SINBAD Consortium 2010

Randomized full-waveform inversion with compressive sensing

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SLIM

Full-waveform inversion

objective function:

$$\min_{\mathbf{m}} \ \Phi(\mathbf{m}) := \left\{ \frac{1}{2K} \sum_{i=1}^{K} \|\mathbf{d}_i - \mathcal{F}_i[\mathbf{m}]\mathbf{q}_i\|_2^2 = \frac{1}{2}\|\mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m}]\mathbf{Q}\|_F^2 \right\}$$

$$\begin{aligned} \mathbf{d}_i &= \text{monochromatic shot records} \\ \mathbf{q}_i &= \text{monochromatic sources} \\ \mathcal{F}_i[\mathbf{m}, \mathbf{q}_i] &= \text{monochromatic nonlinear forward operators} \end{aligned}$$

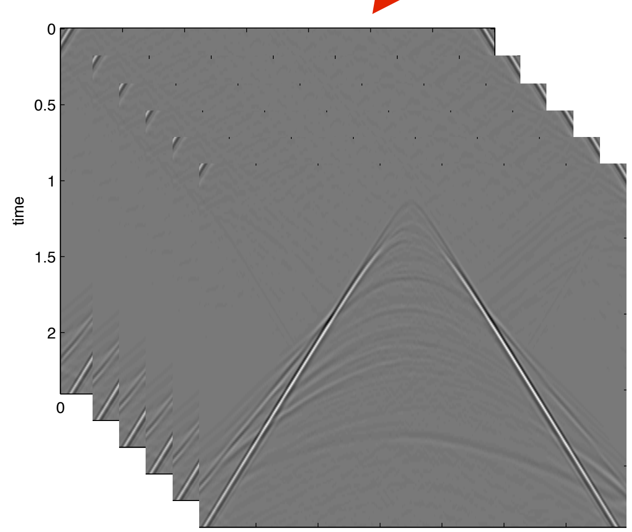
Gauss-Newton

GN subproblem:

$$\min_{\mathbf{m}} \quad \underline{\Phi}(\mathbf{m}) := \frac{1}{2} \| \underbrace{\mathbf{D} - \mathcal{F}[\mathbf{m};\mathbf{Q}]}_{\mathbf{b}} - \underbrace{\nabla \mathcal{F}[\mathbf{m};\mathbf{Q}]}_{\mathbf{A}} \underbrace{\delta \mathbf{m}}_{\mathbf{x}} \|_F^2 .$$

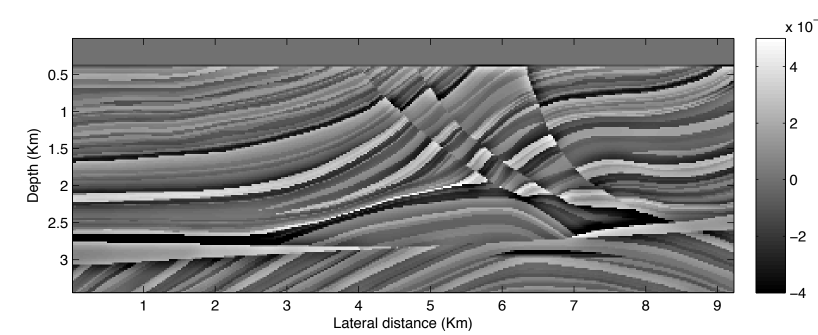
This is a least-squares problem:

$$\min_{\mathbf{x}} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2$$



$n_s \times n_f \times n_r$

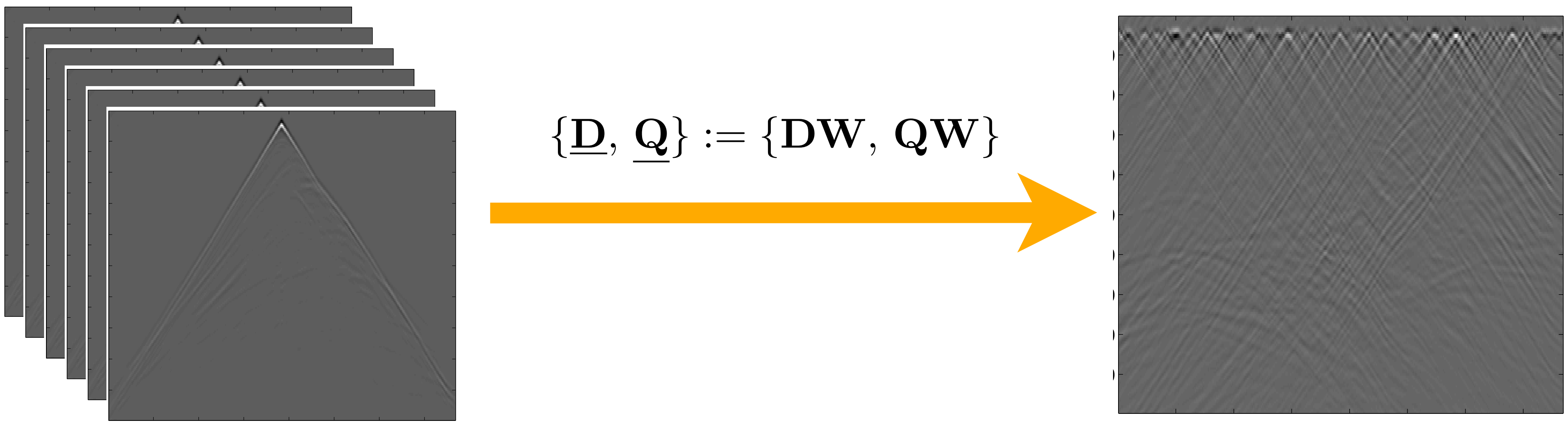
large over-determined linear operator



$n_x \times n_z$

Dimensionality reduction

$$\min_{\mathbf{m}} \ \underline{\Phi}(\mathbf{m}) := \left\{ \frac{1}{2K'} \sum_{i=1}^{K'} \|\mathbf{D}\mathbf{w}_i - \mathcal{F}_i[\mathbf{m}]\mathbf{w}_i\|_2^2 = \frac{1}{2}\|\underline{\mathbf{D}} - \boldsymbol{\mathcal{F}}[\mathbf{m};\underline{\mathbf{Q}}]\|_F^2 \right\}$$



GN subproblem:

$$\min_{\mathbf{m}} \quad \underline{\Phi}(\mathbf{m}) := \frac{1}{2}\|\underline{\mathbf{D}} - \boldsymbol{\mathcal{F}}[\mathbf{m};\underline{\mathbf{Q}}] - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m};\underline{\mathbf{Q}}]\delta\mathbf{m}\|_F^2.$$

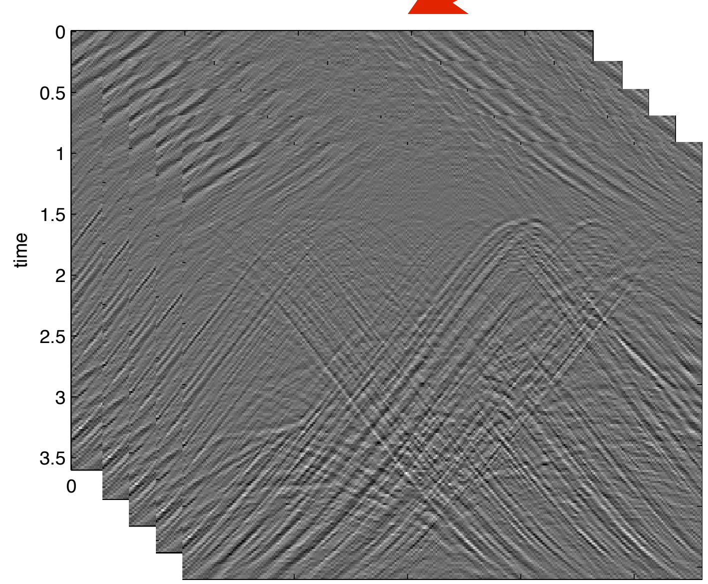
Gauss-Newton

GN subproblem:

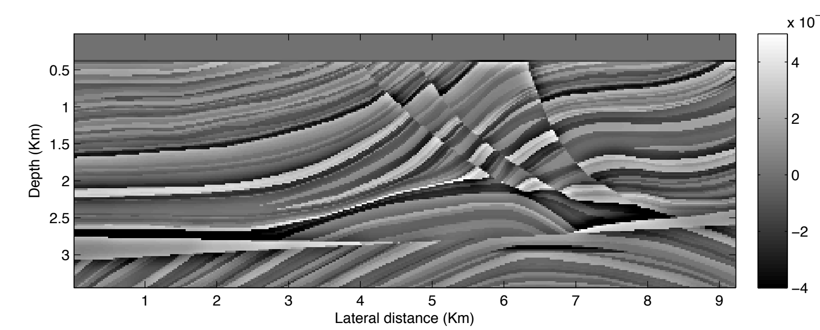
$$\min_{\mathbf{m}} \quad \underline{\Phi}(\mathbf{m}) := \frac{1}{2}\|\underbrace{\underline{\mathbf{D}} - \mathcal{F}[\mathbf{m};\underline{\mathbf{Q}}]}_{\underline{\mathbf{b}}} - \underbrace{\nabla\mathcal{F}[\mathbf{m};\underline{\mathbf{Q}}]}_{\underline{\mathbf{A}}}\underbrace{\delta\mathbf{m}}_{\mathbf{x}}\|_F^2.$$

This is a least-squares problem:

$$\min_{\mathbf{x}} \|\underline{\mathbf{b}} \quad - \quad \underline{\mathbf{A}} \quad \mathbf{x}\|_2^2$$



underdetermined
linear operator



$ns' \times nf' \times nr$

$nx \times nz$

Sparsity promotion

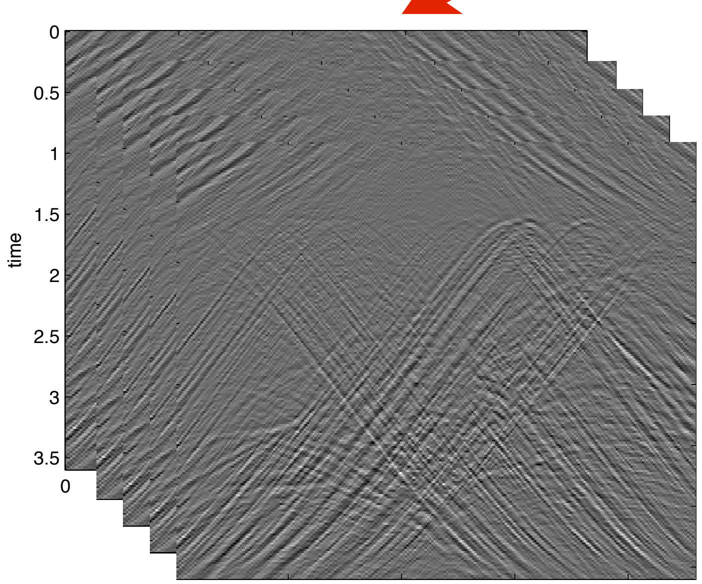
Modified GN subproblem (LASSO):

$$\min_{\mathbf{x}} \frac{1}{2}\|\underline{\boldsymbol{\delta}\mathbf{D}} - \nabla\mathcal{F}[\mathbf{m};\underline{\mathbf{Q}}]\mathbf{S}^H\mathbf{x}\|_F^2 \quad \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau$$

$$\underline{\mathbf{b}} \qquad \underline{\mathbf{A}} \quad \mathbf{S}^H \; \mathbf{x}$$

S is a sparsifying transform.

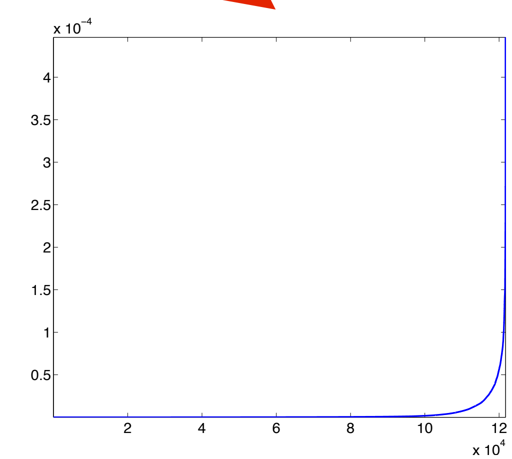
$$\min_{\mathbf{x}} \|\underline{\mathbf{b}} - \underline{\mathbf{A}} \; \mathbf{S}^H \; \mathbf{x}\|_2^2$$

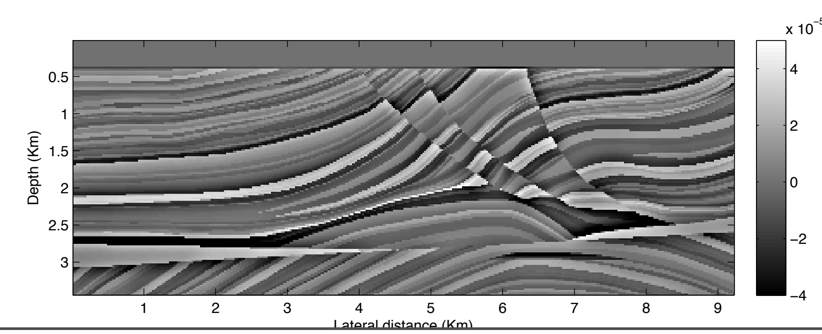


$ns' \times nf' \times nr$

under-determined linear operator

$\mathbf{S}^H$





$nx \times nz$

Sparse promotion

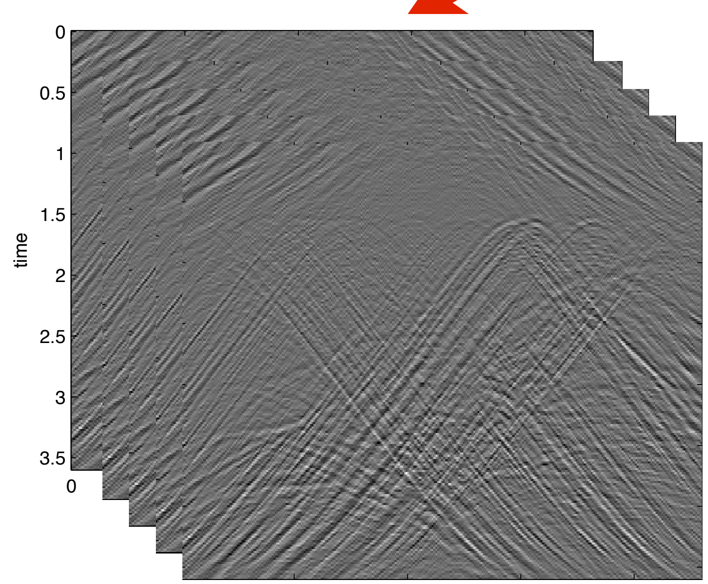
Modified GN subproblem (LASSO)

$$\min_{\mathbf{x}} \frac{1}{2}\|\underline{\boldsymbol{\delta}\mathbf{D}} - \nabla\mathcal{F}[\mathbf{m};\underline{\mathbf{Q}}]\mathbf{S}^H\mathbf{x}\|_F^2 \quad \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau$$

$$\underline{\mathbf{b}} \qquad \underline{\mathbf{A}} \quad \mathbf{S}^H \; \mathbf{x}$$

This is a least-squares problem:

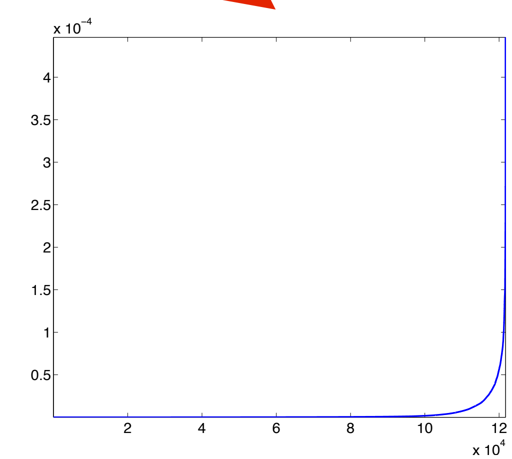
$$\min_{\mathbf{x}} \|\underline{\mathbf{b}} - \underline{\mathbf{A}}\,\mathbf{S}^H\,\mathbf{x}\|_2^2$$

