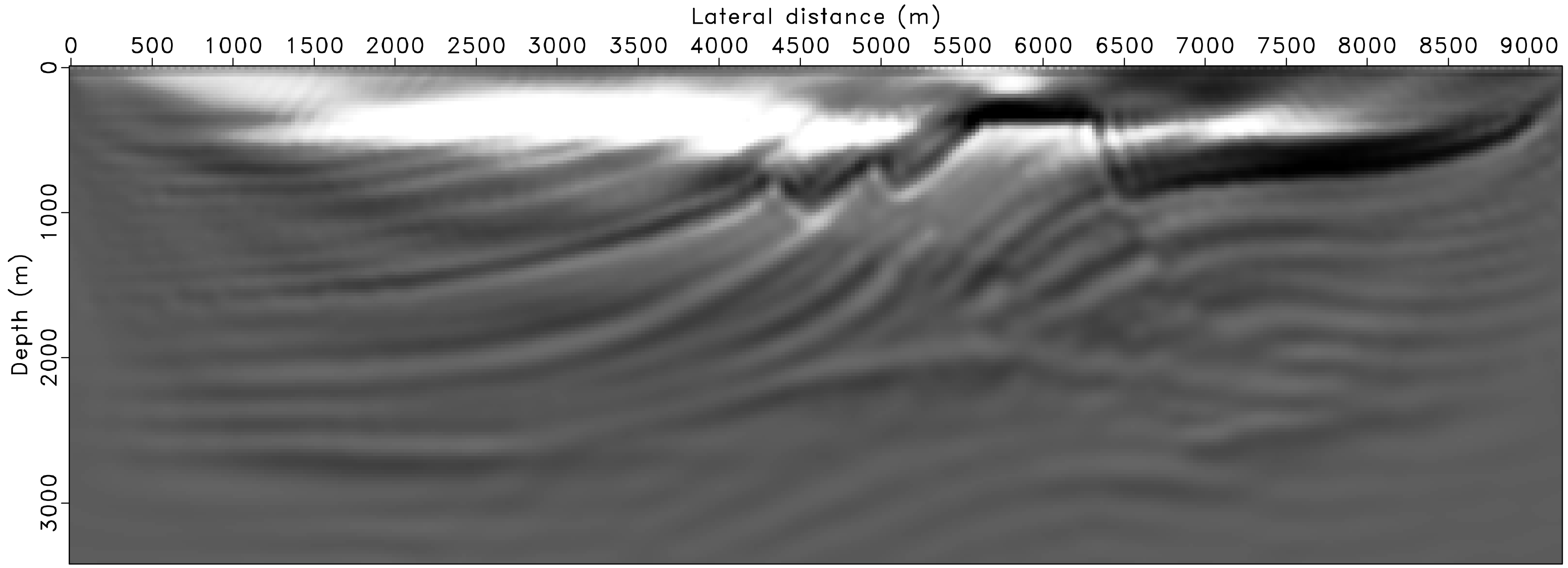
SINBAD Consortium 2010

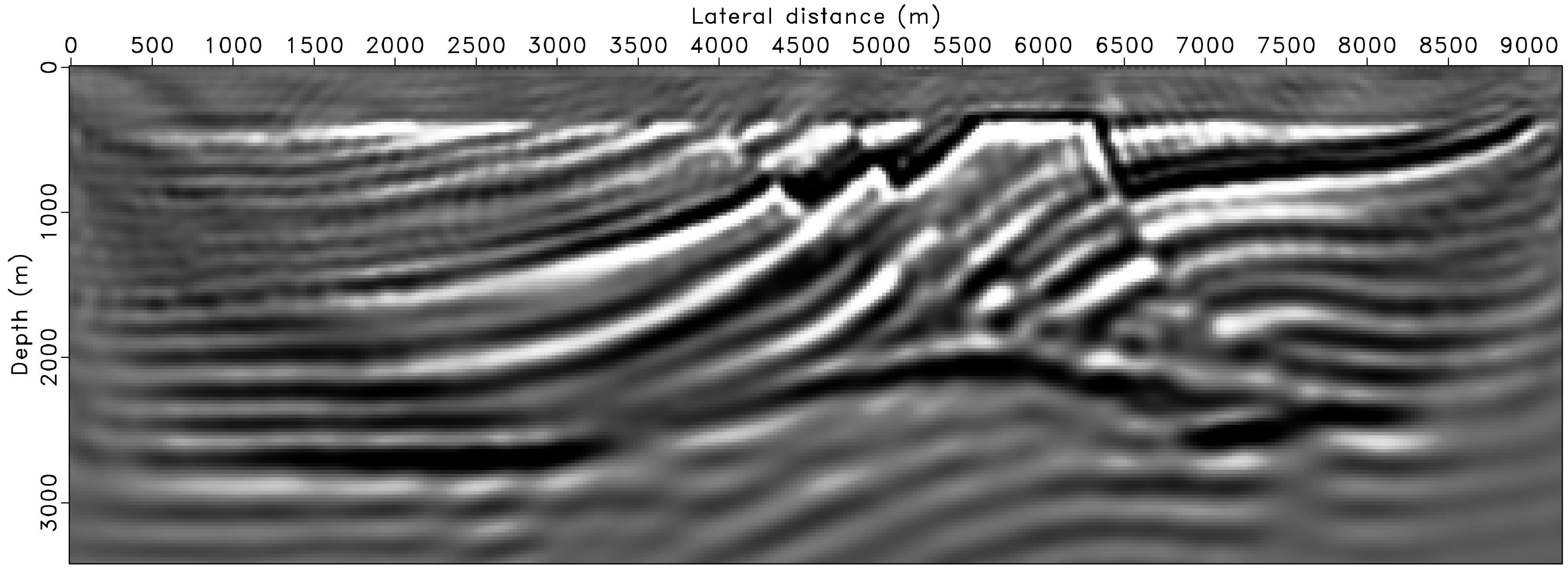
Efficient least-squares imaging with *sparsity* promotion and *compressive sensing*

Xiang Li and Felix J. Herrmann

SLIM
University of British Columbia

Migration vs *linearized* inversion





[Li & FJH et. al. '10-]

Compressive imaging

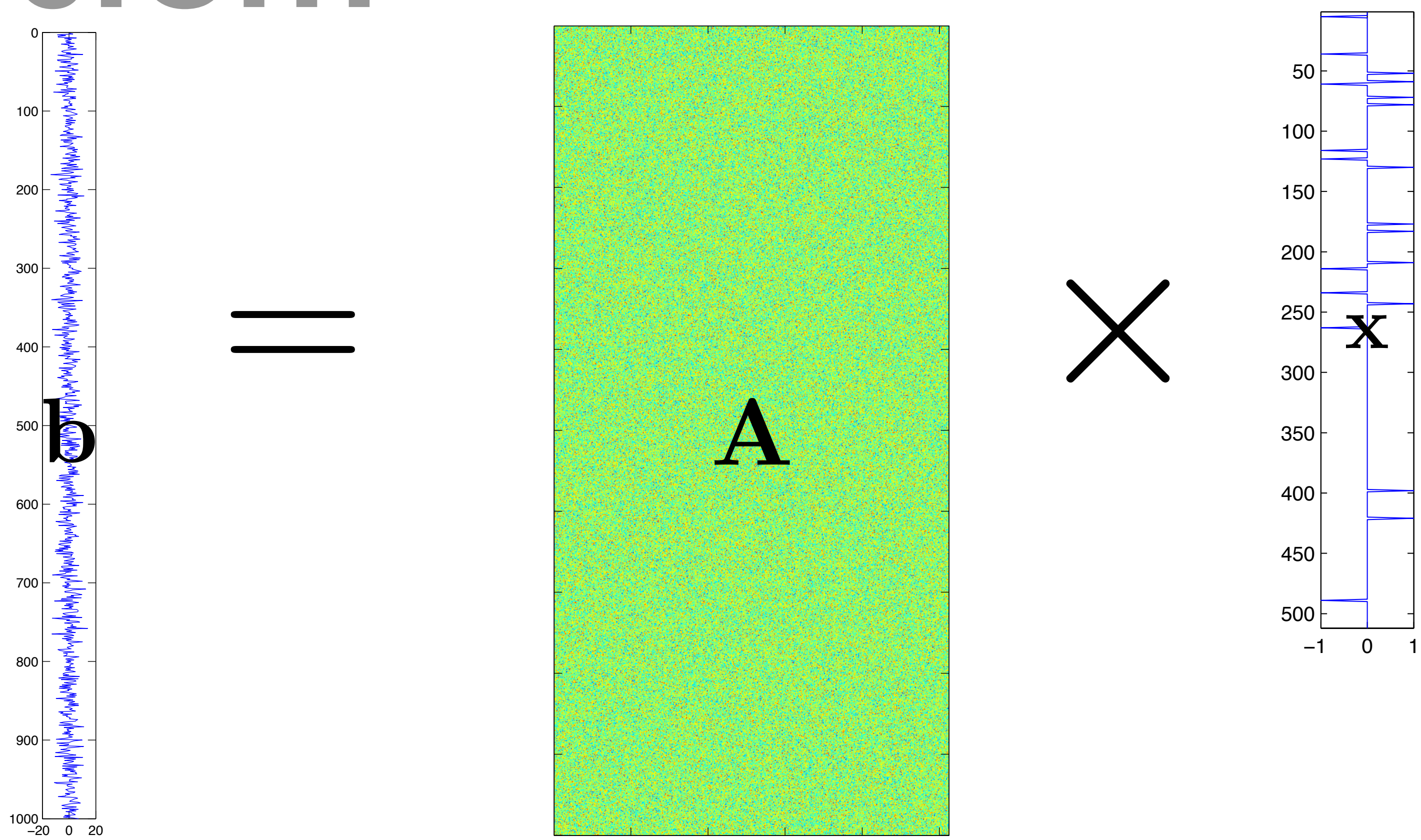
Challenge:

Least-squares migration requires *multiple passes & PDE solves*

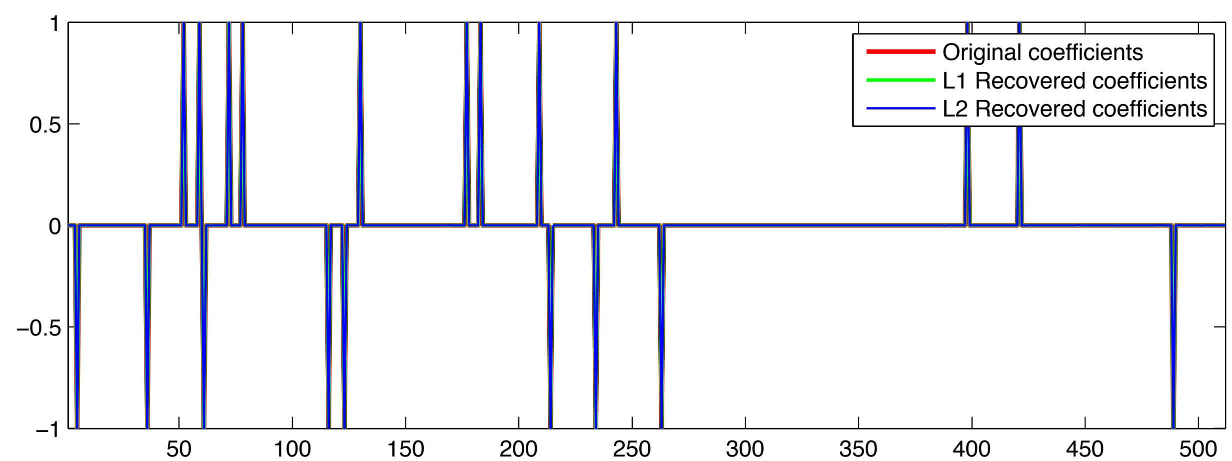
Key idea:

- combine *Compressive Sensing* & '*Phase encoding*'
- turn "*overdetermined*" imaging problem into *underdetermined* problem with *randomized* supershots
- use *curvelet*-based *sparse* recovery to remove *crosstalk*

Overdetermined system

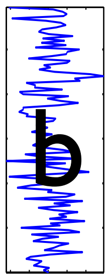


L1/L2 recovery



$$\mathbf{x} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1 or \ell 2} \quad \text{subject to} \quad \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma$$

