Migration from surface-related multiples

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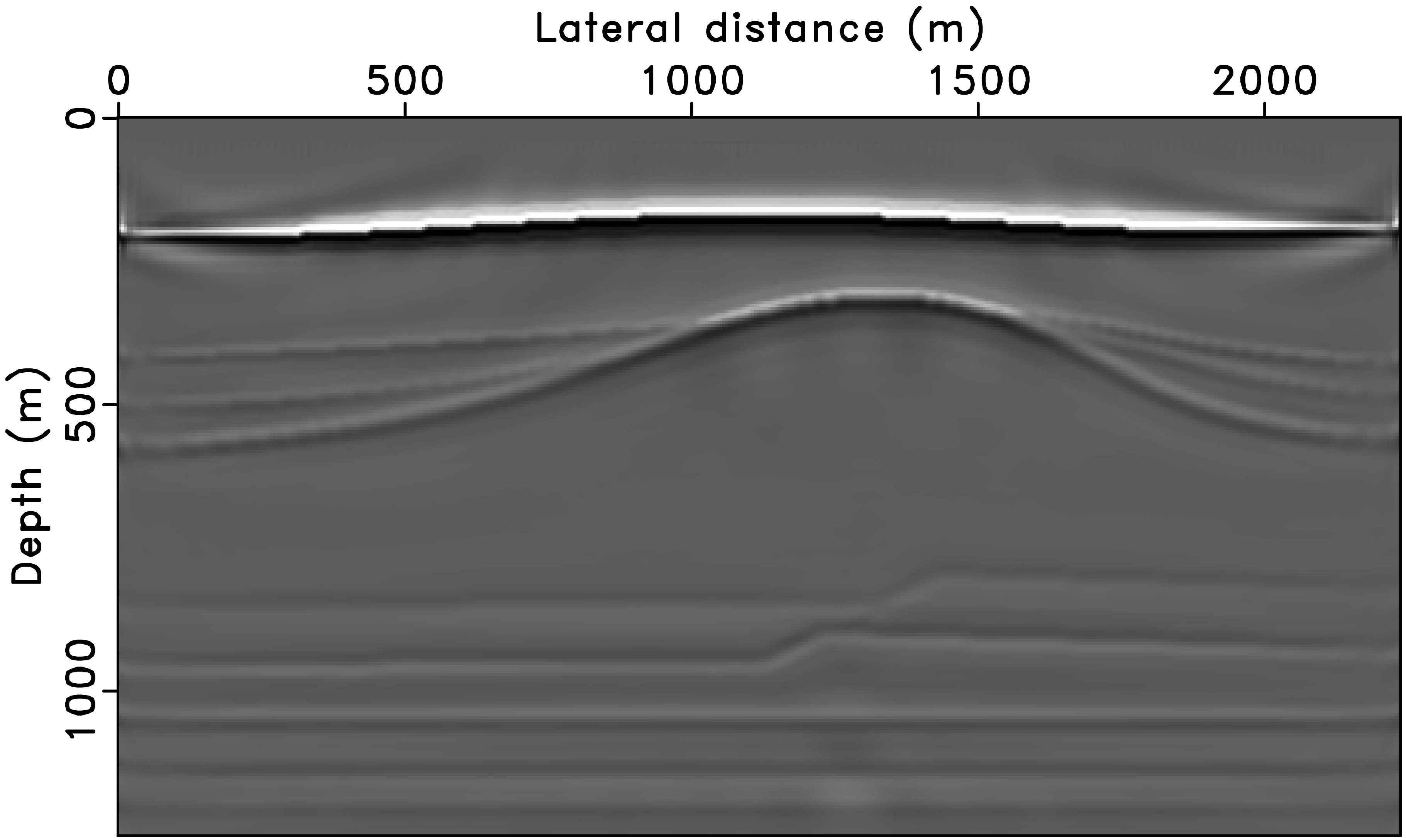
joint work with Tim Lin

SLIM

University of British Columbia

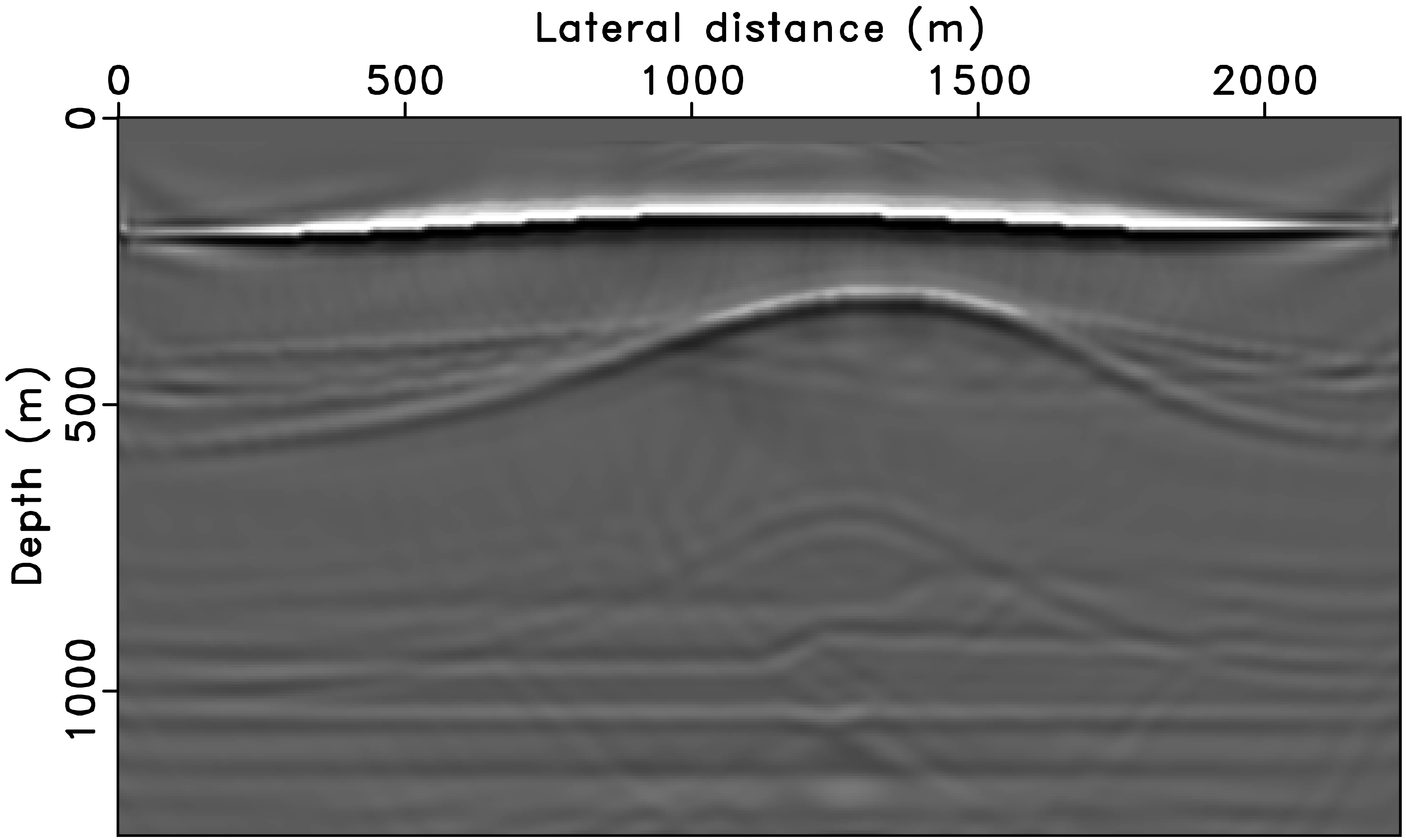
Motivation

[migration from surface-free data]



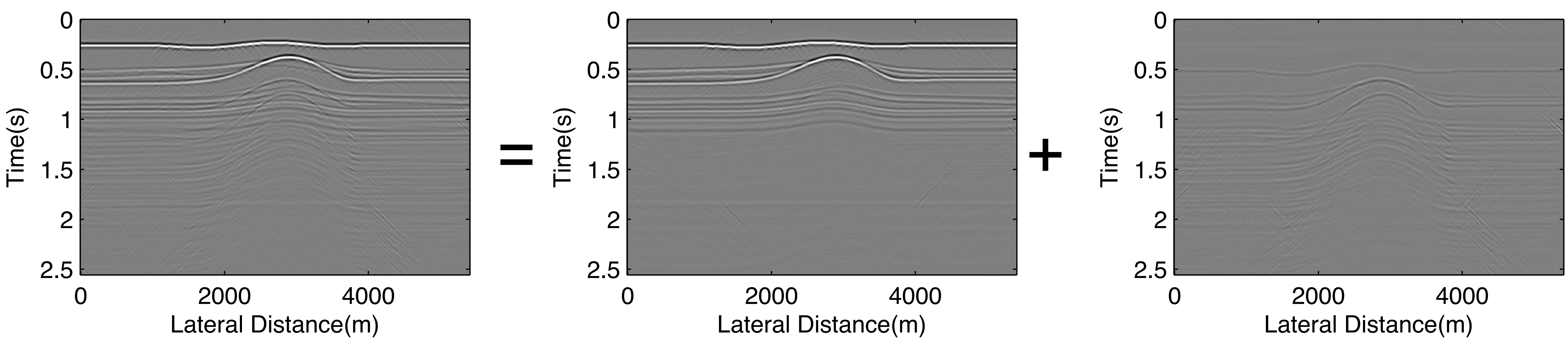
while...

[migration from data with surface multiples]



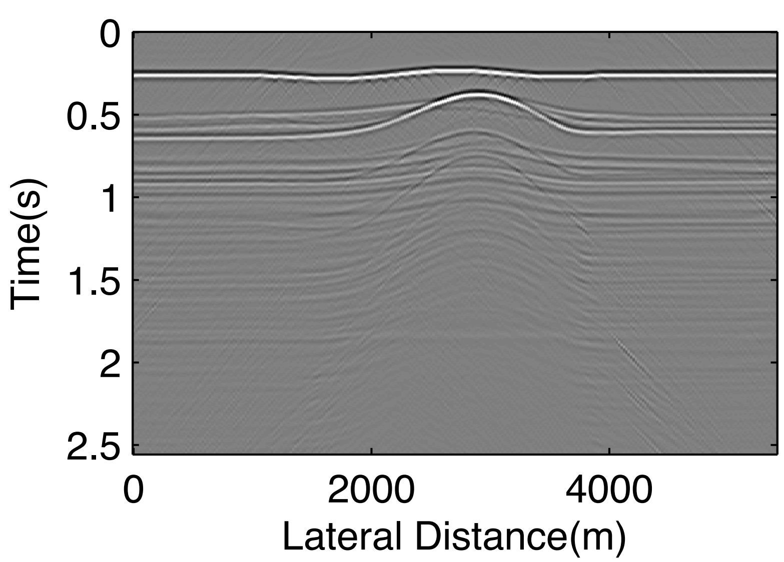
Motivation

So...



Motivation

So...



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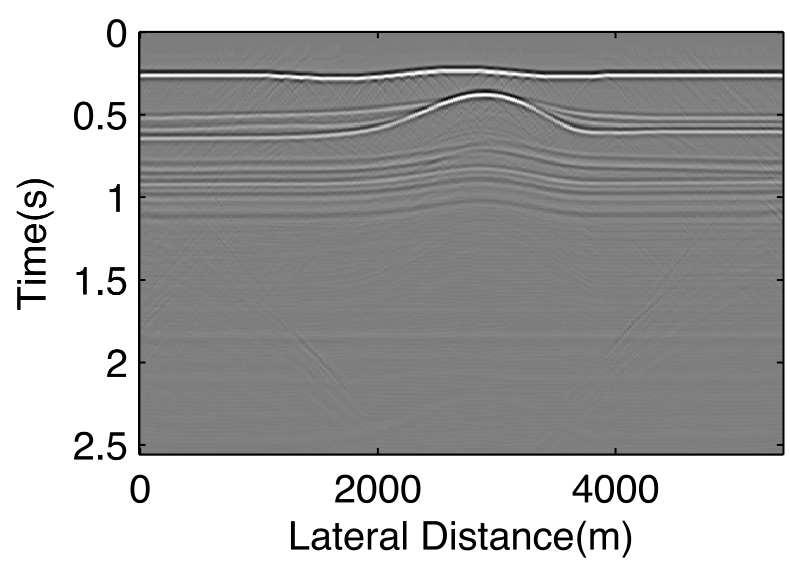
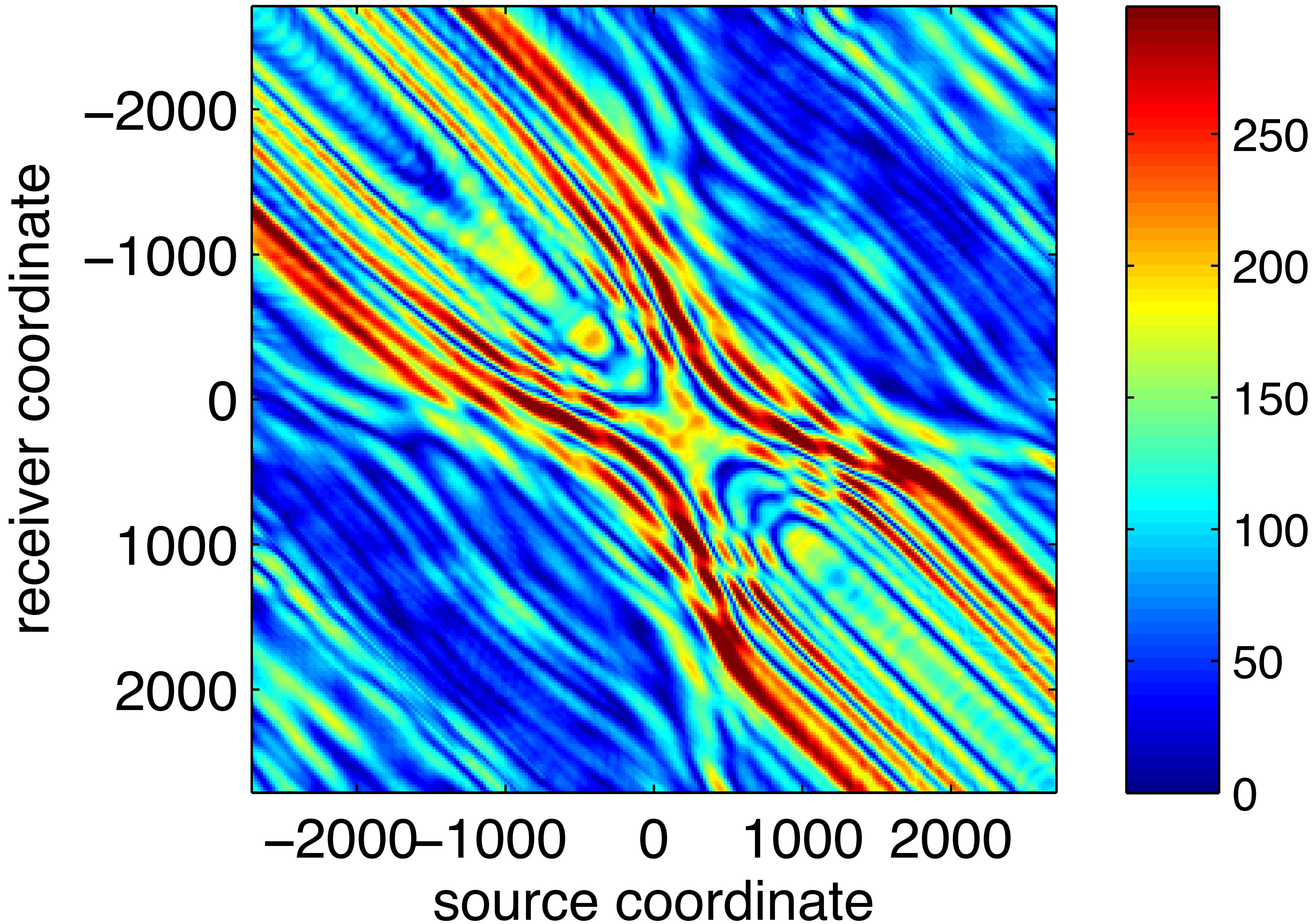


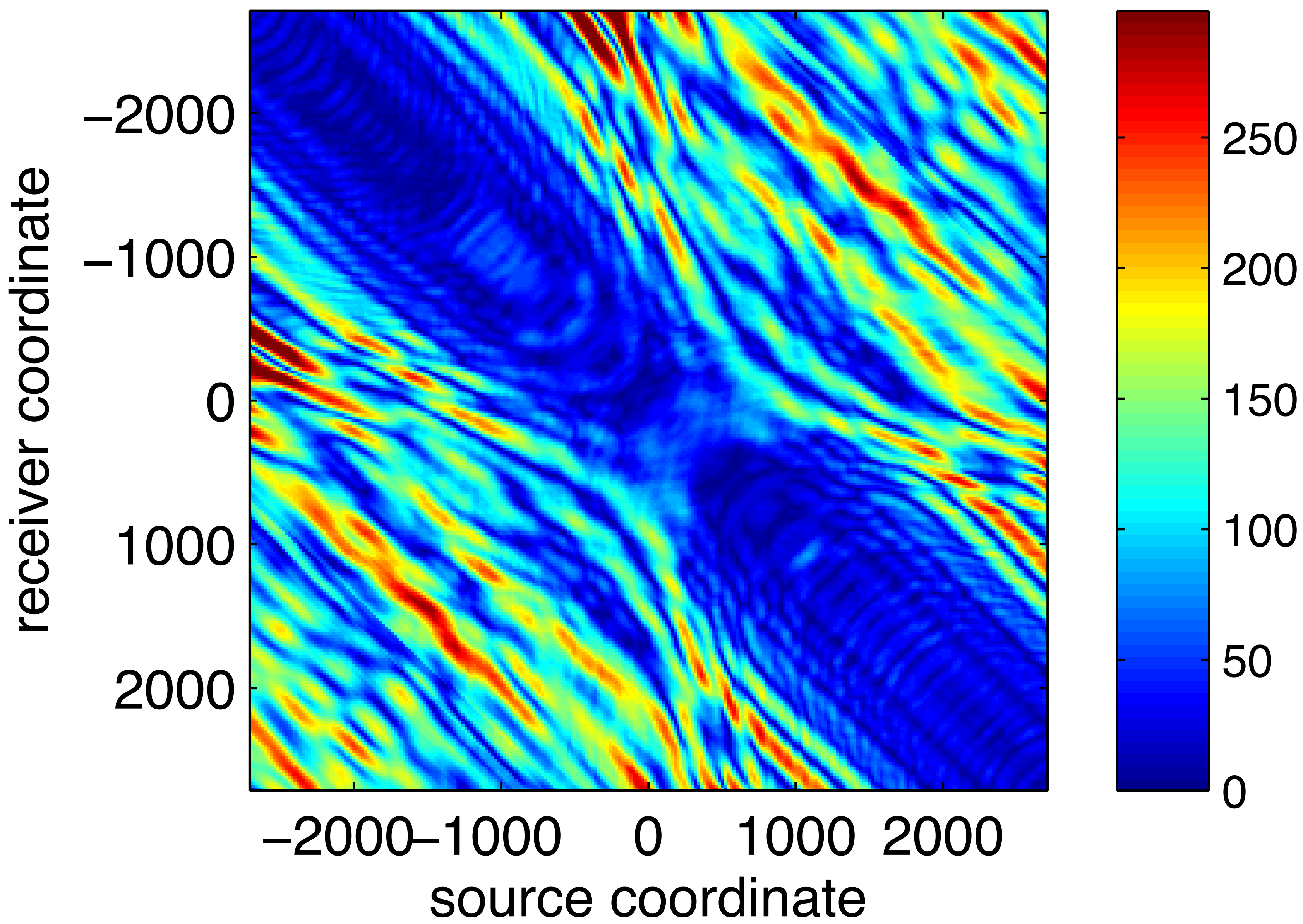
+

Rethink multiples



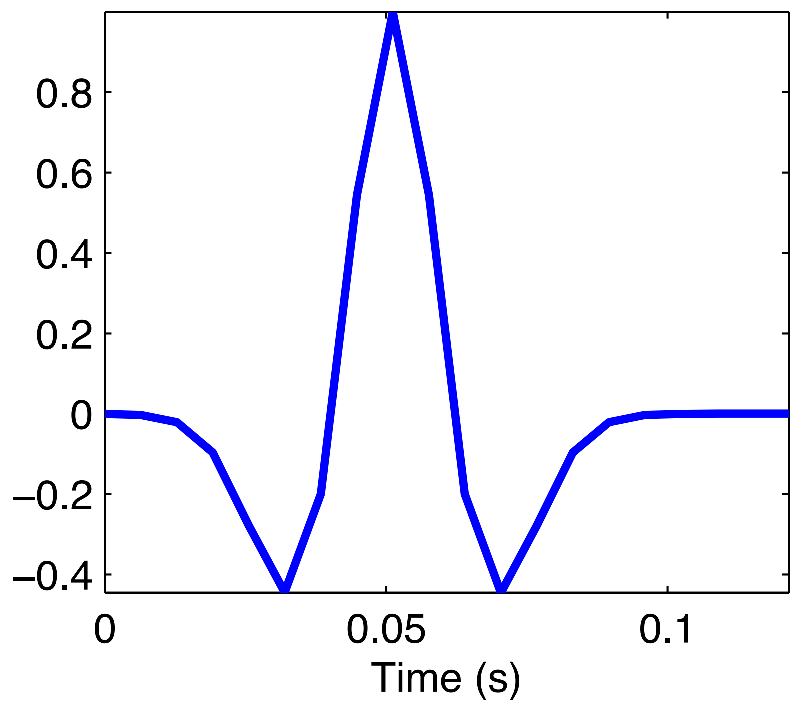
amplitude spectrum: primaries @15Hz

Rethink multiples

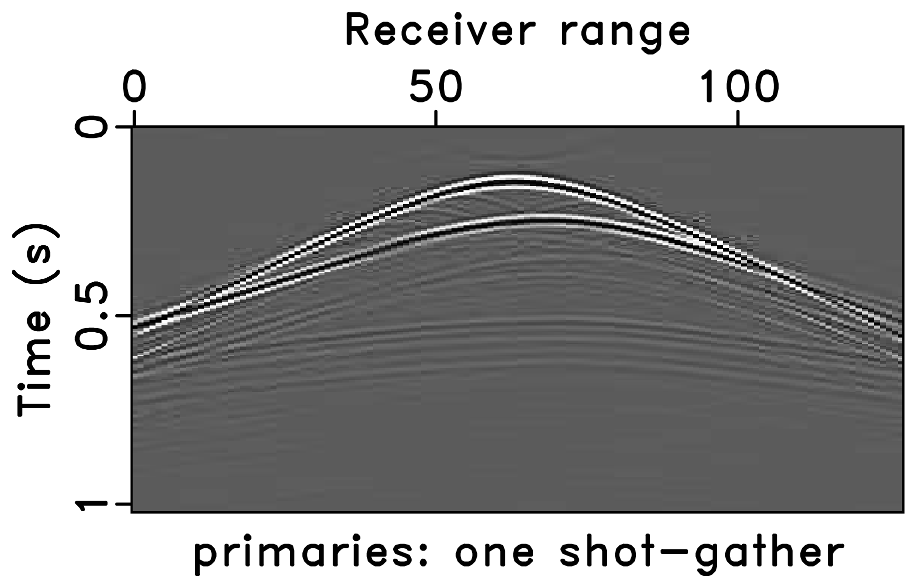


amplitude spectrum: multiples @15Hz

Rethink multiples



Surface-free Green's function



Rethink multiples

Lin, Tu, and Herrmann, 2010

Verschuur and Berkhout, 2011

Whitmore, Valenciano, and Sollner, 2010

Extra illumination from surface multiples

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Sounds reasonable, but…*how*?

Verschuur, 1992

SLIM

Extra illumination from surface multiples

From the monochromatic formulation of SRME:

$$\overbrace{\hat{\mathbf{G}}\hat{\mathbf{Q}}}^{\text{primaries}} + \overbrace{\hat{\mathbf{G}}(-\hat{\mathbf{P}})}^{\text{surface multiples}} = \hat{\mathbf{P}}$$

$\hat{\mathbf{G}}$: surface-free Green's function

$\hat{\mathbf{Q}}$: point-source function

$\hat{\mathbf{P}}$: total up-going wavefield

Groenestijn and Verschuur, 2009

Lin and Herrmann, 2010

Exploit the extra illumination with EPSI

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EPSI - *Estimation of Primary by Sparse Inversion*:

- inverts the Green's function from the total up-going wavefield.
- exploits the sparsity of the Green's function in *data* space.

