Recent results in curvelet-based primary-multiple separation

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4.Department of Mathematics,The University of British Columbia

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Contents

- Introduction
- Curvelet-based primary-multiple separation
- Examples
- Discussion and conclusion
- Acknowledgments

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Introduction

Problems with WE-based multiple elimination

- imperfect multiple predictions
  - amplitude
  - phase
  - timing
- failure of direct subtraction after matched filtering

Exploit the ability of curvelets to

- sparsify the to-be-separated signal components
- separation based on the curvelet parameterization
  - location
  - angle
  - scale

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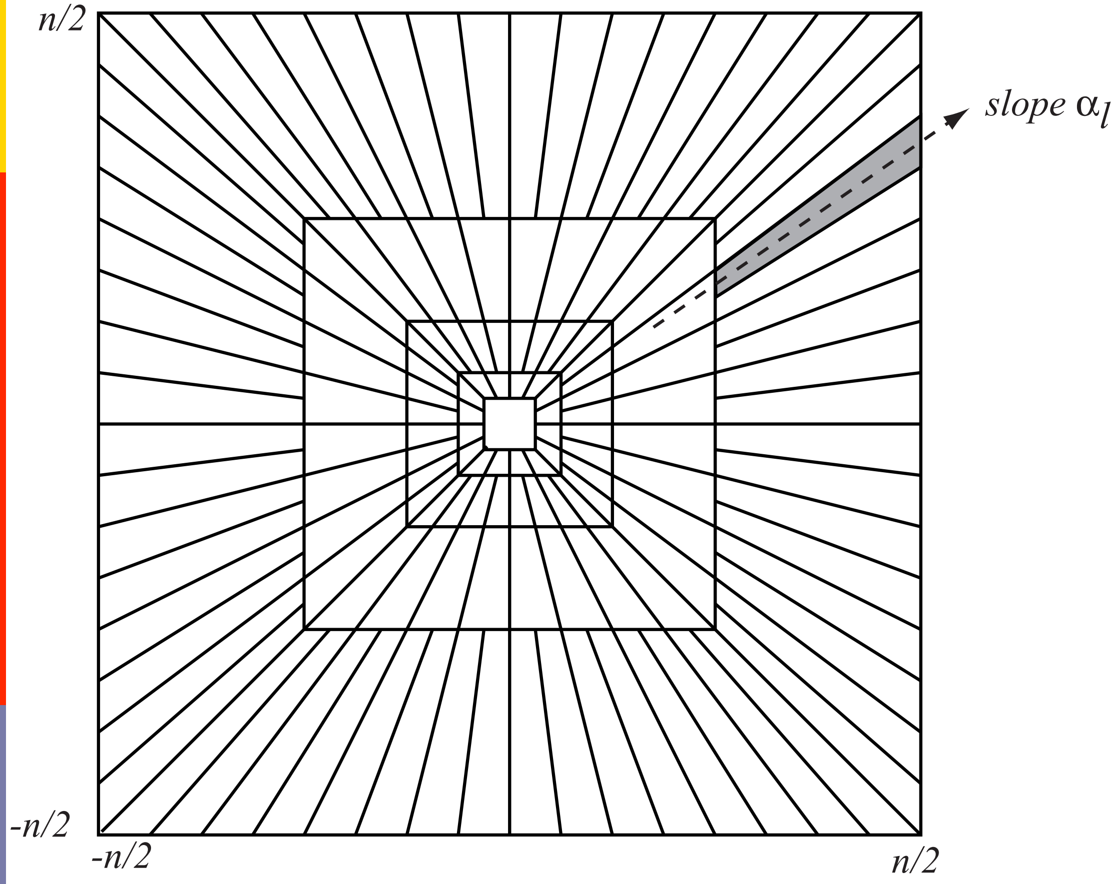
Introduction



Discrete frequency tiling

One curvelet

Ying et al,2005

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Curvelet-based separation

Forward model

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n}$$

$\mathbf{s}_1$: primaries

$\mathbf{s}_2$: multiples

Soft thresholding

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T S_w(\mathbf{C}\mathbf{s})$$

where

$$S_w(x) := \mathrm{sgn}(x) \cdot \max(0, |x| - w)$$

and $w := |\mathbf{C}\breve{\mathbf{s}}_2|$

- predictions may contain moderate
  - amplitude, phase
  - and sign errors

Herrmann et al,2006

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Curvelet-based separation

Nonlinear optimization from a Bayesian perspective

Forward model

$$\mathbf{b} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \qquad \text{(total data)}$$

$$\mathbf{b}_1 = \mathbf{A}\mathbf{x}_1 + \mathbf{n}_1 \qquad \text{(predicted primaries)}$$

$$\mathbf{b}_2 = \mathbf{A}\mathbf{x}_2 + \mathbf{n}_2 \qquad \text{(predicted multiples)}$$

where

$\mathbf{x}_1$ curvelet coefficients of primaries

$\mathbf{x}_2$ curvelet coefficients of multiples

$\mathbf{A}$ inverse curvelet transform

Saab et. al. 2007 & Wang et al,2008

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Curvelet-based separation

Separate by solving the nonlinear problem

$$\mathbf{P_w}: \quad \begin{cases} \tilde{\mathbf{x}} \;=\; \arg\min_{\mathbf{x}} \lambda_1 \|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2 \|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2\|_2^2 + \eta \|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

where

- $\mathbf{b}_2$ predicted multiples
- $\mathbf{A}$ inverse discrete curvelet transforms
- $\tilde{\mathbf{s}}_{1,2}$ estimated primaries(1)and multiples(2)
- $\lambda_{1,2}$ and η are control parameters

Can be solved by iterative soft thresholding.

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Curvelet-based separation

Given initial estimates of $\mathbf{x}_1^0$ and $\mathbf{x}_2^0$, the n^{th} iteration of the algorithm proceeds as follows

$$
\begin{aligned}
\mathbf{x}_1^{n+1} &= \mathbf{T}_{\frac{\boldsymbol{\lambda}_1 \mathbf{w}_1}{2\eta}} \left[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n + \mathbf{x}_1^n\right] \\
\mathbf{x}_2^{n+1} &= \mathbf{T}_{\frac{\boldsymbol{\lambda}_2 \mathbf{w}_2}{2(1+\eta)}} \left[\mathbf{A}^T\mathbf{b}_2 - \mathbf{A}^T\mathbf{A}\mathbf{x}_2^n + \mathbf{x}_2^n + \frac{\eta}{\eta+1}\left(\mathbf{A}^T\mathbf{b}_1 - \mathbf{A}^T\mathbf{A}\mathbf{x}_1^n\right)\right]
\end{aligned}
$$

where $\mathbf{T_u} : \mathbb{R}^{|\mathcal{M}|} \mapsto \mathbb{R}^{|\mathcal{M}|}$ is the elementwise soft-thresholding operator

(Rayan Saab et al.,2007 and his presentation this meeting)

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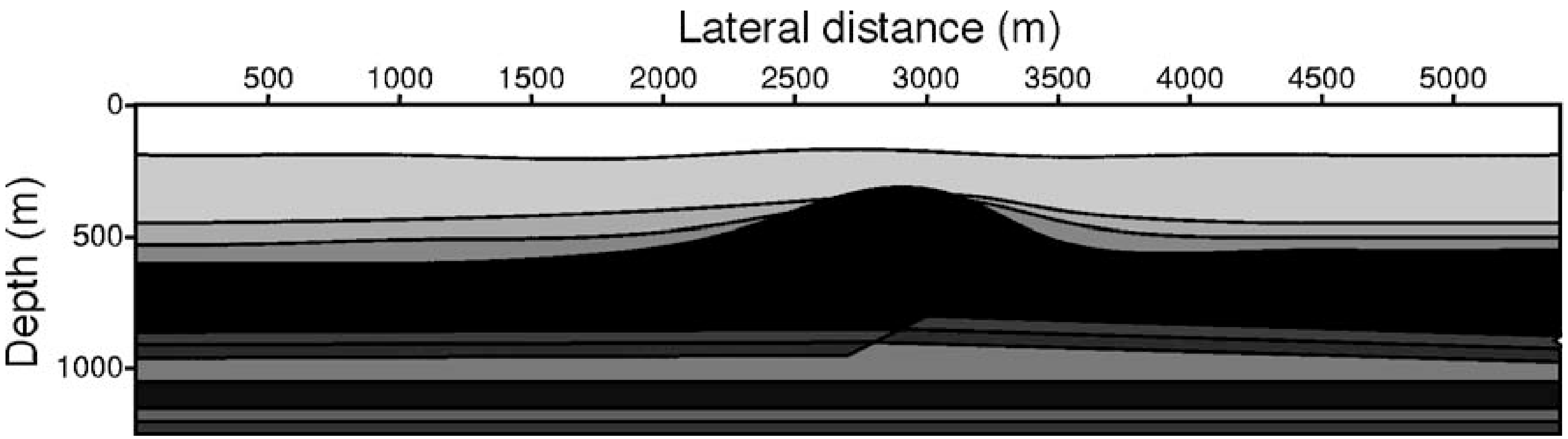
Examples

Example 1

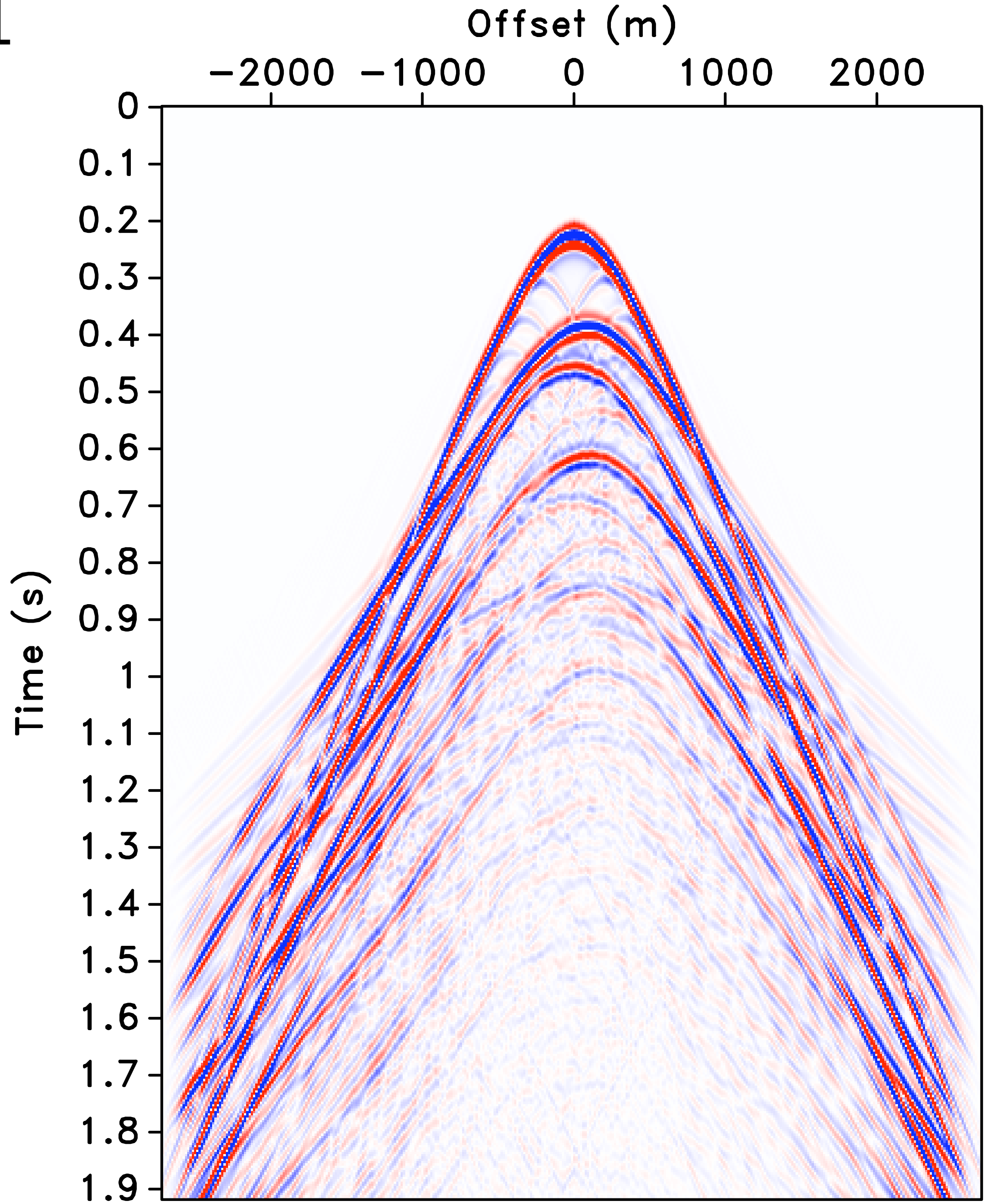
Synthetic data:

361 shots
361 traces/shot
501 samples/trace



Geology model

Example 1



Total data

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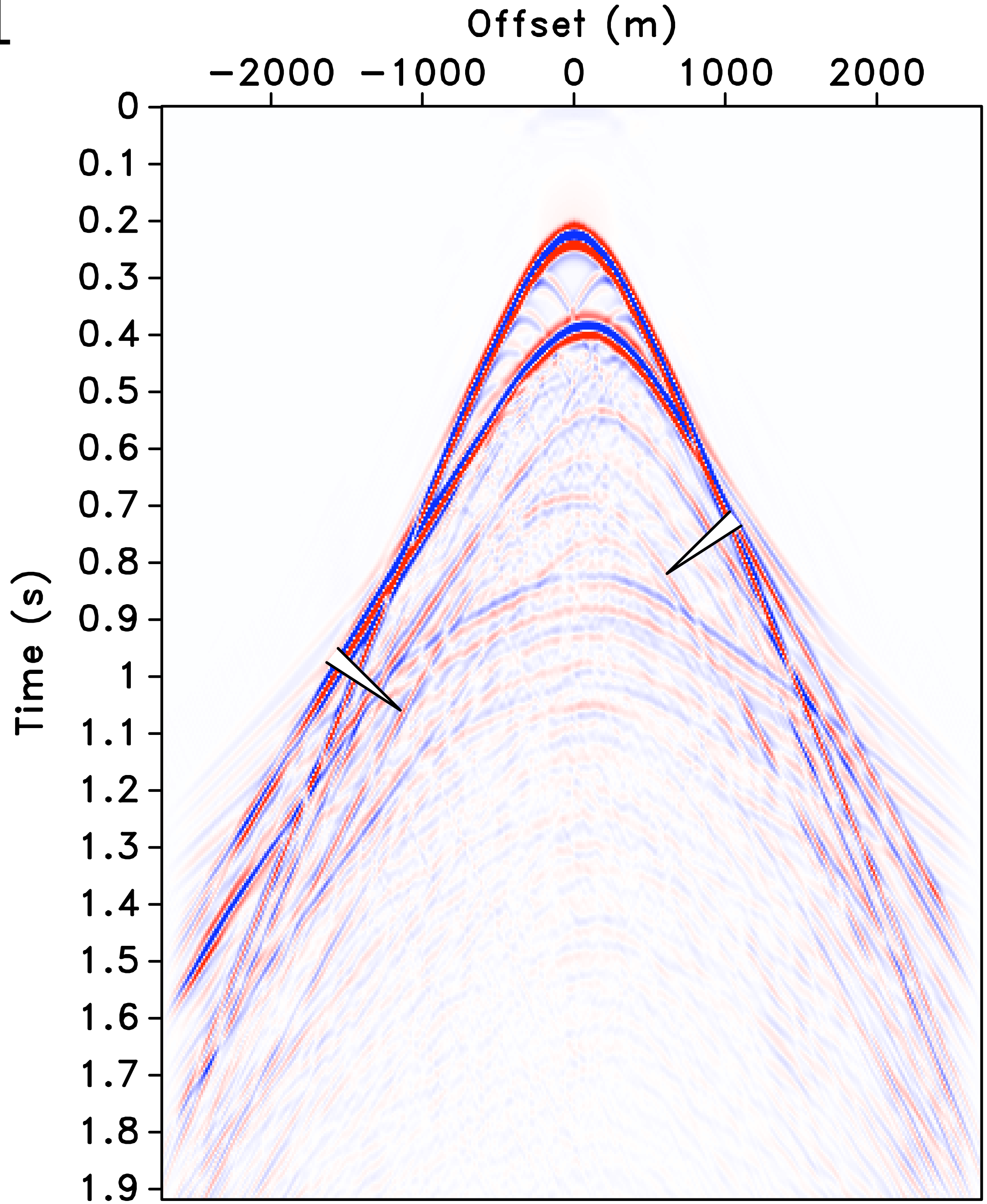
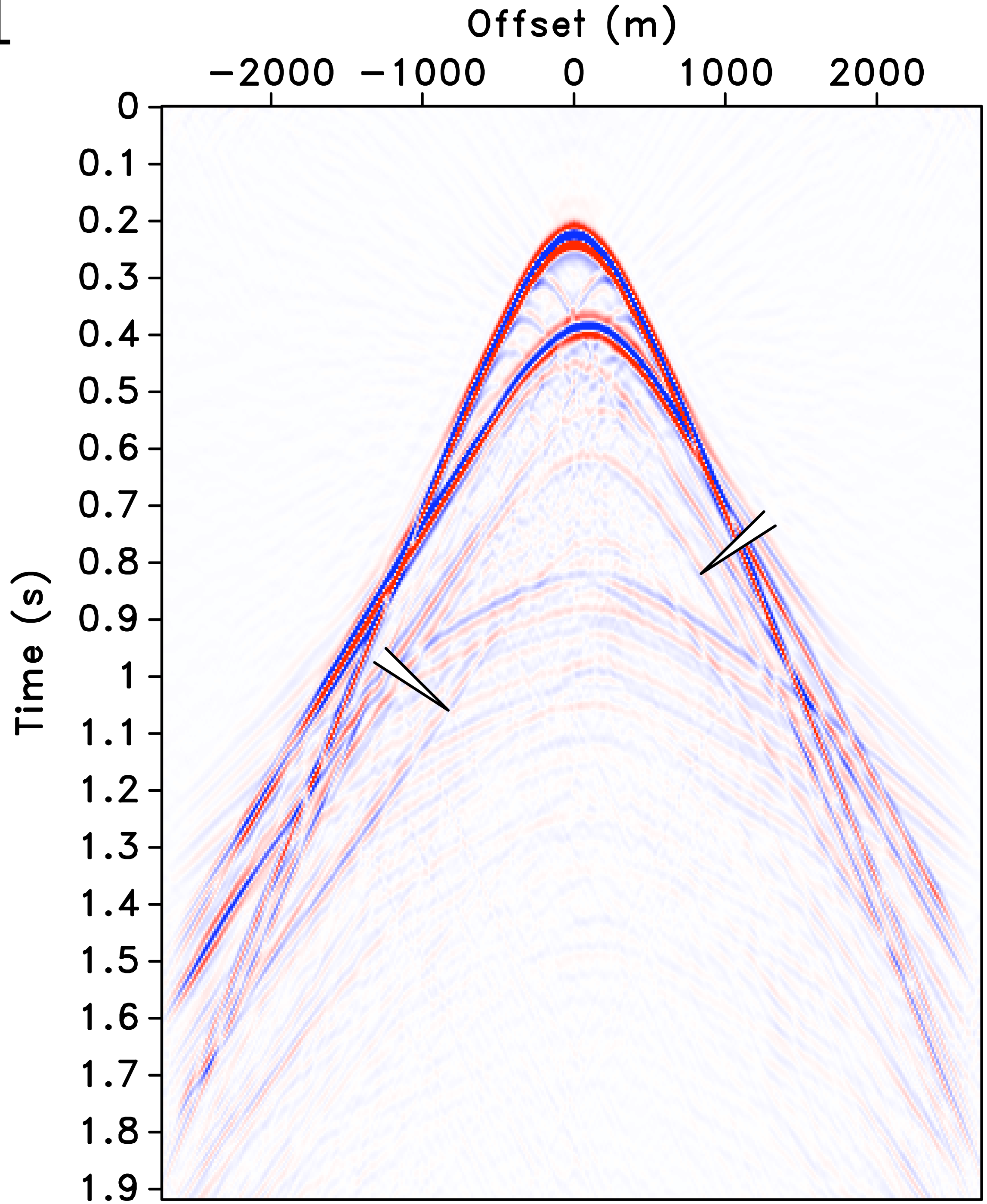
Example 1



SRME primaries

Example 1



3D single threshold

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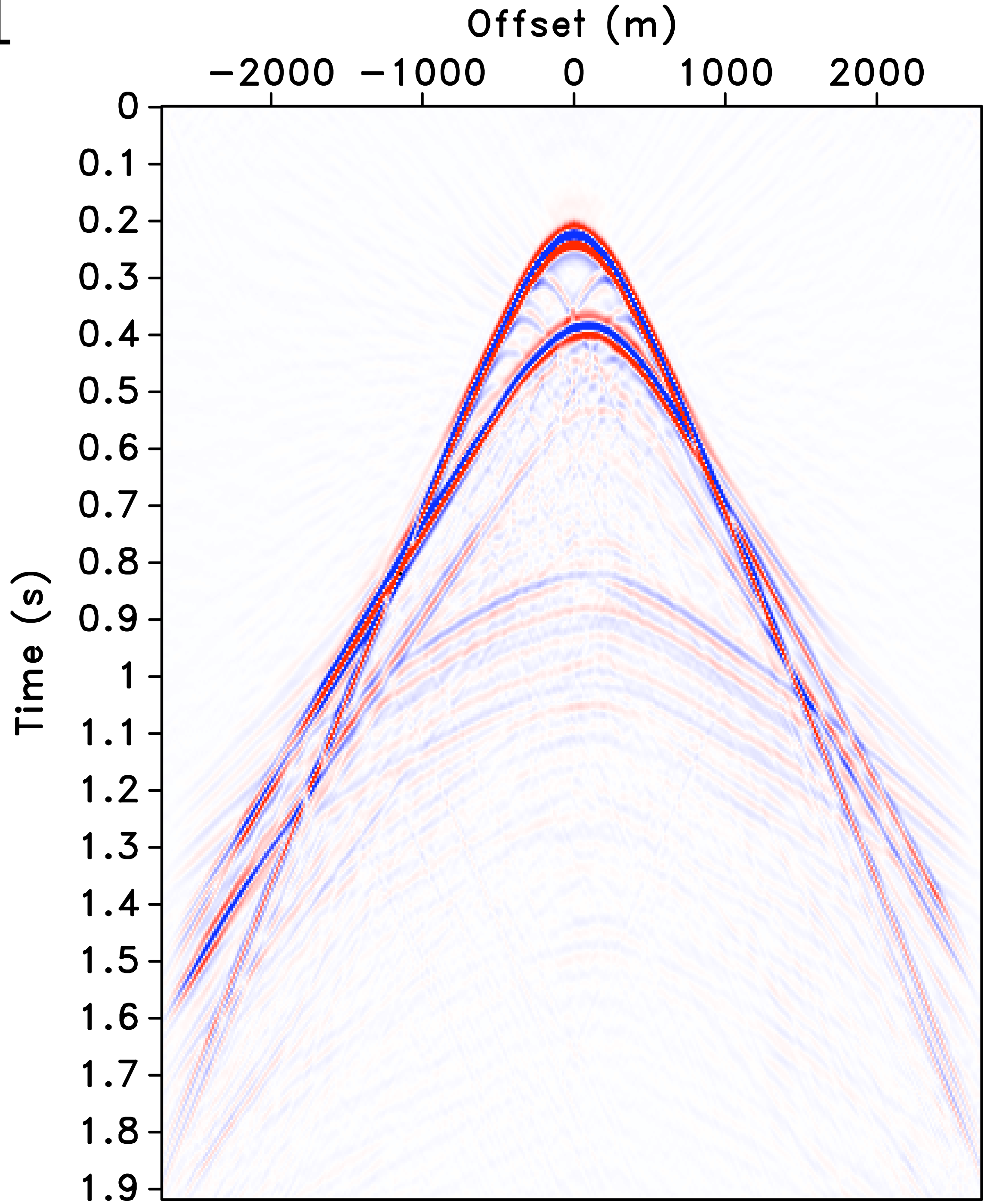
Example 1



