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Convex Composite Optimization

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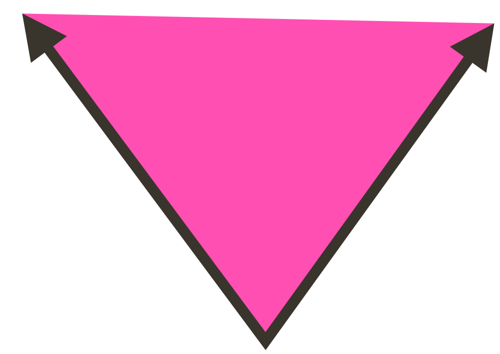
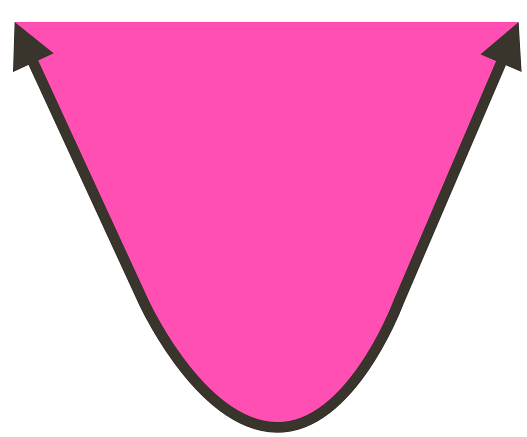
Convexity

- Key Concept in optimization, both in theory and algorithm design
- More important than differentiability
- Powerful algorithms and software for convex problems have been developed

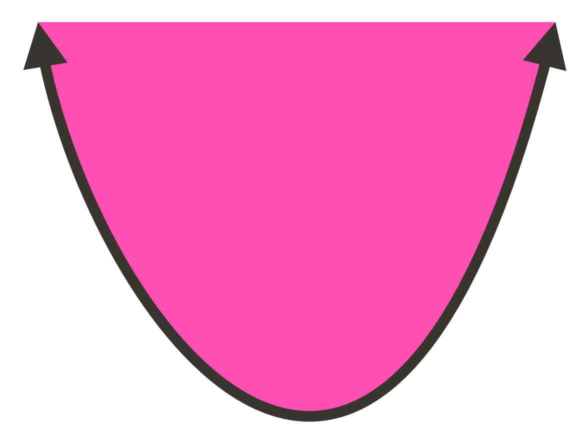
$f(x) = \|x\|_1$



$f(x) = \text{Huber}$



$f(x) = \frac{1}{2}\|x\|_2^2$



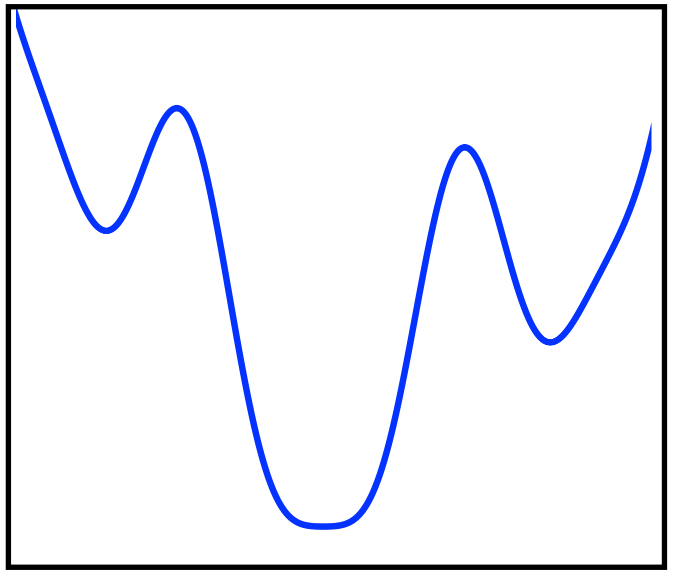
Beyond Convex

- Often we want to minimize functions that are NOT convex
- Example: $f(m) = \|D - RH^{-1}[m]Q\|_F^2$
- Even so, some convex structure remains:

$$f(m) = {\color{red}\|}{\color{blue}D - RH^{-1}[m]Q}{\color{red}\|_F^2}$$

RED: CONVEX
BLUE: DIFFERENTIABLE

SCALAR EXAMPLE



VELOCITY

Convex Composite

- Convex composite optimization:

$$f(m) = h(g(m))$$

h	CONVEX
g	DIFF.

- Includes all smooth functions: $h(x) = x$

- Includes all convex functions: $g(m) = m$

- Includes many non-smooth non-convex problems

- Why is this interesting?

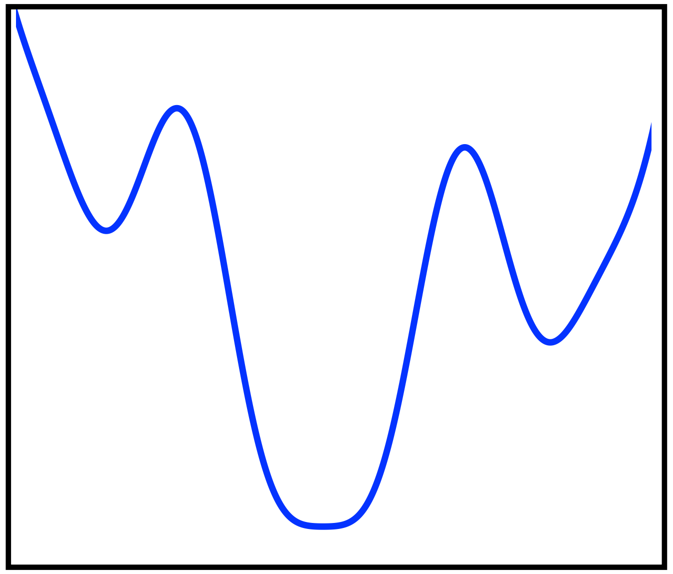
Extensions to FWI

- We would like to consider formulations

$$\min_m \rho_{\text{noise}}(D - RH^{-1}[m]Q) + \rho_{\text{reg}}(m)$$

- Two key applications: Robustness to outliers and sparsity regularization.