Verschuur, 1992

Herrmann, 2008

Lin and Herrmann, 2010

SLIM

EPSI Formulation

EPSI follows the time-harmonic formulation of SRME:

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

Matrix-vector formulation of EPSI:

$$\underbrace{\mathcal{F}_t^* \mathrm{BlockDiag}_{1\ldots\mathrm{n_f}}[(\hat{\mathbf{Q}} - \hat{\mathbf{P}})^* \otimes \mathbf{I}]\mathcal{F}_\mathbf{t}}_{\mathbf{E}}\,\mathbf{g} = \mathbf{p}$$

Lin and Herrmann, 2010

Lin and Herrmann, 2011

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Robust-EPSI

in physical domain:

$$\tilde{\mathbf{g}} = \underbrace{\arg\min_{\mathbf{g}} \|\mathbf{g}\|_1}_{\text{sparsity promoting}} \quad \text{subject to} \quad \underbrace{\|\mathbf{p} - \mathbf{E}\mathbf{g}\|_2 \leq \sigma}_{\text{data fitting}}$$

in sparsifying transform domain:

$$\tilde{\mathbf{g}} = \underbrace{\mathbf{S}_3^* \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity promoting}} \quad \text{subject to} \quad \underbrace{\|\mathbf{p} - \mathbf{E}\mathbf{S}_3^*\mathbf{x}\|_2 \leq \sigma}_{\text{data fitting}}$$

Linearized data examples of *EPSI*

Linearized data

- Linearized Green's function

$$\mathbf{p_1} = \mathbf{K}\delta\mathbf{m}$$

- Linearized total data

$$\mathbf{p_2} = \mathbf{E}\mathbf{K}\delta\mathbf{m}$$

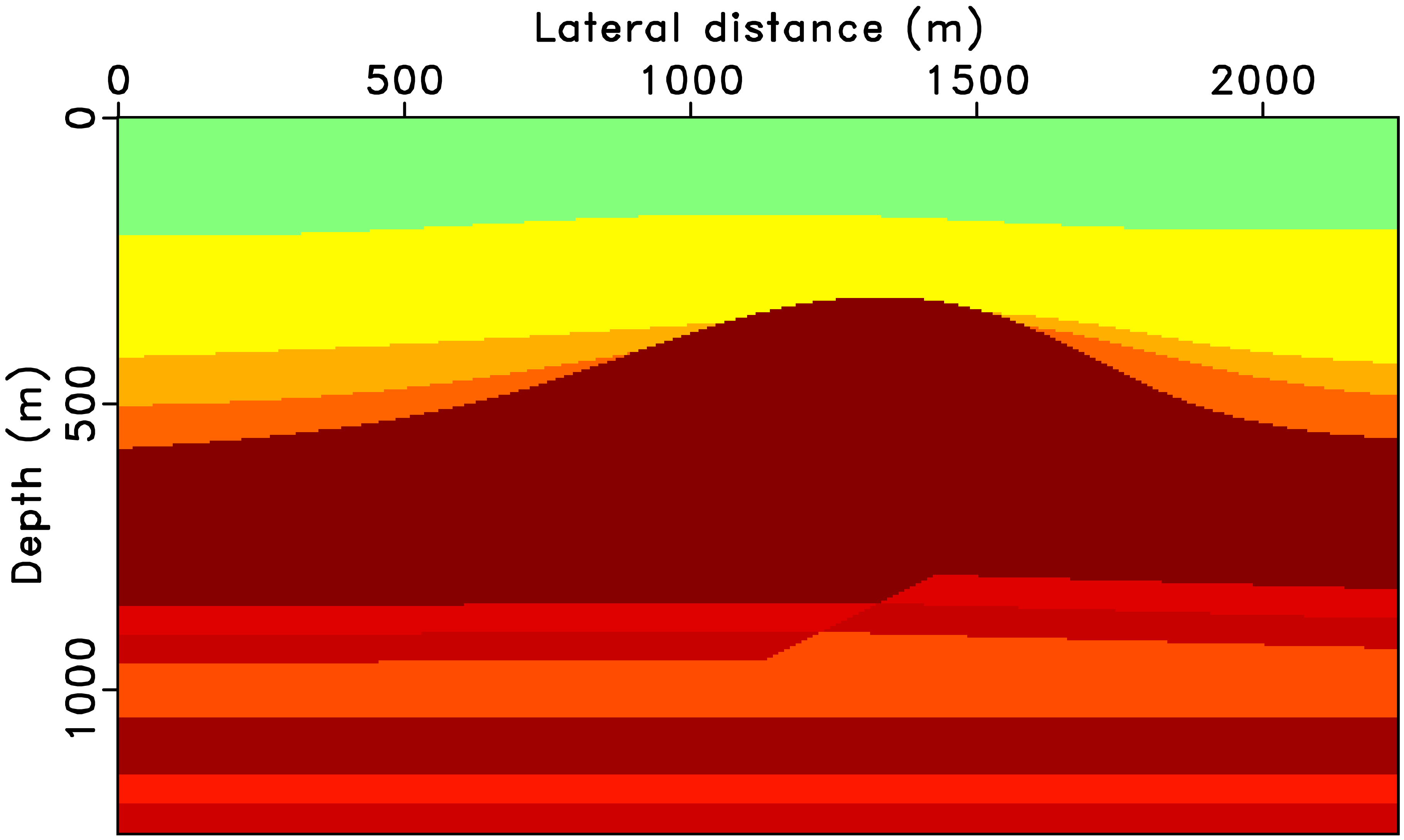
- Brief acquisition geometry:

  15m source/receiver spacing

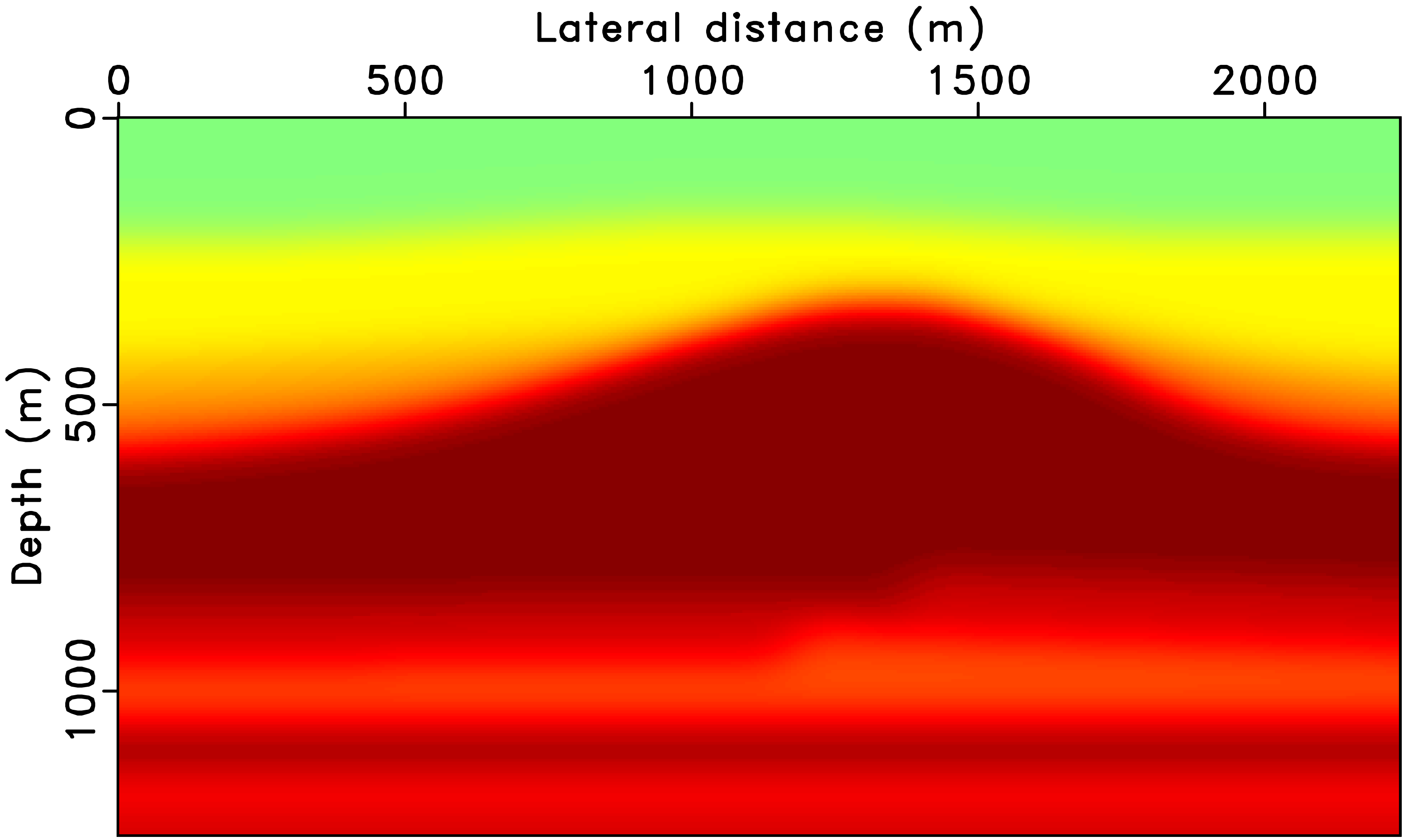
  150 sources/receivers

  512 time samples, 4ms rate

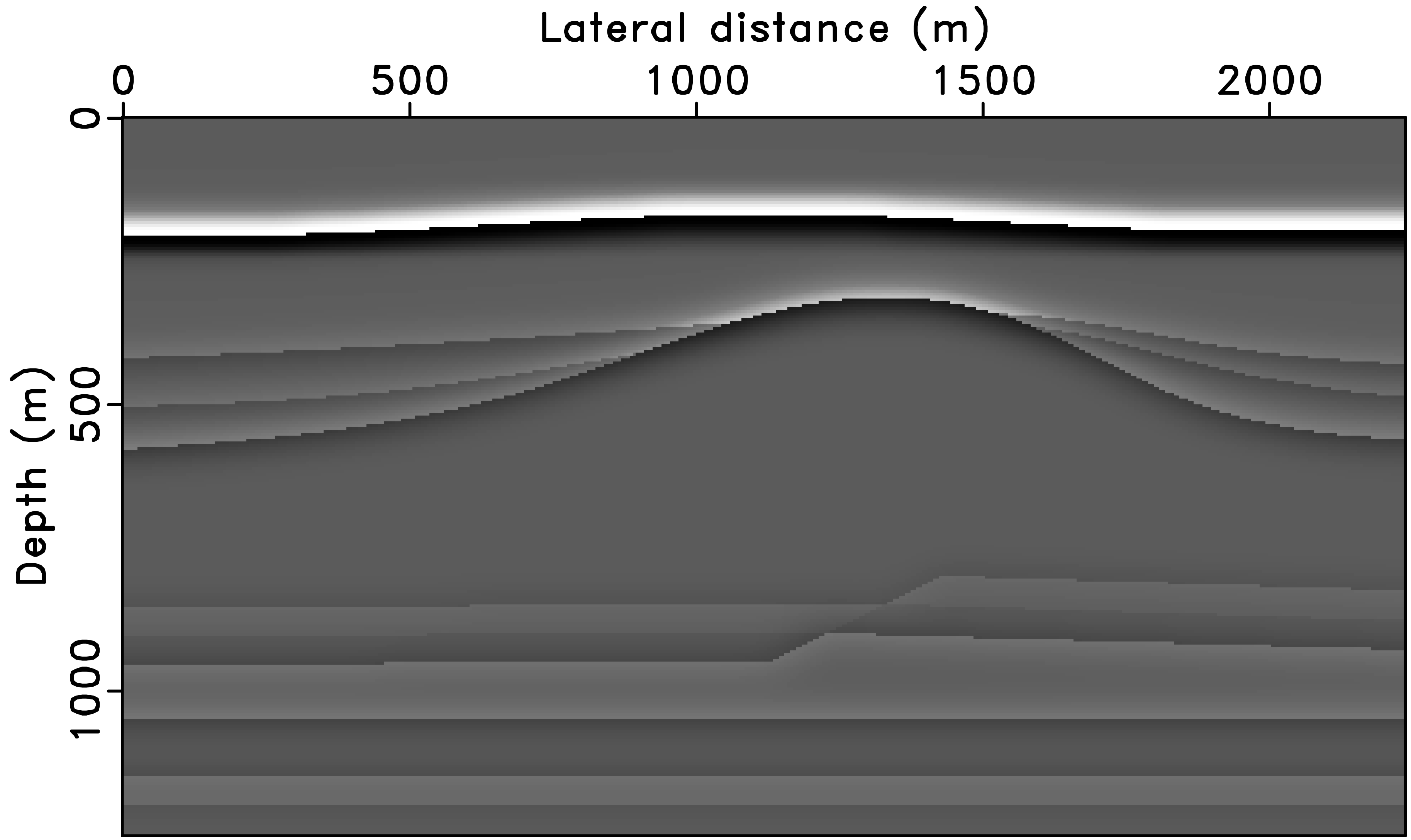
True model [5m grid distance]



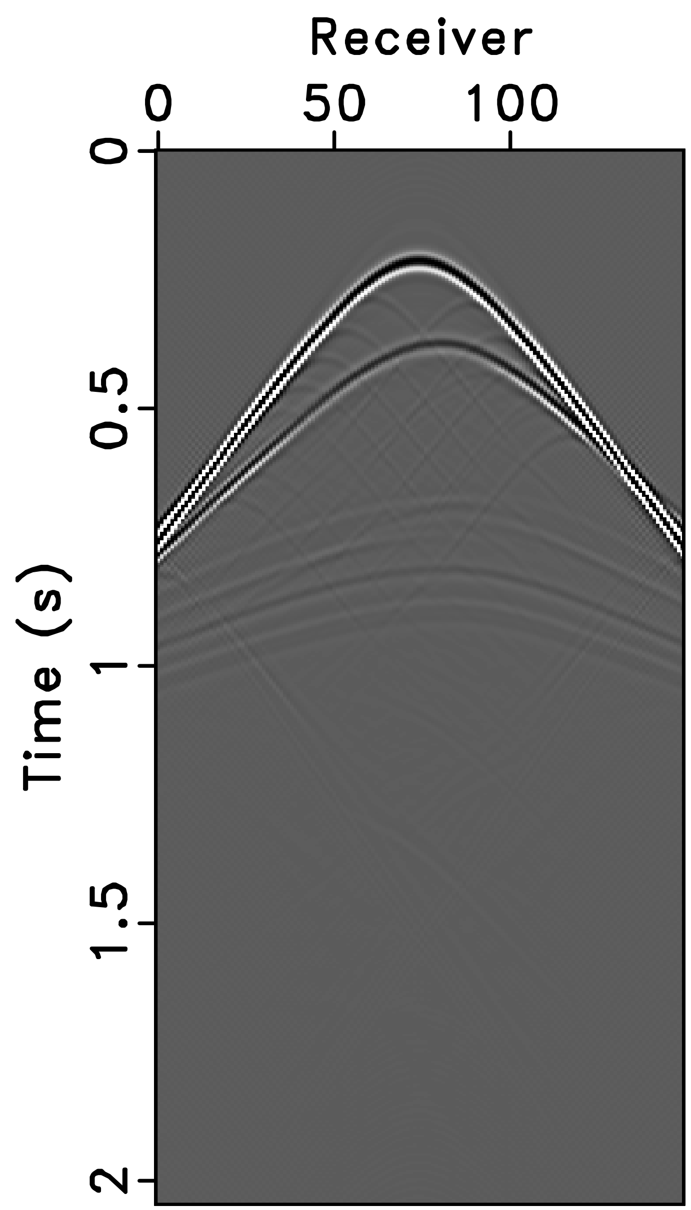
Background model [5m grid distance]