Underdetermined system



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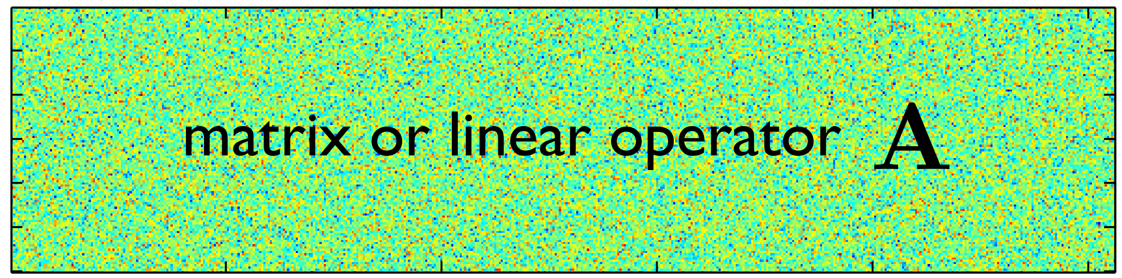
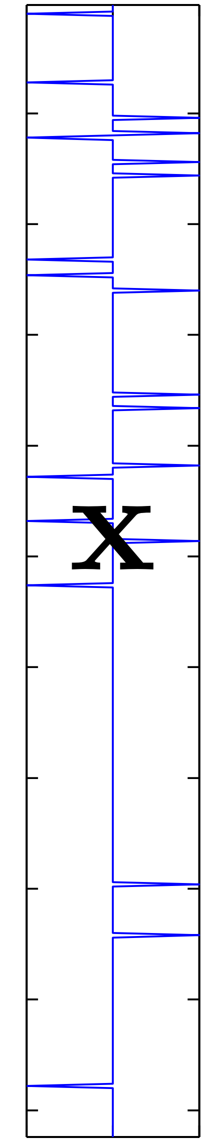
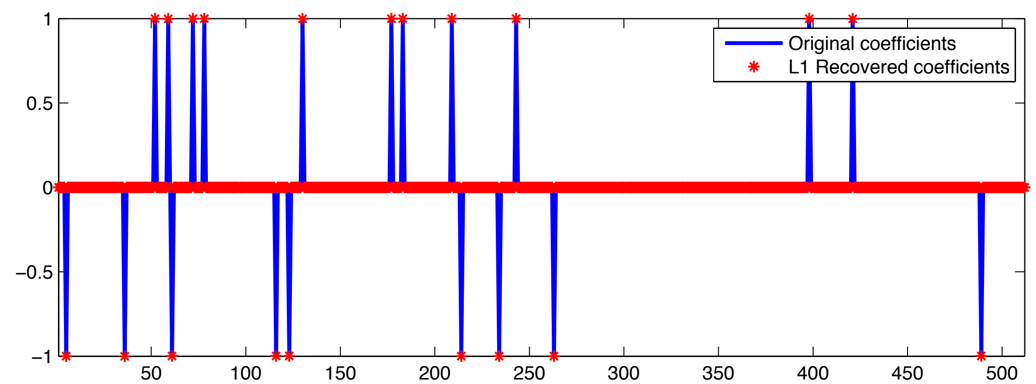


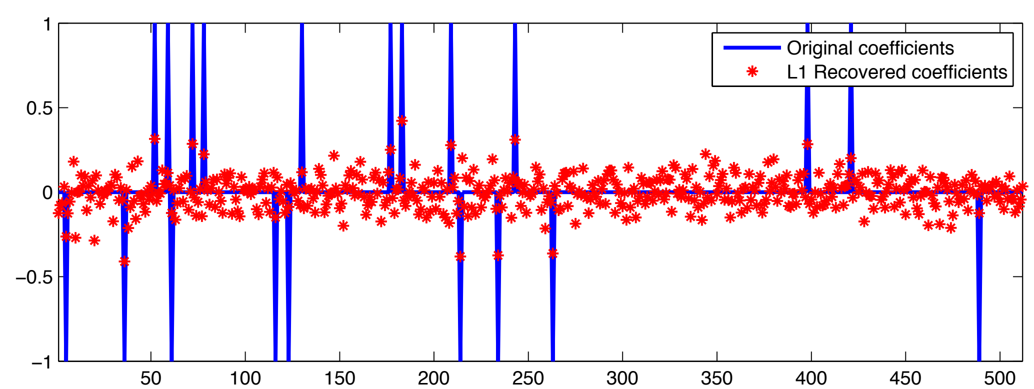
$\times$



L1 recovery



L2 recovery

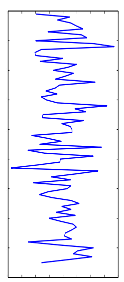


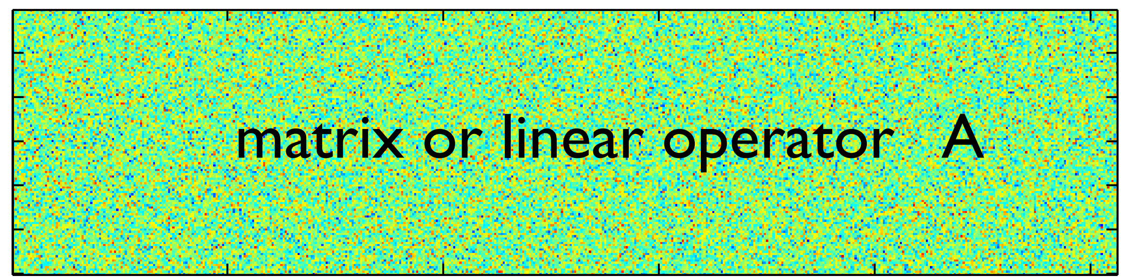
$$\mathbf{x} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1 or \ell 2} \quad \text{subject to} \quad \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma$$

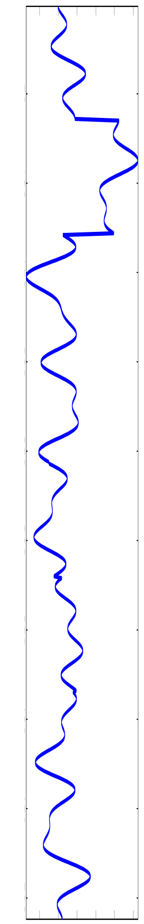
[van den Berg & Friedlander, ’08]

Compressible signals

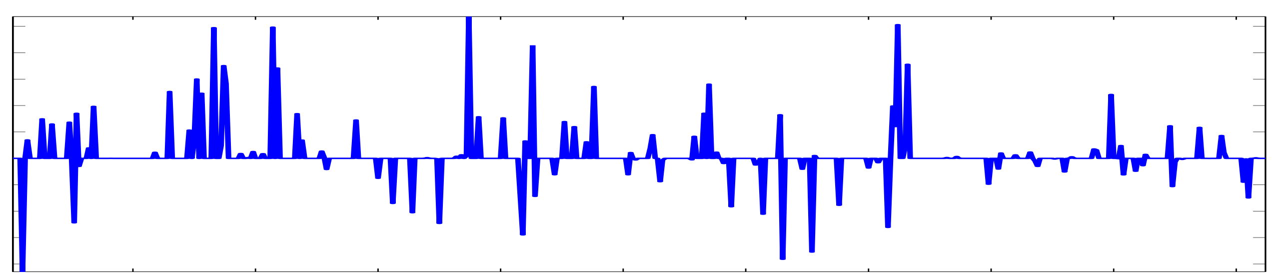
What if the signal is not sparse?







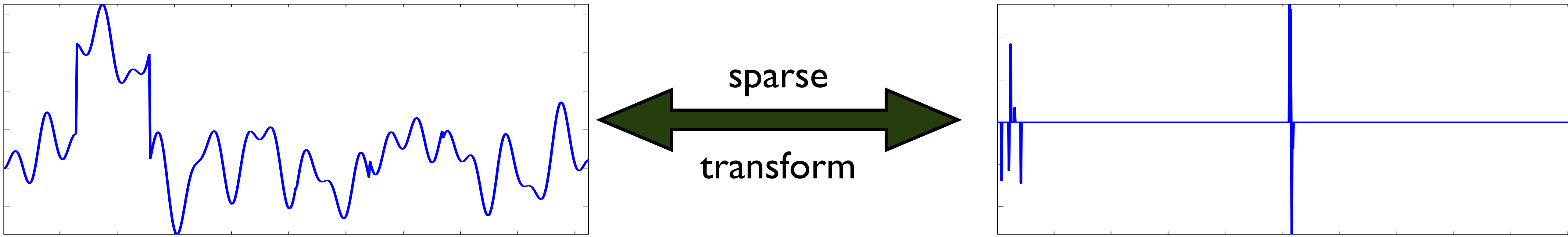
L1 recovery



$$\mathbf{x} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|b - A\mathbf{x}\|_2 \leq \sigma$$

Sparsifying domains

wavelet, curvelet, shearlet, wave atom...

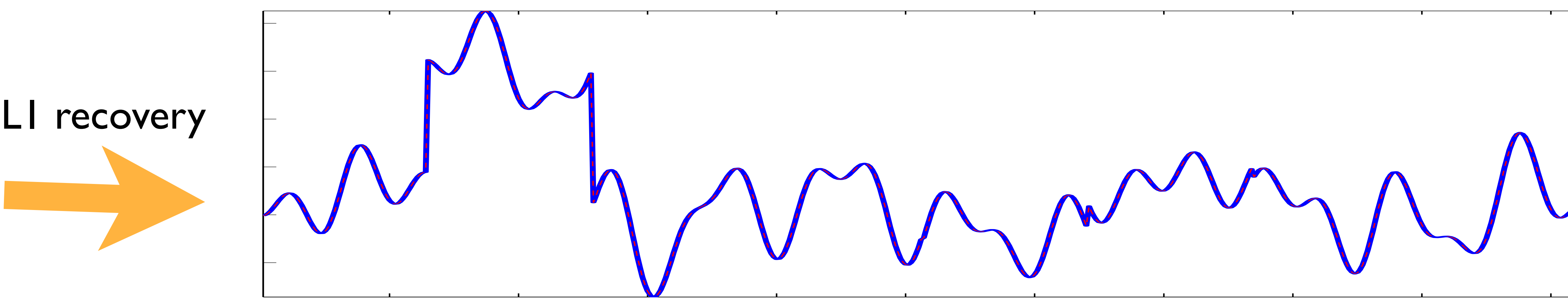


physical domain

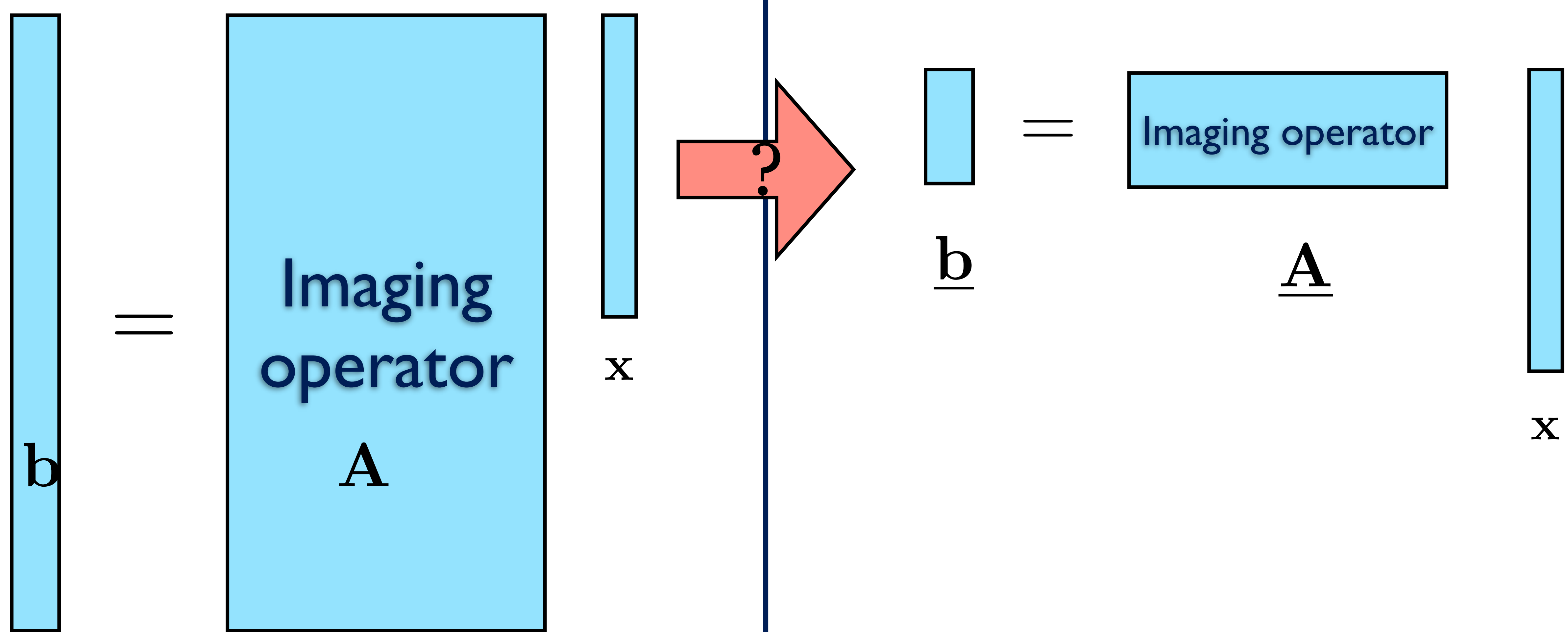
Sparse domain

$$x = \mathbf{S}^* \arg\min_{\delta\mathbf{x}} \|\delta\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|b - A\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma$$

L1 recovery



Dimensionality-reduced imaging



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[Nemeth et. al. '99]

Linearized inversion

Least-squares migration:

$$\delta\widetilde{\mathbf{m}} = \arg\min_{\delta\mathbf{m}} \frac{1}{2}\|\delta\mathbf{d} - \nabla\mathcal{F}[\mathbf{m}_0;\mathbf{Q}]\delta\mathbf{m}\|_2^2$$

$$\begin{aligned}
\delta\mathbf{d} &= \text{Multi-source multi-frequency data residue} \\
\nabla\mathcal{F}[\mathbf{m}_0;\mathbf{Q}] &= \text{Linearized Born-scattering operator} \\
\mathbf{m}_0 &= \text{Background velocity model} \\
\mathbf{Q} &= \text{Sources} \\
\delta\tilde{\mathbf{m}} &= \text{image}
\end{aligned}$$

[Nemeth et. al. '99]

