SINBAD Consortium 2010

Recovery of compressively sampled signals using partial support information

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SLIM
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Collaborators

Joint work with:

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- Özgür Yılmaz
- Michael Friedlander

Outline

Part 1: Introduction and Overview

Part 2: Stability and Robustness of Weighted ℓ_1 Minimization

Part 3: Experimental Results

Motivation

- We address the problem of reconstructing a seismogram Z from incomplete measurements Y.
- Examples where incomplete measurements arise:
  - Dimensionality reduction of extremely high resolution seismic data (HP and Shell sensing system).
  - Simultaneous source/receiver mixing, source/receiver malfunctioning.

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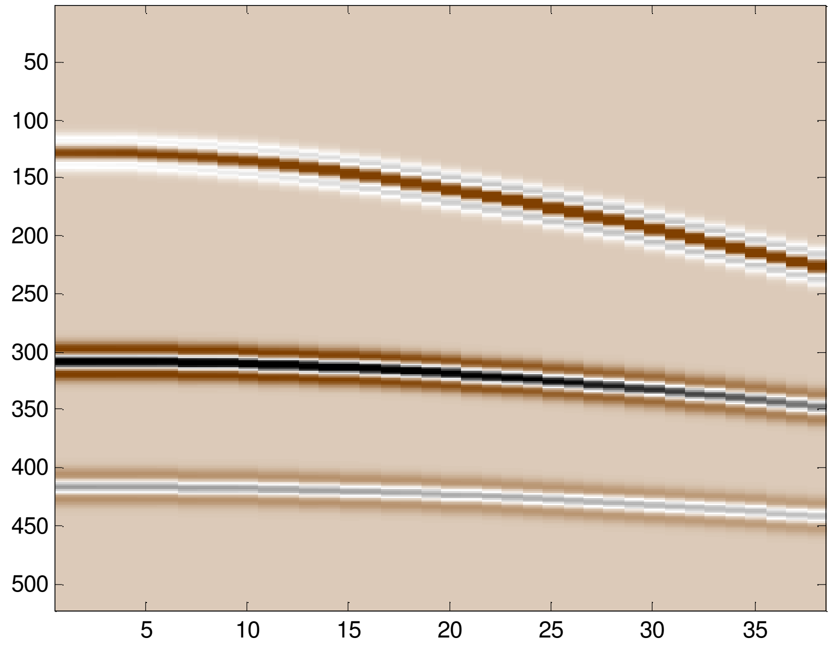

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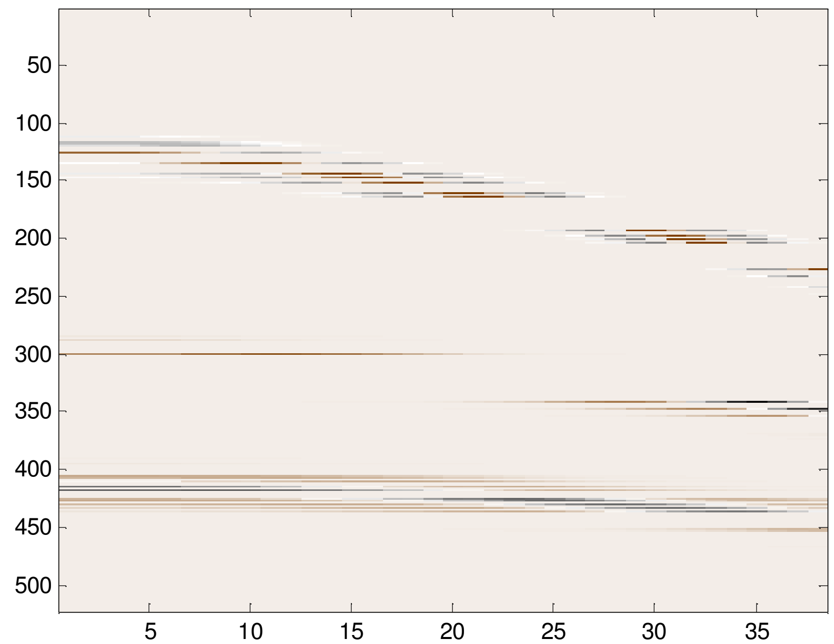

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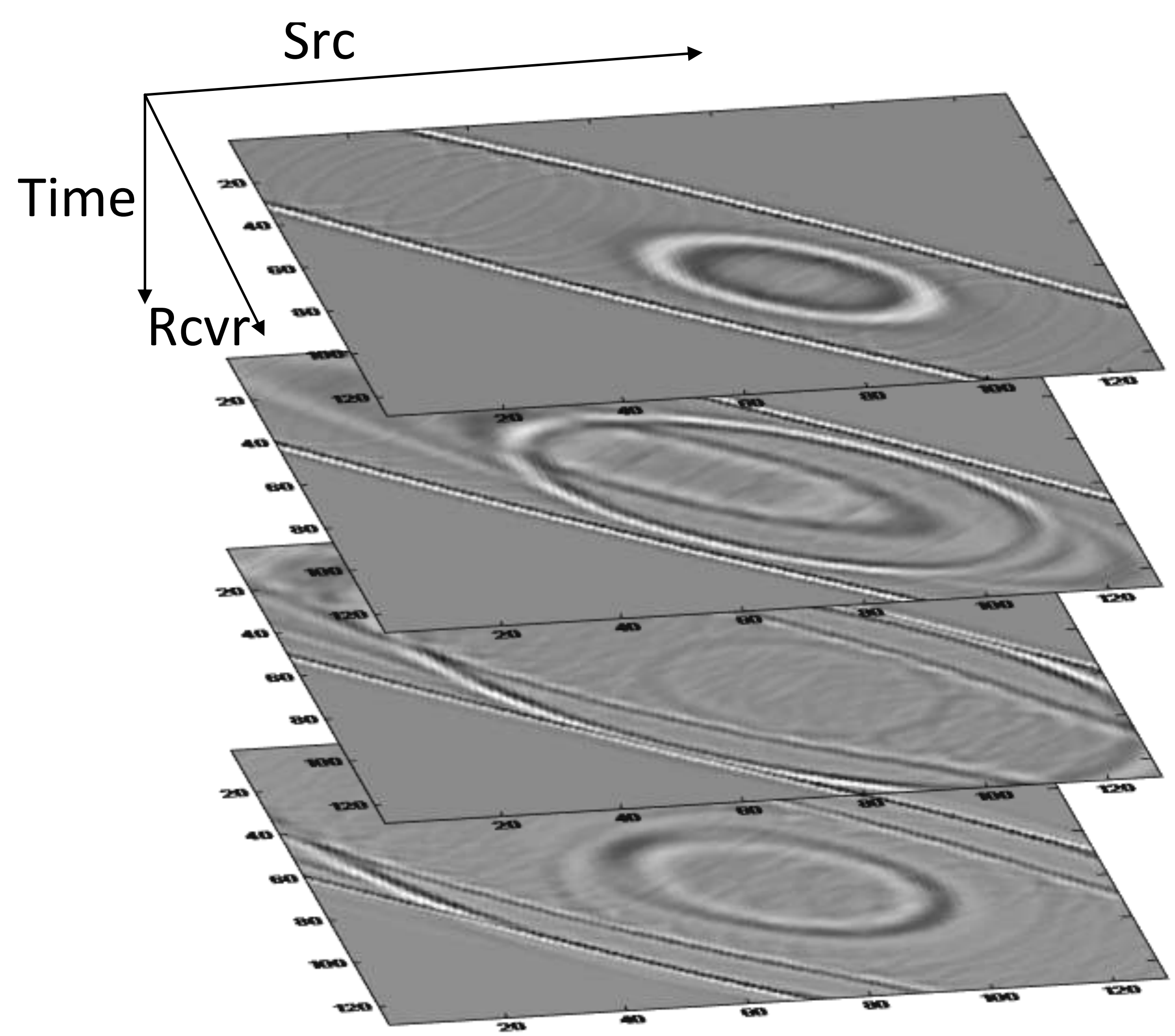

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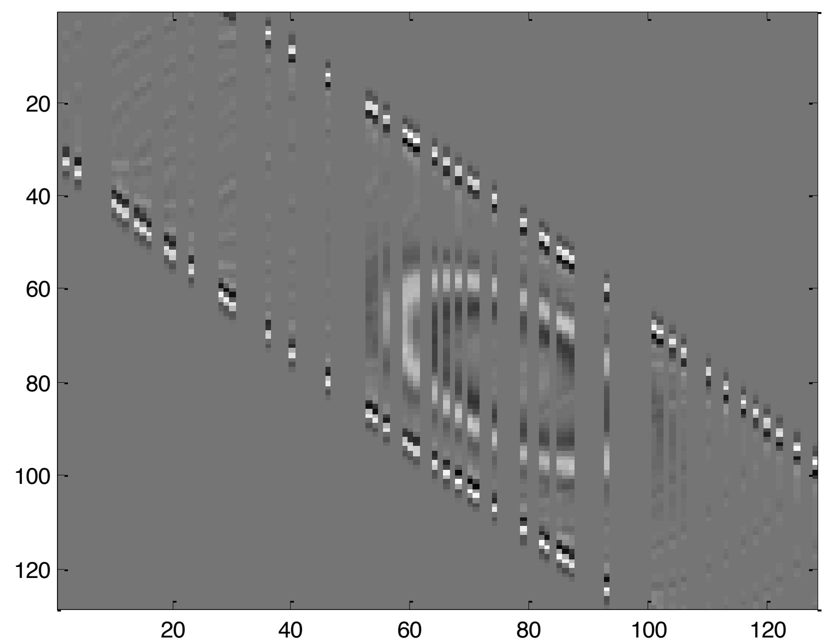

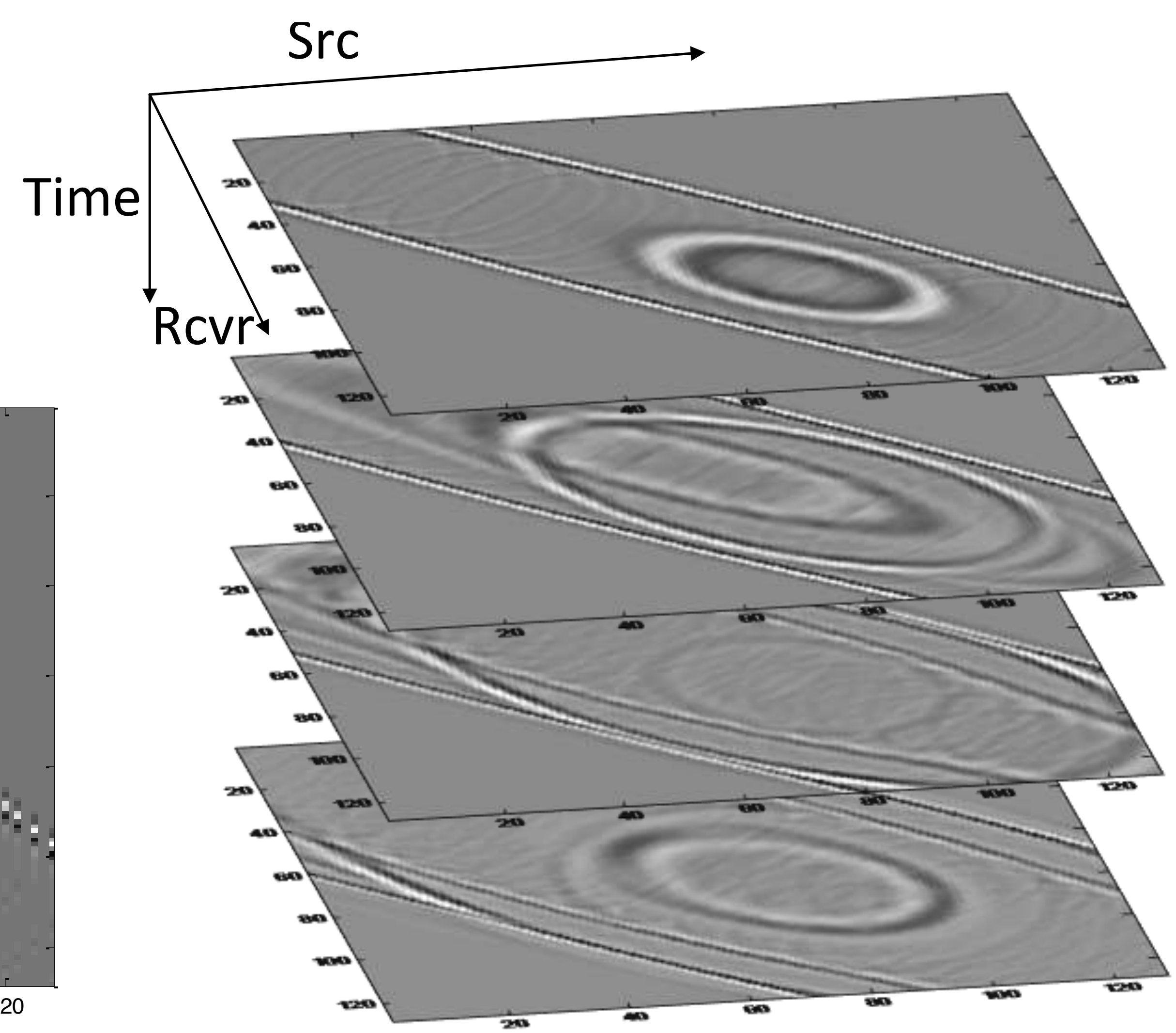

Observations on Sparsity

- Every column/slice z_i of Z admits a (nearly) sparse representation in some transform domain.
- The time axis of the common-receiver gathers are sparse in the wavelet domain.
- Every source receiver slice is sparse in the curvelet domain.
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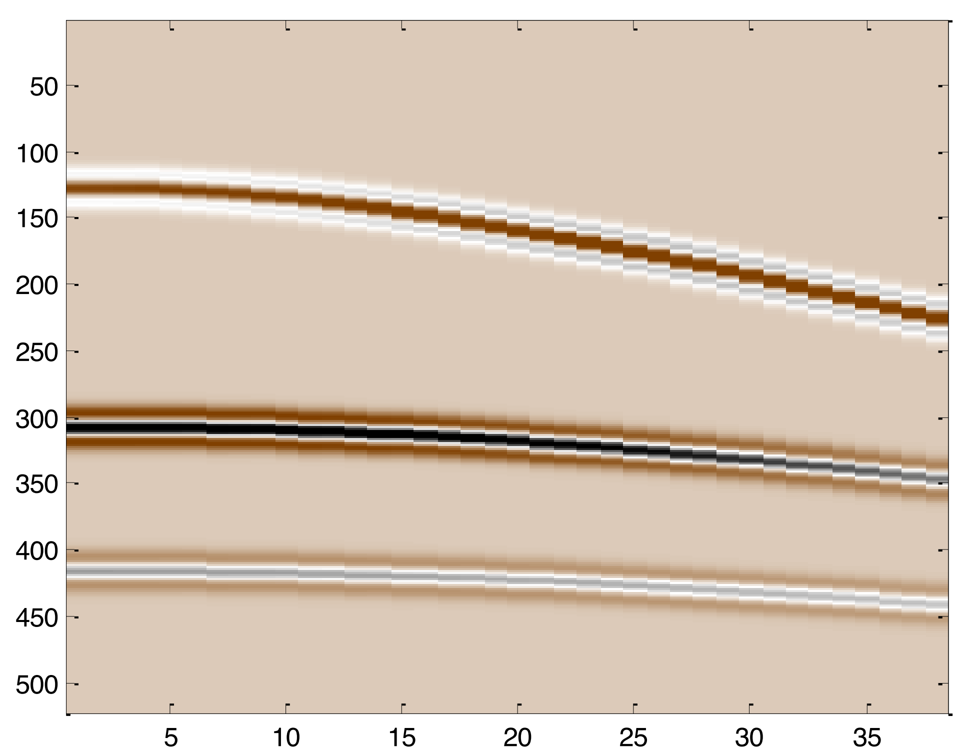

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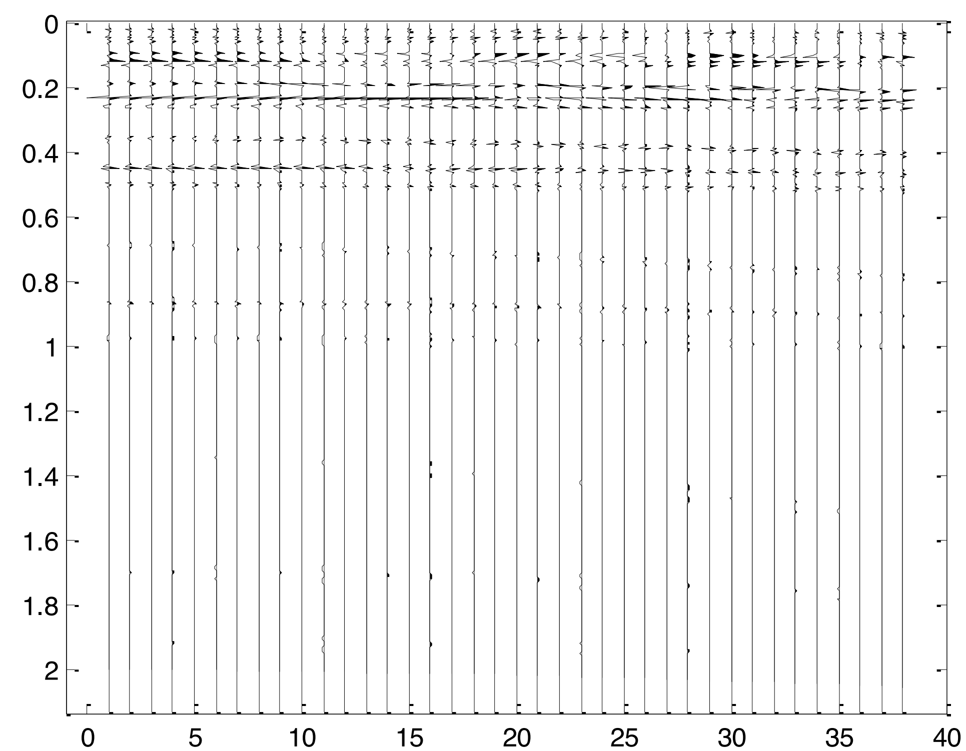

