SINBAD Consortium 2010

Randomized Full Waveform Inversion

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Motivation

- Cost of the FWI is proportional to the number of shots and it requires hundreds of RTM (Reverse Time Migration).
- Dimensionality reduction with compressive sensing aims at compressing the data volume in inversion.
- Stochastic optimization provides better solution to randomized inversion problem than conventional optimization.

Overview

- Full Waveform Inversion (FWI)
- Simultaneous Source Experiment
- Randomized FWI
- Stochastic Optimization Methods
- Examples
- Conclusion
- Future Plans

Full Waveform Inversion

- Mathematically FWI can be formed as

$$\min_{\sigma} J(\sigma) = \frac{1}{2}||\mathbf{d} - \mathbf{D}\mathbf{u}(\sigma)||_2 \text{ subject to } \mathbf{F}(\mathbf{u}, \sigma) = \mathbf{0}$$

$$\mathbf{F}(\mathbf{u}, \sigma) = (\omega^2\sigma^2 + \nabla^2)\mathbf{u} + q = 0$$

$\mathbf{d} : data$

$\mathbf{D} : \text{ detection operator}$

$\sigma : sloweness$

$\mathbf{u}(\sigma) : wavefield$

$\omega : \text{ angular frequency}$

$q : \text{ source}$

$\nabla^2 : Laplacian$

(Ben Hadj Ali 08)

Full Waveform Inversion

- Gradient of the cost function with respect to slowness is defined as,

$$\partial J(\sigma)/\partial\sigma = -\Re\{(\partial \mathbf{u}/\partial\sigma)^H \mathbf{D}^T[\mathbf{d} - \mathbf{D}\mathbf{u}(\sigma)]\}$$

$$\Re : \ \text{real part}$$

$$(.)^H : Hermitian$$

  - with $\partial \mathbf{u}/\partial\sigma$ defined as,

$$\partial \mathbf{u}/\partial\sigma \quad = \quad -2\omega^2(\omega^2\sigma^2 + \nabla^2)^{-1}\sigma\mathbf{u} = \mathbf{K}$$

$$\mathbf{K} : \ \text{de-migration operator, linearized Born}$$

(Plessix 2009)

Conventional Optimization

- Limited-memory BFGS (Plessix 2009)

$$\sigma_{k+1} = \sigma_k - \tau \mathbf{H}_k \nabla J(\sigma_k)$$

$$\mathbf{H}_k: \text{ inverse Hessian}$$

$$\tau: \text{ line search value}$$

- Preconditioned Gradient method (Ravaut 2004)

$$\sigma_{k+1} = \sigma_k - \tau diag(\mathbf{K}^T\mathbf{K} + \epsilon\mathbf{I})^{-1} \nabla J(\sigma_k)$$

- Conjugate Gradient method (Virieux 2009)

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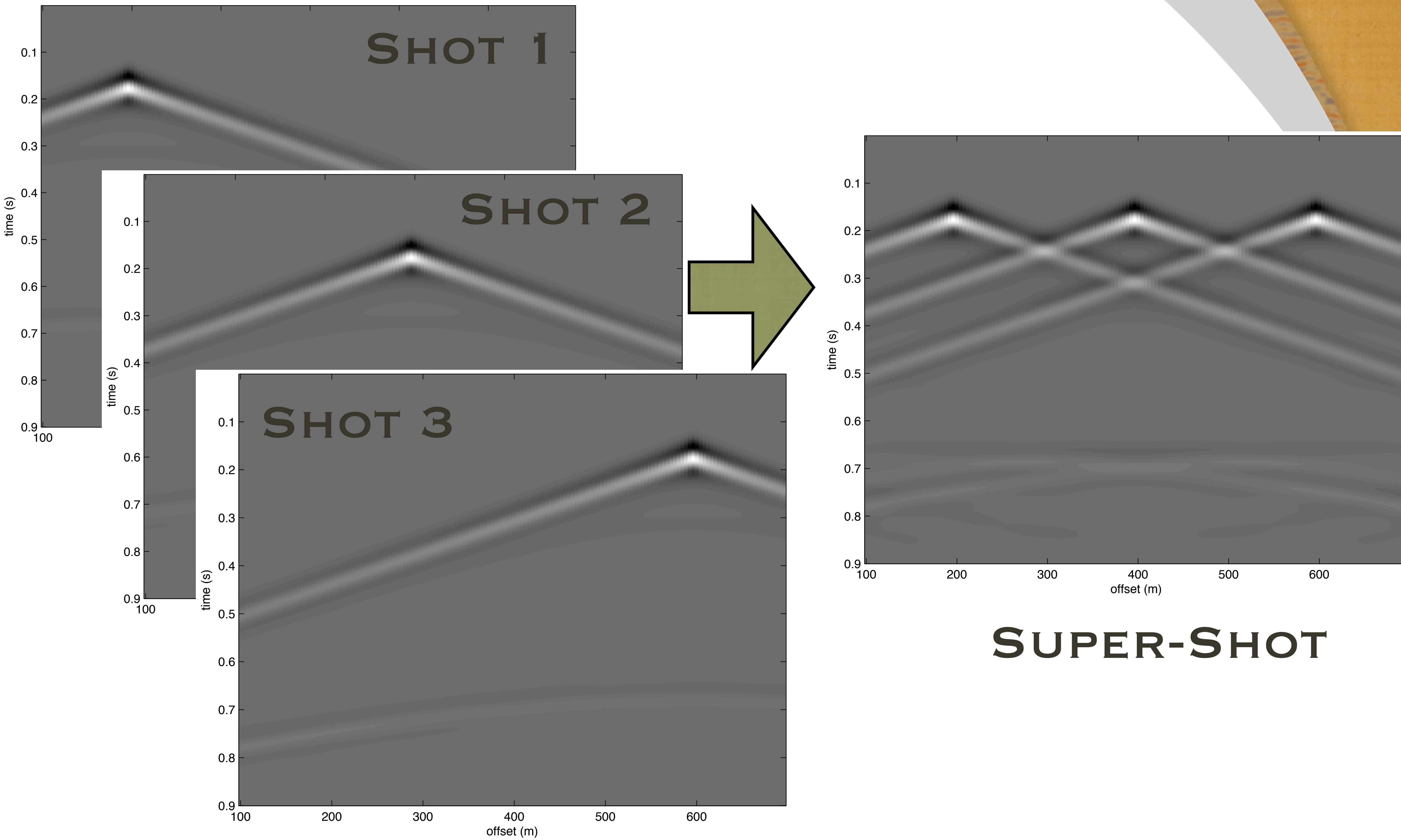
Simultaneous Source Experiment



(Krebs 2009)

Simultaneous Source Experiment

updates read

$$\sigma_{k+1} = \sigma_k - \tau \nabla J(\sigma_k, \mathbf{Q})$$

$\mathbf{Q}$: all sources

with $$\nabla J(\sigma_k, \mathbf{Q}) \approx \frac{1}{N_s} \sum_{i=1}^{i=N_s} \nabla J(\sigma_k, \mathbf{Q}_i)$$

$\mathbf{Q_i}$: a simultaneous source

$$\mathbf{E}\left(\frac{1}{N_s} \sum_{i=1}^{i=N_s} \nabla J(\sigma_k, \mathbf{Q}_i) \right) \rightarrow \nabla J(\sigma_k, \mathbf{Q})$$

$\mathbf{E}(.)$: *expectation*

Randomized FWI

$\sigma \longleftarrow \sigma_0$ initial model

$\{J(\sigma, \mathbf{Q_i}), \nabla_\sigma J(\sigma, \mathbf{Q}_i)\} \longleftarrow$ new randomized super-shot

$\textbf{While} \|\nabla_\sigma J(\sigma)\| \geq \epsilon$

$\sigma \longleftarrow$ update model with $J(\sigma, \mathbf{Q}_i), \nabla_\sigma J(\sigma, \mathbf{Q}_i)$

