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Full-waveform inversion with randomized L1 recovery for the model updates

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Motivation

Curse of dimensionality for d>2

- *Exponentially* increasing *data volumes*
- *Helmholtz* requires *iteratively* solvers to address *bandwidth*
- Computational complexity grows *linearly* with # RHS's
- Makes *computation* of the misfit functional & gradients prohibitively *expensive*

Wish list

An *inversion* technology that

- is based on a *time-harmonic* PDE solver, which is easily *parallelizable*, and *scalable* to 3D
- does *not* require *multiple* iterations with *all* data
- removes the *linearly* increasing costs of *implicit* solvers for increasing numbers of frequencies & RHS's
- produces *high-resolution* inversion results

Key technologies

Simultaneous sources & phase encoding [Beasley, '98, Berkhout, '08] [Morton, '98, Romero, '00]

- supershots [Krebs et.al., '09, Operto et. al., '09, Herrmann et.al., '08-10']

Stochastic optimization & machine learning [Bertsekas, '96]

- stochastic gradient decent

Compressive sensing [Candès et.al, Donoho, '06]

- *sparse* recovery & *randomized* subsampling

[Wang & Sacchi, '07]

Sparse recovery

Least-squares migration with *sparsity* promotion

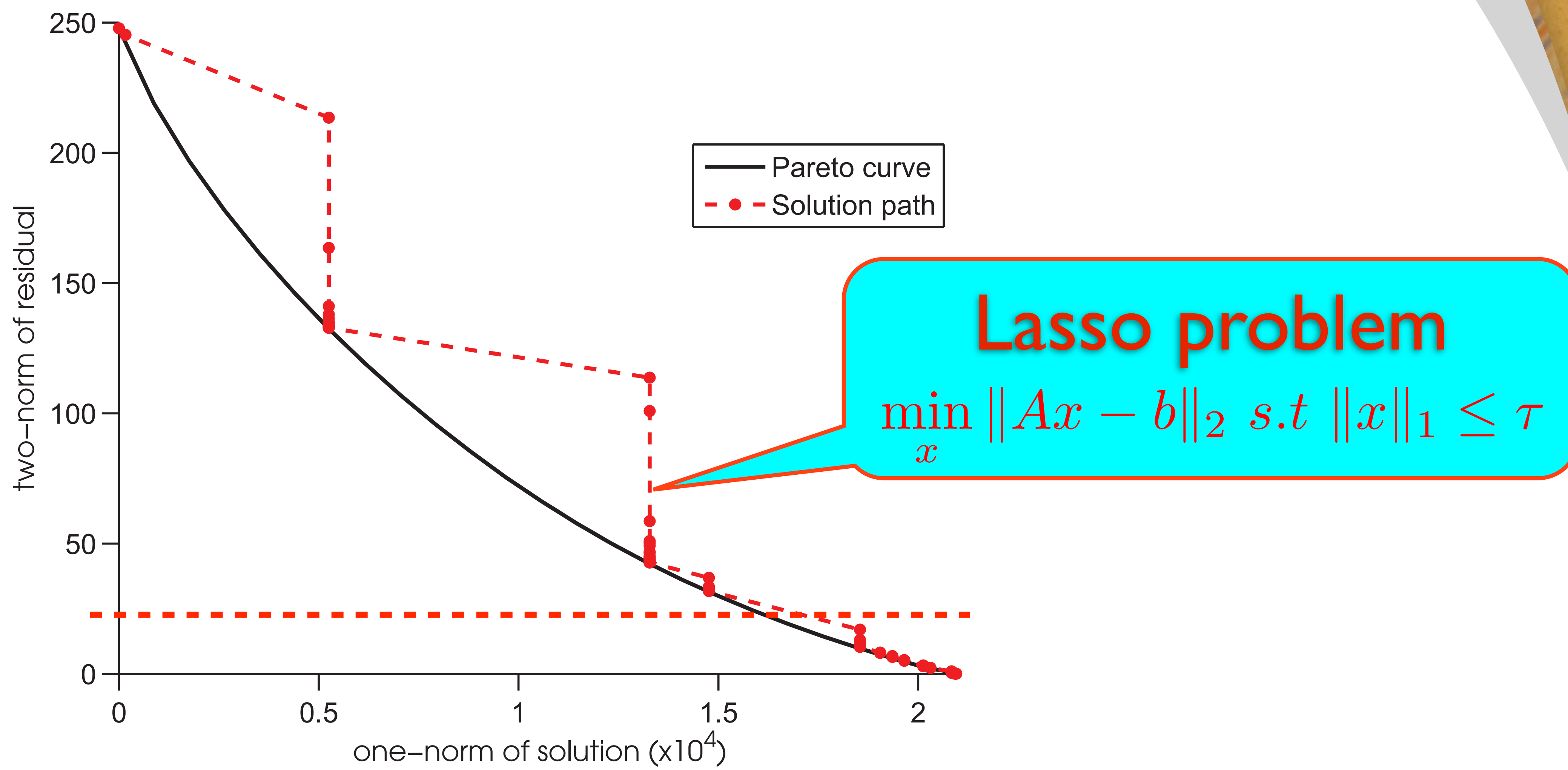
$$\delta\widetilde{\mathbf{m}} = \mathbf{S}^* \arg\min_{\delta\mathbf{x}} \frac{1}{2}\|\delta\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\delta\underline{\mathbf{d}} - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0; \underline{\mathbf{Q}}]\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma$$

$$\begin{aligned} \delta\mathbf{x} &= \text{Sparse curvelet-coefficient vector} \\ \mathbf{S}^* &= \text{Curvelet synthesis} \end{aligned}$$

leads to *significant* speedup as long as

$$n^{\ell_1}_{PDE} \times K \ll n^{\ell_2}_{PDE} \times n_f \times n_s$$

Pareto Curve



$$\delta\widetilde{\mathbf{m}} = \mathbf{S}^* \arg\min_{\delta\mathbf{x}} \frac{1}{2}\|\delta\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\delta\underline{\mathbf{d}} - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0; \underline{\mathbf{Q}}]\mathbf{S}^*\delta\mathbf{x}\|_2 \leq \sigma$$

