# Parameter-selection strategy for density in frequency-domain elastic waveform inversion

Dong-Joo Min*, Woodon Jeong, and Felix J. Herrmann
(Seoul National University)

SLIM
University of British Columbia

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# Motivation

- Needs for elastic full waveform inversion (FWI)
  - *multicomponent* data are *commonly* acquired
  - more *reliable* subsurface *information* is needed

# Motivation

- Limitations of elastic FWI
  - *more* parameters than *acoustic* FWI
    --> (velocities and density
    or Lame constants and density)
  - easily stuck in *local* minima
    --> it is assumed that Poisson's ratio and density are constant (Brossier et al. 2009; Brossier et al. 2010; Bae et al. 2010; Lee et al. 2010).

# Motivation

- Density
  - for *acoustic* or *elastic* impedance (Connolly, 1999), *density* is needed
  - in *conventional* elastic FWI, velocities are *properly* restored, but *density* is *very* difficult to recover (Forgues and Lambare, 1997, Choi et al., 2008, Virieux and Operto, 2009).

# Objective

- Develop an inversion strategy for density

  - Tarantola (1986) proposed a parameter-selection strategy based on *sensitivity* analysis.
  - propose a new parameter-selection strategy

    --> inversion is performed over two stages

    --> velocities and density are recovered *sequentially*

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# Contents

- FWI algorithm
- Conventional FWI & examples
- Parameter-selection strategy & examples
- Conclusions

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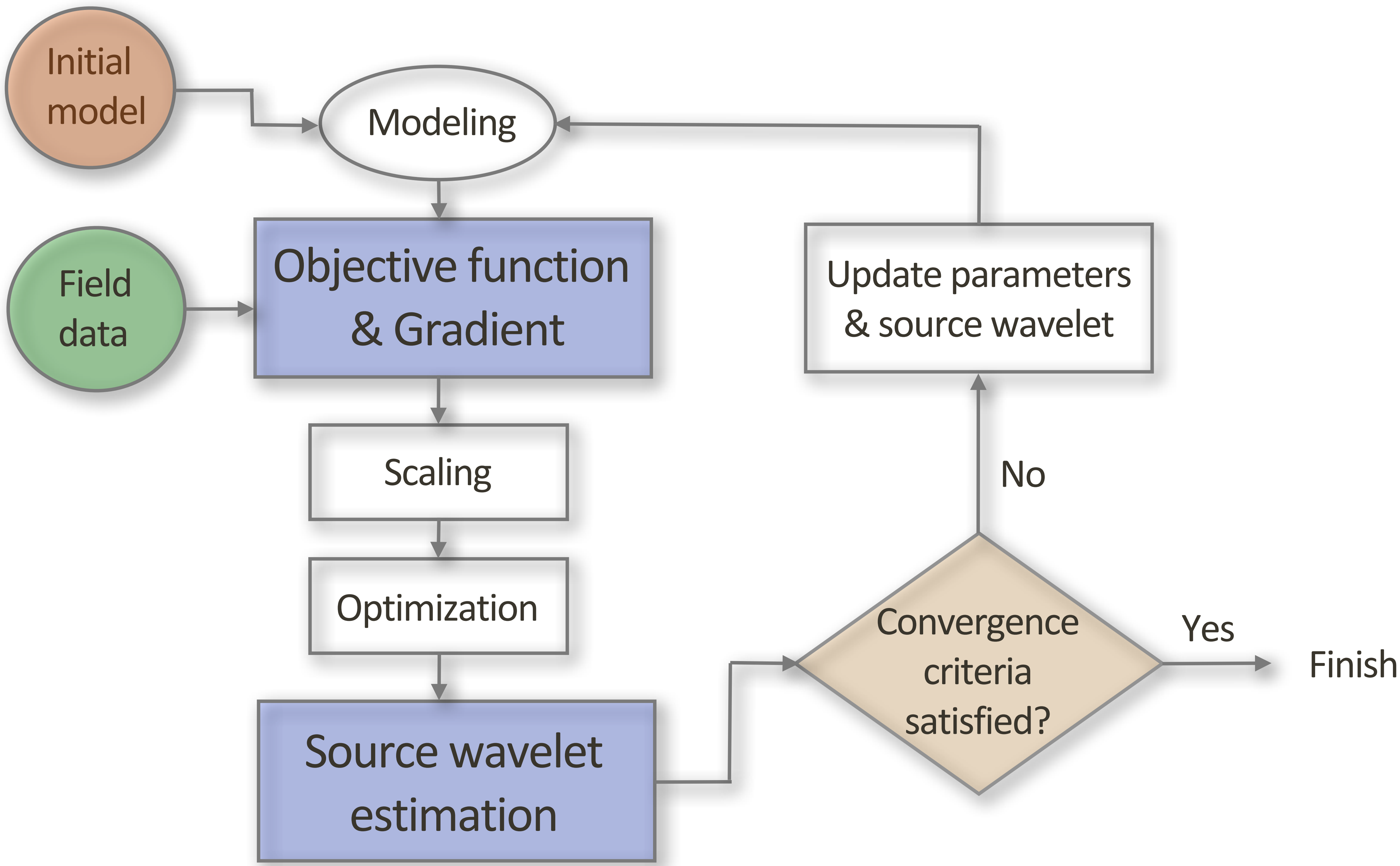
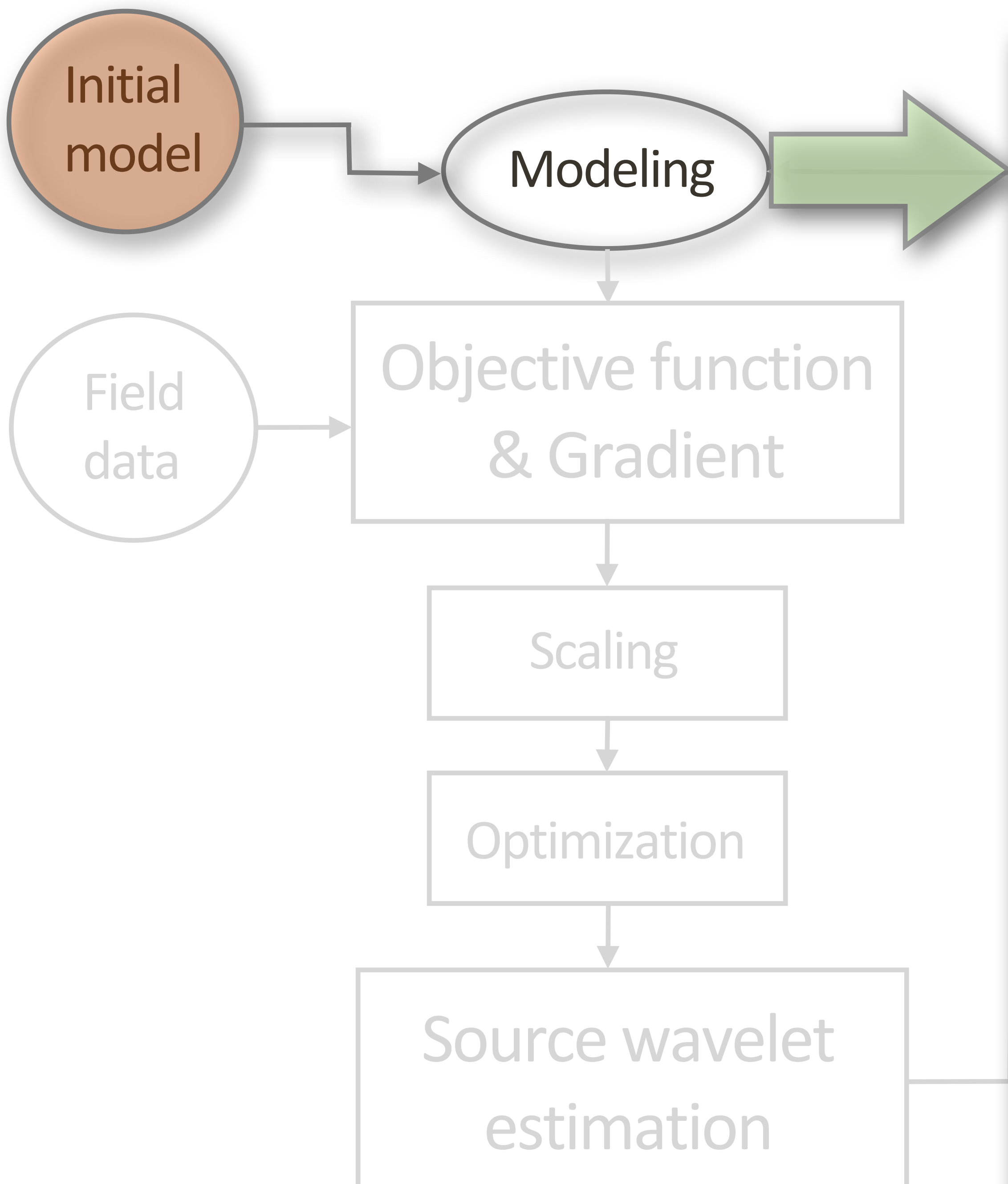
# FWI algorithm



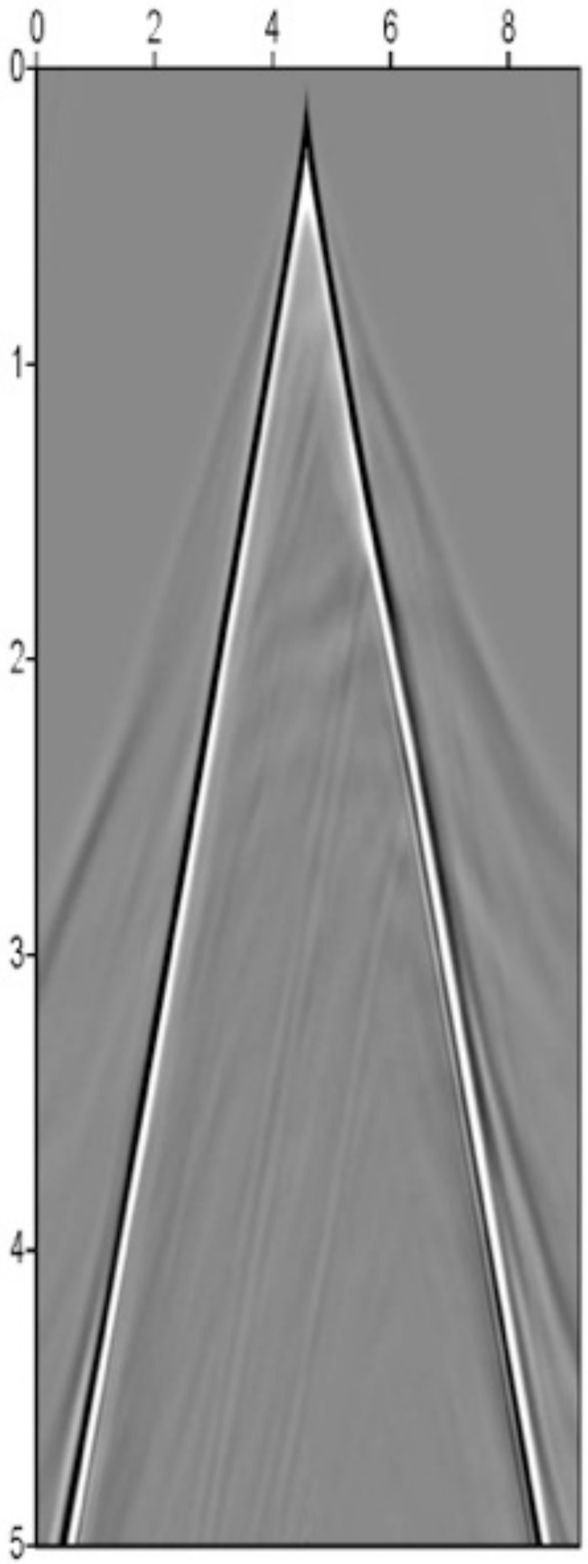
# FWI algorithm

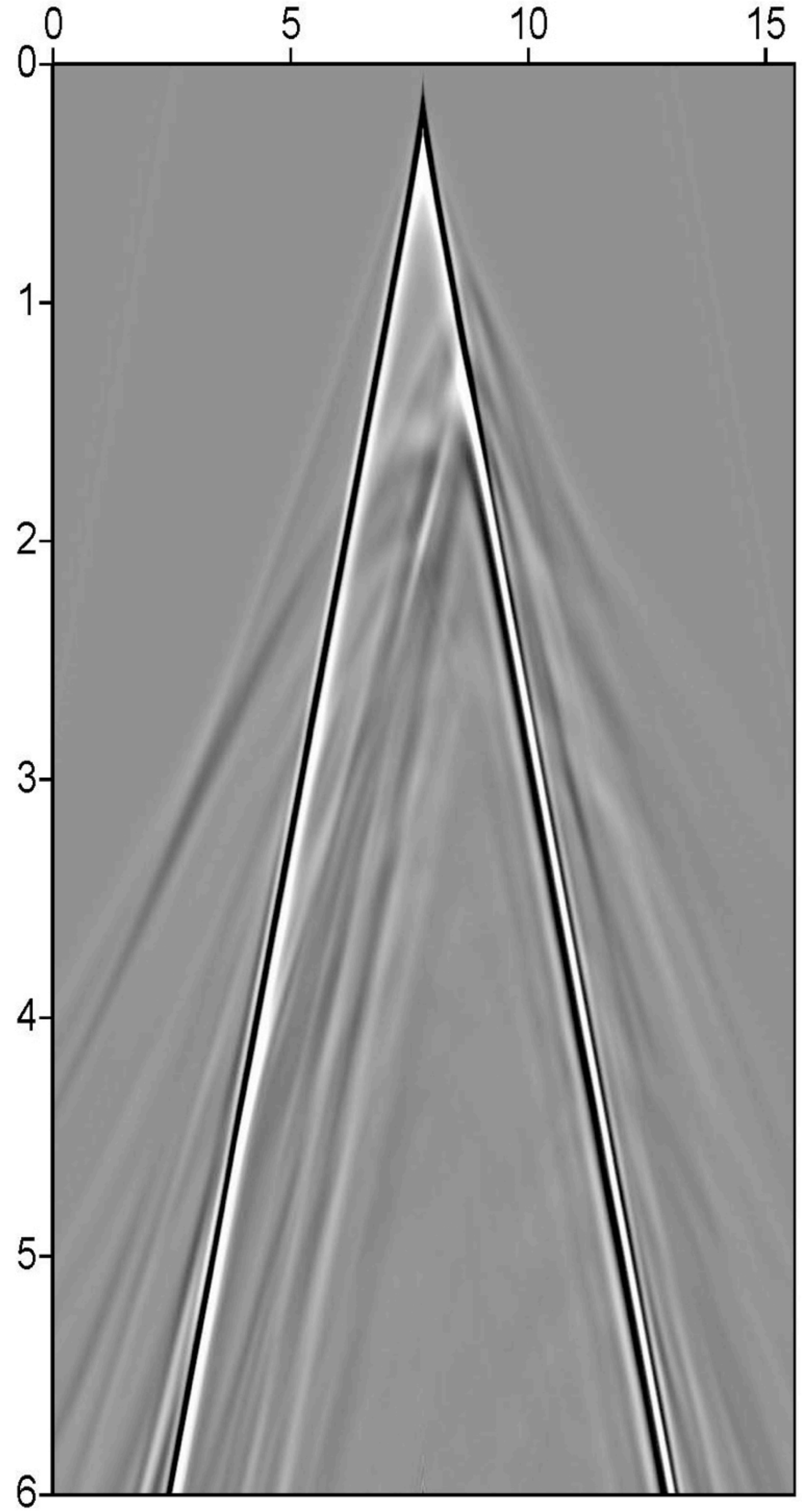



## Forward modeling scheme

- finite-element method with Galerkin's method
- PML boundary condition (Cohen, 2002)
- consider all kinds of waves including Rayleigh waves

# FWI algorithm




## Forward modeling scheme

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- PML boundary condition
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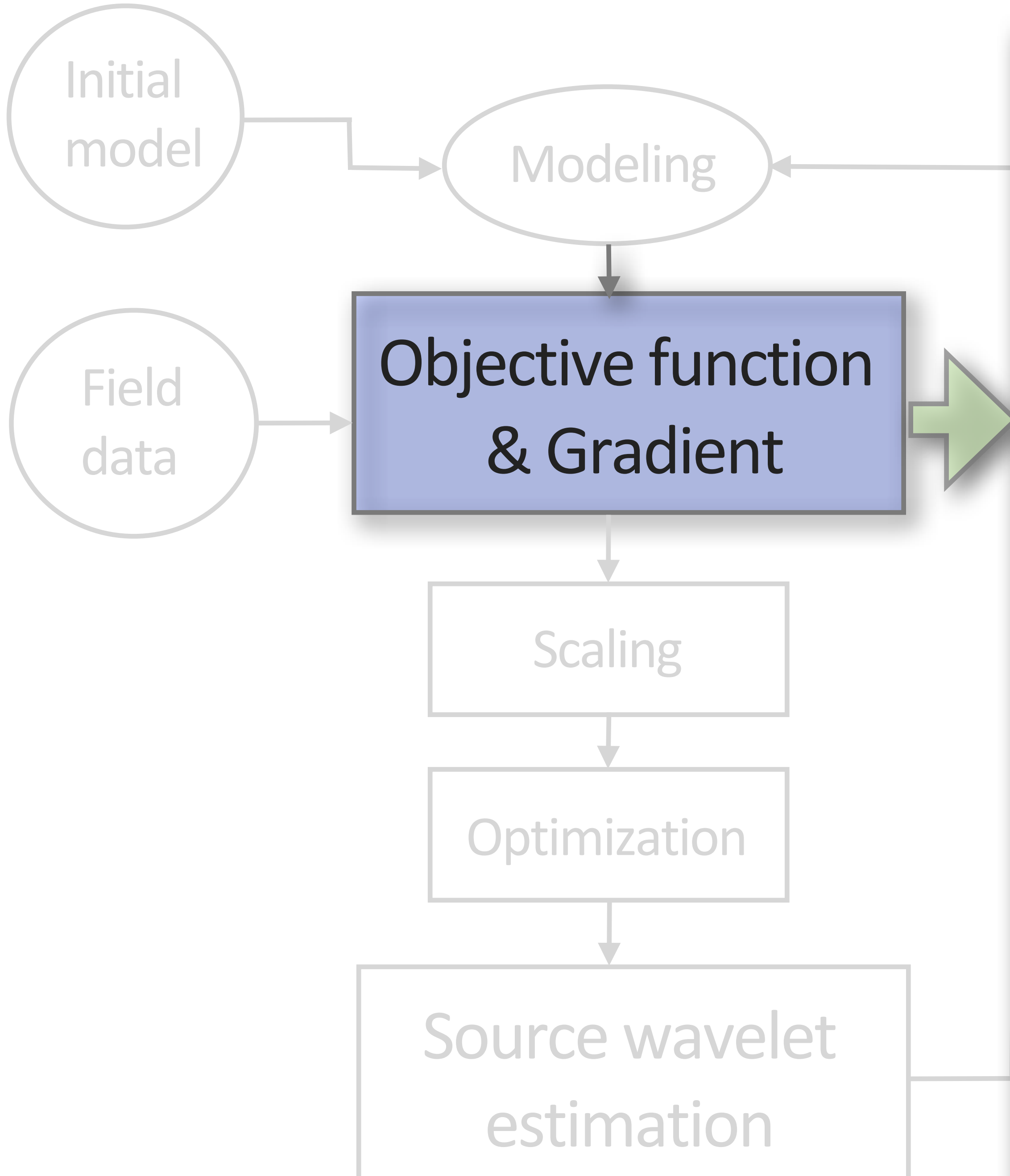
# FWI algorithm




