Wavefield Reconstruction Using Simultaneous Denoising Interpolation VS Denoising after Interpolation

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2008 Consortium Meeting

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Outline

- Introduction
- General Problem Formulation
  - Denoising after Interpolating
  - Interpolating and Denoising Simultaneous
- Results
  - comparison with different noise levels
  - comparison with different missing trace percentages
- Conclusion & Future Work

Introduction

- Seismic Interpolation
  - Seismic data with missing traces due to physical or economic constrains
  - Require Interpolation
- Seismic Denoising
  - Seismic data Corrupted by noise
  - Require Denoising

Problem formulation by sparsity-promoting inversion

- Interpolation

$$\min \|\mathbf{x}\|_1$$
$$\text{s.t.} \mathbf{A}\mathbf{x} = \mathbf{y}$$

where

$$\mathbf{A} = \mathbf{R}\mathbf{C}^T$$

- Denoising

$$\min \|\mathbf{x}\|_1$$
$$\text{s.t.} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

where

$$\mathbf{A} = \mathbf{C}^T$$
$$\epsilon \sim \text{noise level}$$

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Processing Incomplete & Noisy Data

- Strategy 1:
  - First Interpolate

$$\mathbf{y}_0 \sim \text{Input data}$$

$$\widetilde{\mathbf{x}} = \arg \min \|x\|_1 \quad \text{s.t. } \mathbf{y}_0 = \mathbf{R}\mathbf{C}^T\mathbf{x}$$

  - Then Denoise $\mathbf{f} = \mathbf{C}^T\widetilde{\mathbf{x}}$

$$\min \|\mathbf{x}\|_1 \quad \text{s.t.} \|\mathbf{C}^T\mathbf{x} - \mathbf{f}\|_2 \leq \epsilon_1$$

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Processing Incomplete & Noisy Data

- Strategy 2: Interpolate and Denoise Simultaneously

$$\mathbf{y}_0 \sim \text{Input data}$$

$$\begin{aligned} min\ &\|\mathbf{x}\|_1 \\ \text{s.t.} &\|\mathbf{R}\mathbf{C}^T\mathbf{x} - \mathbf{y}_0\|_2 \leq \epsilon_2 \end{aligned}$$

Processing Incomplete & Noisy Data

- Choice of ϵ
  - denoise problem assumes white noise (Gaussian, standard deviation σ), N measurements

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

data noise

$$\frac{\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2}{\sigma^2} \sim \chi^2(N)$$

chi square distribution

$$mean \sim N$$
$$SD \sim \sqrt{2N}$$

  - choose $\epsilon^2 = \sigma^2[N + 2\sqrt{2N}]$ then,

$$\text{pr } (\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \ > \epsilon) \text{ is small}$$

Processing Incomplete & Noisy Data

- Choice of ϵ
  - Interpolating then Denoising

$$\epsilon 1 = \sigma \sqrt{N}$$

~ need to fit full interpolated data

  - Combined

$$\epsilon 2 = \sigma \sqrt{N * (1 - miss\%)}$$

~ need to fit incomplete data

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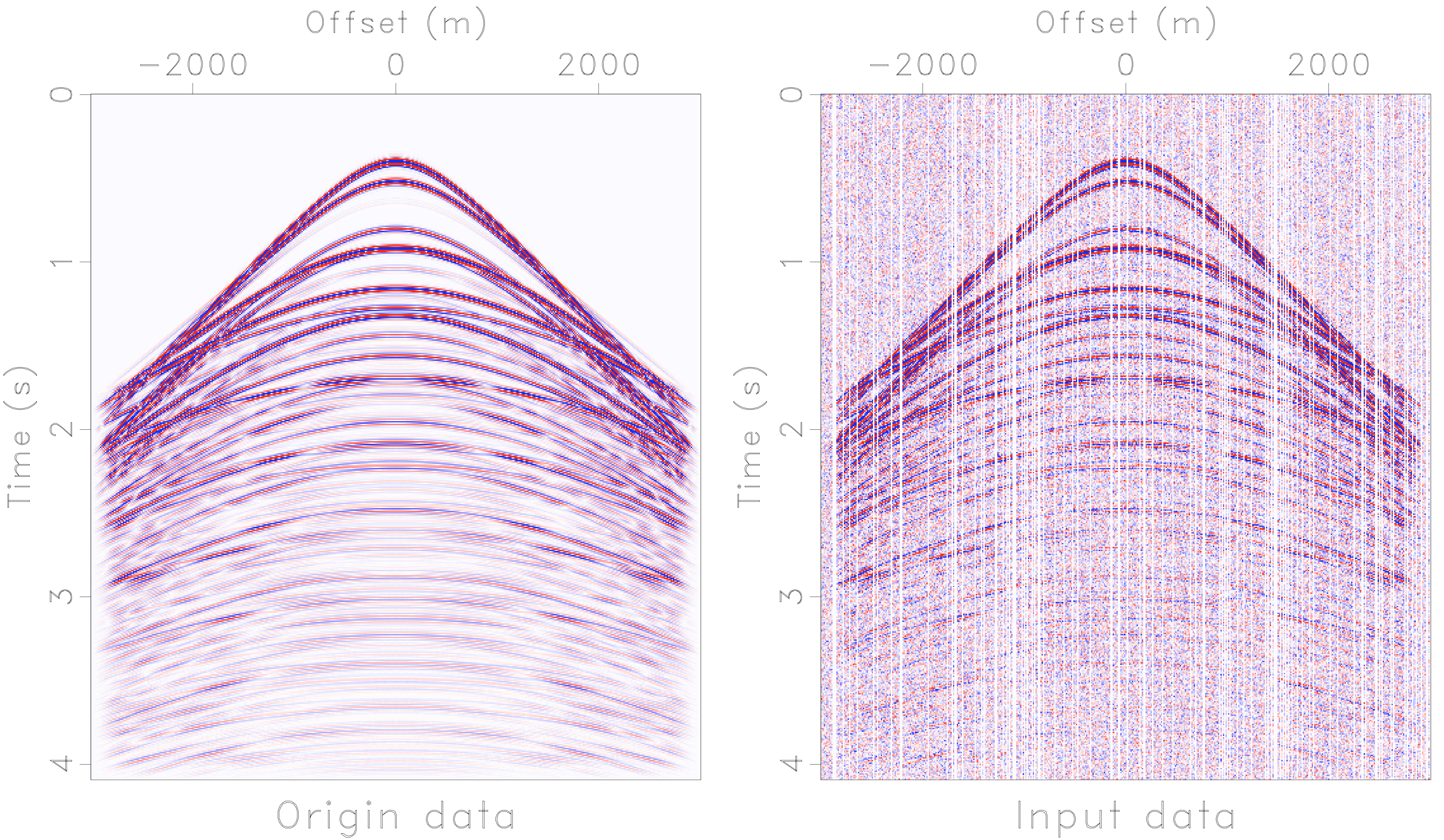