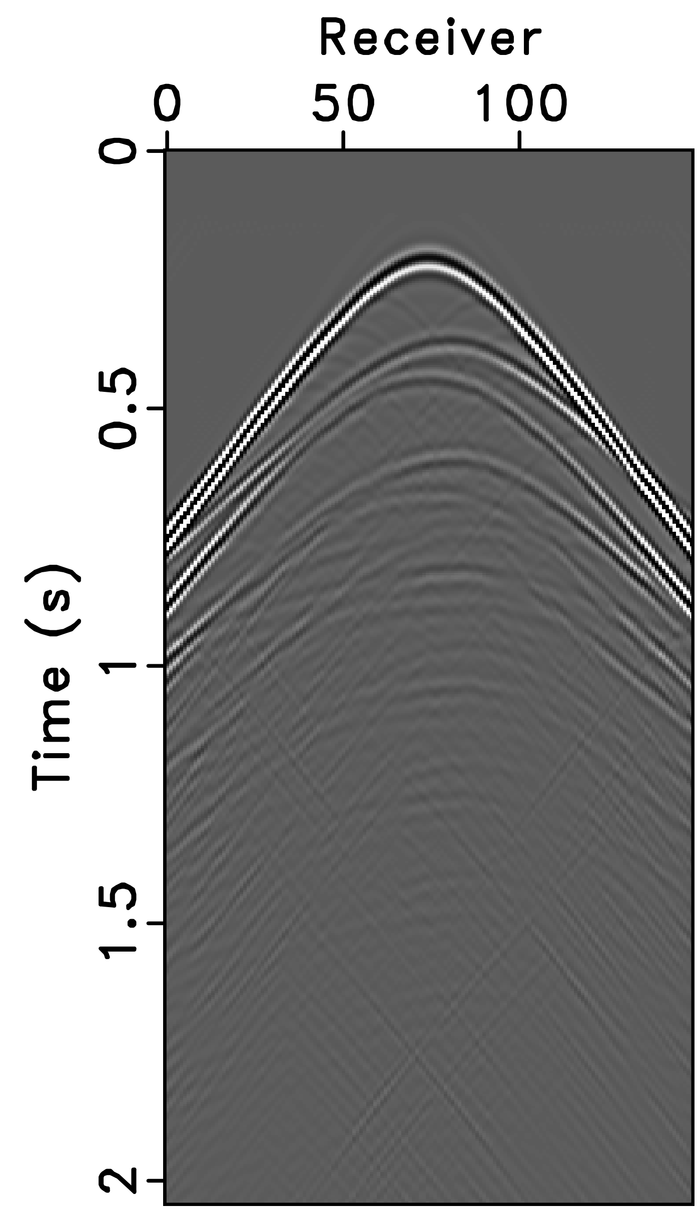
Model perturbation



Linearized data

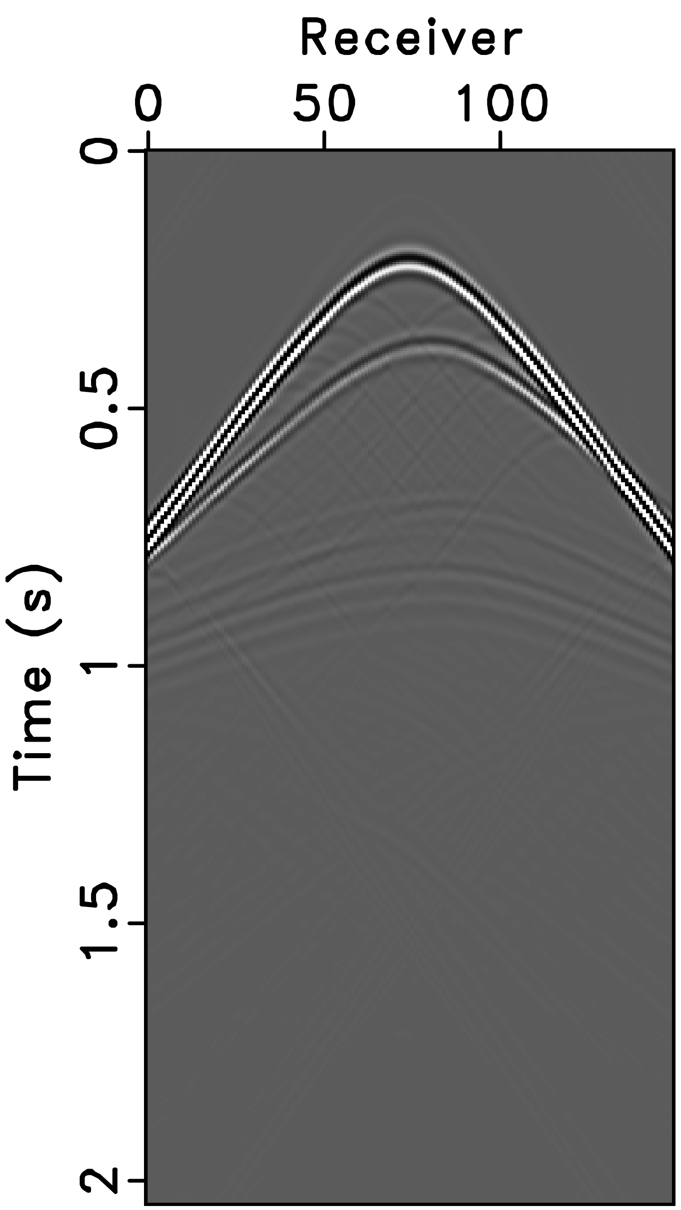


Green’s function

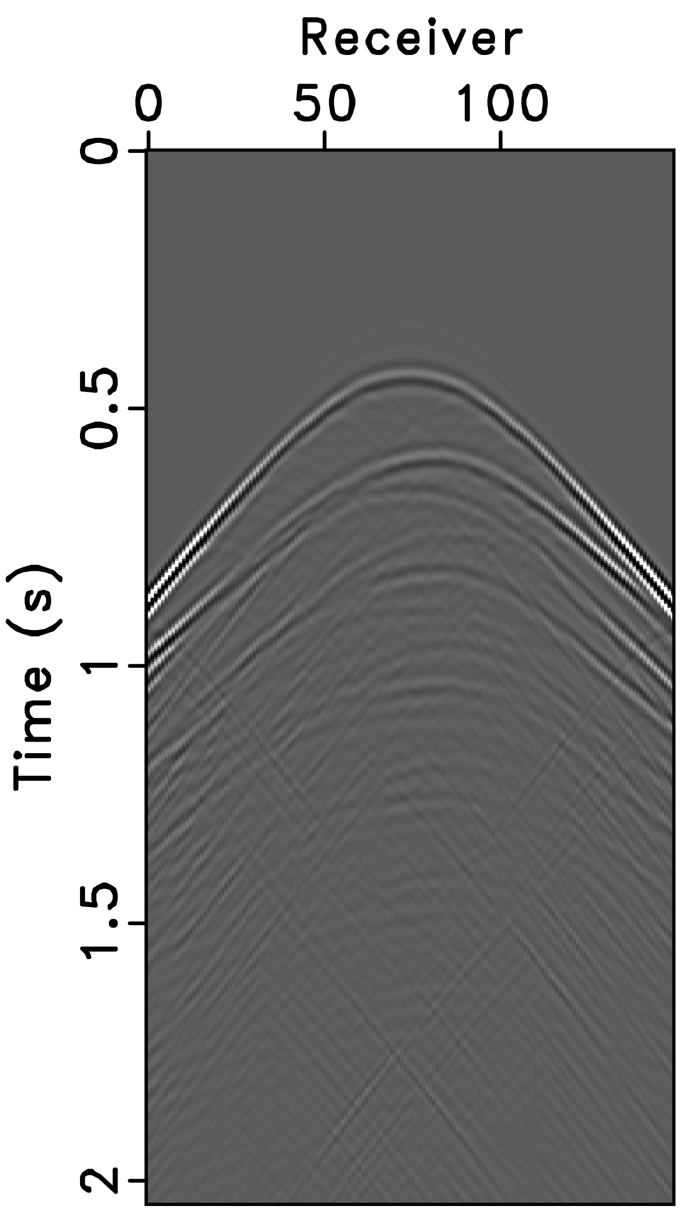


Total data

Linearized data

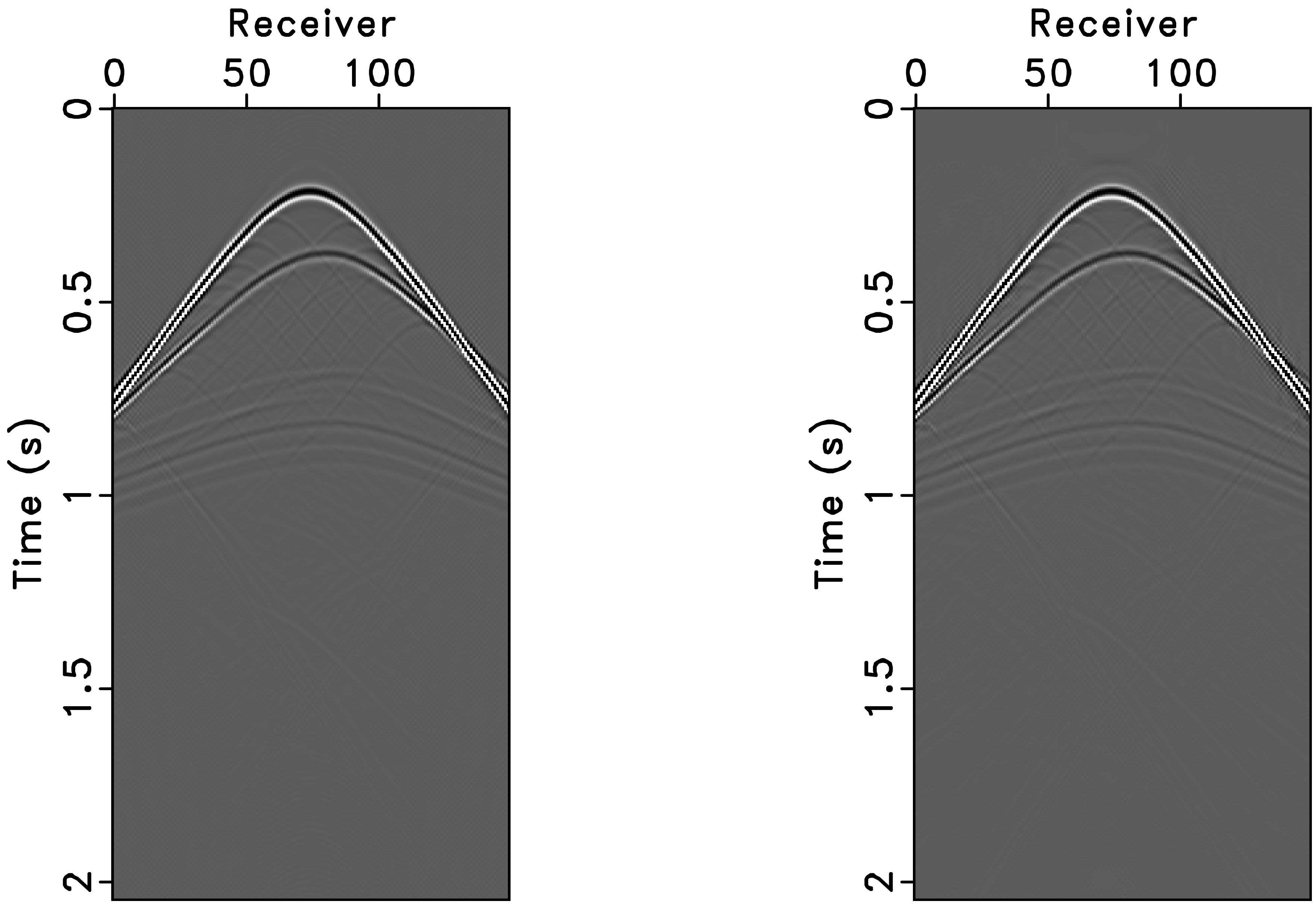


Primaries



Multiples

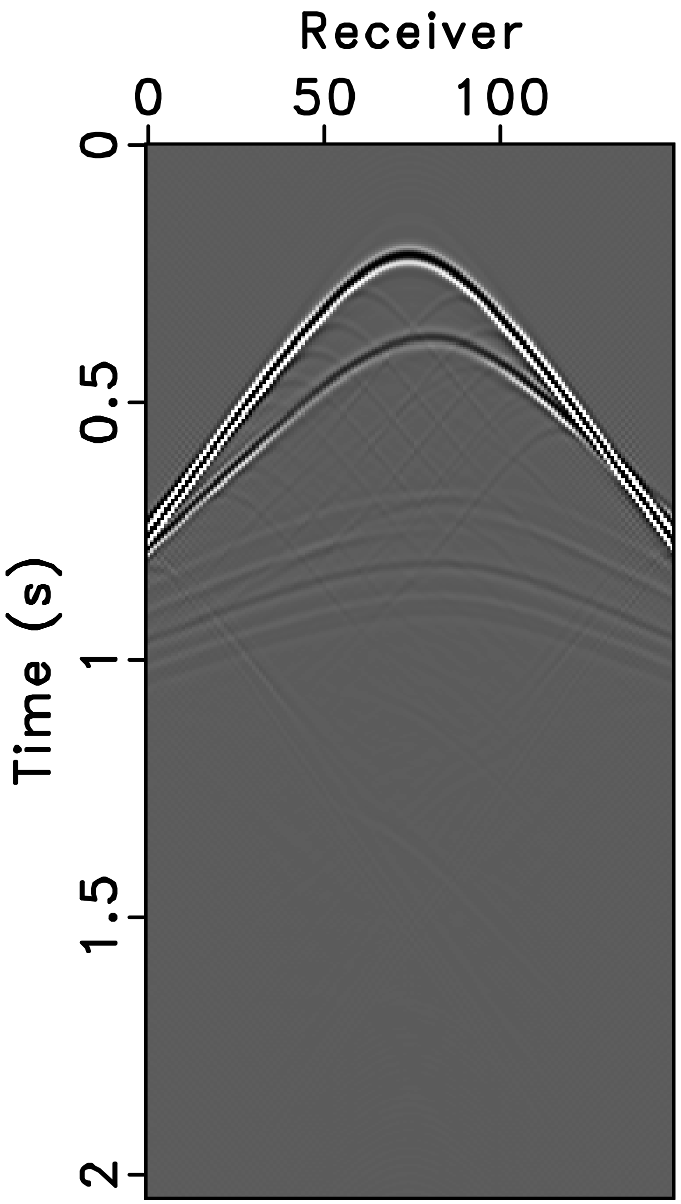
Inverted Green's function



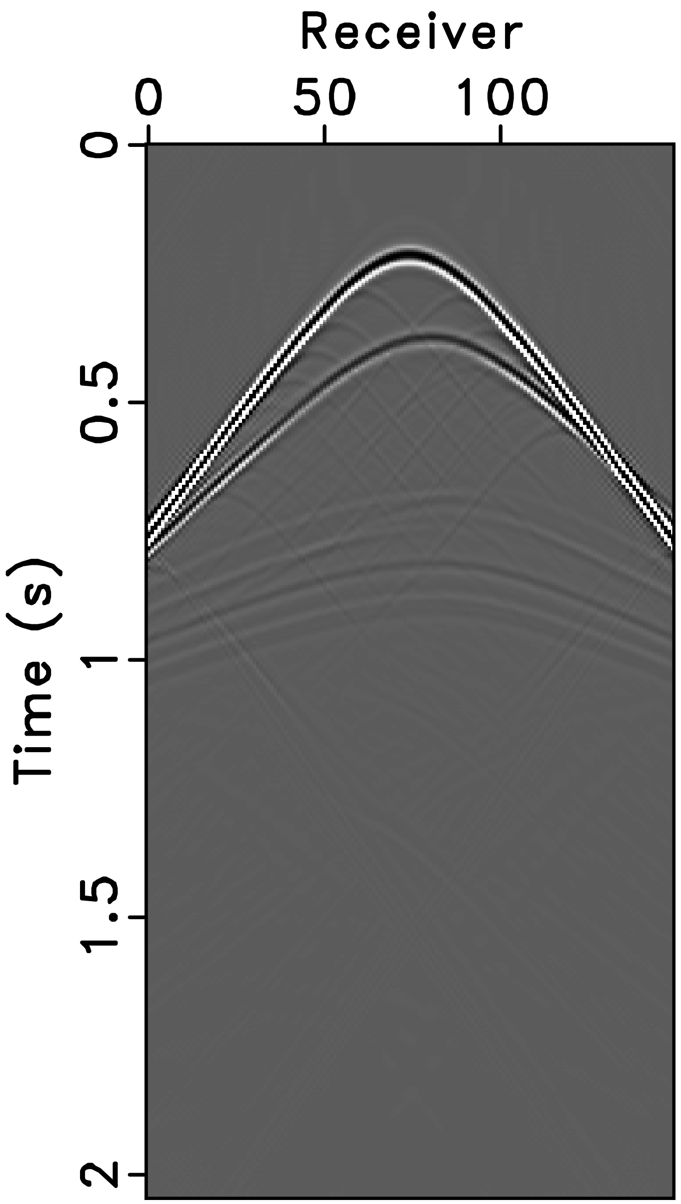
True Green's function

From total data, SNR 16.9dB

Inverted Green's function

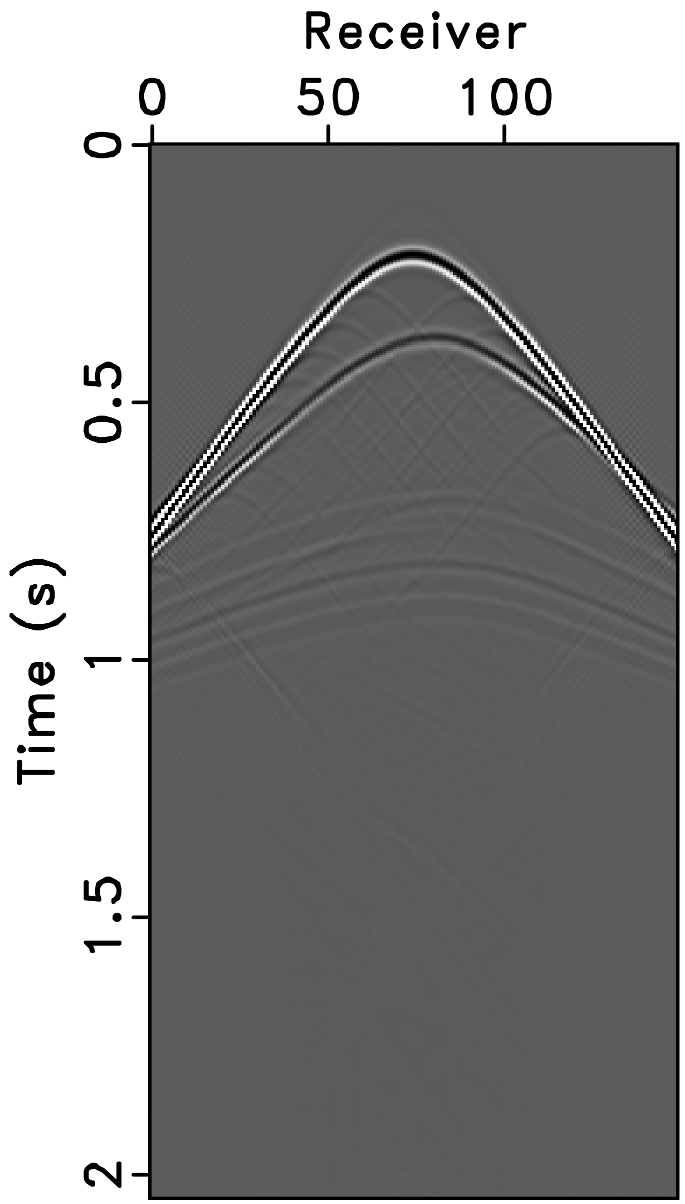


True Green's function

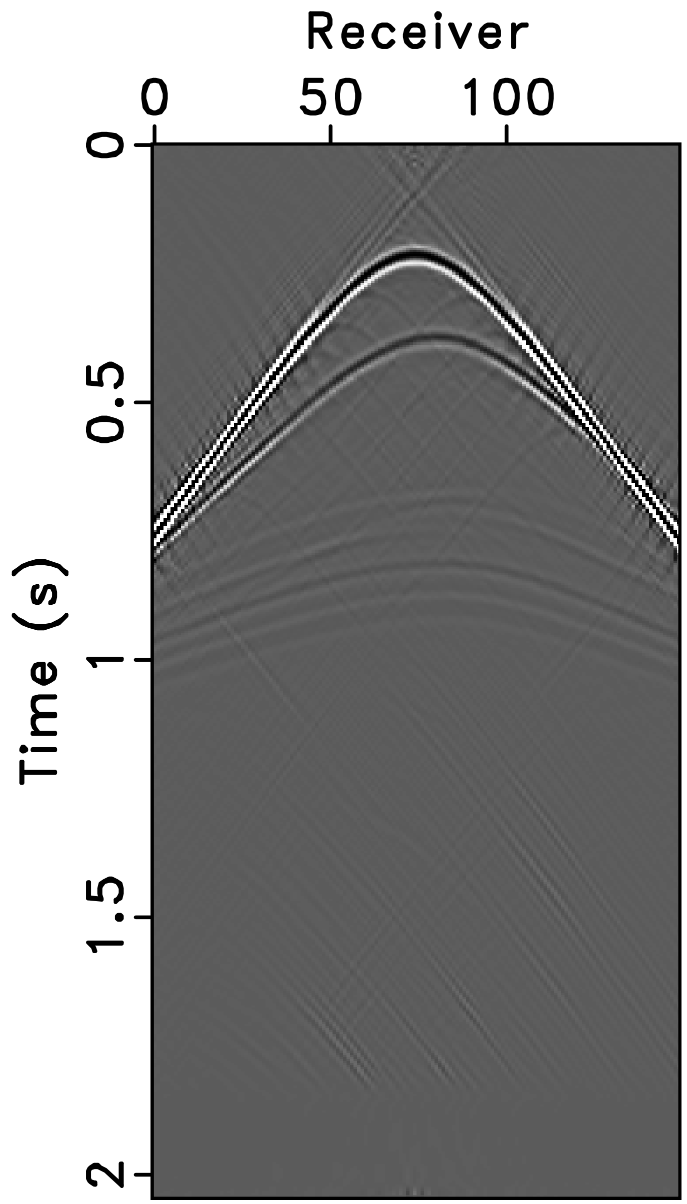


From primaries, SNR 17.9dB

Inverted Green's function



True Green's function



From multiples, SNR 4.3dB

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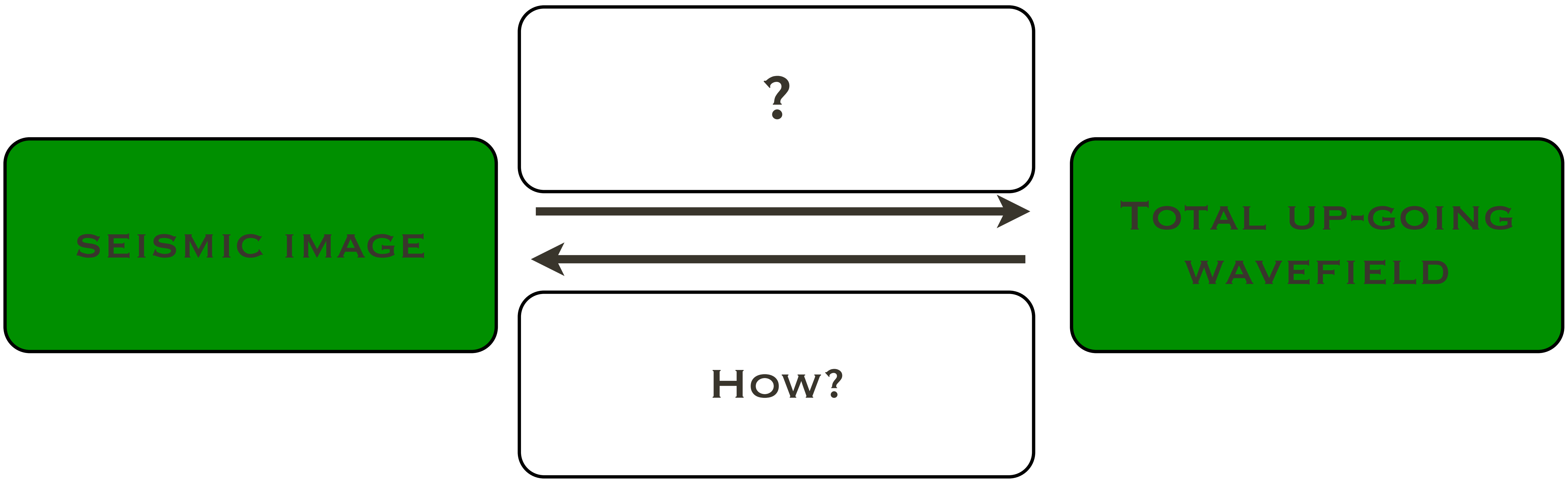
Motivation

- How to exploit this extra illumination in seismic imaging?
- How to exploit the sparsity in the image space instead of data space?

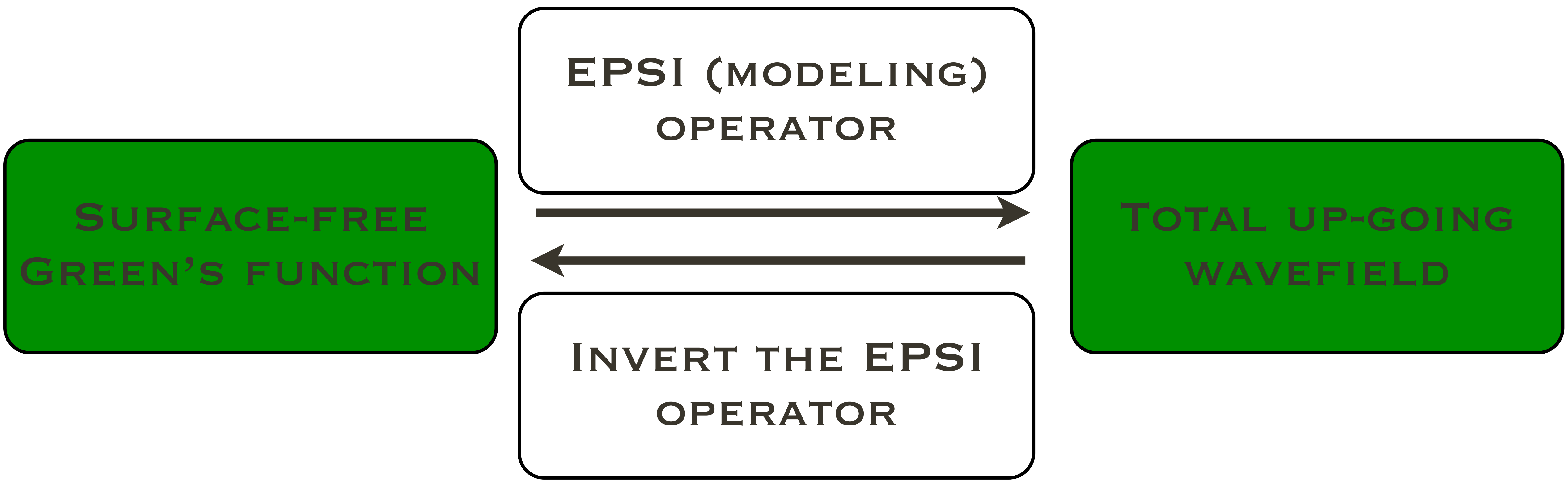
Relate data space and image space



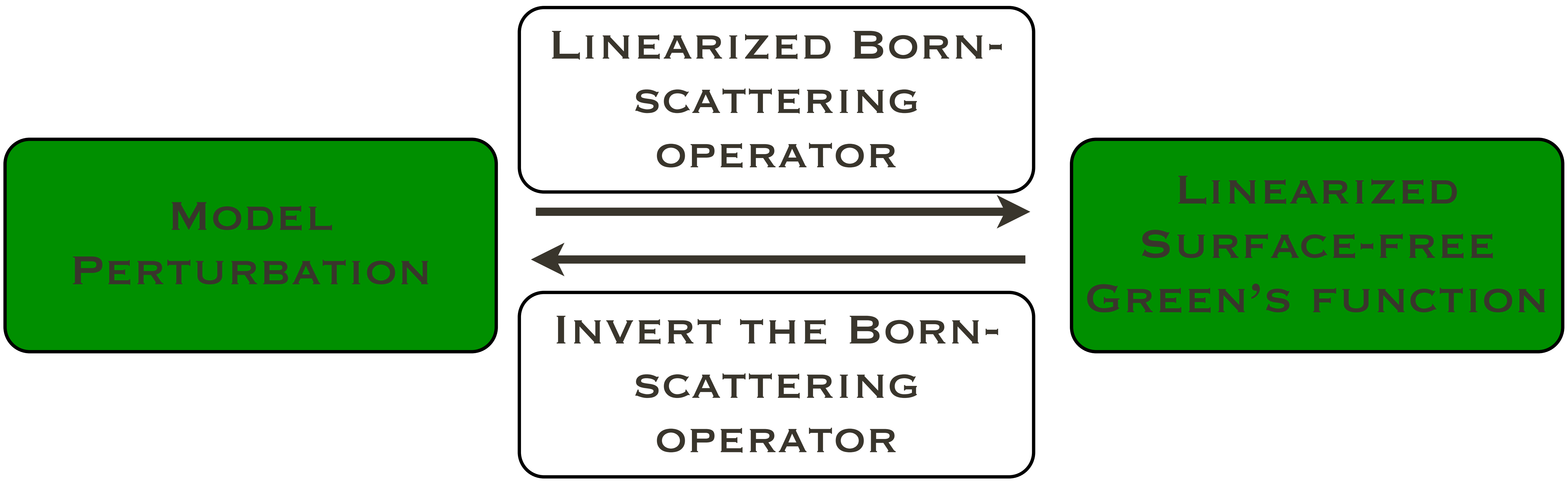
Groenestijn and Verschuur, 2009

Lin and Herrmann, 2010

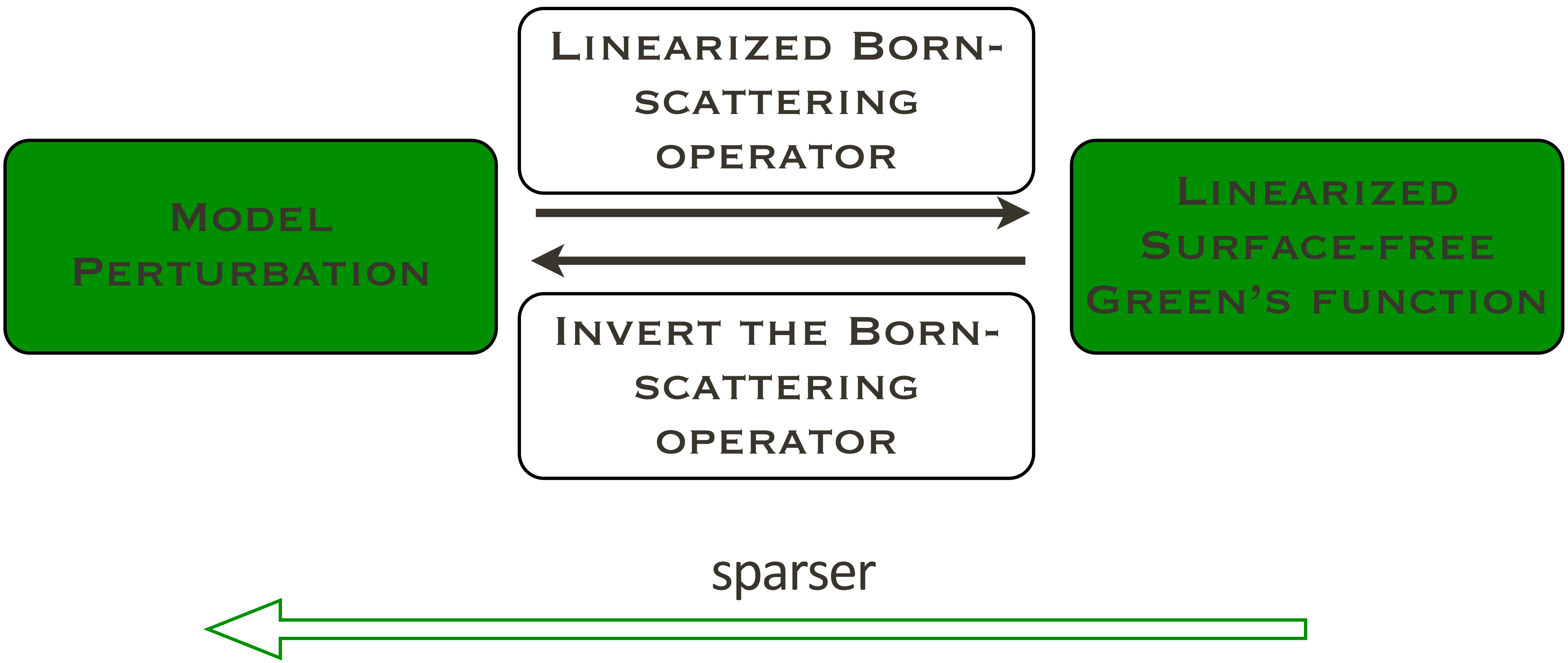
EPSI operator relates...



Migration operator relates...



Migration operator relates...



Migration operator relates...

Model Perturbation

Linearized Born-scattering operator →

← Invert the Born-scattering operator

Linearized Surface-free Green's function

sparser

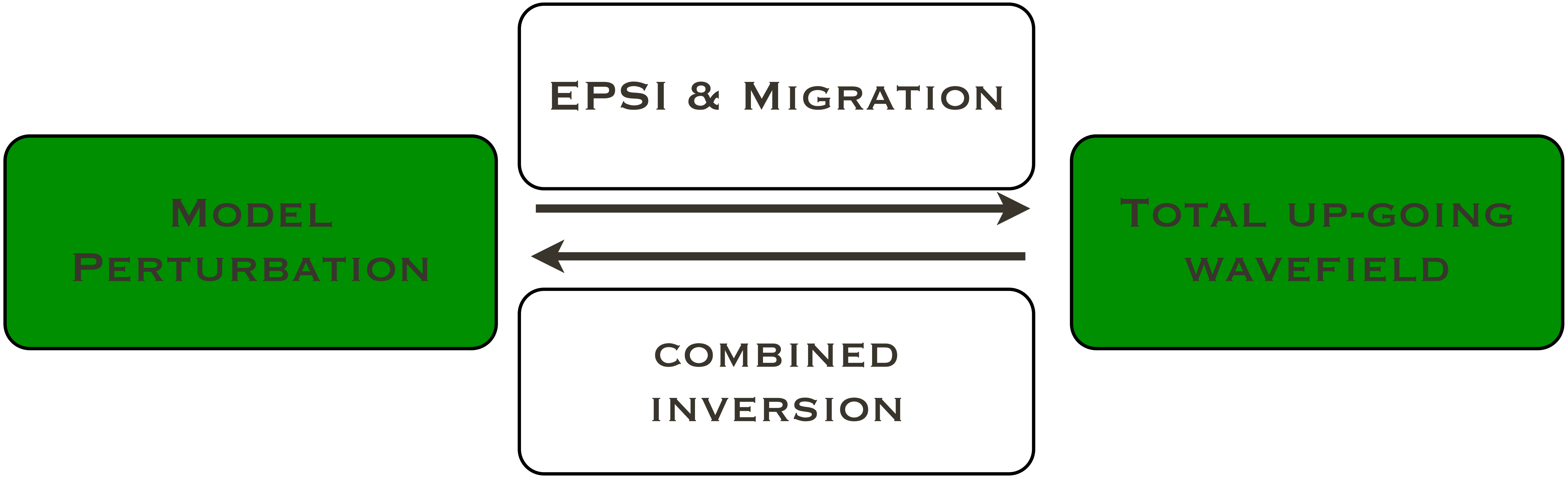
← physical information of wave propagation

Lin, Tu, and Herrmann, 2010

Tu, Lin, and Herrmann, 2011(1)

Tu, Lin, and Herrmann, 2011(2)

What about combining...



EPSI linearization

Approximate by linearization:

$$\hat{\mathbf{P}} \approx \delta\hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

Approximate Robust-EPSI in sparsifying domain:

$$\delta\tilde{\mathbf{g}} = \underbrace{\mathbf{S_3^*} \arg\min_{\mathbf{x}} \|\mathbf{x}\|_\mathbf{1}}_{\text{sparsity promoting}} \text{ subject to } \underbrace{\|\mathbf{p} - \mathbf{E}\mathbf{S_3^*}\mathbf{x}\|_\mathbf{2} \leq \sigma}_{\text{data fitting}}$$

Combine EPSI and migration

Migration as sparsifying operator for EPSI:

$$\delta\tilde{\mathbf{g}} = \mathbf{K}\mathbf{S}_2^* \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \ \text{subject to} \ \|\mathbf{p} - \mathbf{E}\mathbf{K}\mathbf{S}_2^*\mathbf{x}\|_2 \leq \sigma$$

Imaging from the total upgoing wavefield:

$$\delta\tilde{\mathbf{m}} = \mathbf{S}_2^* \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \ \text{subject to} \ \underbrace{\|\mathbf{p} - \mathbf{E}\mathbf{K}\mathbf{S}_2^*\mathbf{x}\|_2 \leq \sigma}$$

Linearized data examples of *EPSI+imaging*

Two inversion schemes

- EPSI, then migration

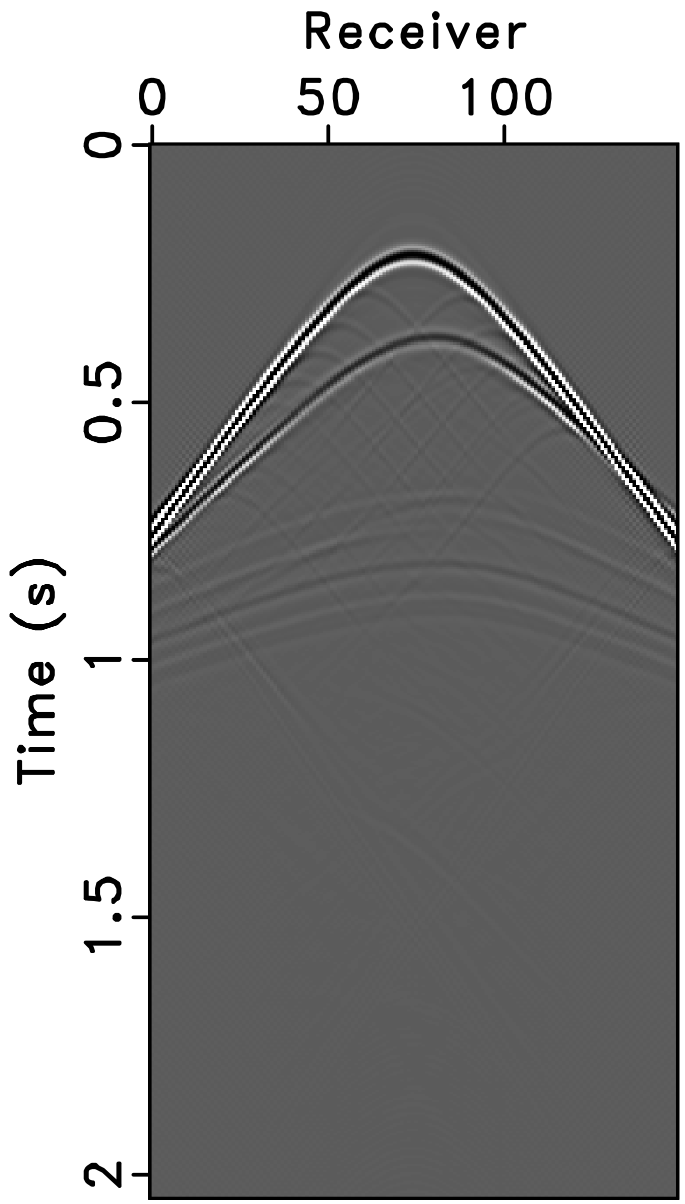
$$\delta\tilde{\mathbf{g}} = \mathbf{S_3^*} \arg\min_{\mathbf{x}} \|\mathbf{x}\|_\mathbf{1} \text{ subject to } ||\mathbf{p} - \mathbf{ES_3^*x}||_\mathbf{2} \leq \sigma$$

$$\delta\tilde{\mathbf{m}} = \mathbf{S_2^*} \min \|\mathbf{x}\|_\mathbf{1} \text{ subject to } ||\delta\tilde{\mathbf{g}} - \mathbf{KS_2^*x}||_\mathbf{2} \leq \sigma$$

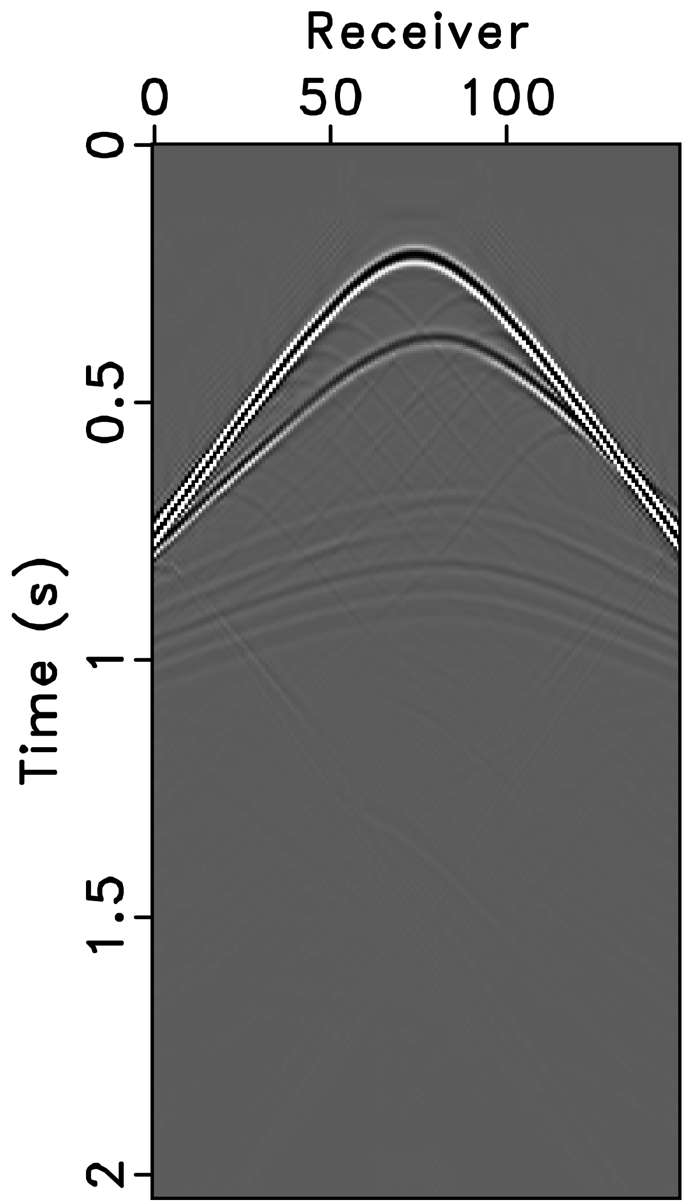
- EPSI+migration

$$\delta\tilde{\mathbf{m}} = \mathbf{S_2^*} \min \|\mathbf{x}\|_\mathbf{1} \text{ subject to } ||\mathbf{p} - \mathbf{EKS_2^*x}||_\mathbf{2} \leq \sigma$$