$\{J(\sigma, \mathbf{Q_i}), \nabla_\sigma J(\sigma, \mathbf{Q}_i)\} \longleftarrow$ new randomized super-shot

end

(Moghaddam 2010, Krebs 2009)

Stochastic Optimization Approaches

- Stochastic Gradient Descent

$$\sigma_{k+1} = \sigma_k - \tau \nabla J(\sigma_k, \mathbf{d}_k)$$

- Integrated Stochastic Gradient Descent (iSGD)

$$\sigma_{k+1} = \sigma_k - \eta_k \overline{\nabla J(\sigma_k)}$$

with $\overline{\nabla J(\sigma_k)}$ is averaging on the past numbers of gradients with weights,

$$\overline{\nabla J(\sigma_k)} = \frac{\sum_{i=k-m}^{k} e^{\alpha[i-(k-m)]} \nabla J(\sigma_i, \mathbf{d}_i)}{\sum_{i=k-m}^{k} e^{\alpha[i-(k-m)]}}$$

(Moghaddam 2010)

Stochastic Optimization

- Limited Memory BFGS (quasi-Newton methods),

$$\sigma_{k+1} = \sigma_k - \eta_k \mathbf{H}_k \nabla J(\sigma_k, \mathbf{d}_k)$$

- with updates:

$$\mathbf{H}_{k+1} = \mathbf{V}_k^T \mathbf{H}_k \mathbf{V}_k + \rho_k \mathbf{s}_k \mathbf{s}_k^T$$

$$\mathbf{s}_k = \sigma_{k+1} - \sigma_k \qquad \mathbf{y}_k = \nabla J(\sigma_{k+1}) - \nabla J(\sigma_k)$$

$$\mathbf{V}_k = \mathbf{I} - \rho_k \mathbf{y}_k \mathbf{s}_k^T$$

- On-line Limited Memory BFGS

$$\mathbf{H}_0 = \sum_{i=1}^{m} \frac{\mathbf{s}_{k-i}^T \mathbf{y}_{k-i}}{\mathbf{y}_{k-i}^T \mathbf{y}_{k-i}}$$

(Schraudolph 2007)

Stochastic Optimization

- Regular Limited Memory BFSG (quasi-Newton methods),

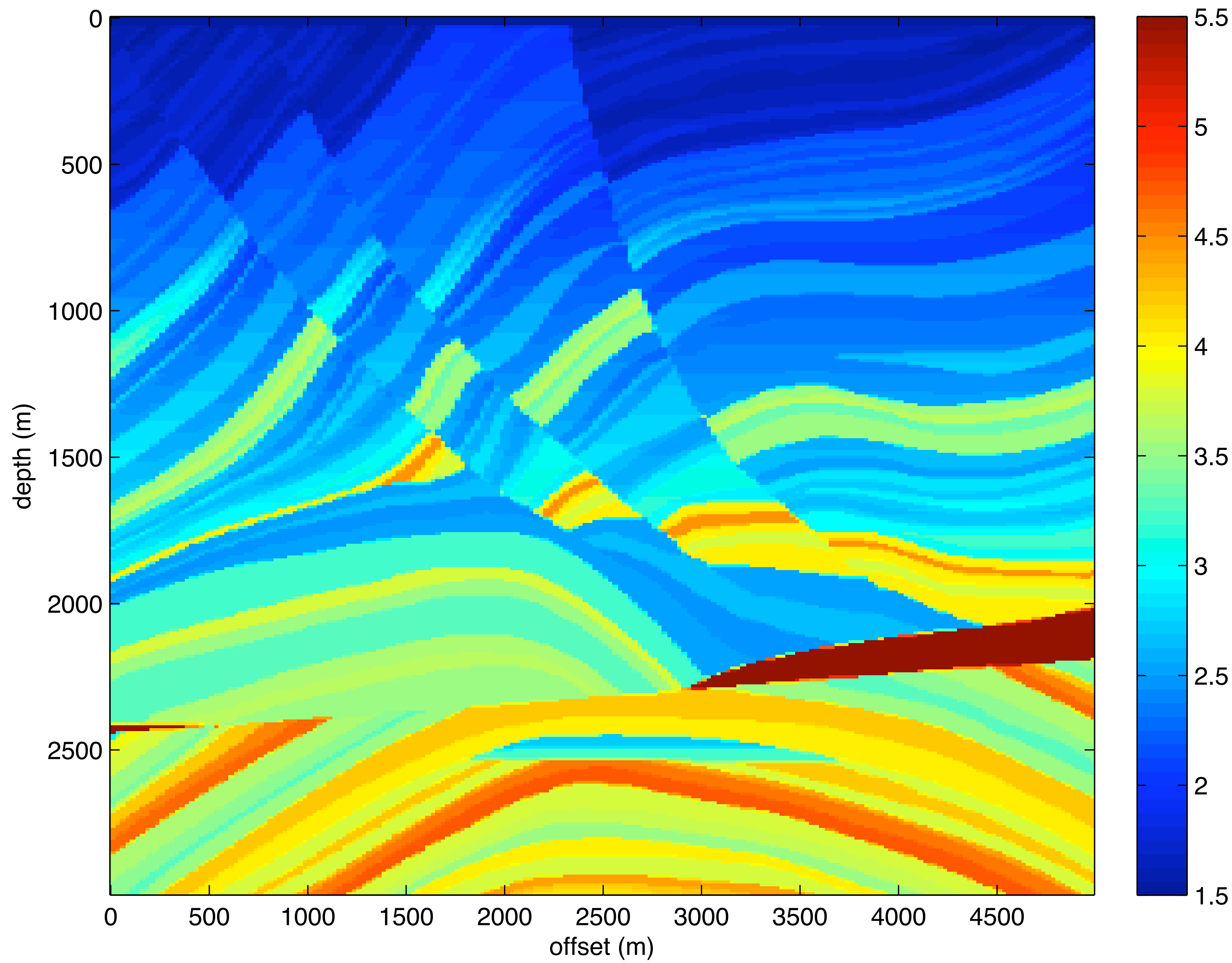
$$\begin{array}{ll} \min_H & ||\mathbf{H}_{k+1} - \mathbf{H}_k||_F \\ \text{subject to} & \mathbf{H}_{k+1}^T = \mathbf{H}_{k+1}, \mathbf{H}_{k+1}^T \mathbf{y}_k = \mathbf{s}_k \end{array}$$

$$\mathbf{s}_k = \sigma_{k+1} - \sigma_k \qquad \mathbf{y}_k = \nabla J(\sigma_{k+1}) - \nabla J(\sigma_k)$$

- Integrated Limited Memory BFSG (iLBFGS) ,

$$\begin{array}{ll} \min_H & ||\mathbf{H}_{k+1} - \sum_k \mathbf{H}_k||_F \\ \text{subject to} & \mathbf{H}_{k+1}^T = \mathbf{H}_{k+1}, \mathbf{H}_{k+1}^T \mathbf{y}_k = \mathbf{s}_k \end{array}$$

Examples (Marmoussi Model)



- 113 shots with 40 (m) spacing, 249 receivers with 20 (m) spacing, WAZ survey with 5 (km) max. aperture, Ricker source with 10 Hz central frequency, 3.6 second recording time with .9 (ms) time sampling.