## Objective function

$$\min_{\mathbf{p}} \frac{1}{2}\sum_{\omega}\sum_{s}\left\|\tilde{\mathbf{u}}_s(\mathbf{p})-\tilde{\mathbf{d}}_s\right\|_2^2$$

## Gradient

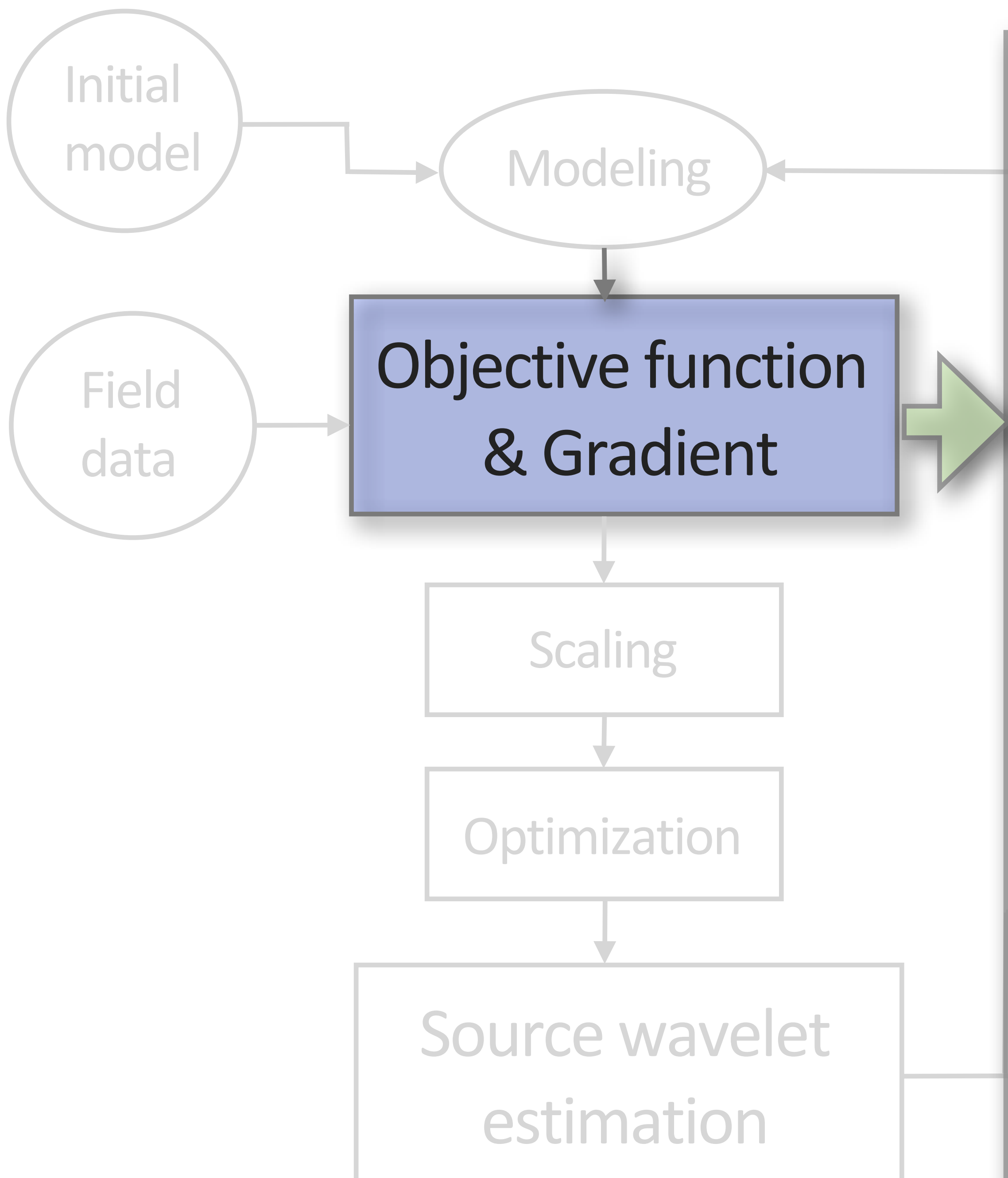
$$\frac{\partial E}{\partial p_k}=\sum_{\omega}\sum_{s}\mathrm{Re}\left[\left(\frac{\partial \tilde{\mathbf{u}}_s}{\partial p_k}\right)^T\left(\tilde{\mathbf{u}}_s-\tilde{\mathbf{d}}_s\right)^*\right]$$

$$\mathbf{S}\tilde{\mathbf{u}}_s=\mathbf{f}_s$$

$$\frac{\partial \tilde{\mathbf{u}}_s}{\partial p_k}=\mathbf{S}^{-1}\left(-\frac{\partial \mathbf{S}}{\partial p_k}\tilde{\mathbf{u}}_s\right)=\mathbf{S}^{-1}\left(\mathbf{f}^v_{s,k}\right)$$

$$\therefore\ \frac{\partial E}{\partial p_k}=\sum_{\omega}\sum_{s}\mathrm{Re}\left[\left(\mathbf{f}^v_{s,k}\right)^T\left(\mathbf{S}^{-1}\right)^T\left(\tilde{\mathbf{u}}_s-\tilde{\mathbf{d}}_s\right)^*\right]$$

# FWI algorithm




## Objective function

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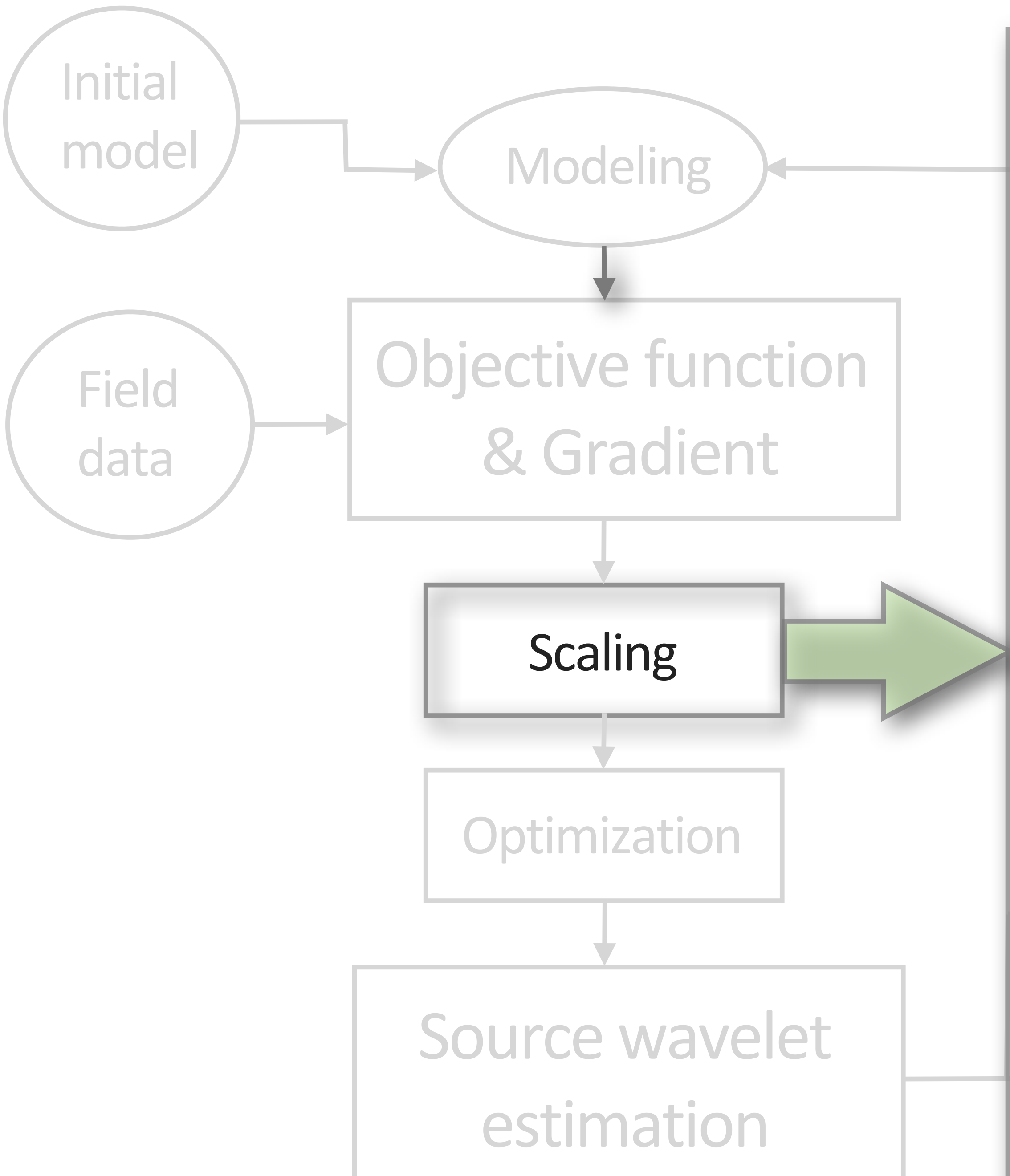
$$\mathbf{S}\tilde{\mathbf{u}}_s=\mathbf{f}_s$$

$$\frac{\partial \tilde{\mathbf{u}}_s}{\partial p_k}=\mathbf{S}^{-1}\left(-\frac{\partial \mathbf{S}}{\partial p_k}\tilde{\mathbf{u}}_s\right)=\mathbf{S}^{-1}\left(\mathbf{f}_{s,k}^{v}\right)$$

$$\therefore \frac{\partial E}{\partial p_k}=\sum_{\omega}\sum_{s}\mathrm{Re}\left[\left(\mathbf{f}_{s,k}^{v}\right)^T\left(\mathbf{S}^{-1}\right)^T\left(\tilde{\mathbf{u}}_s-\tilde{\mathbf{d}}_s\right)^*\right]$$

# FWI algorithm




## Approximate-Hessian matrix

$$\mathbf{H}_a = \sum_s \left(\frac{\partial \tilde{\mathbf{u}}_s}{\partial \mathbf{p}}\right)^{\mathrm{T}} \left(\frac{\partial \tilde{\mathbf{u}}_s}{\partial \mathbf{p}}\right)^{*} = \sum_s \left(\mathbf{F}_s^v\right)^T \left(\mathbf{S}^{-1}\right)^T \left(\mathbf{S}^{-1}\right)^* \left(\mathbf{F}_s^v\right)^*$$

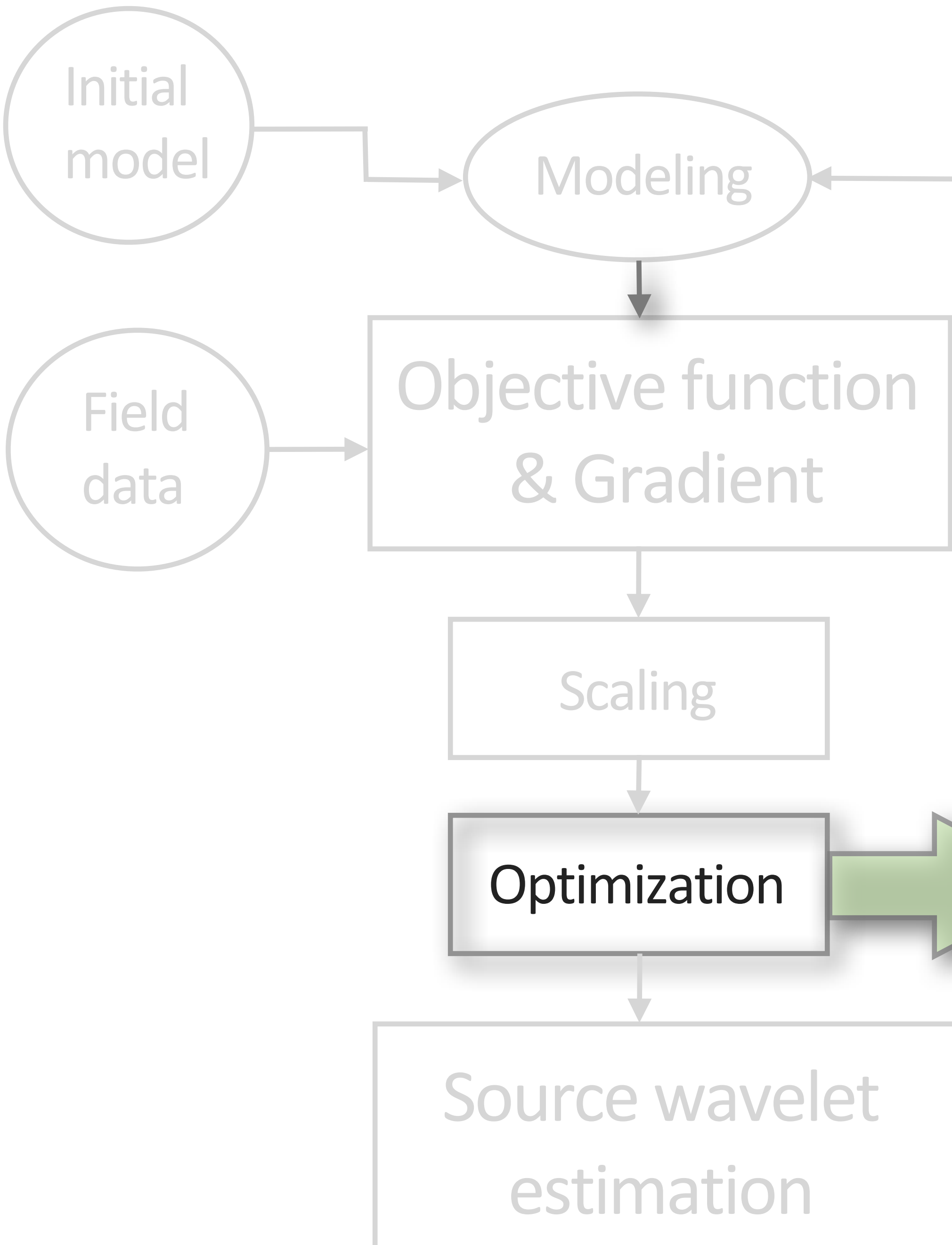
## Pseudo-Hessian matrix

$$\mathbf{H}_p = \sum_s \left(\mathbf{F}_s^v\right)^T \left(\mathbf{F}_s^v\right)^*$$

## For scaling

$$\nabla E = \sum_\omega \left( \frac{\sum_s \mathrm{Re}\left[\left(\mathbf{F}^v\right)^T \left(\mathbf{S}^{-1}\right)^T \left(\tilde{\mathbf{u}} - \tilde{\mathbf{d}}\right)^*\right]}{\sum_s diag\left\{\left(\mathbf{F}^v\right)^T \left(\mathbf{F}^v\right)^* + \phi \mathbf{I}\right\}} \right)$$

# FWI algorithm




## Conjugate gradient method

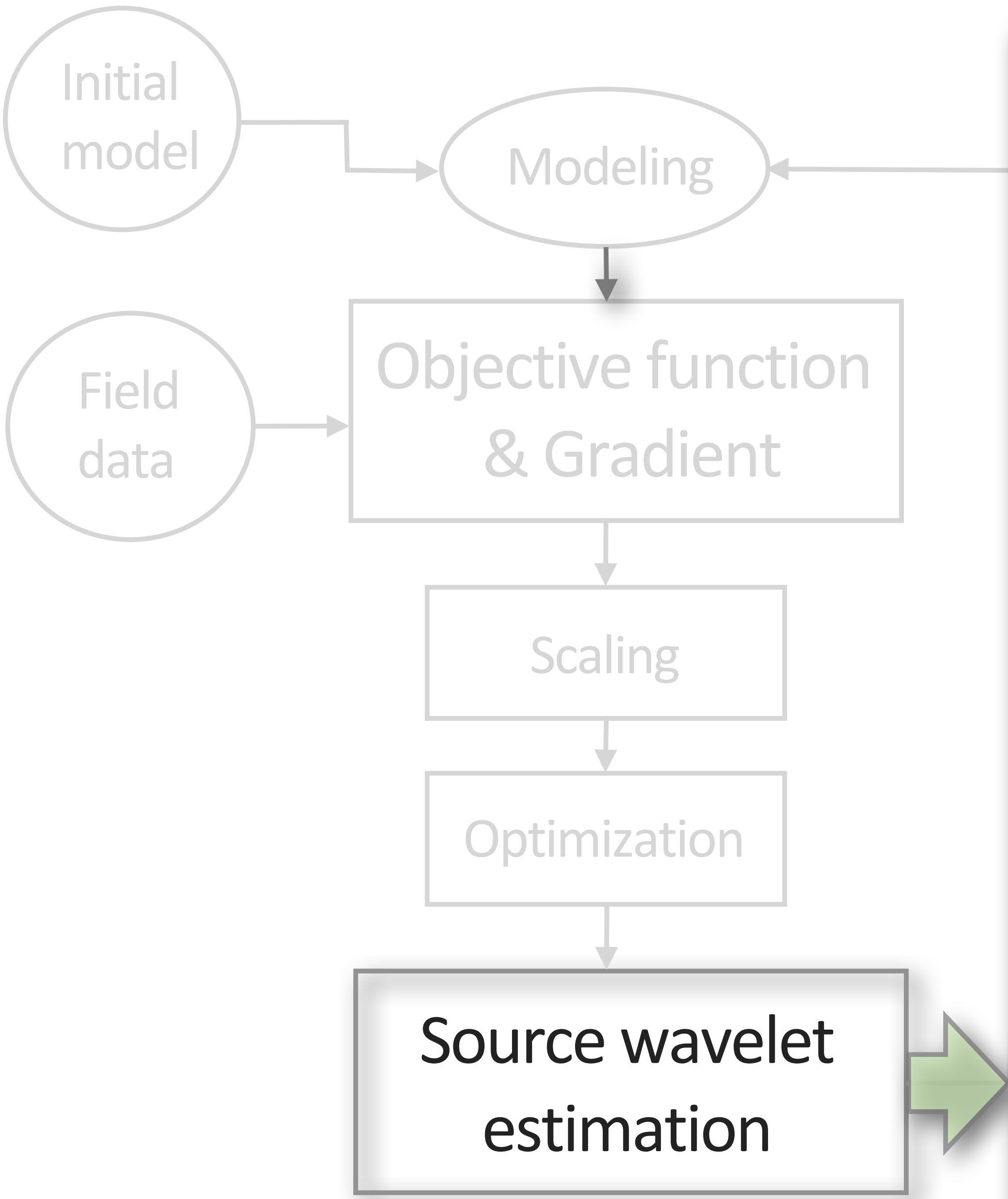
$$\beta^{l+1} = \frac{(\nabla_{l+1} E)^T (\nabla_{l+1} E)}{(\nabla_l E)^T (\nabla_l E)}$$

$$d^{l+1} = -\nabla_{l+1} E + \beta^{l+1} d^l$$

$$p^{l+1} = p^l + \alpha\, d^{l+1}$$

Fletcher and Reeves (1964)

