Reverse-time Migration Amplitude Recovery with Curvelets

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References

- **Curvelets and its invariance**
  - Curvelet and FIOs, Candes and Demanent (2002)
  - Hardy space for FIOs, Smith (1998)
- **Migration amplitude recovery**
  - Optimal scaling for RTM, Symes (2007)
  - Illumination based migration, Rickett and Claerbout (2000)
  - Hessian Approximation based, Mulder (2003)
  - True amplitude migration, Zhang (2003)
  - Least square migration, Kuhl and Sacchi (2001)
- **Optimization methods**
  - Soft-thresholding, Donoho (1995)
  - Iterative thresholding, Daubechies (2005)
  - Gradient based optimization, Nocedal (2001)
  - L1 solver (SPGL1), Friedlander(2007)

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Overview

- Problem definition
- Imaging as an inversion problem
- Curvelets and their properties
- Curvelets and their invariance under the normal operator
- Diagonal approximation of the normal operator
- Inverse problem reformulation
- Optimization
- Results
- Conclusions

Definition of the Problem

- Basic Imaging problem:

$$\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n}$$

- Desired seismic image characteristics:
  - Broad-band
    - Sharp, high resolution
    - 2D curves/3D sheets
  - Continuity along the reflectors
- Noise in seismic images
  - Random noise
    - Instruments distortion
    - Ambient
  - Imaging operator imperfections

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Imaging as an Inverse Problem

- Following inversion problem is introduced

$$\min_{\mathbf{m}} J(\mathbf{m}) \text{ subject to } \|\mathbf{d} - \mathbf{K}\mathbf{m}\|_2^2 \leq \epsilon$$

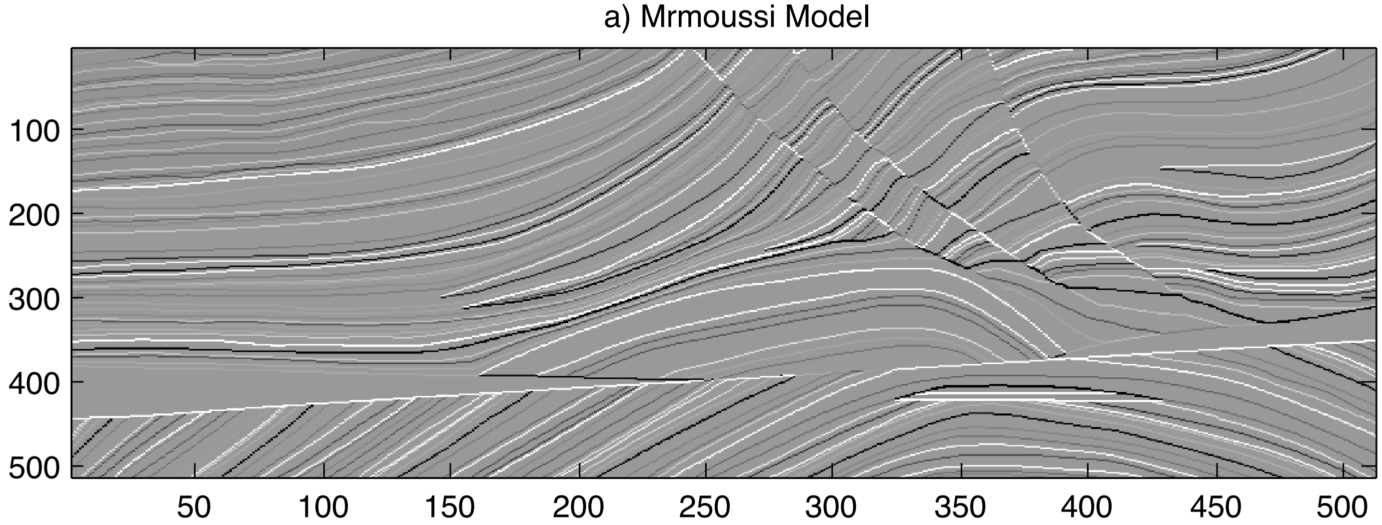
  J(**m**) is a norm or penalty function
- This norm has to
  - promote continuity along reflectors
  - promote sparsity of image in the curvelet domain
  - reduce the artifacts from image
  - enhance the amplitudes of the reflectors
  - remove noise from seismic image

