Inside the Robust EPSI formulation

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EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data predicted data from primary IR

$$\mathbf{P} = \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})$$

$\mathbf{P}$	total up-going wavefield
$\mathbf{Q}$	down-going source signature
$\mathbf{R}$	reflectivity of free surface (assume -1)
$\mathbf{G}$	primary impulse response
	(all monochromatic data matrix, implicit ω)

EPSI Problem

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

recorded data predicted data from primary IR

$$\mathbf{P} = \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - \mathbf{G}(\mathbf{Q} + \mathbf{R}\mathbf{P})\|_2^2$$

EPSI Problem

In time domain (lower-case: whole dataset in time domain)

recorded data predicted data from primary IR

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathcal{M}(\mathbf{g}, \mathbf{q}) := \mathcal{F}_{\mathrm{t}}^{\dagger} \mathrm{BlockDiag}_{\omega_1 \cdots \omega_{nf}} [(q(\omega)\mathbf{I} - \mathbf{P})^{\dagger} \otimes \mathbf{I}] \mathcal{F}_{\mathrm{t}} \mathbf{g}$$

Inversion objective:

$$f(\mathbf{g}, \mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathcal{M}(\mathbf{g}, \mathbf{q})\|_2^2$$

EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

In fact it is bilinear:

$$\mathbf{M}_{\tilde{q}}\mathbf{g} = \mathcal{M}(\mathbf{g}, \tilde{\mathbf{q}}) \qquad \mathbf{M}_{\tilde{g}}\mathbf{q} = \mathcal{M}(\mathbf{q}, \tilde{\mathbf{g}})$$

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EPSI Problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

Associated objectives:

$$f_{\tilde{q}}(\mathbf{g}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2^2 \qquad f_{\tilde{g}}(\mathbf{q}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2^2$$

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EPSI Procedure

Do:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \nabla f_{q_k}(\mathbf{g}_k)$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Alternating updates (Gauss-Seidel) to the linearized problem

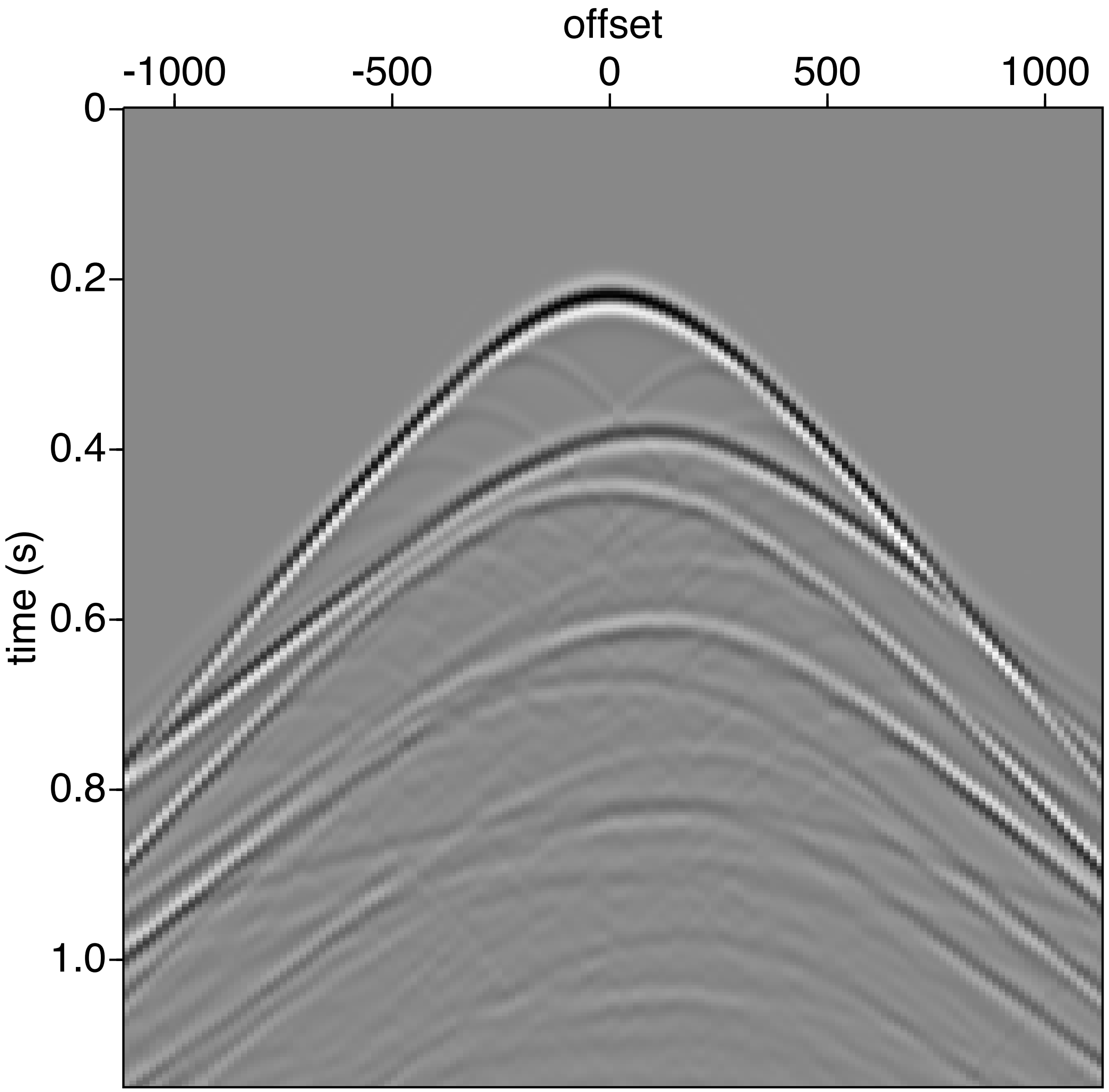
EPSI Procedure

Do:

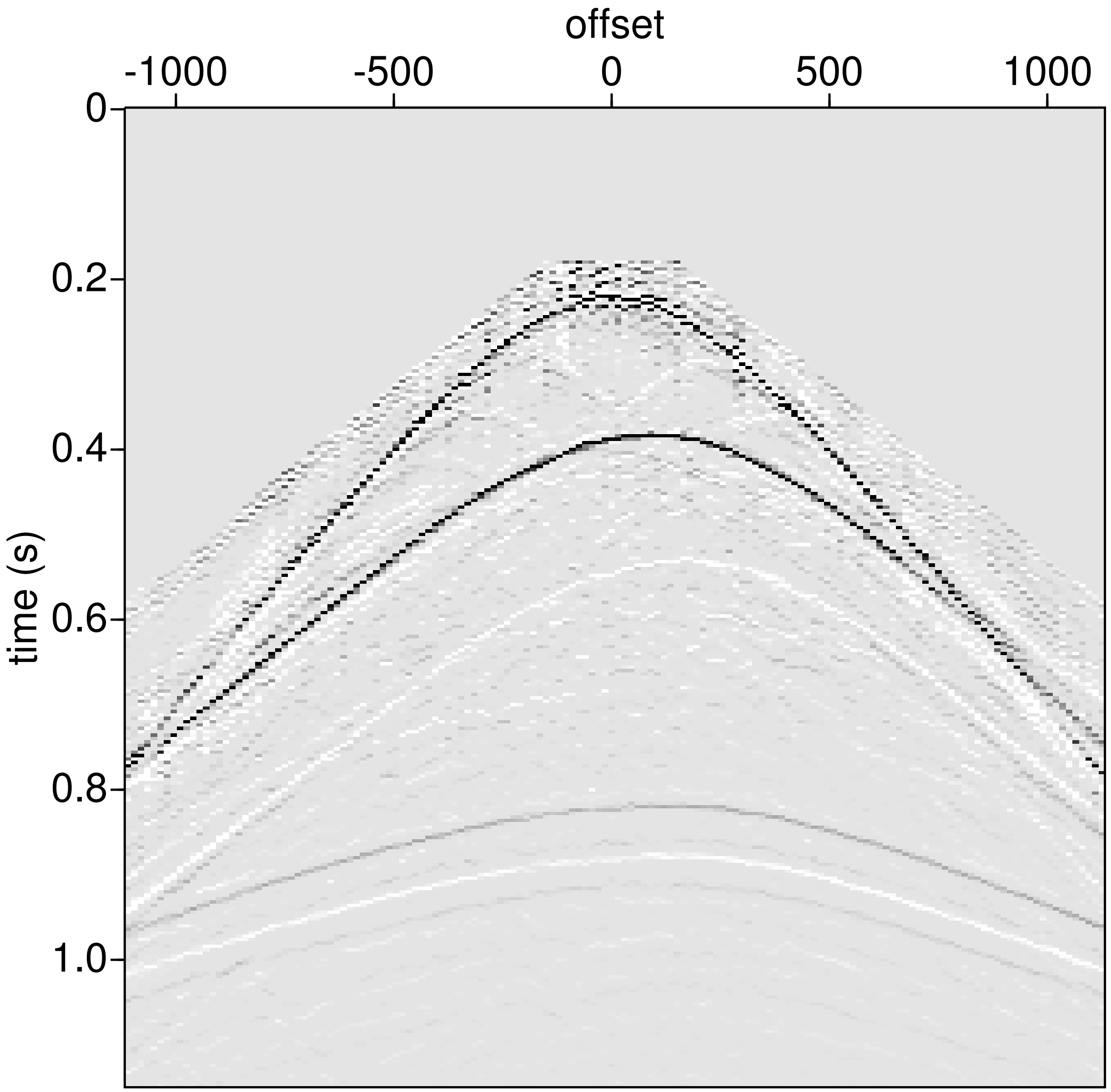
$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \mathcal{S}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Gradient sparsity

$\mathcal{S}$: pick largest ρ elements per trace


offset
-1000
-500
0
500
1000
0
0.2
0.4
0.6
0.8
1.0
time (s)

Data


offset
-1000
-500
0
500
1000
0
0.2
0.4
0.6
0.8
1.0
time (s)

EPSI IR

EPSI Procedure

Related to two underlying sub-problems:

$$\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \quad \text{s.t.} \quad \text{nnz}(\mathbf{g}) \leq \rho$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

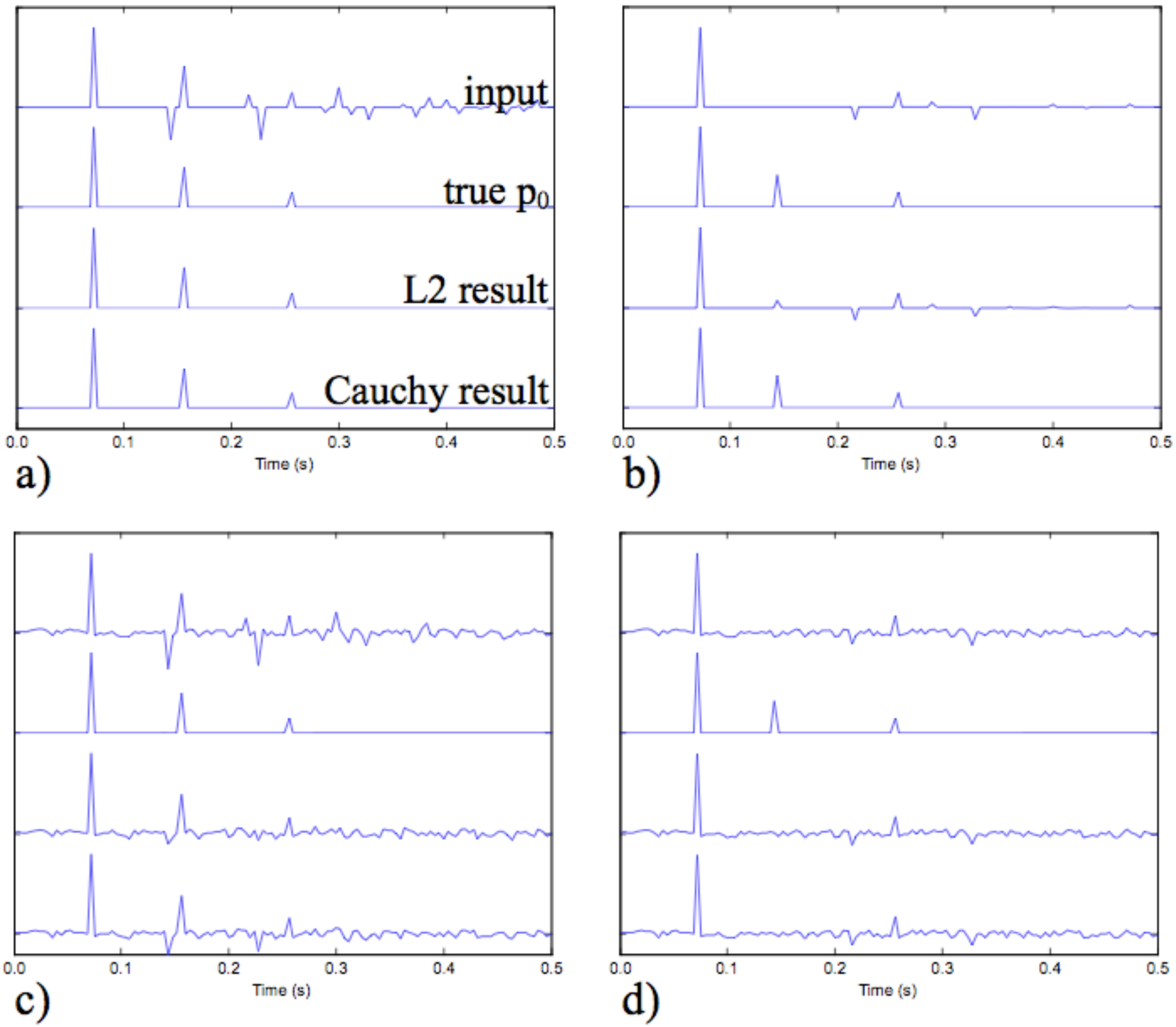
Attempting to approximate:

$$\min_{\mathbf{g}} \text{nnz}(\mathbf{g}) \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

(notion of *sparsest* solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Primary estimation from single trace of plane wave data



Cauchy “norm”:

$$\sum_t \log(1 + \mathbf{p_o}(t)^2/\beta^2)$$

(van Groenestijn and Verschuur 09)

EPSI Procedure

Related to two underlying sub-problems:

$$\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \quad \text{s.t.} \quad \text{nnz}(\mathbf{g}) \leq \rho$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Attempting to approximate:

$$\min_{\mathbf{g}} \text{nnz}(\mathbf{g}) \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

(notion of *sparsest* solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

EPSI Procedure

Can be made non-combinatorial (convex) by:

$$\min_{\mathbf{g}} \|\mathbf{g}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

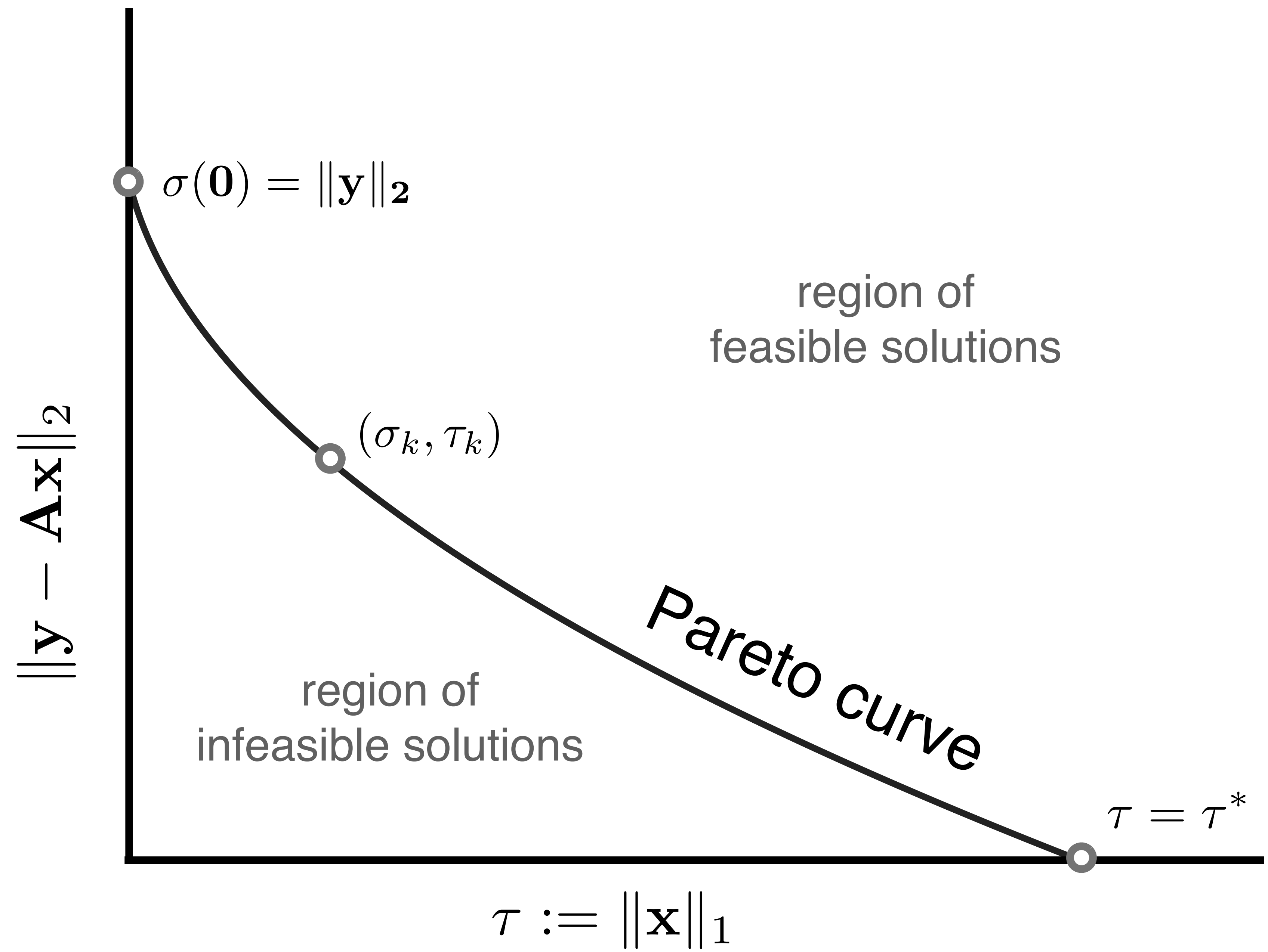
(minimum L1 solution usually the *sparsest* solution)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

Pareto curve

minimize $\|x\|_1$
subject to $\|Ax - b\|_2 \leq \sigma$

Look at the solution space and the line of optimal solutions (Pareto curve)