# FWI algorithm




## Source wavelet estimation

L2-norm objective function and the full Hessian matrix

$$\min_{w} \frac{1}{2}\sum_{\omega}\sum_{s}\left\|\tilde{\mathbf{g}}_s w - \tilde{\mathbf{d}}_s\right\|_2^2$$

$$w = \frac{\sum_{\omega}\sum_{s}\tilde{\mathbf{g}}_s^T\tilde{\mathbf{d}}_s}{\sum_{\omega}\sum_{s}\tilde{\mathbf{g}}_s^T\tilde{\mathbf{g}}_s}$$

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- Conventional FWI & examples
- Parameter-selection strategy & examples
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# Conventional FWI

## Conventional Method I

- elastic wave equations parameterized by Lame constants and density:

$$-\rho\omega^2\tilde{u} = \frac{\partial}{\partial x}\left((\lambda+2\mu)\frac{\partial\tilde{u}}{\partial x} + \lambda\frac{\partial\tilde{v}}{\partial z}\right) + \frac{\partial}{\partial z}\left(\mu\left(\frac{\partial\tilde{v}}{\partial x} + \frac{\partial\tilde{u}}{\partial z}\right)\right)$$

$$-\rho\omega^2\tilde{v} = \frac{\partial}{\partial x}\left(\mu\left(\frac{\partial\tilde{v}}{\partial x} + \frac{\partial\tilde{u}}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial\tilde{u}}{\partial x} + (\lambda+2\mu)\frac{\partial\tilde{v}}{\partial z}\right)$$

- virtual sources for each parameters:

$$\left(\mathbf{f}^v\right)_\lambda = -\frac{\partial\mathbf{S}}{\partial\lambda}\tilde{\mathbf{u}}, \qquad \left(\mathbf{f}^v\right)_\mu = -\frac{\partial\mathbf{S}}{\partial\mu}\tilde{\mathbf{u}}, \qquad \left(\mathbf{f}^v\right)_\rho = -\frac{\partial\mathbf{S}}{\partial\rho}\tilde{\mathbf{u}}$$

# Conventional FWI

## Conventional Method II

- elastic wave equations parameterized by velocities and density:

$$-\rho\omega^2\tilde{u} = \frac{\partial}{\partial x}\left(\rho\alpha^2\frac{\partial\tilde{u}}{\partial x} + \left(\rho\alpha^2 - 2\rho\beta\right)\frac{\partial\tilde{v}}{\partial z}\right) + \frac{\partial}{\partial z}\left(\rho\beta^2\left(\frac{\partial\tilde{v}}{\partial x} + \frac{\partial\tilde{u}}{\partial z}\right)\right)$$

$$-\rho\omega^2\tilde{v} = \frac{\partial}{\partial x}\left(\rho\beta^2\left(\frac{\partial\tilde{v}}{\partial x} + \frac{\partial\tilde{u}}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(\left(\rho\alpha^2 - 2\rho\beta^2\right)\frac{\partial\tilde{u}}{\partial x} + \rho a^2\frac{\partial\tilde{v}}{\partial z}\right)$$

- virtual sources for each parameters:

assumption: parameters are independent of each other

$$\left(\mathbf{f}^v\right)_\alpha = -\frac{\partial\mathbf{S}}{\partial\alpha}\tilde{\mathbf{u}}, \qquad \left(\mathbf{f}^v\right)_\beta = -\frac{\partial\mathbf{S}}{\partial\beta}\tilde{\mathbf{u}}, \qquad \left(\mathbf{f}^v\right)_\rho = -\frac{\partial\mathbf{S}}{\partial\rho}\tilde{\mathbf{u}}$$

# Conventional FWI

## Conventional Method II

- virtual sources using the chain rule (Mora, 1987)
  assumption: velocities are dependent on density

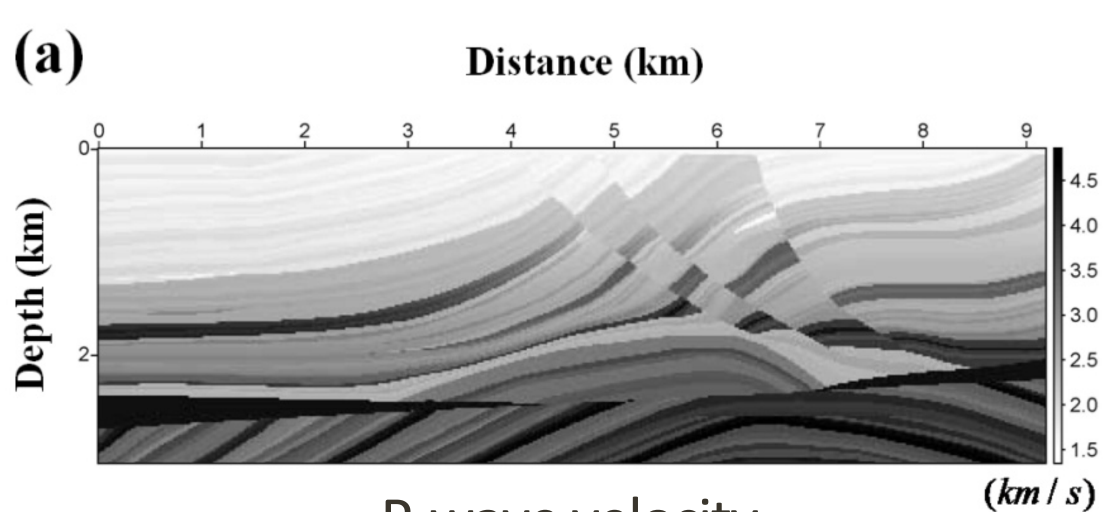
$$\left(\mathbf{f}^{v}\right)_{\alpha} = -\left[\frac{\partial \mathbf{S}}{\partial \lambda}\frac{\partial \lambda}{\partial \alpha} + \frac{\partial \mathbf{S}}{\partial \mu}\frac{\partial \mu}{\partial \alpha} + \frac{\partial \mathbf{S}}{\partial \rho}\frac{\partial \rho}{\partial \alpha}\right]\tilde{\mathbf{u}} = -\left[\frac{\partial \mathbf{S}}{\partial \lambda}2\rho\alpha\right]\tilde{\mathbf{u}}$$

$$\left(\mathbf{f}^{v}\right)_{\beta} = -\left[\frac{\partial \mathbf{S}}{\partial \lambda}\frac{\partial \lambda}{\partial \beta} + \frac{\partial \mathbf{S}}{\partial \mu}\frac{\partial \mu}{\partial \beta} + \frac{\partial \mathbf{S}}{\partial \rho}\frac{\partial \rho}{\partial \beta}\right]\tilde{\mathbf{u}} = -\left[-\frac{\partial \mathbf{S}}{\partial \lambda}4\rho\beta + \frac{\partial \mathbf{S}}{\partial \mu}2\rho\beta\right]\tilde{\mathbf{u}}$$

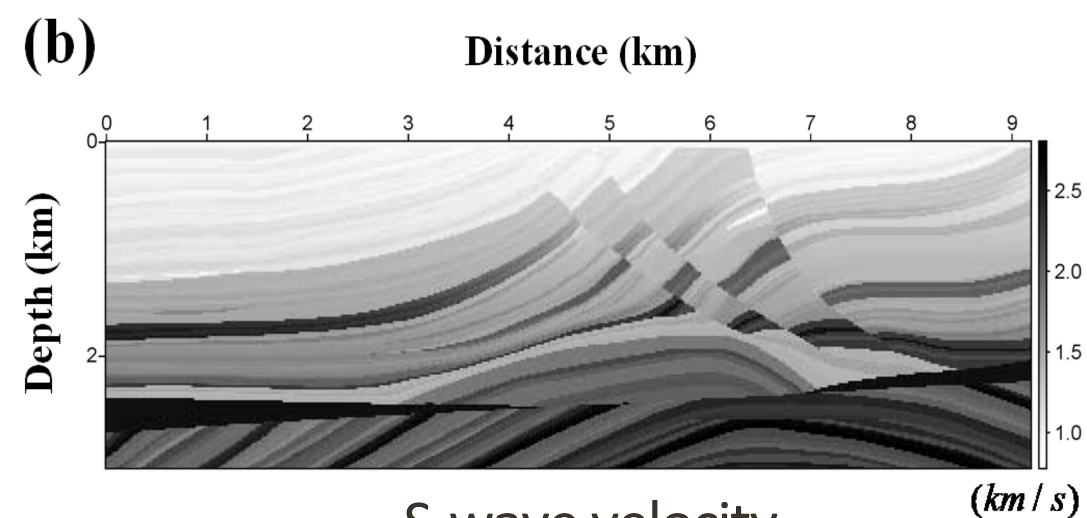
$$\left(\mathbf{f}^{v}\right)_{\rho} = -\left[\frac{\partial \mathbf{S}}{\partial \lambda}\frac{\partial \lambda}{\partial \rho} + \frac{\partial \mathbf{S}}{\partial \mu}\frac{\partial \mu}{\partial \rho} + \frac{\partial \mathbf{S}}{\partial \rho}\right]\tilde{\mathbf{u}} = -\left[-\frac{\partial \mathbf{S}}{\partial \lambda}\left(\alpha^{2} - 2\beta^{2}\right) + \frac{\partial \mathbf{S}}{\partial \mu}\beta^{2} + \frac{\partial \mathbf{S}}{\partial \rho}\right]\tilde{\mathbf{u}}$$

# Elastic Marmousi-2 model

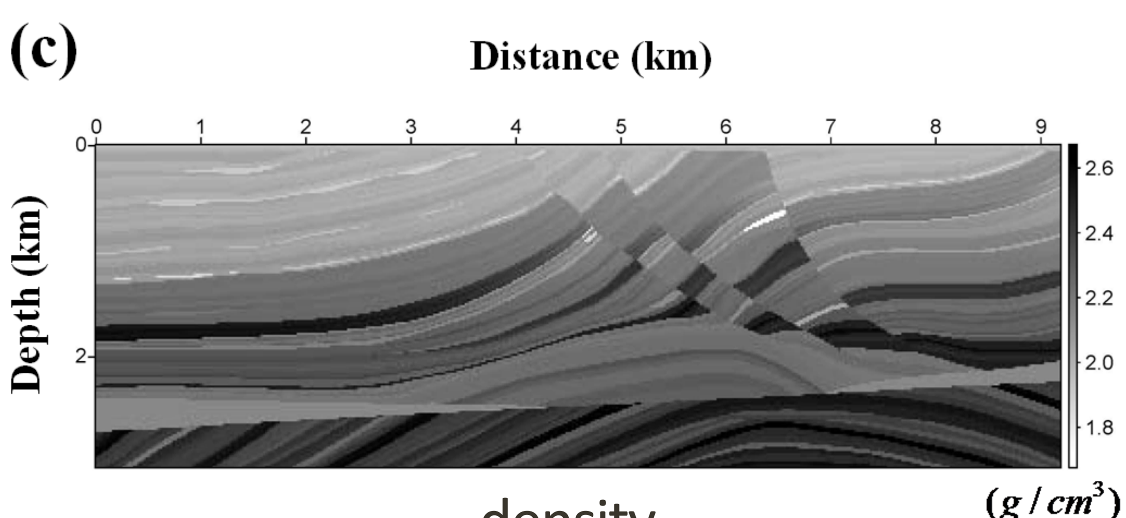
- True model




P-wave velocity




S-wave velocity

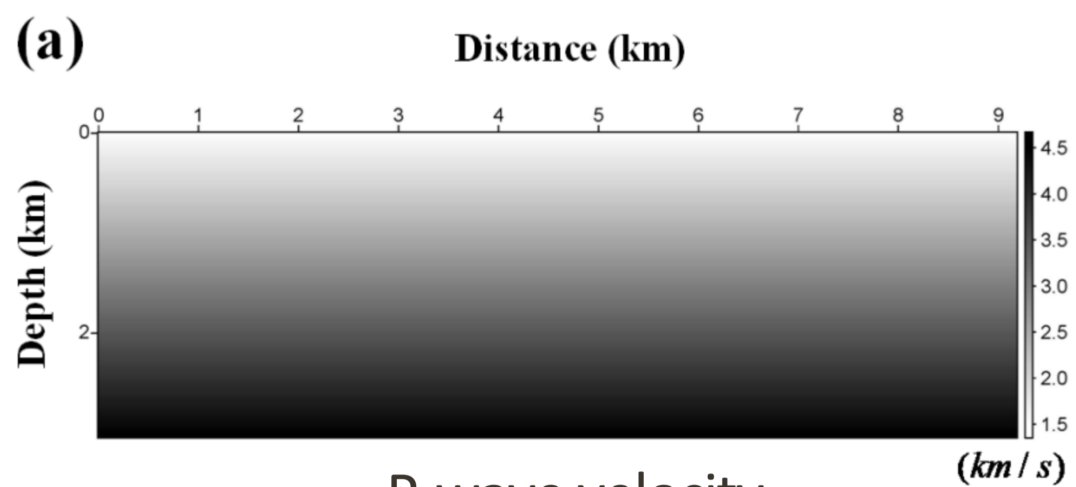



density

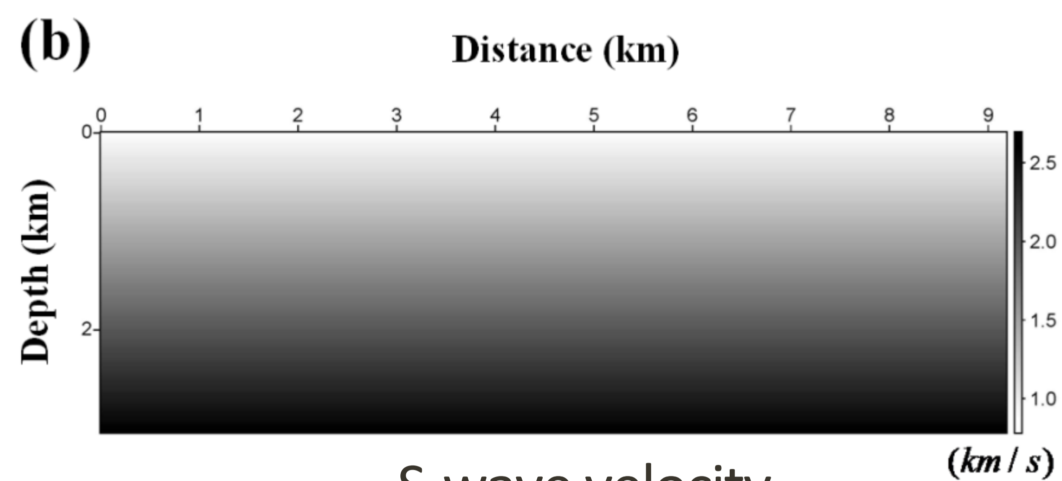
| Parameters | Values |
|---|---|
| dimension | 9.2 km x 3.04 km |
| no. of source | 219 |
| source interval | 0.04 km |
| no. of receiver | 461 |
| receiver interval | 0.02 km |
| recording time | 5 s |
| frequency range | 0.2 - 10 Hz |

# Elastic Marmousi-2 model

- Initial model




P-wave velocity

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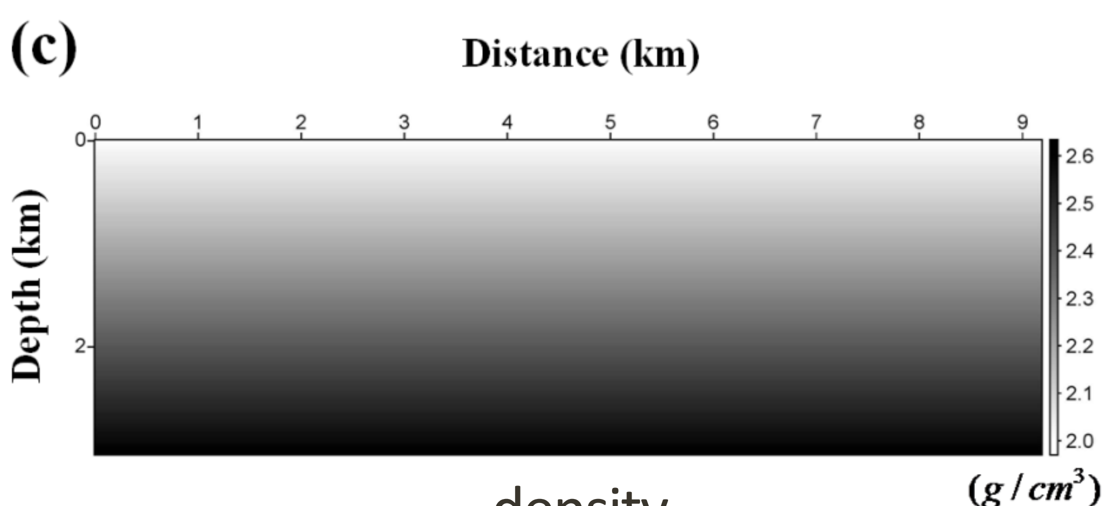
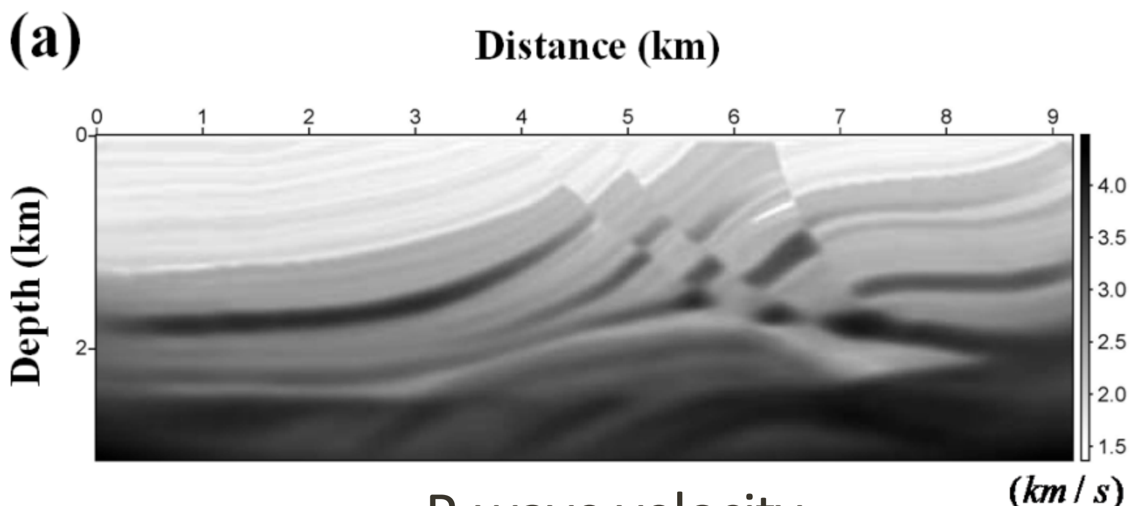


