Dimensionality-reduced estimation of *primaries* by *sparse* inversion

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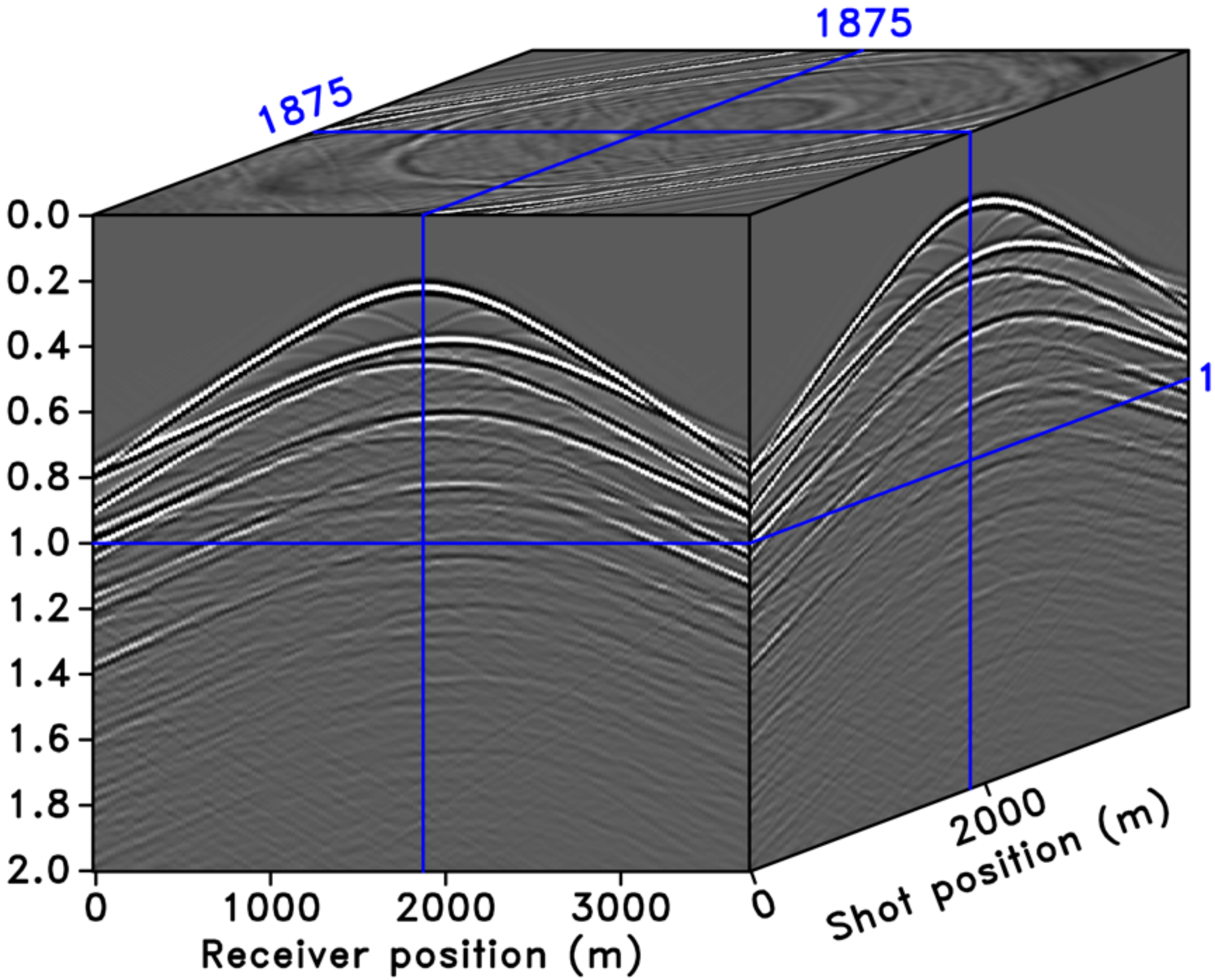


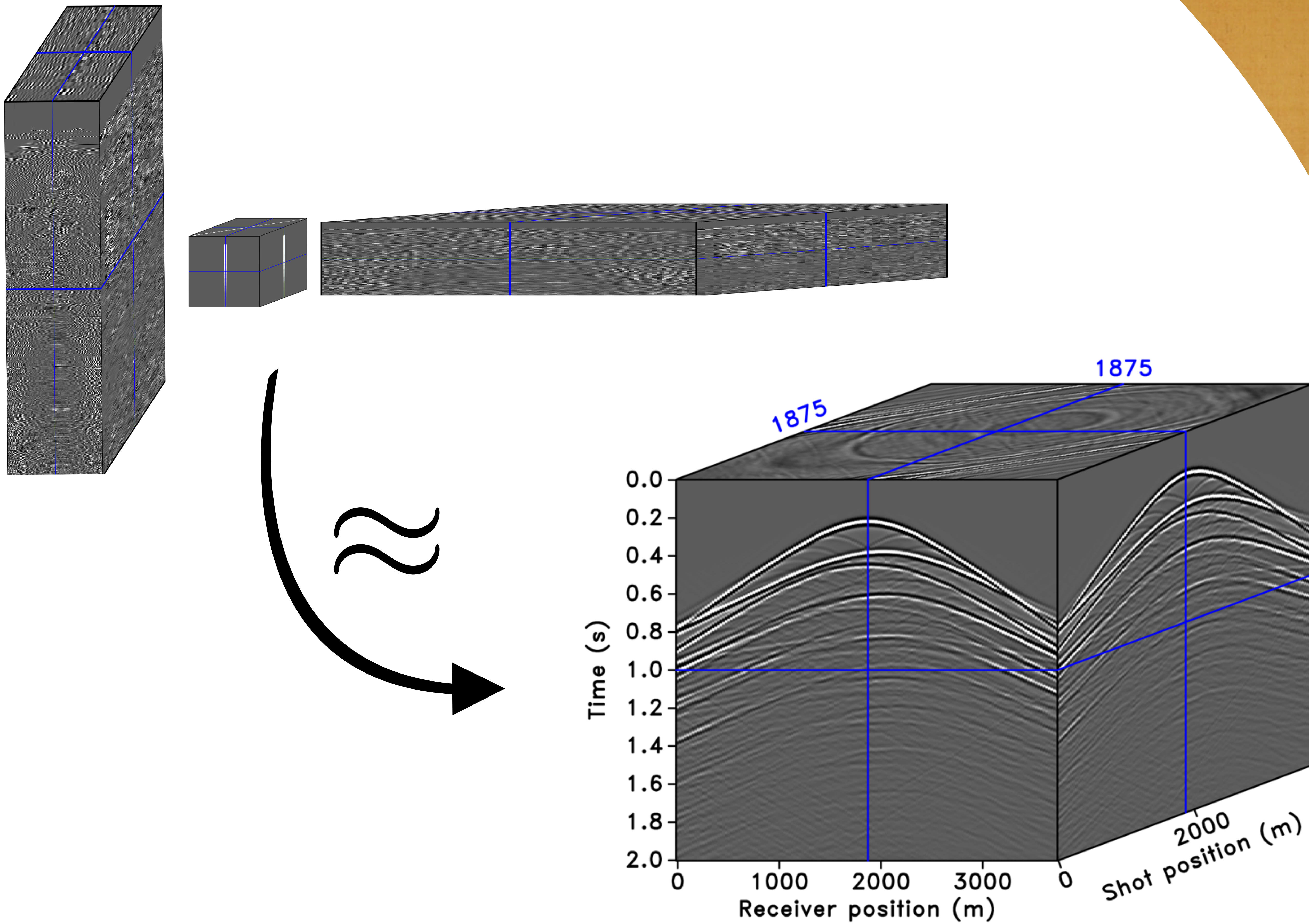
Seismic Data

2D data

- 3D - representations
- dense operations

3D data

- 5D - representations
- giga-bytes to tera-bytes


≈
1875
1875
1
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Time (s)
0
1000
2000
3000
Receiver position (m)
0
2000
Shot position (m)

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Motivation

Data-driven methods

- Estimation of Primaries by Sparse Inversion (EPSI)
- Surface-Related Multiple Elimination (SRME)

Curse of dimensionality

- In 3D these methods *suffer* from *exponential* growth in computational & storage *demands*

[Herrmann, F. J., 2010, Randomized sampling and sparsity]

Objective

Reduction in *computational* and *storage* demands:

- *dimensionality*-reduction technique
- *adaptive* low-rank approximation
- *black-box* implementation

EPSI problem

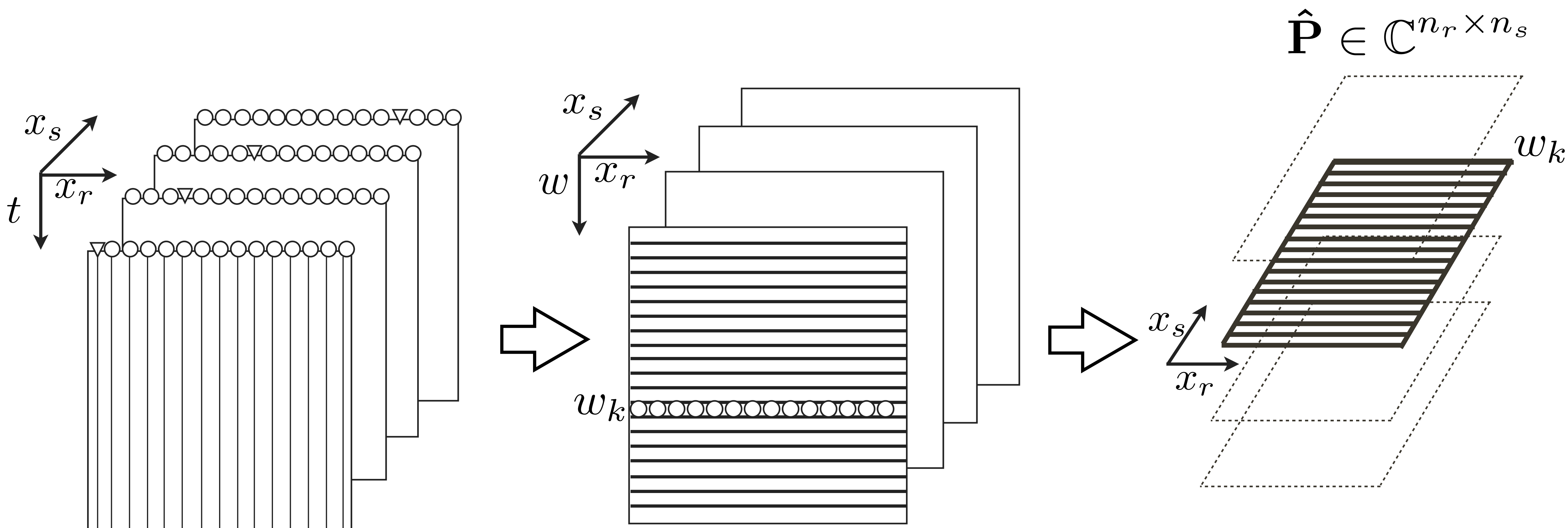
recorded data predicted data

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} + \hat{\mathbf{R}}\hat{\mathbf{P}})$$

$\hat{\mathbf{P}}$	total up-going wave-field
$\hat{\mathbf{Q}}$	down-going source signature
$\hat{\mathbf{R}}$	reflectivity of free surface
$\hat{\mathbf{G}}$	surface-free Green's function

[van Groenestijn and Verschuur, 2009]

Monochromatic "data matrices"



Fourier transform

Extract monochromatic shot records

[Ewoud van Dedem, 2002]

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EPSI problem

recorded data

predicted data

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

$\hat{\mathbf{P}}$ matrix (known)

$\hat{\mathbf{Q}}$ full-rank diagonal matrix (known)

$\hat{\mathbf{R}}$ assume $-\mathbf{I}$

$\hat{\mathbf{G}}$ unknown

EPSI problem

$$\mathbf{Ax} \approx \mathbf{b}$$

$$\tilde{\mathbf{x}} = \underbrace{\arg\min_{\mathbf{x}} ||\mathbf{x}||_{\mathbf{1}}}_{\text{sparsity promoting part}} \quad \text{subject to} \quad \underbrace{||\mathbf{Ax} - \mathbf{b}||_{\mathbf{2}} \leq \sigma}_{\text{data fitting part}}$$

σ : residual between the recorded & predicted data

EPSI problem

EPSI *linear* algebra format:

$$\underbrace{\mathbf{F}_t^* \begin{bmatrix} \left((\widehat{\mathbf{Q}} - \widehat{\mathbf{P}})_1^* \otimes \mathbf{I}\right) & & \\ & \ddots & \\ & & \left((\widehat{\mathbf{Q}} - \widehat{\mathbf{P}})_{n_f}^* \otimes \mathbf{I}\right) \end{bmatrix} \mathbf{F}_t}_{\mathbf{U}} \begin{bmatrix} \mathrm{vec}\,(\mathbf{G}_1) \\ \vdots \\ \mathrm{vec}\,(\mathbf{G}_{n_t}) \end{bmatrix} \approx \begin{bmatrix} \mathrm{vec}\,(\mathbf{P}_1) \\ \vdots \\ \mathrm{vec}\,(\mathbf{P}_{n_t}) \end{bmatrix}$$

$\mathbf{A} = \mathbf{U}\mathbf{C}^*$ $\mathbf{C}$ is curvelet-wavelet transform

$\mathbf{x}$: discrete curvelet-wavelet representation of $\mathbf{G}$

$\mathbf{b}$: vectorized representation of $\mathbf{P}$

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EPSI problem

Data matrix $\hat{\mathbf{P}}$

- dense
- extremely large

  for each 3D frequency is $10^6 \times 10^6$ where $n_r = n_s = 1000$
- expensive to access & store
- high mat-mat multiplication cost $O(N^3)$

EPSI problem

Challenges in solving the optimization problem

- multiple iterations
- multiple evaluations of $\mathbf{A}$, $\mathbf{A}^*$ and $\mathbf{A}^*\mathbf{A}$

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Dimensionality-reduction via SVD

Approximate data matrix $\hat{\mathrm{P}}$ with *low-rank* factorization:

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

$$\hat{\mathbf{P}} \approx \mathbf{USV}^*$$

$\mathbf{U_{n_r \times k}}$ left singular vectors

$\mathbf{S_{k \times k}}$ singular values

$\mathbf{V^*_{k \times n_s}}$ right singular vectors

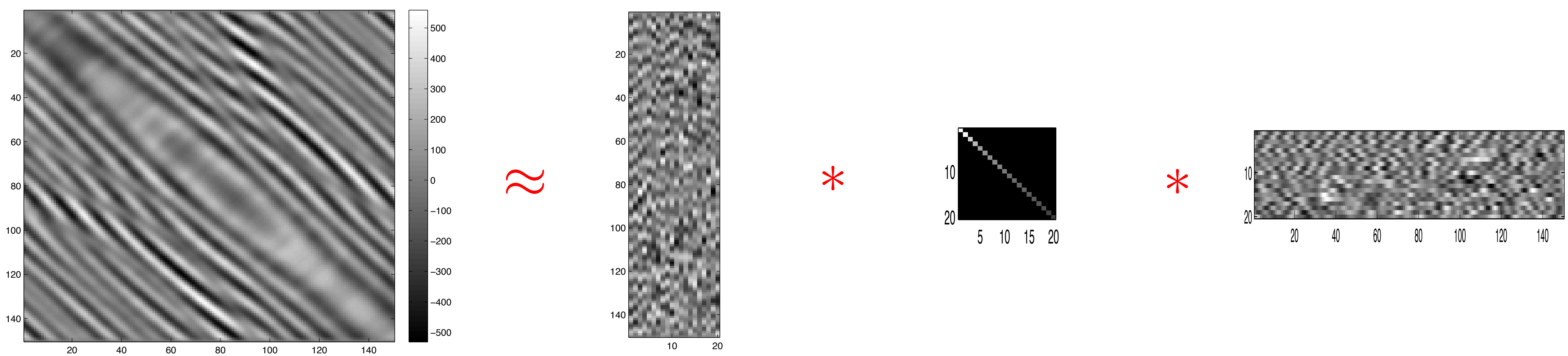
k : approximate rank

$k << min(n_r, n_s)$

Dimensionality-reduction via SVD

Approximate data matrix $\hat{\mathbf{P}}$ with *low-rank* factorization:

$\hat{\mathbf{P}}$ $\qquad$ $\mathbf{U}$ $\qquad$ $\mathbf{S}$ $\qquad$ $\mathbf{V}^*$



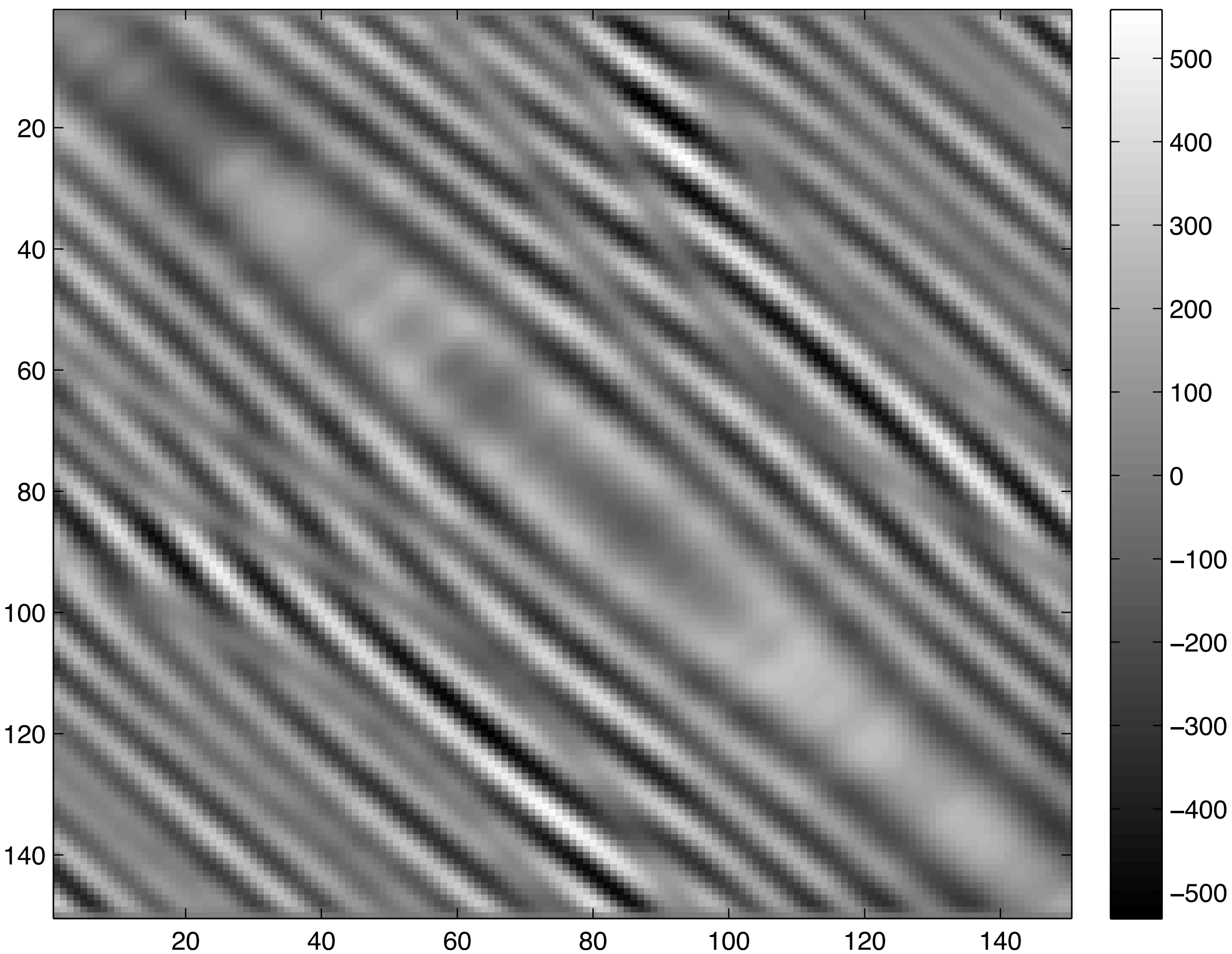
$n_r \times n_s$ $\qquad$ $n_r \times k$ $\qquad$ $k \times k$ $\qquad$ $k \times n_s$