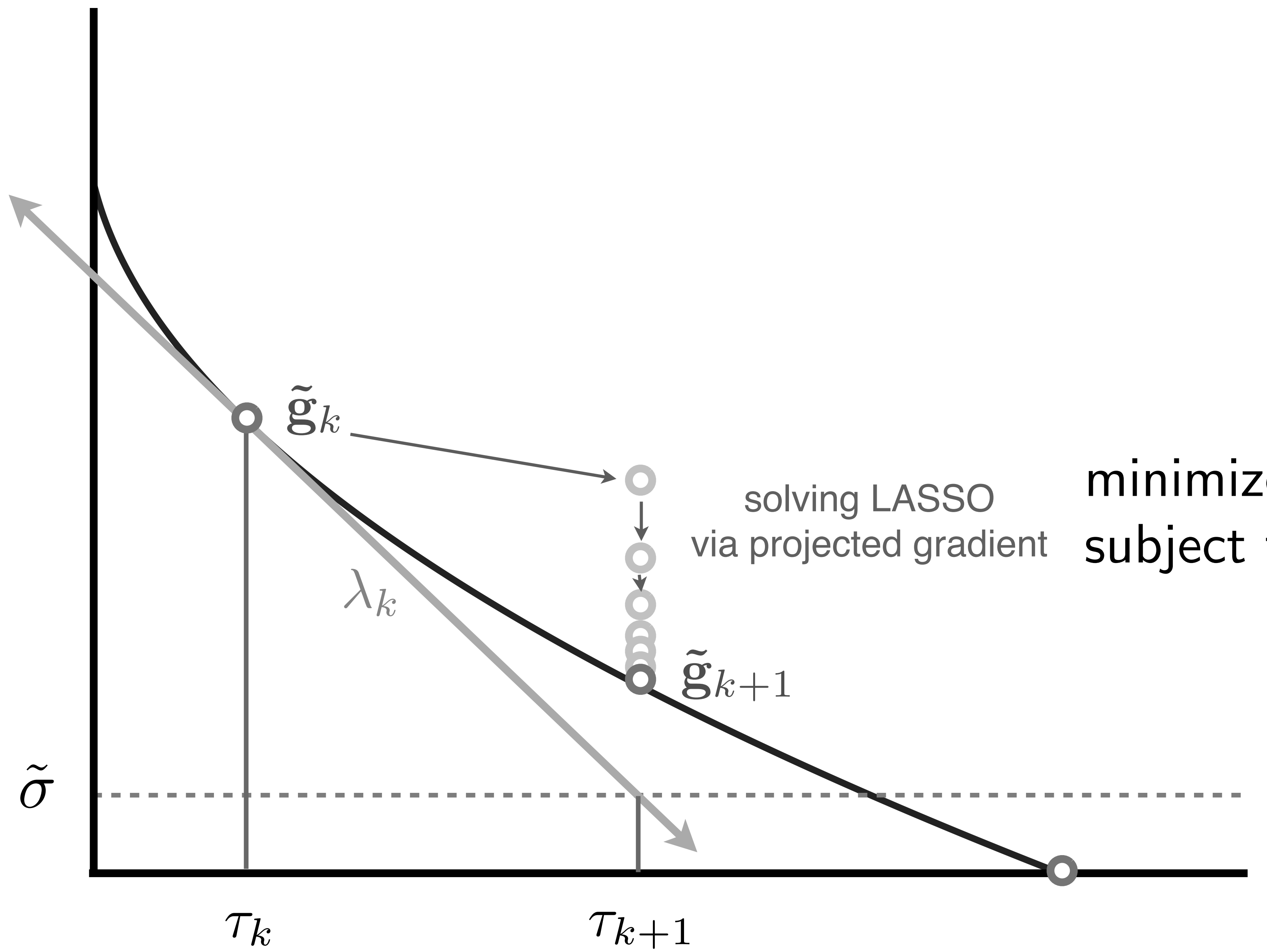


(van den Berg, Friedlander, 2008)

Pareto curve

minimize $\|x\|_1$
subject to $\|Ax - b\|_2 \leq \sigma$

Look at the solution space and the line of optimal solutions (Pareto curve)

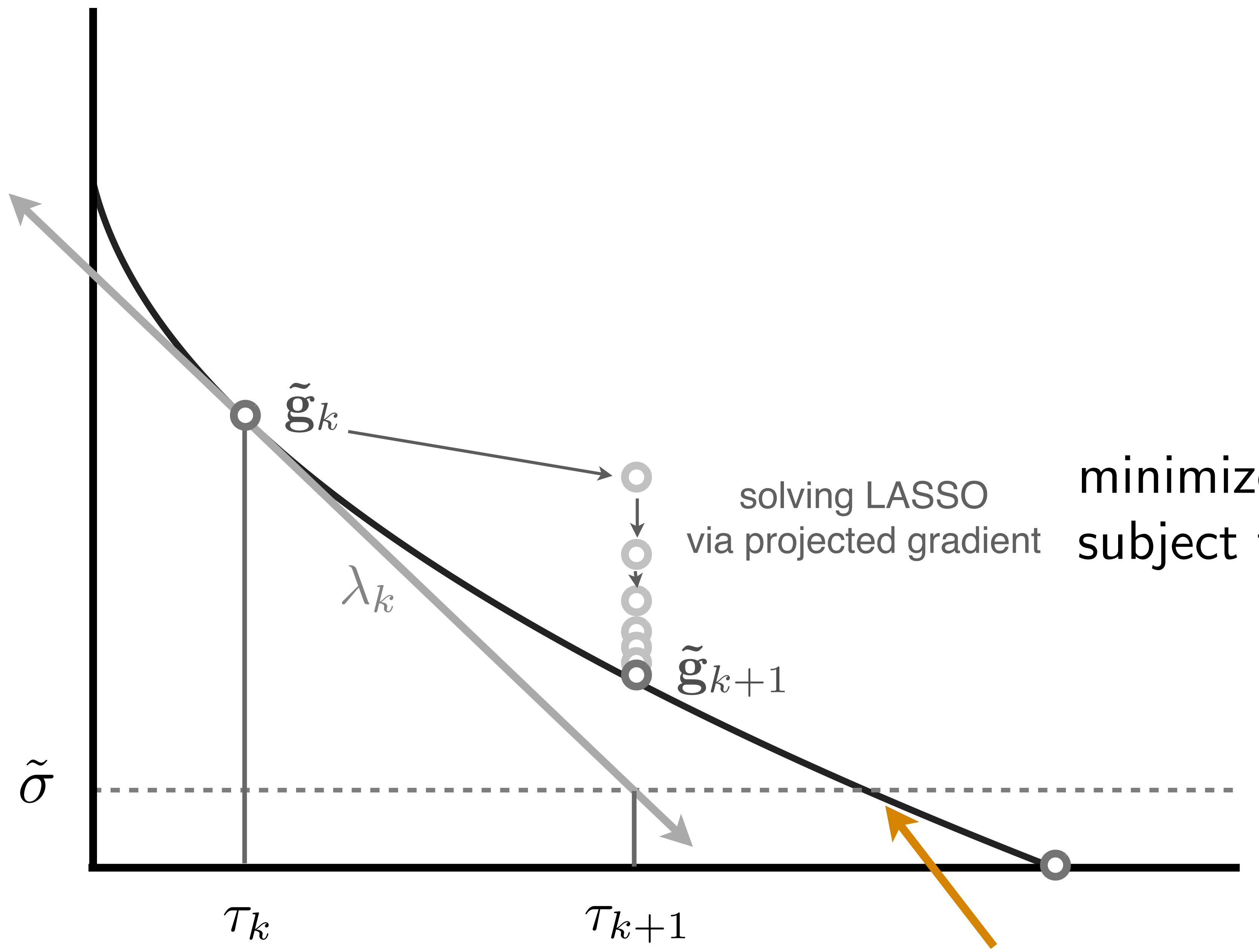


minimize $\|Ax - b\|_2$
subject to $\|x\|_1 \leq \tau$

Pareto curve

minimize $\|x\|_1$
subject to $\|Ax - b\|_2 \leq \sigma$

Look at the solution space and the line of optimal solutions (Pareto curve)



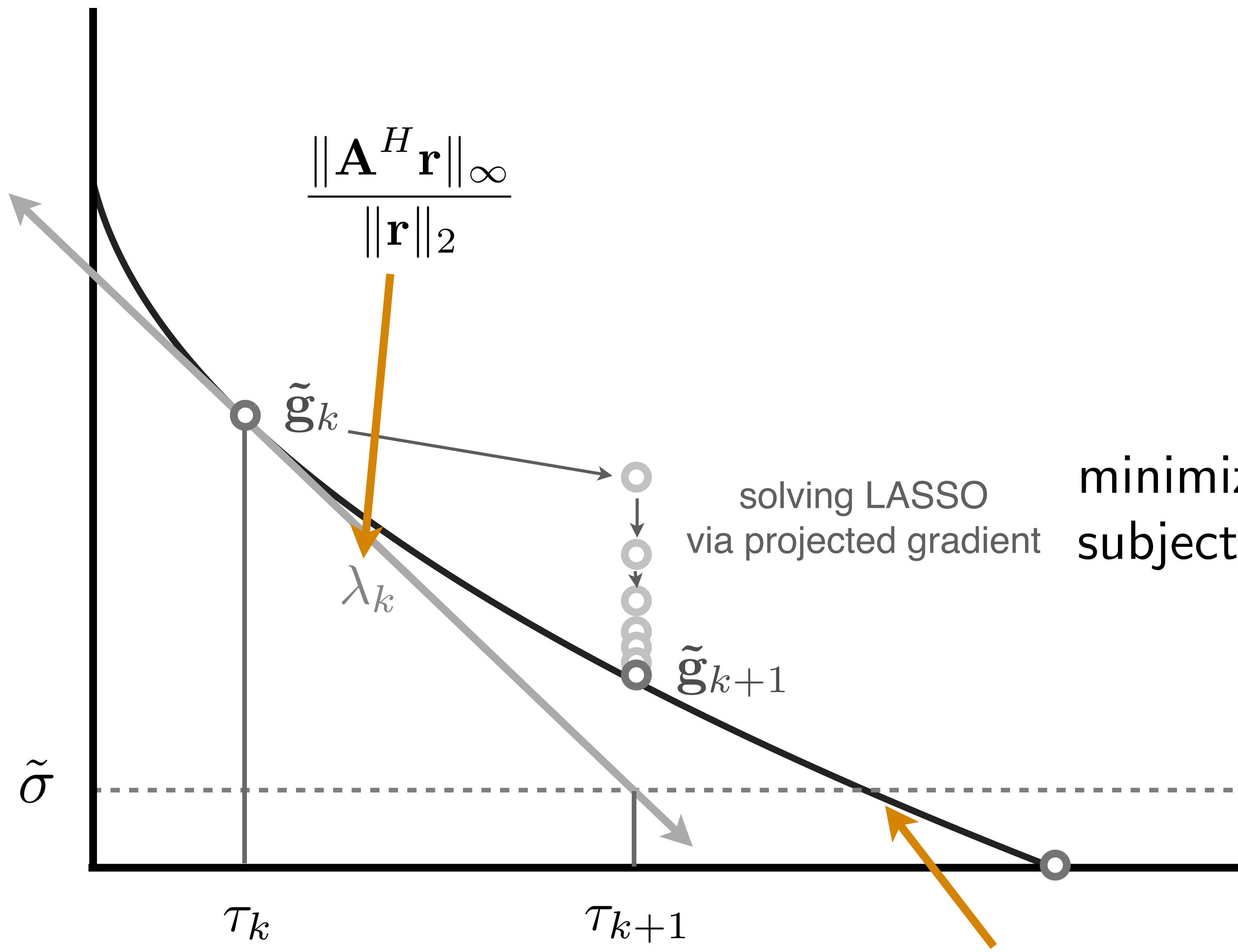
minimize $\|Ax - b\|_2$
subject to $\|x\|_1 \leq \tau$

feasible solution with smallest $\|x\|_1$

Pareto curve

minimize $\|x\|_1$
subject to $\|Ax - b\|_2 \leq \sigma$

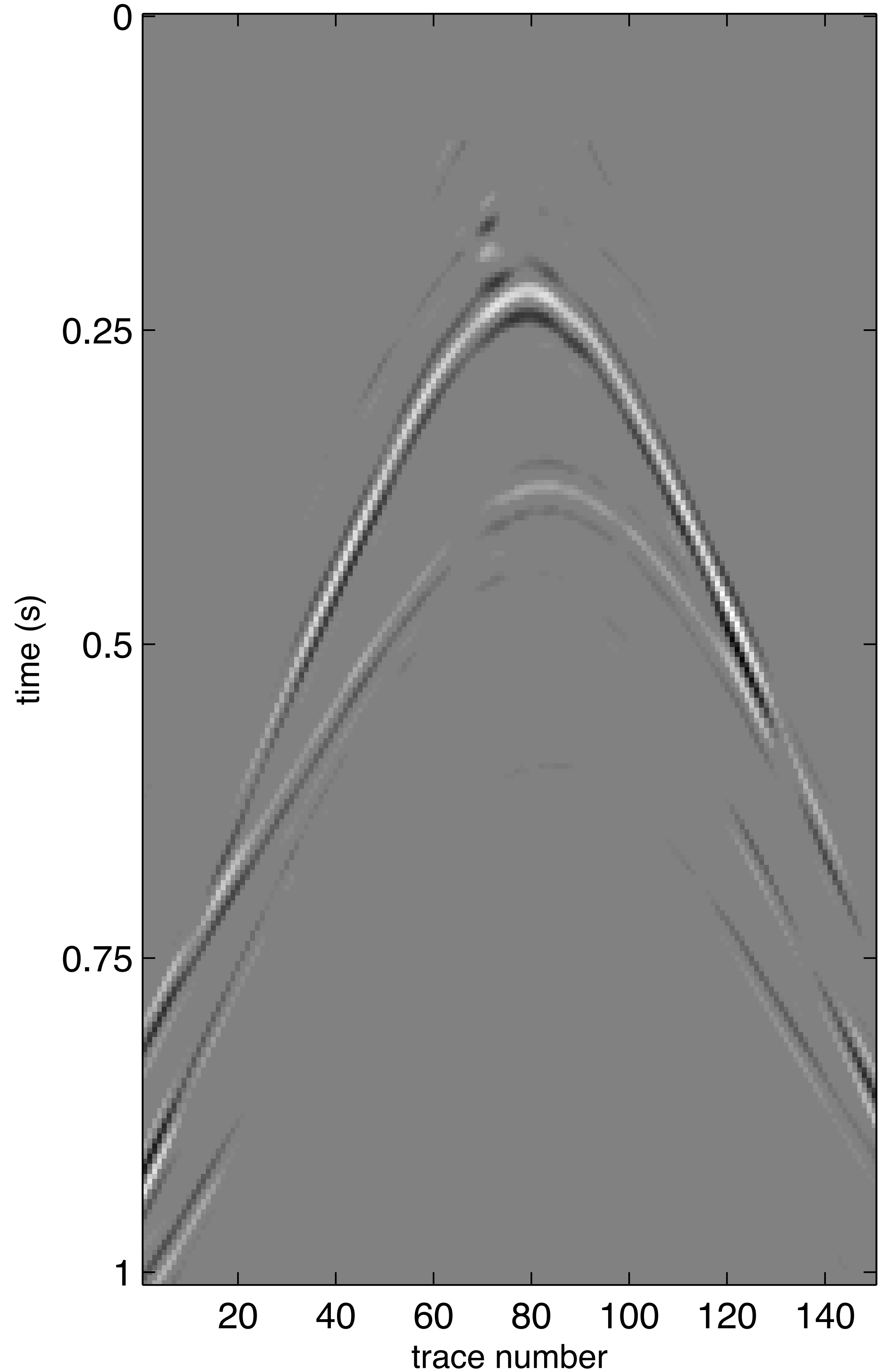
Look at the solution space and the line of optimal solutions (Pareto curve)



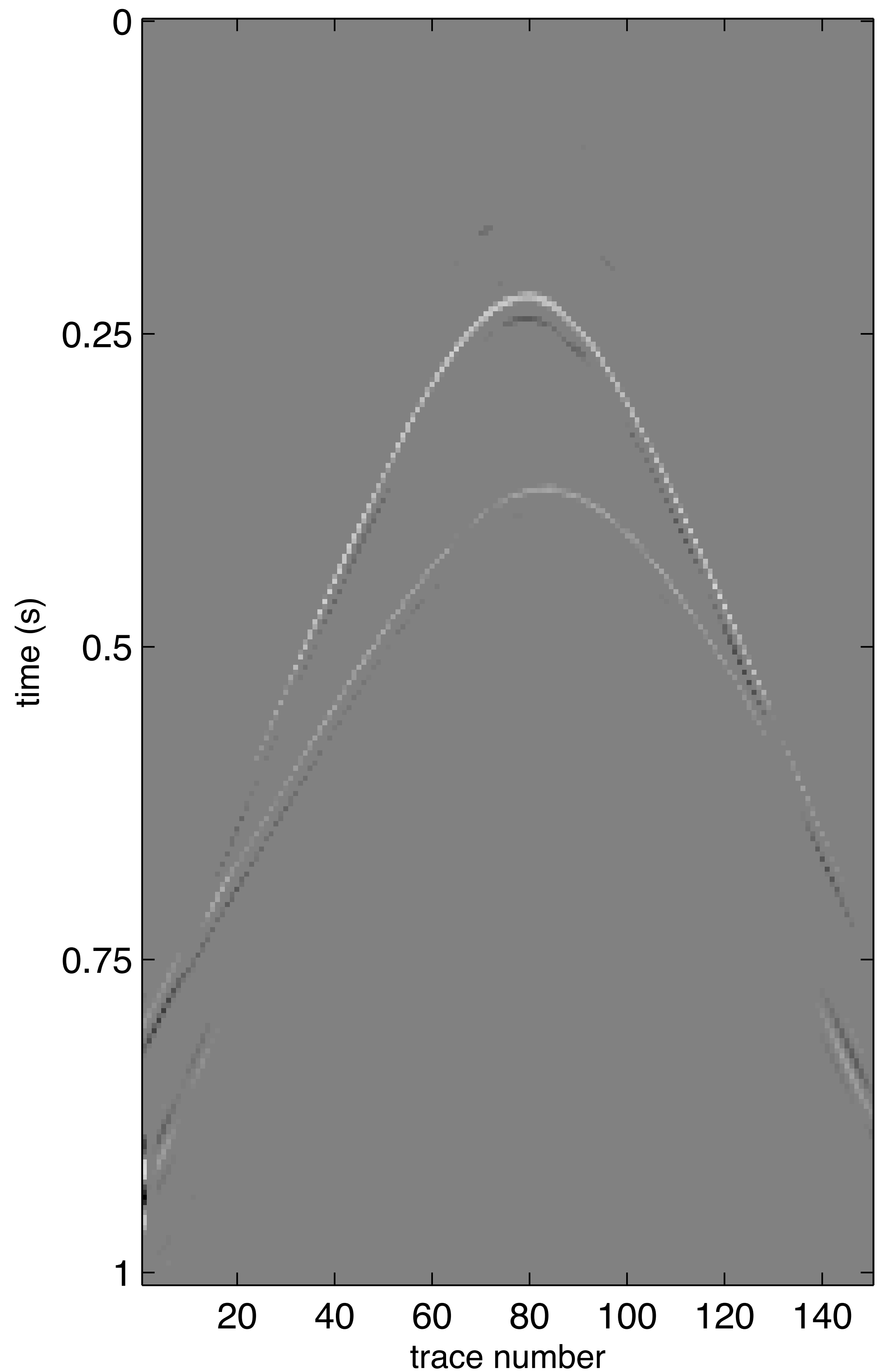
minimize $\|Ax - b\|_2$
subject to $\|x\|_1 \leq \tau$

feasible solution with smallest $\|x\|_1$

SPG start



SPG at Pareto curve



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Robust EPSI procedure

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

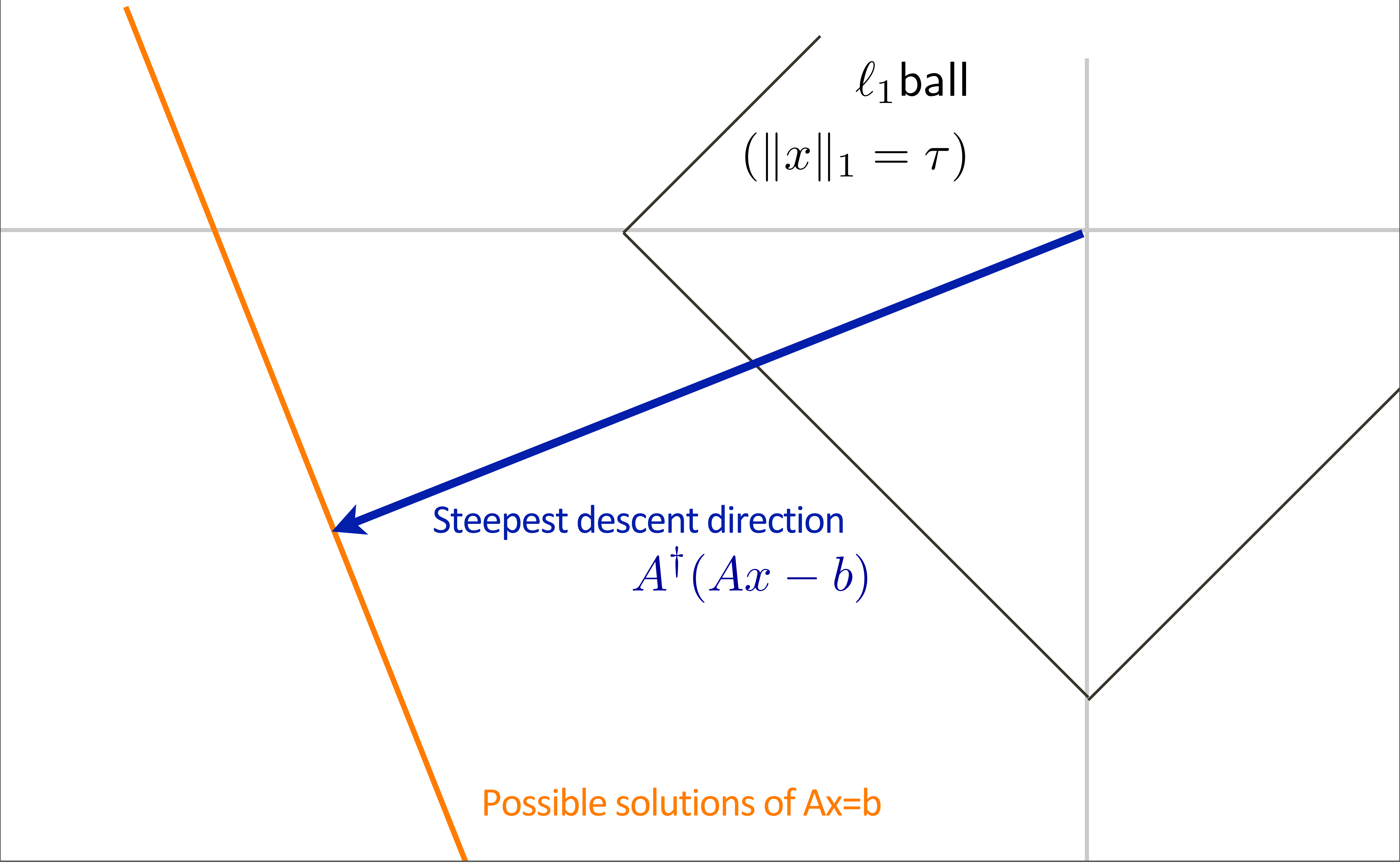
(Solve with SPG part of SPGL! until Pareto curve reached)