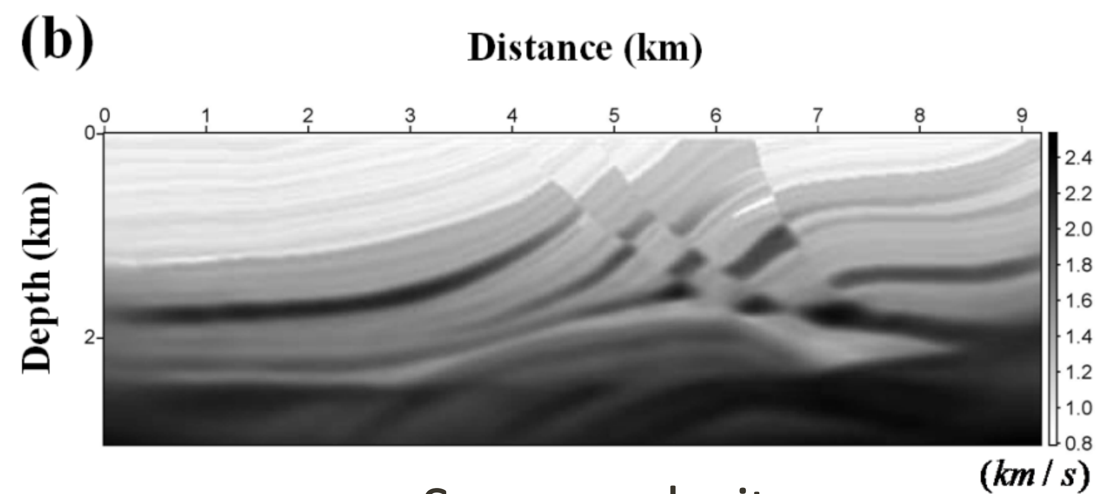

density

# Conventional FWI - Examples

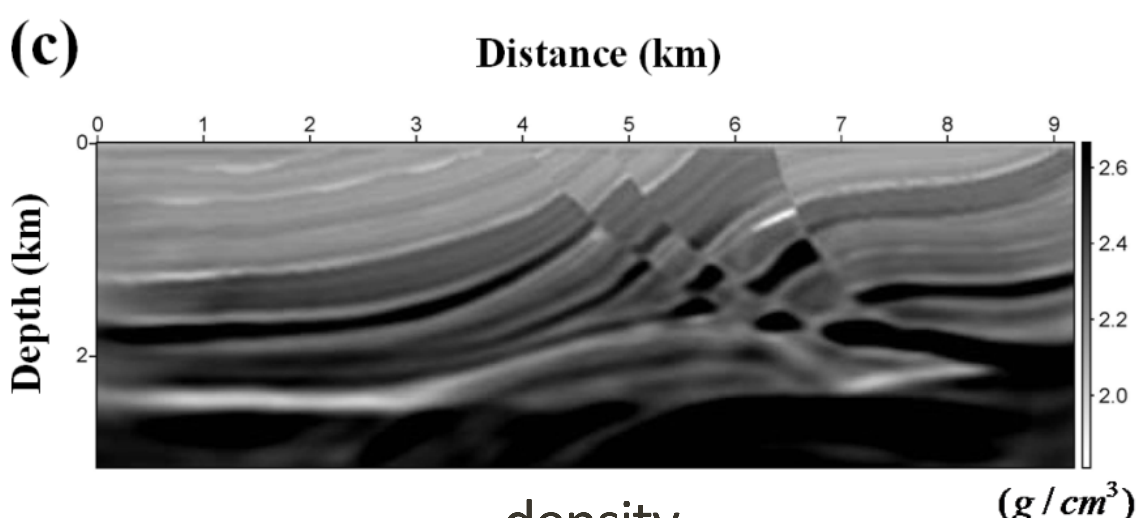
- Inversion results - Conventional methods I




P-wave velocity




S-wave velocity



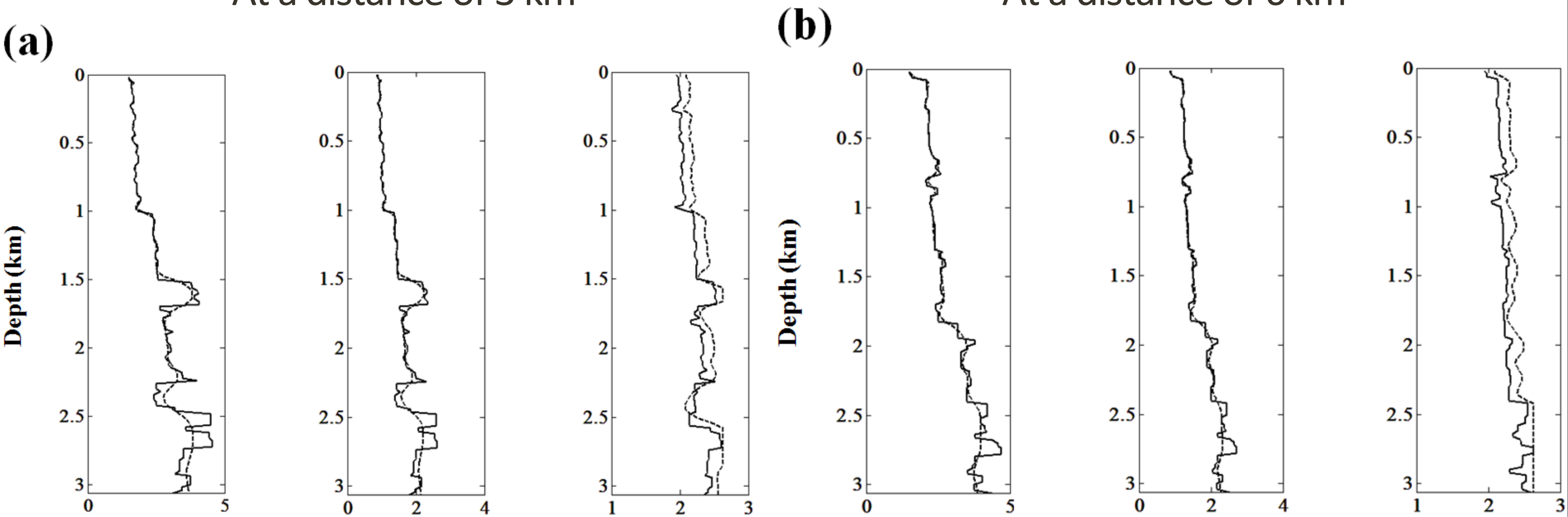

density

# Conventional FWI - Examples

- Depth profiles - Conventional methods I

At a distance of 3 km

At a distance of 6 km

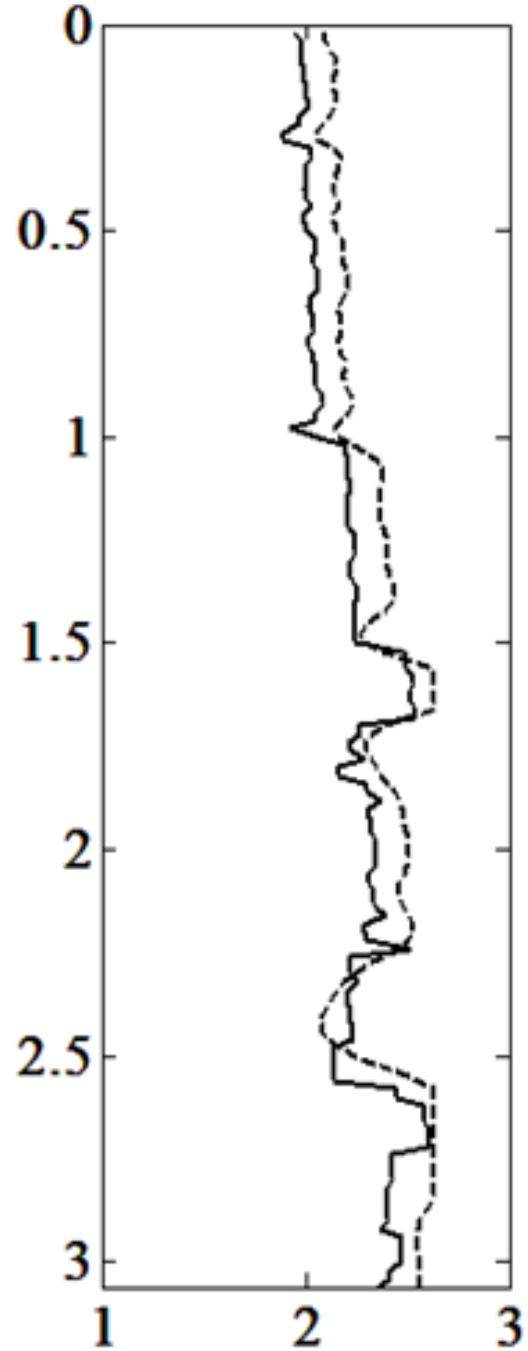



P velocity | S velocity | density | P velocity | S velocity | density

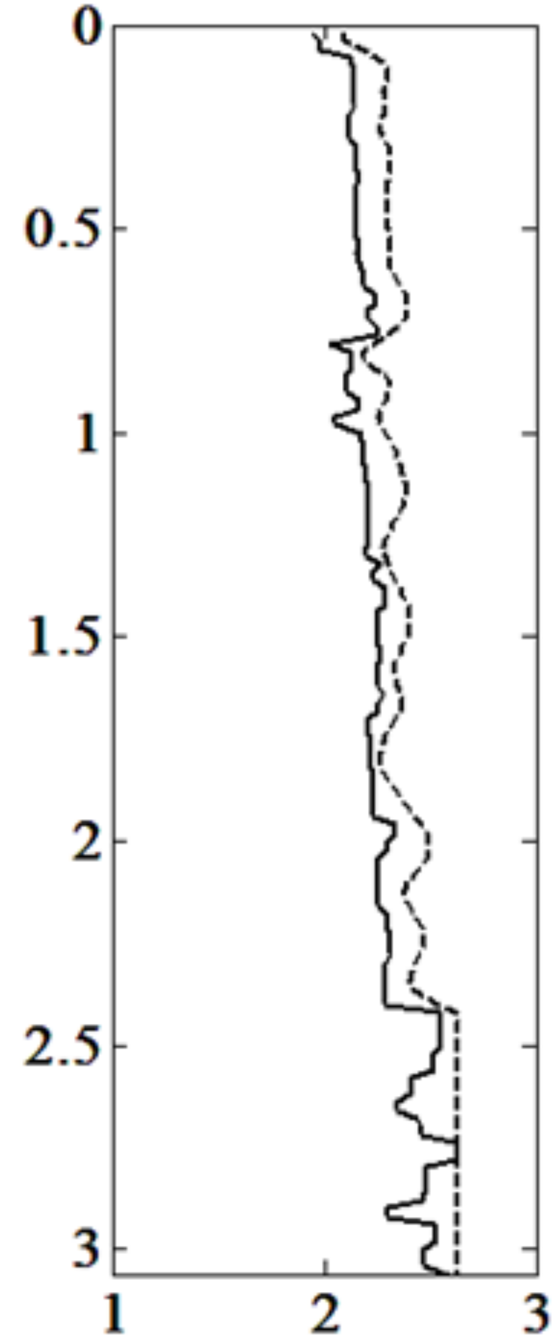
# Conventional FWI - Examples

- Density profiles - Conventional methods I

at a distance of 3 km

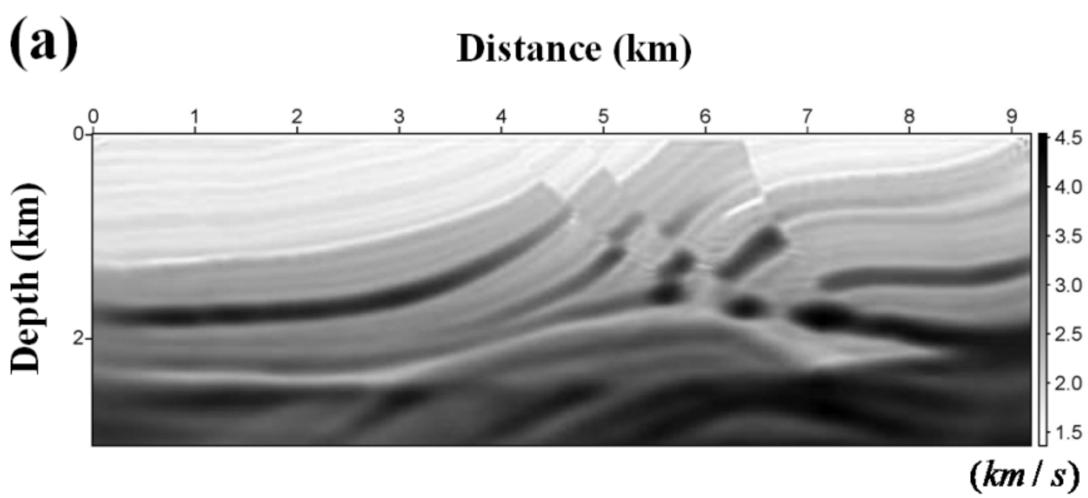



at a distance of 6 km

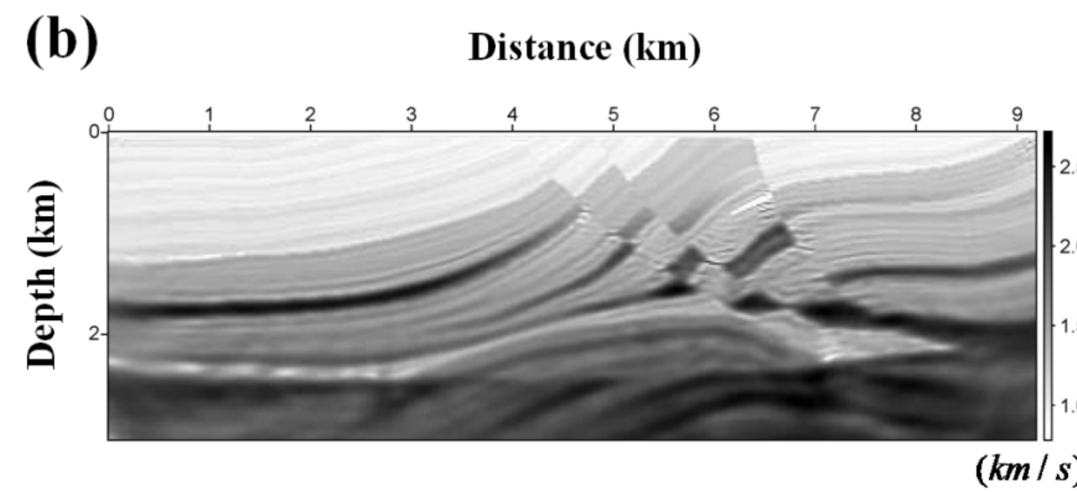


# Conventional FWI - Examples

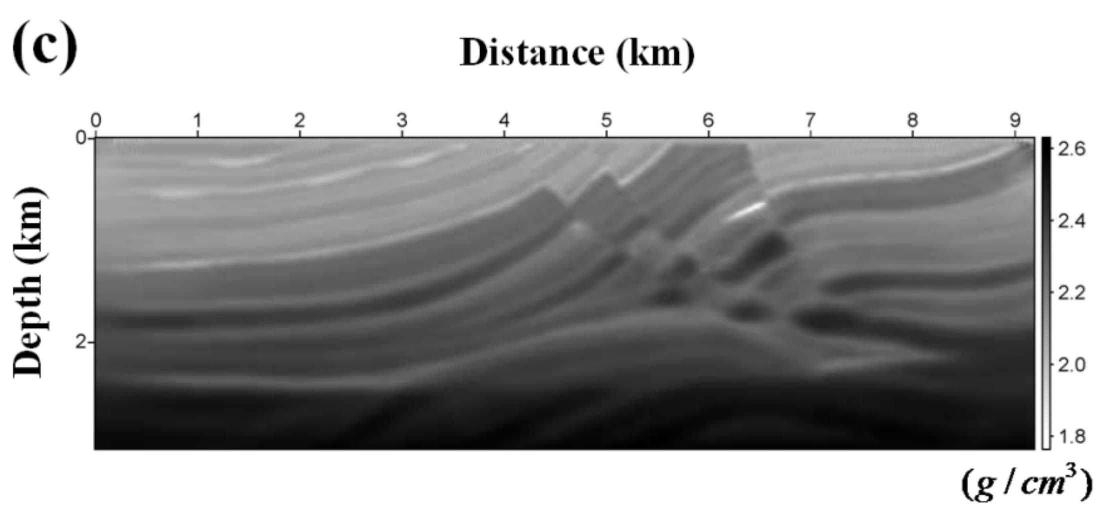
- Inversion results - Conventional methods II