Experiment

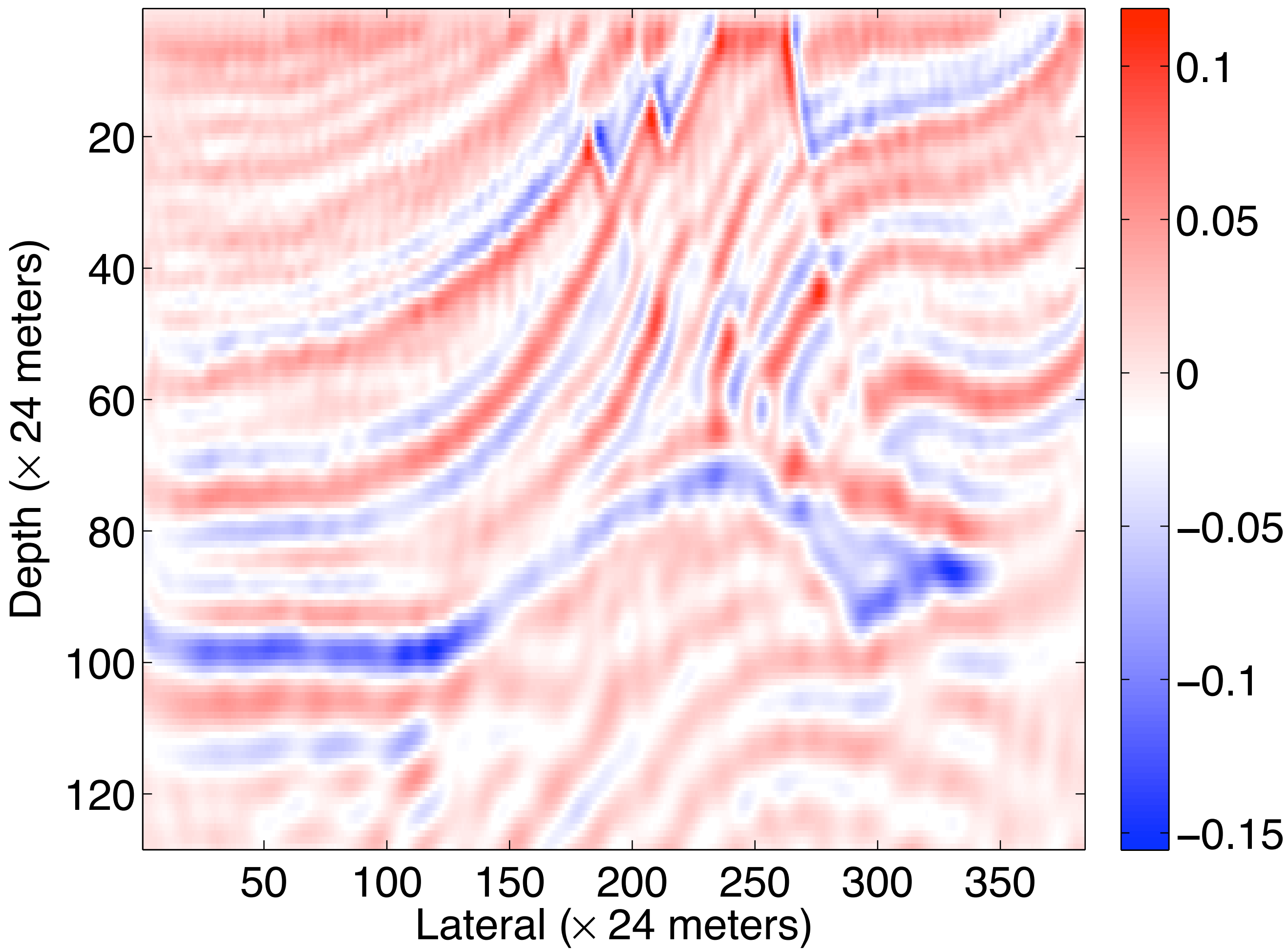
Linearized *sparsity promoting* least-squares migration

- Marmousi model (128x384) with grid size 24 m
- 12 Hz ricker wavelet
- use different
  - # of simultaneous shots
  - # of frequencies

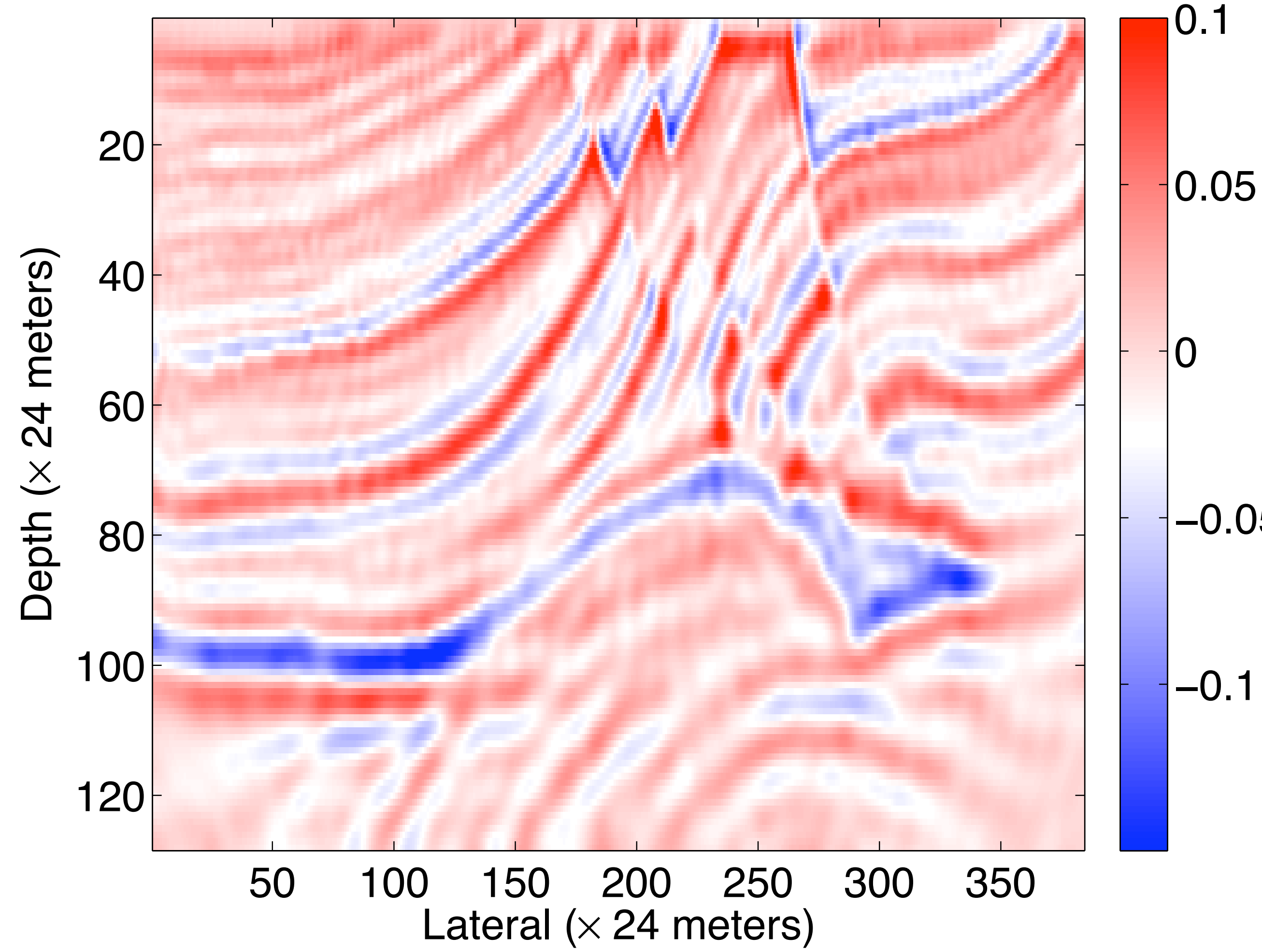
Linearized sparse inversion

14 simultaneous shots 7 random frequencies

L2 recovery with all data



***sparse* recovery with curvelet**



Speed up: x8.3

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FWI formulation

Multiexperiment unconstrained optimization problem:

$$\min_{\mathbf{m}\in\mathcal{M}} \frac{1}{2}\|\mathbf{D}-\mathcal{F}[\mathbf{m};\mathbf{Q}]\|_{2,2}^2 \quad \text{with} \quad \mathcal{F}[\mathbf{m};\mathbf{Q}] := \mathbf{P}\mathbf{H}^{-1}\mathbf{Q}$$

- requires large number of PDE solves
- linear in the sources
- apply *randomized* dimensionality reduction

[Tarantola, 84; Pratt, ’98; Plessix, 06]
[Haber, Chung, and Herrmann, ’10]

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Gauss-Newton

Algorithm 1: Gauss Newton

Result: Output estimate for the model $\mathbf{m}$

$\mathbf{m} \longleftarrow \mathbf{m}_0;\ k \longleftarrow 0$; `// initial model`

while not converged **do**

$\mathbf{p}^k \longleftarrow \arg\min_{\mathbf{p}} \frac{1}{2}\|\delta\mathbf{d} - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k;\mathbf{Q}]\mathbf{p}\|_2^2 + \lambda^k\|\mathbf{p}\|_2^2$; `// search dir.`

$\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k\mathbf{p}^k$; `// update with linesearch`

$k \longleftarrow k+1$;

end

FWI with phase encoding

Multiexperiment unconstrained optimization problem:

$$\min_{\mathbf{m}\in\mathcal{M}} \frac{1}{2}\|\underline{\mathbf{D}} - \mathcal{F}[\mathbf{m};\underline{\mathbf{Q}}]\|_{2,2}^2 \quad \text{with} \quad \mathcal{F}[\mathbf{m};\underline{\mathbf{Q}}] := \mathbf{P}\underline{\mathbf{H}}^{-1}\underline{\mathbf{Q}}$$

- requires *smaller* number of PDE solves
- exploits *linearity* in the *sources* & *block-diagonal* structure of the *Helmholtz system*
- uses *randomized* frequency *selection* and *phase encoding*

[Krebs et.al., '09, Operto et. al., '09 ; Herrmann et. al. '08-'10]

Renewals

Use *different* simultaneous shots for each *subproblem*, i.e.,

$$\underline{\mathbf{Q}} \quad \mapsto \quad \underline{\mathbf{Q}}^k$$

Requires *fewer* PDE solves for each GN *subproblem*...

