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Sequential source data recovery from simultaneous acquisition through transform domain sparsity promotion - Curvelet and Shearlet Transform

Haneet Wason

SLIM
University of British Columbia

Outline

- Motivation
- Curvelets and Shearlets
- Recovery Problem and Algorithm
- Experiments and Observations
- Conclusions
- Future Work
- Acknowledgements

Motivation

Question: Why adopt simultaneous source data acquisition ?

- Use fast, multiscale and multidirectional transforms to convert simultaneous data into sequential data
- Recover sequential data by promoting transform-domain sparsity
- Economize survey time, processing efforts and acquisition costs

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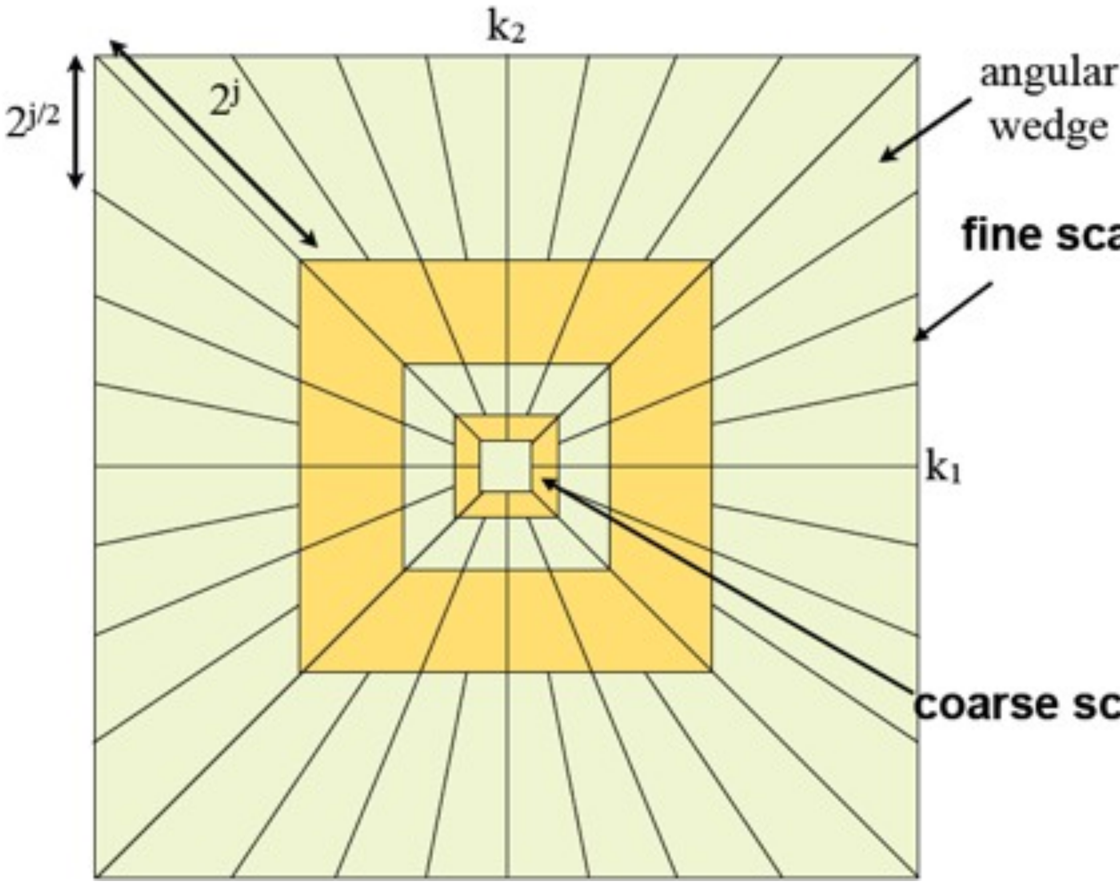
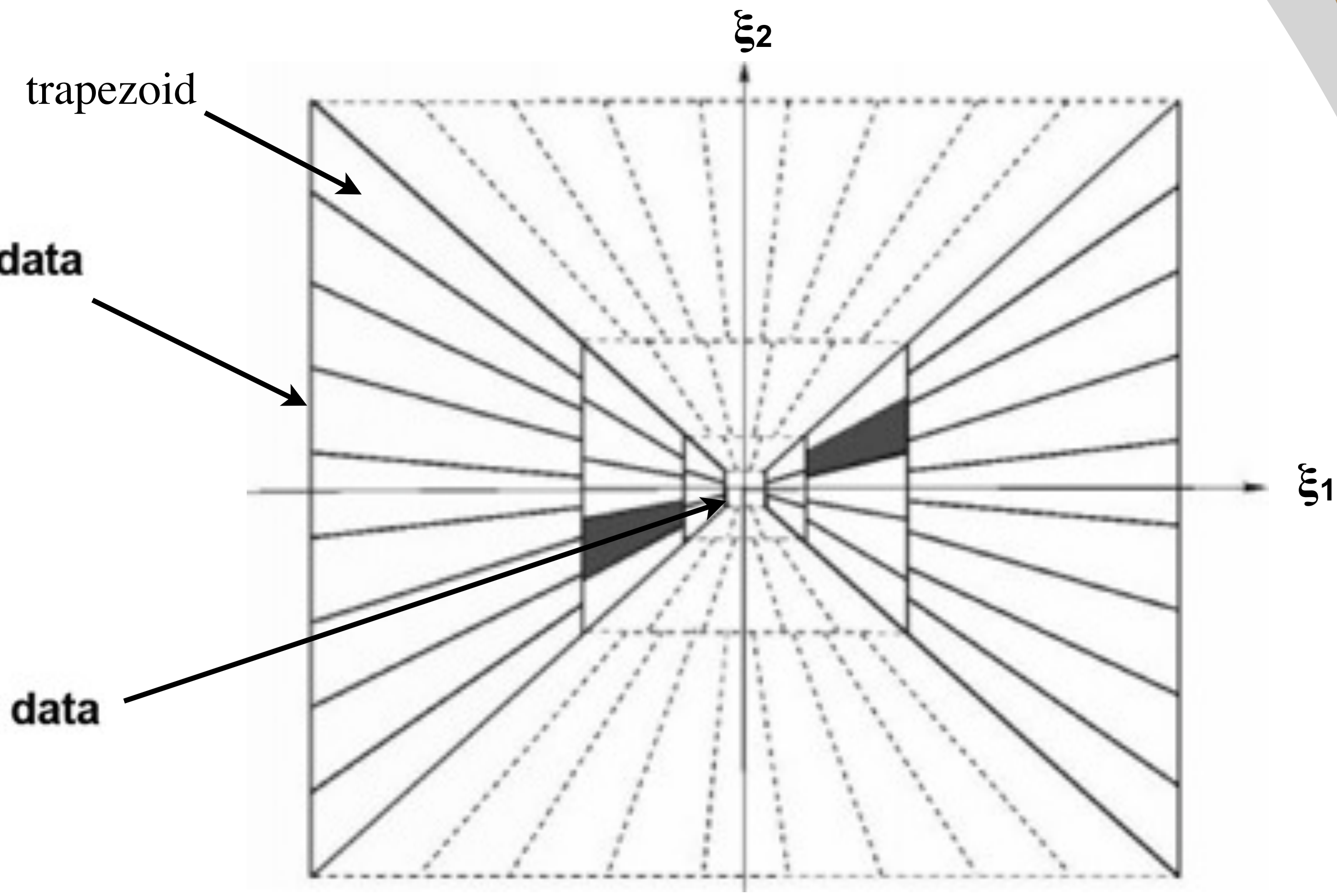
Curvelets and Shearlets

- Non-separable wavelet-like constructions
- Multiscale and multidirectional representation systems
- Involve parabolic dilations, translations and rotations (for curvelets), shearing (for shearlets)
- Obey parabolic scaling principle, length $\approx$ width^2
- Oscillatory in one direction and smooth in the other
- Rapid decay in space and strictly localized in frequency domain