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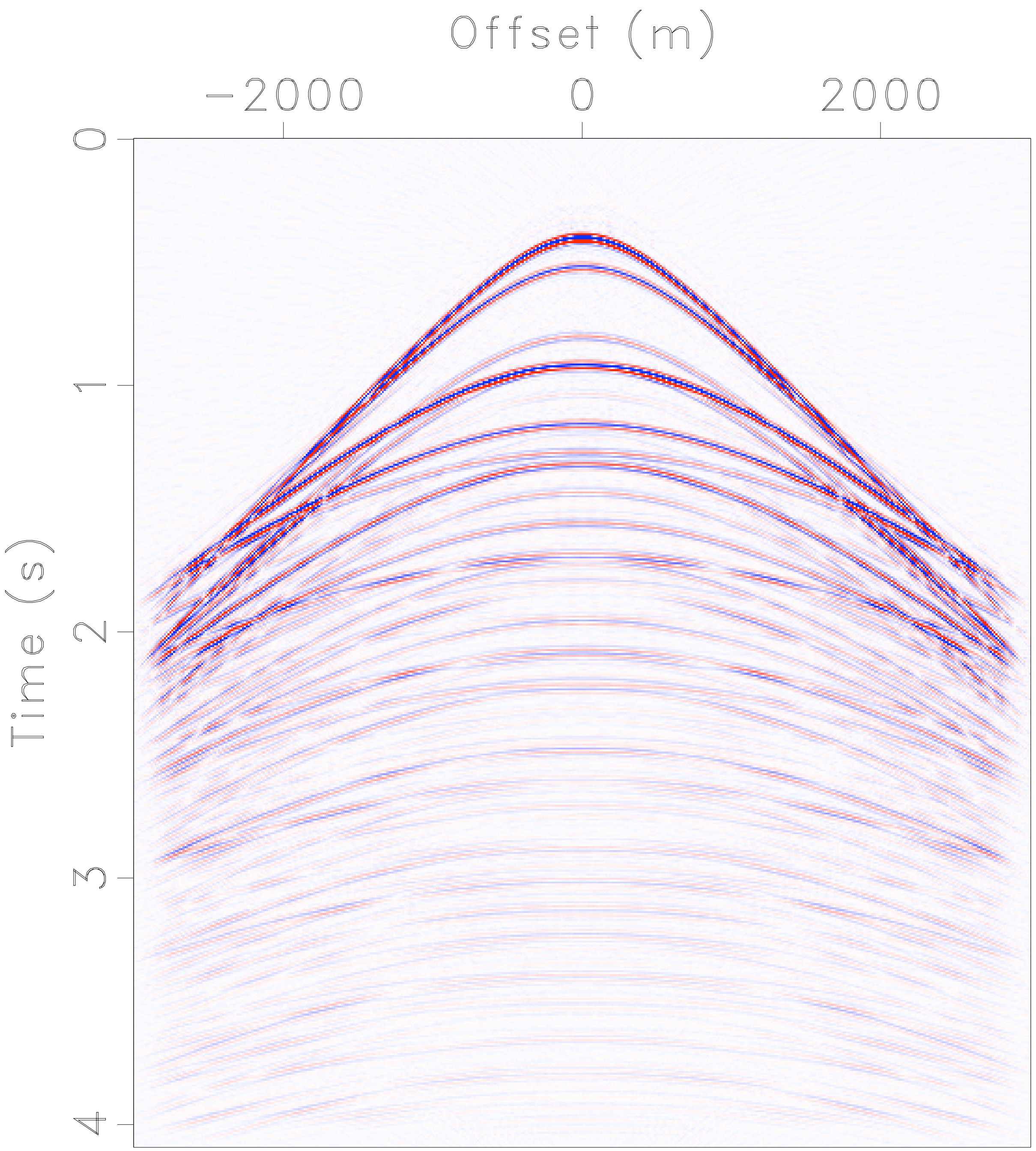