3D Bayesian

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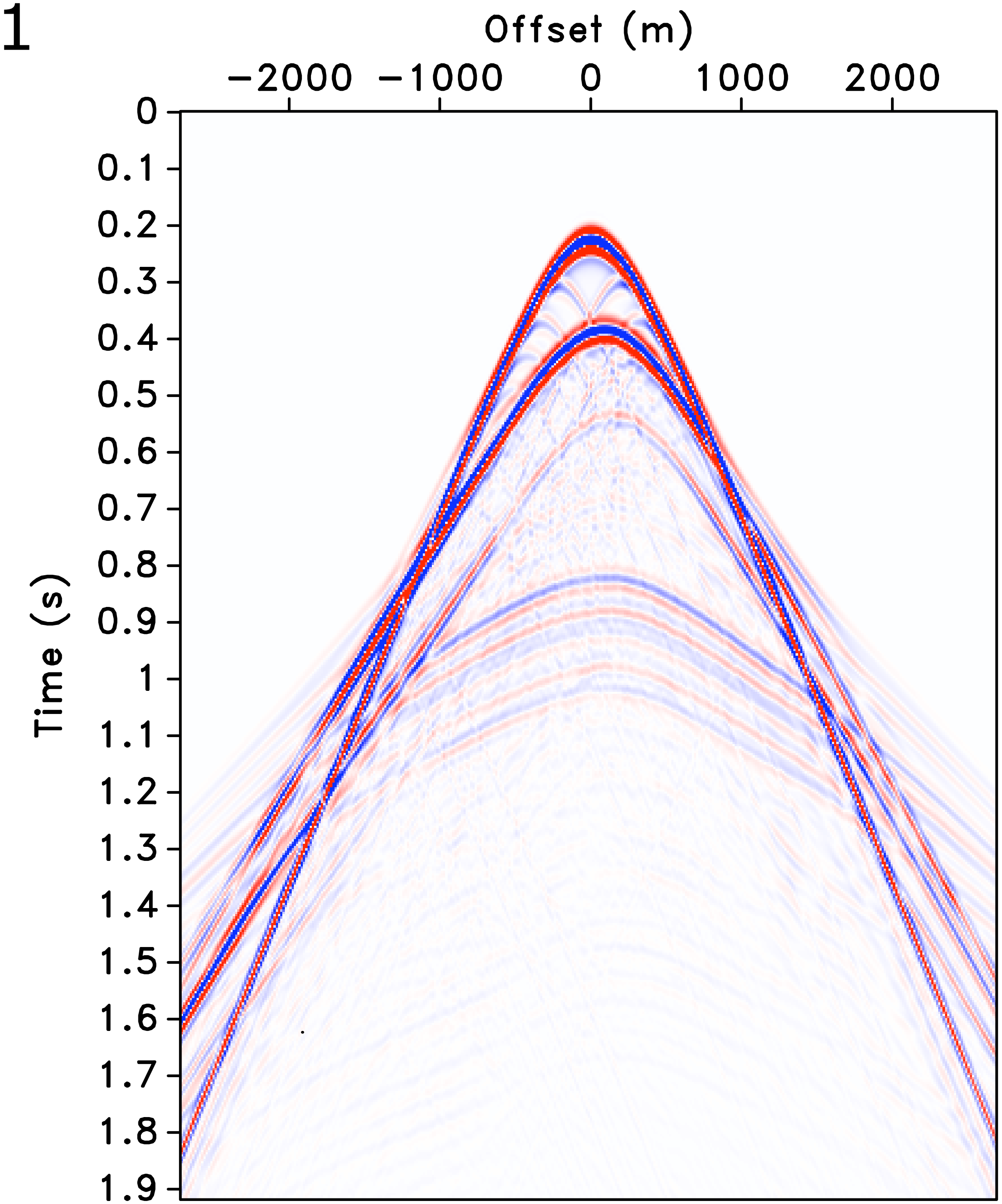
Example 1



True primaries

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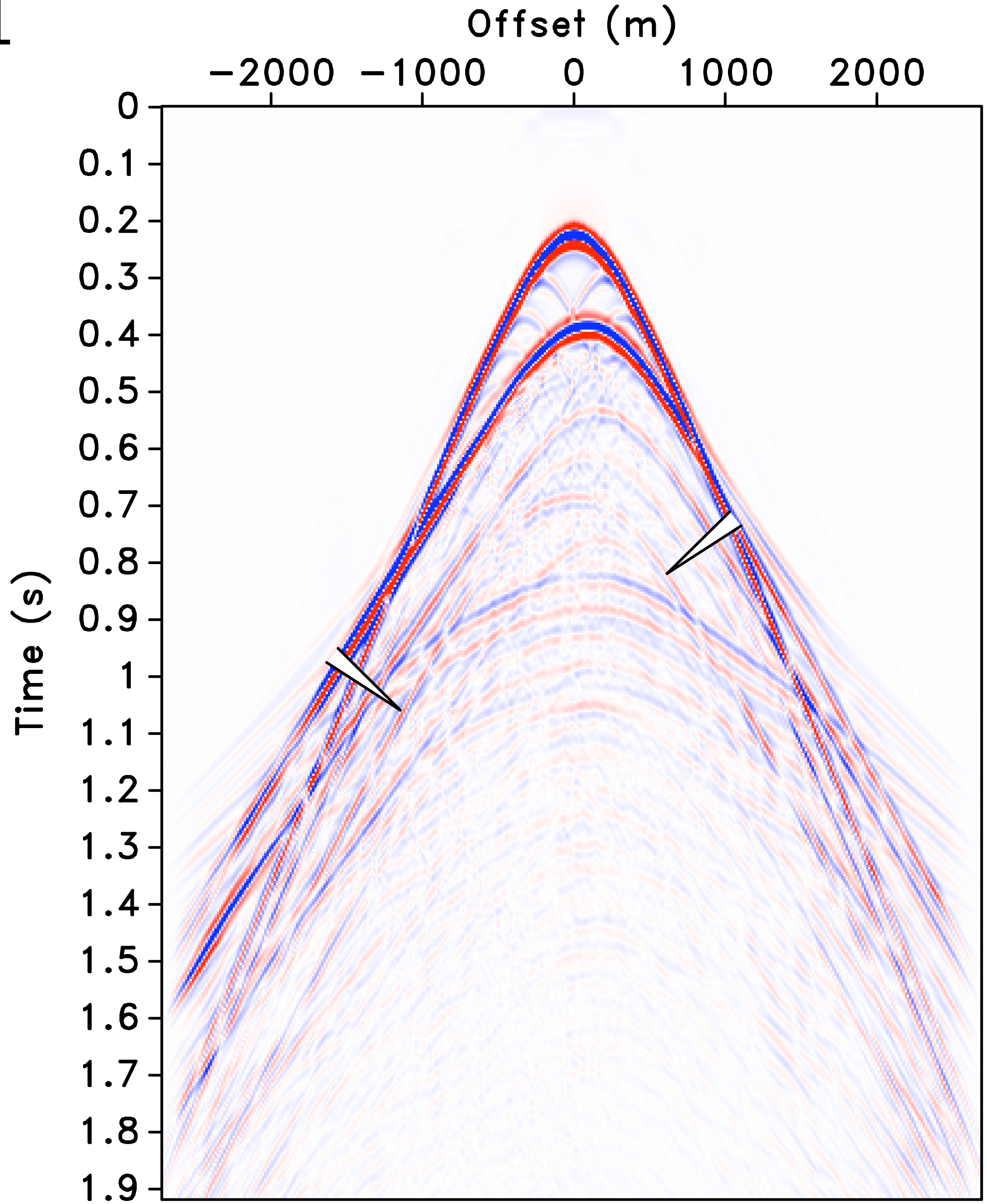
Example 1



SRME primaries

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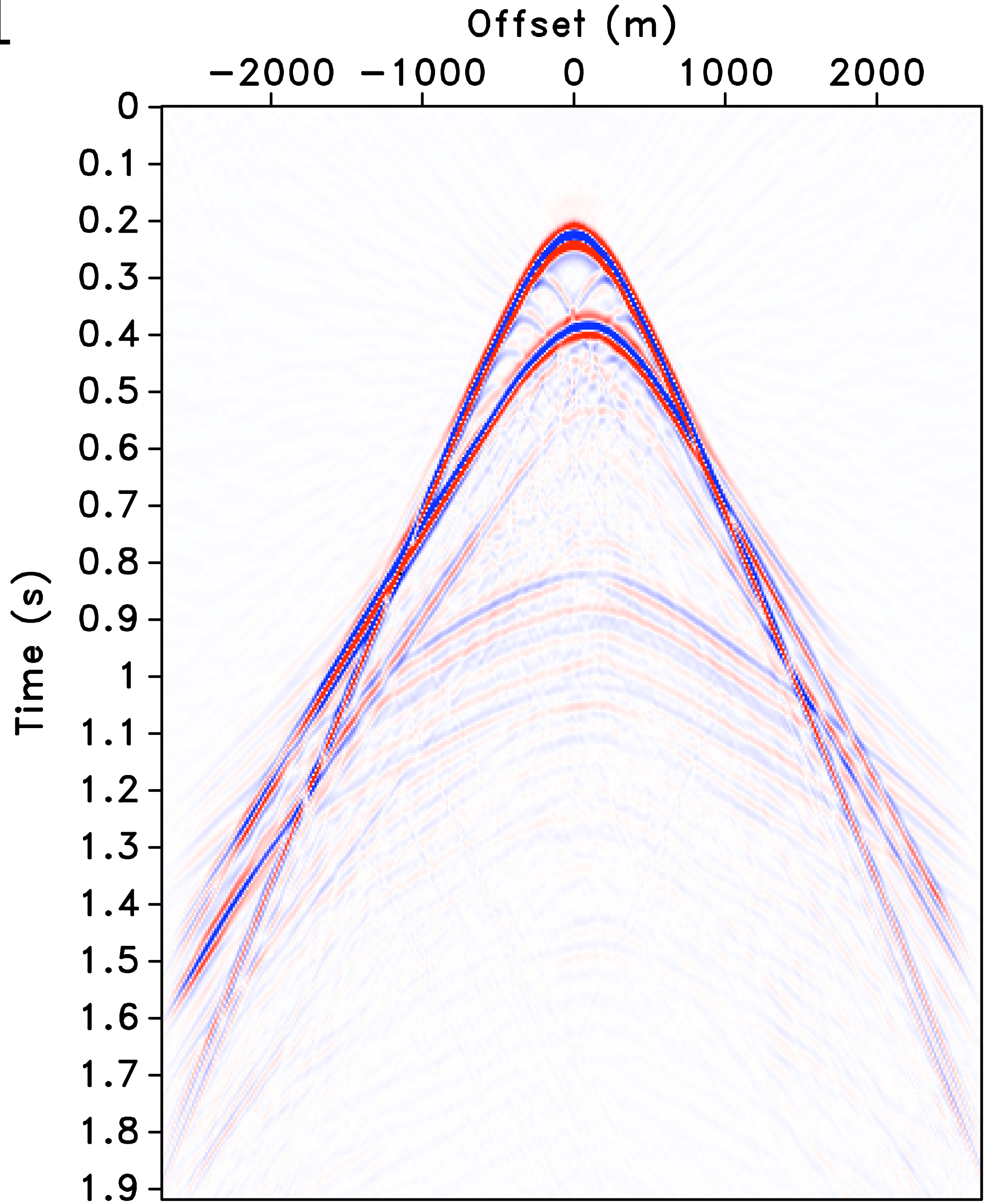
Example 1



3D Bayesian threshold

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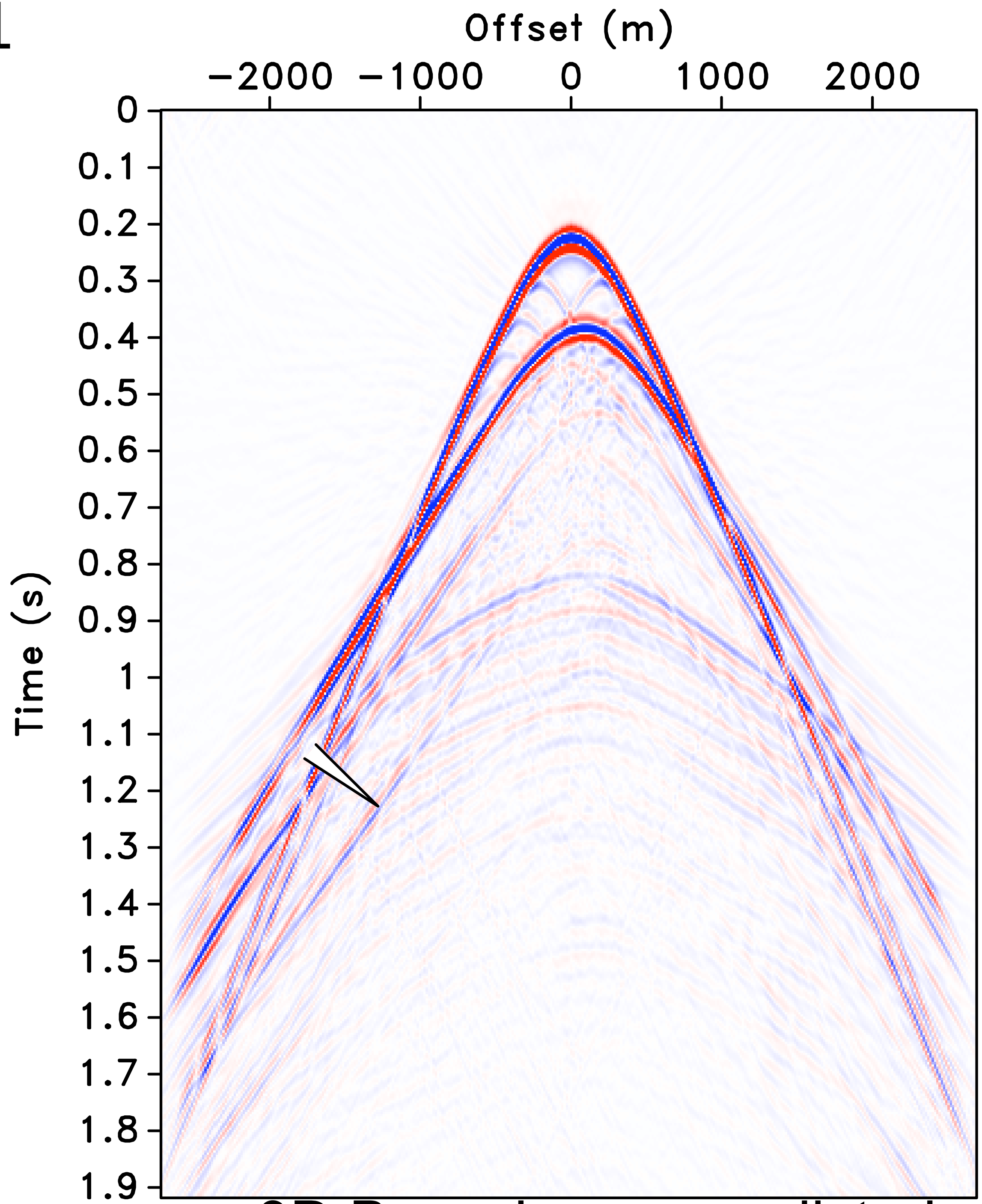
Example 1



3D Bayesian no predicted multiple control

Curvelet-based separation

Separate by solving the nonlinear problem

$$\mathbf{P_w}: \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda_1 \|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2 \|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \underline{\|\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2\|_2^2} + \eta \|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

where

$\mathbf{b}_2$ predicted multiples

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Can be solved by iterative soft thresholding.

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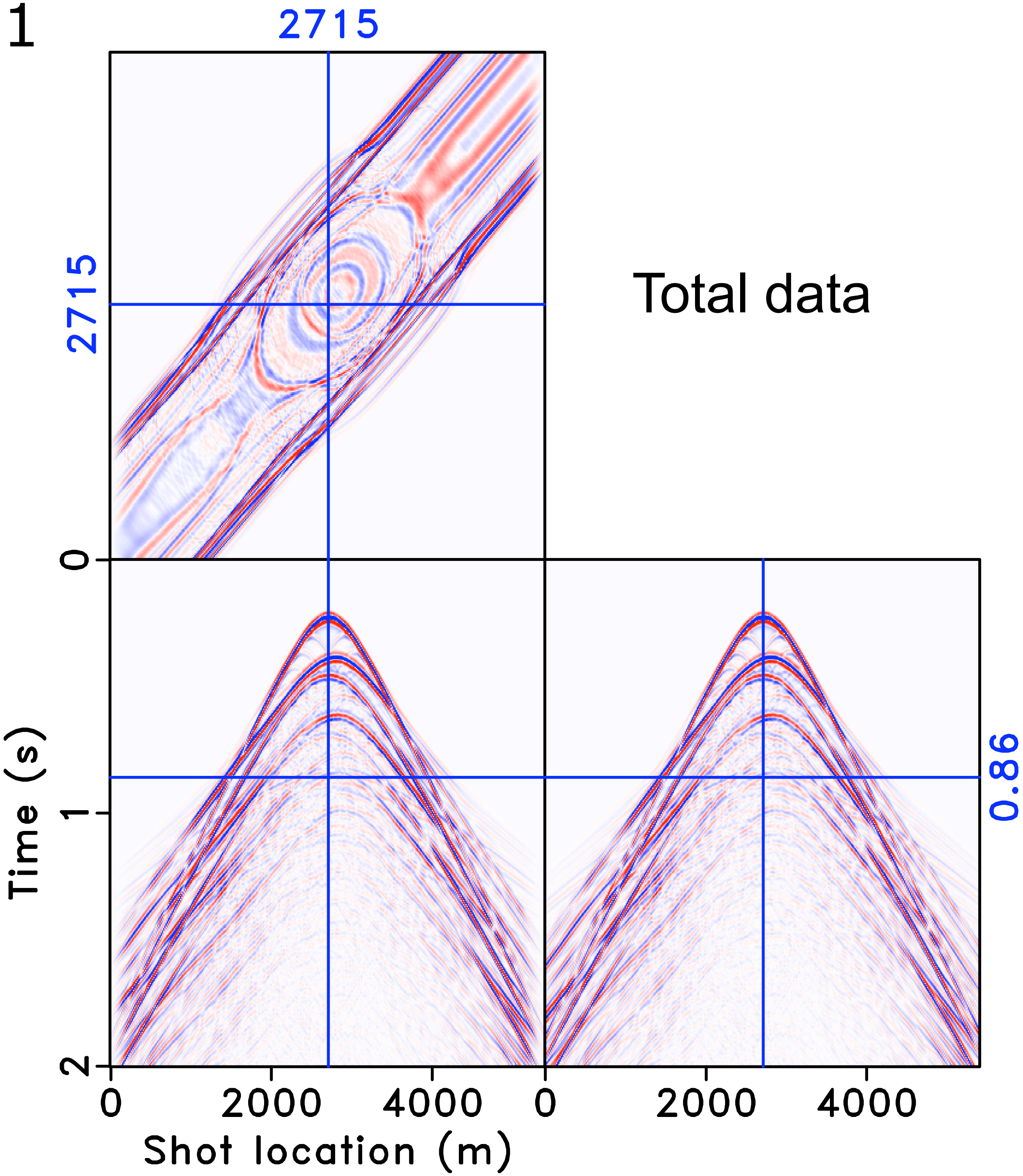
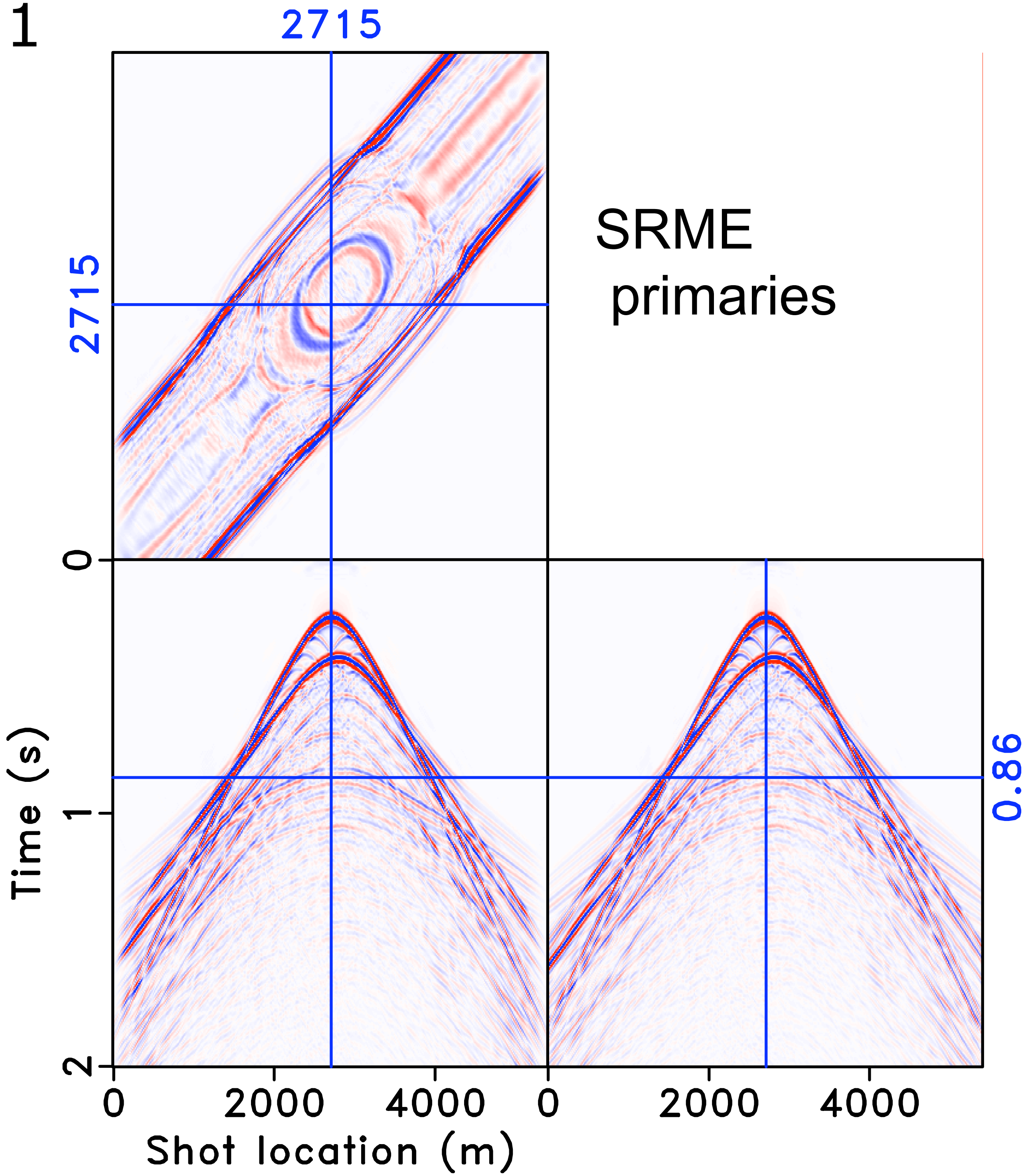
Example 1