- motivated by *stochastic approximation* [Nemirovski, '09]
- related to Kaczmarz ('37) method applied by Natterer, '01
- *supersedes ad hoc* approach by Krebs *et.al.*, 2009

Phase encoding

Algorithm 1: Gauss Newton with renewed phase encodings

Result: Output estimate for the model $\mathbf{m}$

$\mathbf{m} \longleftarrow \mathbf{m}_0$; $k \longleftarrow 0$; `// initial model`

while not converged **do**

$\mathbf{p}^k \longleftarrow \arg\min_{\mathbf{p}} \frac{1}{2}\|\delta\underline{\mathbf{d}}^k - \nabla\mathcal{F}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\mathbf{p}\|_2^2 + \lambda^k\|\mathbf{p}\|_2^2$; `// search dir.`

$\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k\mathbf{p}^k$; `// update with linesearch`

$k \longleftarrow k + 1$;

end

Observations

Stochastic optimization

- introduces noisy search directions
- interferences go down *slowly* as batch size *increases*
- requires *averaging* over *previous* model *updates*

Formulation does not exploit *sparsity* on the *model*

[Bertsekas, '96]

[Krebs et.al, '09]

Our approach

Leverage findings from *sparse recovery* & *compressive sensing*

- consider each *phase-encoded* Gauss-Newton update as separate *compressive-sensing* experiment
- remove *interferences* by *curvelet-domain sparsity* promotion
- exploit properties of the Pareto curve

[Candes et al., '06; Donoho, '06]

[Demanet et. al. '07; Herrmann & Li, '08-'09]

Compressive updates

Algorithm 1: Gauss Newton with sparse updates

Result: Output estimate for the model $\mathbf{m}$

$\mathbf{m} \longleftarrow \mathbf{m}_0;\ k \longleftarrow 0$; `// initial model`

while not converged **do**

$\mathbf{p}^k \longleftarrow \mathbf{S}^* \arg\min_{\mathbf{x}} \frac{1}{2}\|\delta\underline{\mathbf{d}}^k - \nabla\mathcal{F}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\mathbf{S}^*\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \tau^k$

$\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k \mathbf{p}^k$; `// update with linesearch`

$k \longleftarrow k + 1$;

end

[van den Berg & Friedlander, ’08]

Phase encoding

Algorithm 1: Gauss Newton with renewed phase encodings

Result: Output estimate for the model $\mathbf{m}$

$\mathbf{m} \longleftarrow \mathbf{m}_0$; $k \longleftarrow 0$; `// initial model`

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  $\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k\mathbf{p}^k$; `// update with linesearch`

  $k \longleftarrow k+1$;

end

Example

Marmousi model and BP model:

- 128x384 with a mesh size of 24 meters, 100m for BP
- 384 co-located shots and receivers with offset = 3 X depth
- 2.4s recording time for Marmousi, 12s for BP

Explicit Time-harmonic Helmholtz solver

- 9-point finite difference
- Absorbing boundary condition
- 12 Hz Ricker source wavelet and 7 Hz for BPmodel