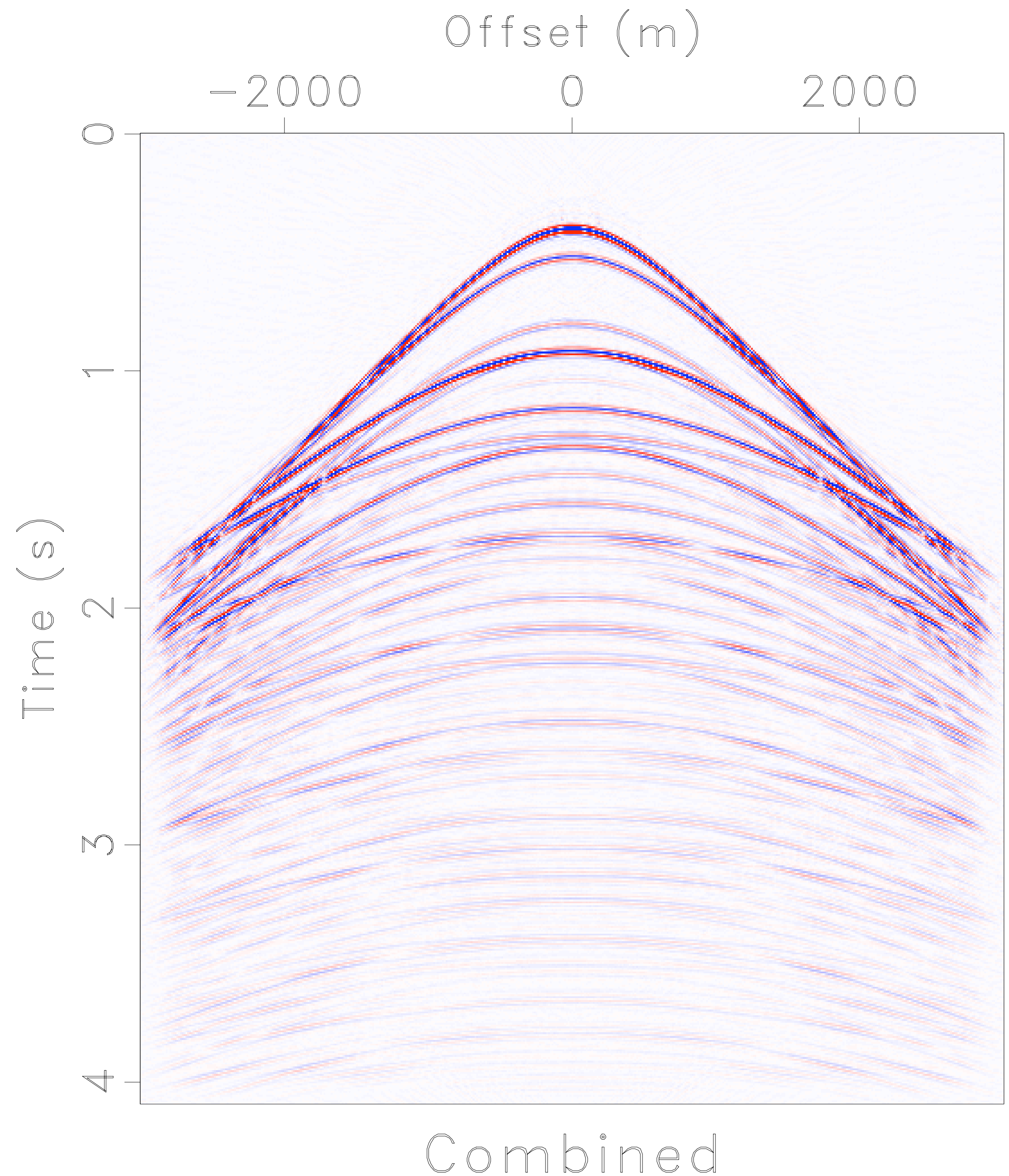
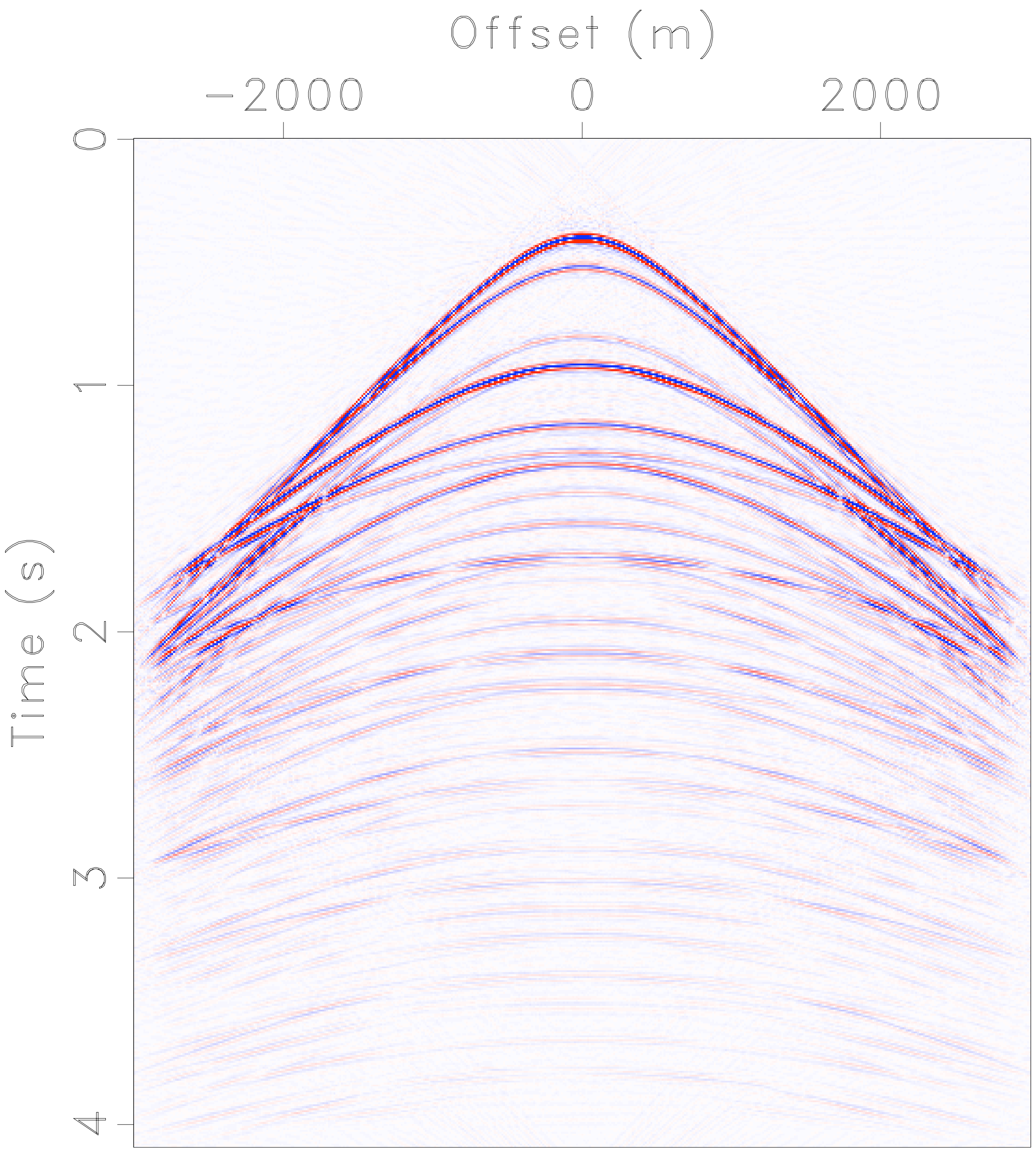


