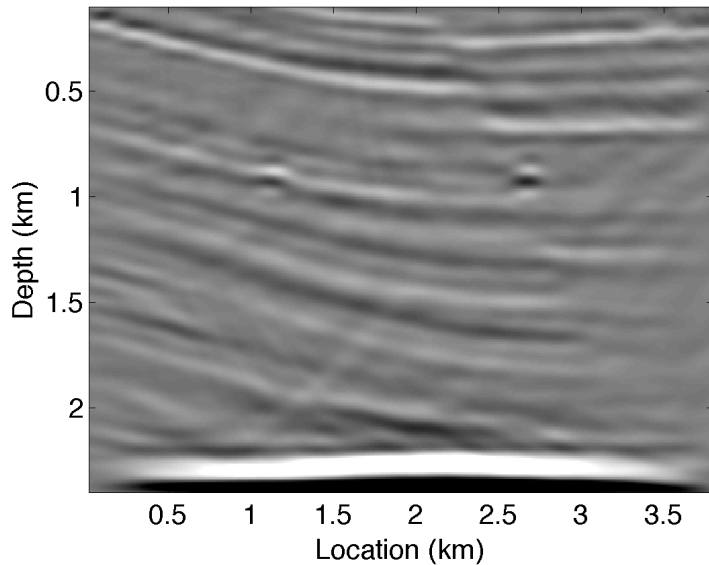
True medium perturbation:



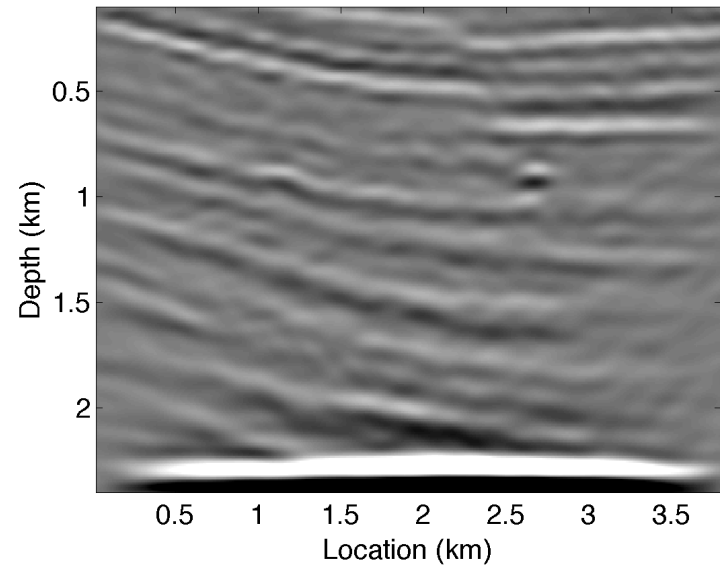
Observations:

- Improved focusing / broadening frequency spectrum.
- **Additional high frequency artifacts.**
- Not much difference for the case with noise (depends on parameters).

Without noise:



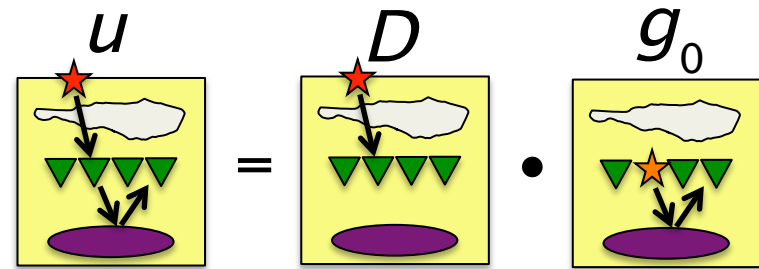
With noise:



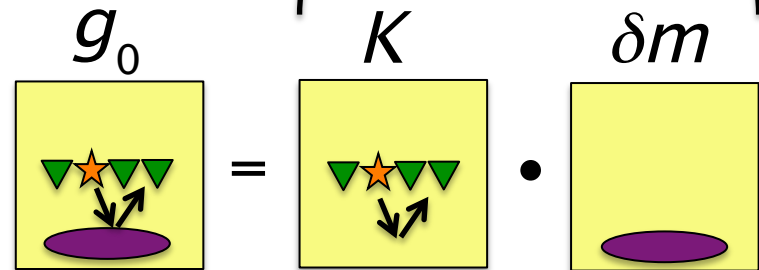
4. Ongoing research

Joint interferometric redatuming and imaging

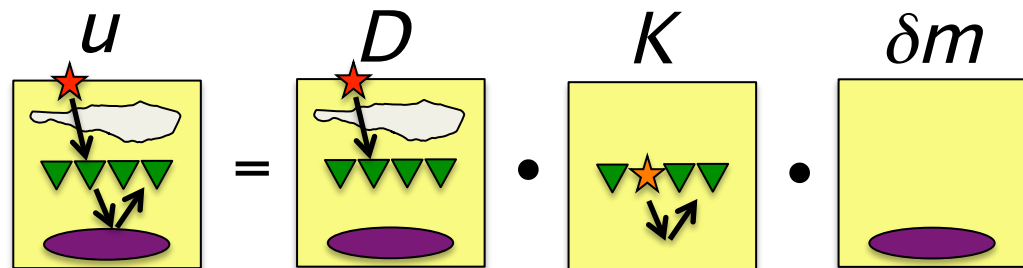
Redatuming problem:



Imaging problem:



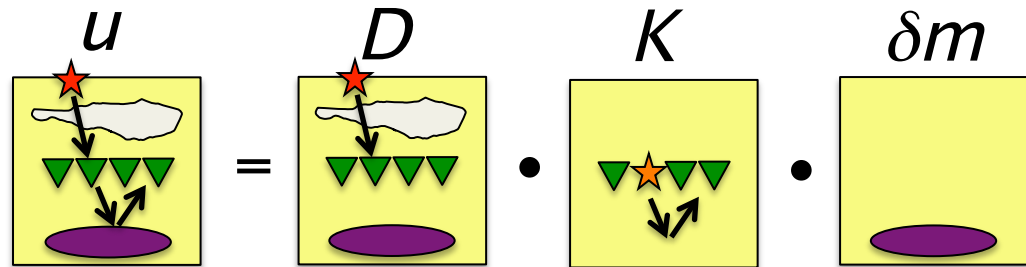
➔ Joint problem:



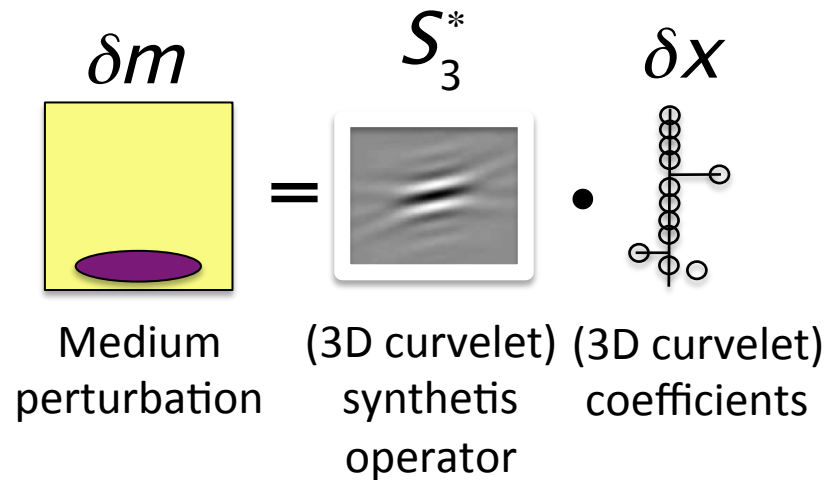
4. Ongoing research

Sparse redatuming and imaging

Forward problem:



Synthesis:



Sparse inversion:

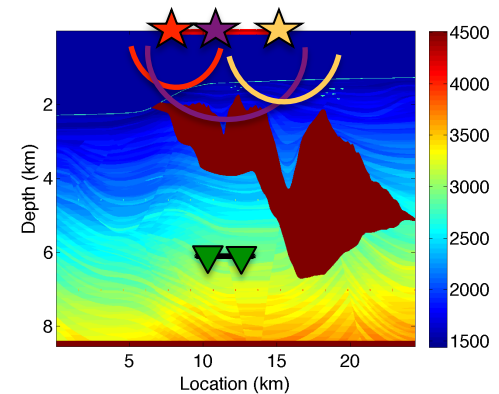
$$\min_{\delta x} \|\delta x\|_1 \text{ subject to } \|u - DK S_3^* \delta x\|_2 \leq \sigma$$

Noise level: σ

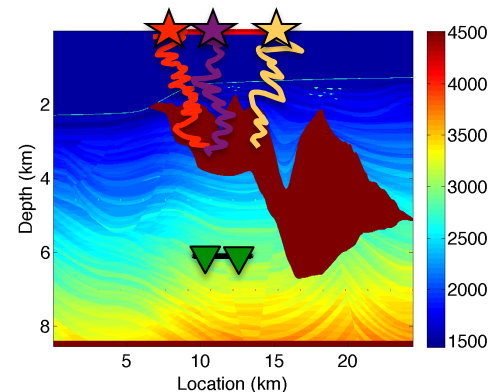
4. Ongoing research

Other ideas

- Dimensionality reduction by randomized low-rank approximation of operator P (*see presentation by Bander Jumah*)
- Simultaneous sources with randomized excitation times and positions:



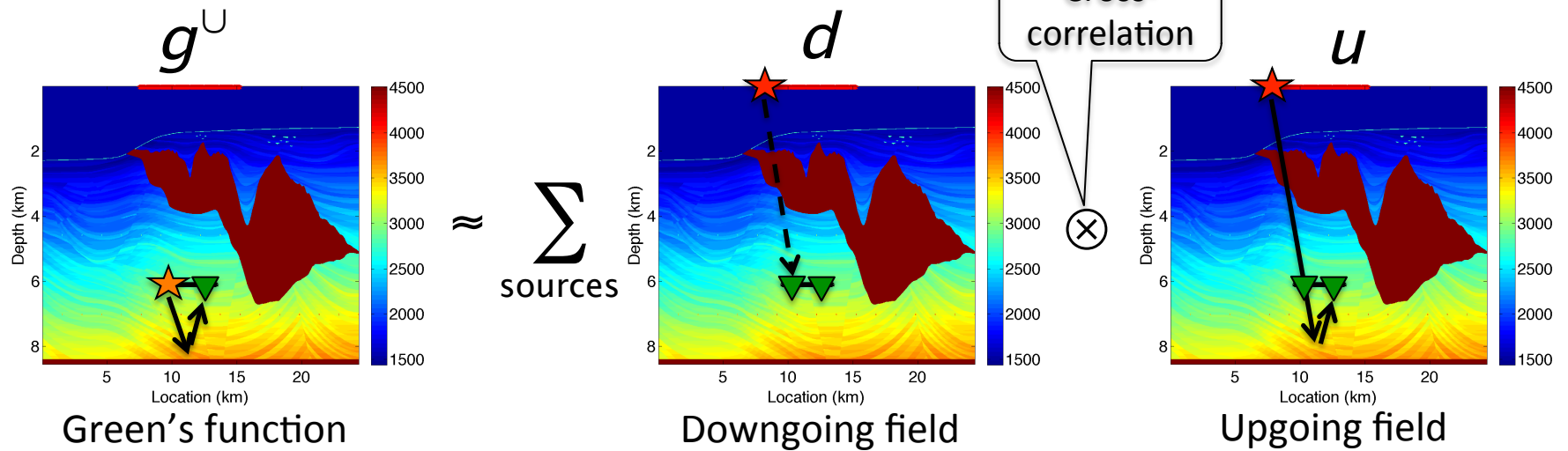
- Passive seismic interferometry:



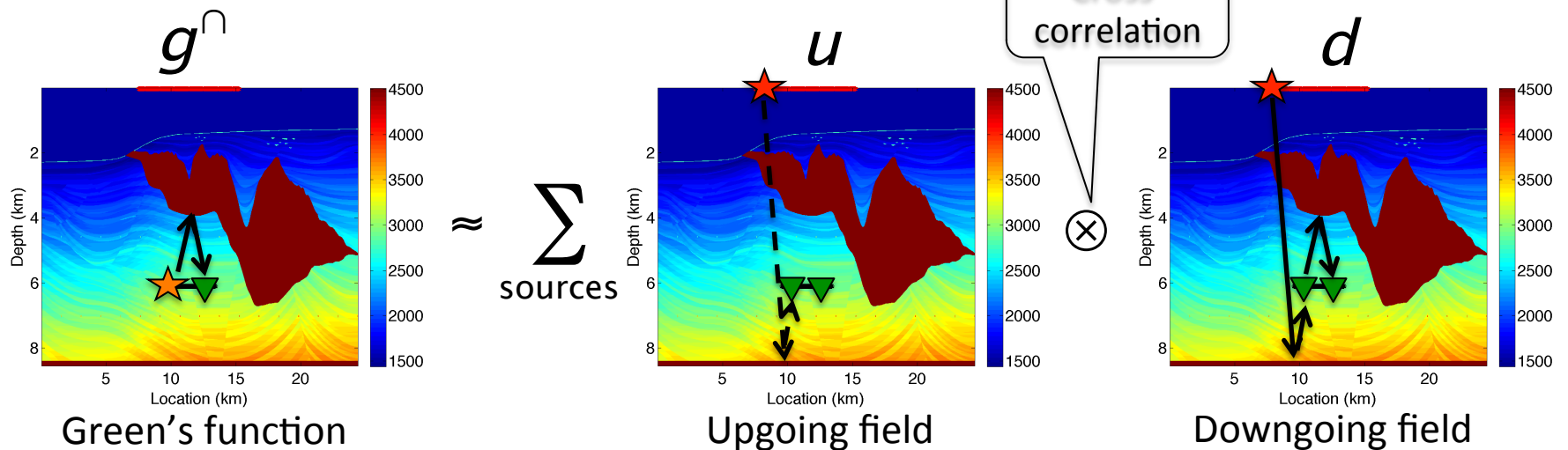
4. Ongoing research

Upward radiating virtual sources

Downward radiating virtual sources:

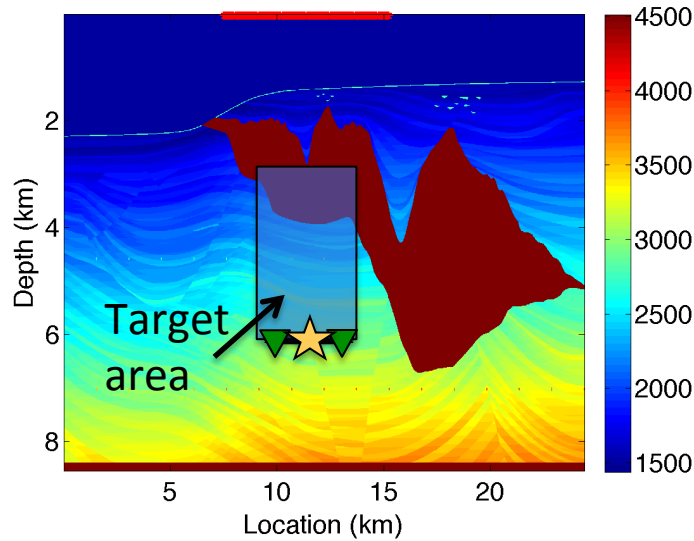


Upward radiating virtual sources:

