Why do Curvelets work?

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Outline

Part 1: Sparsity and randomized acquisition

- Joint work with Haneet Wason, Tim Lin, and Felix Herrmann

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Part 3: Extensions in FWI, 3D acquisition and time-lapse imaging

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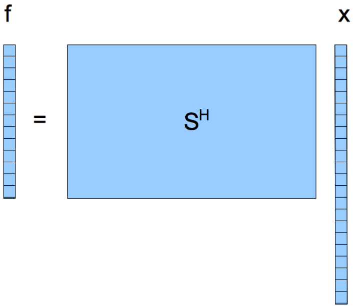
Part 3: Applications in FWI, 3D acquisition and time-lapse imaging

Curvelets and sparsity

- Consider a signal $f \in \mathbb{R}^N$, $f = S^H x$, where S is a curvelet transform matrix and x is the synthesis coefficient vector.
- We can approximate f by the signal $\bar{f}$ using the k-largest coefficients of x.
- For example: a 512×128 shot gather
  - Largest 4598 (0.95%) of curvelet coefficients produce SNR = 10.68dB .
  - Largest 6511 (9.93%) of Fourier coefficients produce SNR = 10.11dB.



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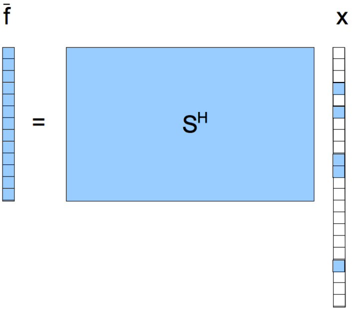

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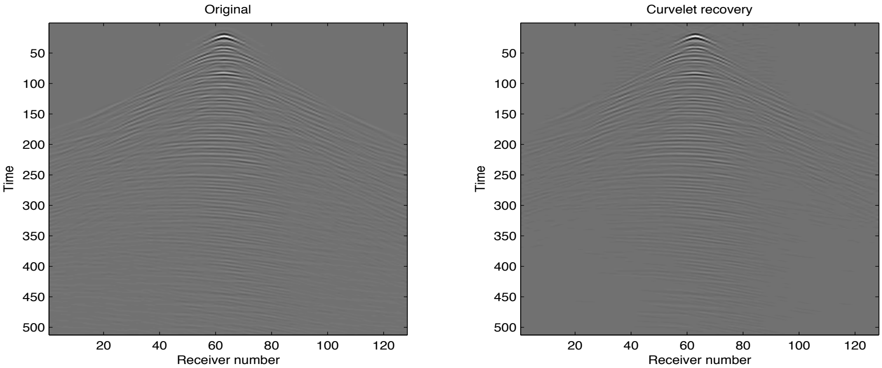

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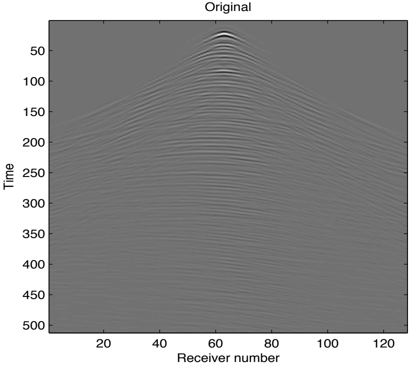

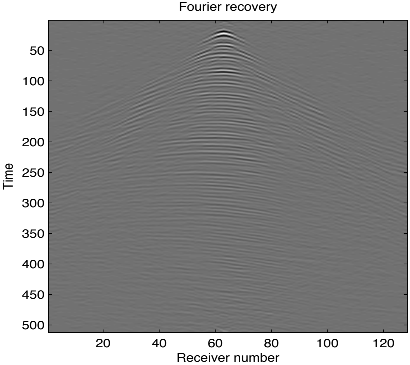

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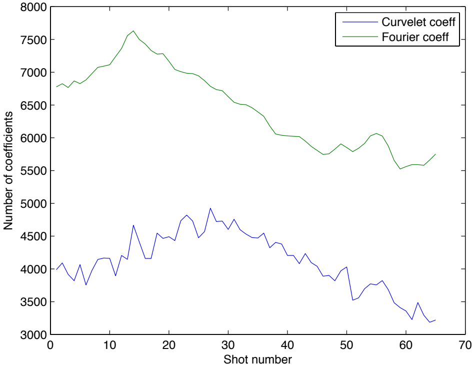

