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Exploiting Sparsity in FWI: Nonlinear BPDN

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Full Waveform Inversion

- The Full Waveform Inversion (FWI) problem is to find solutions to the Helmholtz PDE that match data from source experiments observed on the surface

- Commonly formulated as nonlinear least-squares problem:

$$\min_{m} \|D - RH^{-1}[m]Q\|_F^2$$

- FWI is ill-posed -- the observed data is not sufficient to recover the solution.
  1) Need to start close
  2) Can't iterate 'too long'

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Ill-posed Problems

Several techniques are used to deal with ill-posed problems

1) Small fixed iteration count

2) Regularization with respect to initial guess, e.g.:

$$\min_{m} \|D - RH^{-1}[m]Q\|_F^2 + \lambda \|m - m_0\|_2^2$$

3) Our approach: use sparsity by imposing L1 penalty

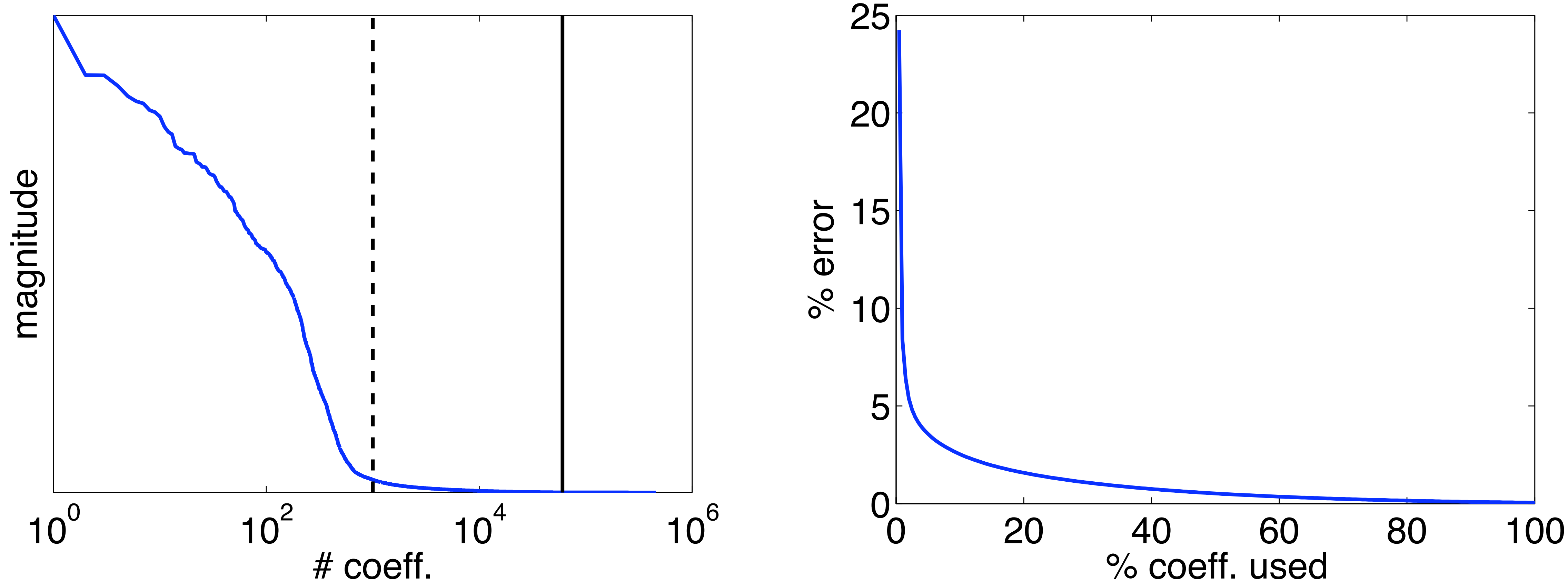
- Images and velocity models are sparse in Curvelets
- Time-lapse difference images are sparse

Sparsity in Curvelets

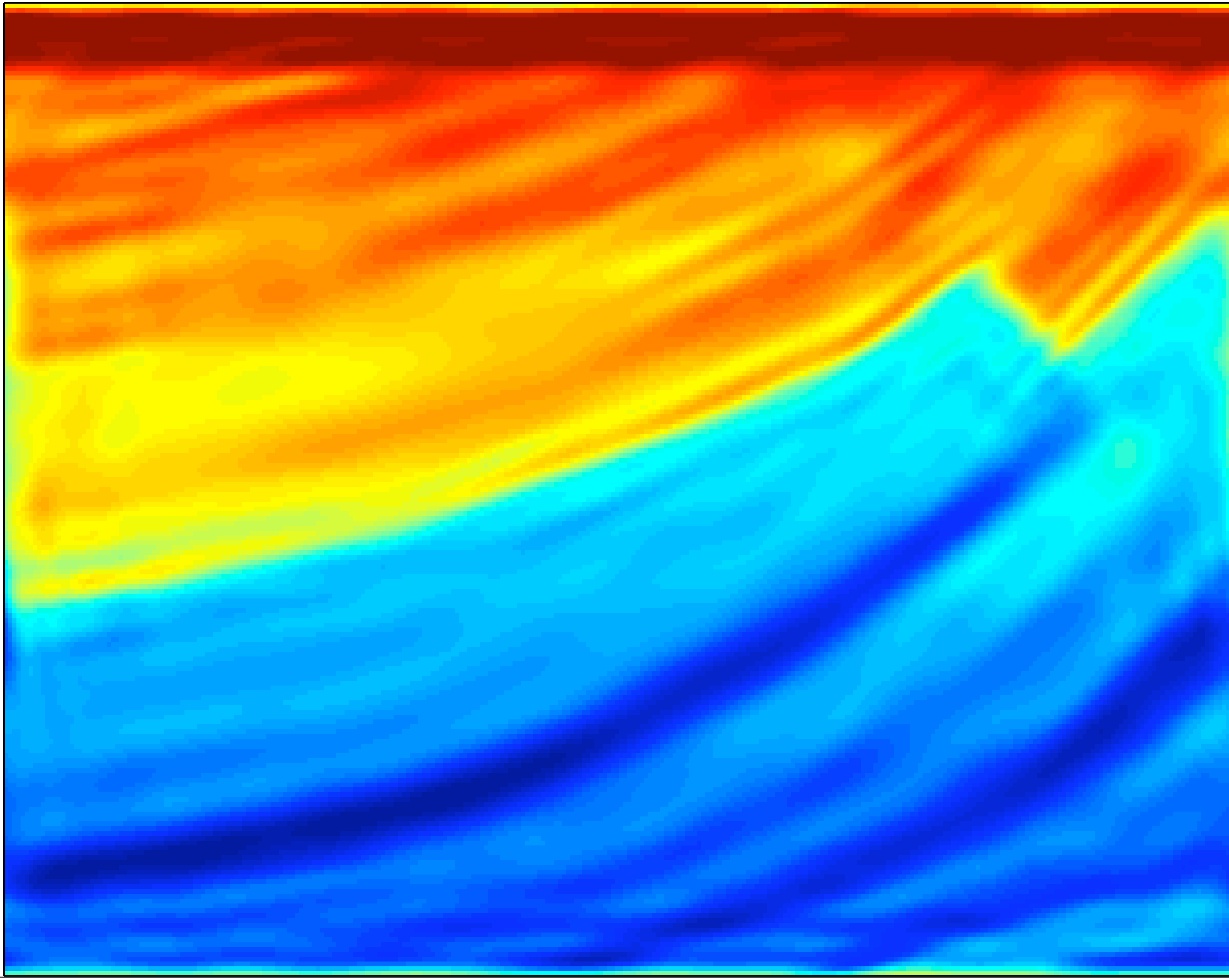
- Typical **velocity model** is sparse (compressible) in Curvelets:

$$\bar{m} = C^* \bar{x}$$

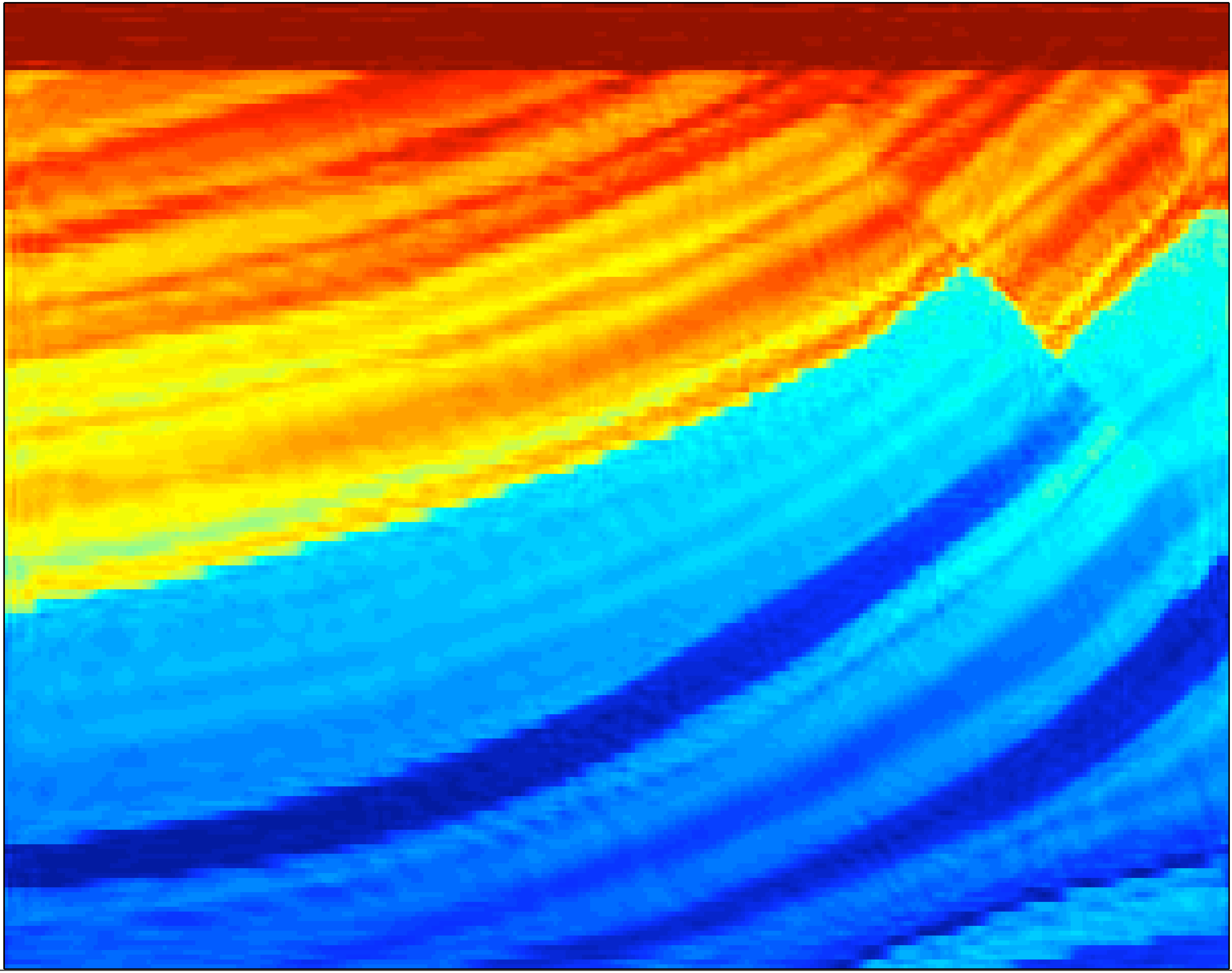
- This means we need very few coefficients to capture the model:



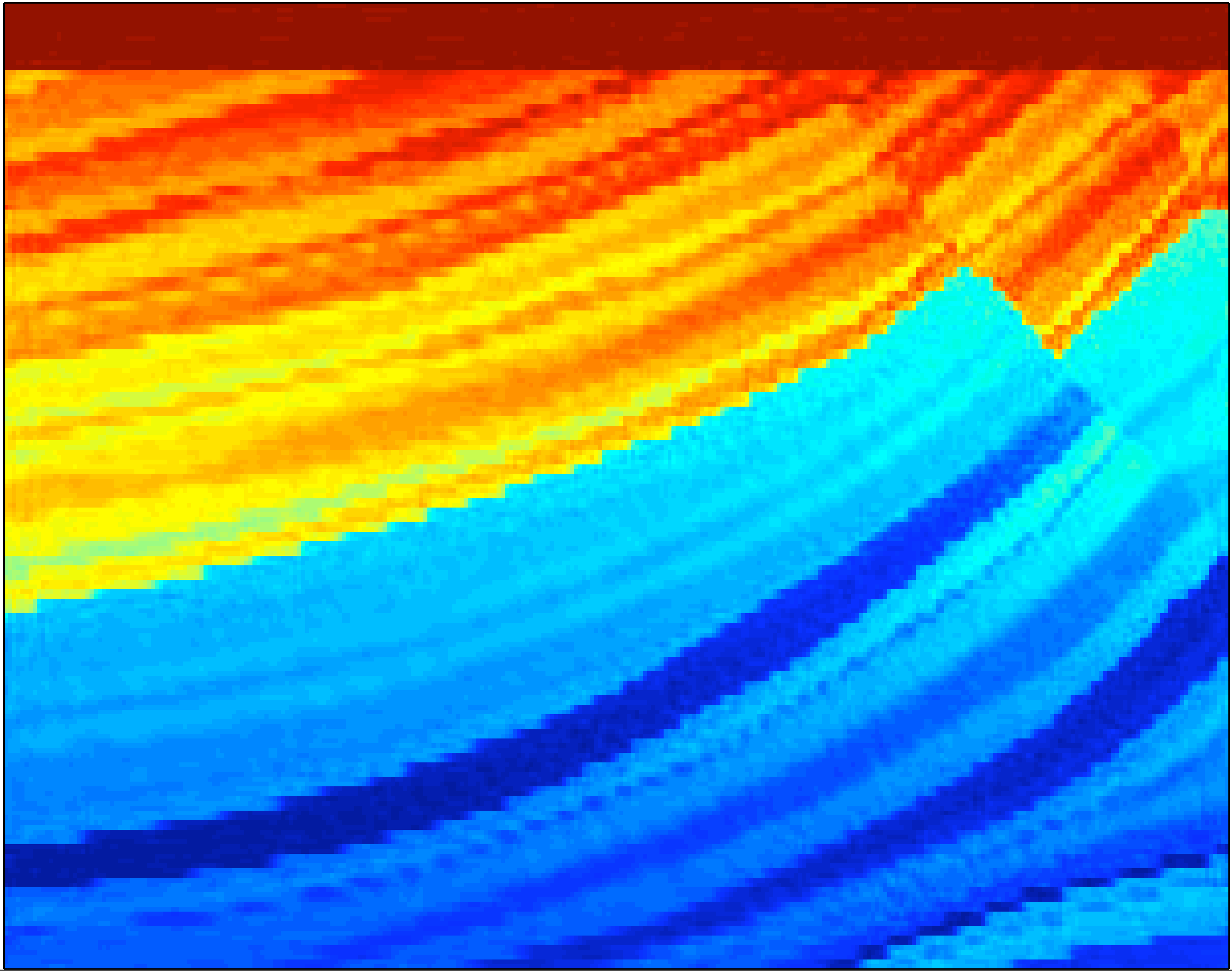
1% of coeff.



