Spot — A linear operator toolbox for Matlab

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Sparco operator toolbox (SPOT)

- Sparco: test problem suite for compressed sensing
- Combinations of linear operators; $A = MB$, $B = [D, I]$
- Developed operator toolbox

```
D = opDCT(256);
I = opEye(256);
B = opDictionary(D,I);
M = ...;
A = opFoG(M,B);
```

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- Application of operator (function)

```
y = A(x,1);
x = A(y,2);
C = opClass(A);  % Rudimentary class wrapper
y = C * x;
x = C' * y;
```

Spot

- Leverages classes in Matlab
- Factors out the operator toolbox part from Sparco
- Largely improved in usability, flexibility
- Main purpose: rapid prototyping

Aim of Spot

- Combine the advantages of functions and matrices
- Keep the matrix feel in operators

```
>                    > D = opDCT(16);    > D = dct(eye(16));
> y = dct(x);        > y = D  * x;       > y = D  * x;
> z = idct(y);       > z = D' * y;       > z = D' * y;
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Advantage

- Fast operation
- Low memory requirements

Disadvantage

- Cannot pass to functions
- Difficult to manipulate

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Disadvantage

- Does not scale well

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- Easy to manipulate

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Outline

1. Instantiate elementary Spot operators
2. Manipulate and combine operators
3. Apply operators
4. Adding operators

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`y'*D` $\longrightarrow$ `(D' * y)'`

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Non-linear operators are possible but not recommended!

Basic operators

Elementary

- opEye, opZeros, opOnes, opDiag
- opMatrix
- opGaussian, opBinary

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Fast transforms

- opCurvelet, opSurfacelet
- opDCT, opDCT2, opDFT, opDFT2, opWavelet

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D = opDFT2(m,n); % 2D discrete Fourier transform
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```
D = opDFT2(m,n); % 2D discrete Fourier transform
```

- opHeaviside, opHadamard, opToeplitz
- opConvolve (regular, truncated, cyclic)

```
C = opConvolve(m,n,kernel,offset,type);
```

Meta operators (1/5)

Multiplication

```
C = B * A;      % Overloaded *:  C = opFoG(B,A);
C = 3 * A;
C = -B;
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x = (B + C + D) * y;
A = B + C + D;
x = A * y;      % Equivalent to first statement
C = M + A;
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Transposition and conjugation

```
A = B';
A = B.';
A = conj(B);
```

Meta operators (2/5)

Dictionaries

```
A = [B, C, M];   % Horizontal concatenation
A = [B; C; M];   % Vertical concatenation
A = [B, C; C', D];
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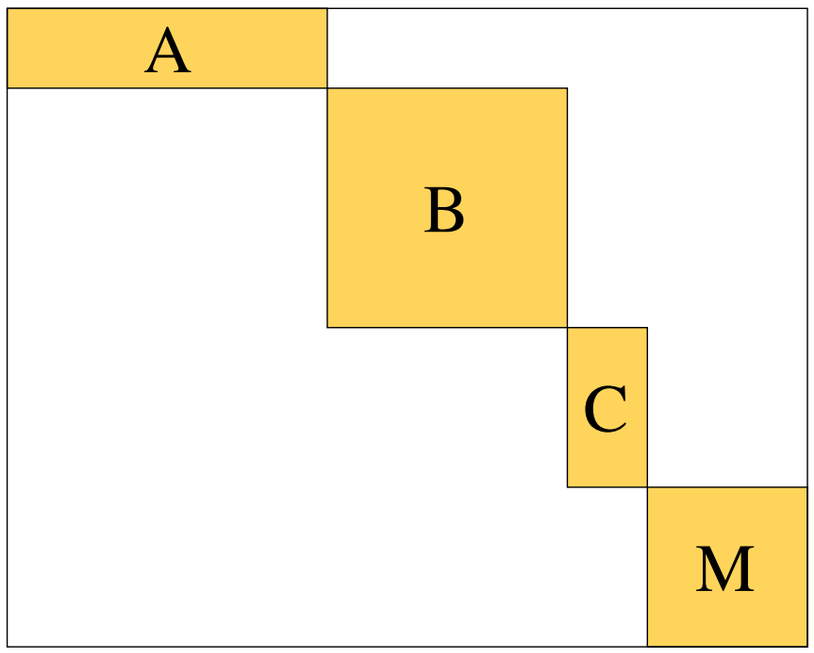
Block diagonal

```
D = blkdiag(A,B,C,M);
D = opBlockDiag([weight],op1,...,opn,[overlap]);
```