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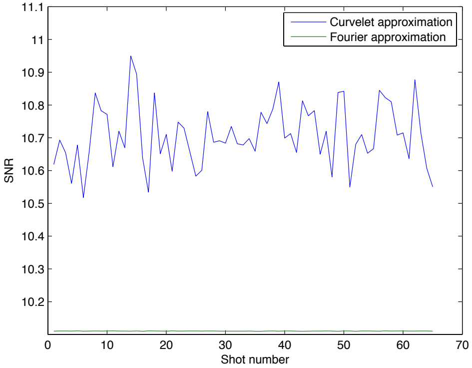

Compressed sensing

- Compressed Sensing is an acquisition paradigm for signals that admit *sparse* or nearly sparse representations in some transform domain.
- Given $n \ll N$ linear and noisy measurements $b = RMf + e$.
- Let $A = RMS^H$, it is possible to approximate x from the measurements b if
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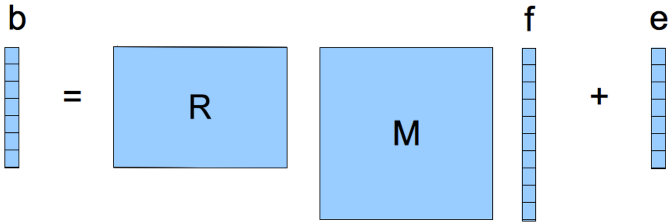

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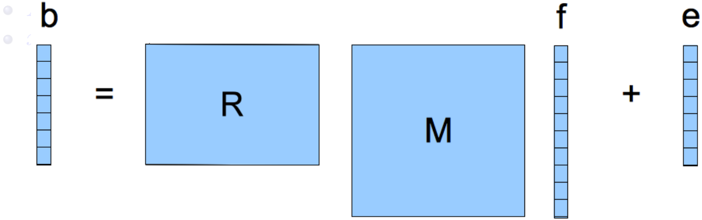

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CS example: simultaneous/randomized acquisition

- Simultaneous acquisition is a perfect example of a compressed sensing problem.
- The objective is to recover the high-dimensional sequential shot record f from the lower-dimensional "supershot" record $b = RMf$.
- Formulate the acquisition process in terms of the sampling operator RM.
- We want to construct the sampling operator RM such that:
  1. $A = RMS^H$ satisfies the CS recovery conditions.
  2. RM is physically realizable.
- Recover the sequential shot record by finding $\tilde{f} = S^H\tilde{x}$, where

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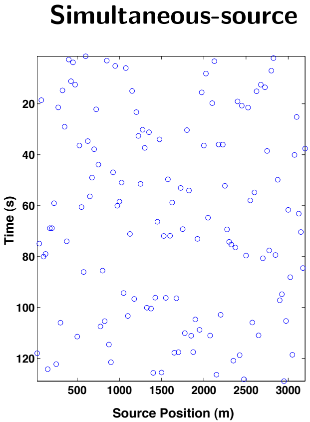
Simultaneous-source acquisition

- Typically, we would like RM to be a matrix with i.i.d Gaussian random entries.
- In the marine impulsive airgun setting, only binary matrices with $0-1$ entries are possible.



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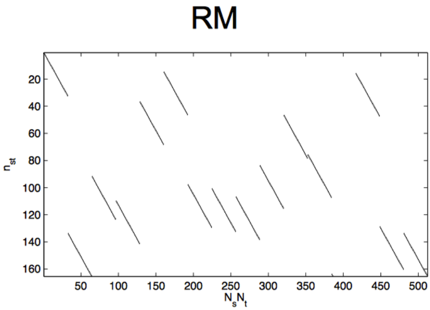

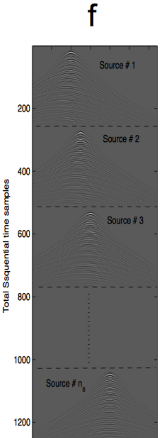

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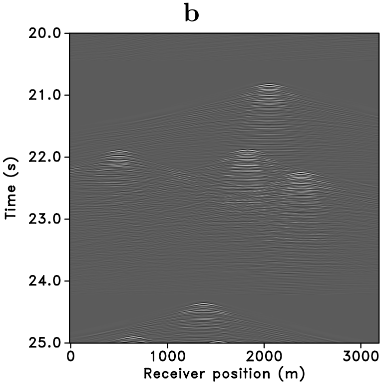

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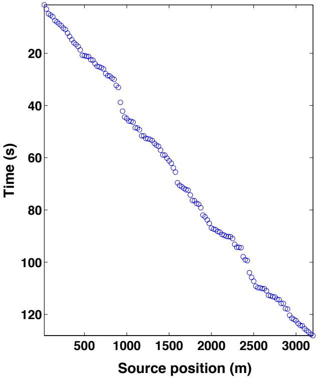
Random time-dithering

- Sort the random source positions so that we only dither in time ("jitter blending").
- It turns out the resulting measurement matrix is almost as good as "ideal" simultaneous-source acquisition (SNR = 8.06 dB).