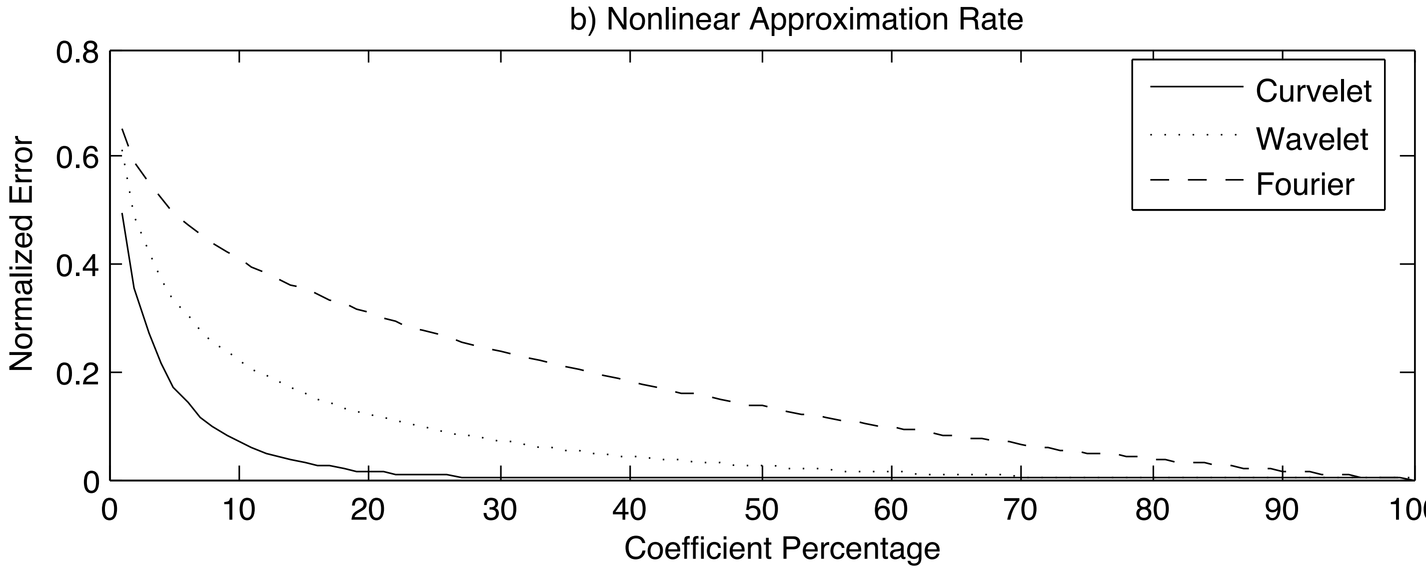
Curvelets and their properties

Curvelets:

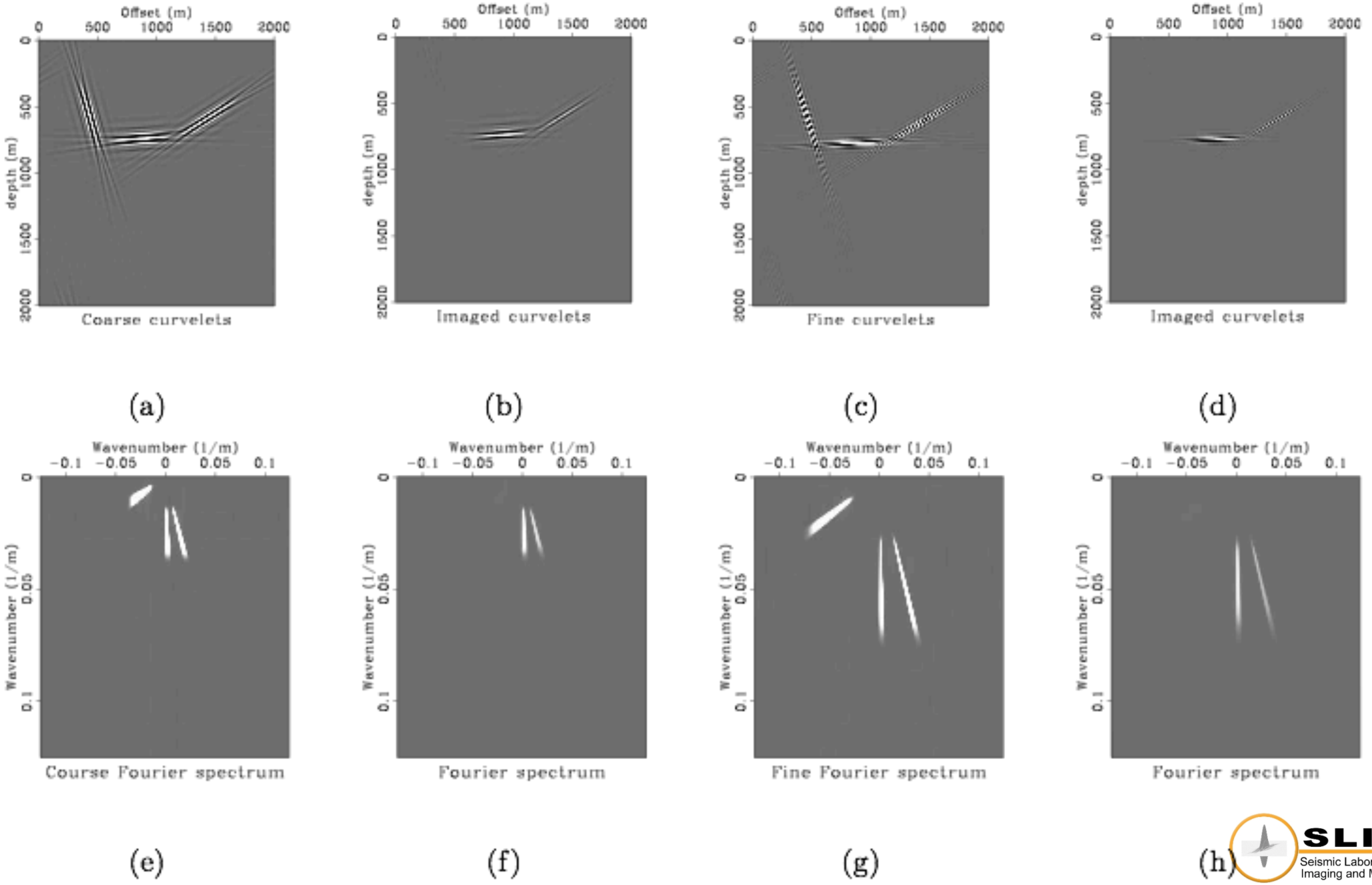
- are multiscale and multi-directional
- sparsely represent seismic images
- are invariant under the action of idealize normal operator
- are constructed as tight frames
- permit a fast transformation
- are used reliably for denoising in image processing applications