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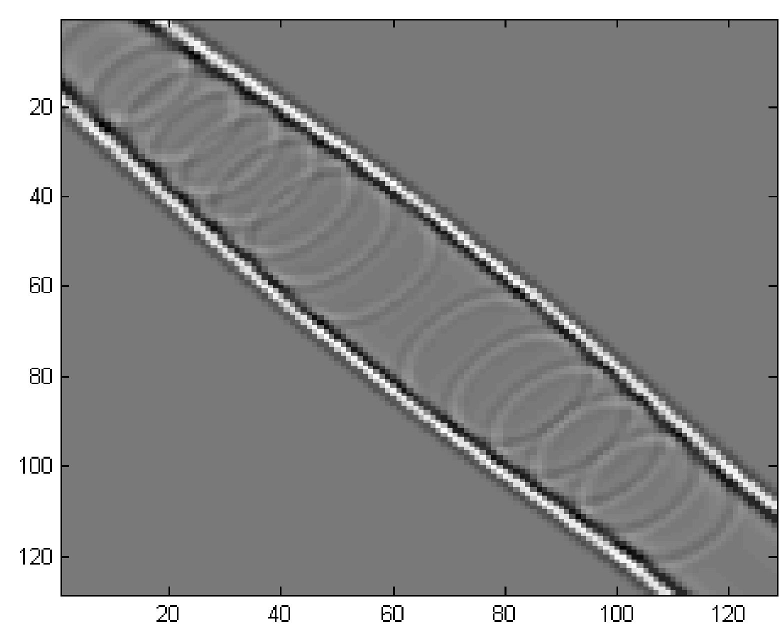

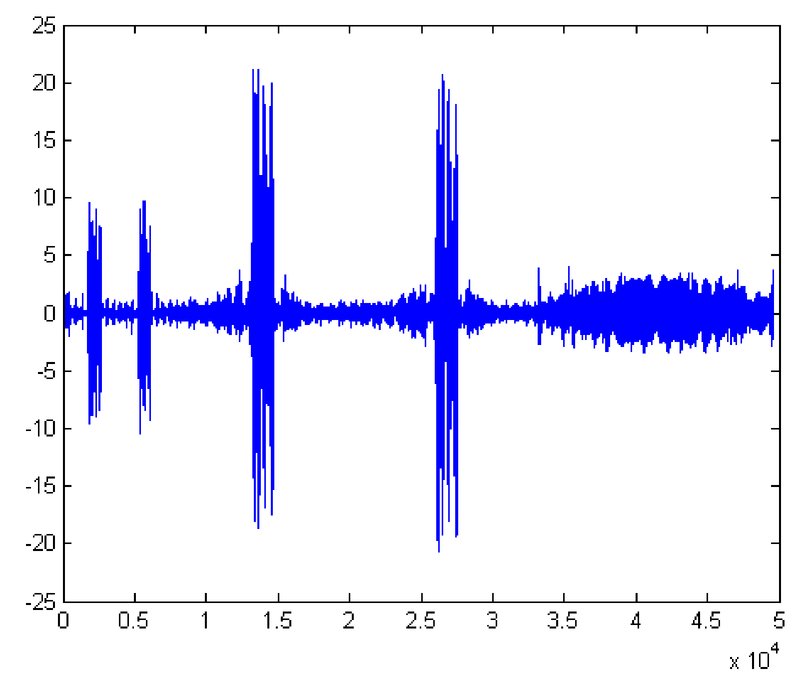

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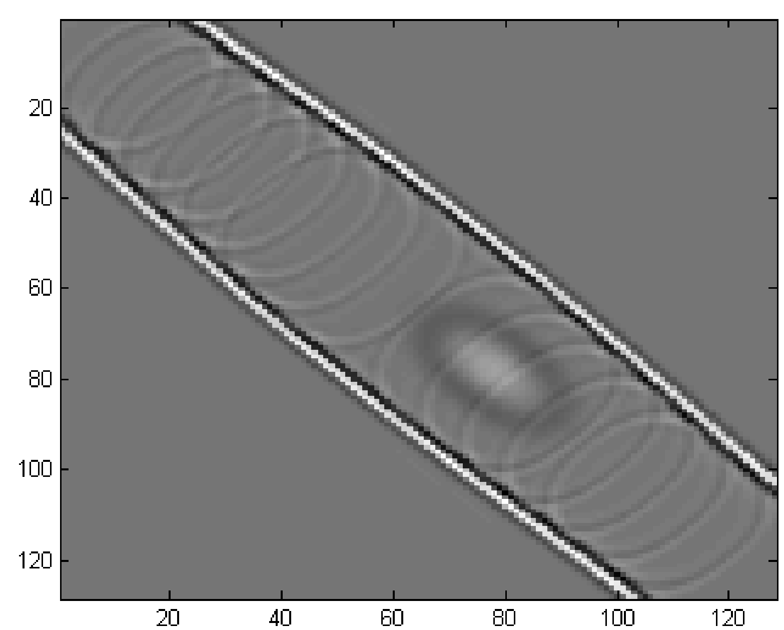

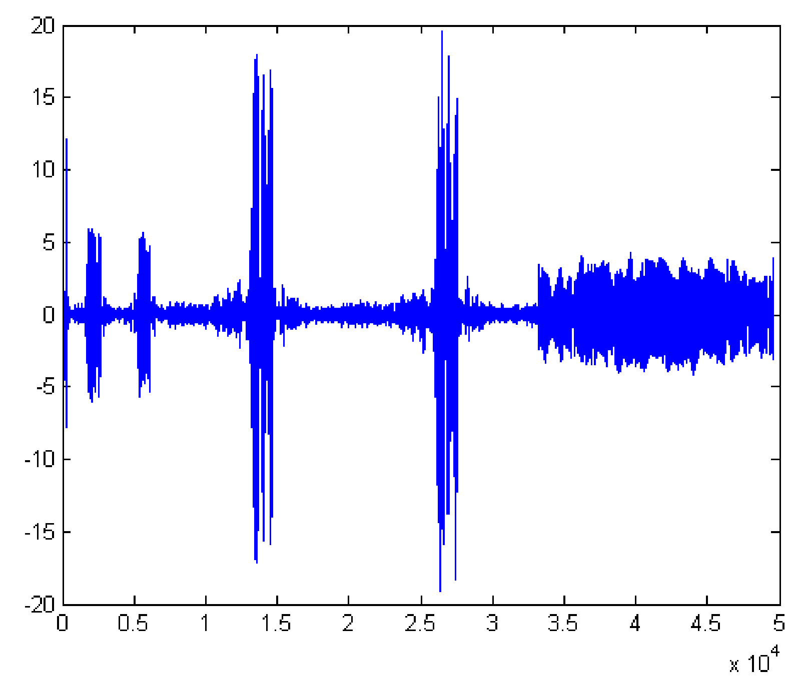

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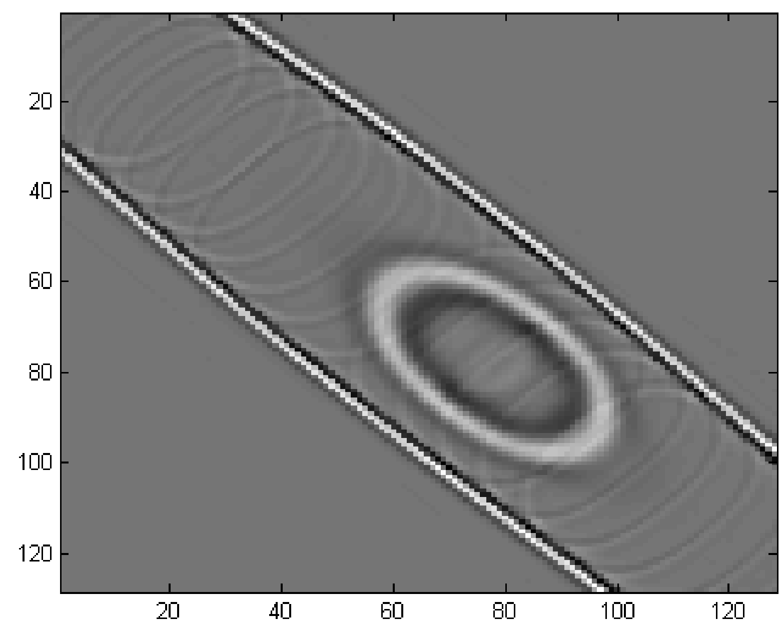

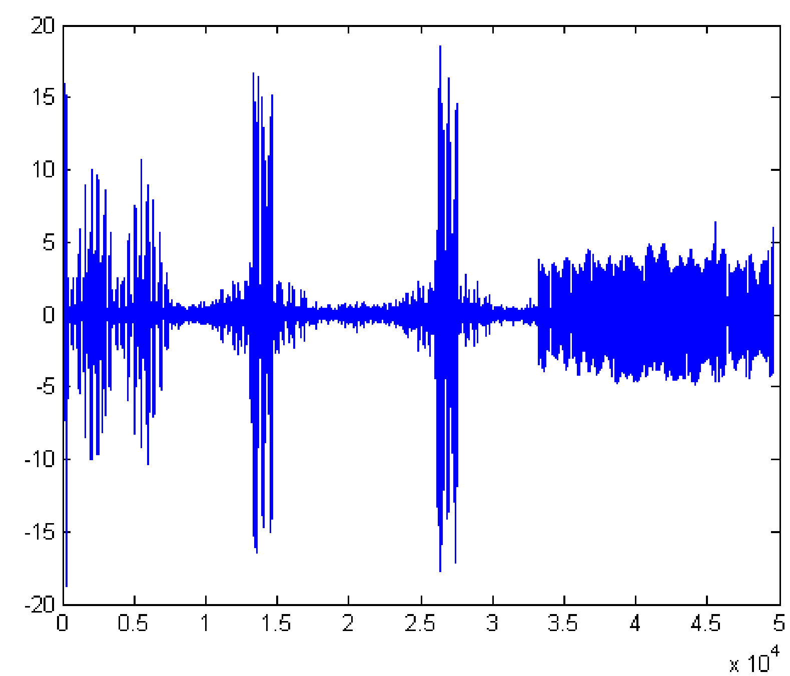

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Compressed Sensing

- *Compressed Sensing* is an acquisition paradigm for signals that admit *sparse* or nearly sparse representations in some transform domain.
- Consider a signal $z \in \mathbb{R}^N$, $z = Dx$, where D is a transform matrix and x is a k-sparse coefficient vector.
- Given $n \ll N$ linear and noisy measurements $y = \Psi Dx + e$.
- Let $A = \Psi D$, it is possible to approximate x from the measurements y if
  - A obeys certain conditions
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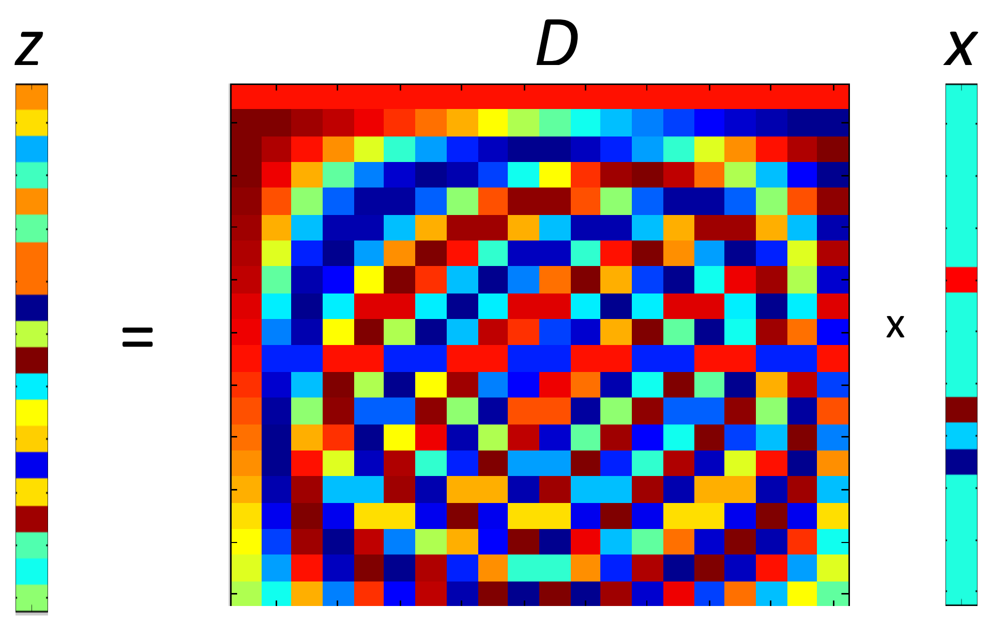

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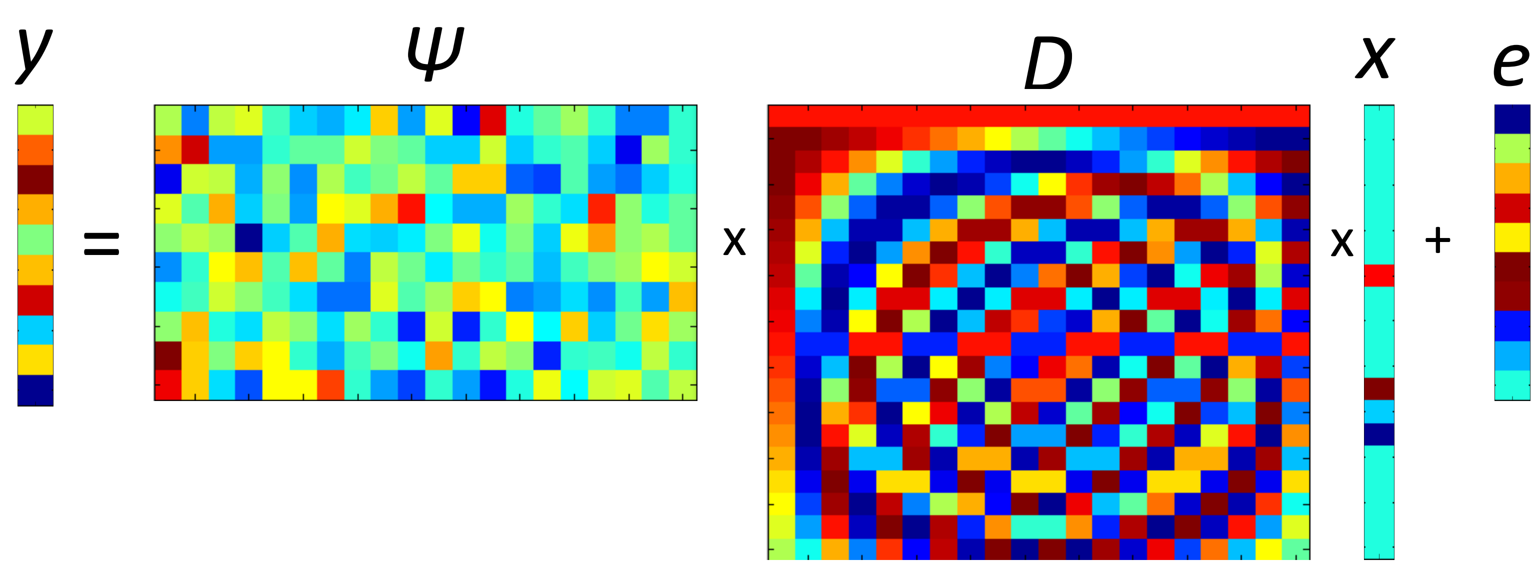