Random time-shifting



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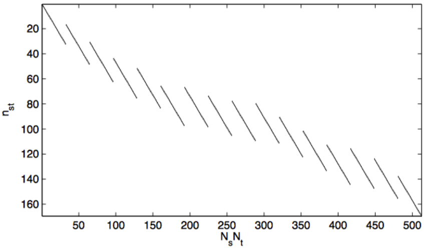

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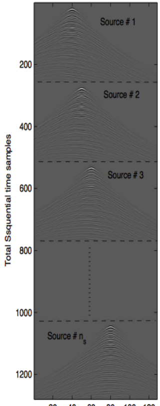

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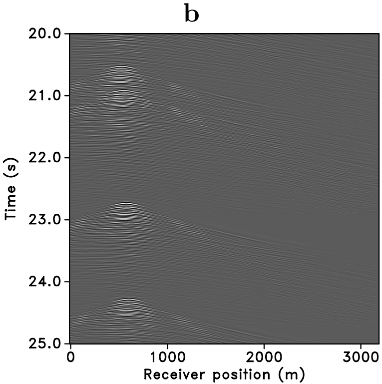

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- The restricted isometry property (RIP) of order k indicates whether every group of k columns of A form a well conditioned submatrix.

Definition: Restricted Isometry Property (RIP) (Candés and Tao)

The RIP constant $\delta_k \in (0, 1)$ is defined as the smallest constant such that $\forall x \in \Sigma_k^N$

$$(1 - \delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_k)\|x\|_2^2,$$

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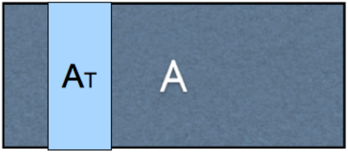

- RIP is equivalent to saying that for any set T of size k, the symmetric matrix $A_T^H A_T$ is positive definite with eigenvalues in $[1 - \delta_k, 1 + \delta_k]$.

Measurement matrix conditions

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Stable recovery condition (Candés, Romberg, and Tao)

If A has RIP with constant $\delta_{(a+1)s} < \frac{a-1}{a+1}$ for some $a > 1$, then the signal x can be recovered using ℓ_1 minimization to produce an estimate $\tilde{x}$ with an error bounded by

$$\|x - \tilde{x}\|_2 \leq \frac{C}{\sqrt{s}} \|x - x_s\|_1.$$

Why do curvelets work?

- Monte Carlo estimation of the RIP constant δ_k of $A = RMS^H$, for some $k = |T|$

$$\delta_k = \sup_{T \in \{1, \ldots P\}} \max\{1 - \sigma_{\min}(A_T), \sigma_{\max}(A_T) - 1\}$$

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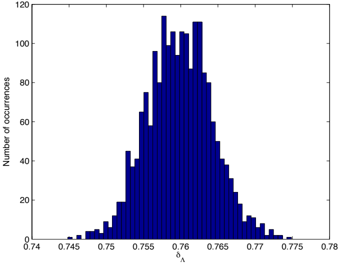
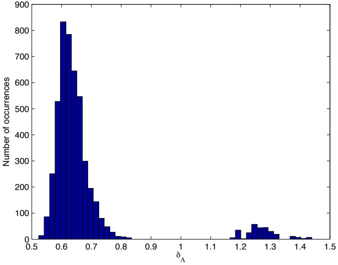
Curvelet



Fourier



Importance of randomization

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- To emphasize the importance of randomization, we include the case where we simply decrease the intershot time delays.

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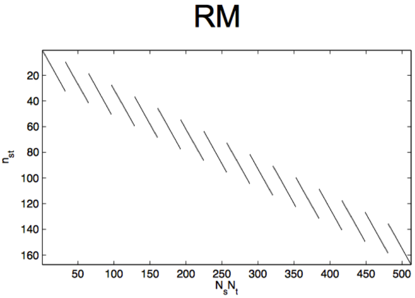

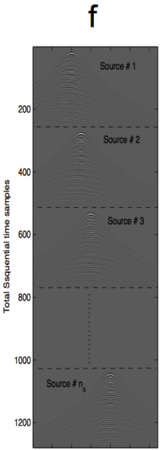

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Measurement matrix conditions

- The mutual coherence measures the orthogonality of all columns of A.

Definition: Mutual Coherence (Donoho and Elad; Bruckstein et al.)