k : approximate rank

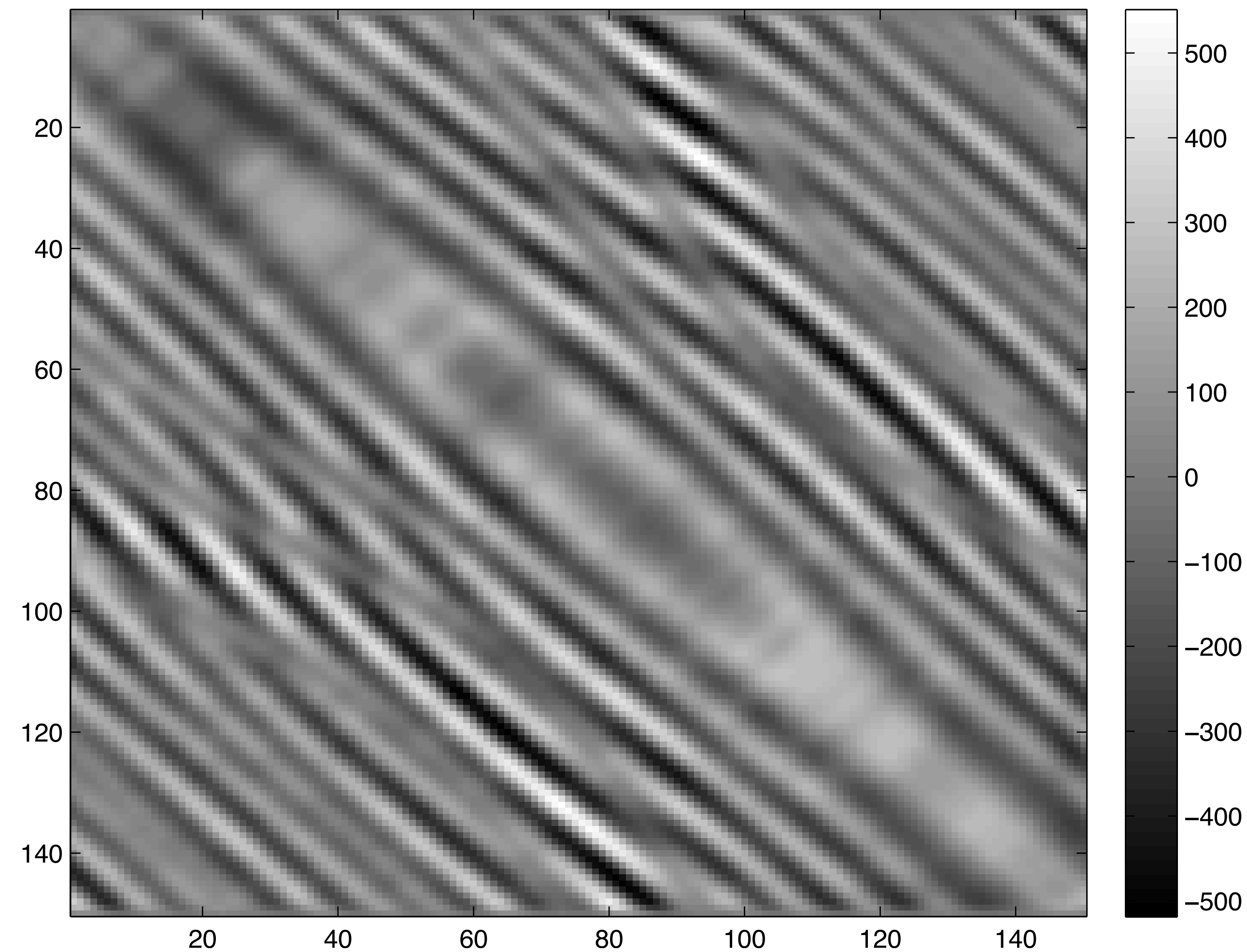
$k << min(n_r, n_s)$

Full vs approximated data

$\hat{\mathbf{P}}$



Approximated $\hat{\mathbf{P}}$



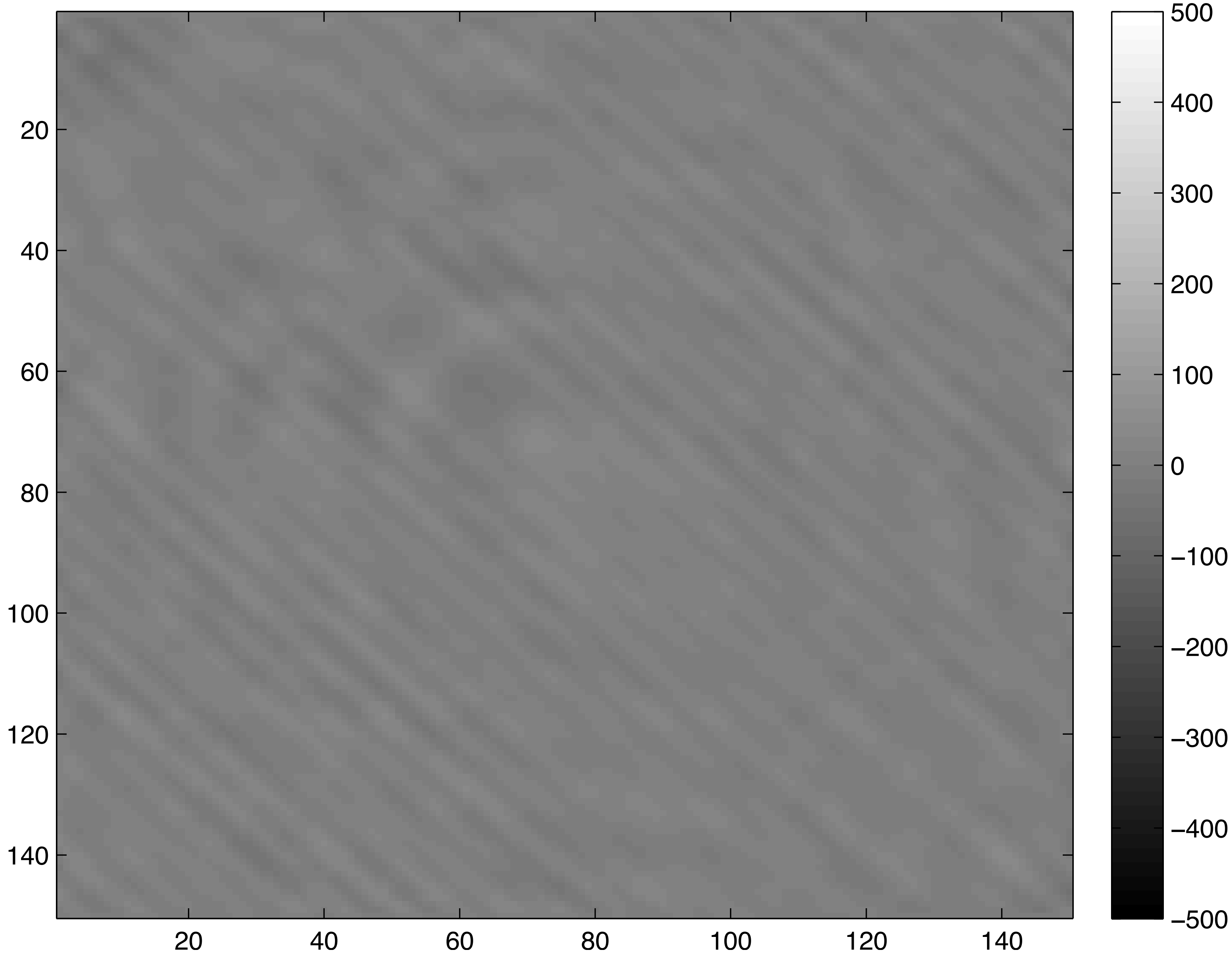
$n_s = n_r = 150$

$k = 20 = 14\%$

$SNR = 16dB$

Full vs approximated data

$$\widehat{\mathbf{P}} - \text{ approximated } \widehat{\mathbf{P}}$$



SNR = 16 dB

Multiplication speed up

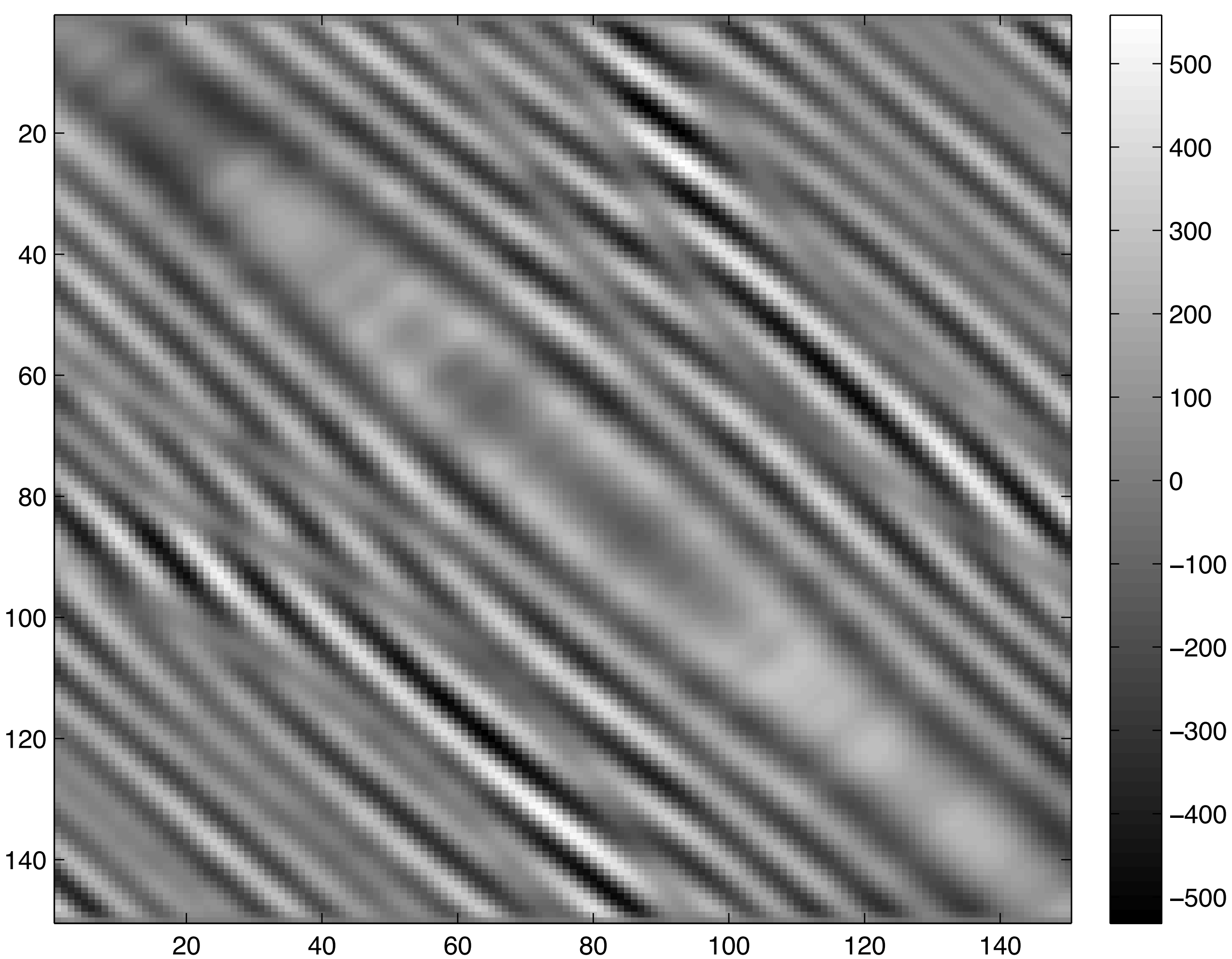
7.5x

Memory usage

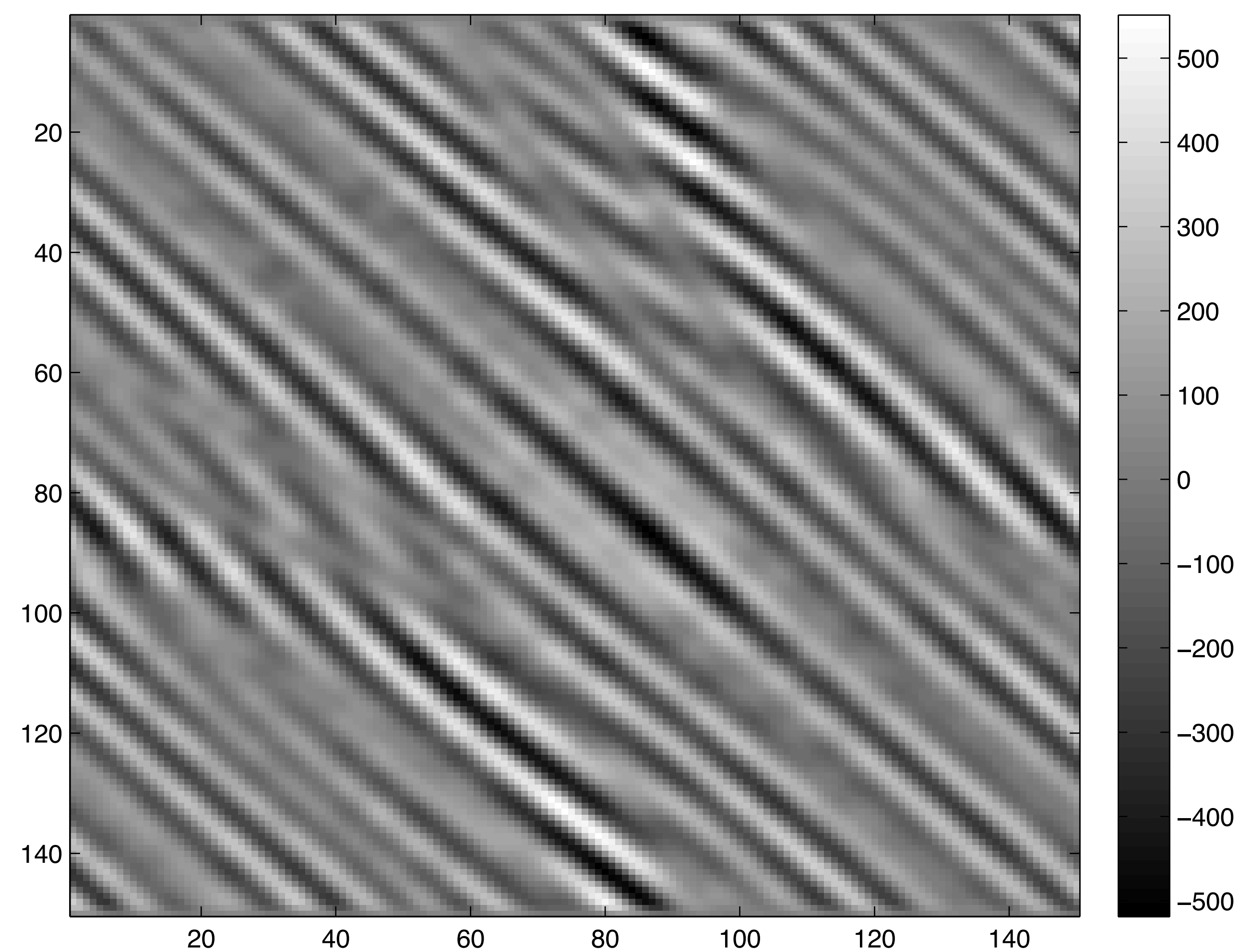
70% less

Full vs approximated data

$\hat{\mathbf{P}}$

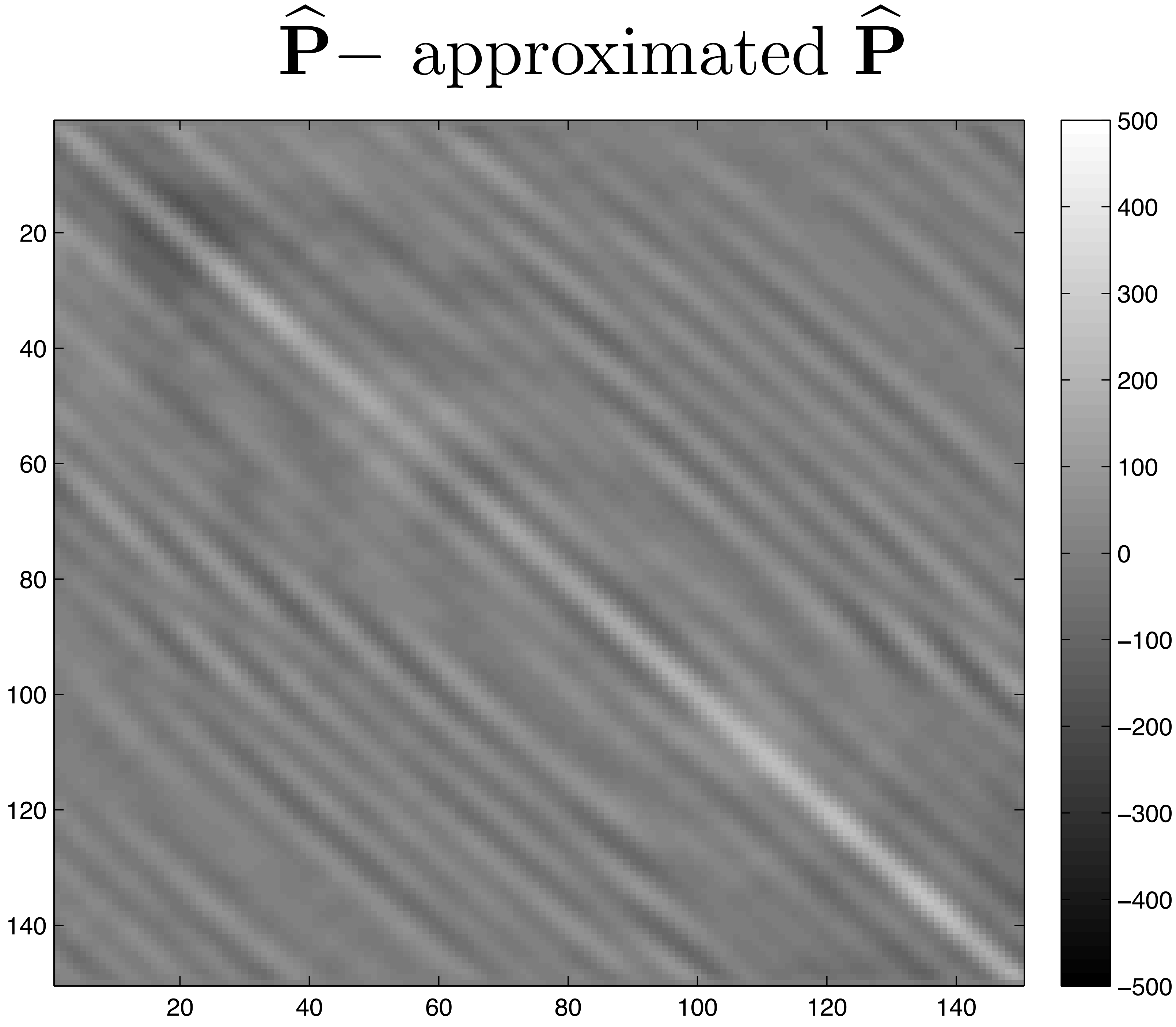


Approximated $\hat{\mathbf{P}}$



$$n_s = n_r = 150$$
$$k = 8 = 5\%$$
$$SNR = 8dB$$

Full vs approximated data



SNR = 8 dB

Multiplication speed up

18x

Memory usage

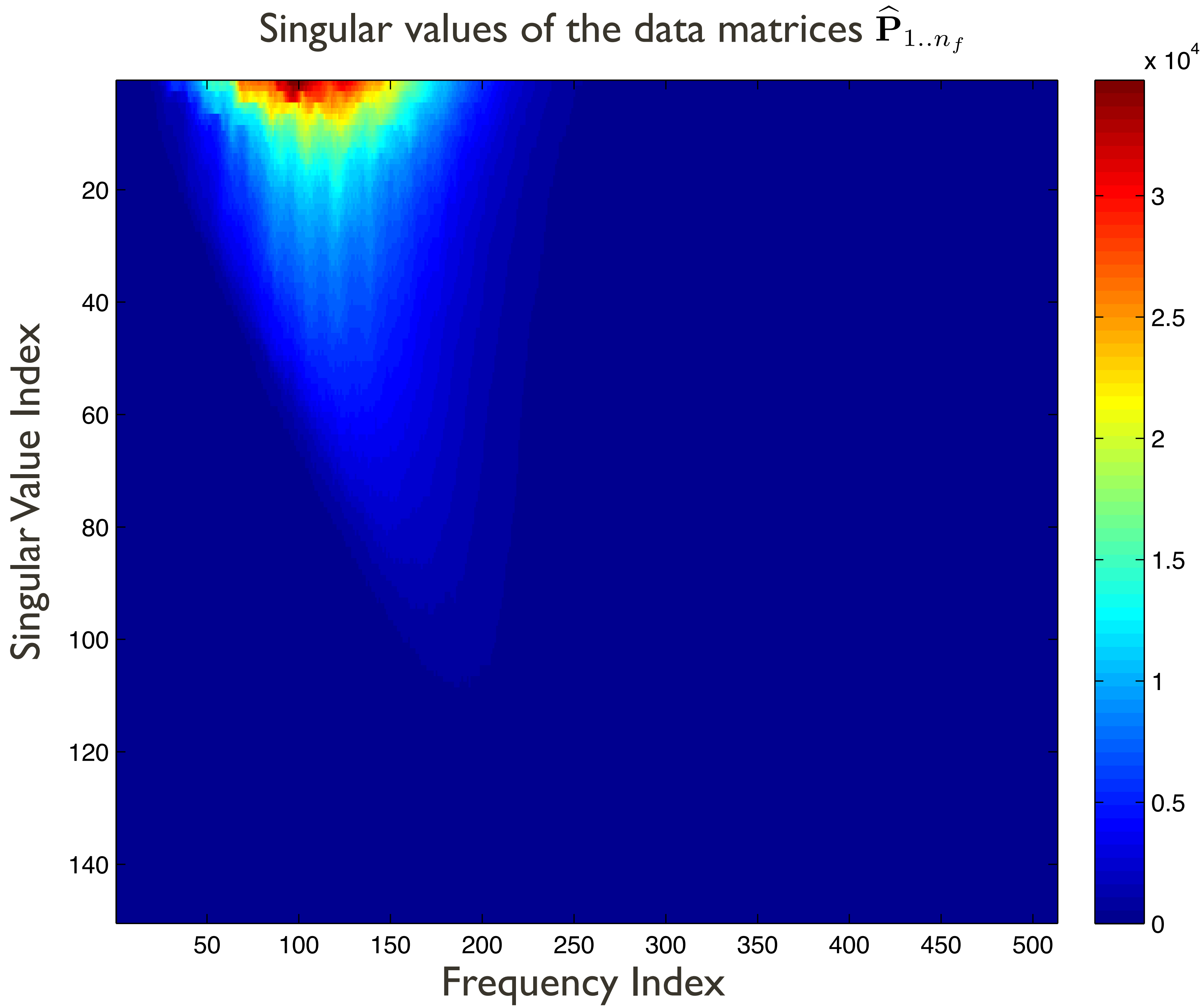
90% less

Dimensionality-reduction via SVD

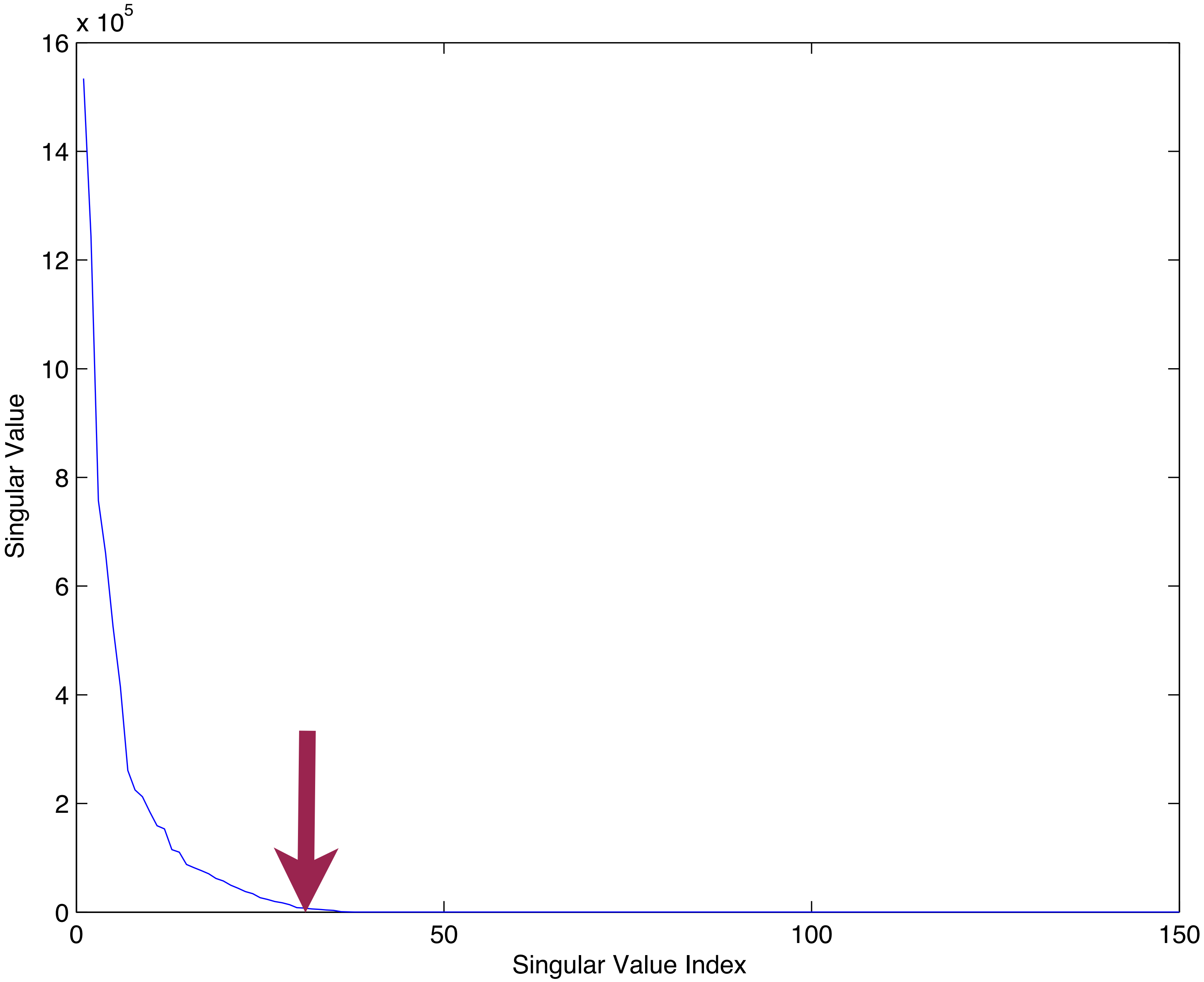
Advantages of using low-rank factorization

	Full data	**Low-rank approximation**
Matrix-Matrix multiplication	$O(N^3)$	$O(kN^2)$
Storage (bytes)	$O(N^2)$	$O(2Nk + k^2)$

Singular values of the data matrix

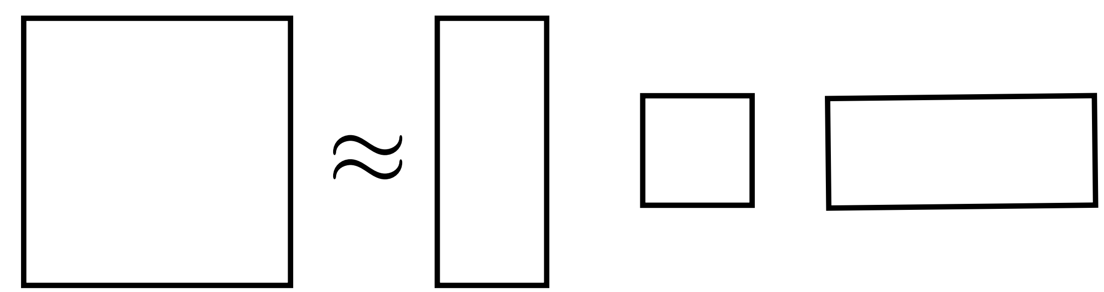


Decay of singular-values



full matrix size$= 150 \times 150$

rank $k = 30 = 20\%$

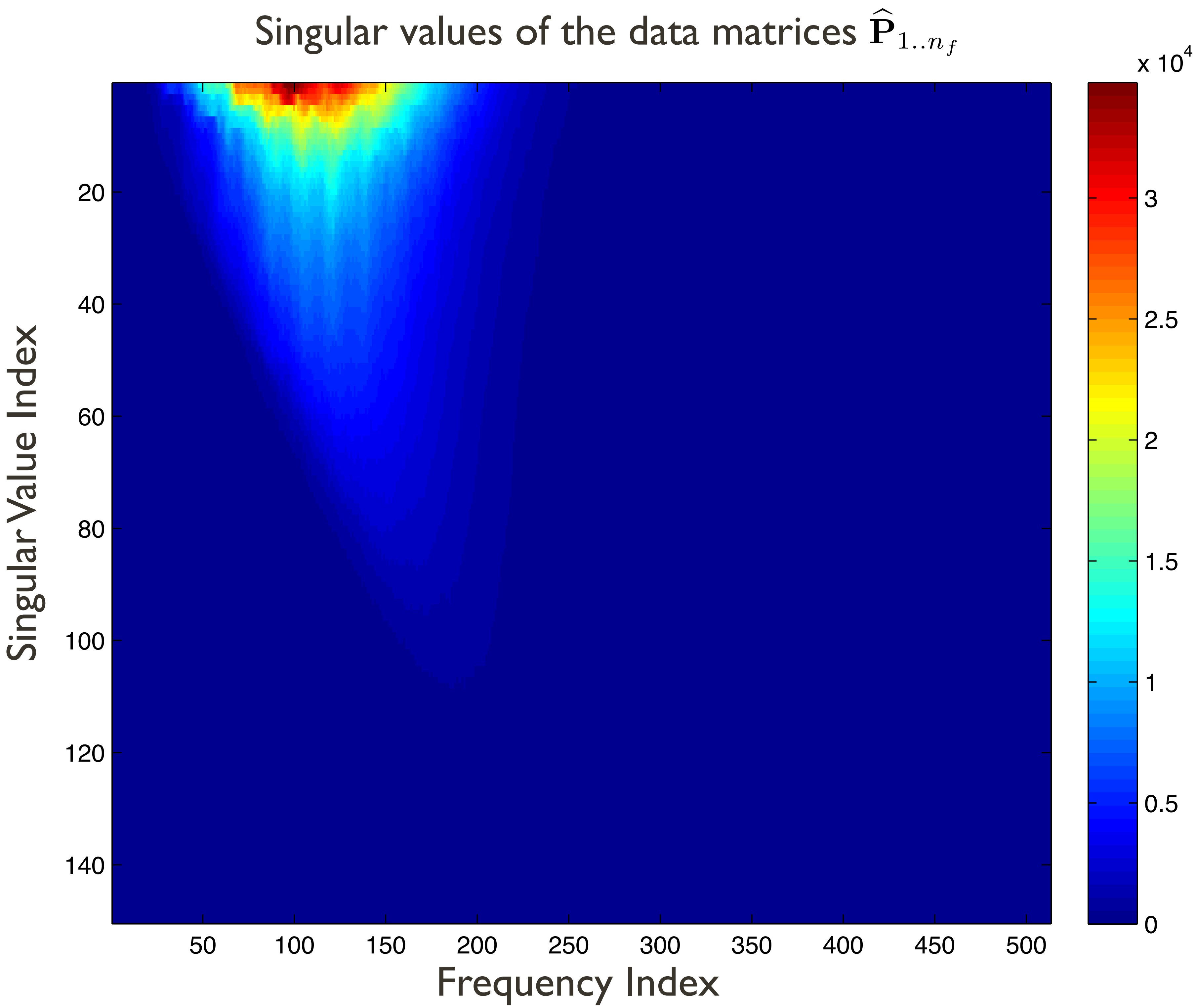