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)

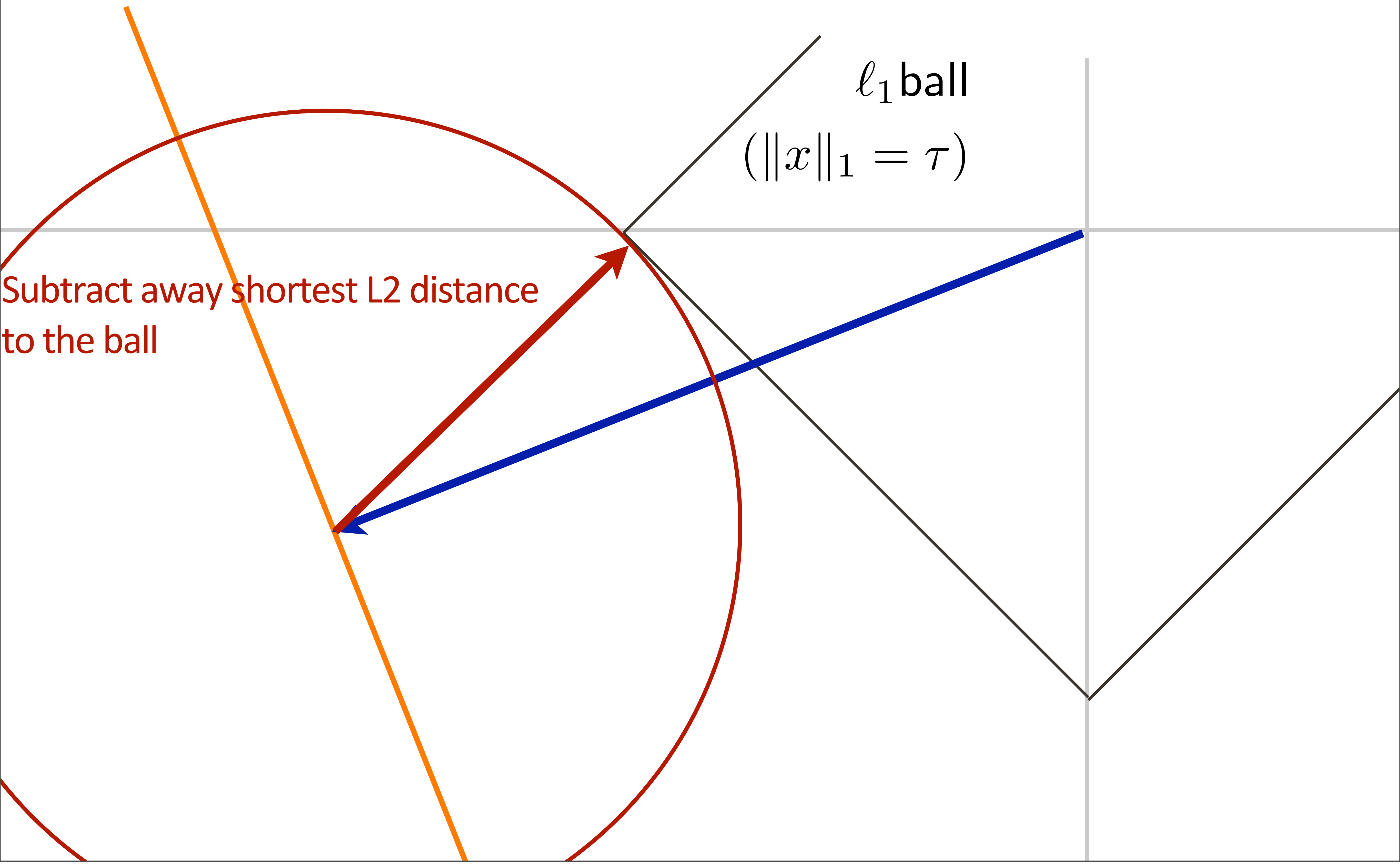
Solving with SPG

$$\begin{aligned} \text{minimize} \quad & \|Ax - b\|_2 \\ \text{subject to} \quad & \|x\|_1 \leq \tau \end{aligned}$$



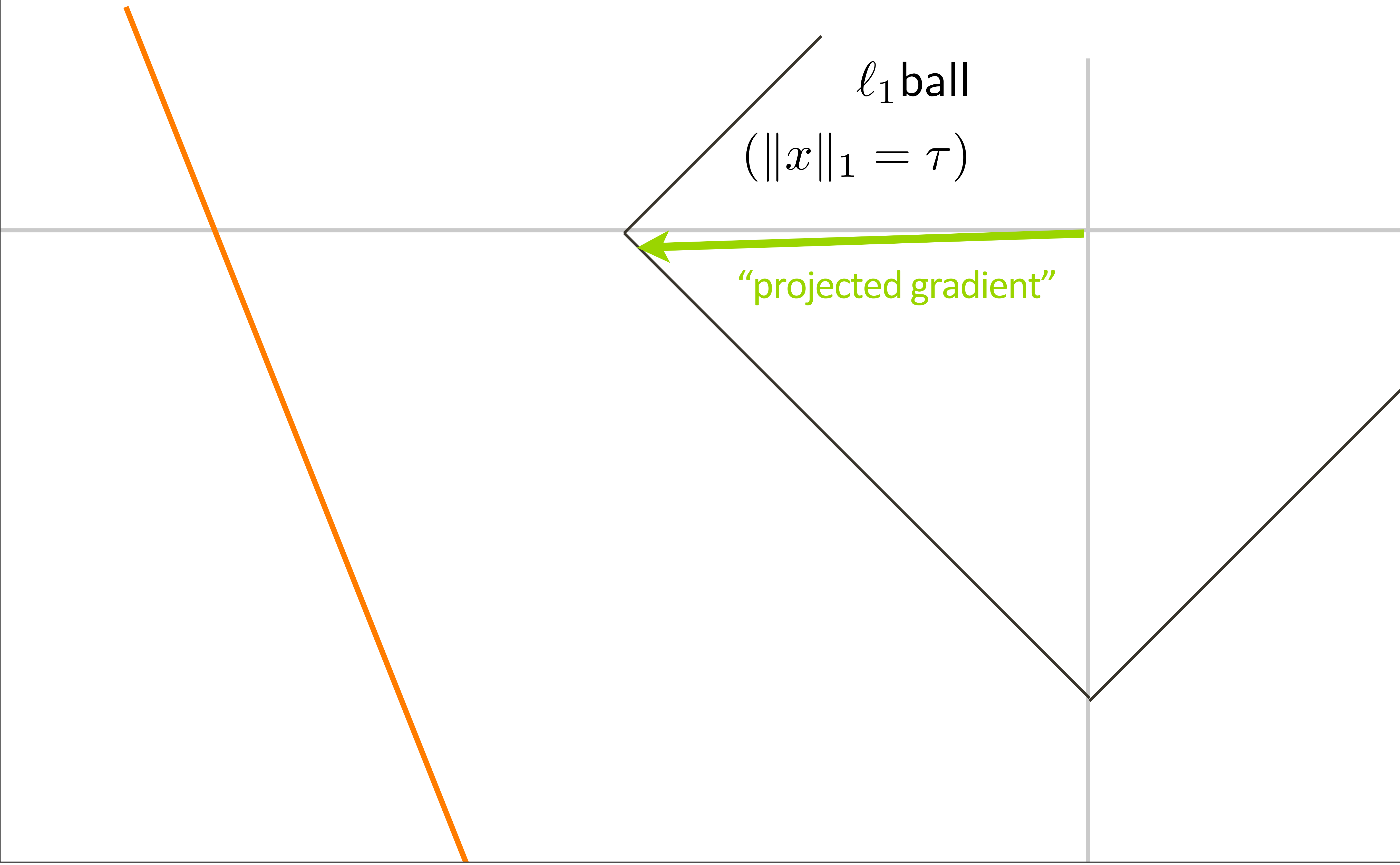
Solving with SPG

$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \leq \tau \end{array}$$



Solving with SPG

$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \le \tau \end{array}$$



Solving with SPG

$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \leq \tau \end{array}$$

Projecting onto 1-ball

1) Find $\Delta\tau := \|\mathbf{x}\|_1 - \tau$

2) Subtract each element of $\mathbf{x}$ by $\dfrac{\Delta\tau}{N}$

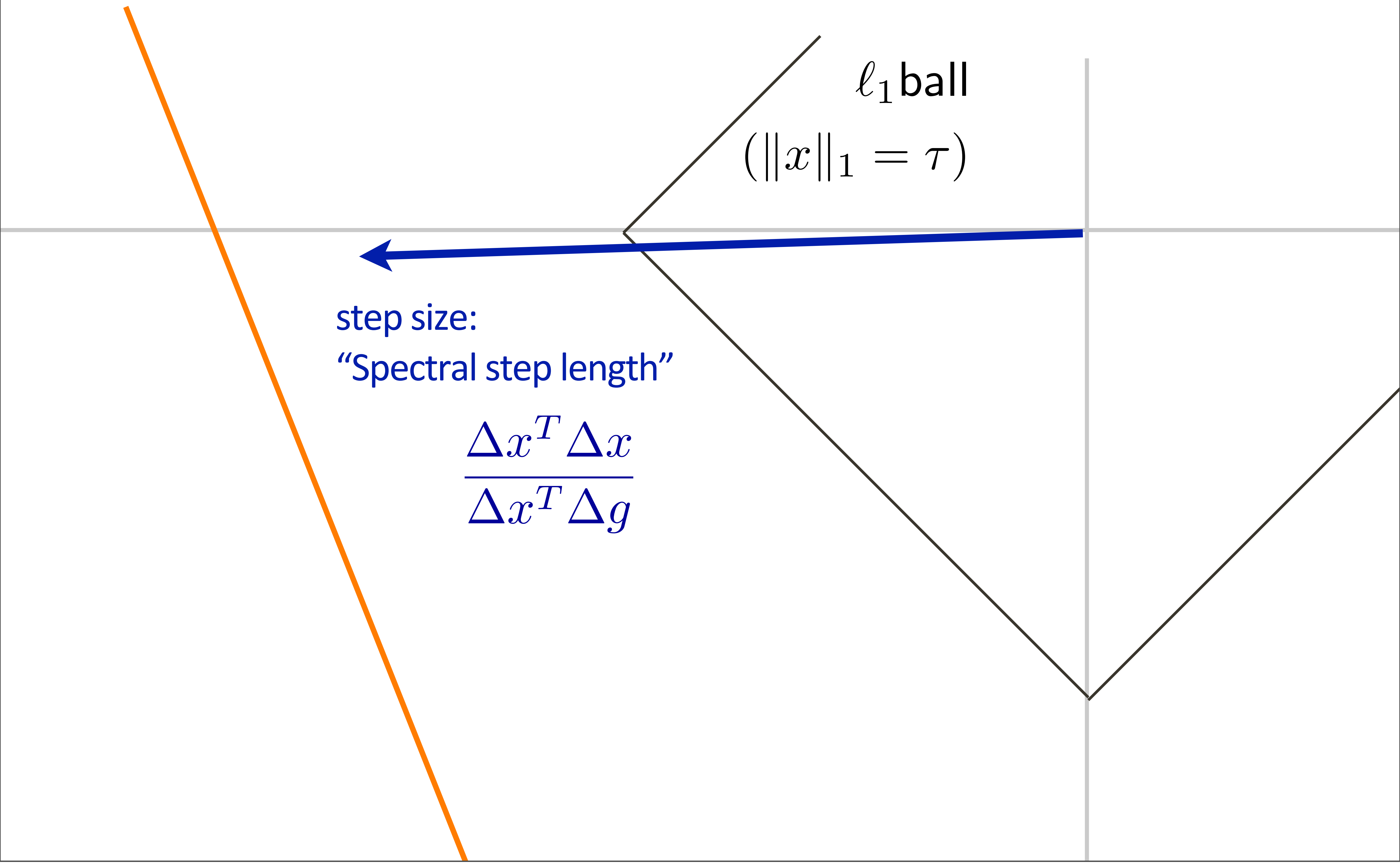
3) If any element < 0, set them to 0

3) Repeat until $\|\mathbf{x}\|_1 \leq \tau$

*** Easily parallelizeable**

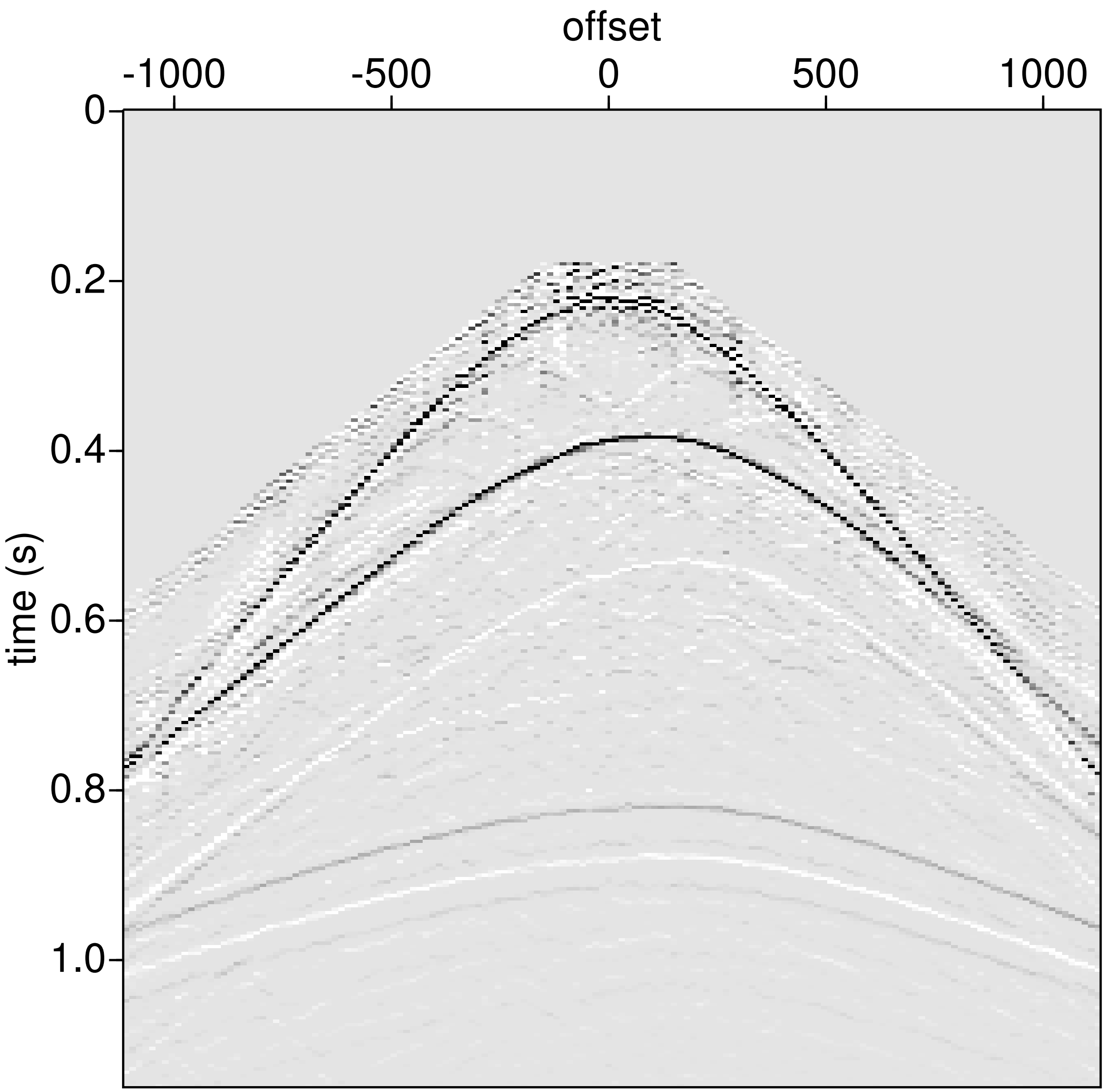
Solving with SPG

$$\begin{array}{ll} \text{minimize} & \|Ax - b\|_2 \\ \text{subject to} & \|x\|_1 \le \tau \end{array}$$

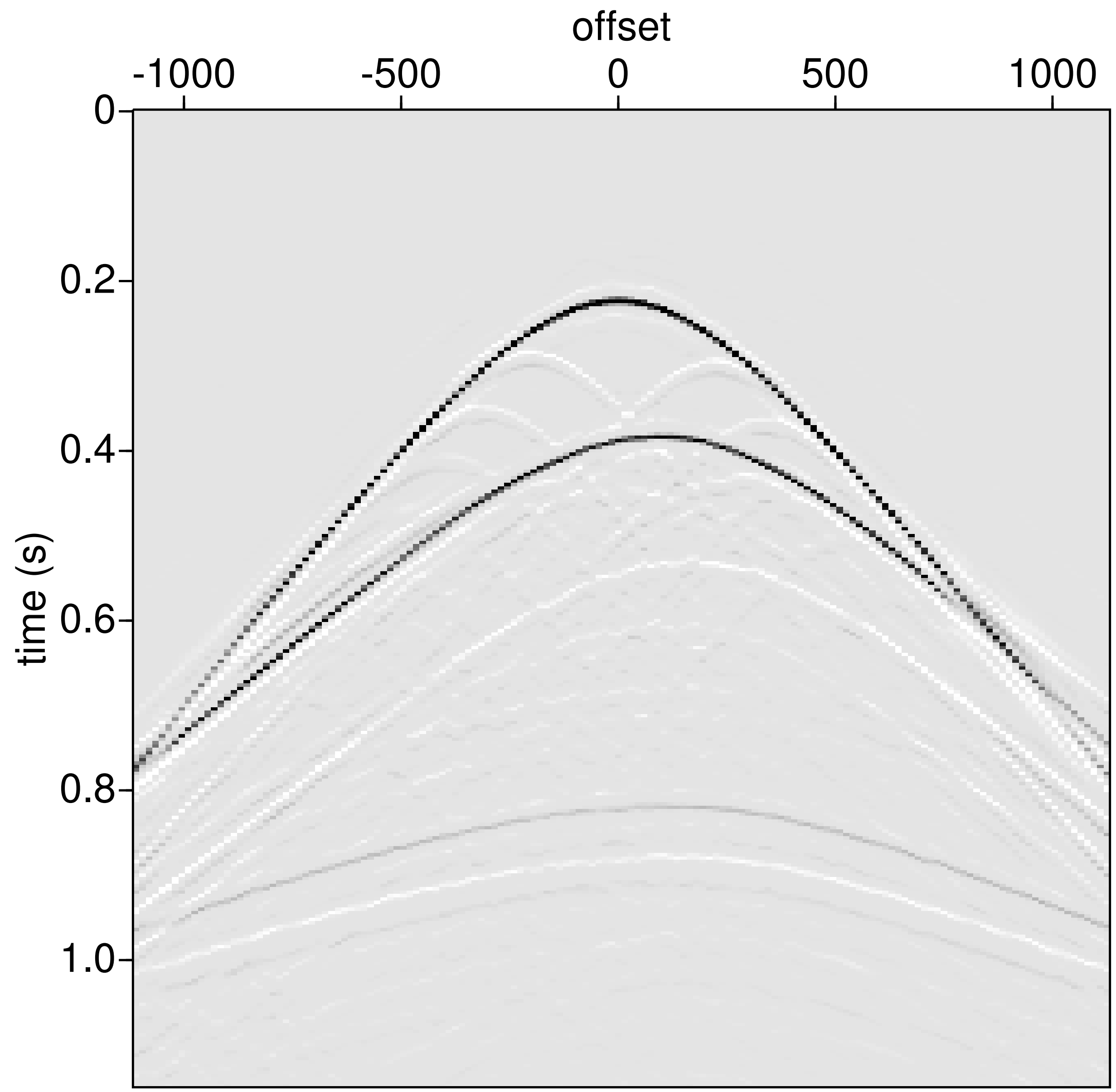


step size:
"Spectral step length"