The mutual coherence is equal to the largest inner product between between the normalized columns of A

$$\mu(A) = \max_{1 \leq i \neq j \leq P} \frac{|a_i^H a_j|}{(\|a_i\|_2 \cdot \|a_j\|_2)}.$$

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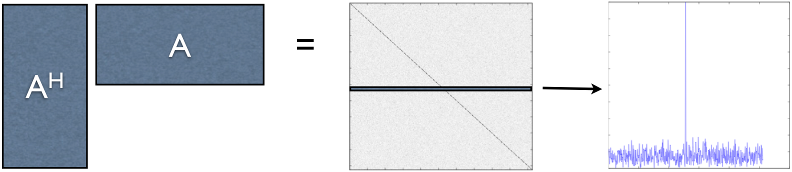
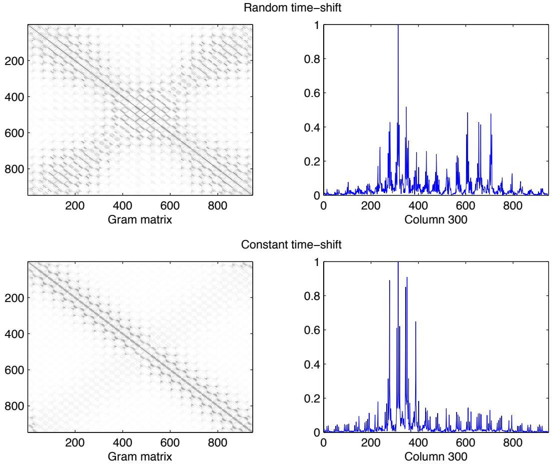


Why does the random time-dithering operator work?

Mutual coherence of $A = RMS^H$ (curvelet), $k < \frac{1}{2}(1 + \frac{1}{\mu(A)})$



Steinhaus sequences

- A Steinhaus random variable v is a unit norm complex random variable uniformly distributed on the complex unit circle: $v = e^{i\theta}, \theta \in U(-\pi, \pi)$.
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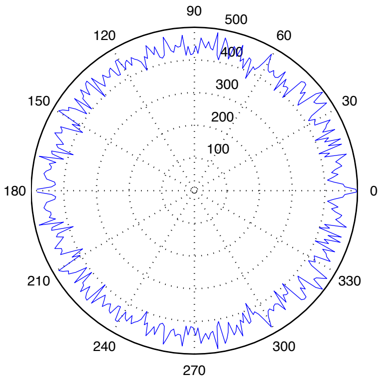
- Do the curvelet coefficients of seismic shot gathers form instances of Steinhaus sequences? Distribution of $\mathrm{sgn}(x) = e^{i\theta}$, where $x = re^{i\theta}$.



Why may this be useful?

- An old result by Fuchs(2004) and Tropp(2005) states that if

$$|\langle A_T^{\dagger} a_j, \mathsf{sgn}(x_T) \rangle| < 1$$

then ℓ_1 minimization is guarantees to recovery a signal x supported on T.

Why may this be useful?

- Tropp (2007) then showed that random signals x that are supported on a set T and whose $\mathsf{sgn}(x)$ form Steinhaus sequences, satisfy the condition

$$|\langle A_T^\dagger a_j, \mathsf{sgn}(x_T)\rangle| < 1$$

with probability $1 - 2\rho$, when:

  - A has coherence μ
  - $\sigma_{\min}(A_T) > 1/\sqrt{2}$
  - $8\mu^2 k \leq \log(N/\rho)$

- This condition guarantees recovery and is less strict than the coherence condition for general signals.
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Part 3: Applications in FWI, 3D acquisition and time-lapse imaging

Redundancy in seismic data

- Seismic data are highly redundant.
- Examples from seismic lines:
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  - the support of curvelet coefficients of offset slices
- How do we take advantage of this redundancy to reduce the acquisition cost?

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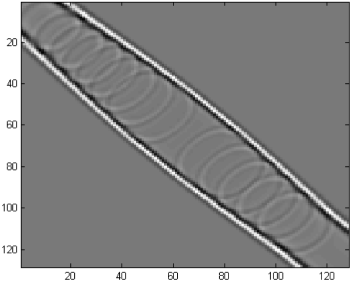

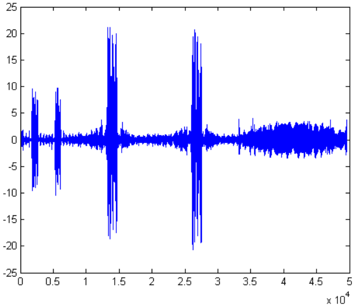

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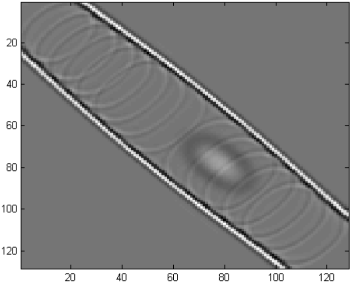