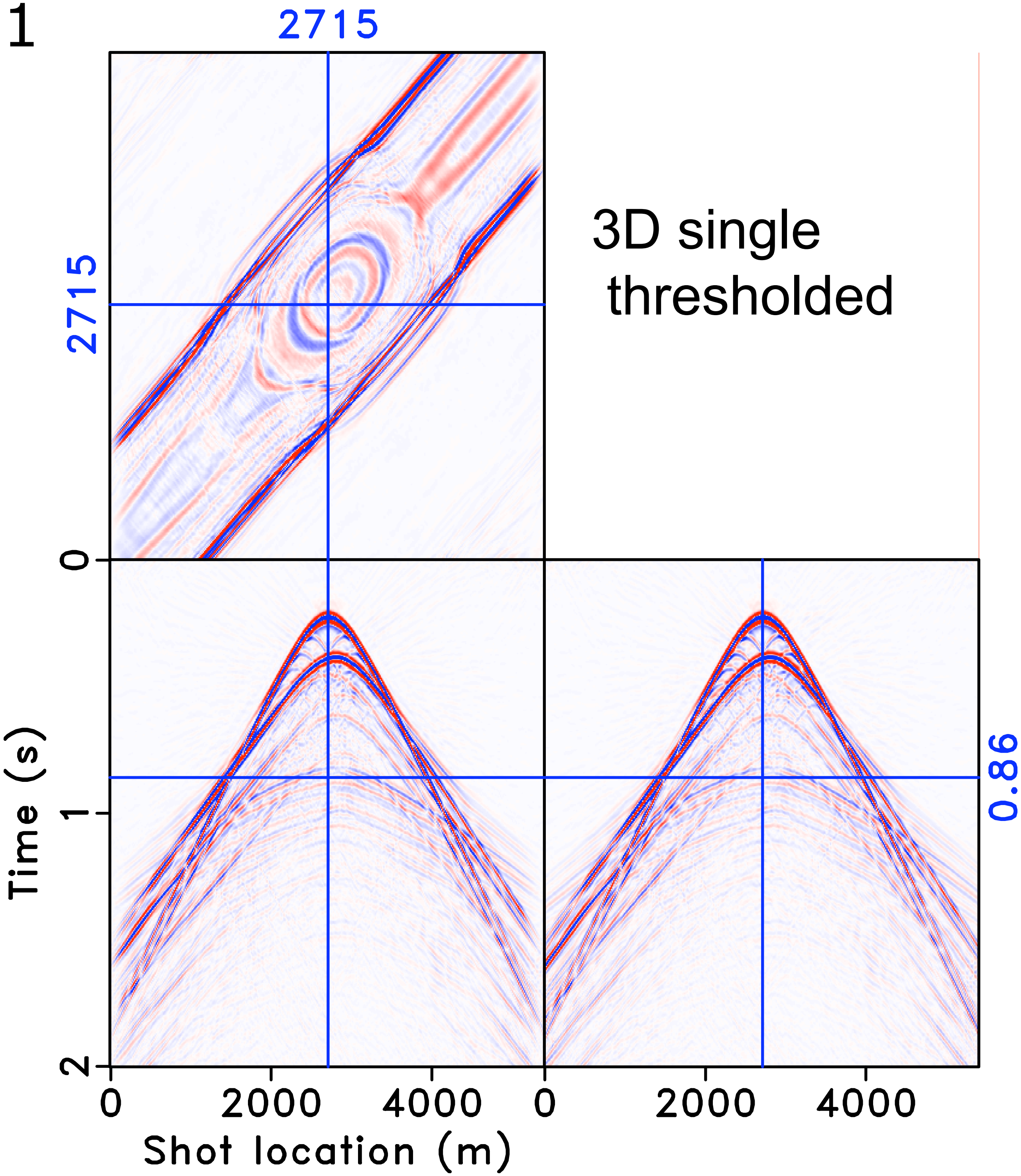
Total data

Example 1



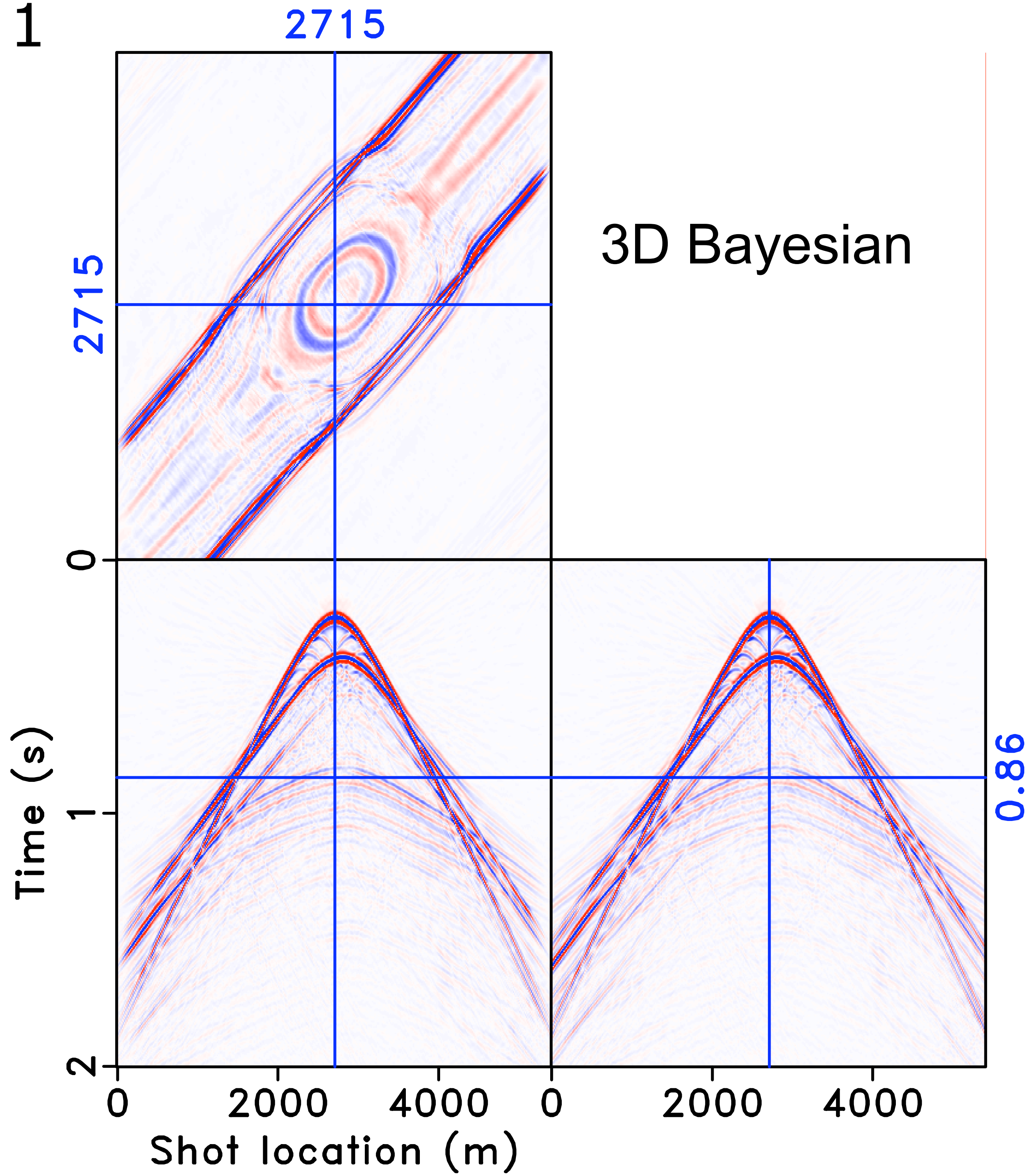
SRME primaries

Example 1

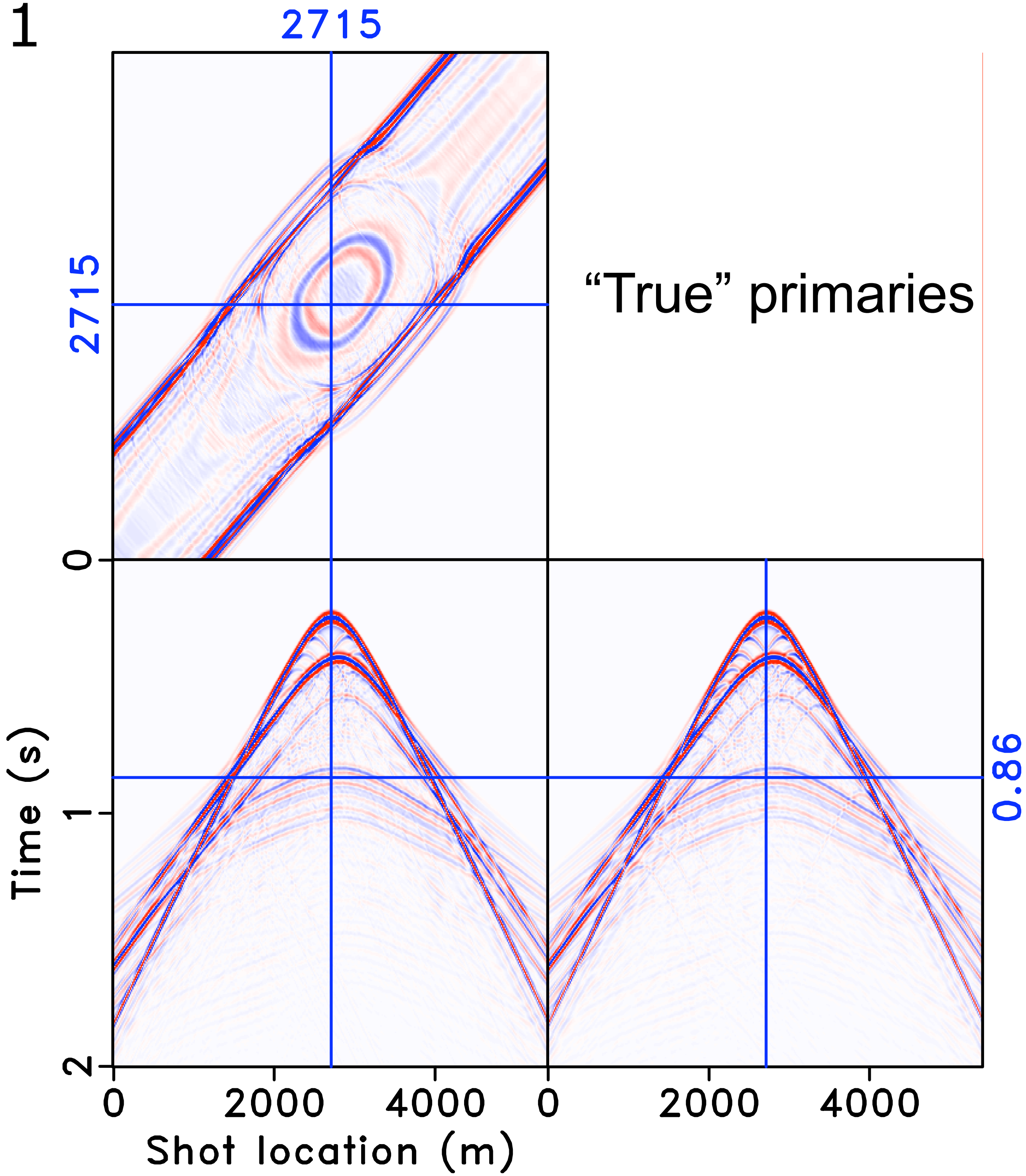


3D single thresholded

Example 1

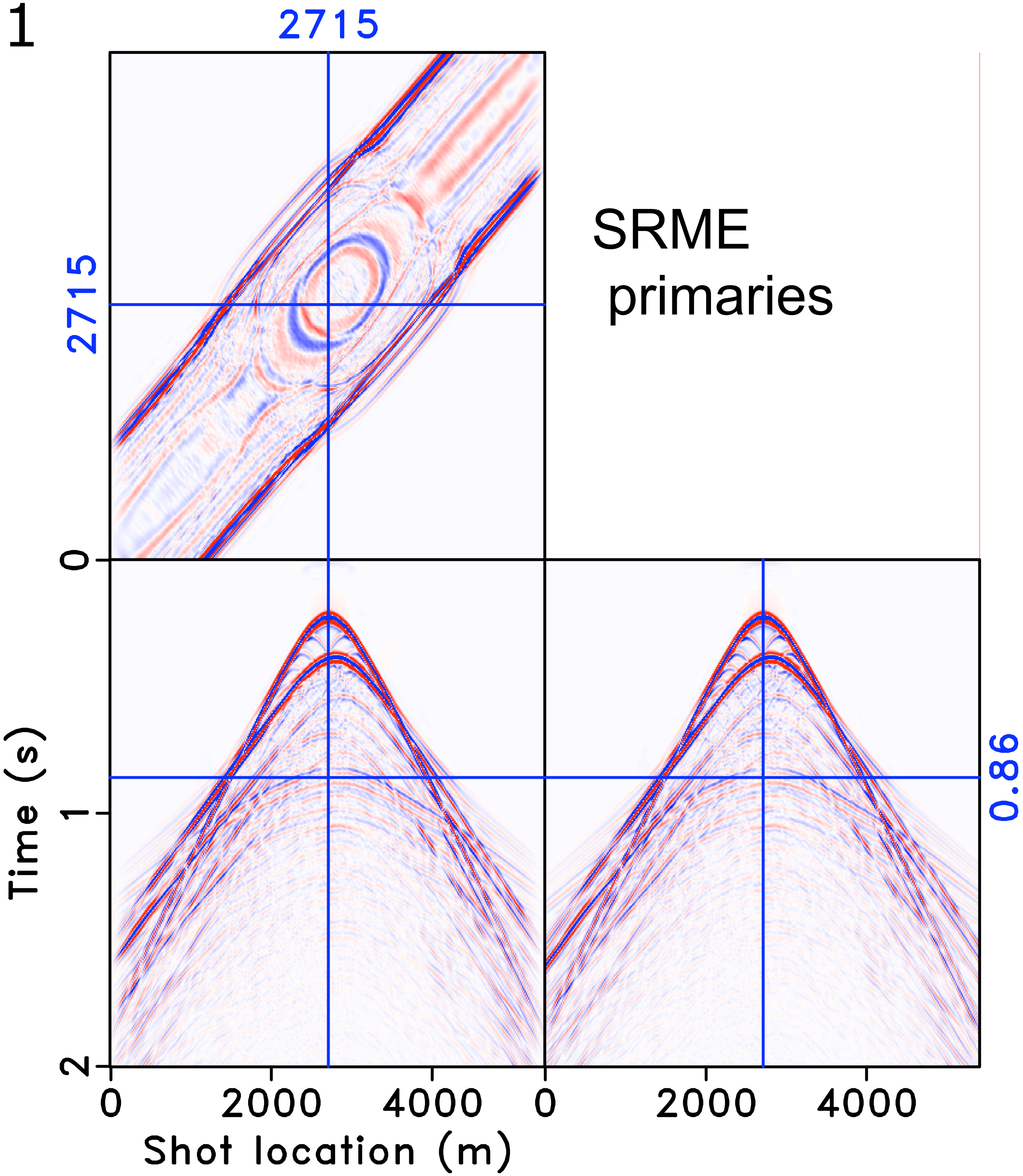

2715
2715
0
1
2
Time (s)
0.86
0
2000
4000
0
2000
4000
Shot location (m)
3D Bayesian

Example 1



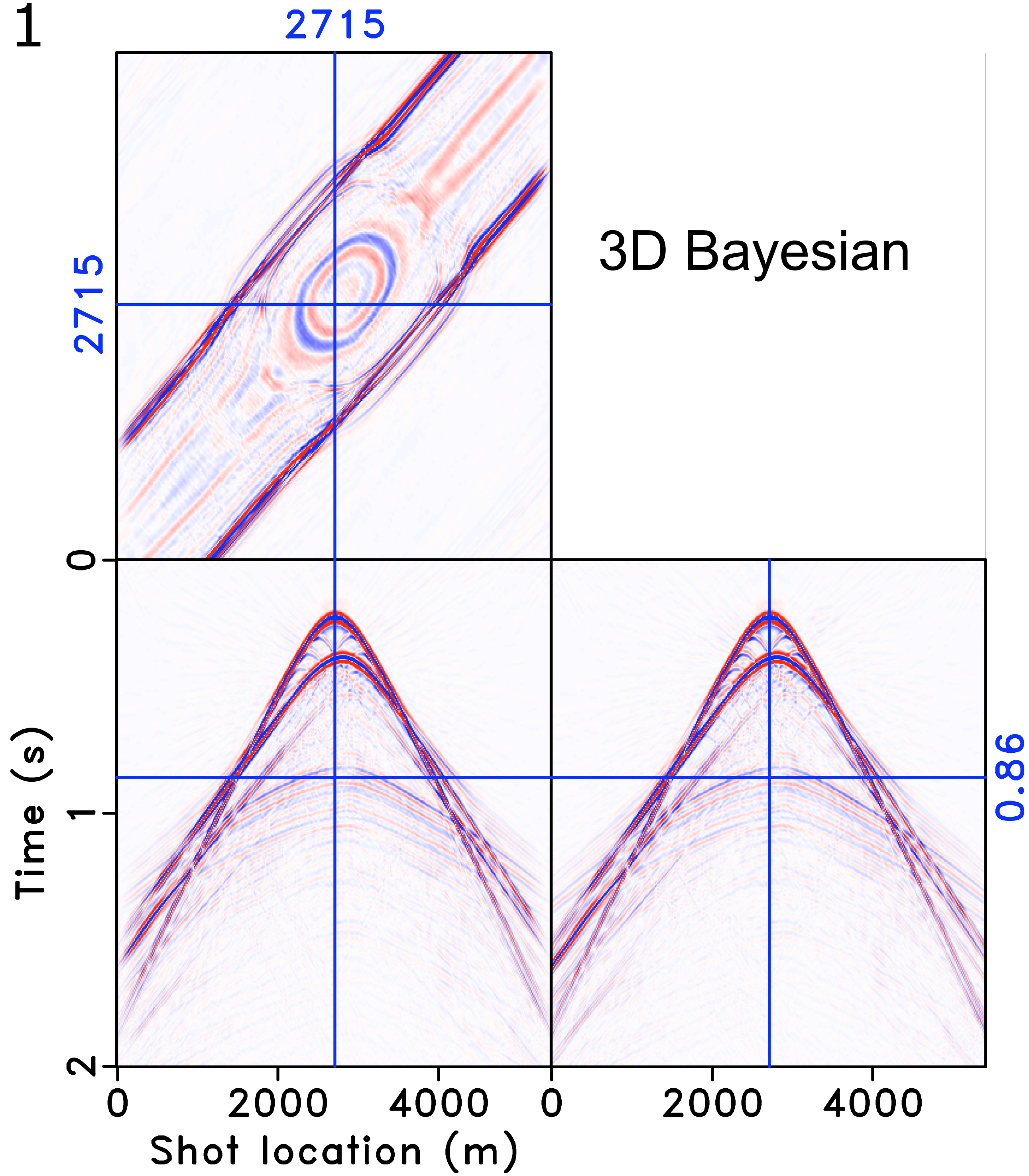
"True" primaries

Example 1



2715
2715
SRME
primaries
0
1
2
Time (s)
0.86
0
2000
4000
0
2000
4000
Shot location (m)

Example 1



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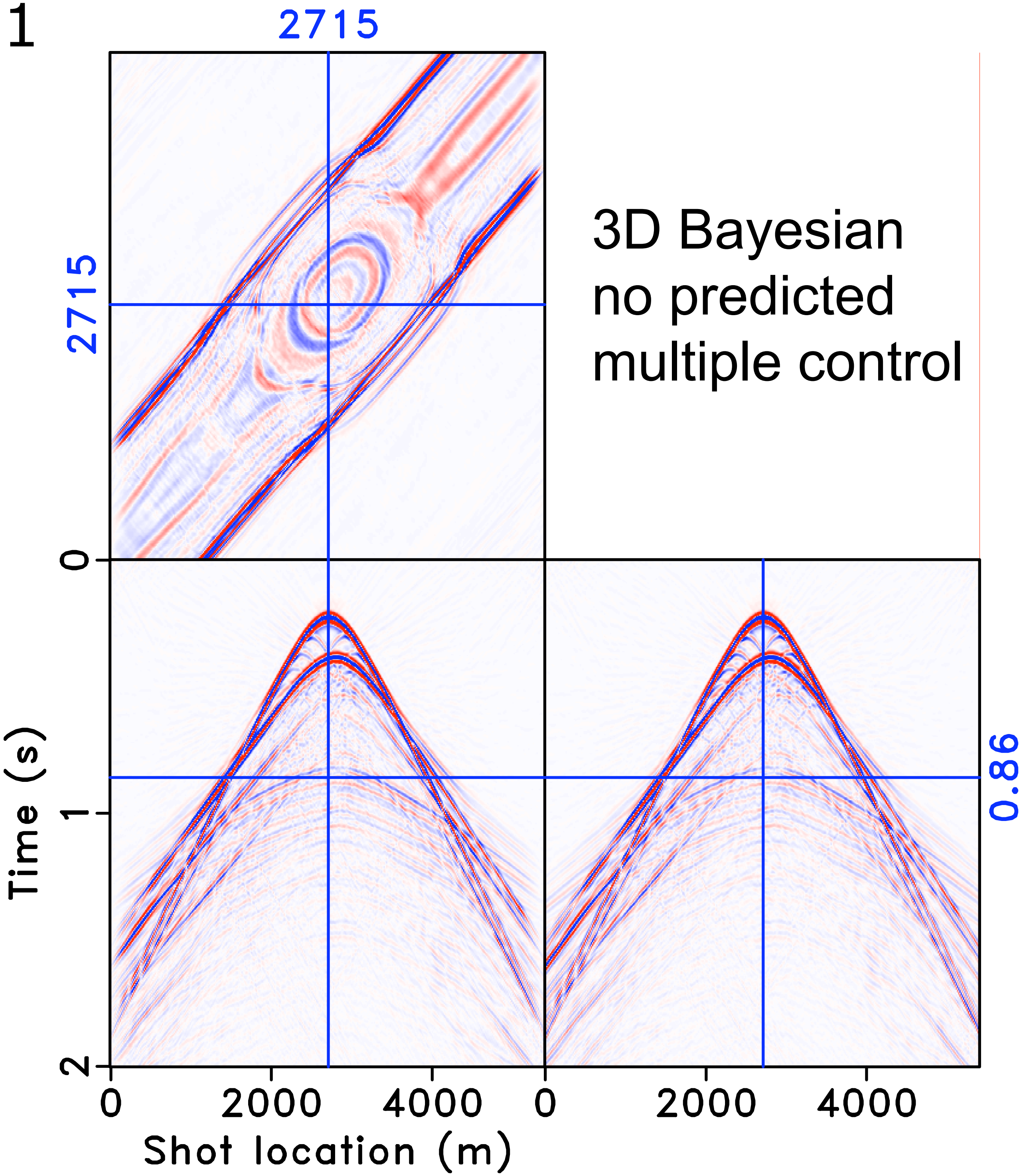
Example 1