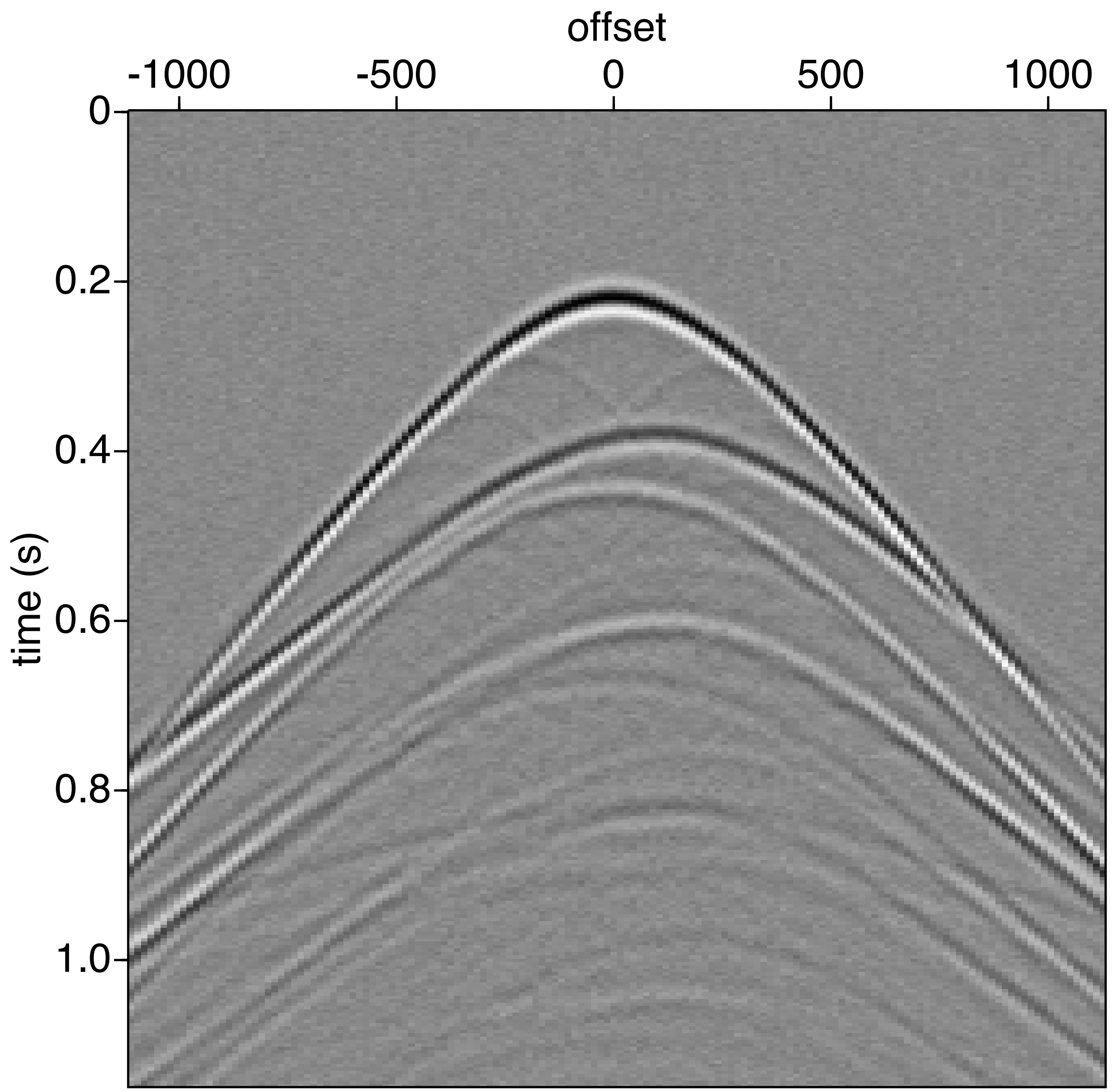
$$\frac{\Delta x^T \Delta x}{\Delta x^T \Delta g}$$



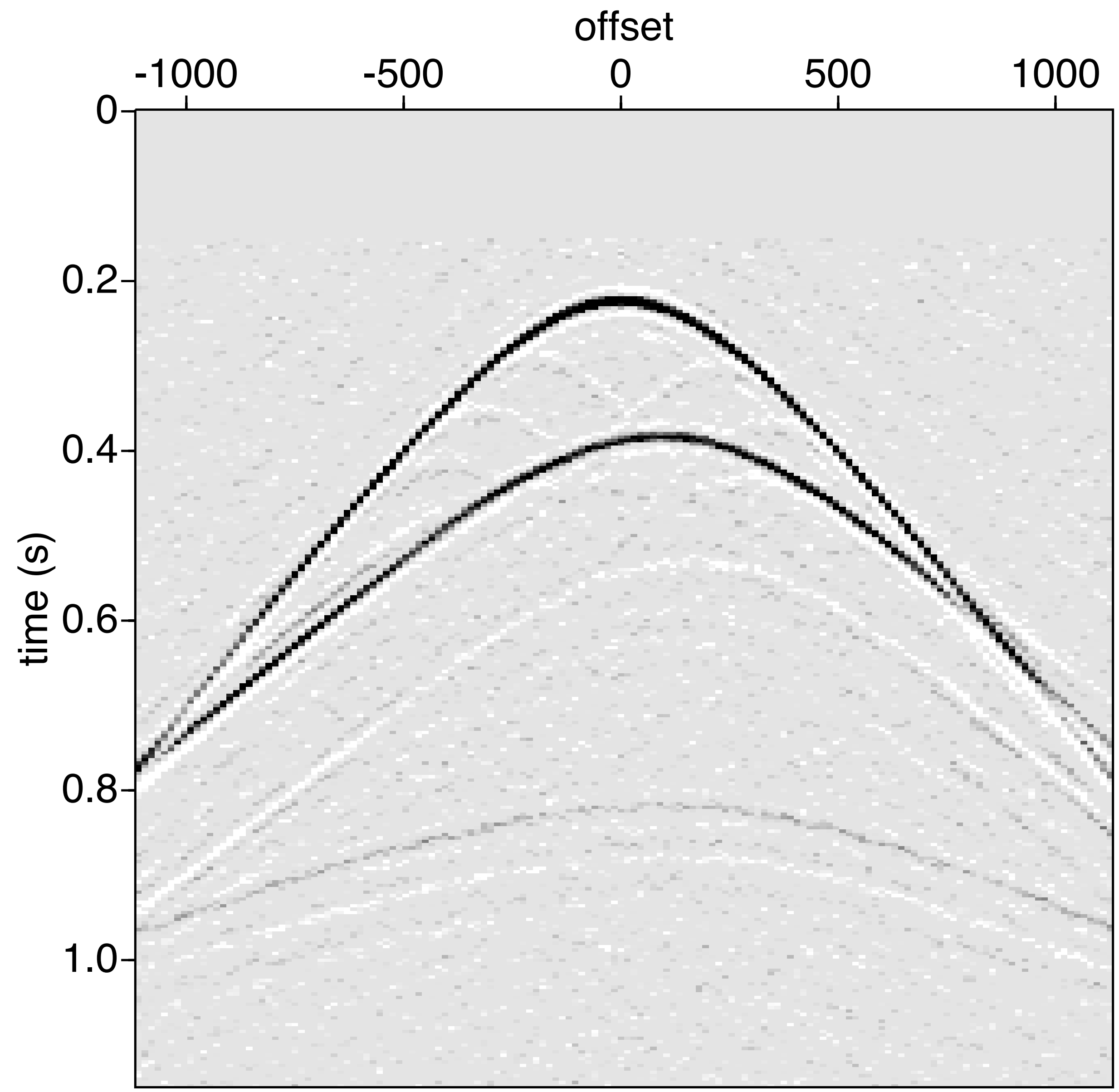
EPSI IR



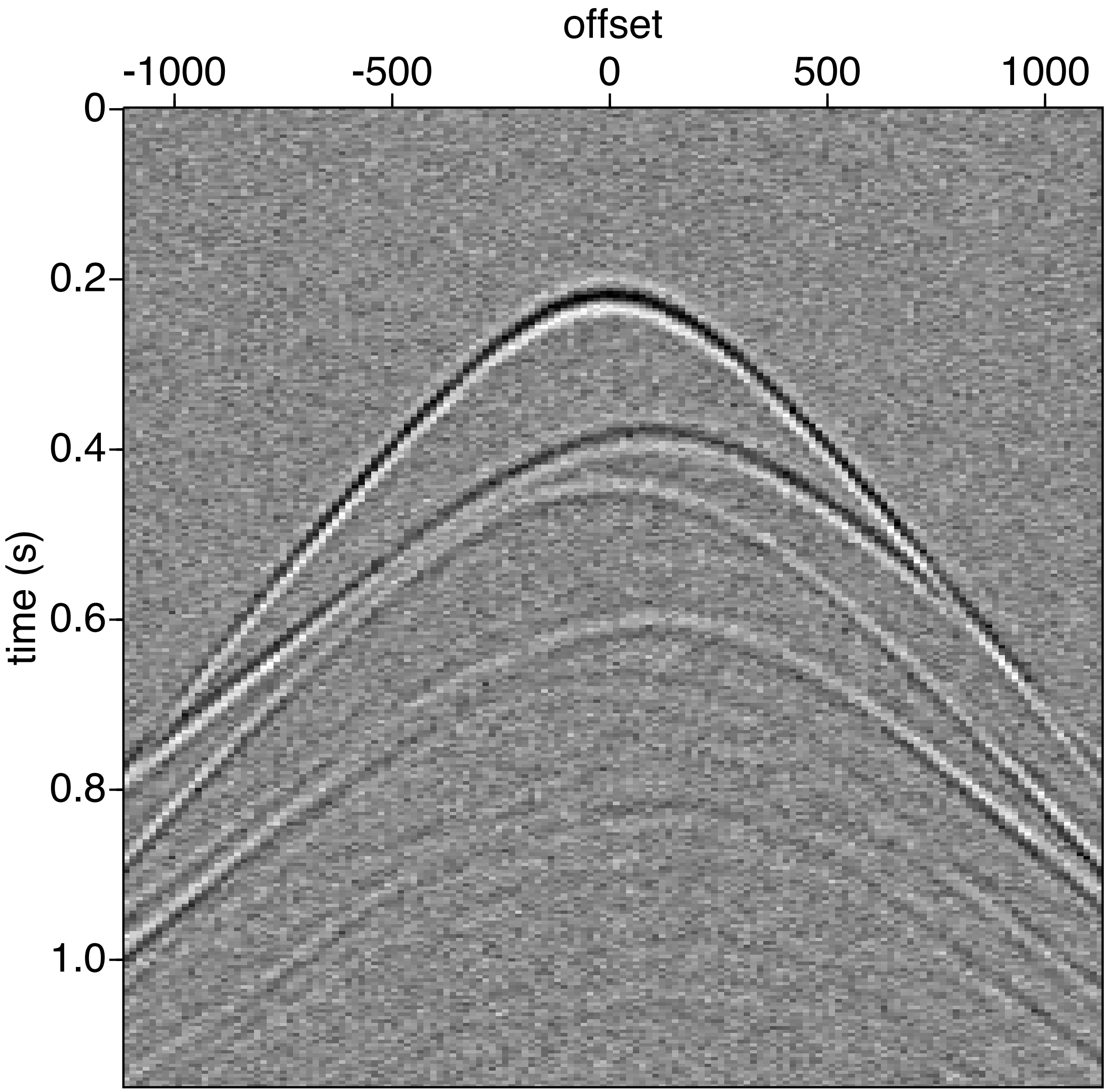
Robust EPSI IR



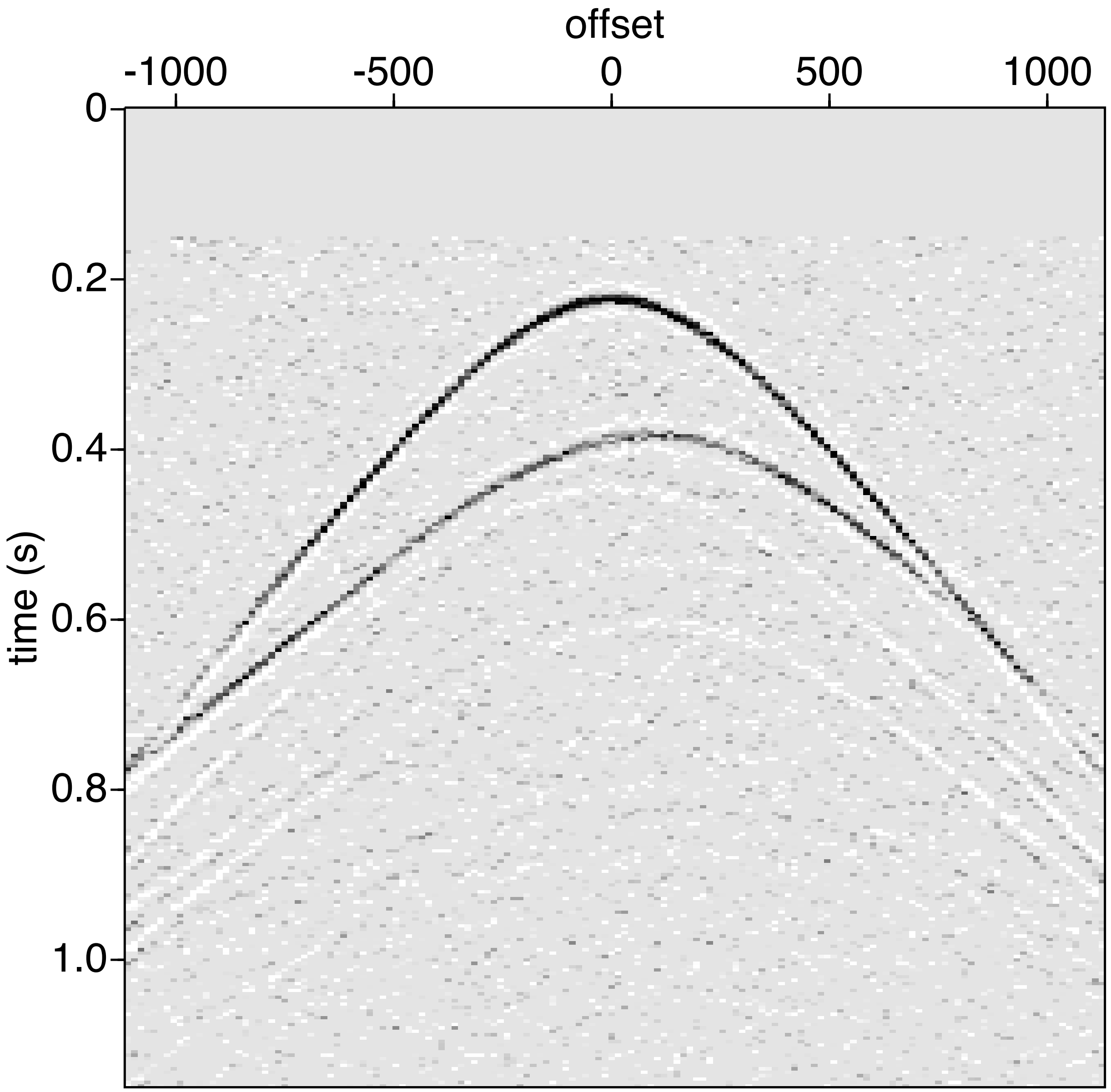
Data + 40% noise (SNR 7)



Robust EPSI IR



Data + 100% noise (SNR 0)



Robust EPSI IR

Convolution model

Convolution Model

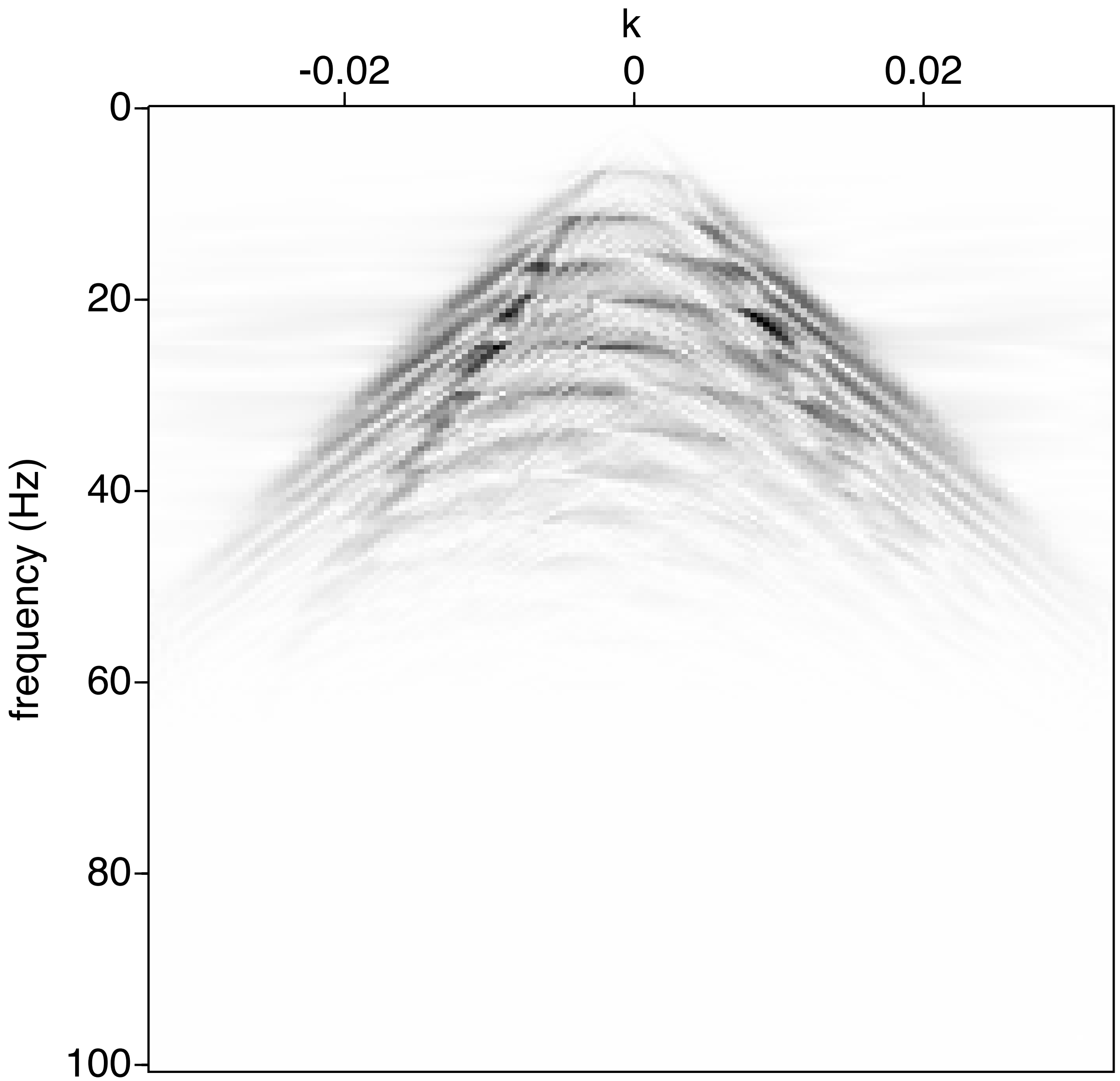
$$\text{Up-going Primary} = \mathbf{GQ}$$