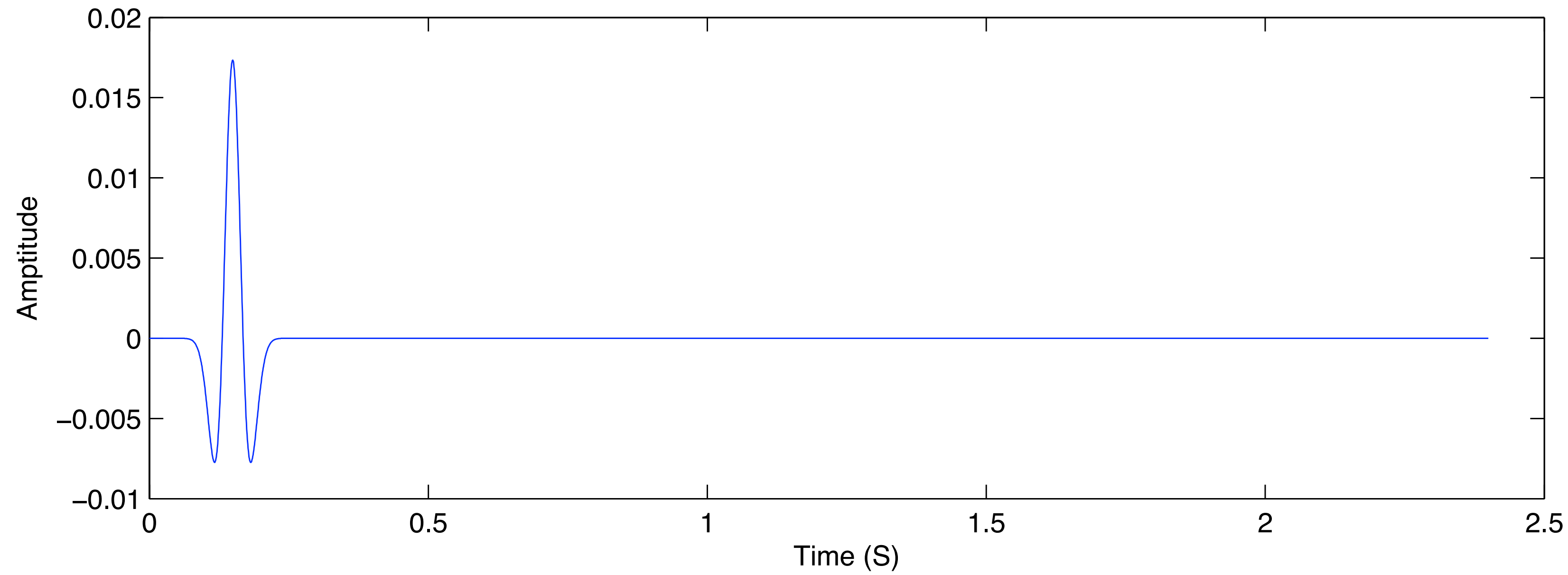
Example

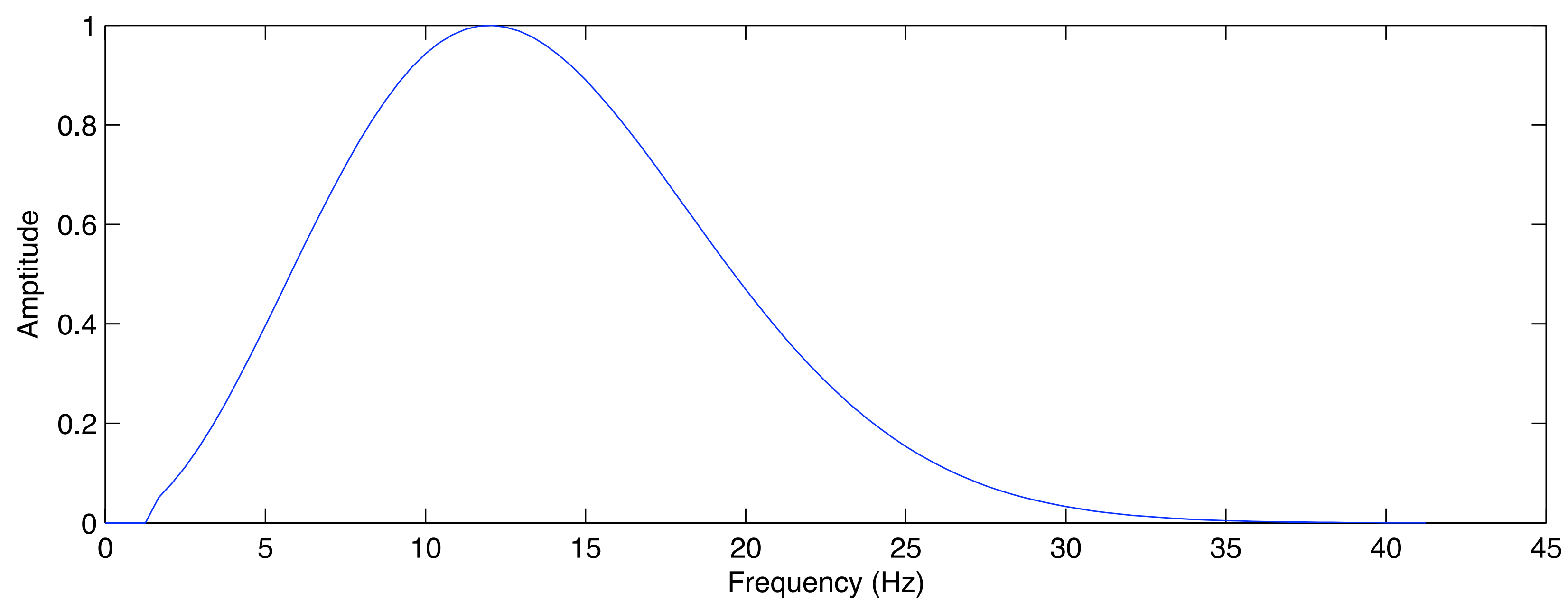
FWI specs:

- Committed *inversion crime*
- Frequency continuation over 10 bands
- 15 *simultaneous* shots with 10 *frequencies* each