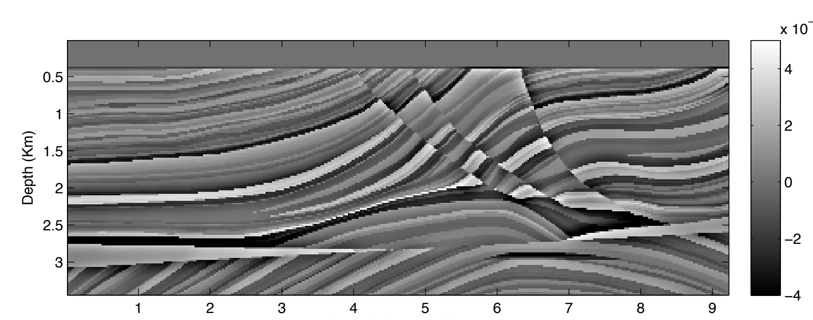


$ns' \times nf' \times nr$

under-determined linear operator

$\mathbf{S}^H$

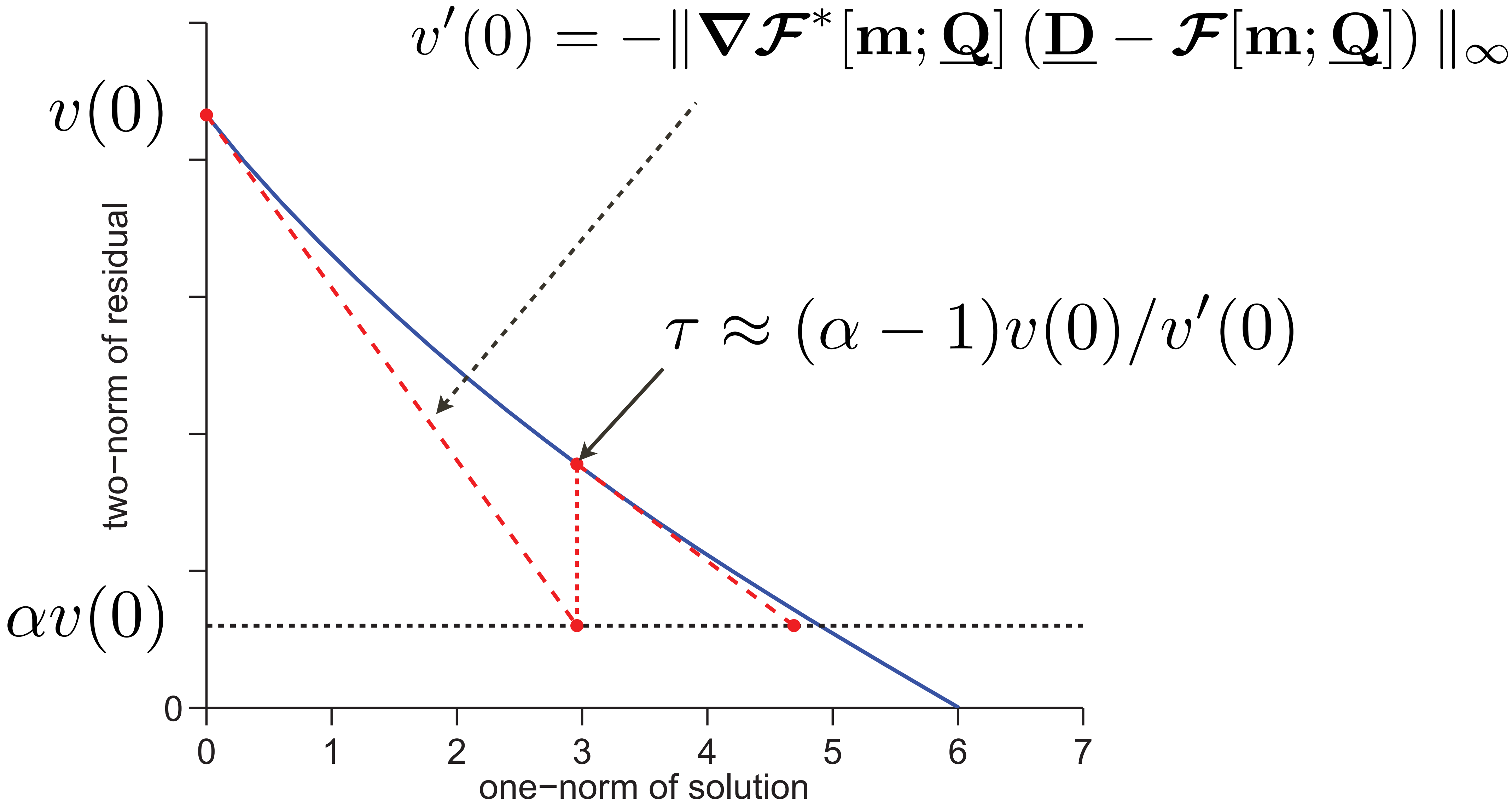


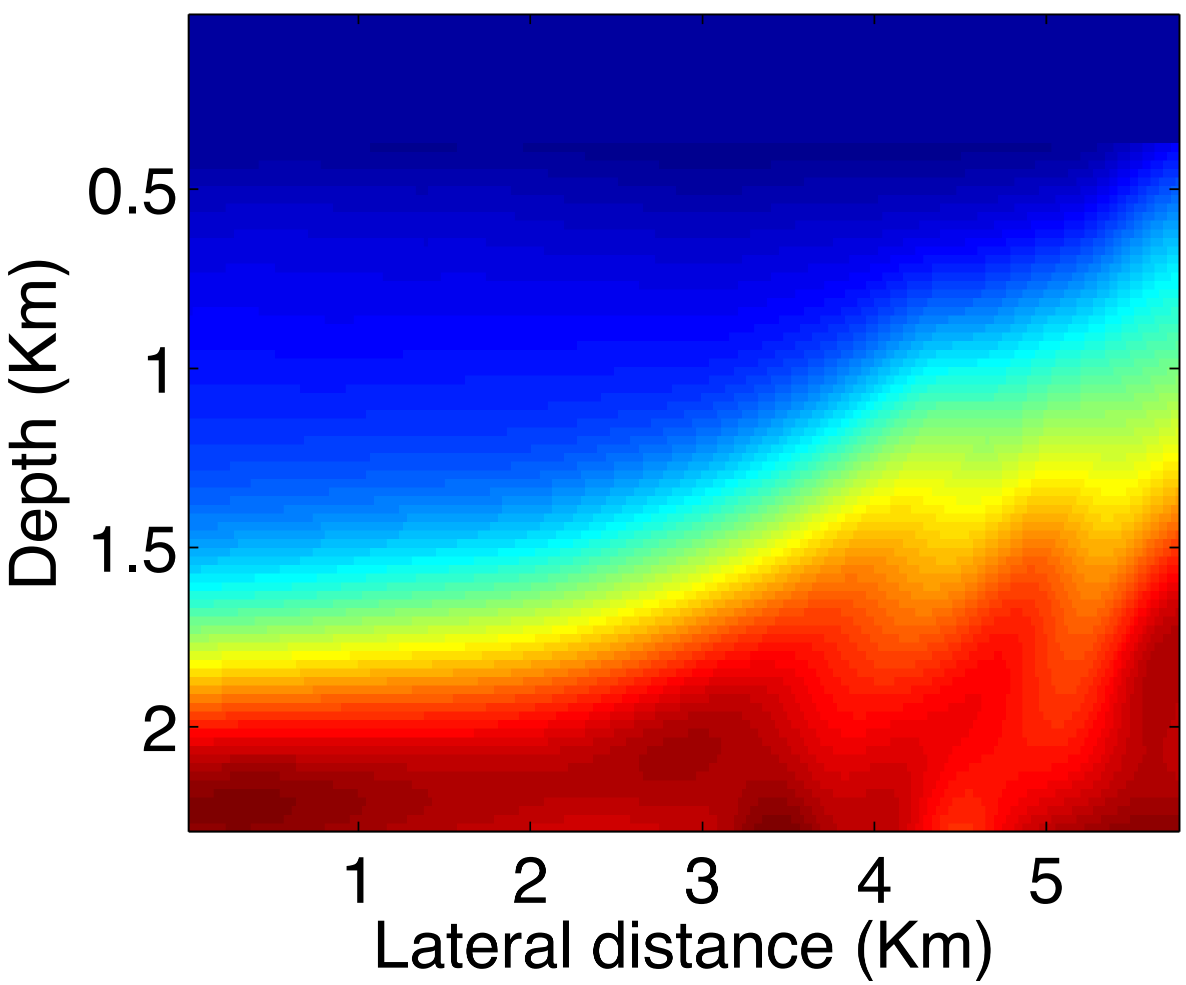


$nx \times nz$

Pareto curve

Compute τ using Pareto curve [van den Berg & Friendlander, '08]





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Depth (Km)
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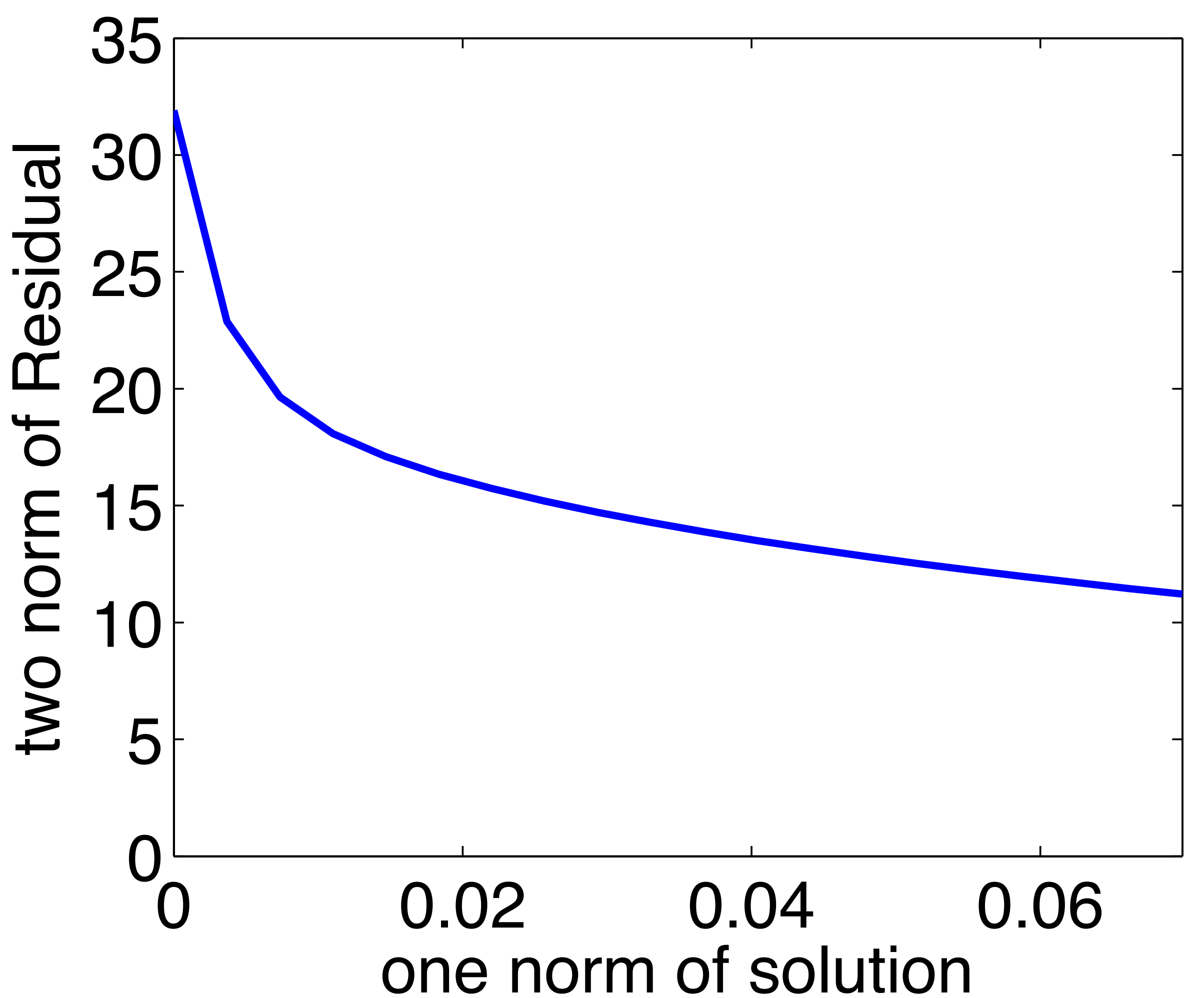
Lateral distance (Km)