EPSI Model

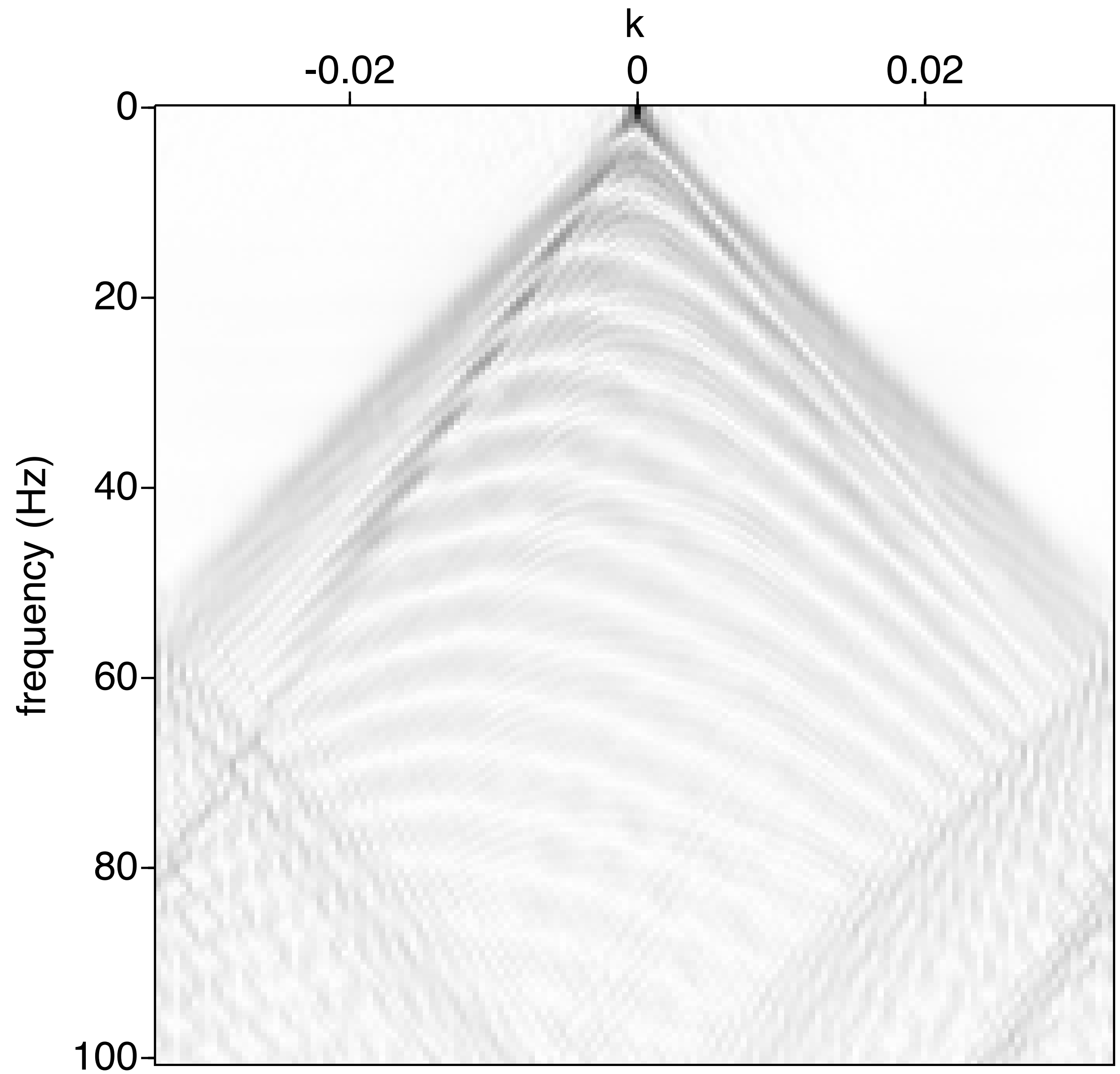
$$\text{Up-going Primary} + \text{Multiples} = \mathbf{GQ} + \mathbf{GRP}$$

additional info on G

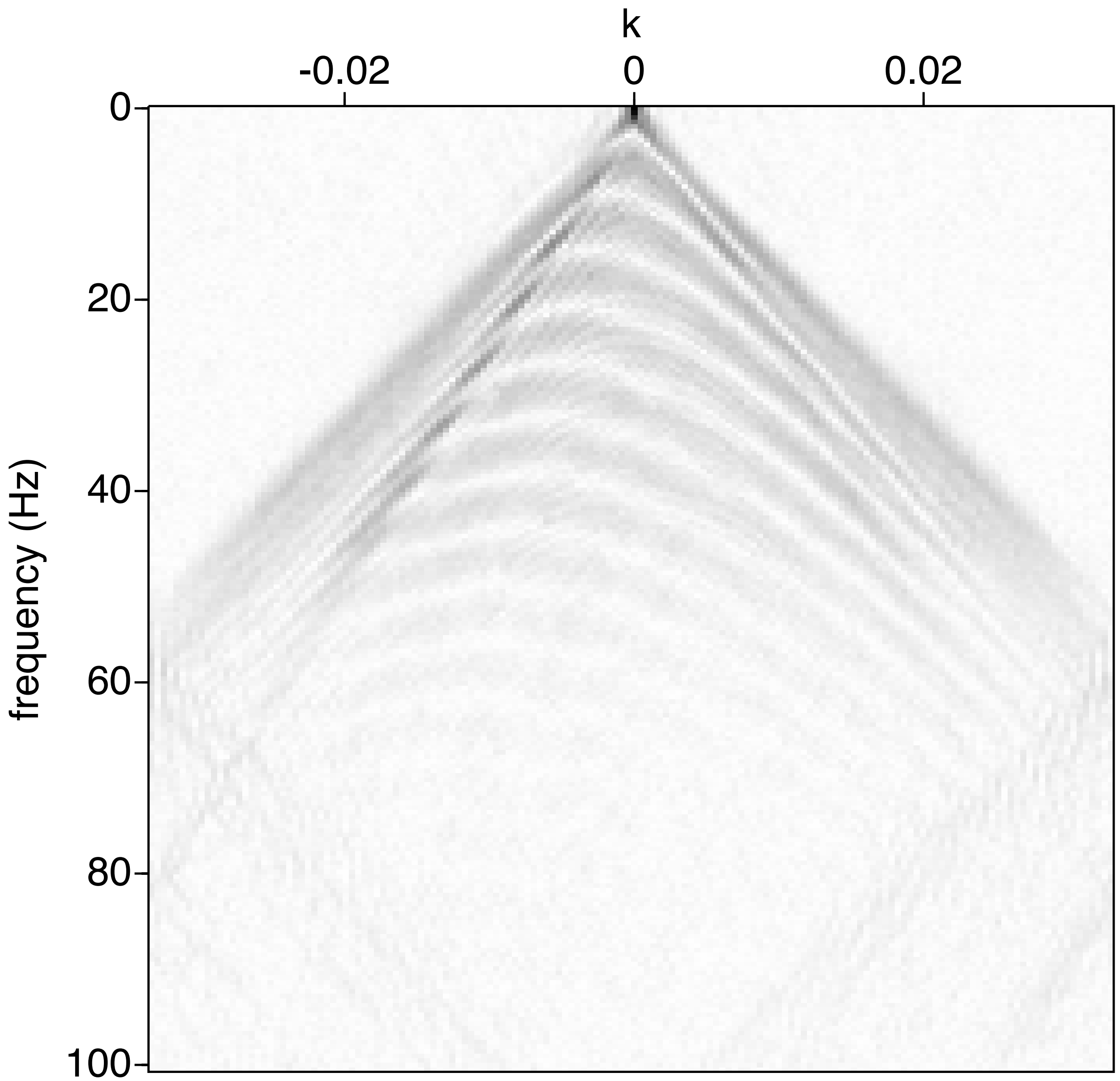
$\mathbf{P}$	total recorded up-going wavefield
$\mathbf{Q}$	source signature (incl. src ghosts)
$\mathbf{R}$	reflectivity of free surface (assume -1)
$\mathbf{G}$	primary impulse response
	(all monochromatic data matrix, implicit ω)