3D Bayesian
no predicted
multiple control

Example 1

Sensitivity analysis for the performance of Bayesian

SNR (dB)	$\{\lambda_1^*, \lambda_2^*\}$	$\{2\cdot\lambda_1^*, \lambda_2^*\}$	$\{\lambda_1^*, 2\cdot\lambda_2^*\}$	$100\cdot\{\lambda_1^*, \lambda_2^*\}$
η^*	12.13	11.21	11.46	-
$\frac{1}{2}\cdot\eta^*$	11.36	9.43	11.46	-
$2\cdot\eta^*$	11.44	12.13	9.92	-
$100\cdot\eta^*$	-	-	-	10.65

with $\lambda_1^* = 0.7, \lambda_2^* = 2.0, \eta^* = 0.5$

SNRs are computed with respect to the "true" primaries are relative robust against changes. The inclusion of control on the estimated multiples adds 1.48 dB. (SRME : 9.82 dB)

$$\mathbf{P_w}: \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda_1\|\mathbf{x}_1\|_{1,\mathbf{w}_1} + \lambda_2\|\mathbf{x}_2\|_{1,\mathbf{w}_2} + \\ \|\mathbf{A}\mathbf{x}_2 - \mathbf{b}_2\|_2^2 + \eta\|\mathbf{A}(\mathbf{x}_1 + \mathbf{x}_2) - \mathbf{b}\|_2^2 \\ \tilde{\mathbf{s}}_1 = \mathbf{A}\tilde{\mathbf{x}}_1 \quad \text{and} \quad \tilde{\mathbf{s}}_2 = \mathbf{A}\tilde{\mathbf{x}}_2. \end{cases}$$

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Examples

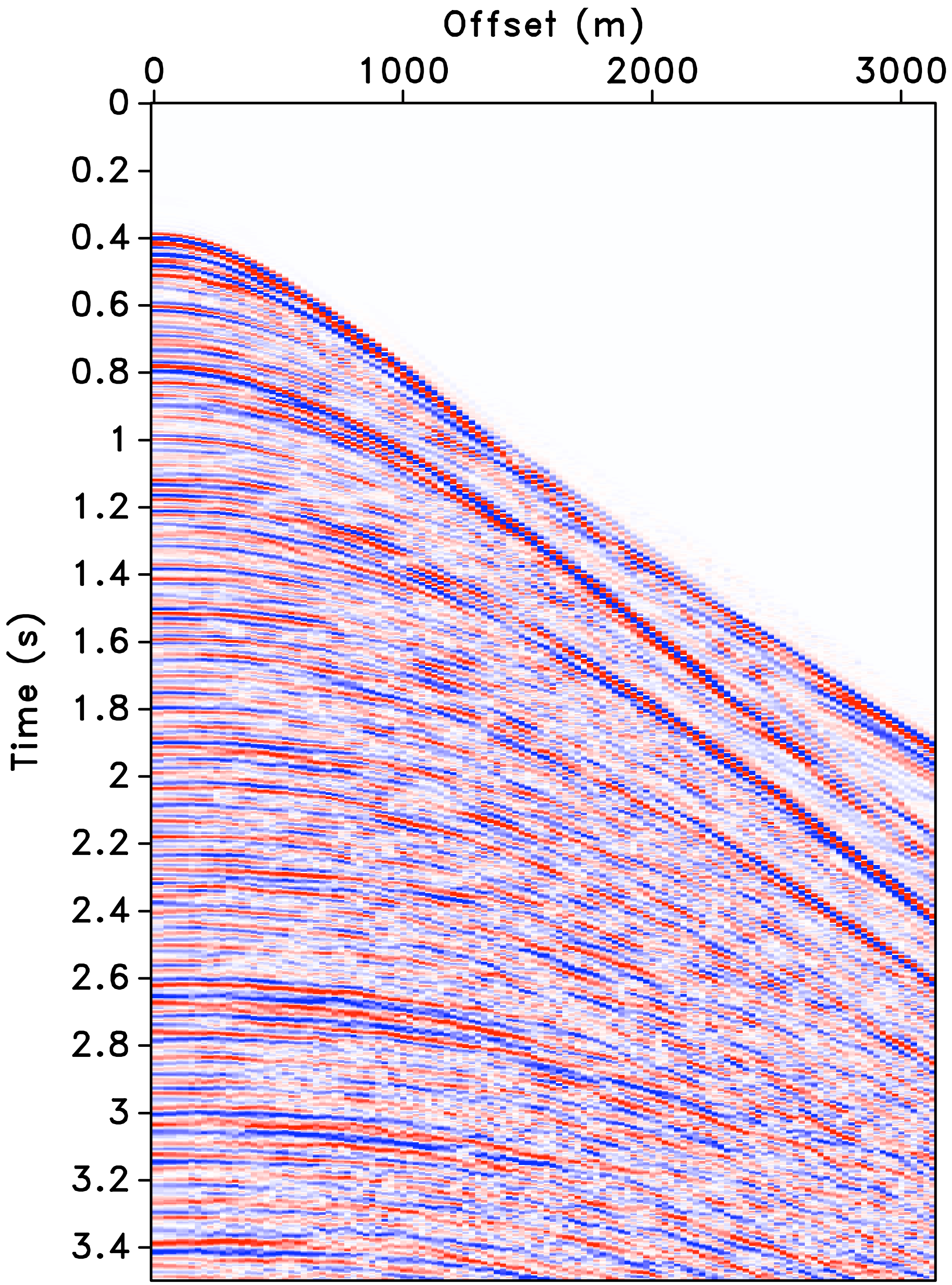
Example 2

Saga data:

261 shots
126 traces/shot
1024 samples/trace

The original data contains many strong surface-related multiples

Example 2



Data

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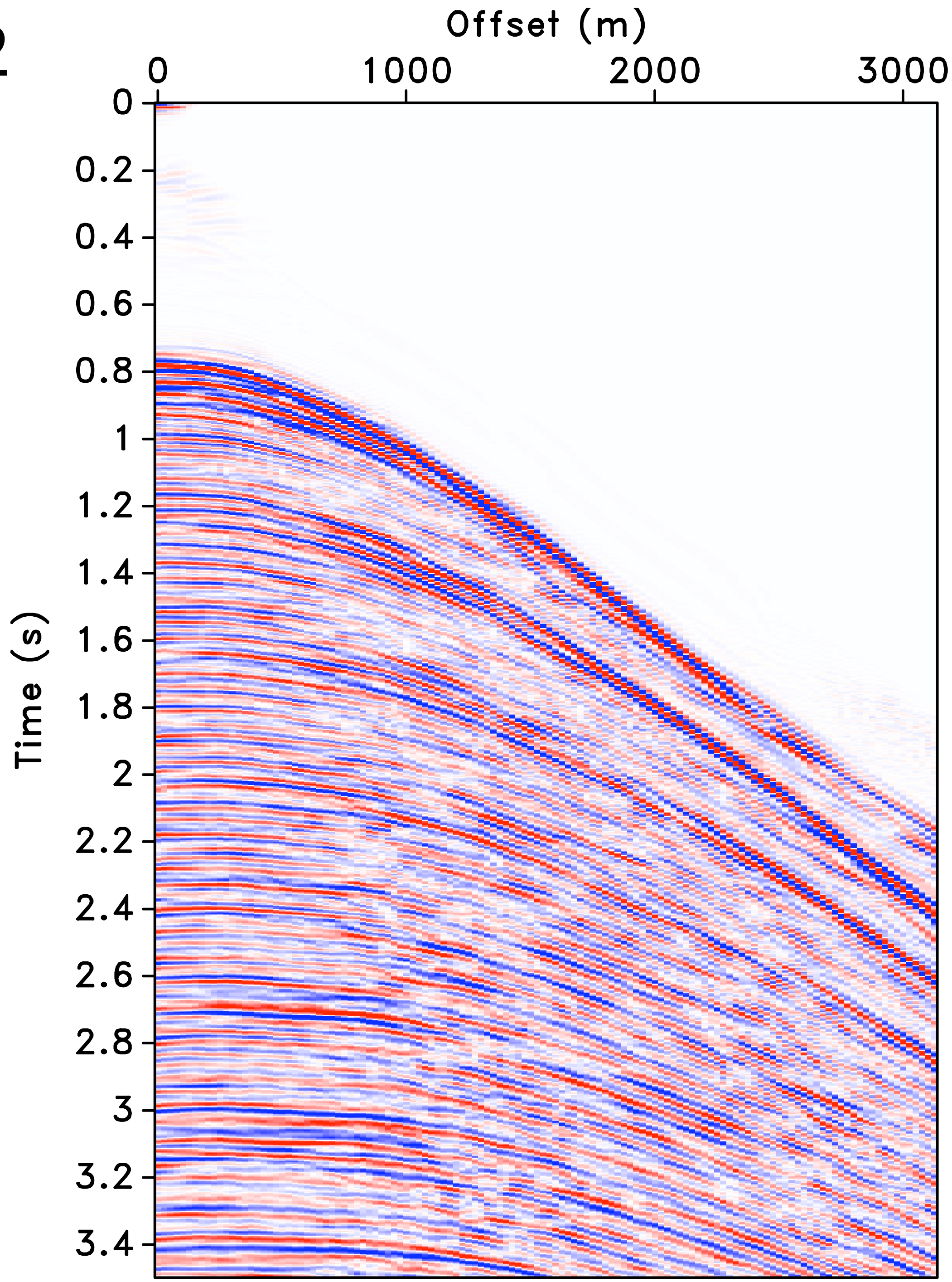
Example 2



Multiples

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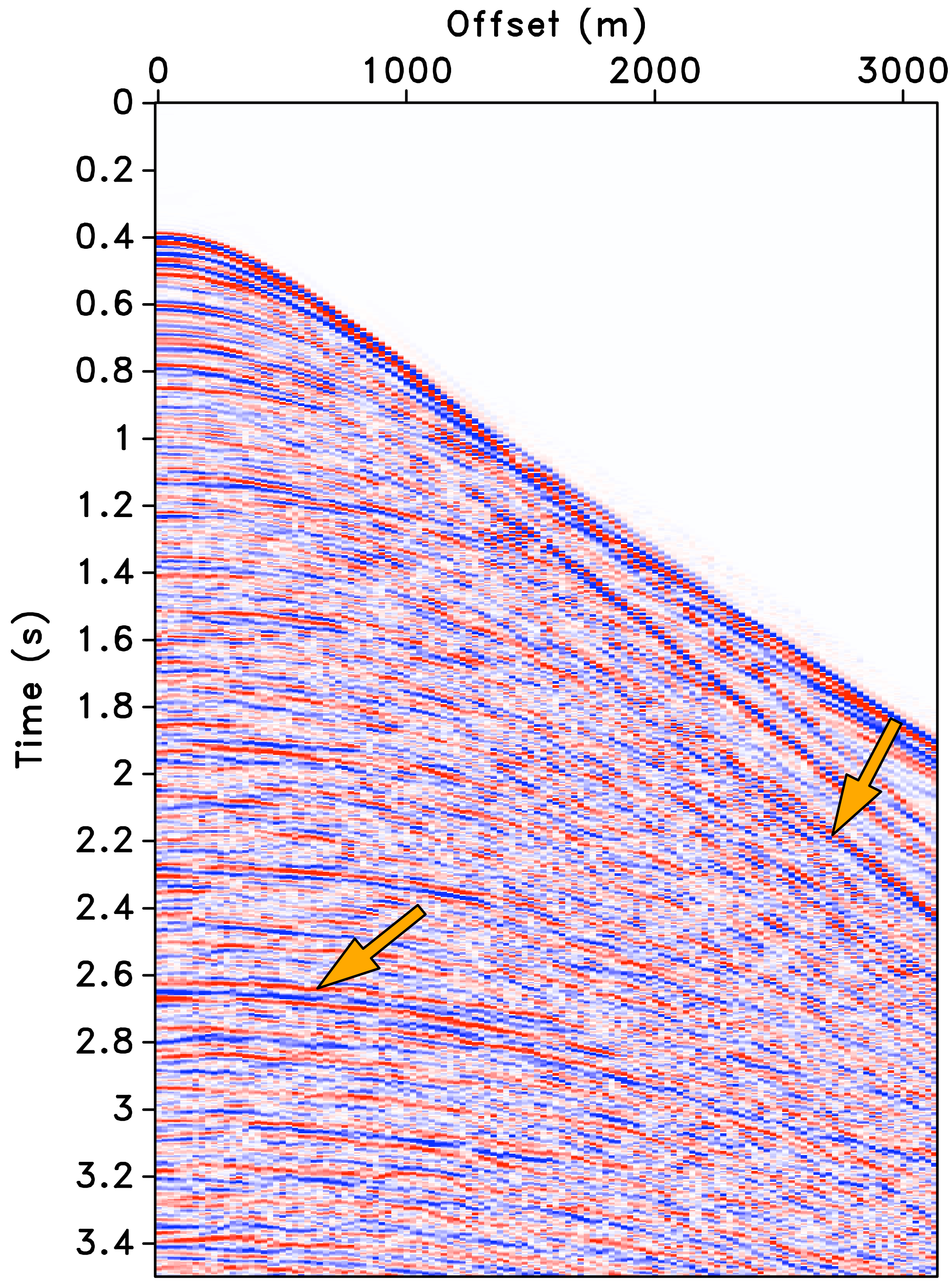
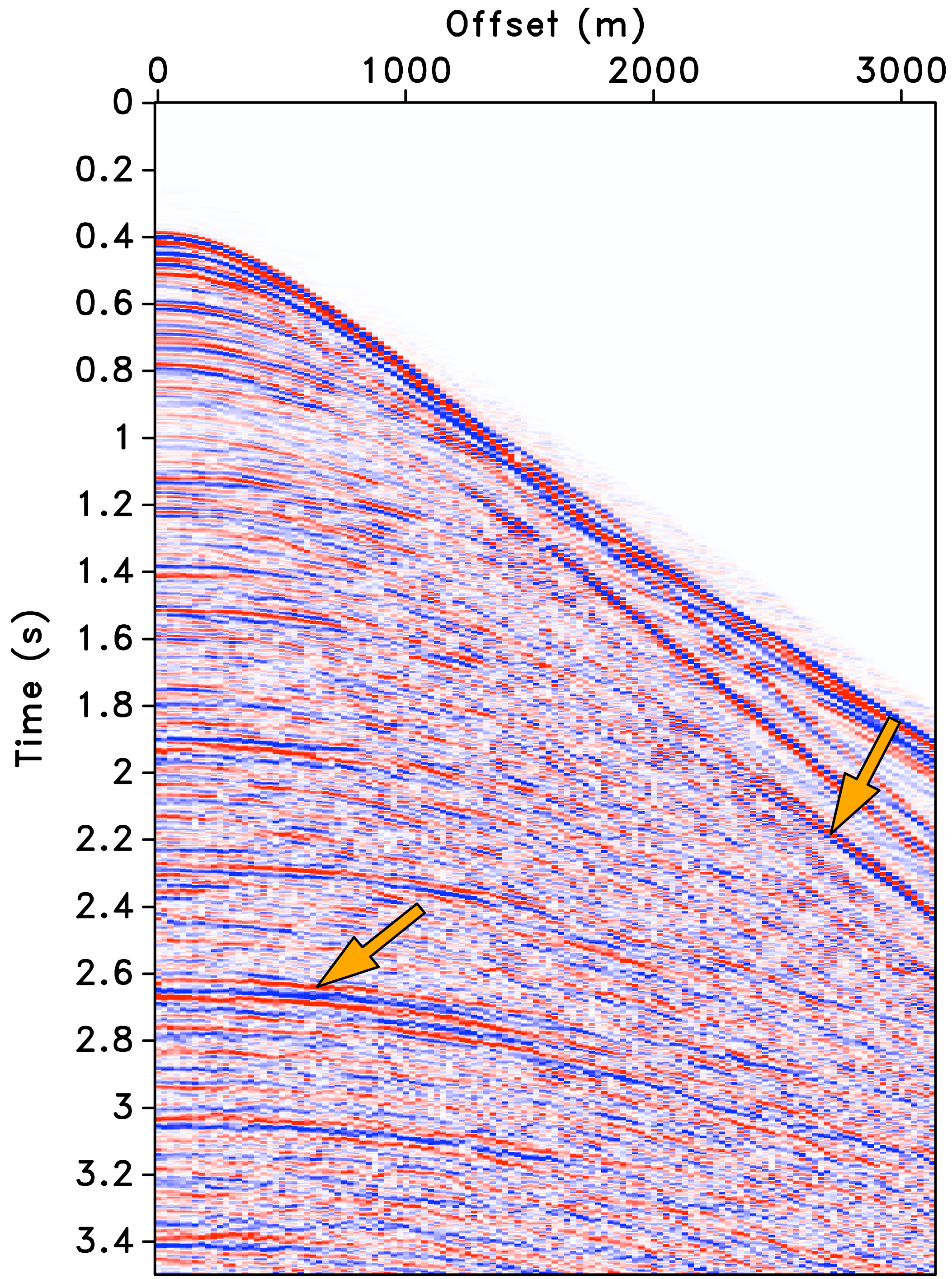
Example 2



SRME primaries

Example 2



Single threshold

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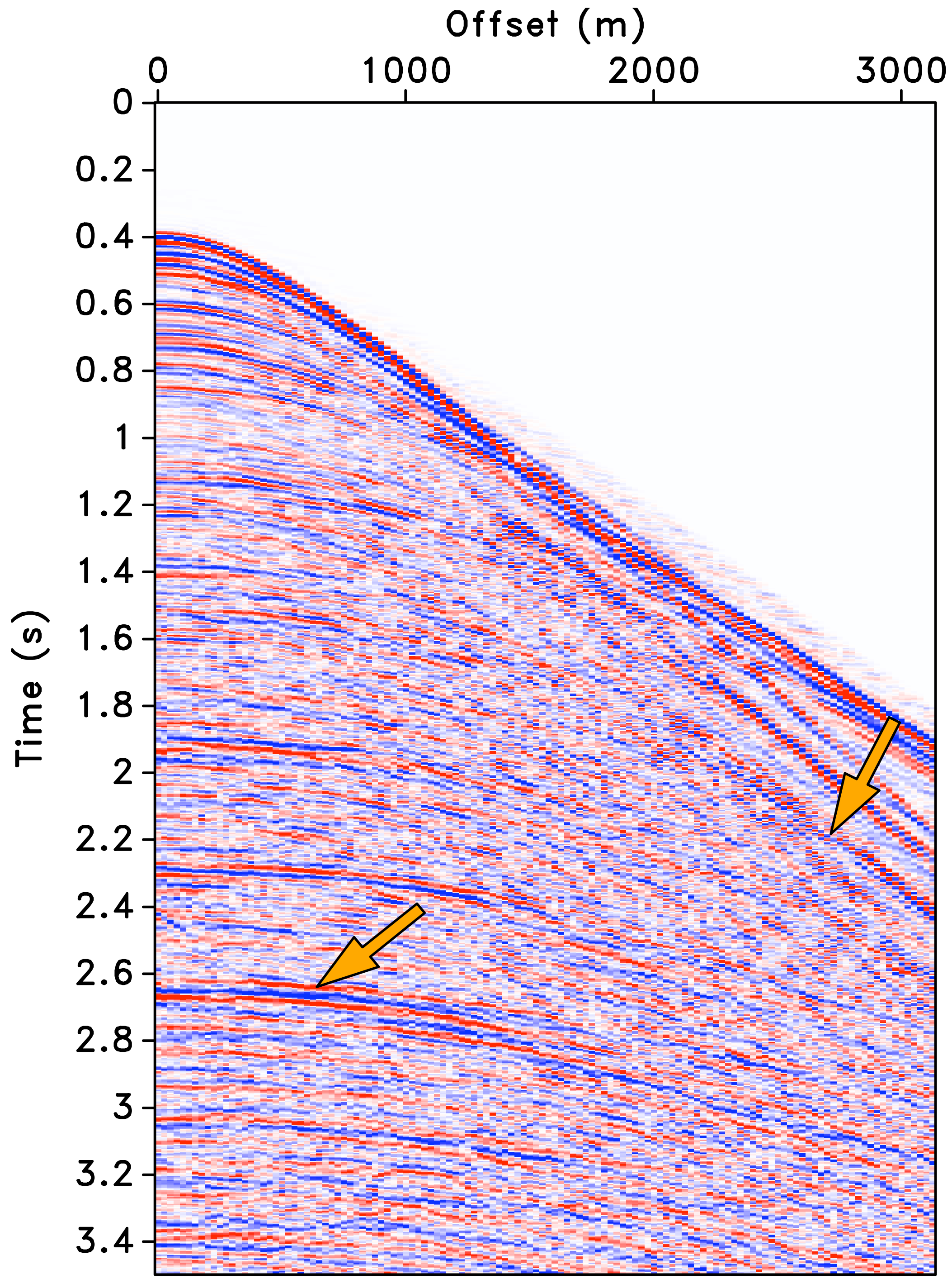
Example 2