$$\hat{\mathbf{P}}_{150\times150} \approx \mathbf{U}_{150\times30} \; \mathbf{S}_{30\times30} \; \mathbf{V^*}_{30\times150}$$

60% compression

$5\times$ faster mat-mat product

Singular values of the data matrix



Objective:
Approximate *all* frequency slices

Problem:
SVD is expensive

Randomized SVD

More efficient in handling large data

Parallel computing environments

- *fast* matrix-vector products

Full data accessed *only* (1-2) times

- *slow* communication and secondary storage

[Halko, N., P. G. Martinsson, and J. A. Tropp, 2011]

Dimensionality-reduction via *R*-SVD

Two-stage approach:

1. *Capture* action of the data $\hat{\mathbf{P}}$ matrix on $k+l$ *random vectors*

$$\hat{\mathbf{Y}} = \hat{\mathbf{P}}\hat{\mathbf{W}}$$

$\hat{\mathbf{W}}$: Gaussian random matrix

l is a small over sampling parameter (1-3)

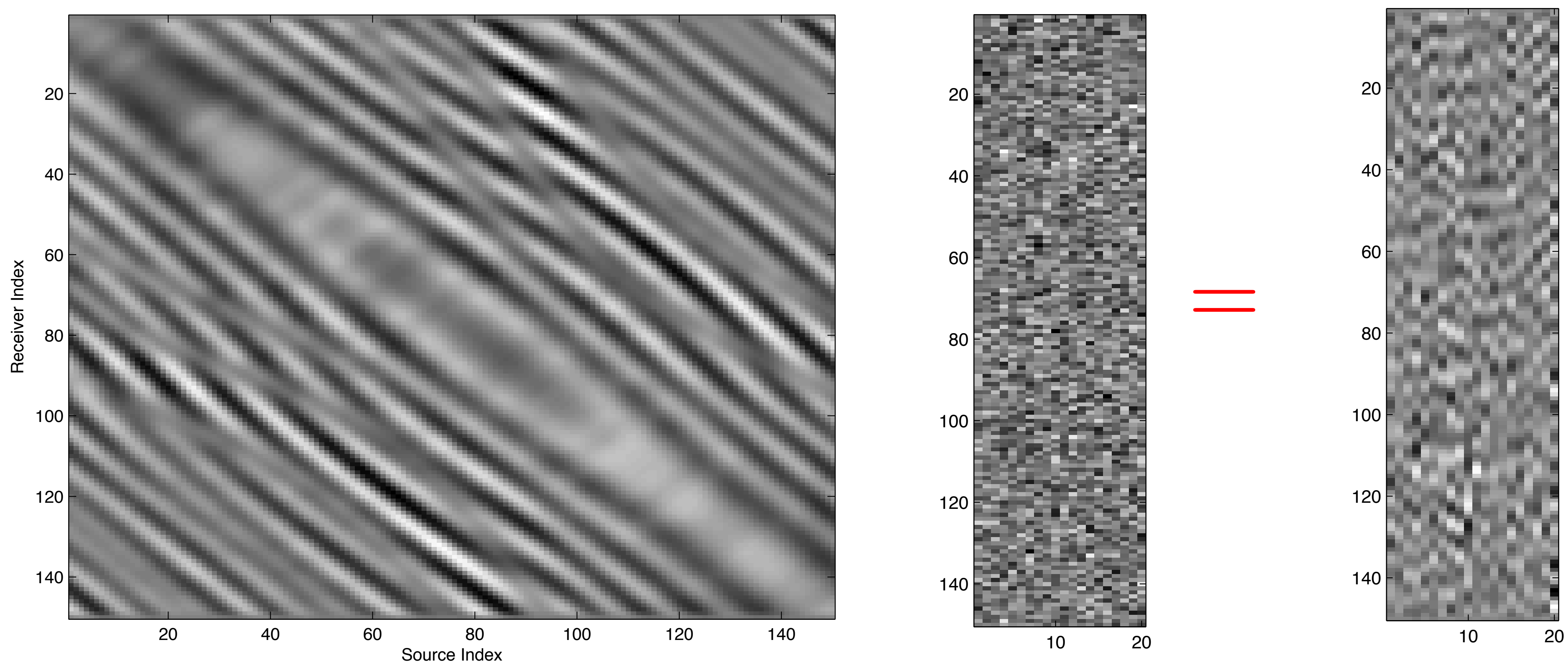
2. *Form* a SVD on $\hat{\mathbf{Y}}$

[Halko, N., P. G. Martinsson, and J. A. Tropp, 2011]

Dimensionality-reduction via *R*-SVD

Stage 1: Capturing the action of $\hat{\mathbf{P}}$

$\hat{\mathbf{P}}$ $\hat{\mathbf{W}}$ $\hat{\mathbf{Y}} = \hat{\mathbf{P}}\hat{\mathbf{W}}$



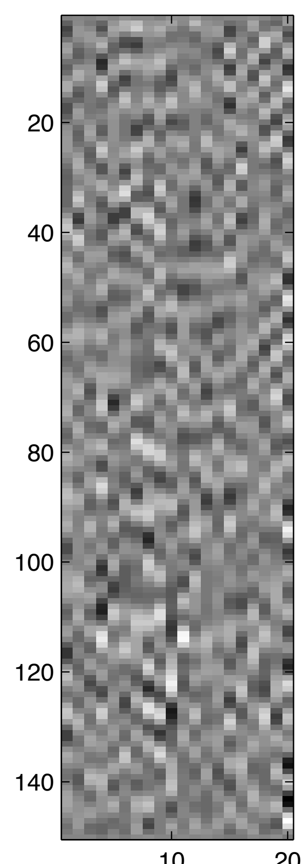
$n_r \times n_s$ $n_s \times k$ $n_r \times k$