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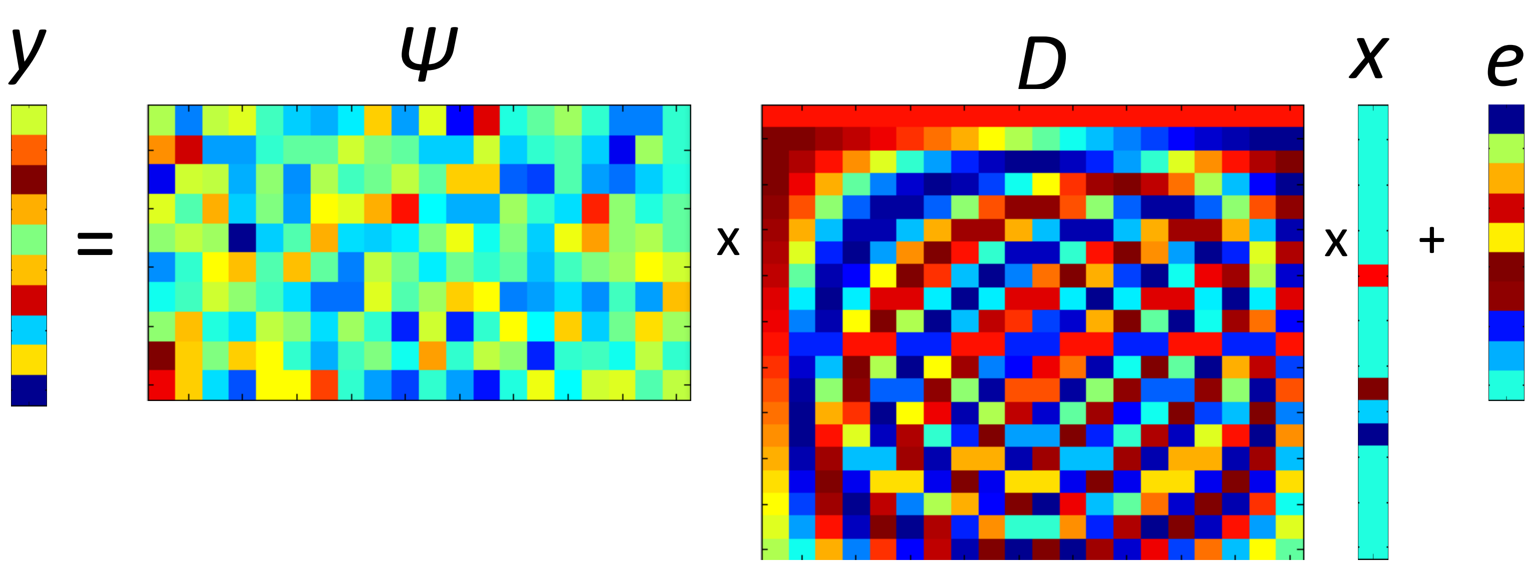

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Definition: Restricted Isometry Property (RIP)

The RIP constant δ_k is defined as the smallest constant such that $\forall x \in \Sigma_k^N$

$$(1 - \delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_k)\|x\|_2^2$$

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Recovery Algorithms (optimization)

- $\min_{\tilde{x} \in \mathbb{R}^N} \|\tilde{x}\|_0 \quad \text{subject to } \|Ax - y\|_2 \leq \|e\|_2, \qquad k < n/2$
- $\min_{\tilde{x} \in \mathbb{R}^N} \|\tilde{x}\|_1 \quad \text{subject to } \|Ax - y\|_2 \leq \|e\|_2, \qquad k \lesssim n/\log(N/n)$

Stability and Robustness

- Candés, Romberg, and Tao, and Donoho showed that ℓ_1 minimization

$$\min_{\tilde{x}\in\mathbb{R}^N} \|\tilde{x}\|_1 \quad \text{subject to } \|A\tilde{x} - y\|_2 \leq \|e\|_2$$

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- If the matrix A has $\delta_{(a+1)k} < \frac{a-1}{a+1}$, then x can be recovered with the approximation error:

$$\|x^* - x\|_2 \leq C_0\epsilon + C_1 k^{-1/2}\|x - x_k\|_1.$$

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- But, the ℓ_1 minimization formulation is non-adaptive, i.e., no prior information on x is used in the recovery.

Prior Information in Seismic Imaging

- The columns of NMO corrected common-receiver gathers are jointly sparse in the wavelet domain.
- Typically in seismic lines, the measurement matrix is a Kronecker between 2D-curvelet and 1D-wavelet transforms.
- The curvelet coefficients of source receiver slices are highly correlated in time.

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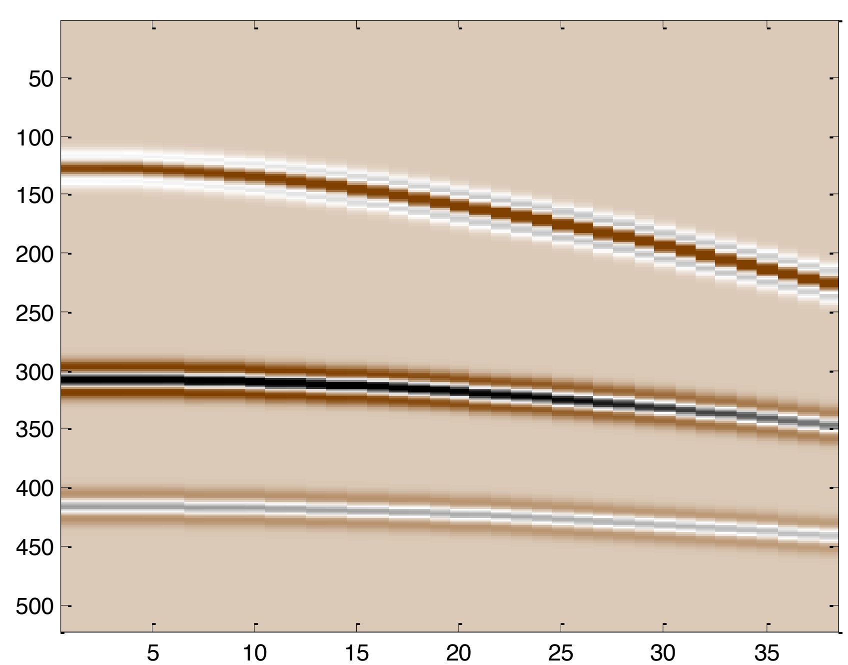

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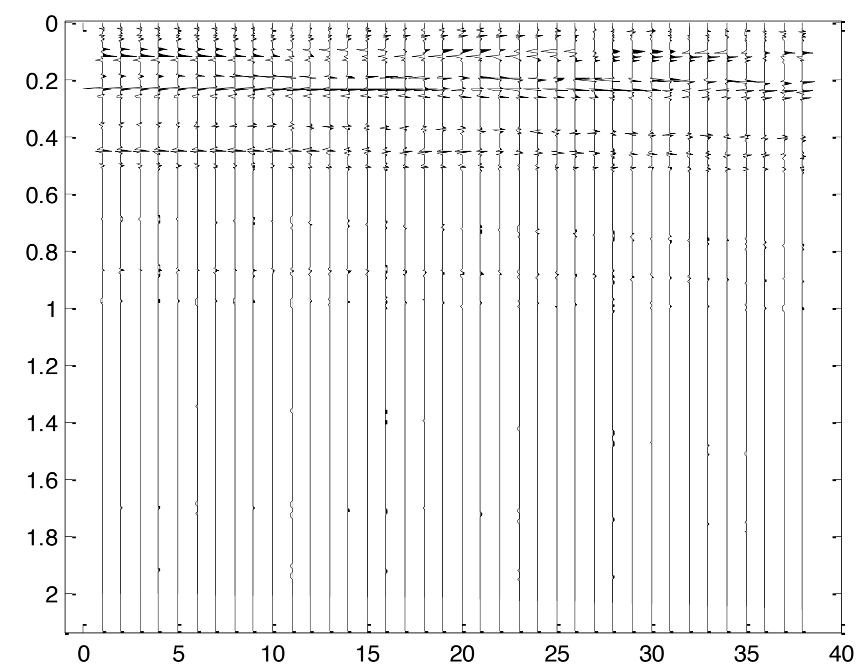

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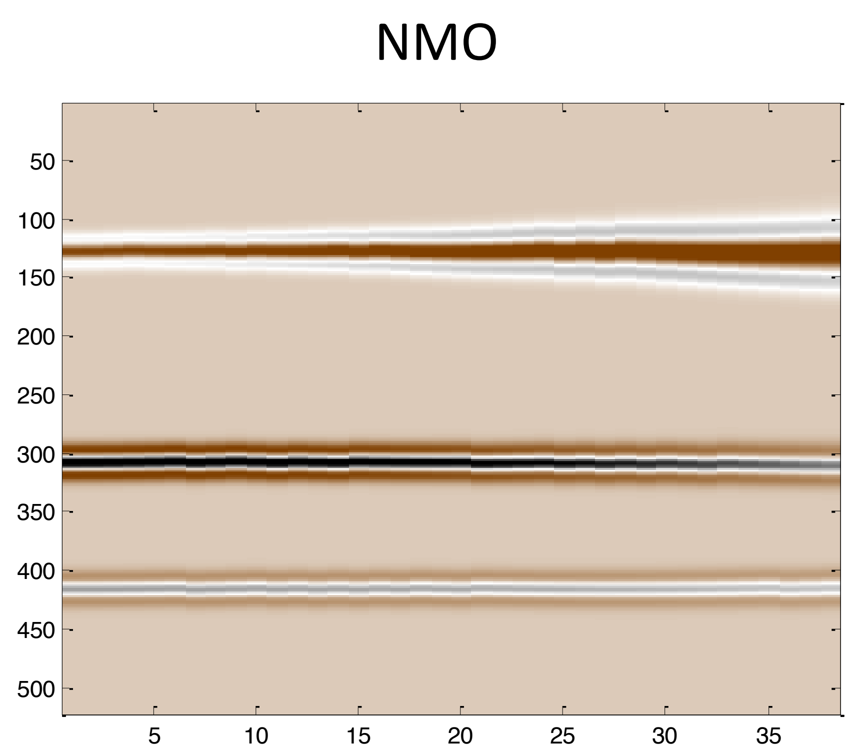

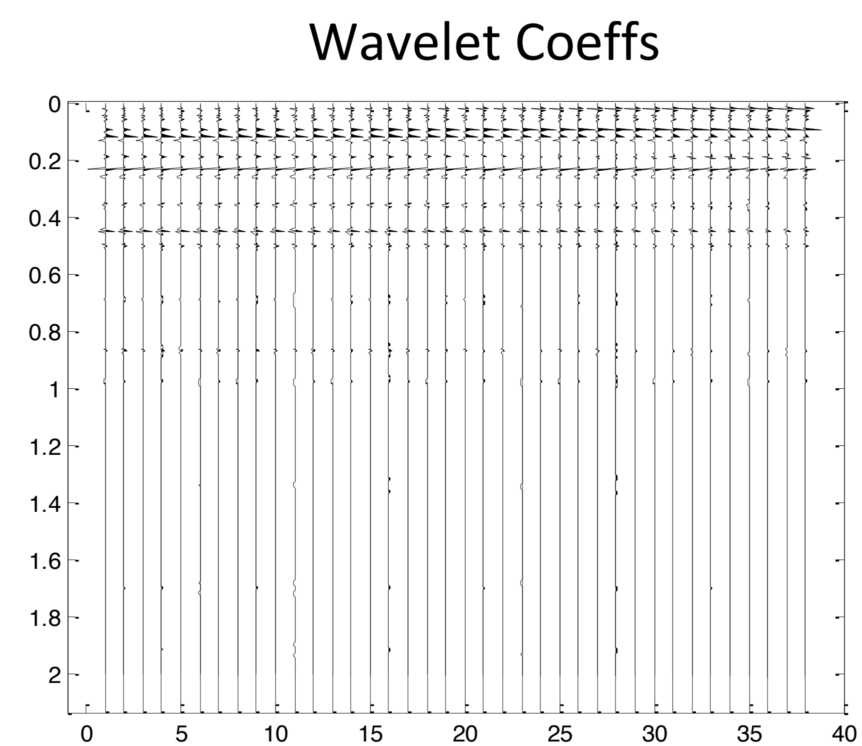

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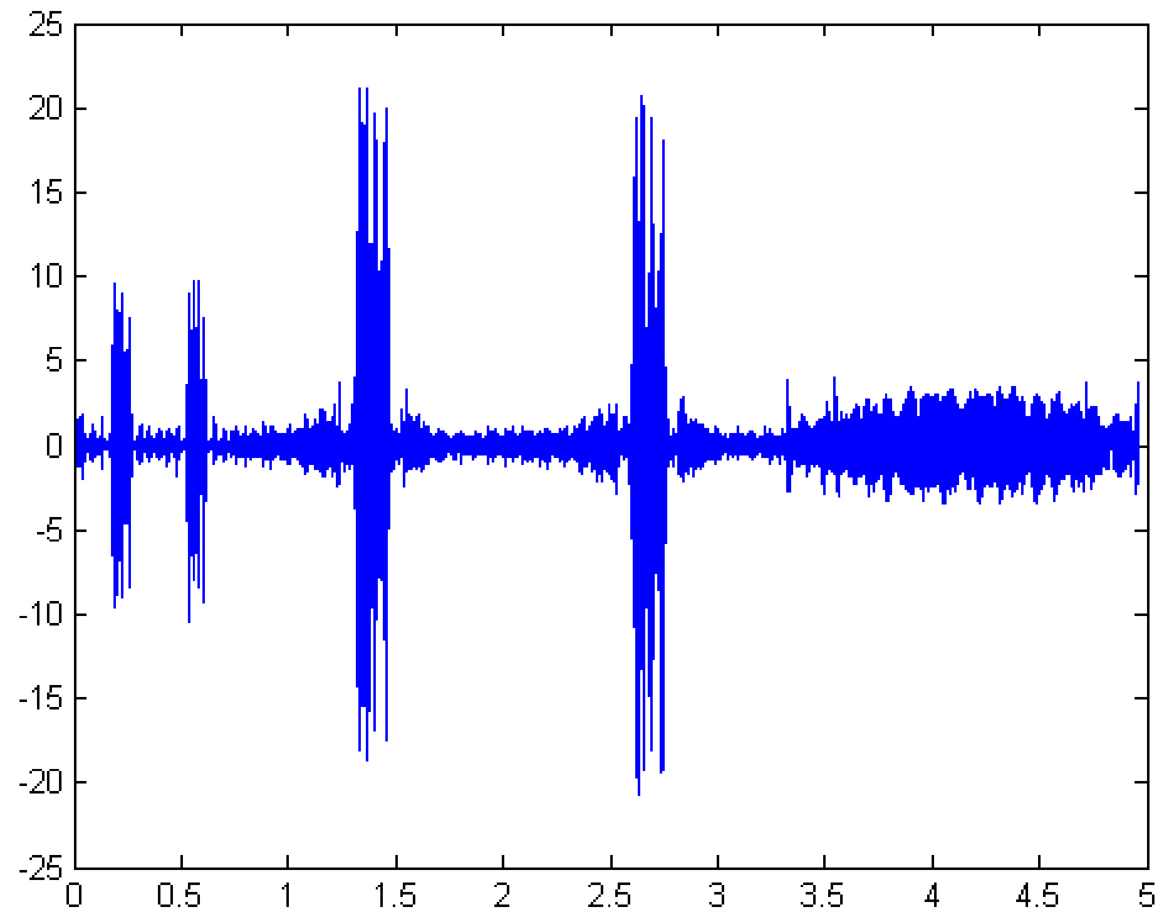

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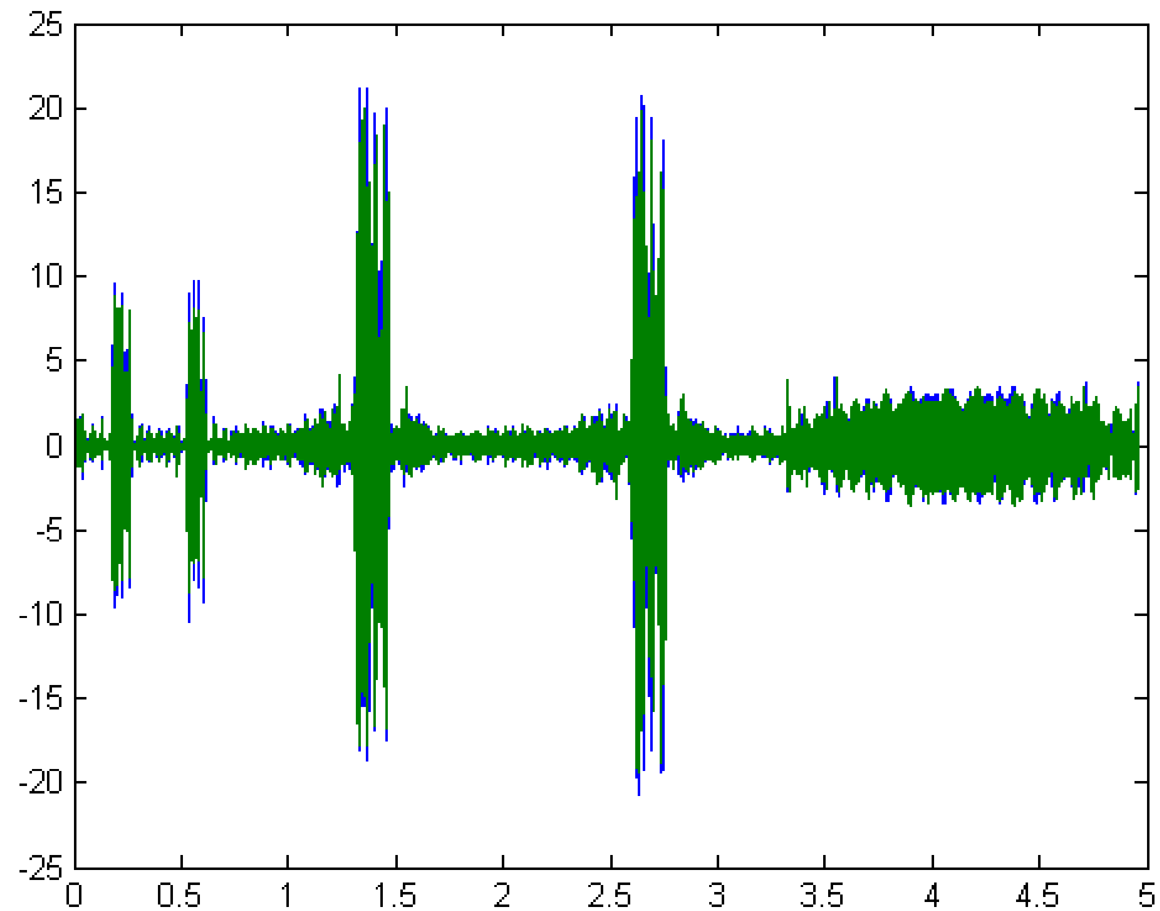