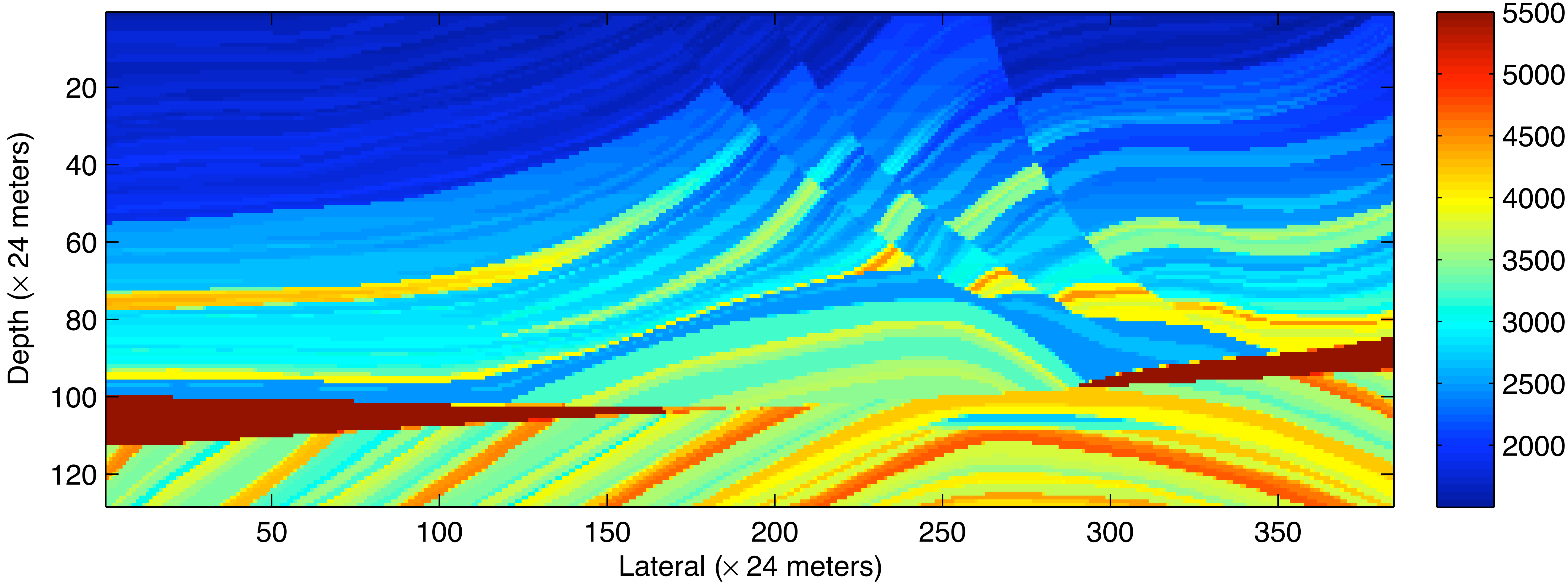
$$K = 10 \times 15 \ll 100 \times 384$$

Source wavelet

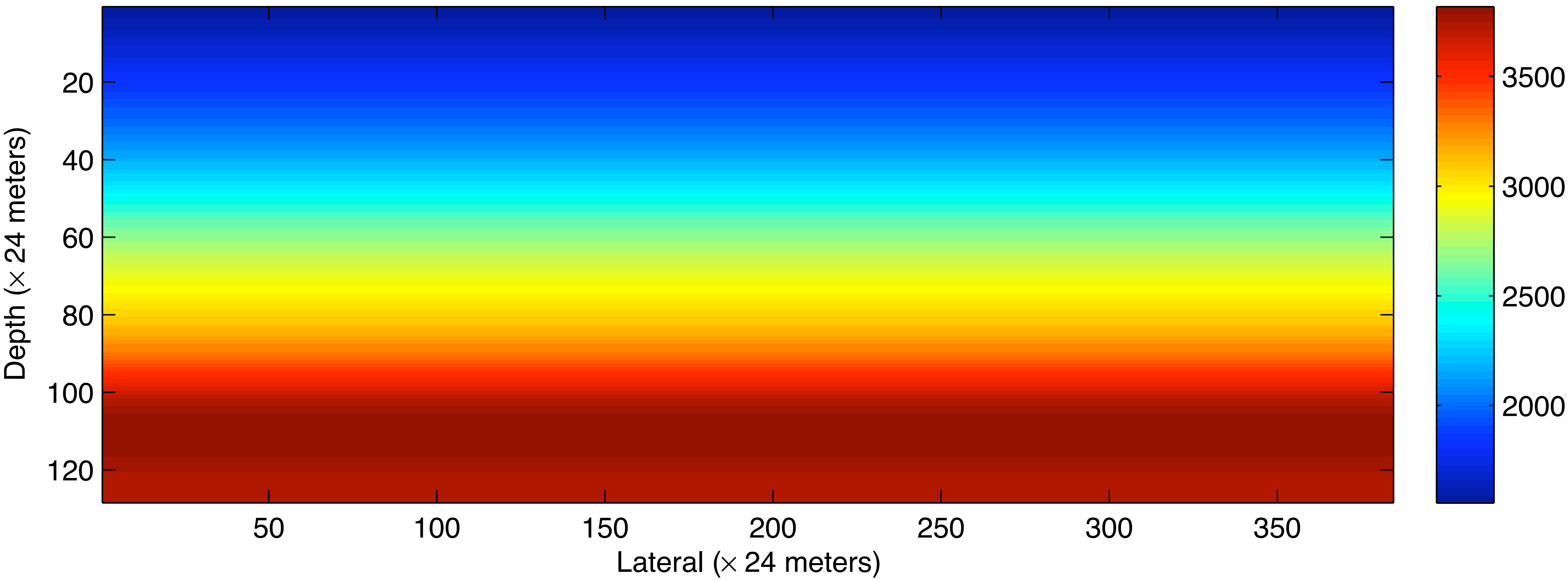




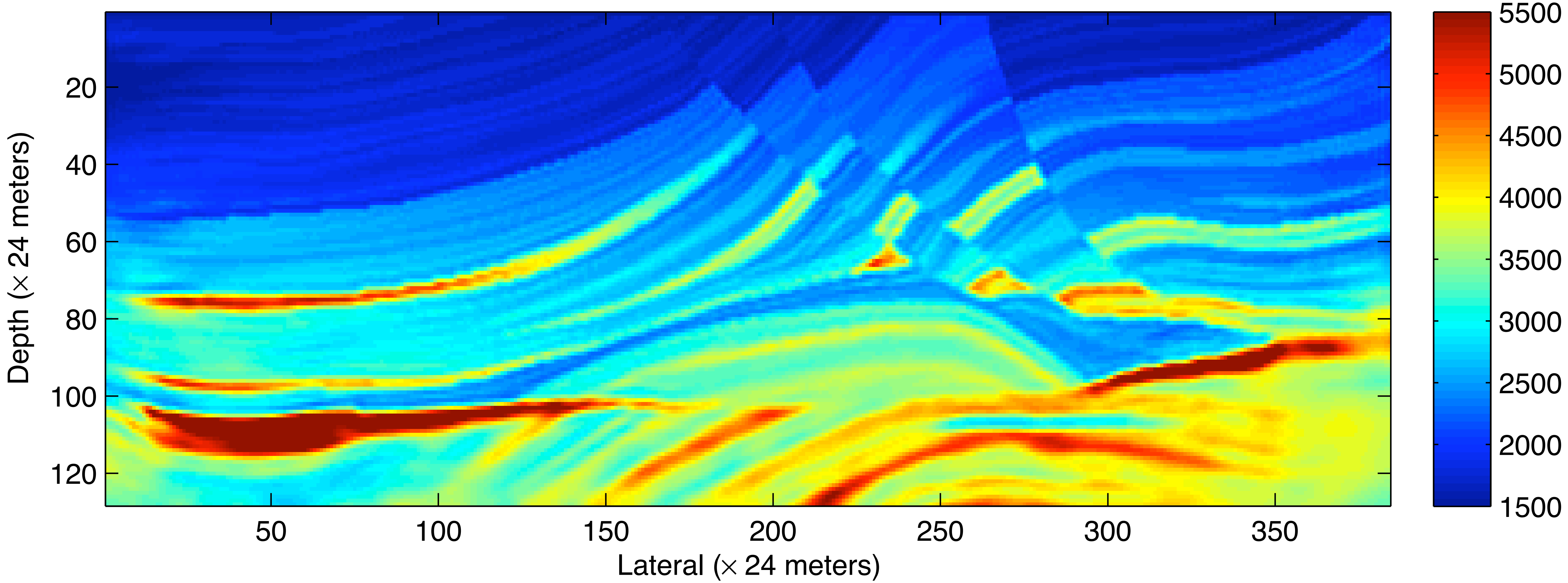
True model


Depth (× 24 meters)
20
40
60
80
100
120
50
100
150
200
250
300
350
Lateral (× 24 meters)
5500
5000
4500
4000
3500
3000
2500
2000

