# Sparco: A testing framework for sparse reconstruction

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with
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February 18, 2008

This presentation was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).

# Background

- Central theme: sparse recovery
- Numerous solvers
  - Linear Programming, Iterative Soft Thresholding
  - PDCO, GPSR, L1LS, SPGL1, ...
- Ad hoc problem evaluation
  - Problems often easy to solve
  - Comparison between results is difficult
  - Lacks objectivity
- Standard set of test problems to provide a common ground
- Sparco (after common denominator: sparse coefficients)

# Sparco

- Software toolbox written in Matlab
- Prototyping compressed sensing scenarios
- Provides small/medium scale test problems
- Not intended as replacement for SlimPy

# Design criteria

- General problem format
  - Independent of solver
  - Independent of formulation (BP, BPDN, QP, TV)
- Easy to use and understand
  - Getting more people to use it
  - Simplifies comparing results
- Easy to extend
  - Community can contribute problems
  - We don't have to do all the work ourselves
- Easy to maintain
  - Self documenting (webpage is automatically generated)
  - Minimize dependencies between files

# Outline

1. Problem formulation
2. Operators
3. Sparco internals
4. Application

# Problem Formulation

- All problems are given in general form:

$$b = Ax_0 + n$$

- $x_0$ is a sparse coefficient vector
- $n$ is additive noise
- $b$ is observed signal
- $A = MB$
  - $M$ measurement matrix
  - $B$ sparsity 'basis'

# Problem Formulation: Example

- Example from MRI (Shepp-Logan phantom)
  - M: restricted Fourier,
  - B: wavelet transform

- Independent of sparsity formulation:

- Basis pursuit denoise: $\underset{x}{\text{minimize}} \ \|x\|_1 \ \text{ s.t. } \ \|Ax - b\|_2 \le \sigma$
- Total variation: $\underset{x}{\text{minimize}} \ \|x\|_1 + \gamma \mathrm{TV}(Bx) + \lambda \|Ax - b\|_2$

# OPERATORS

# Overview

- Operators provide routines for
  - Multiplication; $v = Ax$, $w = A^T y$ `v=A(x,1), w=A(y,2)`
  - Querying its properties `info = A([],0)`
  - Conversion to class object `C = opClass(A); C*x, C'*y`
- Most operators are implemented based on fast routines
- Different types of operators
  - Basic operators
  - Meta operators
  - Other operators

# Basic operators

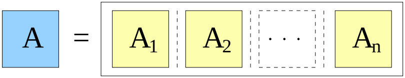
- Identity `A = opDirac(...);`
- General matrix `A = opMatrix(...);`
- Diagonal matrix `A = opDiag(...);`

- DCT/FFT `A = opDCT(...), opFFT(...);`
- Curvelet `A = opCurvelet2d(...);`
- Wavelet `A = opWavelet(...);`
- Hadamard `A = opHadamard(...);`
- Heaviside `A = opHeaviside(...);`

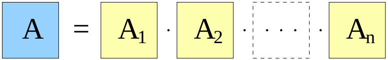
- Gaussian ensemble `A = opGaussian(...);`
- Sign ensemble `A = opSign(...);`
- Binary ensemble `A = opBinary(...);`

# Meta operators

- Transpose `AT = opTranspose(A);`
- Dictionary `A = opDictionary(A1,A2,...,An);`

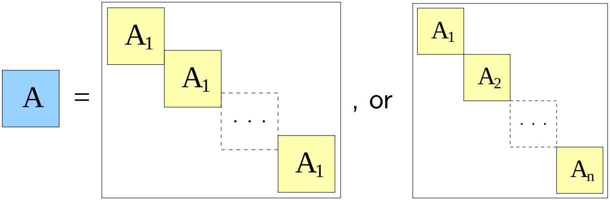



- Compound functions `A = opFoG(A1,A2,...,An);`

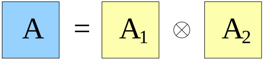


# Meta operators *(continued)*

- Block diagonal `A = opBlockDiag(A1,A2,...,An)`
- Windowed with overlap `A = opWindowedOp(...)`