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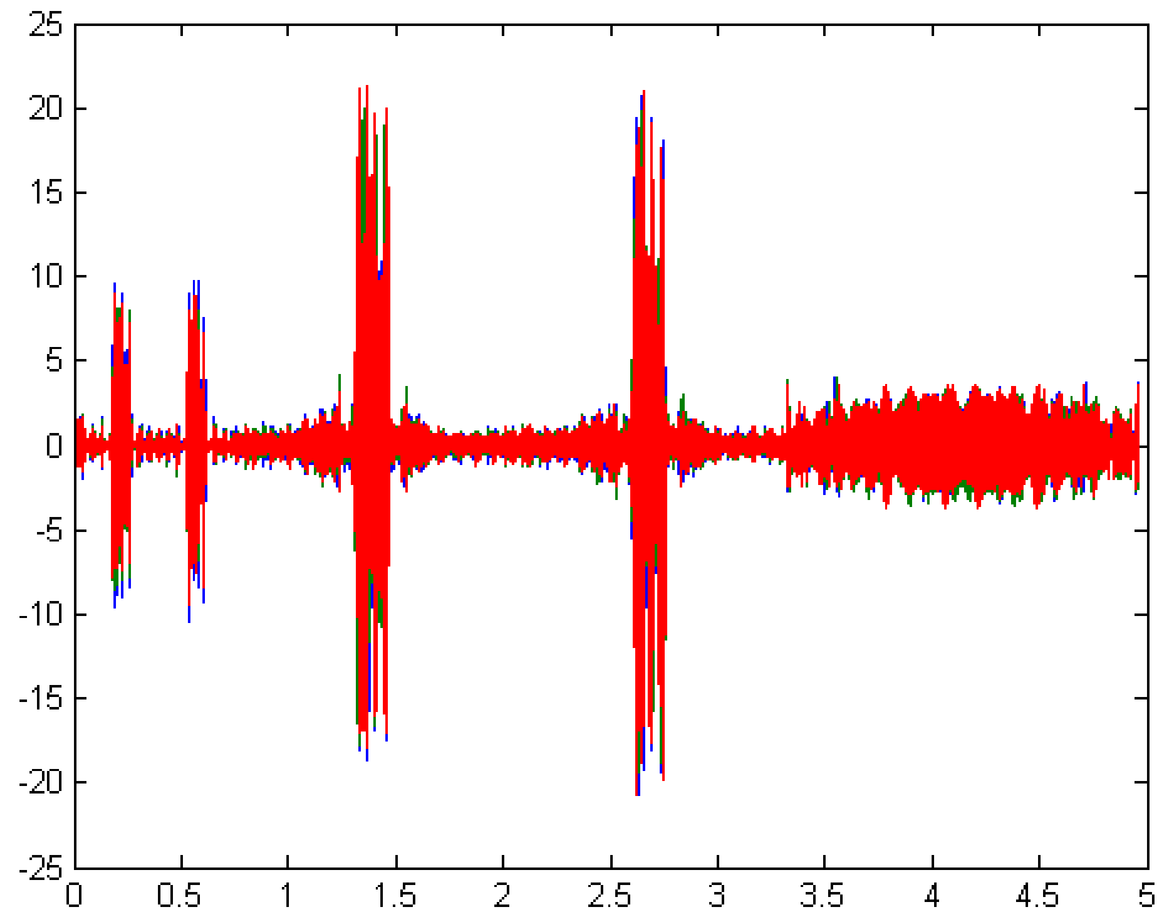

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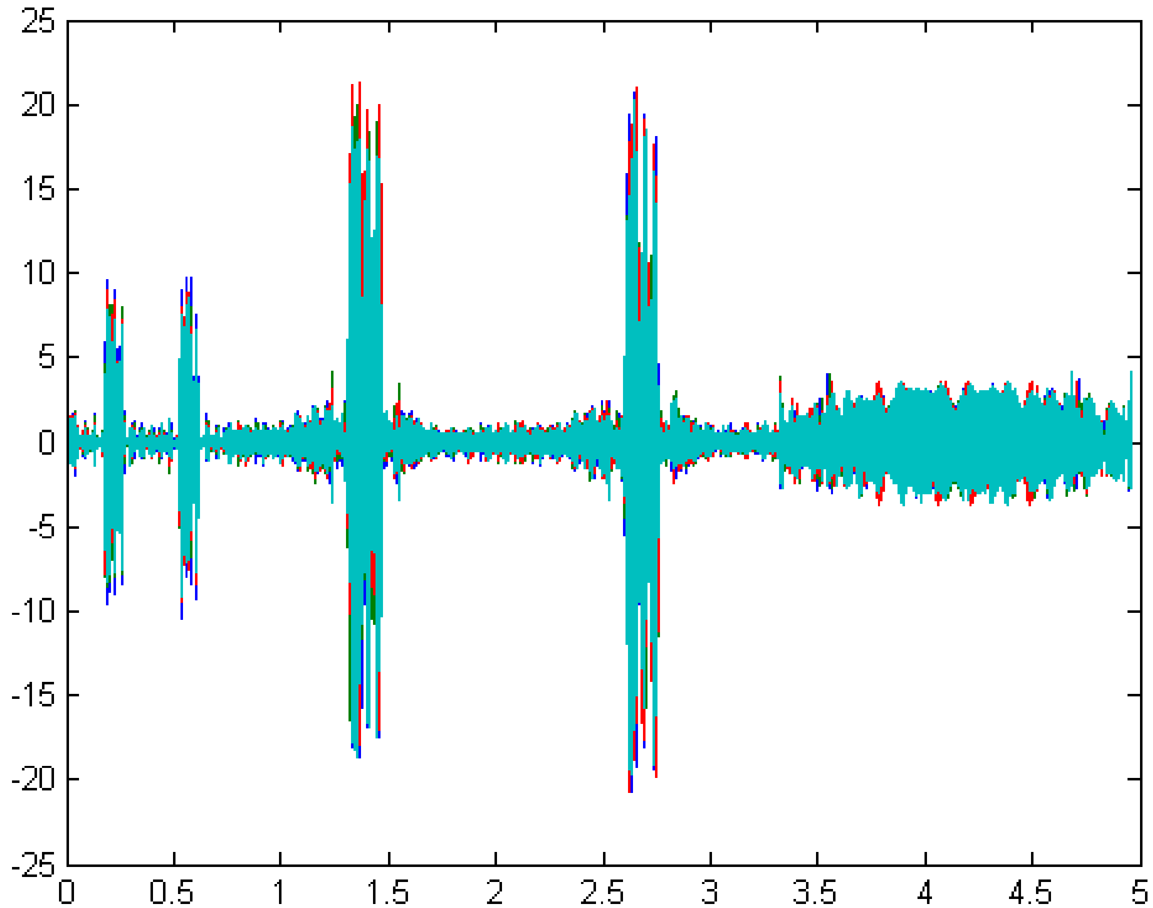

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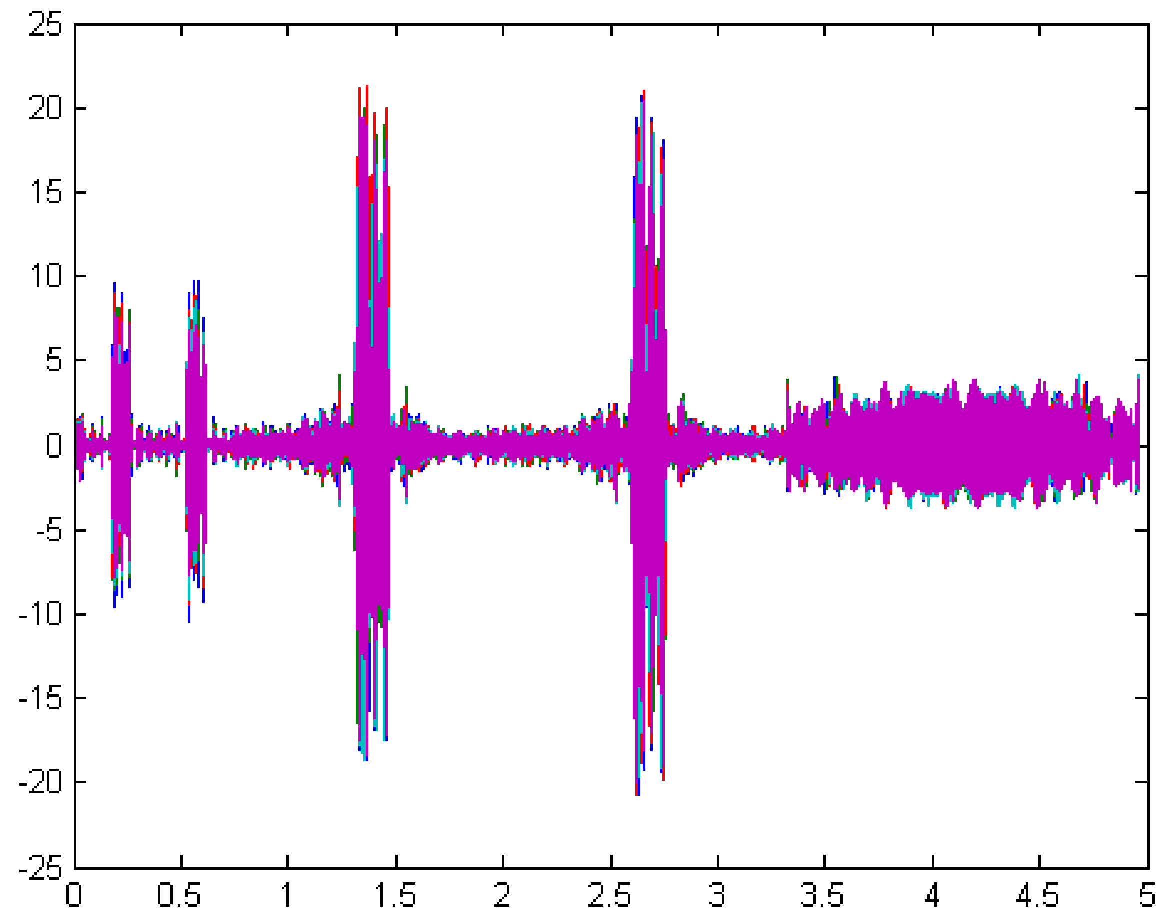

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- The curvelet coefficients of source receiver slices are highly correlated in time.
- How do we “bias” the recovery algorithm to use the prior information while keeping the measurement process nonadaptive?

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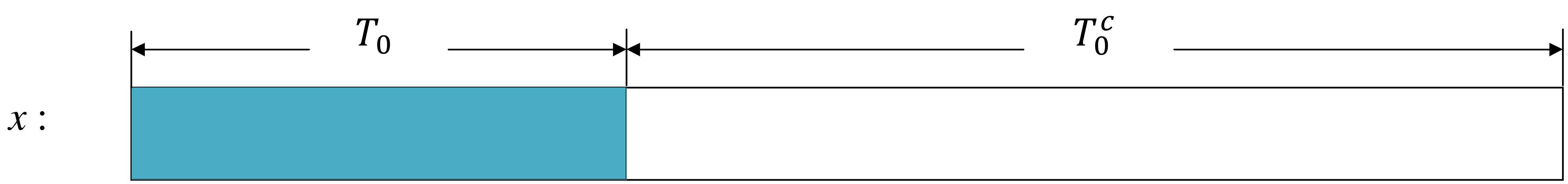
Part 2: Stability and Robustness of Weighted ℓ_1 Minimization

Part 3: Experimental Results

Problem Setup

- Suppose that x is a k-sparse signal supported on an unknown set T_0.
- Let $\widetilde{T}$ be a known support estimate that is partially accurate.
- We want to:
  1. Recover x by incorporating $\widetilde{T}$ in the recovery algorithm.
  2. Obtain recovery guarantees based on the size and accuracy of $\widetilde{T}$.
- Our approach: weighted ℓ_1 minimization.



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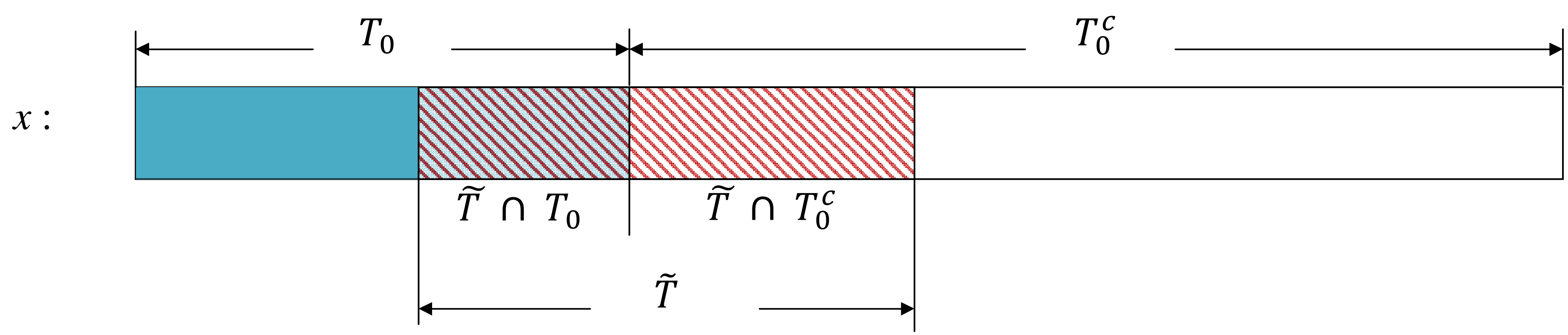

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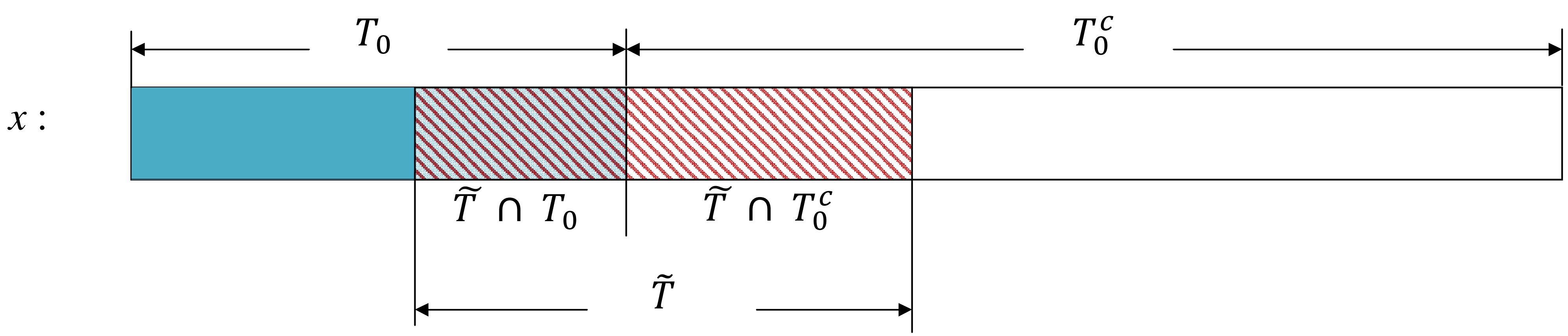