Linearized inversion

Least-squares migration:

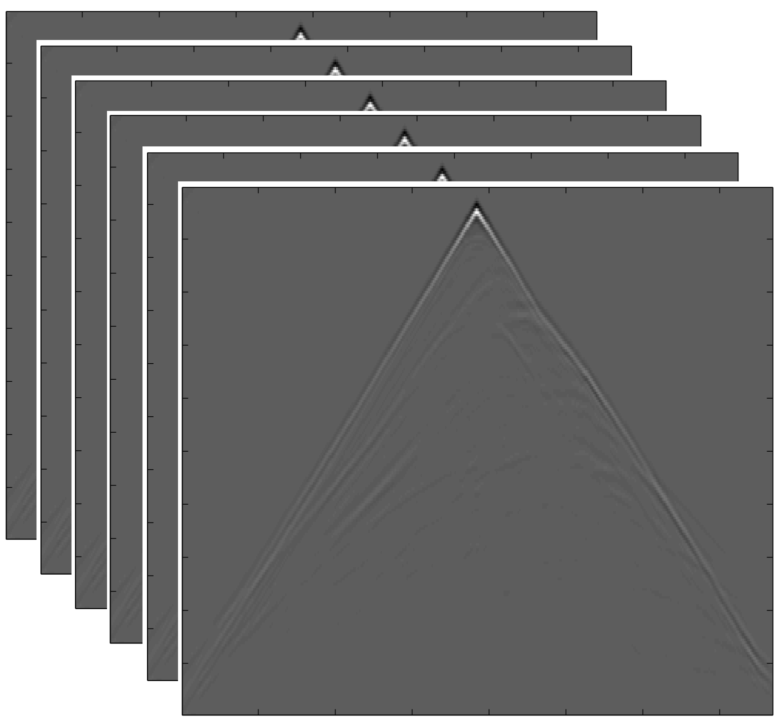
$$\delta\widetilde{\mathbf{m}} = \arg\min_{\delta\mathbf{m}} \frac{1}{2}\|\underbrace{\delta\mathbf{d}}_{\mathbf{b}} - \underbrace{\nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0;\mathbf{Q}]}_{\mathbf{A}}\underbrace{\delta\mathbf{m}}_{\mathbf{x}}\|_2^2$$

- overdetermined system, ($n_f \times n_s \times n_r$, $n_x \times n_z$)
- multiple passes trough all data

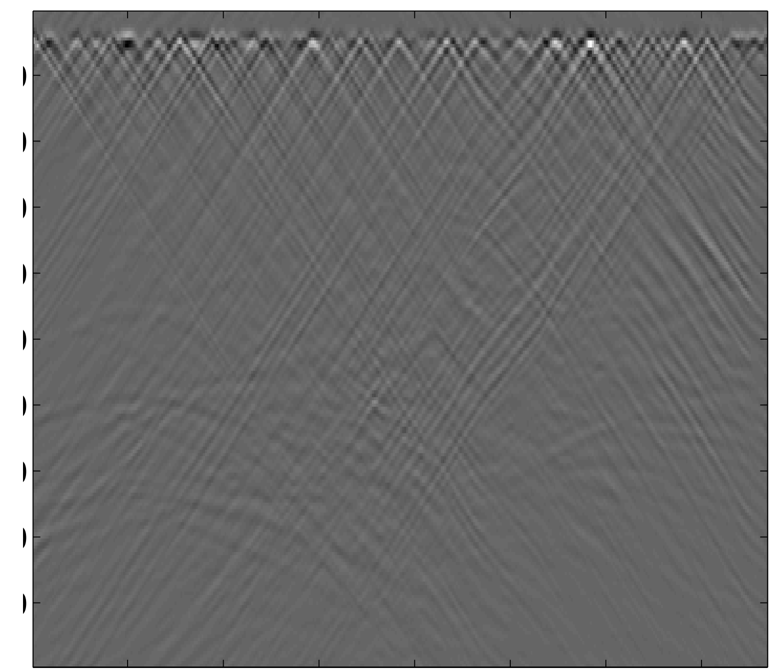
[Herrmann et. al. ’08-’10]

Dimensionality reduction

adapted from Herrmann et. al. ,09



$\mathbf{Q}$

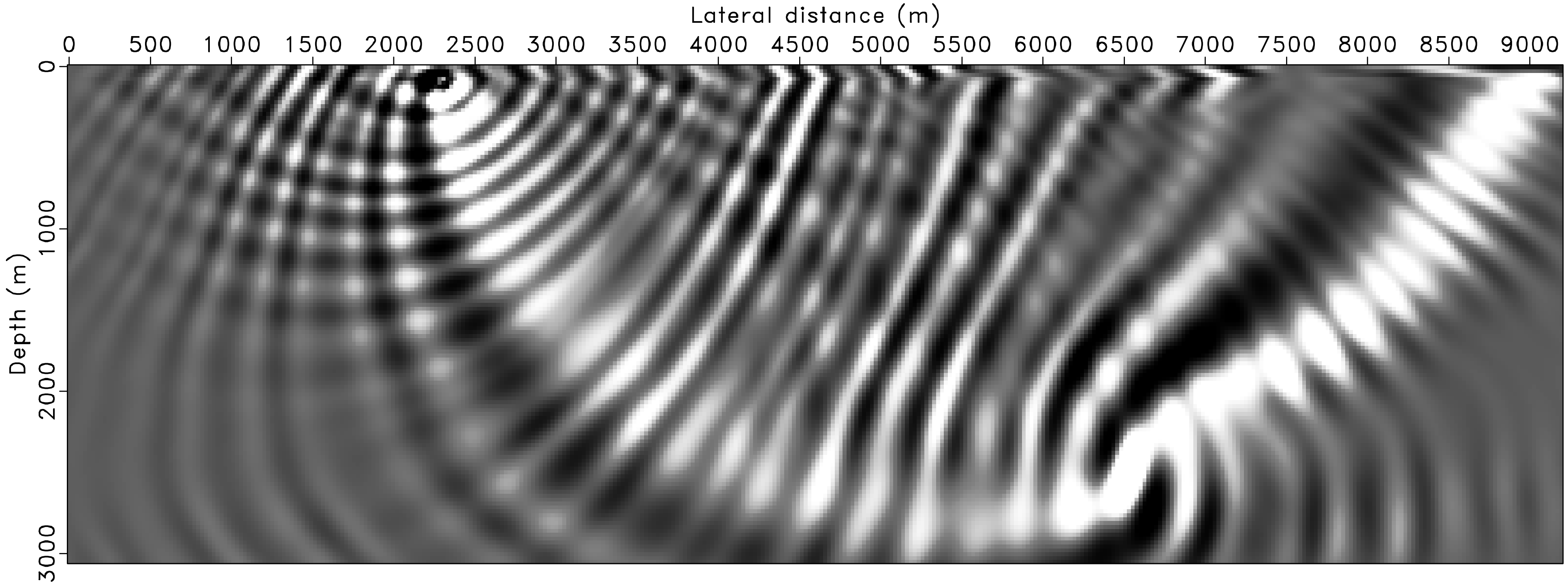


$\underline{\mathbf{Q}} = \mathbf{Q}\mathbf{W}$

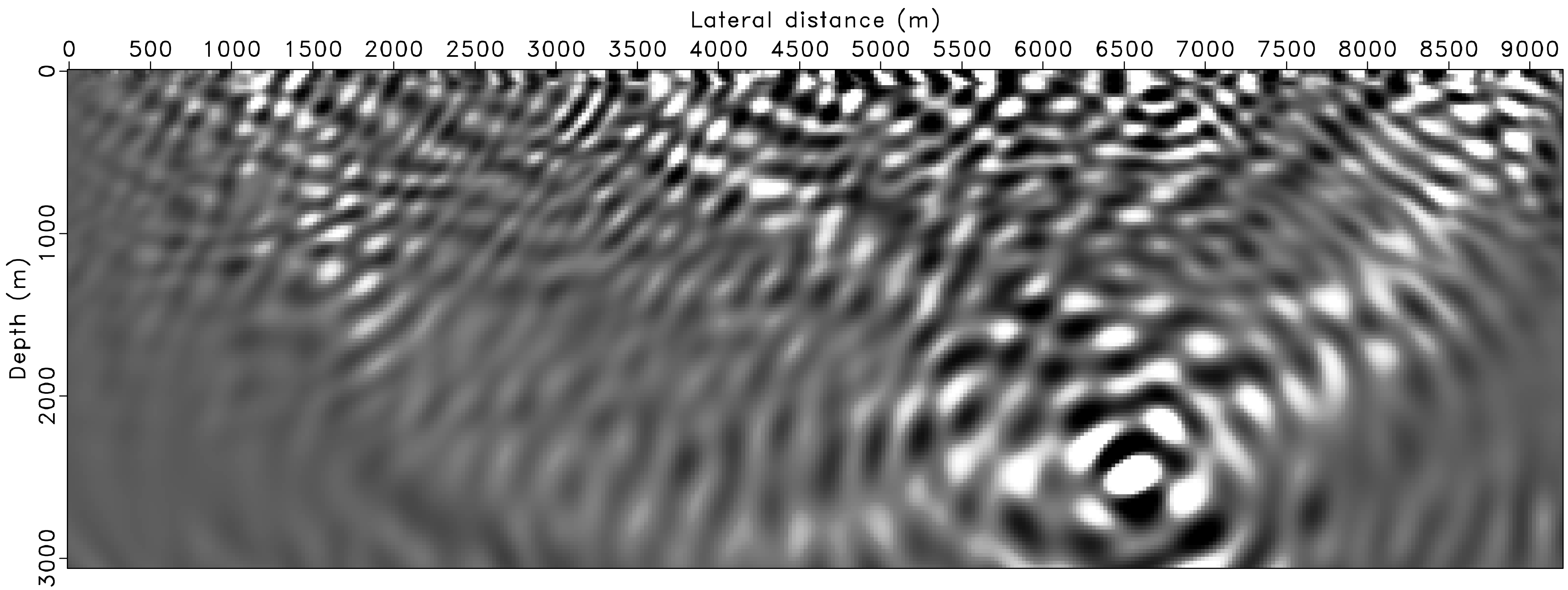
Collection of *K simultaneous-source* experiments with batch size $K \ll n_f \times n_s$

Single-shot image

one shot



Sequential shot image



Simultaneous shot image

Phase encoding

Least-squares migration:

$$\delta\widetilde{\mathbf{m}} = \arg\min_{\delta\mathbf{m}} \frac{1}{2}\|\delta\underline{\mathbf{d}} - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0;\underline{\mathbf{Q}}]\delta\mathbf{m}\|_2^2$$

$$\begin{aligned}\delta\underline{\mathbf{d}} &= \delta\mathbf{d}\mathbf{W} \ (\text{Simultaneous-source data residue})\\ \underline{\mathbf{Q}} &= \mathbf{Q}\mathbf{W} \ (\text{Simultaneous sources})\end{aligned}$$

[Wang & Sacchi, '07]

Sparse recovery

Least-squares migration with *sparsity* promotion

$$\min_{\delta\mathbf{x}} \frac{1}{2}\|\delta\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\delta\underline{\mathbf{d}} - \nabla\mathcal{F}[\mathbf{m}_0;\underline{\mathbf{Q}}]\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma$$

$$\delta\widetilde{\mathbf{m}} = \mathbf{S}^*\delta\mathbf{x}$$

$\delta\mathbf{x}$ = Sparse curvelet-coefficient vector

$\mathbf{S}^*$ = Curvelet synthesis

Migration results

Time-harmonic Helmholtz:

- 409 X 1401 with mesh size of 5m
- 9 point stencil [C. Jo et. al., ’96]
- absorbing boundary condition with damping layer with thickness proportional to wavelength
- solve wavefields on the fly with direct solver

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Migration results

Split-spread surface-free 'land' acquisition:

- 350 sources with sampling interval 20m
- 701 receivers with sampling interval 10m
- maximal offset 7km (3.5 X depth of model)
- Ricker wavelet with central frequency of 30Hz
- recording time for each shot is 3.6s

Migration results

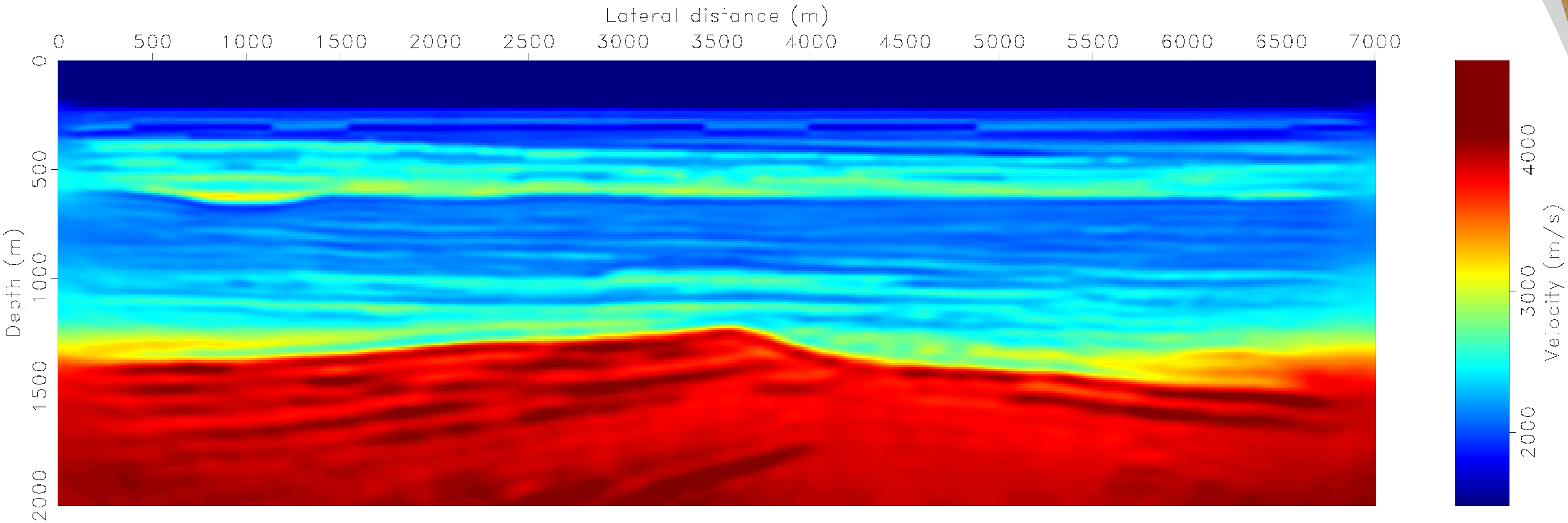
Migration:

- 10 random frequencies (20Hz-50Hz)
- 17 *simultaneous* shots (*versus 350 sequential shots*)
- LASSO problems determined by SPGL1

[Herrmann & Xi, 2011]

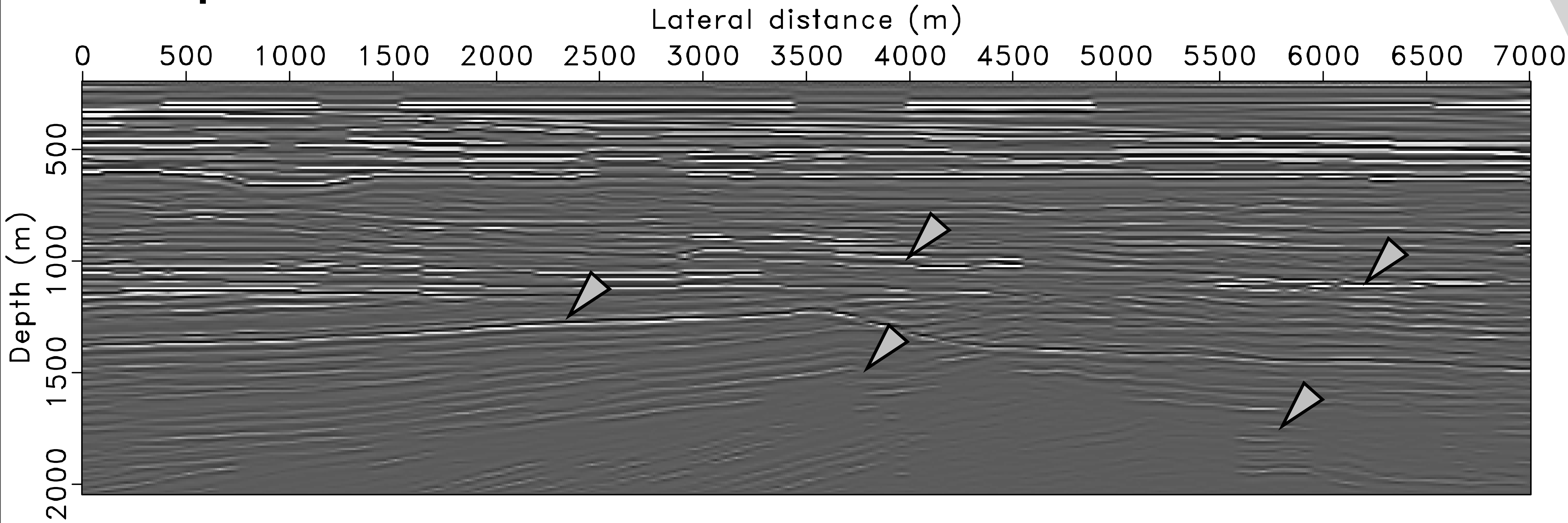
Migration results

true perturbation



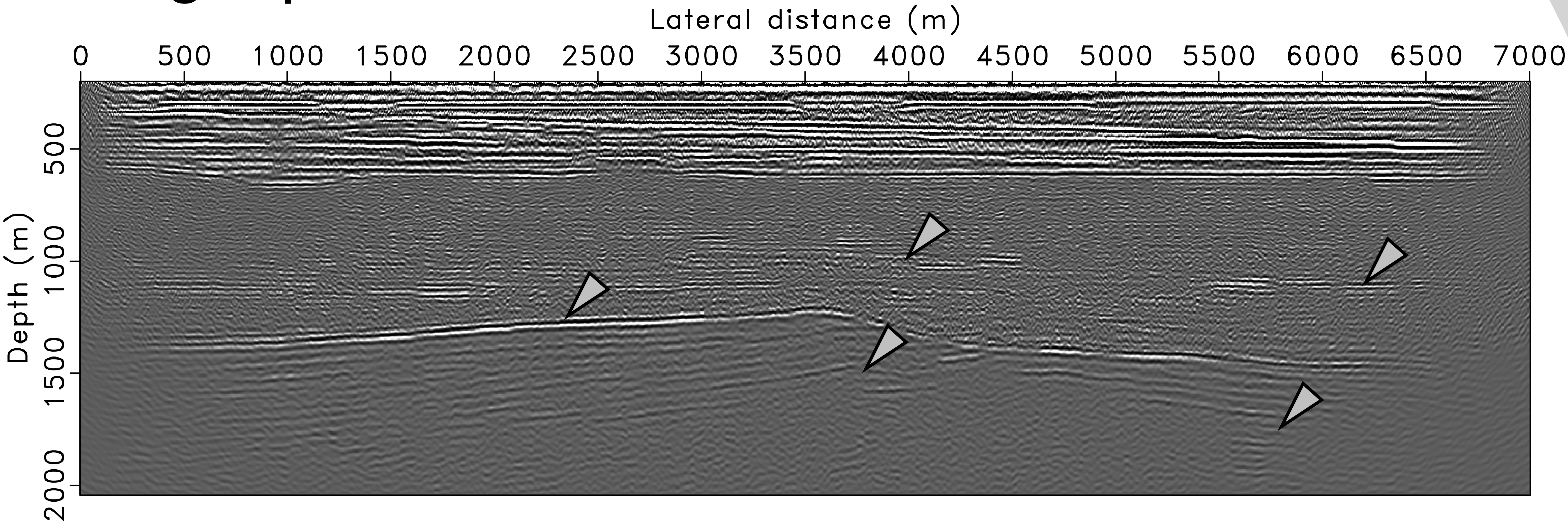
Migration results

true perturbation



Migration results
overdetermined

imaged perturbation with L2

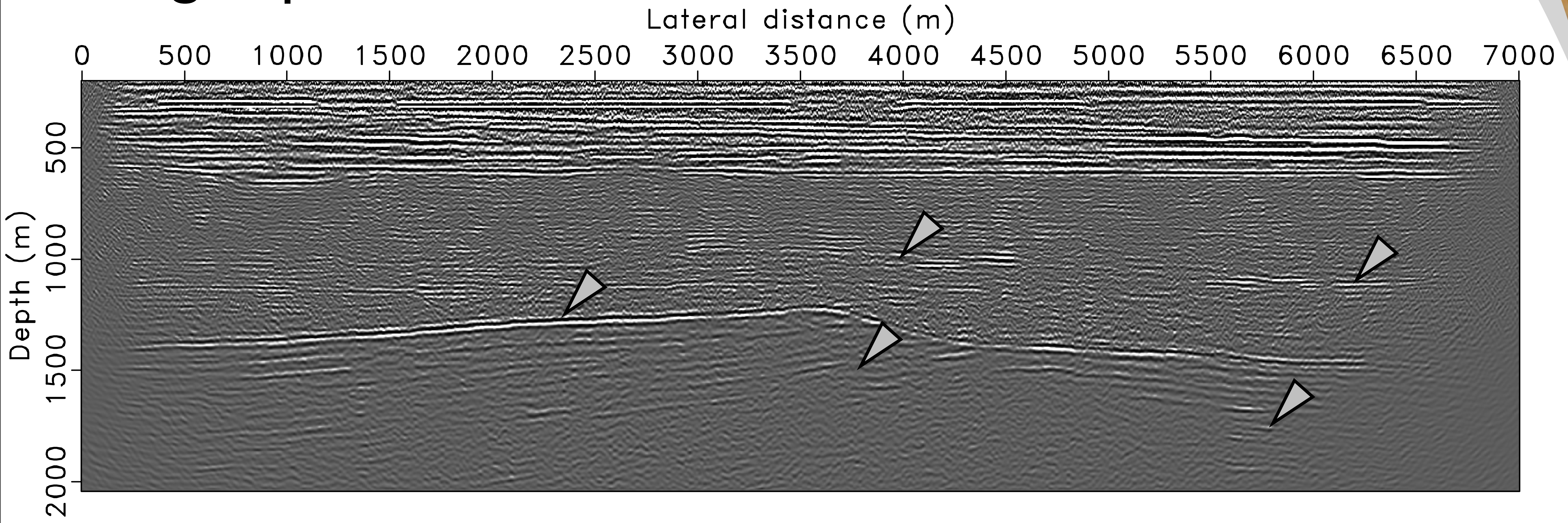


71 *sequential* shots

# of PDEs: 56800

Migration results
underdetermined

imaged perturbation with L1

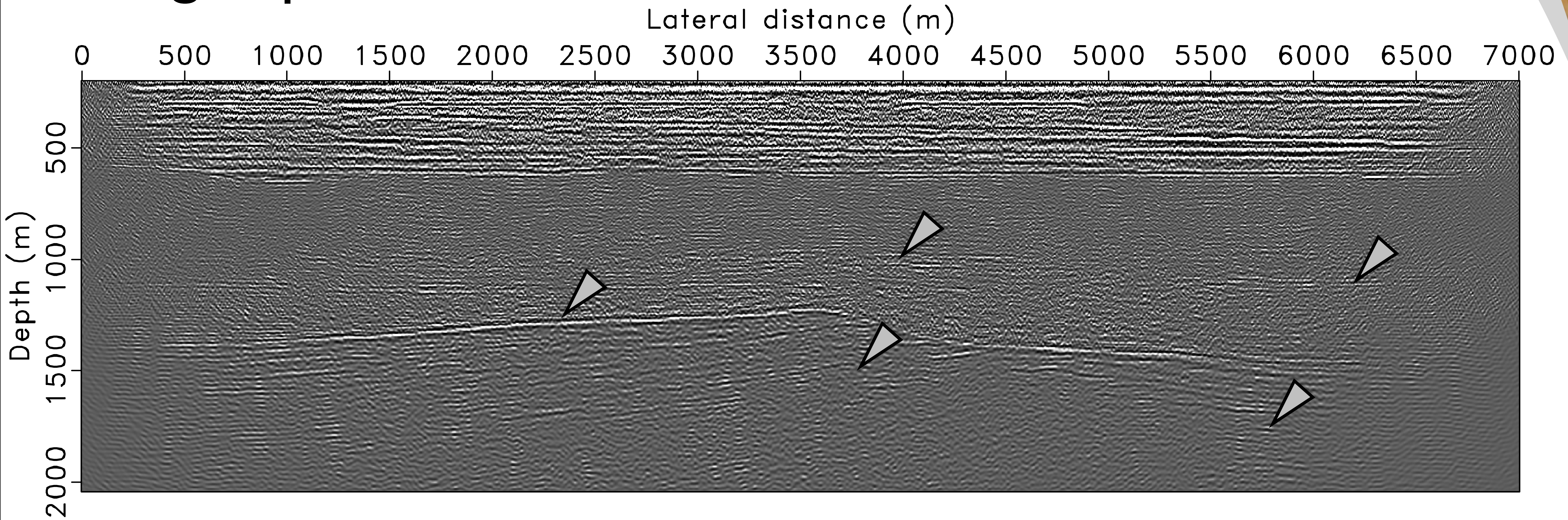


with 17 *simultaneous* shots

# of PDEs: 34000

Migration results

imaged perturbation with L2

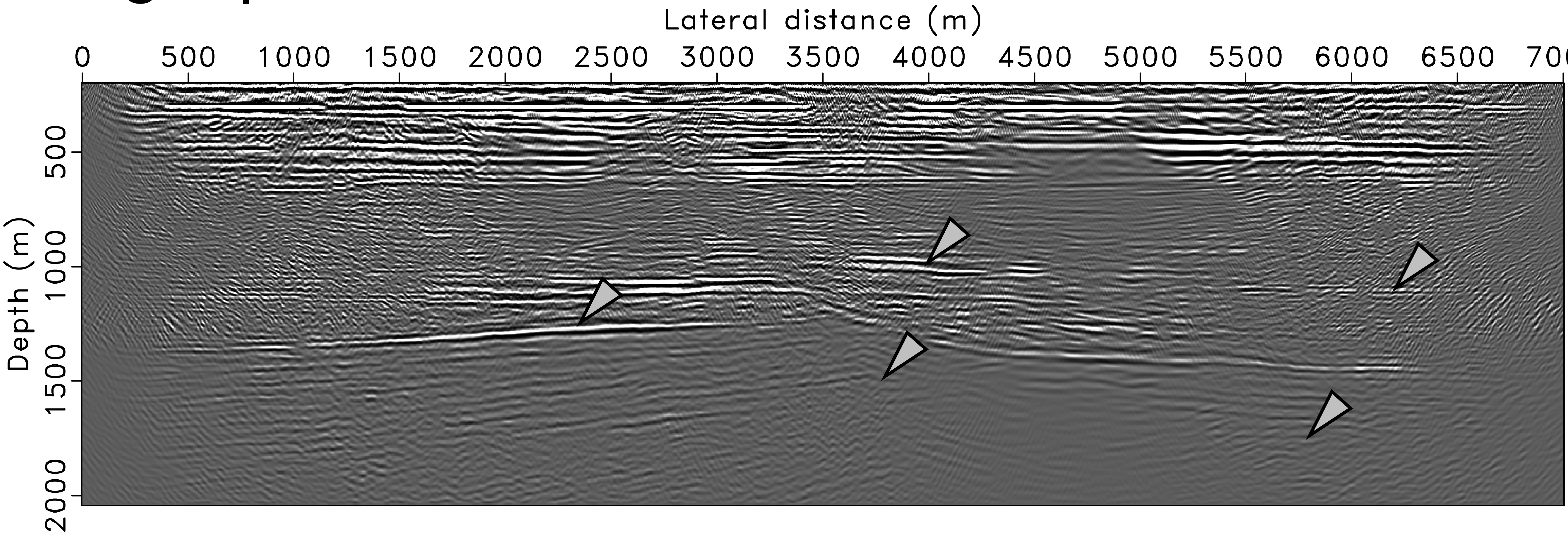


with 17 simultaneous shots

# of PDEs: 34000

Migration results
underdetermined

imaged perturbation with L1



with 17 *sequential* shots

# of PDEs: 34000

Observations

- $\ell 1$ regularization works better than $\ell 2$ for *underdetermined* systems
- Since $\ell 1$ relies on the *sparsity*, it is more *efficient* in removing Gaussian type *noise*
- Migration artifact can be reduced by using *simultaneous* shots instead of *sequential* shots
- By turning *overdetermined* system into *underdetermined* system, we can save on demans for computational resources

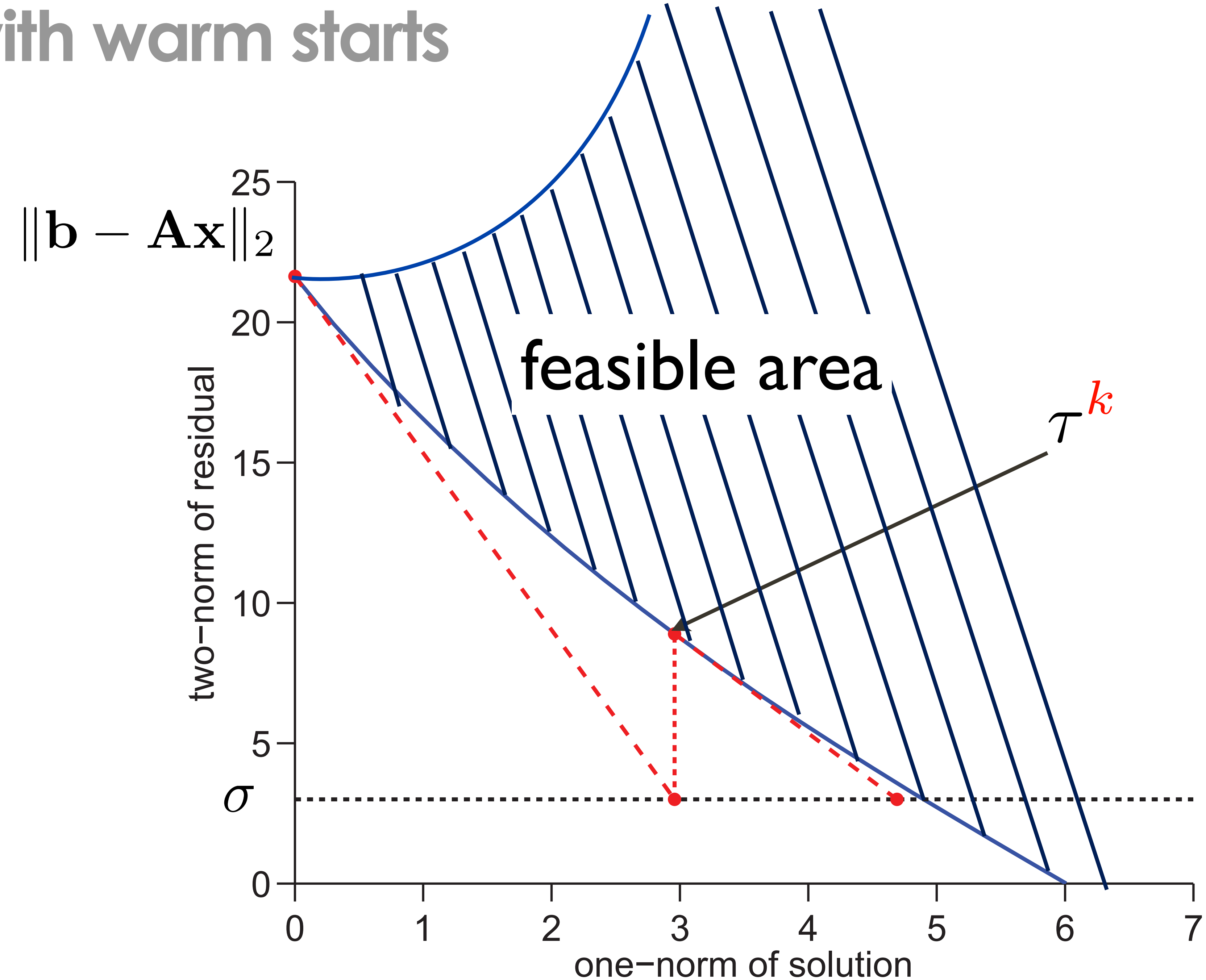
Is this all we can do?

Continuation

Large-scale *sparsity*-promoting solvers *limit* the number of *matrix-vector* multiplies by