P-wave velocity



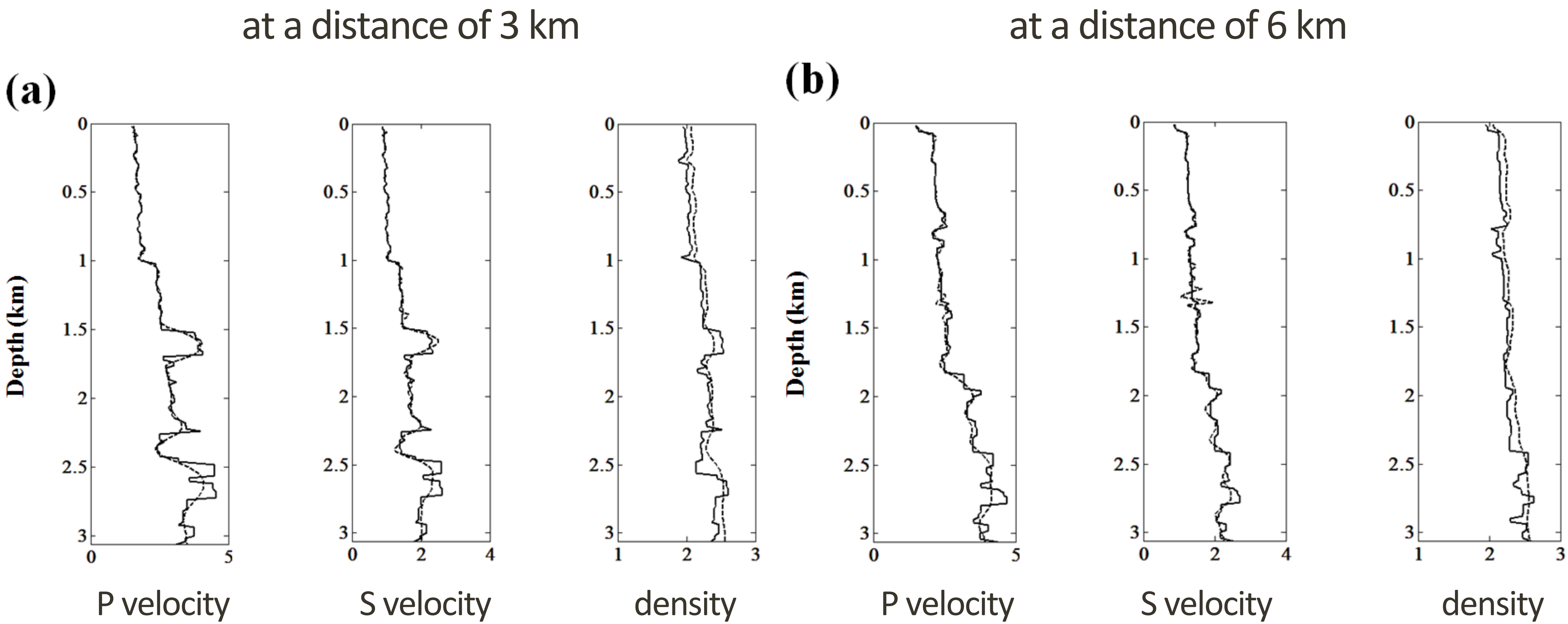

S-wave velocity




density

# Conventional FWI - Examples

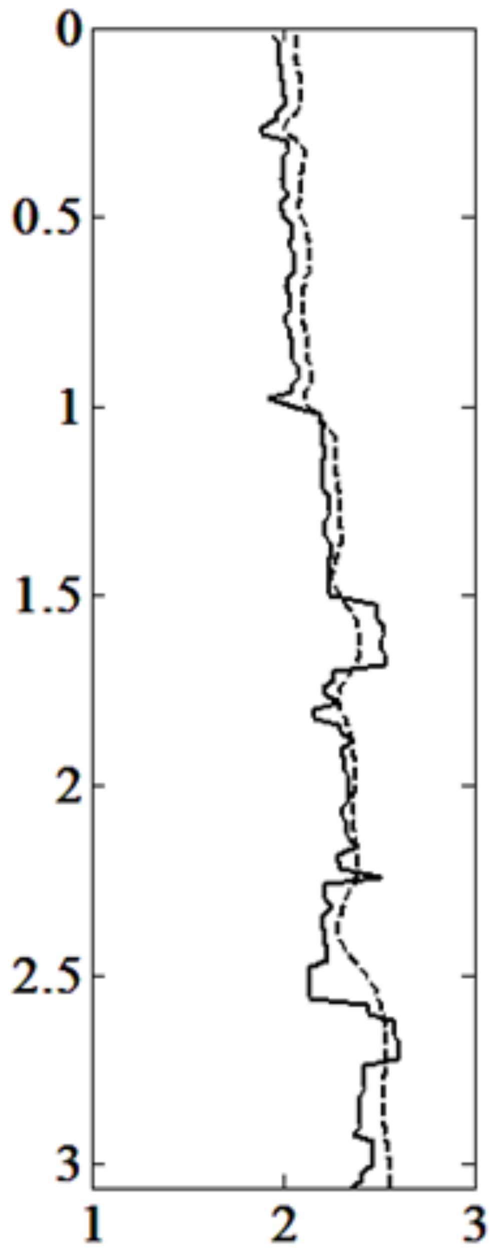
- Depth profiles - Conventional methods II



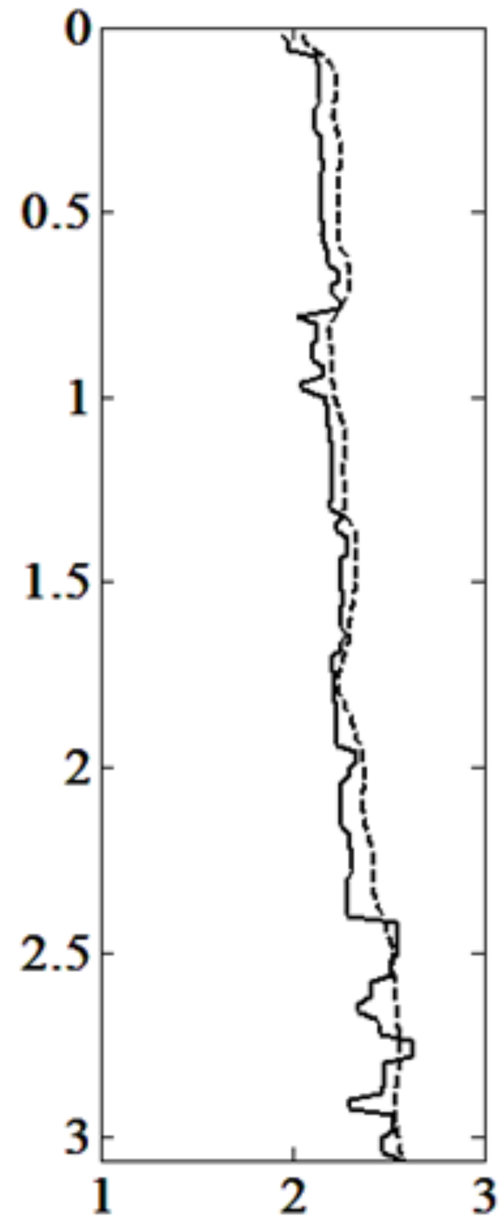
# Conventional FWI - Examples

- Density profiles - Conventional methods II

at a distance of 3 km



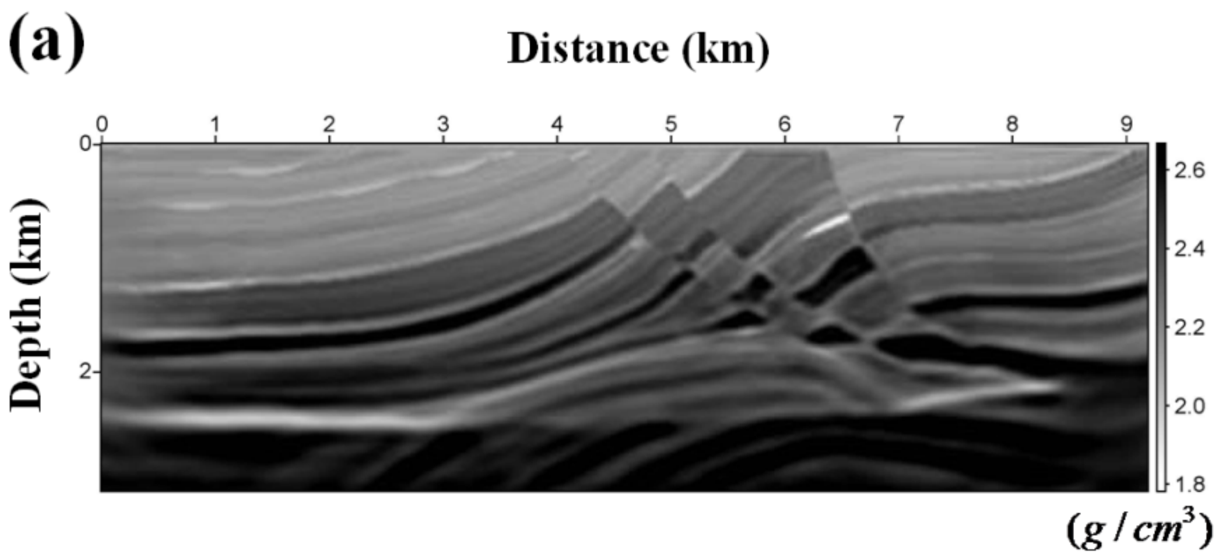

at a distance of 6 km



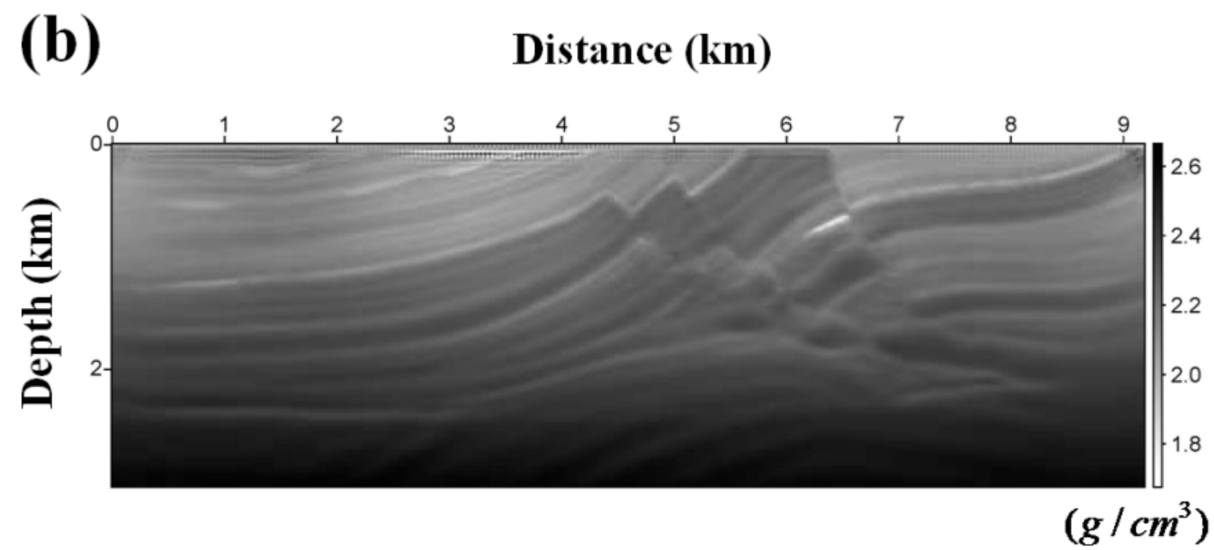
# Conventional FWI - Examples

- Density results in the second stage

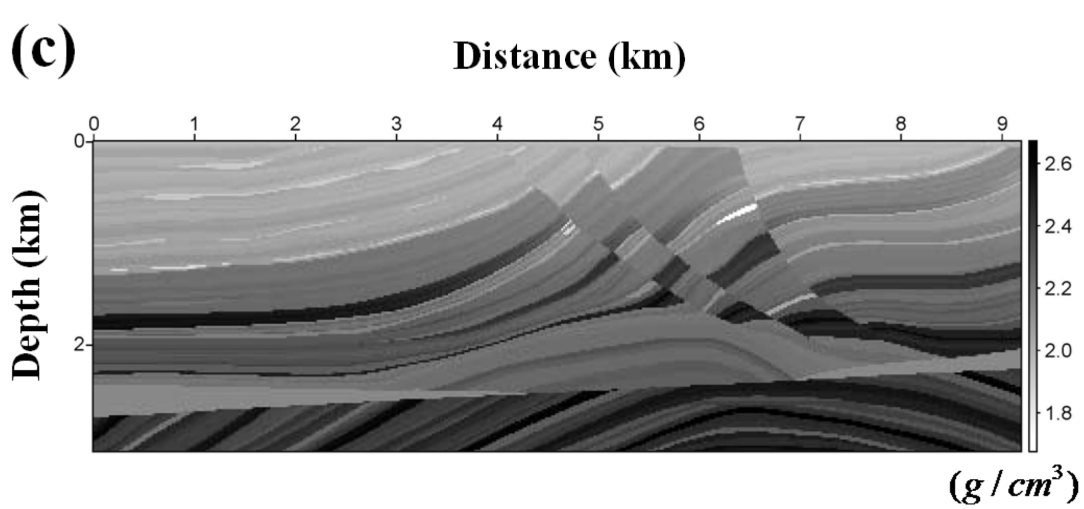
Conventional method I




Conventional method II



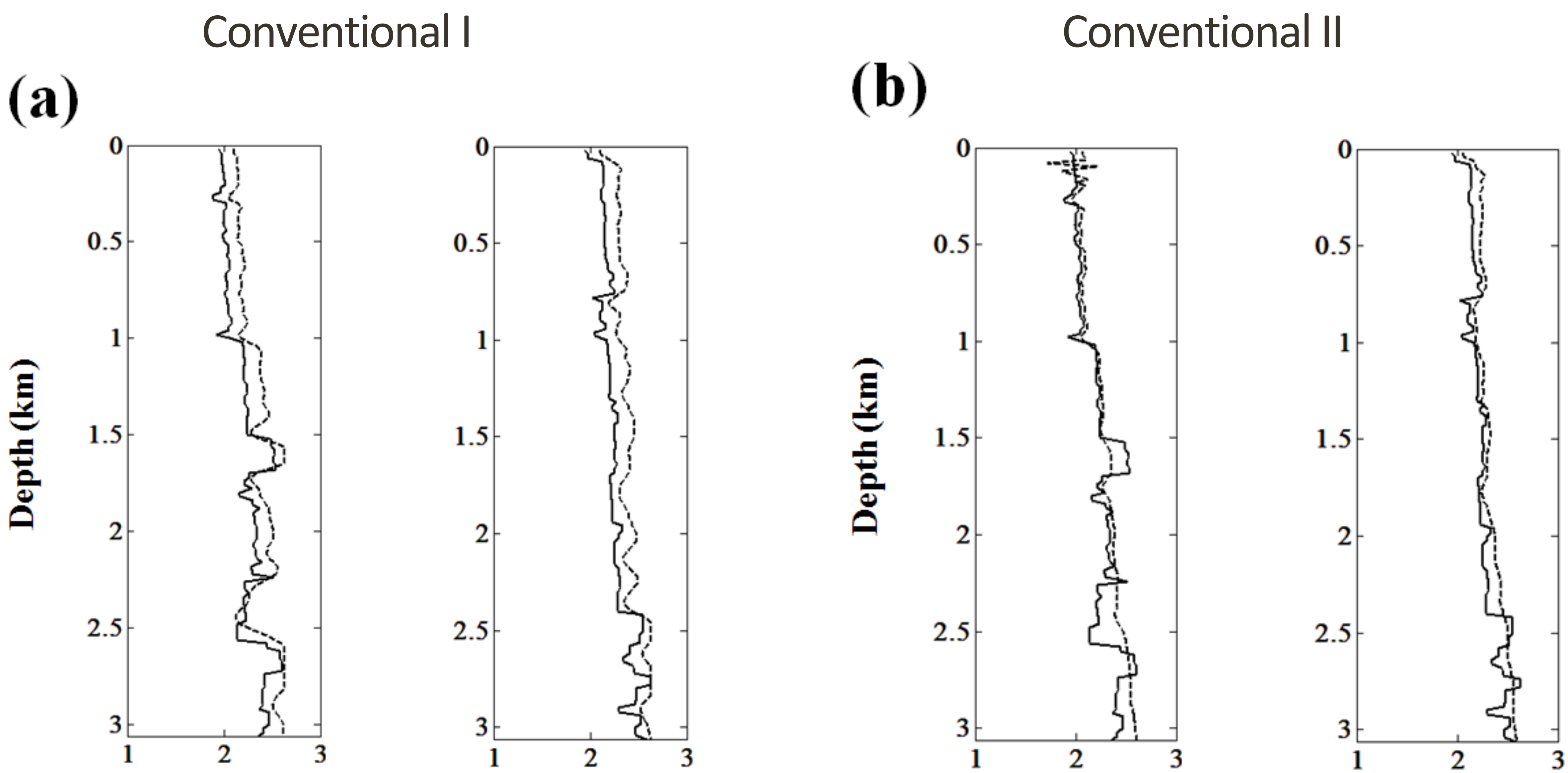

The initial guess for density is a gradually increasing model.




true density

# Conventional FWI - Examples

- Density profiles in the second stage



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# New strategy for density

- Parameter-selection strategy
  - consider that velocities are properly recovered but density is distorted in the conventional inversion results.
  - inversion is performed over two stages.
    --> first stage: velocities are described
    --> second stage: Lame constants and density are inverted

# New strategy for density

## Parameter-selection strategy

- First stage
  --> inversion is conducted for Lame constants with density fixed based on the wave equations parameterized by Lame constants and density
  --> wrong Lame constants and density
  --> but reliable velocities can be extracted from wrong information

$$\alpha = \sqrt{\frac{\lambda_V + 2\mu_V}{\rho_C}} \qquad \beta = \sqrt{\frac{\mu_V}{\rho_C}}$$

V: virtual, C: constant

- Virtual sources

$$\left(\mathbf{f}^{v}\right)_{\lambda} = -\frac{\partial \mathbf{S}}{\partial \lambda}\tilde{\mathbf{u}}, \qquad \left(\mathbf{f}^{v}\right)_{\mu} = -\frac{\partial \mathbf{S}}{\partial \mu}\tilde{\mathbf{u}},$$

# New strategy for density

- Parameter-selection strategy
  - Second stage

    --> velocities obtained in the first stage are used as initial guesses

    --> for initial guess for density, a linearly increasing model is used

    --> both Lame constants and density are inverted based on the wave equation parameterized by velocities and density