Dimensionality-reduction via *R*-SVD

Stage 1: Capturing the action of $\hat{\mathbf{P}}$

2. Form a low-rank QR factorization $\hat{\mathbf{Y}} \approx \mathbf{QR}$

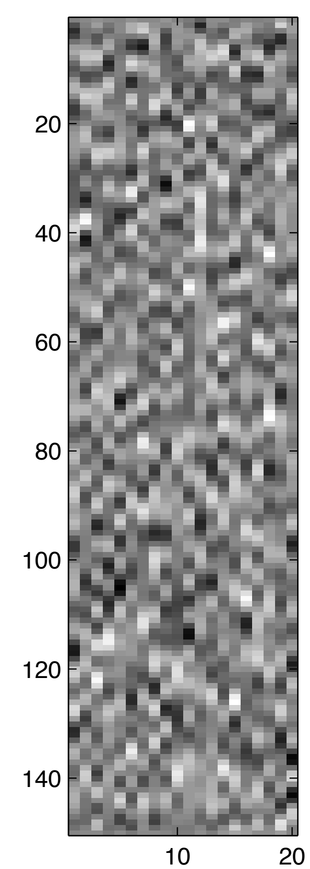
$\hat{\mathbf{Y}}$



$n_r \times k$

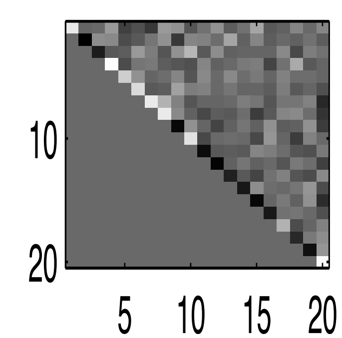
$=$

$\mathbf{Q}$



$n_r \times k$

$\mathbf{R}$



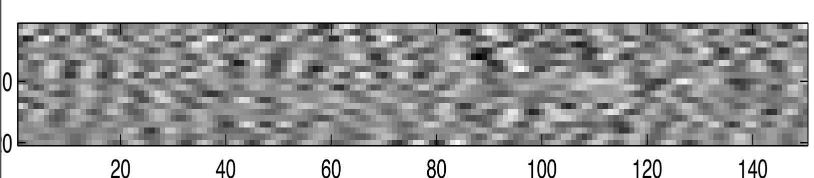
$k \times k$

Dimensionality-reduction via *R*-SVD

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

1. Form $\mathbf{B} = \mathbf{Q}^*\hat{\mathbf{P}}$

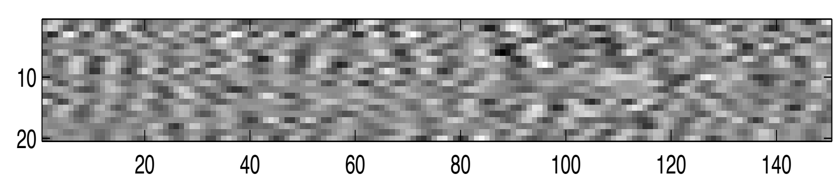
$\mathbf{B}$



$k \times n_s$

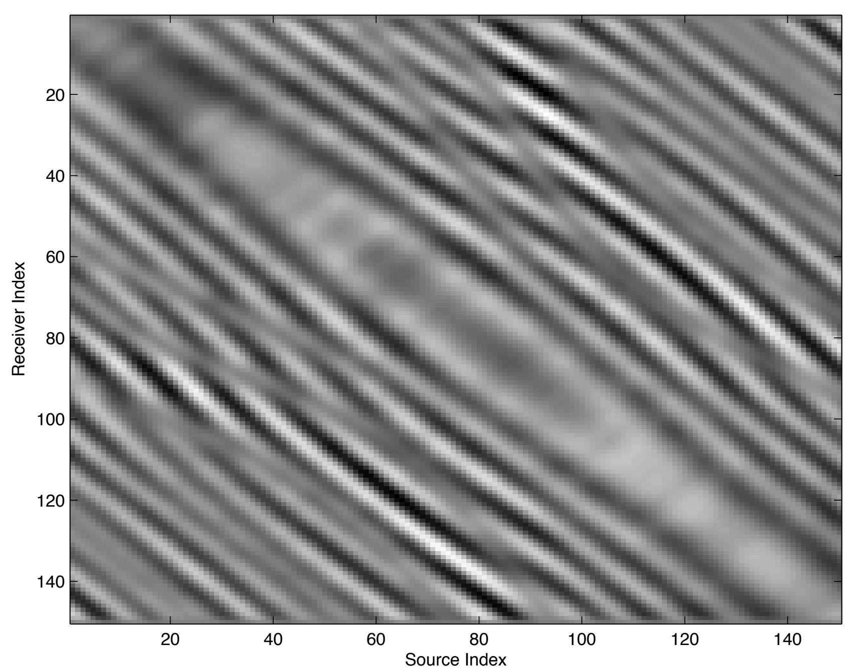
=

$\mathbf{Q}^*$



$k \times n_r$

$\hat{\mathbf{P}}$



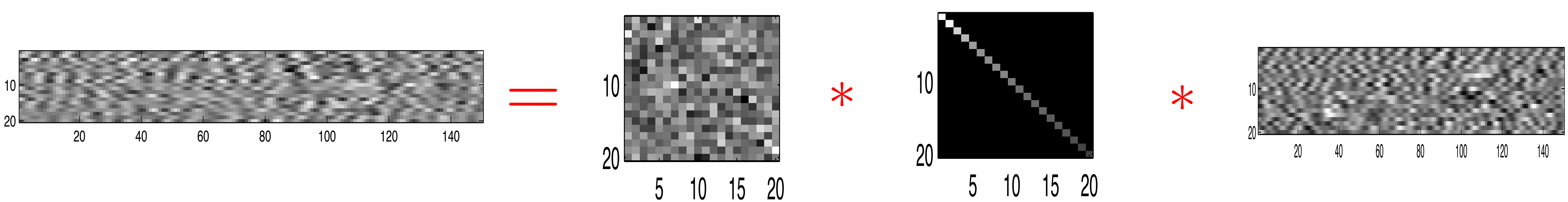
$n_r \times n_s$

Dimensionality-reduction via *R*-SVD

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

2. Compute **SVD** of the small matrix $\mathbf{B} = \widetilde{\mathbf{U}}\mathbf{\Sigma}\mathbf{V}^*$

$\mathbf{B}$ $\qquad$ $\widetilde{\mathbf{U}}$ $\qquad$ $\mathbf{S}$ $\qquad$ $\mathbf{V}^*$



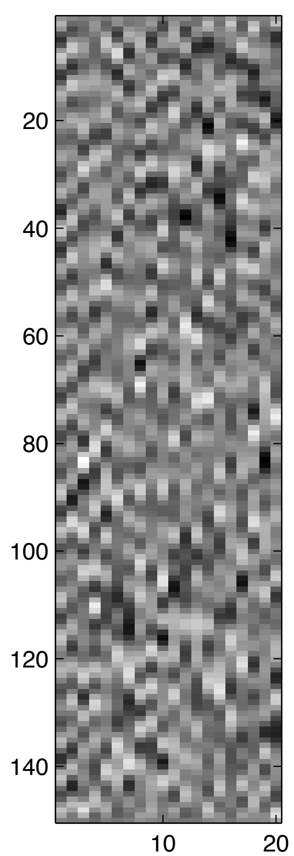
$k * n_s$ $\qquad$ $k * k$ $\qquad$ $k * k$ $\qquad$ $k * n_s$

Dimensionality-reduction via *R*-SVD

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

3. Compute $\mathbf{U} = \mathbf{Q}\widetilde{\mathbf{U}}$

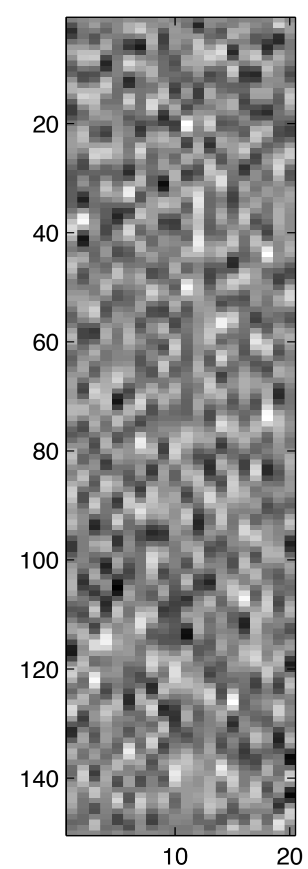
$\mathbf{U}$



$n_r \times k$

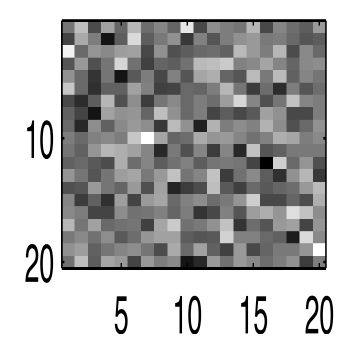
$=$

$\mathbf{Q}$



$n_r \times k$

$\widetilde{\mathbf{U}}$

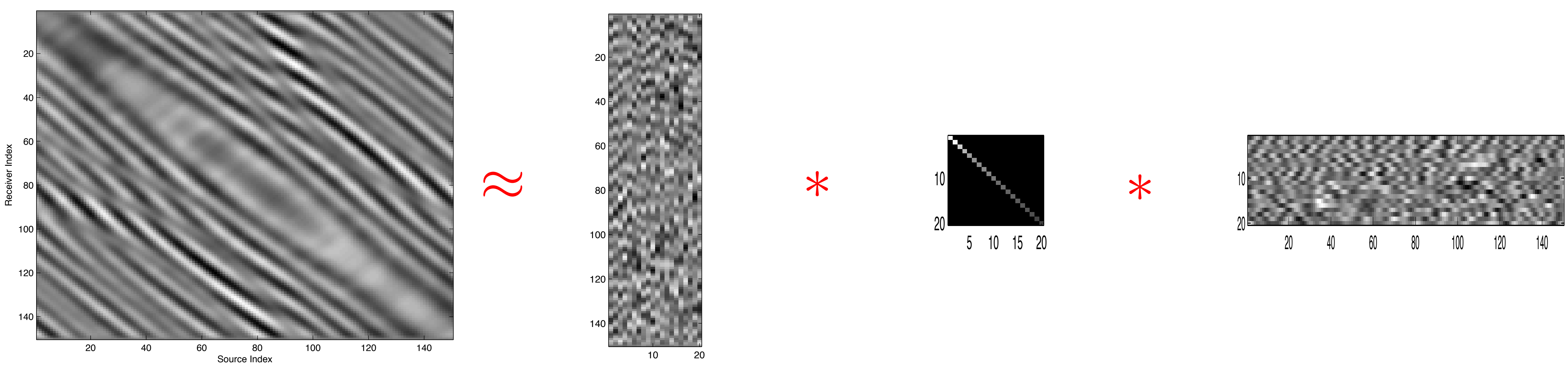


$k \times k$

Dimensionality-reduction via *R*-SVD

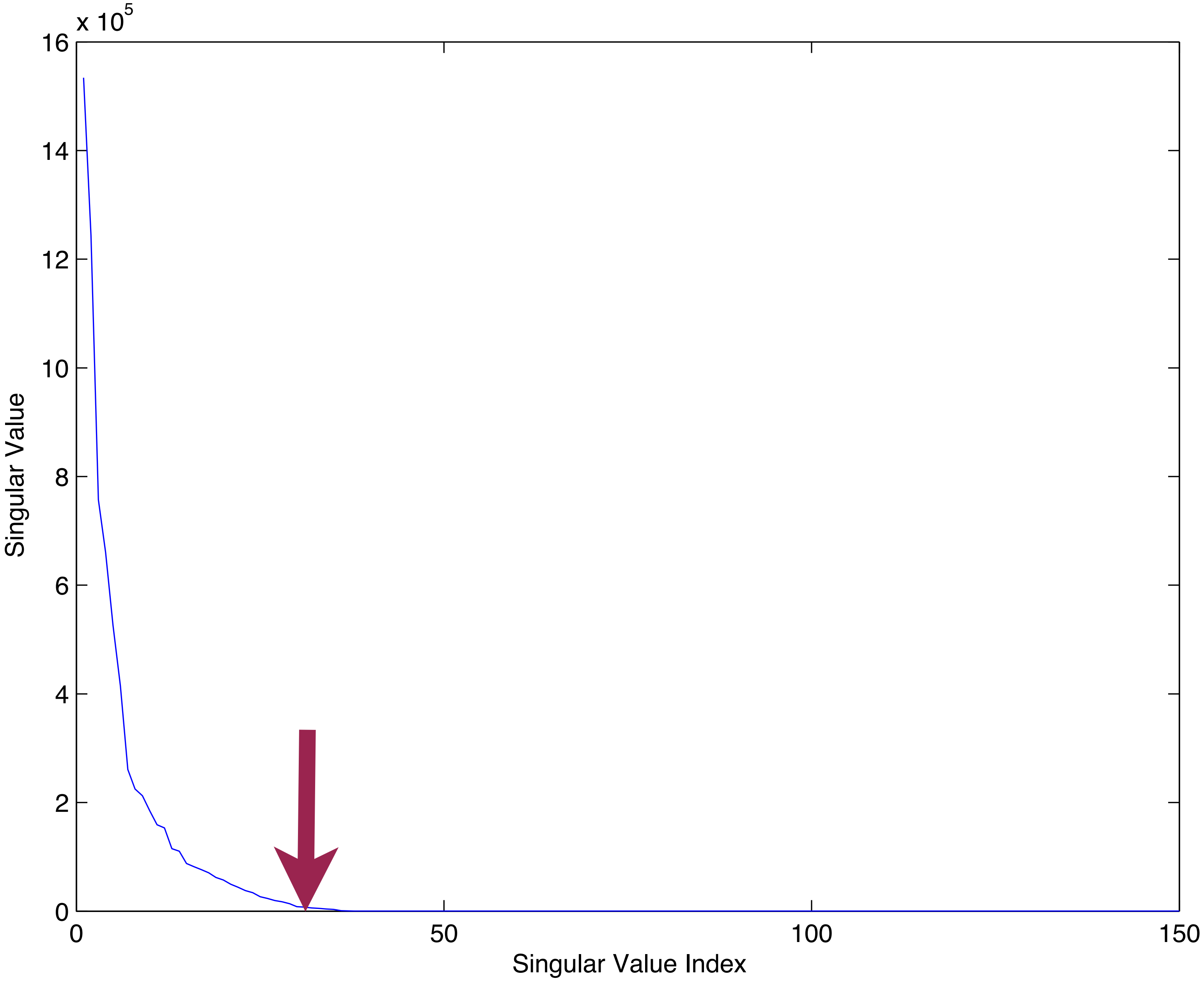
Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

$$\hat{\mathbf{P}} \approx \mathbf{U} * \mathbf{S} * \mathbf{V}^*$$



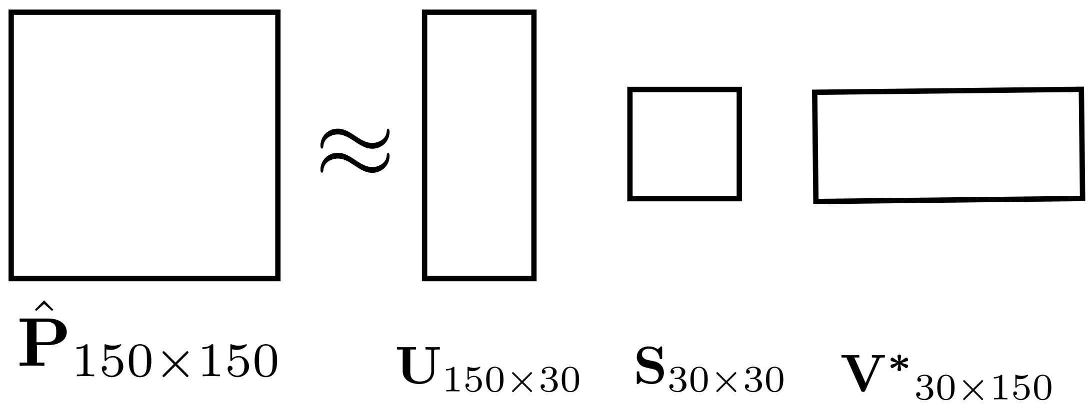
$n_r \times n_s$ | $n_r \times k$ | $k \times k$ | $k \times n_s$

Decay of singular-values



full matrix size$= 150 \times 150$

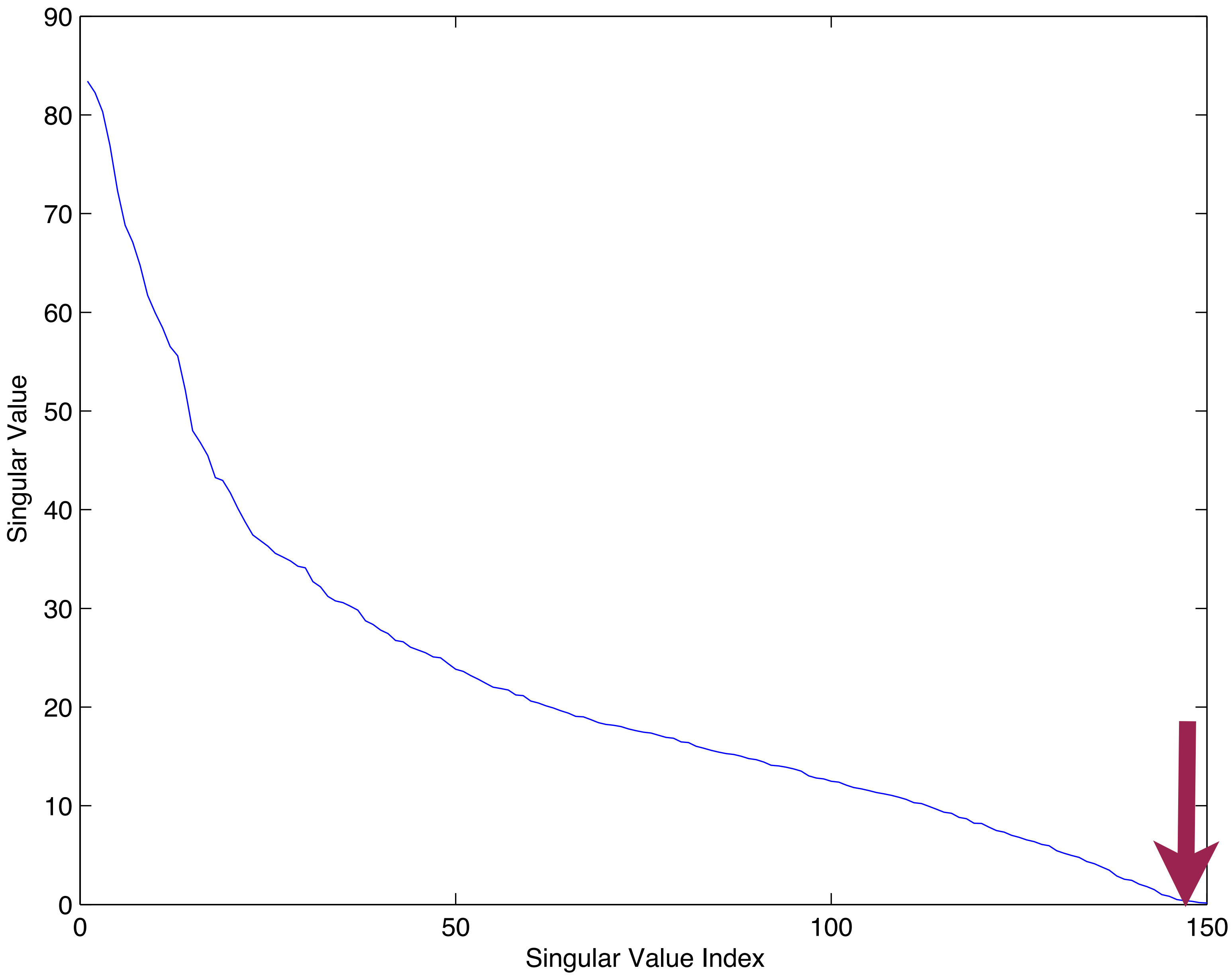
rank $k = 30 = 20\%$



60% compression

$5\times$ faster mat-mat product

Slow decay of singular-values



full matrix size$= 150 \times 150$
rank $k \approx 150$

$$\hat{\mathbf{P}}_{150\times150} \approx \mathbf{U}_{150\times150}\,\mathbf{S}_{150\times150}\,\mathbf{V^*}_{150\times150}$$