Scalar Example



Robust FWI

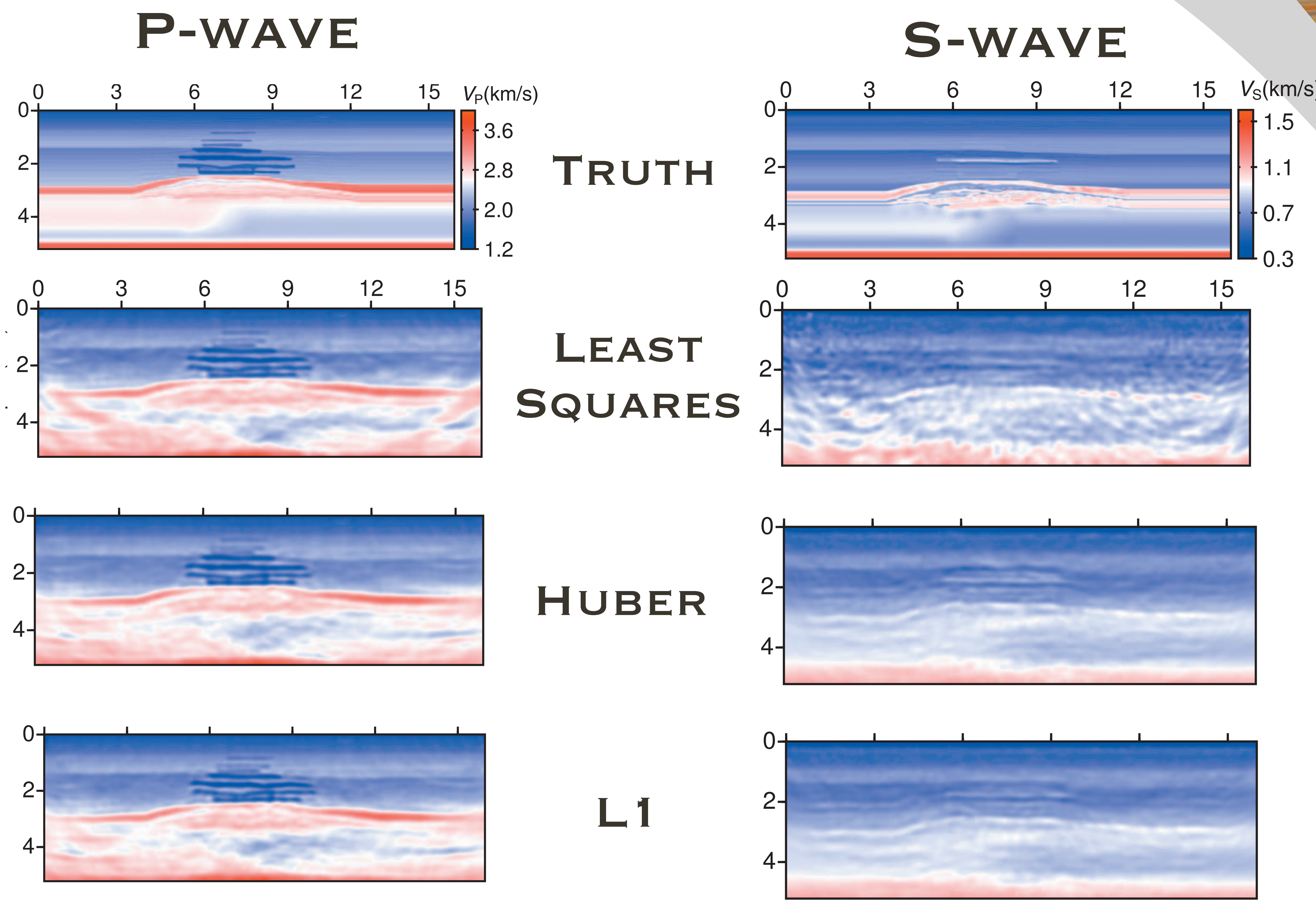
- The standard FWI formulation $(\rho_{\text{noise}} = \| \cdot \|_F^2)$ is equivalent to a normal error model for measurement errors:

$$D = RH^{-1}[m]Q + \epsilon$$

$$\epsilon \sim \mathcal{N}(0, I)$$

- Data error in the real world may not have this distribution, may have large outliers, or may have large systematic features we cannot model.

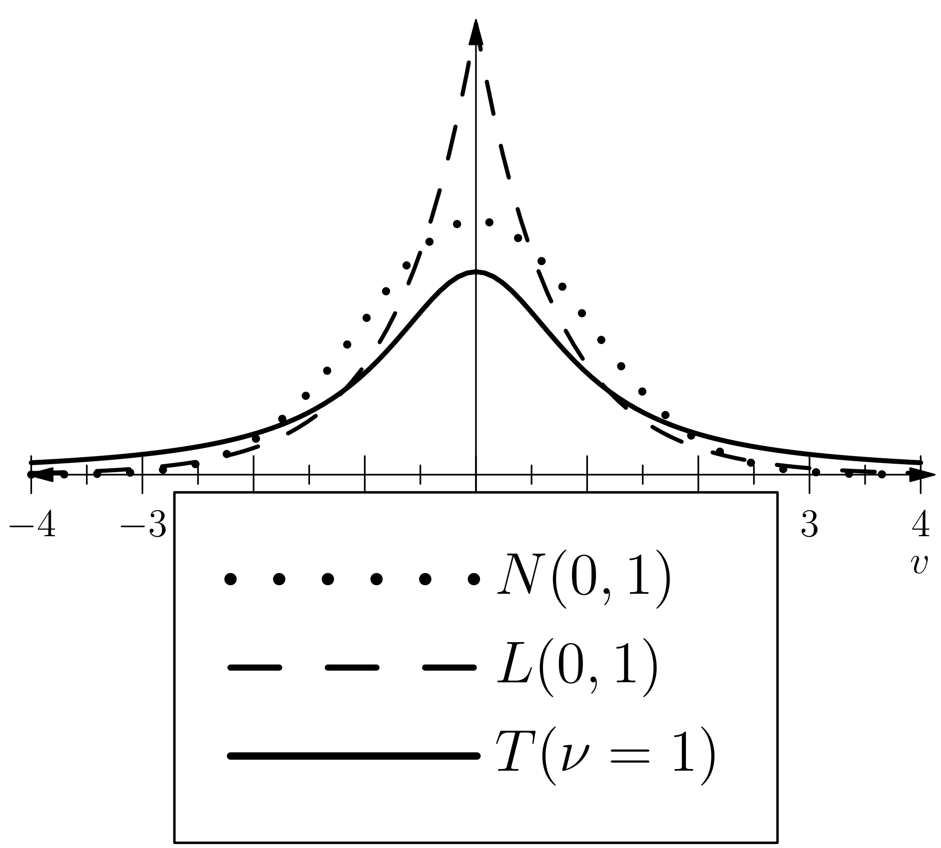
[Brossier et al. 2010, Guitton & Symes 2003]

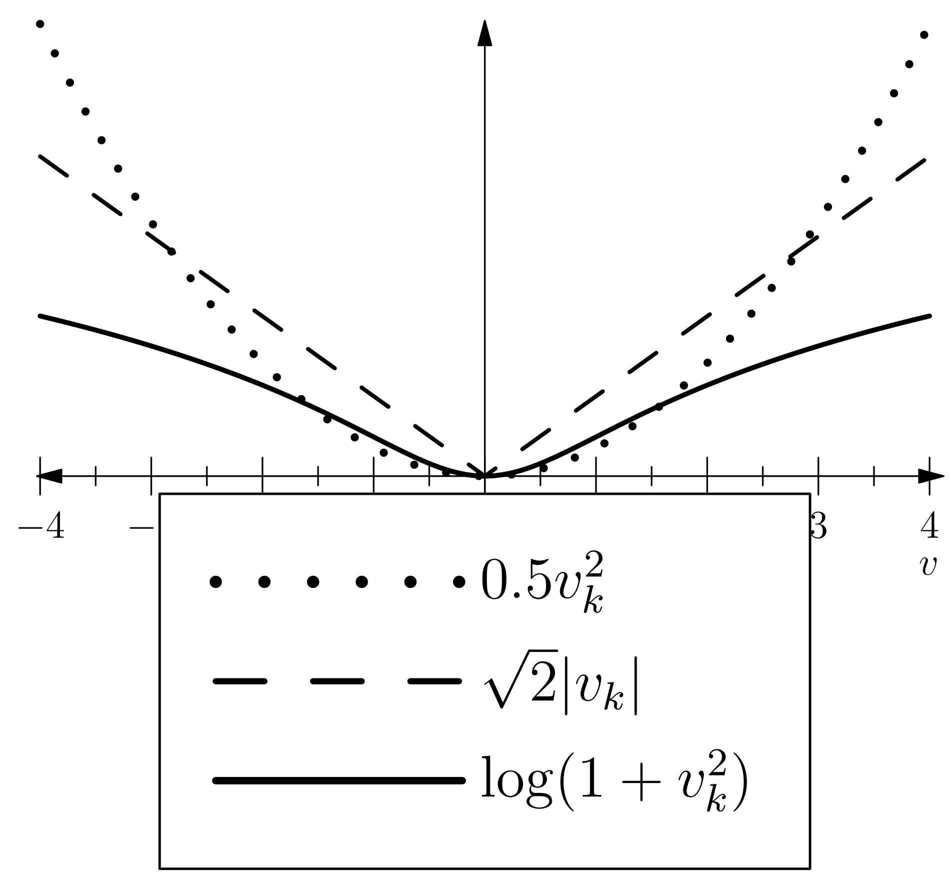


- When there are outliers in the data, Huber and L1 recover better.