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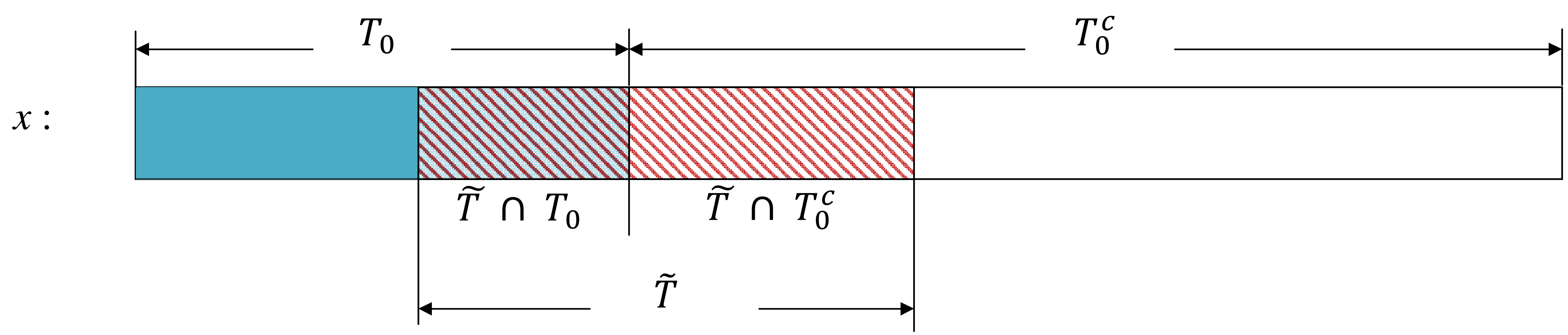
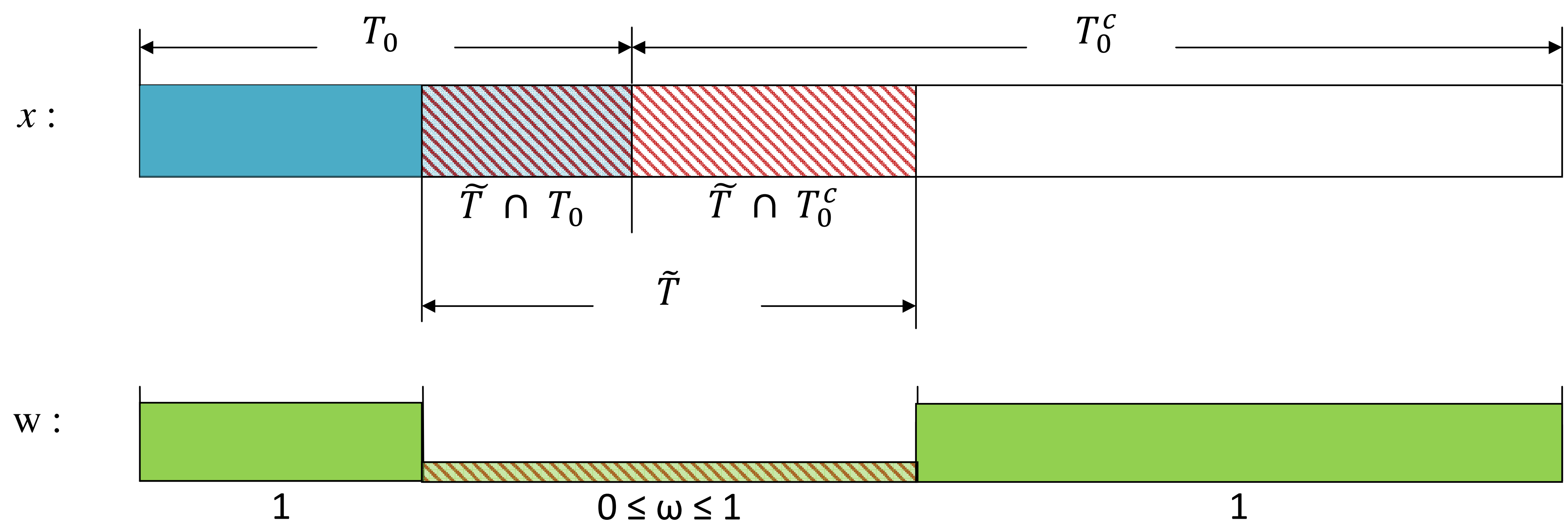


Weighted ℓ_1 Minimization

Given a set of measurements y, solve

$$\min_x \ \|x\|_{1,\mathrm{w}} \text{ subject to } \|Ax-y\|_2 \le \epsilon \quad \text{with} \quad w_i = \begin{cases} 1, & i \in \widetilde{T}^c, \\ \omega, & i \in \widetilde{T}. \end{cases}$$

where $0 \le \omega \le 1$ and $\|x\|_{1,\mathrm{w}} := \sum_i \mathrm{w}_i |x_i|$.



Weighted ℓ_1 Minimization in Seismic Imaging

- Let $Y = \Phi Z$, where Z is the common-receiver gather.
- Vectorize the system to make the NMO operator linear.
- Solve the ℓ_1 minimization problem using the support of x as the estimate set $\widetilde{T}$.

Φ Z

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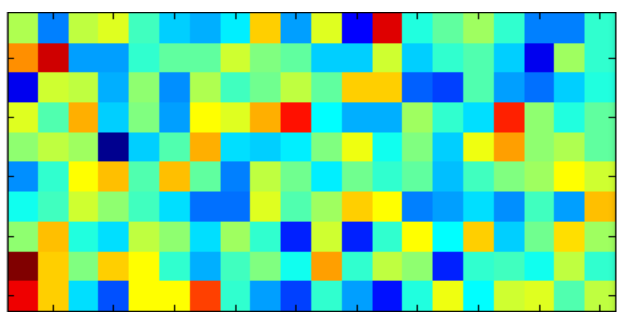

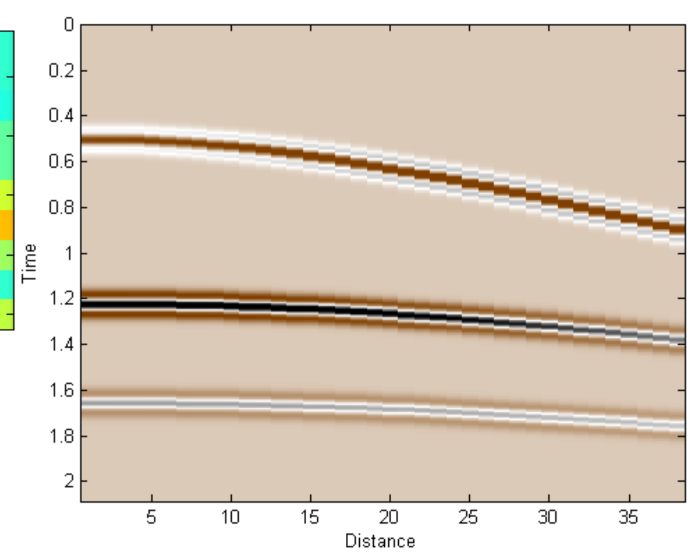

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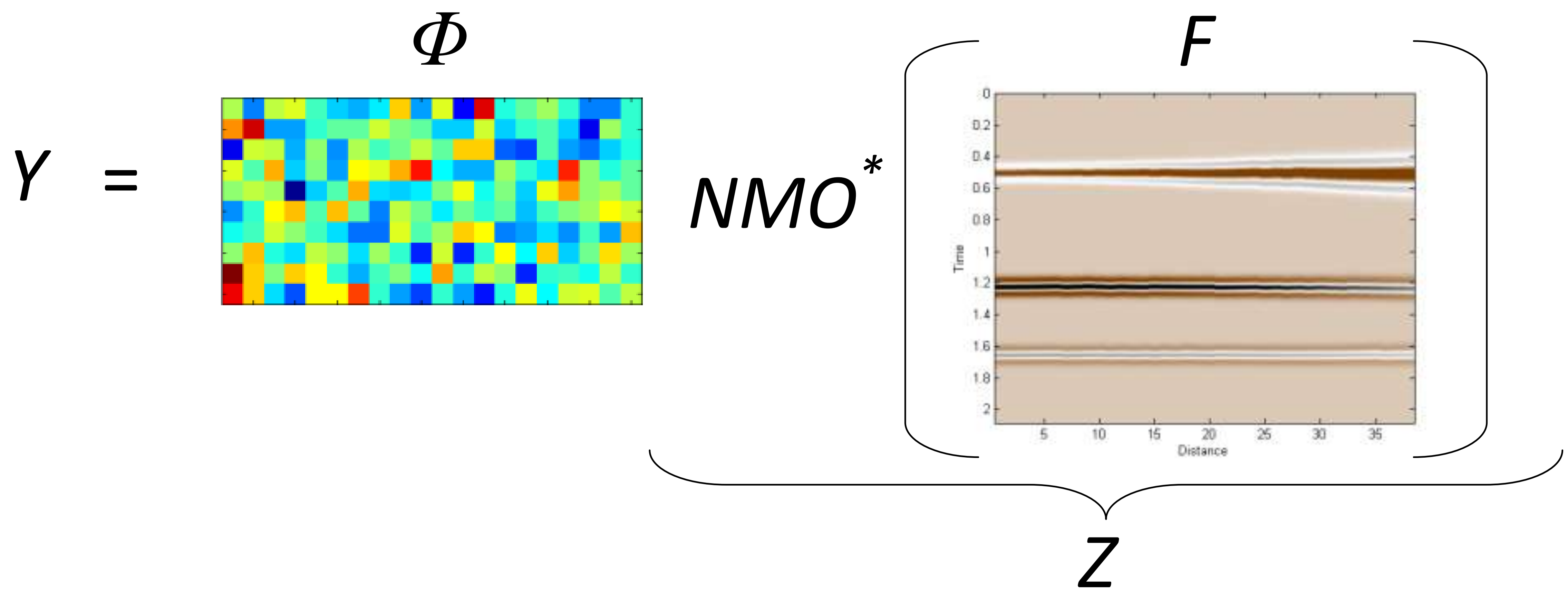

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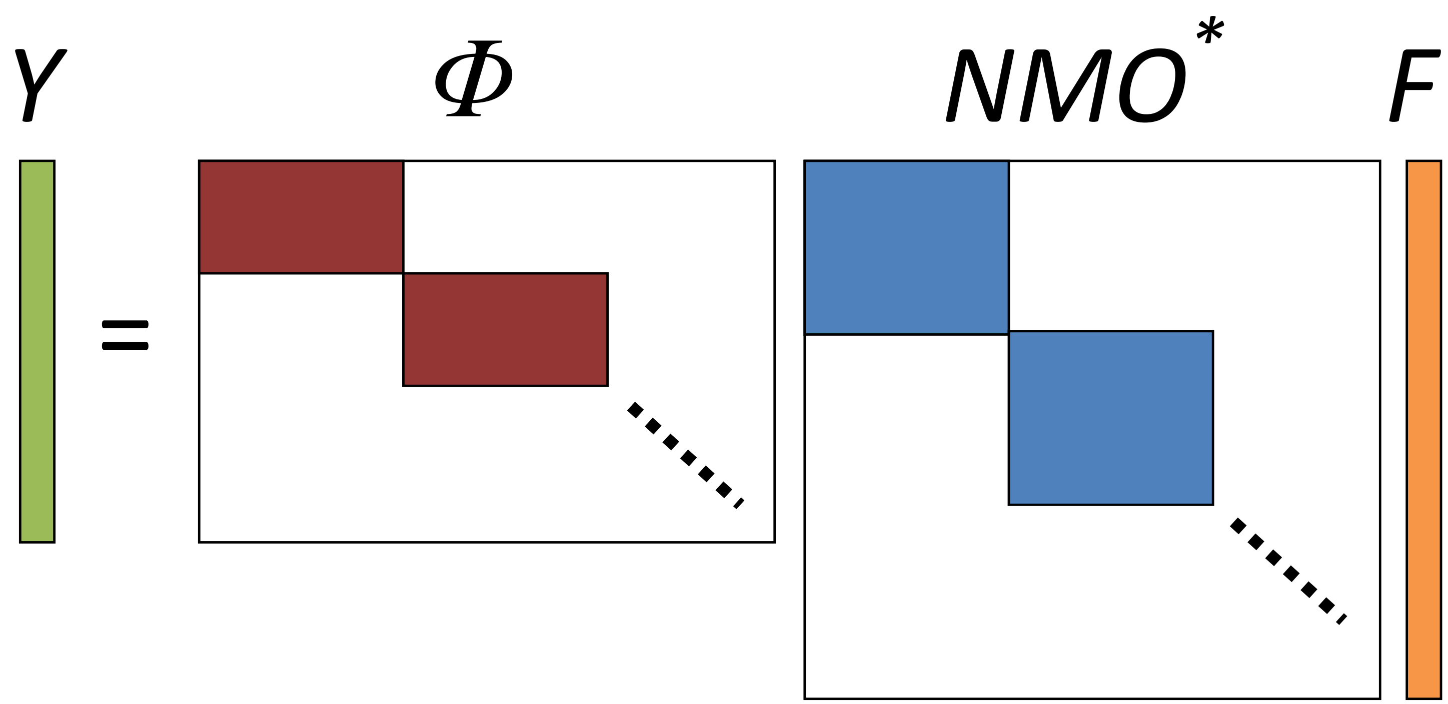

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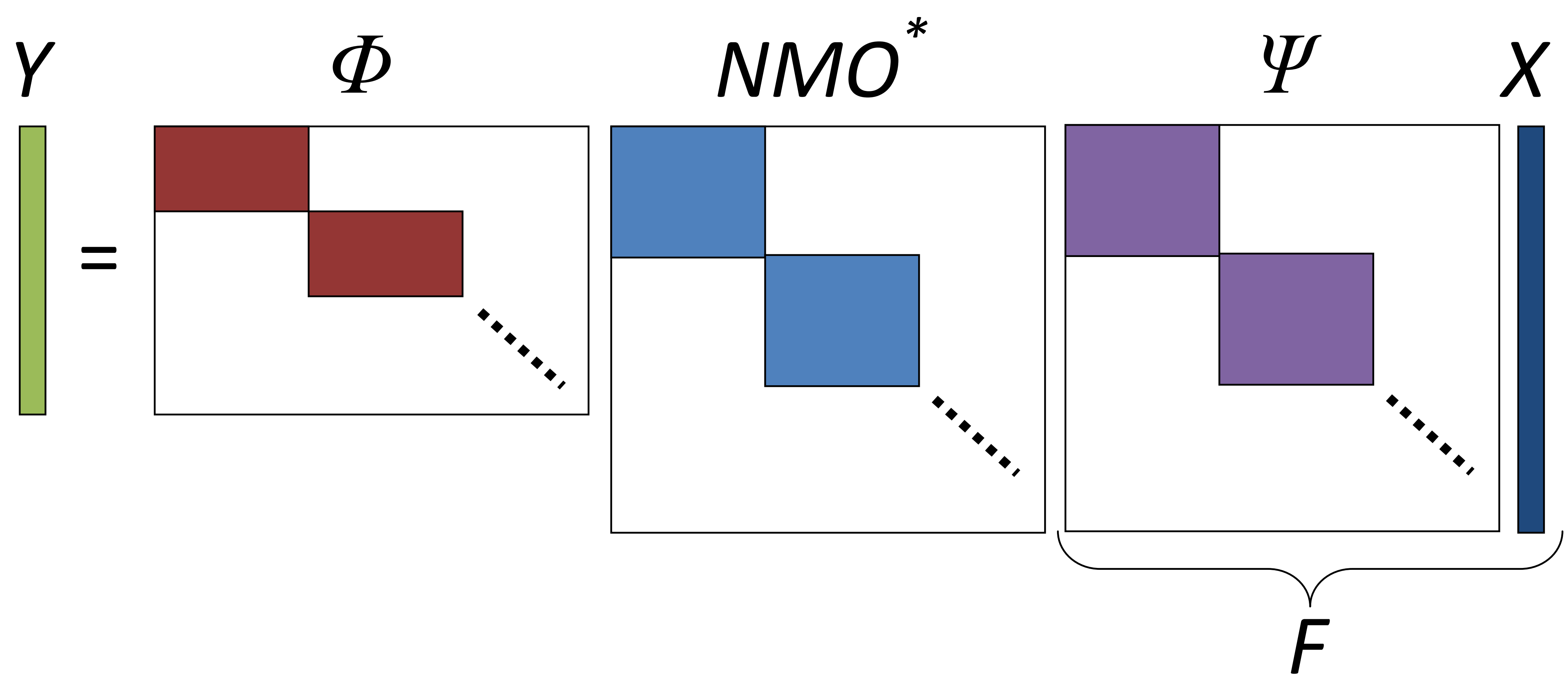

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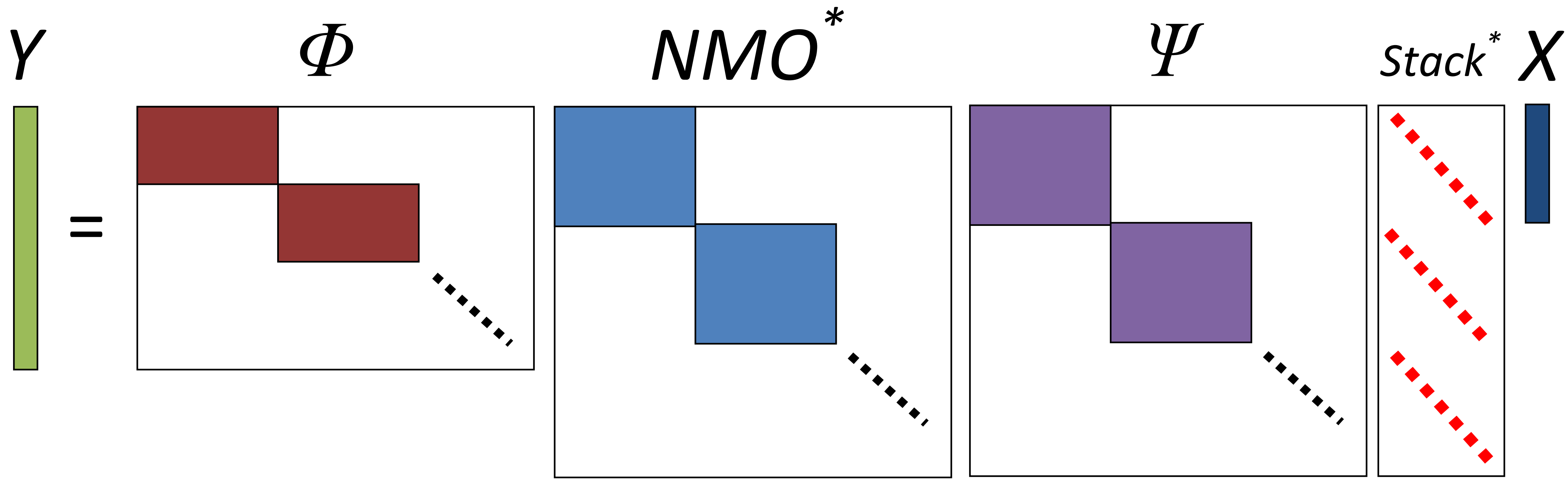