- solving an *intelligent* series of LASSO *subproblems* for *decreasing* sparsity levels
- *exploring* properties of the *Pareto* trade-off curve
- slowly allowing *components* to *enter* into the *solution*

Picking Lasso parameter
with warm starts

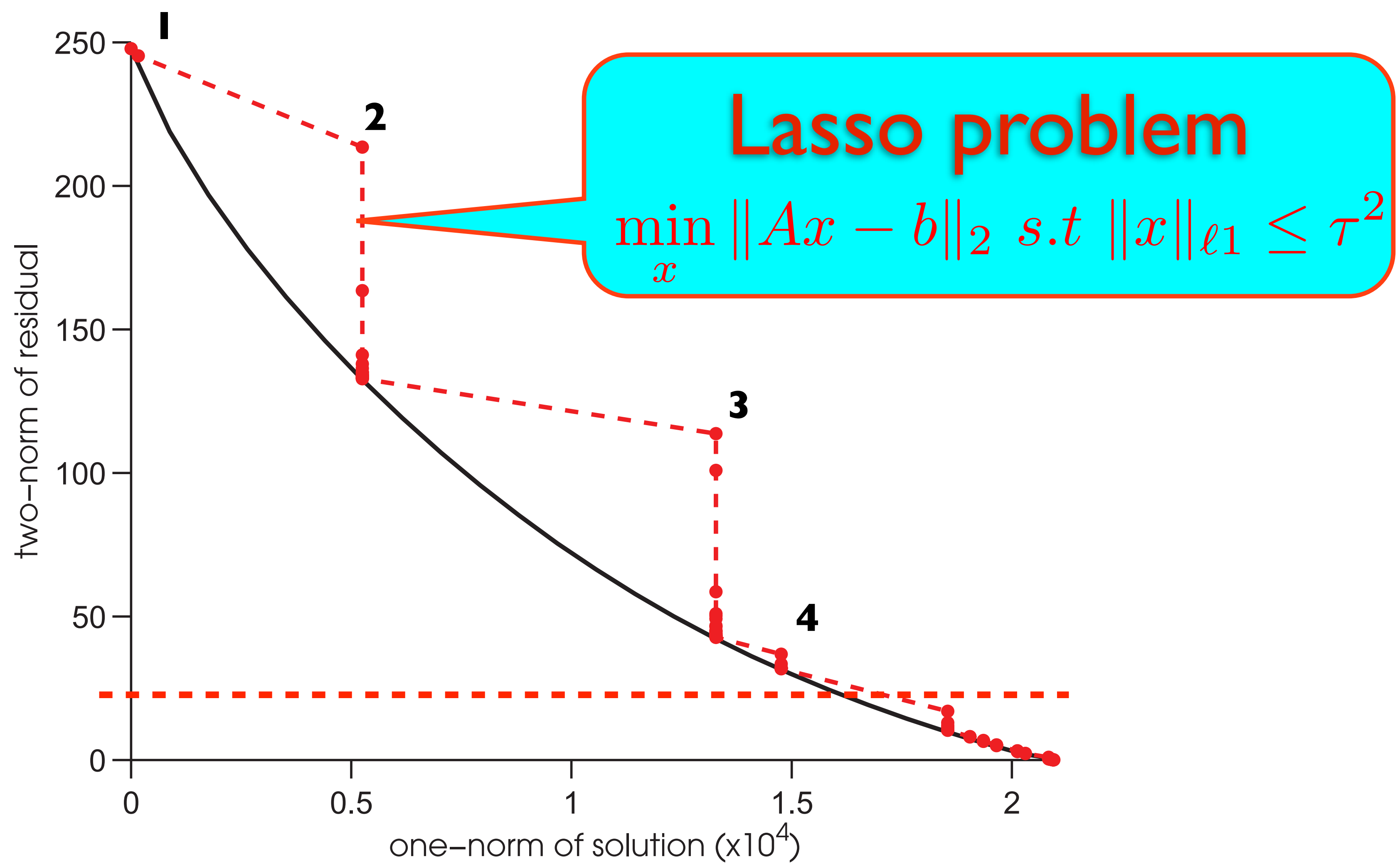


Root finding

$$\text{basis pursuit denoise:} \quad \min \|\mathbf{x}\|_1 \quad s.t \quad \|\mathbf{b}-\mathbf{Ax}\|_2 \leq \sigma$$

Pareto curve

subproblems



[Wang & Sacchi, '07]

Sparsity recovery

This is a BPDN problem:

$$\min_{\delta\mathbf{x}} \tfrac{1}{2}\|\delta\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\delta\underline{\mathbf{d}} - \nabla\mathcal{F}[\mathbf{m}_0;\underline{\mathbf{Q}}]\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma$$

$$\delta\widetilde{\mathbf{m}} = \mathbf{S}^*\delta\mathbf{x}$$

BPDN problem is a *series* of LASSO *subproblems:*

$$\min_{\delta\mathbf{x}} \tfrac{1}{2}\|\delta\underline{\mathbf{d}} - \nabla\mathcal{F}[\mathbf{m}_0;\underline{\mathbf{Q}}]\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma \quad \text{subject to} \quad \|\delta\mathbf{x}\|_{\ell_1} \leq \tau^k$$

Renewals & warm starts

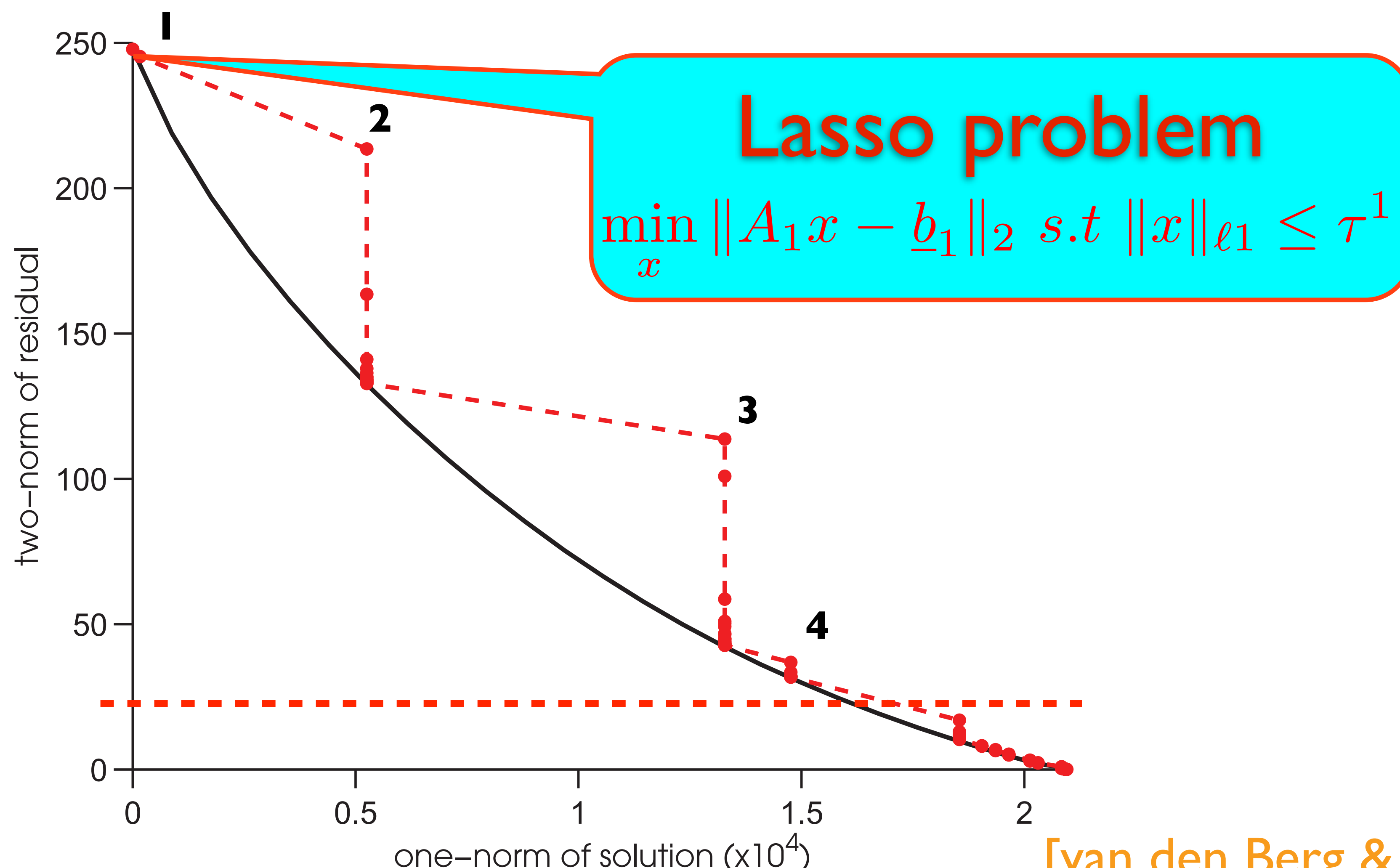
For *each* subproblem:

$$\min_{\delta \mathbf{x}} \frac{1}{2}\|\delta\underline{\mathbf{d}}^{k} - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0; \underline{\mathbf{Q}}^{k}]\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma \quad \text{subject to} \quad \|\delta\mathbf{x}\|_{\ell_1} \leq \tau^{k}$$