[Brossier et al. 2010]

Heavy Tailed Modeling



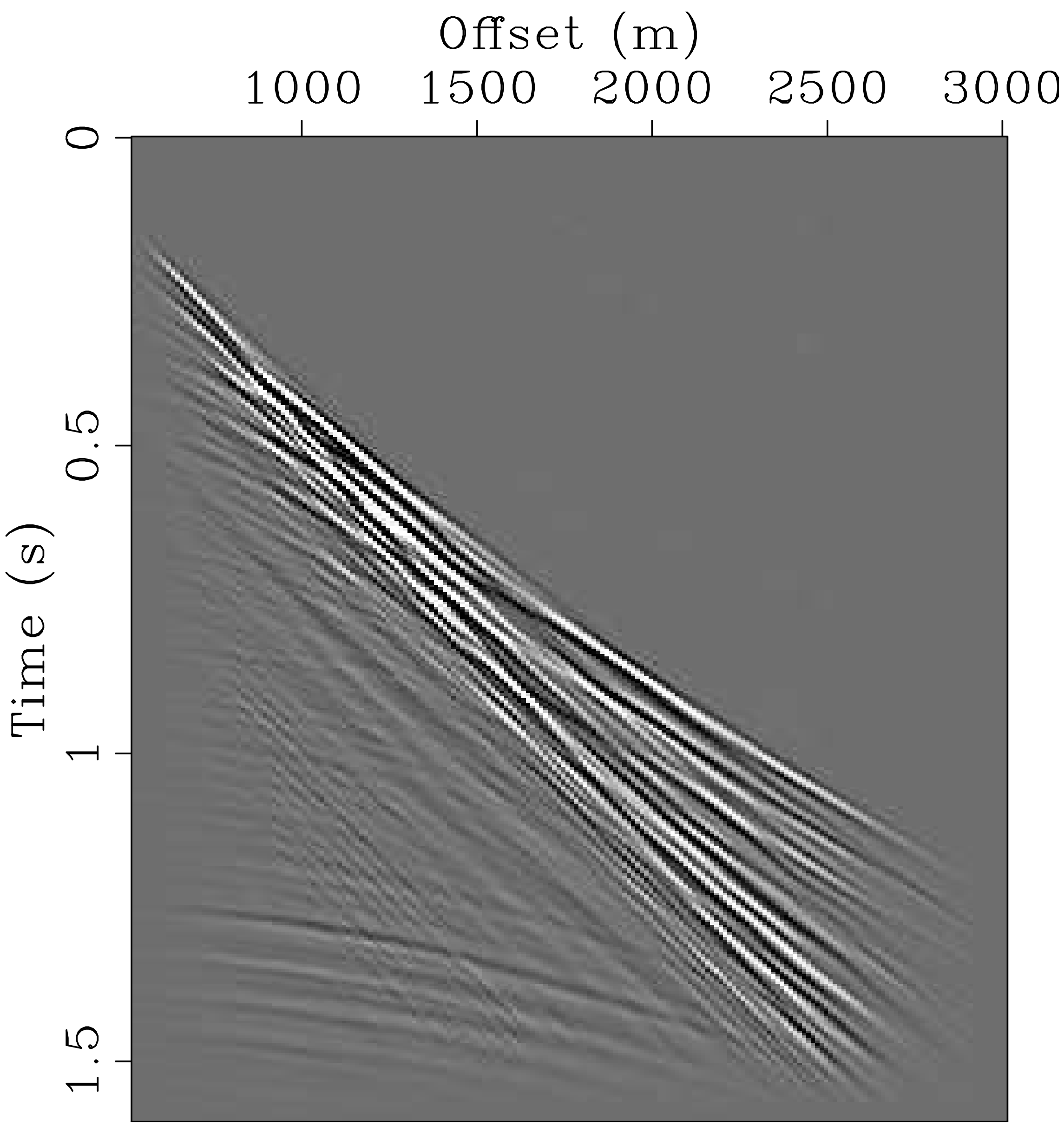


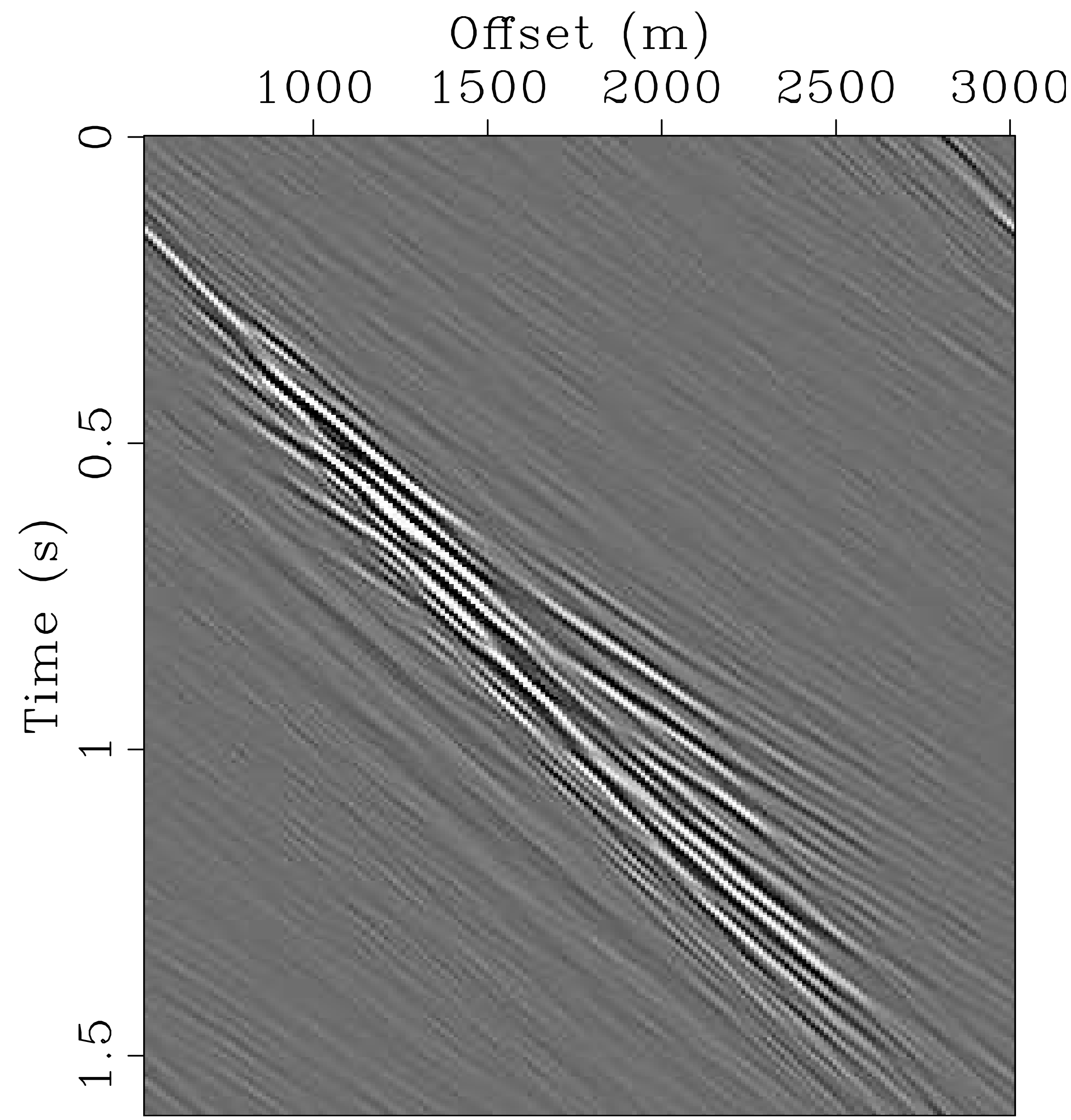
[Aravkin 2010]

Regularized FWI

- The standard FWI formulation $(\rho_{\text{error}} = 0)$ does not exploit the fact that the solutions to FWI are sparse in Fourier, Wavelets, and Curvelets
- Adding a sparsity regularization term $(\rho_{\text{error}} = \|C * x\|_1)$ is a good regularization strategy for FWI
- Resulting problems are again Convex Composite!

