A Linear-Operator Toolbox for Matlab

Michael P. Friedlander
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$$Ax \approx b$$

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Signal and image processing

- b is observed signal
- x is "true" signal or its representation in some basis/dictionary

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Spot Toolbox

Keeping the "mat" in Matlab

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A = F'*diag(F*v)*F;
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Implementing matrix-vector products

$$y = Ax = F^* \Lambda F x \qquad \Longleftrightarrow \qquad y = \texttt{ifft2(fft2(}a_1\texttt{).*fft2(x))}$$

Blurring Demo

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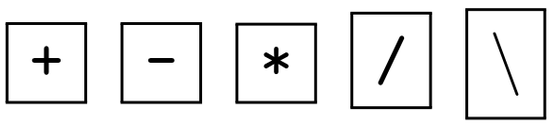

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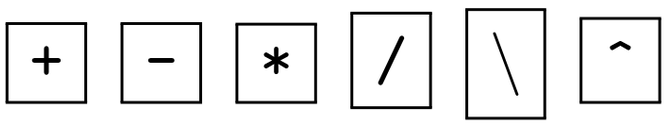

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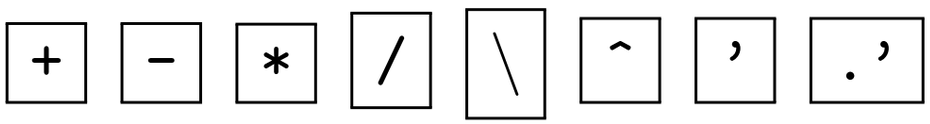

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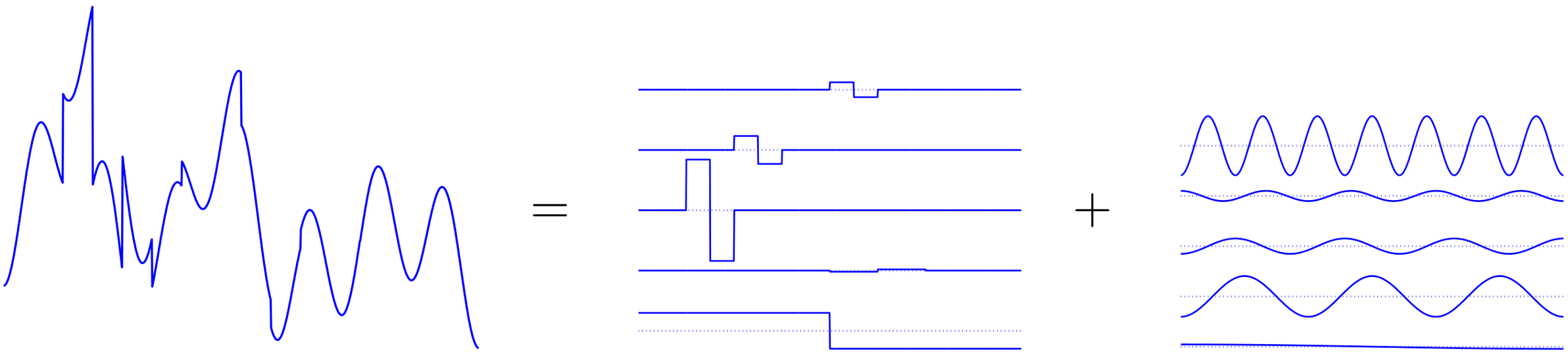
Example 2: Sparse Recovery



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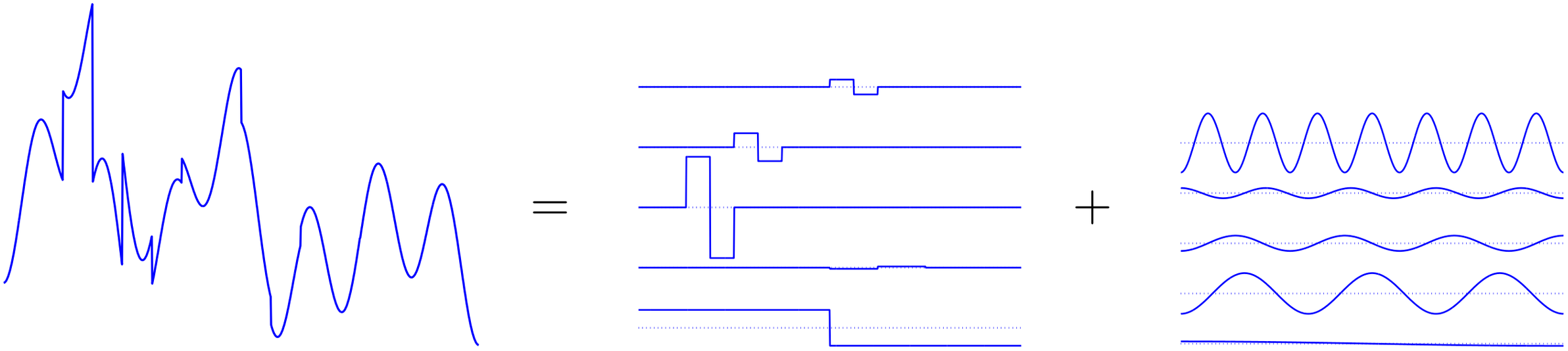