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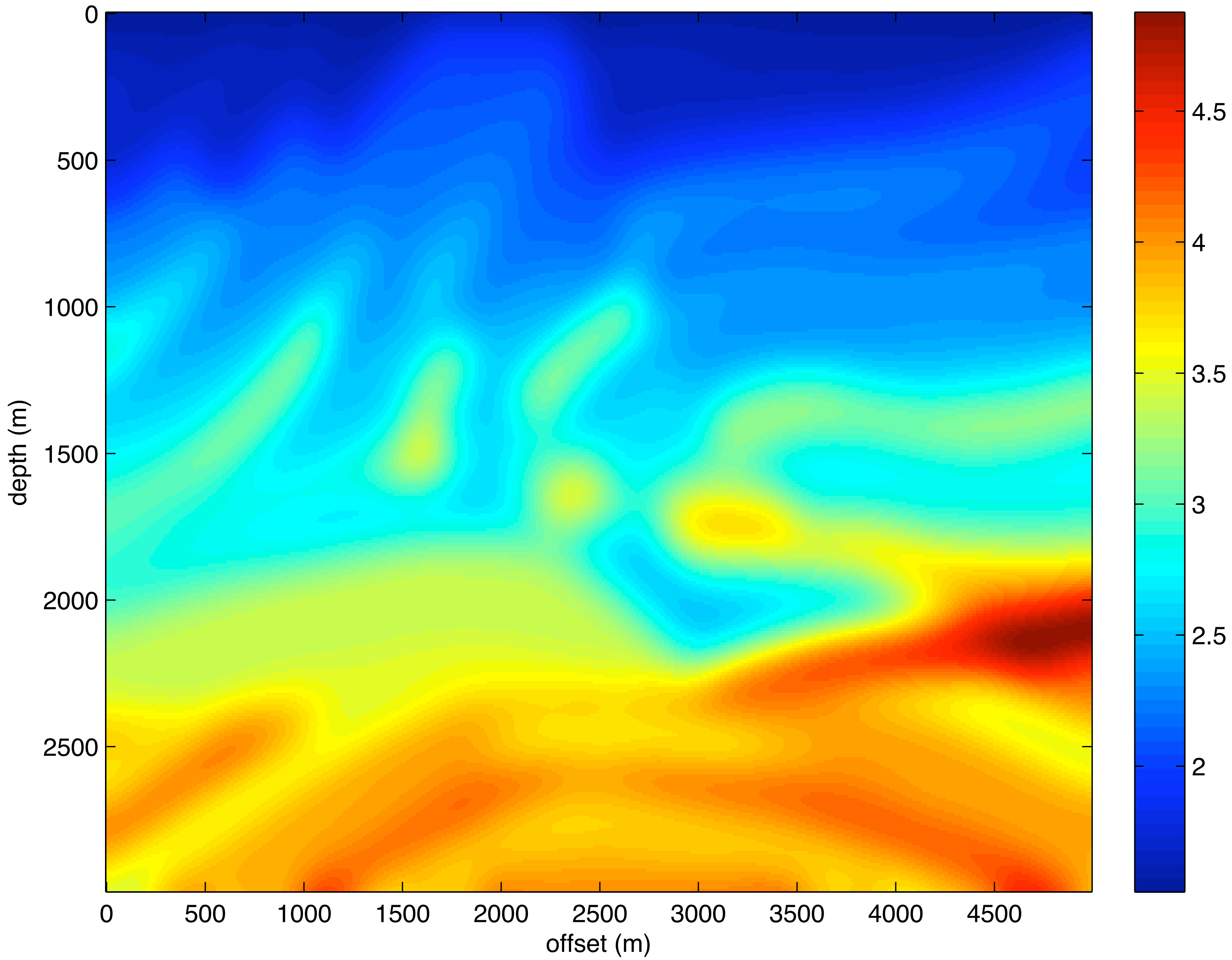
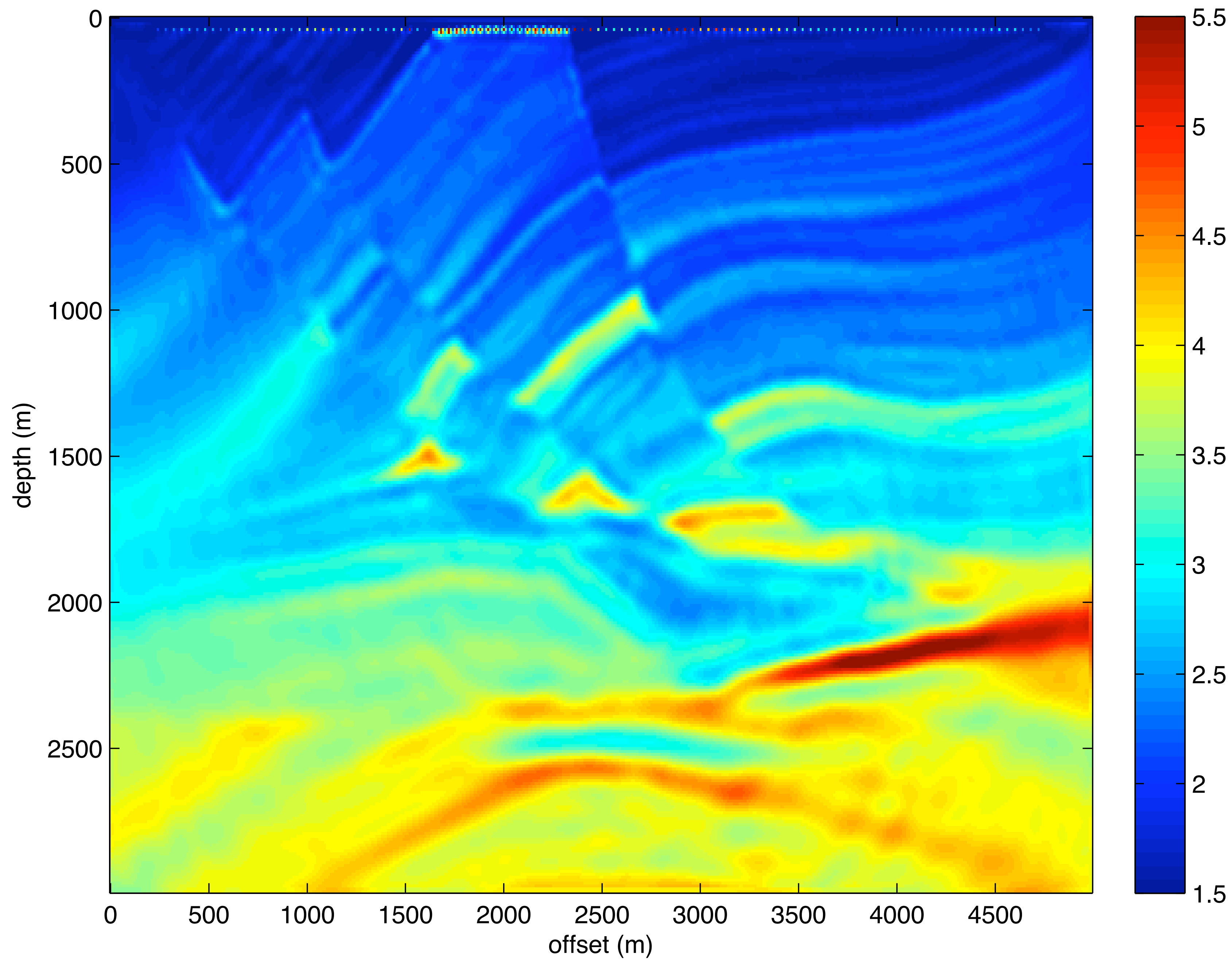
Examples (Initial Model)