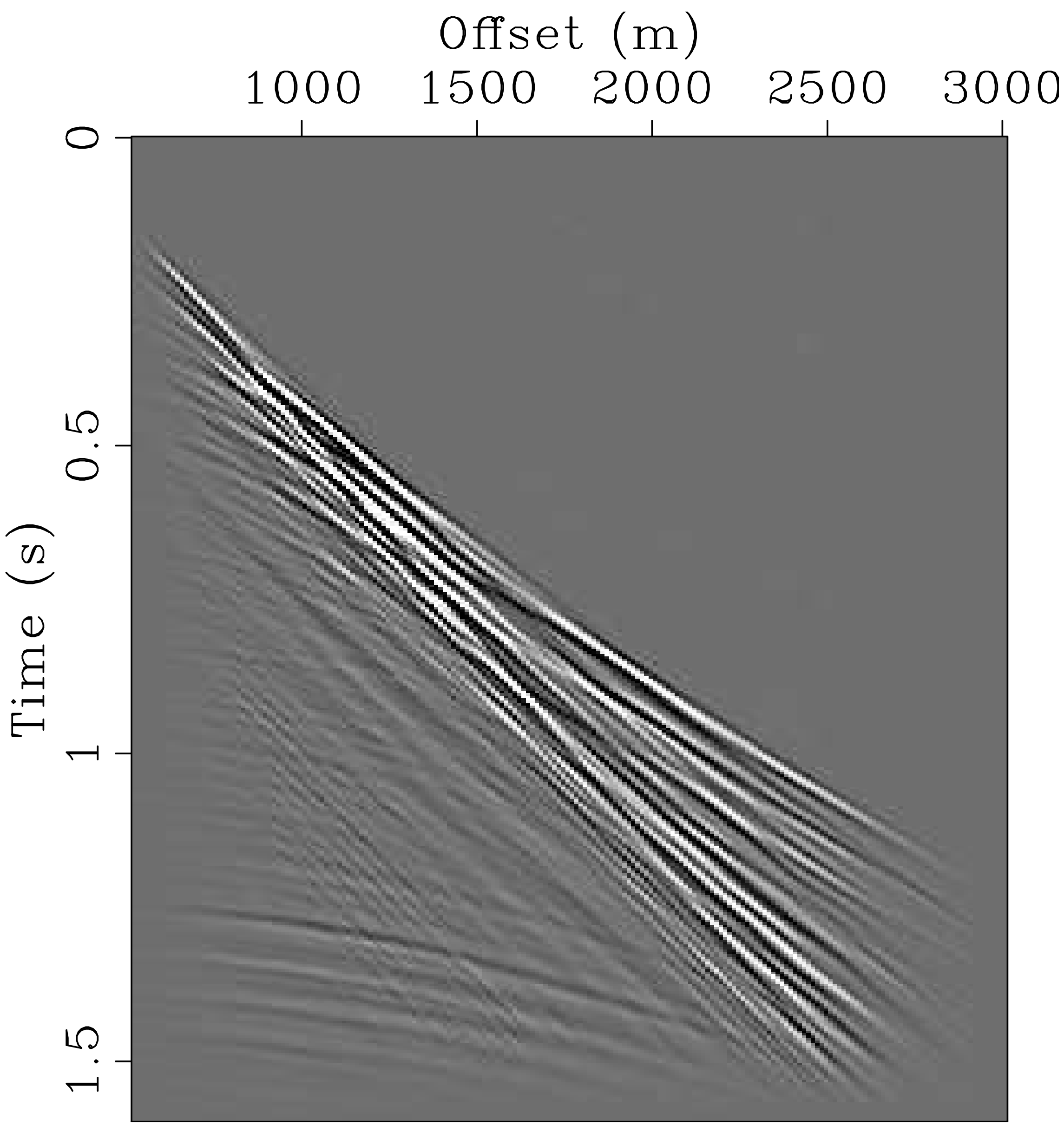
Fourier reconstruction

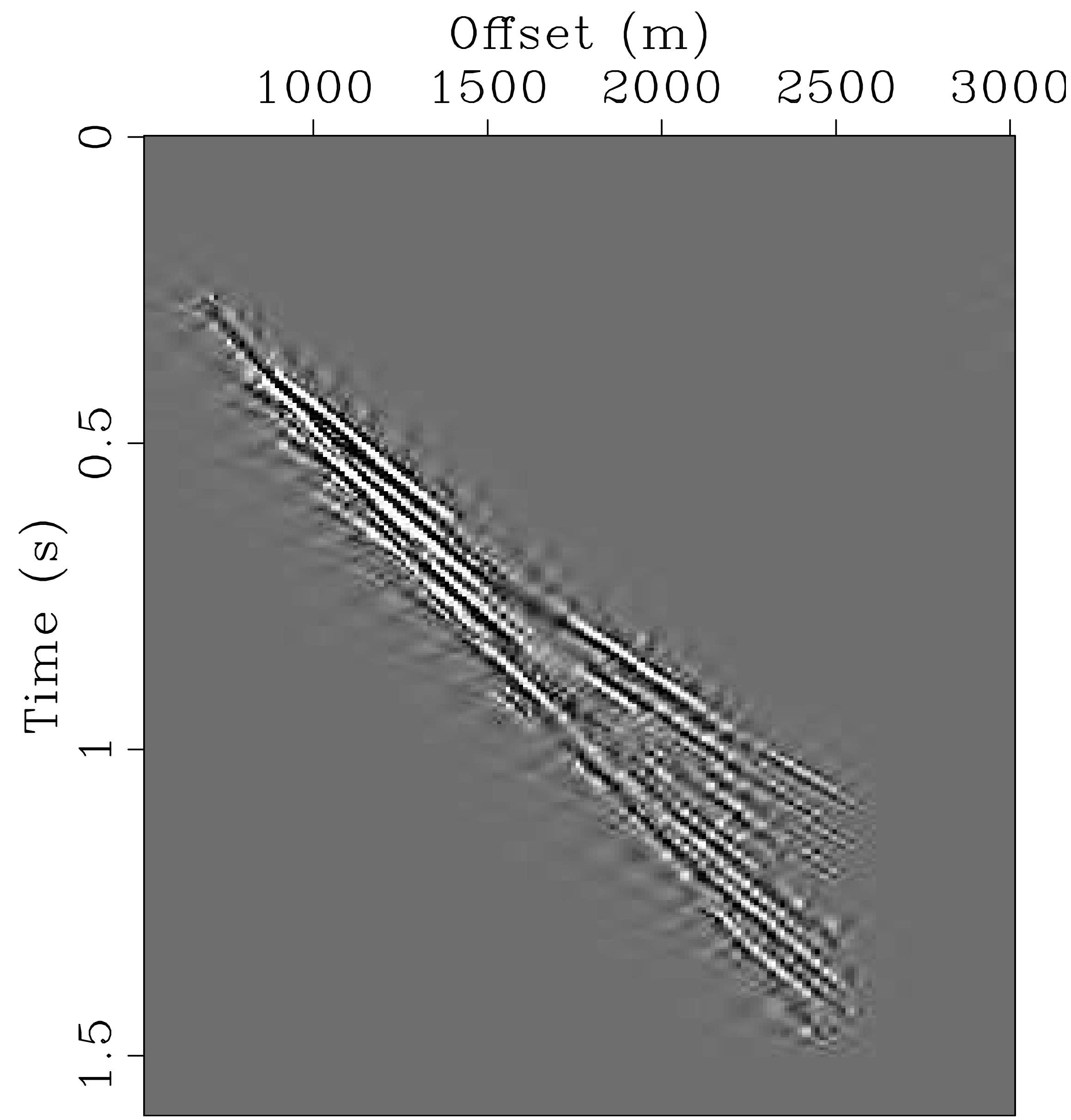




1 % of coefficients

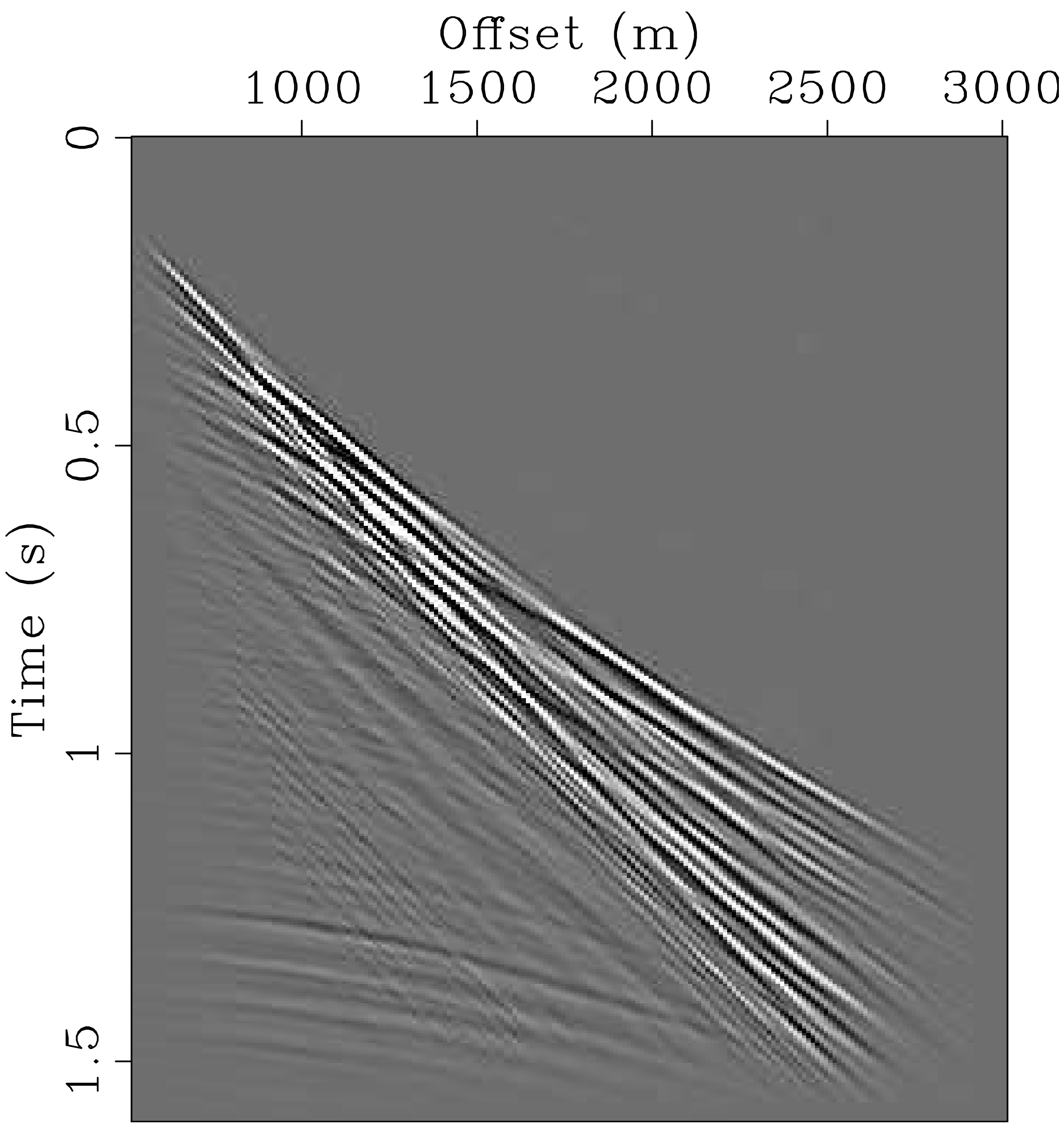
Wavelet reconstruction

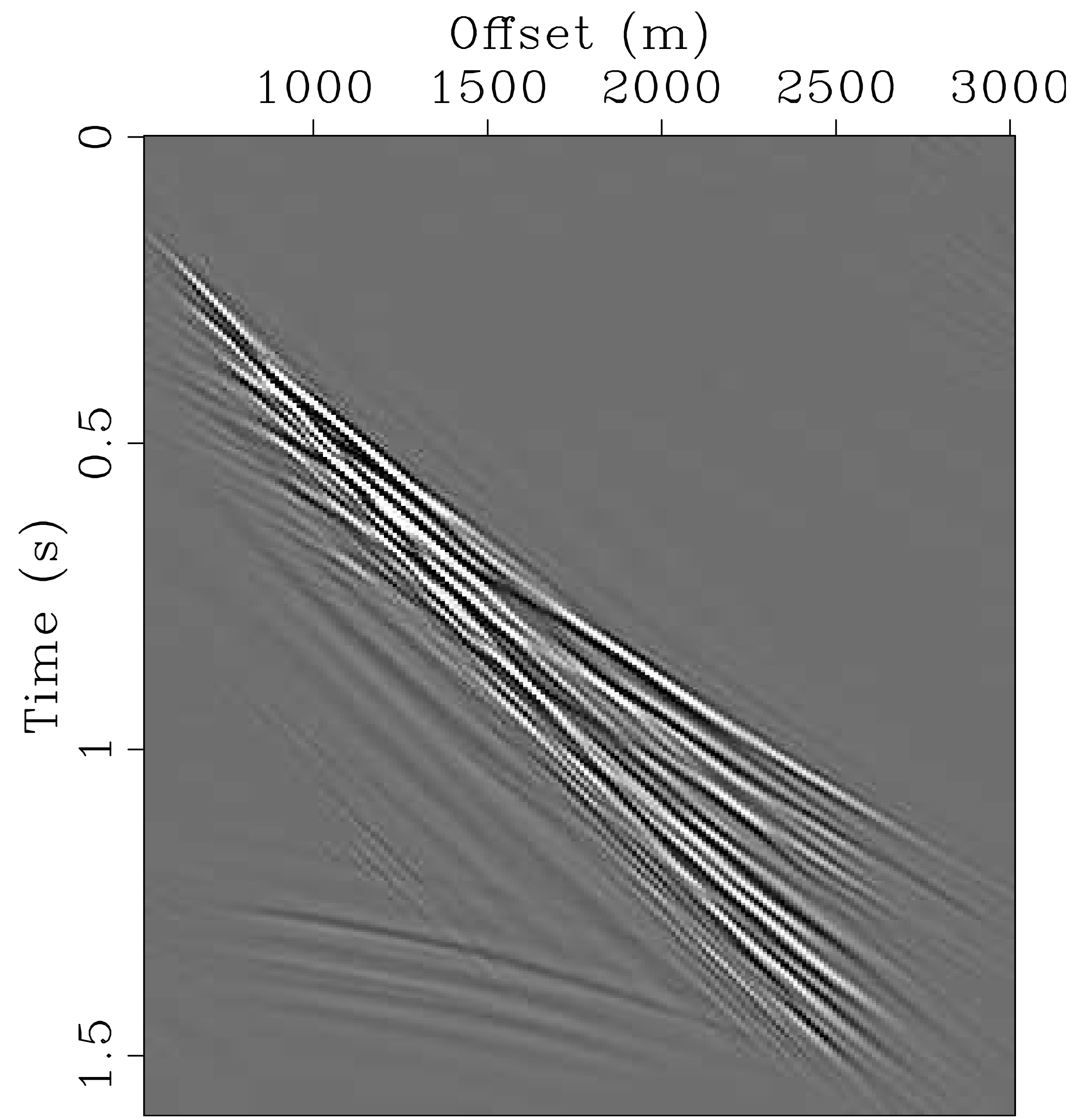




1 % of coefficients

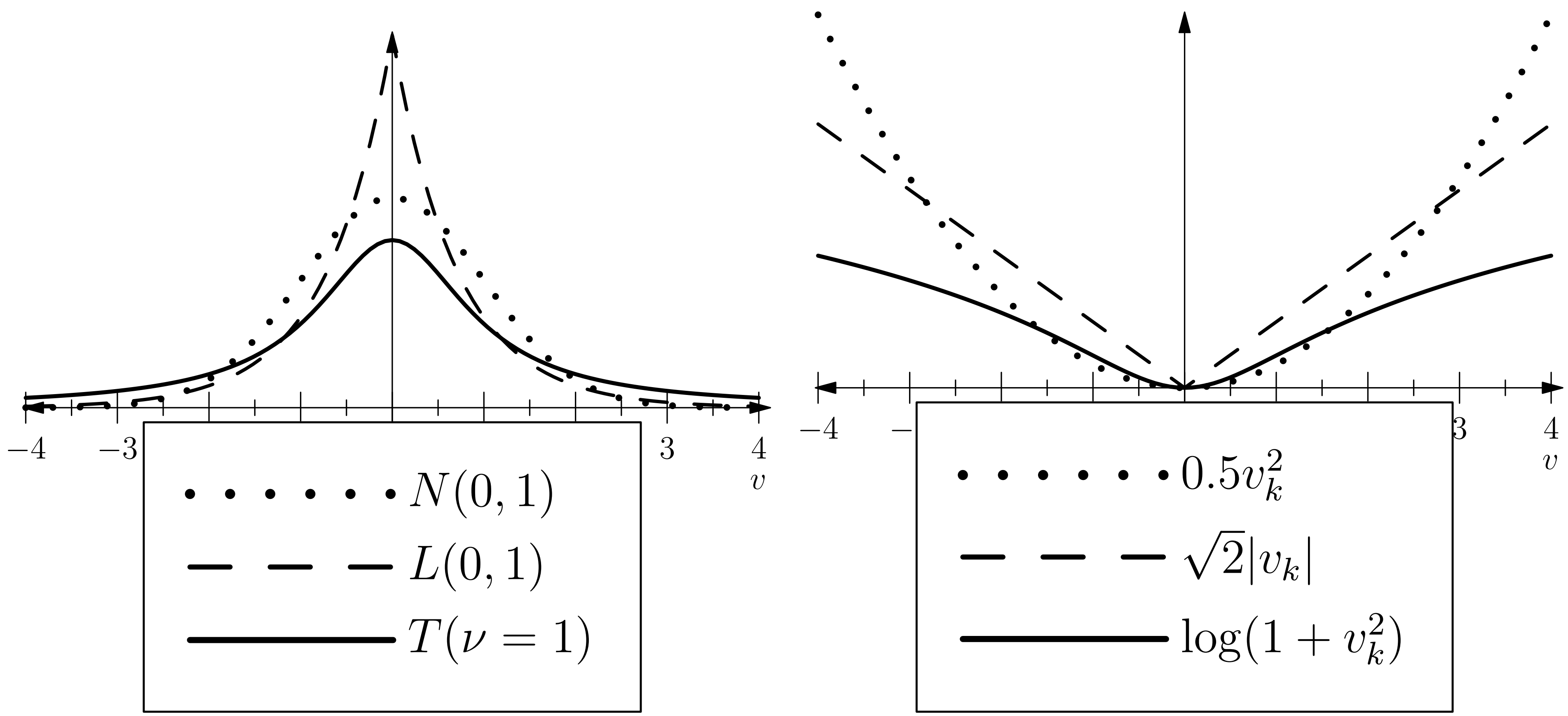
Curvelet reconstruction





1 % of coefficients

Behavior at 0



[Aravkin 2010]

Convex Composite

- Convex composite optimization:

$$f(m) = h(g(m))$$

h	Convex
g	Diff.

- Includes many non-smooth non-convex problems, e.g. Robust/Sparsity Regularized FWI