Weighted ℓ_1 Minimization in Seismic Imaging

- Let $Y = \Phi Z$, where Z is the common-receiver gather.
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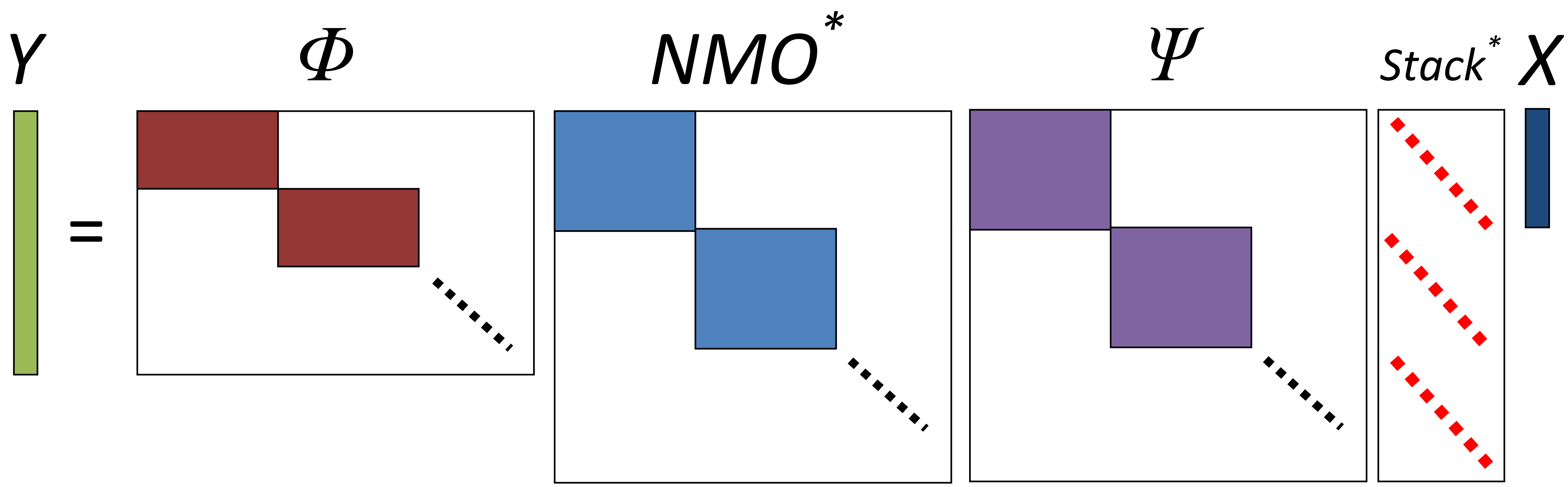

Main Results

- We adopt weighted ℓ_1 minimization and derive stability and robustness guarantees for the recovery of a signal x with partial support estimate $\widetilde{T}$.
- We show that if at least 50% of $\widetilde{T}$ is accurate, then weighted ℓ_1 minimization guarantees better recovery conditions and tighter error bounds.
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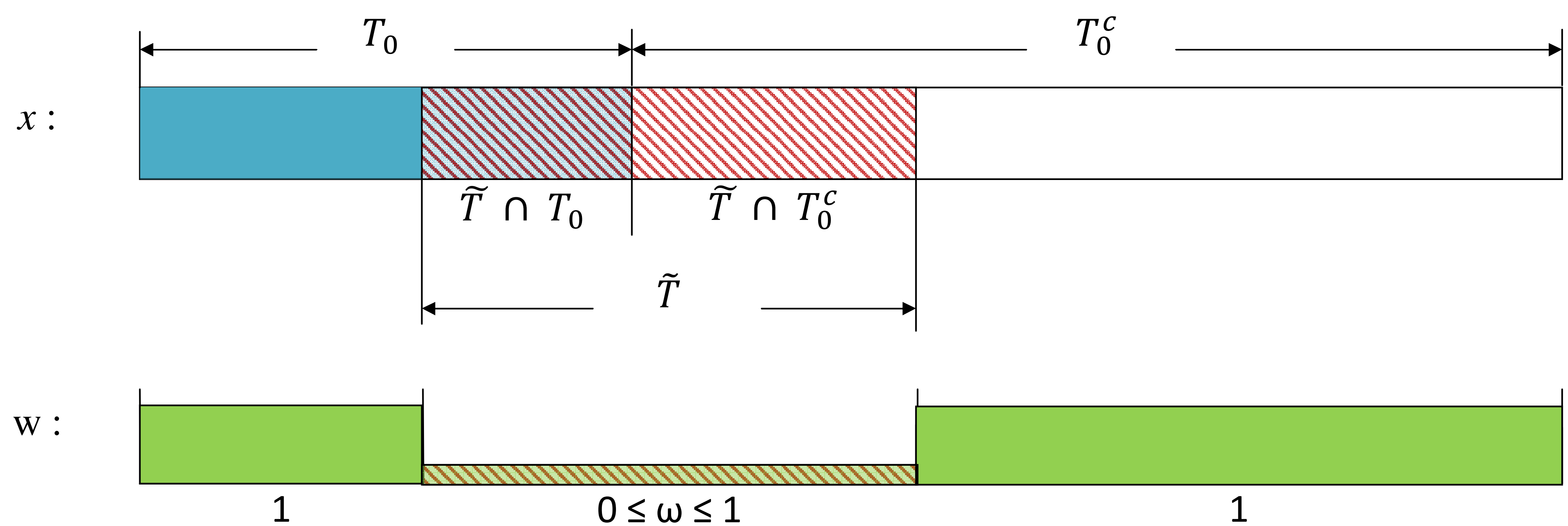
Weighted ℓ_1 Minimization

Find the vector x from a set of measurements y using the support estimate $\widetilde{T}$ by solving

$$\min_x \ \|x\|_{1,\mathrm{w}} \text{ subject to } \|Ax - y\|_2 \leq \epsilon \quad \text{with} \quad w_i = \begin{cases} 1, & i \in \widetilde{T}^c, \\ \omega, & i \in \widetilde{T}. \end{cases}$$

where $0 \leq \omega \leq 1$ and $\|x\|_{1,\mathrm{w}} := \sum_i \mathrm{w}_i |x_i|$.



Stability and Robustness

- Let x be in $\mathbb{R}^N$ and let x_k be its best k-term approximation, supported on T_0.
- Let $|\widetilde{T}| = \rho k$ and define $\alpha = \frac{|\widetilde{T} \cap T_0|}{|\widetilde{T}|}$, and $0 \leq \omega \leq 1$.
- If A satisfies

$$\delta_{(a+1)k} < \frac{a - \left(\omega + (1-\omega)\sqrt{1+\rho-2\alpha\rho}\right)^2}{a + \left(\omega + (1-\omega)\sqrt{1+\rho-2\alpha\rho}\right)^2},$$

then the recovery error is bounded by

$$\|x^* - x\|_2 \leq C_0'\epsilon + C_1' k^{-1/2}\left(\omega\|x_{T_0^c}\|_1 + (1-\omega)\|x_{\widetilde{T}^c \cap T_0^c}\|_1\right).$$

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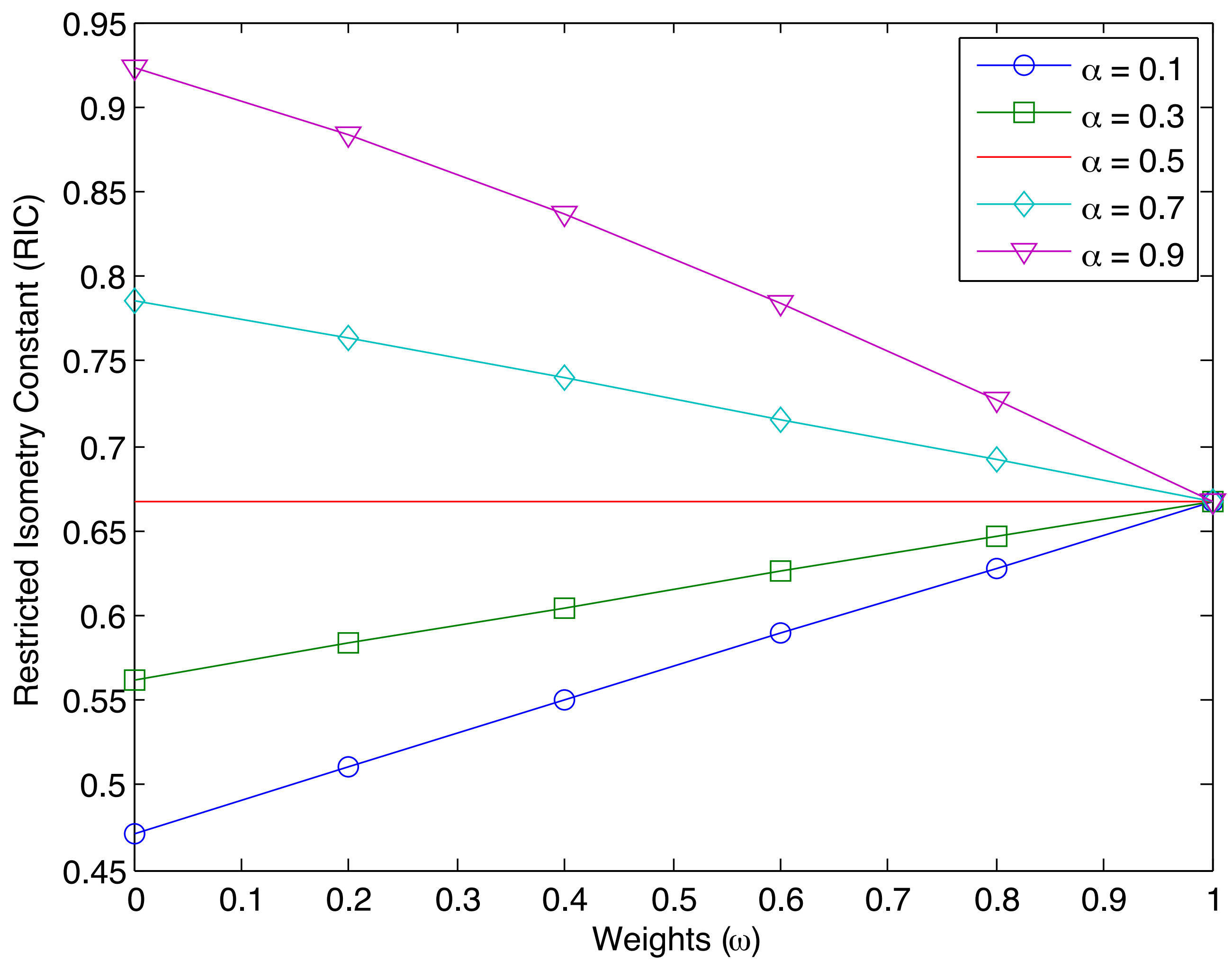
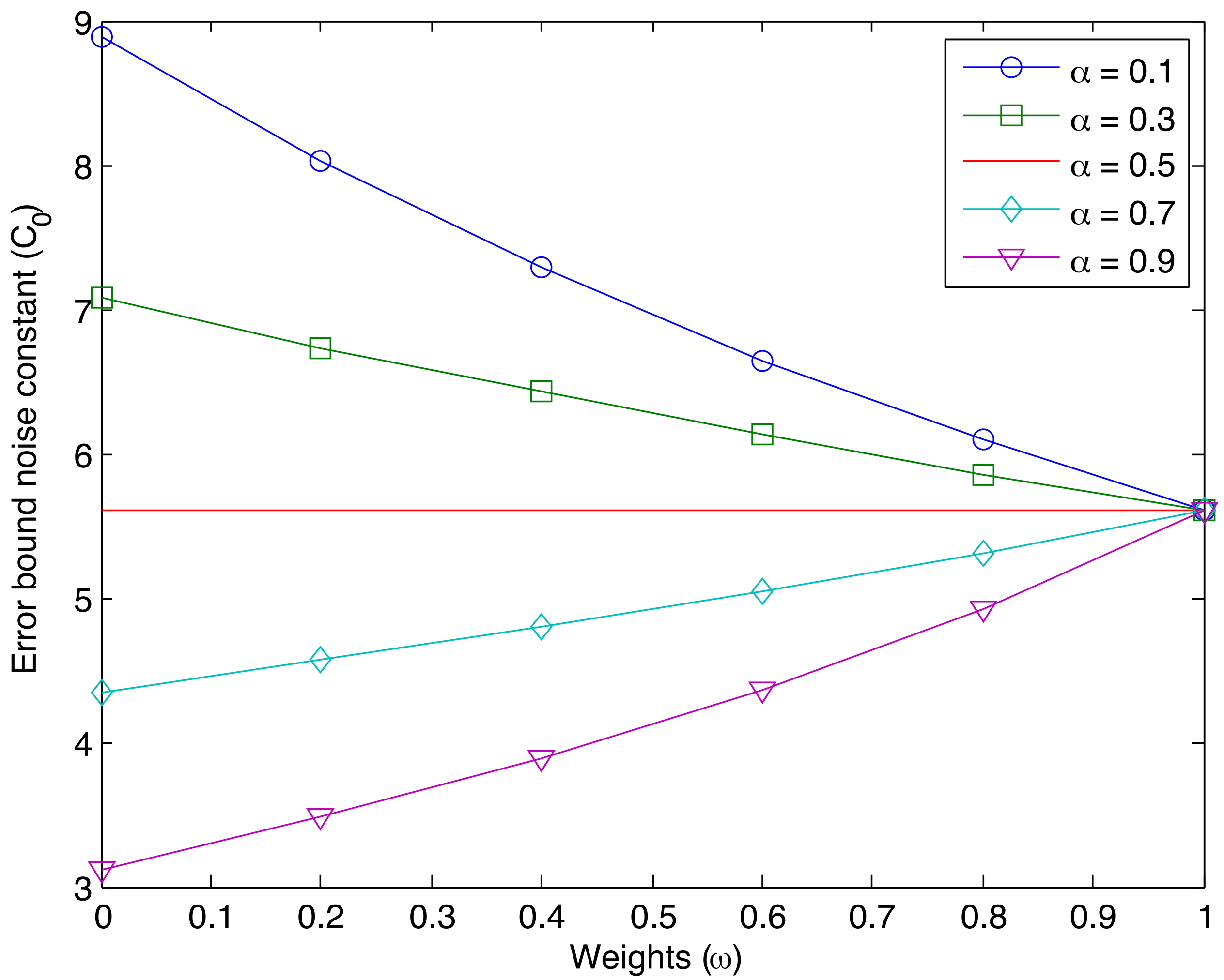
Sufficient Recovery Condition

Comparison with ℓ_1 sufficient recovery condition.



Error Bound Constants

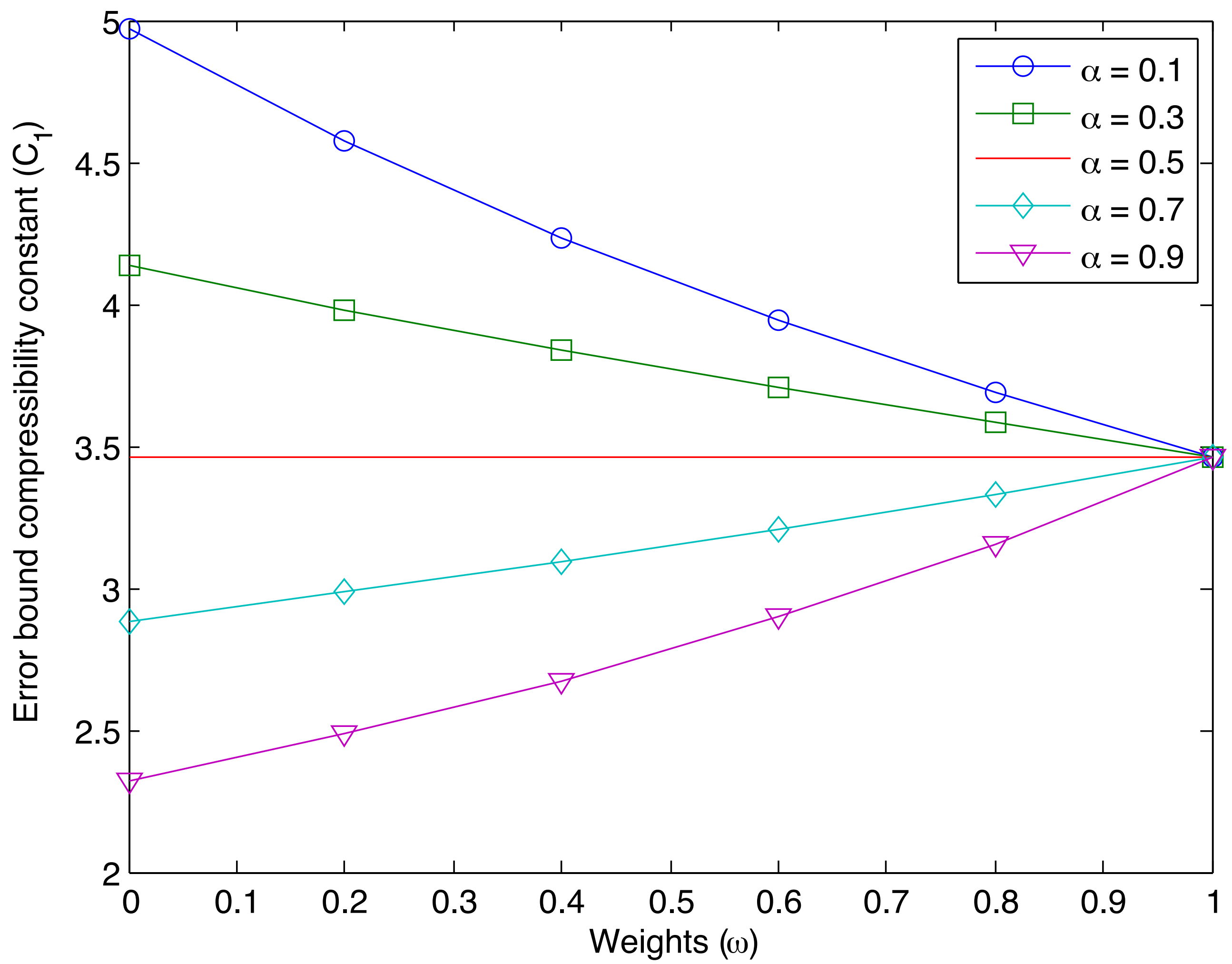
Measurement noise constant C_0':



Thursday, December 9, 2010

Error Bound Constants

Signal compressibility constant C_1':



Part 1: Introduction and Overview

Part 2: Stability and Robustness of Weighted ℓ_1 Minimization

Part 3: Experimental Results

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- SNR averaged over 20 experiments for k-sparse signals x with $k = 40$, and $N = 500$.