Bayesian primaries

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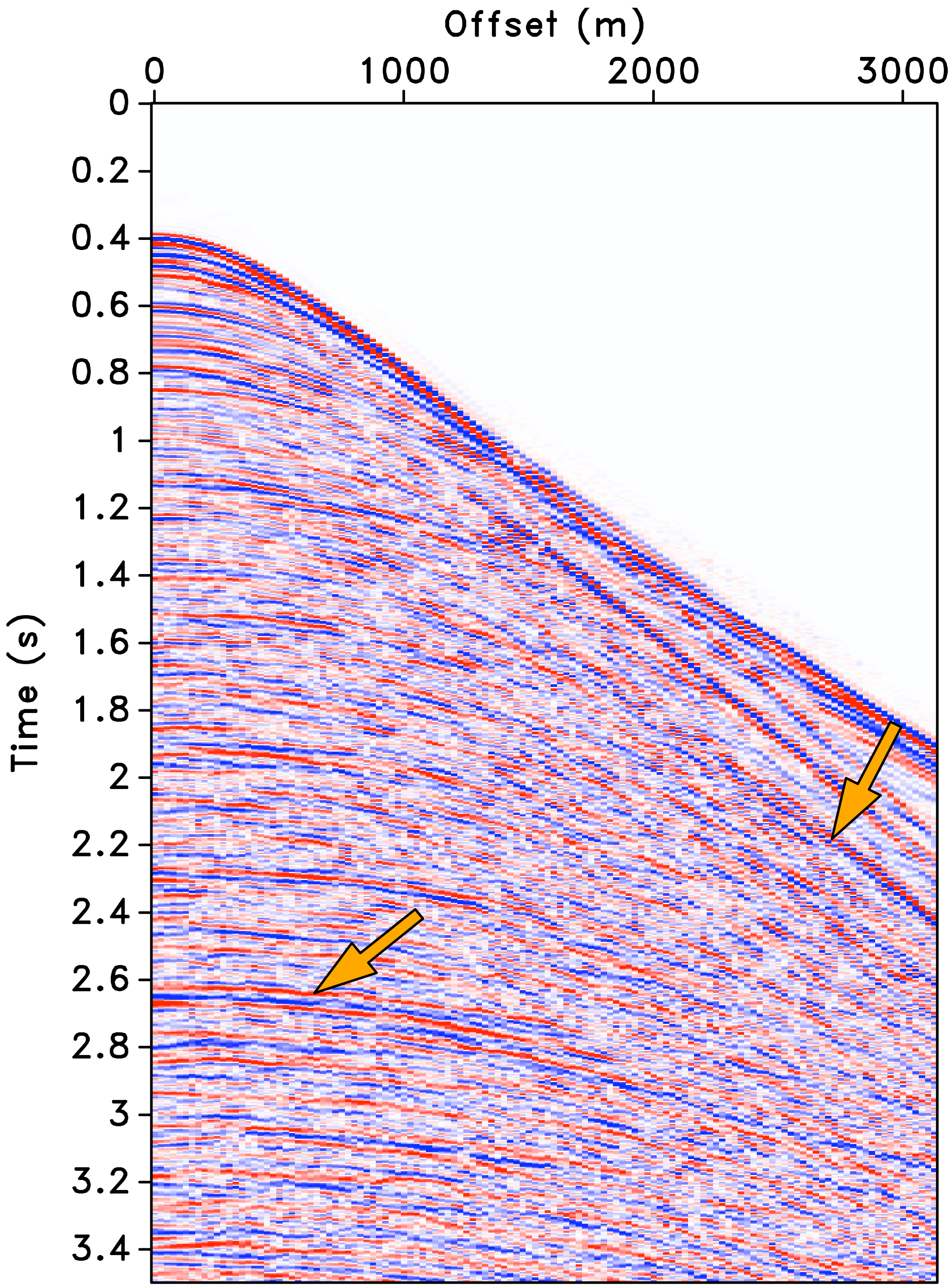
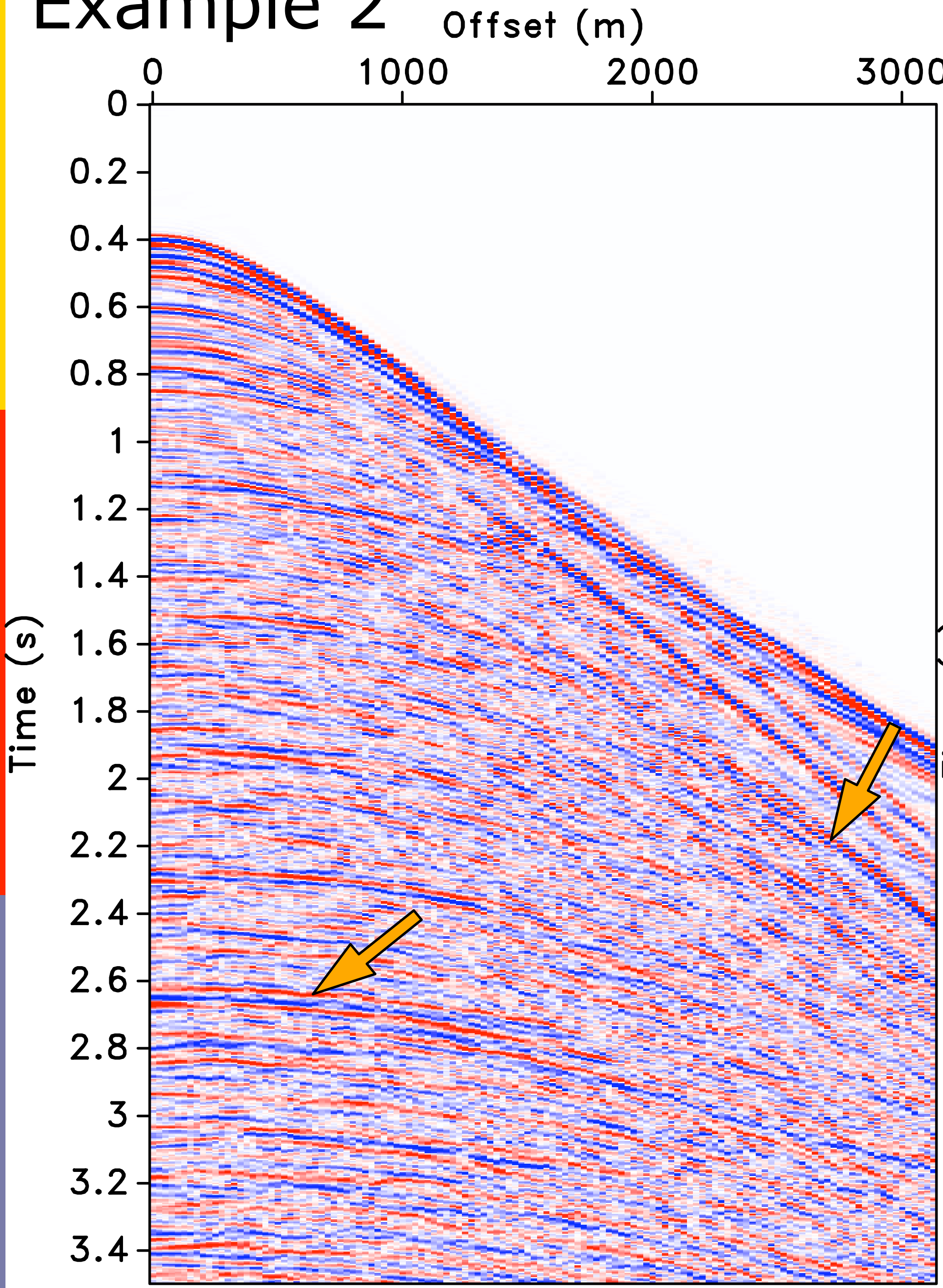
Example 2

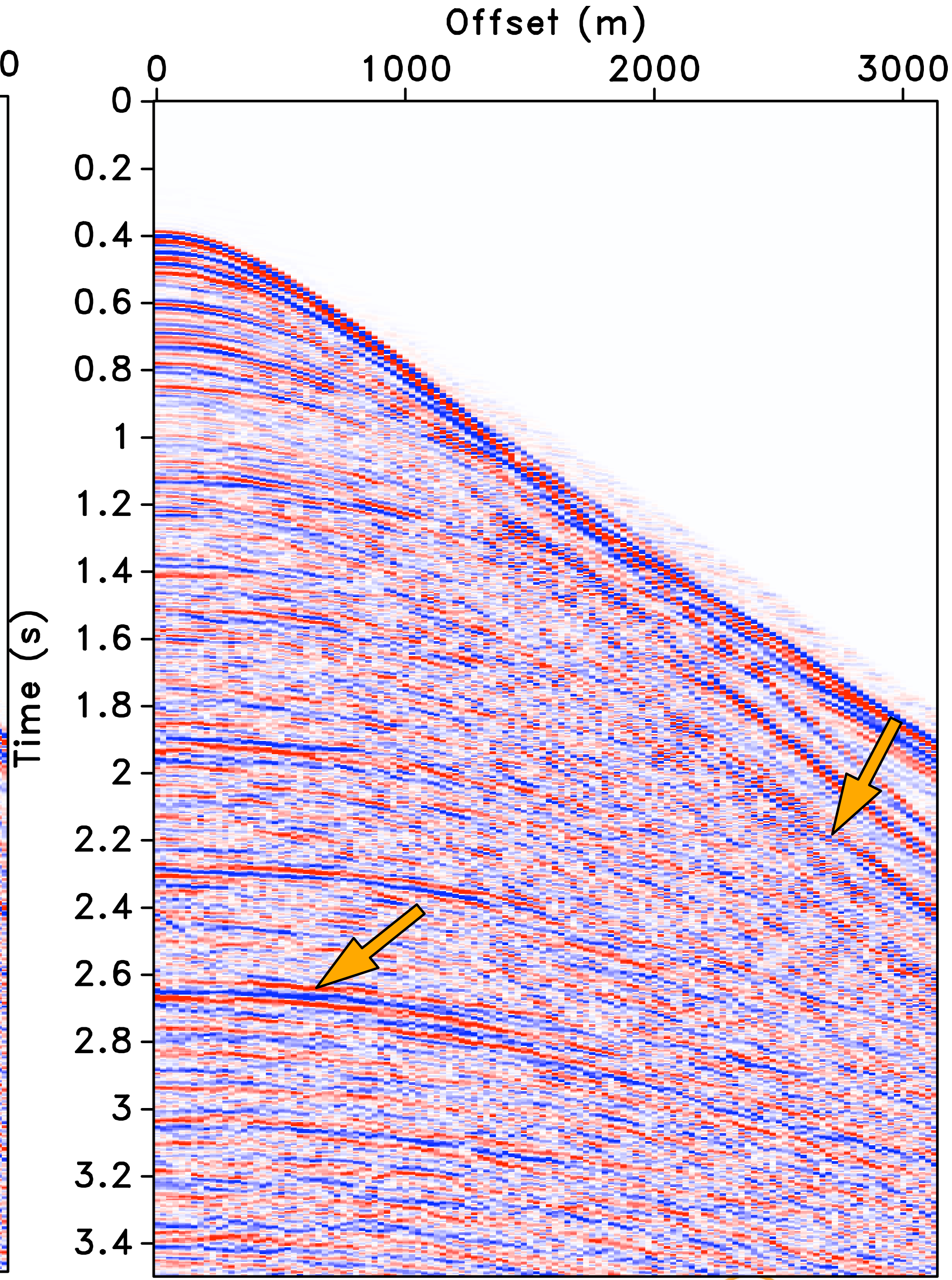


SRME primaries

Example 2



SRME primaries



Bayesian primaries

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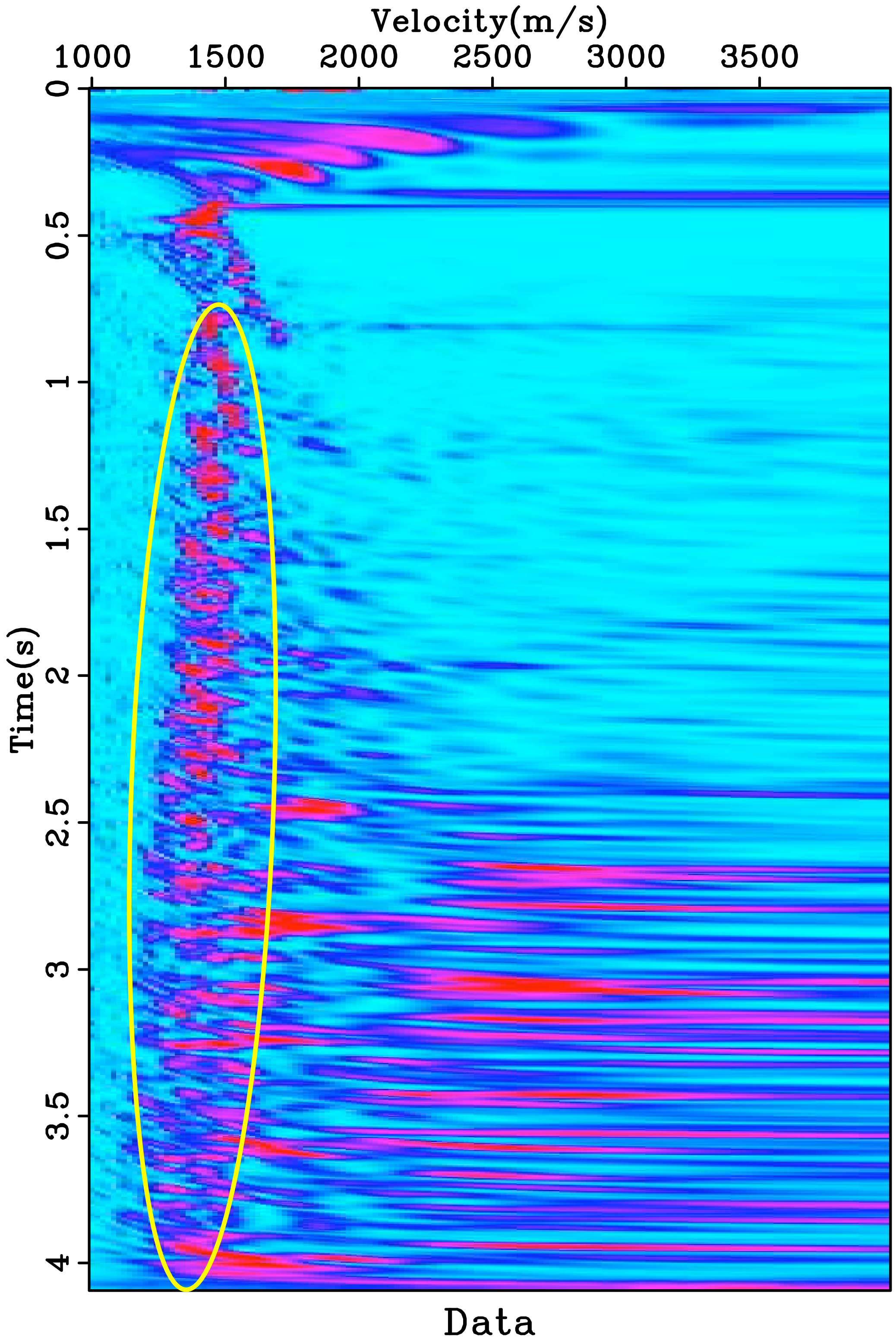
Example 2



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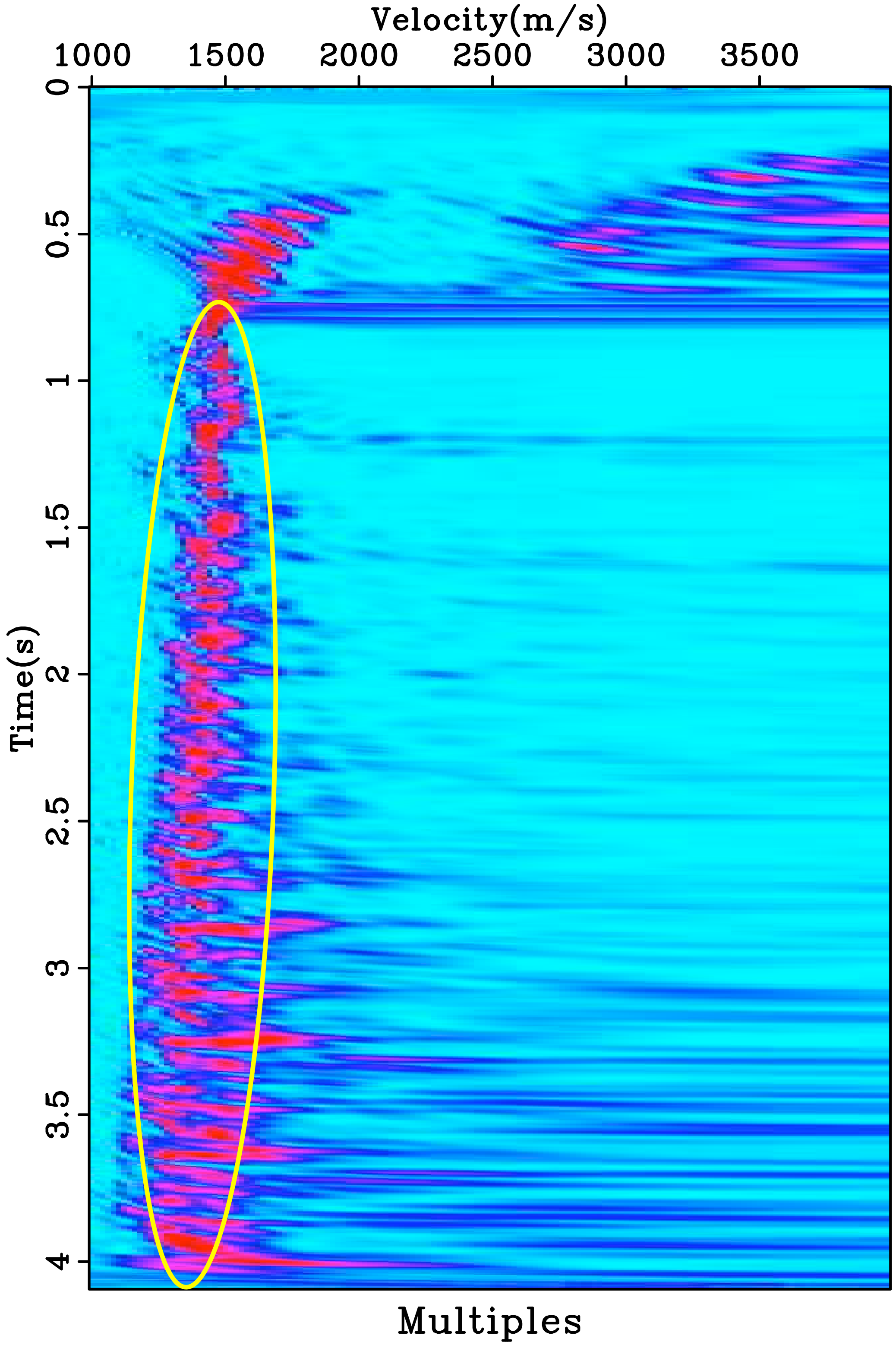
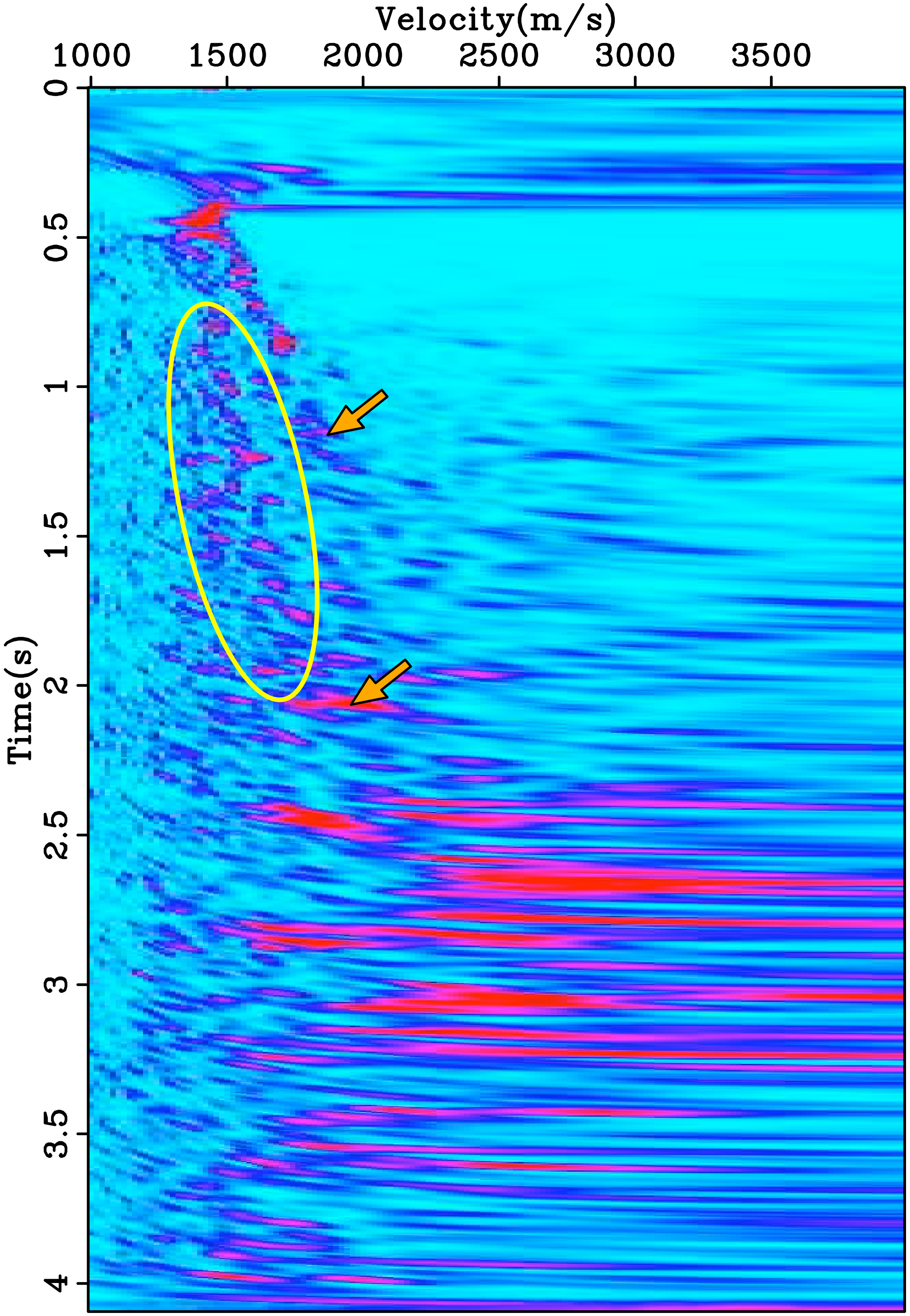
Example 2