- How do we exploit this structure to design algorithms?

Gauss-Newton Method

- Objective: $$\min_{m} \|d - g(m)\|_2^2$$
- Iterative algorithm: $$m^{\nu+1} = m^{\nu} + \gamma_{\nu} s^{\nu}$$
- Direction s^{ν} solves $$\min_{\delta m} \|d - g(m^{\nu}) - \nabla g(m^{\nu})\delta m\|_2^2$$
- The subproblem for s^{ν} is convex!

CONVEX IN δm

SLIM

Extension of Gauss-Newton

- Objective: $$\min_{m} h(d - g(m))$$
- Iterative algorithm: $$m^{\nu+1} = m^{\nu} + \gamma_{\nu} s^{\nu}$$
- Direction s^{ν} solves $$\min_{\delta m} h(d - g(m^{\nu}) - \nabla g(m^{\nu})\delta m)$$
- The subproblem for s^{ν} is convex! **Convex in δm**

[Burke & Ferris 1993]

Examples

Convex Composite Objective	Convex Subproblem	
$\|d - g(m)\|_2^2 + \lambda\|m\|_1$	$\|d - g(m^\nu) - \nabla g(m^\nu)\delta m\|_2^2 + \lambda\|m + \delta m\|_1$	Sparsity promotion
$\rho_H(d - g(m))$	$\rho_H(d - g(m^\nu) - \nabla g(m^\nu)\delta m)$	Robust Optimization
$\rho_H(d - g(m)) + \lambda\|m\|_1$	$\rho_H(d - g(m^\nu) - \nabla g(m^\nu)\delta m) + \lambda\|m + \delta m\|_1$	Sparse & Robust