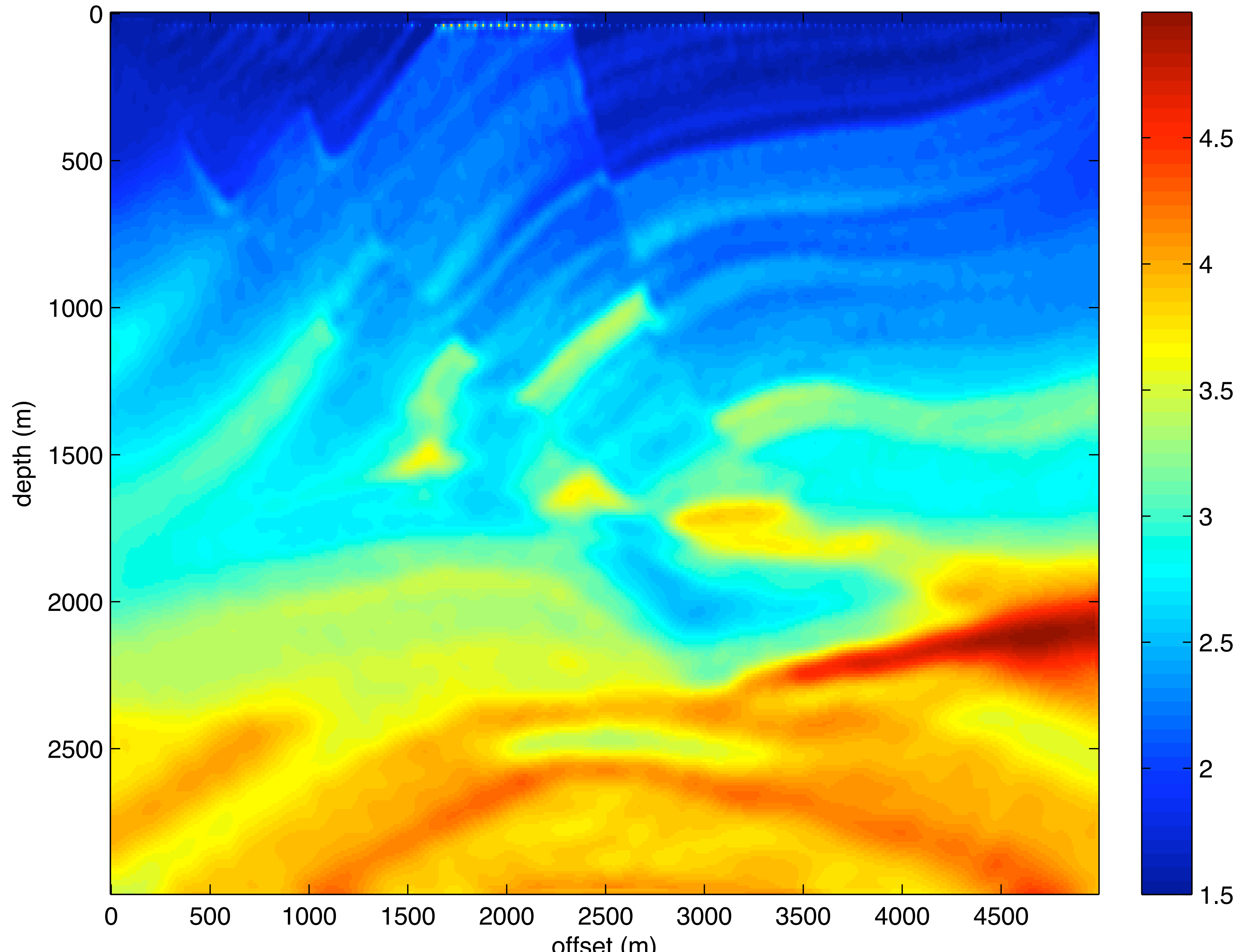


Examples (Inverted Model)



- inverted model after 18 iterations of LBFGS, 113 sequential shots, 50 frequency components has been used from 5 to 33 Hz with .55 Hz resolution

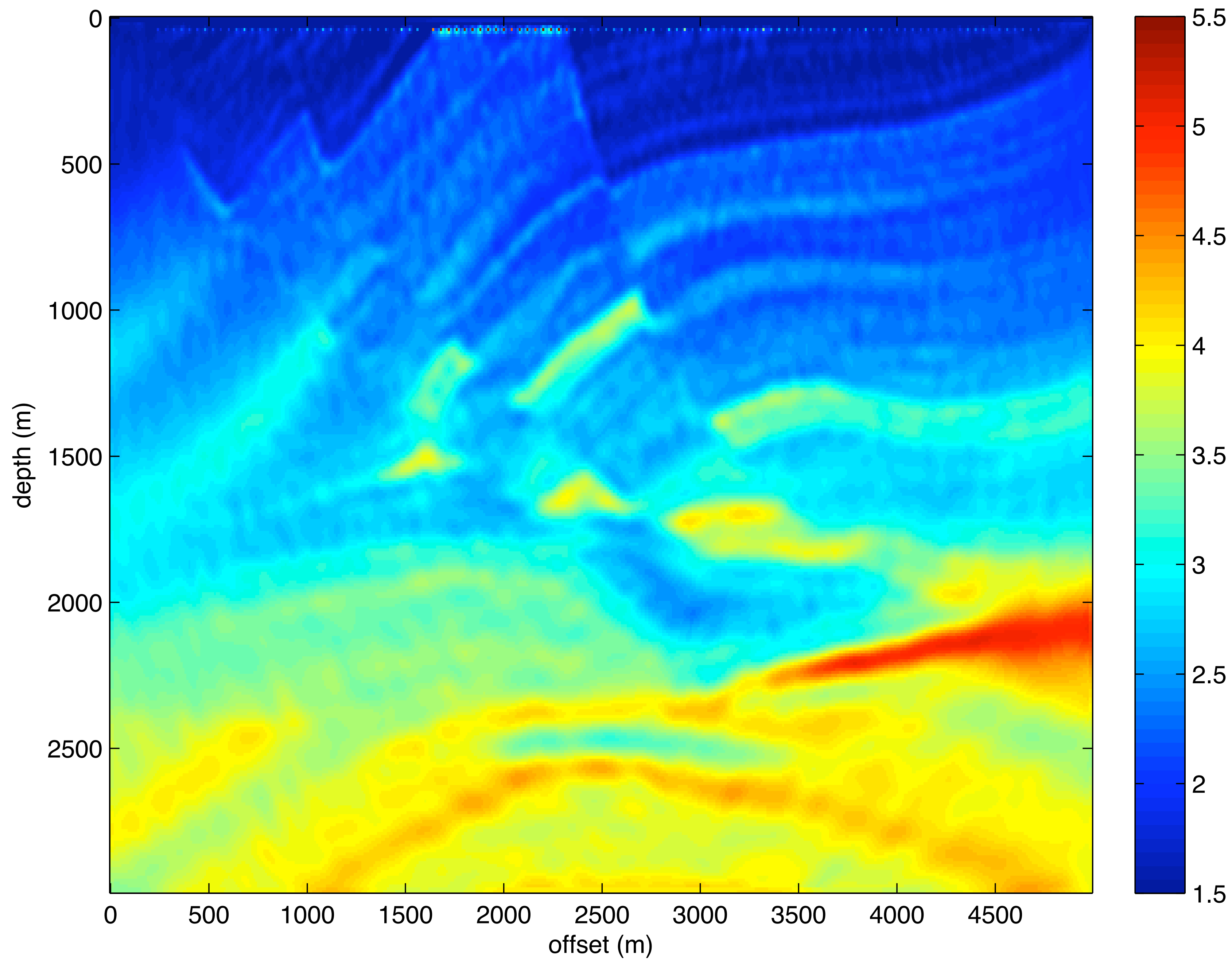
Randomized FWI (Inverted Model)



- Stochastic Gradient Descent, SNR= 4.65 dB, $\mathbf{SNR} = 20\log_{10}(\frac{\|\delta\mathbf{m} - \delta\tilde{\mathbf{m}}\|_2}{\|\delta\mathbf{m}\|_2})$