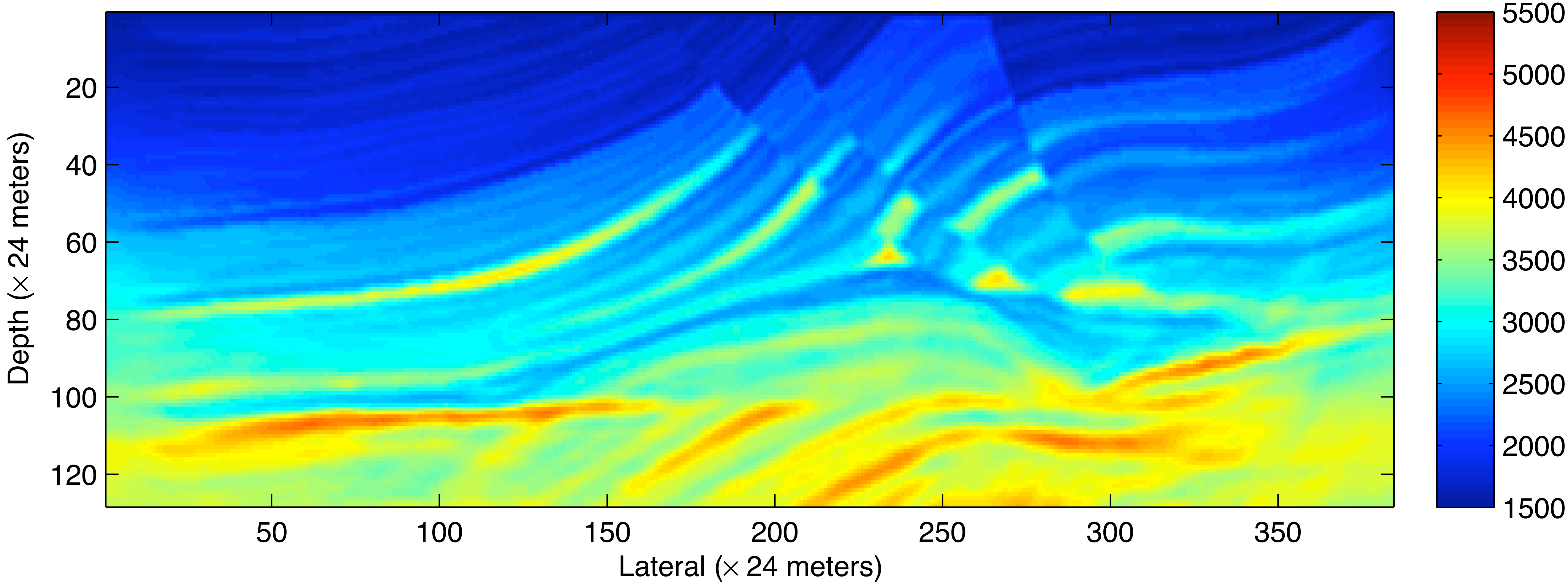
Initial model



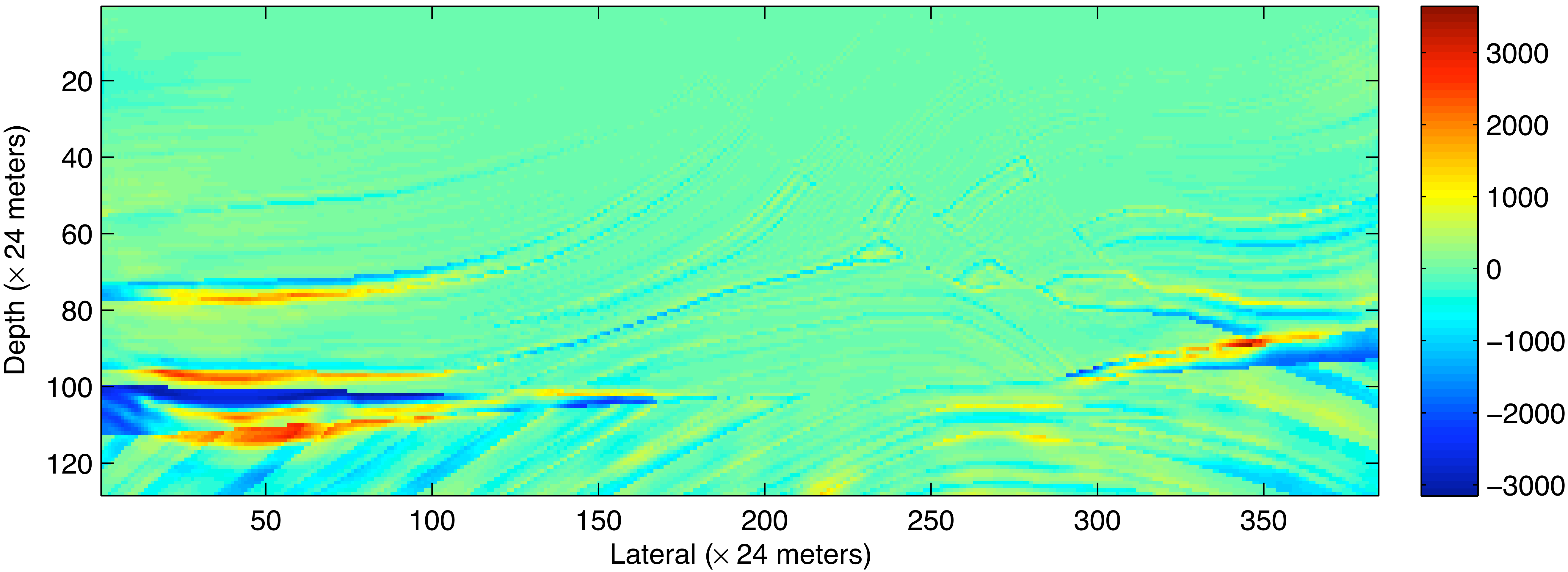
Inverted model



Inverted model

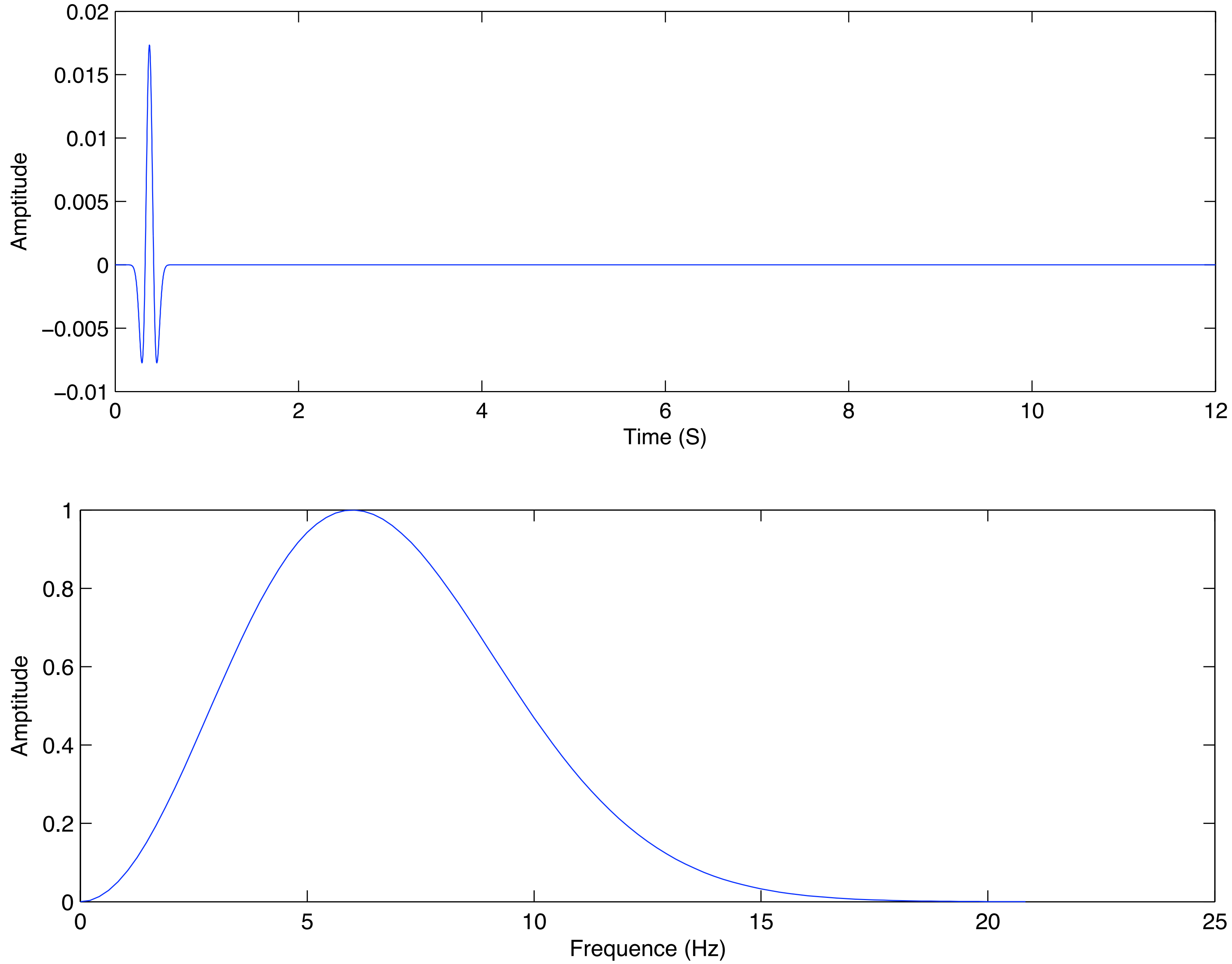


Difference

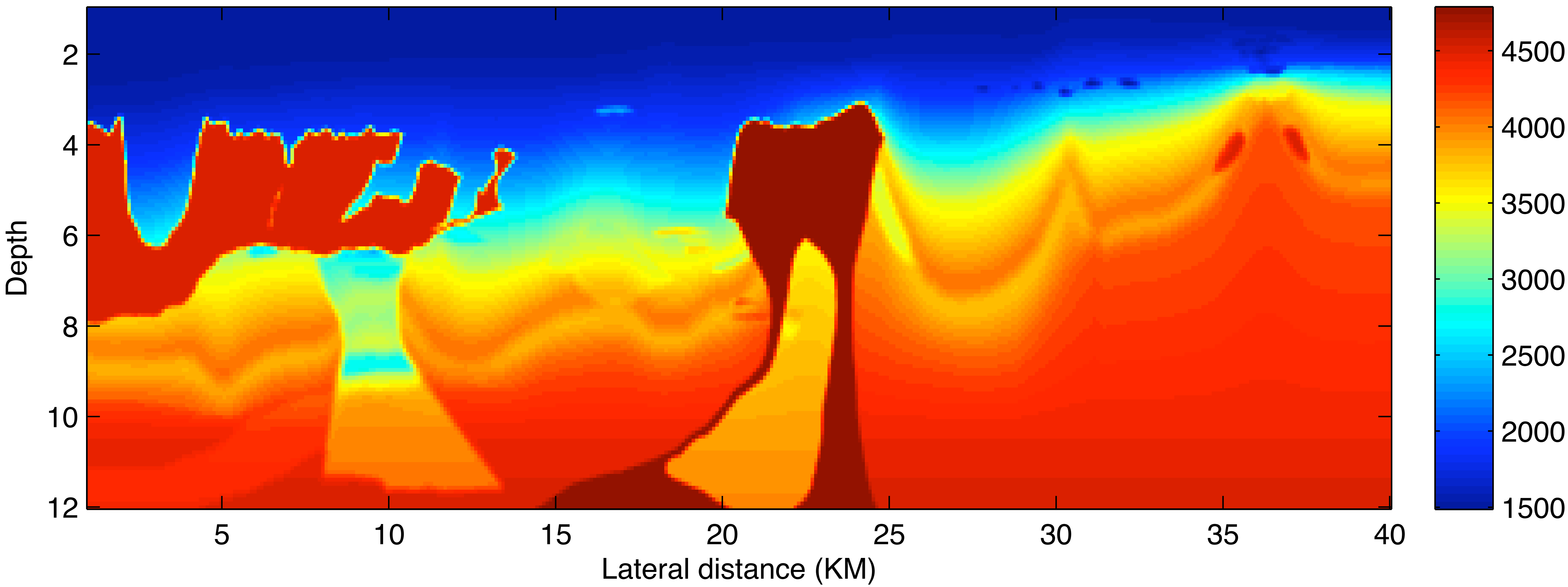


Depth (× 24 meters)
20
40
60
80
100
120
50
100
150
200
250
300
350
Lateral (× 24 meters)
3000
2000
1000
0
−1000
−2000
−3000