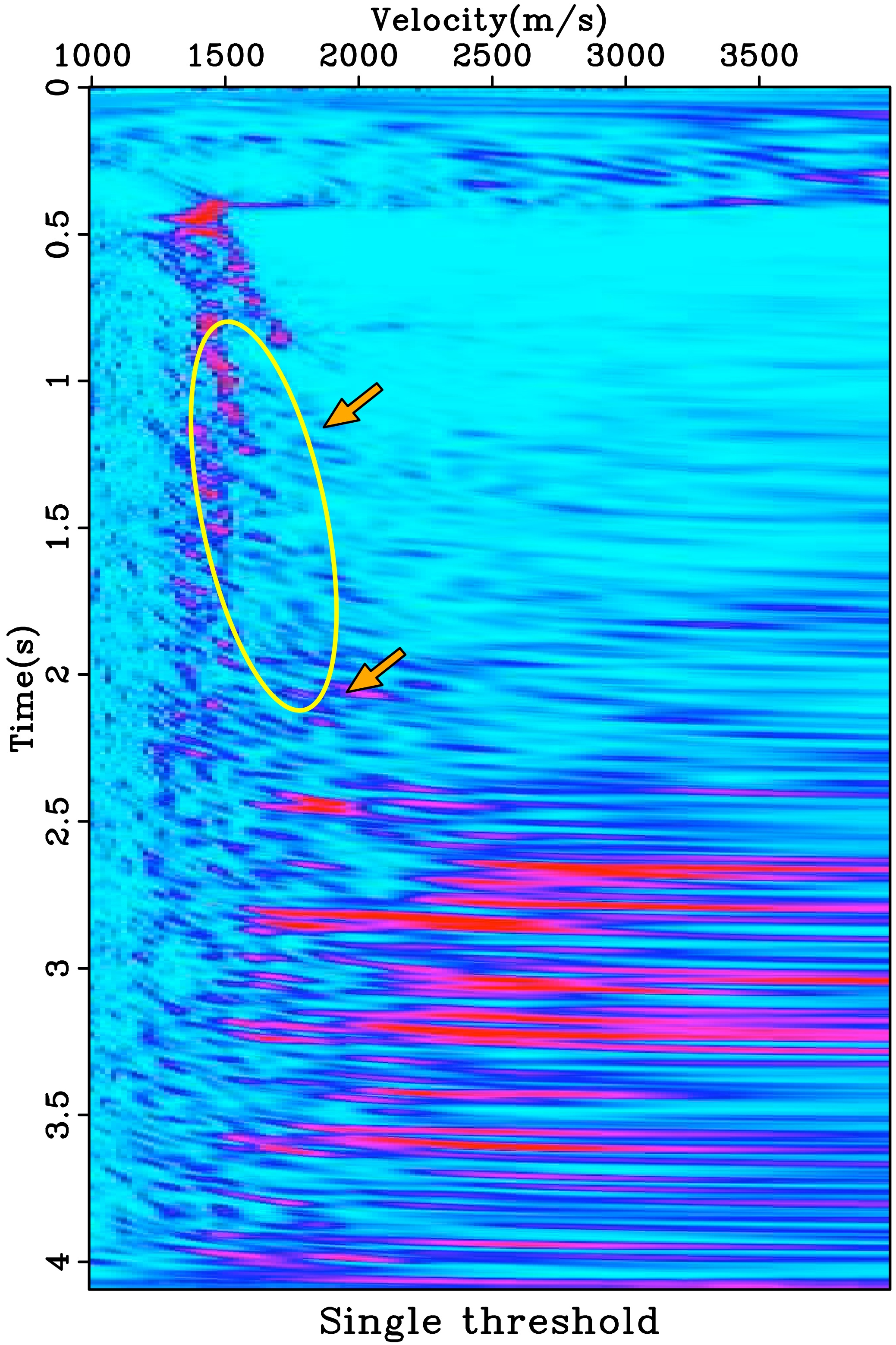

Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
Multiples

Example 2



SRME

Example 2



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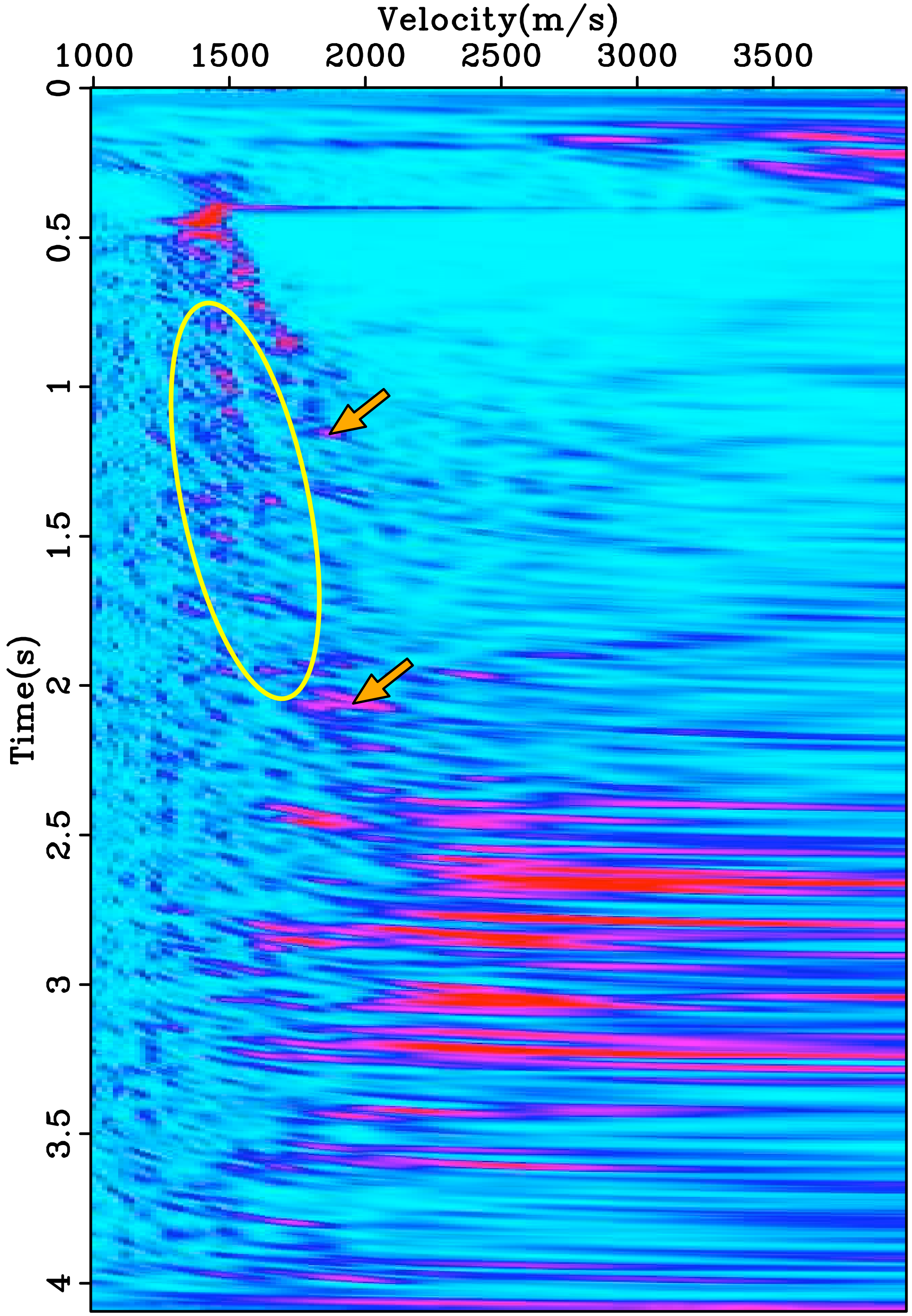
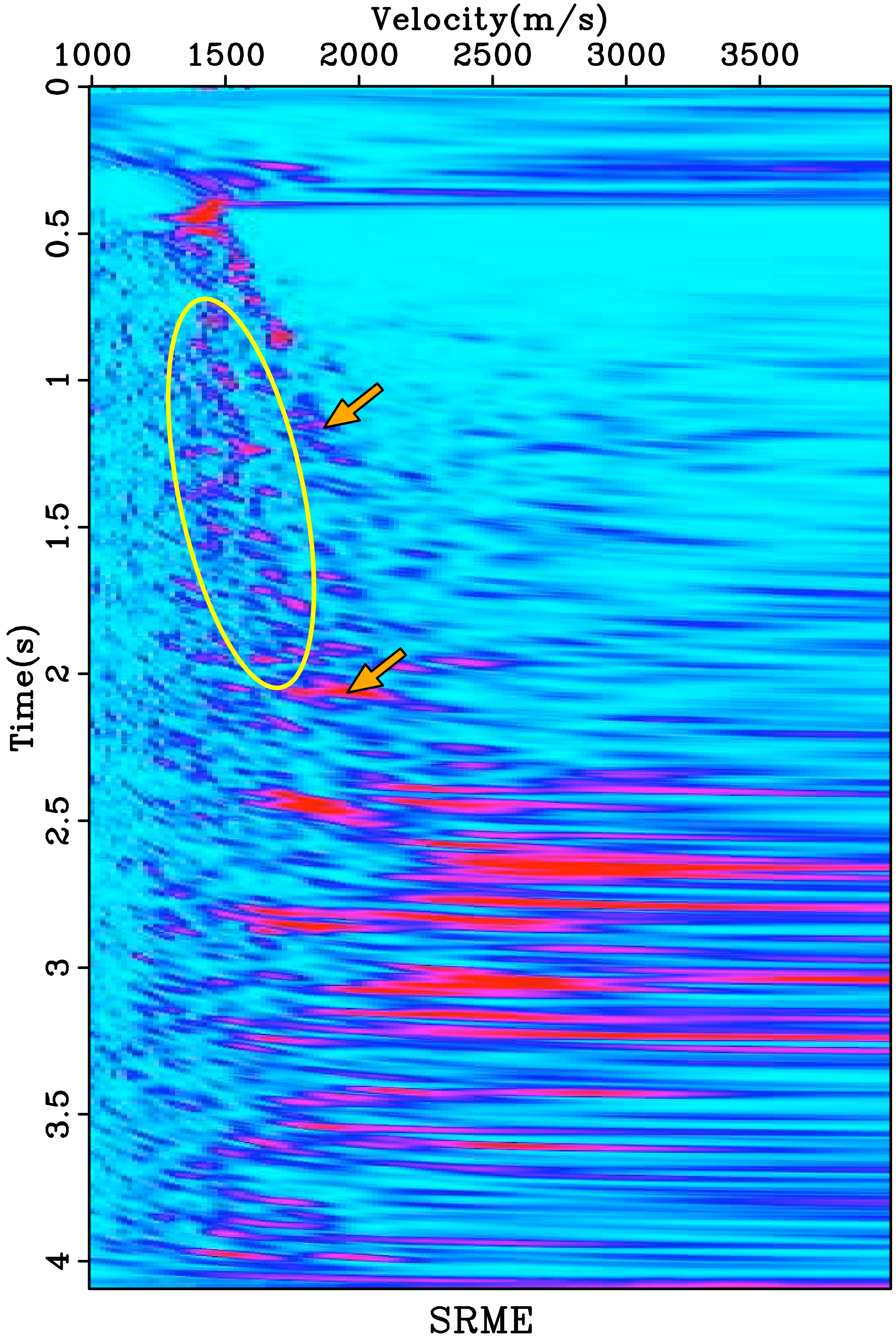
Example 2



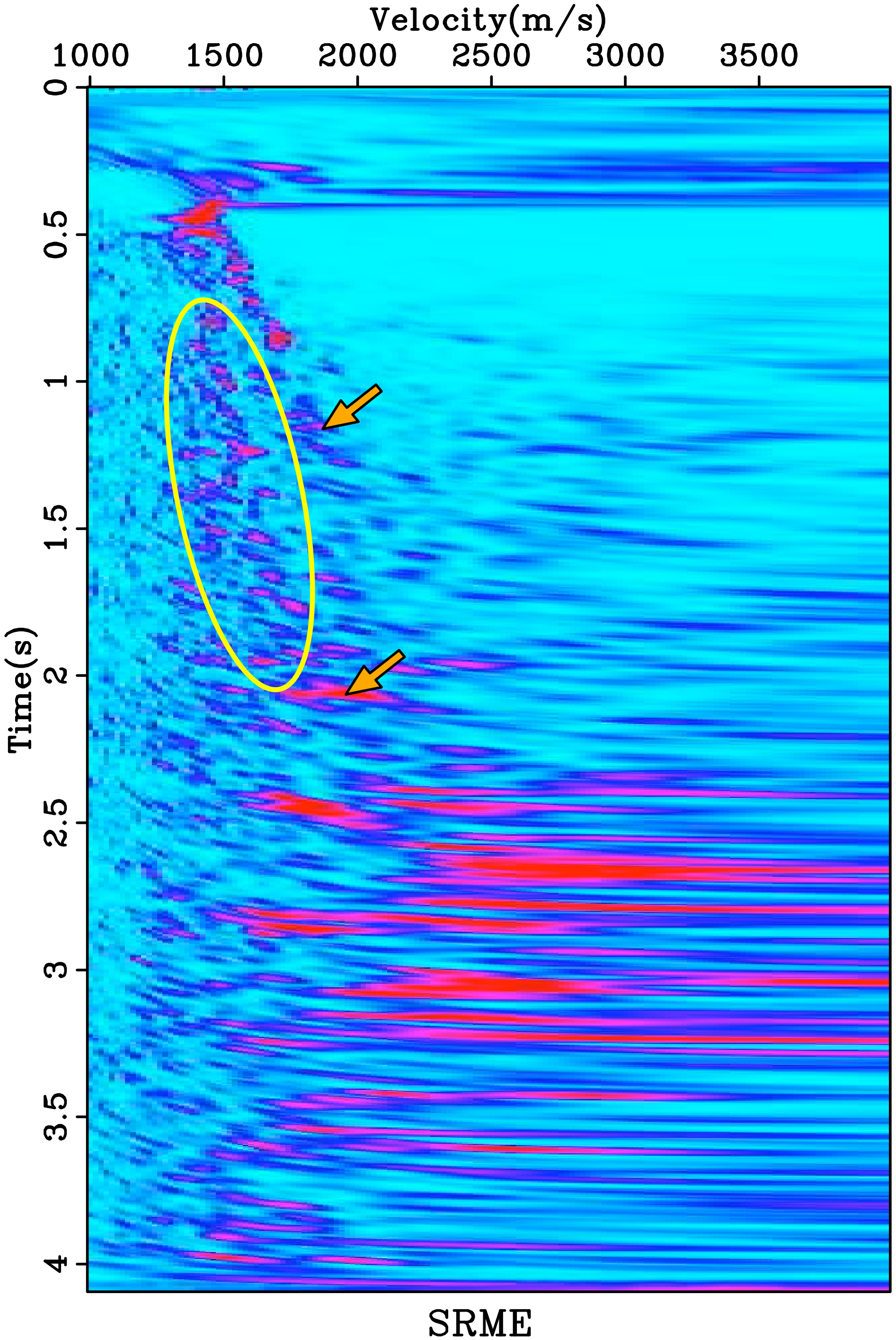
Bayesian

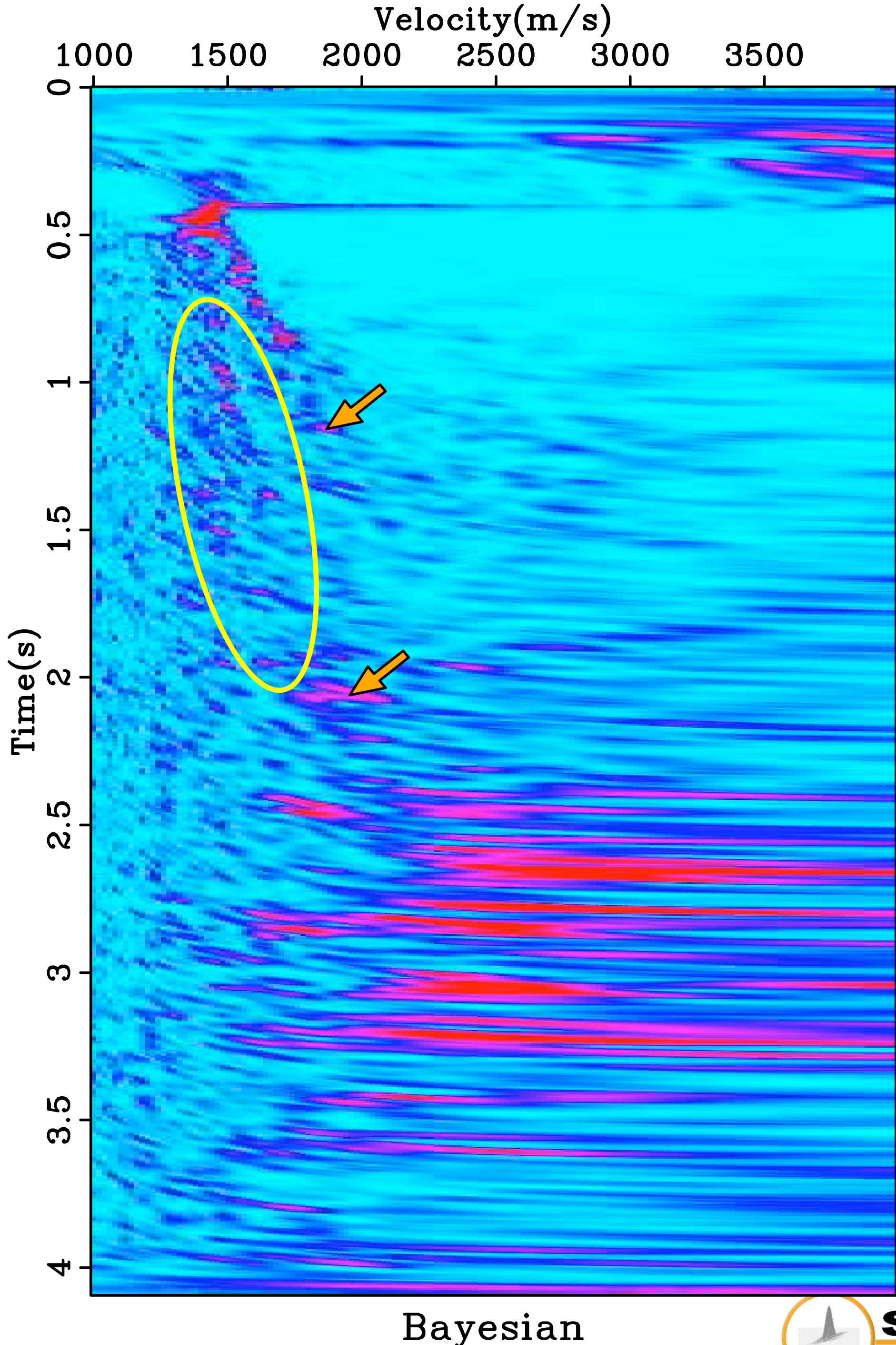
Example 2



Velocity(m/s)
1000
1500
2000
2500
3000
3500
Time(s)
0
0.5
1
1.5
2
2.5
3
3.5
4
SRME

Example 2





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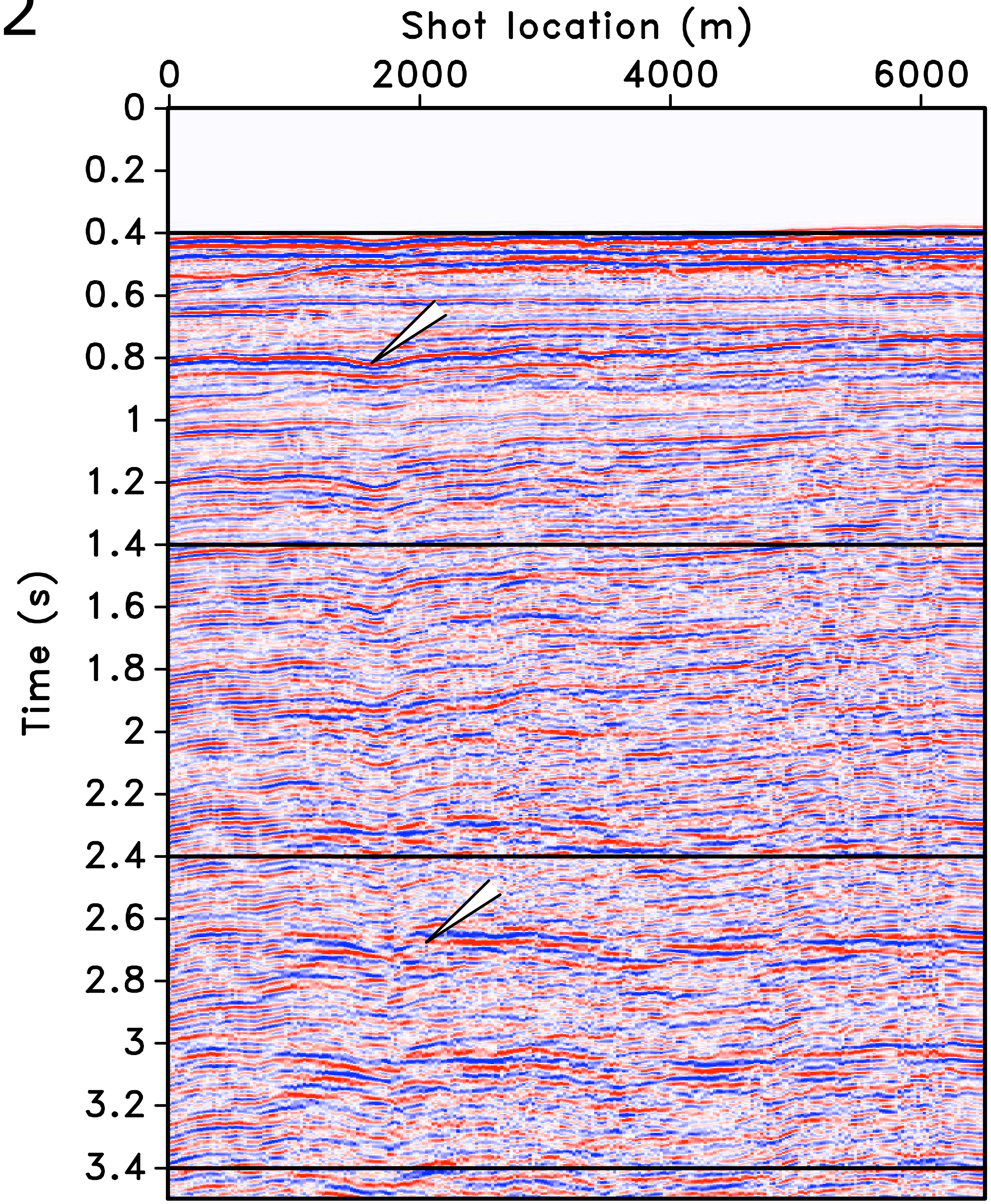
Example 2