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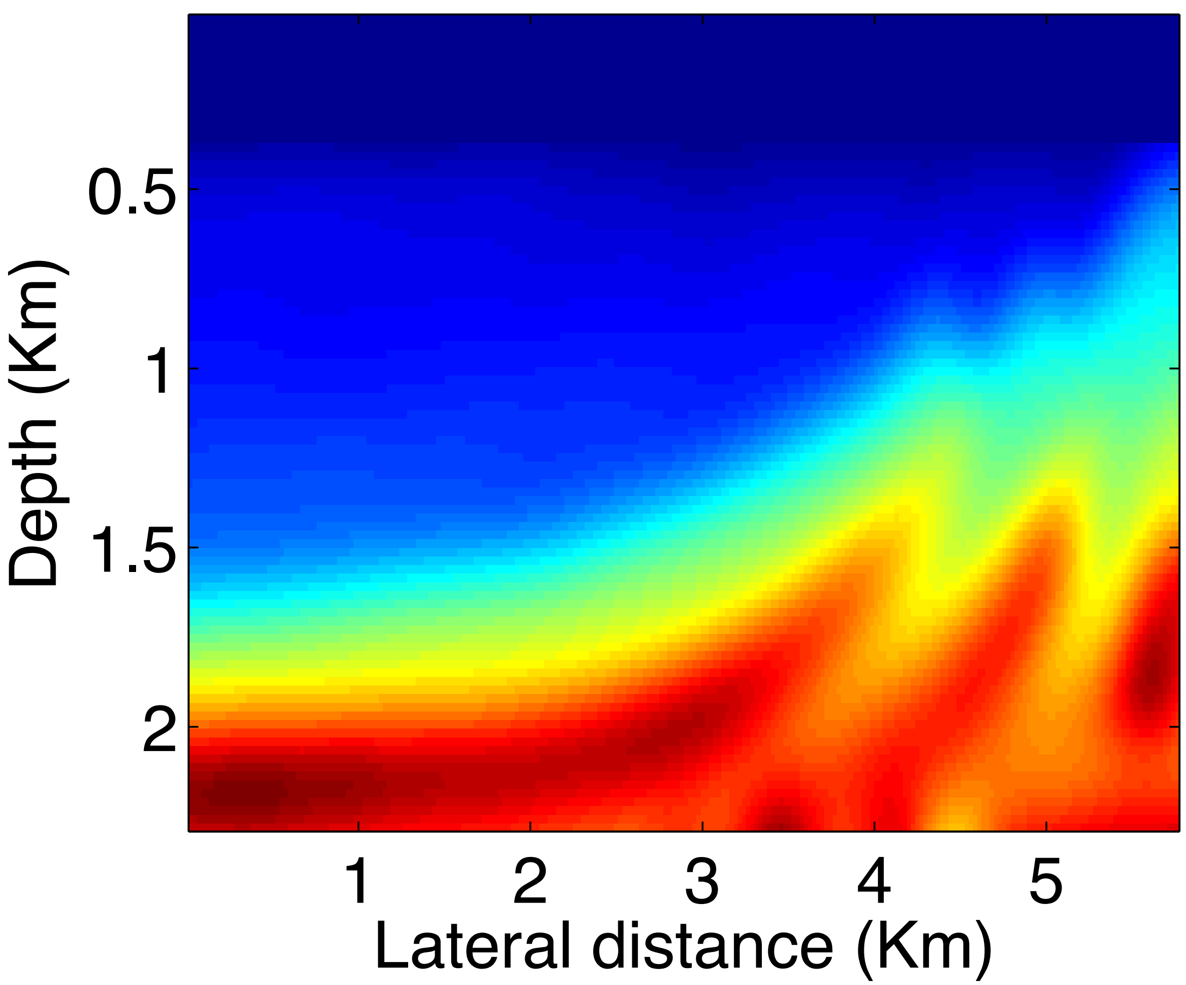
one norm of solution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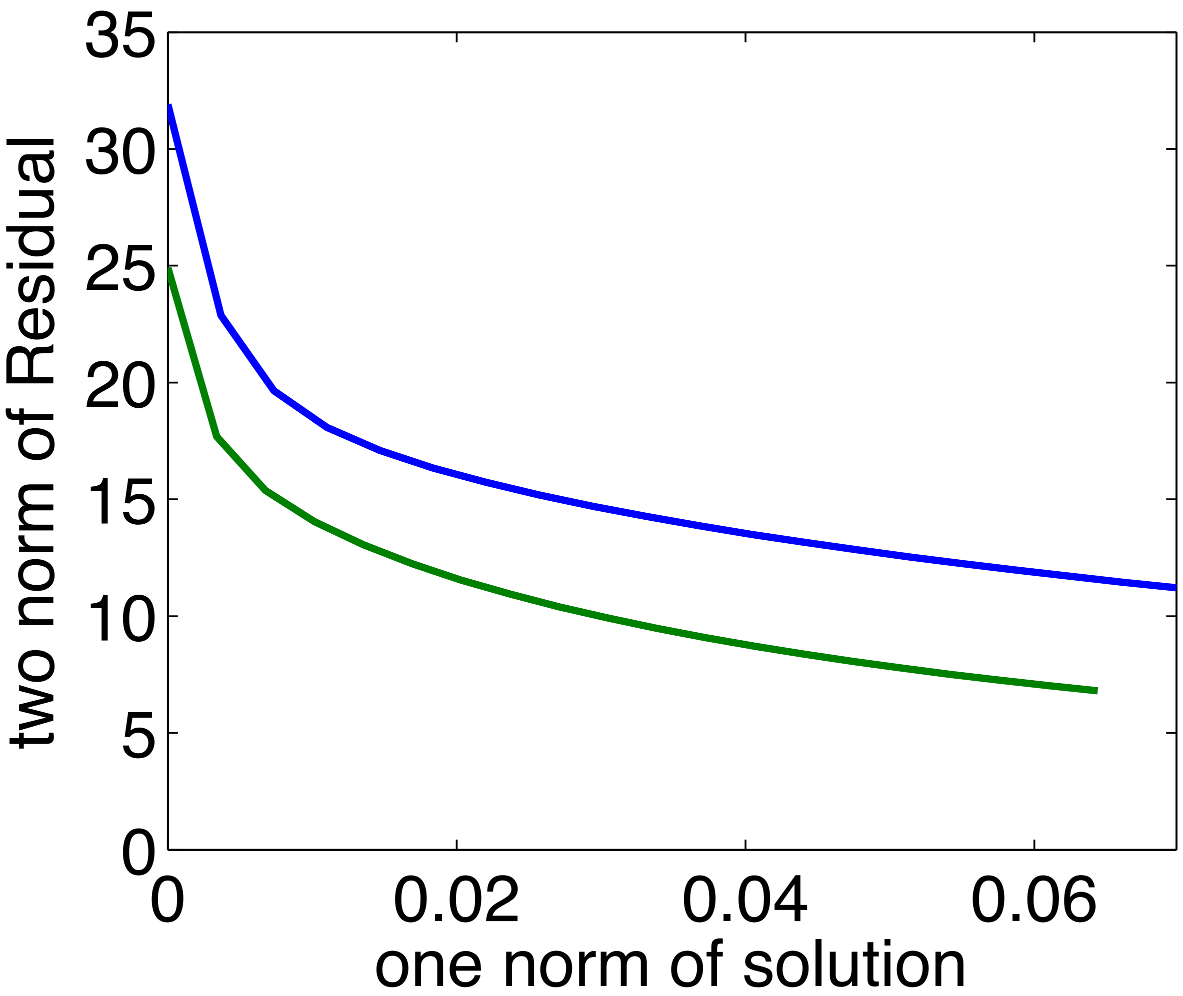
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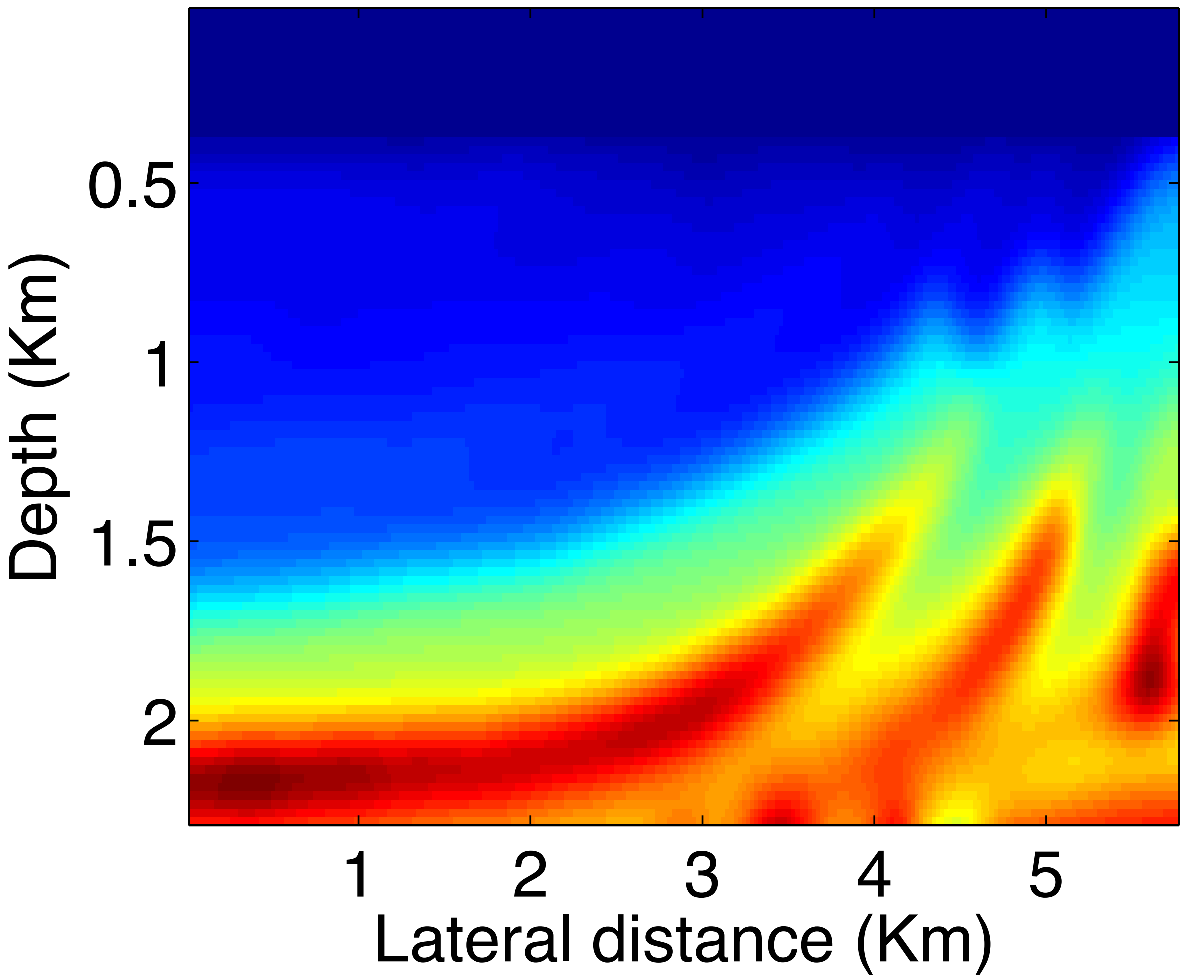
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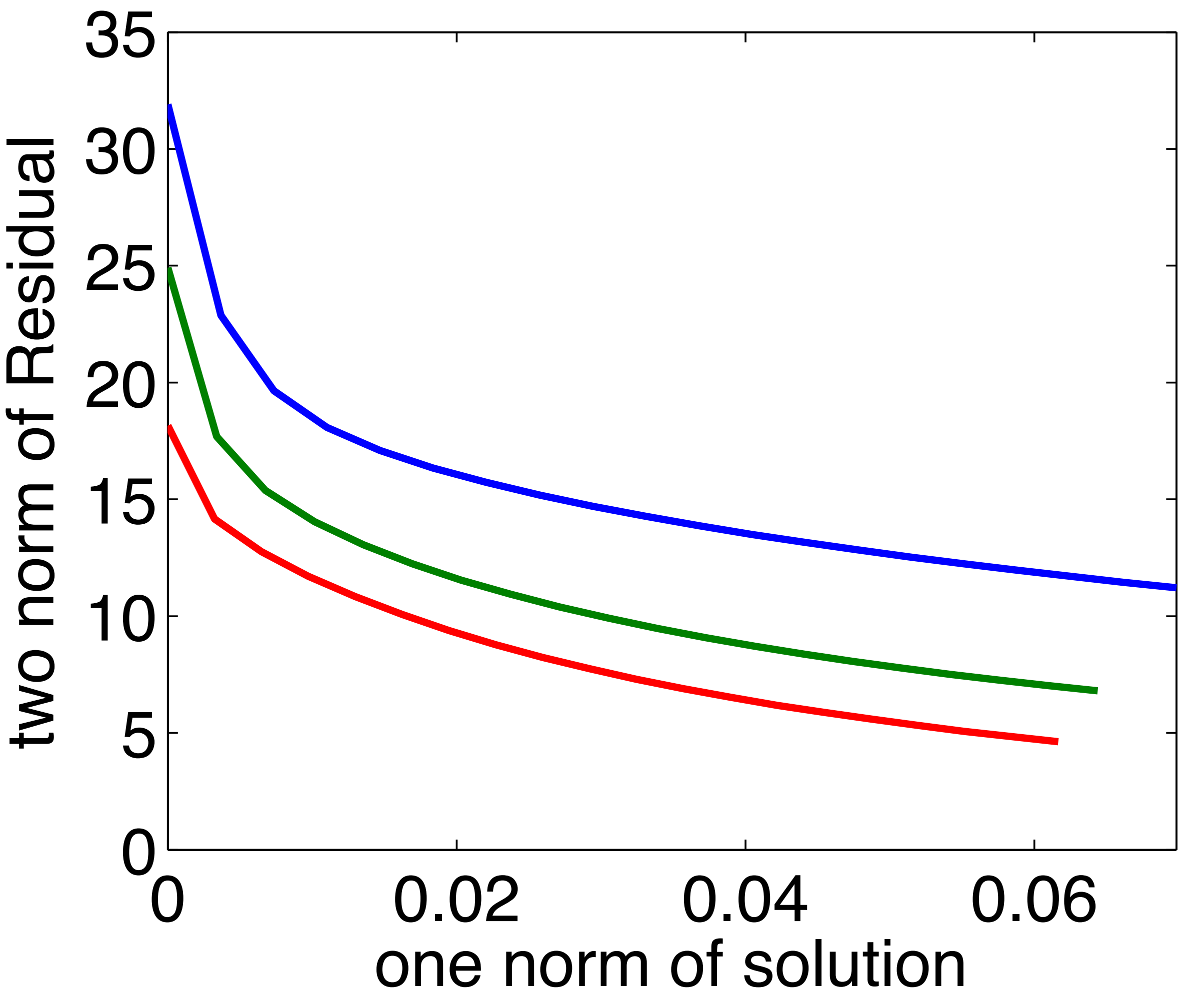
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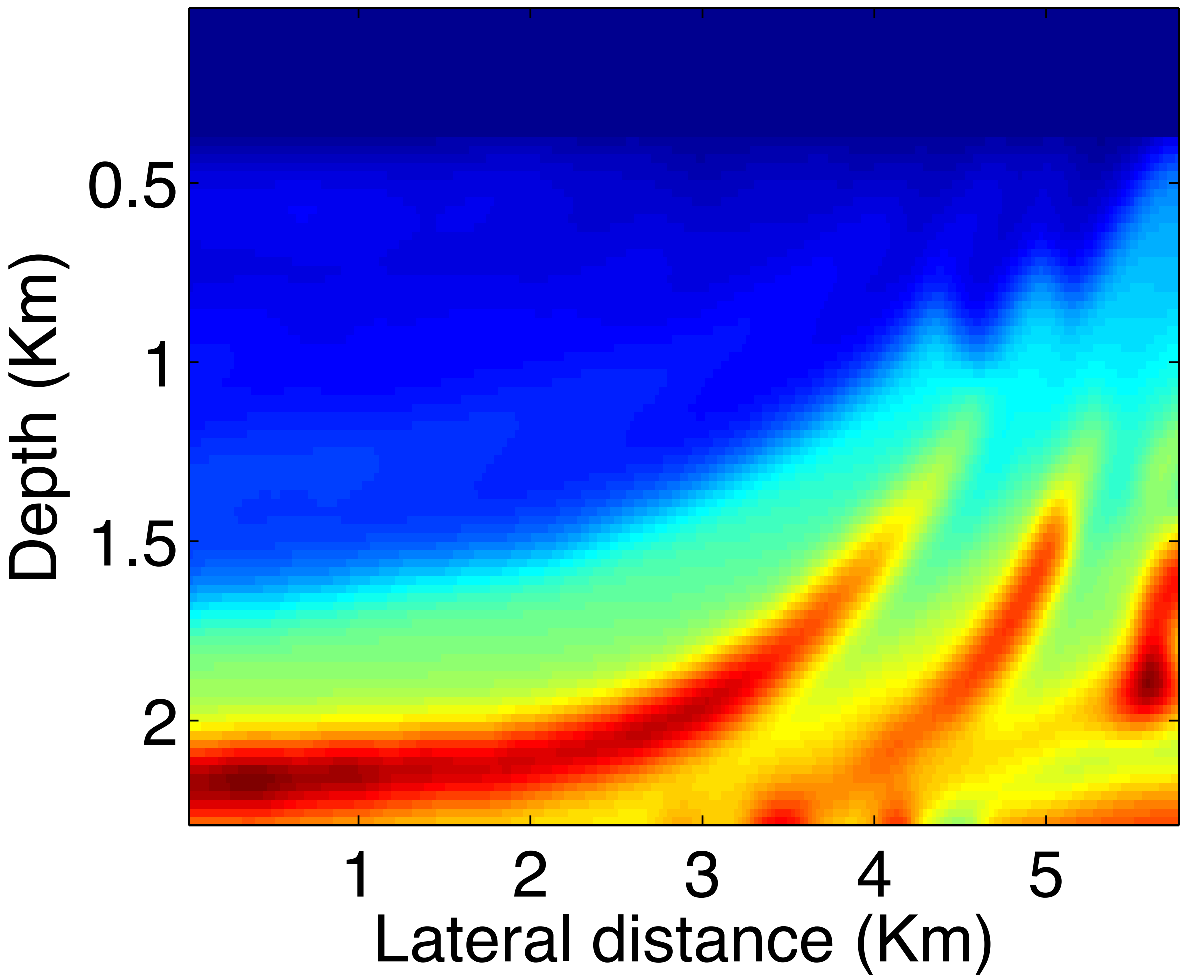

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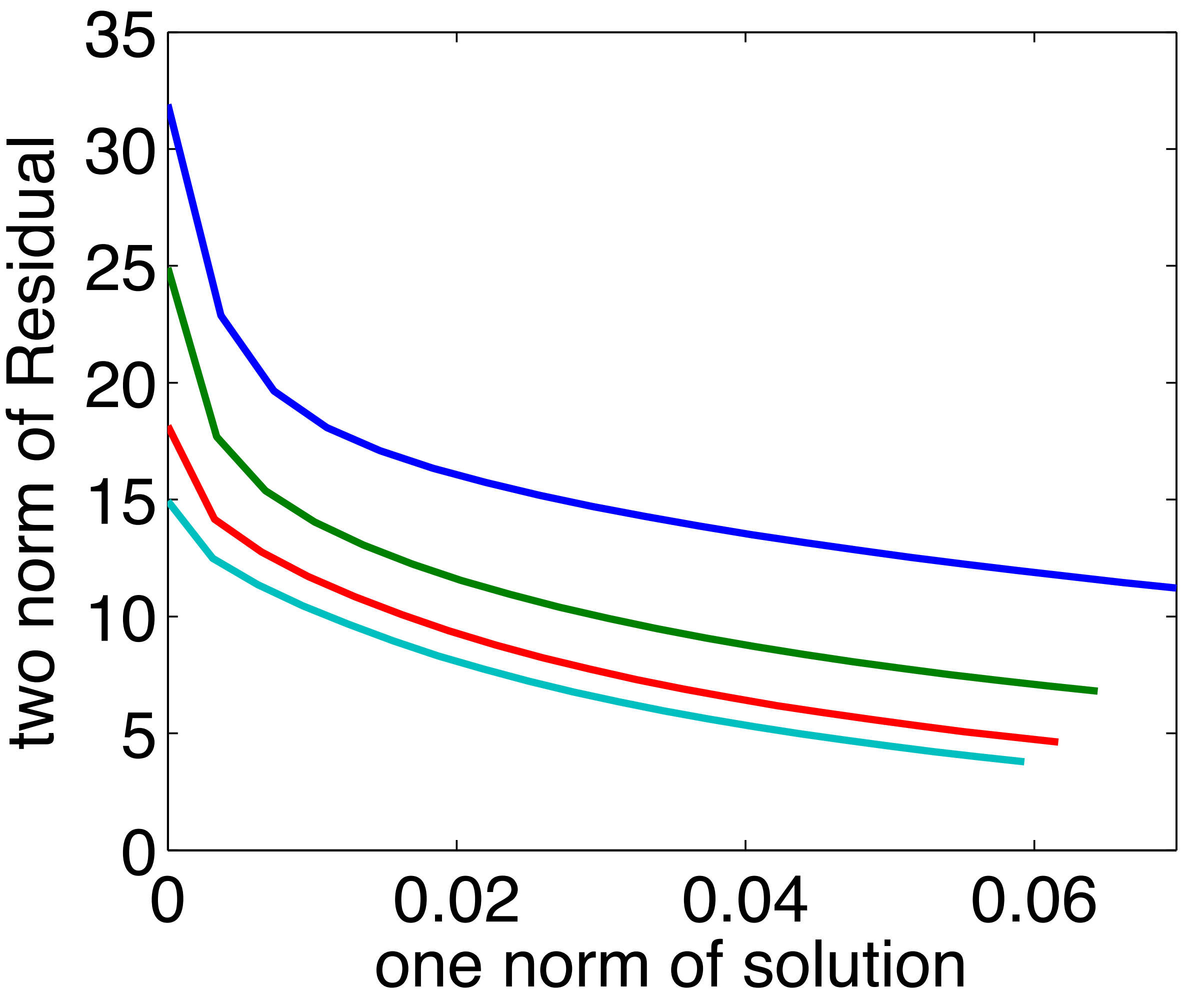

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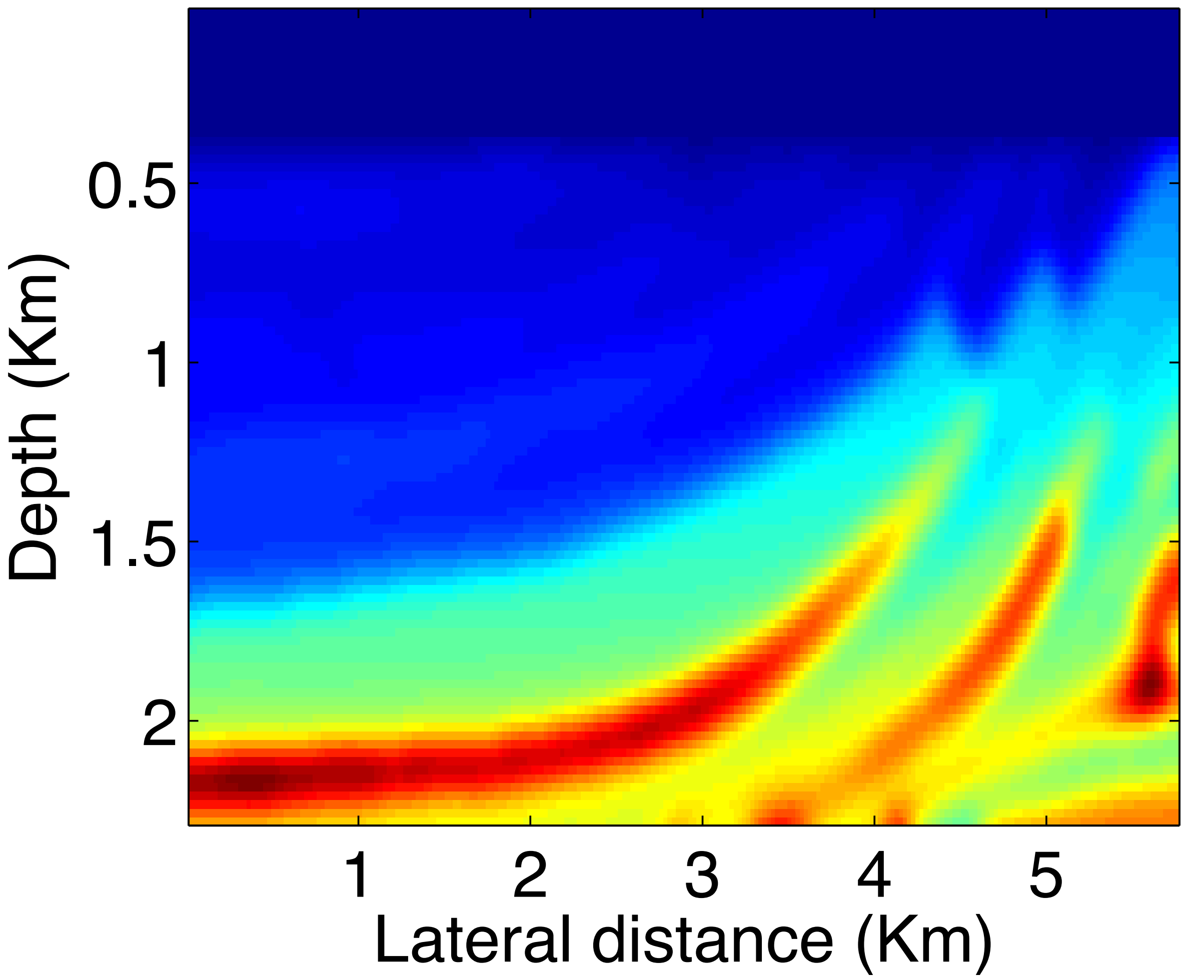