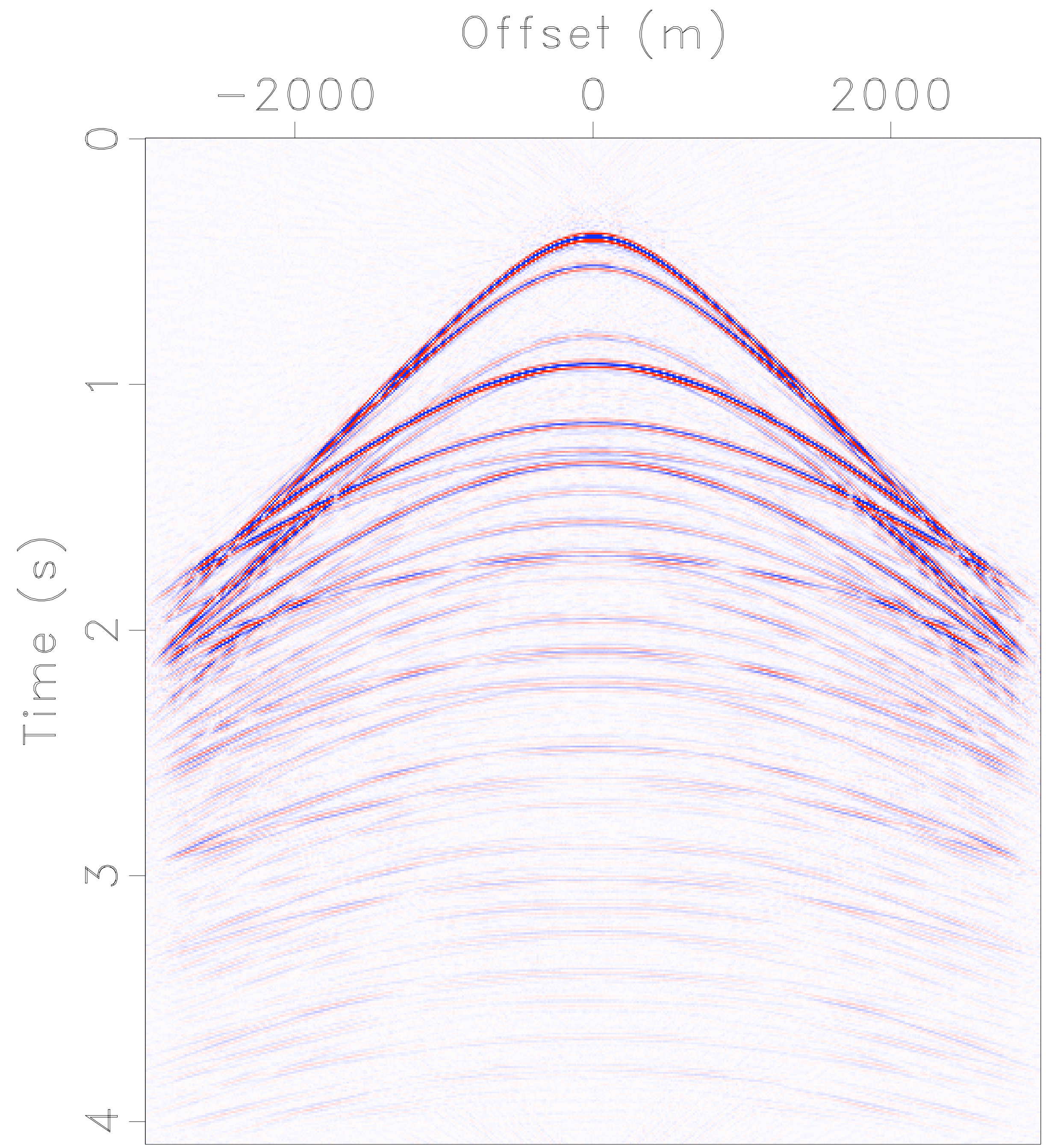
Denoising after Interpolating



Input SNR=3.35 dB



Denoising after Interpolating



Combined