Approximation Rate Comparison





Example (three curvelets)



Normal operator approximation

- Approximation with curvelet eigenvalue-like decomposition:

$$\mathbf{C}^T \mathbf{D}_\Psi \mathbf{C} \approx \mathbf{K}^T \mathbf{K}$$

- Diagonal matrix is smooth in phase space (in the curvelet domain) for smooth background velocity models
- Computationally cheap, requires only “one” evaluation of the normal operator to estimate the diagonal scaling

Diagonal Approximation

- Approximation with curvelet regularization,

$$\mathbf{C}^T \mathbf{D}_\Psi \mathbf{C} \approx \mathbf{K}^T \mathbf{K}$$

- Positivity constraint on diagonal estimation,

$$\min_{\mathbf{u}} (||\mathbf{K}^T \mathbf{K} \mathbf{r} - \mathbf{C}^T \mathrm{diag}(e^{\mathbf{u}}) \mathbf{C} \mathbf{r}||_2^2 + \lambda ||L e^{\mathbf{u}}||_2^2)$$

$$\mathbf{L} = [\mathbf{D_x}^T \; \mathbf{D_y}^T \; \mathbf{D}_{\boldsymbol{\theta}}^T]^T \qquad \mathbf{D}_{\boldsymbol{\Psi}} = \mathrm{diag}(e^{\mathbf{u}})$$

- Solve using quasi-Newton method
- Explore smoothness along the curvelet symbol
- Computationally cheap, requires only “one” evaluation of normal operator

Problem reformulation

- Forming the normal equation,

$$\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n} \Rightarrow \mathbf{K}^T\mathbf{d} = \mathbf{K}^T\mathbf{K}\mathbf{m} + \mathbf{K}^T\mathbf{n}$$
$$\Rightarrow \mathbf{y} \approx \mathbf{A}\mathbf{A}^T\mathbf{m} + \mathbf{e}$$
$$= \mathbf{A}\mathbf{x_0} + \mathbf{e}$$

- with

$$\mathbf{y} = \mathbf{K}^T\mathbf{d}, \qquad \mathbf{A} = \mathbf{C}^T\sqrt{\mathbf{D}_{\Psi}}$$

- $\mathbf{A}$ is *scaled* the inverse curvelet transform

Recovery Problem Formulation

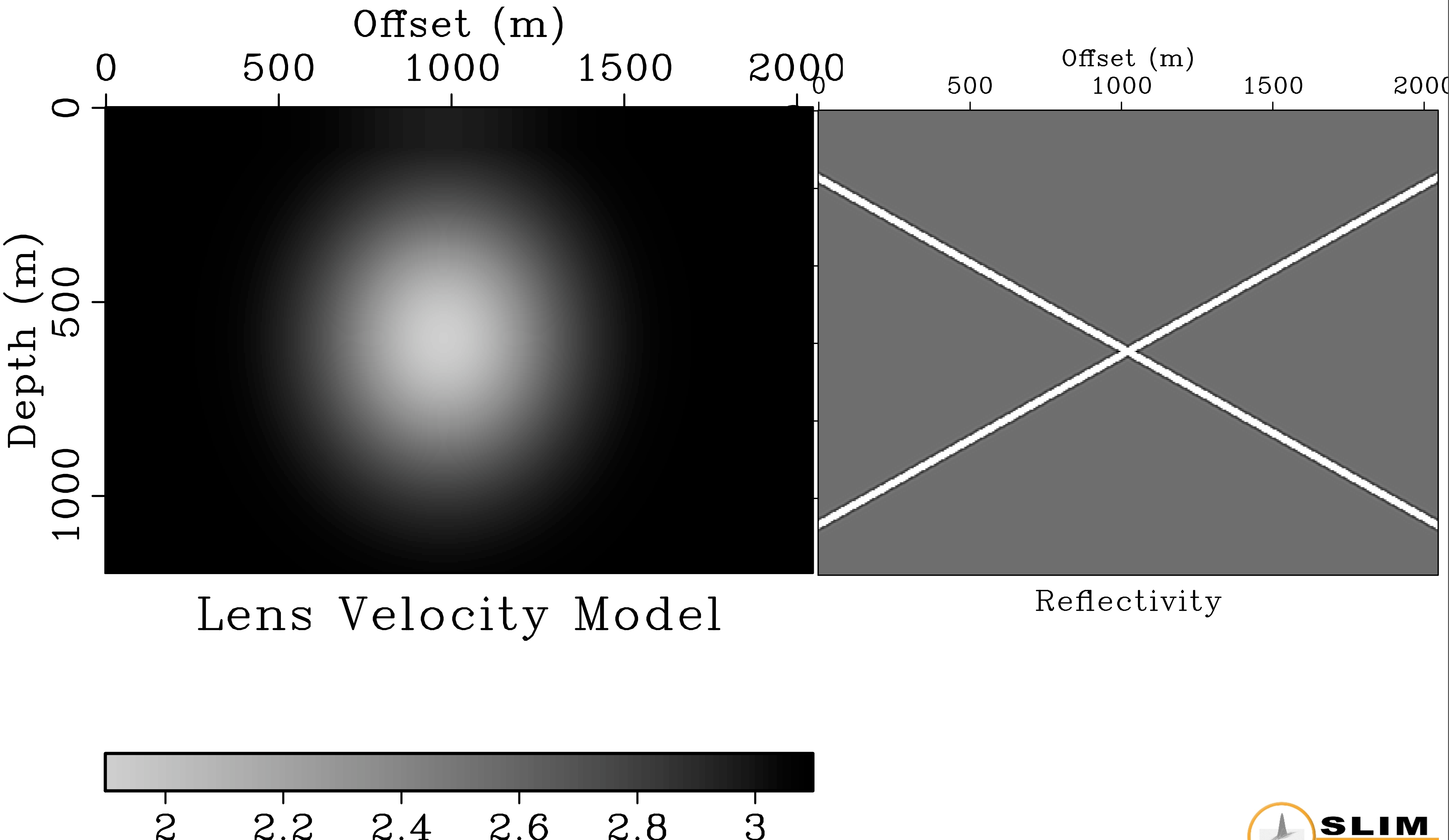
$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \left(\mathbf{A}^T\right)^{\dagger} \tilde{\mathbf{x}}. \end{cases}$$