Inverted Green's function in separate inversions

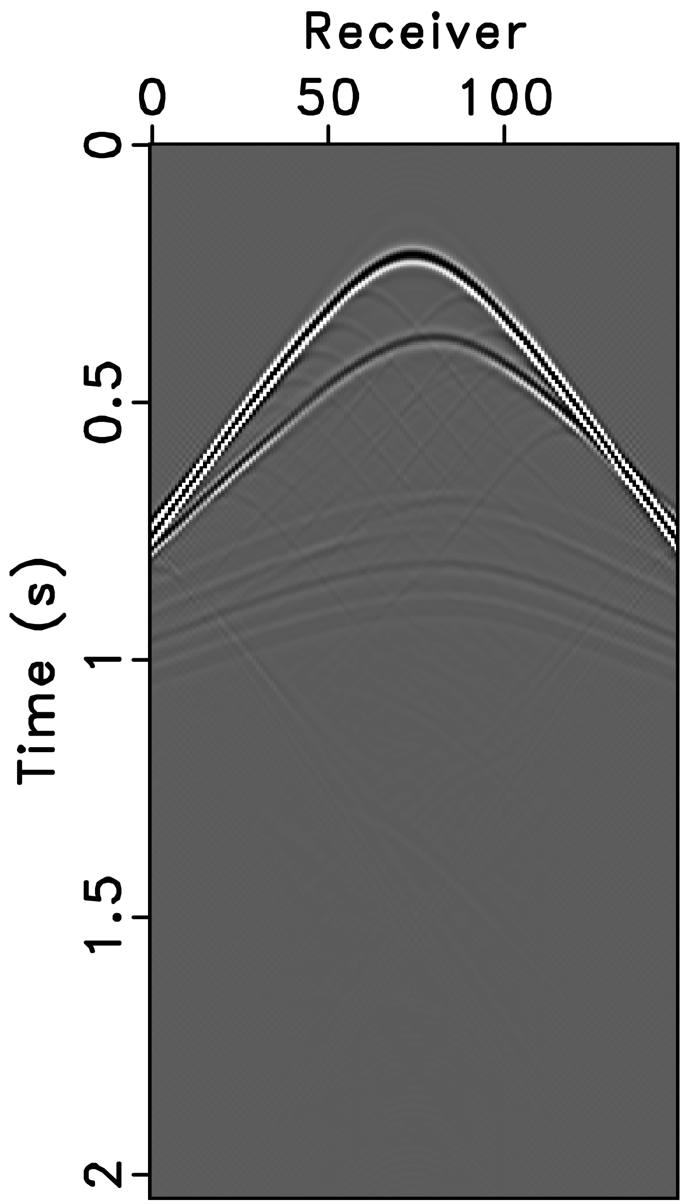


True Green's function

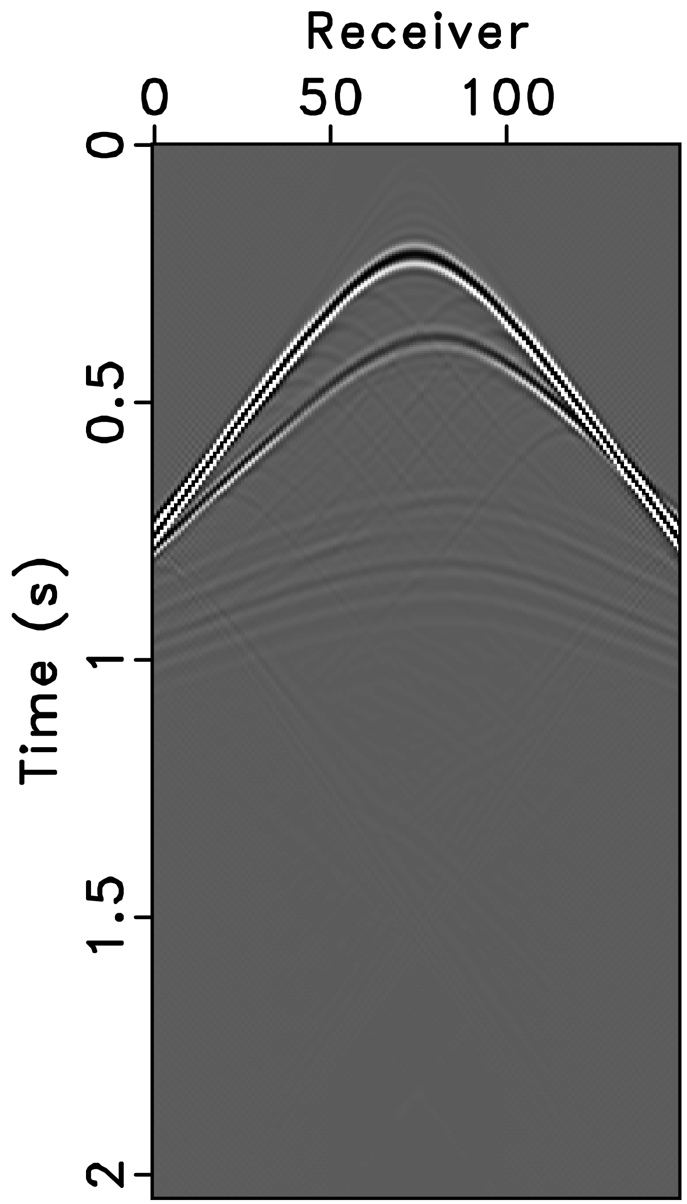


EPSI in data space, SNR 16.9dB

Inverted Green's function in combined inversions

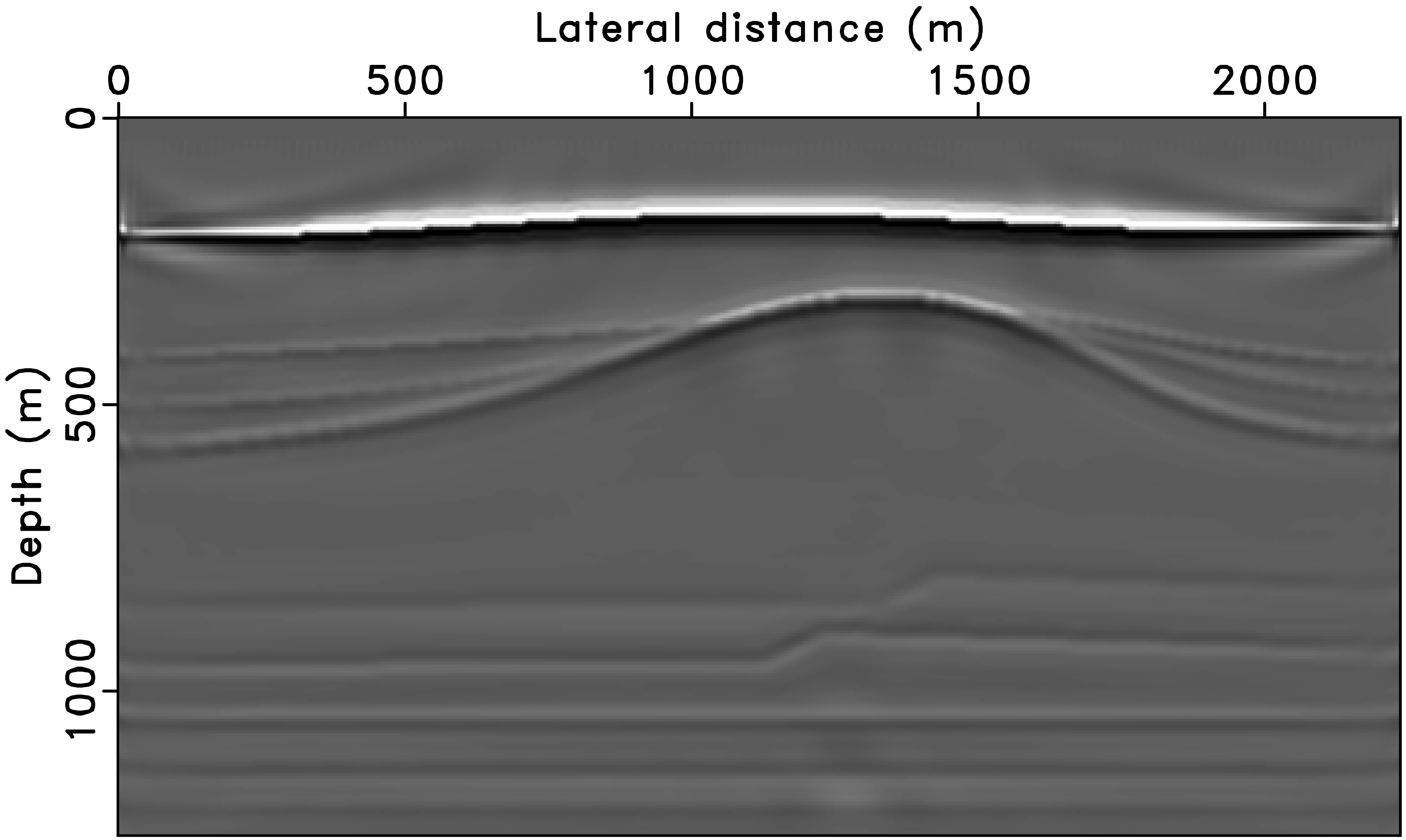


True Green's function



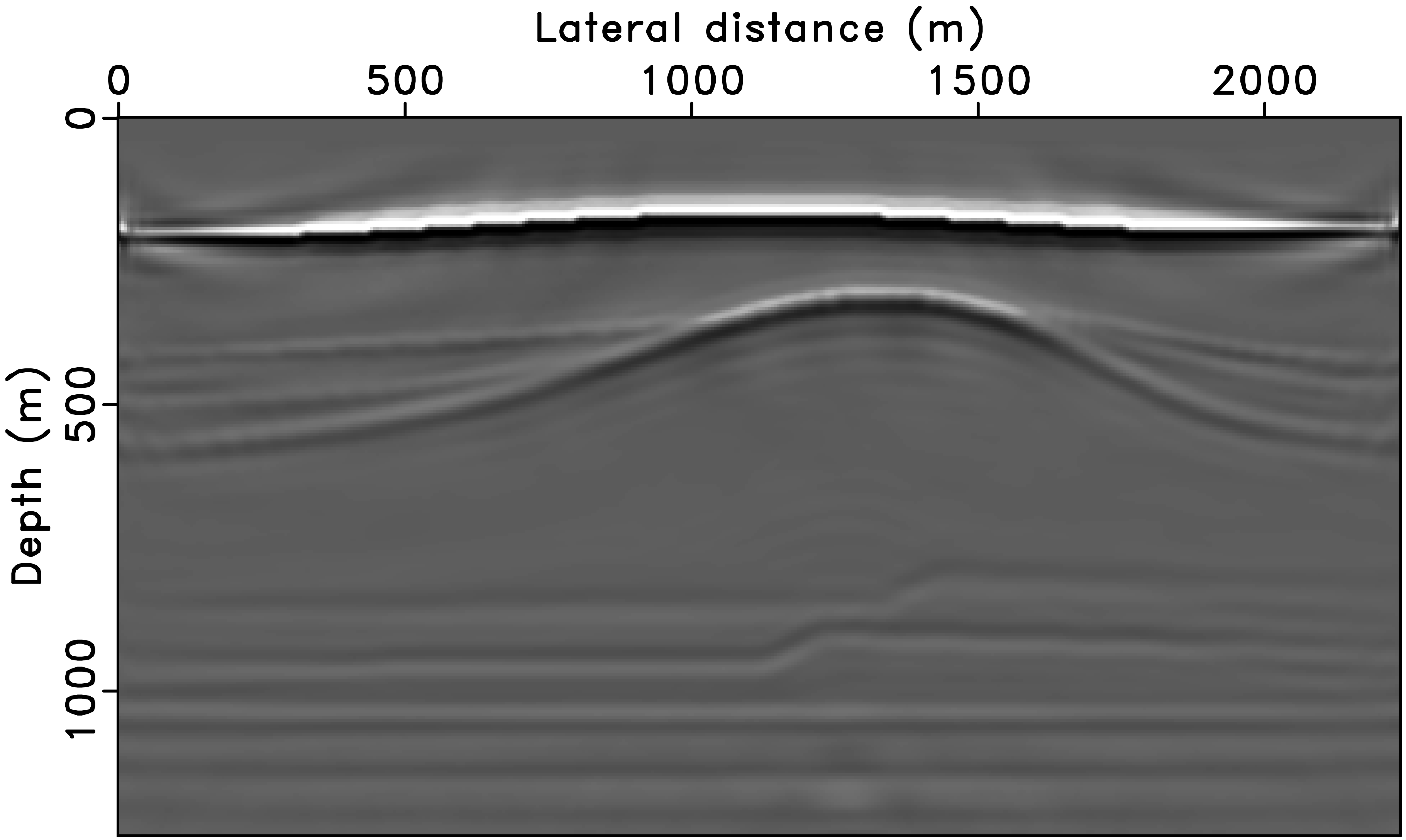
EPSI in image space, SNR 25.7dB

Inverted model perturbation in separate inversions



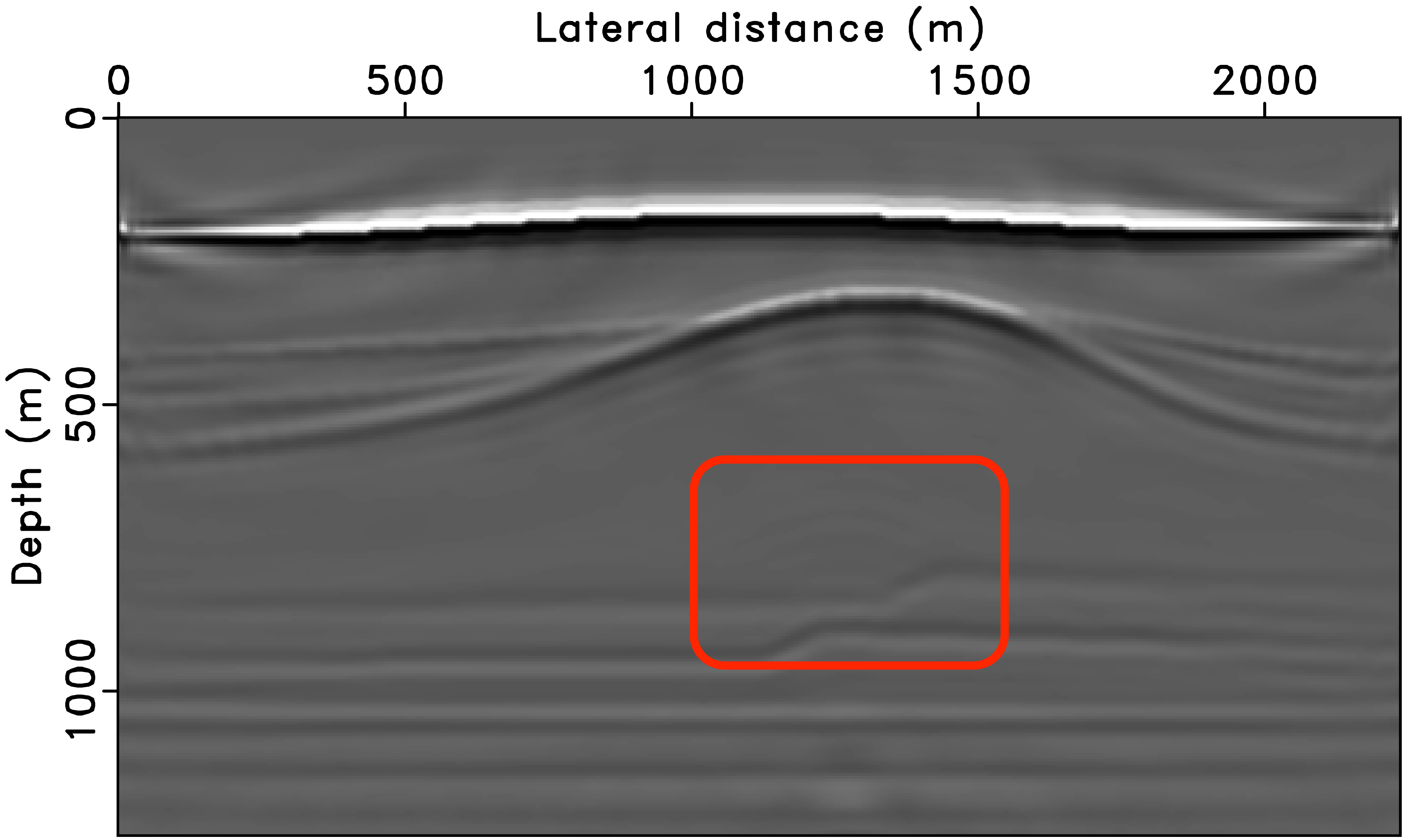
SNR: 6.59dB

Inverted model perturbation in combined inversion



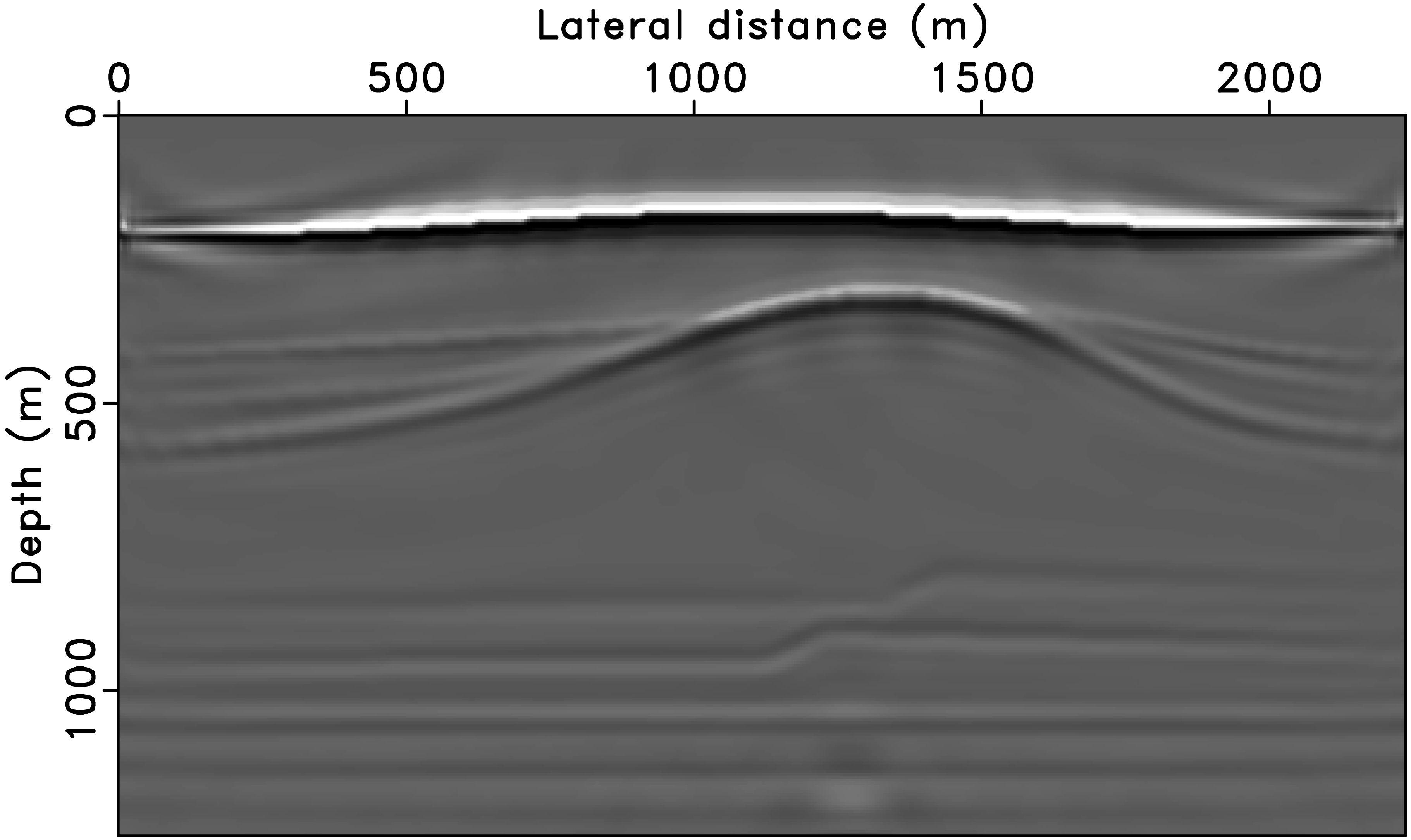
SNR: 5.78dB

Inverted model perturbation in combined inversion



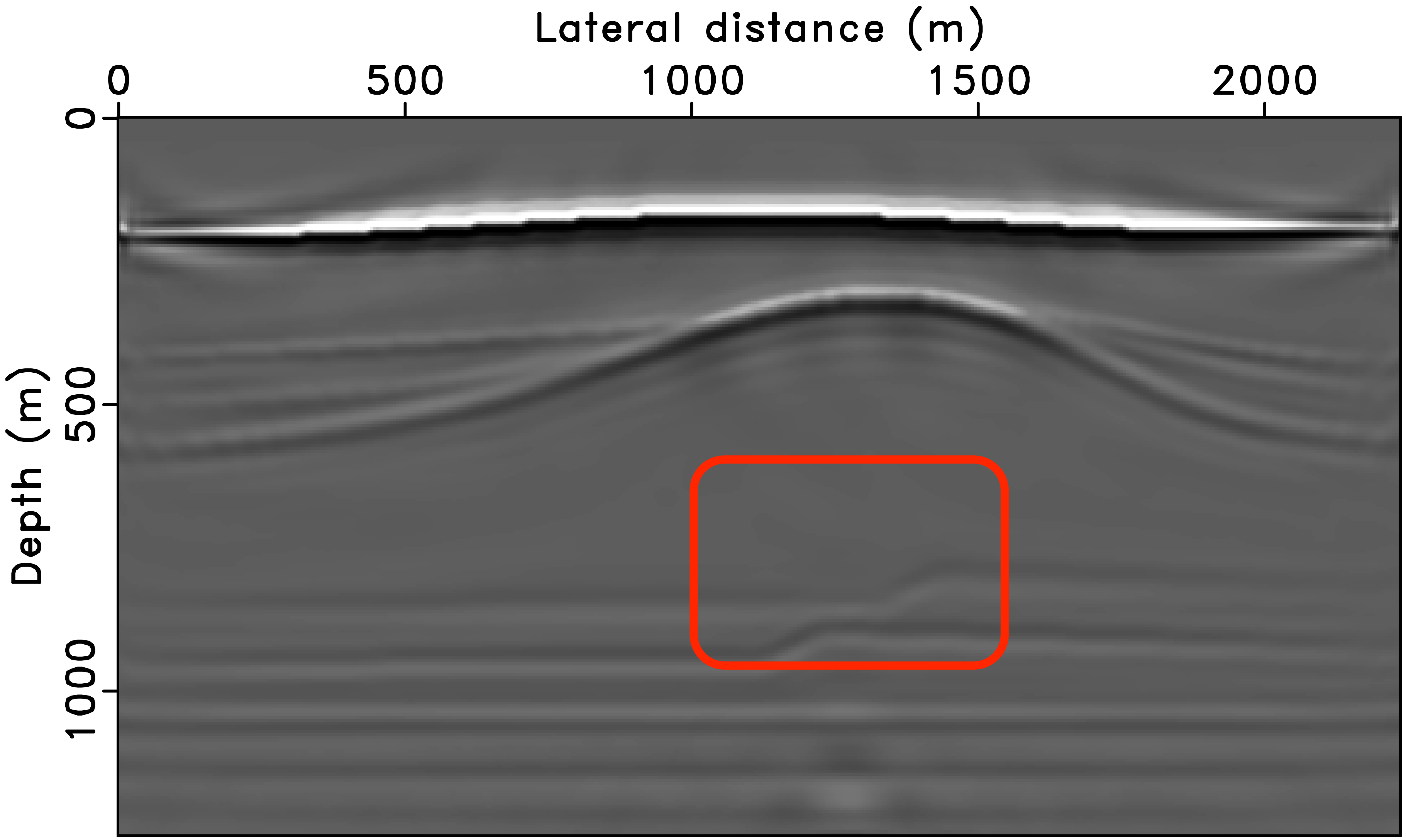
SNR: 5.78dB

Inverted model perturbation from primaries



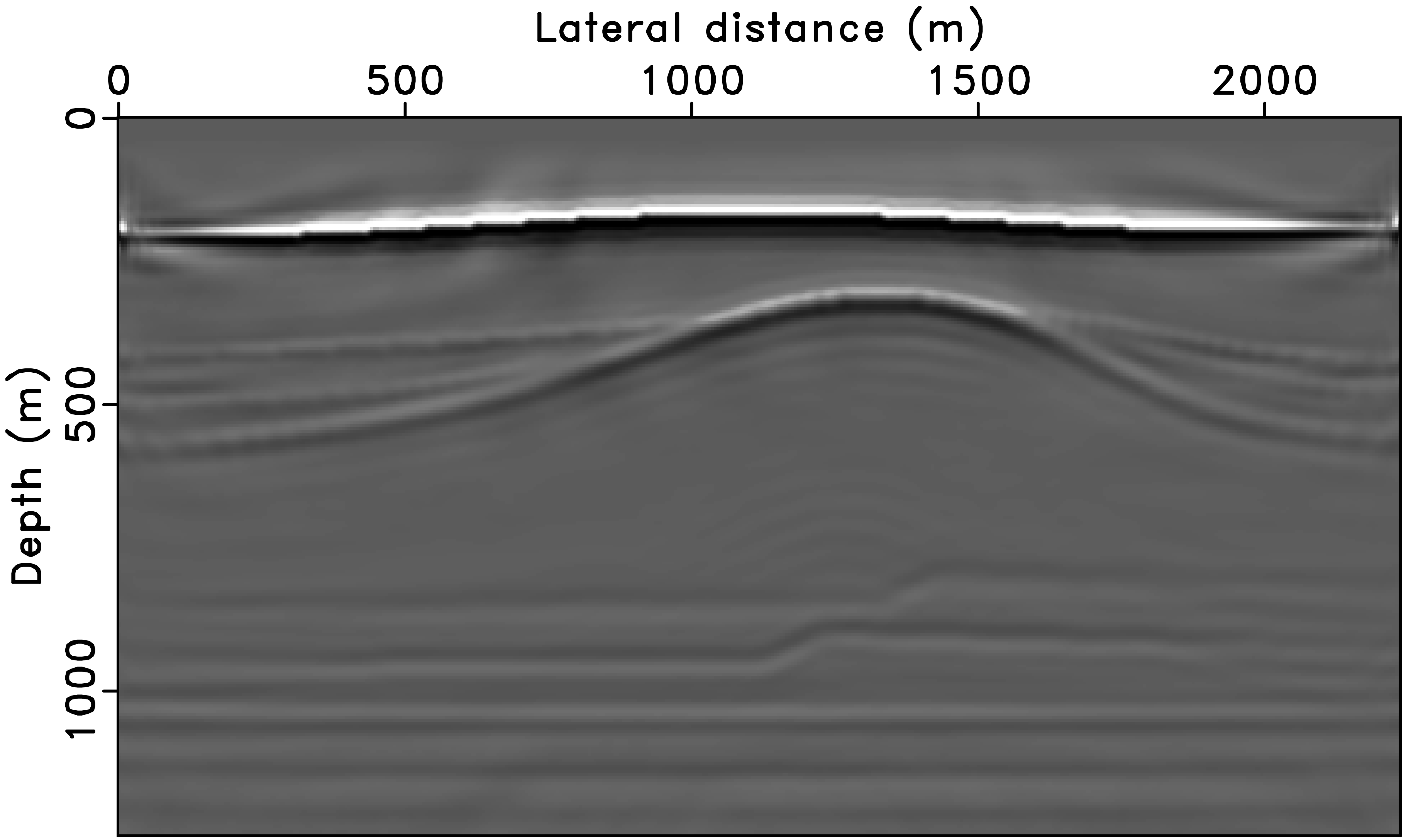
SNR: 5.73dB

Inverted model perturbation from primaries



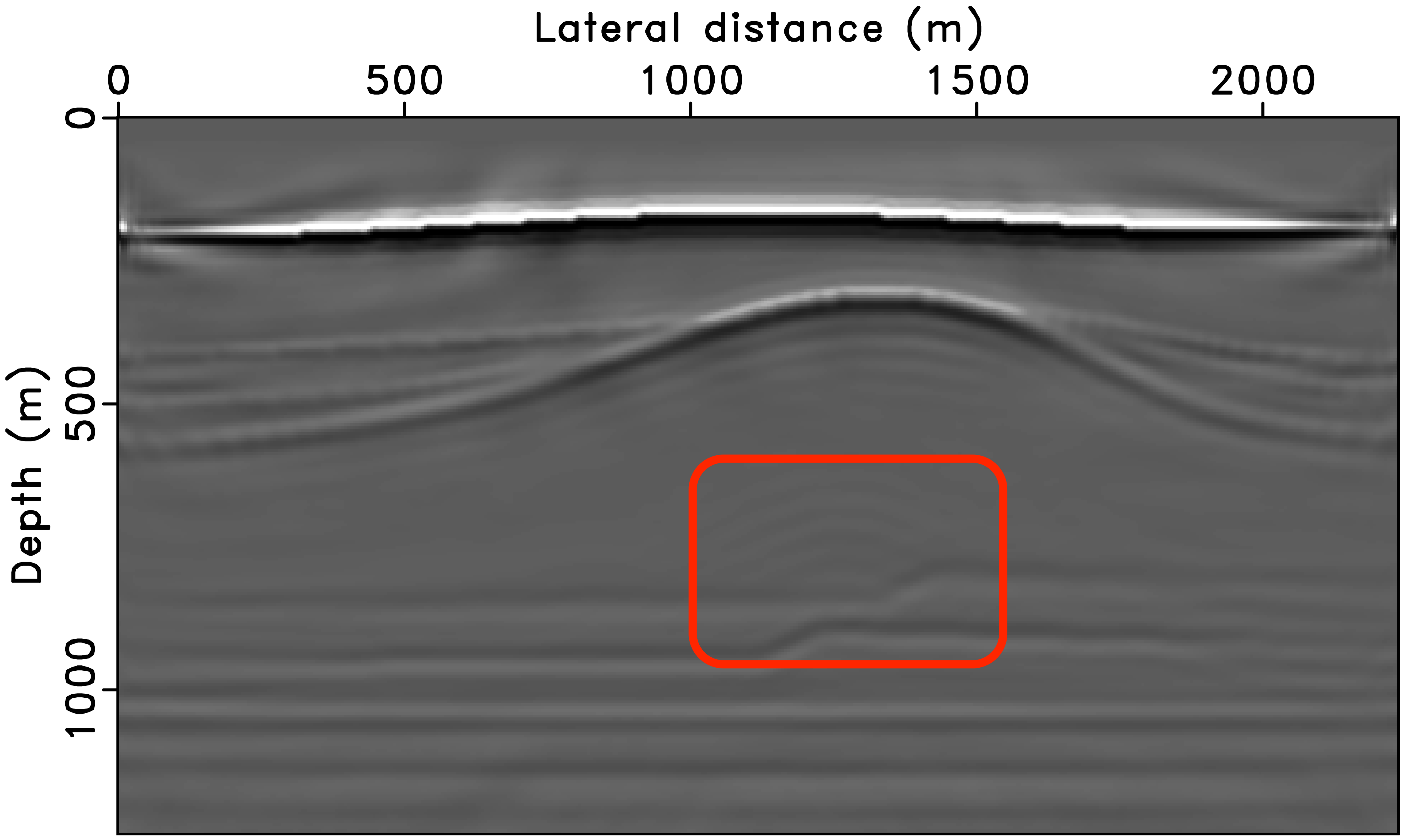
SNR: 5.73dB

Inverted model perturbation from multiples



SNR: 5.68dB

Inverted model perturbation from multiples



SNR: 5.68dB

Marine simultaneous acquisition examples

Linearized data

- Linearized Green's function

$$\mathbf{p_1} = \mathbf{K}\delta\mathbf{m}$$

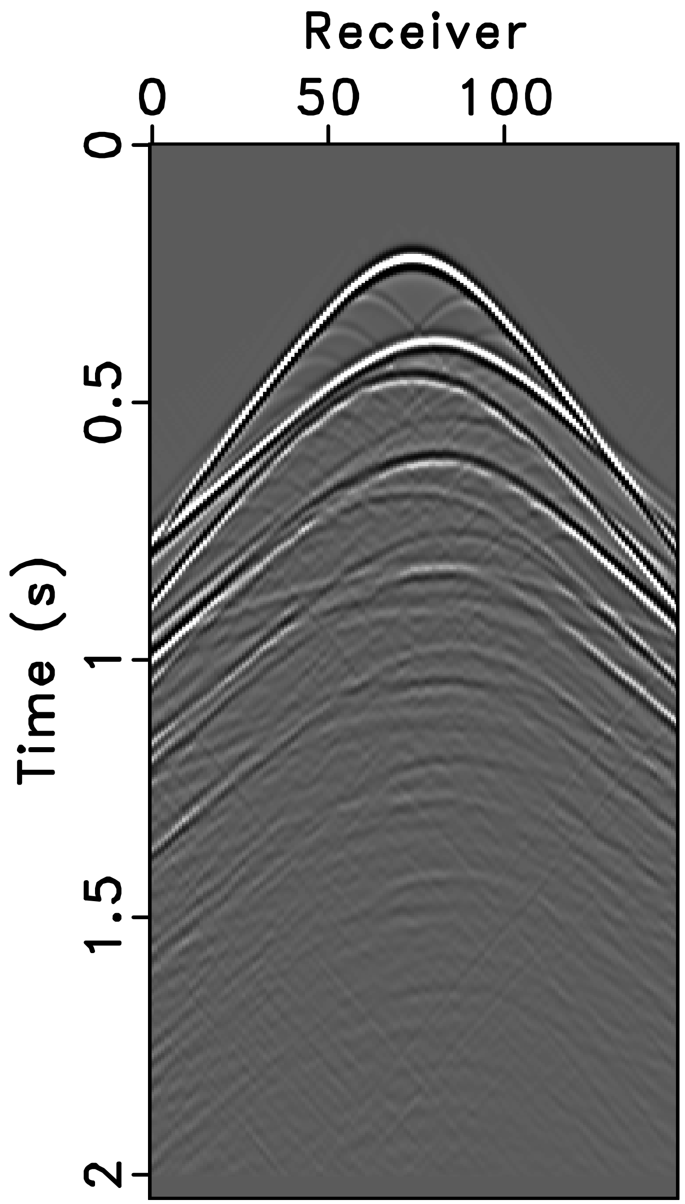
- Linearized total data, $\mathbf{RM}$ is the simultaneous acquisition operator*

$$\mathbf{p_2} = \mathbf{RMEK}\delta\mathbf{m}$$

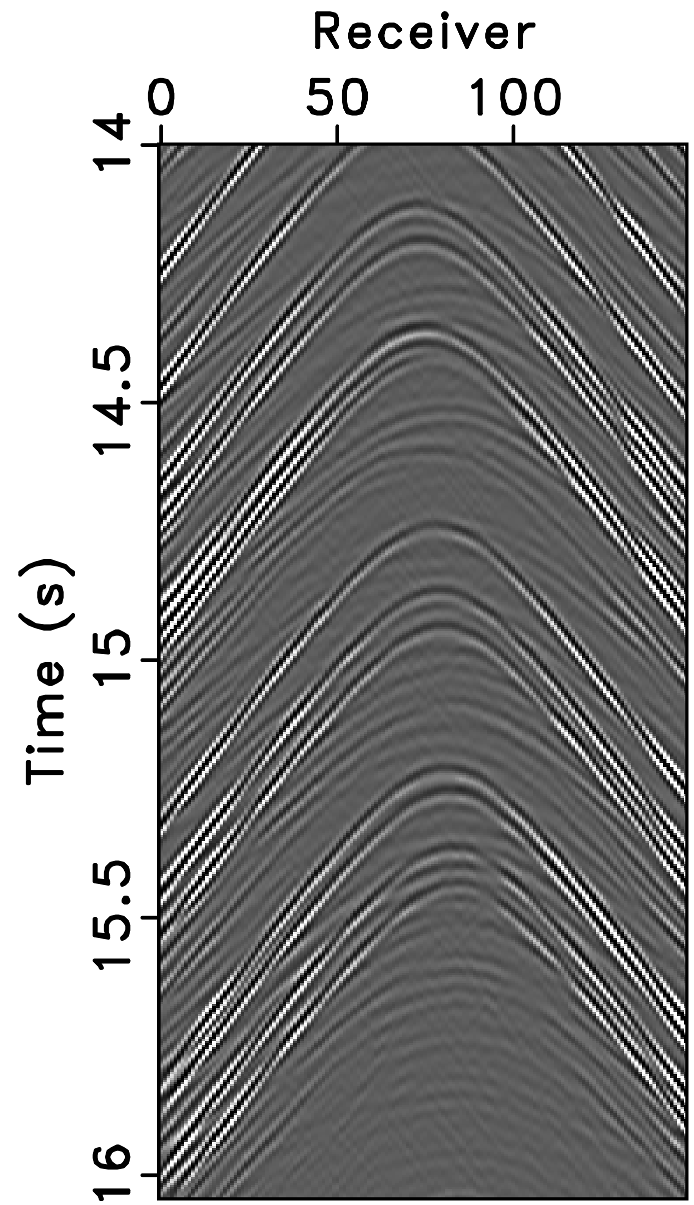
- Brief acquisition geometry: mimicking ocean bottom nodes 150 receivers, 7680 samples, 4ms rate

*: we for now assume that we have the full data P for the EPSI operator

Marine simultaneous data



Sequential shot



Simultaneous shots*

* with time dithering only

Two inversion schemes

- EPSI, then migration

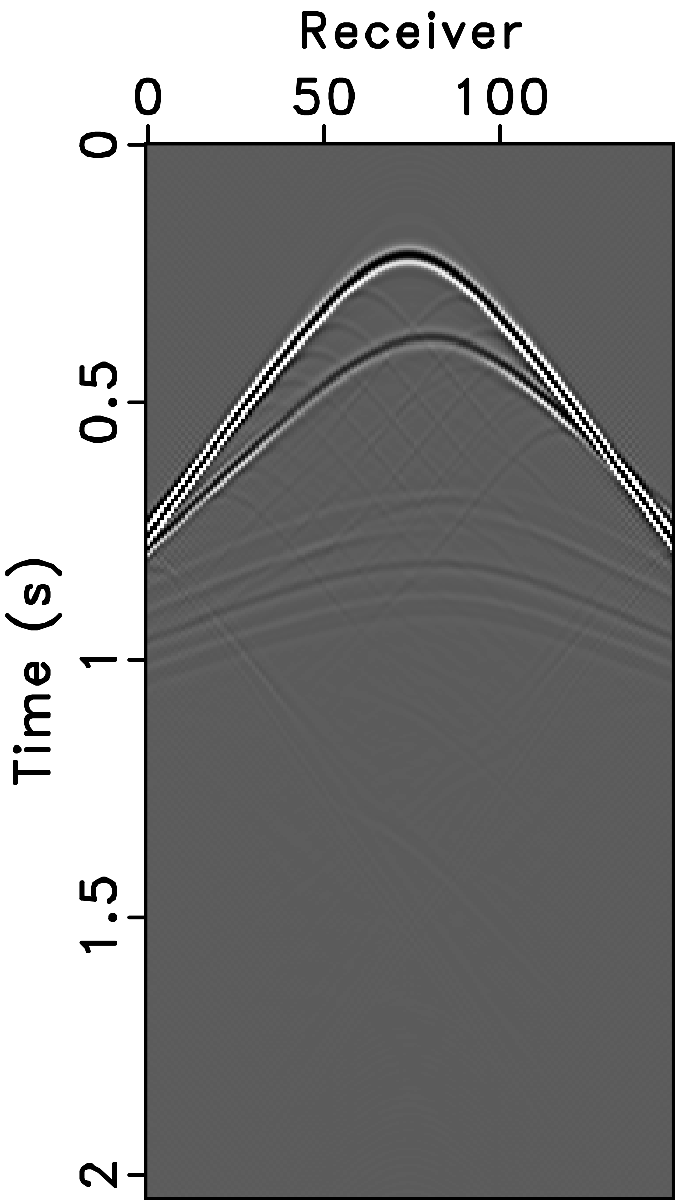
$$\delta\tilde{\mathbf{g}} = \mathbf{S}_3^* \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \text{ subject to } ||\mathbf{RMp} - \mathbf{RMES}_3^*\mathbf{x}||_2 \leq \sigma$$

$$\delta\tilde{\mathbf{m}} = \mathbf{S}_2^* \min \|\mathbf{x}\|_1 \text{ subject to } ||\delta\tilde{\mathbf{g}} - \mathbf{KS}_2^*\mathbf{x}||_2 \leq \sigma$$