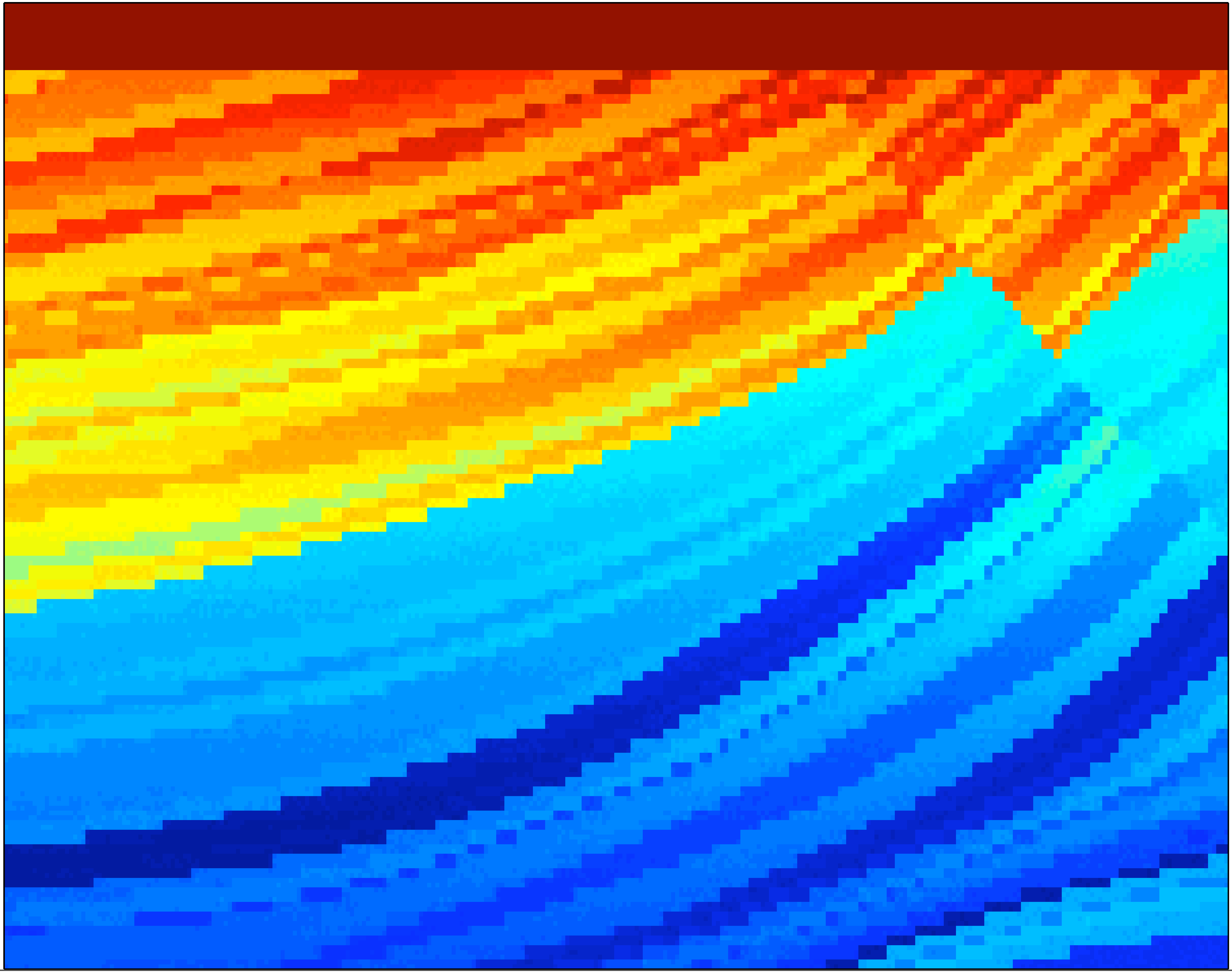
5% of coeff.



10% of coeff.



50% of coeff.



FWI: Sparsity promotion

Sparsity-exploiting formulations

1. $\min_{x} ||D - RH^{-1}[C^* x]Q||_F^2 + \lambda ||x||_1$

2. $\min_{x} ||D - RH^{-1}[C^* x]Q||_F^2 \quad \text{s.t.} \quad ||x||_1 \leq \tau$

3. $\min_{x} ||x||_1 \quad \text{s.t.} \quad ||D - RH^{-1}[C^* x]Q||_F^2 \leq \sigma$

Option 3. has a big advantage: we may be able to derive σ from the data *and* we don't need to guess the sparsity of the solution in Curvelets.

FWI: Sparsity promotion

- Sparsity-exploiting formulation:

$$\begin{array}{ll} \min_x & \|x\|_1 \\ \text{s.t.} & \|D - RH^{-1}[C^*x]Q\|_F^2 \leq \sigma \end{array}$$

$$\bar{m} = C^*\bar{x}$$

- This is a Convex-Composite optimization problem.

Strategy

- Consider a toy model problem:

$$\min_{m} \quad \|m\|_1 \quad \text{s.t.} \quad \|d - g(m)\|_2 \leq \sigma$$

- Implement iterated algorithm:

$$m^{\nu+1} = m^{\nu} + \gamma_{\nu} s^{\nu}$$

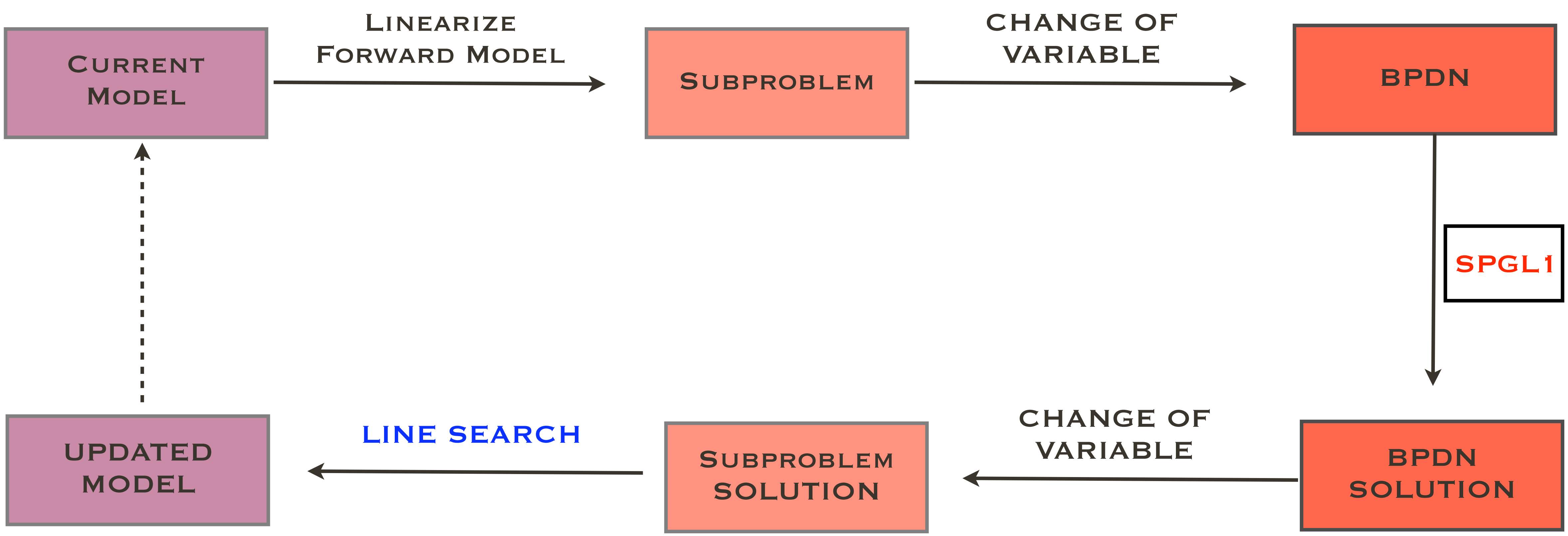
- Direction s^{ν} solves subproblem:

$$\min_{\delta m} \quad \|m^{\nu} + \delta m\|_1 \quad \text{s.t.} \quad \|d - g(m^{\nu}) - \nabla g(m^{\nu})\delta m\|_2 \leq \sigma$$

- Subproblem equivalent to BPDN, which is solved with SPGL1!