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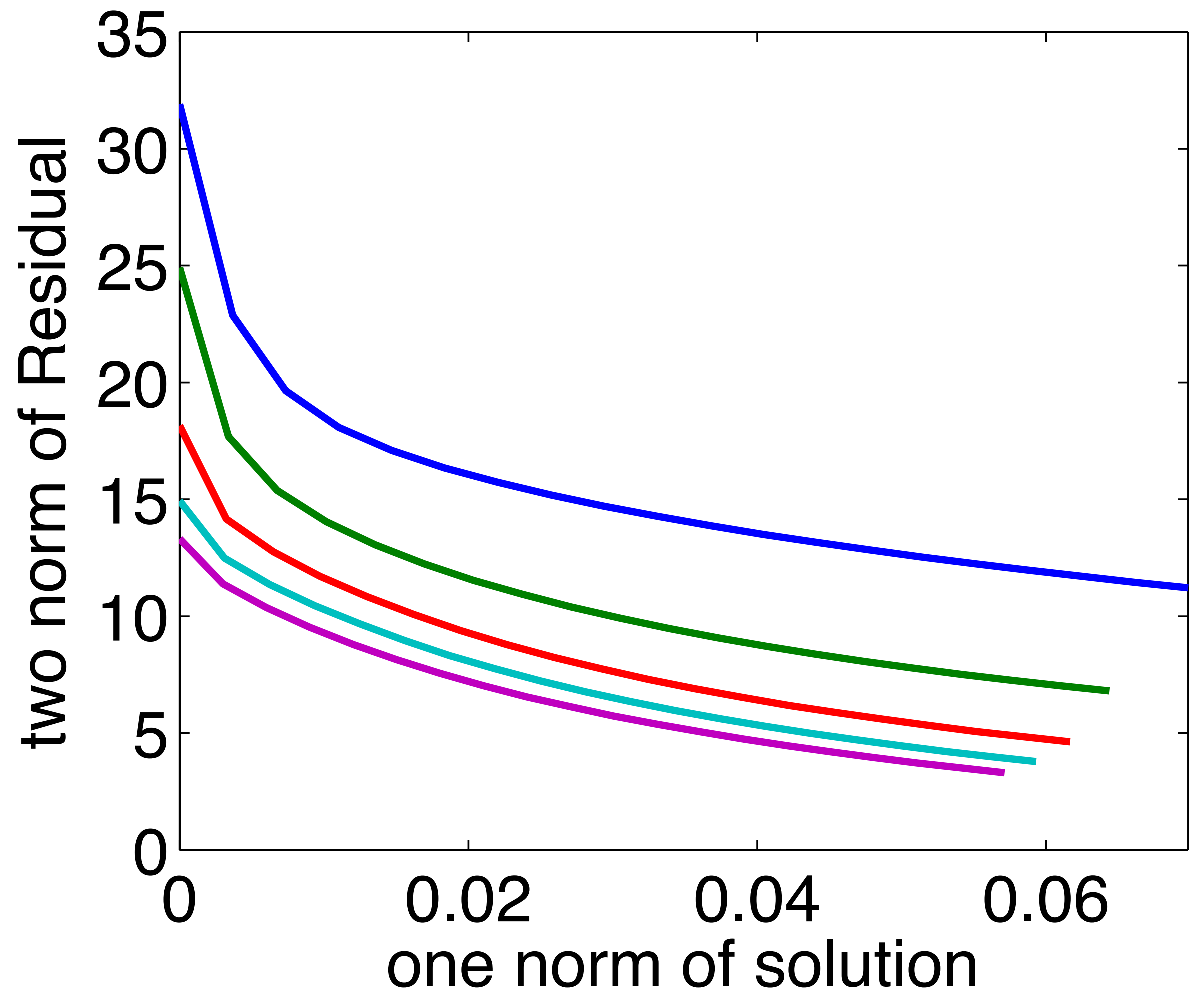

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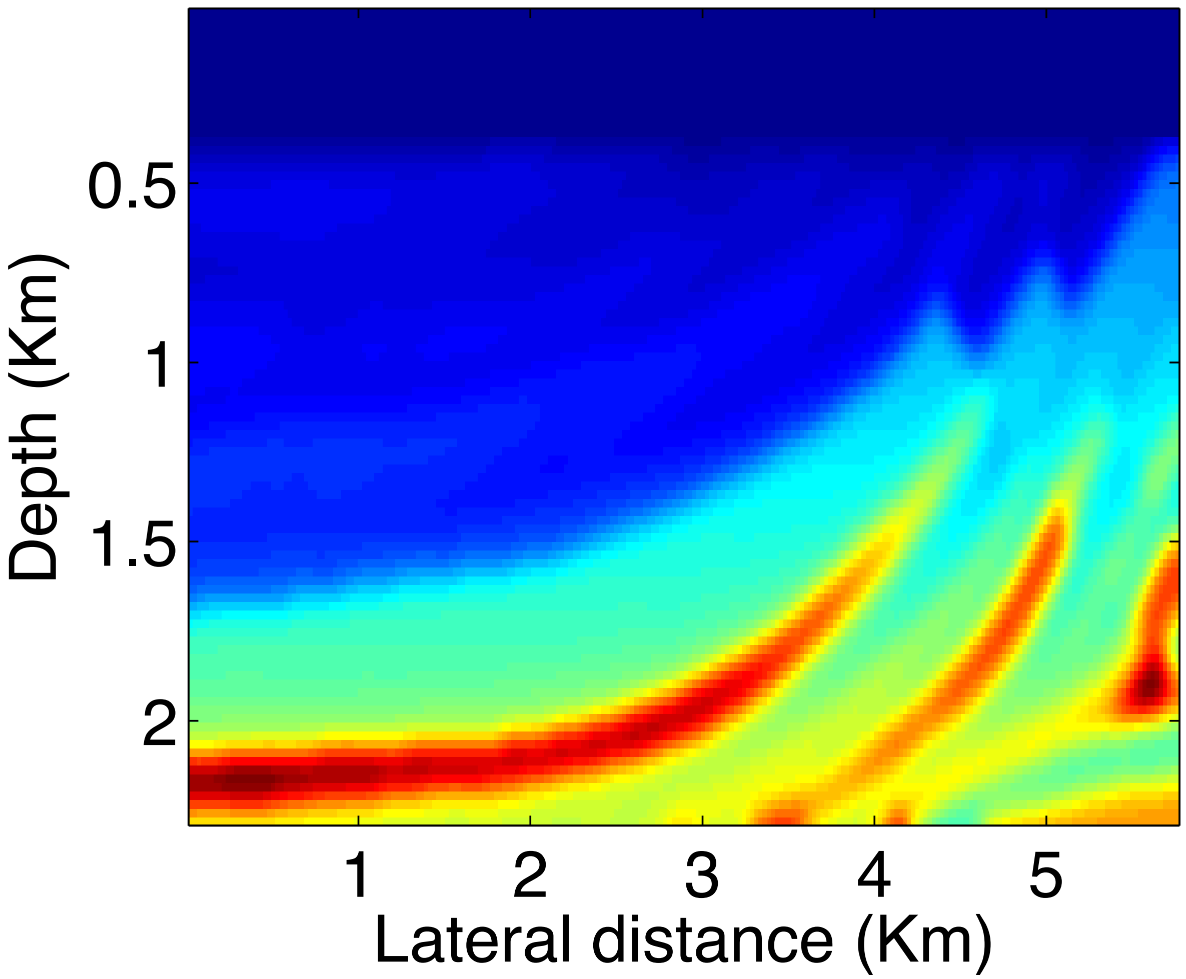
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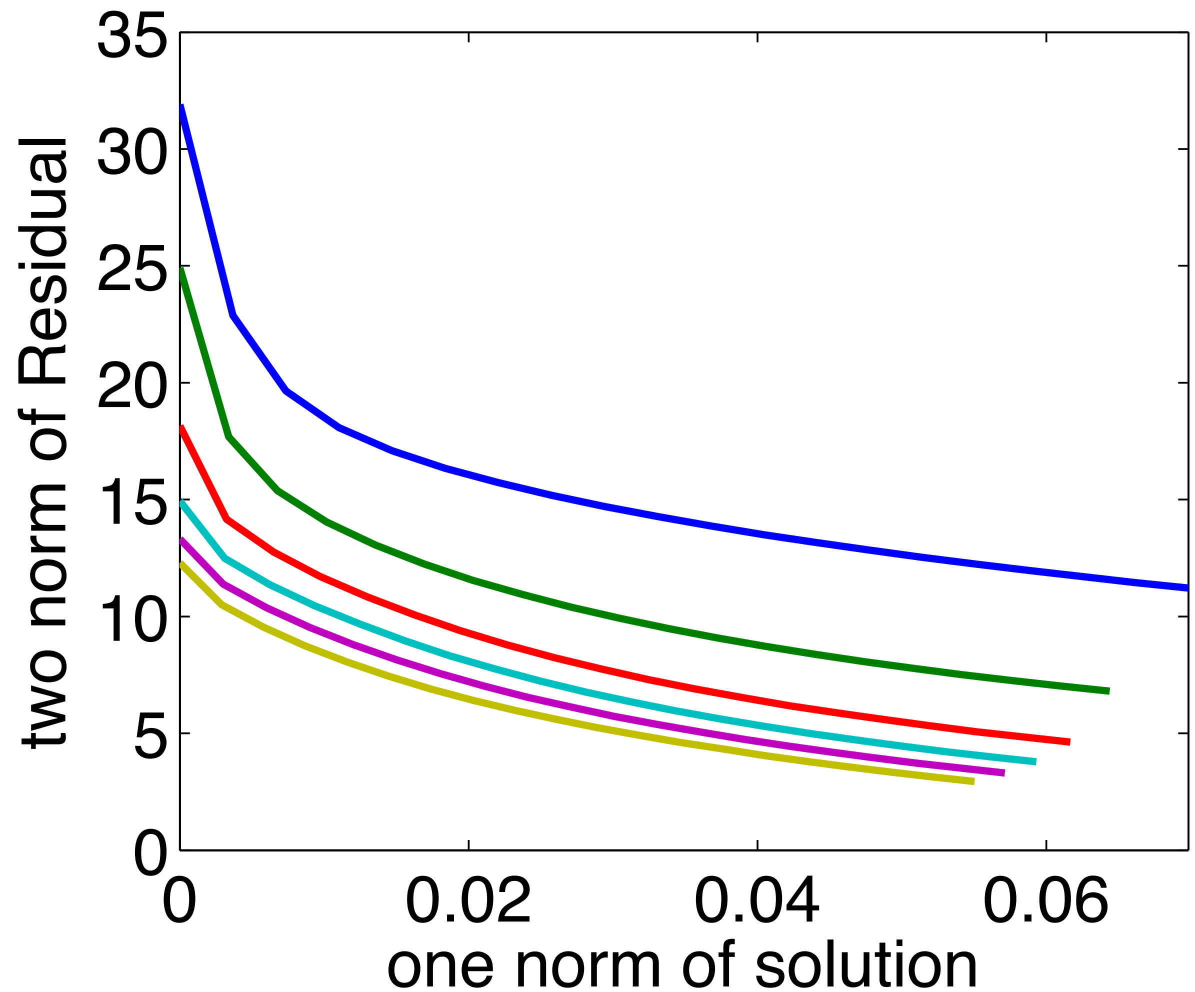
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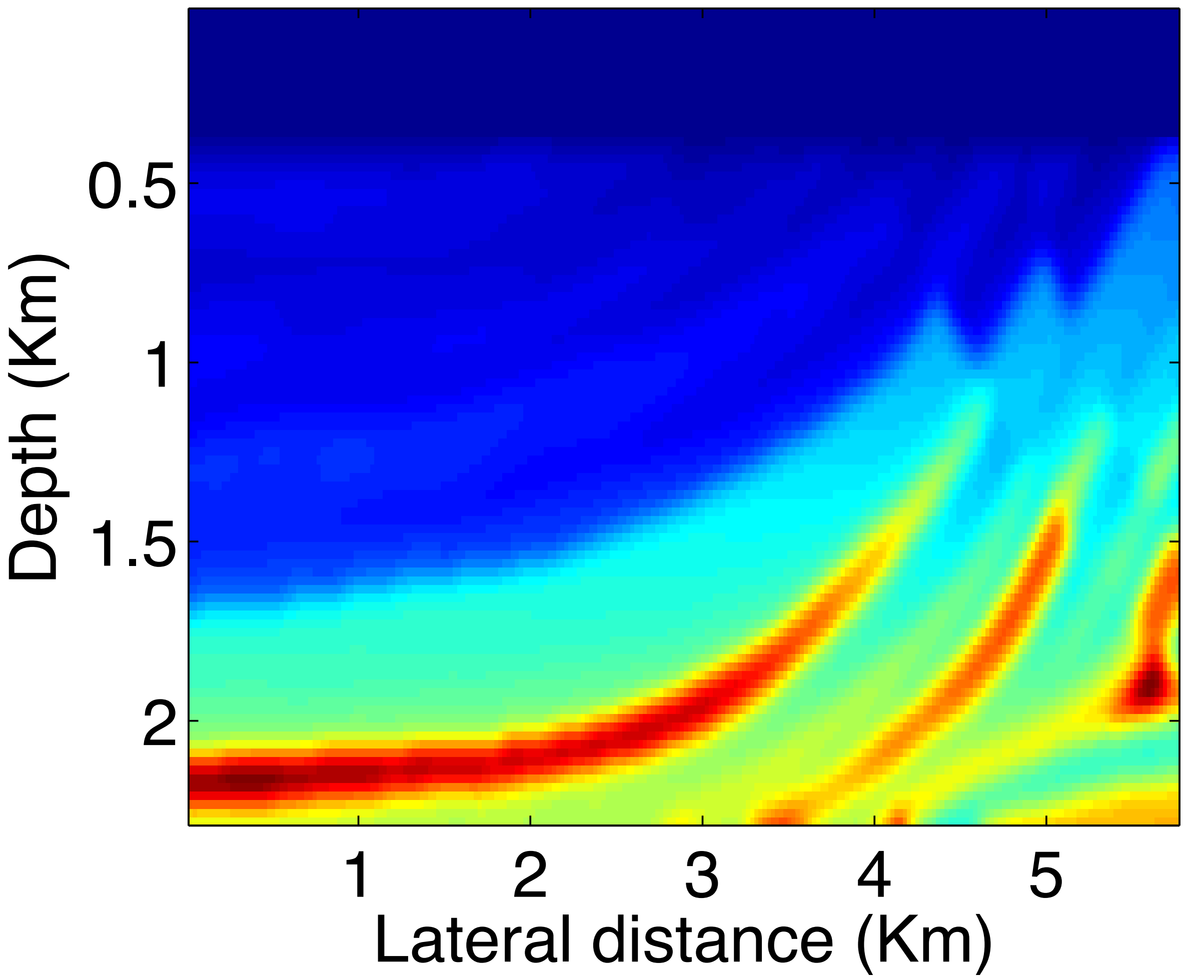

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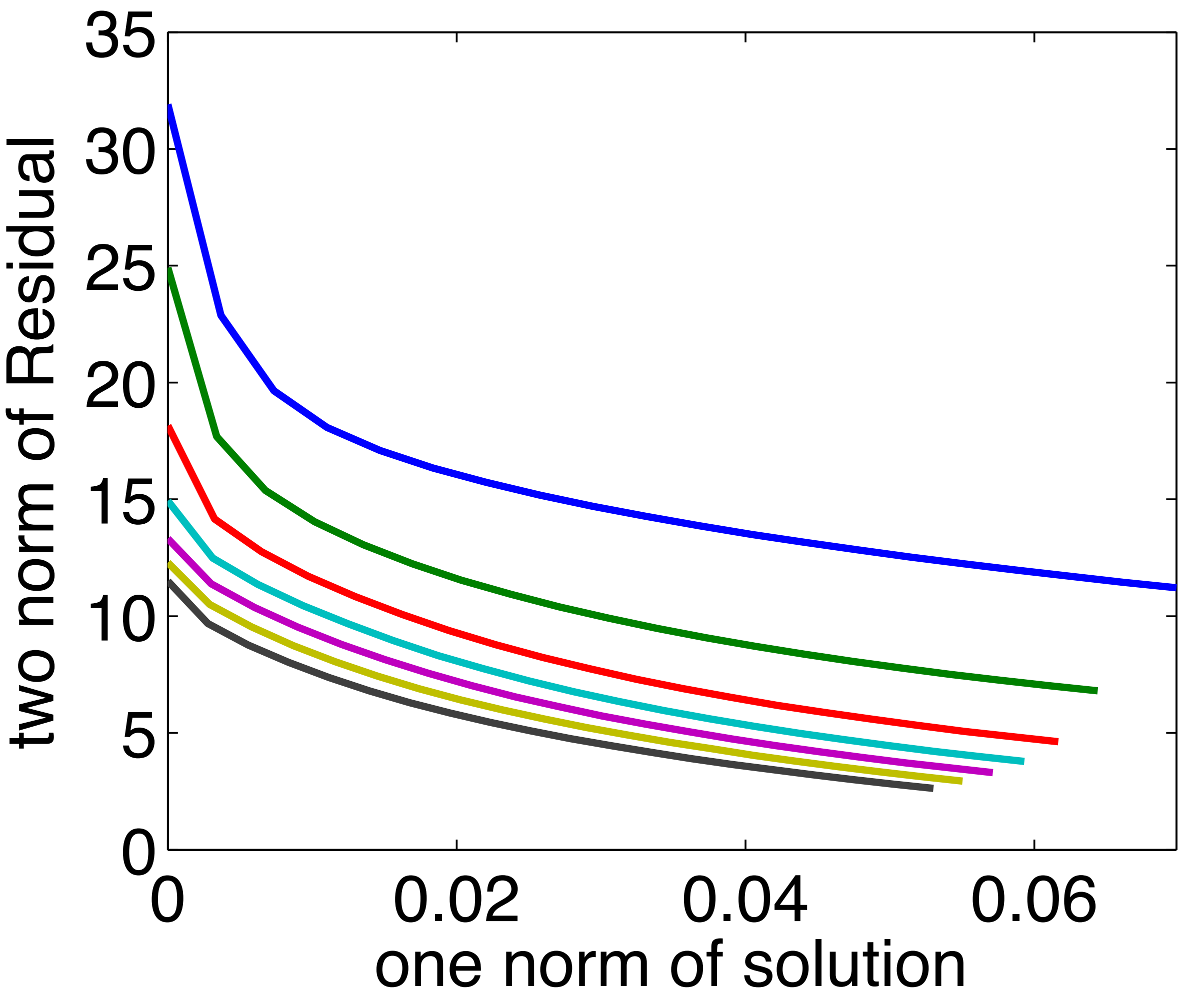