- 16 Randomized simultaneous shots, 4 frequencies, 40 times speed up

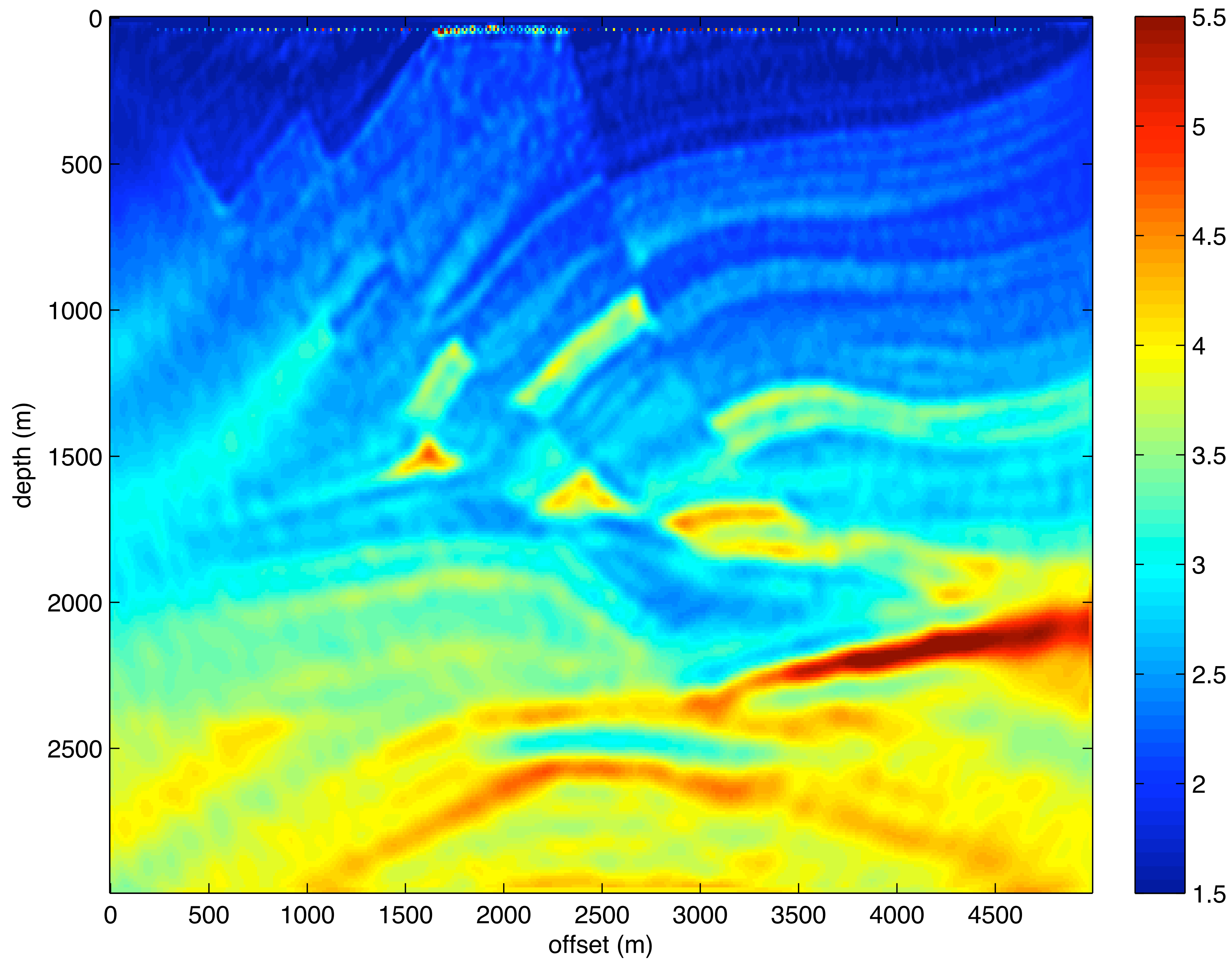
Randomized FWI (Inverted Model)



- online-LBFGS, SNR= 7.17 dB

- 16 Randomized simultaneous shots, 4 frequencies, 40 times speed up

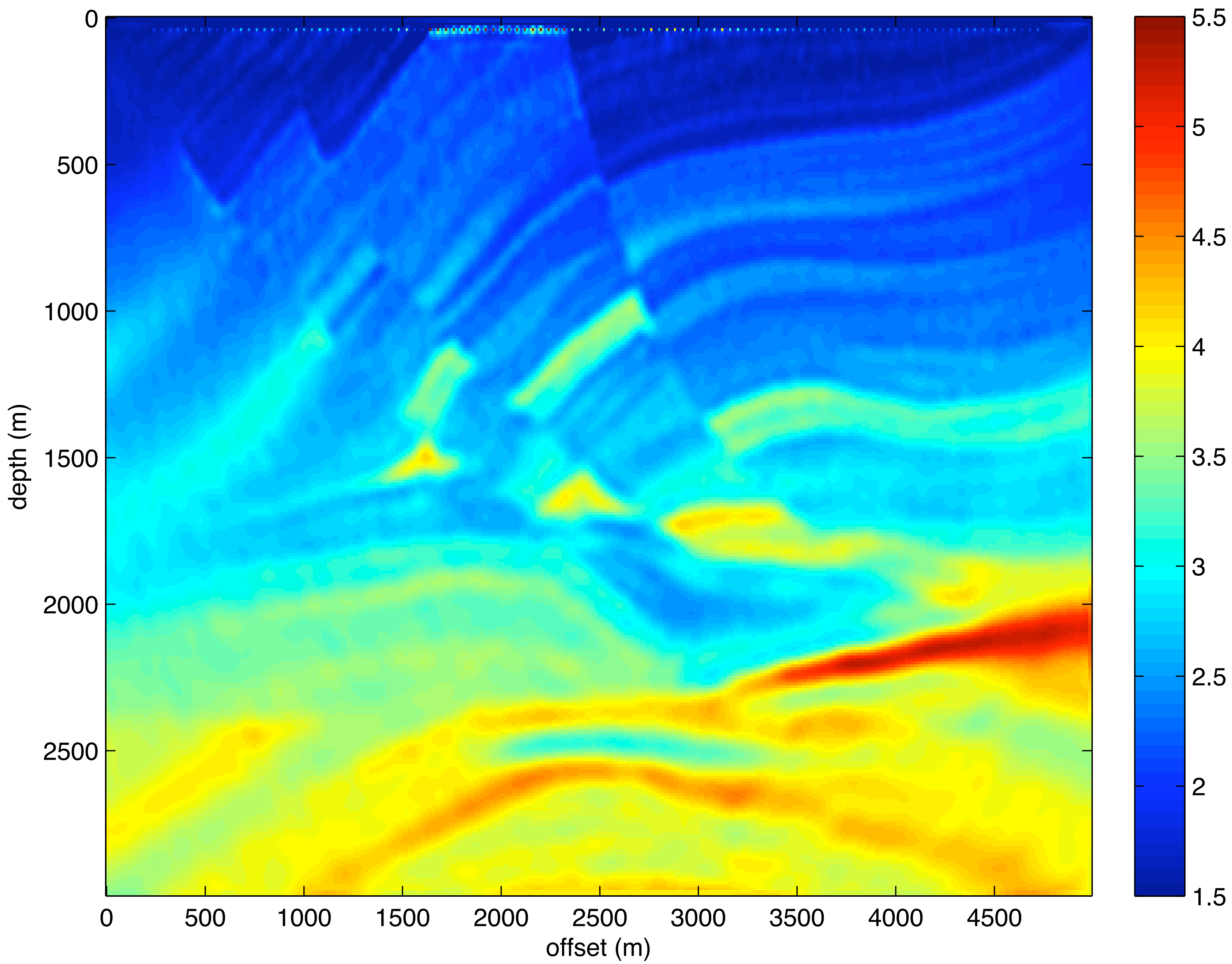
Randomized FWI (Inverted Model)