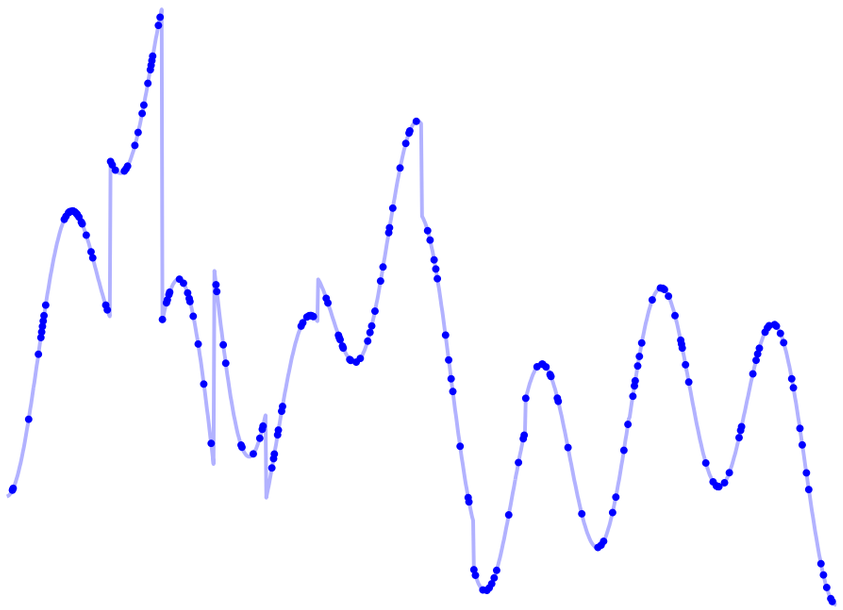

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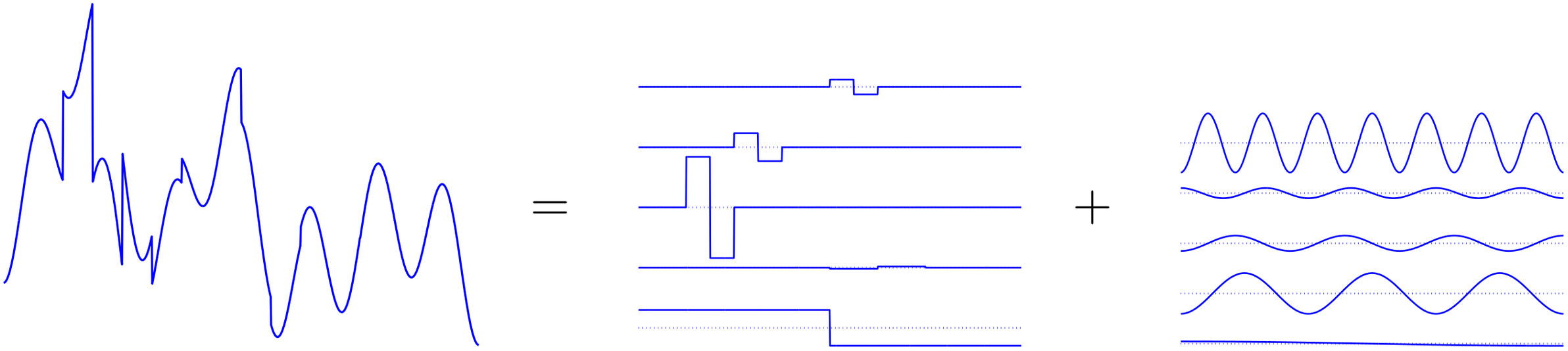