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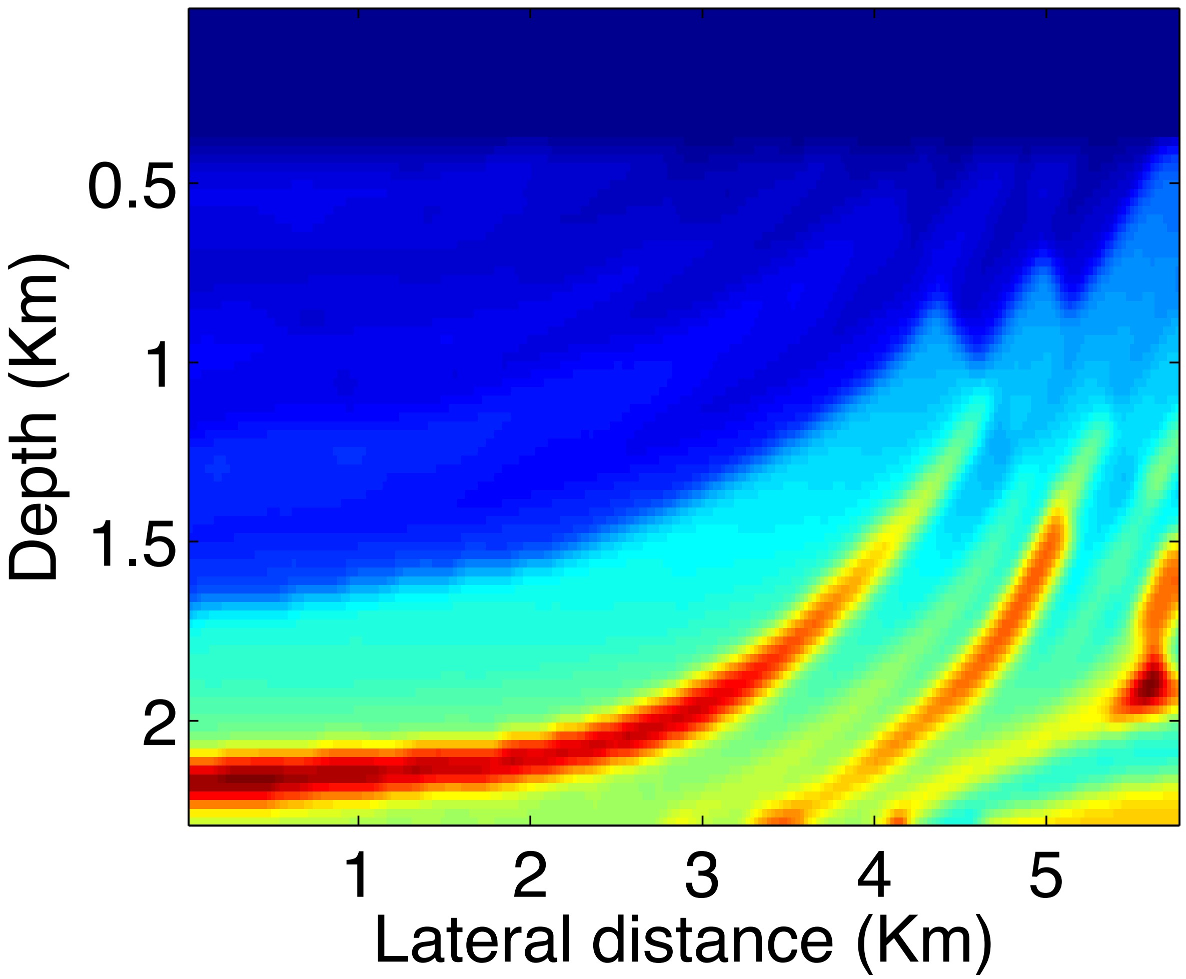
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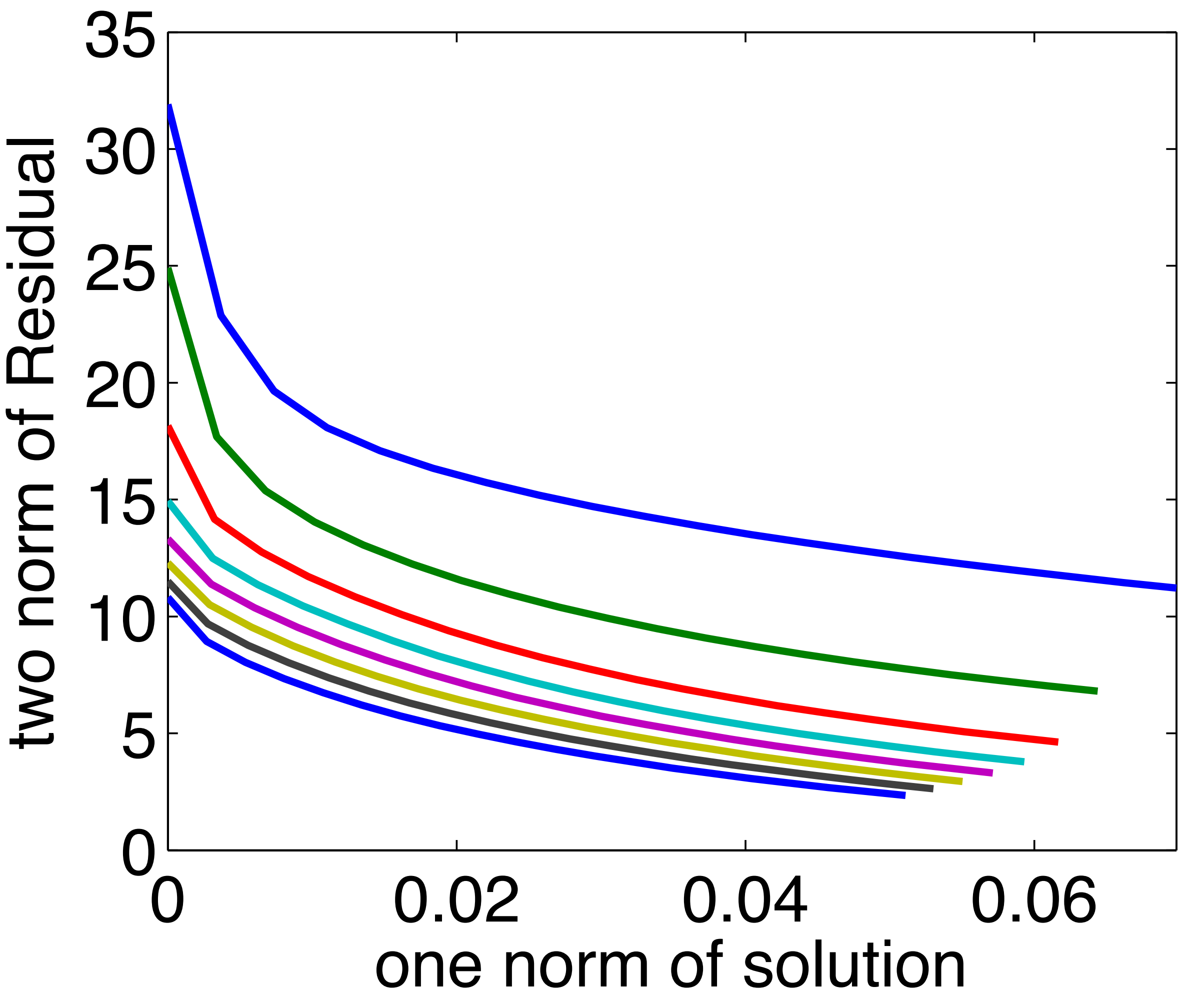
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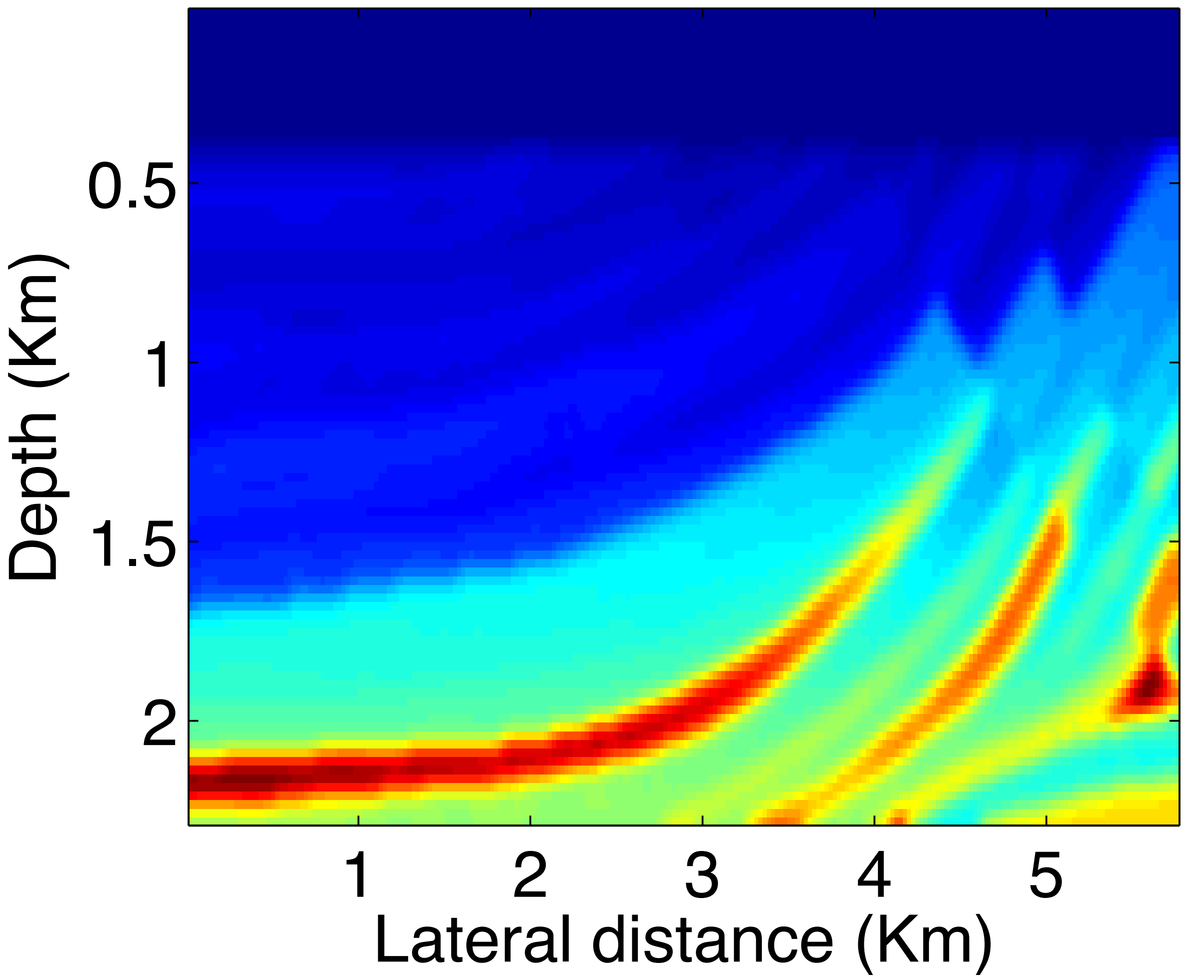
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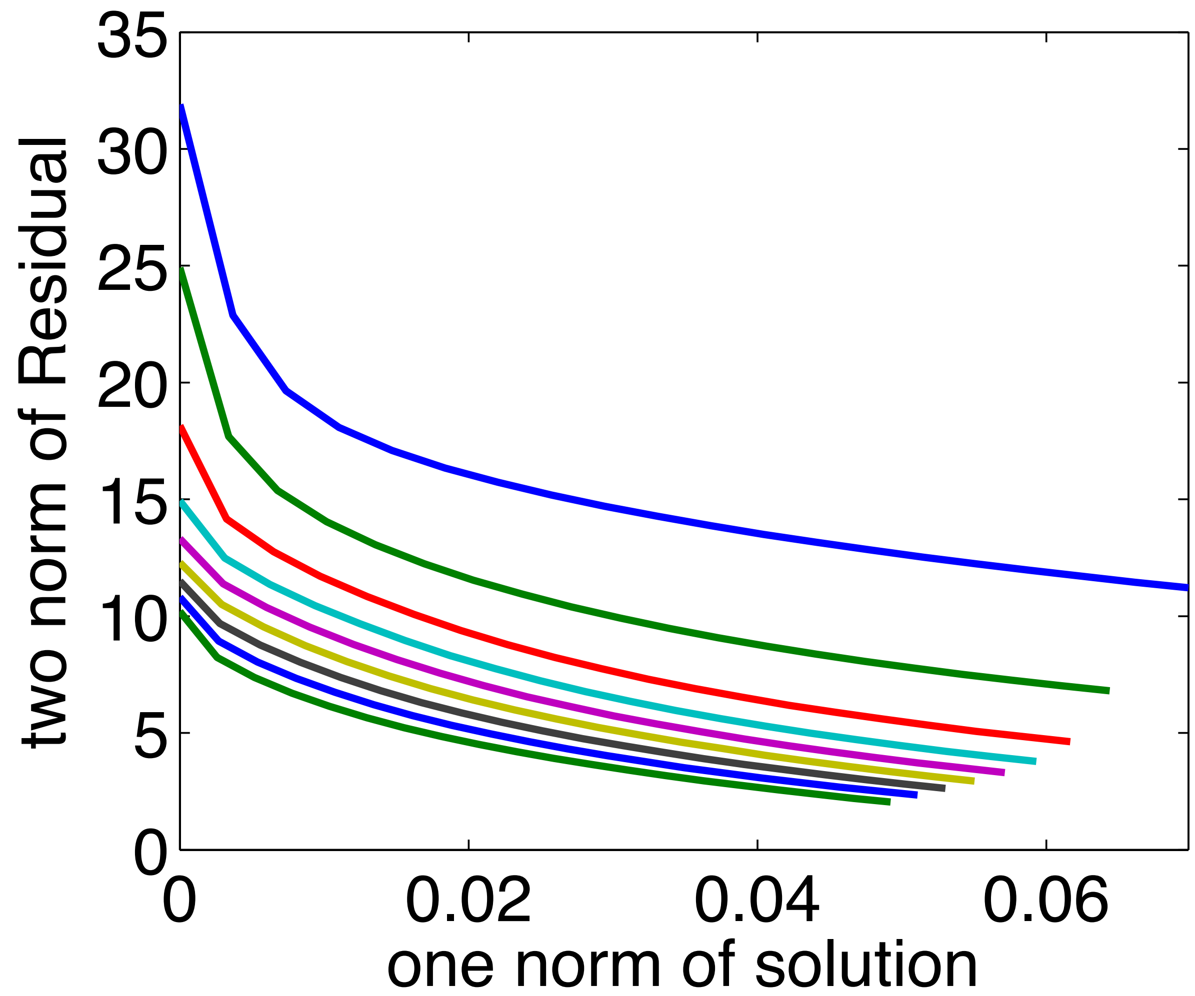
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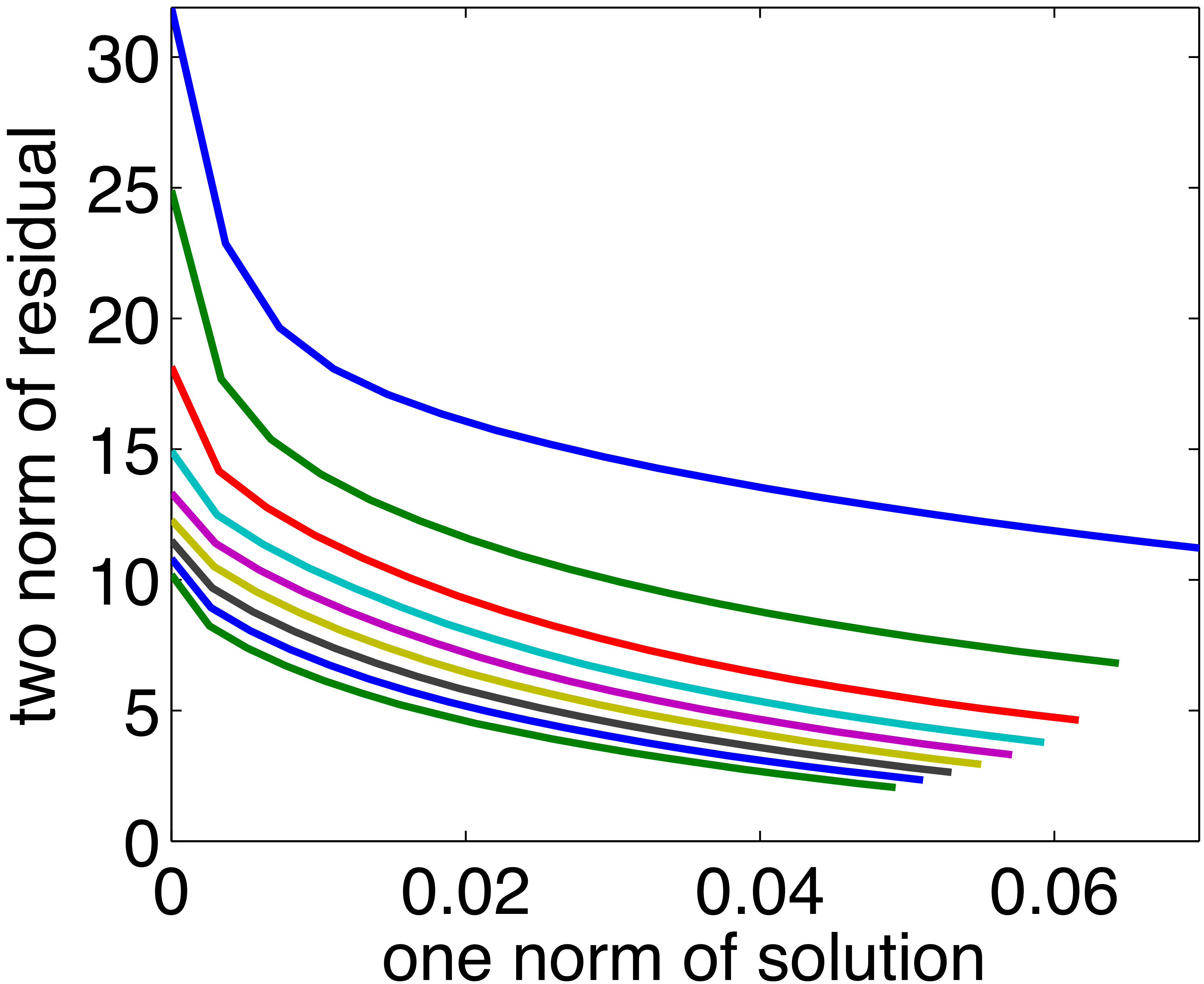
Pareto curves



Modified Gauss Newton

Instead of solving the *full* GN subproblem, we solve a series of *dimensionality*-reduced LASSO problems.