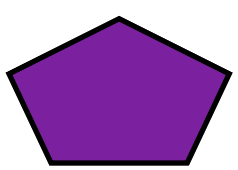
Constrained Optimization

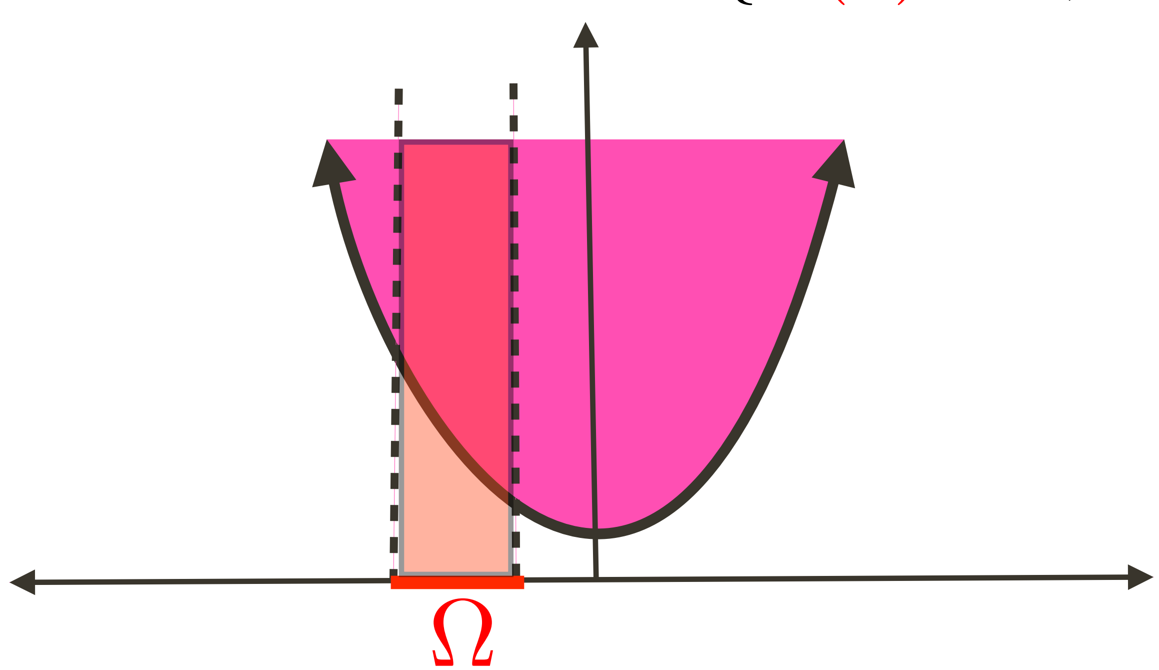
- Convex programs:

$$\begin{aligned} \min \quad & h(x) \\ \text{s.t.} \quad & x \in \Omega \end{aligned}$$

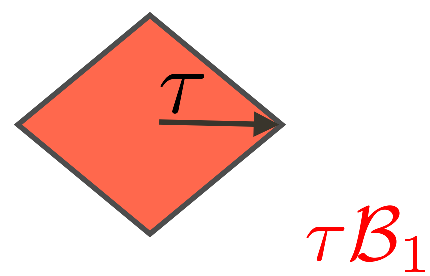
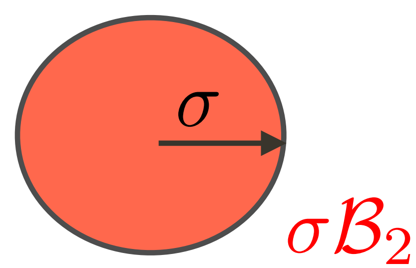
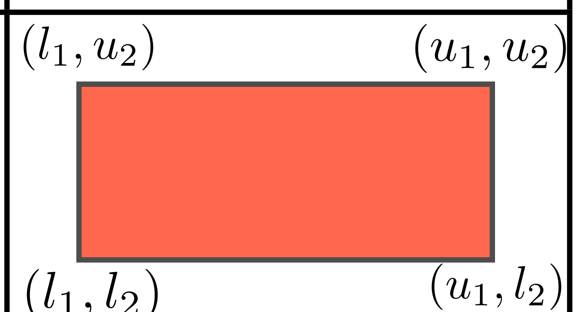
- More explicit representation: $\Omega = \{c_i(x) \leq 0,\ Ax = b\}$

- Polyhedral constraints: $\Omega = \{C(x) \leq c,\ Ax = b\}$





Examples

Formulation	Constraint Region	
$\min \; \|Ax-b\|_2^2$ s.t. $\|x\|_1 \leq \tau$ $(x \in \tau\mathcal{B}_{L_1})$		Lasso (Statistics)
$\min \; \|x\|_1$ s.t. $\|Ax-b\|_2 \leq \sigma$ $(Ax-b \in \sigma\mathcal{B}_2)$		Basis Pursuit Denoise
$\min \; \|Ax-b\|_2^2$ s.t. $l \leq x \leq u$		Regression w/Box Constr.

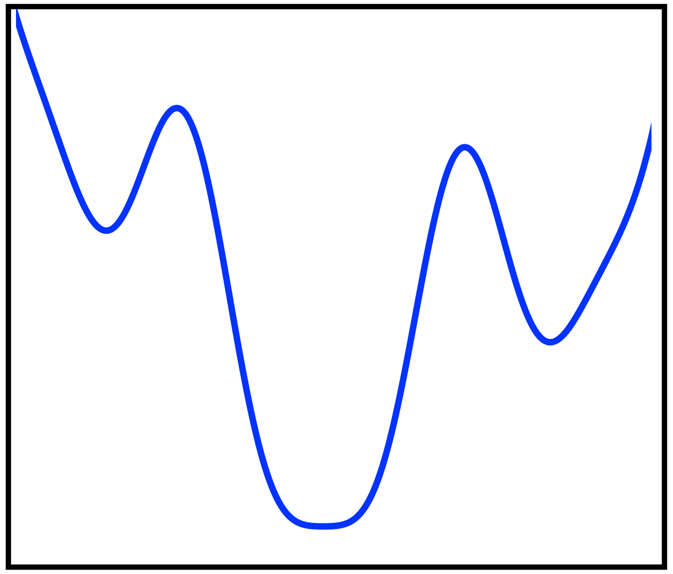
Back to FWI

- Suppose we wanted a LASSO type constraint for FWI in the Curvelet frame:

$$\begin{array}{ll} \min & \|D - RH^{-1}[C^* x]Q\|_F^2 \\ \text{s.t.} & \|x\|_1 \leq \tau \\ & (x \in \tau\mathcal{B}_1) \end{array}$$

- We are still in the convex composite setting!

Scalar Example



More Convex Composite

- Objective:

$$\begin{array}{ll} \min & h(d - g(m)) \\ \text{s.t.} & f(m) \in \Omega \end{array}$$

- Iterative algorithm:

$$m^{\nu+1} = m^{\nu} + \gamma_{\nu} s^{\nu}$$

- Direction s^{ν} solves

$$\begin{array}{ll} \min_{\delta m} & h(d - g(m^{\nu}) - \nabla g(m^{\nu})\delta m) \\ \text{s.t.} & f(m^{\nu}) + \nabla f(m^{\nu})\delta m \in \Omega \end{array}$$

- The subproblem for s^{ν} is convex!

CONVEX IN δm

[Burke 1989]

More Examples

Convex Composite Objective	Convex Subproblem	
$\min_{m} \; \|d - g(m)\|_2^2$ s.t. $\|m\|_1 \leq \tau$	$\min_{\delta m} \; \|d - g(m^\nu) - \nabla g(m^\nu)\delta m\|_2^2$ s.t. $\|m^\nu + \delta m\|_1 \leq \tau$	Nonlinear Lasso
$\min_{m} \; \|m\|_1$ s.t. $\|d - g(m)\|_2 \leq \sigma$	$\min_{\delta m} \; \|m^\nu + \delta m\|_1$ s.t. $\|d - g(m^\nu) - \nabla g(m^\nu)\delta m\|_2 \leq \sigma$	Nonlinear BPDN
$\min_{m} \; \rho_H(d - g(m))$ s.t. $l \leq m \leq u$	$\min_{\delta m} \; \rho_H(d - g(m^\nu) - \nabla g(m^\nu)\delta m)$ s.t. $l \leq m^\nu + \delta m \leq u$	Robust & Box Constr.