Source wavelet

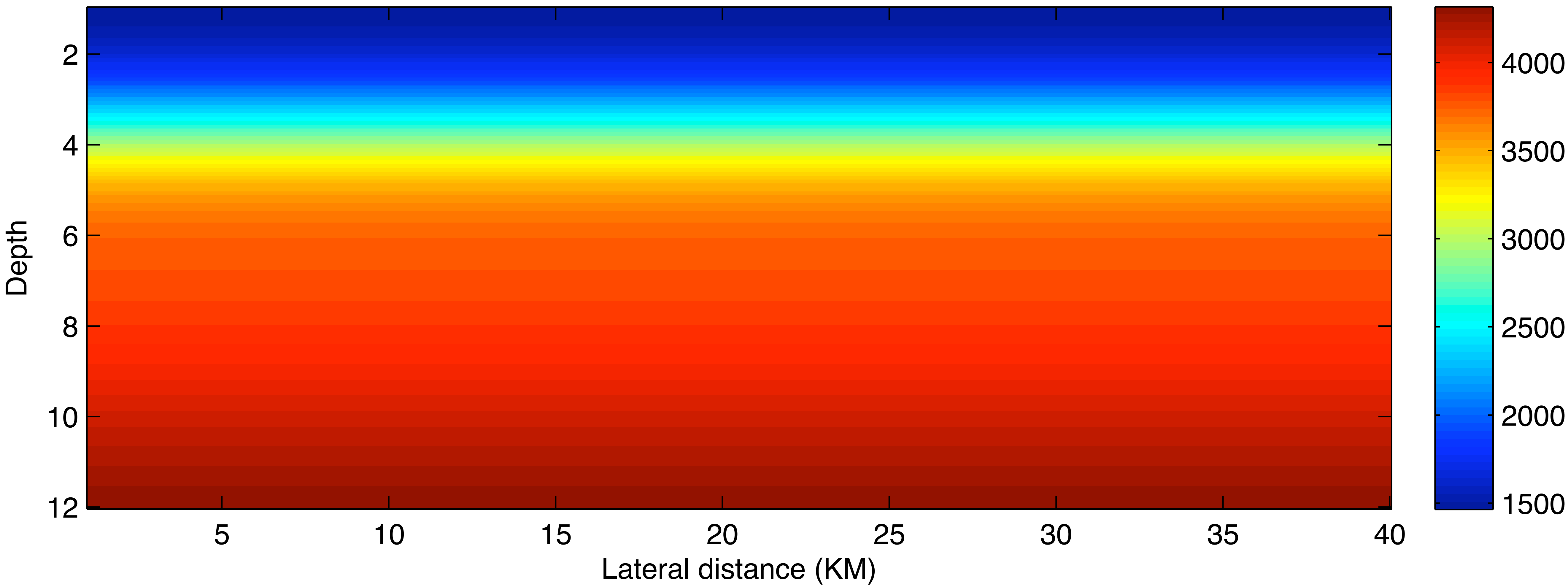


True model

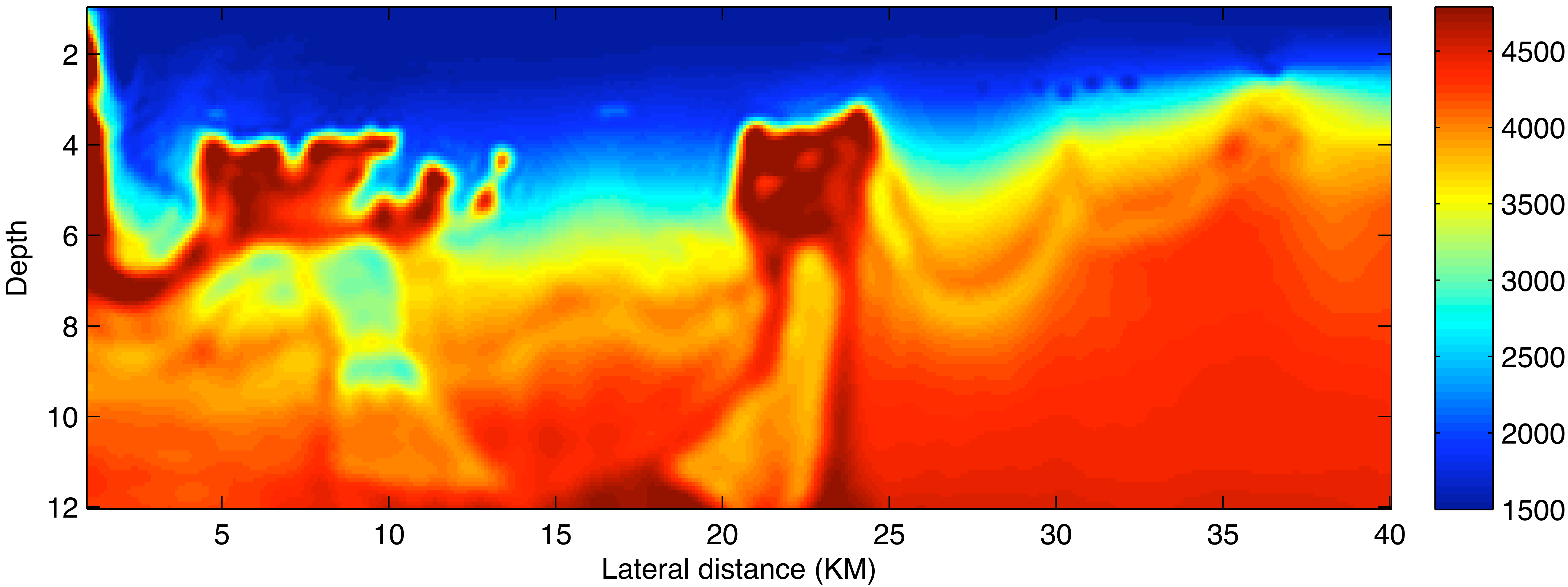


Depth
2
4
6
8
10
12
5
10
15
20
25
30
35
40
Lateral distance (KM)
4500
4000
3500
3000
2500
2000
1500