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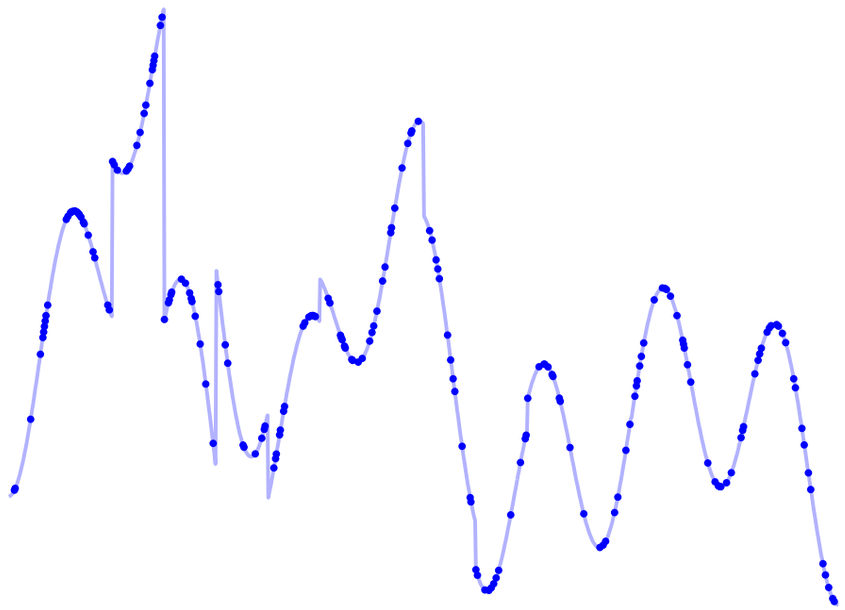

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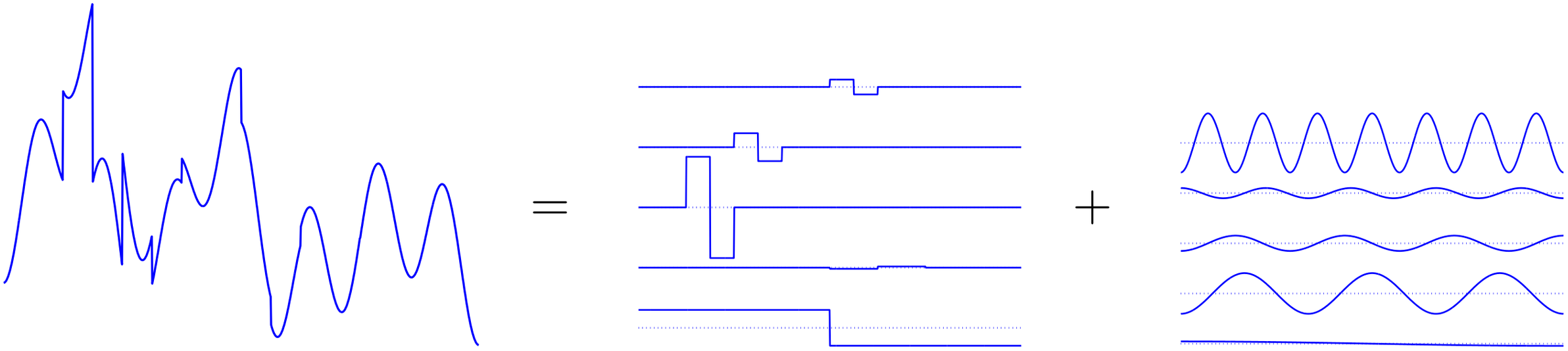

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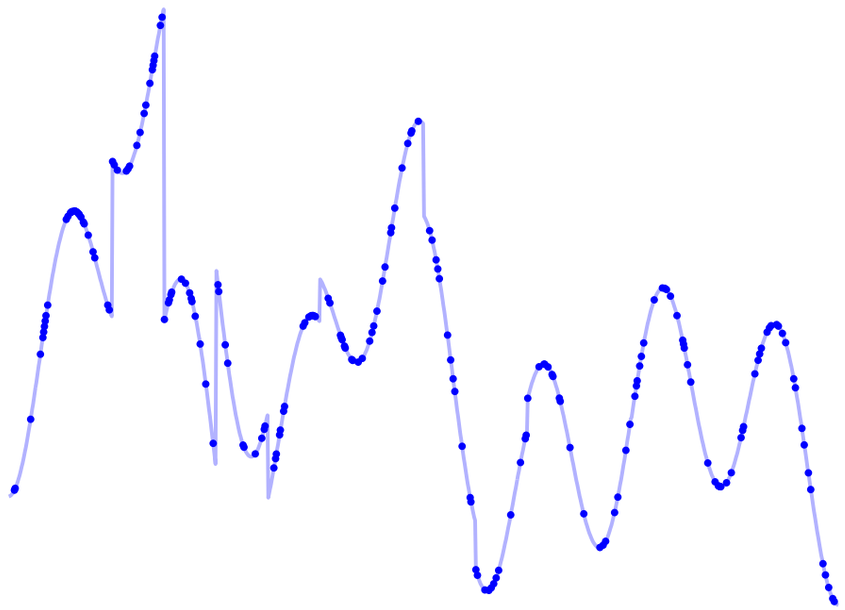