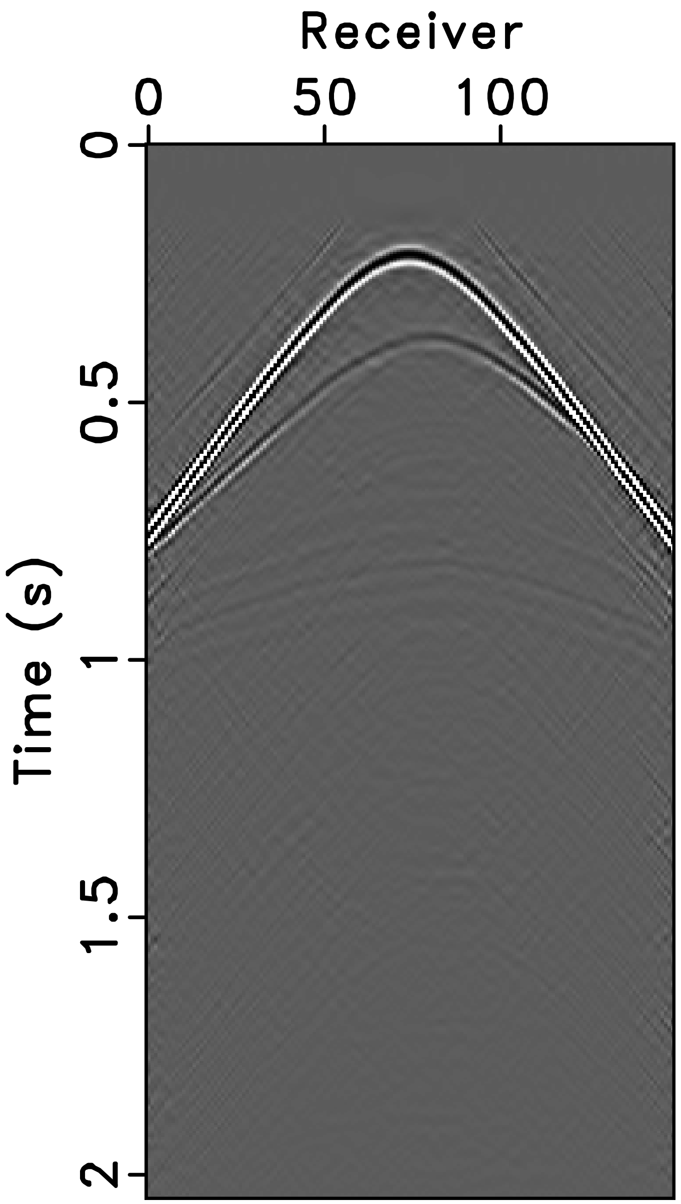
- EPSI+migration

$$\delta\tilde{\mathbf{m}} = \mathbf{S}_2^* \min \|\mathbf{x}\|_1 \text{ subject to } ||\mathbf{RMp} - \mathbf{RMEKS}_2^*\mathbf{x}||_2 \leq \sigma$$

Inverted Green's function in separate inversions

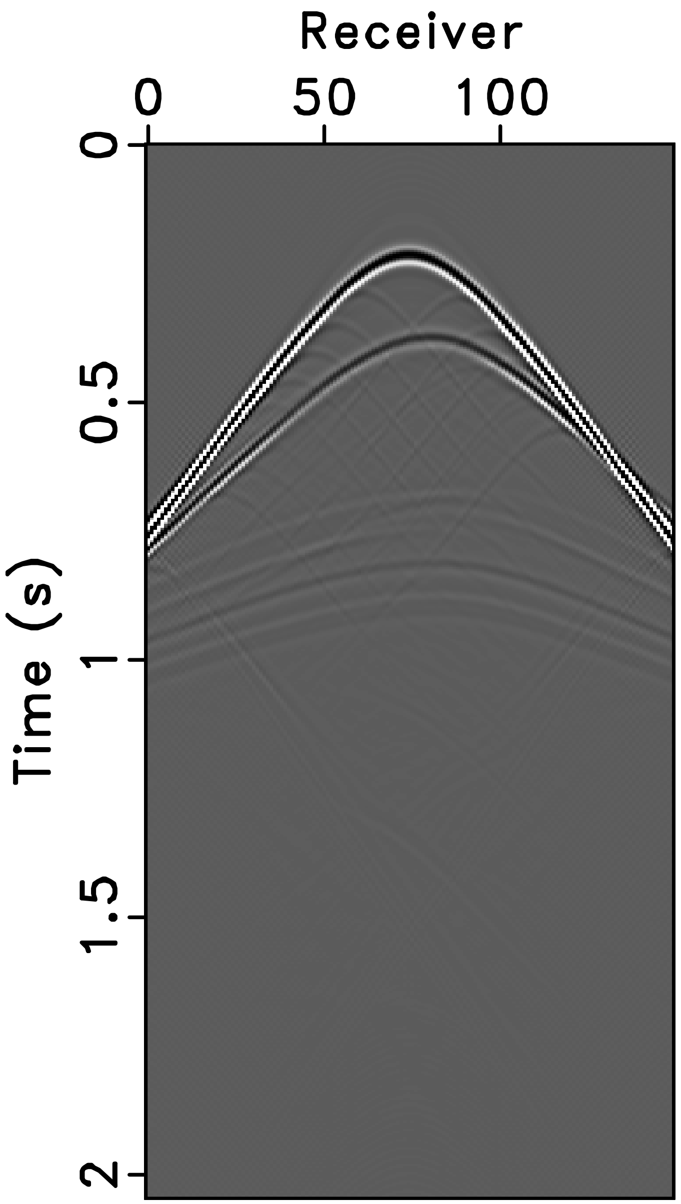


True Green's function

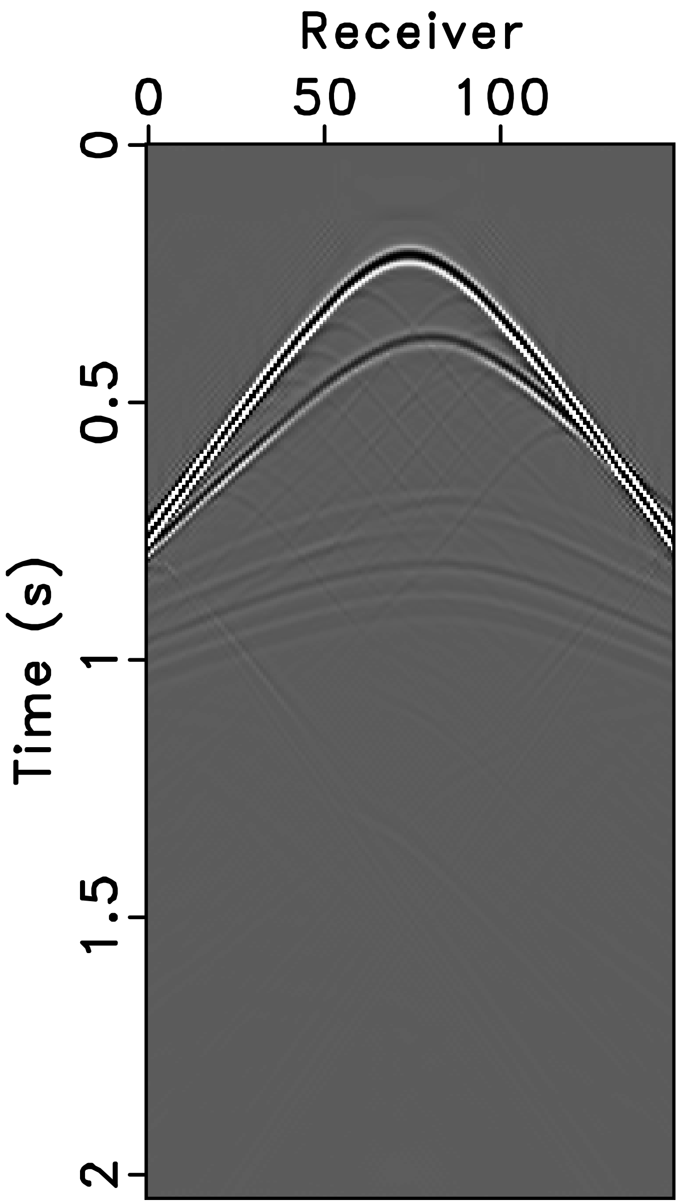


EPSI in data space, SNR 10.8dB

Inverted Green's function in separate inversions

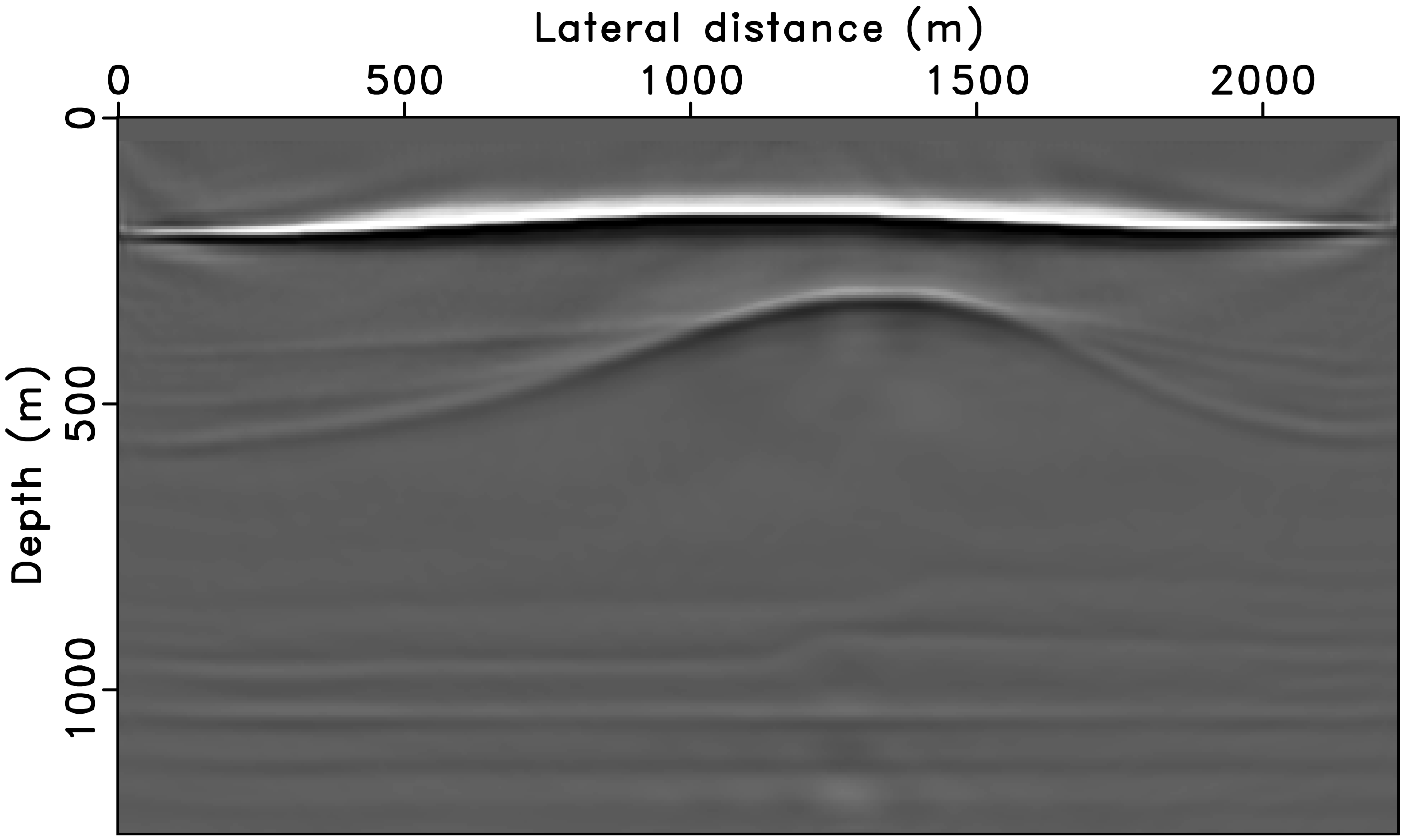


True Green's function



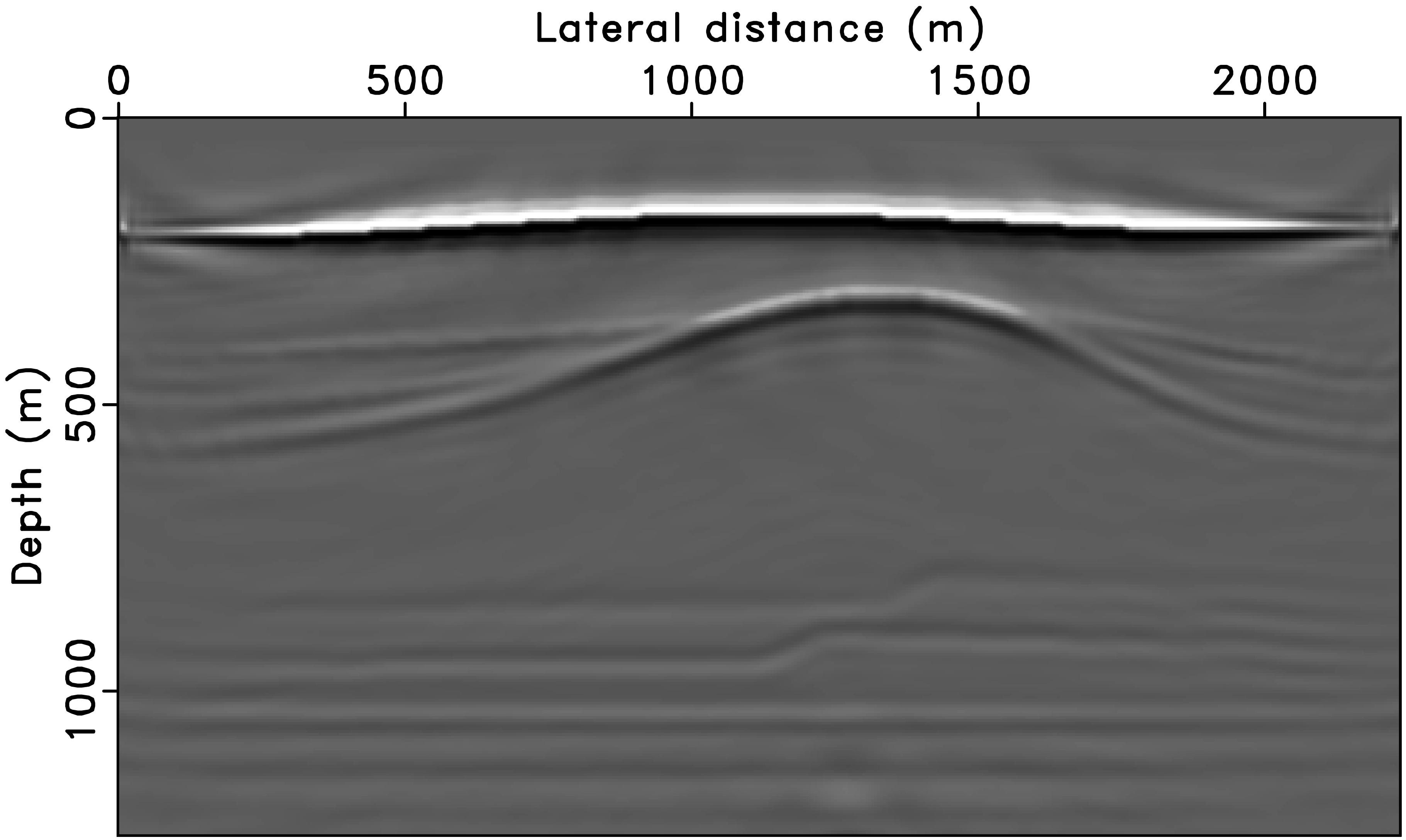
EPSI in image space, SNR 23.4dB

Inverted model perturbation in separate inversions



SNR: 5.04dB

Inverted model perturbation in combined inversion



SNR: 5.37dB

Rethink EPSI+migration

- Great idea, conceptually
- Heavy computation, in practice

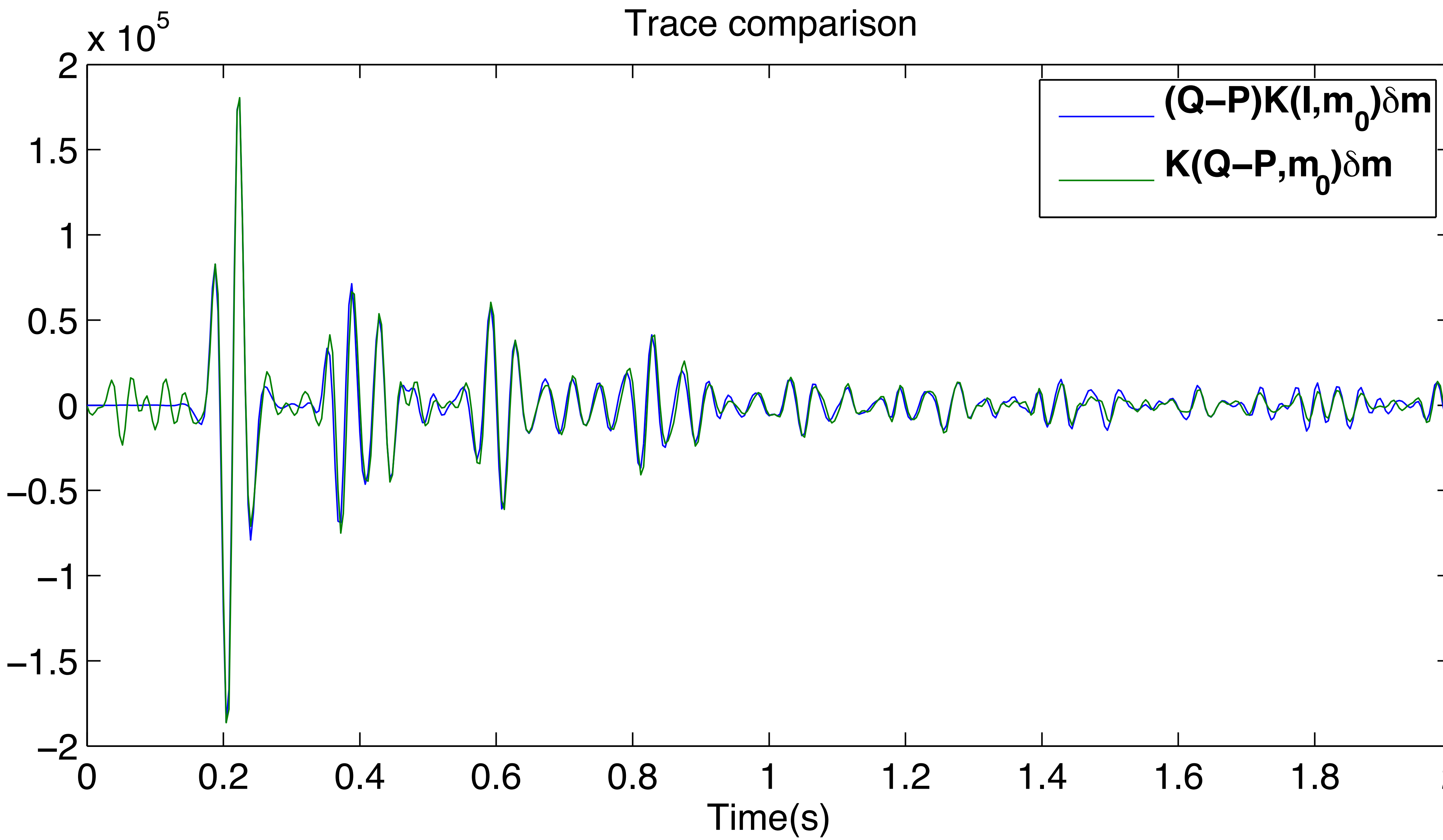
1) Data driven EPSI needs full data matrices, e.g. P and G, and their multiplications

2) Model driven migration needs to solve the same number of PDEs as the number of sources to get full G

Source association ?

$$(\mathbf{Q}-\mathbf{P})\mathbf{K}(\mathbf{I},\mathbf{m_0})\delta\mathbf{m} \quad ? \quad \mathbf{K}(\mathbf{Q}-\mathbf{P},\mathbf{m_0})\delta\mathbf{m}$$

Trace comparison



Monochromatically...

$$\hat{\mathbf{H}}^{-1}(\hat{\mathbf{Q}} - \hat{\mathbf{P}}, \mathbf{m}) = \mathbf{D_r}\hat{\mathbf{H}}^{-1}(\mathbf{m})\mathbf{D_s^*}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

+

$$\hat{\mathbf{H}}^{-1}(\mathbf{I}, \mathbf{m})(\hat{\mathbf{Q}} - \hat{\mathbf{P}}) = \mathbf{D_r}\hat{\mathbf{H}}^{-1}(\mathbf{m})\mathbf{D_s^*}\mathbf{I}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

↓

$$\hat{\mathbf{H}}^{-1}(\mathbf{I}, \mathbf{m})(\hat{\mathbf{Q}} - \hat{\mathbf{P}}) = \hat{\mathbf{H}}^{-1}(\hat{\mathbf{Q}} - \hat{\mathbf{P}}, \mathbf{m})$$

Guitton, 2002

Berkhout 2005

Whitmore 2010

Ning et. al. 2010

Verschuur 2011

Source association !