No Compression
Slower operations

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Slow decay of singular-values

Power Iteration

$$\mathbf{Y} = (\widehat{\mathbf{P}}\widehat{\mathbf{P}}^*)^{q}\widehat{\mathbf{P}}\widehat{\mathbf{W}}$$

Small singular values

Interfere with the approximation

Solution

Reduce their weight

Cost

q passes over the data

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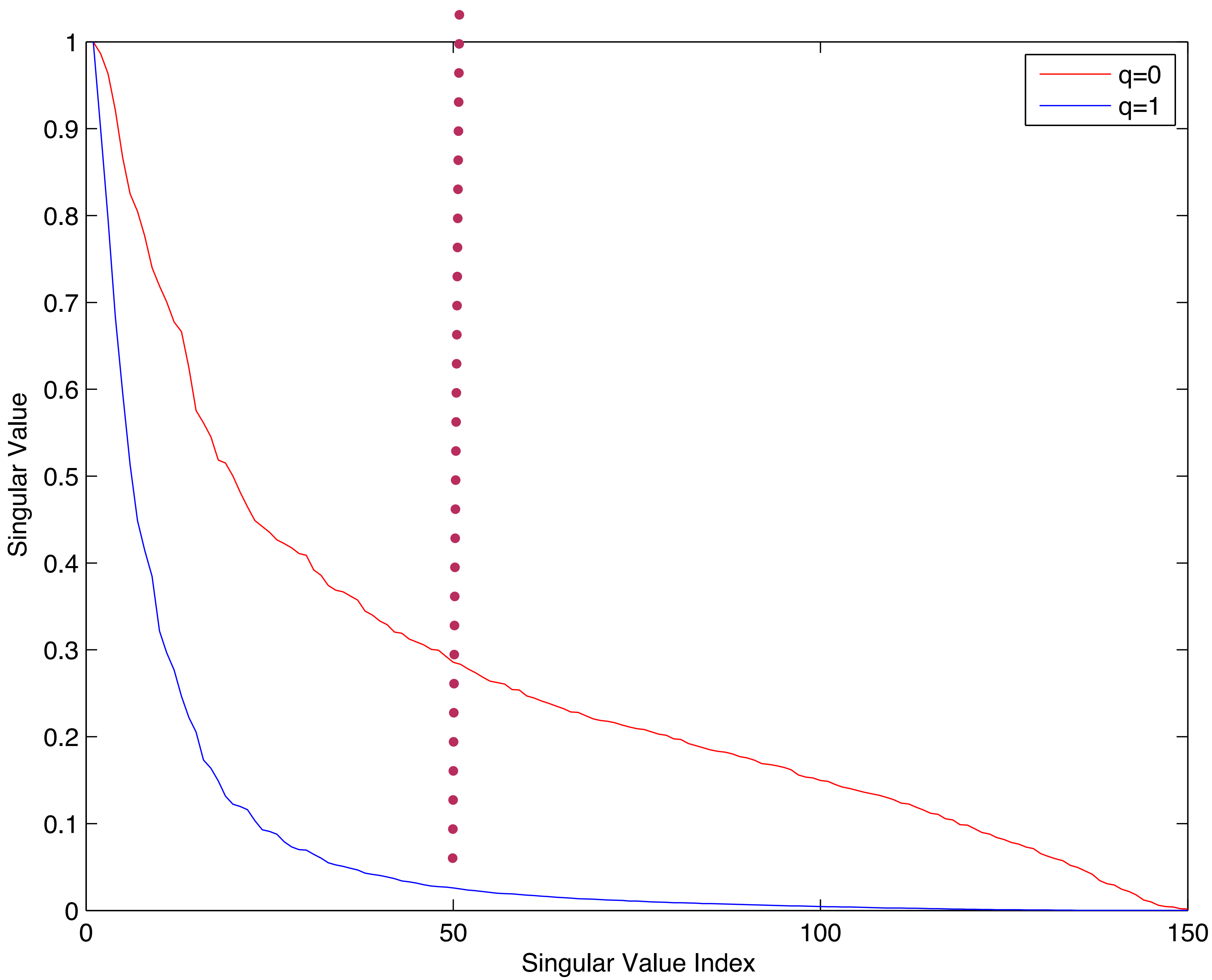
Power Iteration



$q = 0$

$$\text{SNR} = 4 \text{ dB}$$

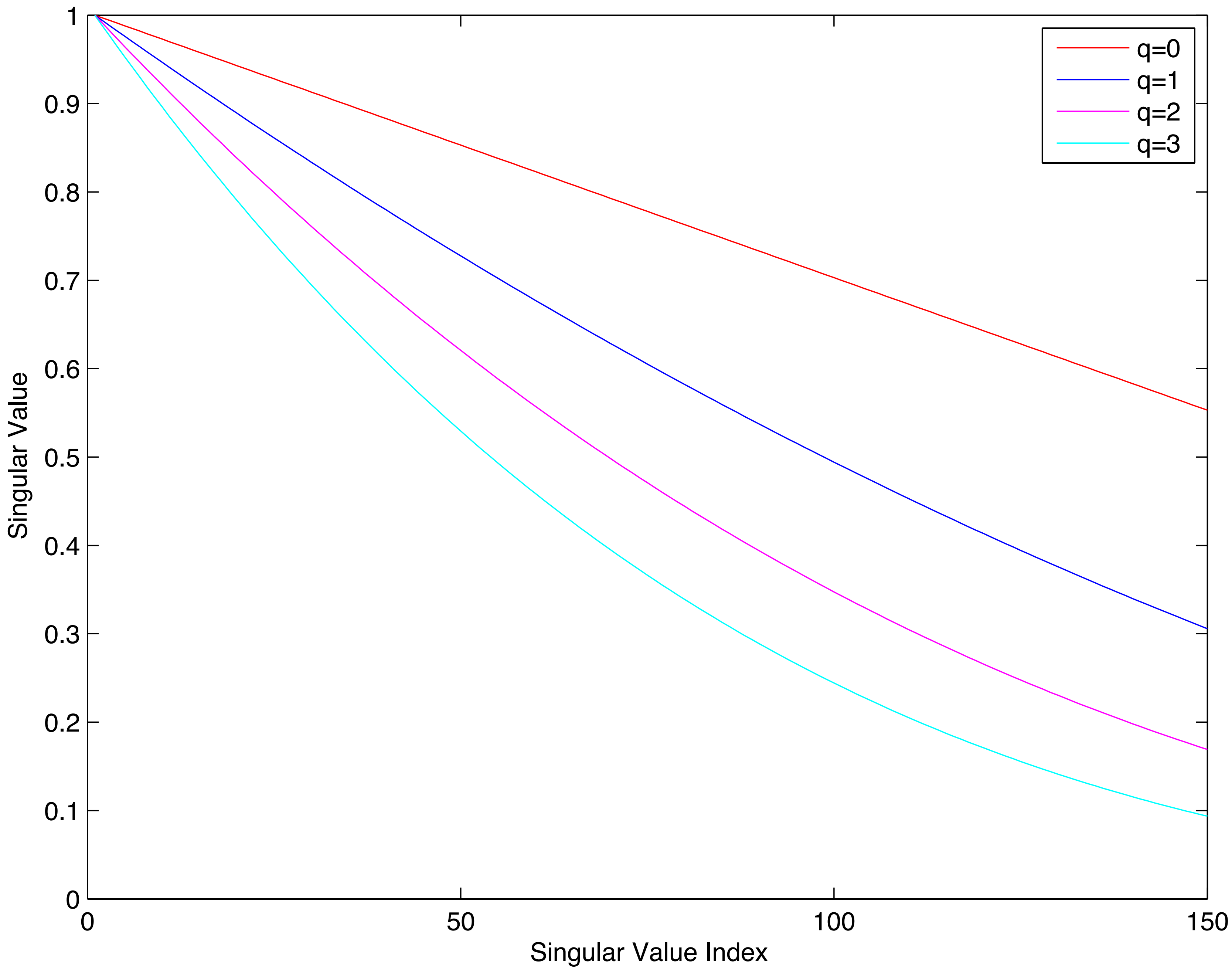
$q = 1$

$$\text{SNR} = 12 \text{ dB}$$

Cost

- Additional q passes

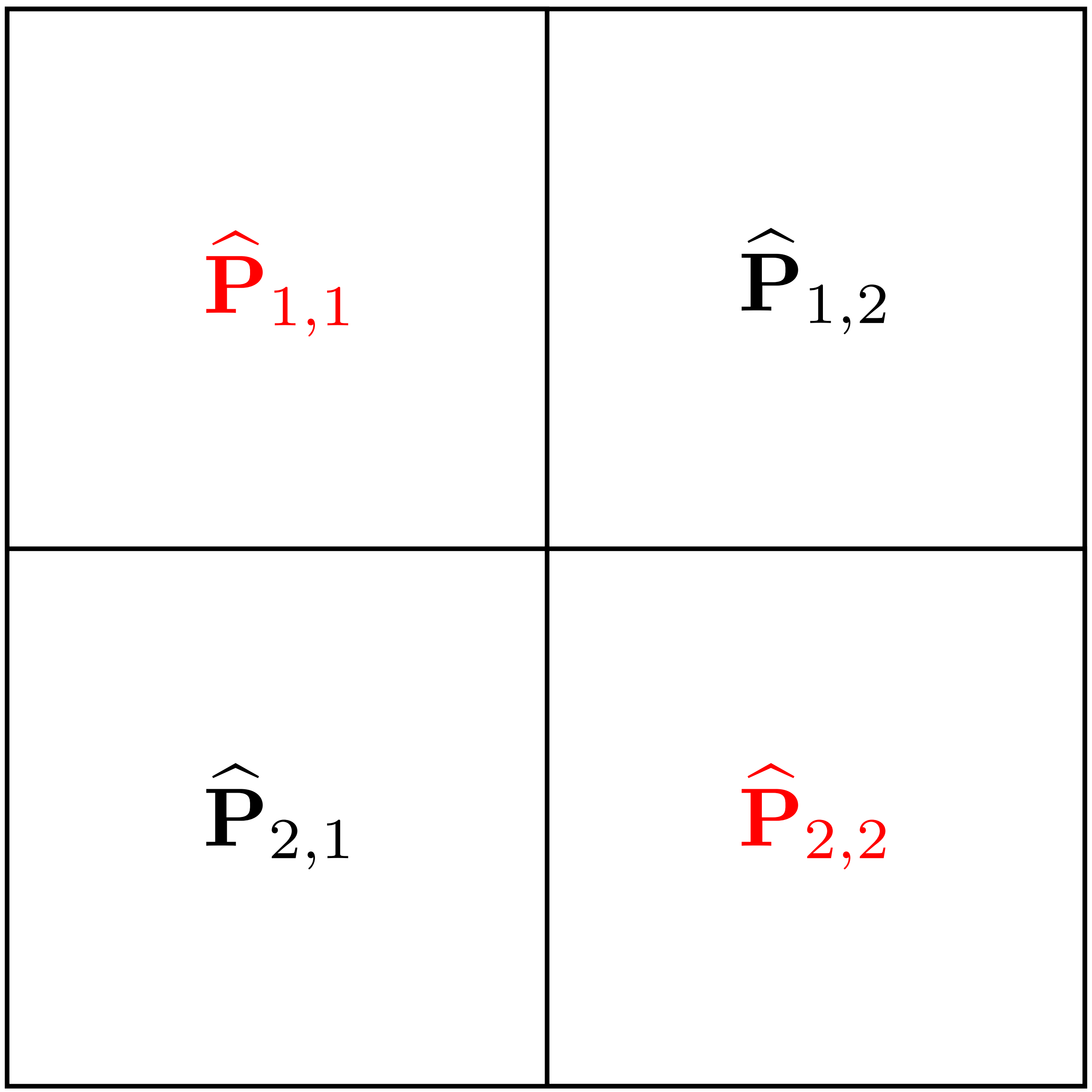
No decay of singular-values



Power Iteration

Not Effective

Hierarchical semi-separable representation



Dense matrices

Level 1 : HSS partitioning

High-rank blocks

Next HSS level

Low-rank blocks

R-SVD

[P.G. Martinsson, 2010]

Hierarchical semi-separable representation

$\widehat{\mathbf{P}}_{1,1}$	$(\mathbf{USV}^*)_{1,2}$
$(\mathbf{USV}^*)_{2,1}$	$\widehat{\mathbf{P}}_{2,2}$