$$\min_{y} \quad \|y\|_1 \quad \text{s.t.} \quad \|b - Ay\|_2 \leq \sigma$$

Algorithm



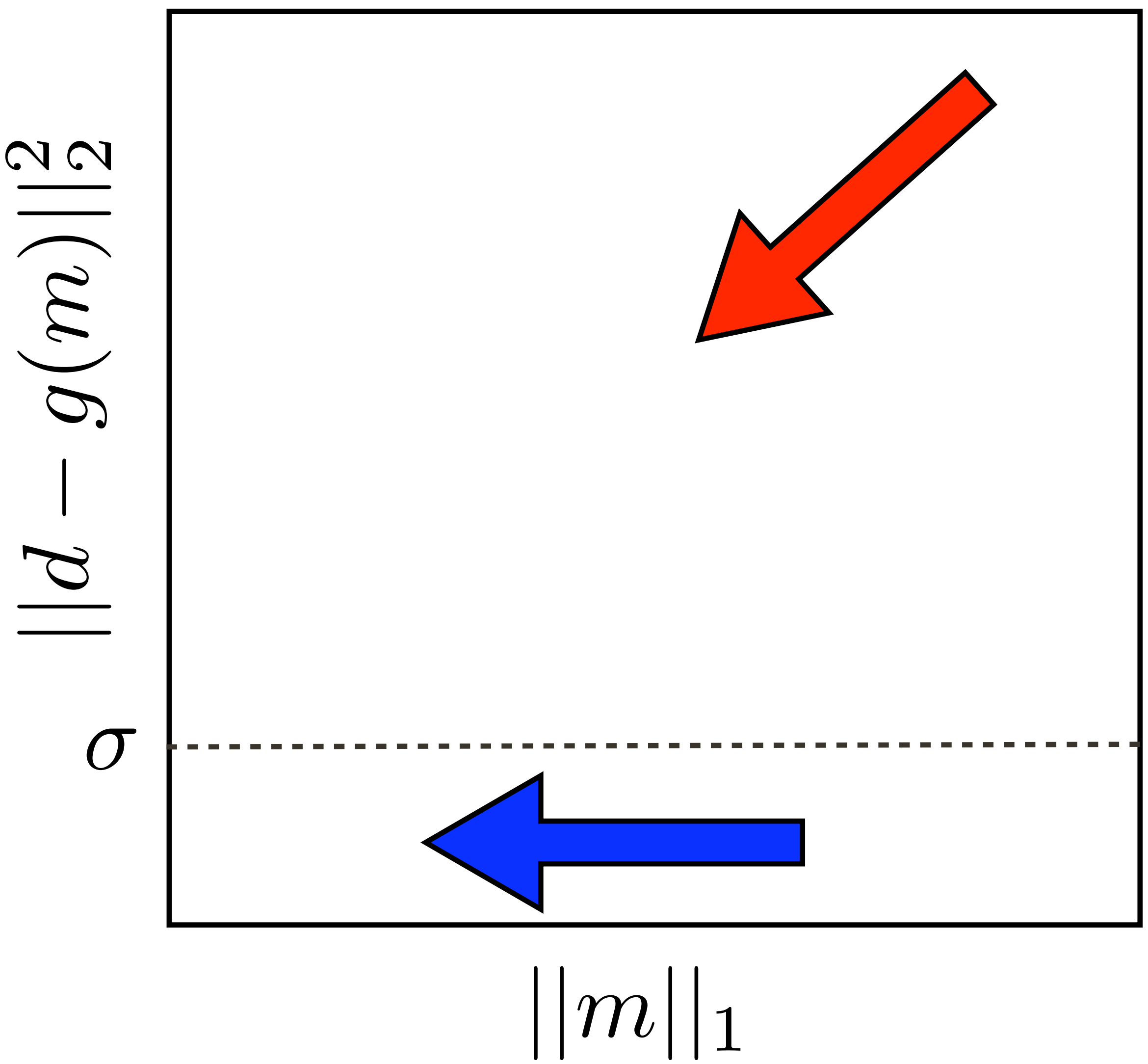
Current Model
Linearize Forward Model
Subproblem
Change of Variable
BPDN
SPGL1
BPDN Solution
Change of Variable
Subproblem Solution
Line Search
Updated Model

Updating the model

$$\min_m \quad \|m\|_1$$
$$\text{s.t.} \quad \|d - g(m)\|_2 \leq \sigma$$

- Competing interests: minimize the 1-norm (sparsity) and decrease misfit.
- Main idea: weigh interests differently as the algorithm proceeds.
- Define merit function $P_\alpha(m) = \|m\|_1 + \alpha(\|d - g(m)\|_2 - \sigma)_+$
- Line search ensures $P_\alpha(m^\nu + \gamma_\nu s^\nu) < P_\alpha(m^\nu)$
- Increase α as the algorithm proceeds.

Updating the model



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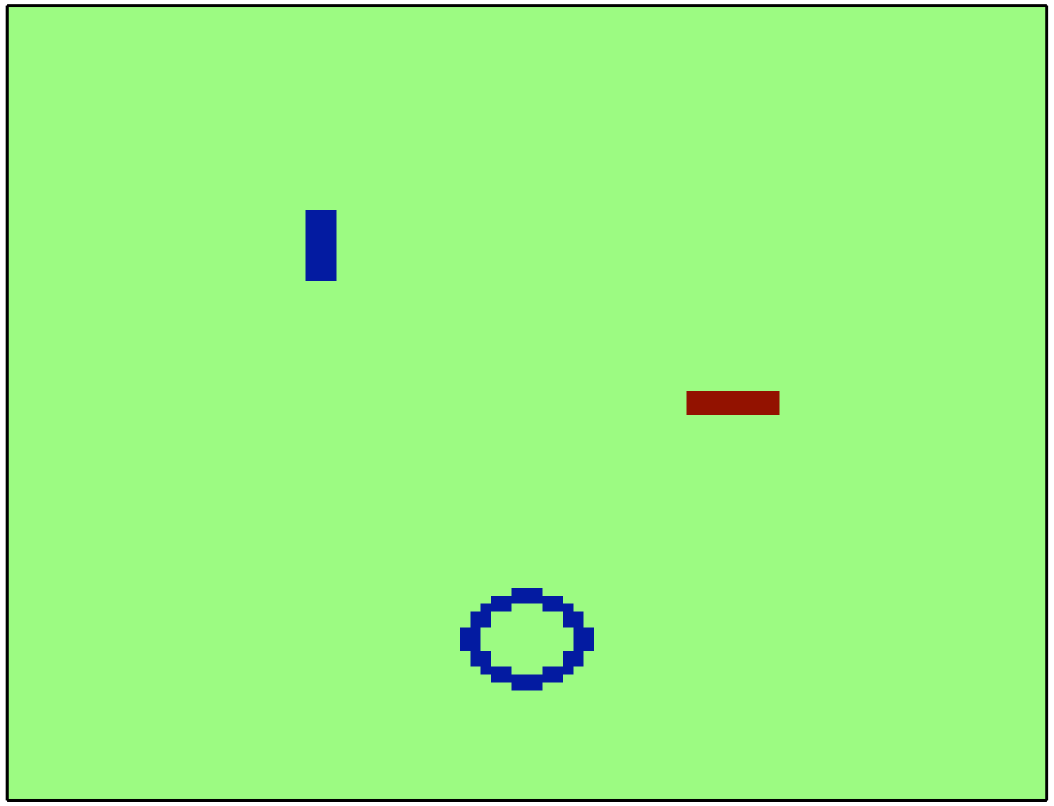
Numerical results

- preliminary tests on model that is sparse in pixels
- cross-well setting, 101 sources and receivers.
- solve

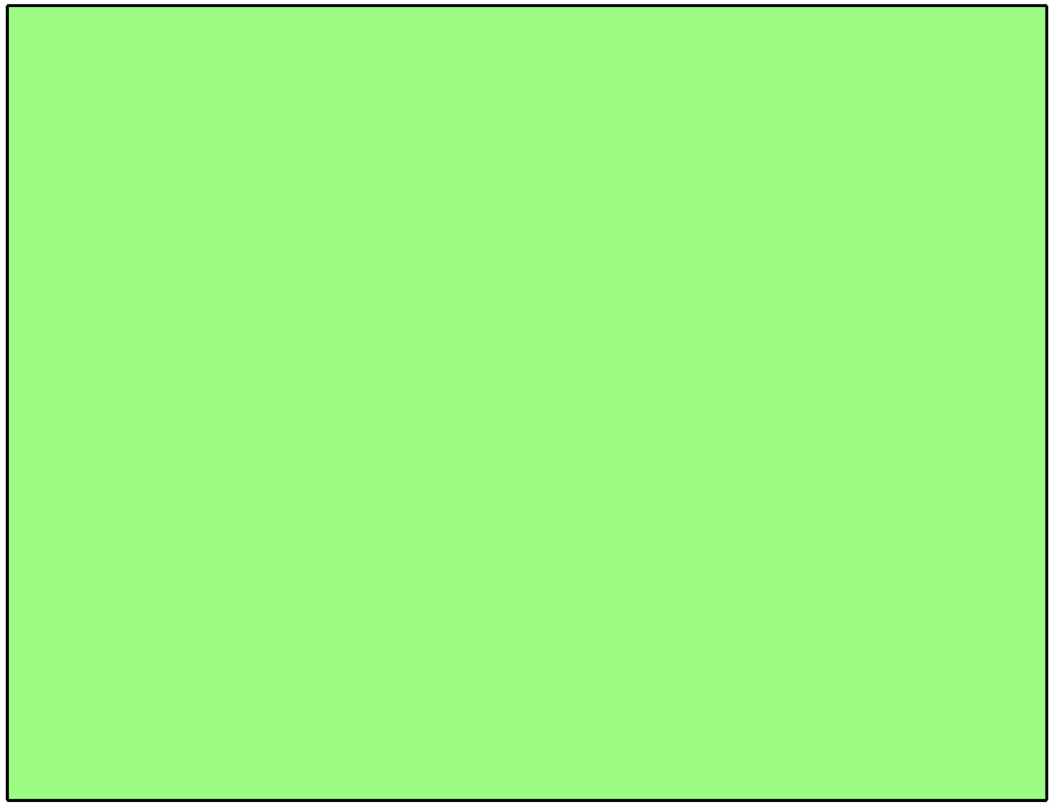
$$\min_{x} \quad \|m\|_1 \qquad \text{s.t.} \quad \|D - RH^{-1}[m_0 + m]Q\|_F^2 \leq \sigma$$