For each LASSO subproblem, we *redraw* a *new* random *subset* of (simultaneous) source experiments.

LASSO problem solved by Spectral Projected Gradient.

Jacobian & Hessian are *never* formed *explicitly*.

We use curvelets to *sparsely* represent *geophysical* model updates.

Modified Gauss-Newton

Algorithm 1: Dimensionality-reduced Gauss Newton with sparsity

Result: Output estimate for the model $\mathbf{m}$

$\mathbf{m} \longleftarrow \mathbf{m}_0$; $k \longleftarrow 0$; `// initial model`

while not converged **do**

$\{\underline{\mathbf{D}}^k, \underline{\mathbf{Q}}^k\} \longleftarrow \{\mathbf{DW}^k, \mathbf{QW}^k\}$ with $\mathbf{W}^k \in N(0,1)$; `// indep. draw.`

$\underline{\delta\mathbf{D}}^k \longleftarrow \underline{\mathbf{D}}^k - \boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]$; `// residual`

$\tau^k \longleftarrow (1-\alpha)\, \|\underline{\delta\mathbf{D}}^k\|_F \,/\, \|\nabla\boldsymbol{\mathcal{F}}^*[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\underline{\delta\mathbf{D}}^k\|_\infty$; `// one-norm LASSO`

$$\delta\mathbf{x} \longleftarrow \begin{cases} \arg\min_{\delta\mathbf{x}} \frac{1}{2}\|\underline{\delta\mathbf{D}}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\delta\mathbf{x}\|_F^2 \\ \text{subject to} \quad \|\delta\mathbf{x}\|_1 \leq \tau^k \end{cases}$$

$\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k \mathbf{C}^* \delta\mathbf{x}$; `// update with linesearch`

$k \longleftarrow k+1$;

end

FWI results

Time-harmonic Helmholtz:

- 205 X 701 with mesh size of 10m
- 9 point stencil [C. Jo et. al., '96]
- absorbing boundary condition with damping layer with thickness proportional to wavelength
- solve wavefields on the fly with direct solver

FWI results

Split-spread surface-free ‘land’ acquisition:

- 350 sources with sampling interval 20m
- 701 receivers with sampling interval 10m
- maximal offset 7km (3.5 X depth of model)
- Ricker wavelet with central frequency of 12Hz
- recording time for each shot is 3.6s

FWI results

FWI:

- 10 overlapping frequency bands with 10 frequencies (2.9Hz-25Hz)
- 10 Gauss-Newton steps for each frequency band (solved with max 20 spectral-projected gradient iterations)

[Bunks '95; Pratt '98]

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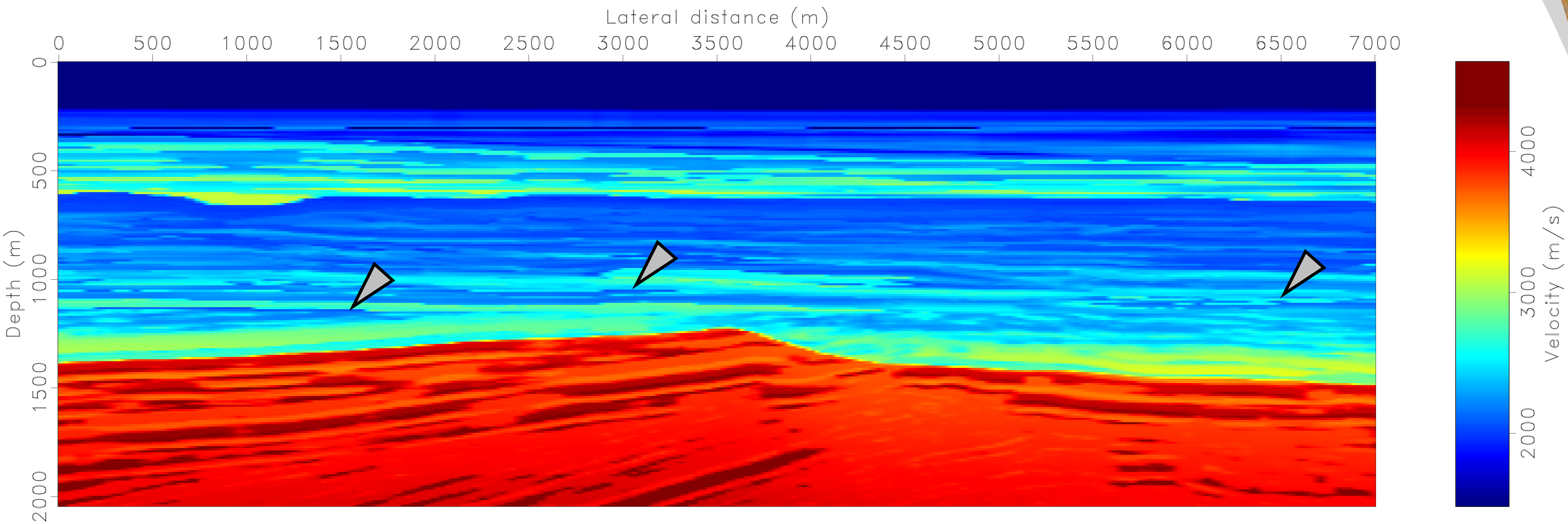
Results

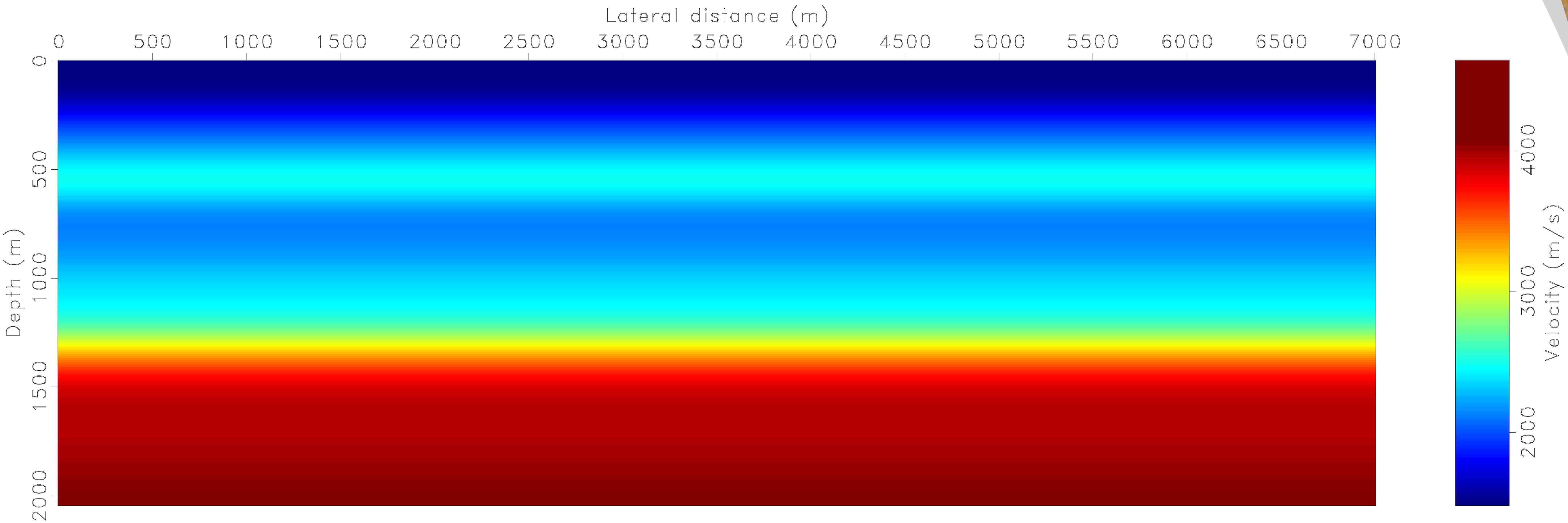
True model



SLIM

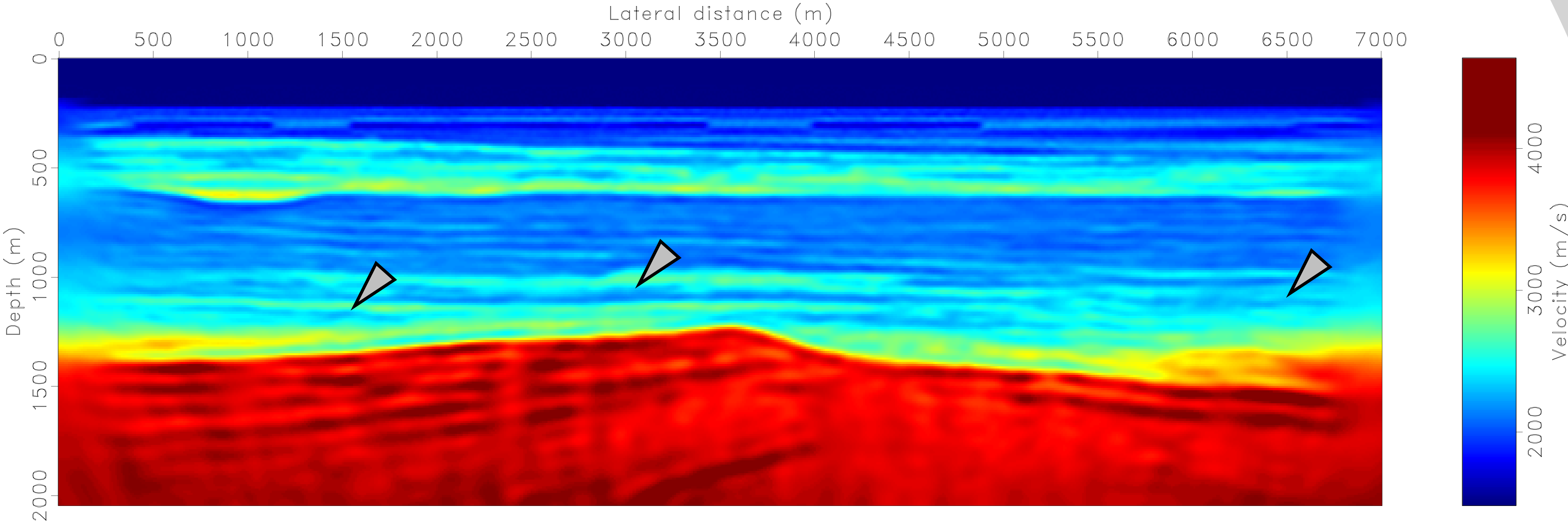
Results

Initial model



Results

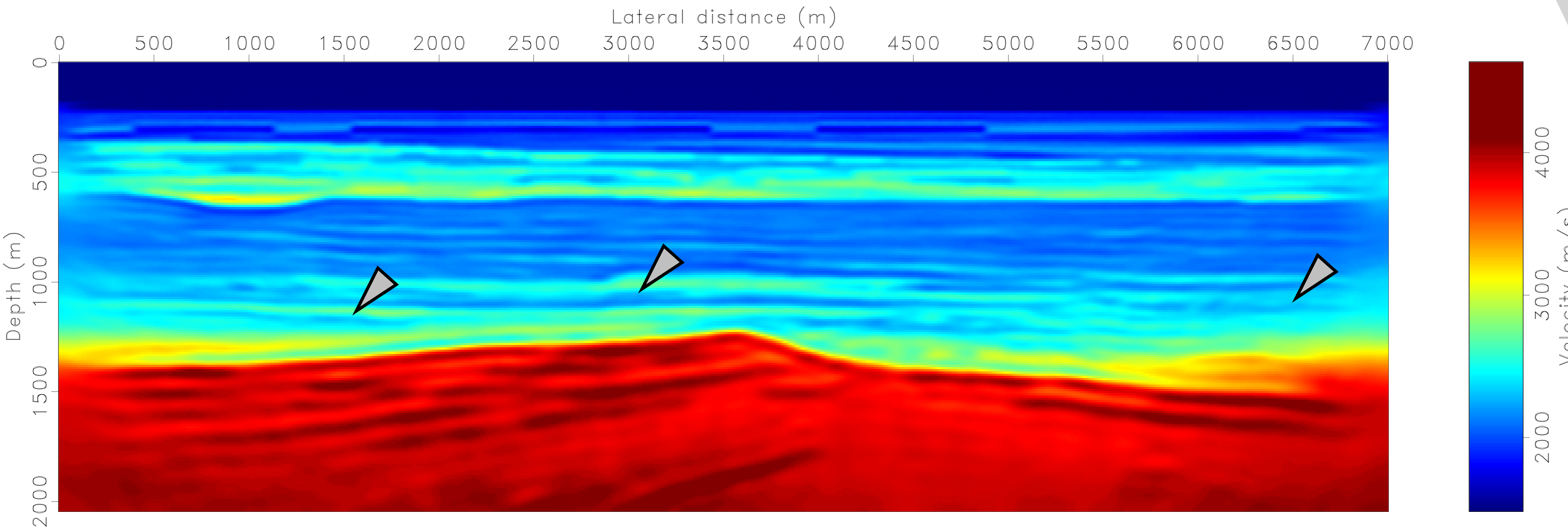
Modified GN 7 sim. shots *without renewals*



25 times speedup compared to full GN

Results

Modified GN 7 sim. shots *with renewals*



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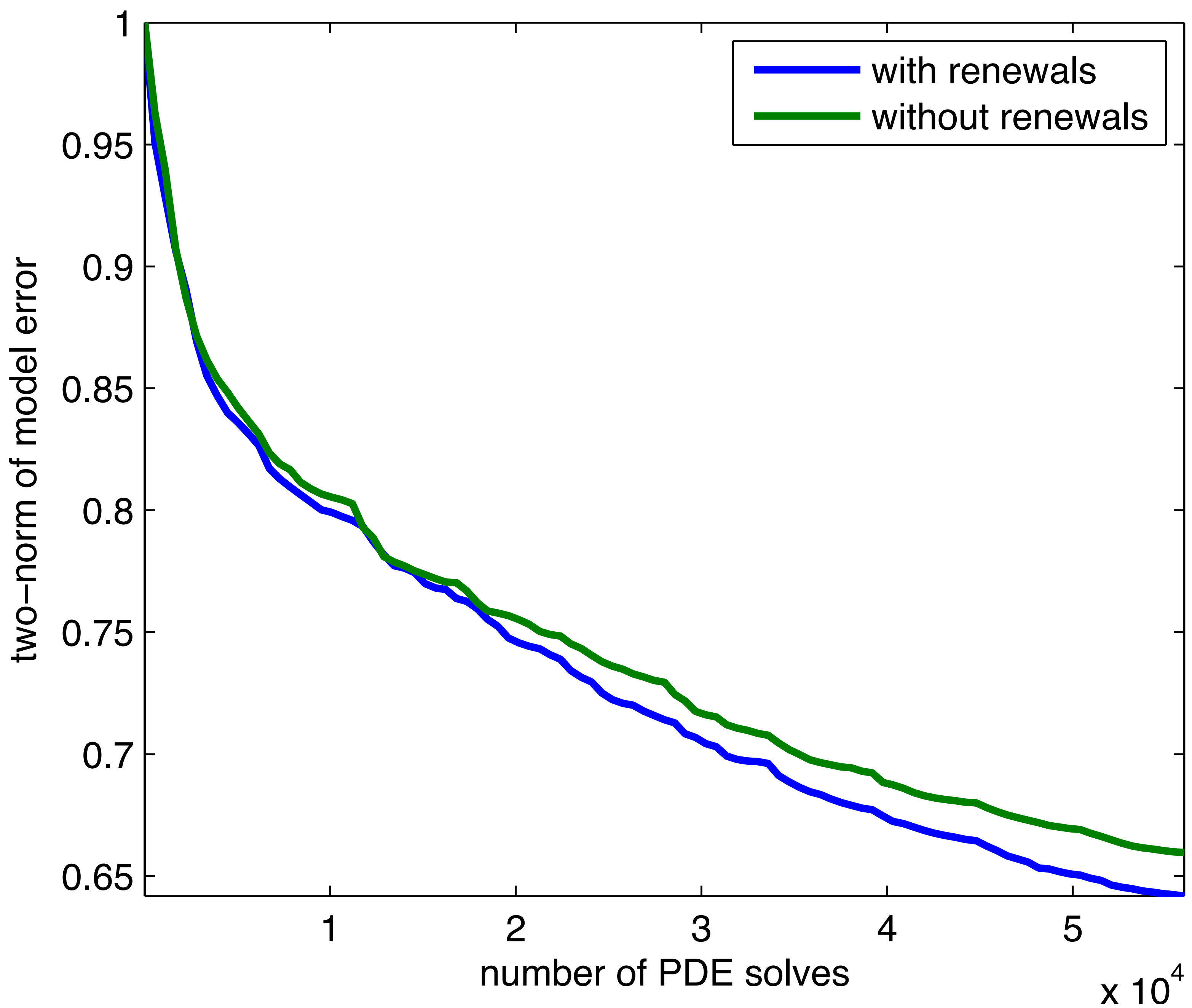
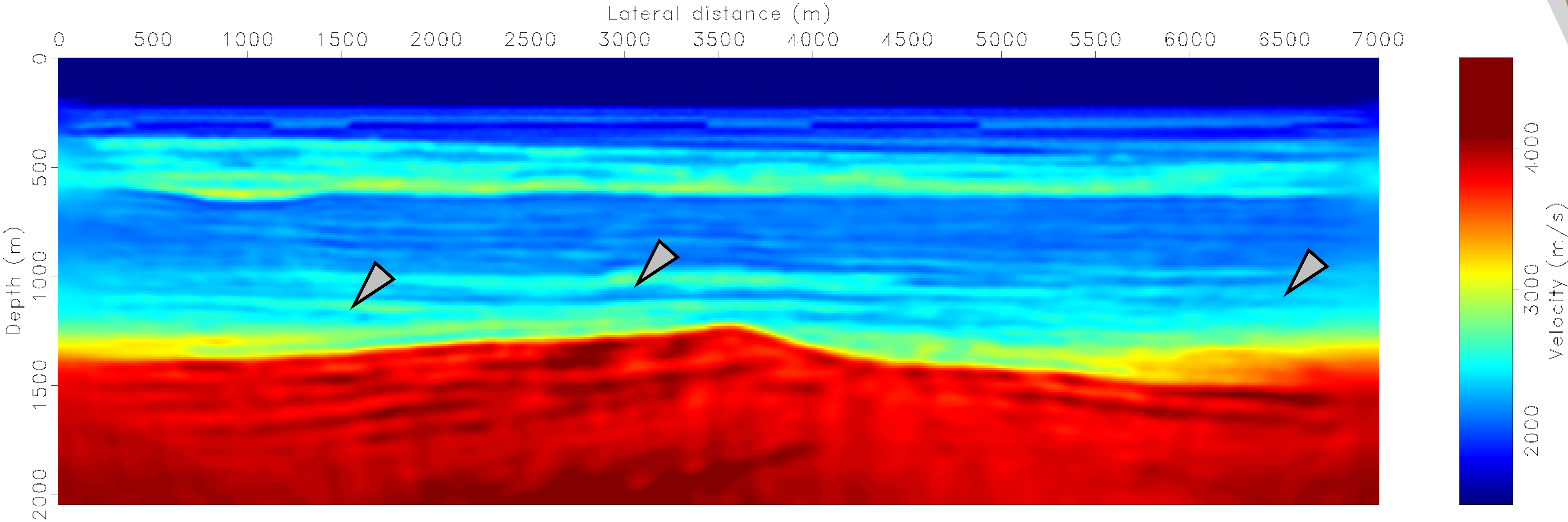
Results