SLIM

$$(\mathbf{Q}-\mathbf{P})\mathbf{K}(\mathbf{I},\mathbf{m_0})\delta\mathbf{m} \quad = \quad \mathbf{K}(\mathbf{Q}-\mathbf{P},\mathbf{m_0})\delta\mathbf{m}$$

SLIM

okay...what does this mean?

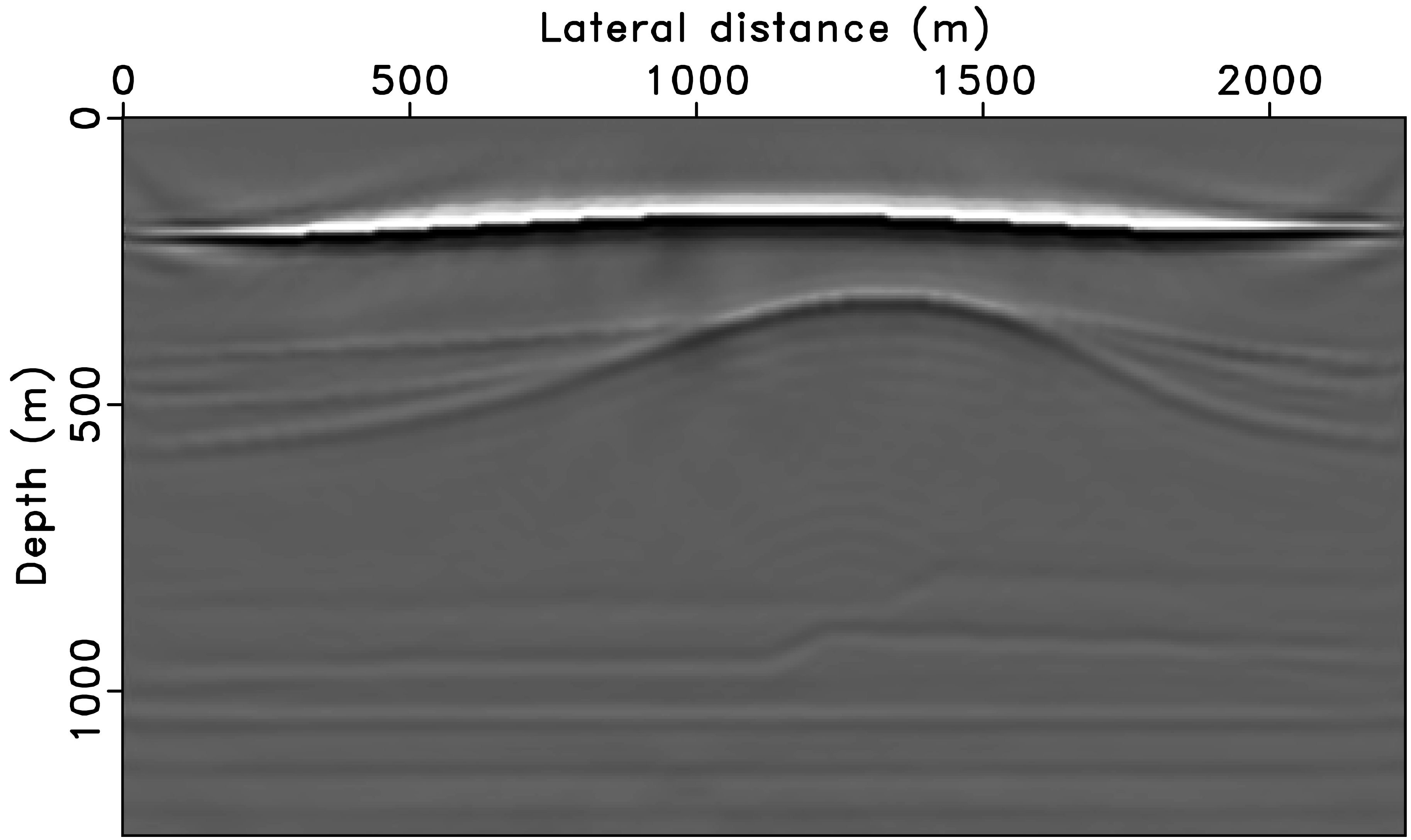
- Multi-dimentional convolution is taken over by wavefield simulation
- A gate is open for CS methodology to speed up “EPSI+migration”!

Speed up EPSI+migration by source-encoding

$$\delta\tilde{\mathbf{m}} = \mathbf{S_2^*} \min \|\mathbf{x}\|_\mathbf{1} \text{ subject to } \|\mathbf{RMp} - \mathbf{K}(\mathbf{RM}(\mathbf{q}-\mathbf{p}))\mathbf{S_2^*}\mathbf{x}\|_\mathbf{2} \leq \sigma$$

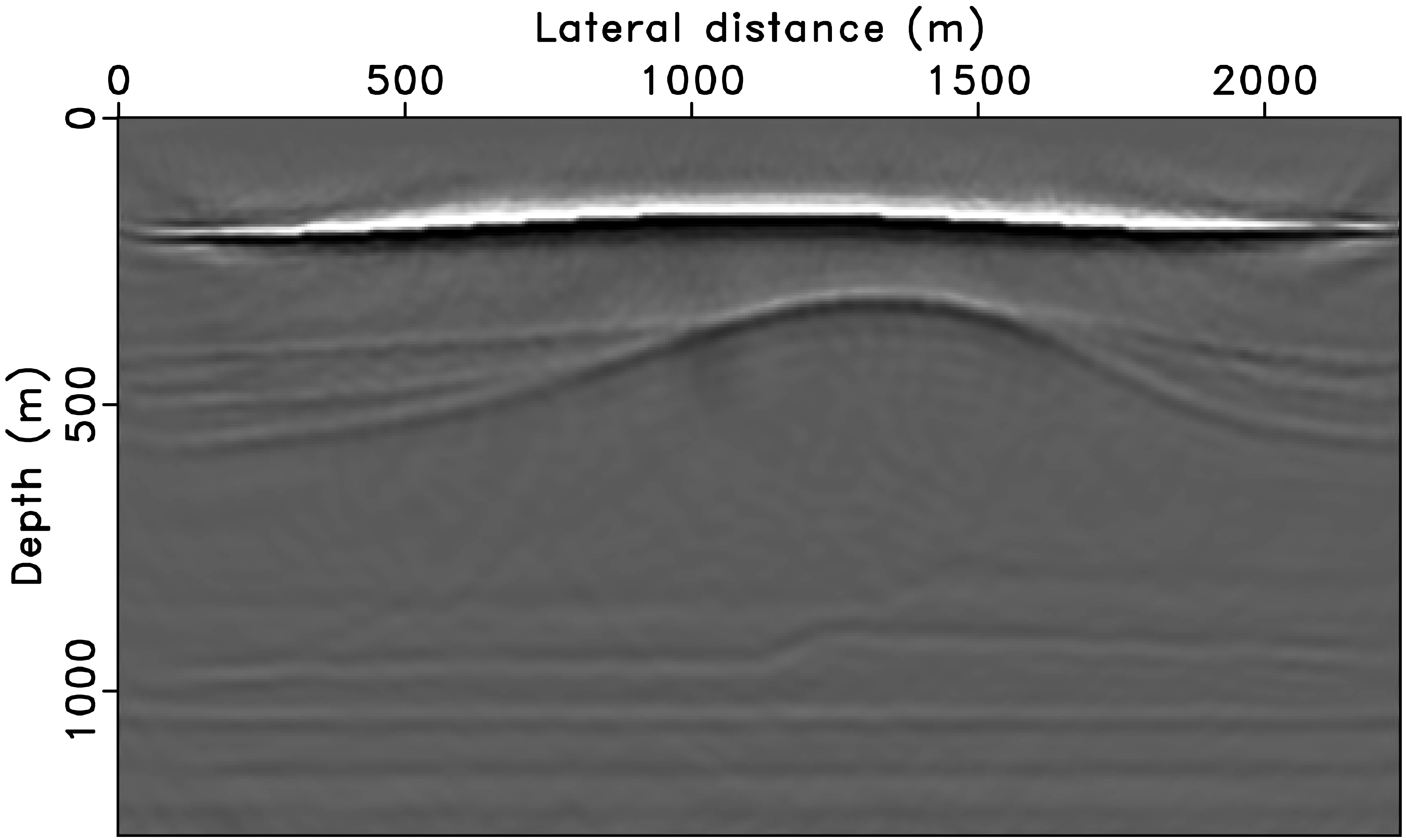
- $\mathbf{RM}$ applies on the source and can greatly reduce the number of sources and thus the system size

Inversion using 10 super-shots



SNR: 5.67dB

Inversion using 2 super-shots

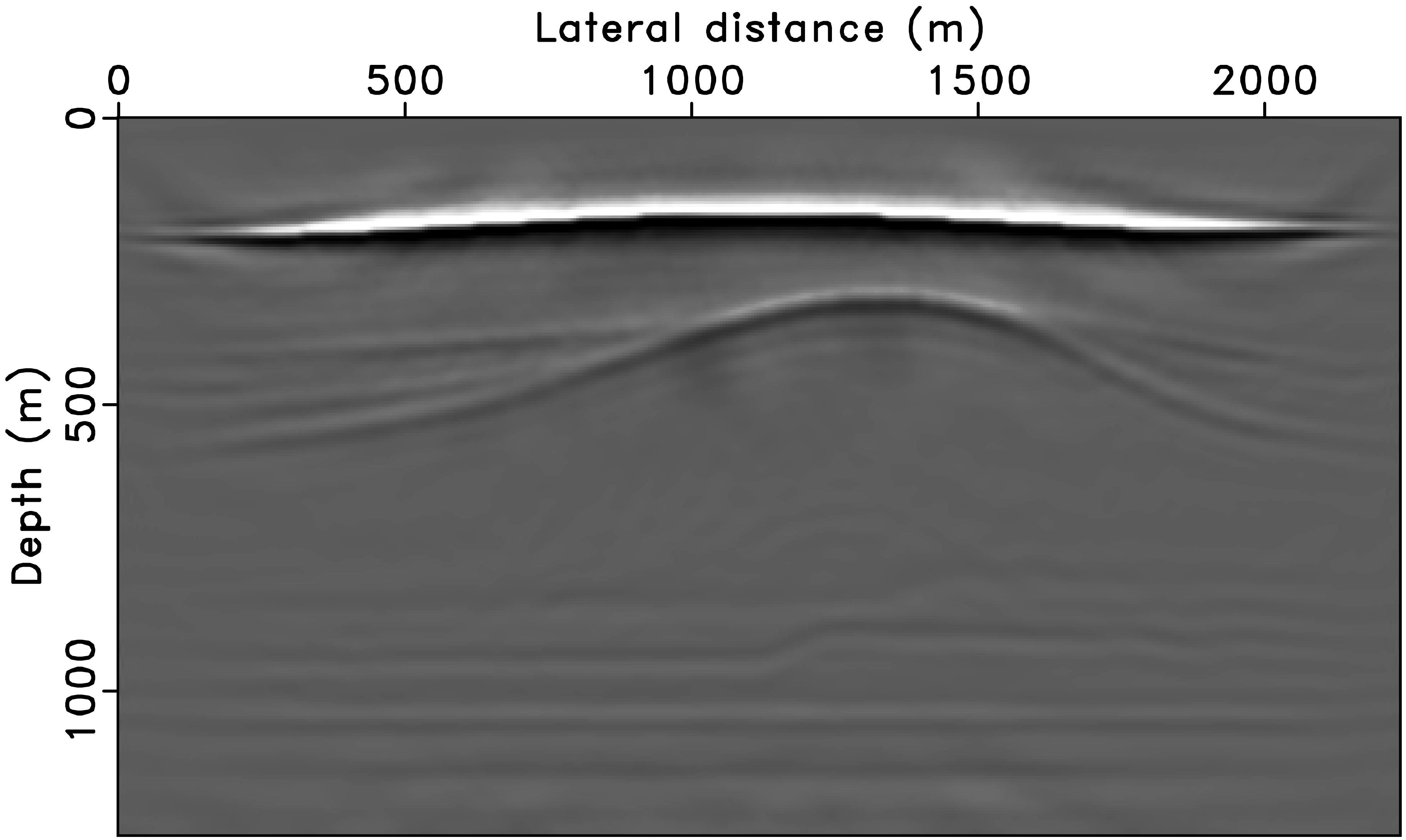


SNR: 5.46dB

Shots renewal

- draw new super-shots after some iterations subject to certain criteria
- add more randomness to the system
- help turn coherent noise to random noise

Inversion using 2 super-shots with *renewal*



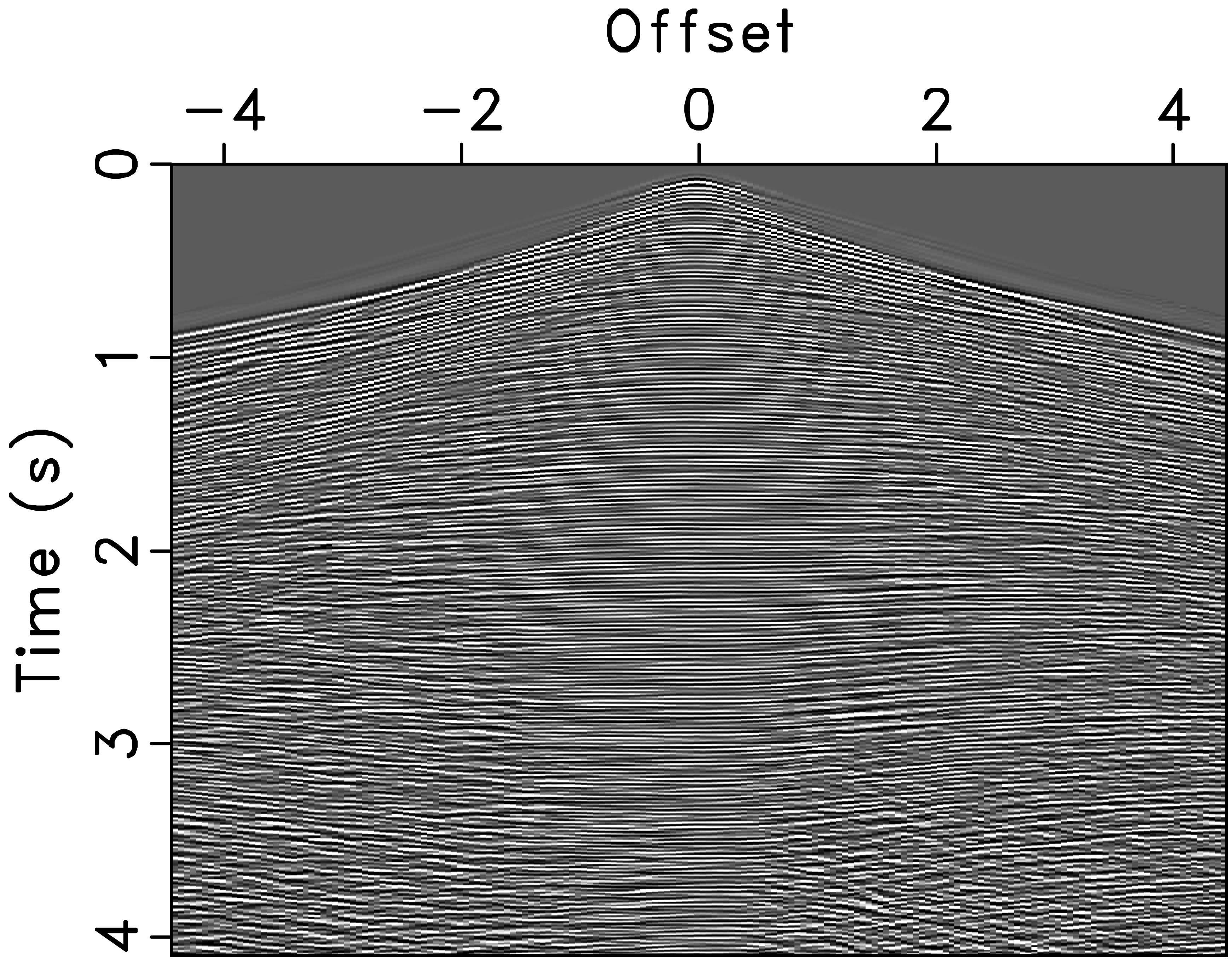
SNR: 4.84dB

Field data examples

Gulf of Suez data

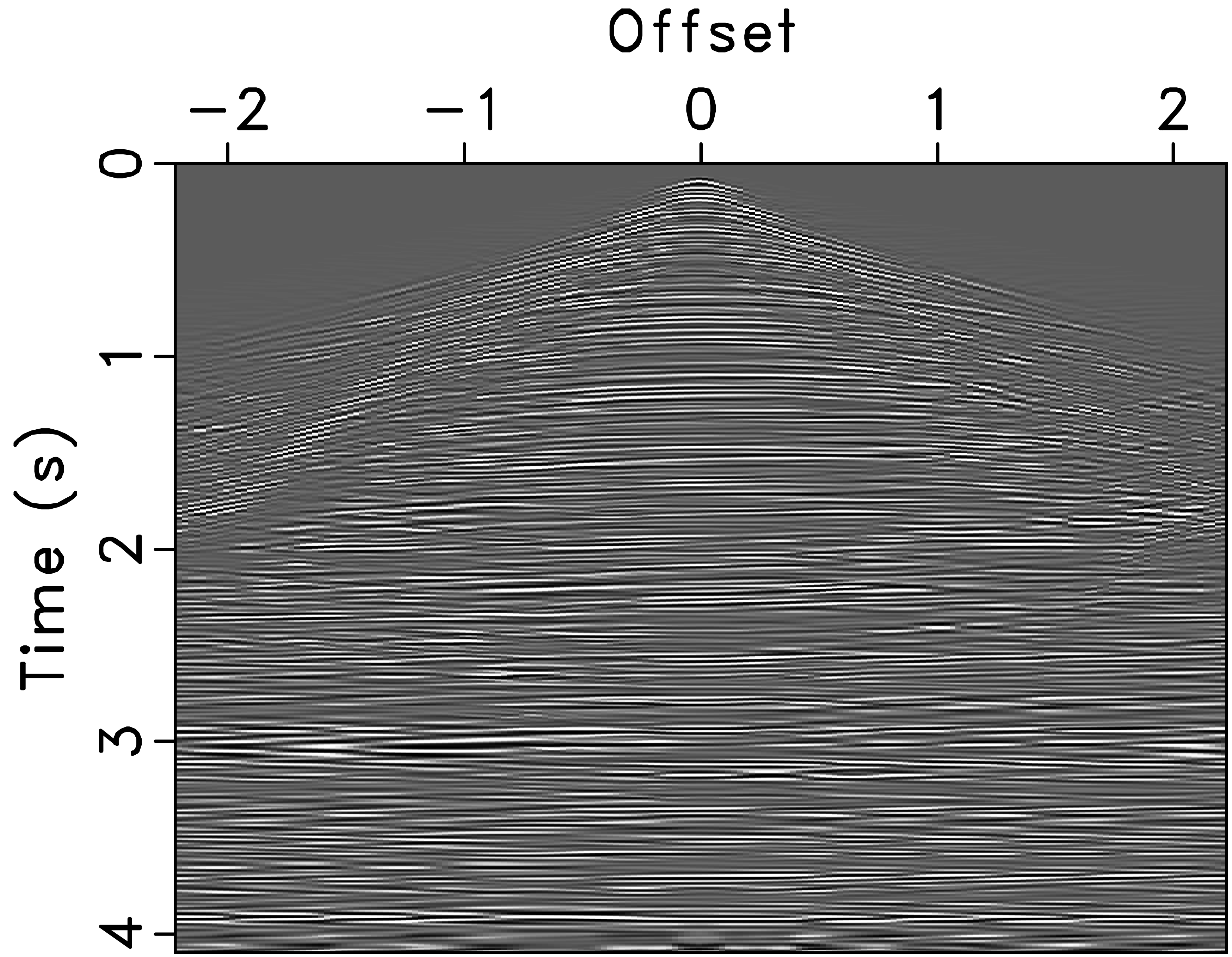
- Very shallow water, strong surface multiples
- Also contains great amount of internal multiples
- About 4s recording time
- 25m source/receiver spacing

Total data: one shot-gather
[shown with AGC]



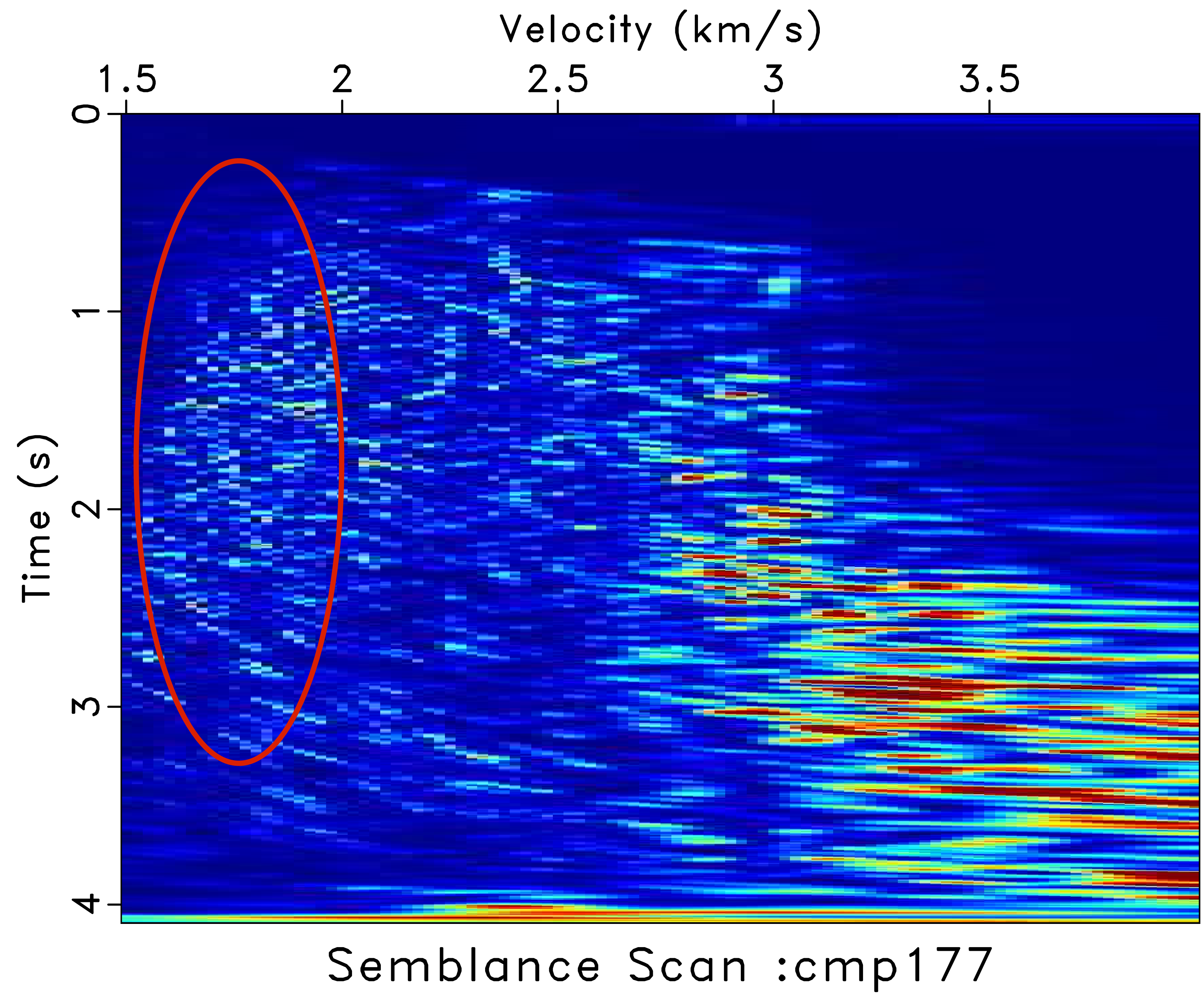
Total data: the 89th shot gather

Primaries: one shot-gather
[shown with AGC and muting]