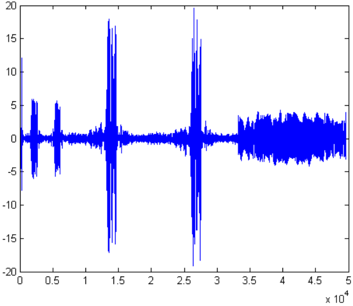

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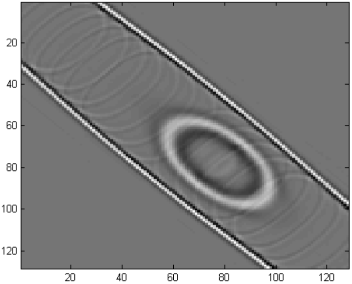

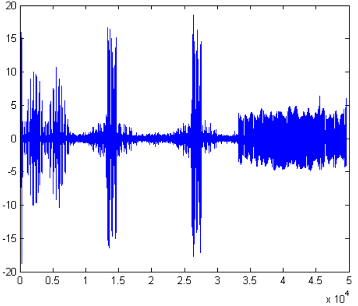

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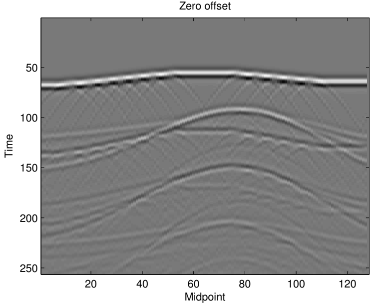

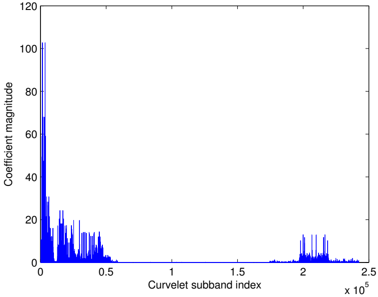

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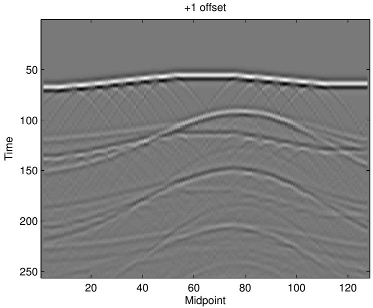

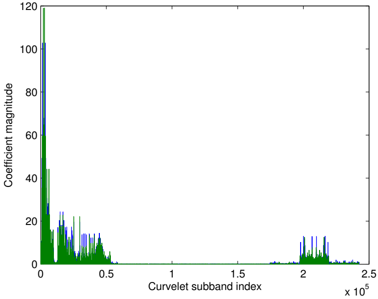

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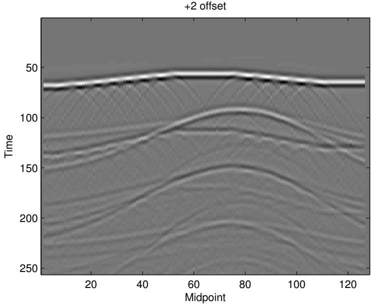

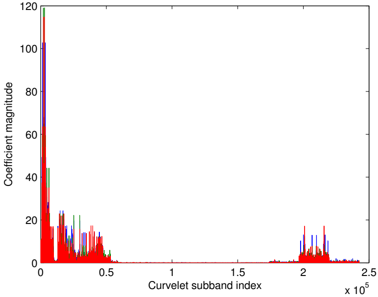

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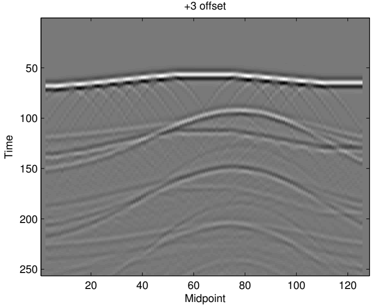

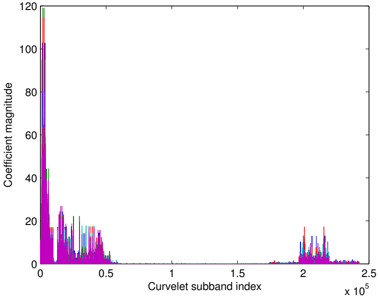

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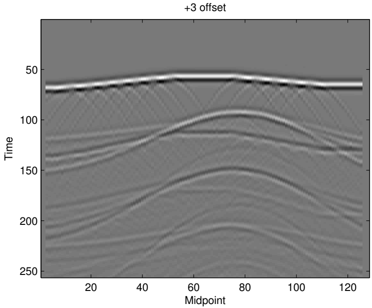