Sparse representation of seismic data

Frequency Domain partitioning





fine scale data

coarse scale data

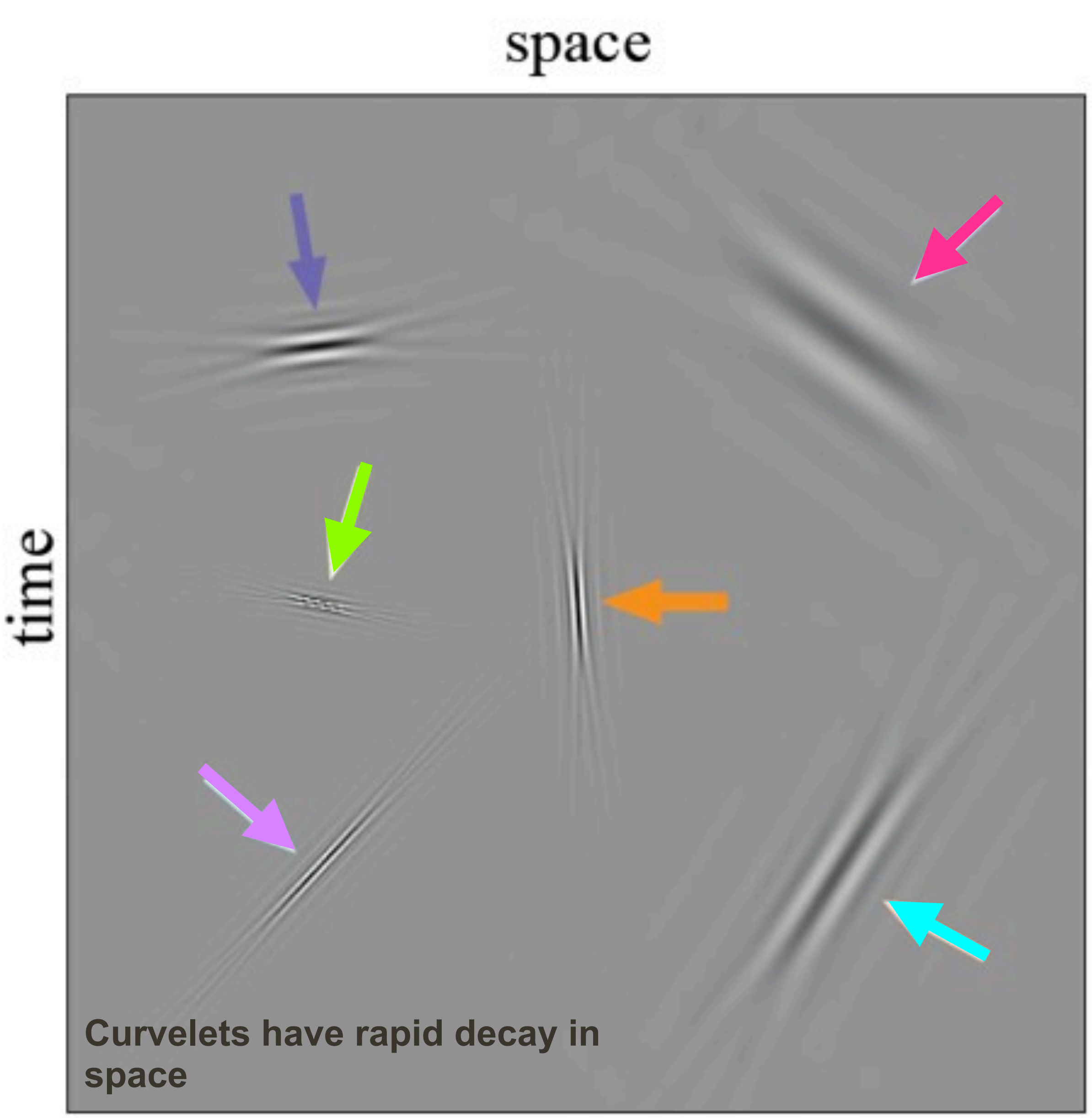
Curvelets

Shearlets

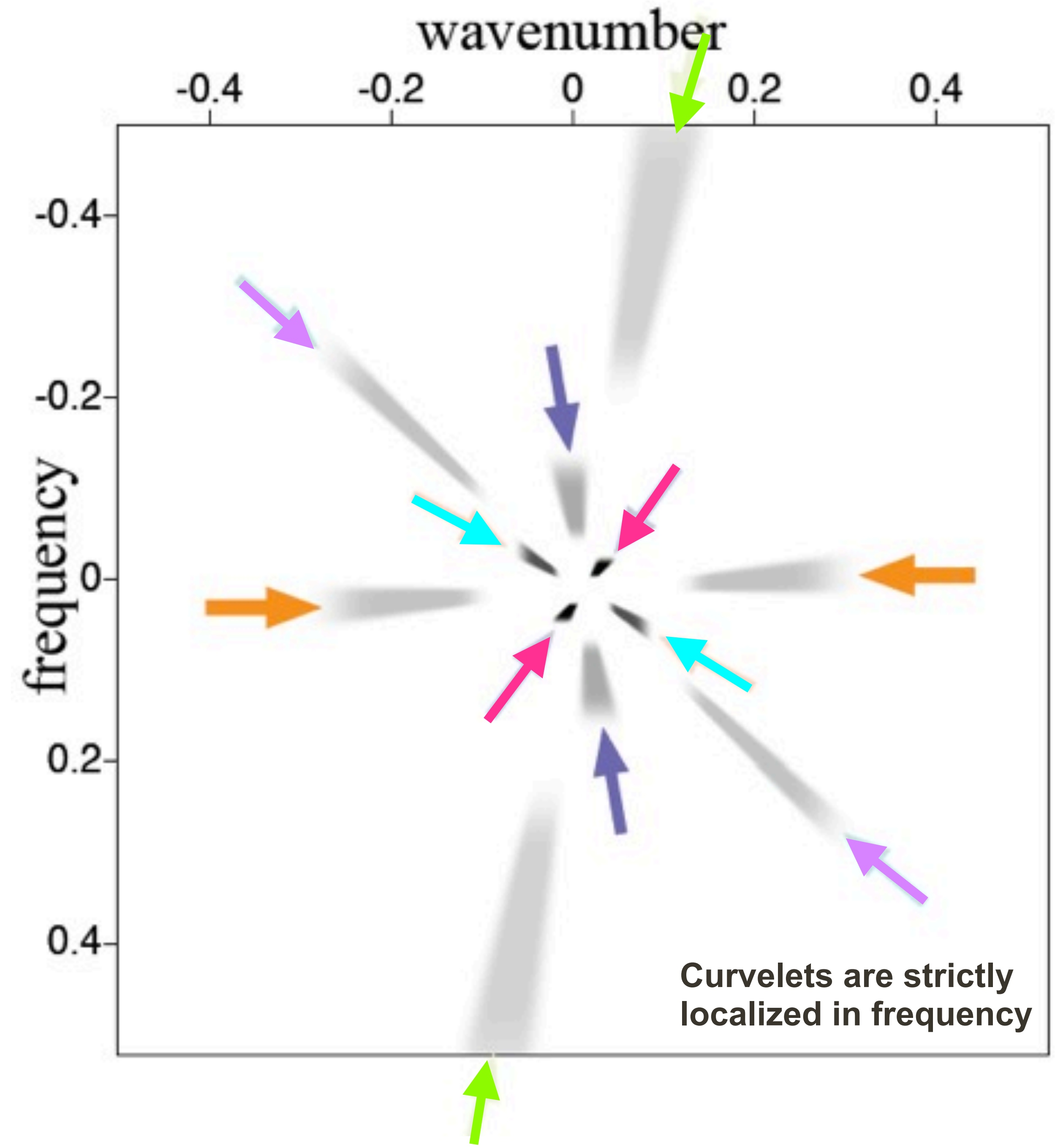
[http://slim.eos.ubc.ca/Publications/Public/
Presentations/2007/herrmann07SEGPRED.pdf]

[http://www.shearlet.org]