Model error



Results

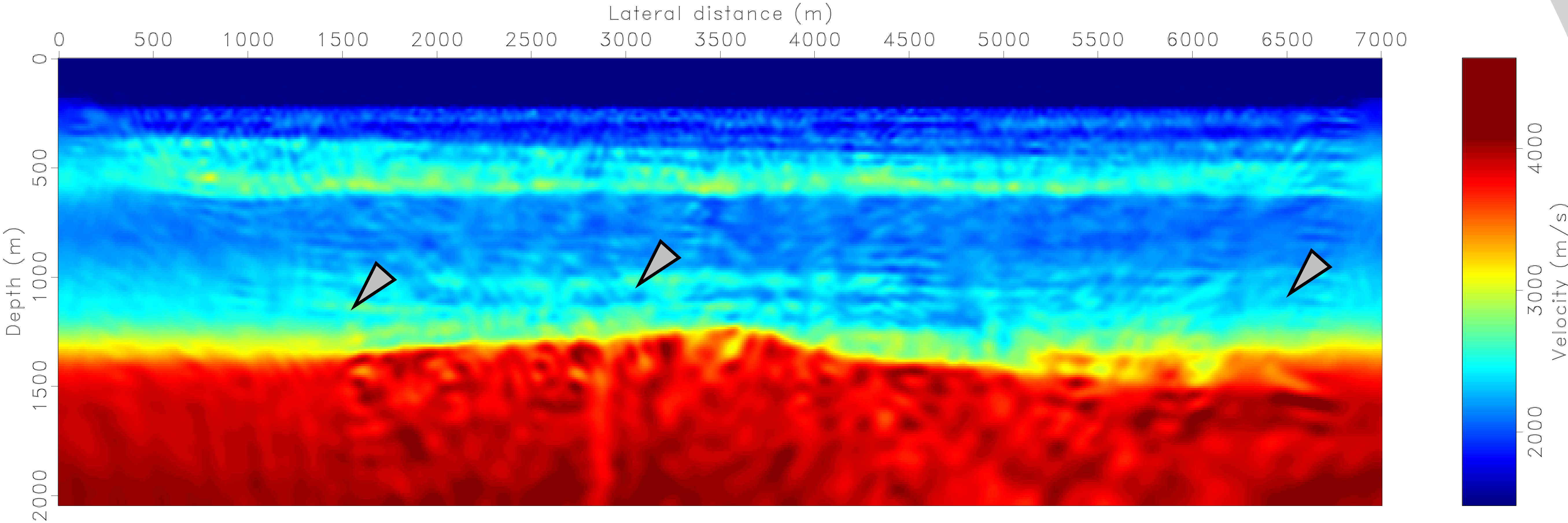
Modified GN 1 sim. shots *with renewals*



175 times speedup compared to full GN

Results

Modified GN 1 sim. shots *without renewals*



175 times speedup compared to full GN

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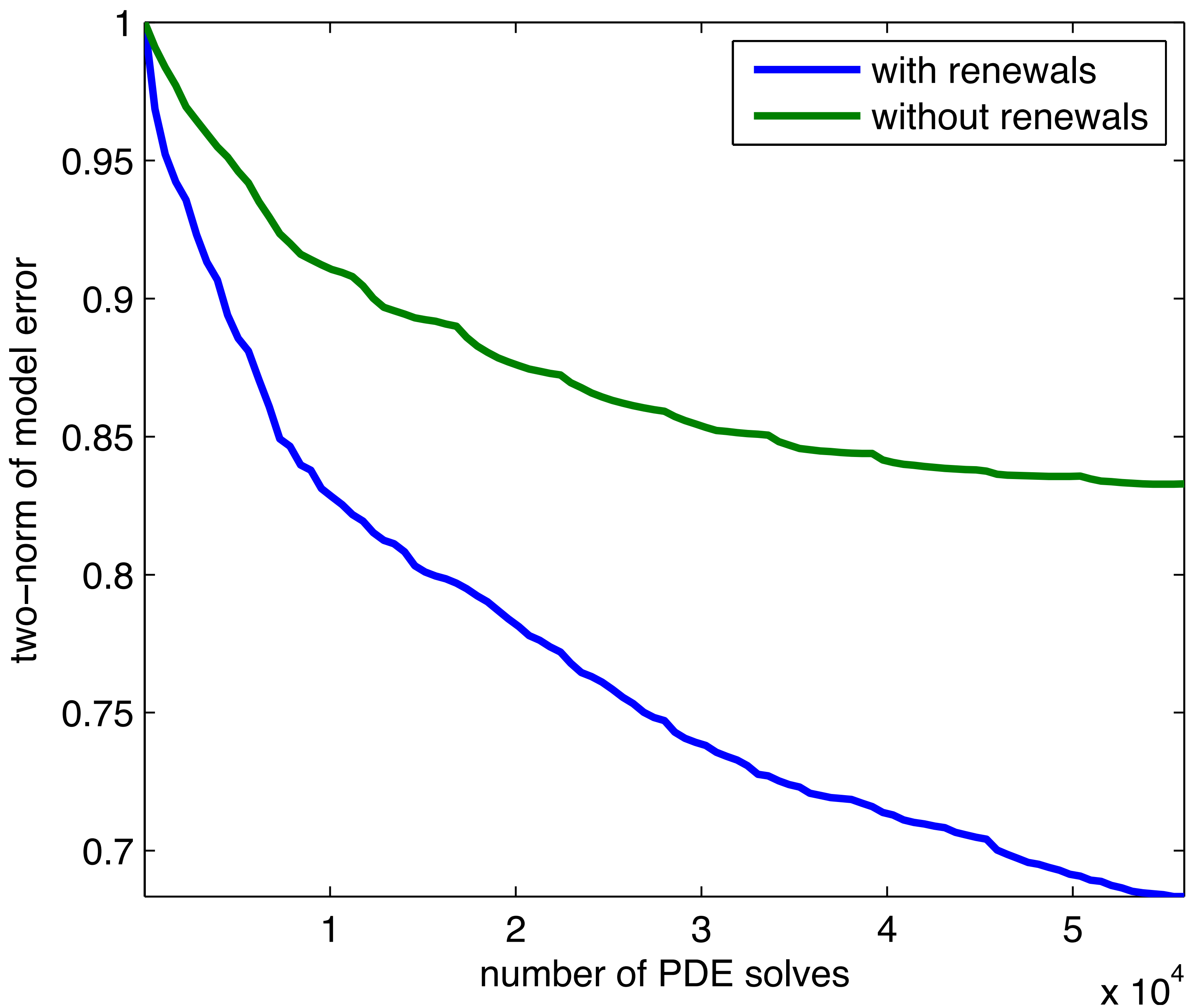
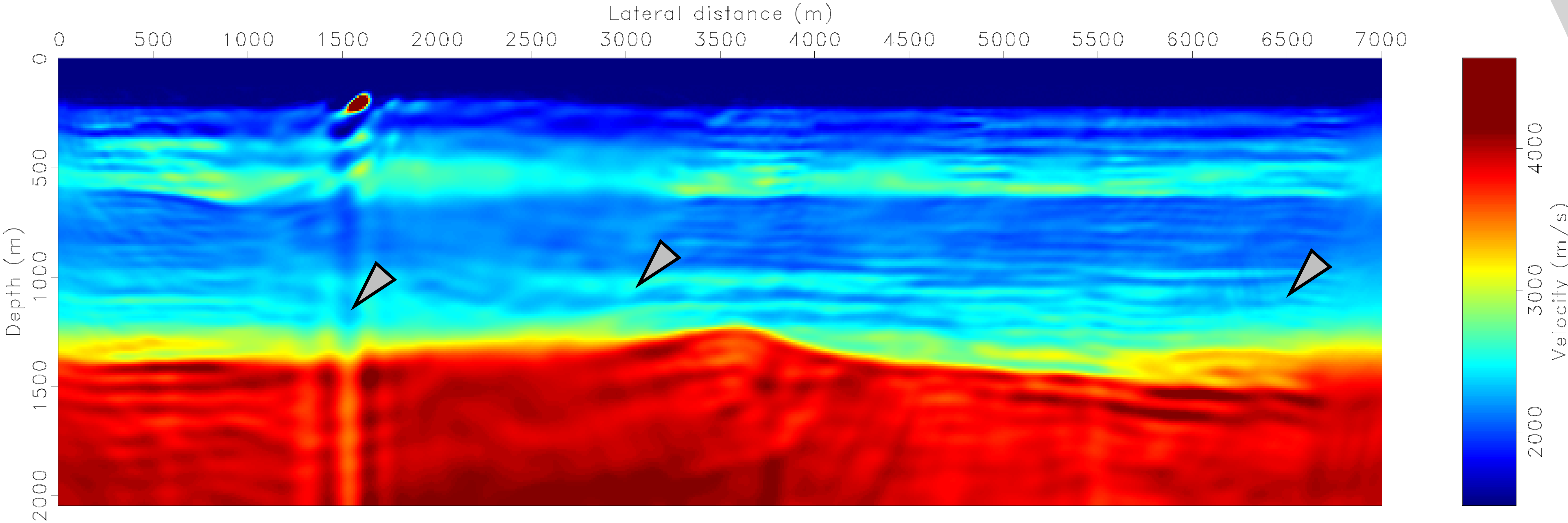


Results

Modified GN 7 seq. shots *without renewals*



25 times speedup compared to full GN

Results

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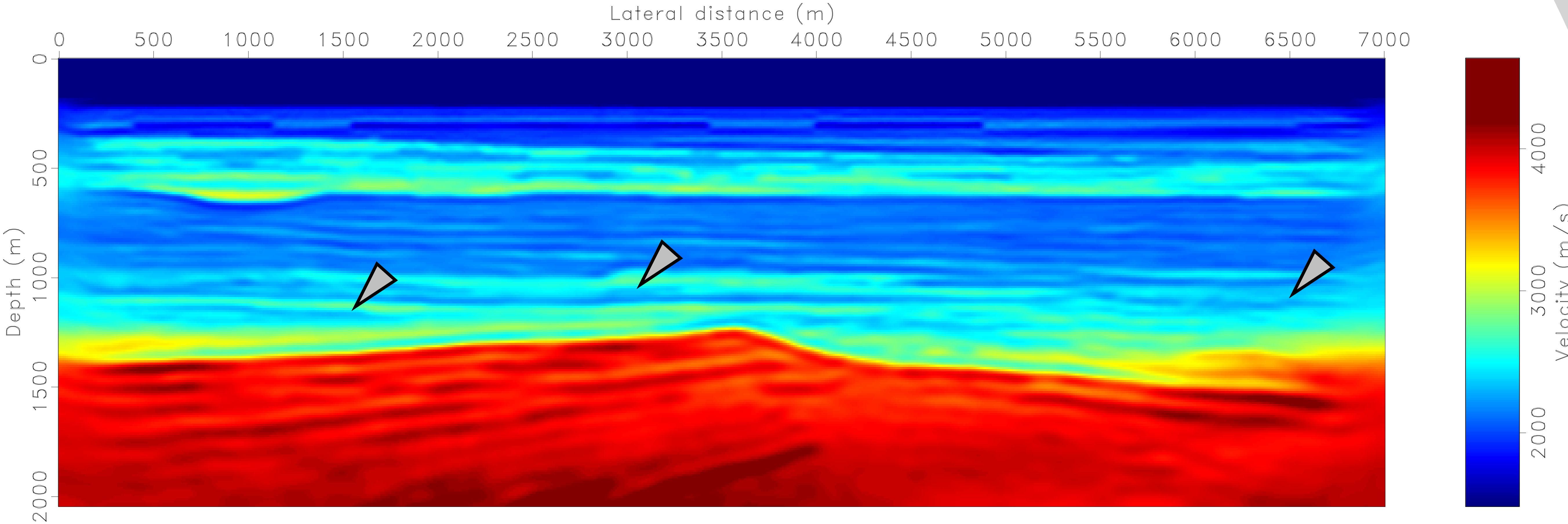

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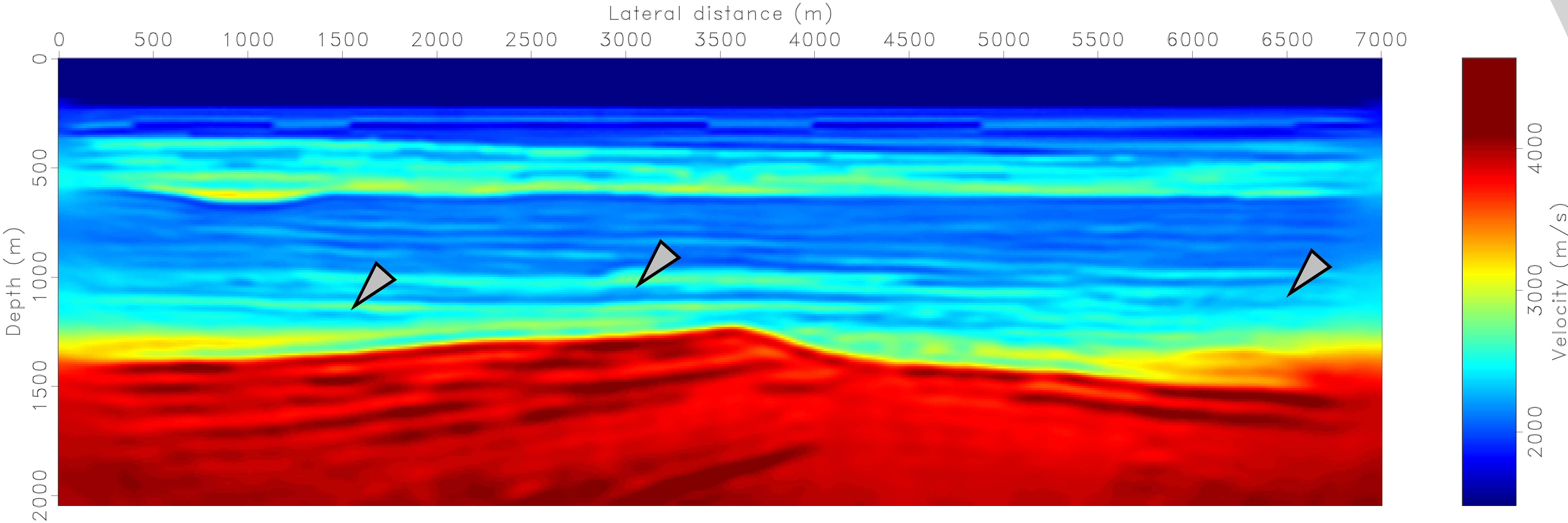