Input SNR=2.37 dB

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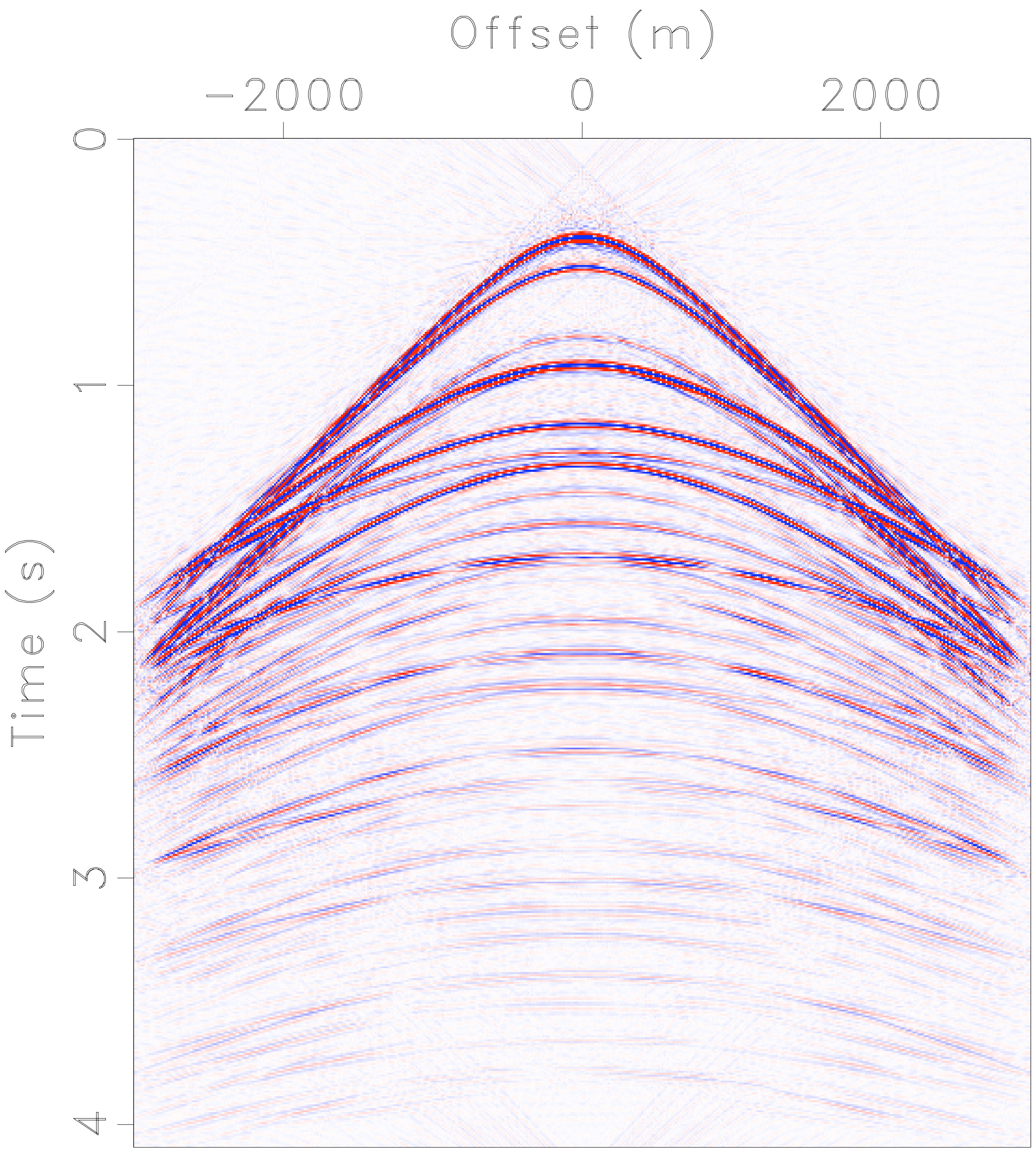
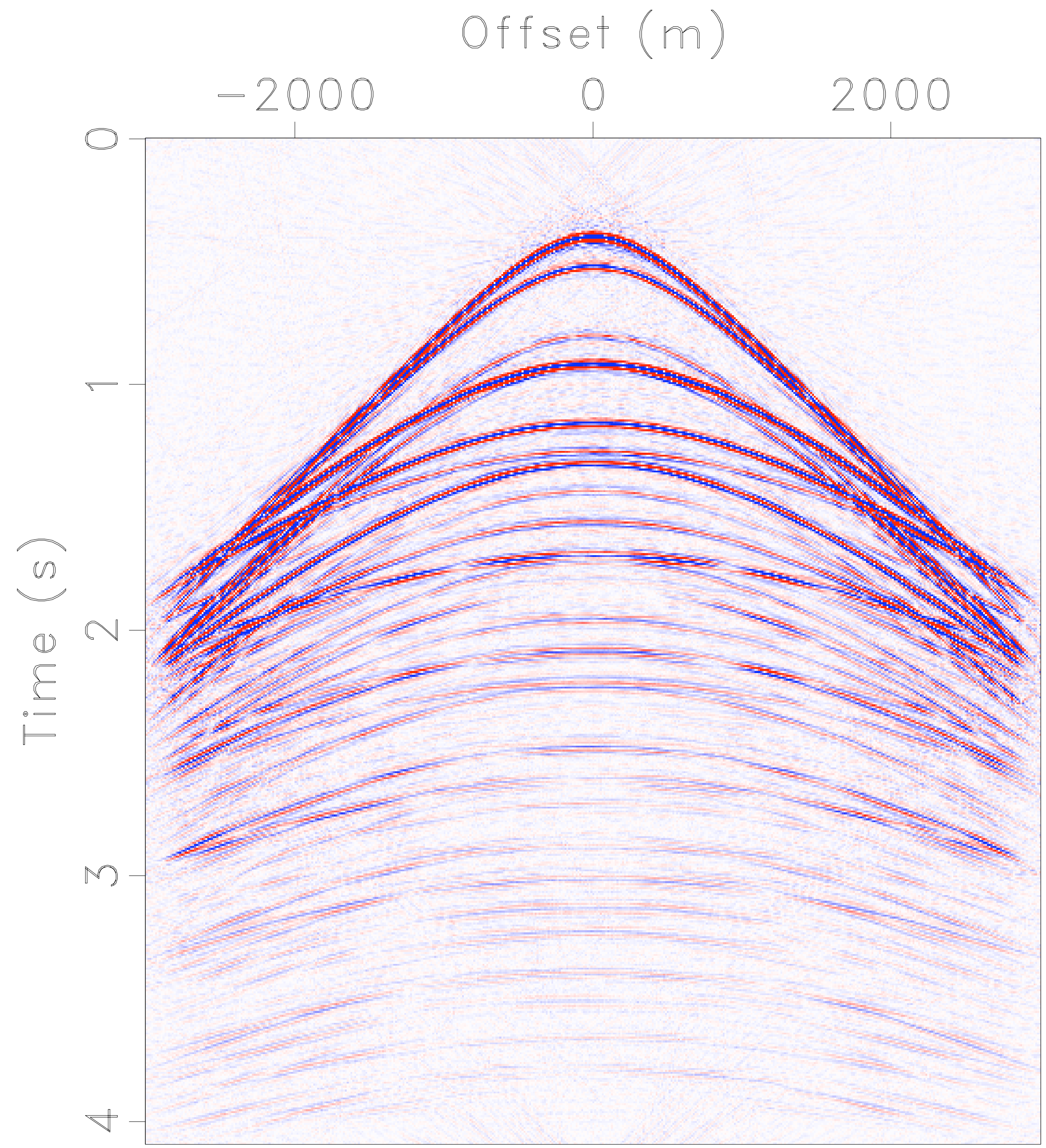


