Sparsity-promoting GN method

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Joint work with

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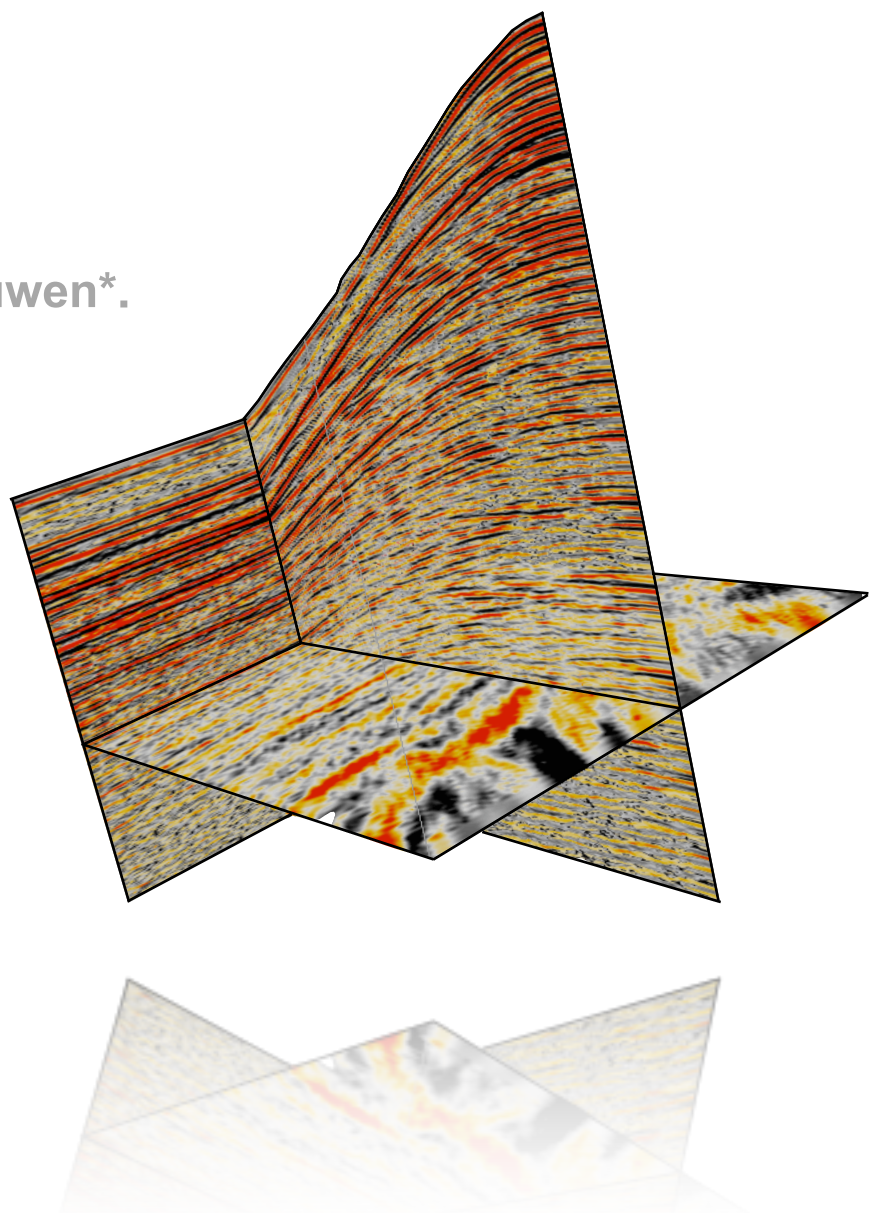

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Outline

- FWI Mathematical Formulation
  - Convex-composite structure of inverse problem
- Quick Review of Gauss-Newton (GN) Method
- Motivations for extensions and modifications
  - Overwhelming data volume
  - Ill-posed problem and need for regularization
- Overview of Modified Gauss-Newton Method

Nonlinear Least Squares Formulation

- We consider inverse problems of the form

$$\mathbf{D} = \boldsymbol{\mathcal{F}}(\mathbf{m}; \mathbf{Q}) + \boldsymbol{\epsilon}$$

$\mathbf{D}$	$n \times m$ matrix of observations
$\mathbf{Q}$	$l \times m$ array of source parameters
$\mathbf{m}$	parameters to be recovered
$\boldsymbol{\mathcal{F}}(\mathbf{m}; \mathbf{Q})$	Forward model (calculated data)
$\boldsymbol{\epsilon}$	Model for error, typically Gaussian i.i.d.