- *redraw* a *new* set of *randomized* source *experiments*
- use $\delta\mathbf{x}^{k-1}$ as *warm* start for next *subproblem*

Pareto curve

subproblems



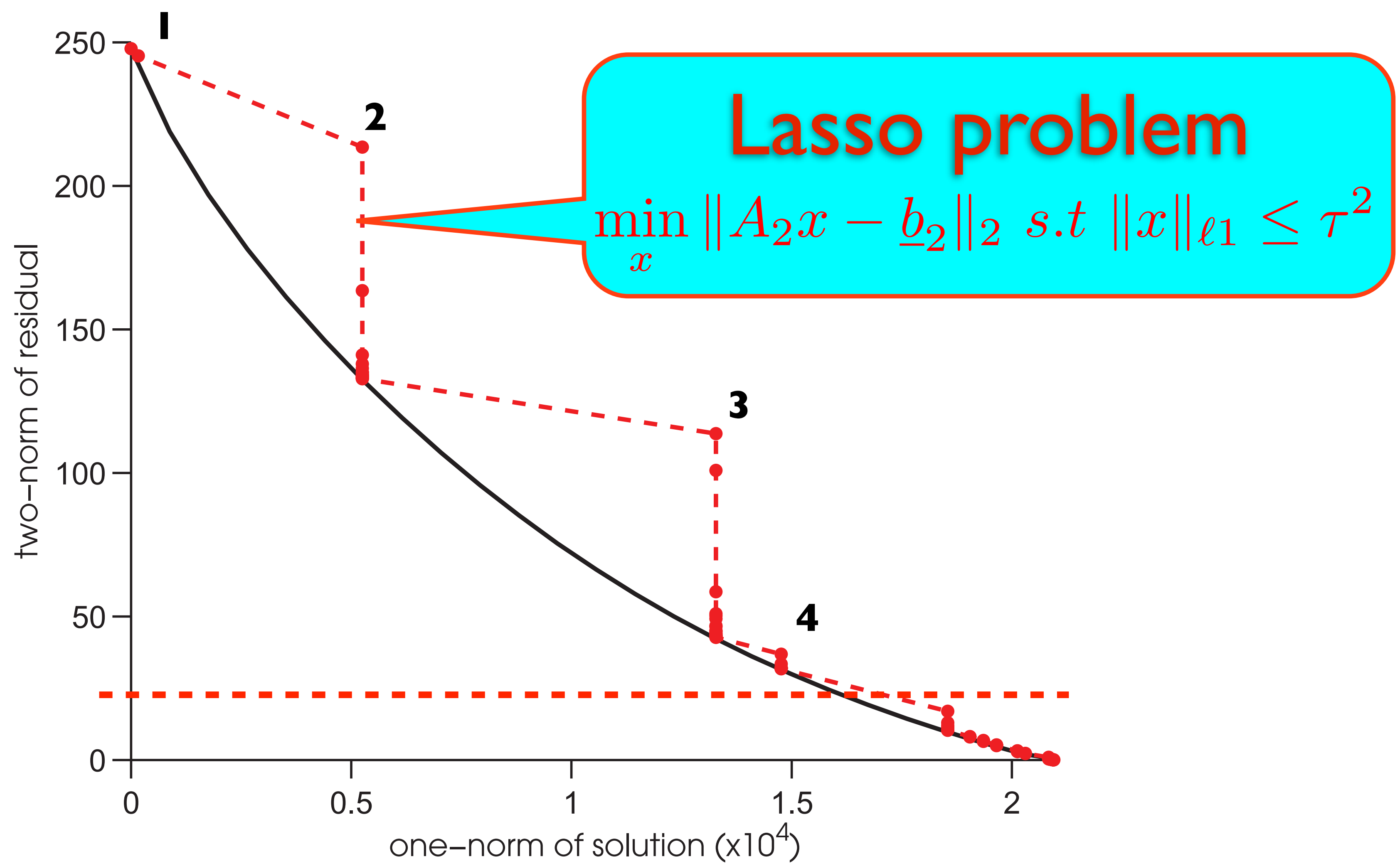
[van den Berg & Friedlander, '08]

[Hennefent et. al., '08]

[Lin & FJH, '09-]