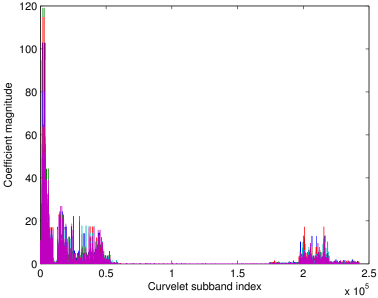

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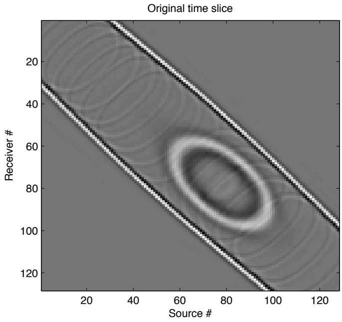

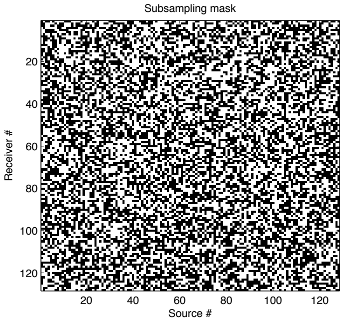

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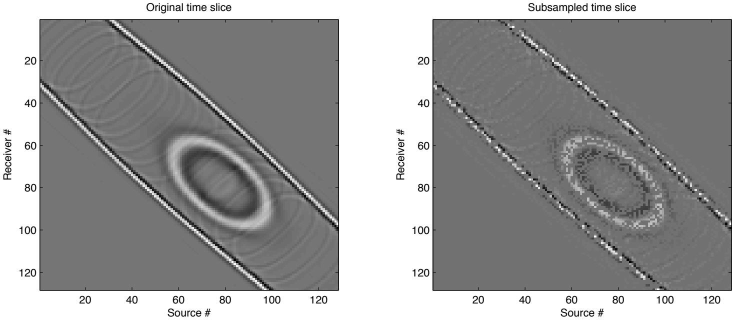

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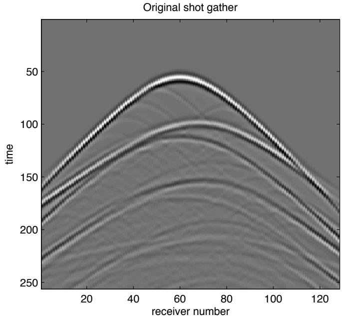

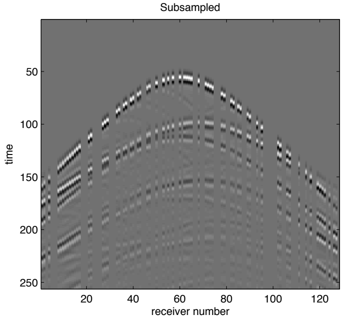

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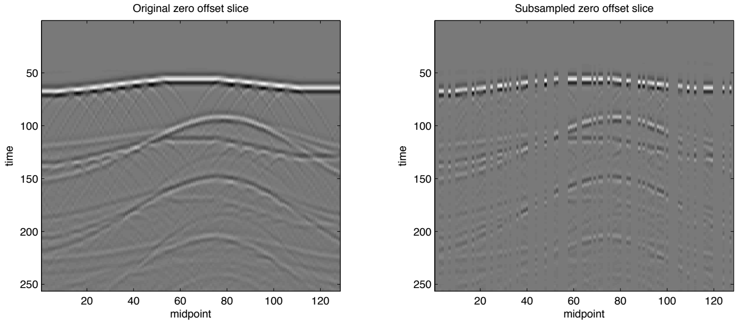

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- Lessons from the morning talks:
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  - Use ℓ_1 minimization to recover the signal.

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where $0 \le \omega \le 1$ and $\|x\|_{1,\mathrm{w}} := \sum_i \mathrm{w}_i |x_i|$.