2D - Curvelets



Spatial representation of six different curvelets at different scales and angles

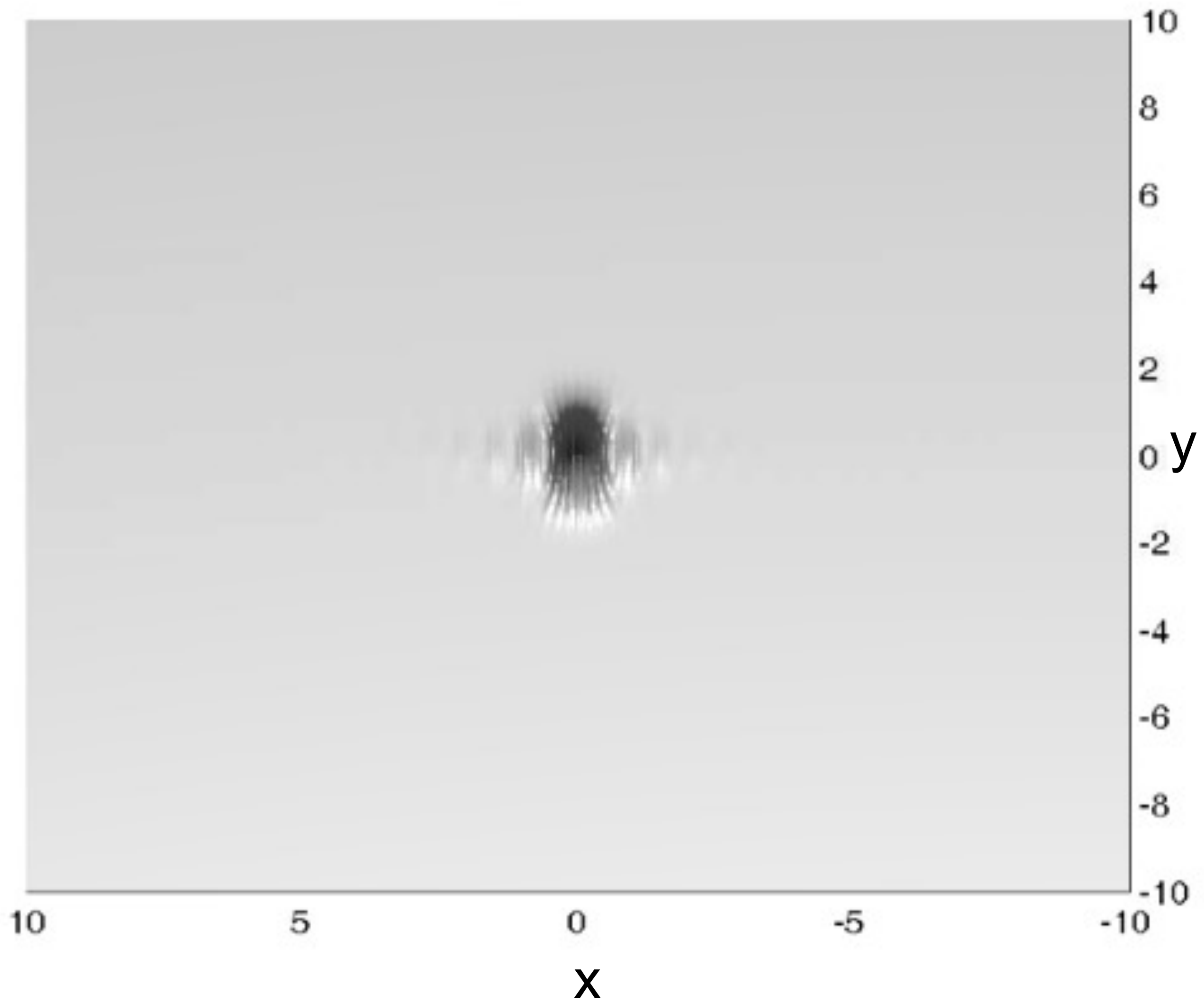


Frequency representation of six different curvelets at different scales and angles

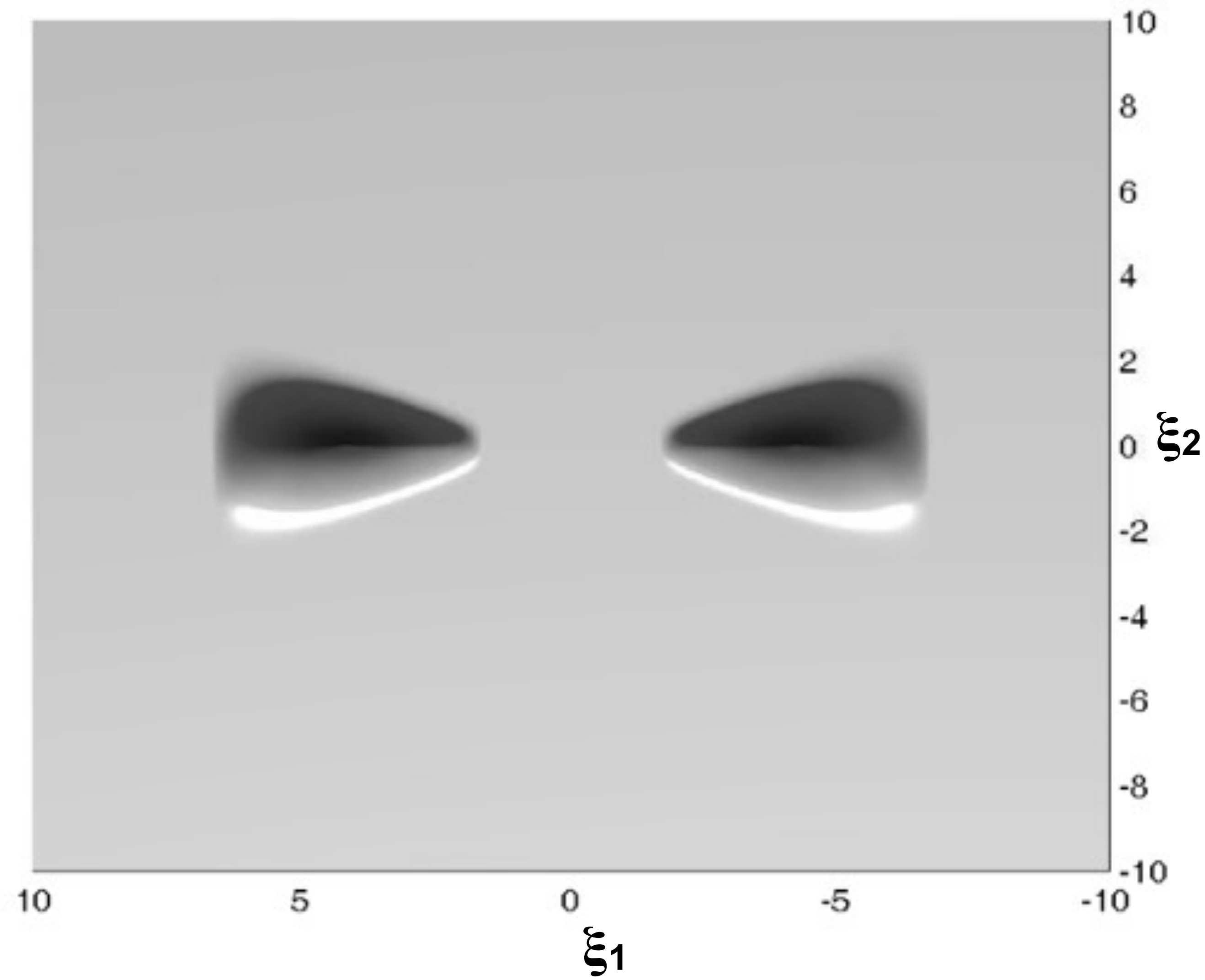
[http://slim.eos.ubc.ca/Publications/Public/Presentations/2007/herrmann07SEGPRED.pdf]