Dense matrices

Level 1 : HSS partitioning

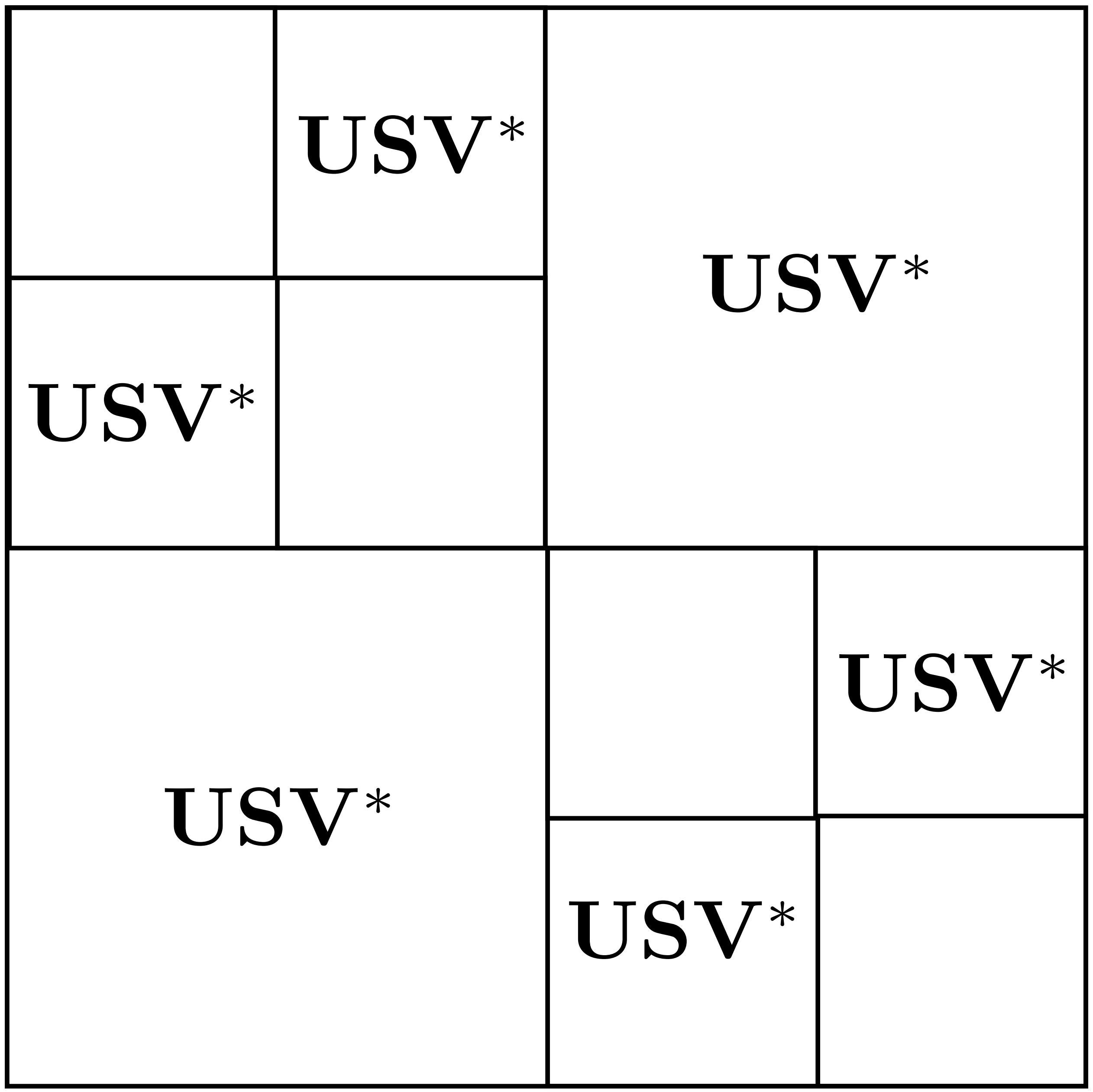
High-rank blocks

Next HSS level

Low-rank blocks

R-SVD

Hierarchical semi-separable representation



Dense matrices

Level 2 : HSS partitioning

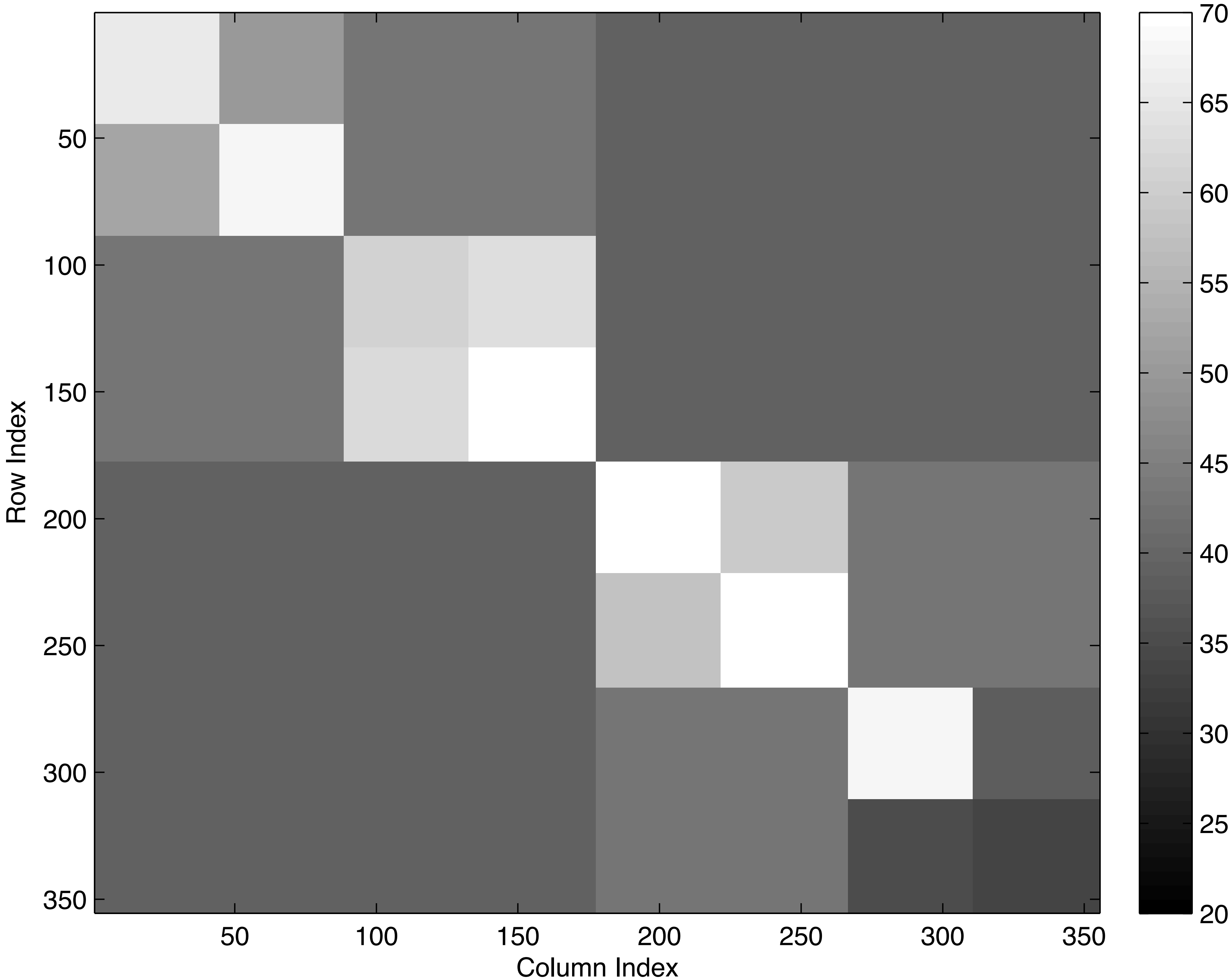
High-rank blocks

Next HSS level

Low-rank blocks

R-SVD

Hierarchical semi-separable representation



Frequency slice

Level 2 : HSS partitioning

High-rank blocks

White

Low-rank blocks

Black

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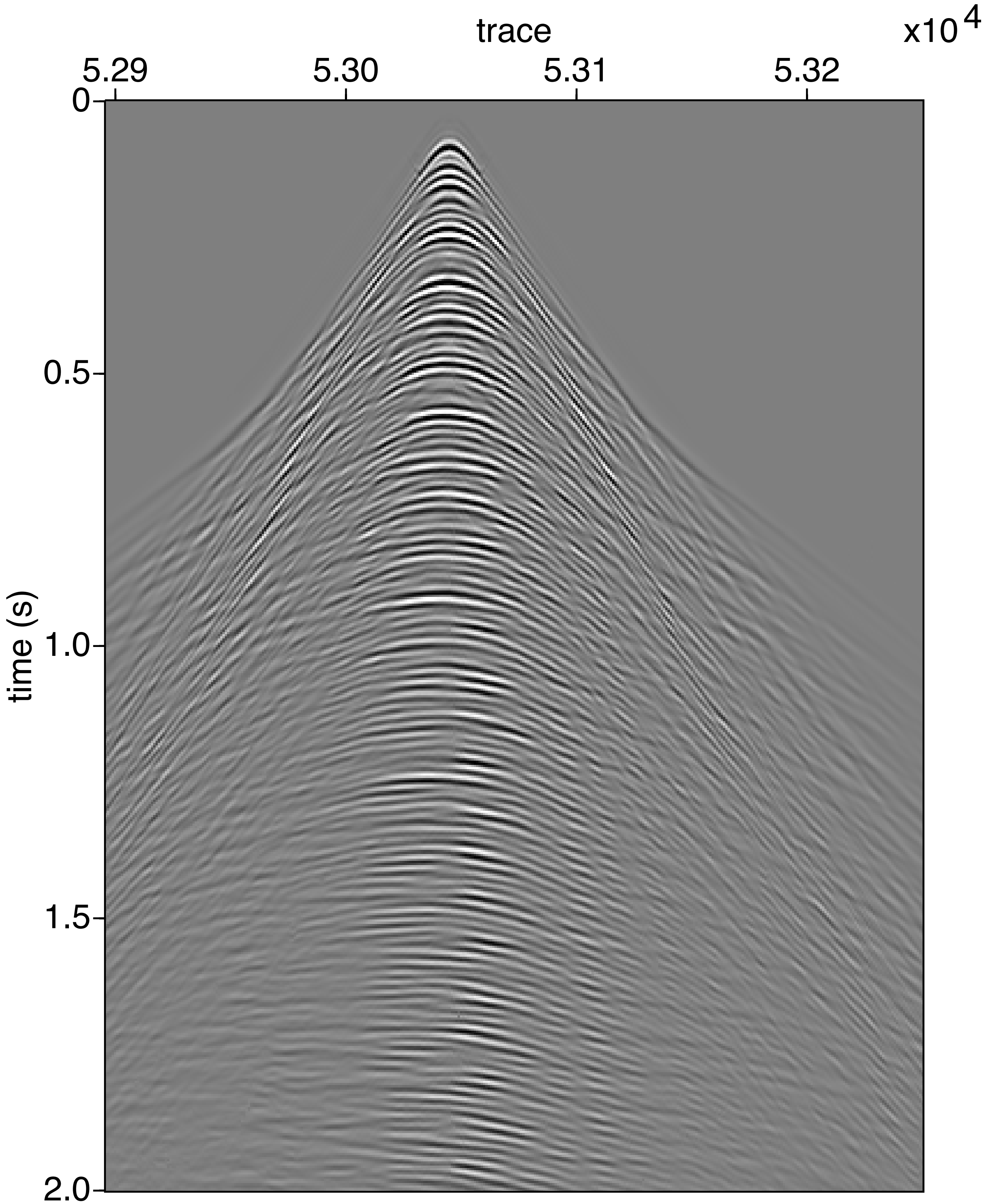
EPSI problem

$$\mathbf{Ax} \approx \mathbf{b}$$

$$\tilde{\mathbf{x}} = \underbrace{\arg\min_{\mathbf{x}} ||\mathbf{x}||_{\mathbf{1}}}_{\text{sparsity promoting part}} \quad \text{subject to} \quad \underbrace{||\mathbf{Ax} - \mathbf{b}||_{\mathbf{2}} \leq \sigma}_{\text{data fitting part}}$$

σ : residual between the recorded & predicted data



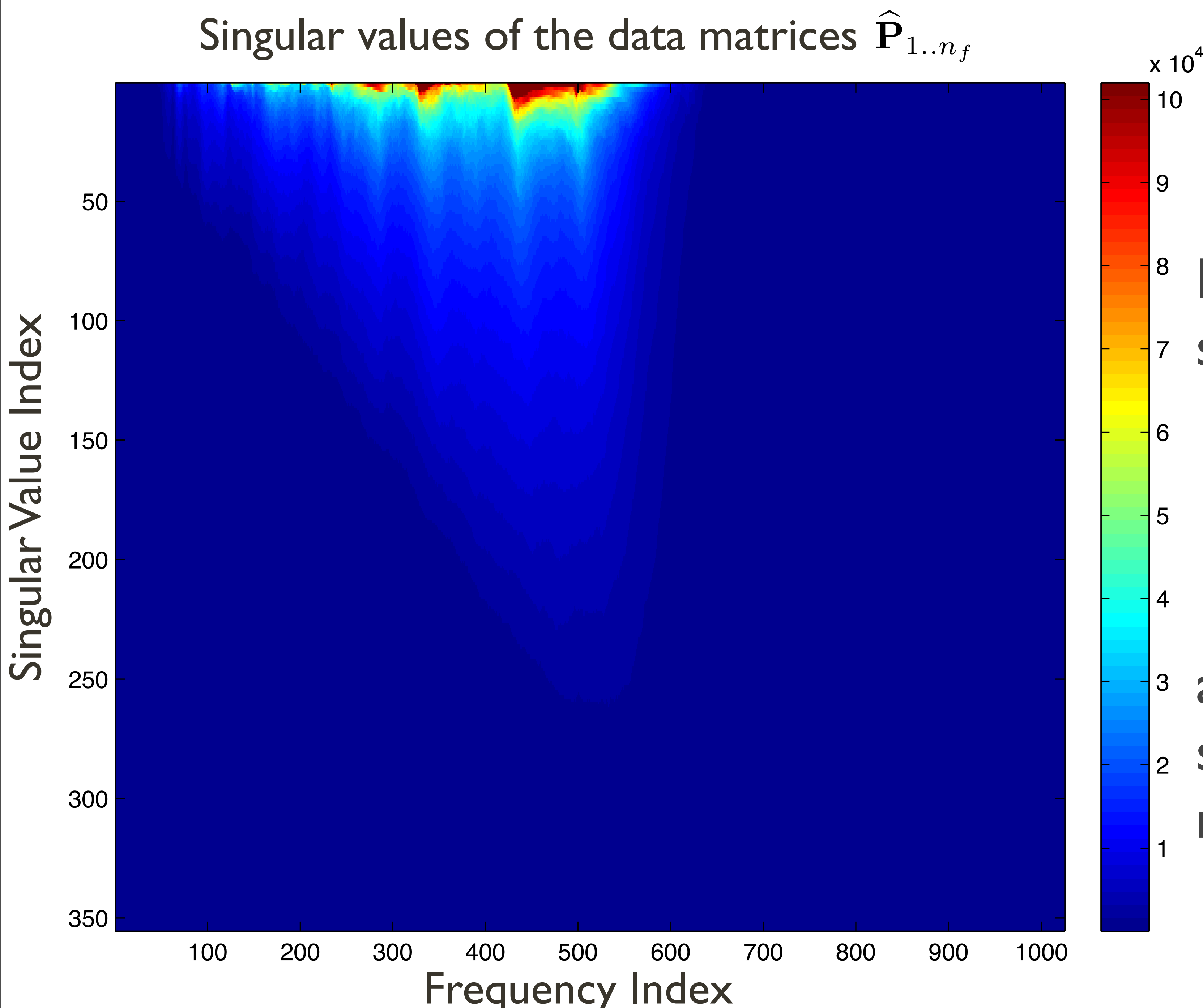
Gulf of Suez

Total Data

shot gather

$$n_r = 355$$
$$n_s = 355$$
$$n_t = 1024$$
$$dt = .004s$$

Singular values of the data matrix



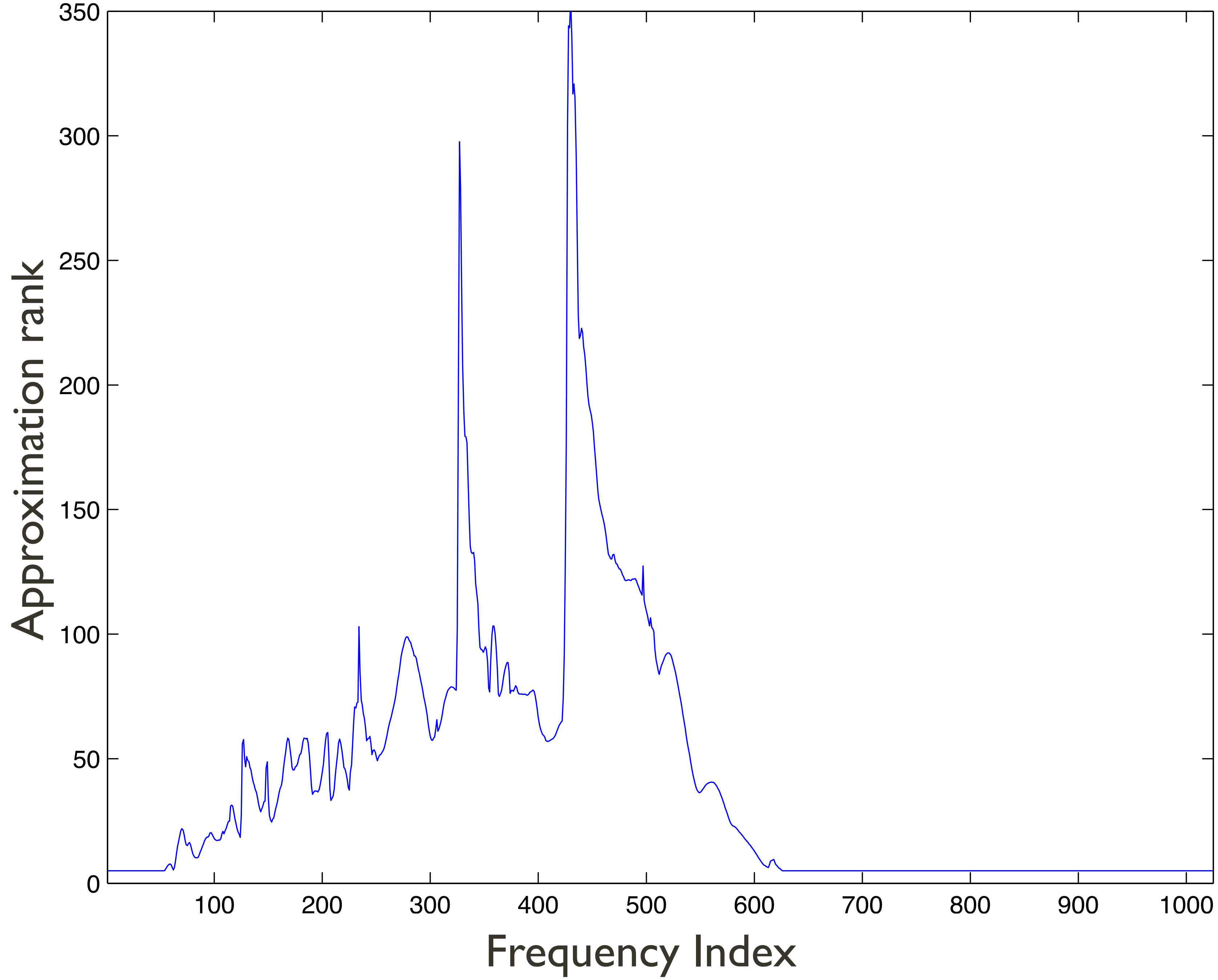
Adaptive approximation

For each frequency find rank k such that

$$\|\mathbf{P} - \mathbf{USV}^*\| \leq \epsilon$$

and the sum of all k ranks used is smaller than some % of total number of columns

Adaptive rank selection



sum of all k ranks used is 9% of total number of columns

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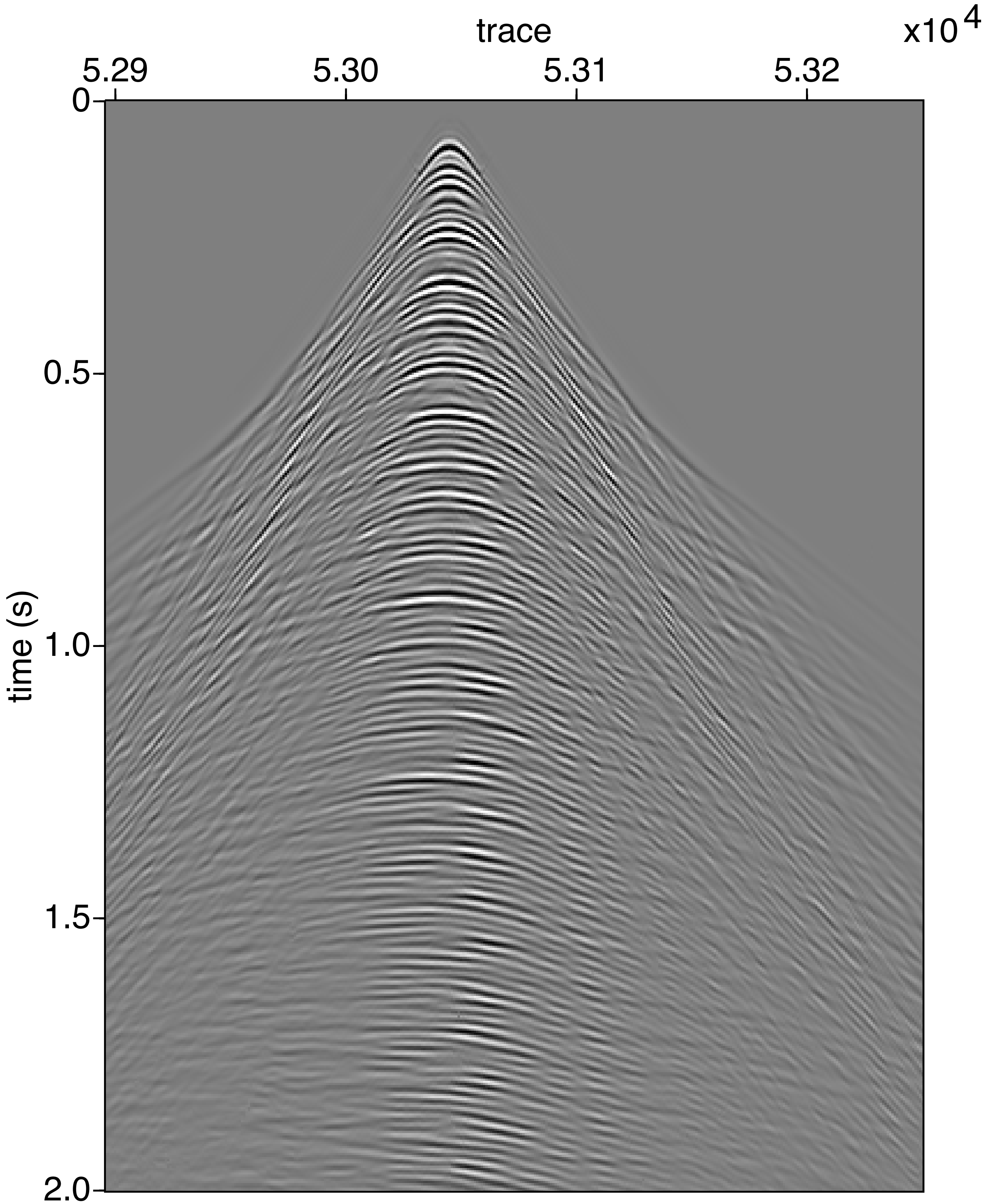

Gulf of Suez

Total Data

shot gather

$$n_r = 355$$
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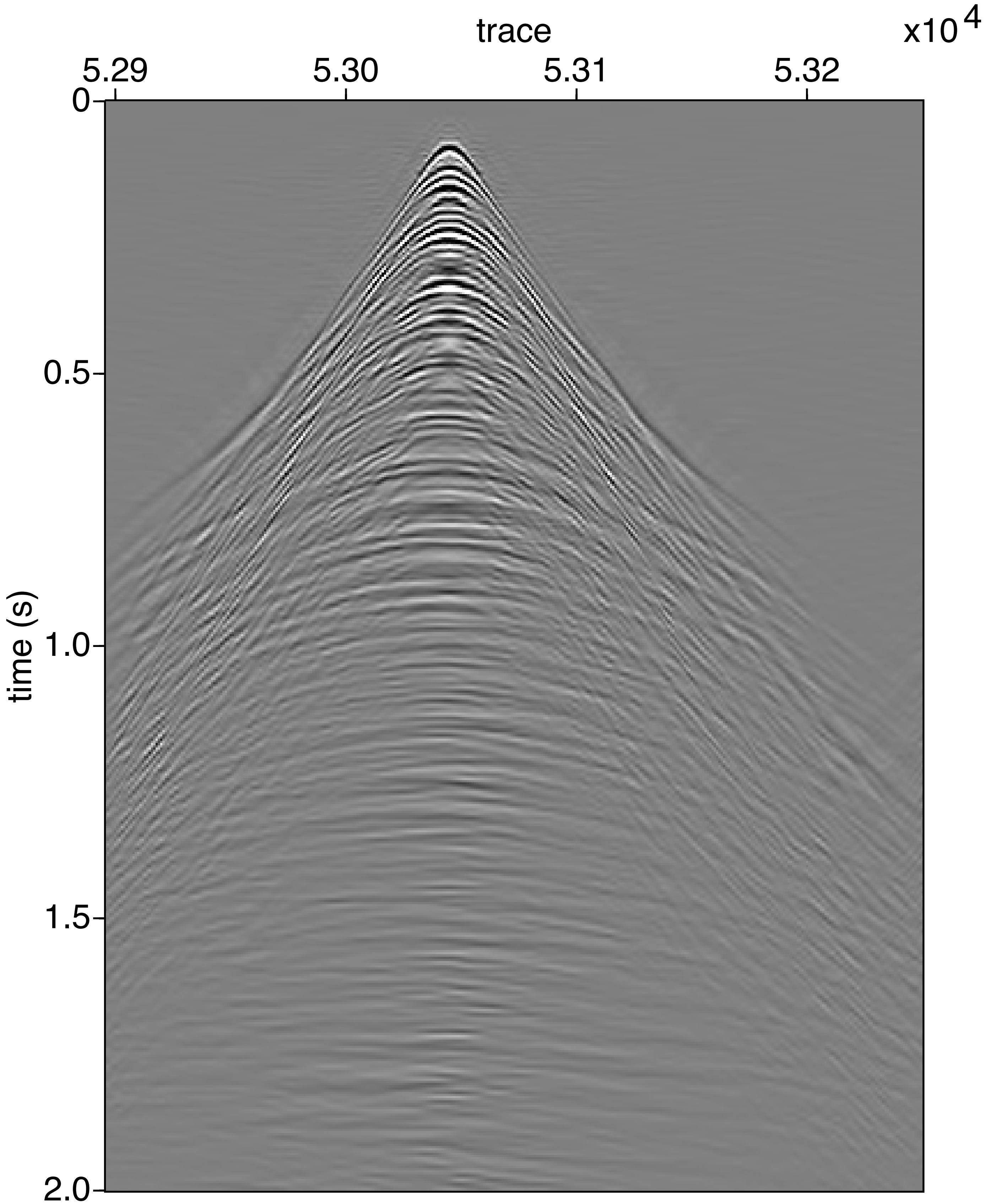