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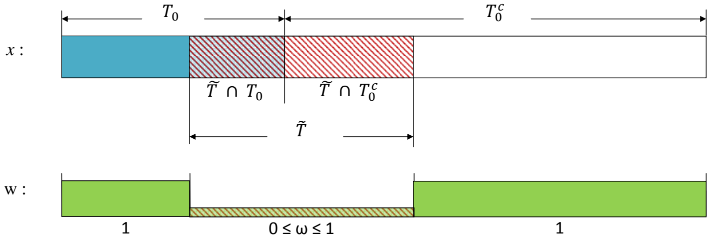

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- How do we find the set $\widetilde{T}$?
  - Solve the standard ℓ_1 problem for the zero-offset slice.
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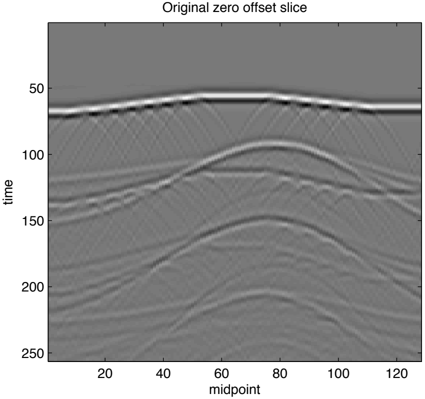
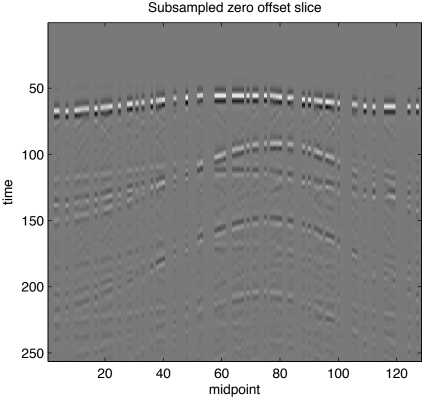
- Offset slices recovered using standard ℓ_1 vs weighted ℓ_1 minimization.
- Average of 2.13dB improvement in shot gather SNR.





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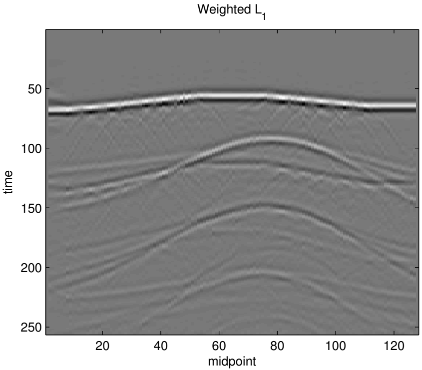

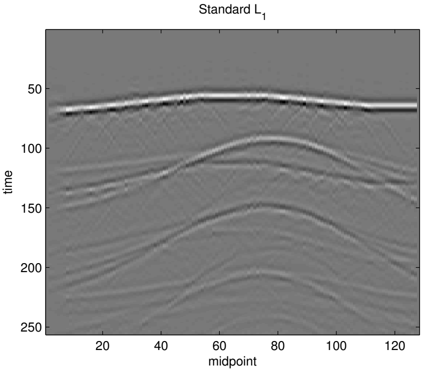

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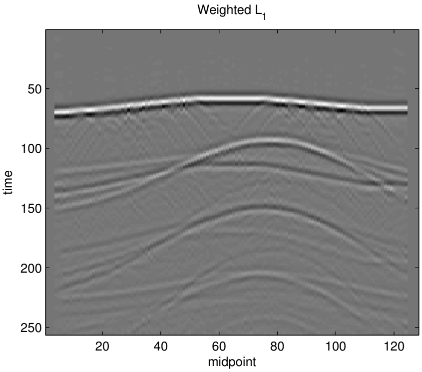

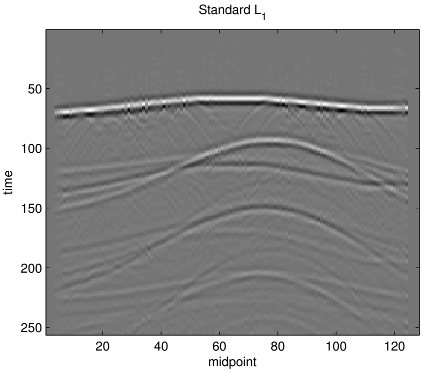

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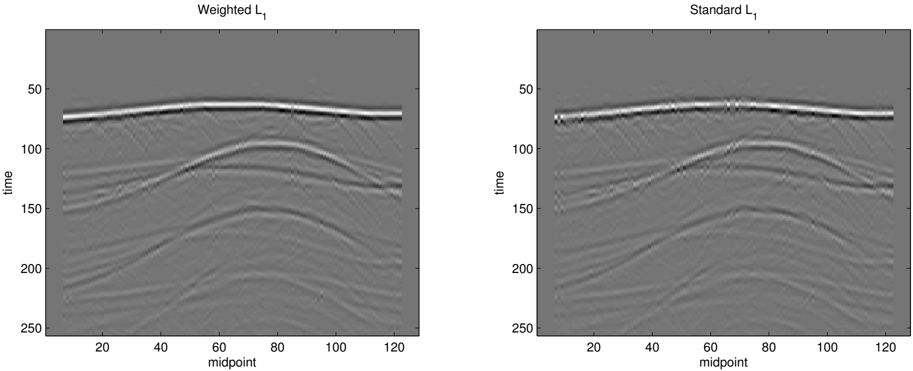