- iLBFGS, SNR= 9.10dB

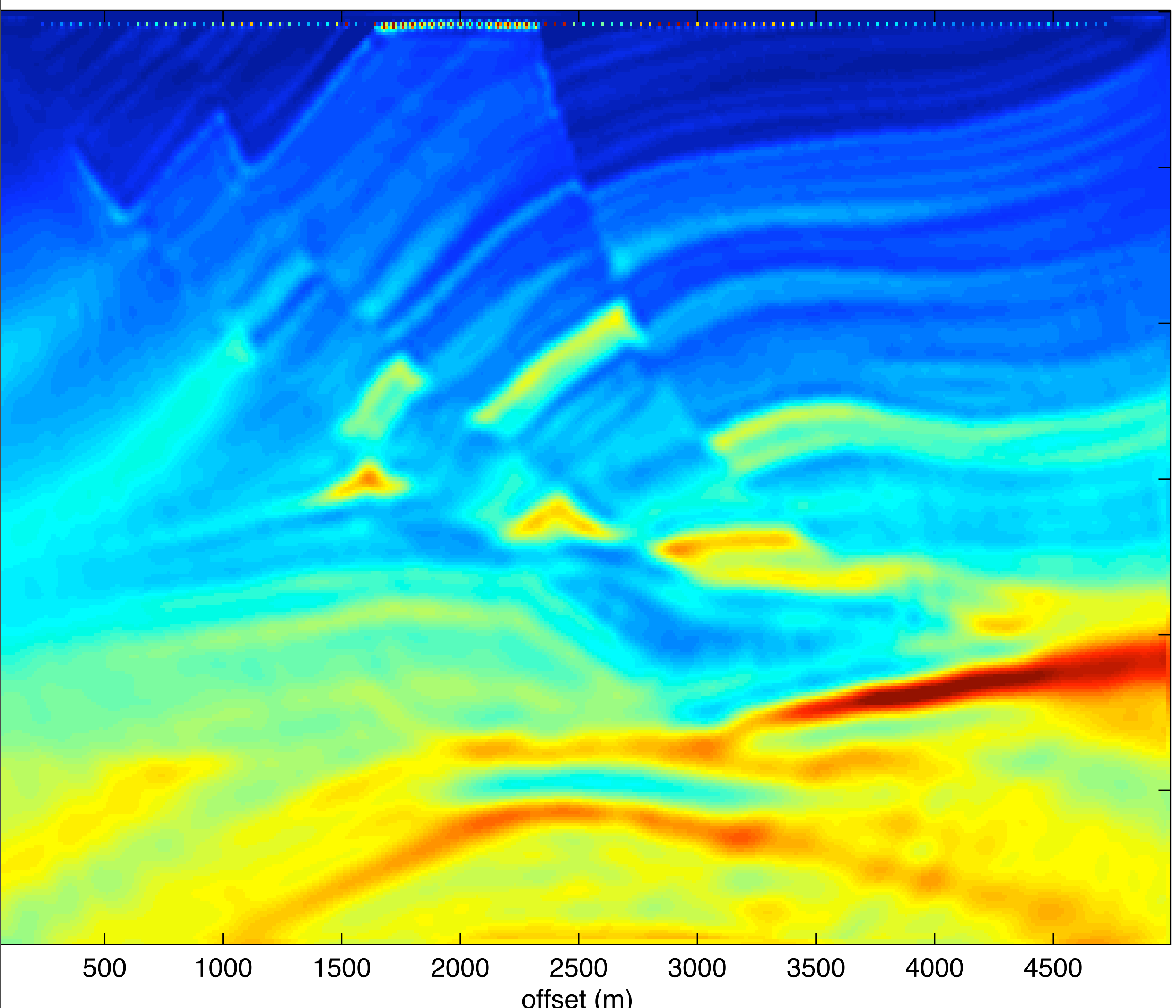
- 16 Randomized simultaneous shots, 4 frequencies, 40 times speed up

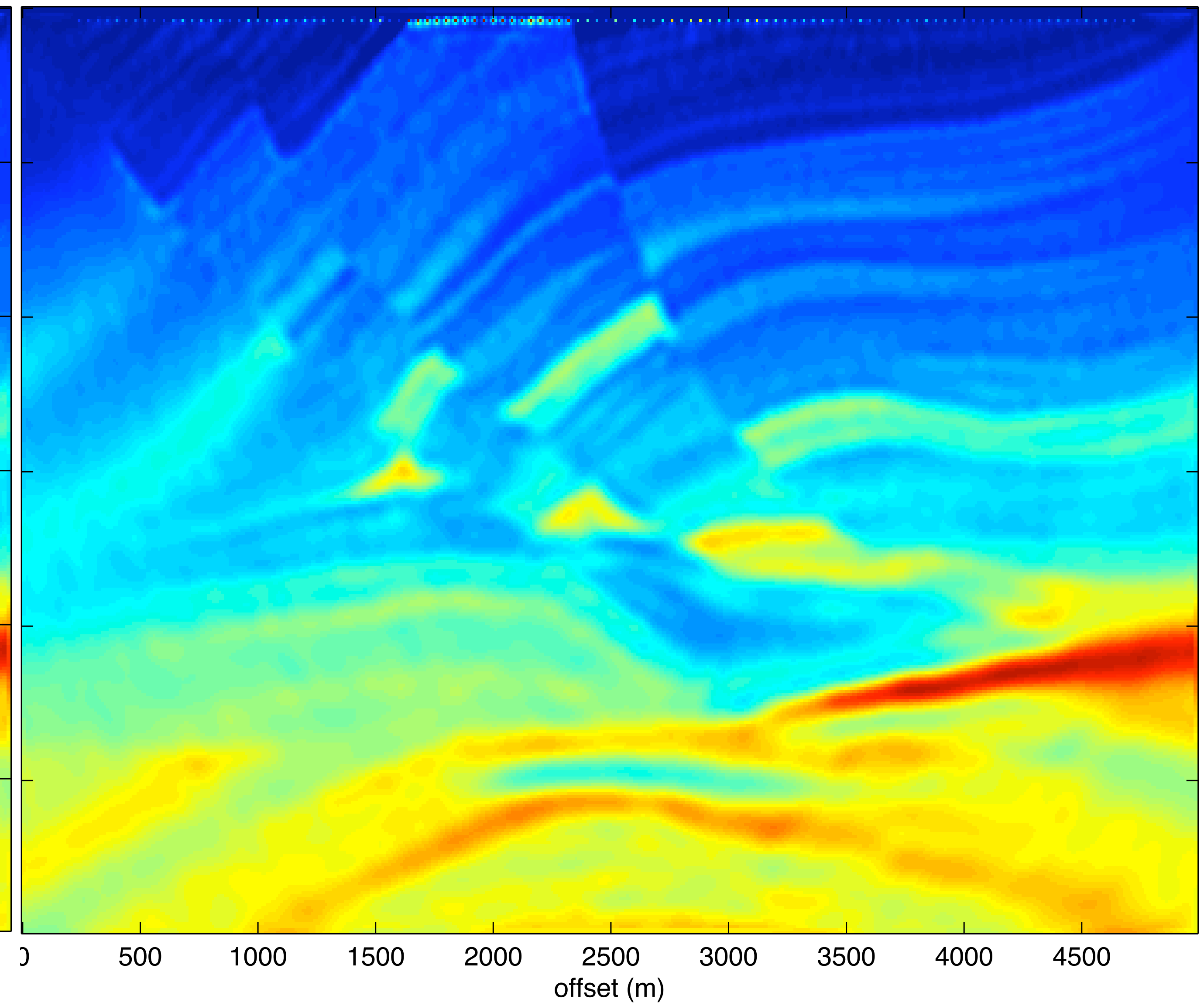
Randomized FWI (Inverted Model)



- iSGD, SNR= 10.85dB
- 16 Randomized simultaneous shots, 4 frequencies, 40 times speed up