- use simultaneous sources to reduce computational load

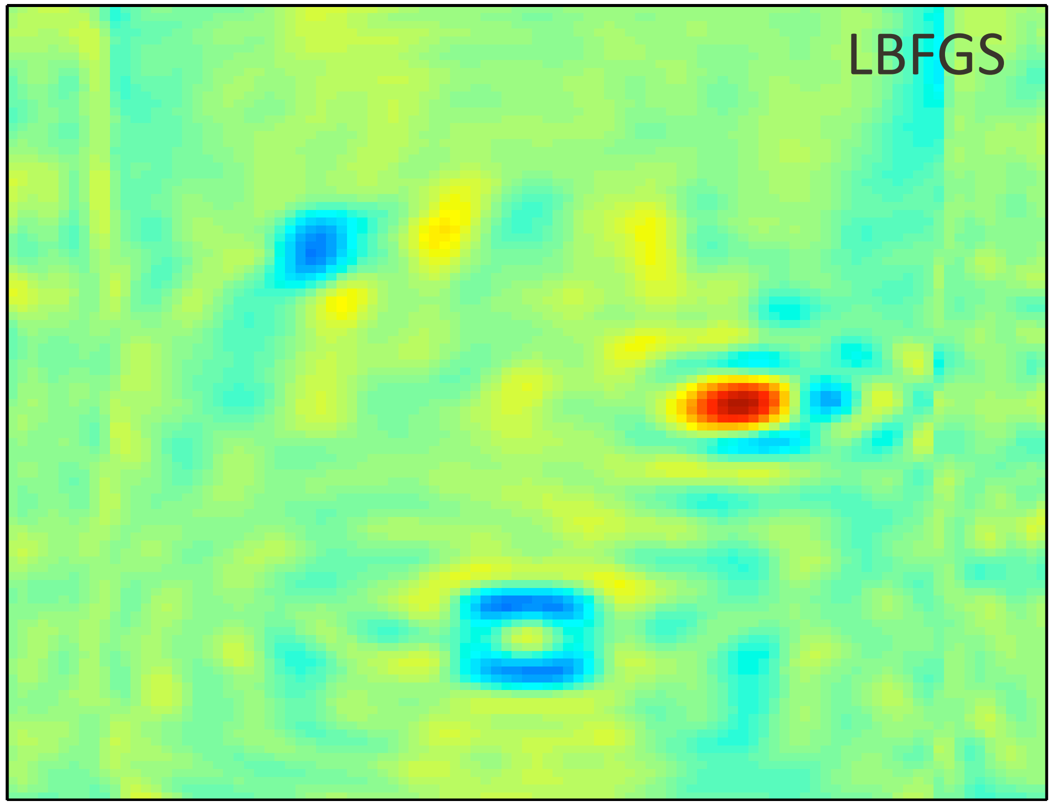
True Model



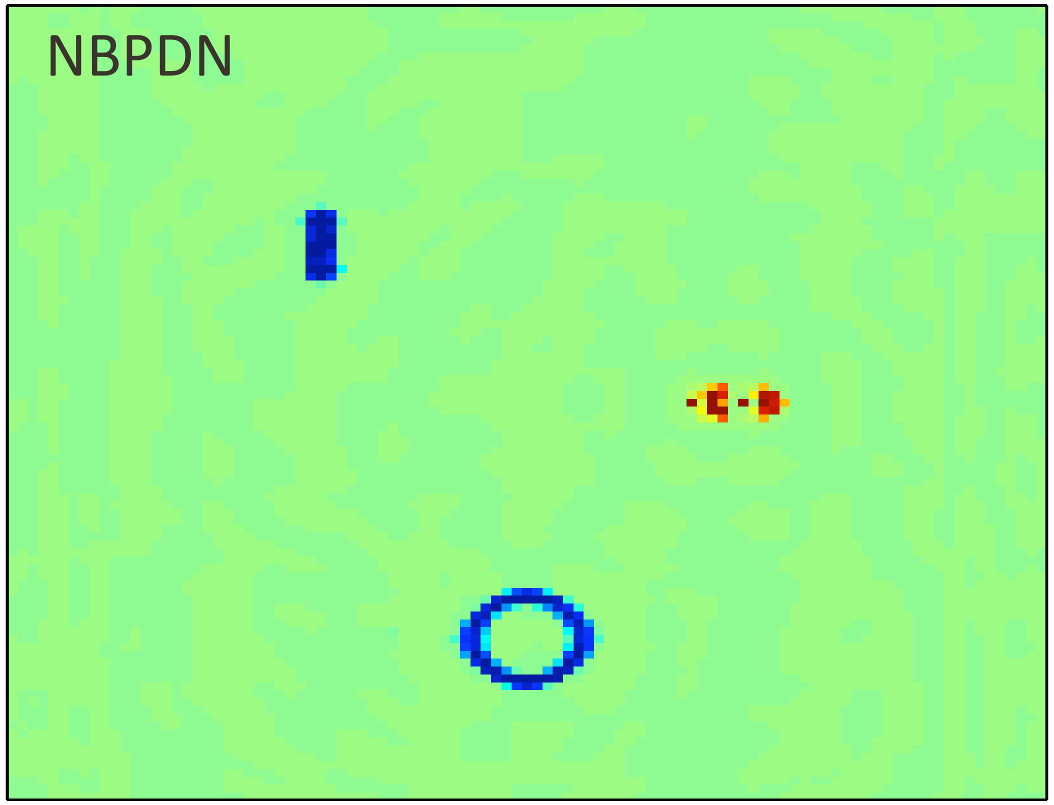
Initial Model

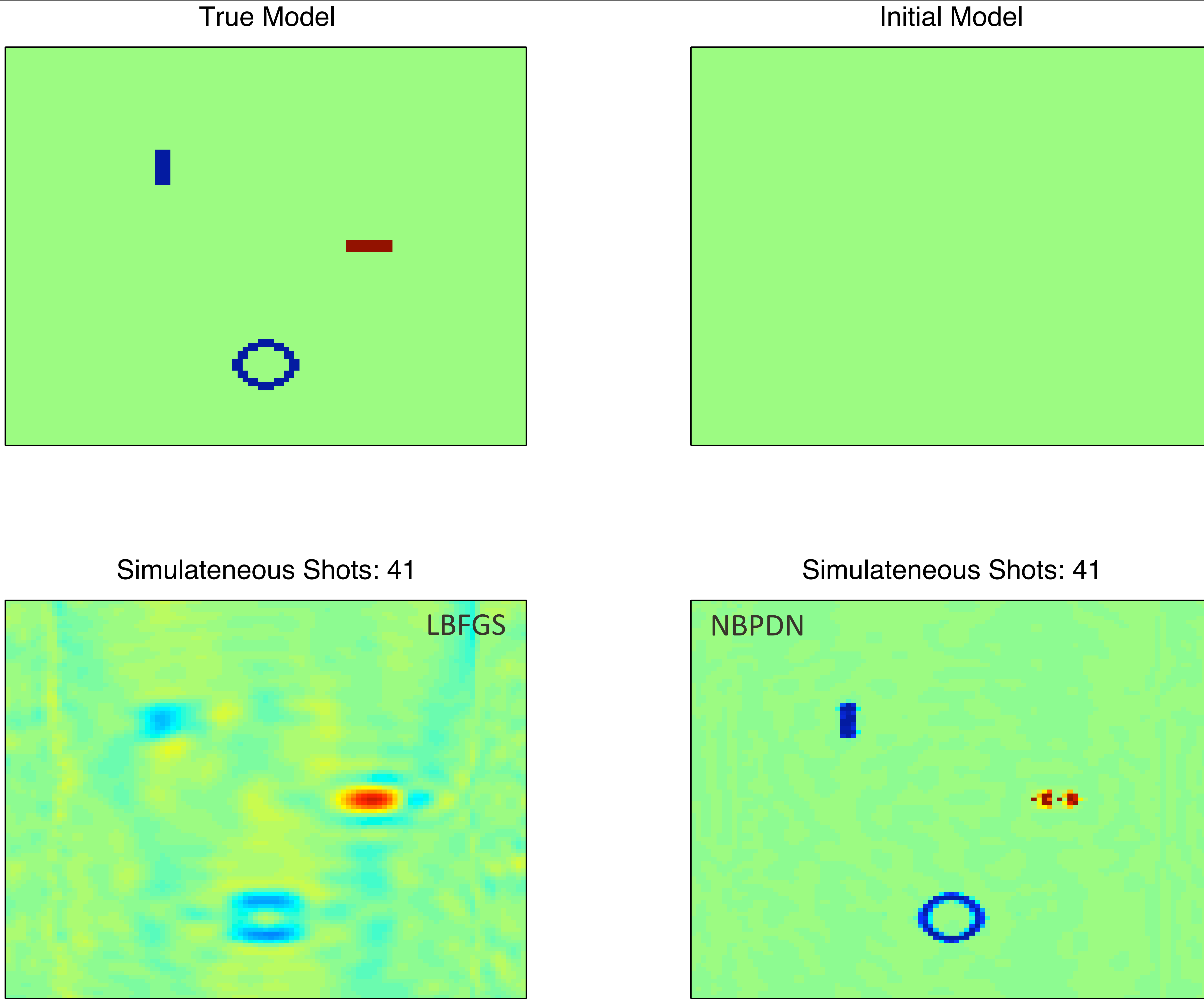