Denoising after Interpolating



Combined

Input SNR=1.77 dB

Comparison under different noises

- miss%=percentage of missing traces=20%
- SNR1 Interpolate and Denoise
  SNR2 Combined

	input SNR	SNR1	SNR2
1	3.50	14.71	14.79
2	3.35	10.01	9.96
3	2.93	7.91	7.93
4	2.37	6.45	6.66
5	1.77	5.31	5.76

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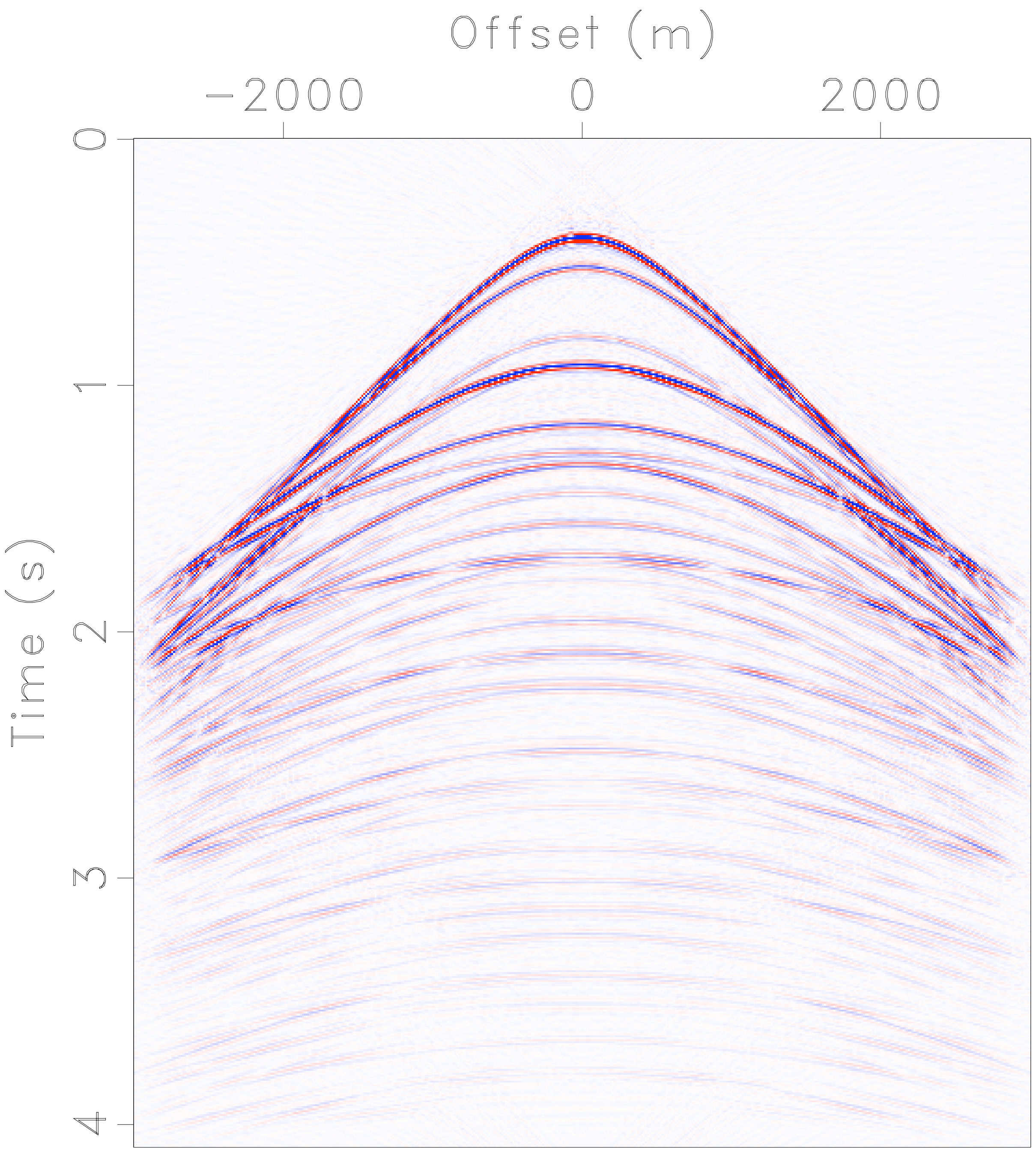
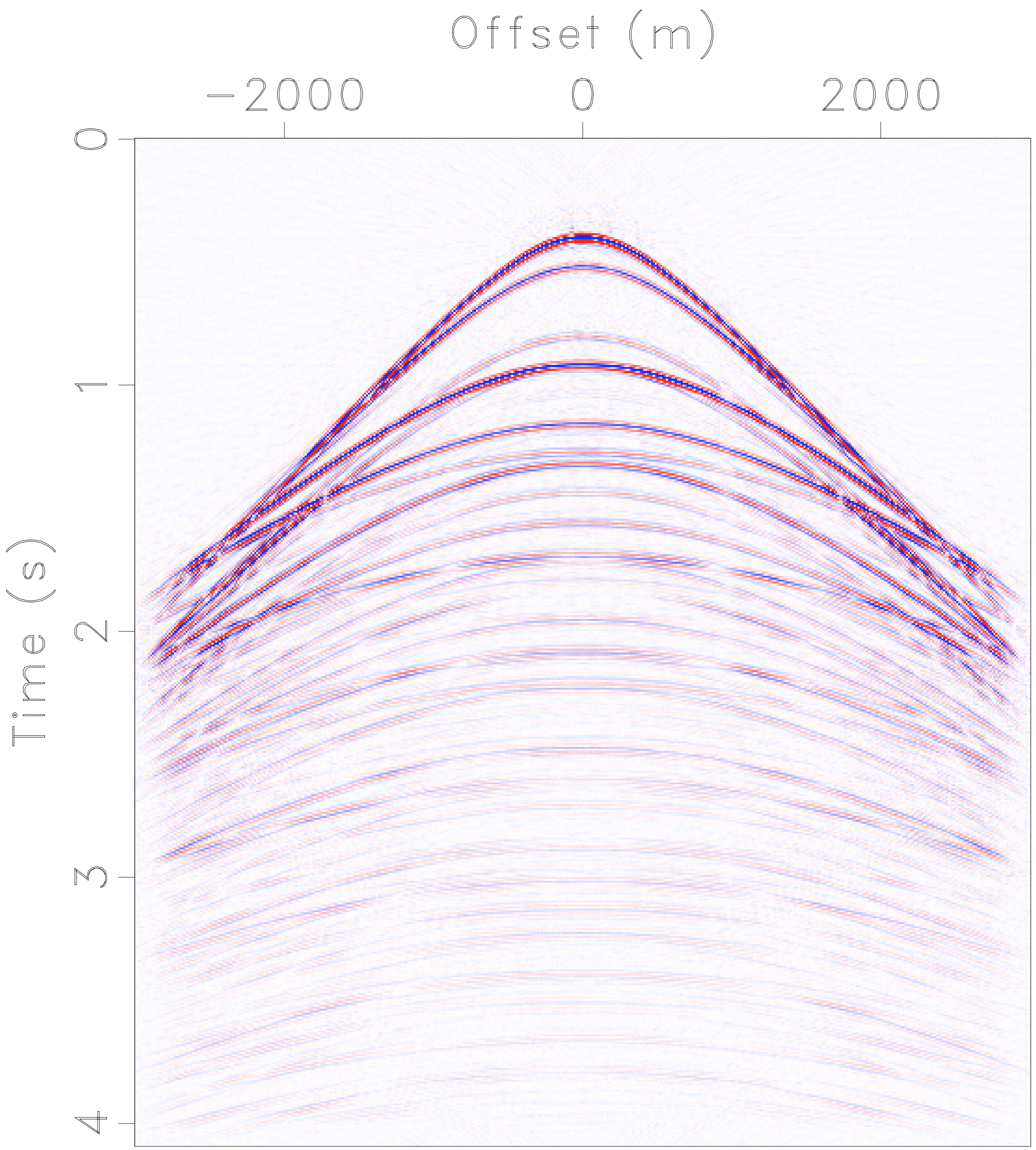
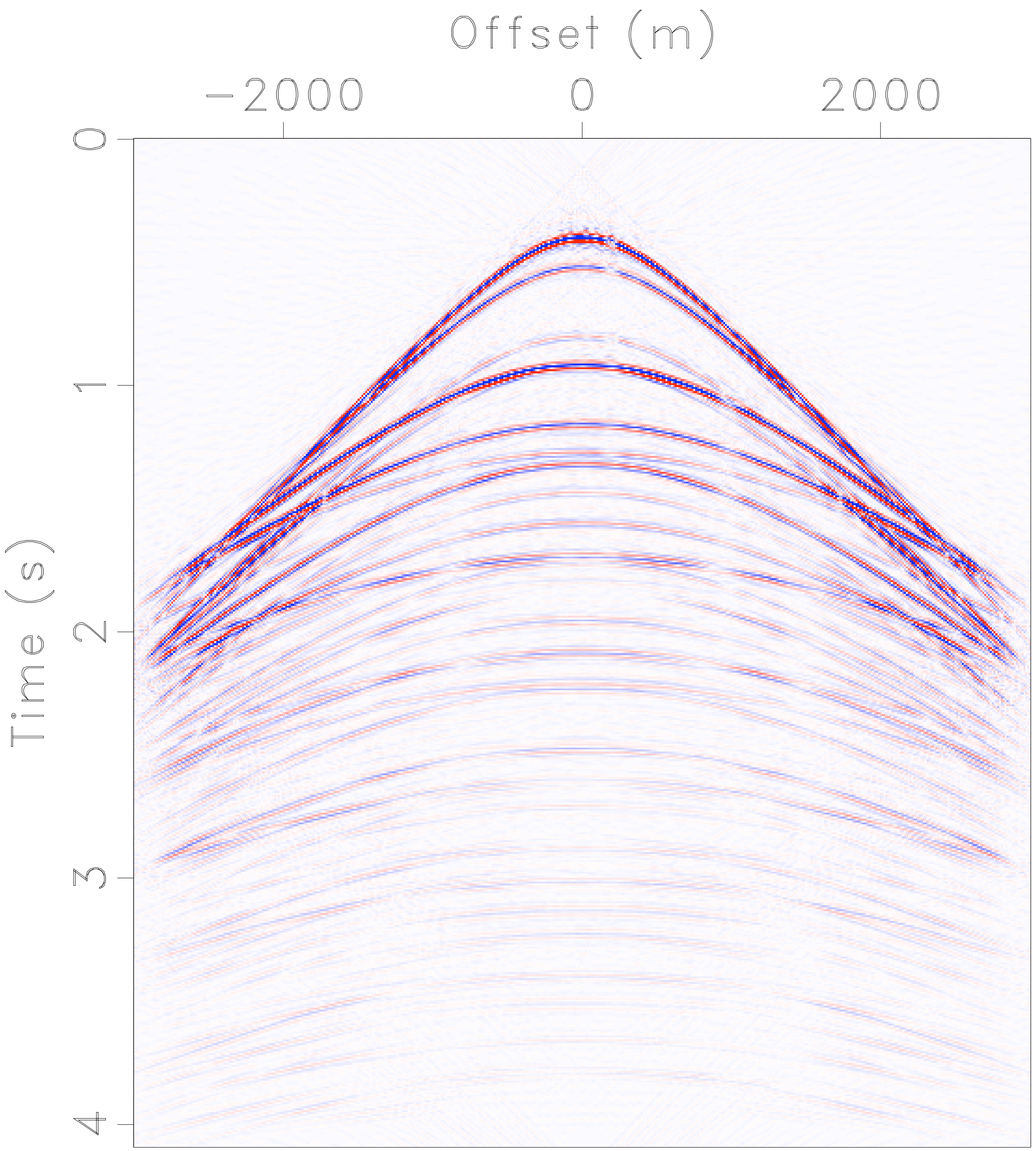