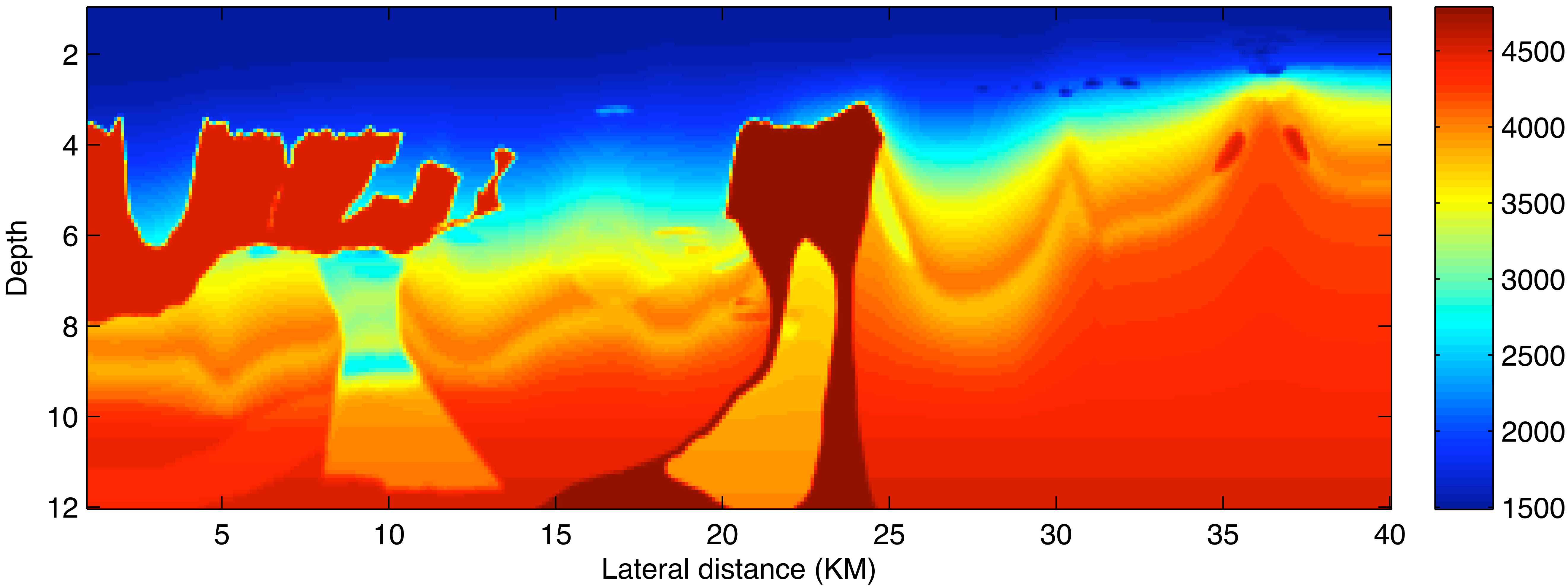
Initial model



Inverted model



True model


Depth
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30
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40
Lateral distance (KM)
4500
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3500
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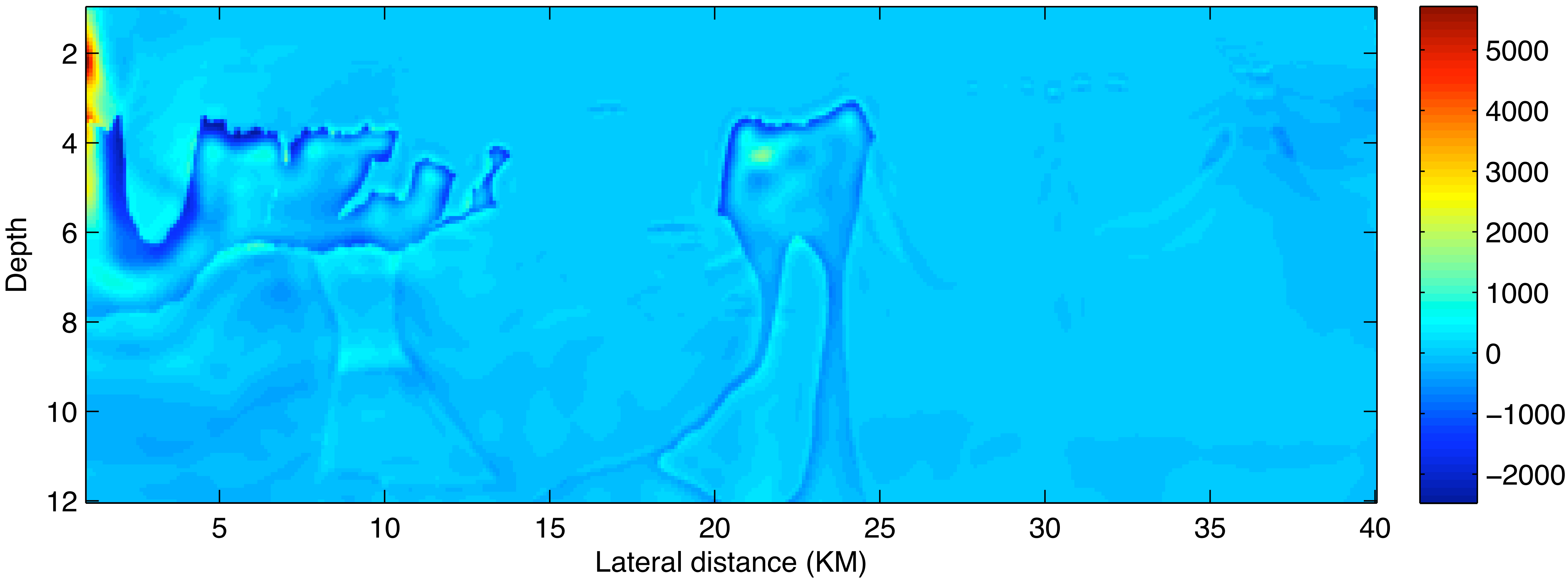
2500
2000
1500

Difference



Depth
2
4
6
8
10
12
5
10
15
20
25
30
35
40
Lateral distance (KM)
5000
4000
3000
2000
1000
0
−1000
−2000

Performance

Remember per *subproblem*

$$n_{PDE}^{\ell_1} \times K \ll n_{PDE}^{\ell_2} \times n_f \times n_s$$

$$\begin{aligned} n_{PDE}^{\ell_1} &\approx 100 \\ K &= 24 \end{aligned}$$

versus

$$\begin{aligned} n_{PDE}^{\ell_2} &\approx 10 \\ K &= 5800 \end{aligned}$$

SPEEDUP of 22 X

Conclusions

Because Compressive Sensing does *not* rely on *averaging* but on *sparsity*, our *approach is a viable* alternative to the *stochastic approximation*

Sparse recoveries offset *random interferences* due to *source encoding*

Hight-quality & high-resolution inversions have been *achieved with significant accelerations*

No need for additional *migration step*

Improvements come from *sparsity promotion & curvelets*

Indications that the *curse of dimensionality* can be removed...

Future plans

Investigate

- *Noise sensitivity*
- *continuation* with batch size
- explore multiscale structure of curvelets
- incomplete data
- extension to 3D

Acknowledgements

- authors of CurveLab.
- My colleagues.

SINBAD

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Thank you

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Software release is soon!

Further reading

Compressive sensing

- *Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information by Candes, 06*.
- *Compressed Sensing* by D. Donoho, '06
- Curvelets and Wave Atoms for Mirror-Extended Images by L. Demanet, L. Ying, 07.

Simultaneous acquisition

- *A new look at simultaneous sources* by Beasley et. al., '98.
- *Changing the mindset in seismic data acquisition* by Berkhout '08.

Simultaneous simulations, imaging, and full-wave inversion:

- *Faster shot-record depth migrations using phase encoding* by Morton & Ober, '98.
- *Phase encoding of shot records in prestack migration* by Romero et. al., '00.
- *Efficient Seismic Forward Modeling using Simultaneous Random Sources and Sparsity* by N. Neelamani et. al., '08.
- *Compressive simultaneous full-waveform simulation* by FJH et. al., '09.
- *Fast full-wavefield seismic inversion using encoded sources* by Krebs et. al., '09
- *Randomized dimensionality reduction for full-waveform inversion* by FJH & X. Li, '10

Stochastic optimization and machine learning:

- *A Stochastic Approximation Method* by Robbins and Monro, 1951
- *Neuro-Dynamic Programming* by Bertsekas, '96
- *Robust stochastic approximation approach to stochastic programming* by Nemirovski et. al., '09
- *Stochastic Approximation and Recursive Algorithms and Applications* by Kushner and Lin
- *Stochastic Approximation approach to Stochastic Programming* by Nemirovski
- *An effective method for parameter estimation with PDE constraints with multiple right hand sides. by* Eldad Haber, Matthias Chung, and Felix J. Herrmann. '10