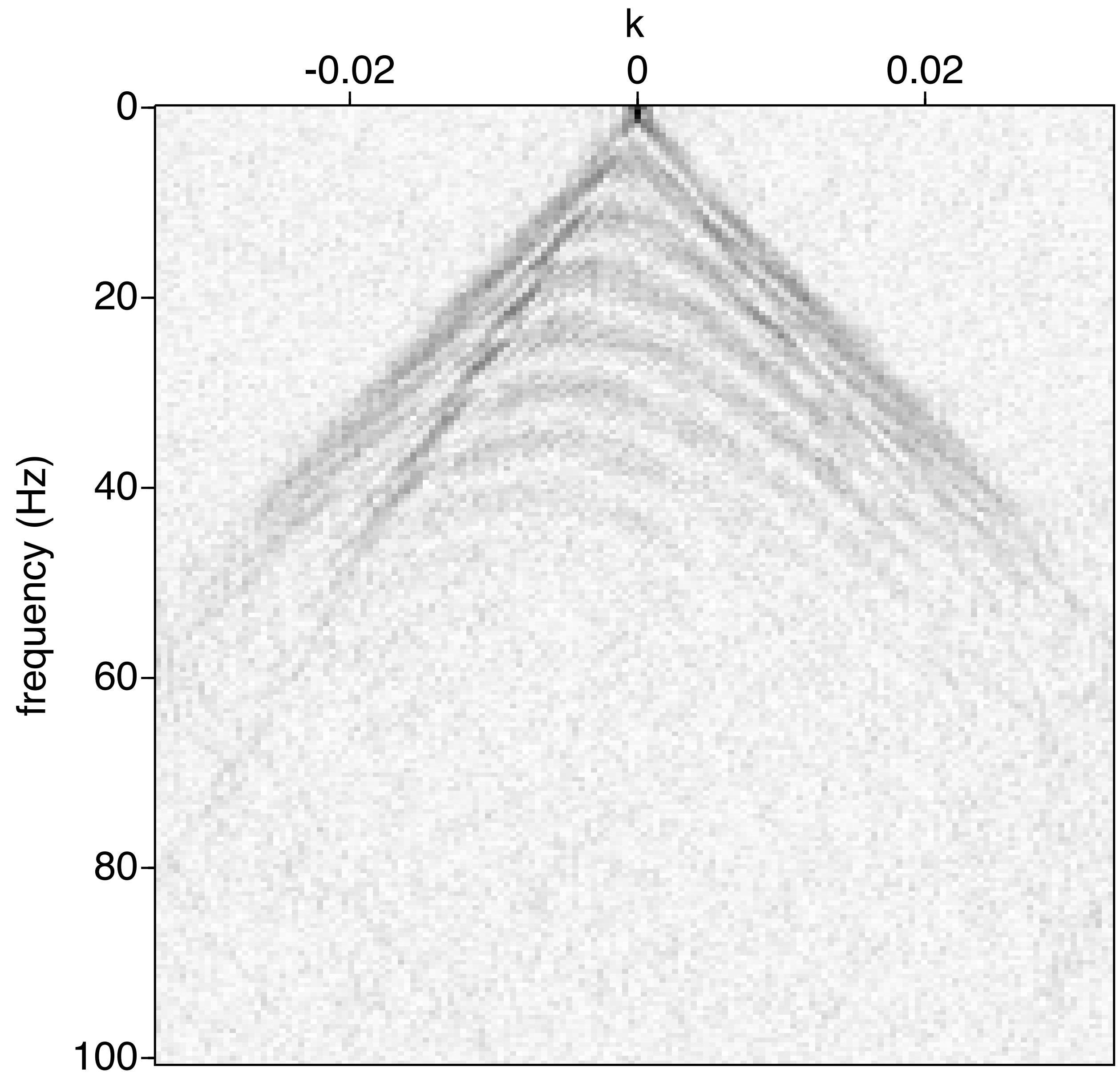
Data FK



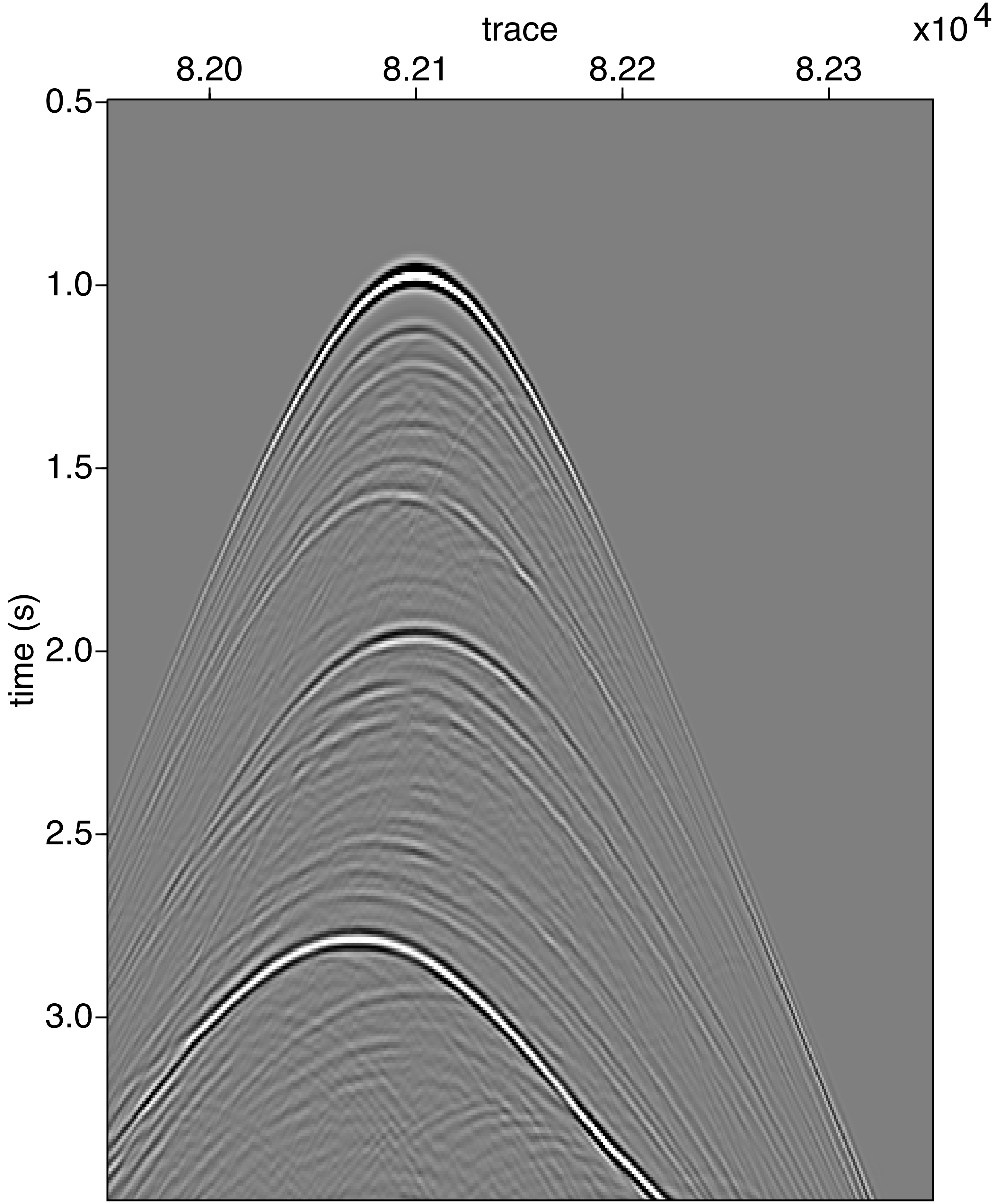
Robust EPSI IR FK



+40% noise Robust EPSI IR FK



+100% noise Robust EPSI IR FK

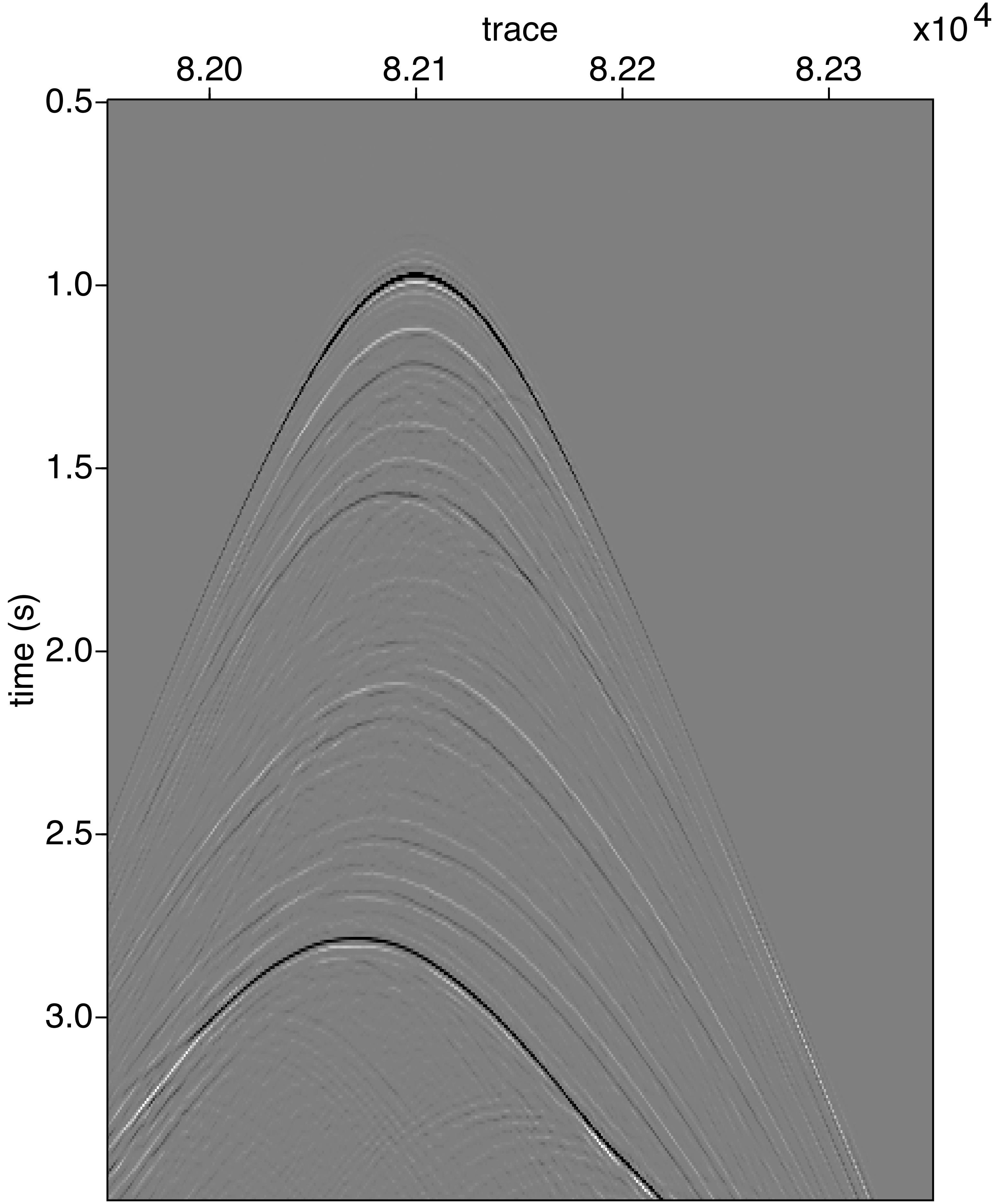


Pluto15
data

Elastic FD Modeling

muted

no deghosting



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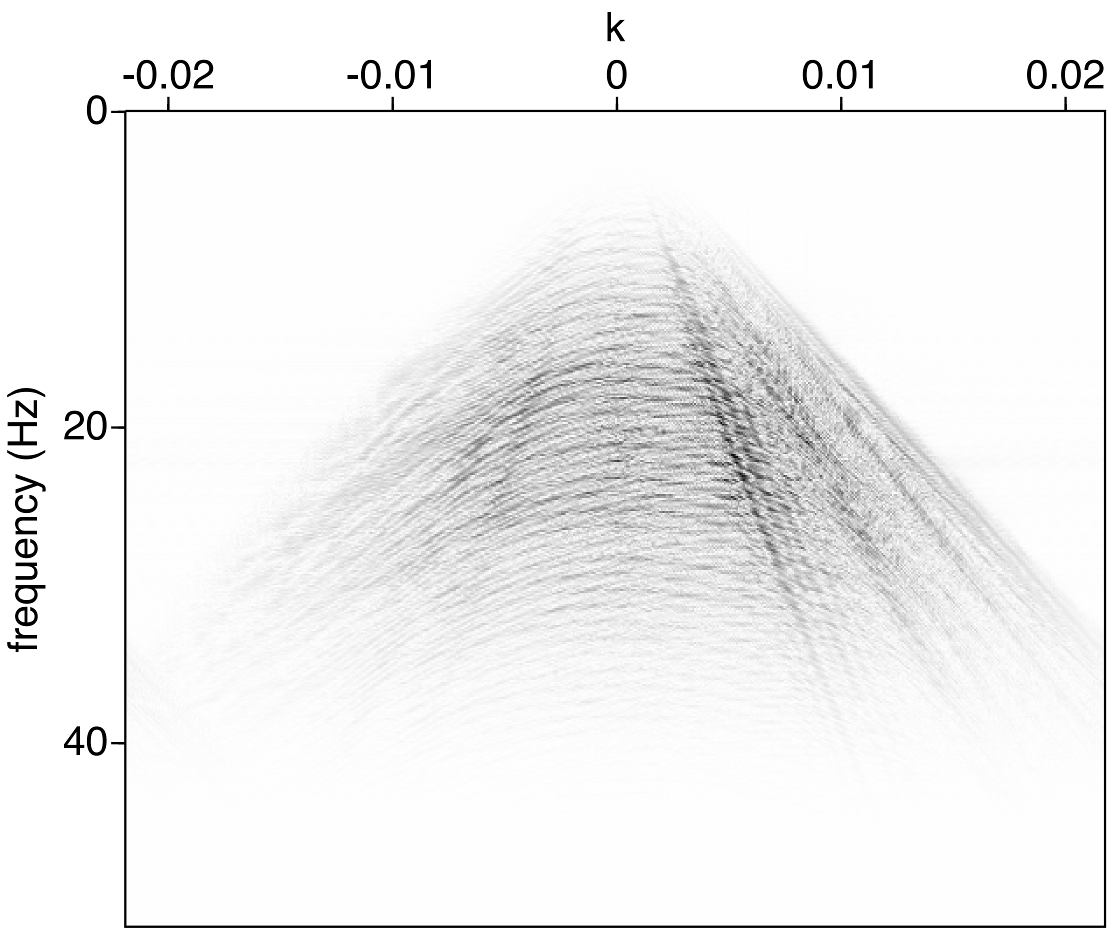
Pluto15

REPSI

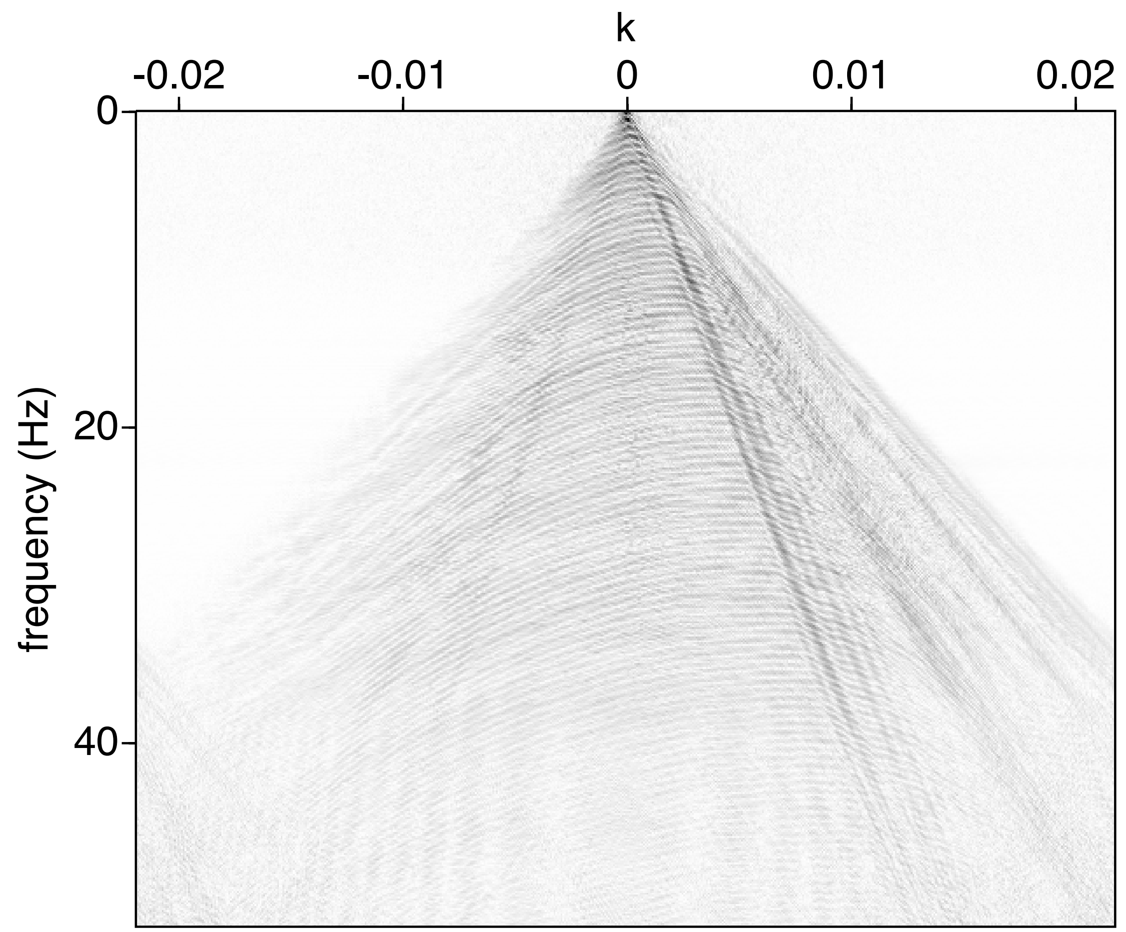
Primary IR (G)

no transform used

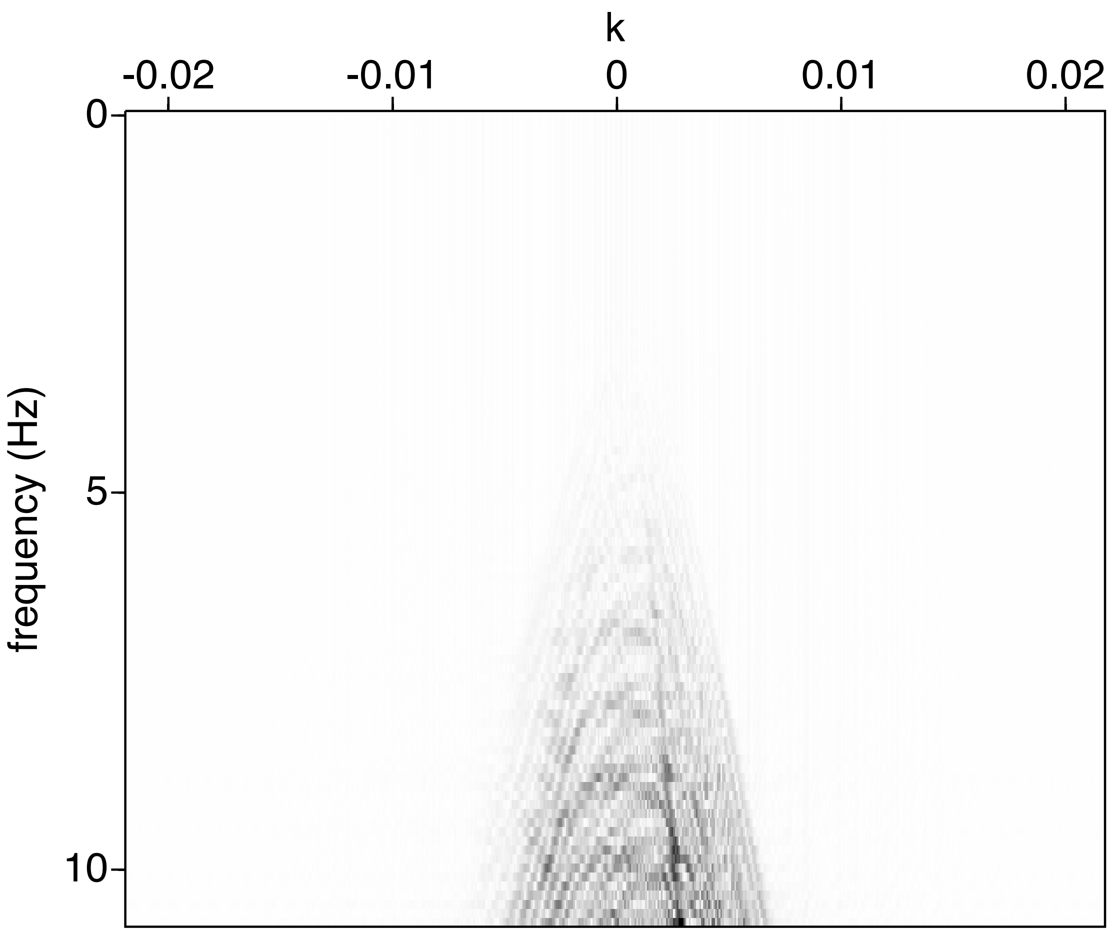
80 iters



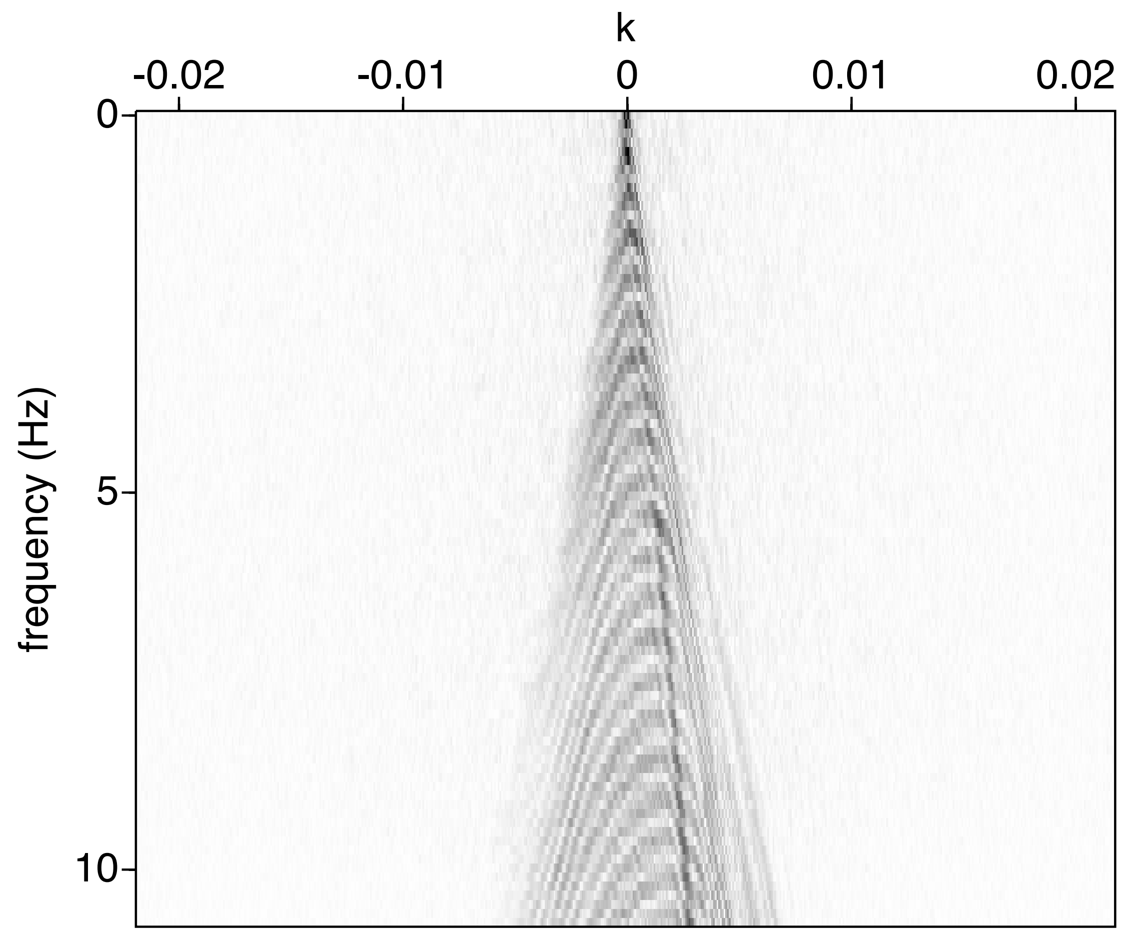
F-K Spectrum of data



F-K Spectrum of REPSI Primary IR



F-K Spectrum of data



F-K Spectrum of REPSI Primary IR

REPSI in transform domain

Modify just the problem for g:

$$\min_{\mathbf{g}} \|\mathbf{g}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2 \leq \sigma$$

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

REPSI in transform domain

Modify just the problem for g:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{S}^{\dagger}\mathbf{x}\|_2 \leq \sigma, \quad \mathbf{g} = \mathbf{S}^{\dagger}\mathbf{x}$$

(basis pursuit + denoise)

$$\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2$$

$\mathbf{S}$: sparsifying representation for seismic signals

- Should have spatially localized support
- ex: nd-Wavelets, Curvelets, etc...

$\mathbf{S}^{\dagger}$: synthesis operator for $\mathbf{S}$

REPSI in transform domain

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

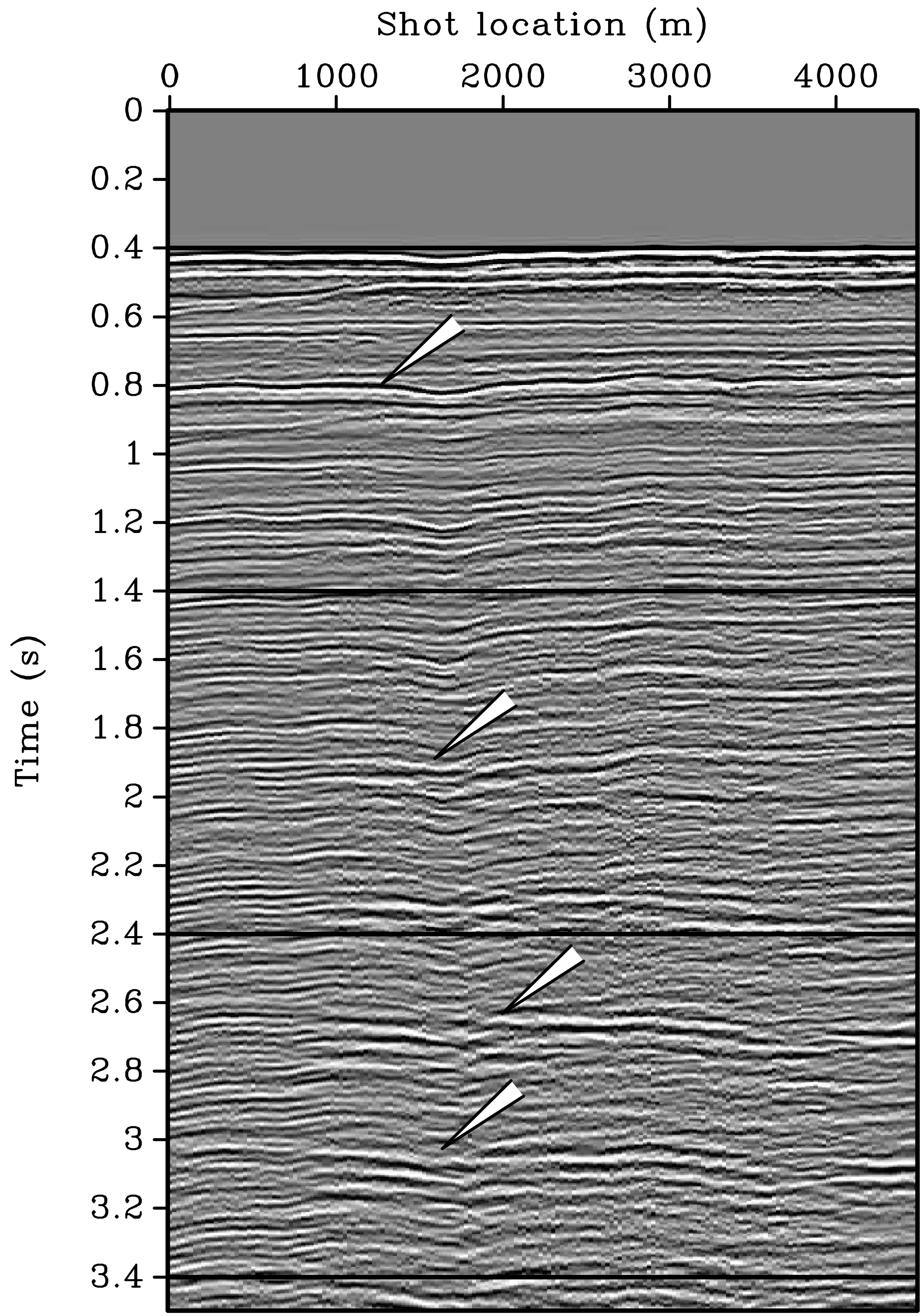
$$\mathbf{x}_{k+1} = \arg\min_{\mathbf{x}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{S}^\dagger\mathbf{x}\|_2 \text{ s.t. } \|\mathbf{x}\|_1 \leq \tau_k$$

(Solve with SPGL1 until Pareto curve reached)

$$\mathbf{g}_{k+1} = \mathbf{S}^\dagger\mathbf{x}_{k+1}$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

(Solve with LSQR)