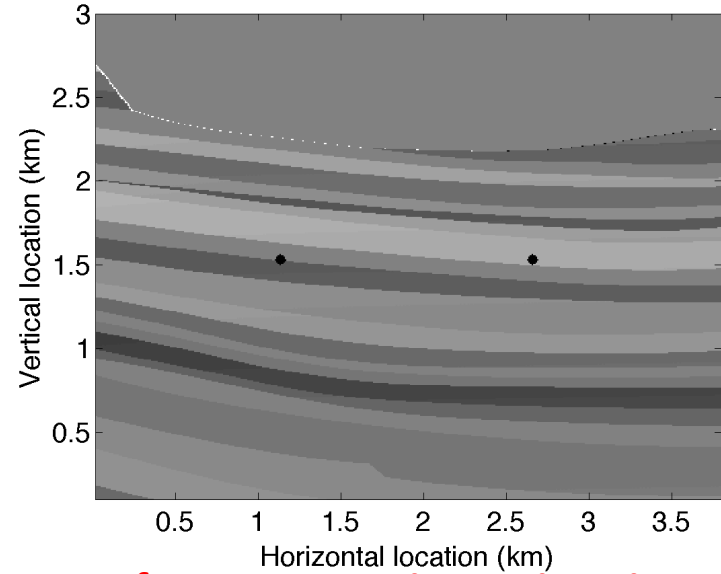


4. Ongoing research

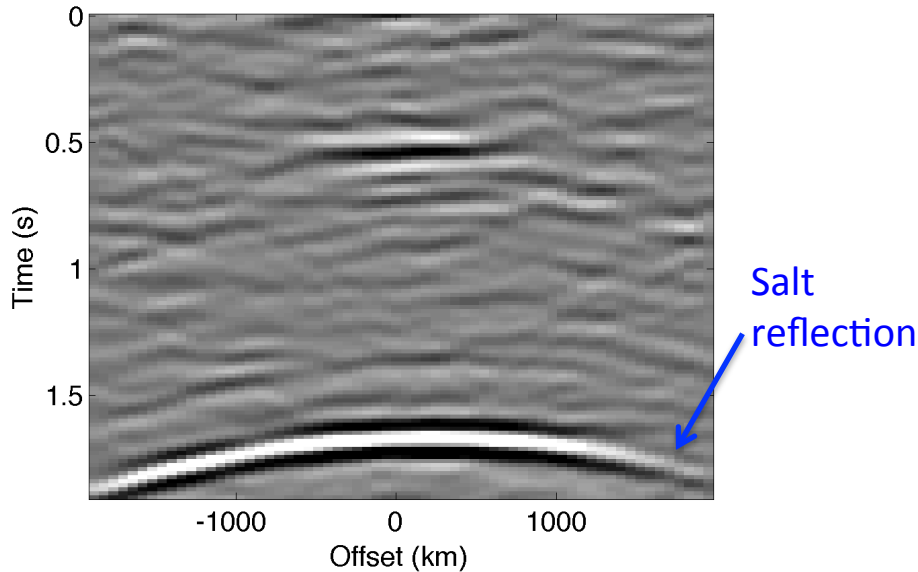
Upward radiating virtual sources



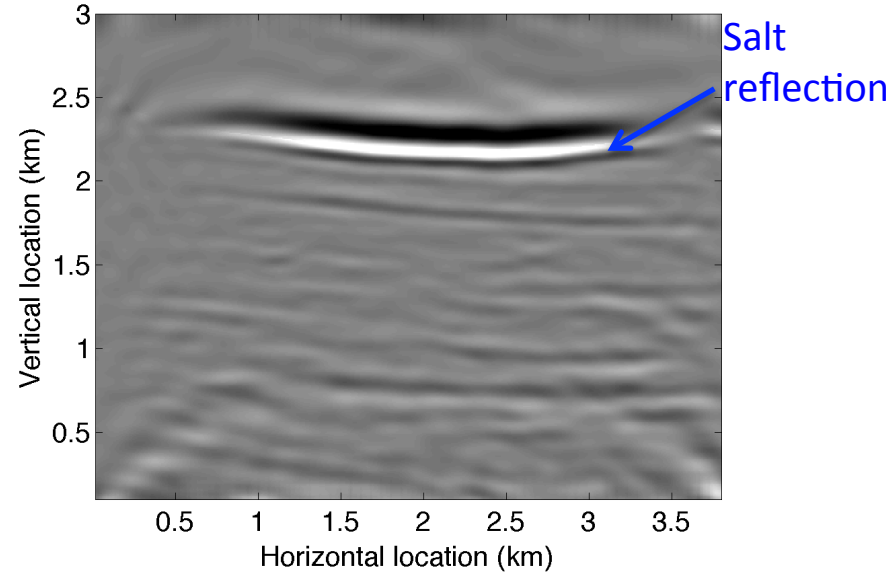
True medium perturbation:



Retrieved virtual shot:



After Reverse Time Migration:



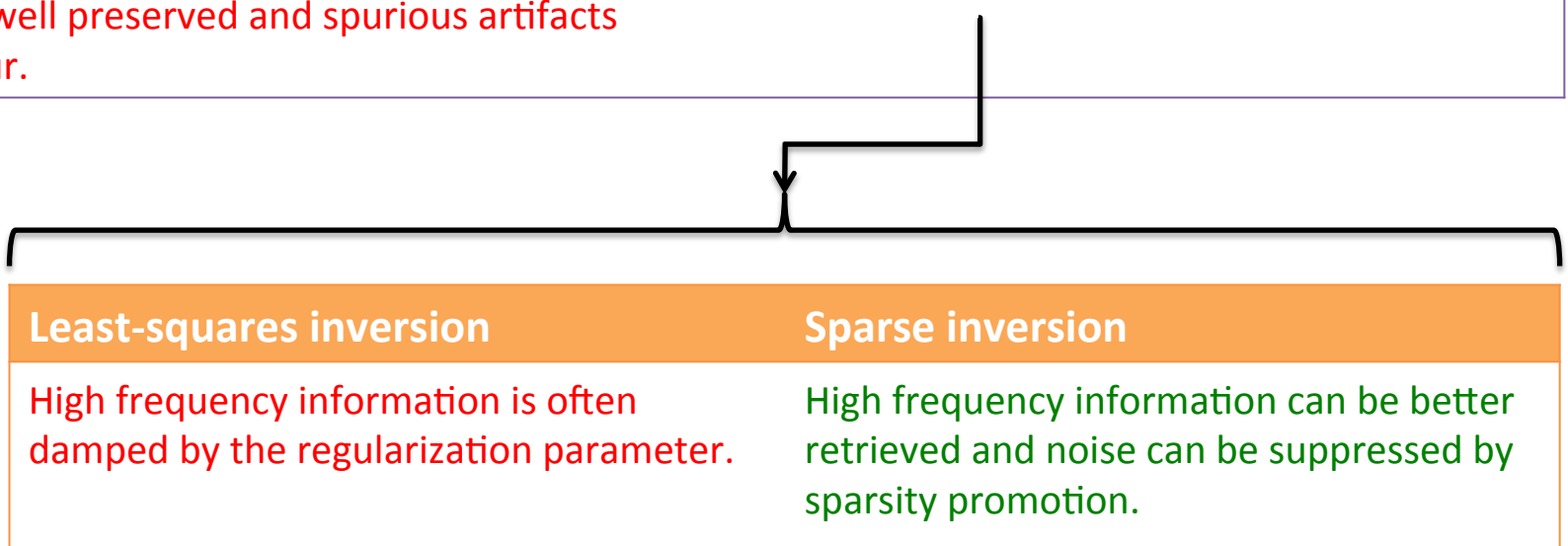
Conclusions

1. Wavefield decomposition

Analytic decomposition	Sparse decomposition
Sensitive for noise, contains singularities at critical angles.	Robust in a noisy environment.
Preserves all events.	Weak events 'hiding' below the noise level can be removed.

2. Interferometric redatuming

Redatuming by cross-correlation	Redatuming by inversion
Virtual source records are blurred, amplitudes are not well preserved and spurious artifacts can occur.	Virtual sources are more accurately focused.

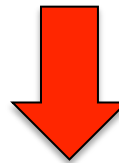
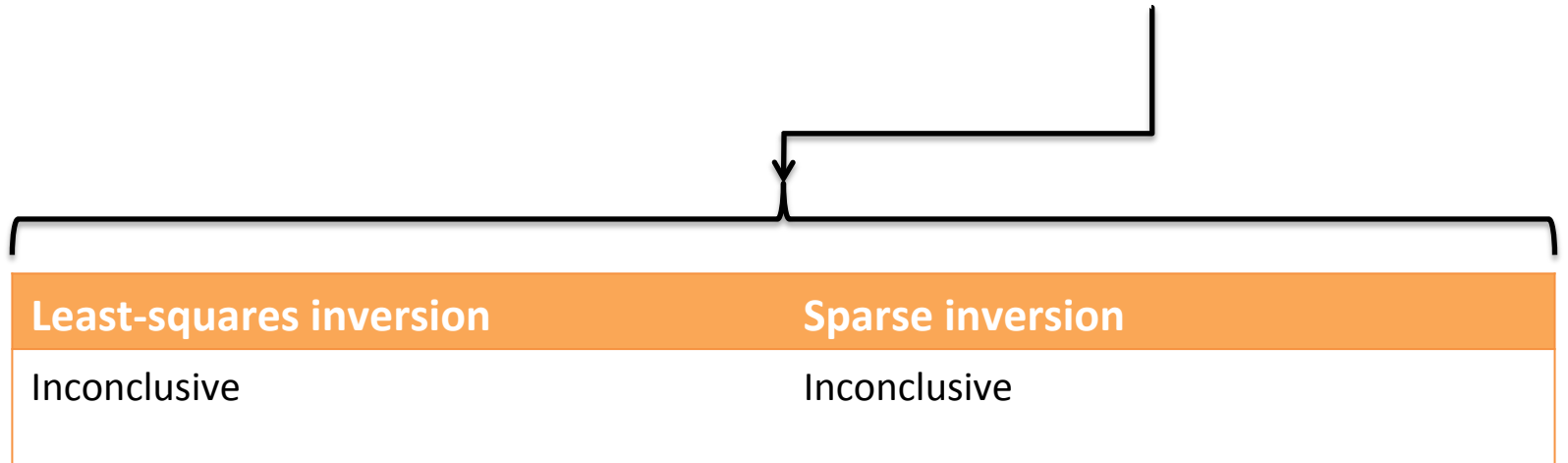


Least-squares inversion	Sparse inversion
High frequency information is often damped by the regularization parameter.	High frequency information can be better retrieved and noise can be suppressed by sparsity promotion.

Conclusions

3. Interferometric imaging


Redatuming by cross-correlation + RTM	Redatuming by inversion + RTM
Blurred image	Deblurred image



Sparse inversion
in the image domain?
(see presentation by Tu Ning)

Acknowledgements

I would like to thank Tim Lin, Ning Tu , Xiang Li, Tristan van Leeuwen and Ian Hanlon for helping me with implementations and for discussions.

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