2D - Shearlets

Scale parameter, a = 0.3
Shear parameter, s = 0
Translation parameter, t = 0



Space Domain

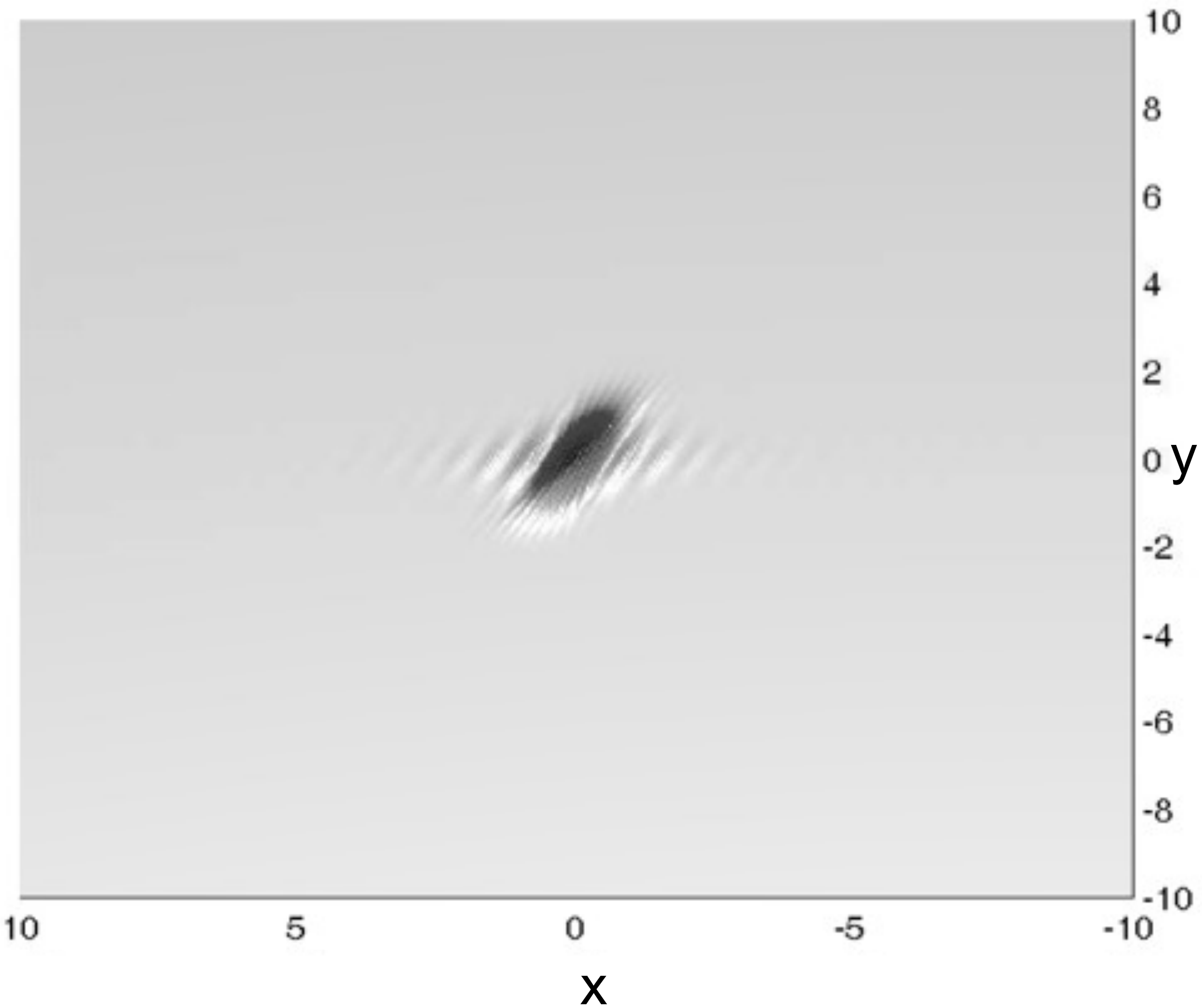


Frequency Domain

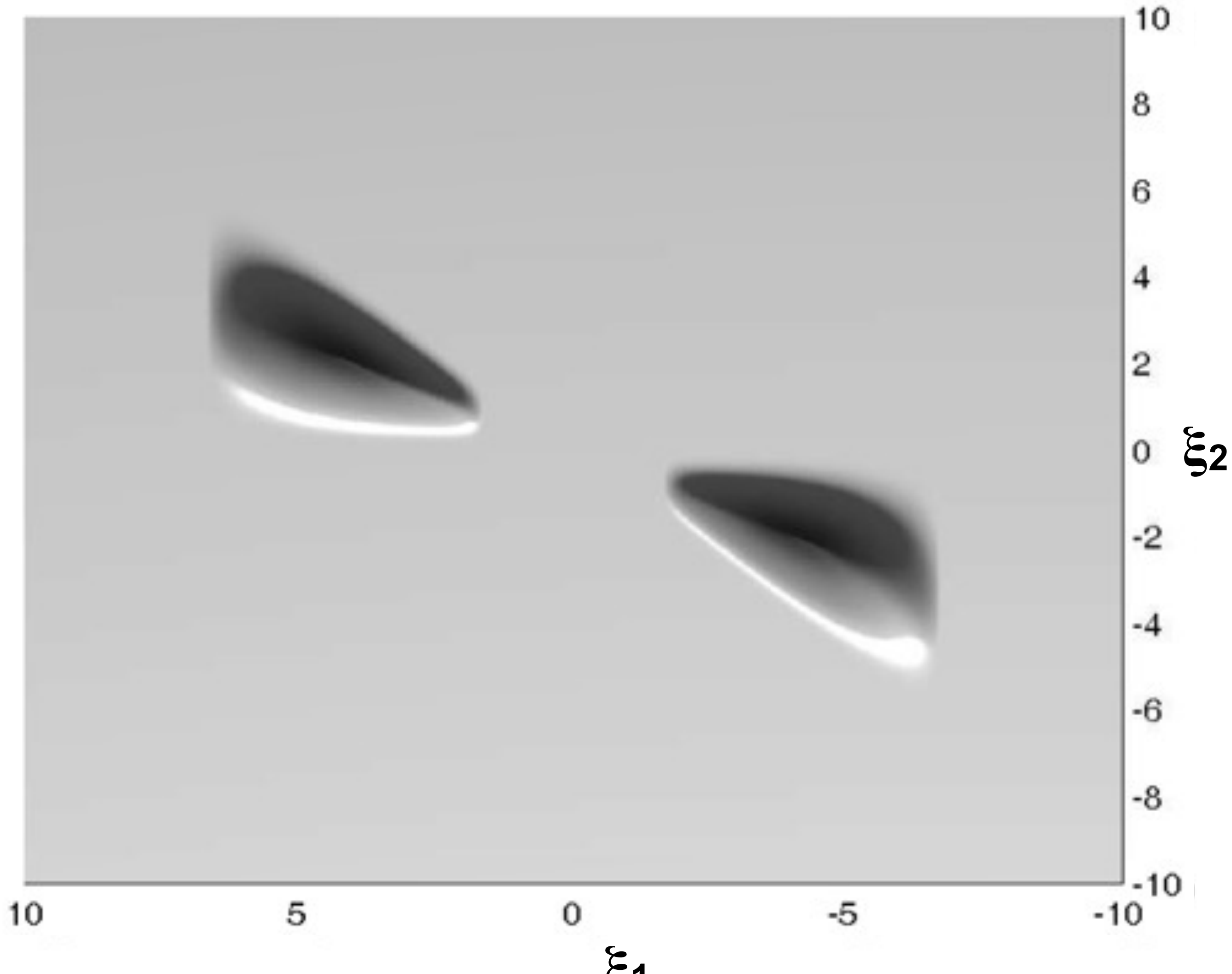
[http://www.shearlet.uni-osnabrueck.de/papers/FWtSaba.pdf]

2D - Shearlets

Scale parameter, a = 0.3
Shear parameter, s = - 0.5
Translation parameter, t = 0



Space Domain



Frequency Domain

[http://www.shearlet.uni-osnabrueck.de/papers/FWtSaba.pdf]

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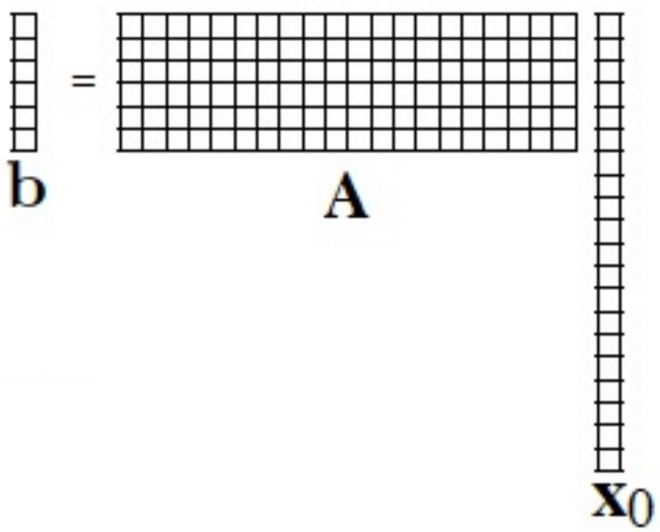
Sparse Recovery Problem

Solve for x_0



where $\mathbf{A} \rightarrow$ **compressive - sampling matrix**

$$\mathbf{A} := \underbrace{\mathbf{R}\ \mathbf{M}}_{\text{Encoding}}\ \mathbf{S}^H$$

Restriction operator (R), Measurement Matrix (M), Sparsifying transform (S^H)

b $\rightarrow$ compressively - sampled data

b = RMD

D = Sampled Sequential Data

$\mathbf{x_0}$ = Sparse representation of D in a transform domain

Sparse Recovery Problem

Resolved by solving :

$$\tilde{x} = \arg\min_{x} ||x||_1 \stackrel{\text{def}}{=} \sum_{i=1}^{N} |x[\,i\,]| \qquad \text{subject to} \quad Ax = b$$