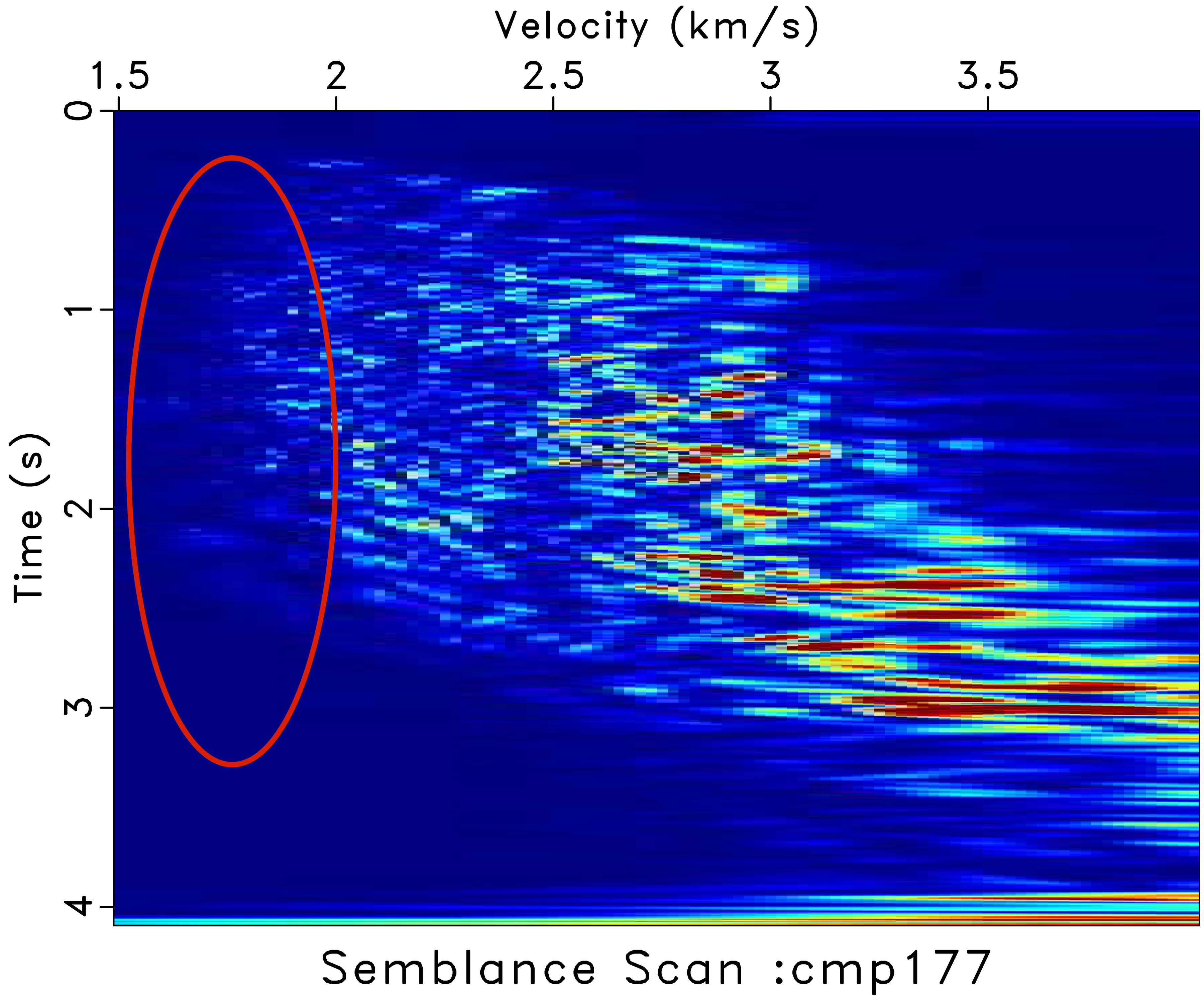


Primary: the 89th shot gather

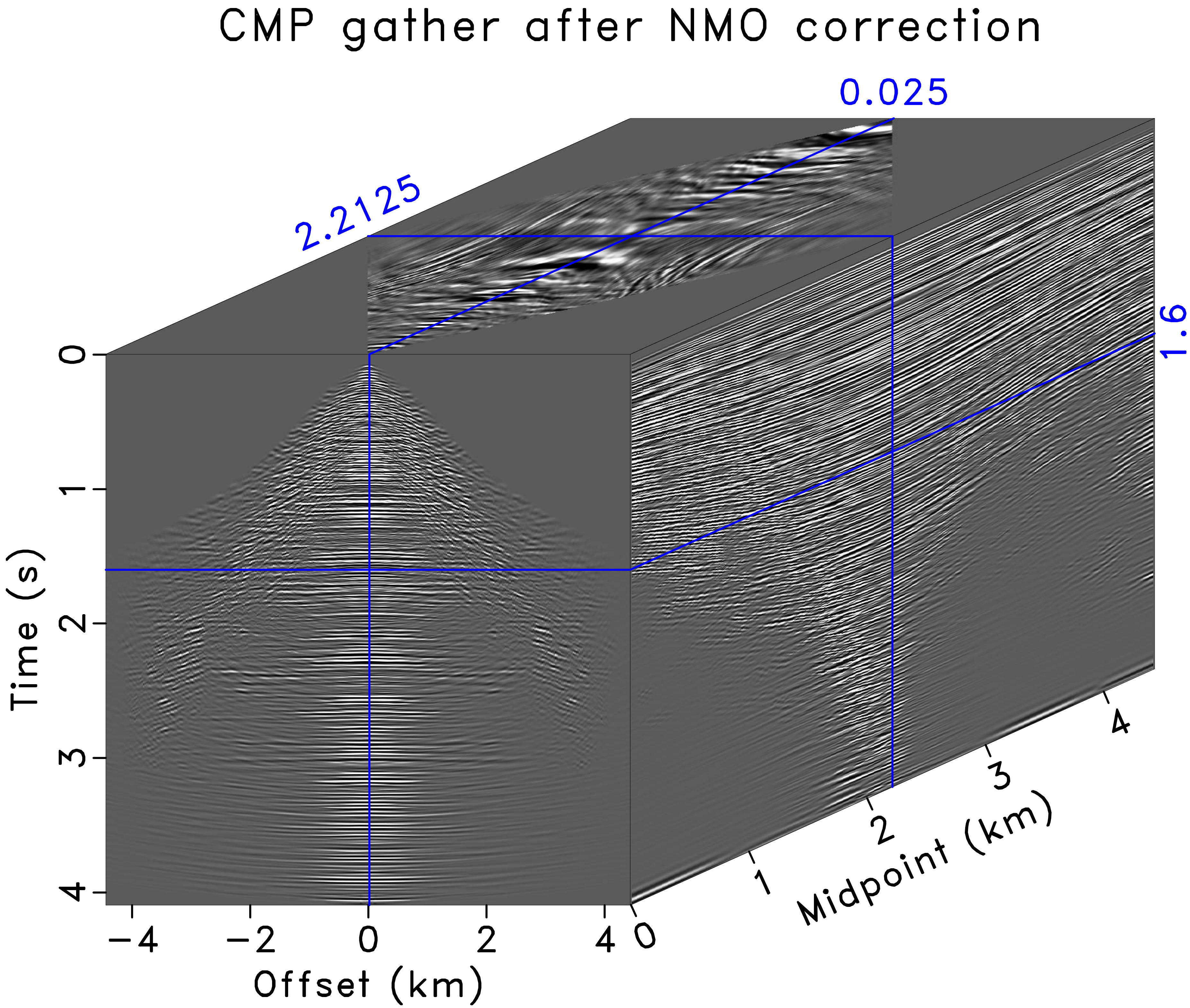
Semblance plot-total data



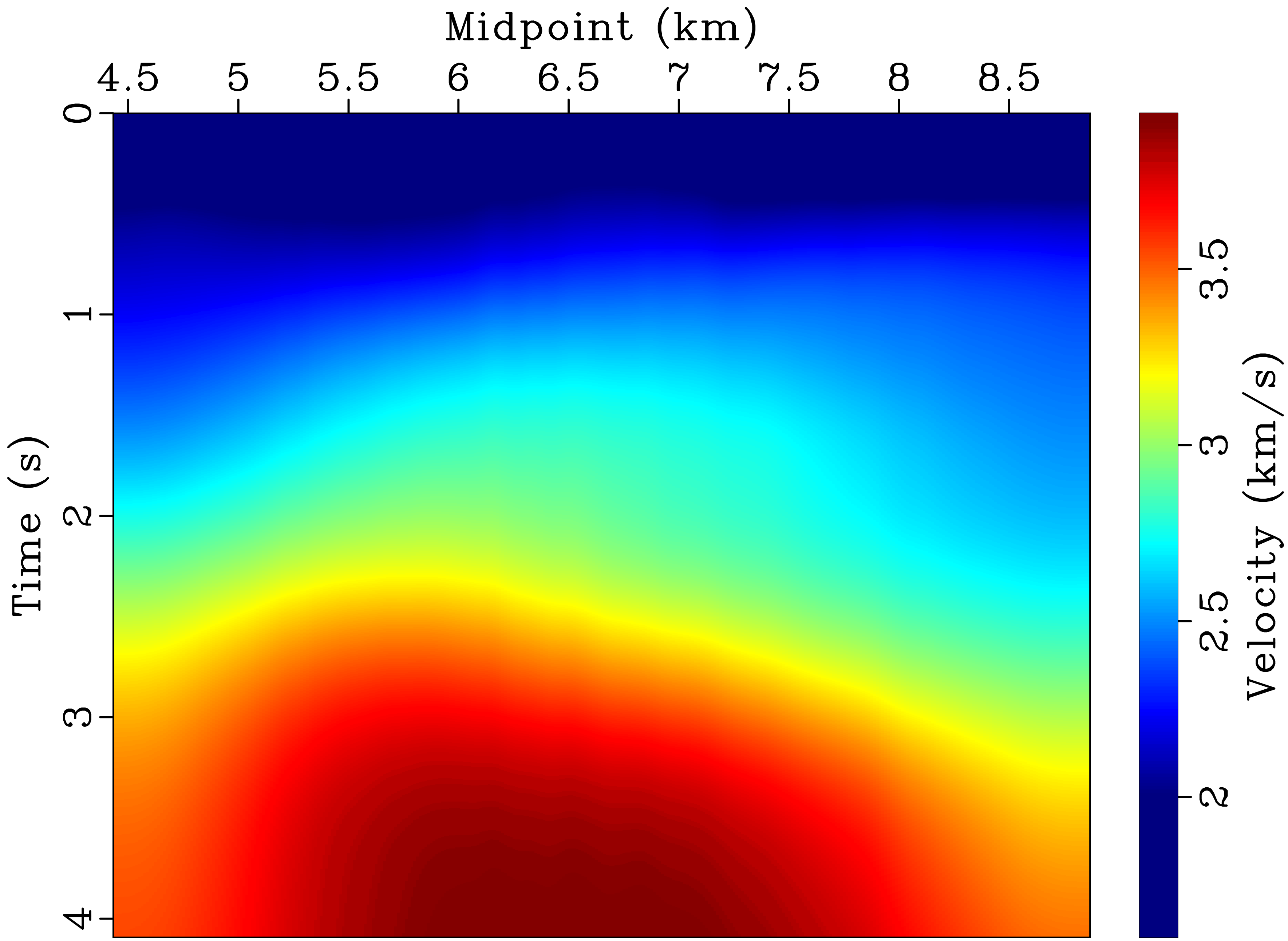
Semblance plot-primaries



NMO correction with stacking velocity [shown with time-weighting]

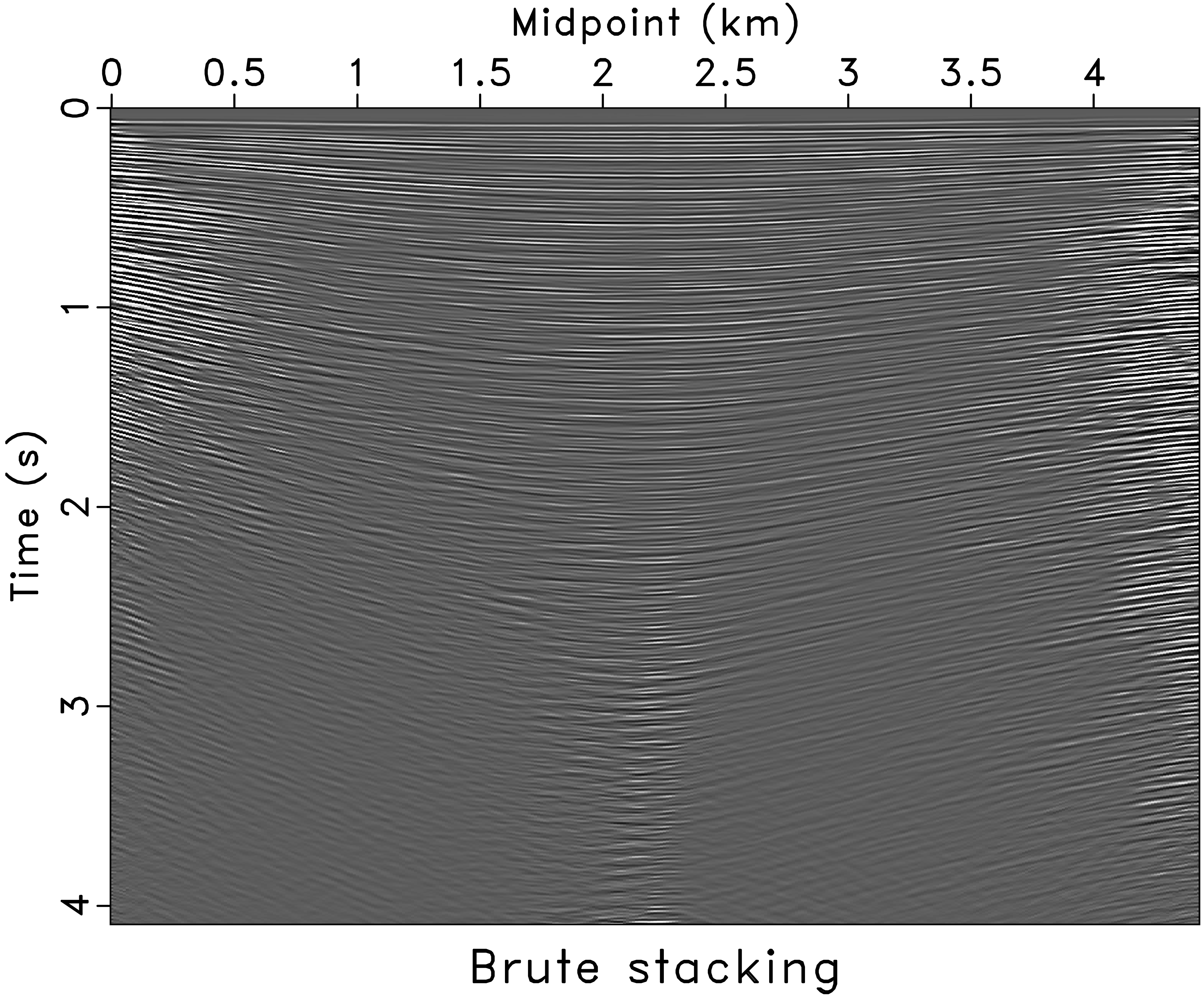


Stacking velocity-surface free data

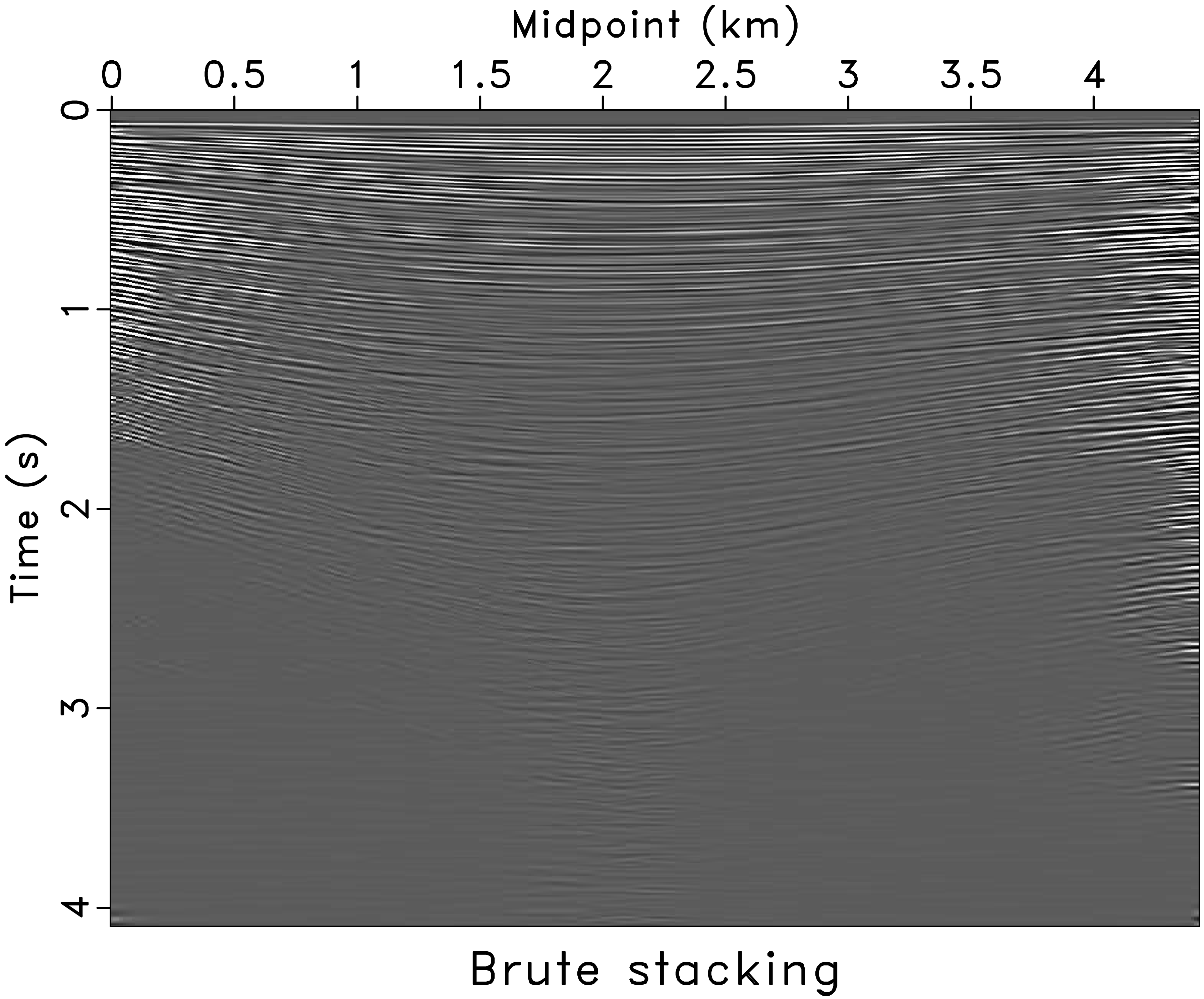


Stacking velocity

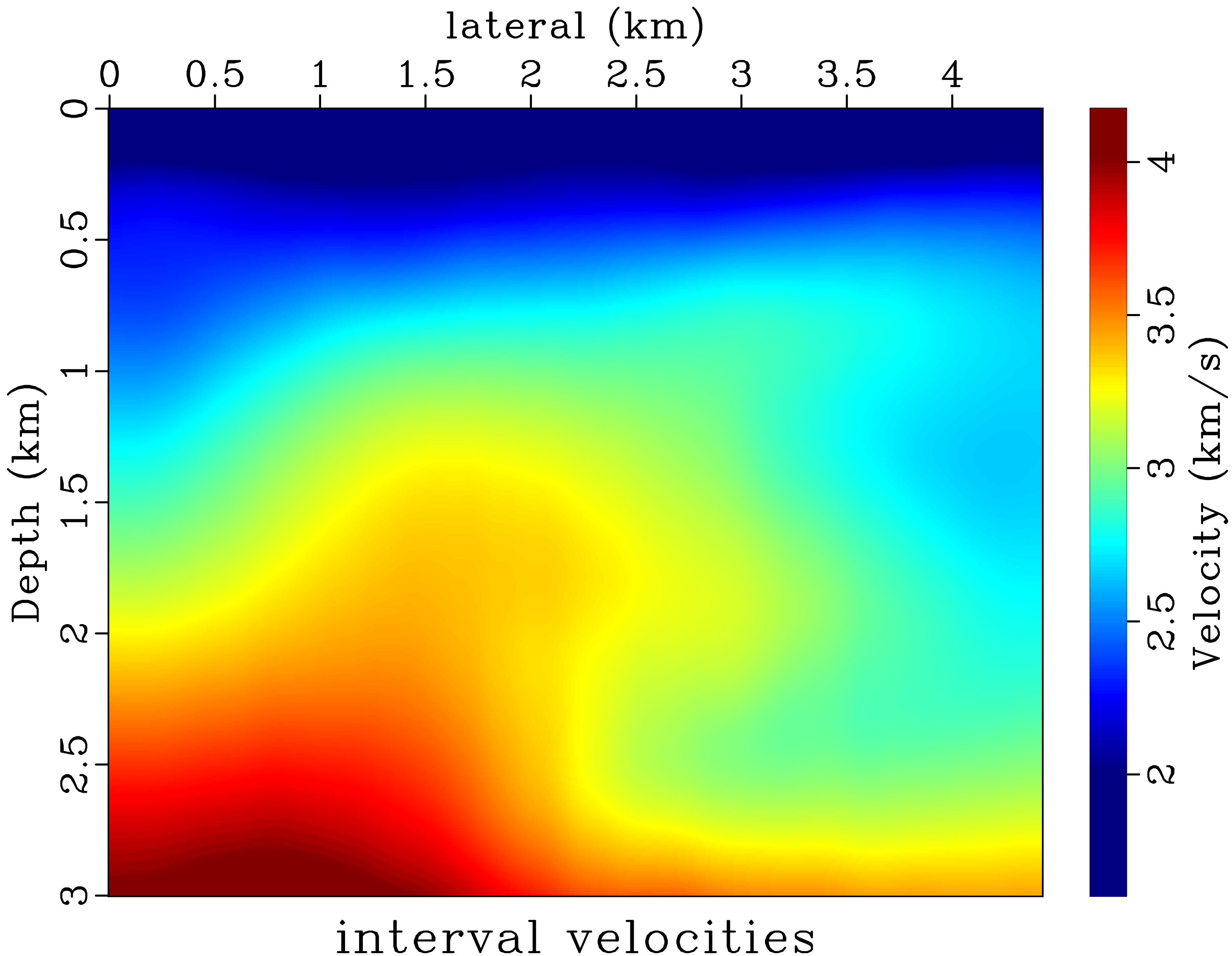
Stacked section-total data
[shown with time-weighting]



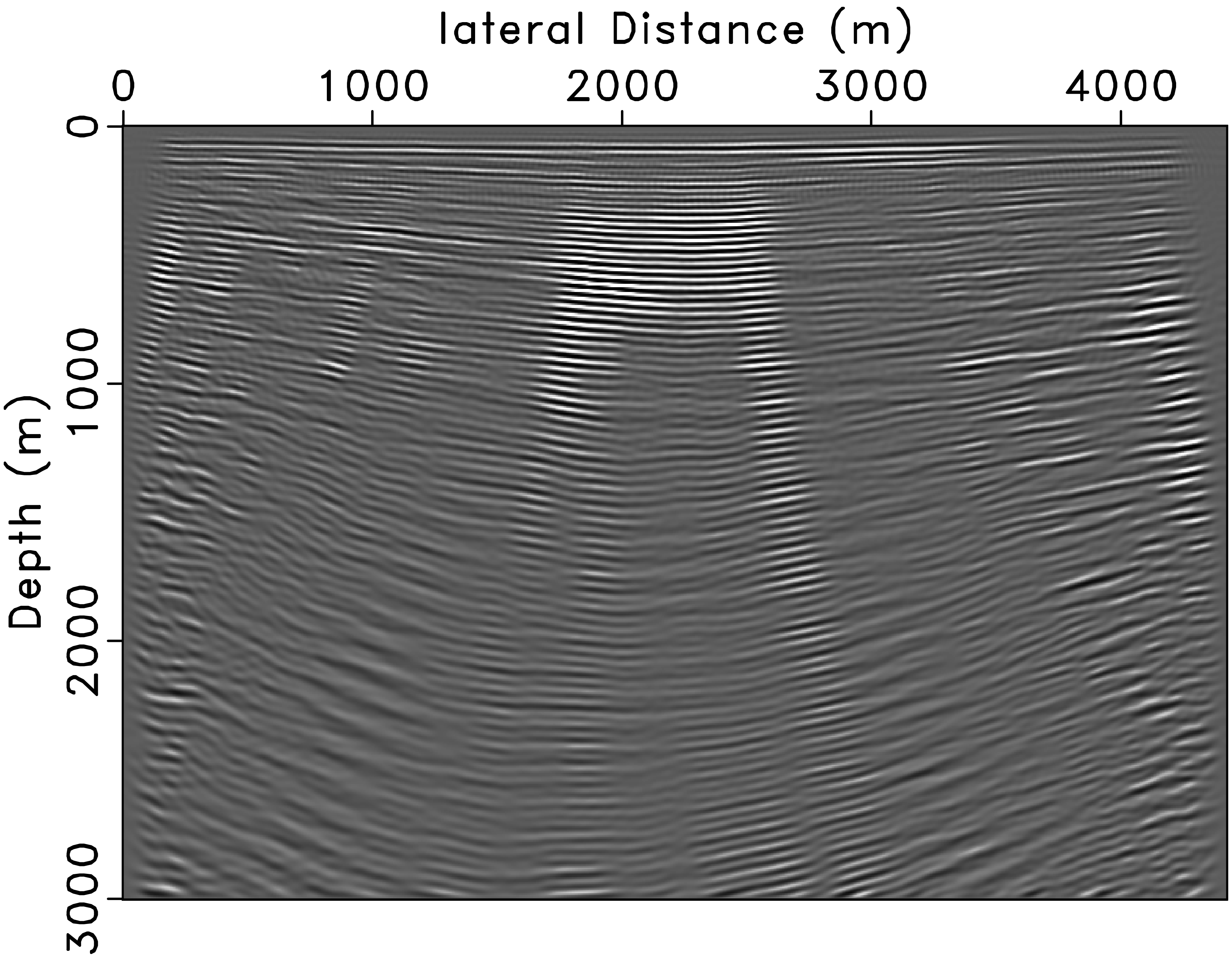
Stacked section-primary
[shown with time-weighting]