Comparison



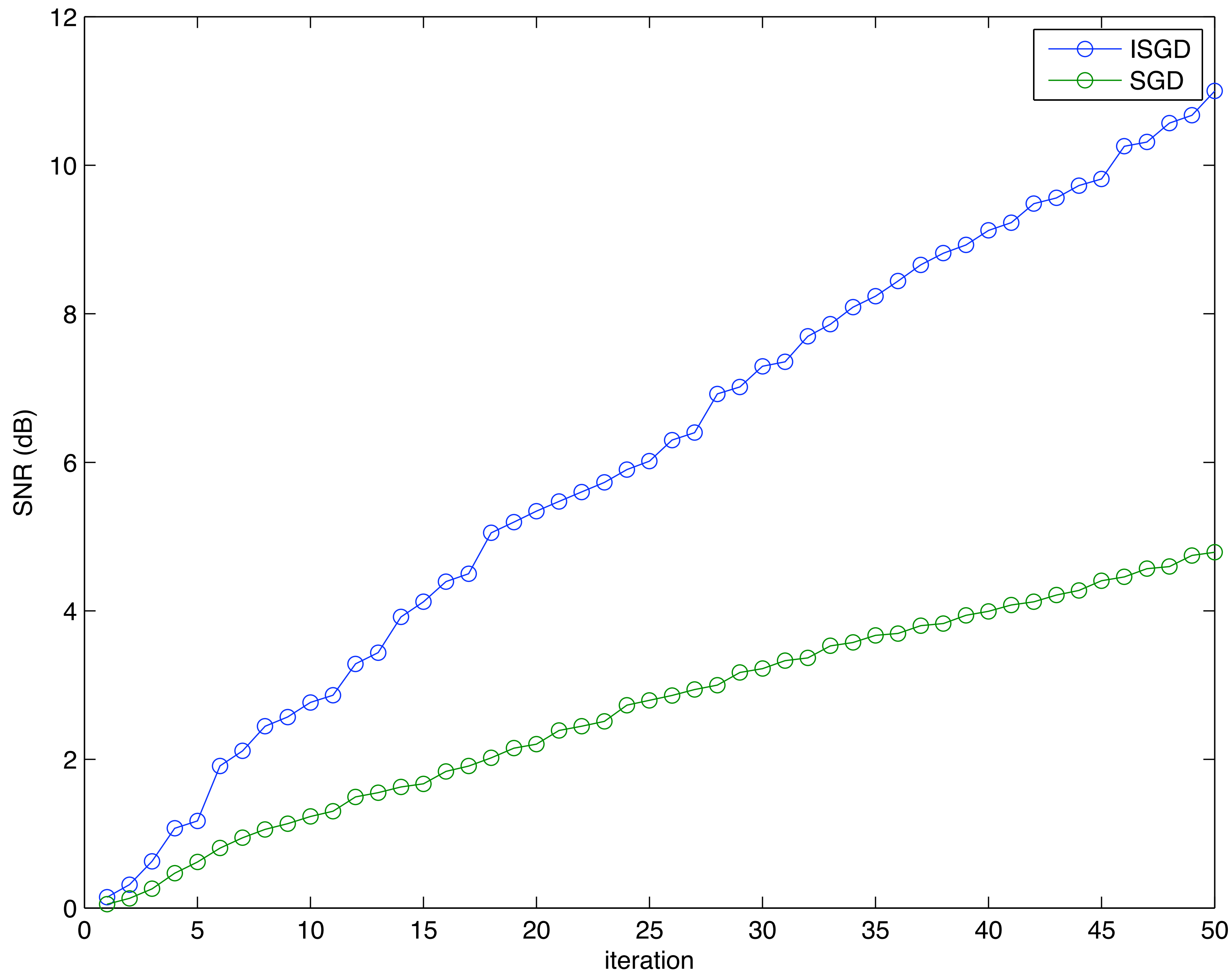


- Inversion for all the shots,
- 1 week on the 32 CPU cluster

- iSGD, SNR= 10.85dB
- 8 hours on the 32 CPU cluster

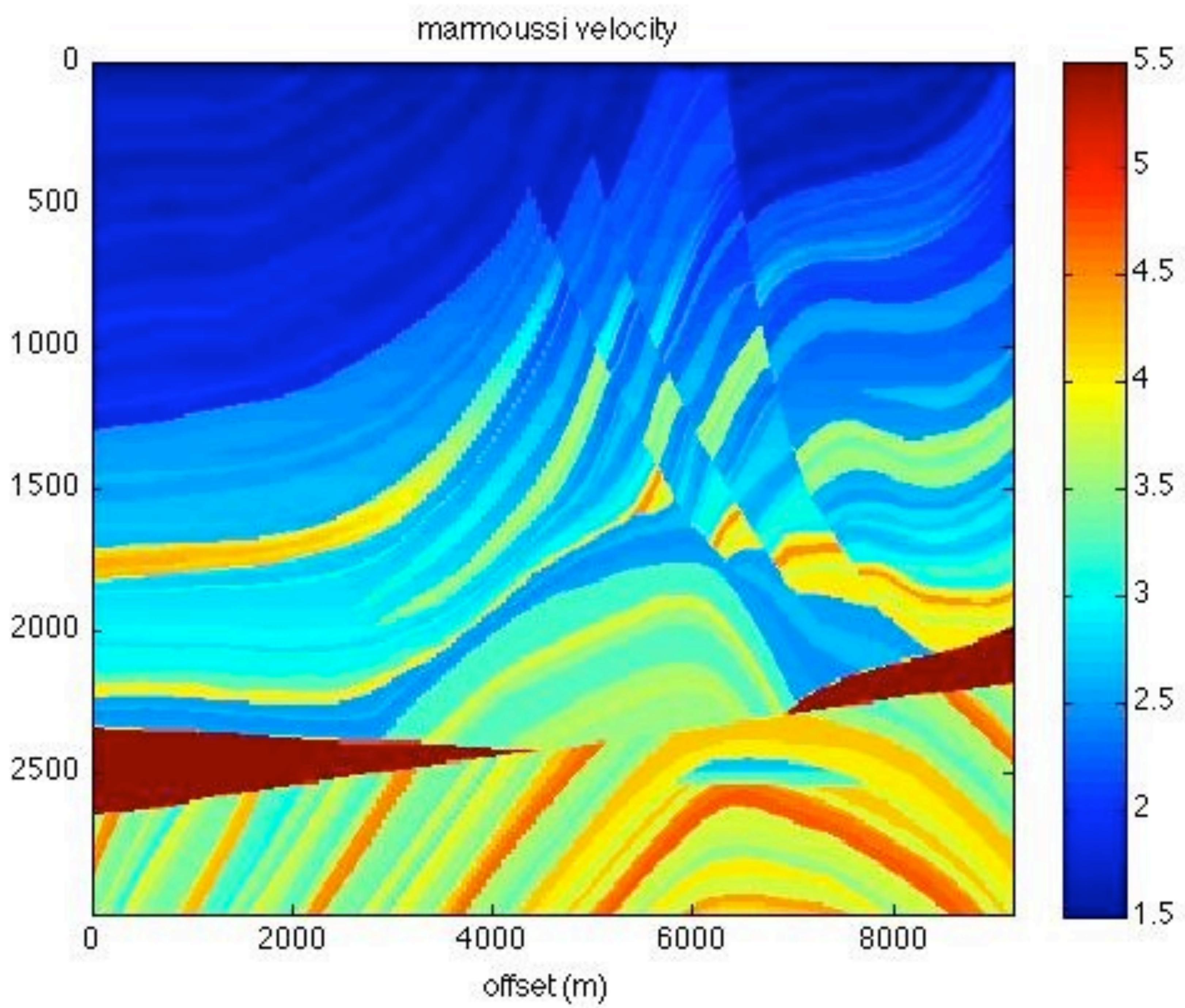
- 16 Randomized simultaneous shots, 4 frequencies, 40 times speed up