Recover sequential data as: $\tilde{\mathbf{D}} = \mathbf{S}^{H} * \tilde{\mathbf{x}}$

**Contribution of Compressive Sensing (CS):

CS combines transformation and encoding in a single linear encoding step:

$$\mathbf{A} = \mathbf{R\,M\,S}^{H} \Rightarrow \textbf{Compressive - sampling matrix}$$

Algorithm

High resolution sampled data - D
(Sequential Shot Record)

↓

Restricted simultaneous acquisition sampling matrix - RM

↓

Sparsifying Transform → Curvelet Operator - C ; Shearlet Operator - S

↓

Compressive sampling matrix
$A = RM^*C^H$

↓

Compressively sampled measurements
$b = RM^*D$

↓

Recover sparsest set of curvelet coefficients
$\tilde{x} = \arg\min_{x} \|x\|_1 \quad \text{s.t.} \quad Ax = b$

→

Sequential data recovery
$\tilde{D} = C^H * \tilde{x}$

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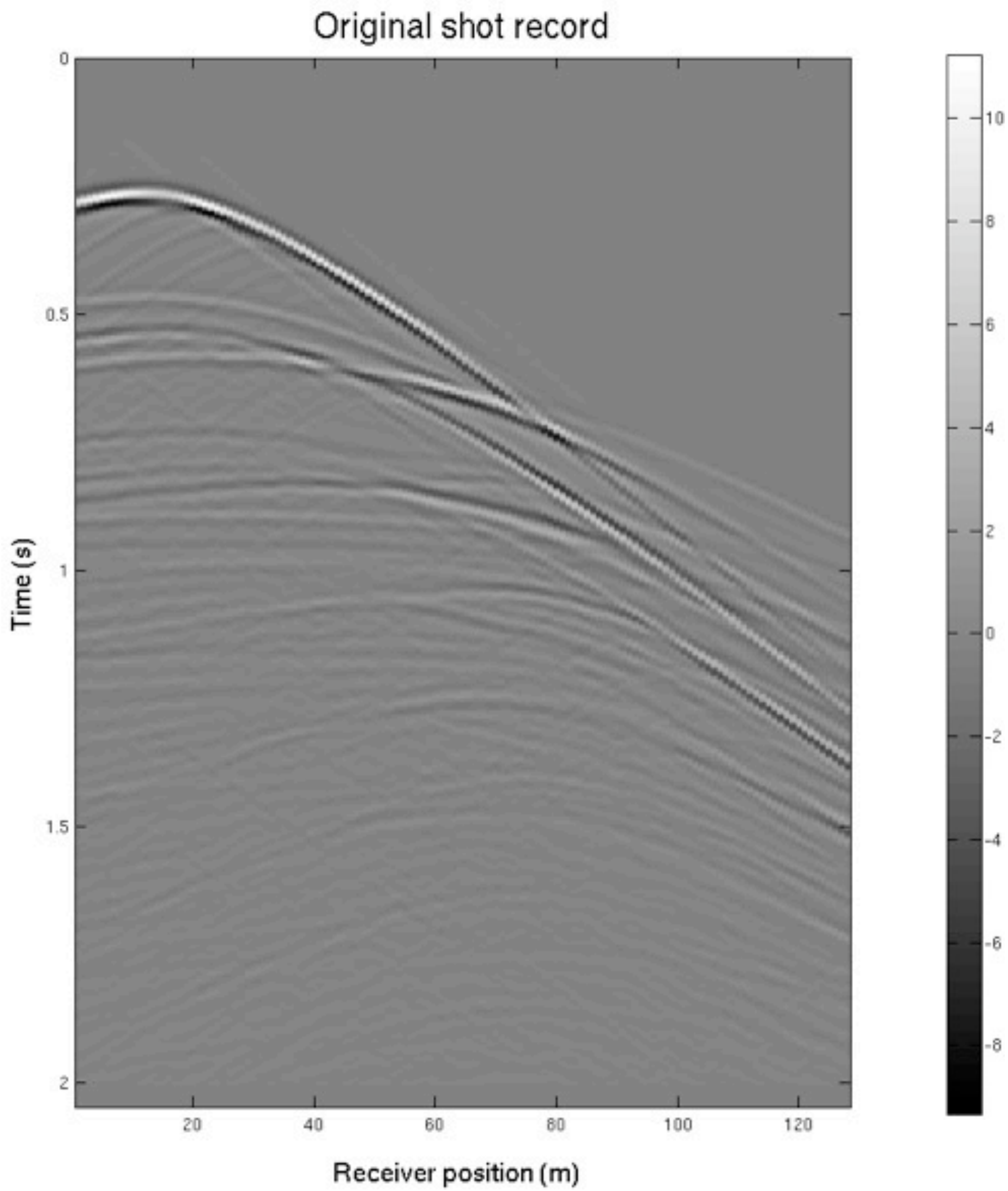
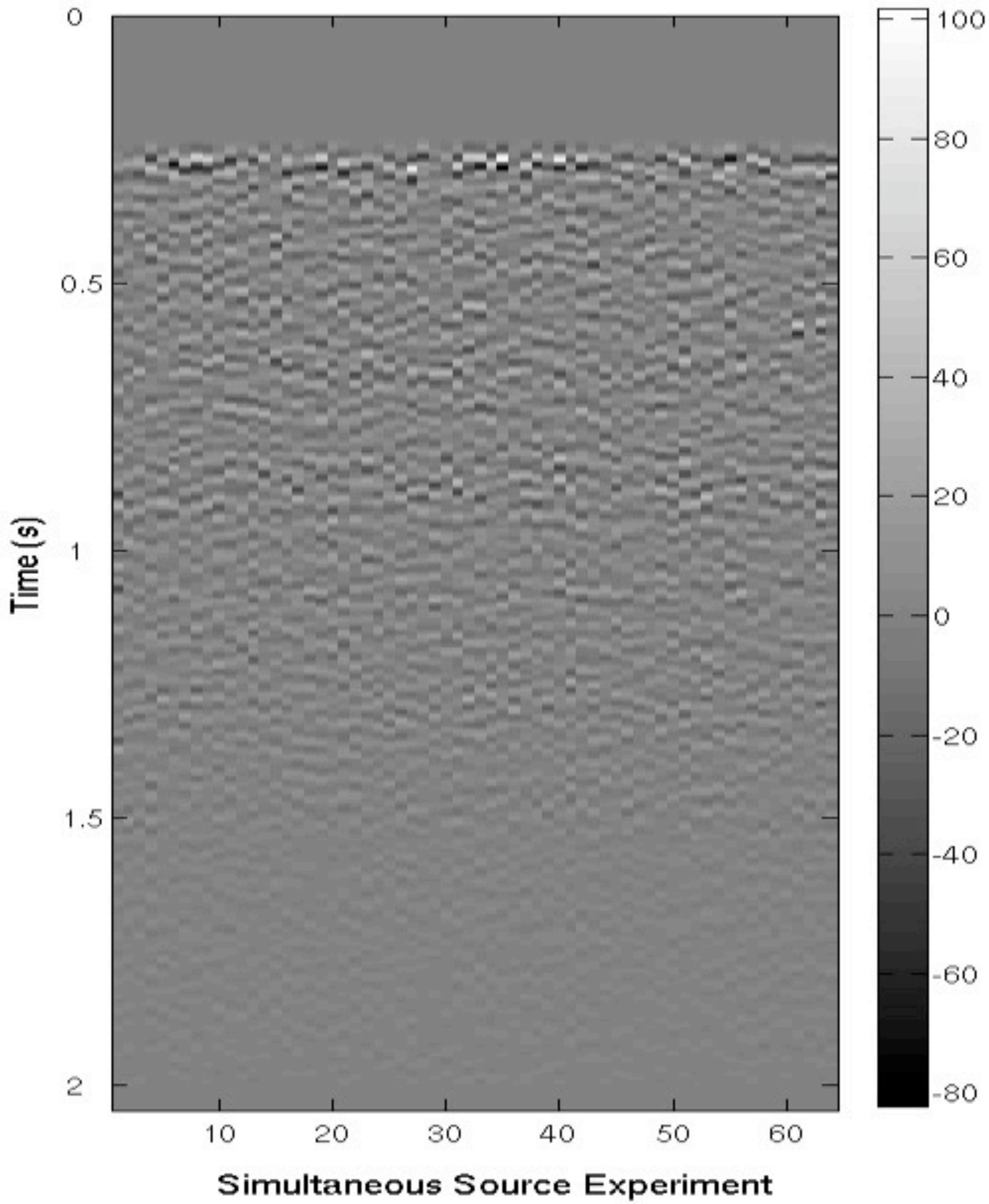
Original Data: Sequential Shot Record