- With $J(\mathbf{x}) = ||\mathbf{x}||_1$
- $J(\mathbf{x})$ promotes sparsity on the weighted curvelet coefficients of the reflectivity

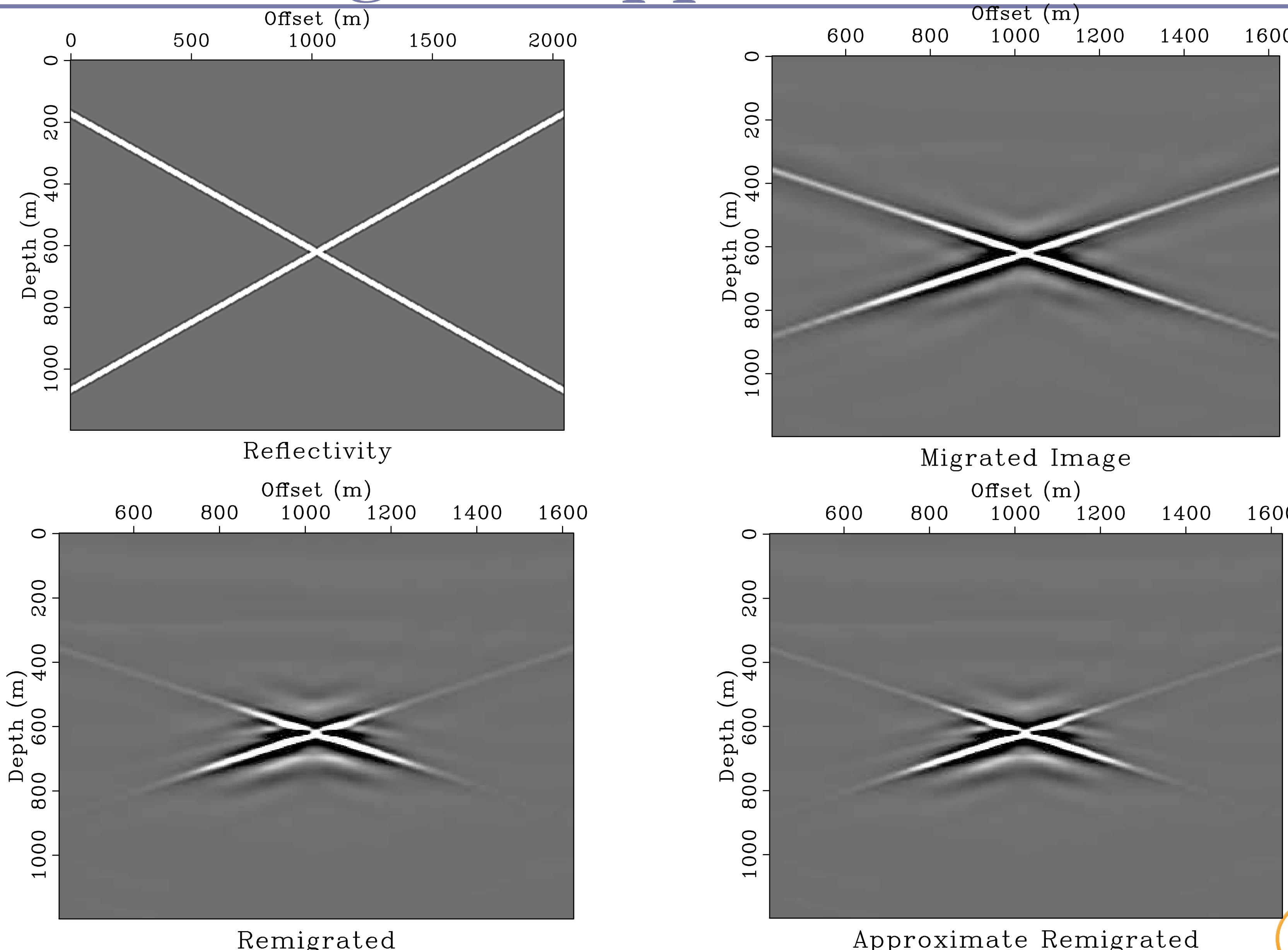
Recovery Method

- Form the normal operator $\mathbf{K}^T\mathbf{K}$.
- Select a reference vector that is close to the unknown image.
- Estimate the diagonal. $(i.e., \mathbf{D}_\mu)$