Data

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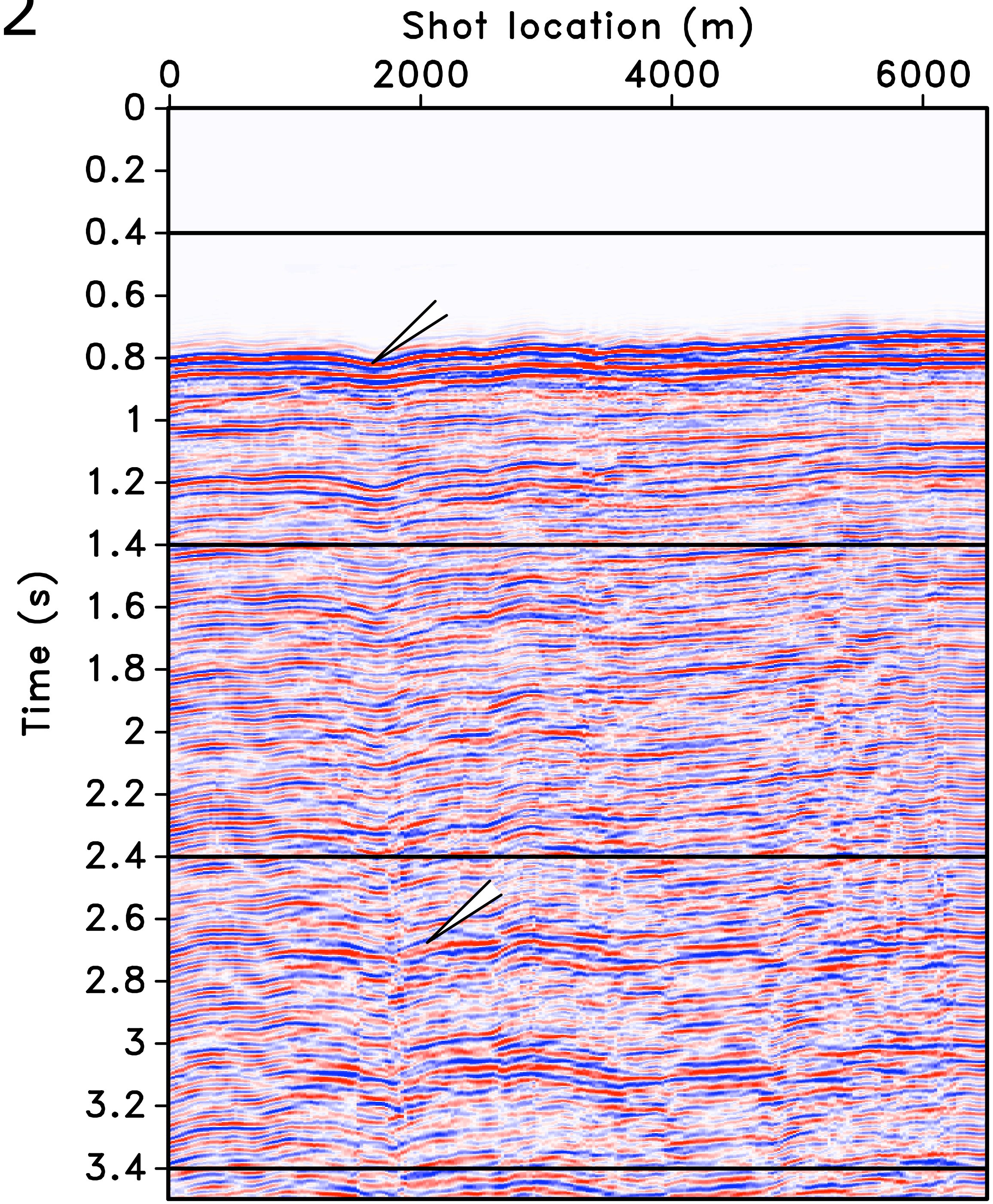
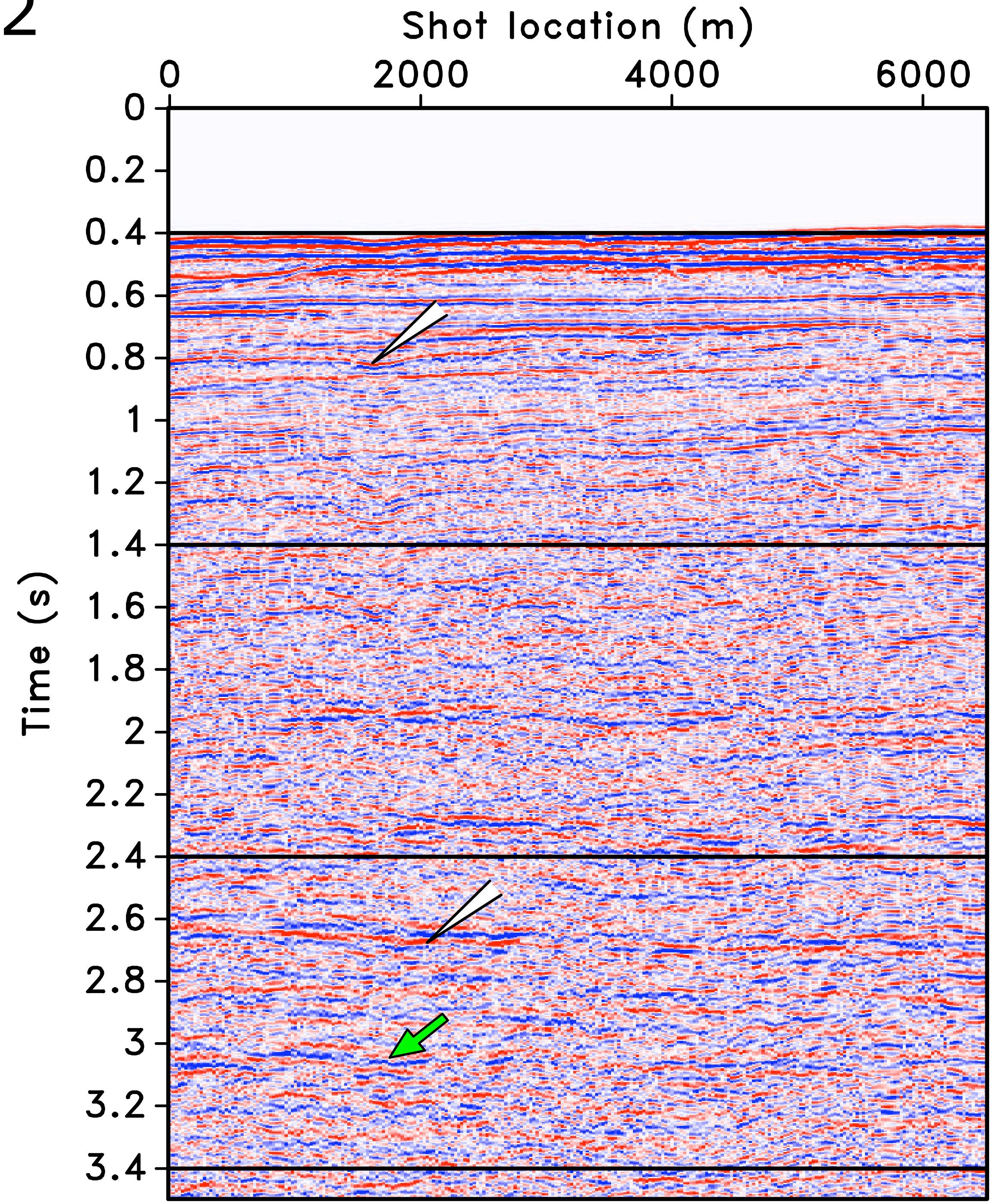
Example 2



Multiples

Example 2



SRME primaries

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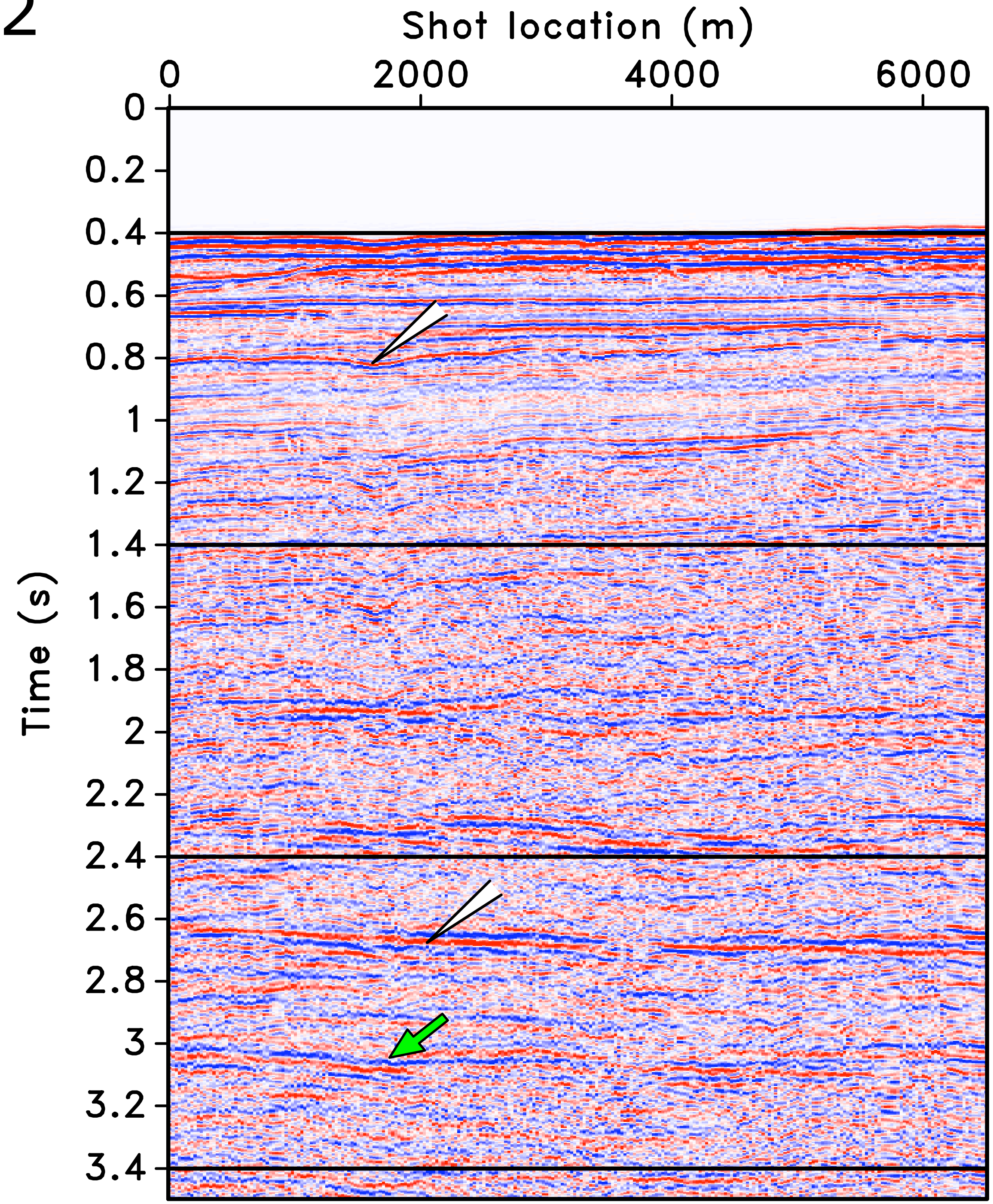
Example 2



Single threshold

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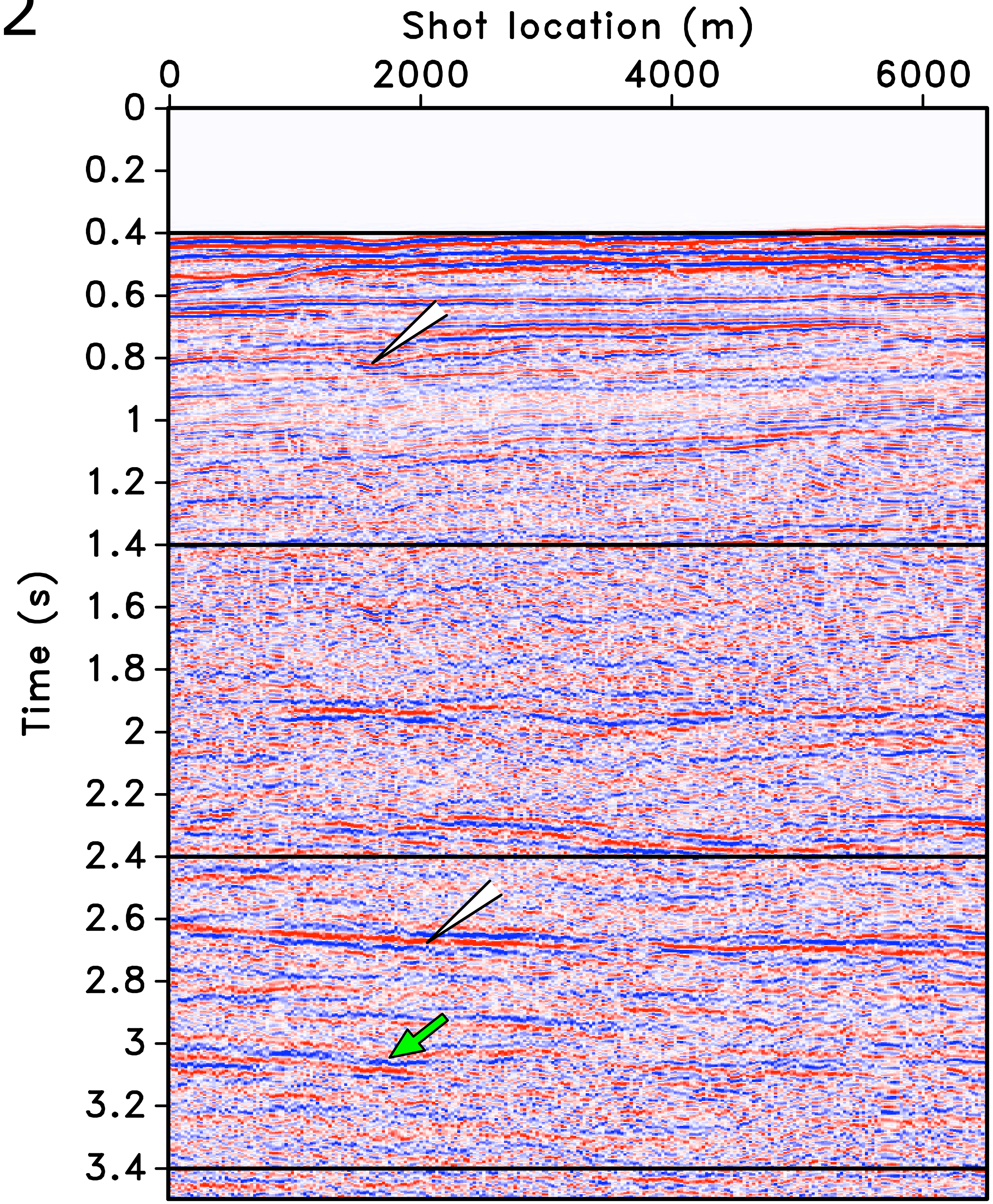
Example 2



Bayesian primaries

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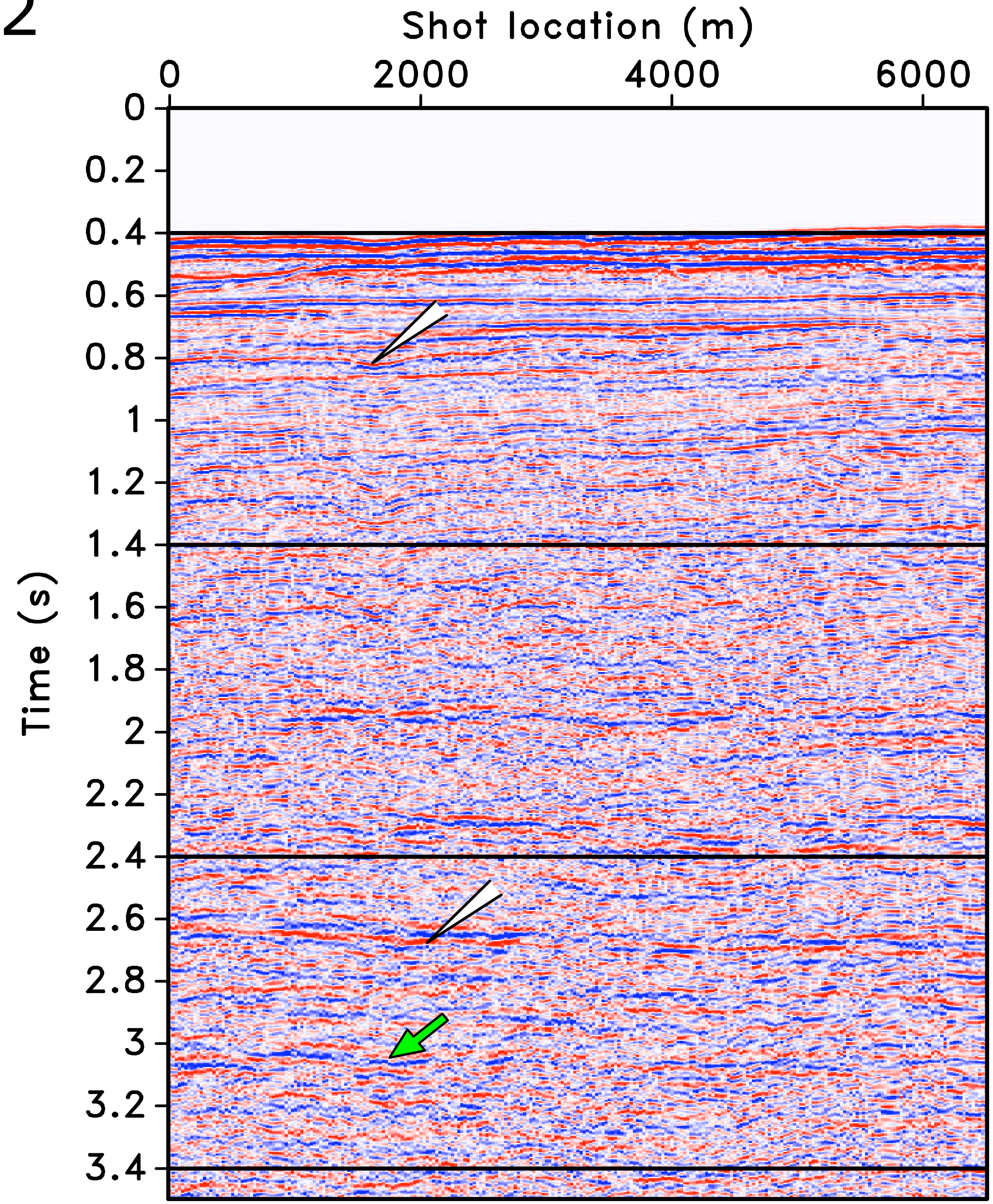
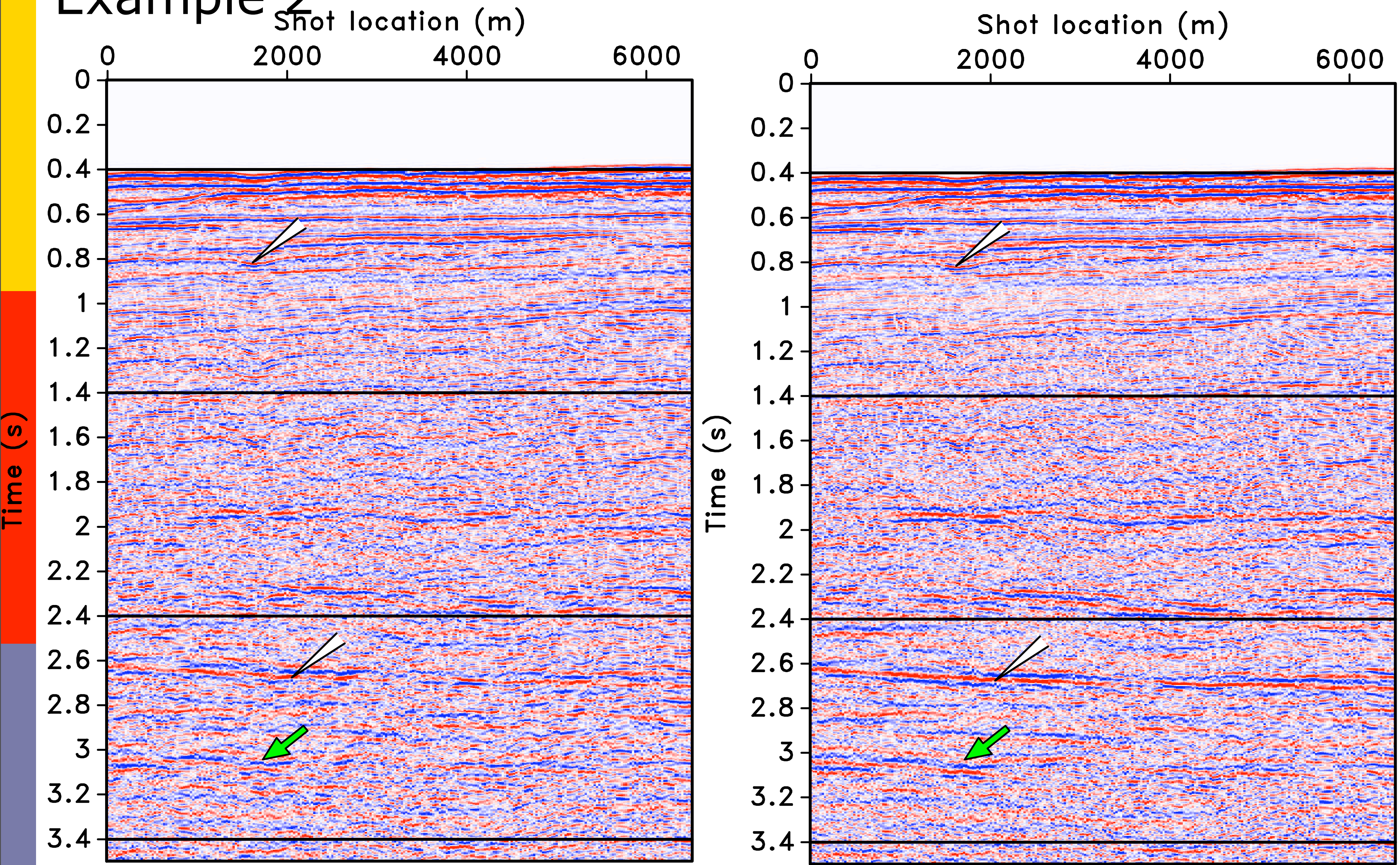
Example 2