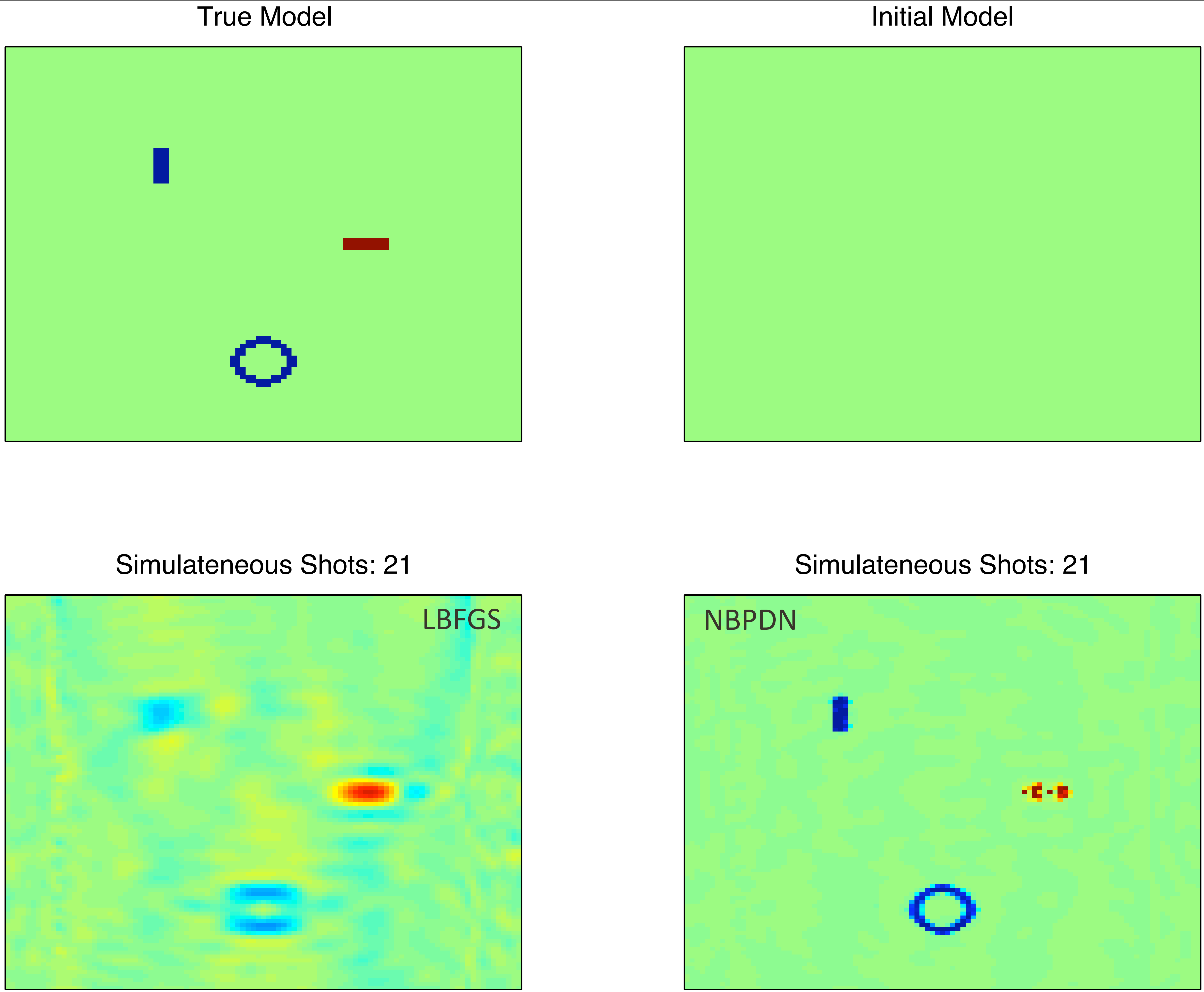
Simulateneous Shots: Full

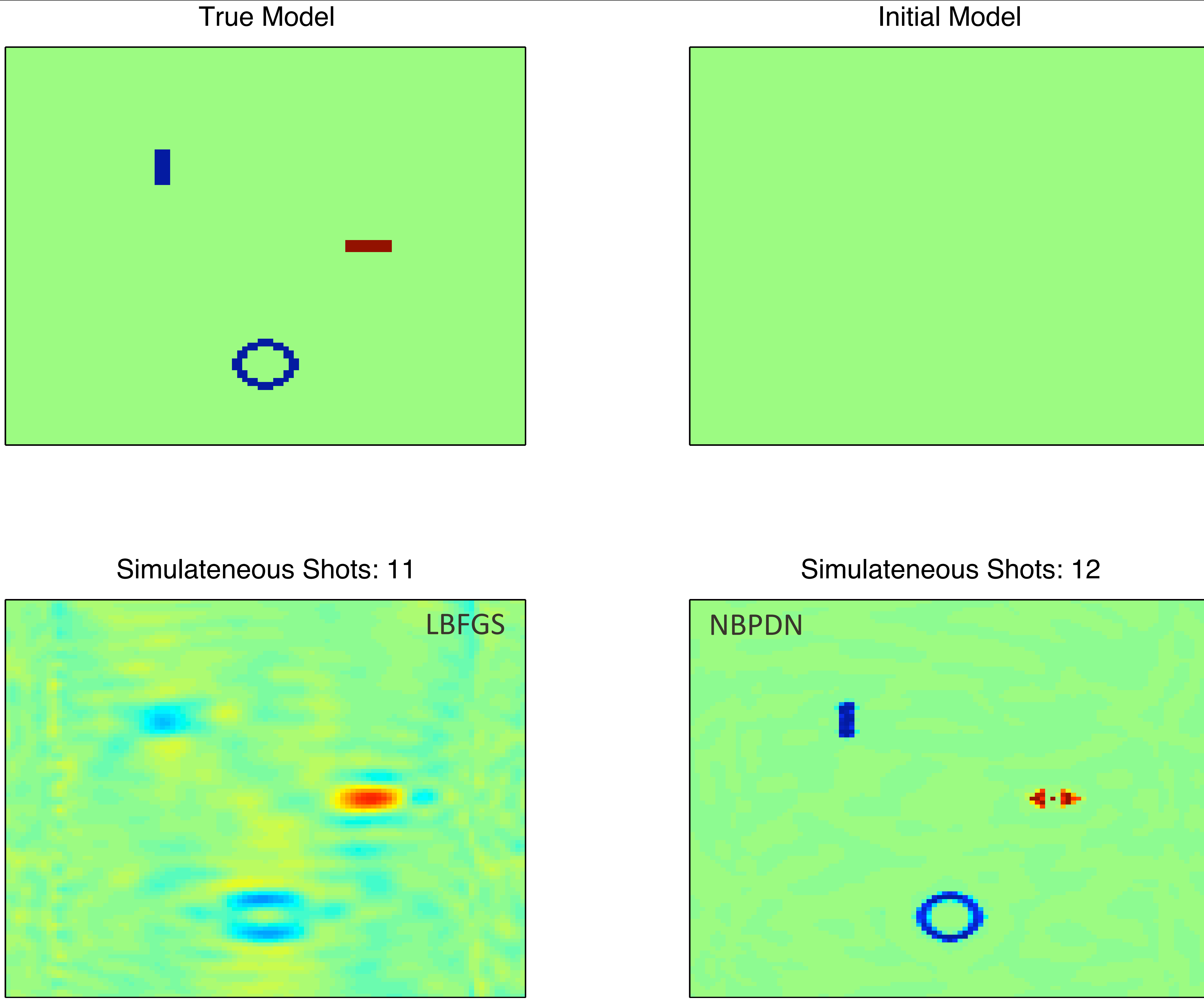


Simulateneous Shots: Full

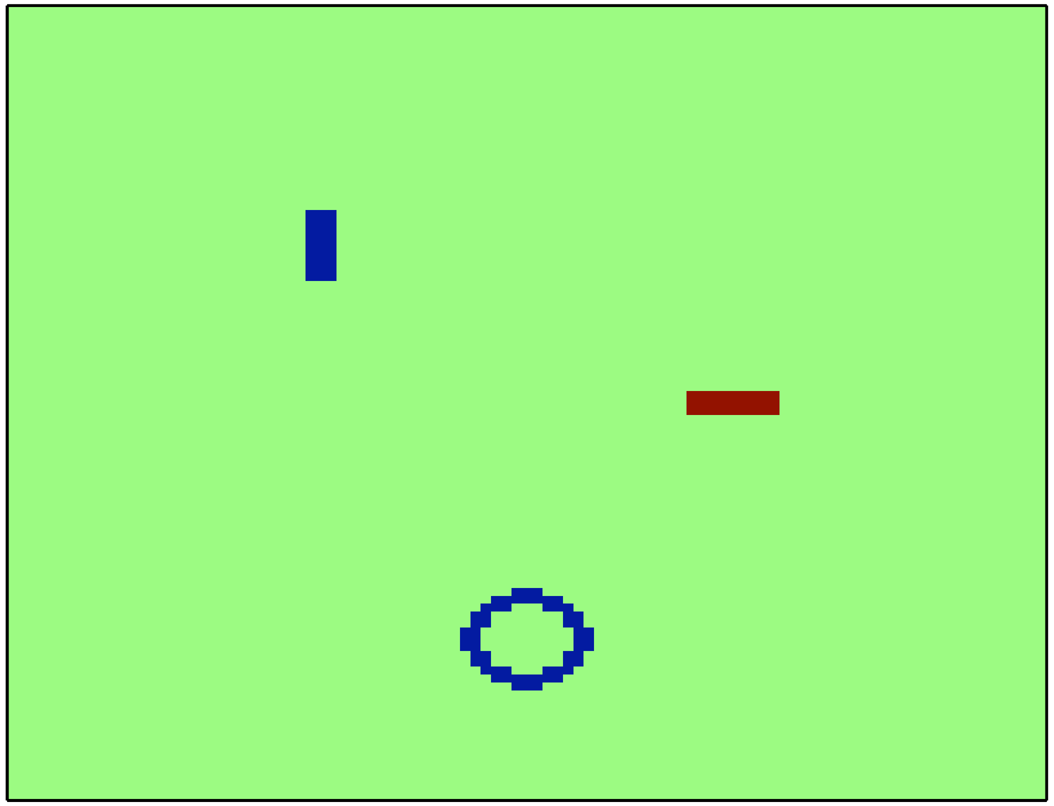



True Model
Initial Model
Simulateneous Shots: 41
LBFGS
Simulateneous Shots: 41