- Construct the matrix $\mathbf{A} = \mathbf{C}^T\sqrt{\mathbf{D}_\mu}$.
- Solve L1 optimization problem to find the solution for the reflectivity.

Conflicting-dips example
Linearized Born modeled Data



Conflicting-dips example
Diagonal approximation



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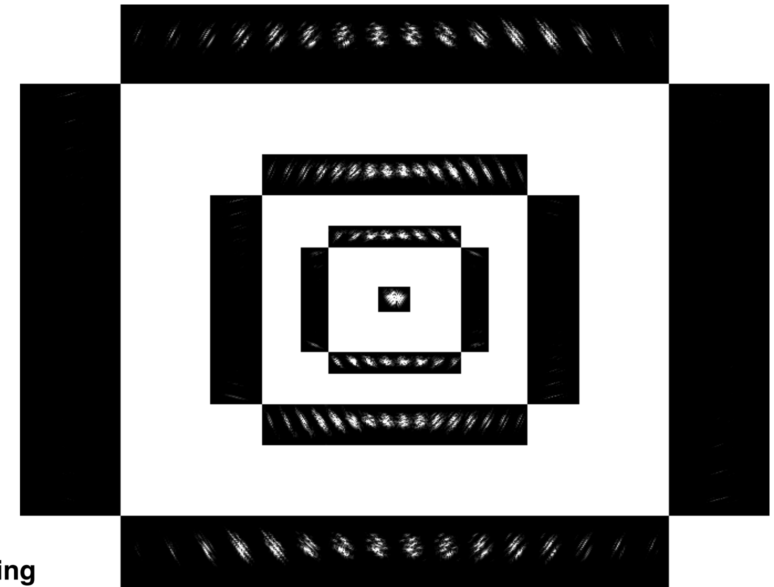
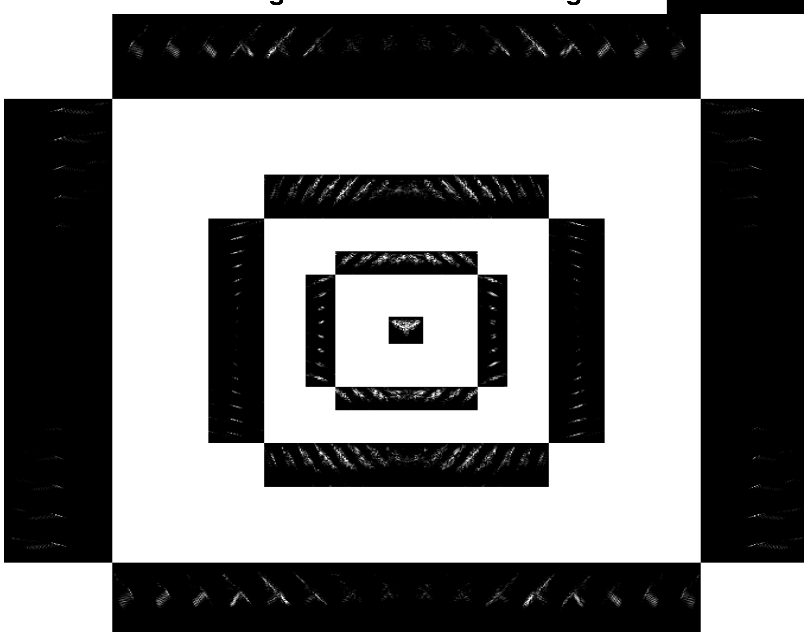
Example
Curvelet Mosaic Plot