- Kronecker `A = opKron(A1,A2);`

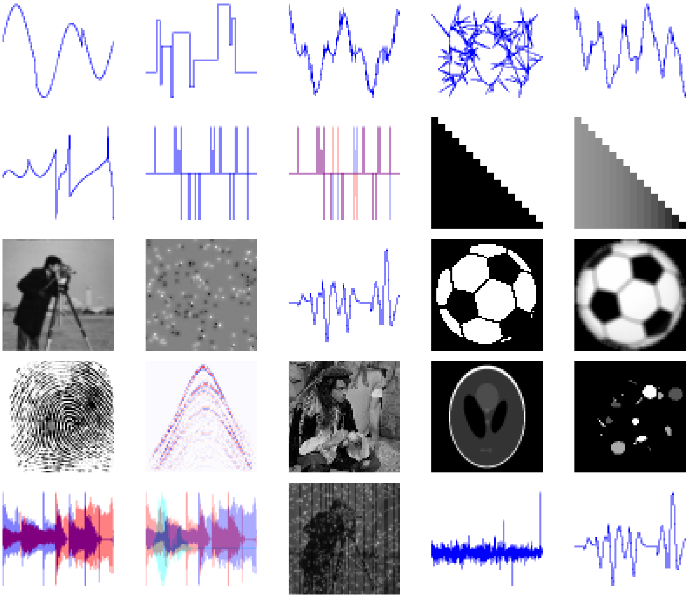



— Demo time —

# Other operators

- Restriction `A = opRestriction(...)`
- Binary mask `A = opMask(...)`
- Column restriction `A = opColumnRestrict(...)`
- 1D Convolution `A = opConvolve1d(...)`
- Non-linear operators
  - Split complex `A = opSplitComplex(...)`
  - Complex to real `A = opReal(...)`

# SPARCO INTERNALS



# Sparco problems

- Currently 27 test problems implemented
- Collected from dozens of papers
- First comprehensive set of test problems

# Instantiating problems

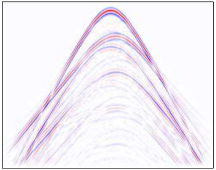
- Problems are identified by their unique ID or name
- `P = generateProblem(5)`, or equivalently
  `P = generateProblem('gcosspike')`.
- `l = generateProblem('list')`

# Problem structure

| P. | **Description** | |
|---|---|---|
| `A` | Operator $A = MB$ | |
| `B` | Sparsity 'basis' $B$ | |
| `M` | Measurement operator $M$ | |
| `x0` | Desired solution | (optional) |
| `b` | Observed signal | |
| `op` | Operators used to construct $M$ and $B$ | |
| `info` | Documentation | |
| ............. | ...................................... | |
| `reconstruct` | Routine to reconstruct signal/image from $x$ | |
| `signal` | Original signal | (optional) |
| `signalSize` | Size of desired signal | (optional) |

# Illustration

Problem 901 – Recovery of missing traces



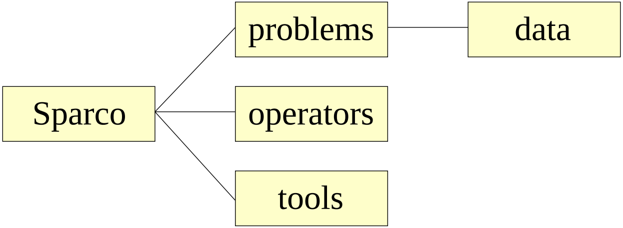
- `P.B` - Curvelet adjoint
- `P.M` - Column restriction operator
- `P.signal` - All traces
- `P.b` - Column restriction of traces (vectorized)

- `[x,r,g,info] = spgl1(P.A,P.b,0,0); % Solve BP`

- `result = P.reconstruct(x); % Resize(B * x)`

— Demo time —

# Directory structure




- Problems are identified by name `prob###.m`
- Adding a file adds a problem
- Operators need additions for documentation only

# Applications of Sparco

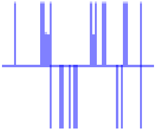
- Testing robustness of solver
- Comparing performance between solvers
- Rapid prototyping