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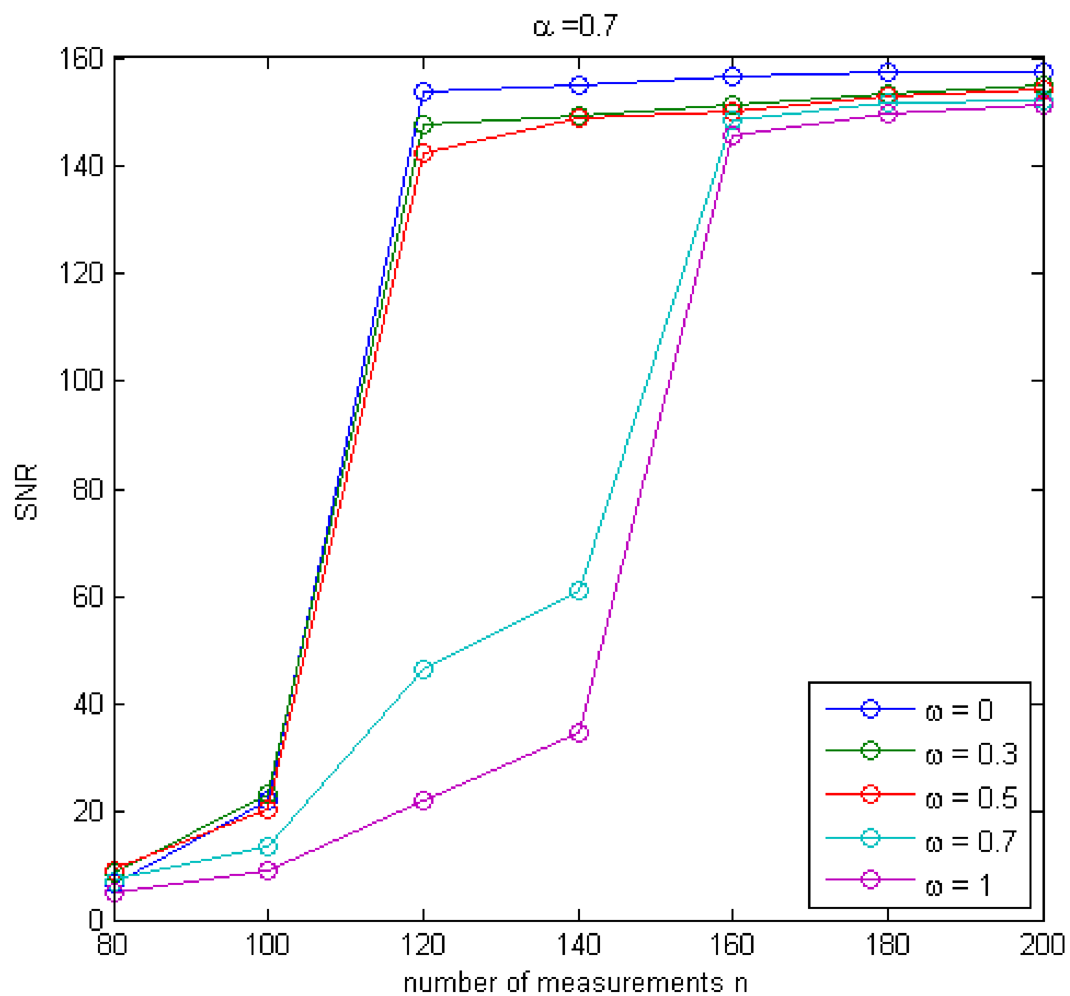

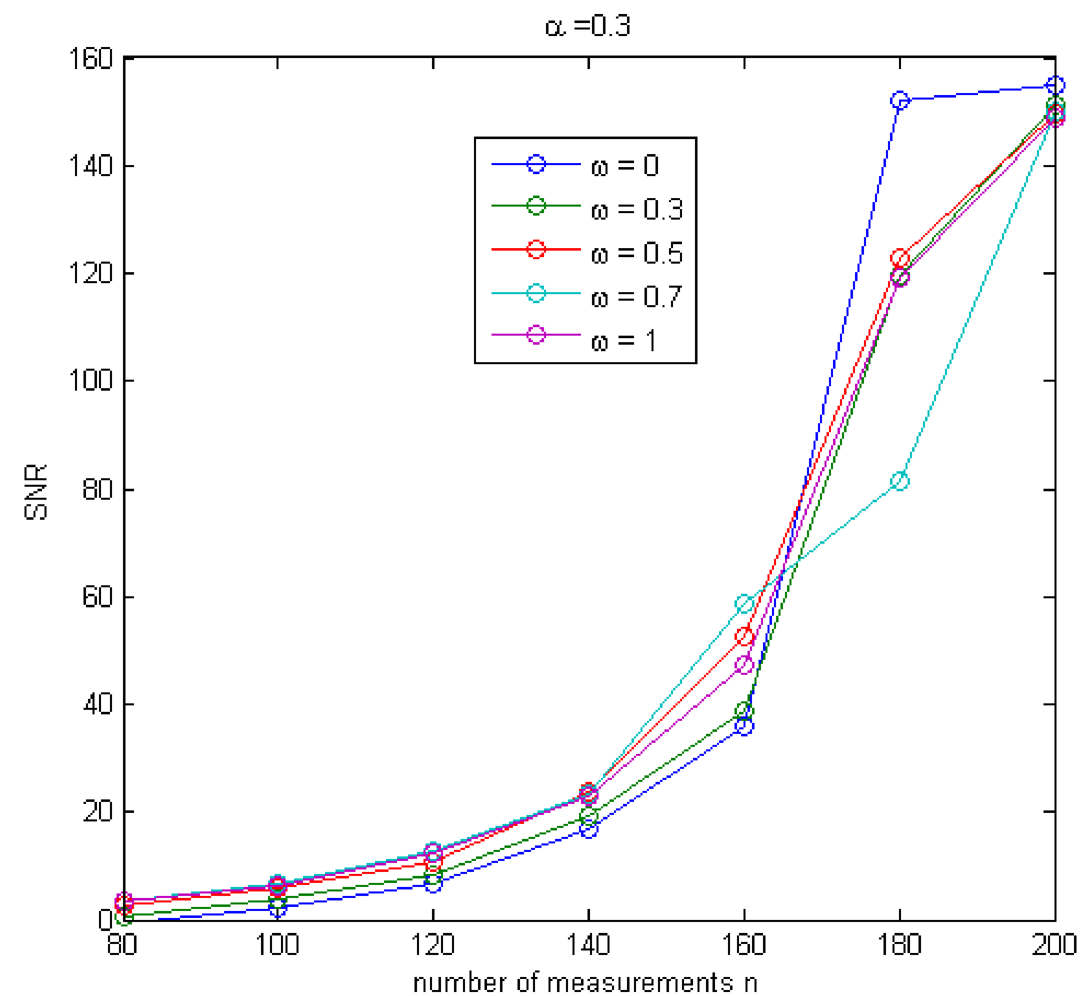

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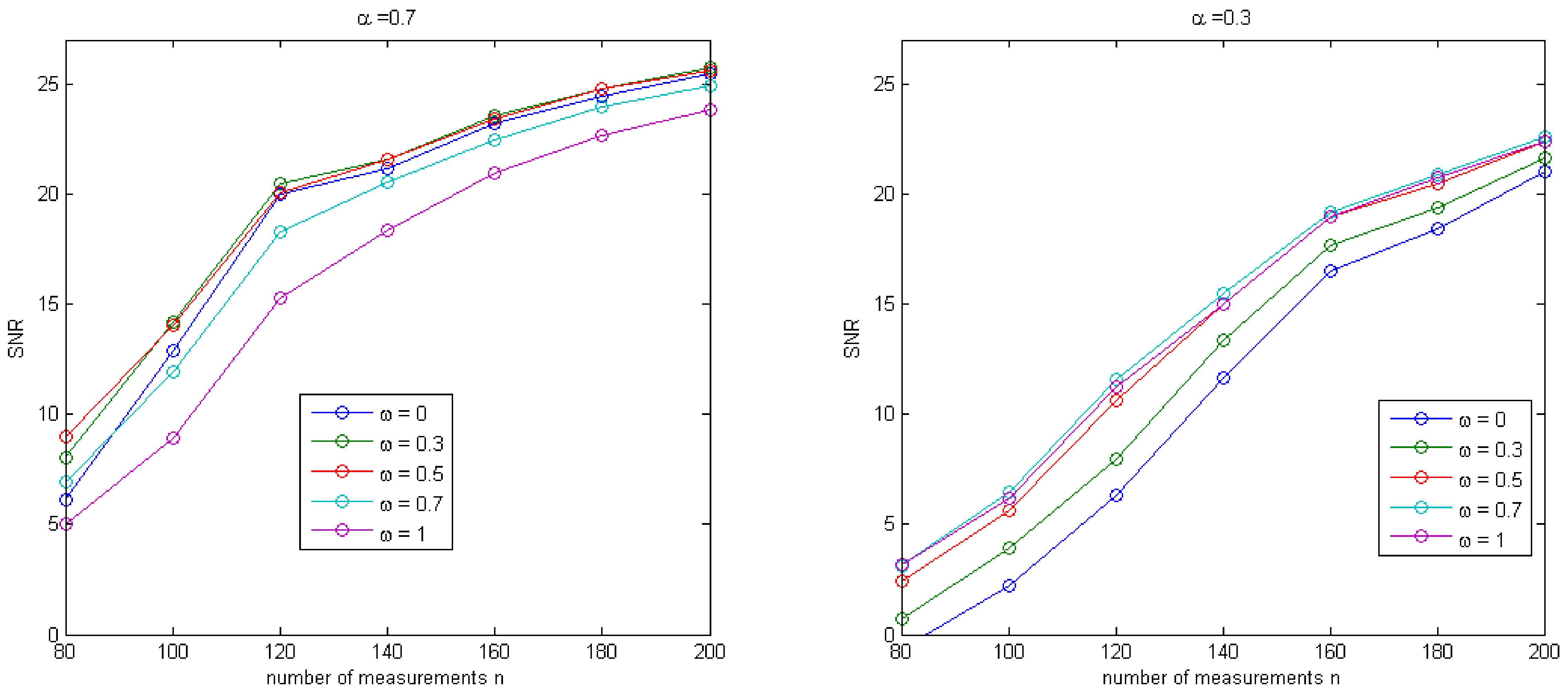

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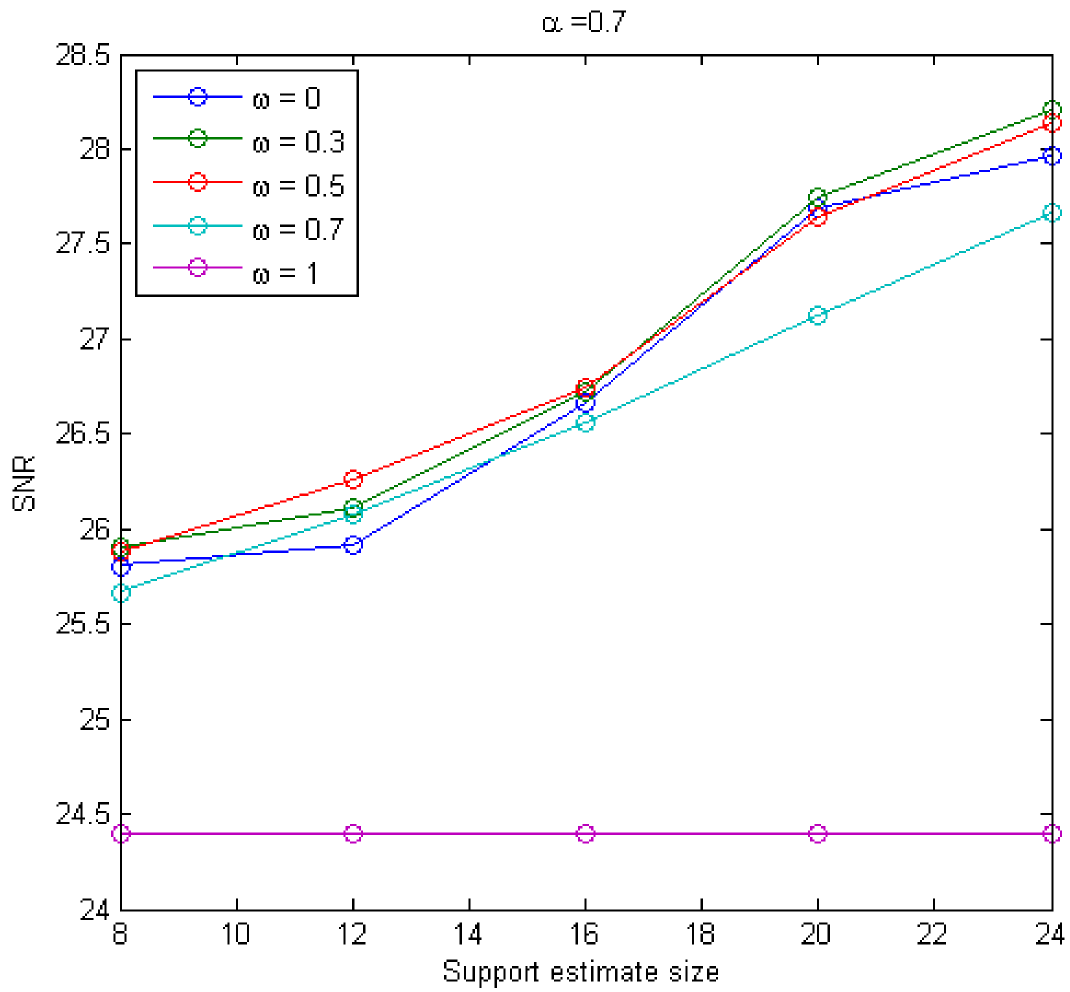