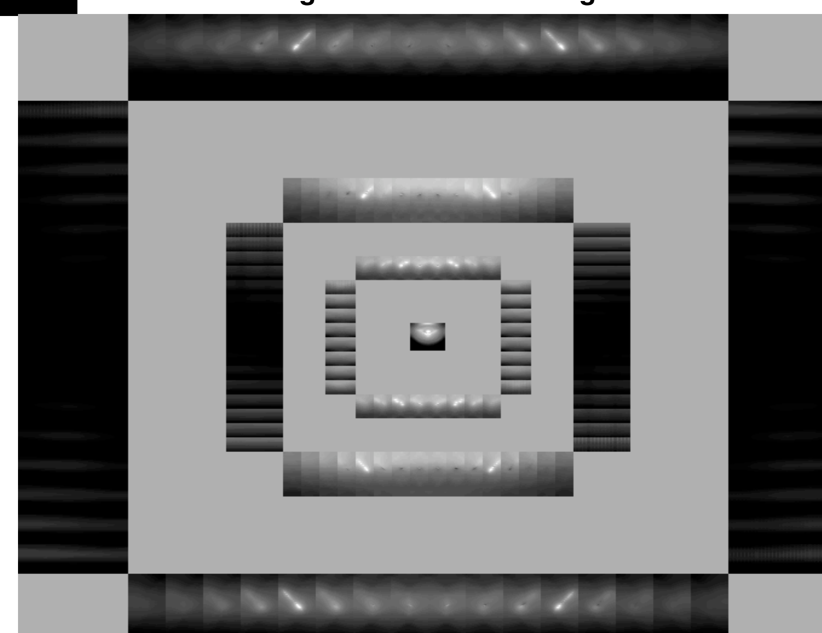
Model in Curvelet Domain



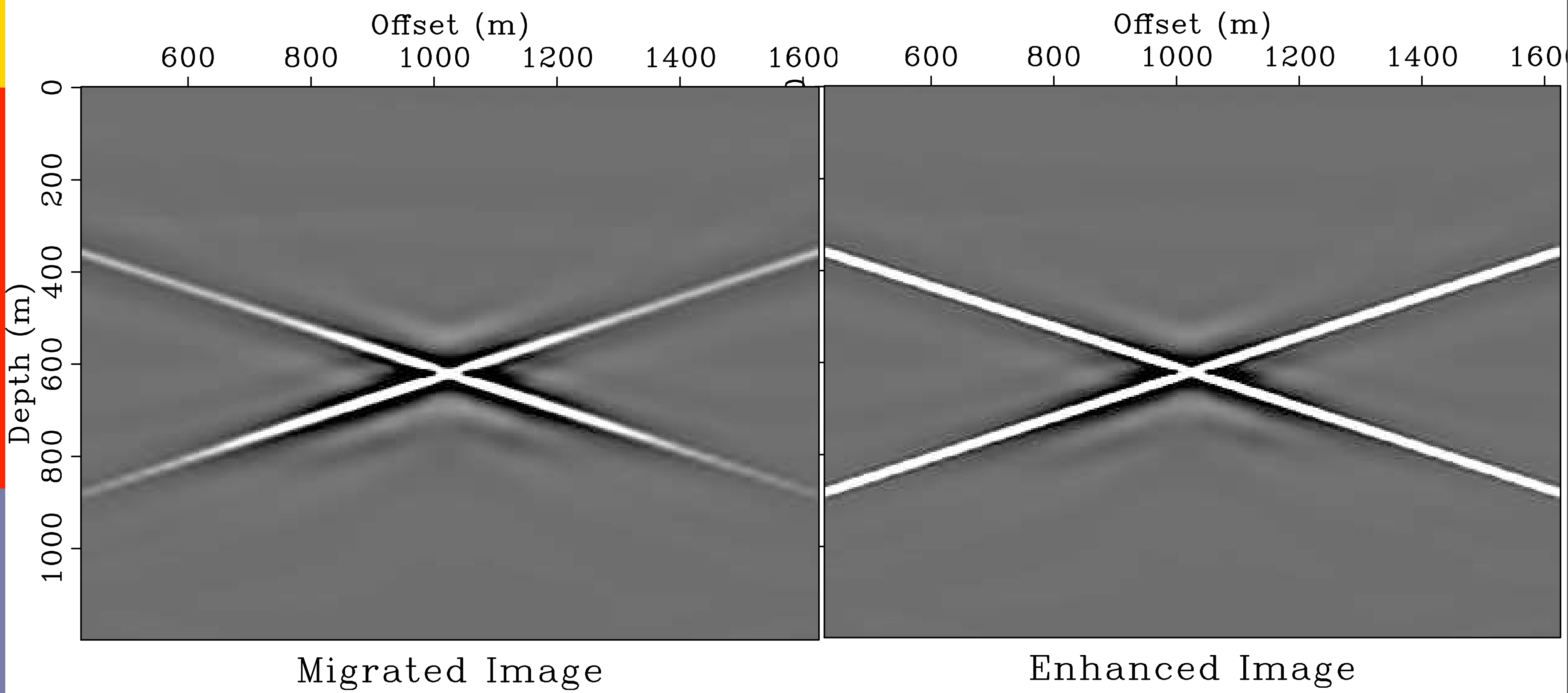
Diagonal Before Smoothing



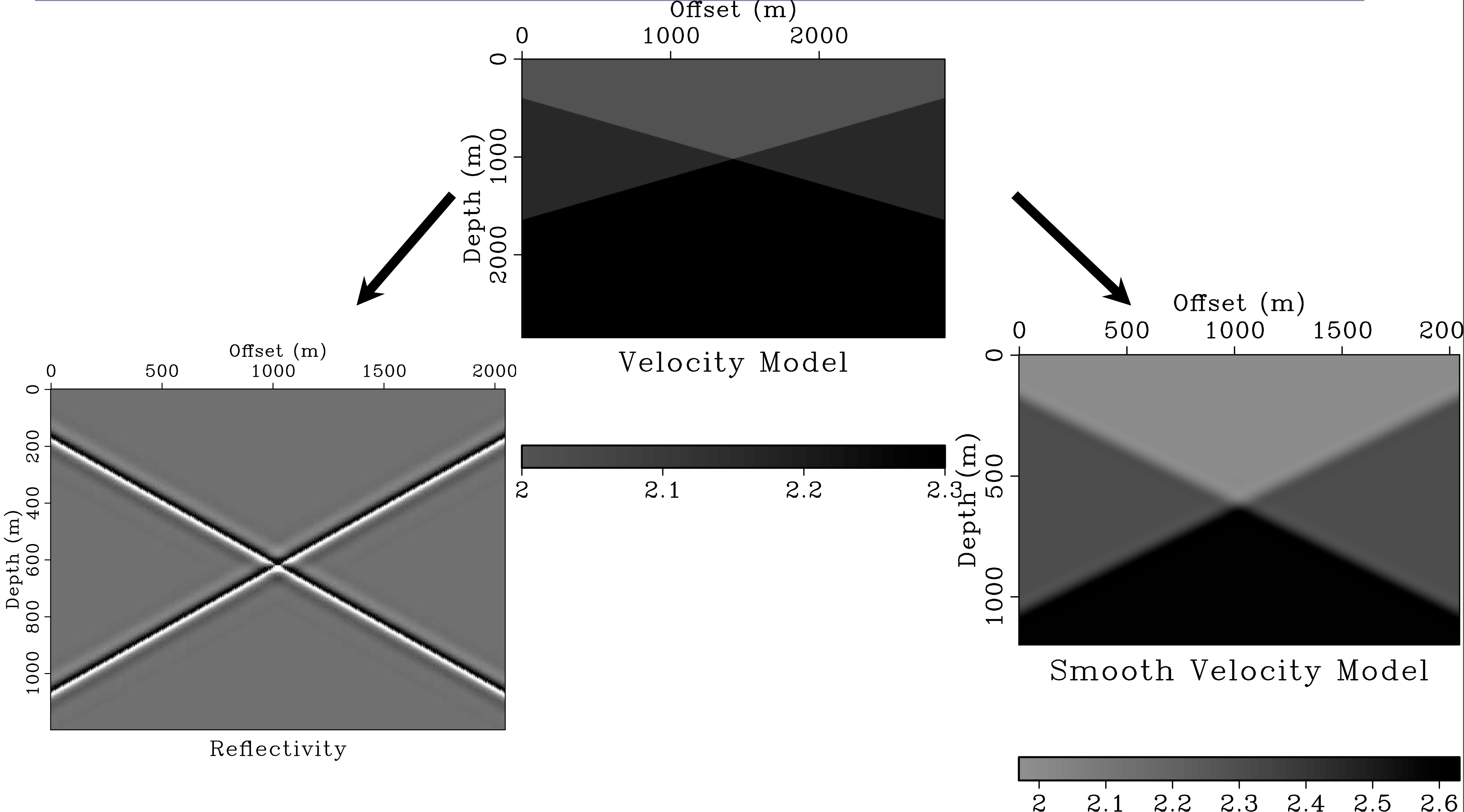
Diagonal After Smoothing