SRME primaries

Example 2



SRME primaries

Bayesian primaries

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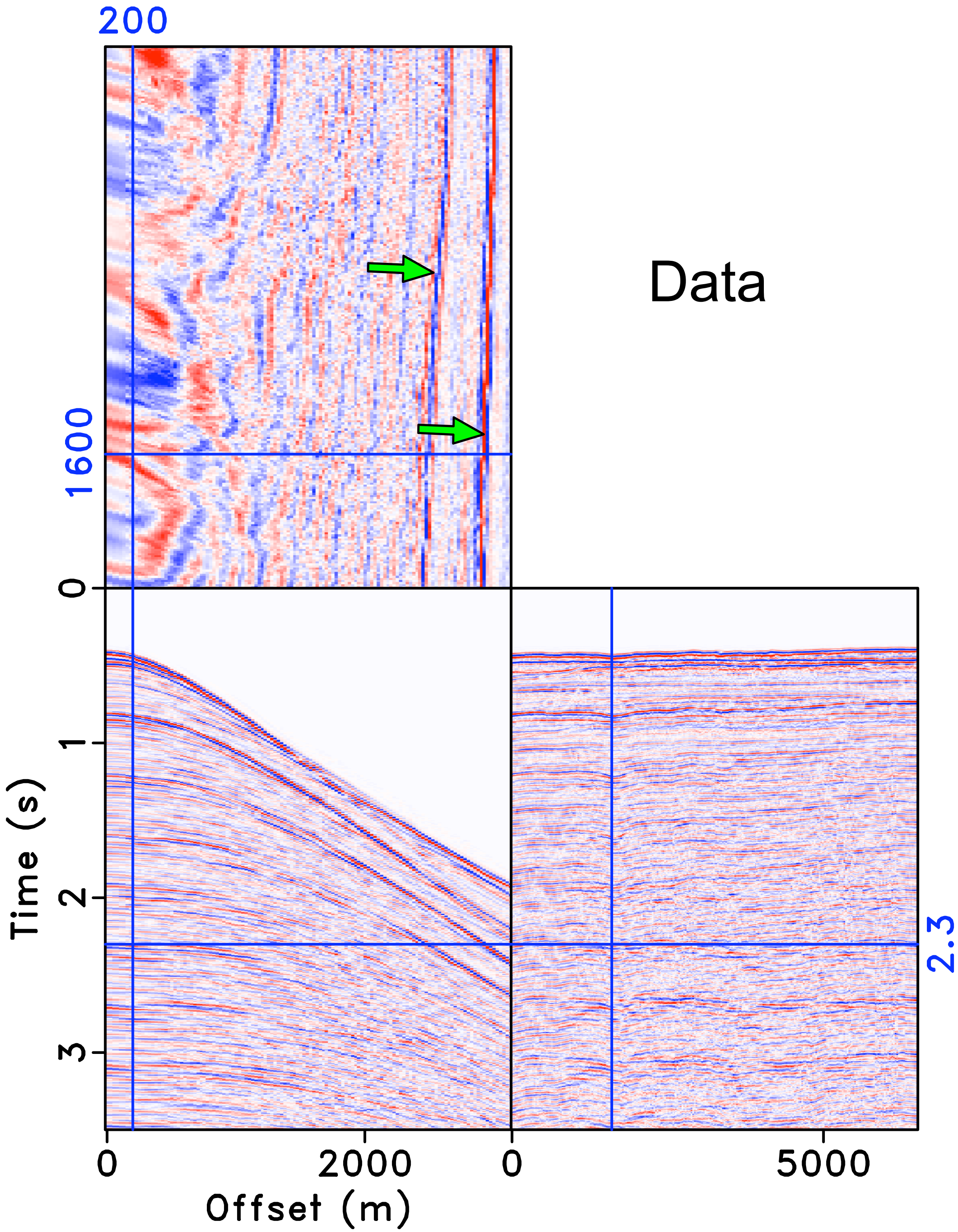
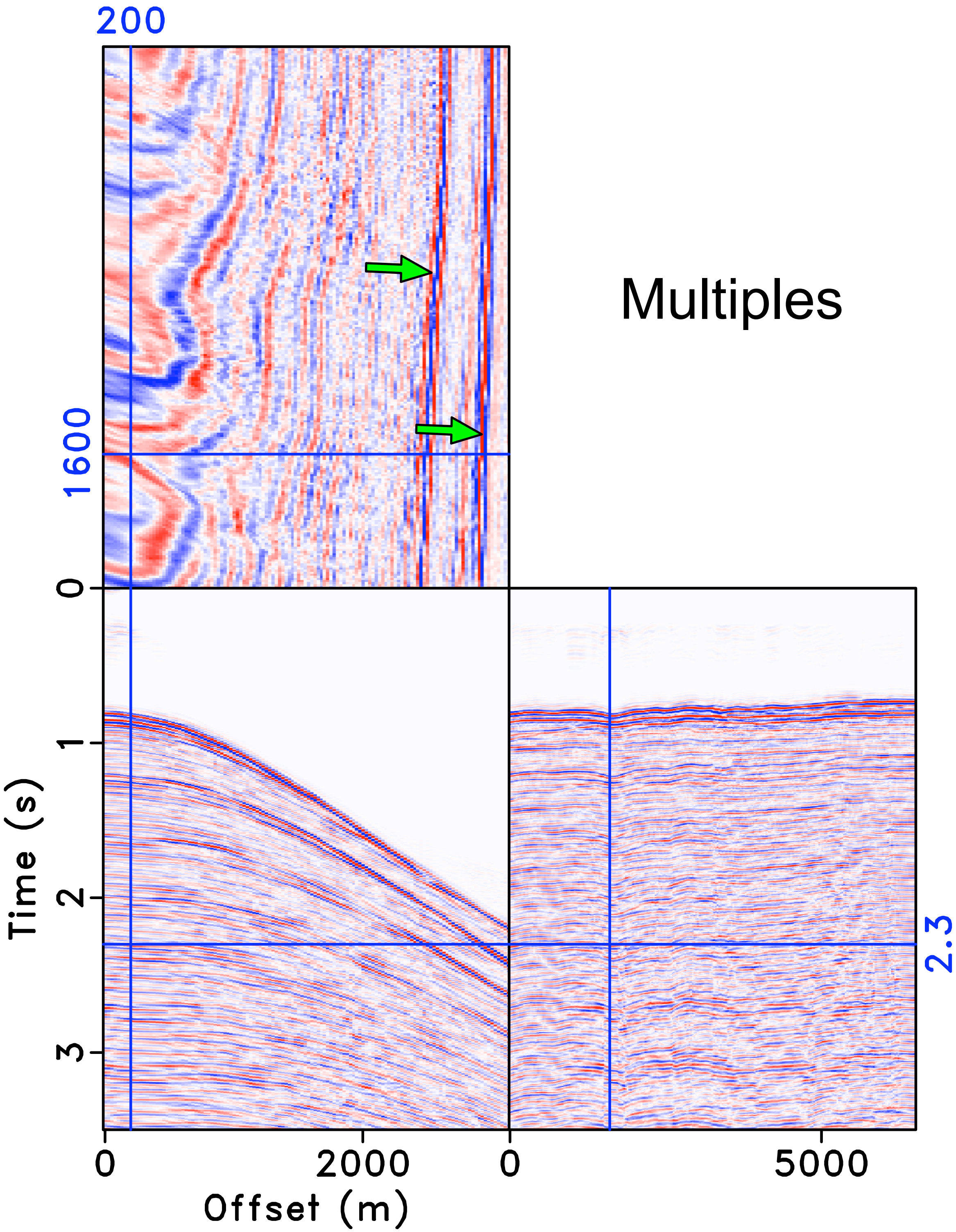
Example 2



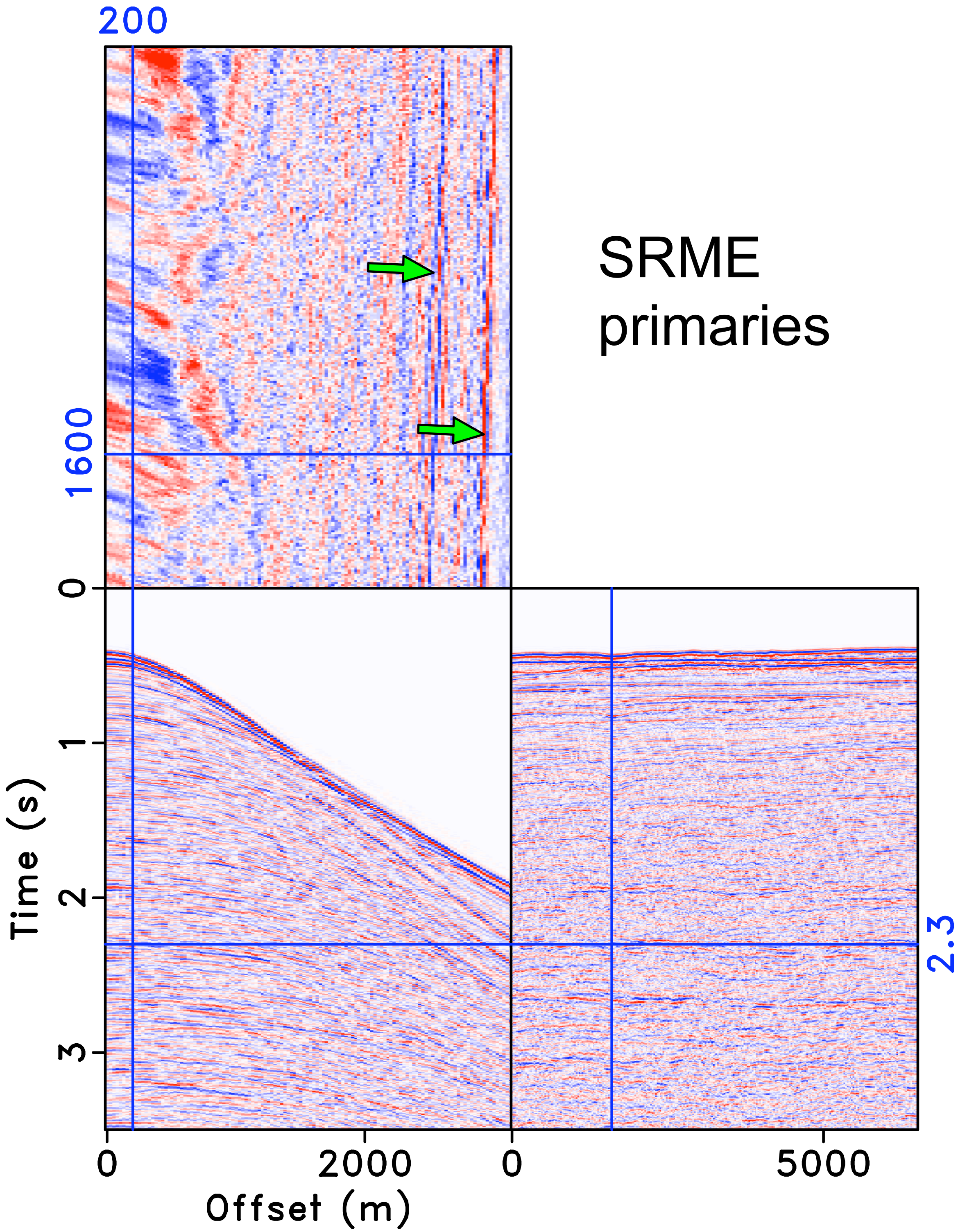
Data

Example 2



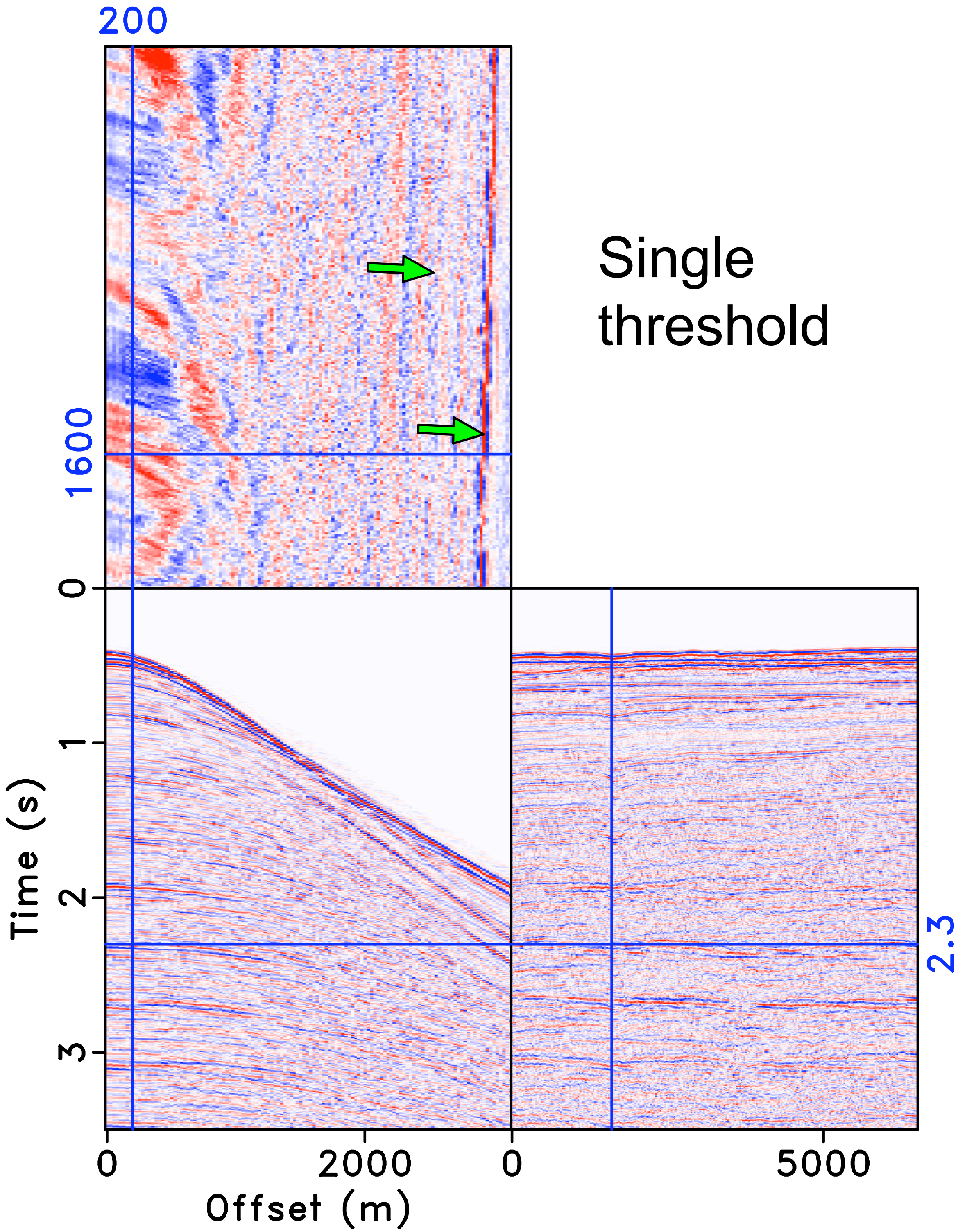
Multiples

Example 2



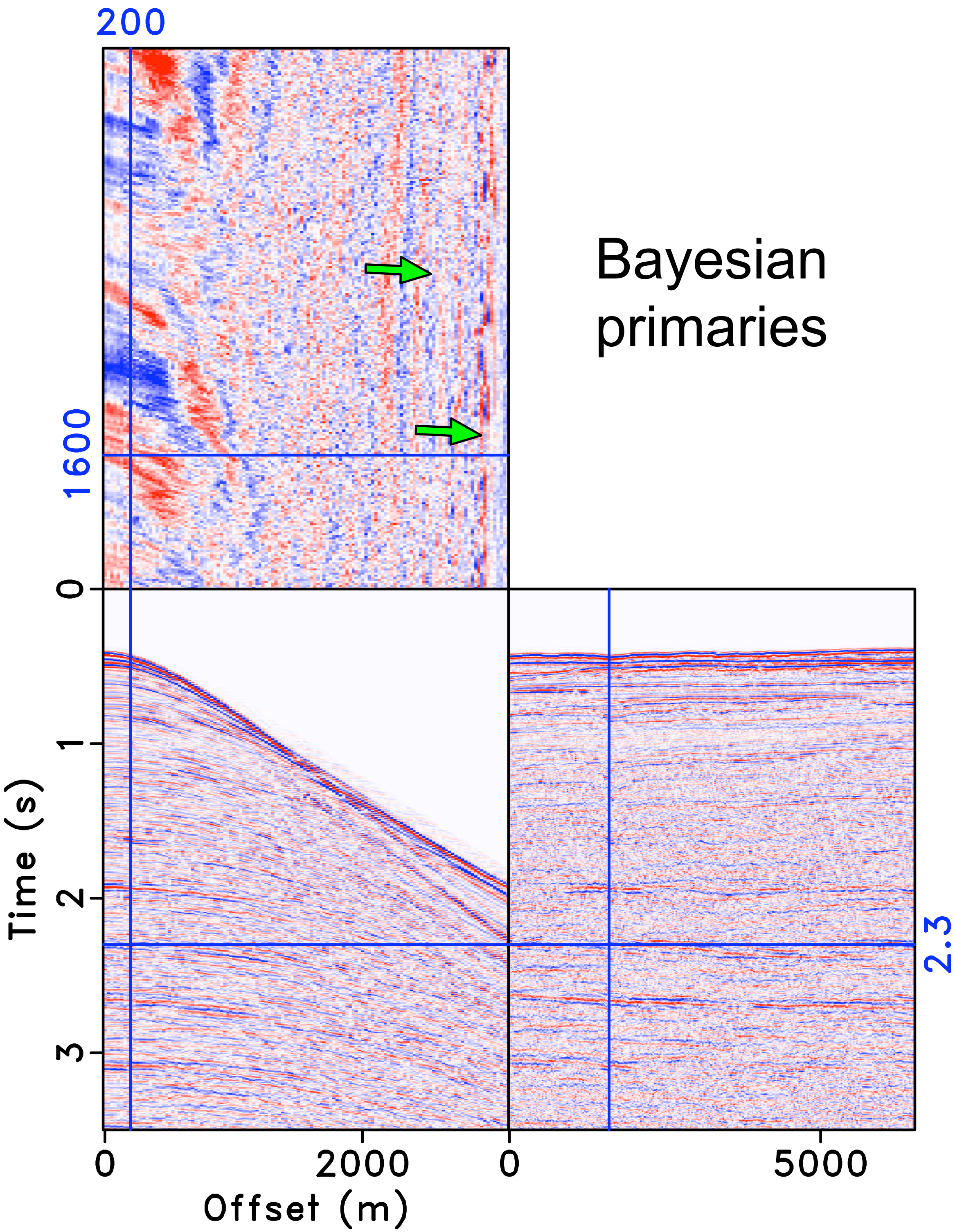
SRME primaries

Example 2



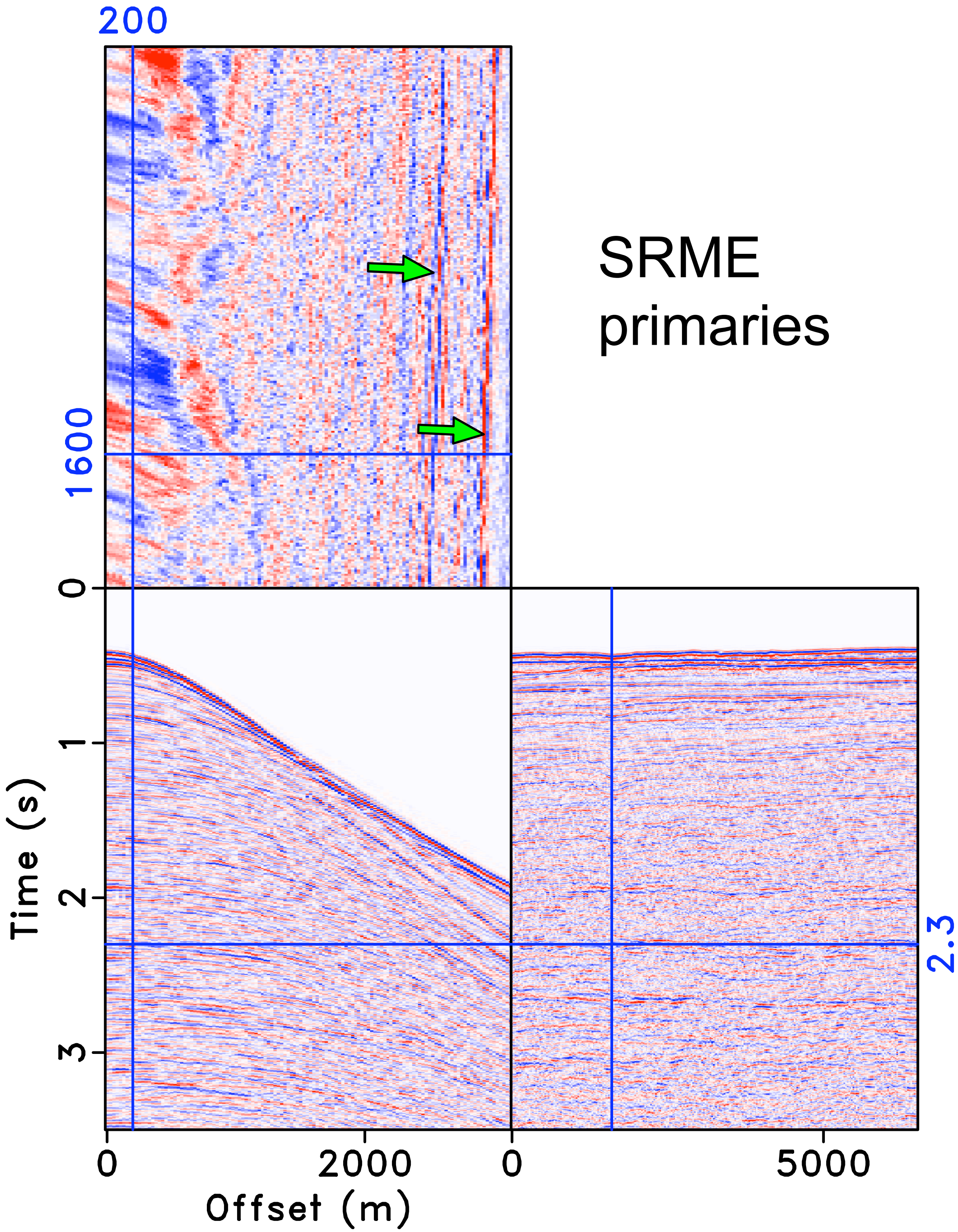
Single threshold

Example 2



Bayesian primaries

Example 2



SRME primaries

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- Introduction
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- Examples
- Discussion and conclusion
- Acknowledgments

Discussion and conclusions

- Curvelets represent the ***ideal*** domain for primary-multiple separation. It'
- The curvelet's multi-angular parameterization helps the separation, even for ***erroneous*** predictions.
- The ***nonlinear*** optimization algorithm shows a clear improvement in the primary-multiple separation and is ***robust*** against parameters changes.
- The convergence and the quality of the separation results both follow from the ability of curvelets to ***sparsely represent*** each signal component
- Results application to synthetic and ***real*** data are encouraging.

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Discussion and conclusions

Future plans:

- Multi-term Bayesian signal separation (in collaboration with Eric Verschuur)
- Parallellization using SLIMPy
- Automatic parameter selection

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