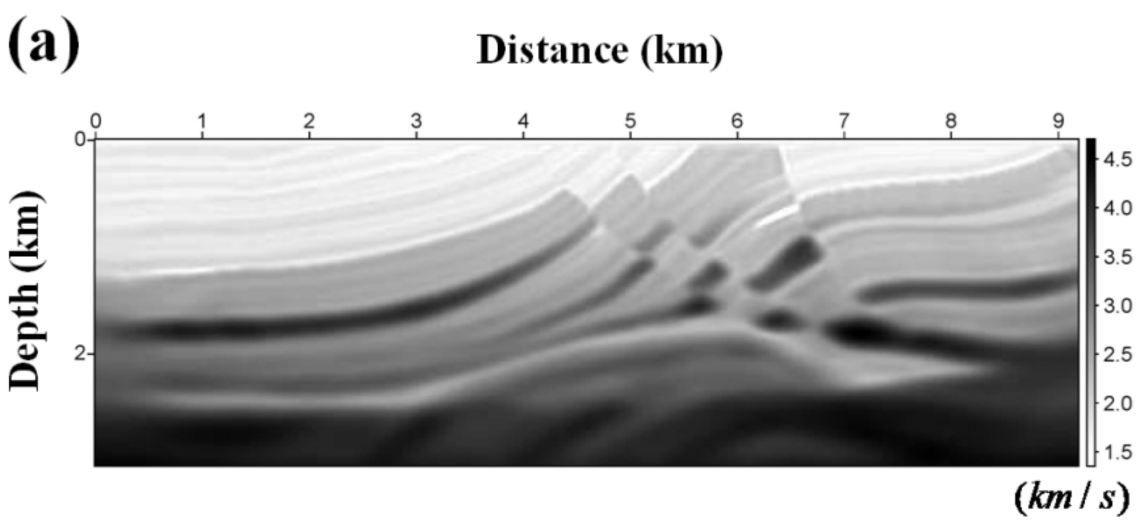
- Virtual sources (using the chain rule reversed to Mora (1987))

$$\left(\mathbf{f}^{v}\right)_{\lambda} = -\left(\frac{\partial \mathbf{S}}{\partial \alpha}\frac{\partial \alpha}{\partial \lambda} + \frac{\partial \mathbf{S}}{\partial \beta}\frac{\partial \beta}{\partial \lambda} + \frac{\partial \mathbf{S}}{\partial \rho}\frac{\partial \rho}{\partial \lambda}\right)\tilde{\mathbf{u}} \qquad \left(\mathbf{f}^{v}\right)_{\rho} = -\frac{\partial \mathbf{S}}{\partial \rho}\tilde{\mathbf{u}}$$

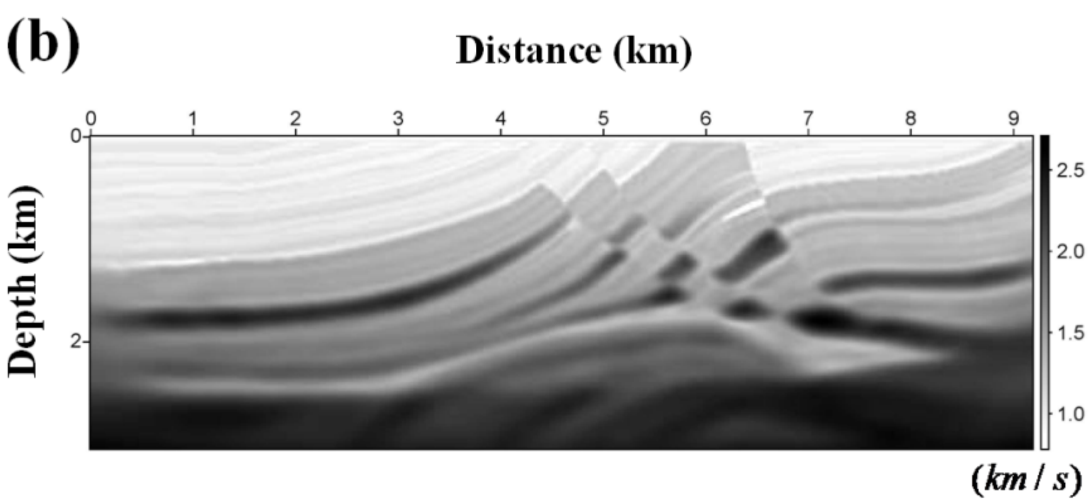
$$\left(\mathbf{f}^{v}\right)_{\mu} = -\left(\frac{\partial \mathbf{S}}{\partial \alpha}\frac{\partial \alpha}{\partial \mu} + \frac{\partial \mathbf{S}}{\partial \beta}\frac{\partial \beta}{\partial \mu} + \frac{\partial \mathbf{S}}{\partial \rho}\frac{\partial \rho}{\partial \mu}\right)\tilde{\mathbf{u}}$$