Conflicting-dips example
Result



Conflicting-dips example
Full synthetic data



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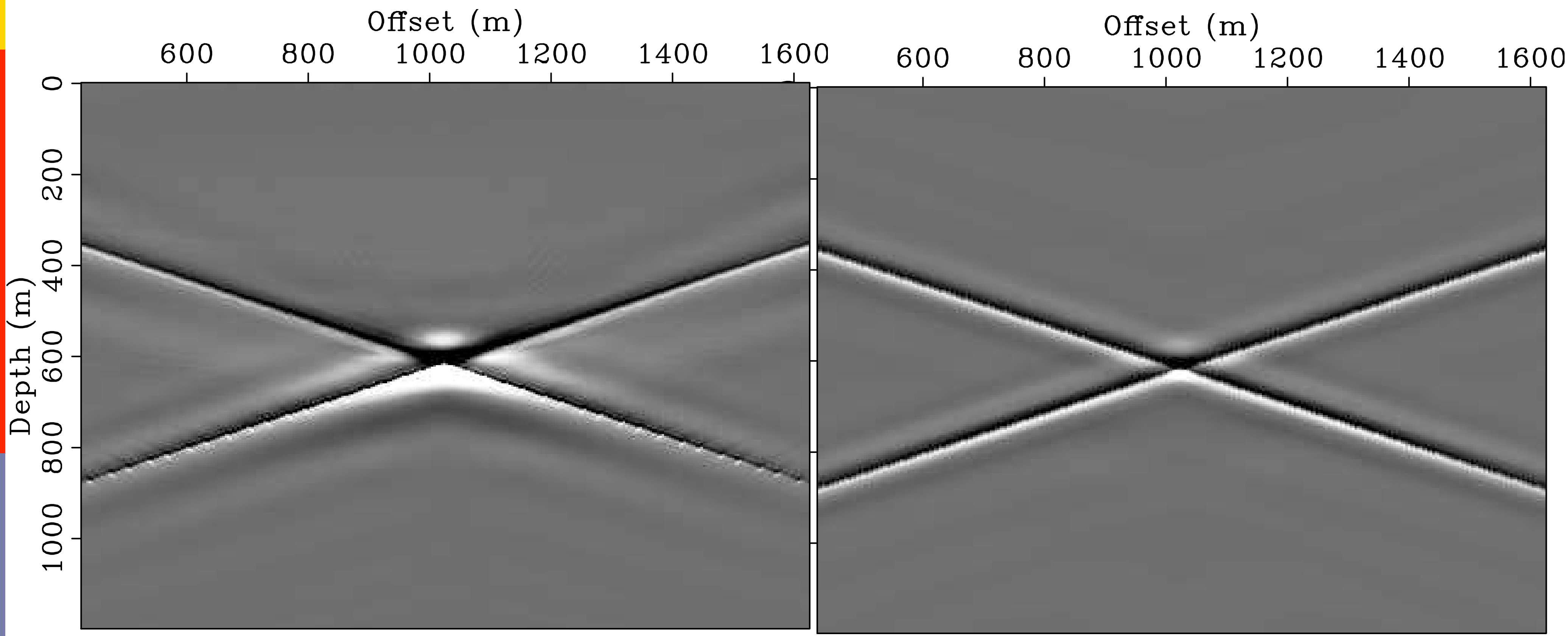
Conflicting-dips example
Full synthetic data