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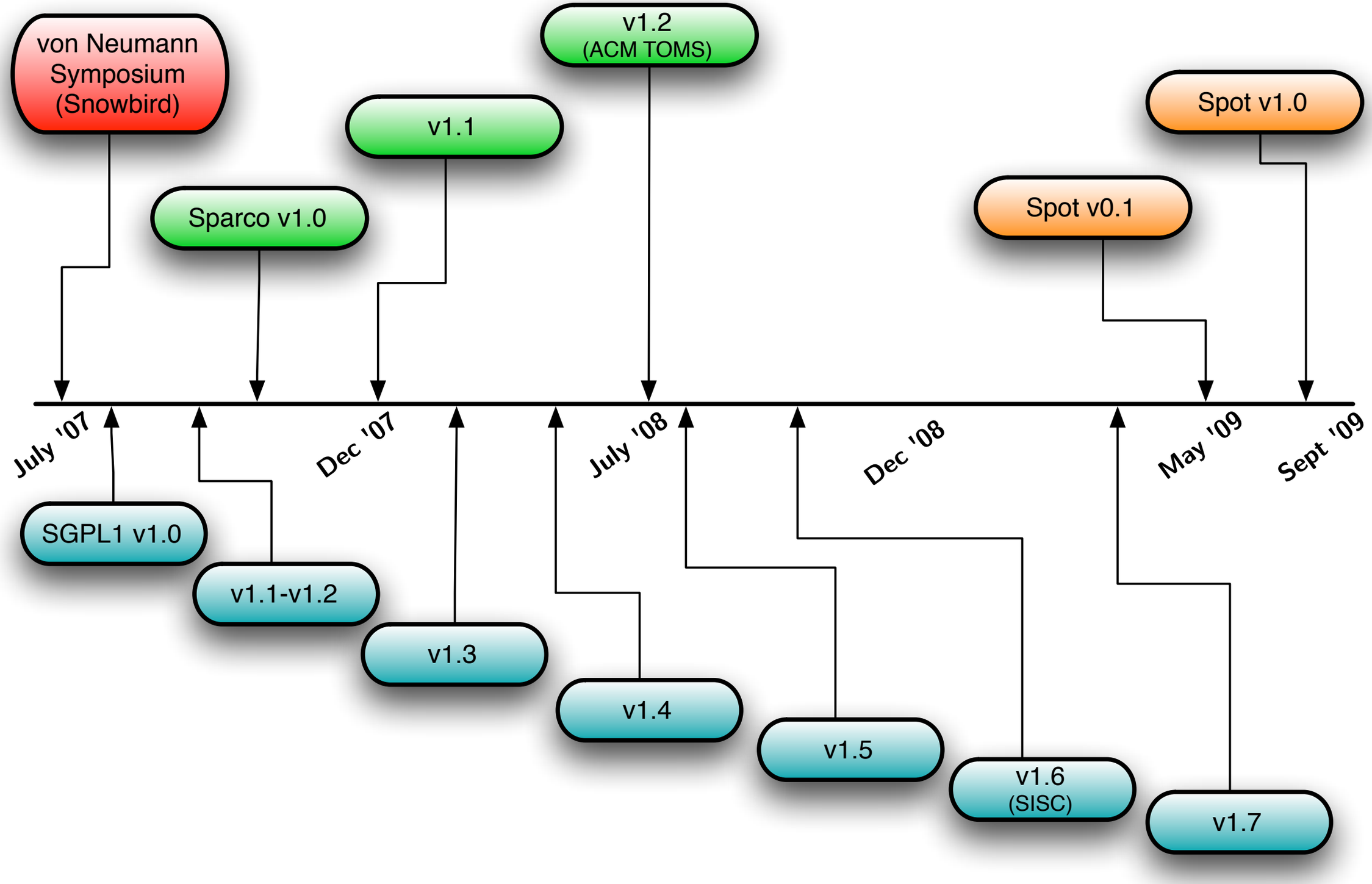
Sparse Recovery Demo

Extensibility

```
classdef opDCT < opSpot
%  D = opDCT(N) creates a 1-D DCT for length-N signals
   methods
      function op = opDCT(n)
      %opDCT constructor.
         op = op@opSpot('DCT',n,n);
      end
      function y = multiply(op,x,mode)
      %multiply  apply the DCT operator.
         if mode == 1
            y = dct(x);    % ie, y = D*x
         else
            y = idct(x);   % ie, y = D'*x
         end
      end
   end % methods
end % classdef
```

Development History



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`http://www.cs.ubc.ca/labs/scl/spot`