# New inversion results

- Velocities inverted in the first stage




P-wave velocity

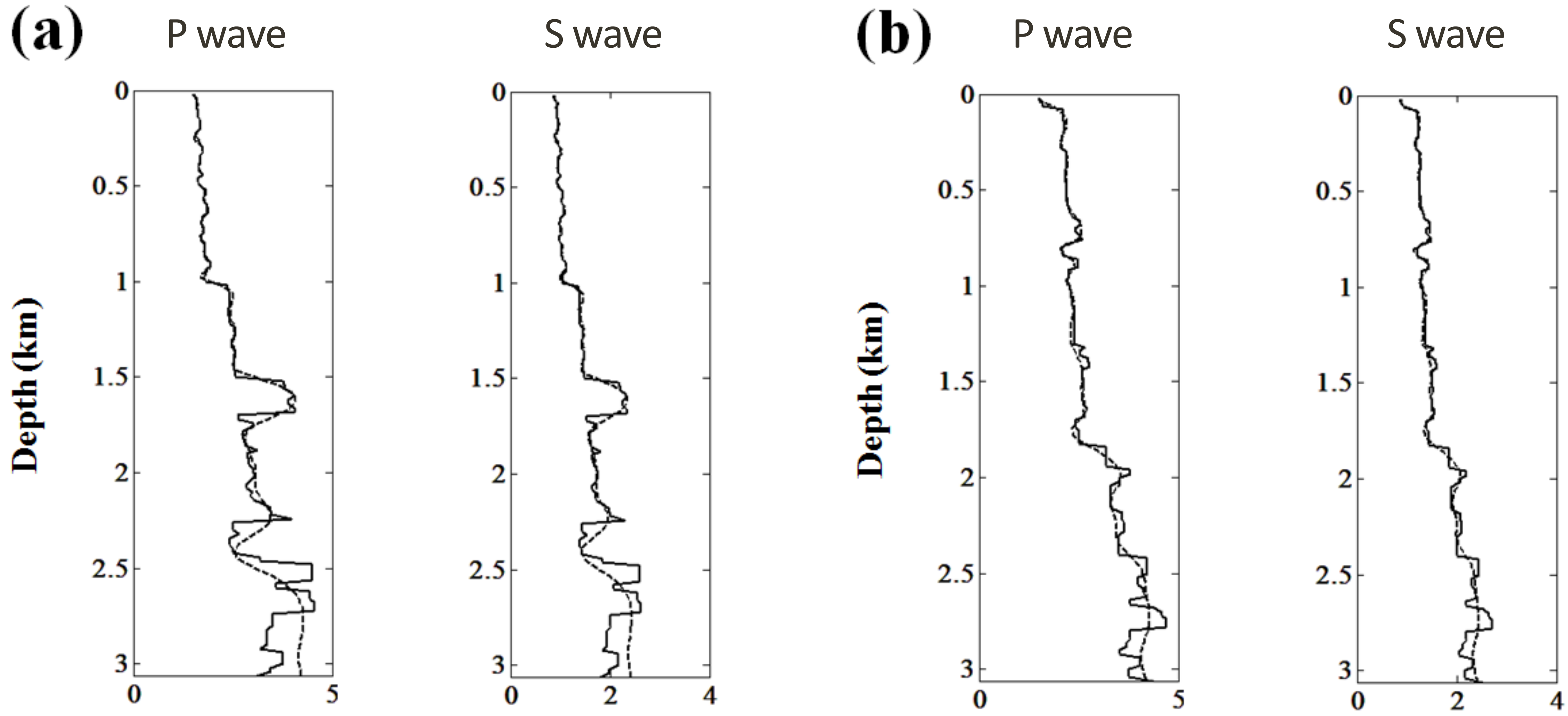



S-wave velocity

The density is fixed as 2 $g/cm^3$

# New inversion results

- Velocity profiles in the first stage

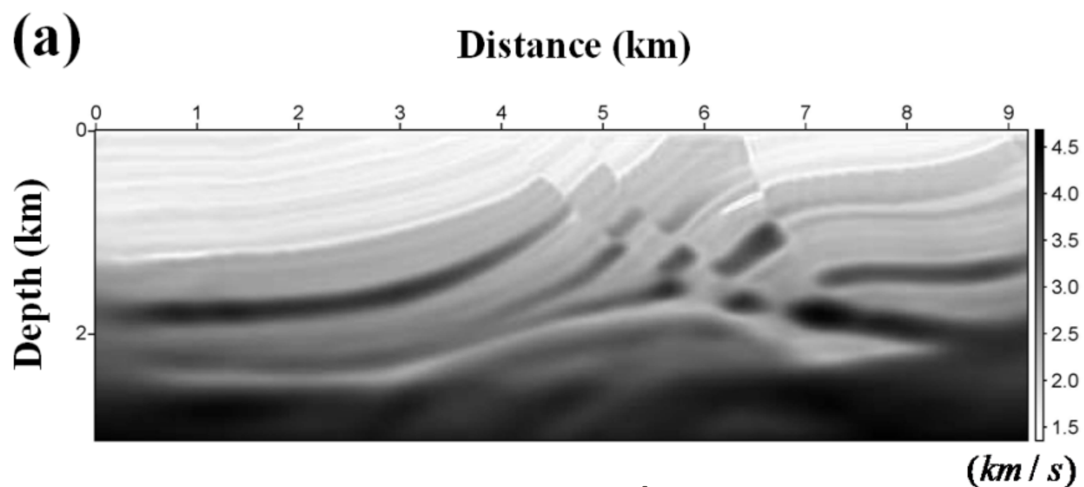



at a distance of 3 km

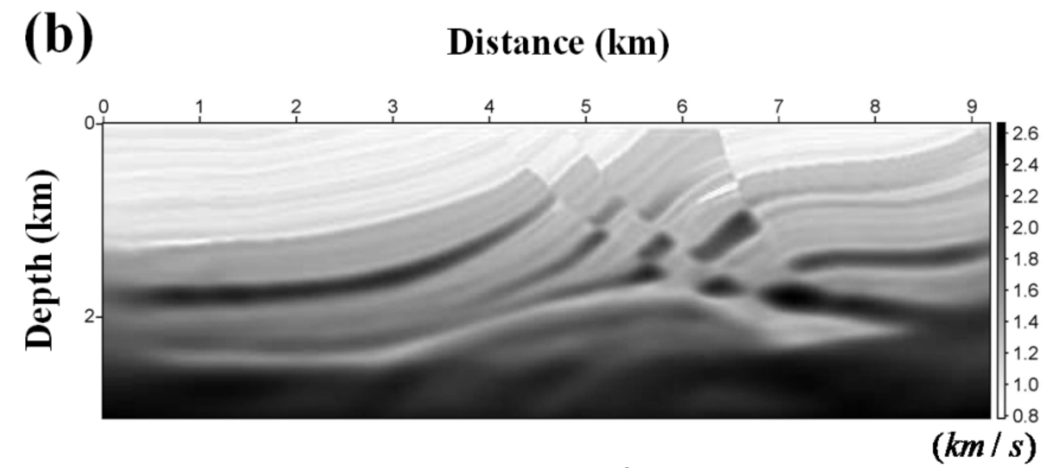
at a distance of 6 km

# New inversion results

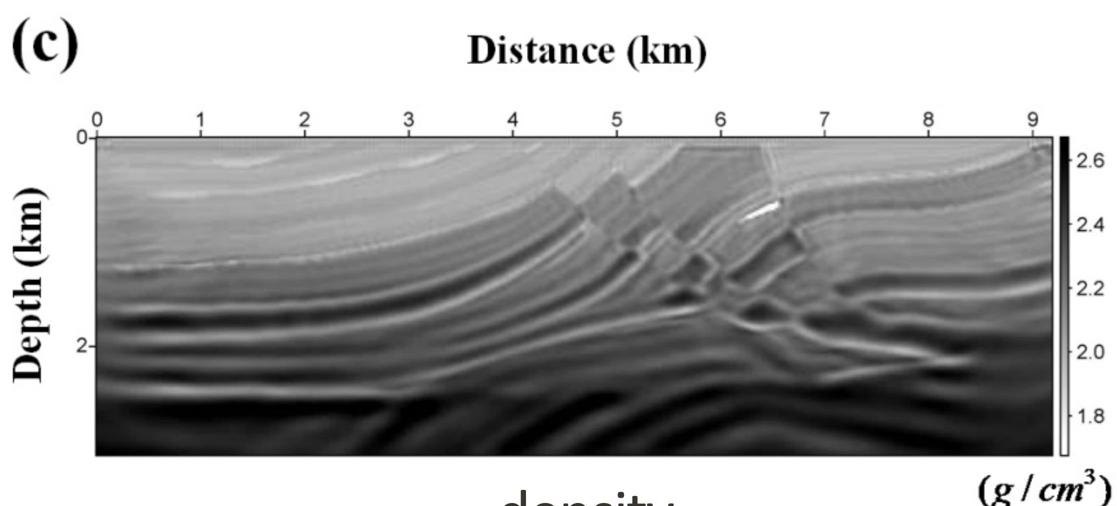
- Inversion results in the second stage




P-wave velocity



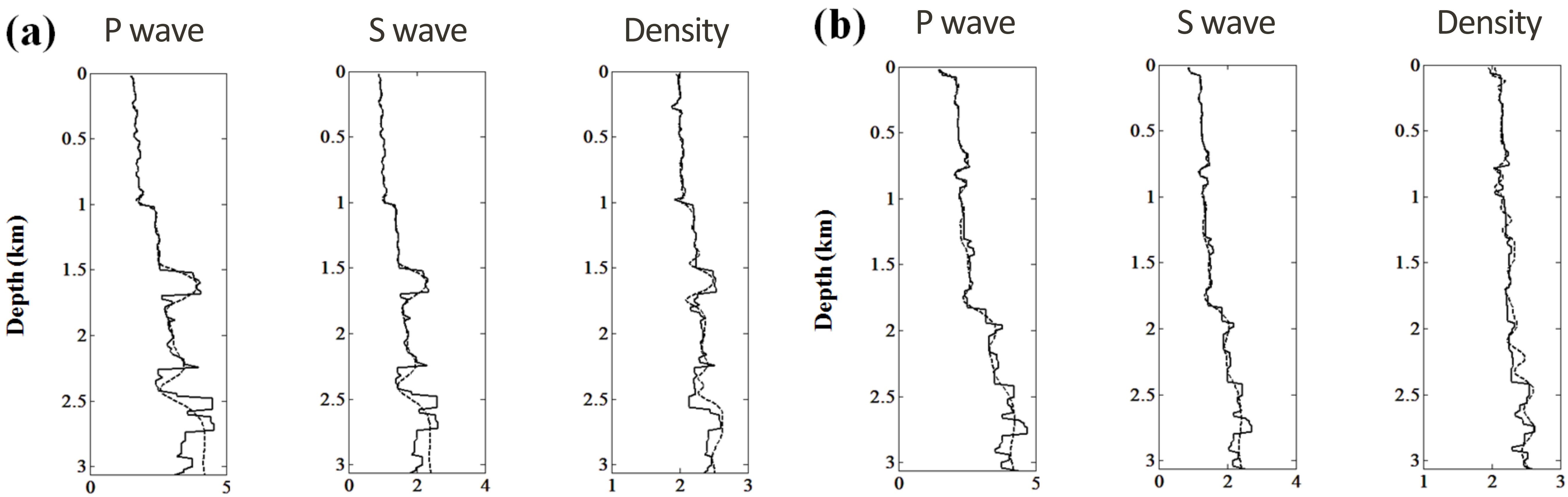

S-wave velocity




density

# New inversion results

- Depth profiles in the second stage

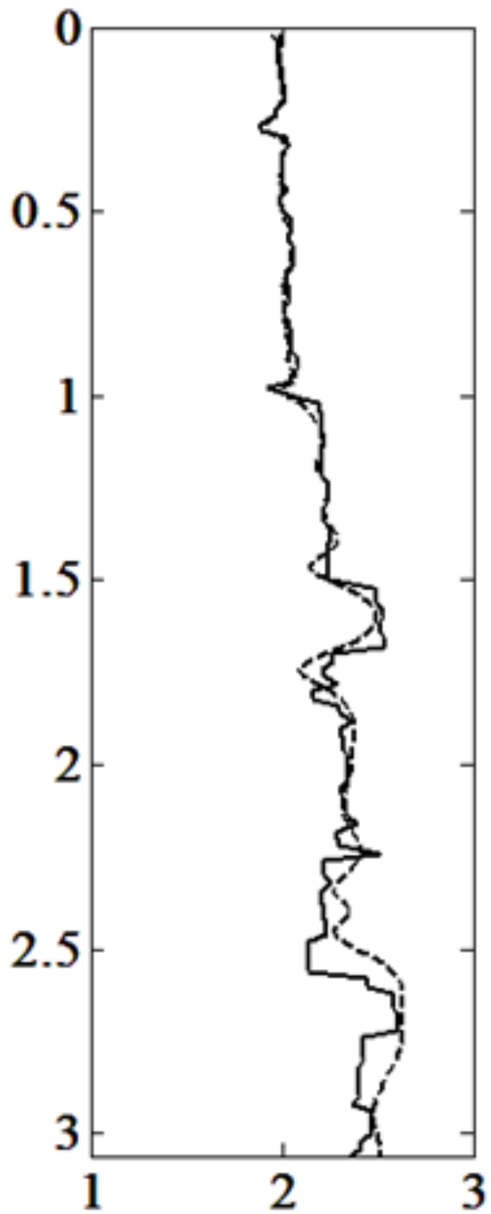



at a distance of 3 km

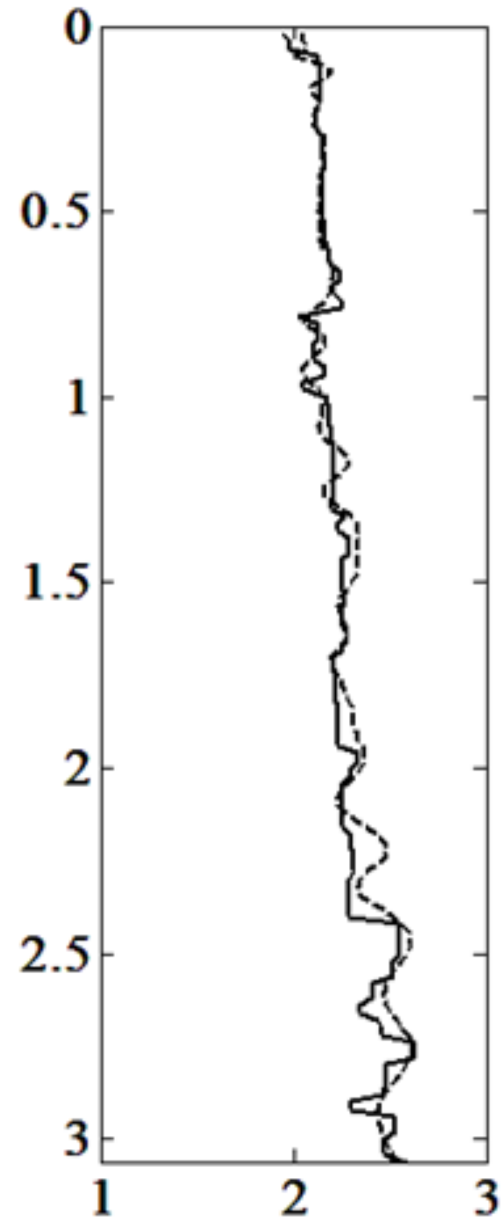
at a distance of 6 km

# New inversion results

- Density profiles in the second stage



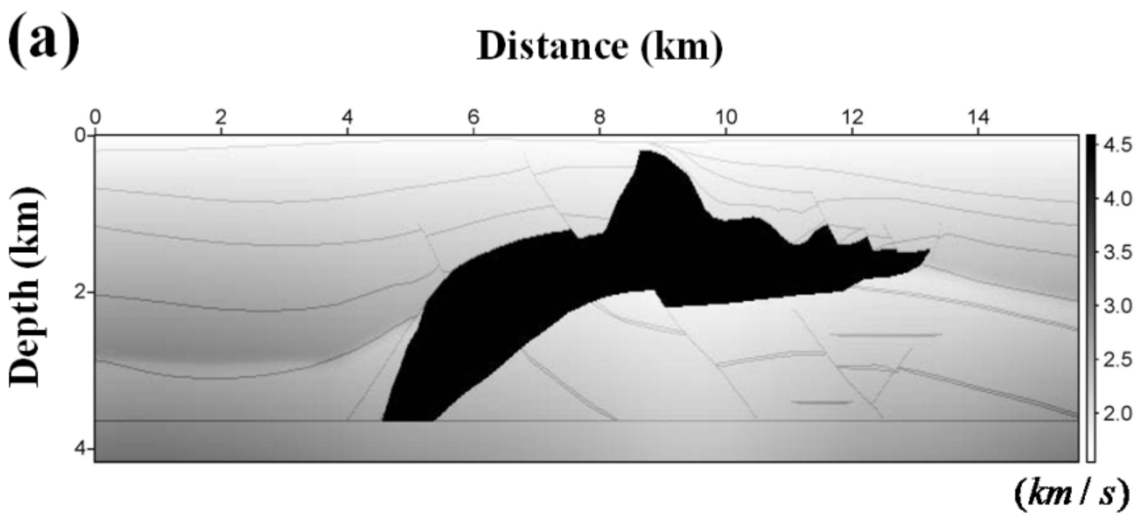

at a distance of 3 km



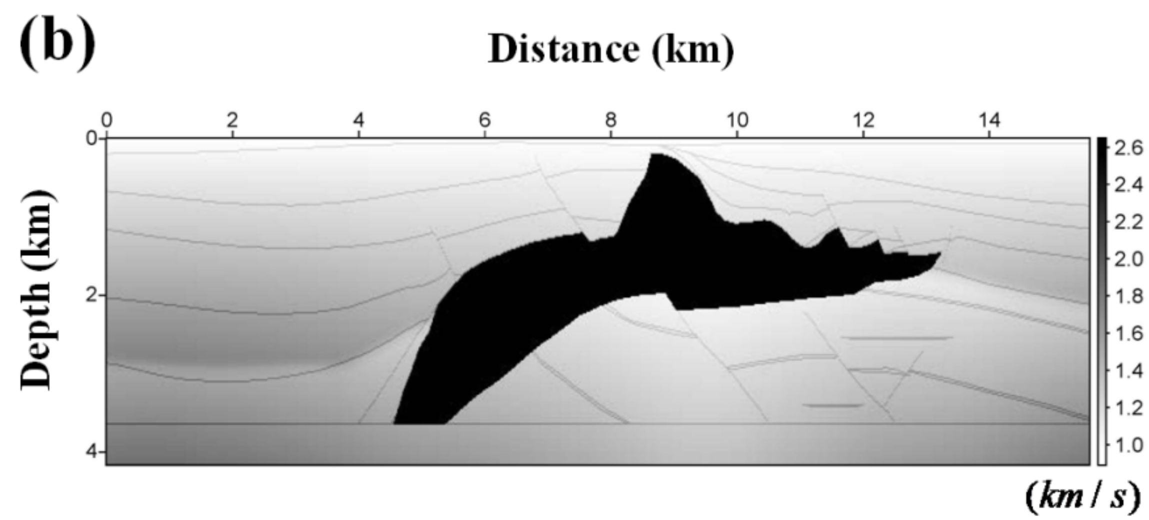

at a distance of 6 km

# SEG/EAGE salt model

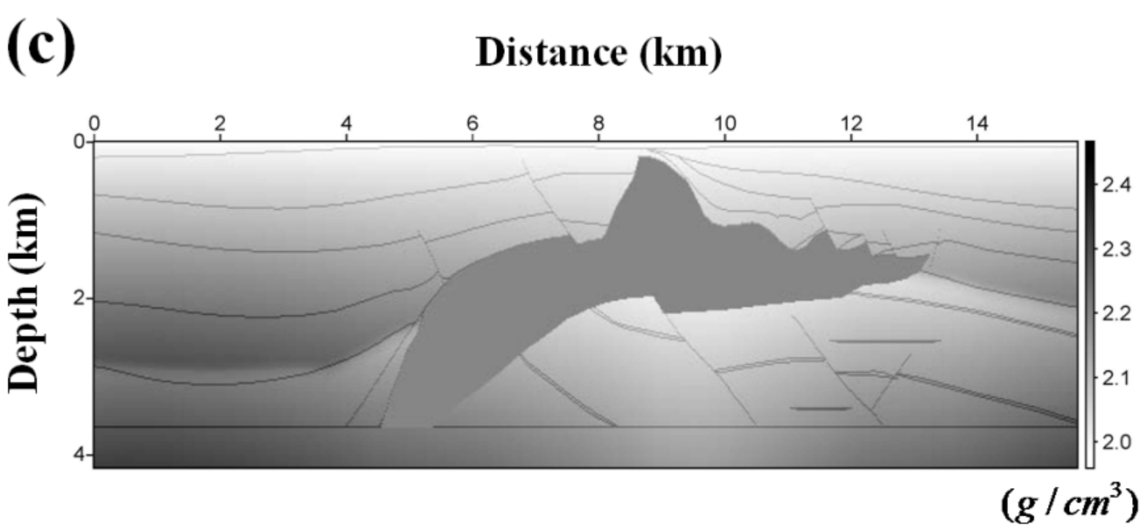
- True model




P-wave velocity




S-wave velocity

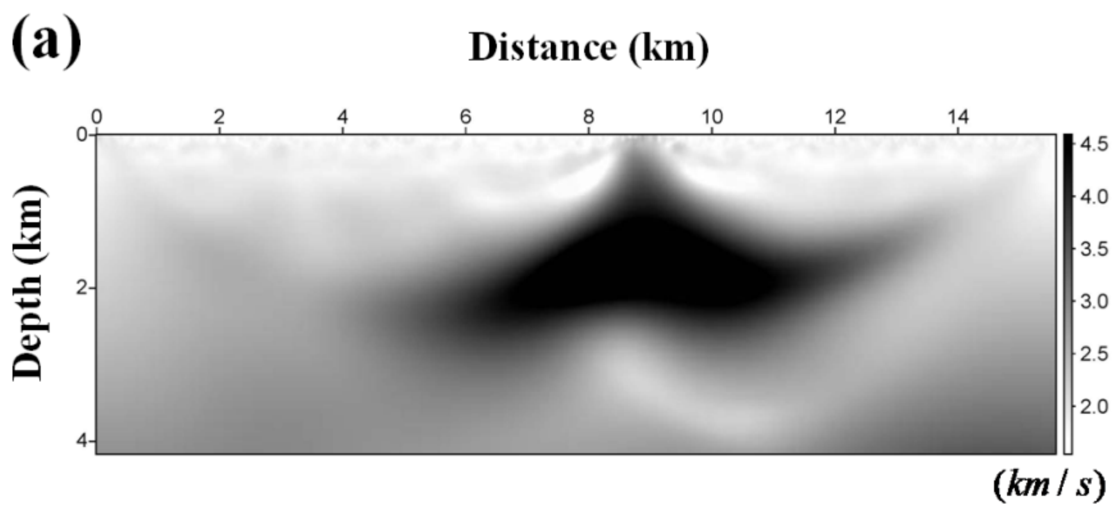



density

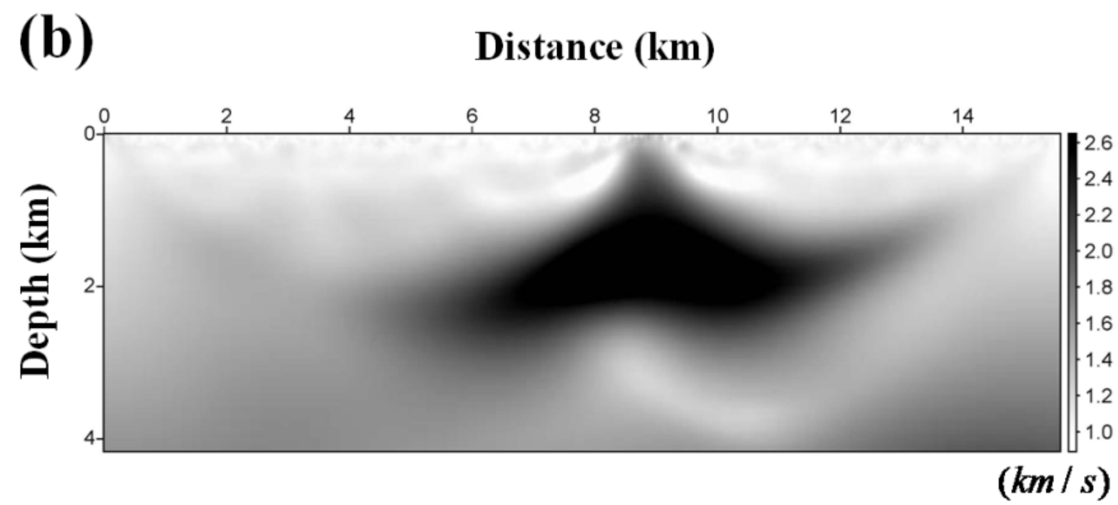
| Parameters | Values |
|---|---|
| dimension | 15.6 km x 4.2 km |
| no. of source | 379 |
| source interval | 0.04 km |
| no. of receiver | 781 |
| receiver interval | 0.02 km |
| recording time | 6 s |
| frequency range | 0.167 - 10 Hz |

# SEG/EAGE salt model

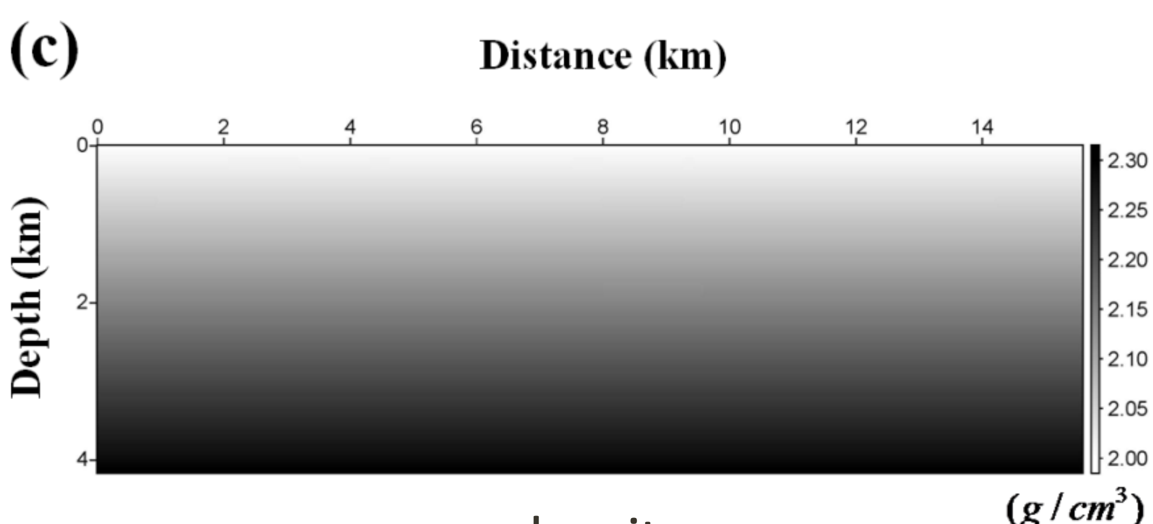
- Initial model




P-wave velocity




S-wave velocity

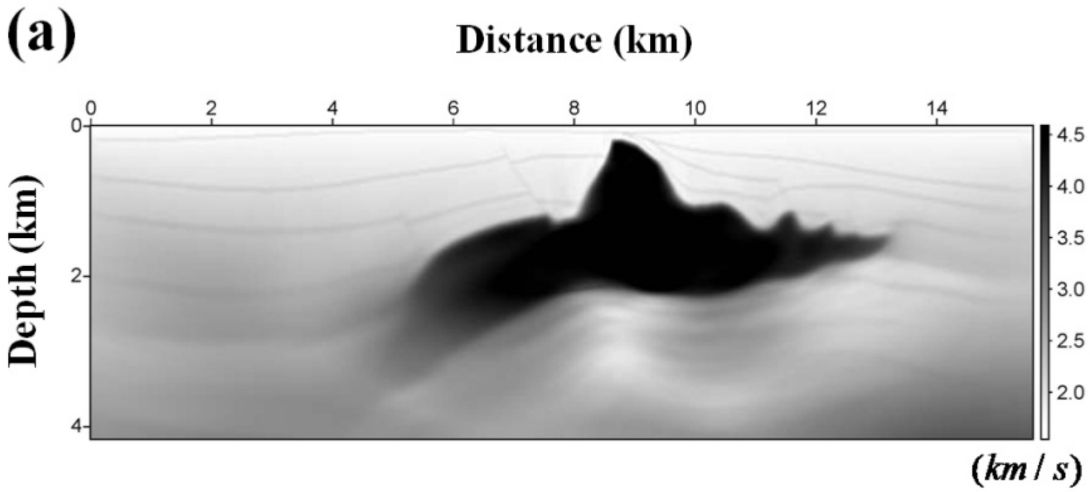



density

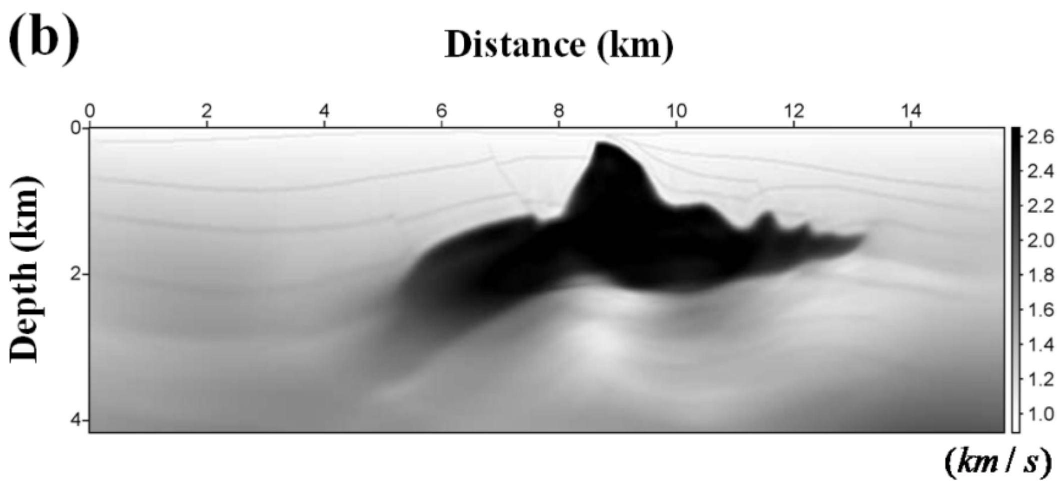
Laplace-domain inversion results
(Chung et al. 2010)

# Conventional inversion results

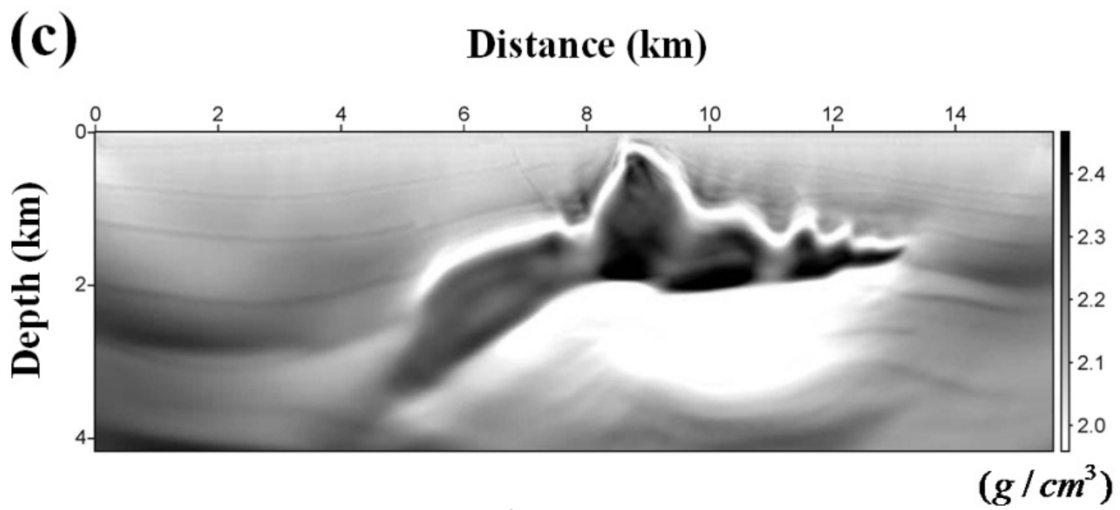
- Inversion results - Conventional method I