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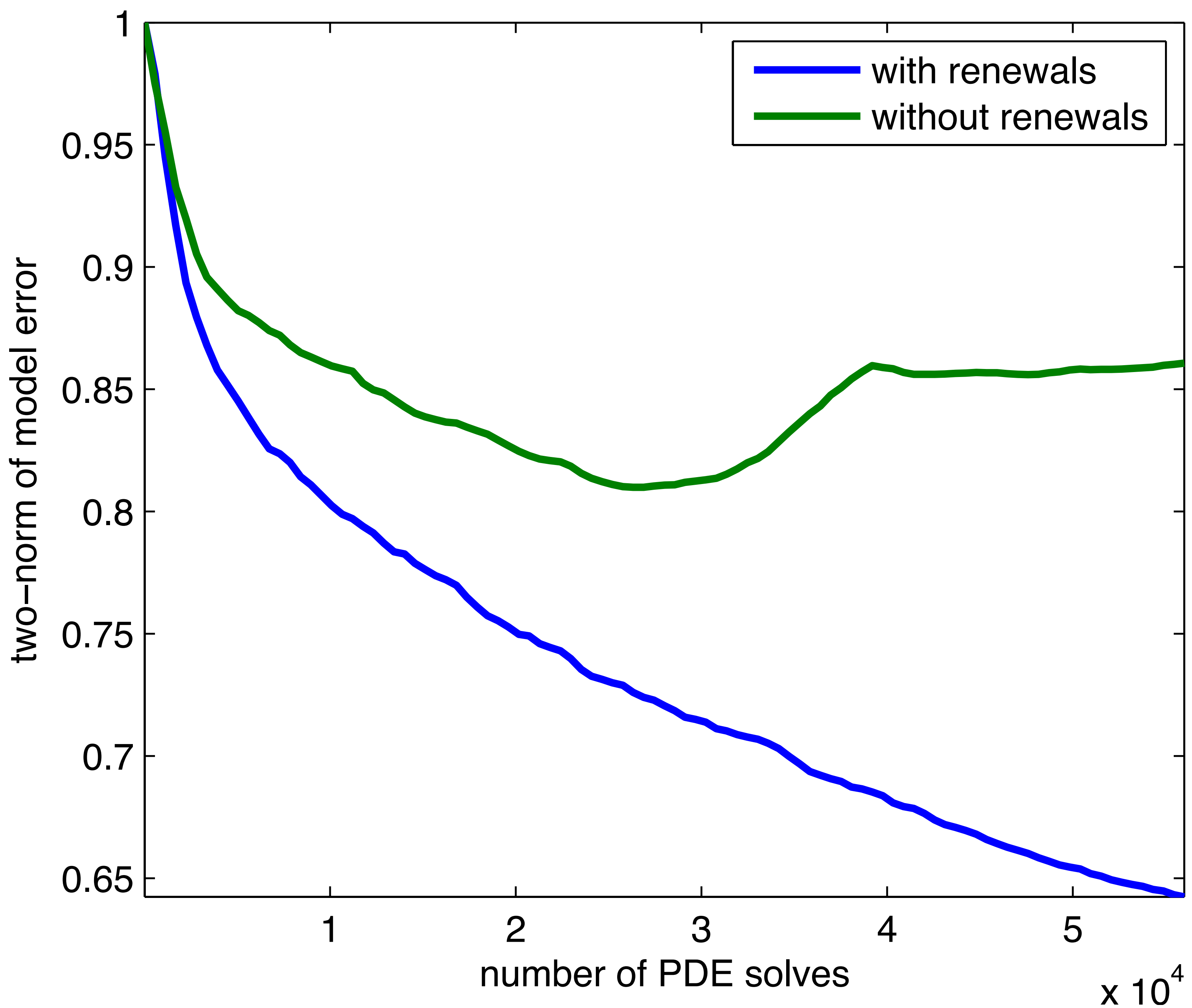

SLIM

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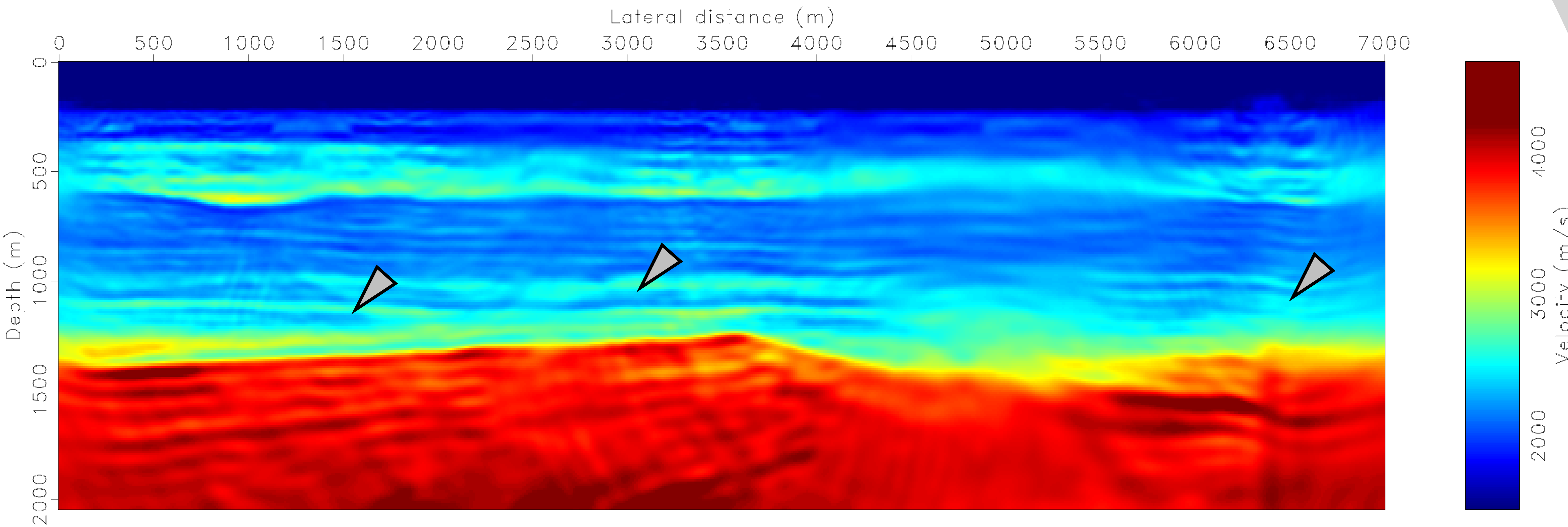
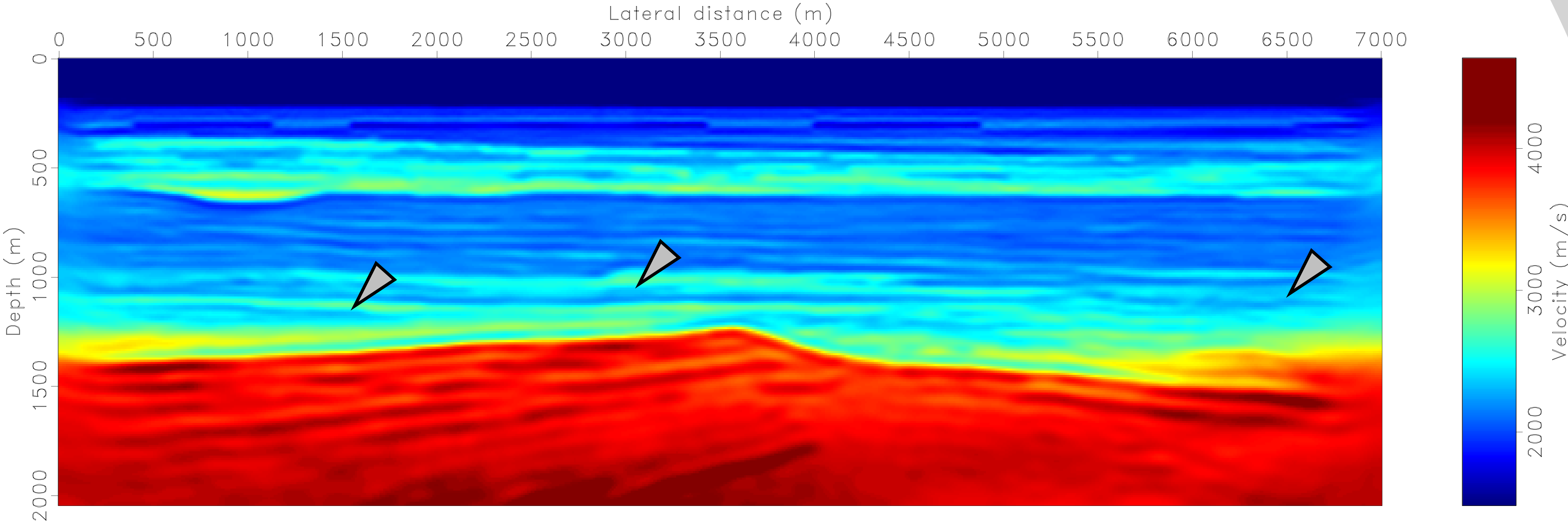


marine acquisition: min offset 100m; max offset 3km

Results

Modified GN 7 seq. shots *with renewals*



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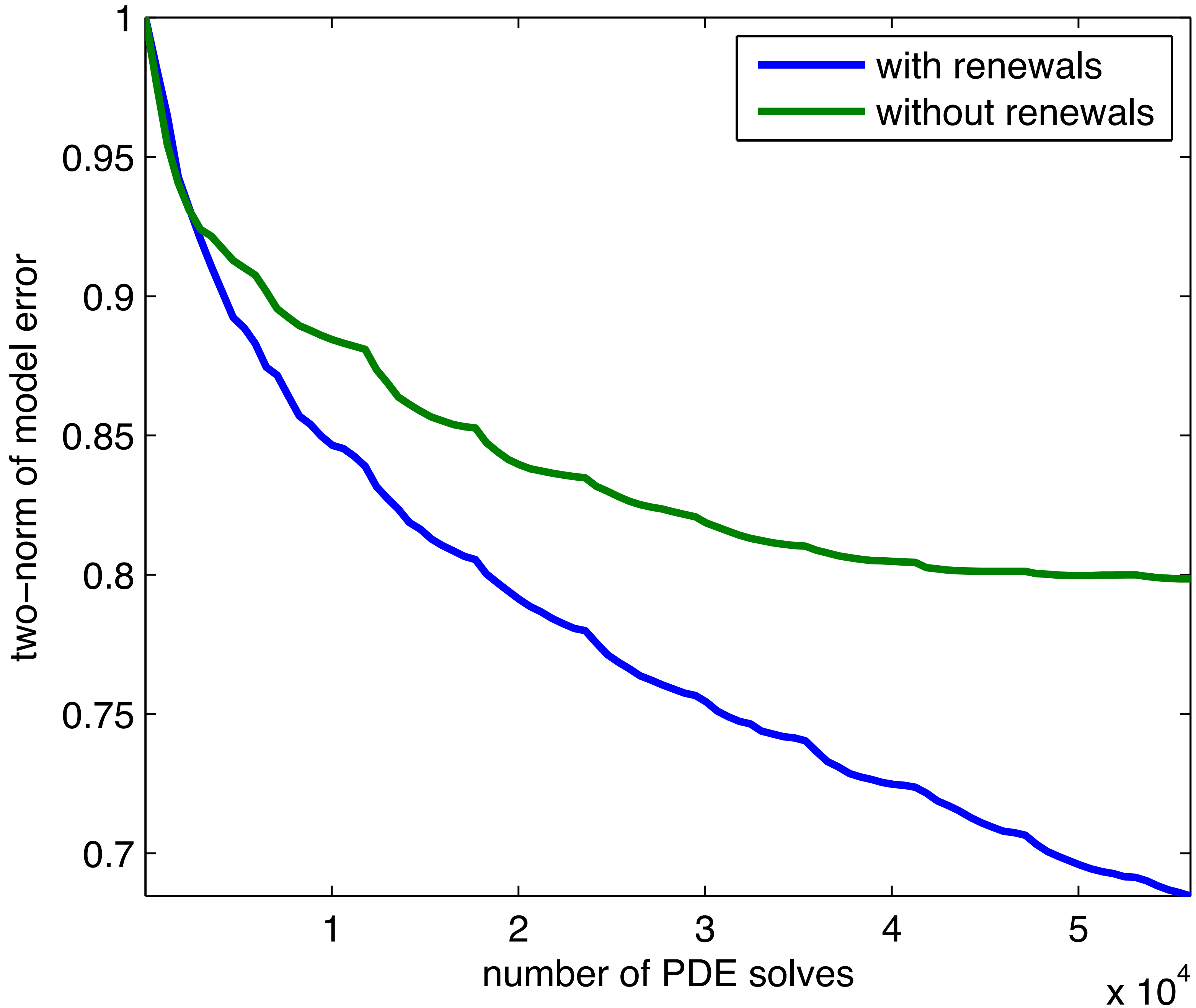
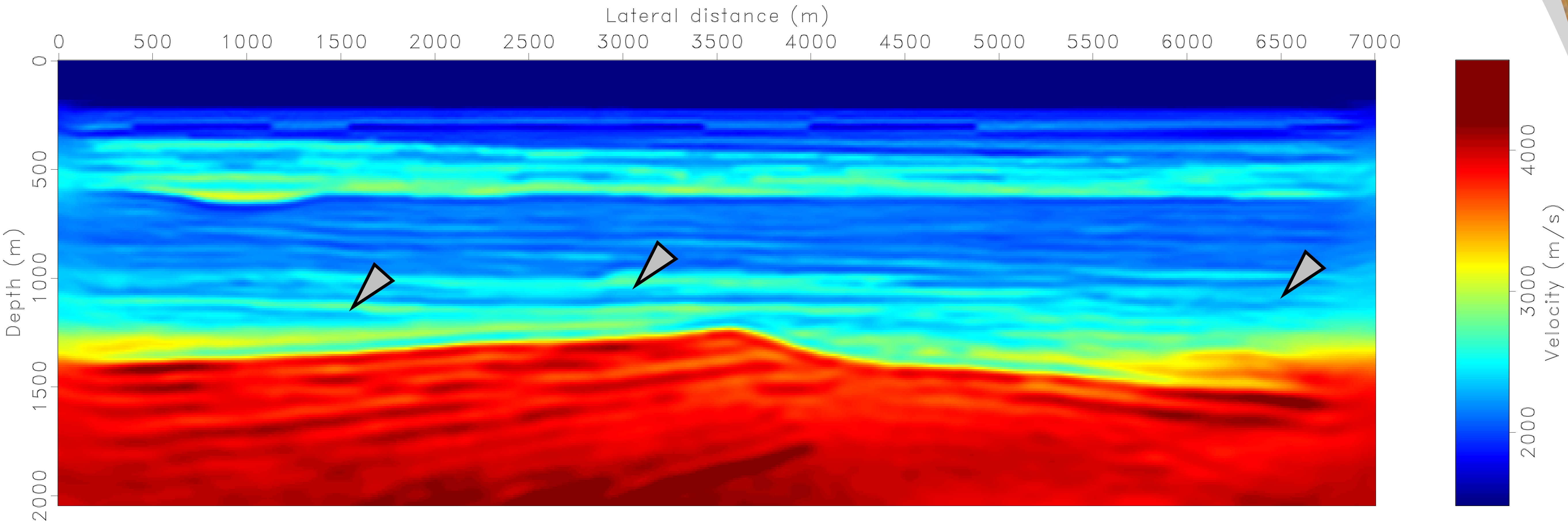


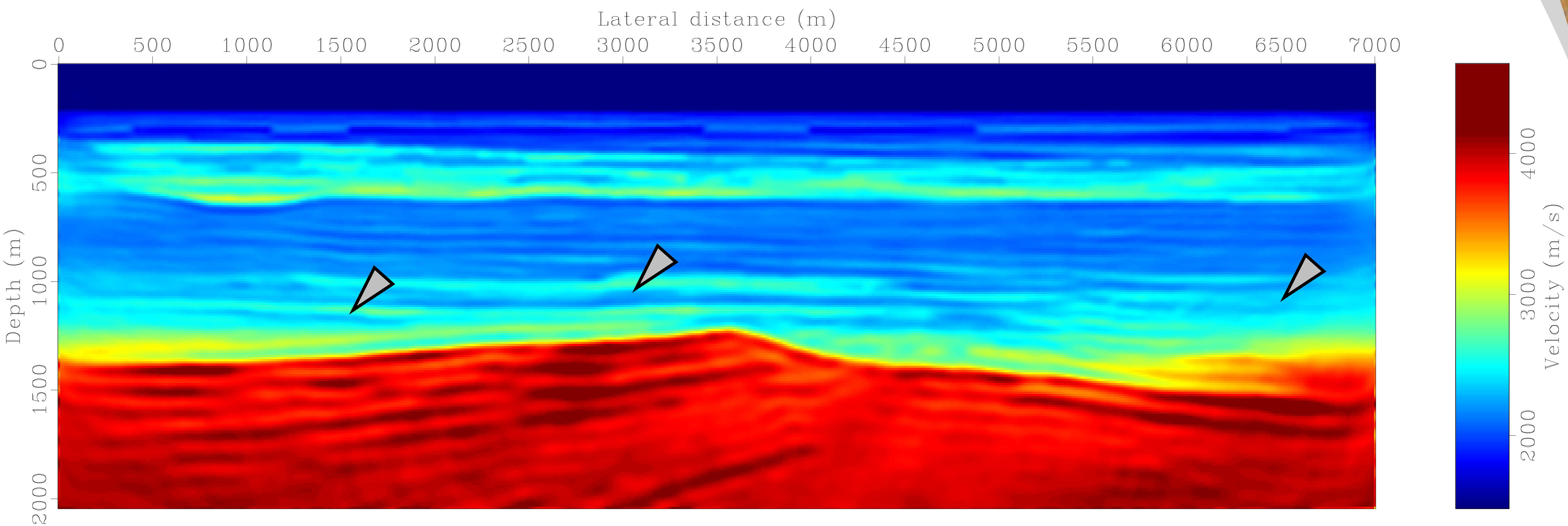
Results

Modified GN 7 seq. shots *with renewals*



Results

Batching LBFGS seq. shots [vanLeeuwen '11]



Conclusions

Computational cost can be *reduced* significantly by using *randomized dimensionality reduction.*

High-resolution *images* can be attainable by using *sparsity* promotion.

LASSO problems maintain fast decrease in residual while preserving *sparsity* of the *solution.*

Simultaneous shots produce *less* source cross-talk *artifacts*.

Renewals lead to significant improvements-especially for *random* subsets of *sequential* shots.

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Thank you

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Further reading

Compressive sensing

- *Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information by Candes, 06*.
- *Compressed Sensing* by D. Donoho, '06
- Curvelets and Wave Atoms for Mirror-Extended Images by L. Demanet, L. Ying, 07.

Simultaneous acquisition

- *A new look at simultaneous sources* by Beasley et. al., '98.
- *Changing the mindset in seismic data acquisition* by Berkhout '08.

Simultaneous simulations, imaging, and full-wave inversion:

- *Faster shot-record depth migrations using phase encoding* by Morton & Ober, '98.
- *Phase encoding of shot records in prestack migration* by Romero et. al., '00.
- *Efficient Seismic Forward Modeling using Simultaneous Random Sources and Sparsity* by N. Neelamani et. al., '08.
- *Compressive simultaneous full-waveform simulation* by FJH et. al., '09.
- *Fast full-wavefield seismic inversion using encoded sources* by Krebs et. al., '09
- *Randomized dimensionality reduction for full-waveform inversion* by FJH & X. Li, '10

Stochastic optimization and machine learning:

- *A Stochastic Approximation Method* by Robbins and Monro, 1951
- *Neuro-Dynamic Programming* by Bertsekas, '96
- *Robust stochastic approximation approach to stochastic programming* by Nemirovski et. al., '09
- *Stochastic Approximation and Recursive Algorithms and Applications* by Kushner and Lin
- *Stochastic Approximation approach to Stochastic Programming* by Nemirovski
- *An effective method for parameter estimation with PDE constraints with multiple right hand sides. by* Eldad Haber, Matthias Chung, and Felix J. Herrmann. '10