NOTE: Total number of shots
N_s = 128

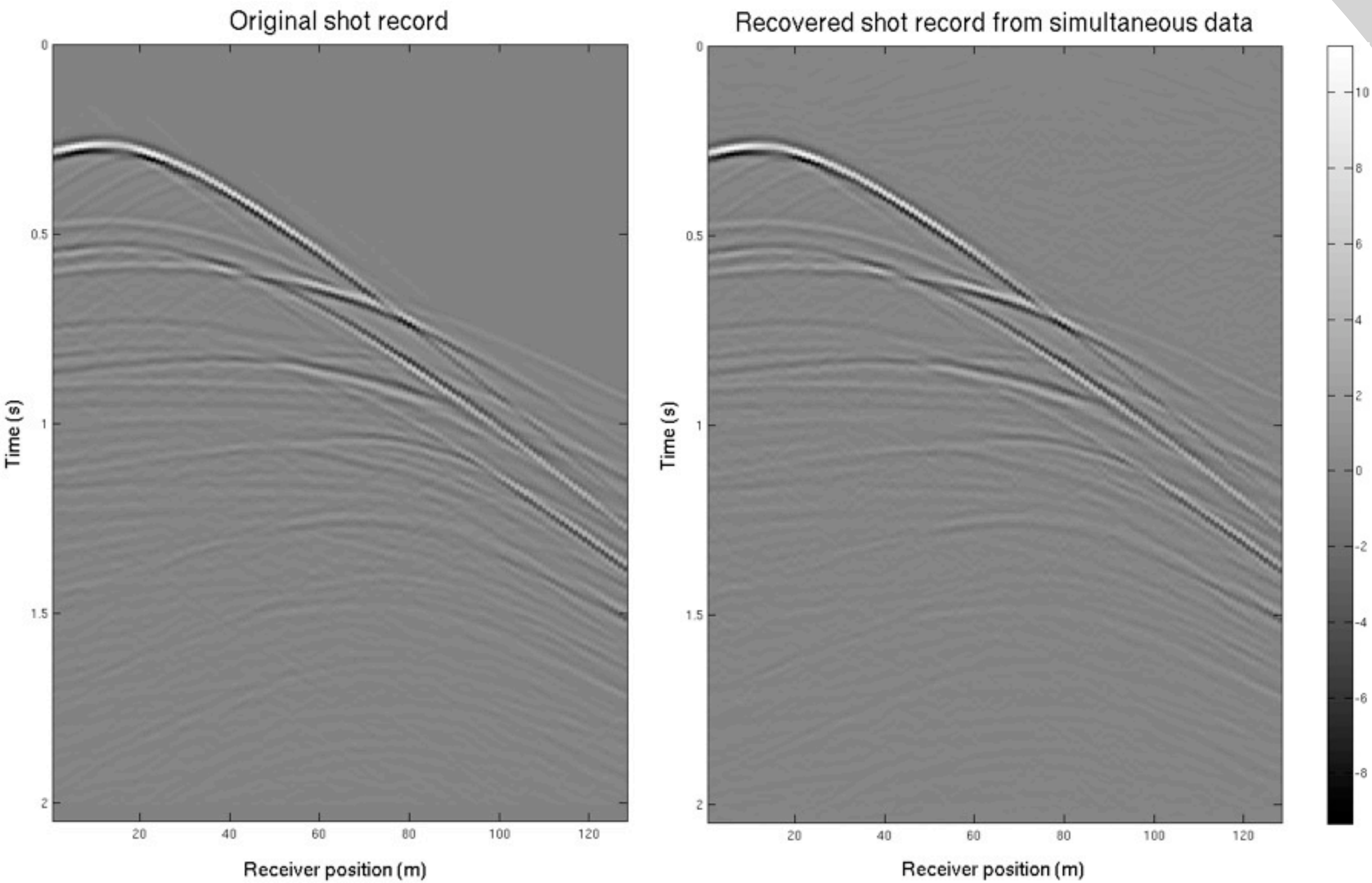
Simultaneous Data



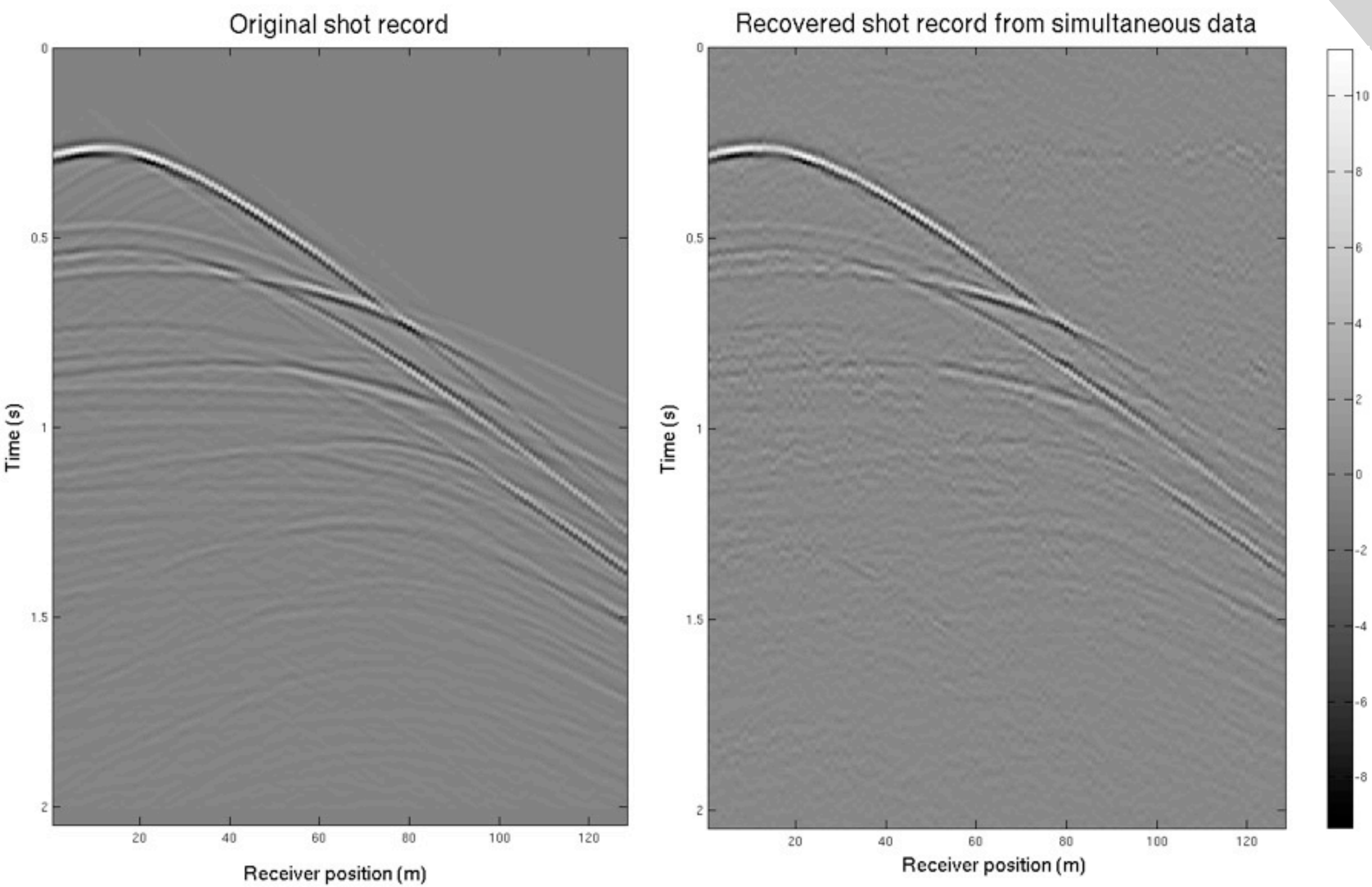
Sampling Ratio, $\delta = n_s / N_s$
where n_s = reduced number of shots