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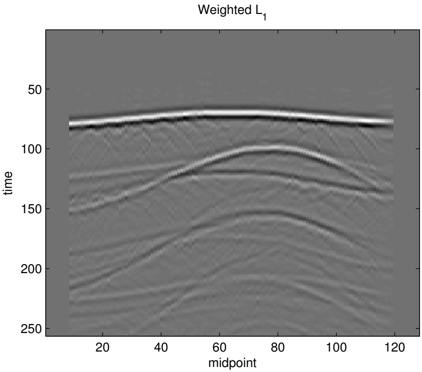

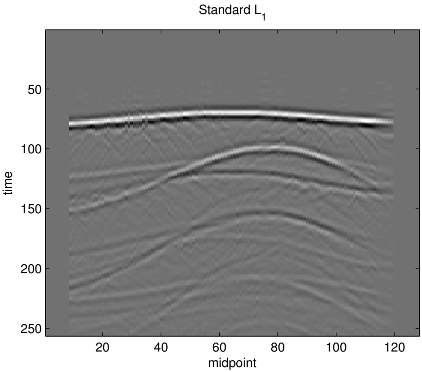

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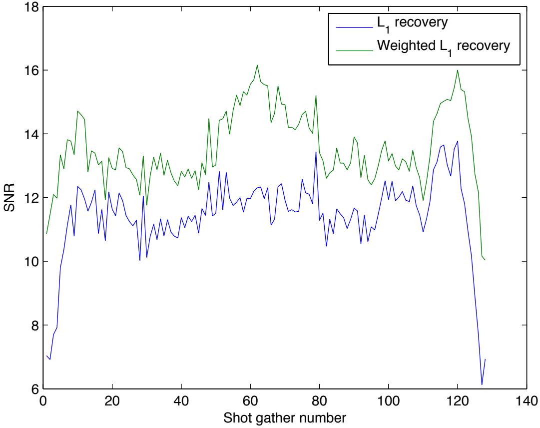

Part 1: Sparsity and randomized acquisition

Part 2: Redundancy in curvelet support information

Part 3: Applications in FWI, 3D acquisition and time-lapse imaging

Where else can we use weighted ℓ_1?

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- Other areas that can benefit from redundant support information:
  - Weighted recovery across adjacent azimuths.
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- Xiang Li will talk about the least-squares migration problem:

$$\delta\tilde{m} = \arg\min_{\delta m} \frac{1}{2}\|\delta d - \nabla\mathcal{F}[m_0, Q]\delta m\|_2^2$$

- Huge overdetermined system, solve over smaller batches iteratively with warm-starting.

δm: model update

δd: multi-source multi-frequency data residue

m_0: background velocity model

Q: sources

$\nabla\mathcal{F}[m_0, Q]$: linearized Born-scattering operator

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$$\delta\tilde{m} = S^*\delta\tilde{x}, \quad S \text{ is the curvelet transform}$$

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$$\delta\tilde{x}_{k+1} = \arg\min_{\delta x} \frac{1}{2}\|W_k\delta x\|_1 \quad \text{subject to} \quad \|\delta\tilde{d} - \nabla\mathcal{F}[m_0, \tilde{Q}]S^*\delta x\|_2^2 \leq \sigma$$

W_k : diagonal weighting matrix using the support of $\delta\tilde{x}_k$

3D marine acquisition

- Recover an initial simultaneous/randomized marine seismic line.
- Use the support of the initial seismic line to recover adjacent lines.

Time-lapse imaging

- Use the support of the past survey to emphasize/de-emphasize the past artifacts from the new survey.

Conclusion

- We saw that curvelets sparsify seismic data more effectively.
- The curvelet transform combined with not-so-random sampling operators maintain good compressed sensing qualities.
- The redundancy that exists in seismic data is preserved in subsets of the support of curvelet coefficients.
- We will investigate how to include redundant support information in other recovery algorithms.

Thank you

Questions?

We would like to thank the authors of CurveLab (curvelet.org), a toolbox implementing the Fast Discrete Curvelet Transform, Madagascar (rsf.sf.net), a package for reproducible computational experiments, SPG'1 (cs.ubc.ca/labs/scl/spgl1), SPOT (http: //www.cs.ubc.ca/labs/scl/spot/), a suite of linear operators and problems for testing algorithms for sparse signal reconstruction, and pSPOT, SLIMs parallel extension of SPOT. The Gulf of Suez dataset was generously provided by Eric Verschuur. This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, WesternGeco (Schlumberger)