Gulf of Suez

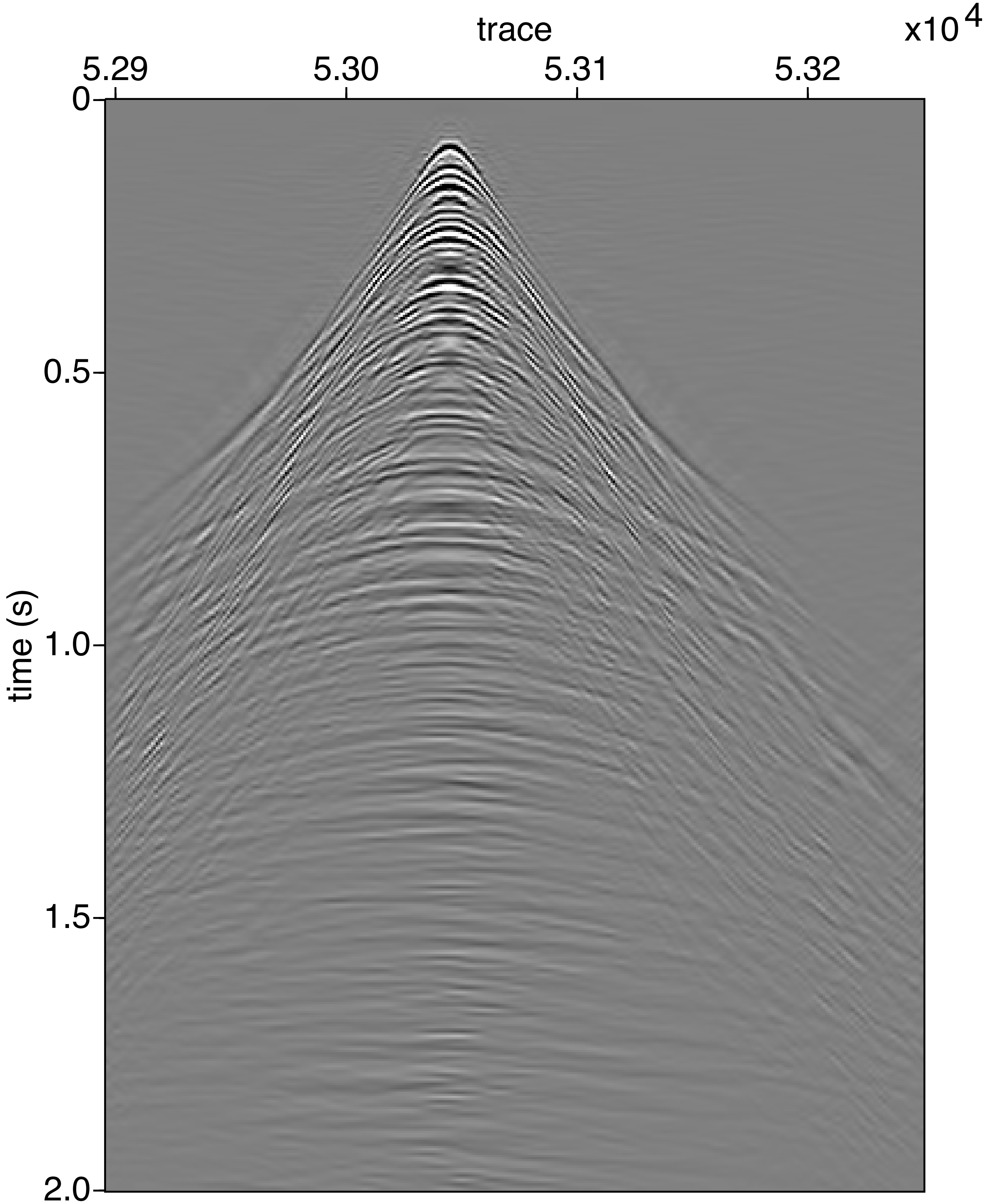
Full Data

Primary IR (G)

shot gather

2D Curvelet (Src-Rcv)

150 iterations



Gulf of Suez

20% of Data

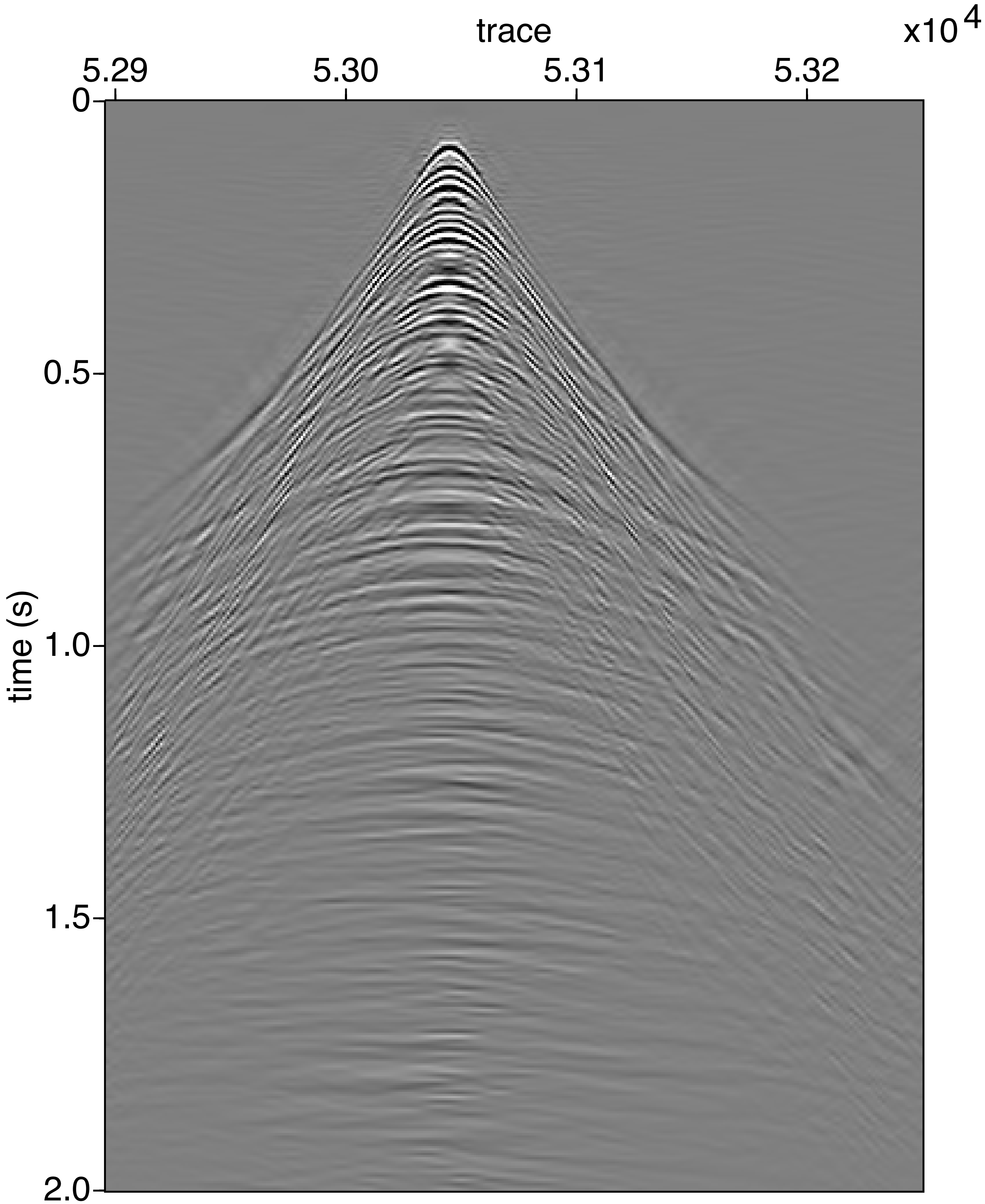
Primary IR (G)

SNR = 27dB

shot gather

2D Curvelet (Src-Rcv)

150 iterations



Gulf of Suez

12% of rank budget

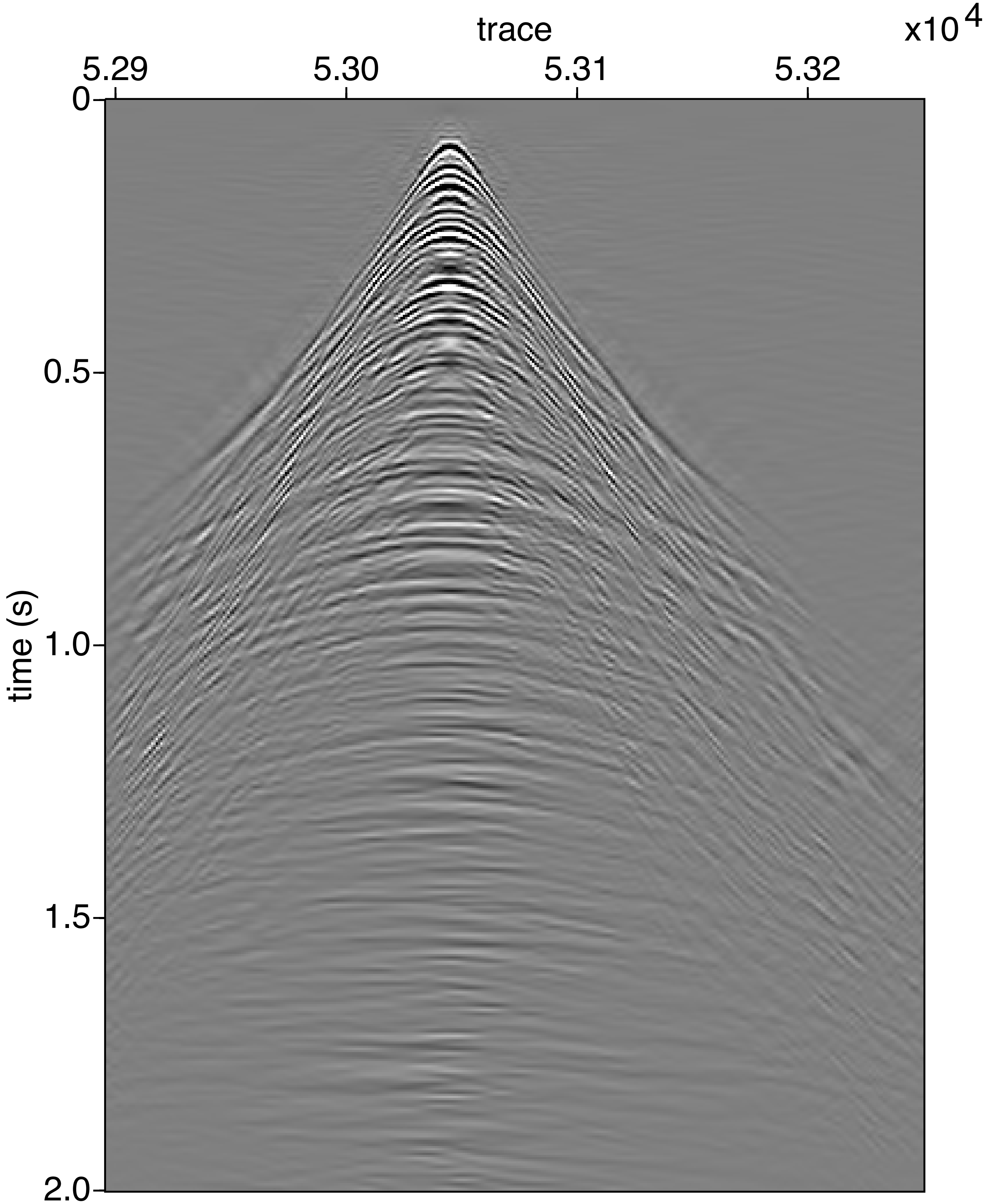
Primary IR (G)

SNR = 17dB

shot gather

2D Curvelet (Src-Rcv)

150 iterations



Gulf of Suez

8% of rank budget

Primary IR (G)

SNR = 12dB

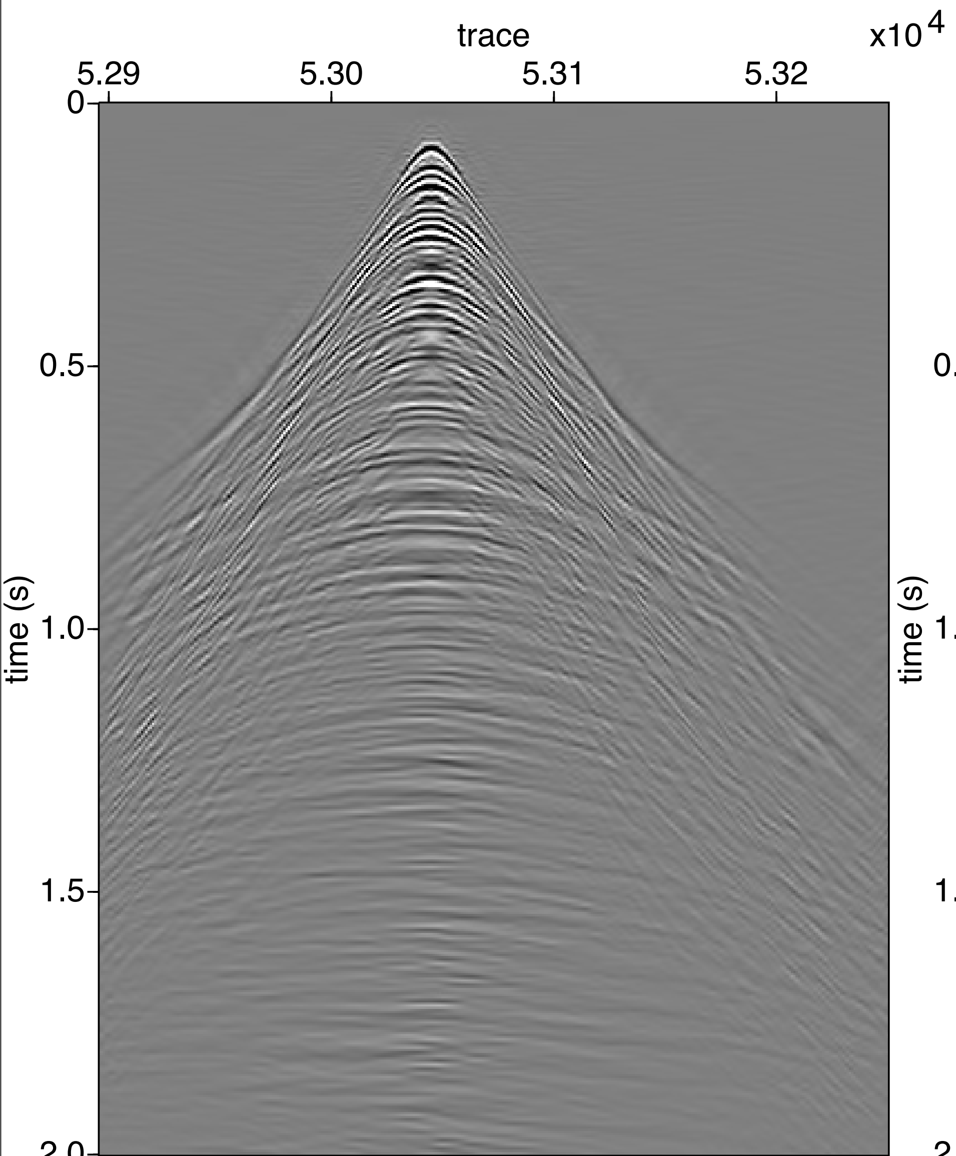
shot gather

2D Curvelet (Src-Rcv)