blkdiag

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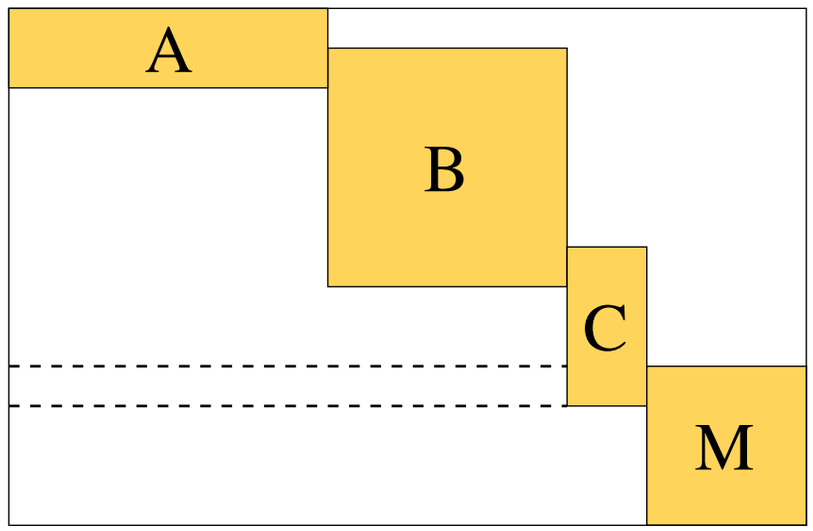

`overlap > 0`

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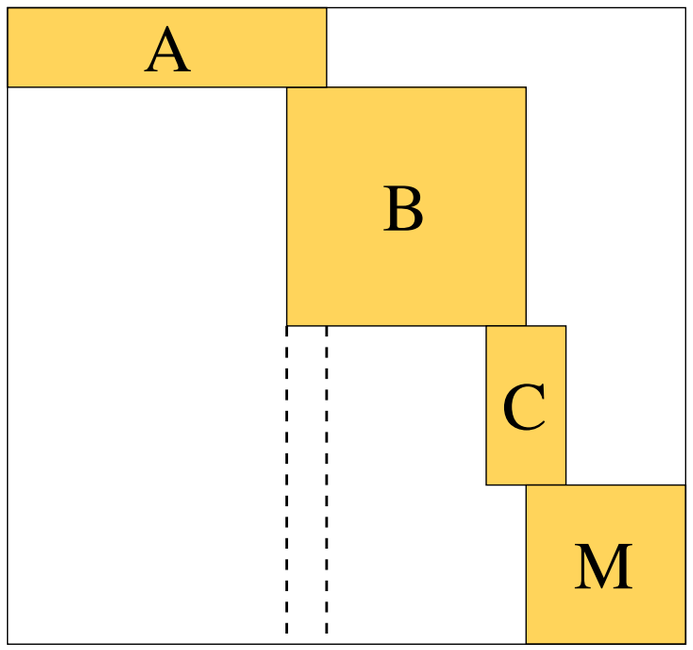

`overlap < 0`

Meta operators (3/5)

Kronecker products

```
A = kron(A,B,C);   % A := A ⊗ B ⊗ C
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Example: n^3 data volume