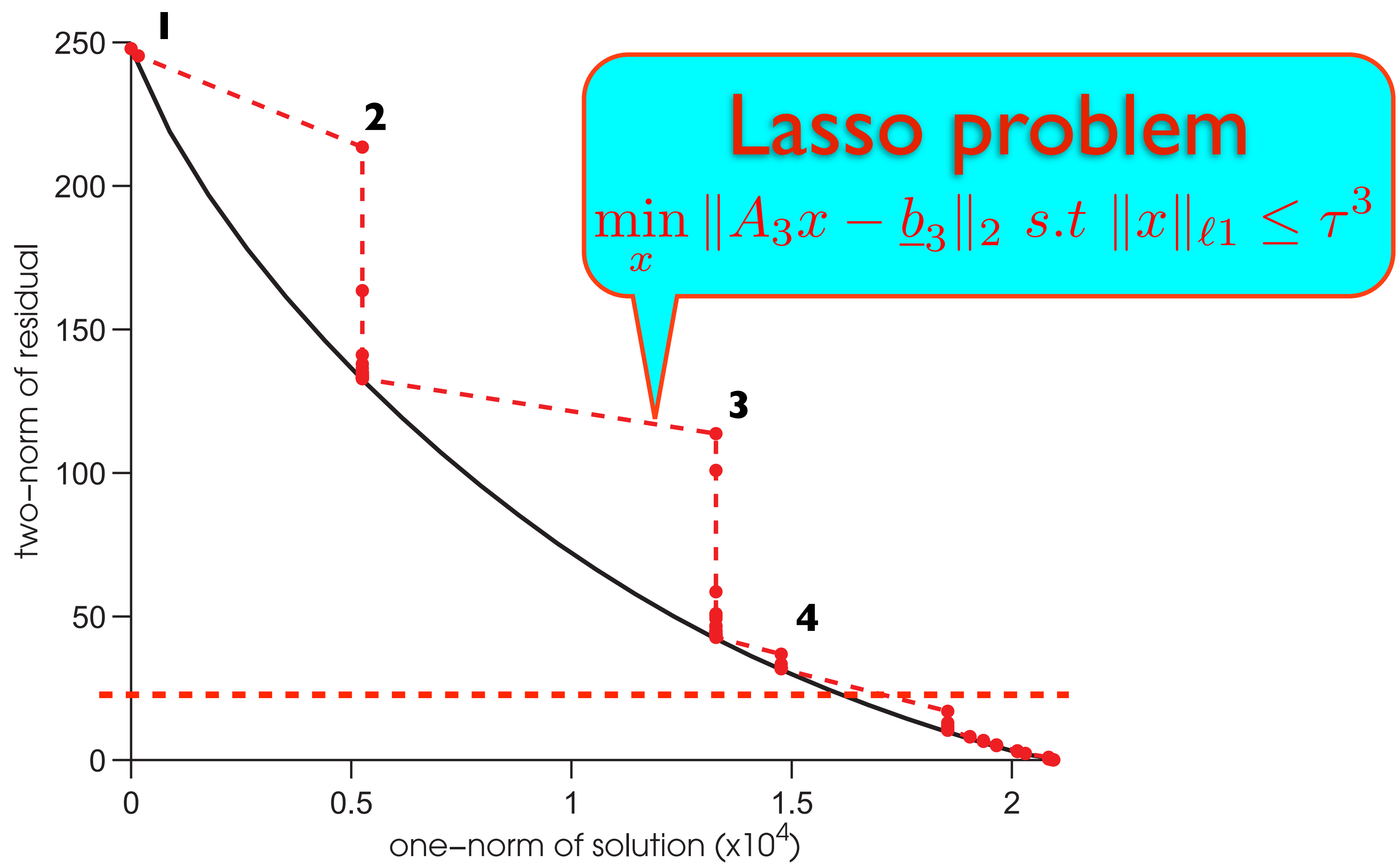
Pareto curve

subproblems



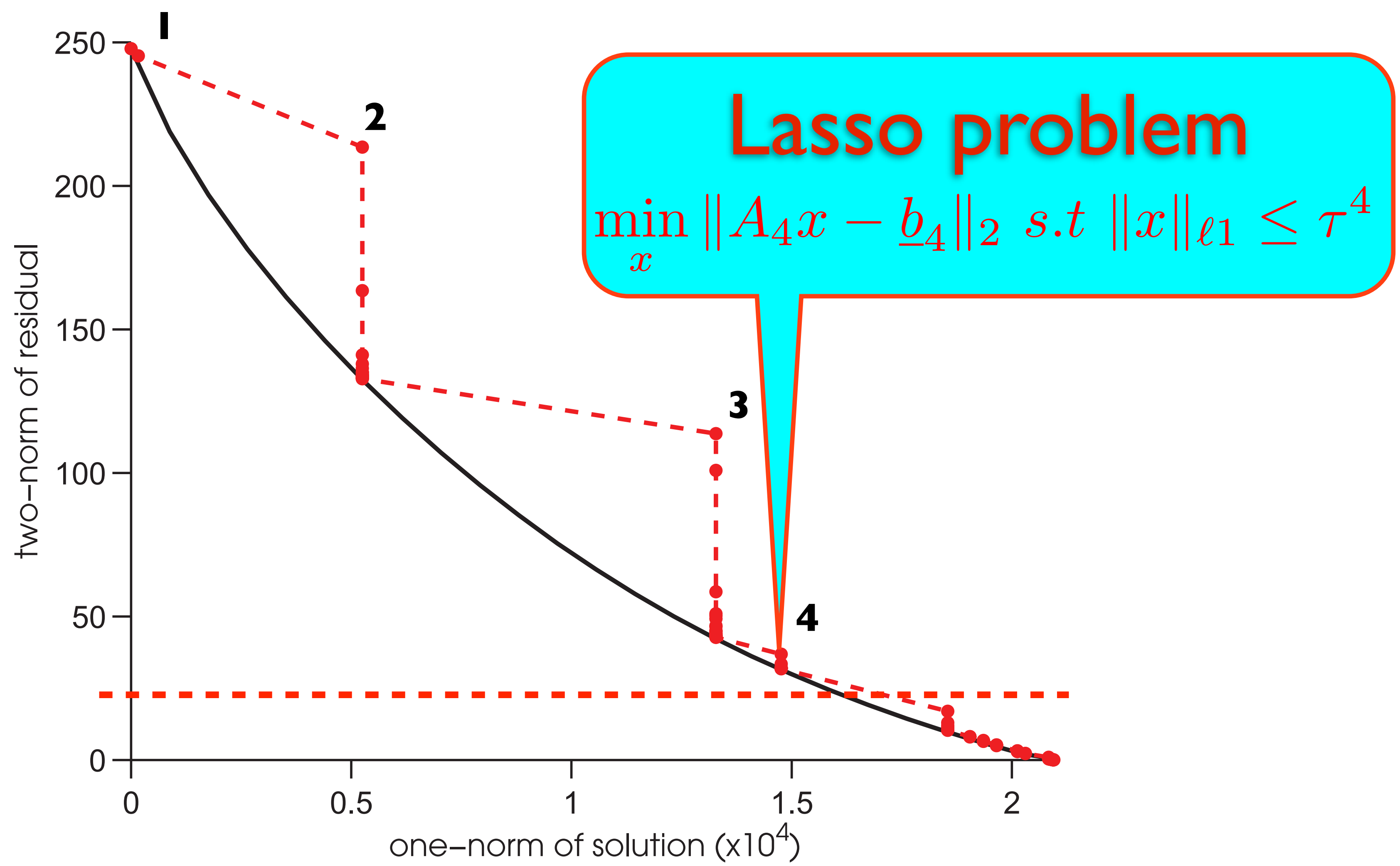
Pareto curve

subproblems

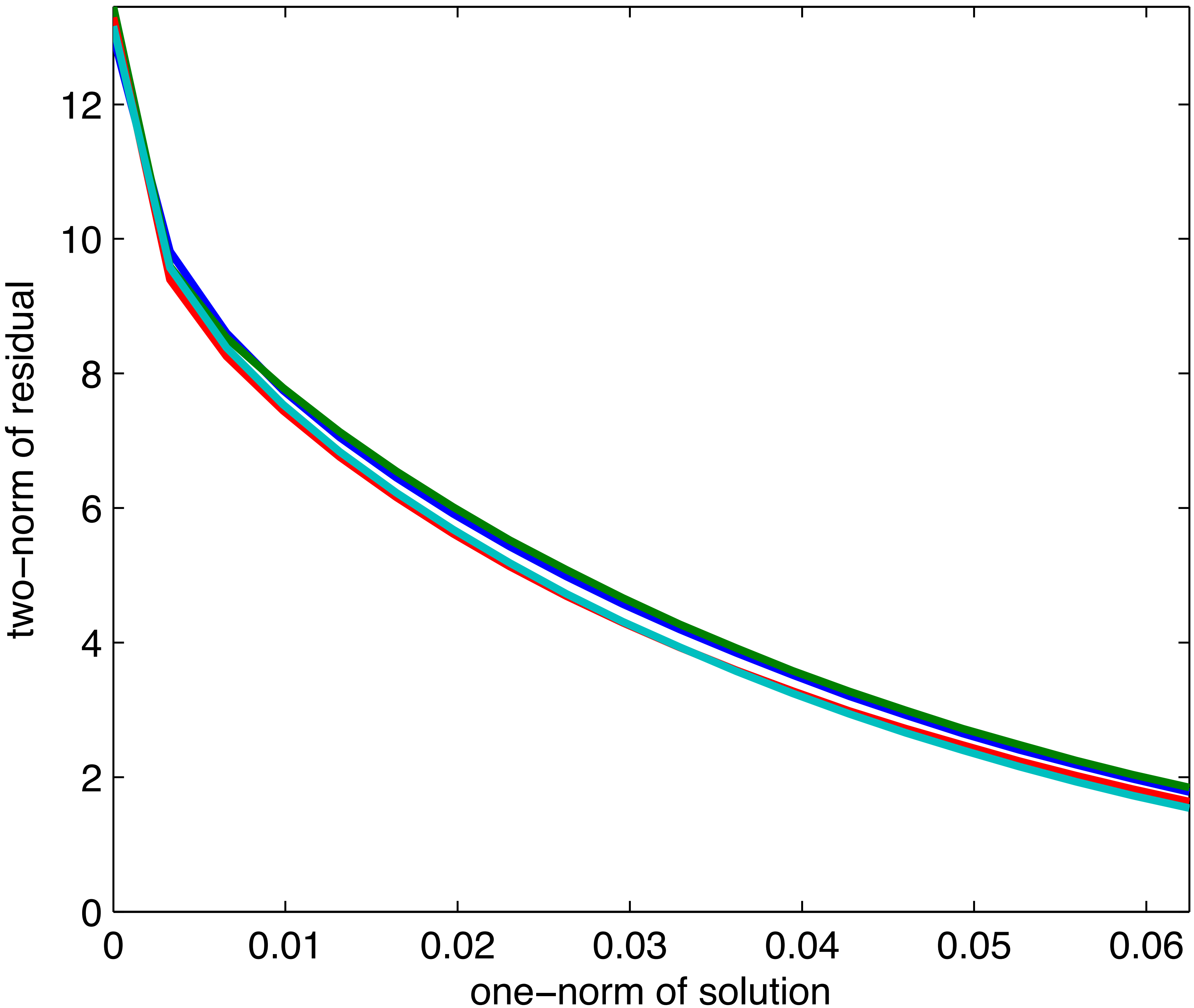


Pareto curve

subproblems



Pareto curves



Continuation methods & *renewals*

Underlying assumption is that Pareto curves are similar

- for large enough batch sizes

In that case the *warm starts* are effective

Renewals remove biases

Migration results

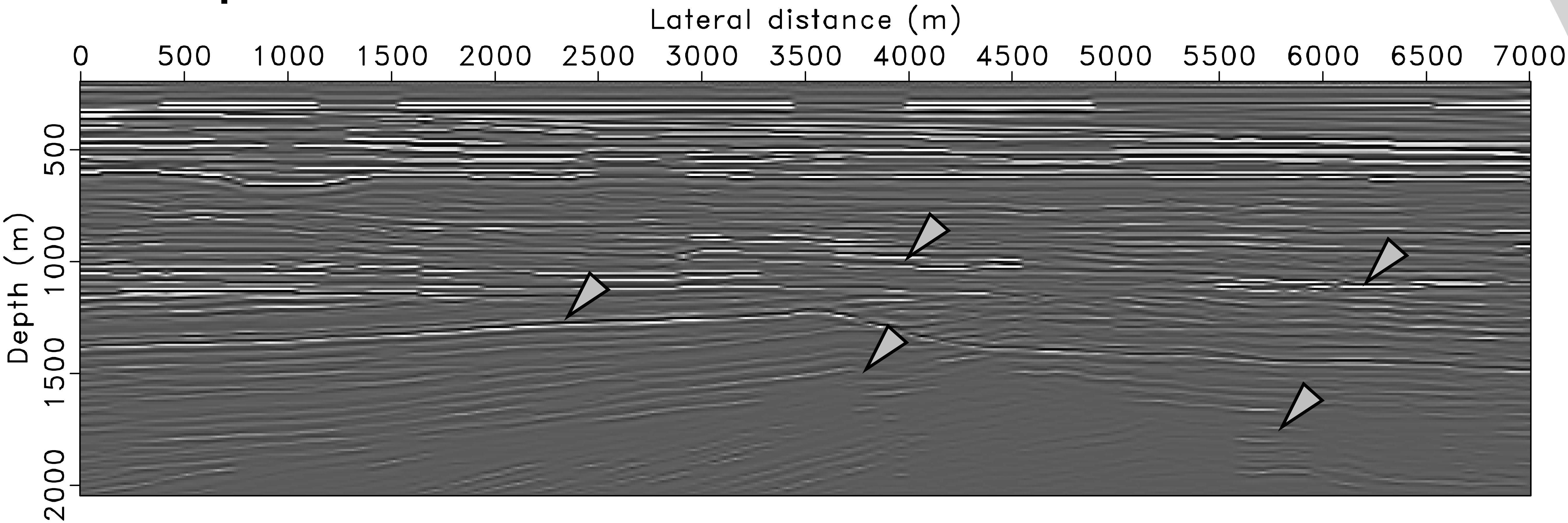
Migration:

- 10 random frequencies (20Hz-50Hz)
- 17 *simultaneous* shots (*versus 350 sequential shots*)
- LASSO problems determined by SPGL1

[Herrmann & Xi, 2011]

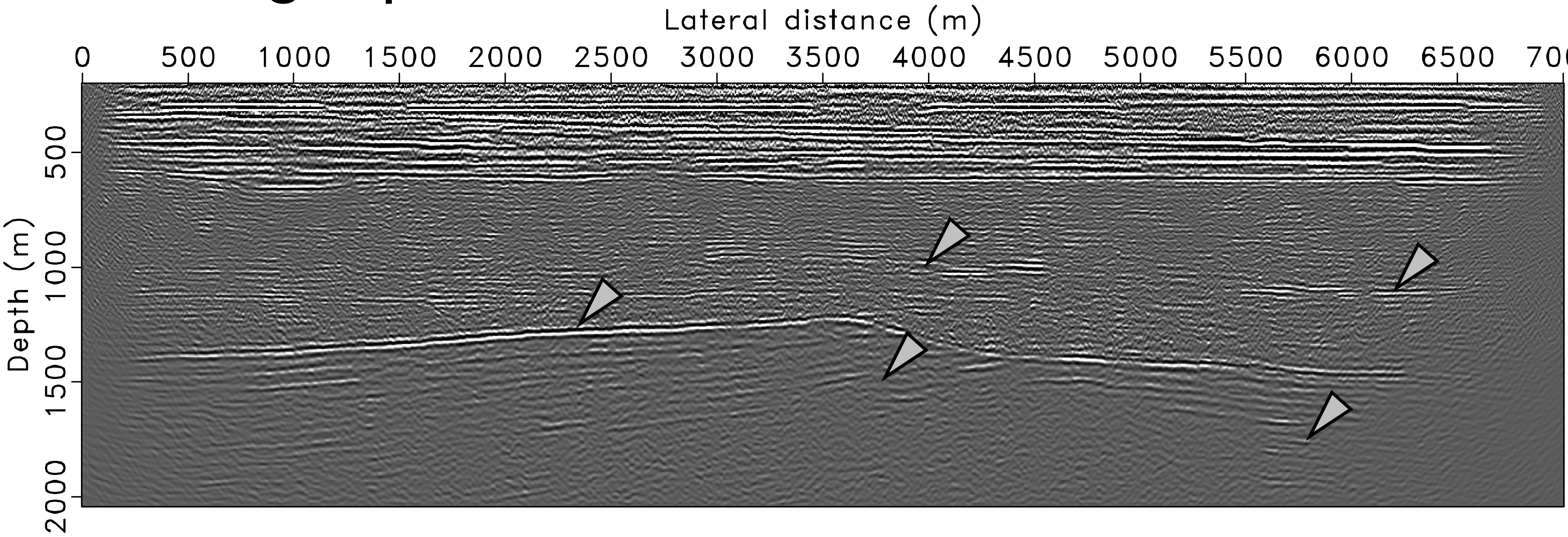
Migration results

true perturbation



Migration results

L1 imaged perturbation *without* renewals



with 17 simultaneous shots

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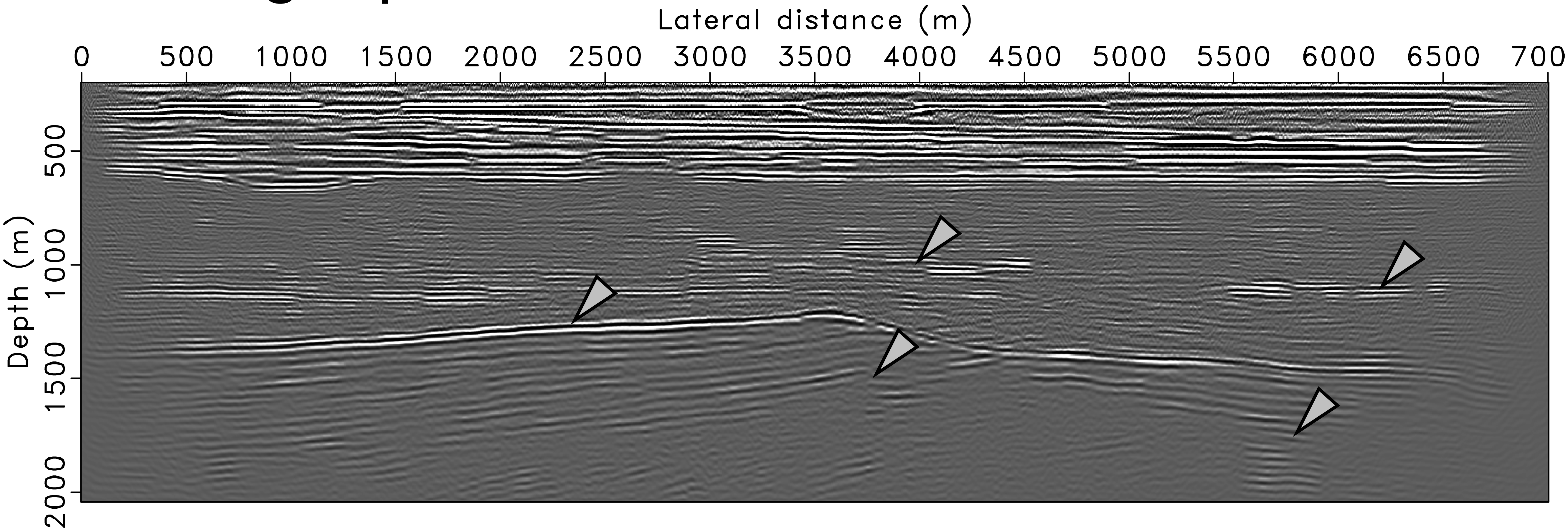
Migration results