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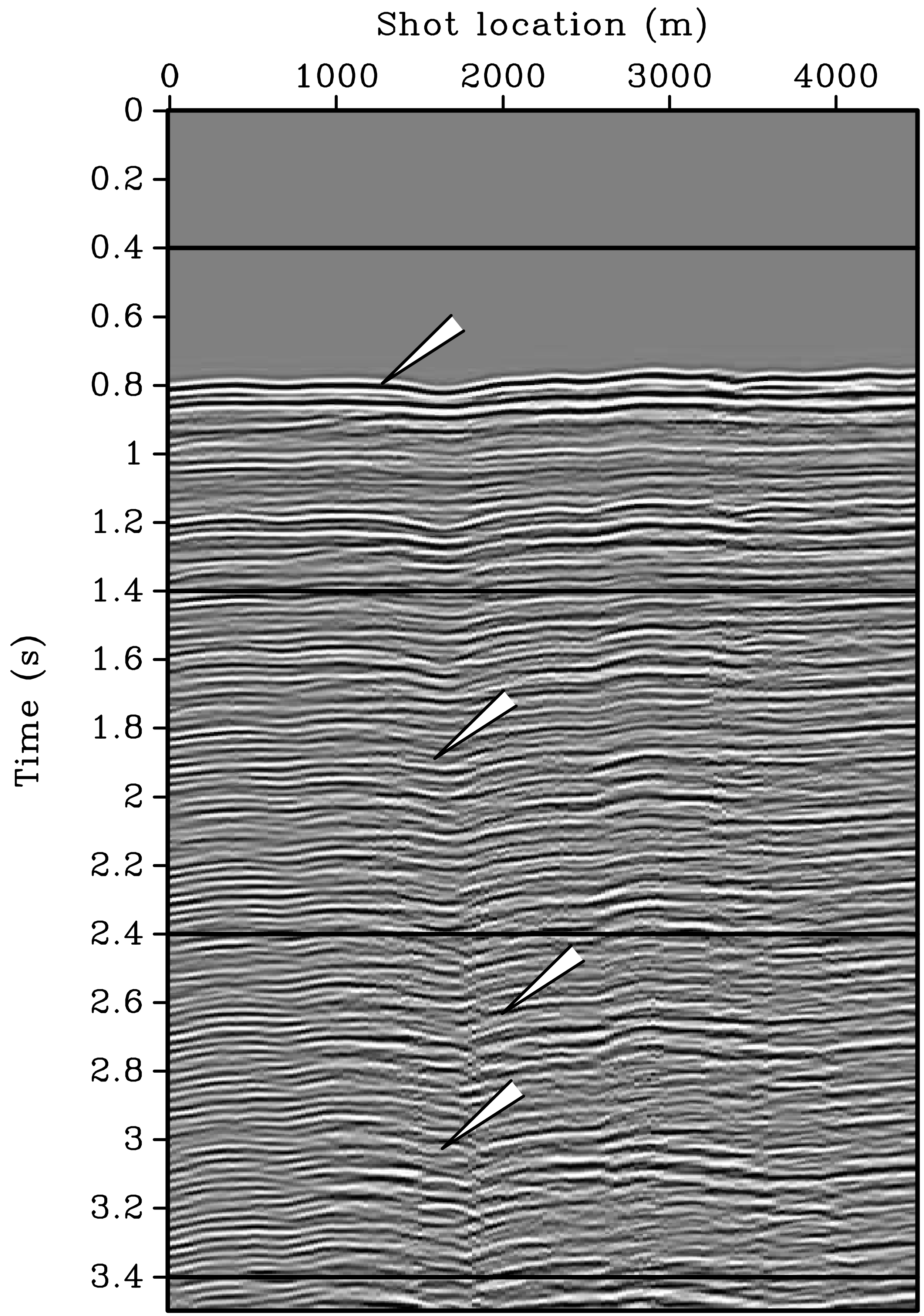
North Sea
data