Denoising after Interpolating

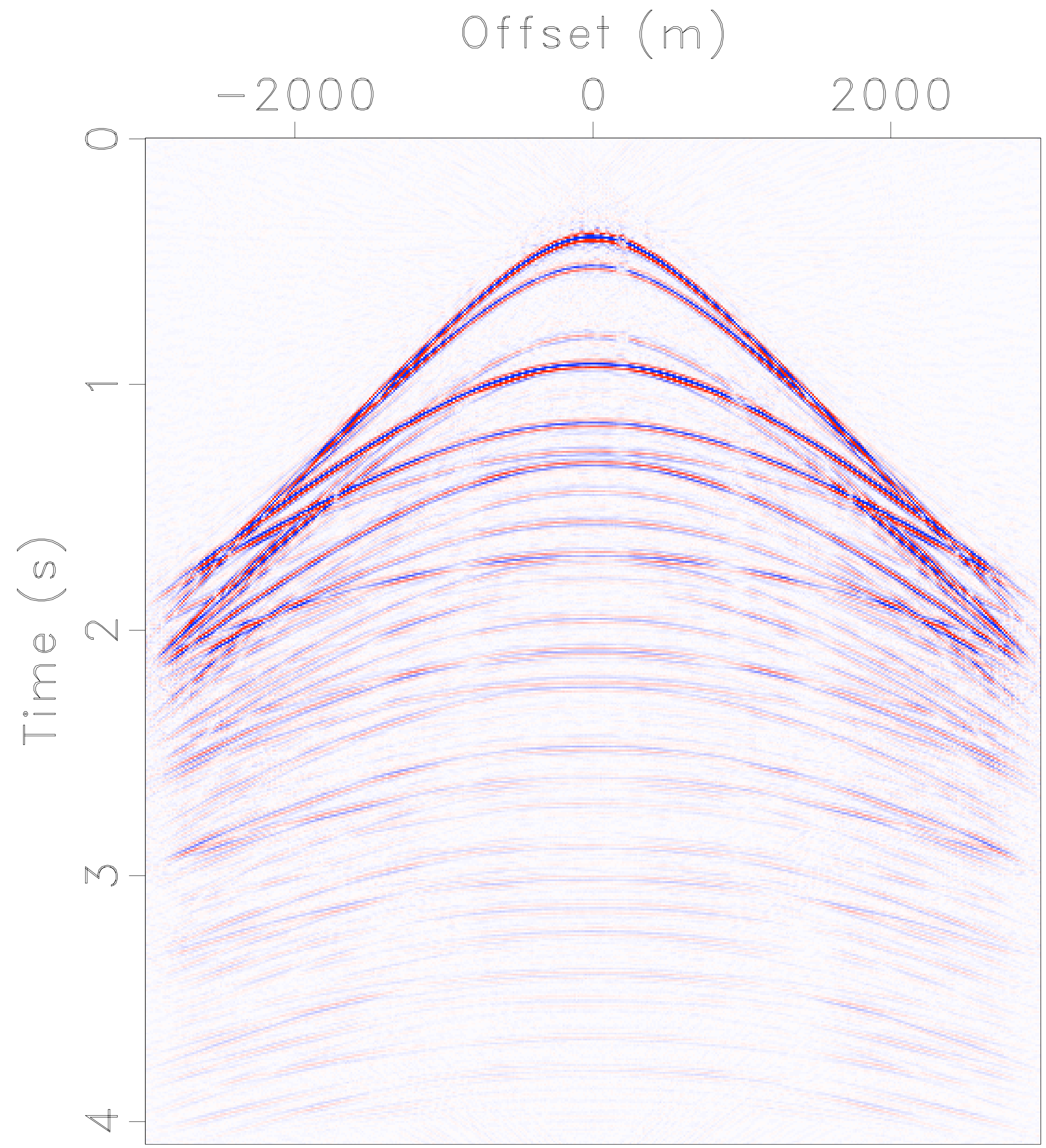


Combined

miss%=30%



Denoising after Interpolating



Combined

miss%=50%

Comparison for different percentages of missing traces

- $\epsilon 1 = \sigma\sqrt{N} \quad \epsilon 2 = \sigma\sqrt{N * (1 - miss\%)}$
- SNR3 ~ the SNR of interpolated data from data with missing traces but without noise
- SNR4 ~ the SNR of denoised data from data with noise but without missing traces is 8.58 dB

	miss%	Input SNR	SNR1(dB)	SNR2(dB)	SNR3(dB)	SNR4(dB)
1	10%	3.95	8.25	8.26	19.49	8.58
2	20%	2.93	7.91	7.93	15.70	8.58
3	30%	2.36	6.68	7.43	12.57	8.58
4	40%	1.84	6.10	6.89	10.45	8.58
5	50%	1.50	5.43	5.97	7.90	8.58

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Conclusion

- Conclusion
  - Synthetic data tests show Combined results slightly better than Denoise after Interpolate
  - small percentage of missing traces, close results; larger percentage of missing traces, larger difference

Acknowledgements

- SLIM team members G. Hennenfent, S. Ross Ross, H. Modzelewski, and C. Brown
- E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying for CurveLab (www.curvelet.org)
- S. Fomel, P. Sava, and the other developers of Madagascar (rsf.sourceforge.net)
- This presentation was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).