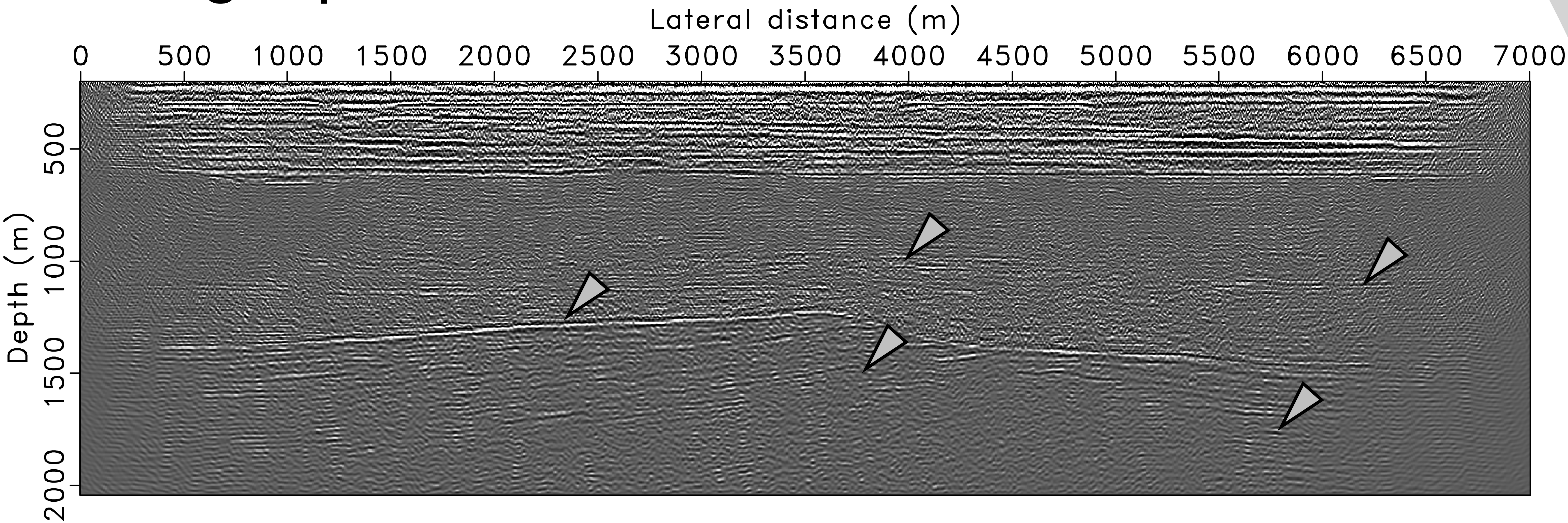
L1 imaged perturbation *with* renewals



with 17 *simultaneous* shots

Migration results

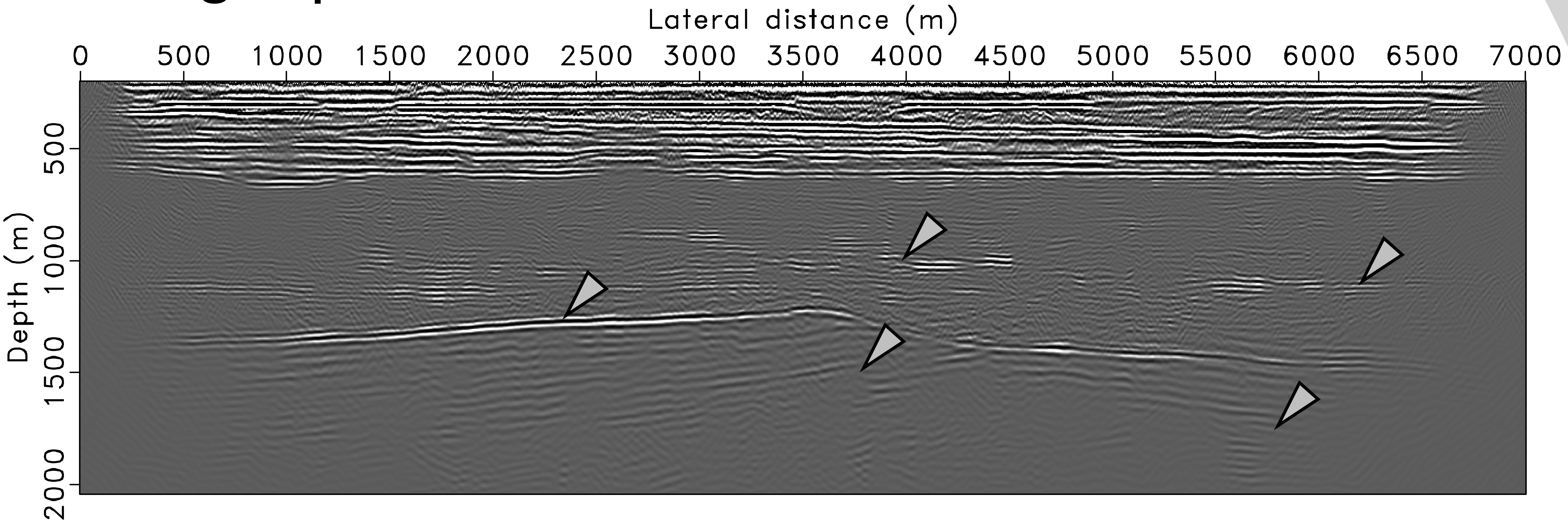
imaged perturbation with L2 *without* renewals



with 17 *simultaneous* shots

Migration results

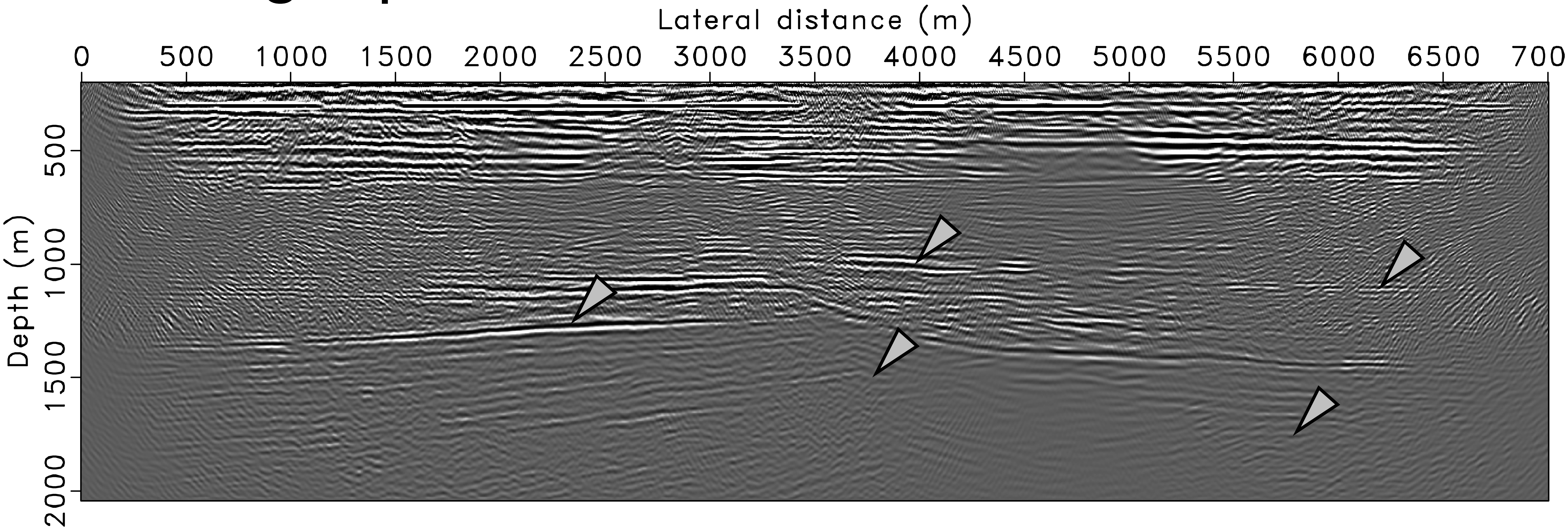
imaged perturbation with L2 *with* renewals



with 17 *simultaneous* shots

Migration results

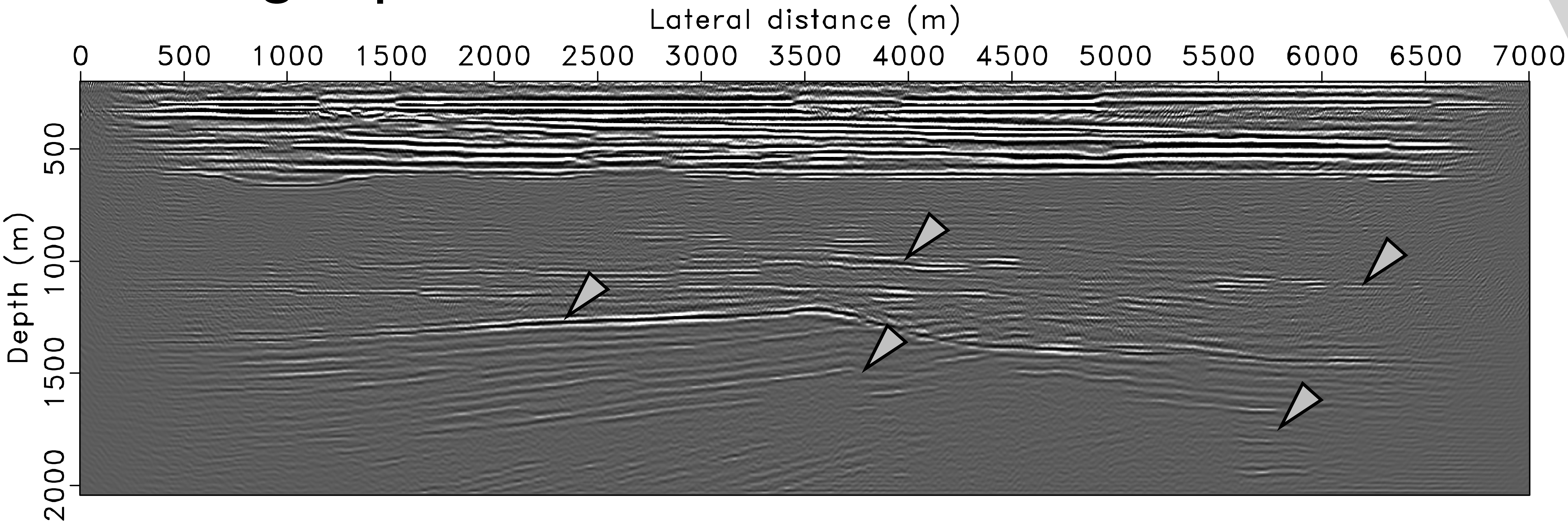
L1 imaged perturbation *without* renewals



with same # of *randomly* selected *sequential* shots

Migration results

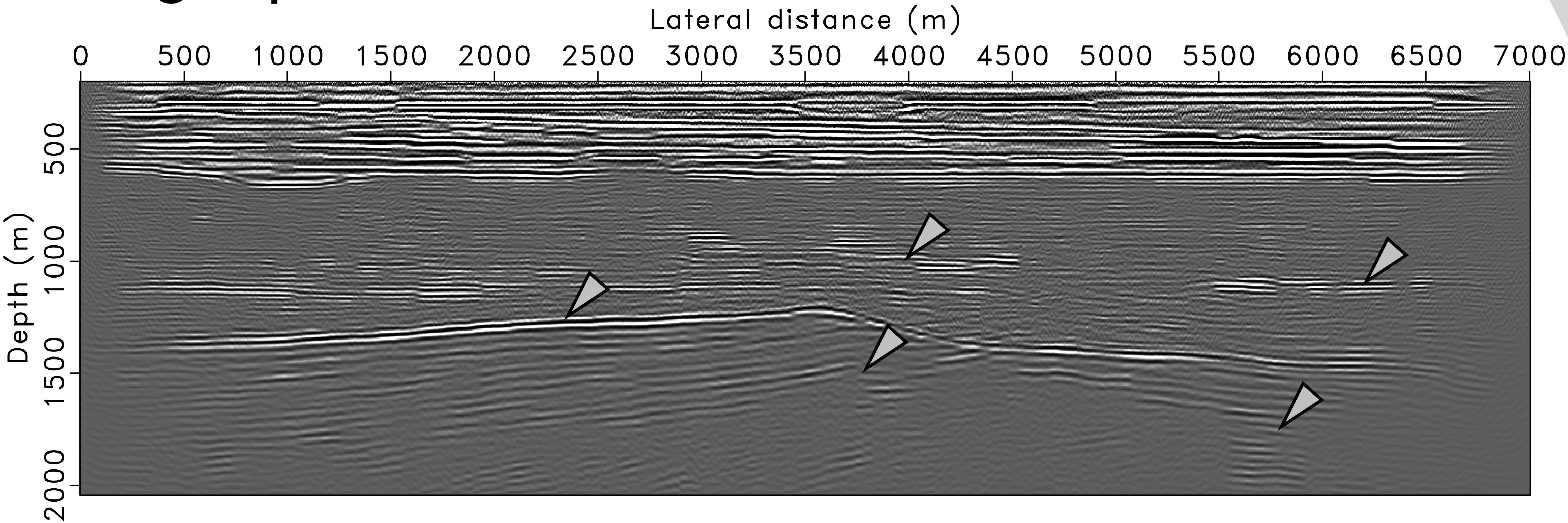
L1 imaged perturbation *with* renewals



with same # of *randomly* selected *sequential* shots

Migration results

imaged perturbation *with renewals*



with 17 *simultaneous* shots

Conclusions

Computational cost can be *reduced* significantly by using *randomized dimensionality reduction*

Underdetermined system can be solved by *sparsity* promotion in a *sparsifying* (e.g curvelet) domain

Within the same *computational* cost, simultaneous shots produce *less* migration *artifacts*

Source cross-talk bias can be *removed* by *renewals* & *warm starts*.

Imaged reflectors in GN updates are *compressible* in the *curvelet* domain

Acknowledgments

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NSERC CRSNG

Thank you

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