offset gather 200m

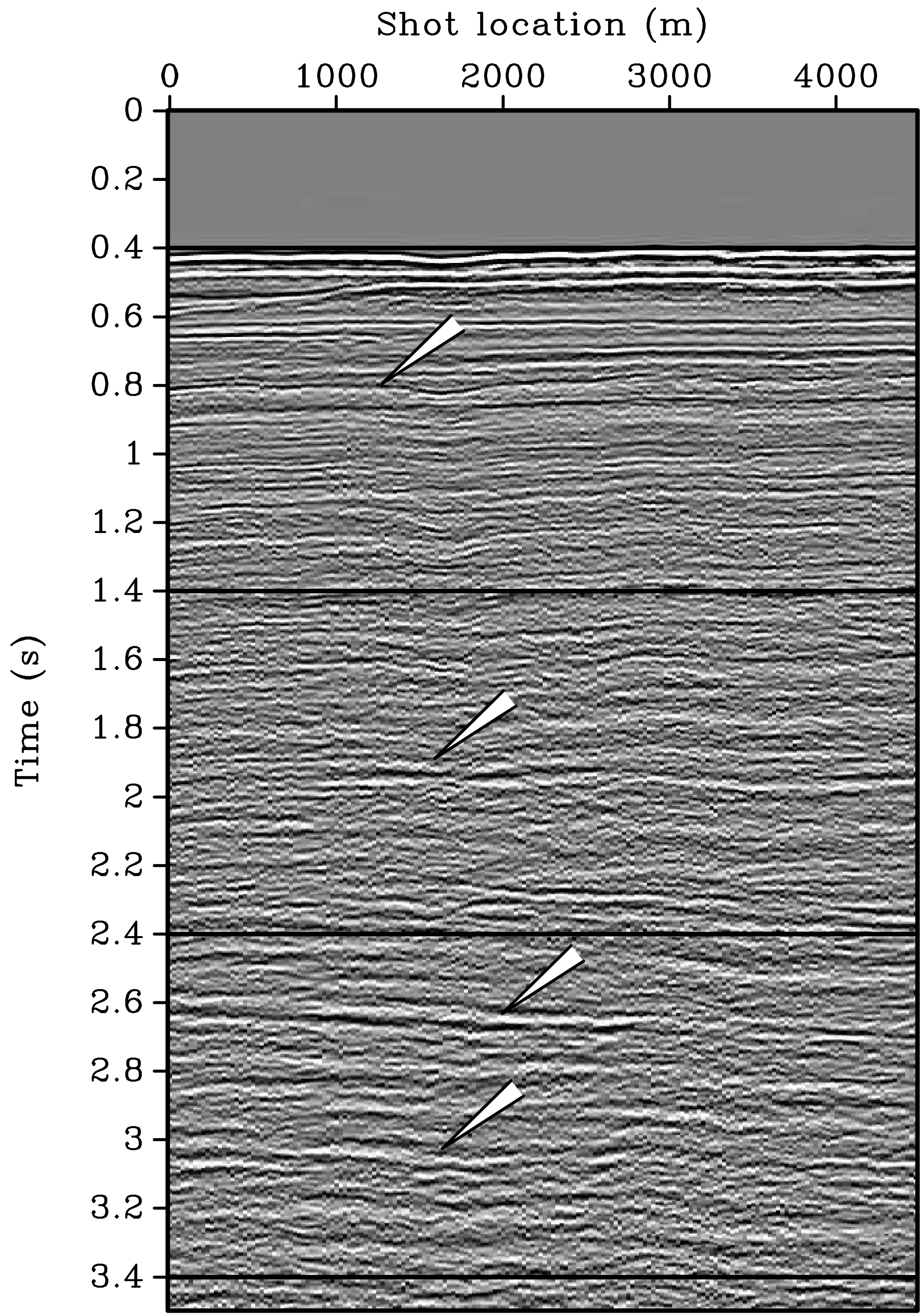
AGC display panels



North Sea
predicted multiples

offset gather 200m

AGC display panels

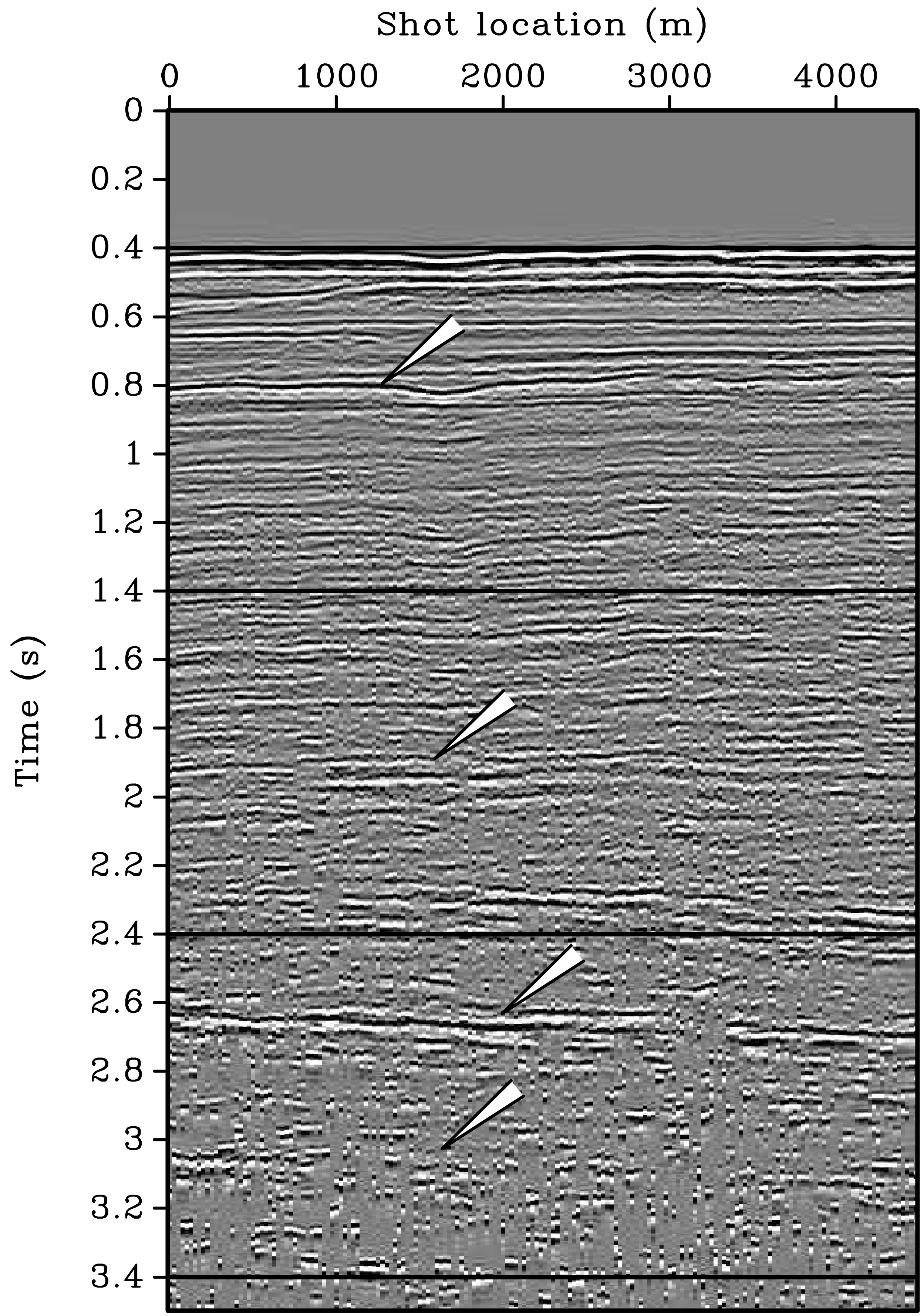


North Sea

SRME + LS subtraction

offset gather 200m

AGC display panels



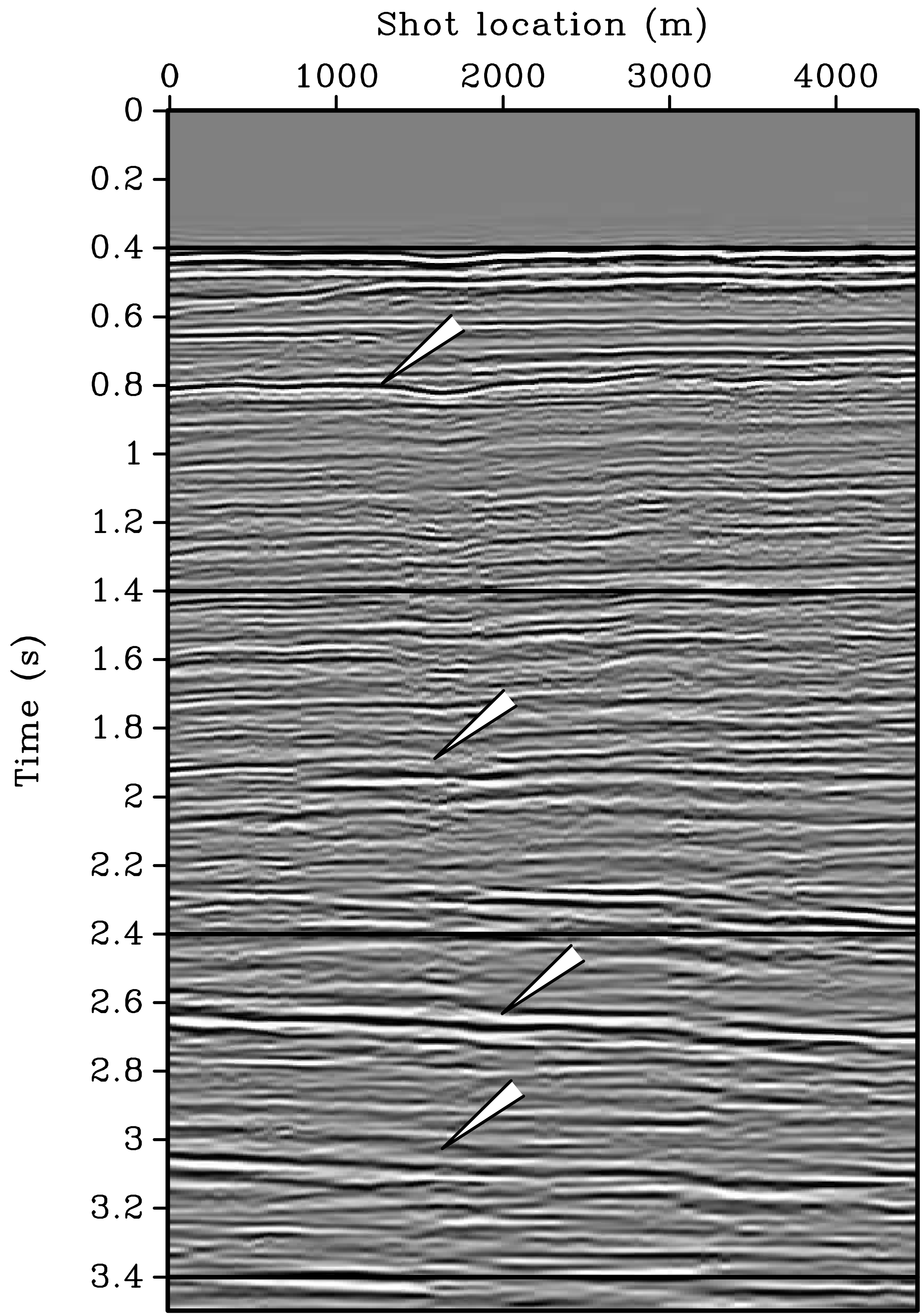
North Sea

REPSI

offset gather 200m

AGC display panels

75 SPG gradient iterations



North Sea

REPSI + Transform

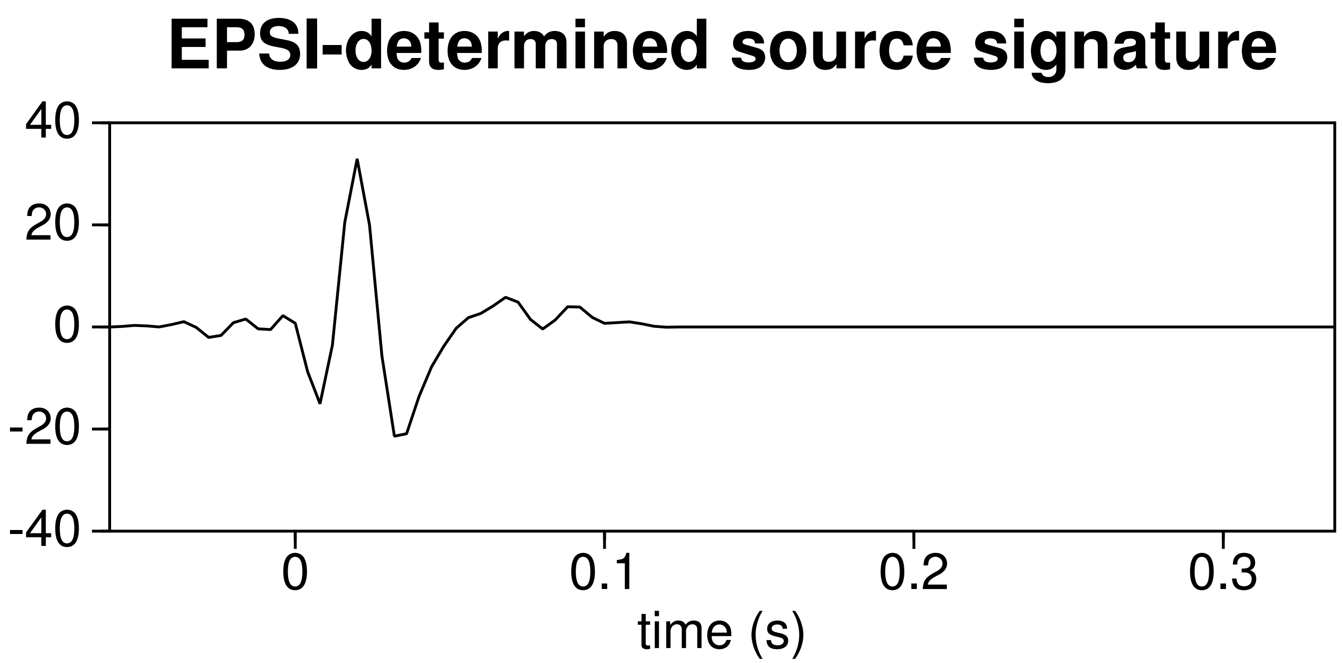
offset gather 200m

AGC display panels

2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

75 SPG gradient iterations



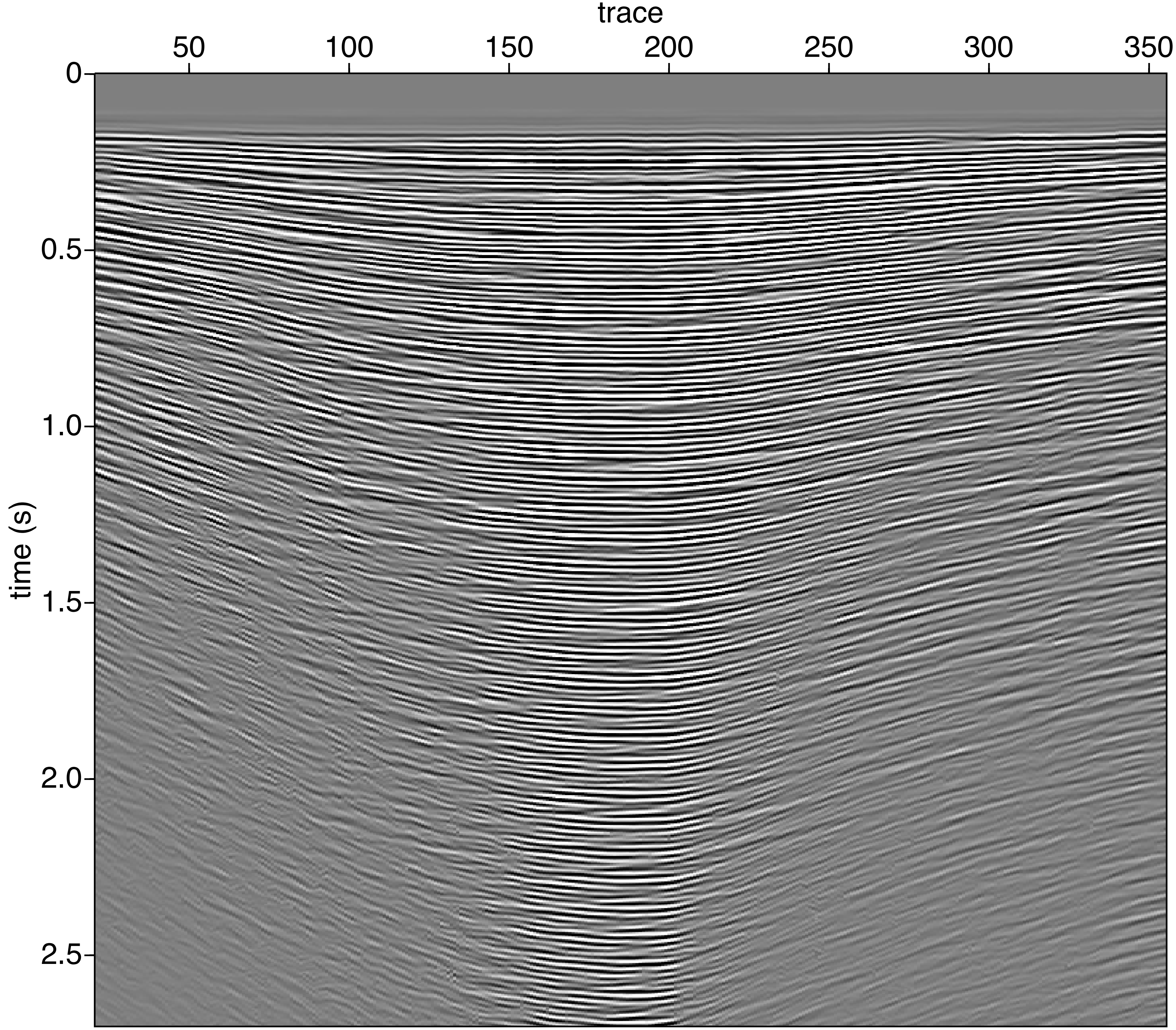
North Sea

REPSI estimated wavelet

offset gather 200m

AGC display panels

75 SPG gradient iterations

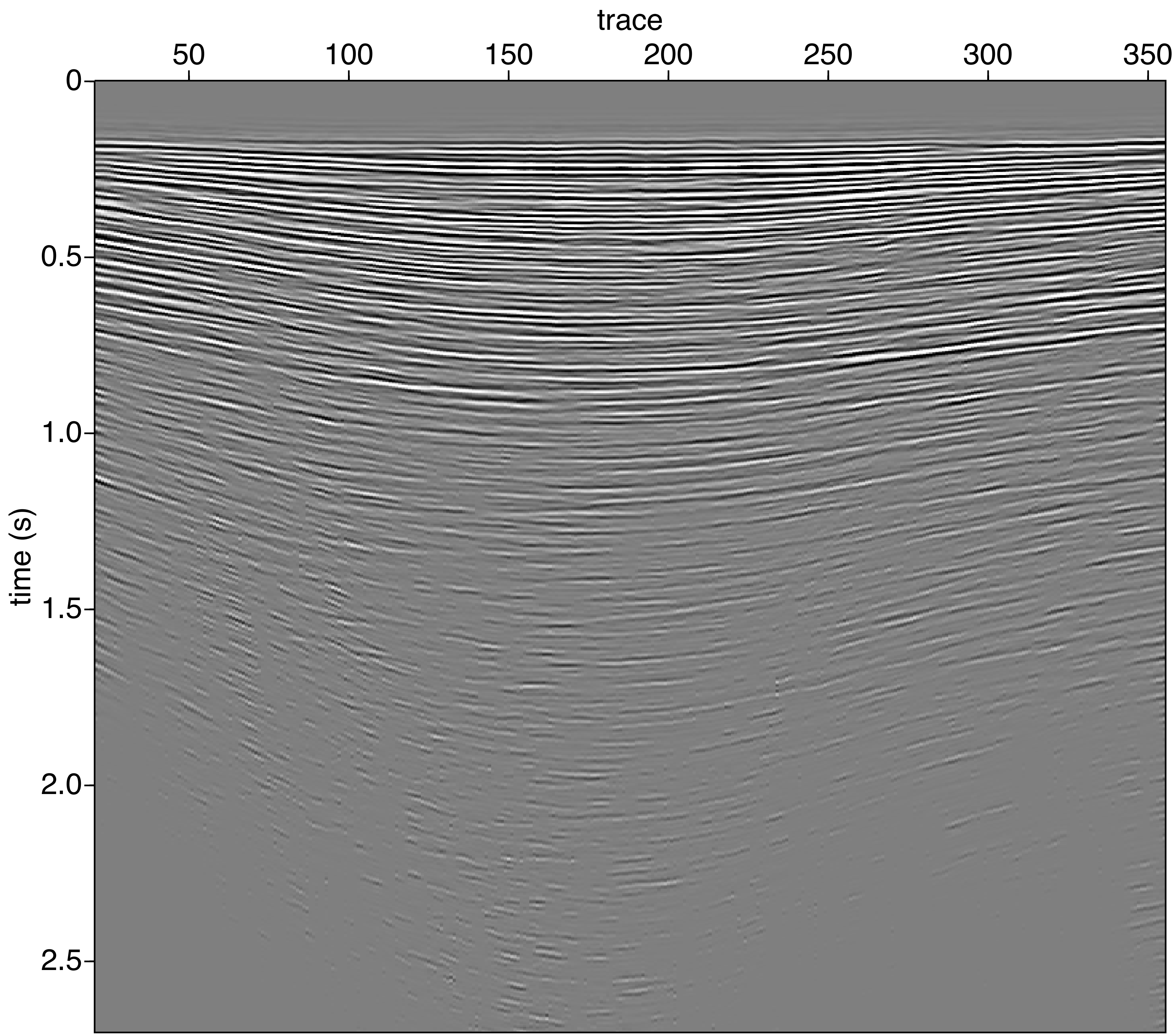


Input data

Gulf of Suez
data

offset gather 250m

t-gain display panels



REPSI primary wavefield

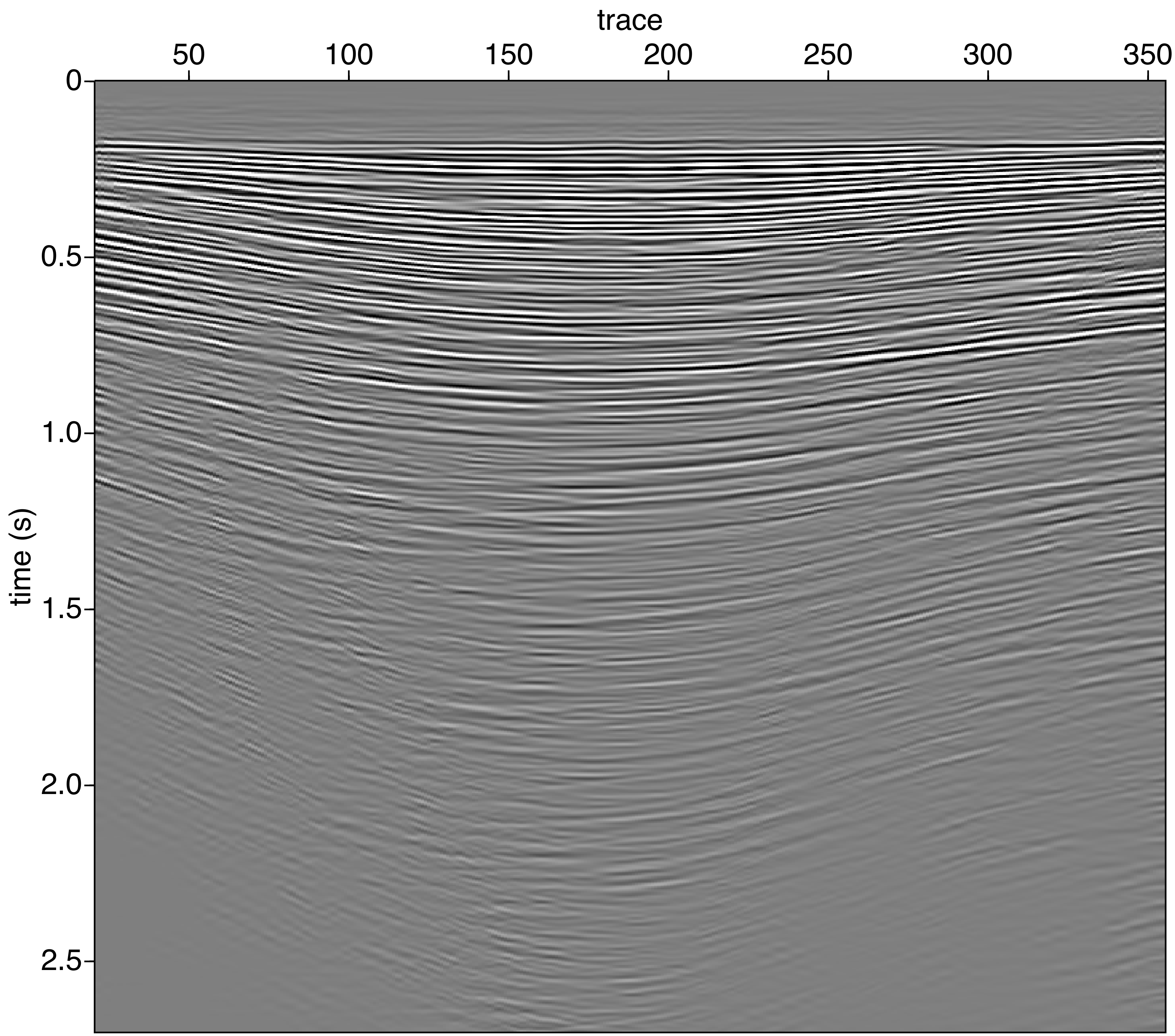
Gulf of Suez

REPSI

offset gather 250m

t-gain display panels

90 SPG grad. iterations



REPSI + transform domain primary wavefield

Gulf of Suez

REPSI + Transform

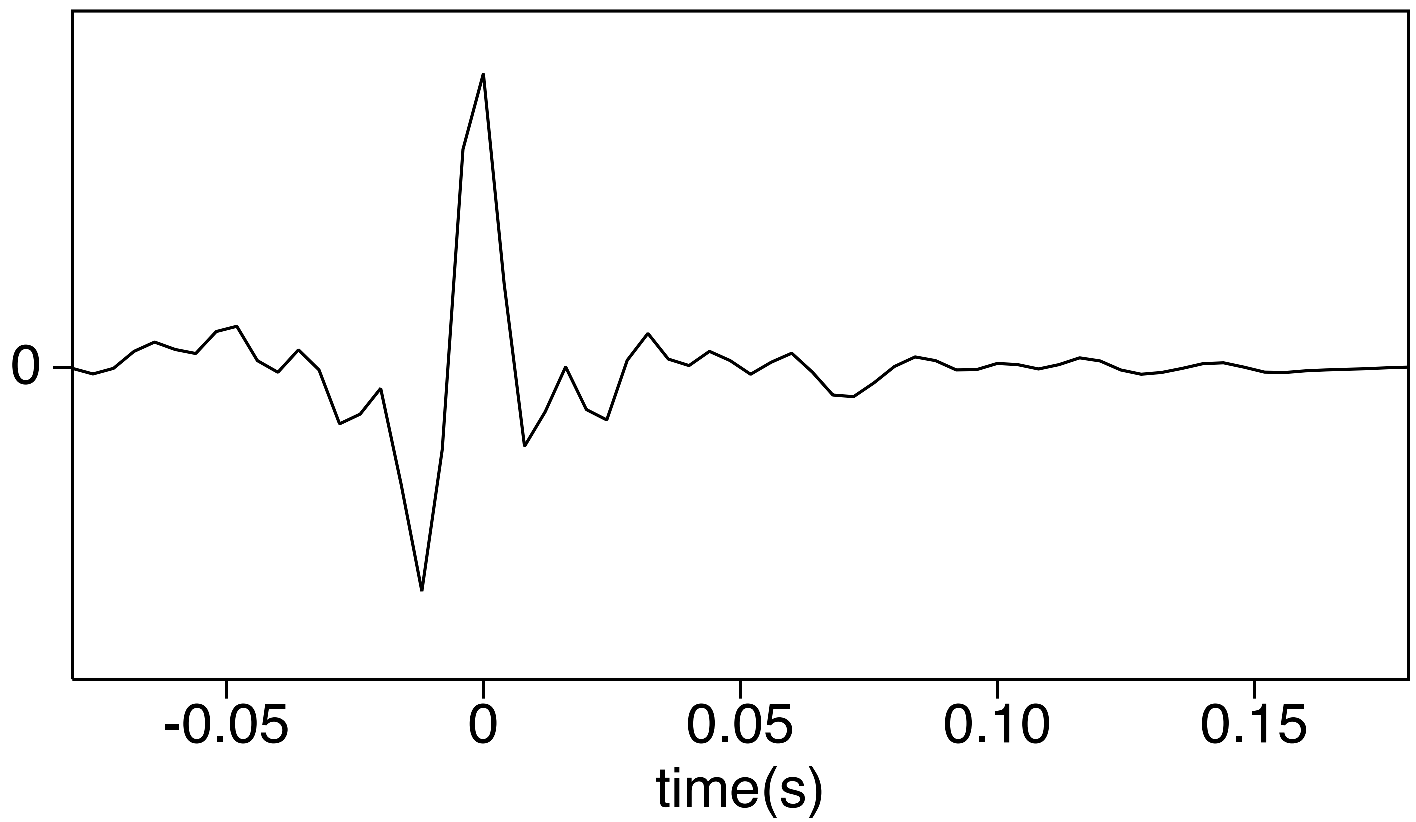
offset gather 250m

t-gain display panels

2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

90 SPG grad. iterations



Gulf of Suez

REPSI estimated wavelet

offset gather 250m

t-gain display panels

2D Curvelet (Src-Rcv)

Spline a=3.0 DWT (Time)

90 SPG grad. iterations

Pathway to 3D

- Sampling issue
- Computation time issue

Sampling Issues

- Cross-line direction poorly sampled in 3D
- Use a representation domain that is coherent across traces

$\mathbf{S}$: sparsifying representation for seismic signals

- Time-domain hyperbolic Radon
- Weighted 3D Curvelets

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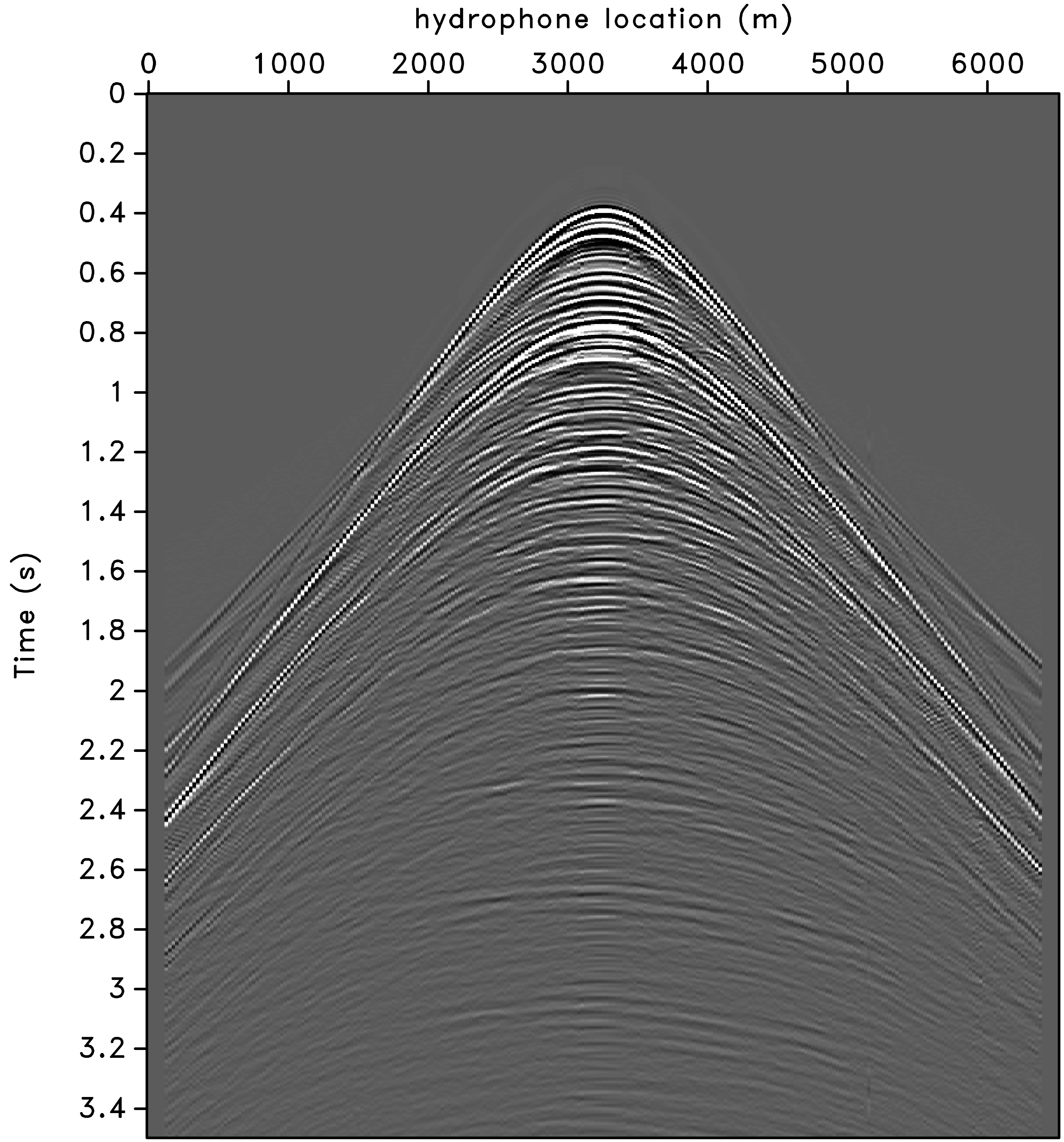
Computation time

- Low-rank decomposition of operator data $\mathbf{P}$
- Use a few randomized input to "probe" the operator
- (see Bander's talk)

Summary

- L1-convexification better behaved than sparse gradients and has few free parameters
- Follows the Pareto curve into a series of projected gradient problems
- Easily incorporates seeking the solution in a transform domain that promotes continuity

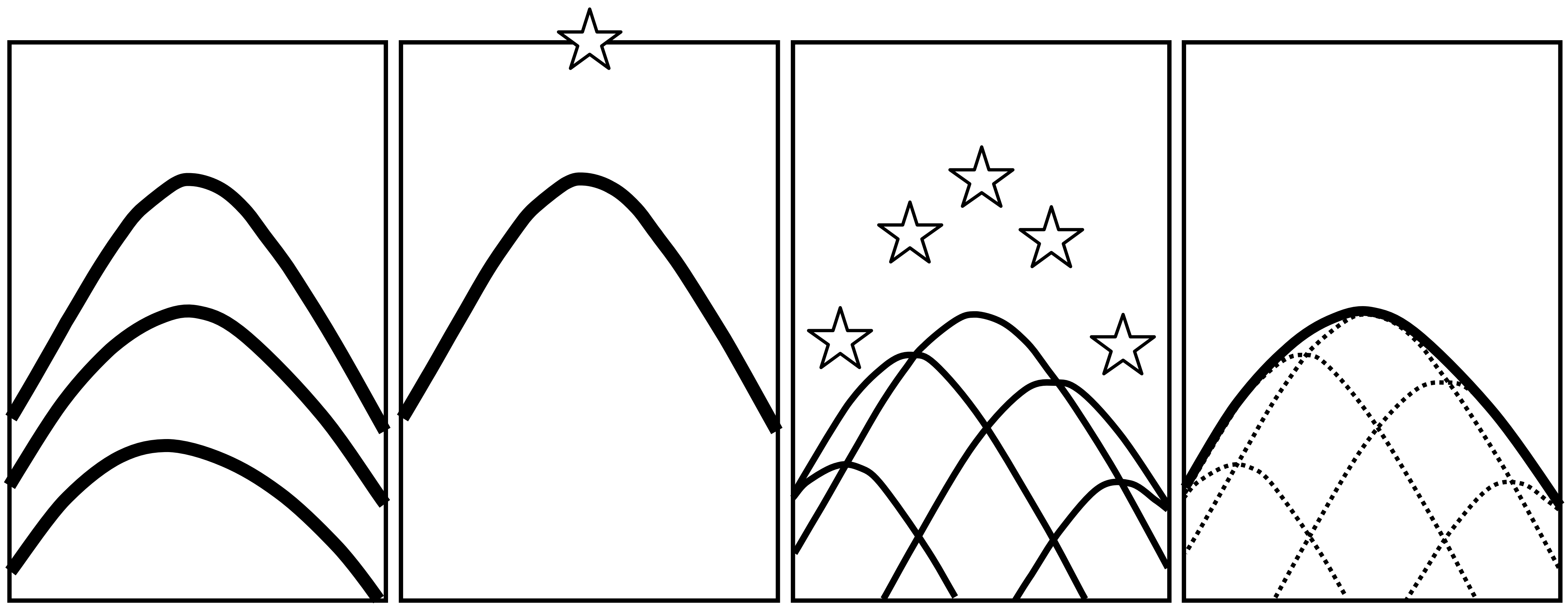
SLIM



North Sea
data

shot gather 3250m

reciprocity + Radon interp



(van Groenestijn and Verschuur 08)