NBPDN

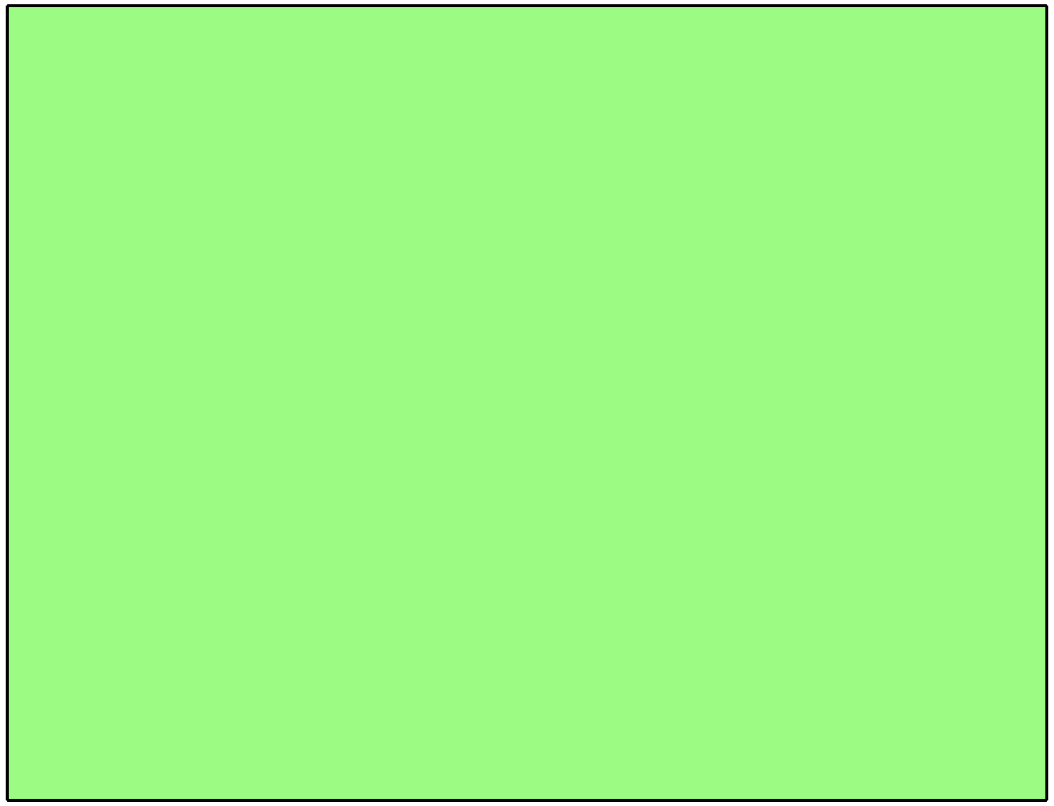

True Model
Initial Model
Simulateneous Shots: 21
LBFGS
Simulateneous Shots: 21
NBPDN


True Model
Initial Model
Simulateneous Shots: 11
LBFGS
Simulateneous Shots: 12
NBPDN