Here: $\delta = 0.5$

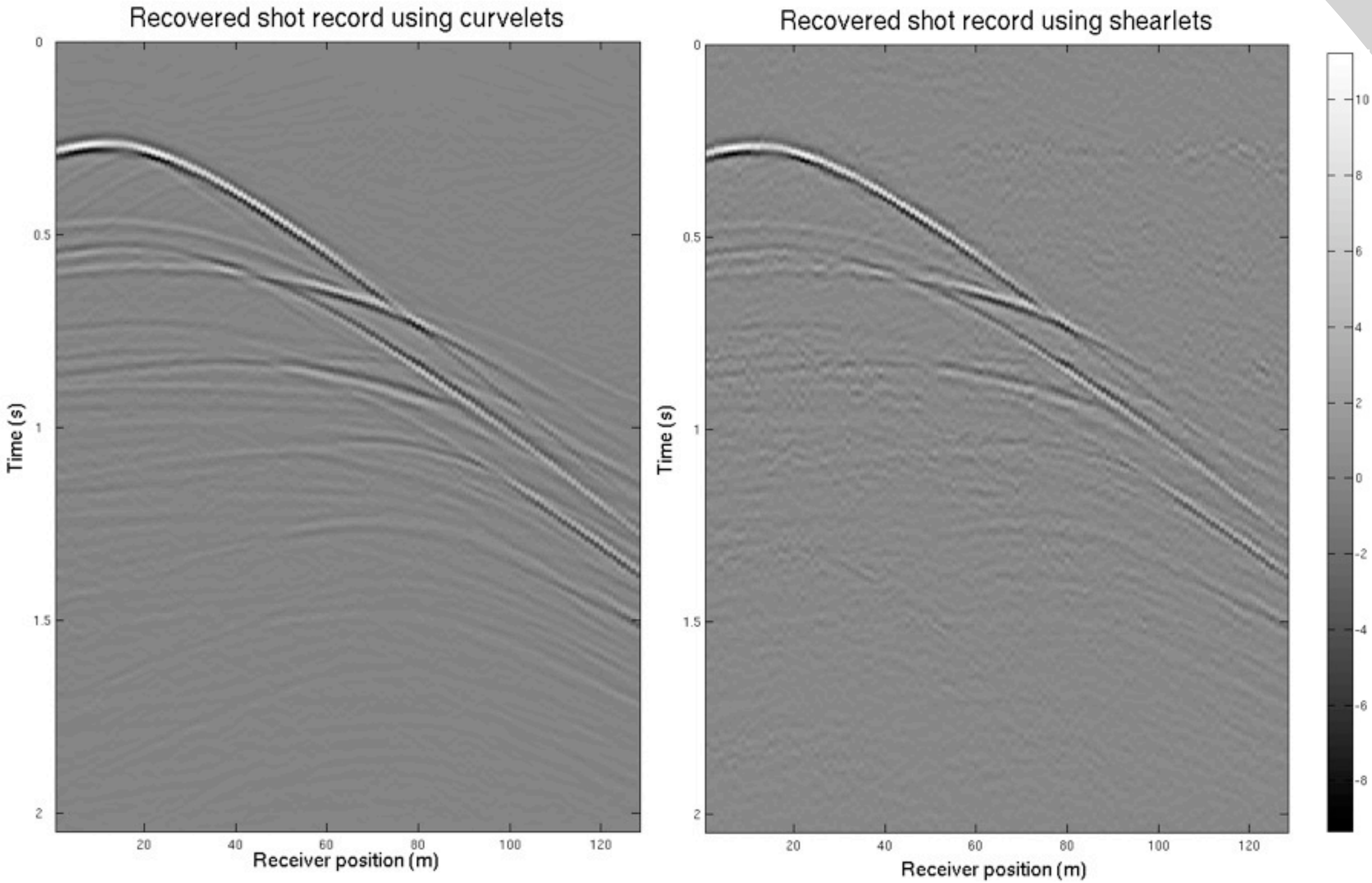
Recovered Sequential Data from Curvelets



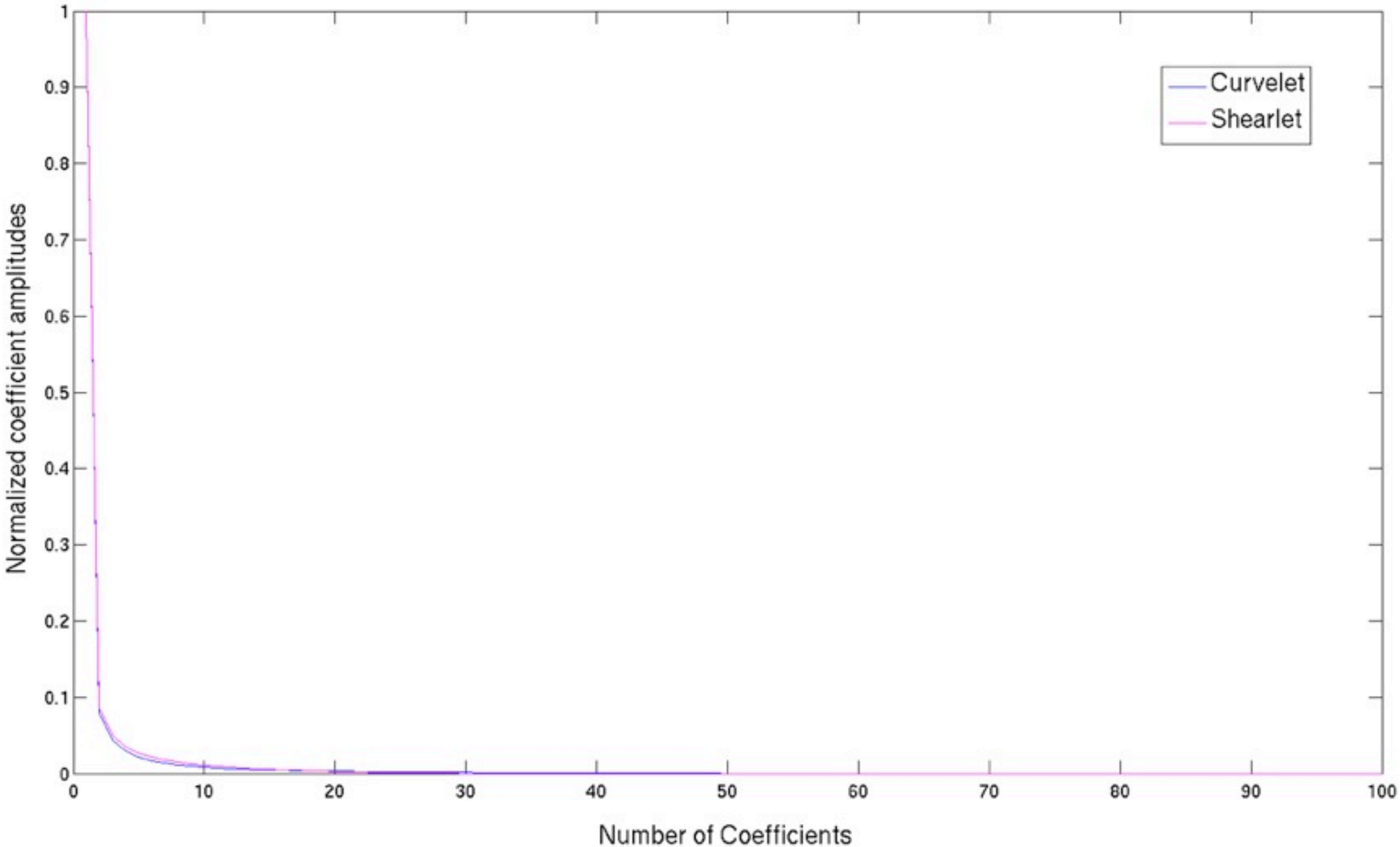
Recovered Sequential Data from Shearlets