```
F  = opDFT(n);
I  = opEye(n);
A1 = kron(I,I,F); % DFT along first dimension
A2 = kron(I,F,I); % DFT along second dimension
A3 = kron(F,I,I); % DFT along third dimension
```

Meta operators (3/5)

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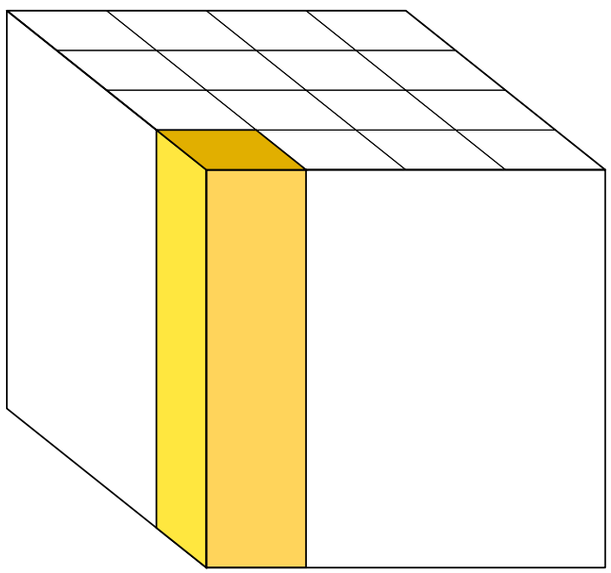
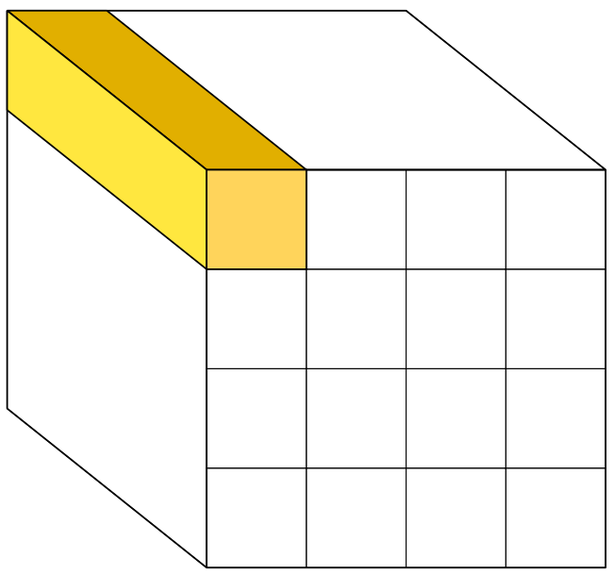
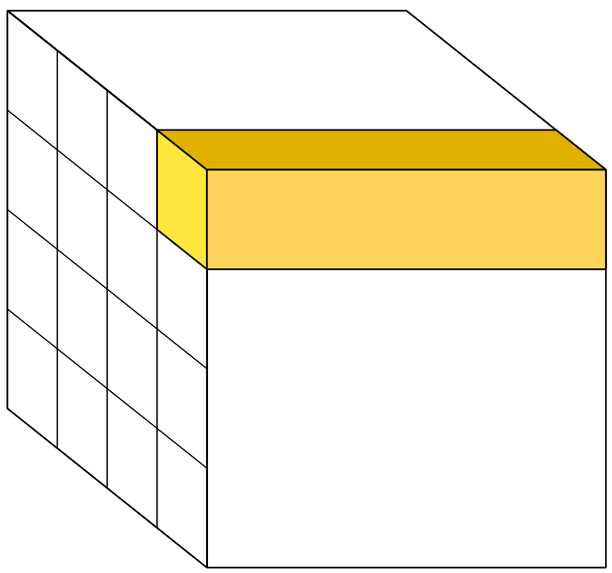


A1



A2



A3

Meta operators (4/5)

Slicing

```
A = B(:,idx);
A = B(idx,:);
A = B(idx1,idx2);
```

Meta operators (4/5)

Slicing

```
A = B(:,idx);
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```

Example: Randomly restricted Fourier operator

```
F = opDFT(128);
p = randperm(128);
A = F(p(1:50),:);
```

Meta operators (4/5)

Slicing

```
A = B(:,idx);
A = B(idx,:);
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```

Example: Randomly restricted Fourier operator

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F = opDFT(128);
p = randperm(128);
A = F(p(1:50),:);
```

Assignment

```
A(idx1,idx2) = B;
A(5,:) = []; % Cut the fifth row
A(:,5) = []; % Cut the fifth column
```

Meta operators (5/5)

Pseudo-inverse and backslash

```
x = A \ b;
P = pinv(A);
x = P * b;
```

Overdetermined system

$$\underset{x}{\text{minimize}} \quad \|Ax - b\|_2$$

Underdetermined system

$$\underset{x}{\text{minimize}} \quad \|x\|_2 \quad \text{s.t.} \quad Ax = b$$

Linear systems are solved using LSQR

Tools

Querying operator information

- `size`
- `disp`
- `isempty`

Counter facilities

- Counters associated with each operator
- Keeps track of number of applications

Adding operators

Function wrapper

```
y = fun(x,1);
x = fun(y,2);
F = opFunction(m,n,fun,cflag);
y = F * x;
x = F'* y
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Class wrapper

```
C = opClass(m,n,obj,cflag,linflag);
```

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Deriving a child class

- Inherit from base class (`opSpot`)
- Write constructor
- Implement `multiply(op,x,mode)` function
- Optionally: overload other operations

http://www.cs.ubc.ca/labs/scl/spot/