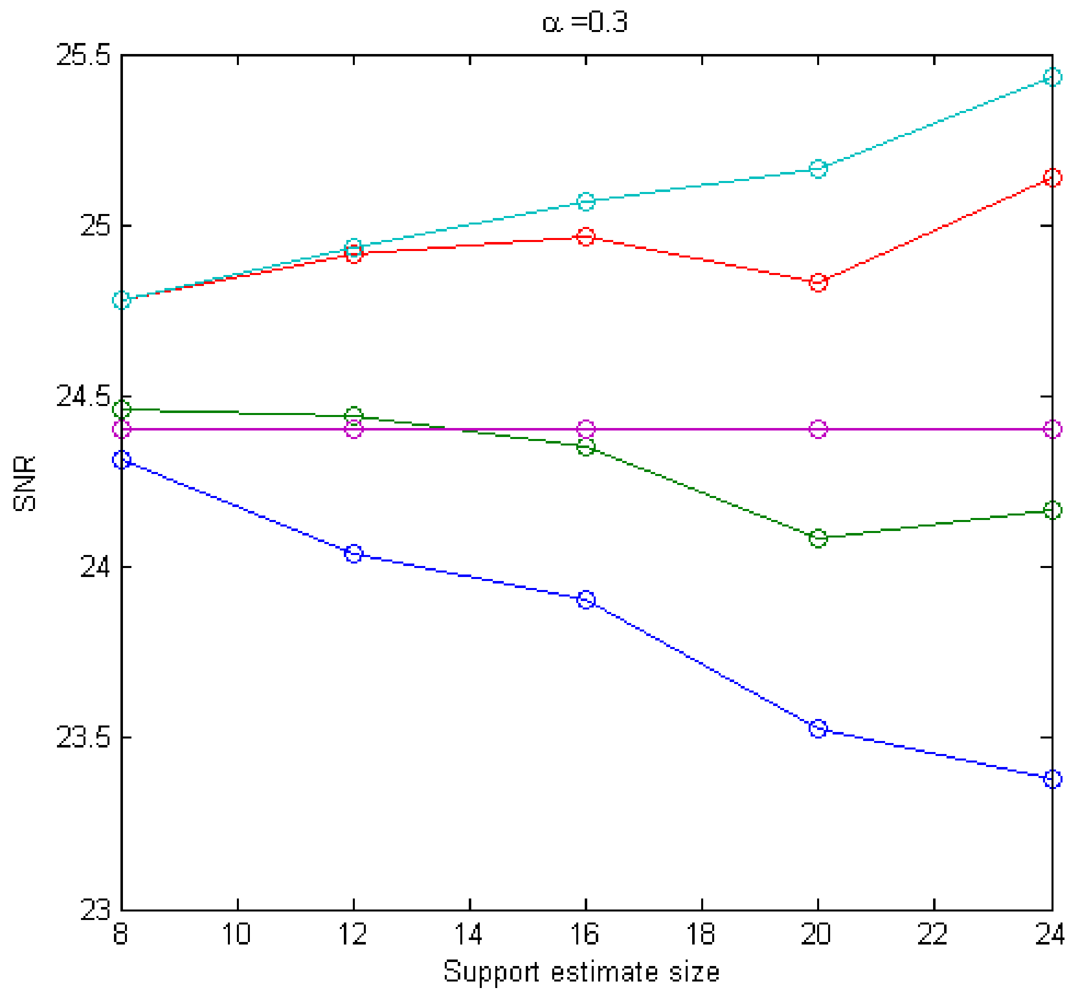

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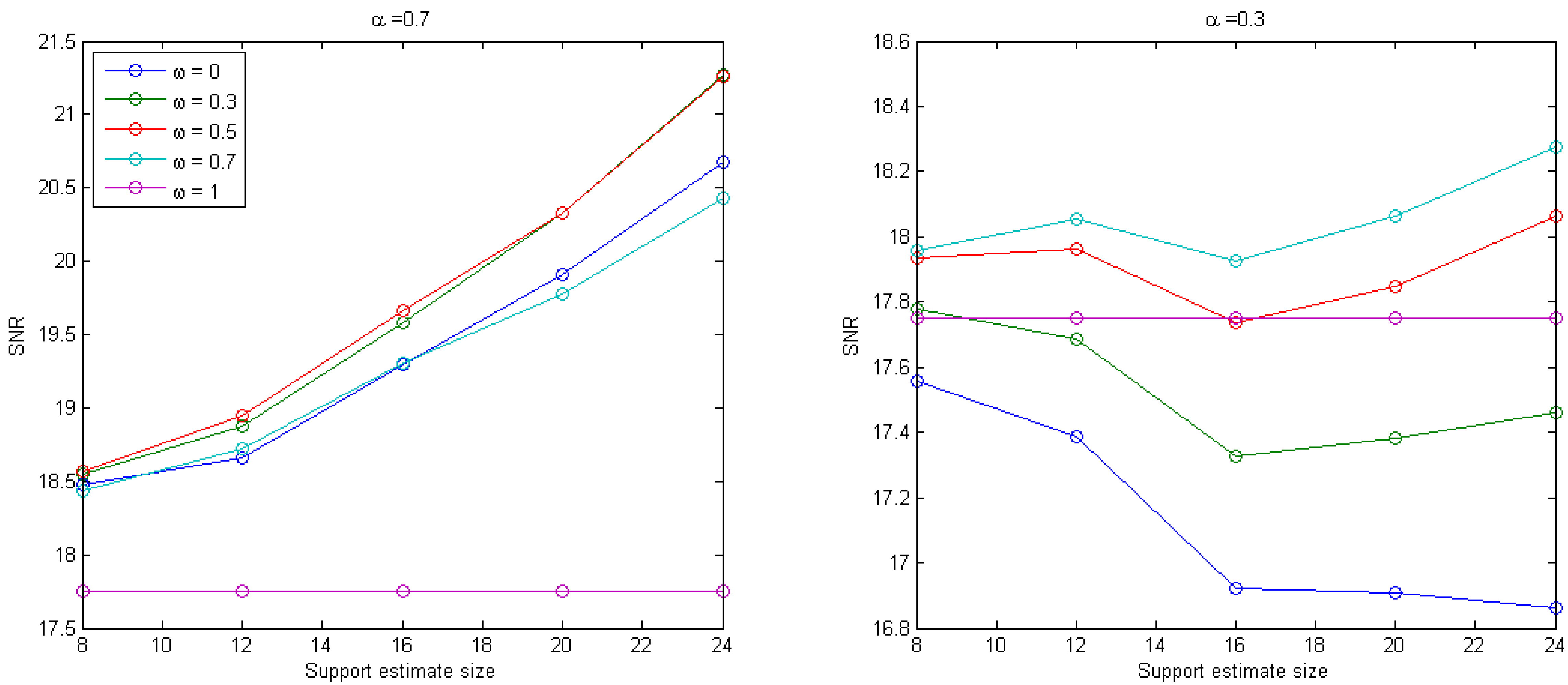

Compressed Sensing of Common-Receiver Gathers

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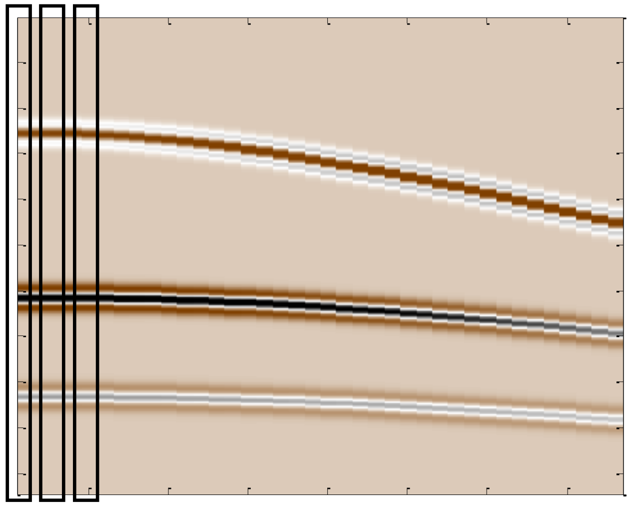
- For each column j, collect n_j measurements sampled randomly.
- Use weighted ℓ_1 minimization to recover x_j with $\widetilde{T}_j = V_{j-1}$.



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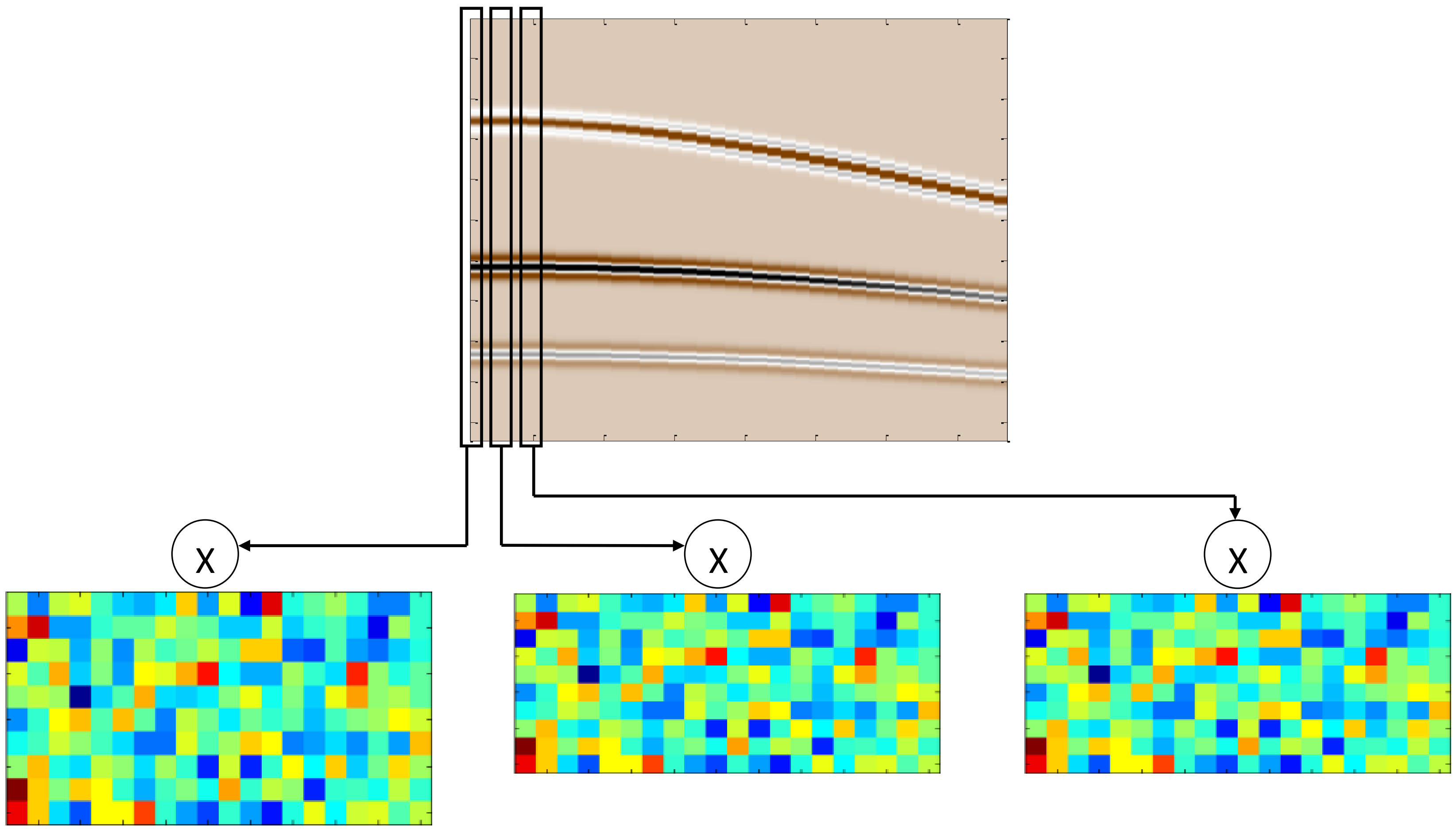

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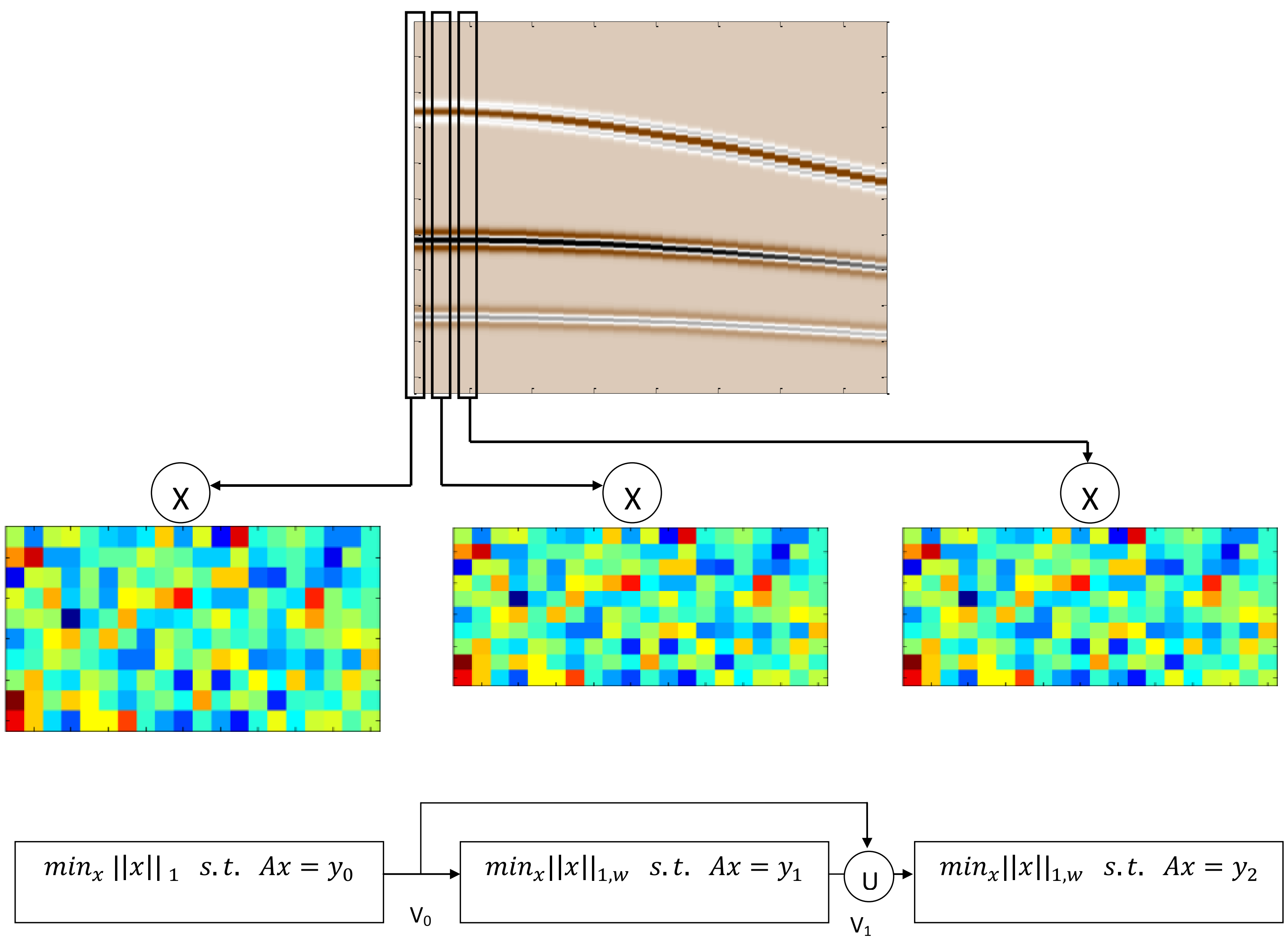

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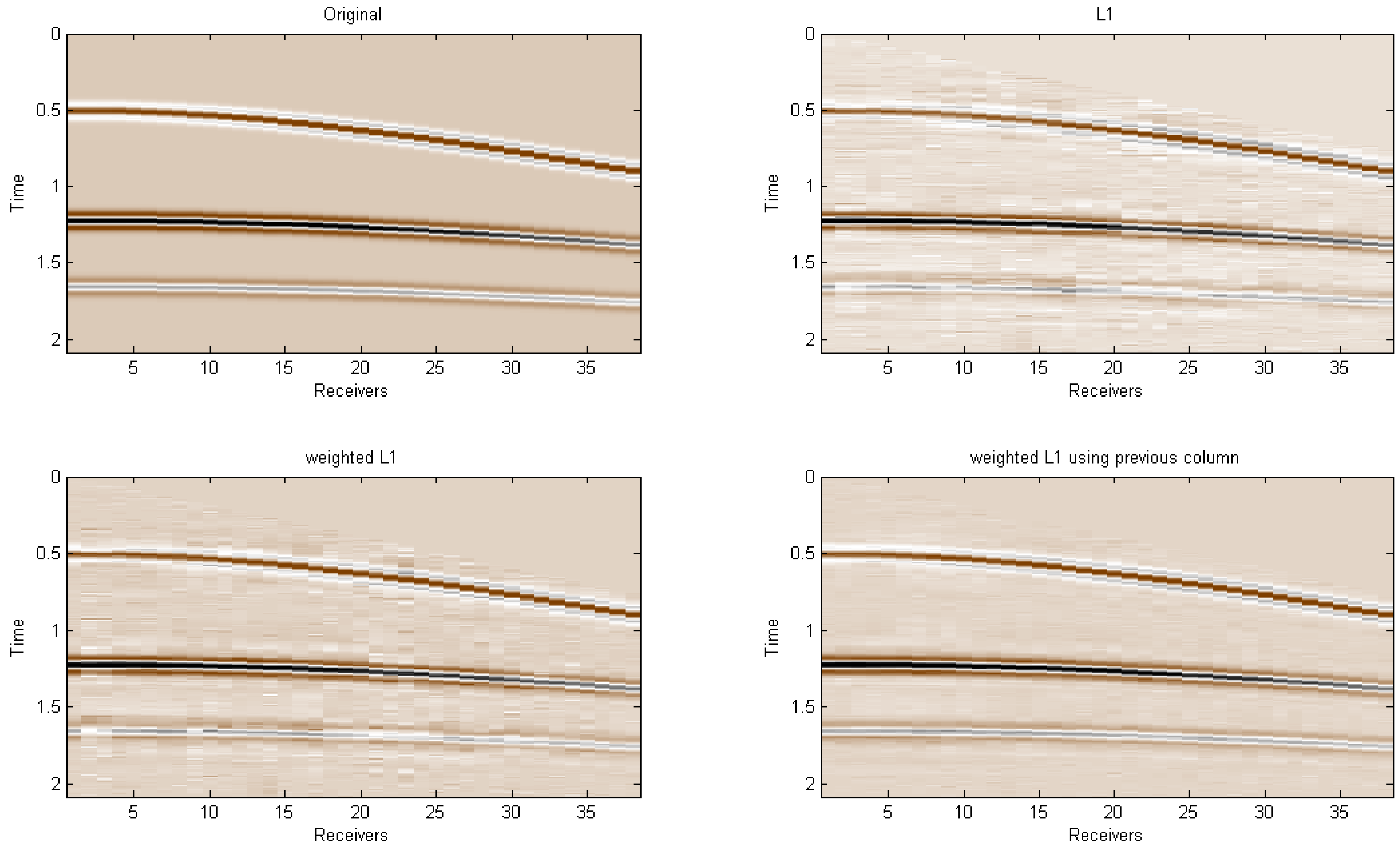

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- We showed that the correlation in seismic images allows us to draw support estimates of the data.
- If at least 50% of the support estimate is accurate, then weighted ℓ_1 minimization guarantees better recovery conditions with smaller recovery error bounds.
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Thank you!

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