- Objective below is **Convex-Composite:**

$$\min_{\mathbf{m}} \Phi(\mathbf{x}) = \|\mathbf{D} - \boldsymbol{\mathcal{F}}(\mathbf{m}; \mathbf{Q})\|_F^2 = \sum_{i=1}^{m} \|\mathbf{d}_i - \boldsymbol{\mathcal{F}}(\mathbf{m})\mathbf{q}_i\|_2^2$$

Overview of Gauss-Newton method

- Gauss-Newton method respects the convex-composite structure:

- Objective: $$\min_{\mathbf{m}} \Phi(\mathbf{m}) = \|\mathbf{D} - \boldsymbol{\mathcal{F}}(\mathbf{m};\mathbf{Q})\|_F^2$$

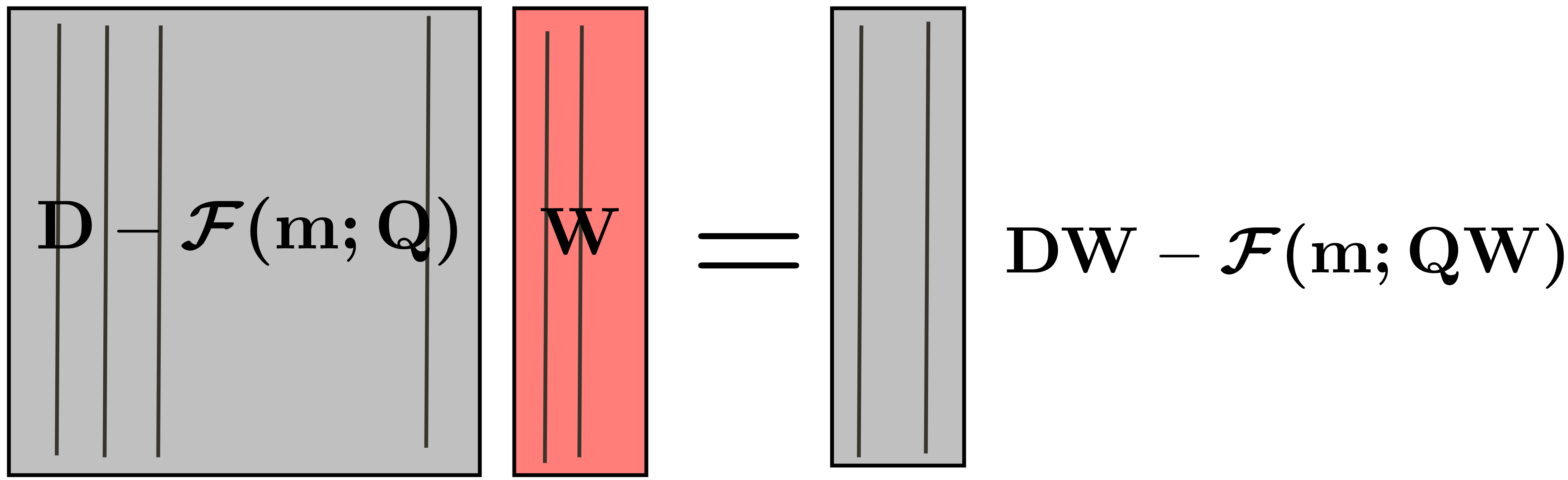
- Iterated algorithm: $$\mathbf{m}^{k+1} = \mathbf{m}^k + \alpha_k \overline{\boldsymbol{\delta}\mathbf{m}}$$