P-wave velocity



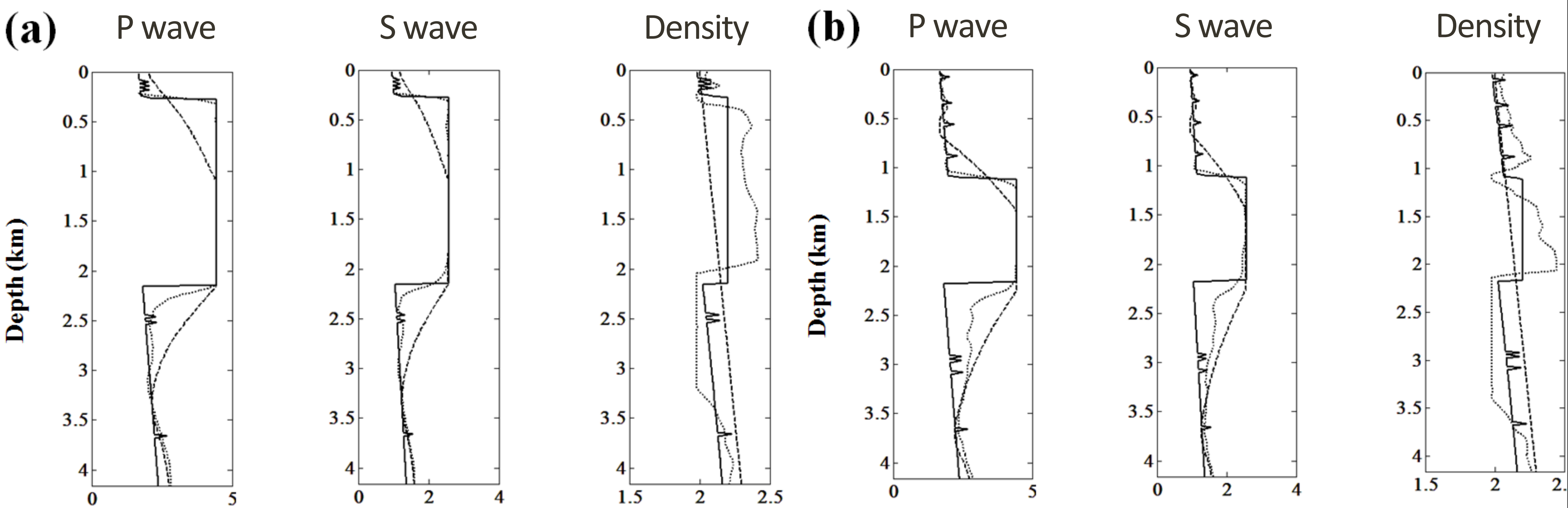

S-wave velocity




density

# Conventional inversion results

- Depth profiles - Conventional method I



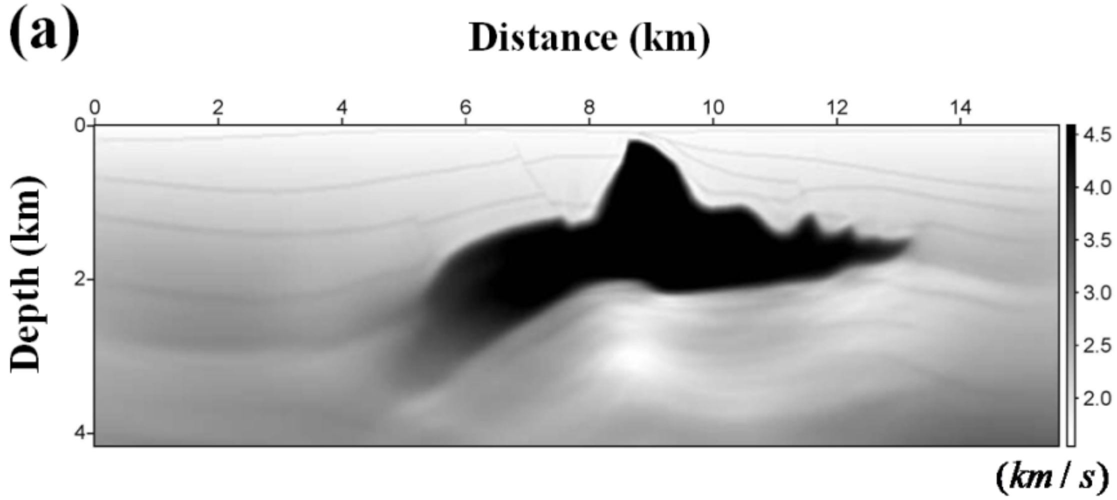

at a distance of 9 km

at a distance of 10 km

# New inversion results

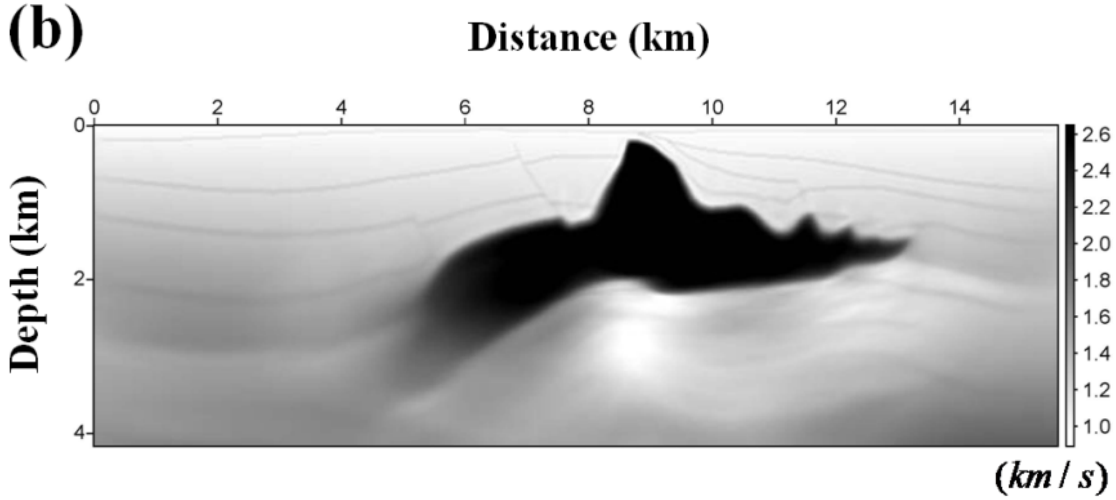
- Inverted velocity models - In the first stage

(a)




P-wave velocity

(b)

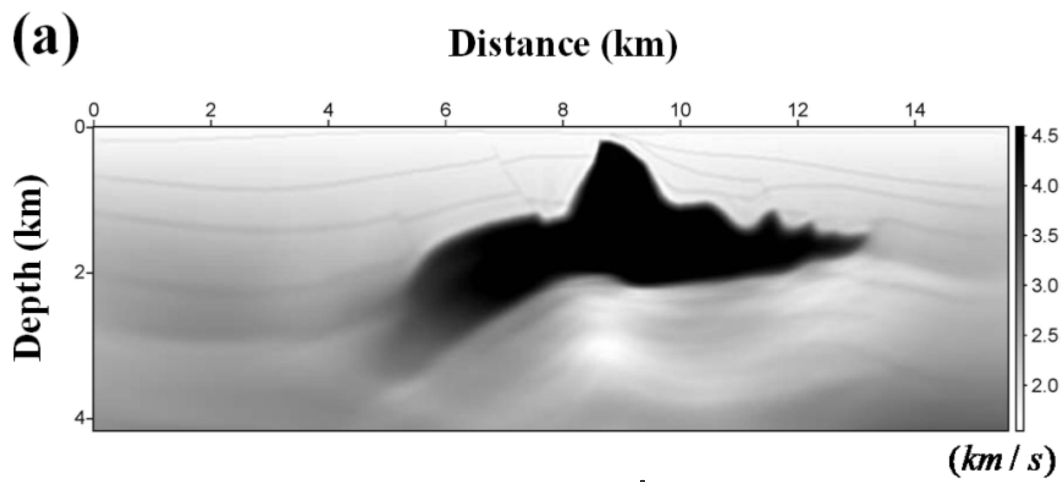



S-wave velocity

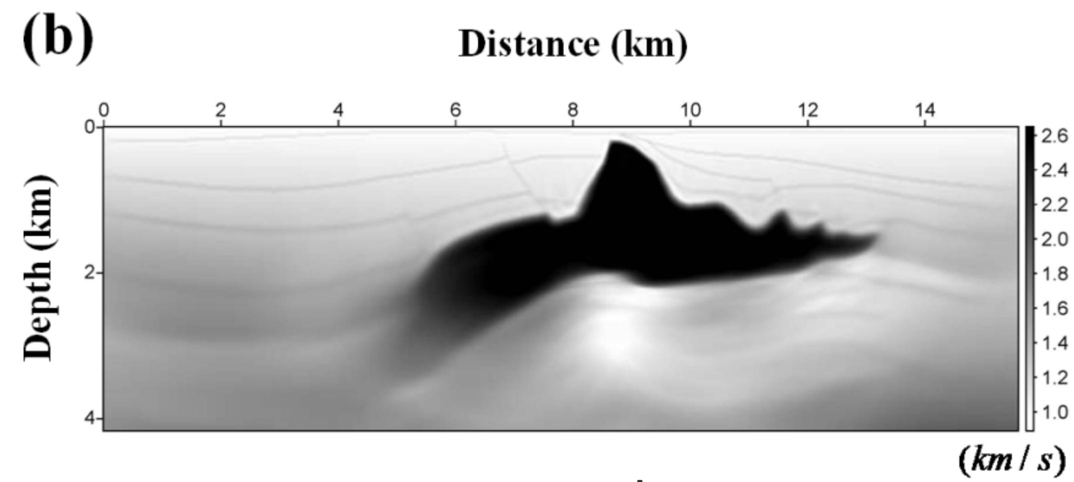
The density is fixed as 2 $g/cm^3$

# New inversion results

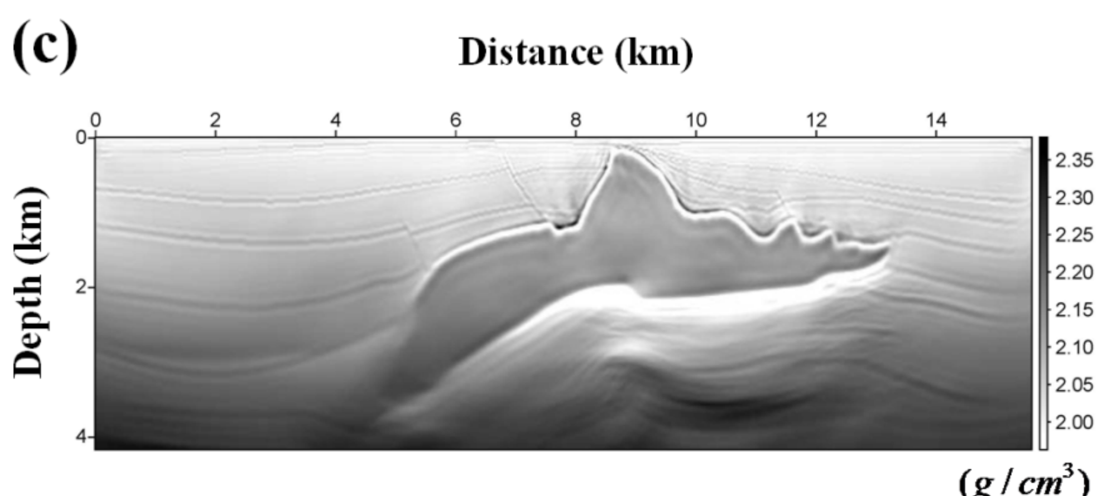
- Inversion results - In the second stage




P-wave velocity



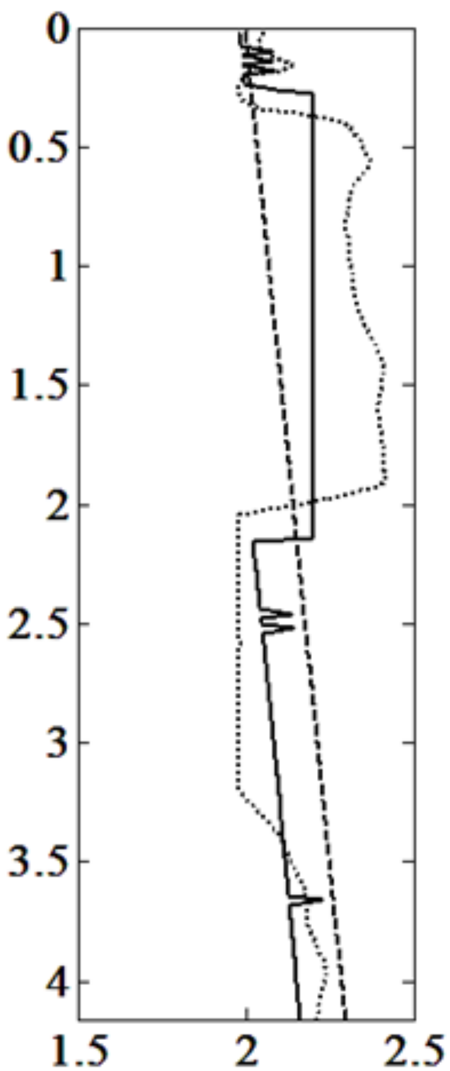

S-wave velocity



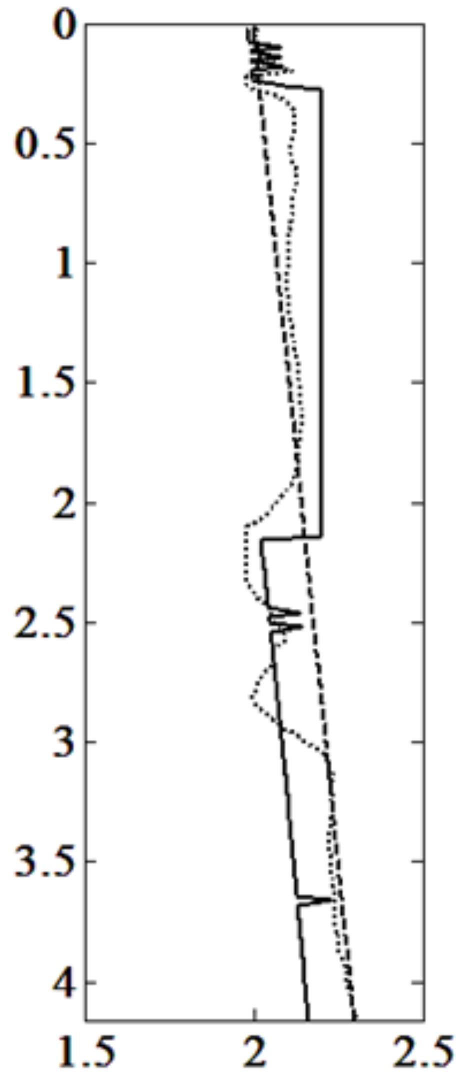

density

# New inversion results

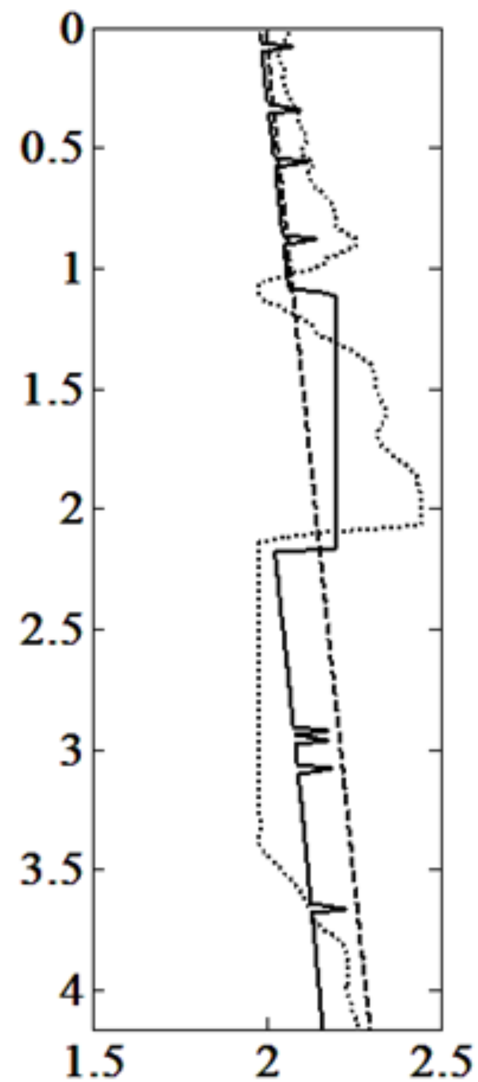
- Density profiles - Conventional method I & new method



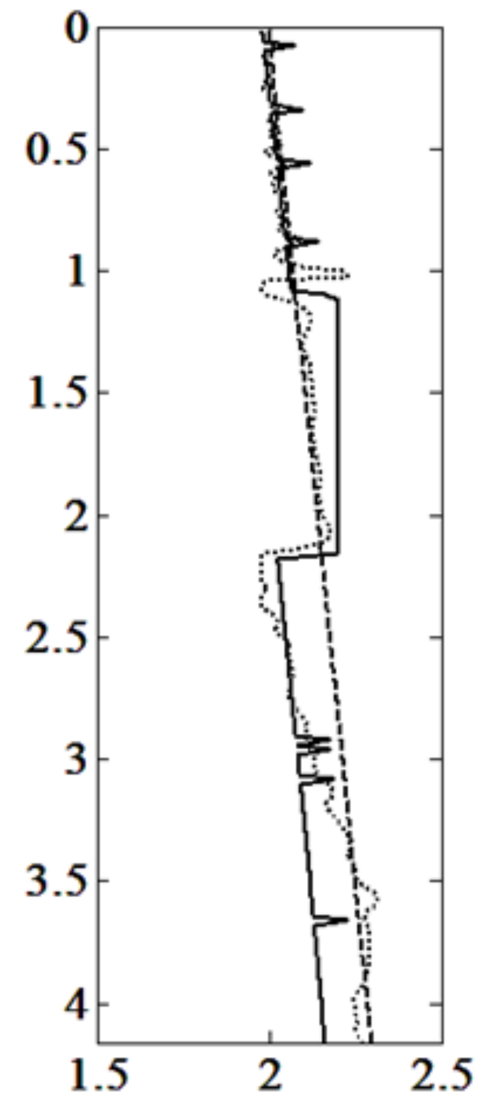

conventional




new




conventional




new

# Contents

# Conclusions

- Parameter selection strategy
  - in the first stage, invert Lame constants with density fixed as a constant
  - in the second stage, invert all parameters simultaneously
- Numerical examples show that
  - the new inversion strategy gives more reliable density models

# Conclusions

- To enhance the accuracy of inverted density model
  - accurate velocity models are necessary, which are obtained with density fixed in the first stage of the new strategy

# Future plans

- Collaboration with SLIM group
  - combine elastic waveform inversion for isotropic and VTI media with the Curvelet transform, simultaneous sources inversion, stochastic inversion, etc.
  - develop inversion techniques to properly recover salt models.

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