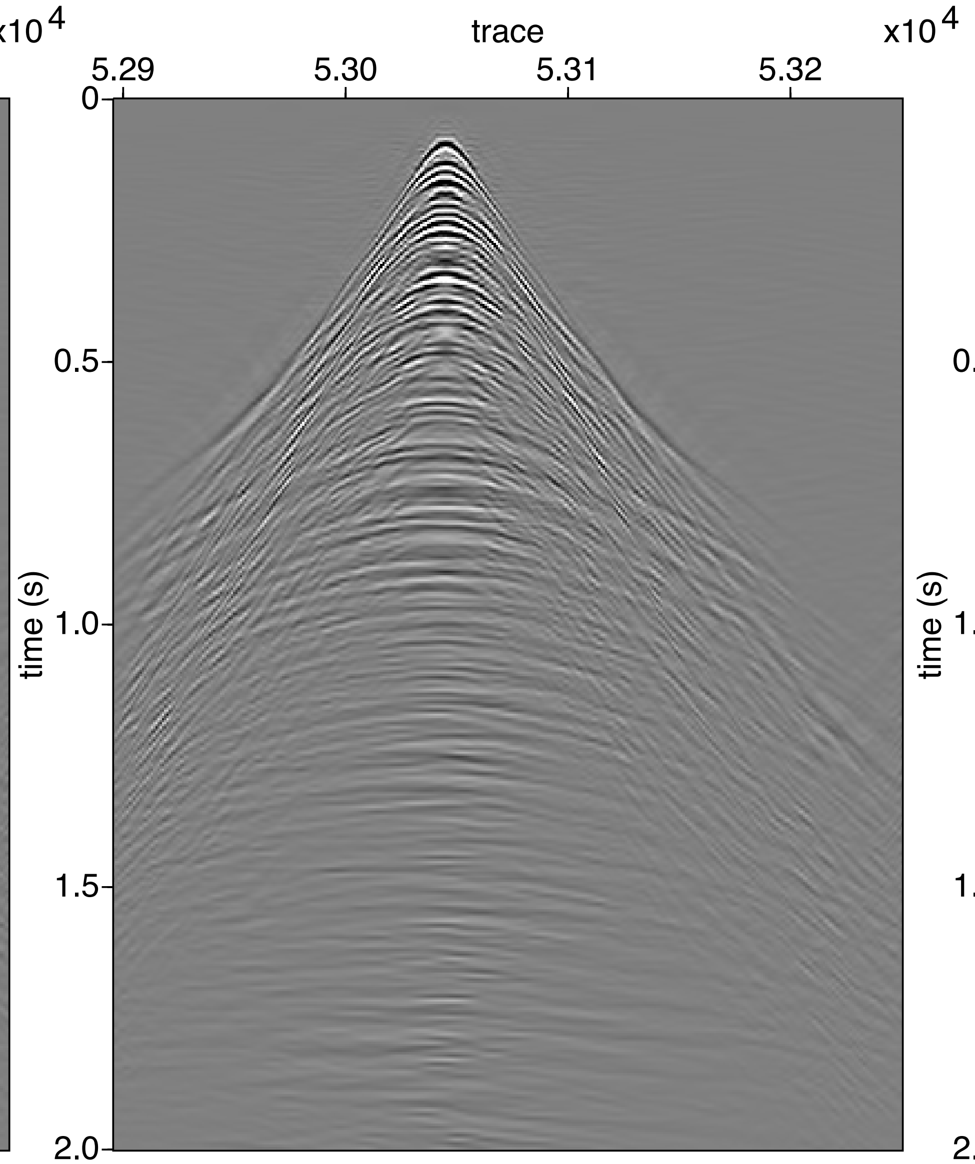
150 iterations

Difference in EPSI Result

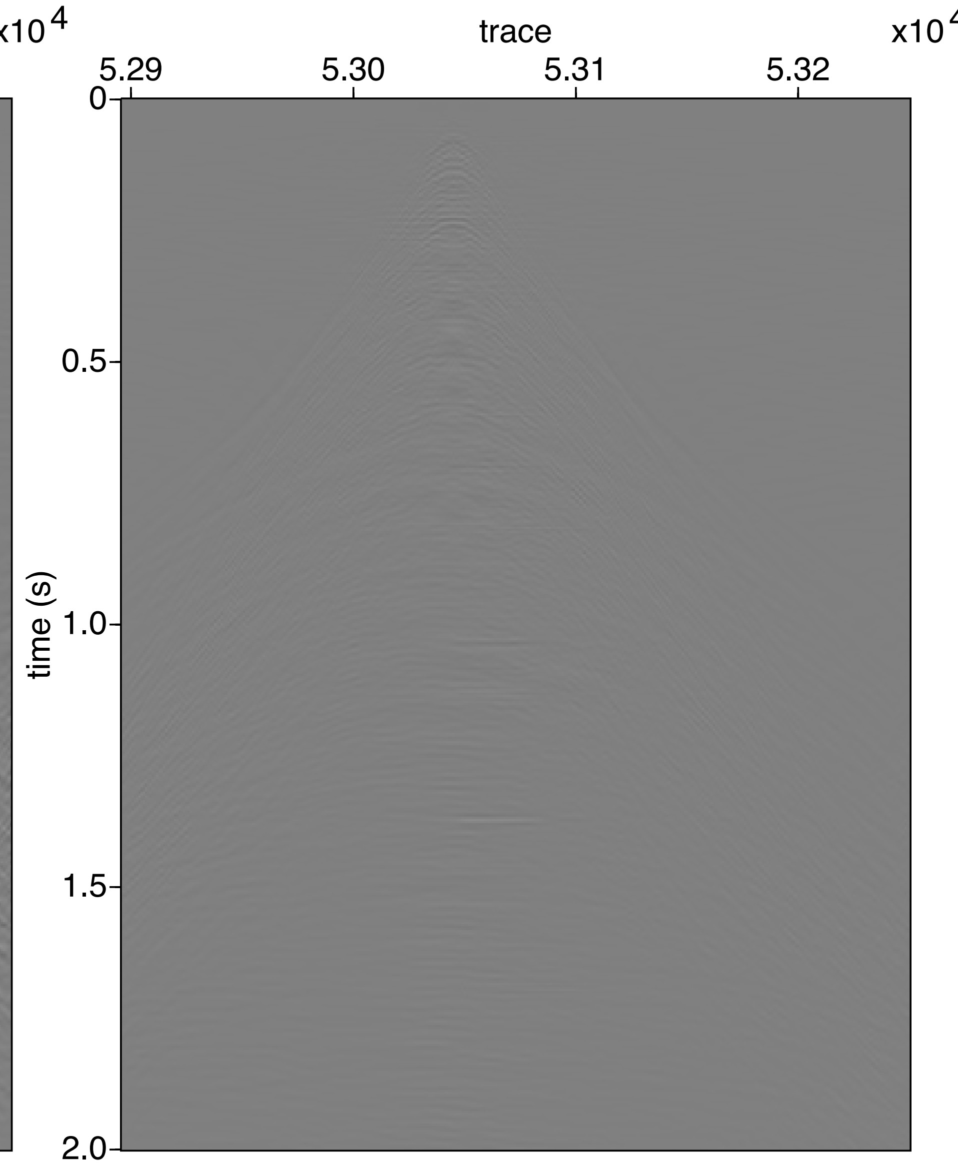
20% rank budget



Primary IR
full data



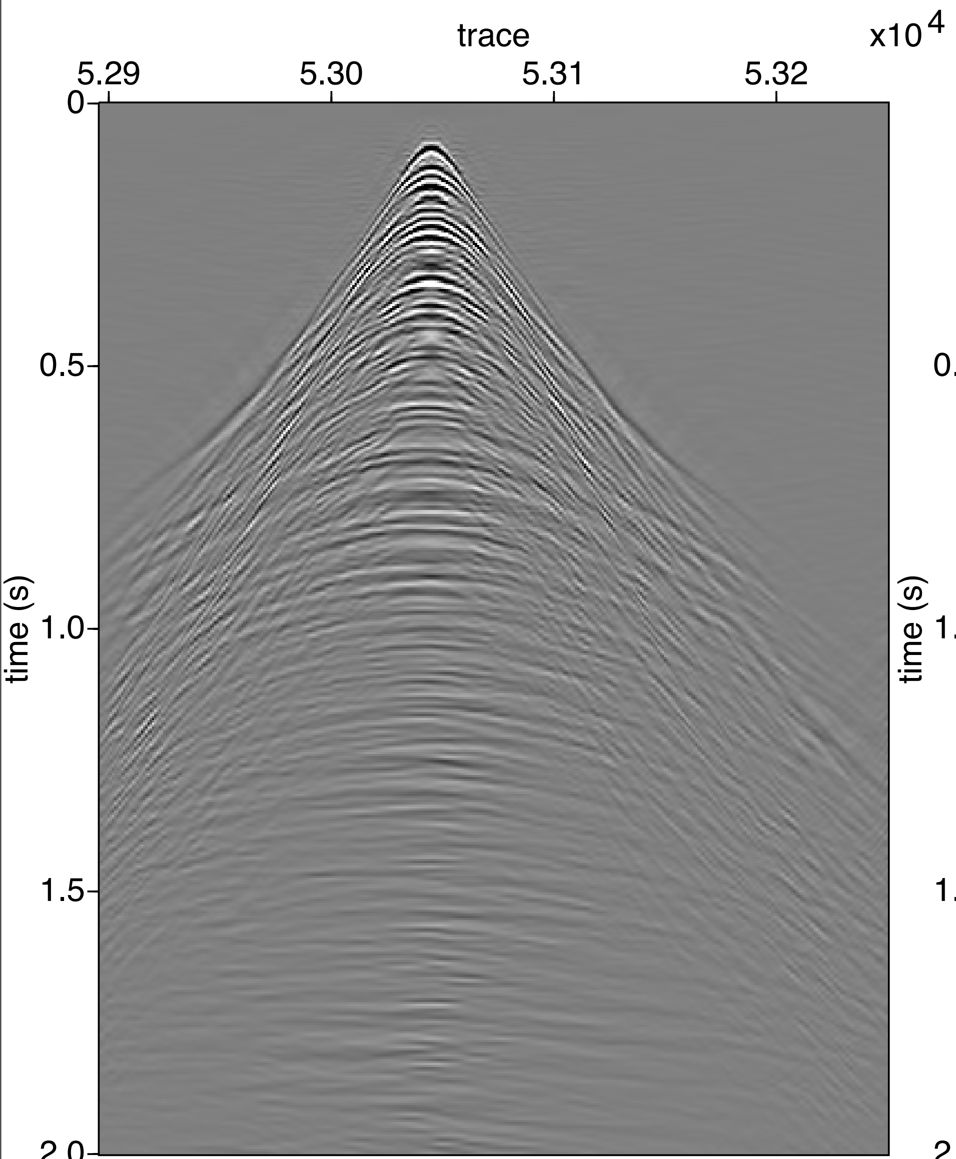
Primary IR
approximated Data



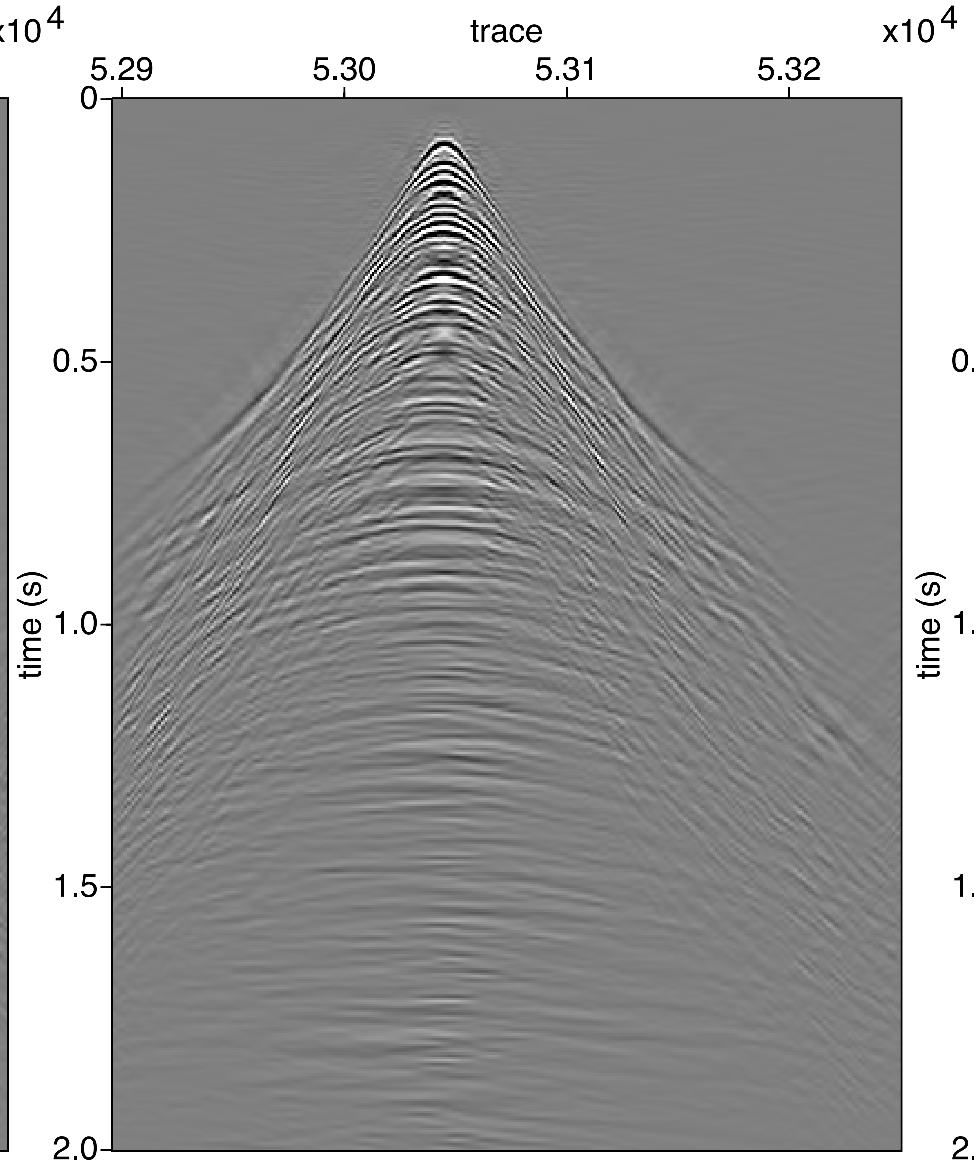
Difference

Difference in EPSI Result

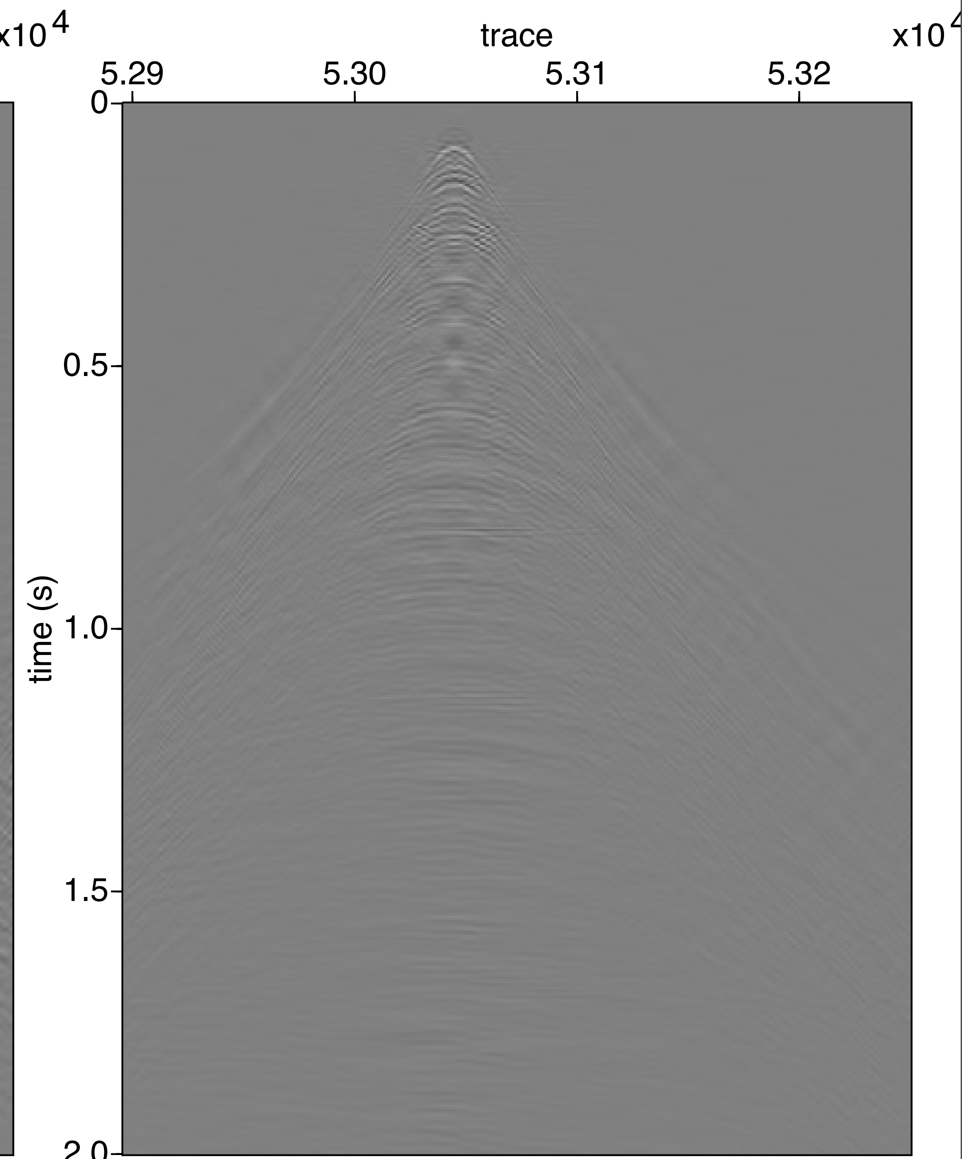
12% rank budget



Primary IR
full data



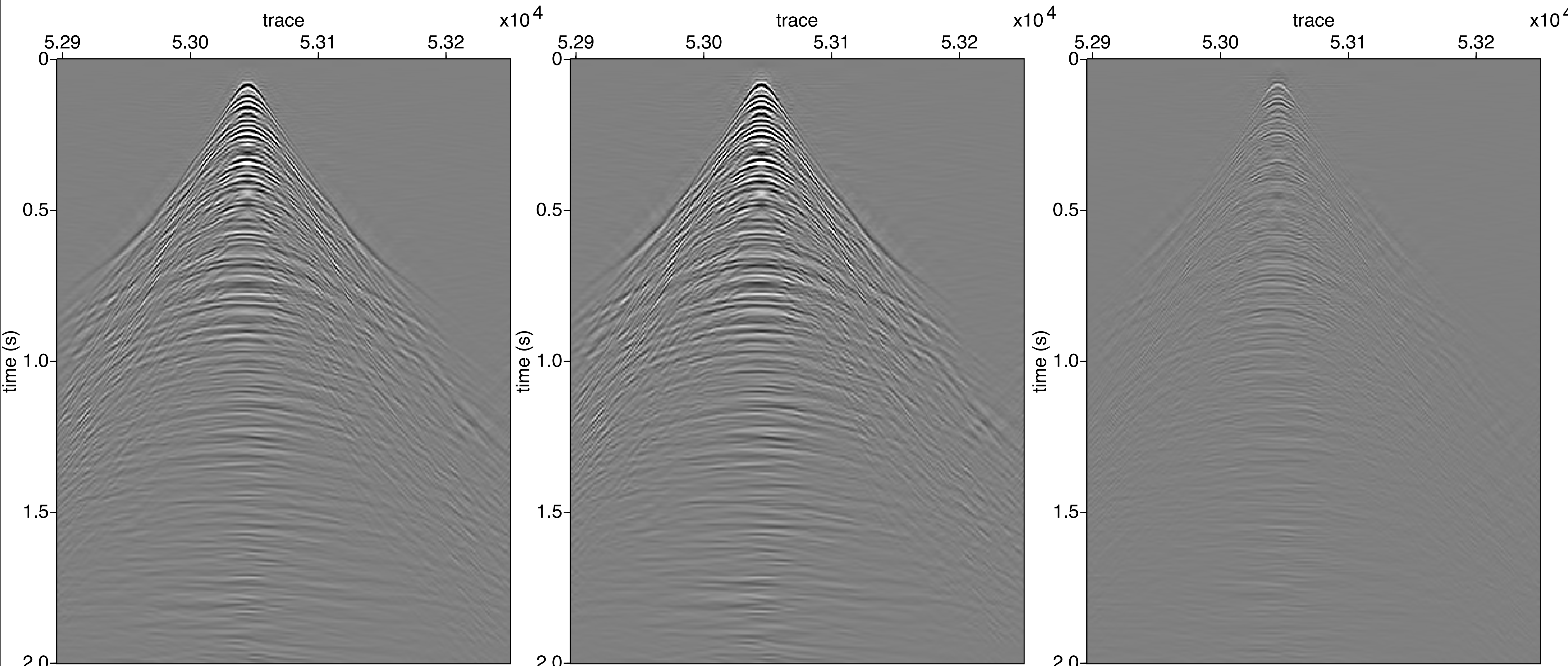
Primary IR
approximated Data



Difference

Difference in EPSI Result

8% rank budget



Primary IR
full data

Primary IR
approximated Data

Difference

Performance Summary

Rank Percentage	50%	20%	12%	8%
SNR (dB)	30	27	17	12
Multiplication Speedup	1.6x	2x	3.5x	5.7x
Memory savings	40%	50%	71%	82%

Conclusions

- Data driven methods - e.g. EPSI - suffers from the 'curse of dimensionality' when moving to 3D
- We utilize insights from *random*-matrix theory to *approximate* action of the *data* matrix
- *Slow* decaying singular values
    - *power* Iterations
    - *HSS* representations
- *Reductions* in *multiplication* and *storage* costs

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Thank you

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