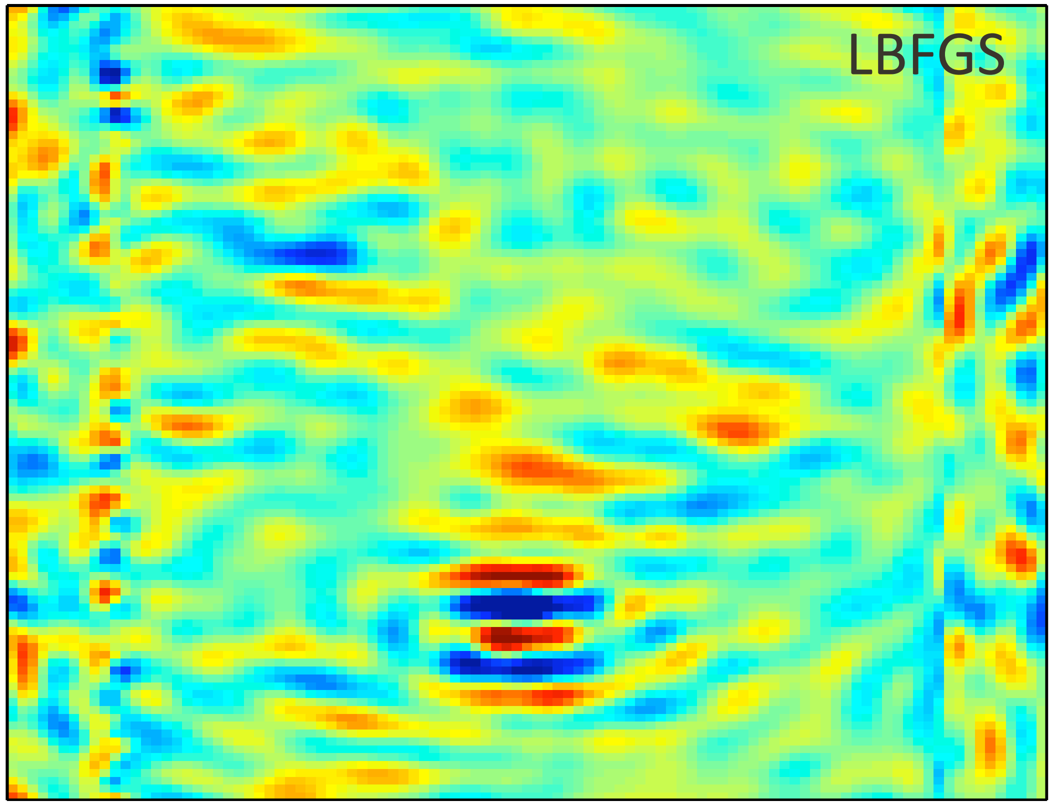
True Model



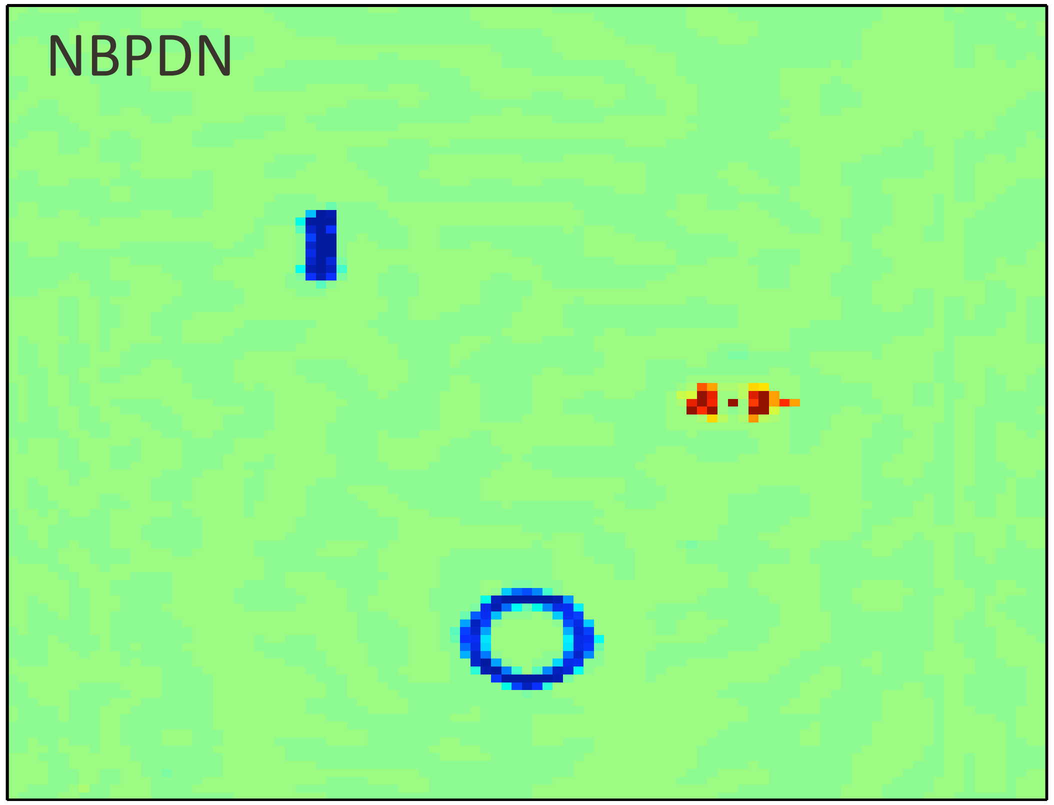
Initial Model

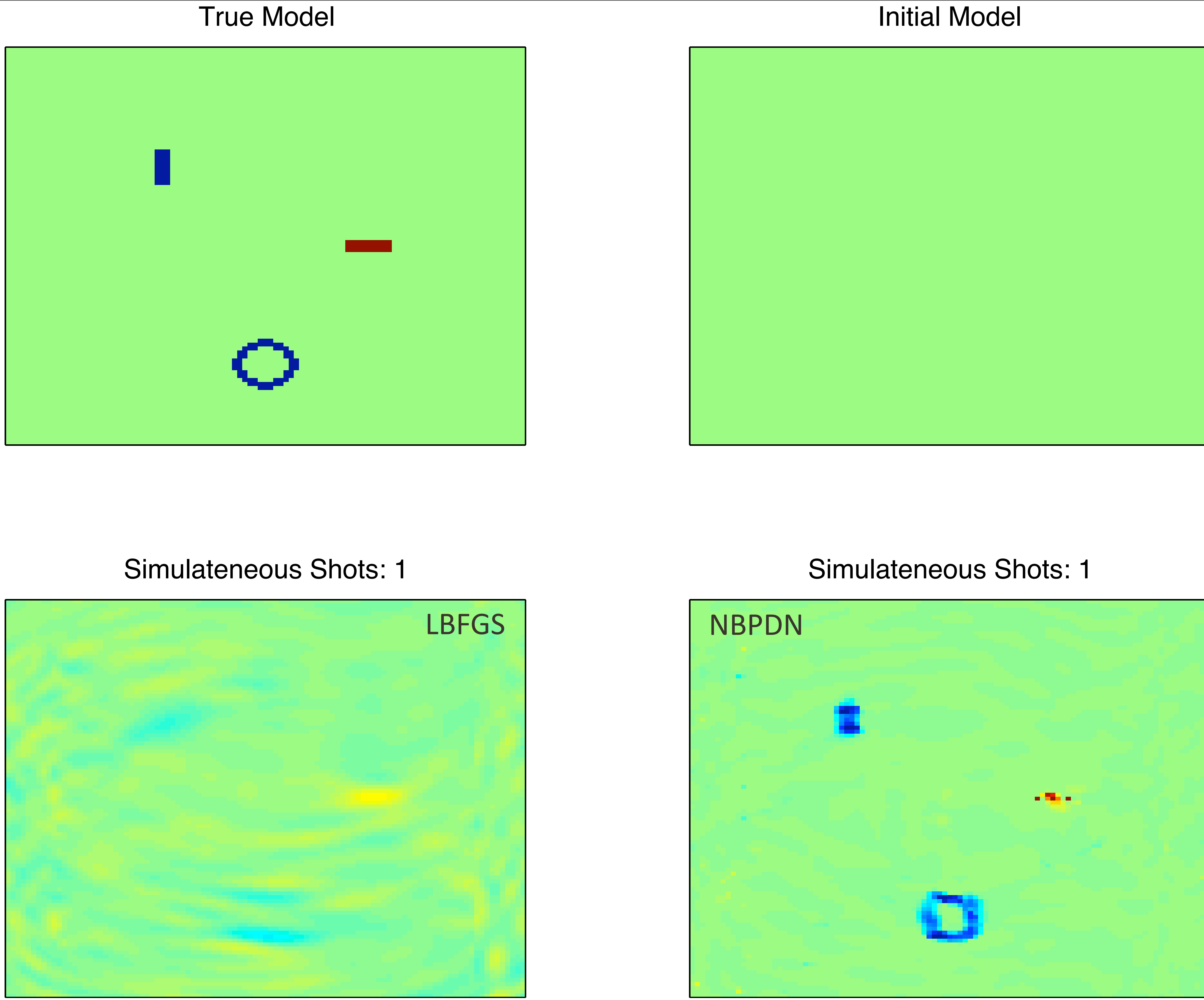


Simulateneous Shots: 5



Simulateneous Shots: 5




True Model
Initial Model
Simulateneous Shots: 1
LBFGS
Simulateneous Shots: 1
NBPDN

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Conclusions

- Way to introduce sparsity constraints into FWI
- Non-linear formulation of BDPN does not require us to guess sparsity level of the solution
- Preliminary results are promising: we can recover a sparse solution from undersampled data.

The Road Ahead

- Test on realistic models with Curvelet sparsity and noise on data
- Apply to time-lapse seismic
- Implement renewal strategy for simultaneous shots
- Extend the entire framework to be robust against outliers (i.e, different norms on the data residual)
- Investigate relation to compressed sensing L1 recovery

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