- Update $\overline{\boldsymbol{\delta}\mathbf{m}}$ solves $$\min_{\boldsymbol{\delta}\mathbf{m}} \|\mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}] - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}]\boldsymbol{\delta}\mathbf{m}\|_F^2$$

Challenges and Solutions

- Overwhelming data volume motivates Dimensionality Reduction
  - Simultaneous shots (random mixtures of right hand sides)
  - Random sequential shots (for marine acquisition)
- FWI is an Ill-Posed problem
  - Many potential solutions explain the same data
  - Different optimization algorithms may go to different solutions from the same initial model.
- Challenge is to design methods that
  - take advantage of prior knowledge to go to the RIGHT solution!
  - work well in a dimensionality reduction regime

Dimensionality Reduction Revisited



- Taking **W** to be Gaussian corresponds to simultaneous shots
- Taking **W** to be random columns of **I** gives sequential shots
- To reduce dimensionality, we can work with:

$$\underline{\Phi}(\mathbf{x}) = \|\underline{\mathbf{D}} - \mathcal{F}(\mathbf{m}; \underline{\mathbf{Q}})\|_F^2 , \quad \underline{\mathbf{D}} = \mathbf{DW} , \quad \underline{\mathbf{Q}} = \mathbf{QW}$$

Gauss-Newton Method Revisited

- Focus on a dimensionality reduced problem:
- Objective: $$\min_{\mathbf{m}} \underline{\Phi}(\mathbf{m}) = \|\underline{\mathbf{D}} - \boldsymbol{\mathcal{F}}(\mathbf{m}; \underline{\mathbf{Q}})\|_F^2$$
- Iterated algorithm: $$\mathbf{m}^{k+1} = \mathbf{m}^k + \alpha_k \overline{\boldsymbol{\delta}\mathbf{m}}$$
- We can modify the Gauss-Newton subproblem in a number ways, and still prove convergence of the algorithm:

$$\overline{\boldsymbol{\delta}\mathbf{m}} \quad \text{solves} \quad \min_{\boldsymbol{\delta}\mathbf{m}} \|\underline{\mathbf{D}} - \boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}] - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}]\boldsymbol{\delta}\mathbf{m}\|_F^2$$

$$\text{subject to } \|\boldsymbol{\delta}\mathbf{m}\| \leq \tau^k$$

Promoting Curvelet Sparsity of UPDATES

- **Idea:** replace usual GN subproblem with a LASSO problem:

$$\min_{\mathbf{y}} \quad \|\underline{\mathbf{D}} - \mathcal{F}[\mathbf{m}^k;\underline{\mathbf{Q}}] - \nabla\mathcal{F}[\mathbf{m}^k;\underline{\mathbf{Q}}]\underbrace{\mathcal{C}^*\mathbf{y}}_{\boldsymbol{\delta}\mathbf{m}}\|_F^2$$

$$\text{s.t.} \qquad \|\mathbf{y}\|_1 \leq \tau$$

- **CS-Perspective (Dim. Red.):** we are trying to recover the TRUE Gauss-Newton update from a subsampled data volume.

- **Regularization perspective (Ill-Posed):** We force the Gauss-Newton subproblem to stop early, and customize it to promote sparsity of updates.

- Both perspectives exploit curvelet sparsity of updates.

Why Curvelet Sparsity of Updates?

- Expression for monochromatic FWI gradient:

$$\partial_{\mathbf{m}_i}\Phi(\mathbf{m}) = \bar{\mathbf{v}}^* \frac{\partial \mathbf{H}[\mathbf{m}]}{\partial \mathbf{m}_i}\bar{\mathbf{u}}$$

  – This gradient can be interpreted as a ‘correlation’ of forward an adjoint wavefields, and hence is still compressible in curvelets, even when the current model estimate is far from the truth.

- Closed form expression for Gauss-Newton update:

$$\boldsymbol{\delta}\mathbf{m} = -\left(\nabla\boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}]^T\boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}]^T\right)^{-1}\nabla\Phi(\mathbf{m})$$

  – Action of Gauss-Newton Hessian or its inverse is diagonal in phase space, so preserves the sparsity of the gradient in curvelets.

SLIM

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