Comparison



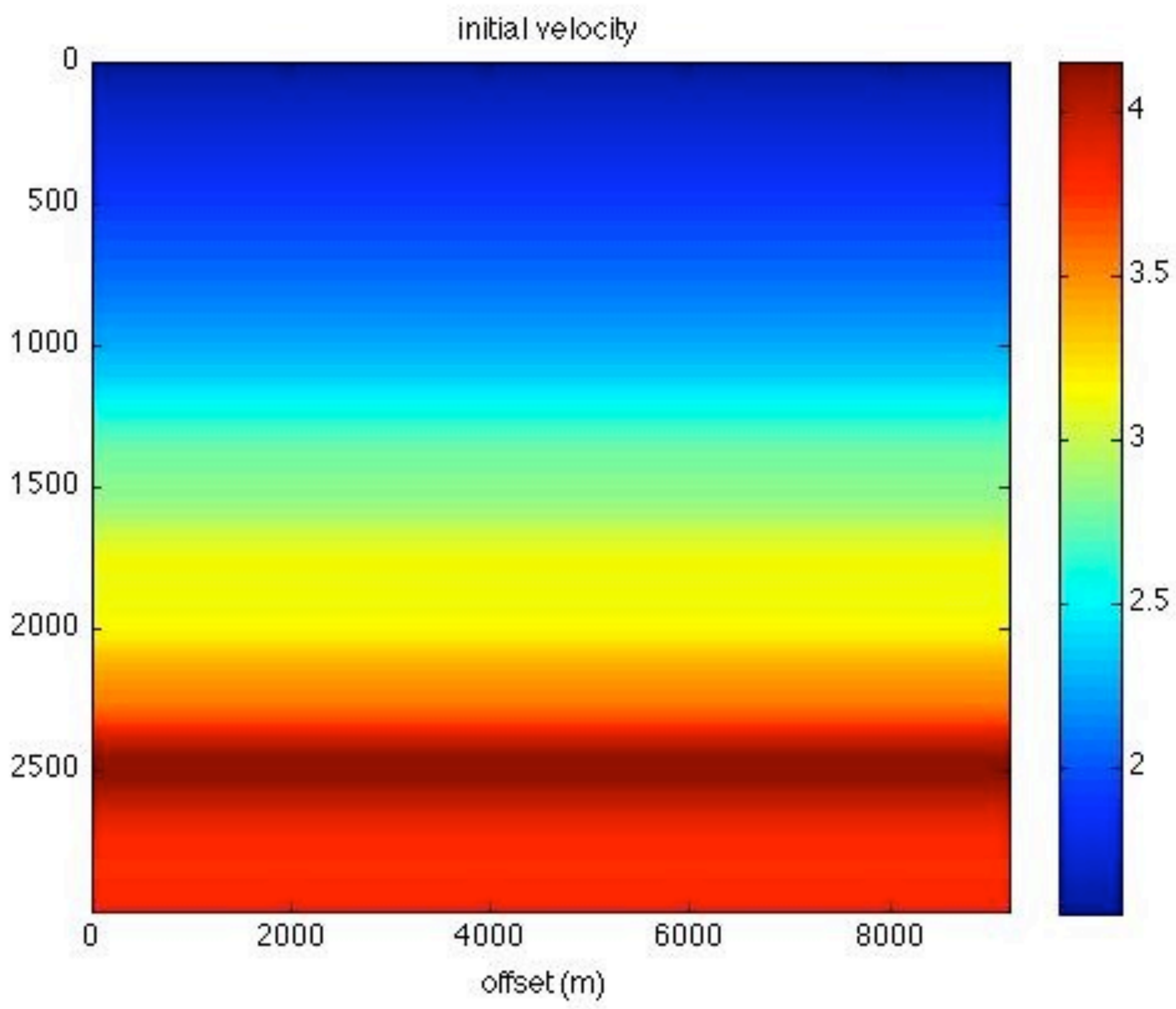
- Comparison between conventional gradient descent and stochastic gradient descent.

Examples (Marmoussi Model)

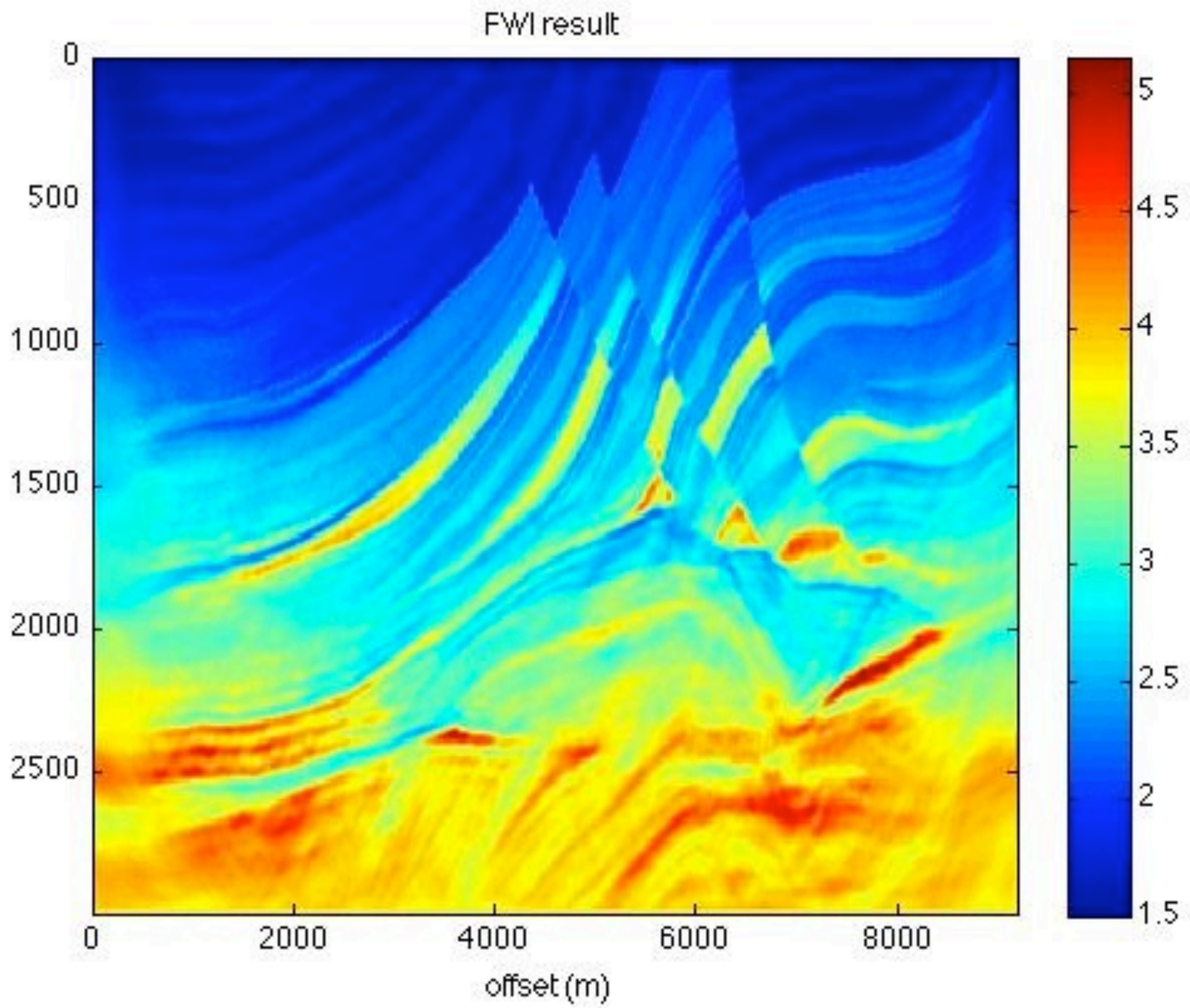


- 900 shots with 10 (m) spacing, 900 receivers with 10 (m) spacing, WAZ survey with 5 (km) max. aperture, Ricker source with 10 Hz central frequency, 3.6 second recording time with .9 (ms) time sampling.

Examples (Initial Model)



Randomized FWI (Inverted Model)



- iSGD method, 1 Randomized simultaneous shots, 900 times speed up!

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Conclusion

- Super-shot experiment combined with stochastic optimization methods produce promising results for solution for FWI
- Randomized FWI greatly increases the performance of the FWI.
- Dimensionality reduction algorithms, open possibility of replacing migration with FWI with no extra cost.

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Future Plans

- Further investigation on the choice of random frequency and super-shot
- Stochastic optimization strategies for FWI, improved iLBFGS, Natural gradient
- Regularization for the FWI
- Solving the uniqueness problem, exploiting the multi-scale nature of the FWI

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