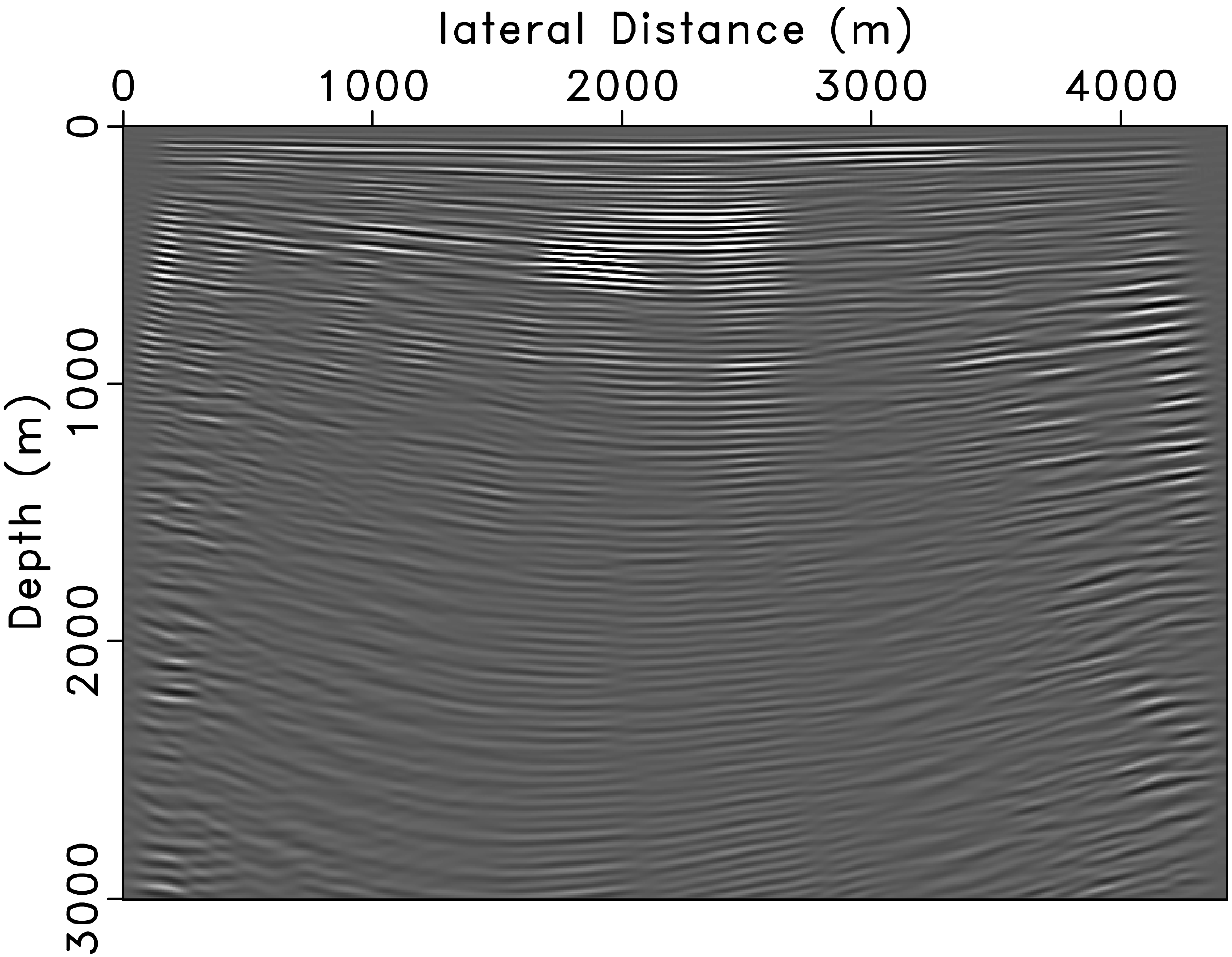
Migration velocity



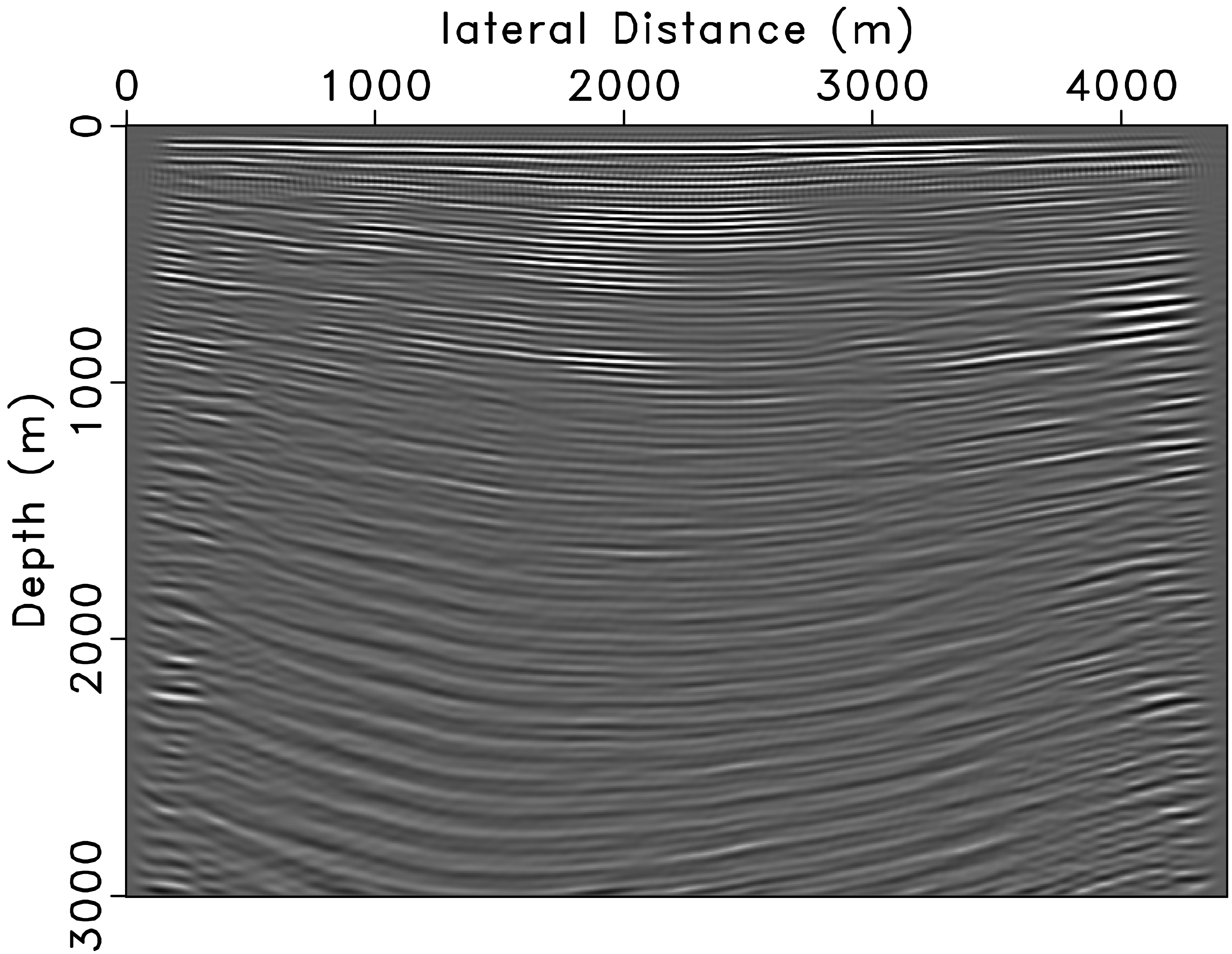
L1 migration of total data without EPSI



L1 migration of total data with EPSI



L1 migration of surface-free data



SLIM

Conclusions

By combining EPSI & migration,

- surface-related multiples are well handled
- for severely subsampled data, e.g., marine simultaneous data, we do gain benefits
- in terms of surface-free data recovery, we gain a lot by optimizing in image space
- we can also get huge efficiency boost by exploiting CS methodology

Future plans

- Alternative formulations of combining EPSI and migration
- EPSI in the image space on spatially under-sampled data, e.g, missing sources/ receivers, by using (de)migration as an "interpolation" tool

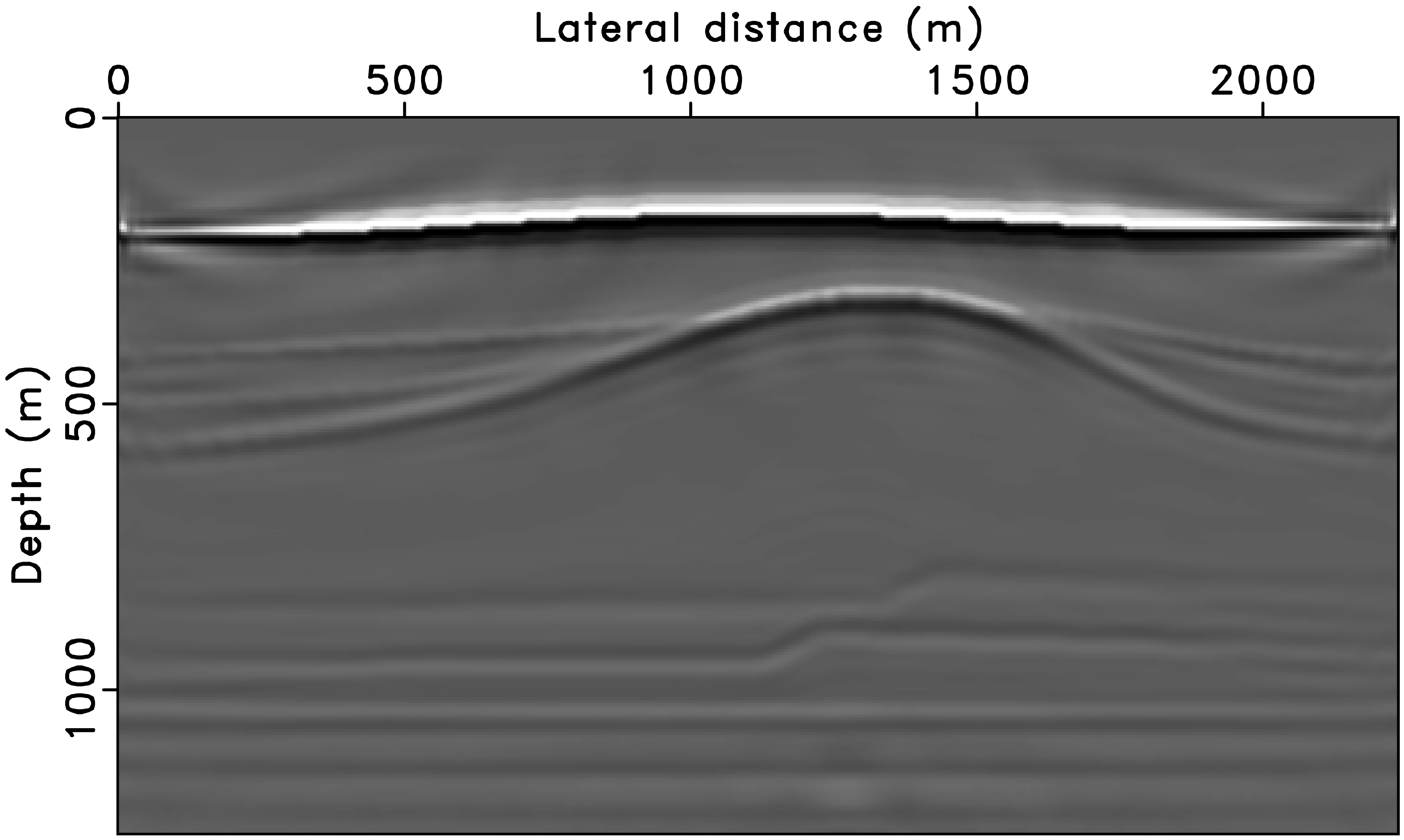
Alternative formulations

- "Collaborative" form

$$\begin{pmatrix} \mathbf{Q} \\ \lambda(-\mathbf{P}) \end{pmatrix} \mathbf{K}\delta\mathbf{m} = \begin{pmatrix} \mathbf{p_0} \\ \lambda\mathbf{u} \end{pmatrix}$$

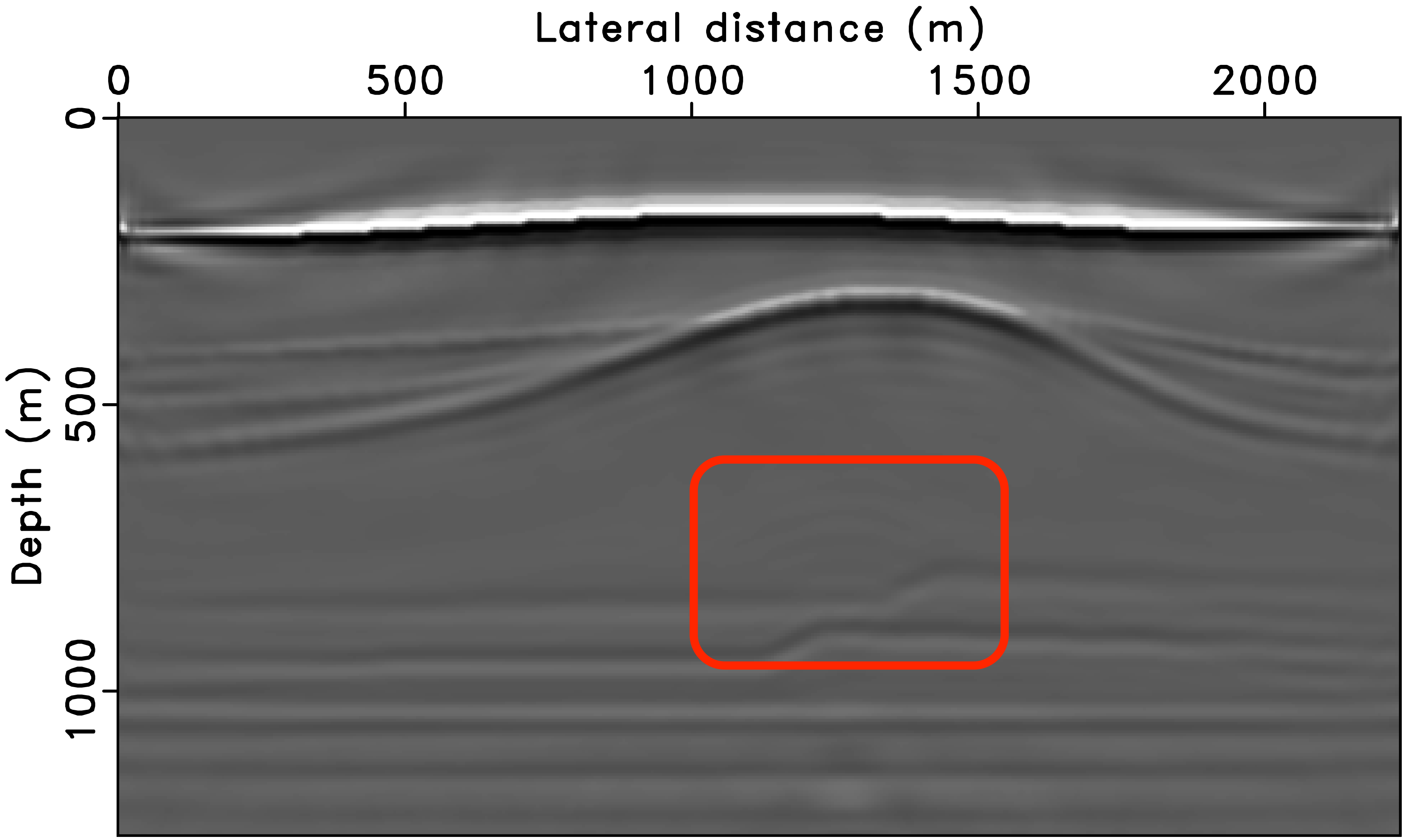
- Multiples are used as a regularization term
- Flexibility to choose how to keep a balance between SNR and illumination
- (*problem*) How to apply source association

Inverted model perturbation in combined inversion



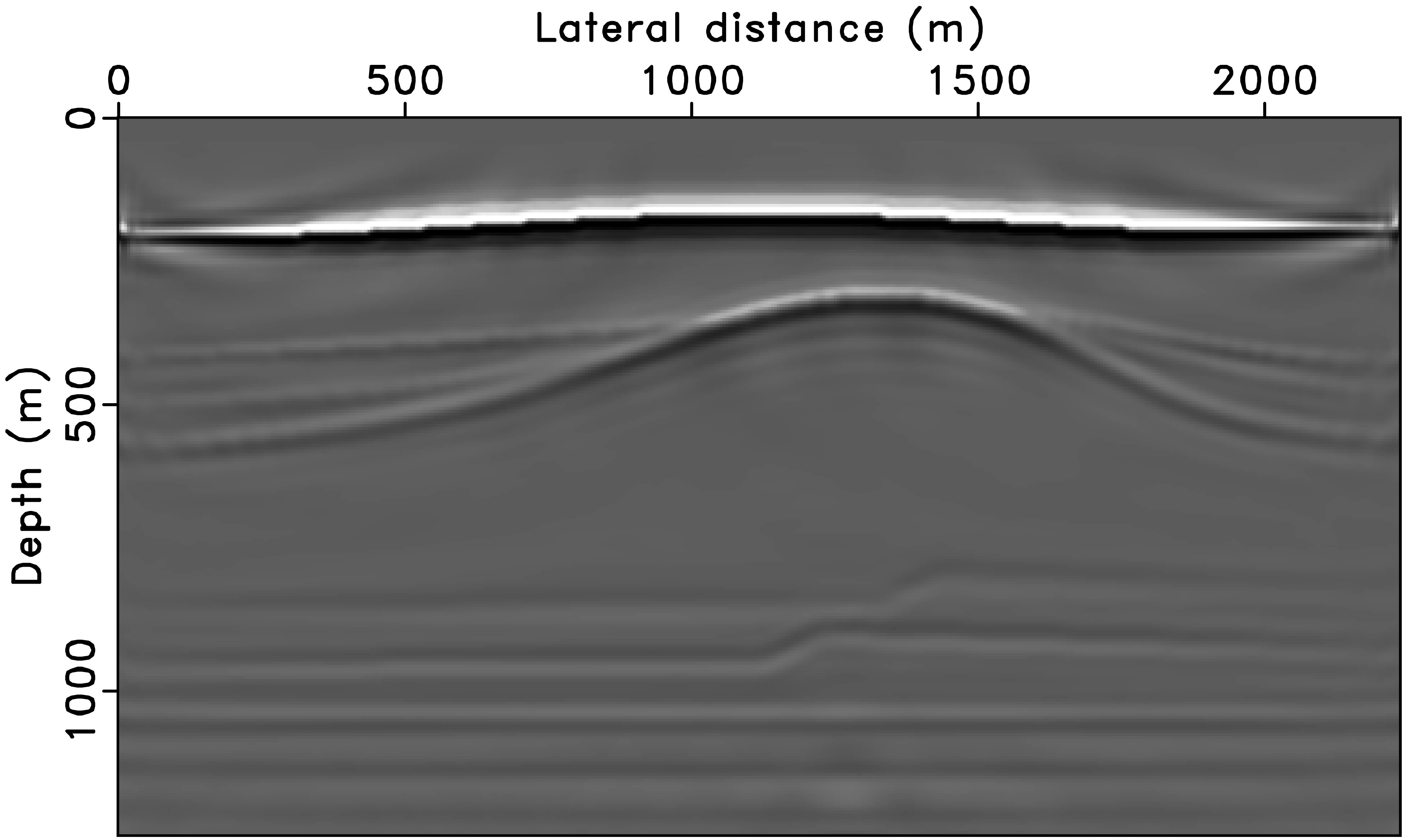
SNR: 5.78dB

Inverted model perturbation in combined inversion



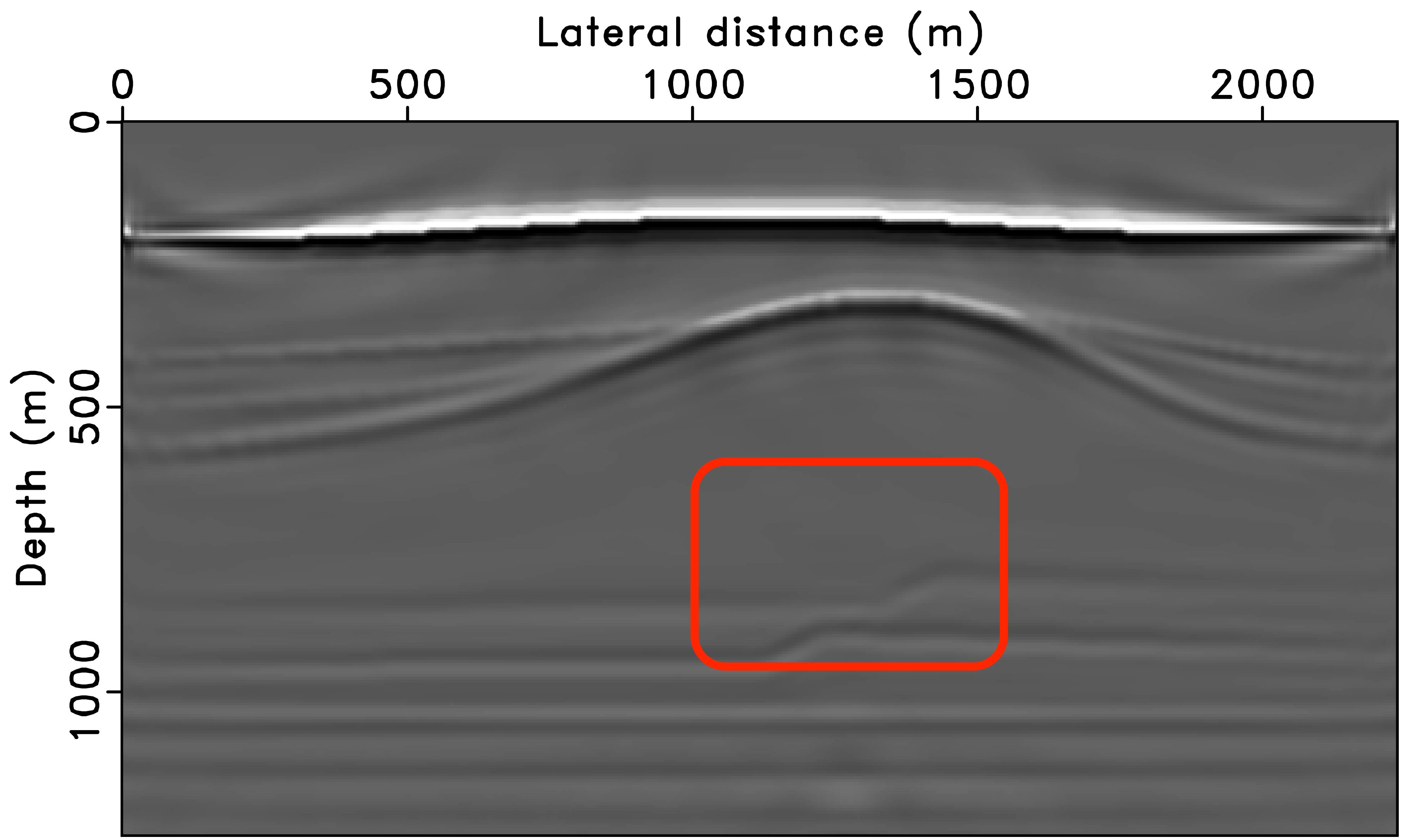
SNR: 5.78dB

Inverted model perturbation in *collaborative* inversion[lambda=1]



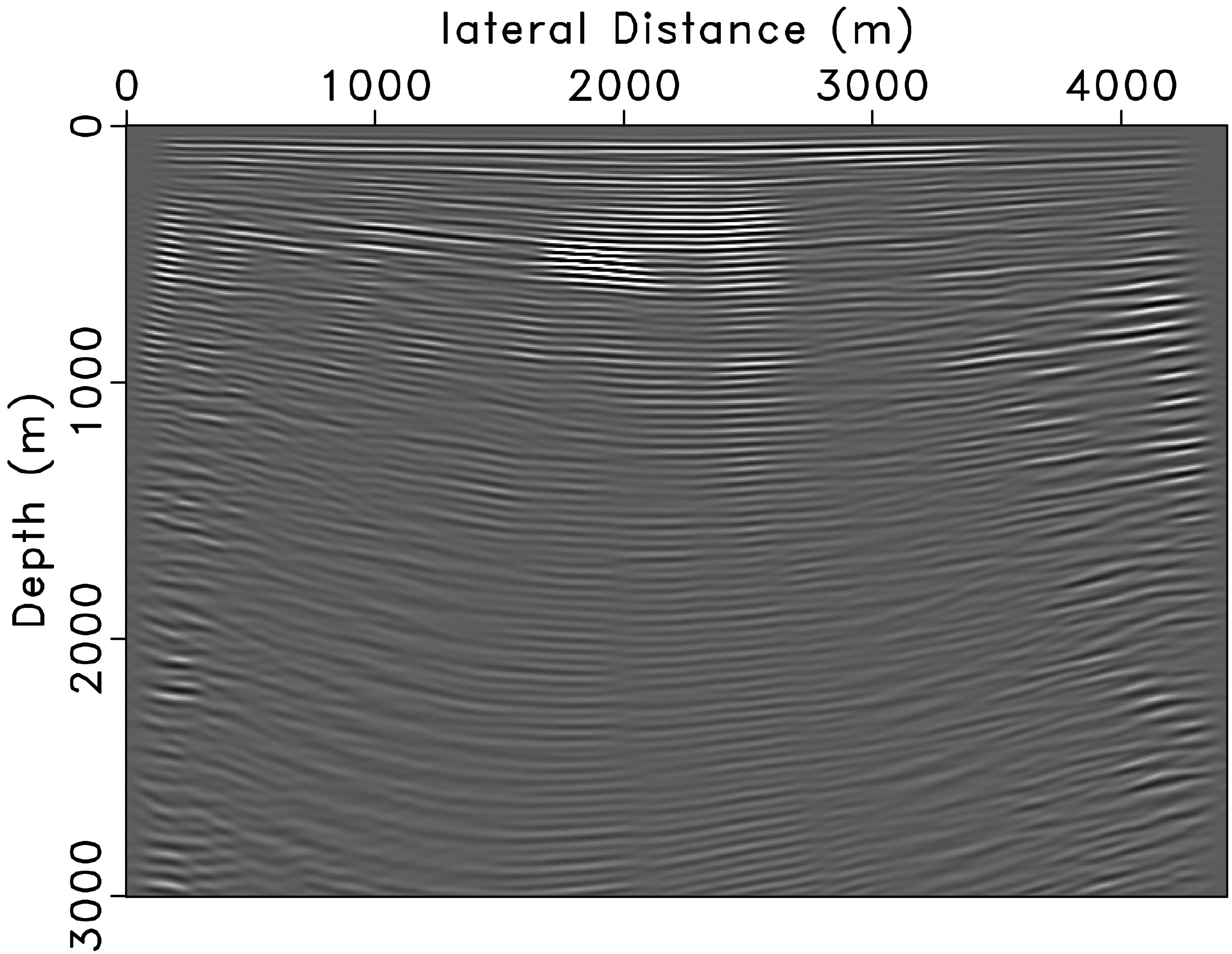
SNR: 5.76dB

Inverted model perturbation in *collaborative* inversion[lambda=1]

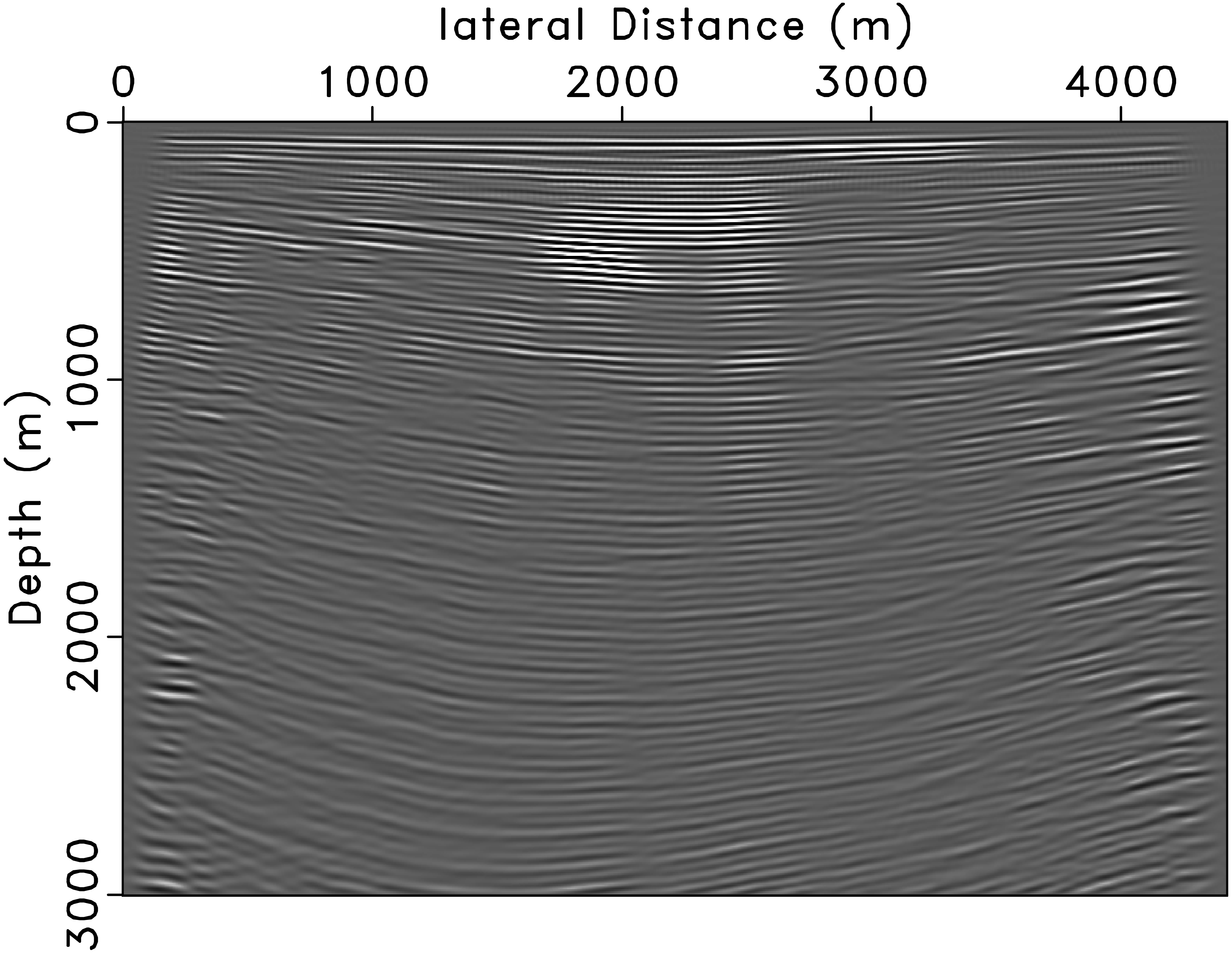


SNR: 5.76dB

L1 migration of total data with EPSI, *combined* inversion



L1 migration of total data with EPSI, *collaborative* inversion



Spatially subsampled data

- poses problems for EPSI due to spatial aliasing
- is still redundant in image space
- (should and *hopefully can*) be effectively “interpolated” with demigration operator by EPSI+migration

Acknowledgements

- Thanks to all my colleagues for the beneficial discussions
- Thanks to Eric for the advice on working with incomplete data, and for sharing with us the Gulf of Suez dataset.

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