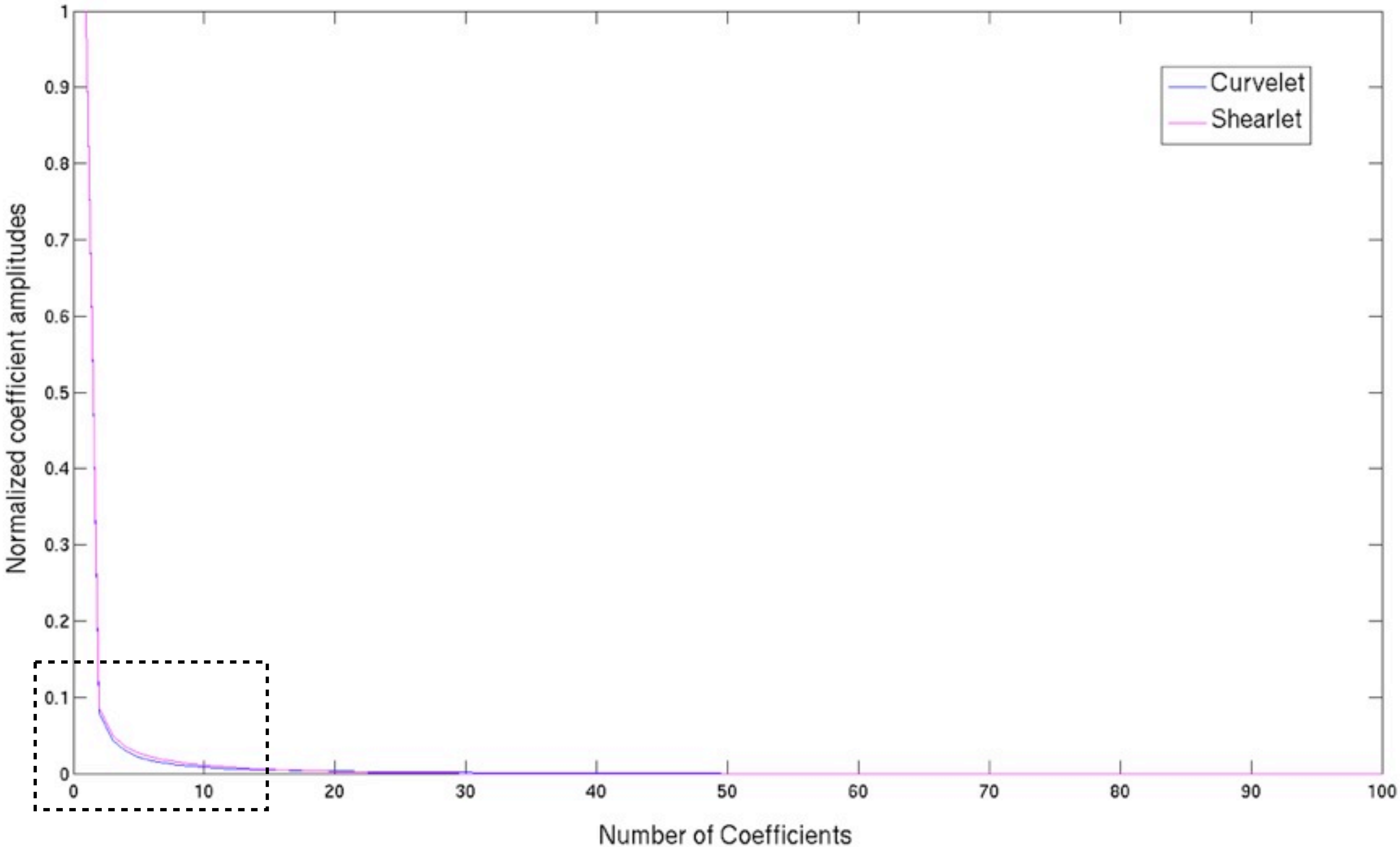
Curvelets and Shearlets



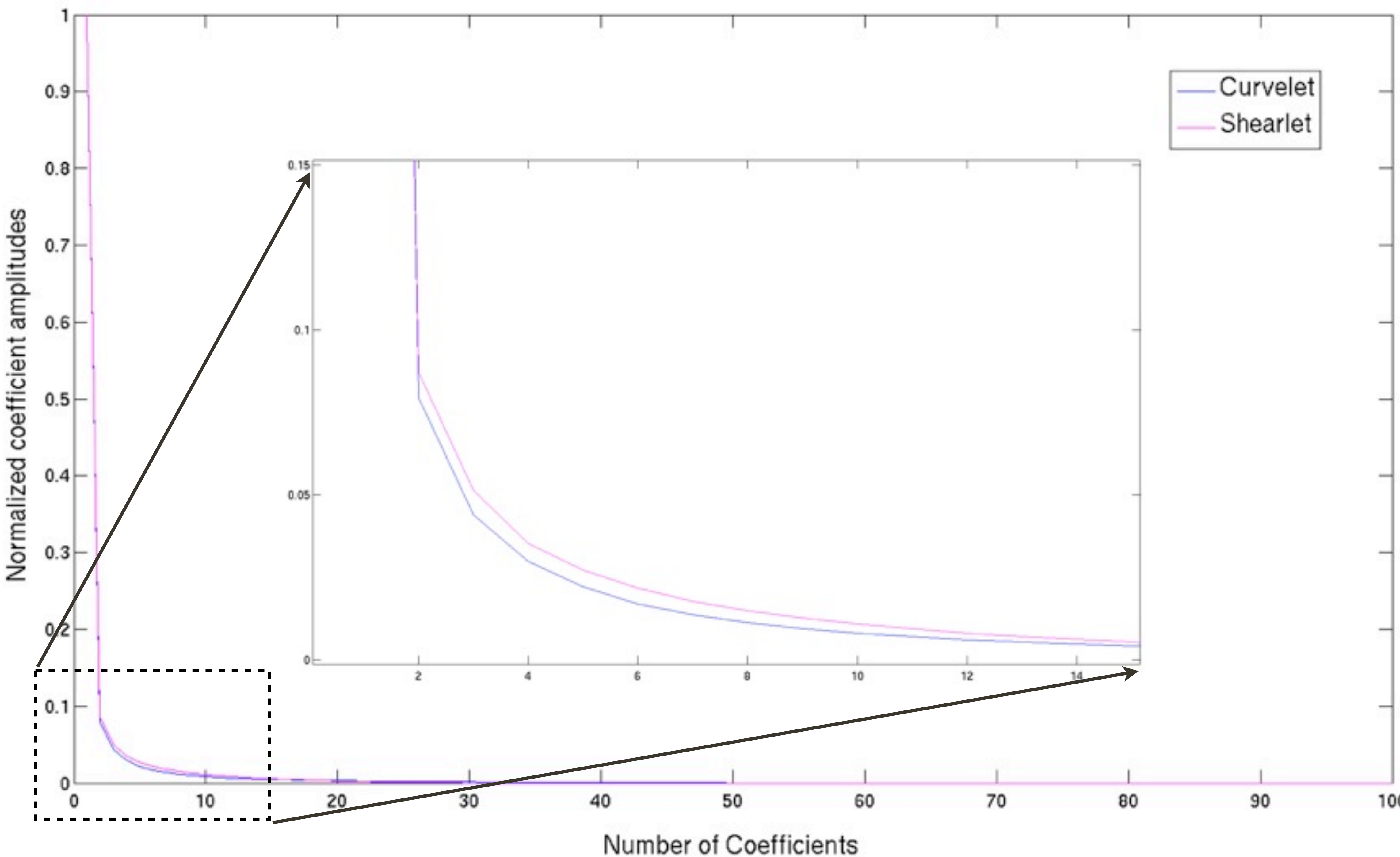
Decay of Coefficients



Decay of Coefficients



Decay of Coefficients



Observations

Sampling Ratio	Simultaneous Source Exp.	SNR* (dB) Curvelets	SNR* (dB) Shearlets
0.3	38	9.1	2.8
0.5	64	16.7	8.3
0.7	90	20.6	10.0

*SNR = Signal-to-noise ratio = $-\,20 \log \frac{\| D - \tilde{D} \|}{\| D \|}$

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Conclusions

- Following ideas from Compressive Sensing, seismic wavefields can be reconstructed from randomized subsamplings

- Acquisition and processing costs scale with transform - domain sparsity of the wavefield

- Recovery from simultaneous simulations depends on transform - domain sparsity

- Curvelet and Shearlet Transforms have sparse representation of seismic data - curvelets being slightly more sparse than shearlets

Future Work

- Full investigation of application of curvelets and shearlets to realistic simultaneous acquisition data

- Extension from 2D to 3D seismic data : Curvelet and Shearlet Transforms for 3D seismic data

- Work with Mirror - Extended Shearlet and Curvelet Transforms to avoid wrapping effect near the image edges

References

Herrmann, F. J., 2010, Randomized sampling and sparsity: getting more information from fewer samples: Technical Report TR-2010-01, 2010-05-19, UBC

Kutyniok, G., Jakob Lemvig, and Wang-Q Lim, 2010, Compactly Supported Shearlets: arXiv:1009.4359v2 [math.FA]

Acknowledgements

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- CurveLab
- ShearLab

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Thank You!