Exploiting Sparsity in FWI: Nonlinear BPDN

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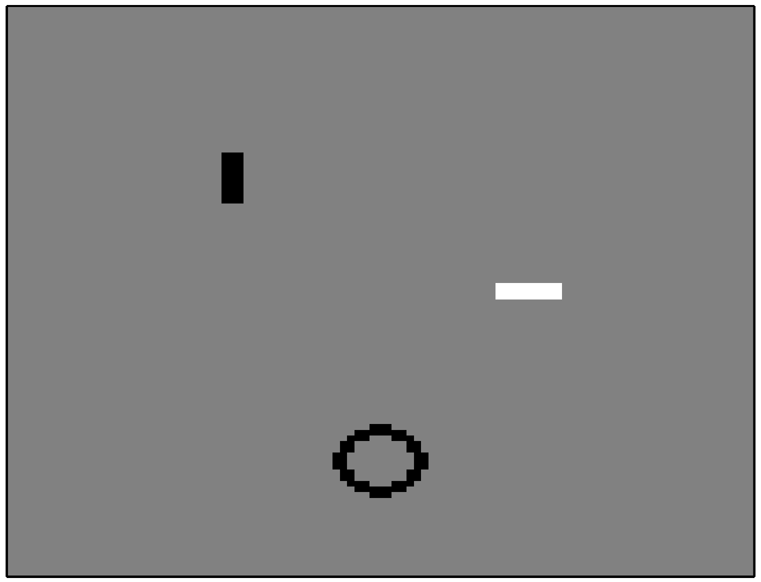
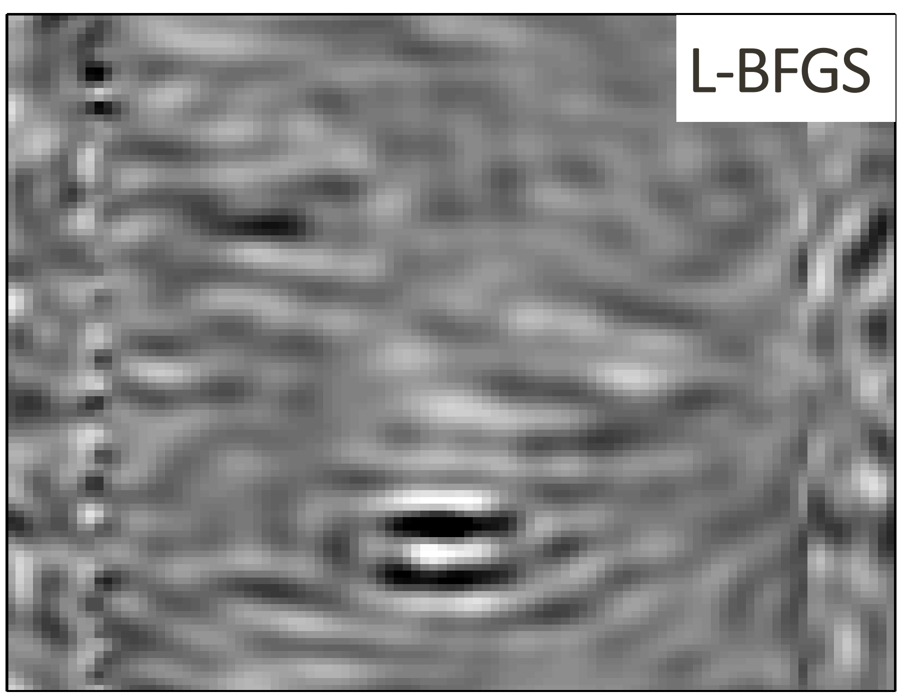
Presenter: Aleksandr Aravkin & Tristan van Leeuwen

True Model

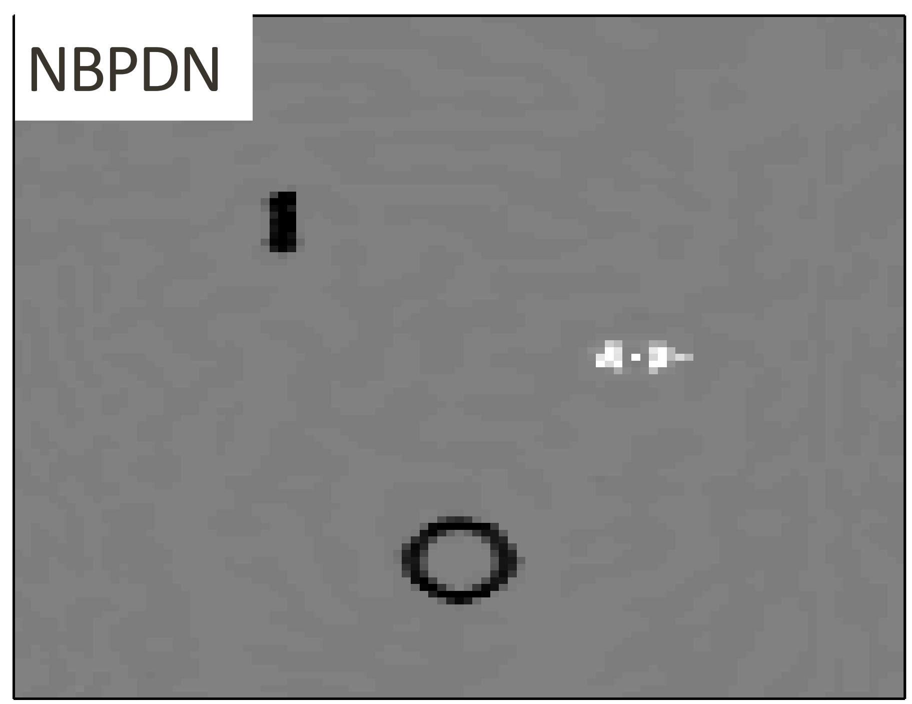


- Cross-well setting, 101 sources and receivers
- True model is sparse w.r.t to a constant reference model in pixel space
- Use of simultaneous shots to reduce computational load
- Compare to L-BFGS recovery without sparsity constraints

Simulateneous Shots: 5



Simulateneous Shots: 5



The Road Ahead

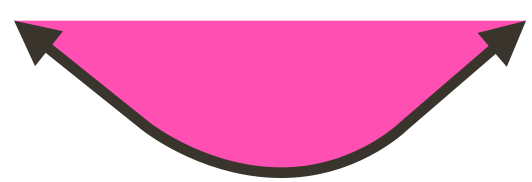
- Convex Composite optimization provides a solid theoretical framework for algorithm development

- Applications to FWI :

    1) Sparsity promotion (NBPDN)

$$\min_{m} \ \|m\|_1 \quad \text{s.t.} \quad \|d - g(m)\|_2 \leq \sigma$$

    2) Outlier-robust misfit (e.g. Huber)



    3) Using prior information (e.g. constraints for velocity modeling).

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References

Aravkin, Robust methods for Kalman filtering/smoothing and Bundle Adjustment, Ph.D. Thesis, University of Washington

Brossier, R., Operto, S., and Virieux, J., Which data residual norm for robust elastic frequency-domain full waveform inversion? *Geophysics, 75,3:37-46, 2010*

Burke, J.V., 1989, A sequential quadratic programming method for potentially infeasible mathematical programs, *Journal of Mathematical Analysis and Applications, 139,2:319-351*

Burke, J.V., 1992, A robust trust region method for constrained nonlinear programming problems, *Siam J. Optimization, 2,2:325-347, 1992*

Burke, J.V., and Ferris, M.C, A Gauss-Newton method for convex composite optimization, *Technical Report, University of Wisconsin-Madison, 1993*

Guitton, A., and Symes, W., Robust inversion of seismic data using the Huber norm, *Geophysics, 68,4:1310-1319, 2003*

R. Fletcher, Second order correction for nondifferentiable optimization, in *G. A. Watson, editor, Numerical Analysis, 85-114, Springer-Verlag, Berlin, 1982*

R. S. Womerseley, Local properties of algorithms for minimizing composite functions, *Mathematical Programming, 32:69-89, 1985*