# Comparing performance

Table: *Basis pursuit comparisons*

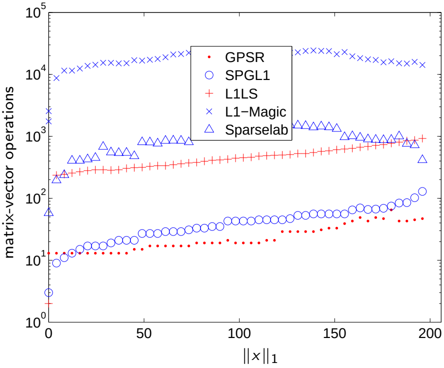
| Problem | PDCO $\|r\|_2$ | PDCO $\|x\|_1$ | Homotopy $\|r\|_2$ | Homotopy $\|x\|_1$ | SPGL1 $\|r\|_2$ | SPGL1 $\|x\|_1$ |
|---|---|---|---|---|---|---|
| blocksig | 3.3e−04 | 4.5e+02 | 1.0e−04 | 4.5e+02 | 2.0e−14 | 4.5e+02 |
| blurrycam | *t* | *t* | *t* | *t* | 9.9e−05 | 1.0e+04 |
| blurspike | 9.1e−03 | 3.4e+02 | *t* | *t* | 9.9e−05 | 3.5e+02 |
| cosspike | 1.6e−04 | 2.2e+02 | 1.0e−04 | 2.2e+02 | 8.6e−05 | 2.2e+02 |
| finger | *t* | *t* | *t* | *t* | 8.2e−05 | 5.5e+03 |
| gcosspike | 1.9e−05 | 1.8e+02 | 1.0e−04 | 1.8e+02 | 9.9e−05 | 1.8e+02 |
| jitter | 1.3e−05 | 1.8e+00 | 1.0e−04 | 1.7e+00 | 5.3e−05 | 1.7e+00 |
| p3poly | 4.6e−02 | 1.7e+03 | 8.5e+01[f] | 1.8e+03 | 9.5e−05 | 1.7e+03 |
| **seismic** | *t* | *t* | *t* | *t* | **8.6e−05** | **3.9e+03** |
| sgnspike | 9.3e−06 | 2.0e+01 | 1.0e−04 | 2.0e+01 | 8.0e−05 | 2.0e+01 |
| soccer1 | *t* | *t* | *t* | *t* | 1.0e−04 | 4.2e+02 |
| spiketrn | 3.6e−03 | 1.3e+01 | 1.0e−04 | 1.3e+01 | 9.9e−05 | 1.3e+01 |
| srcsep1 | 8.2e−03 | 1.1e+03 | *t* | *t* | 8.6e−05 | 1.1e+03 |
| srcsep2 | 5.5e−03 | 1.1e+03 | *t* | *t* | 1.0e−04 | 1.1e+03 |
| yinyang | 1.4e−03 | 2.6e+02 | 2.8e−03[f] | 4.7e+02 | 9.6e−05 | 2.6e+02 |

# Gaussian ensemble, spikes signal

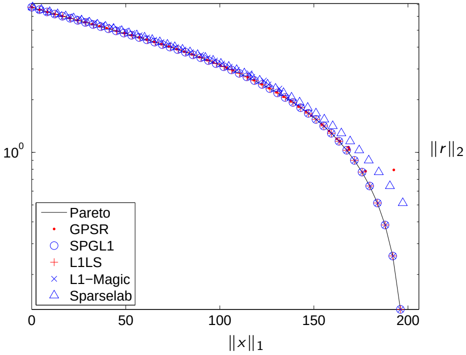
$A =$ Gaussian $\quad x_0 =$ 

$A$: 1200×5120
Candès, Romberg, Tao '05

## Matrix-vector products




## Pareto curve



# Rapid prototyping – Example 1

- Given existing problem with complex $A$, $b$
- People have claimed that complex

$$\underset{z \in \mathbb{C}^n}{\text{minimize}} \|z\|_1 \quad \text{subject to} \quad Az = b$$

can be solved with

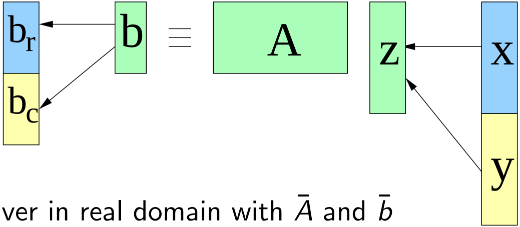
$$\underset{x,y \in \mathbb{R}^n}{\text{minimize}} \|x\|_1 + \|y\|_1 \quad \text{subject to} \quad A(x + iy) = b$$

- Let's find out!

# Rapid prototyping – Example 1

- `opSplitComplex(...);`
- Reformulate operator $A$ and vector $b$

```
S1   = opSplitComplex(m);
S2   = opTranspose(opSplitComplex(n));
Abar = opFoG(S1,A,S2);
bbar = S1(b,1);
```




- Run solver in real domain with $\bar{A}$ and $\bar{b}$

— Demo time —

# Rapid prototyping – Example 2

- Signal sparse in DCT-Dirac dictionary
- Measure with Gaussian ensemble

```
B = opDictionary(opDCT(256),opDirac(256));
M = opGaussian(192,256);
A = opFoG(M,B)
x = sparse(512,32);
n = randn(192,1);
b = A(x,1) + n;
```

- Recover coefficients by Basis Pursuit Denoise

```
xs = spgl1(A,b,0,norm(n,2));
```

http://www.cs.ubc.ca/labs/scl/sparco/