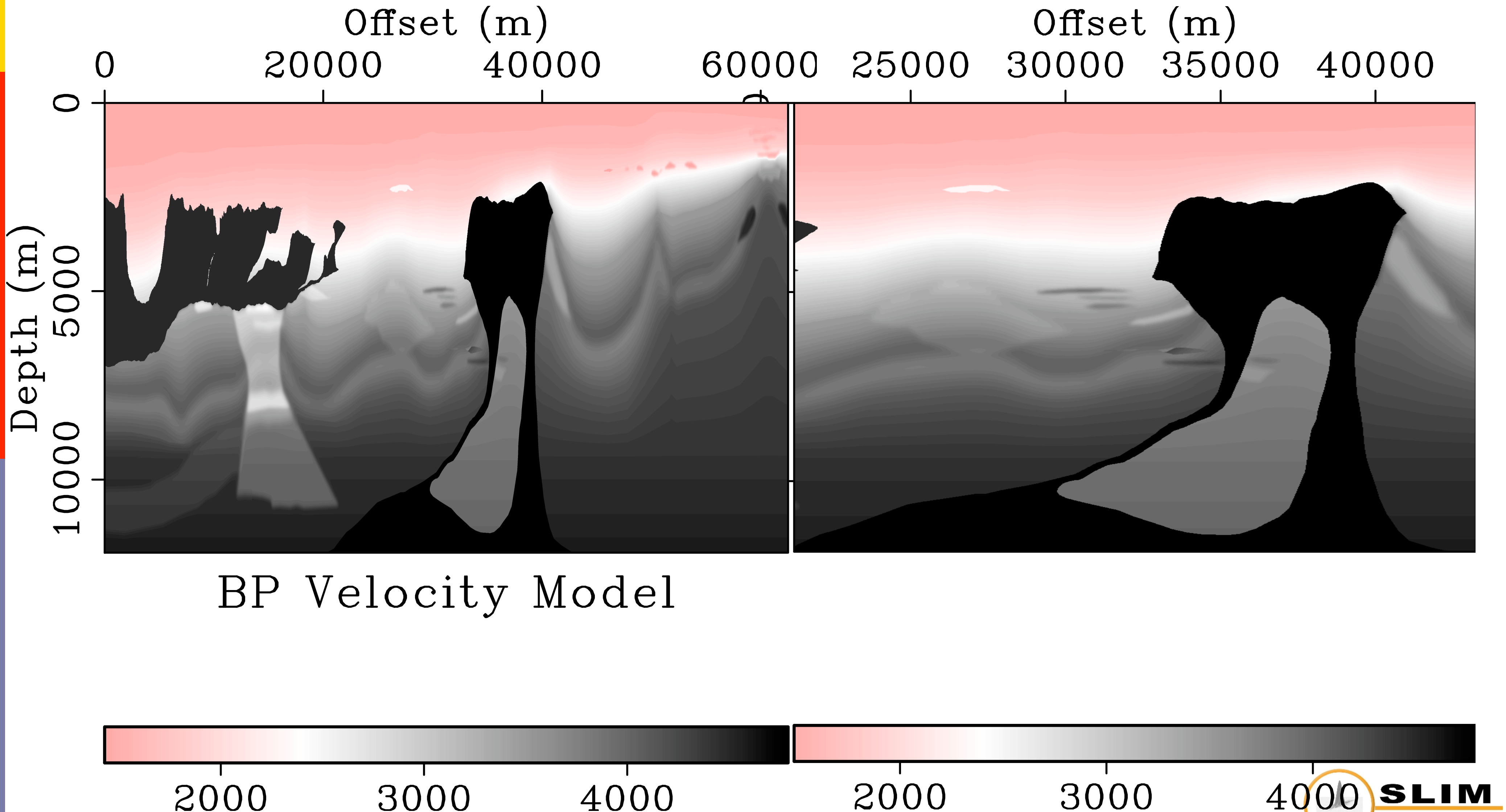


Migrated

Enhanced

BP-dataset example

Velocity Model



BP Velocity Model

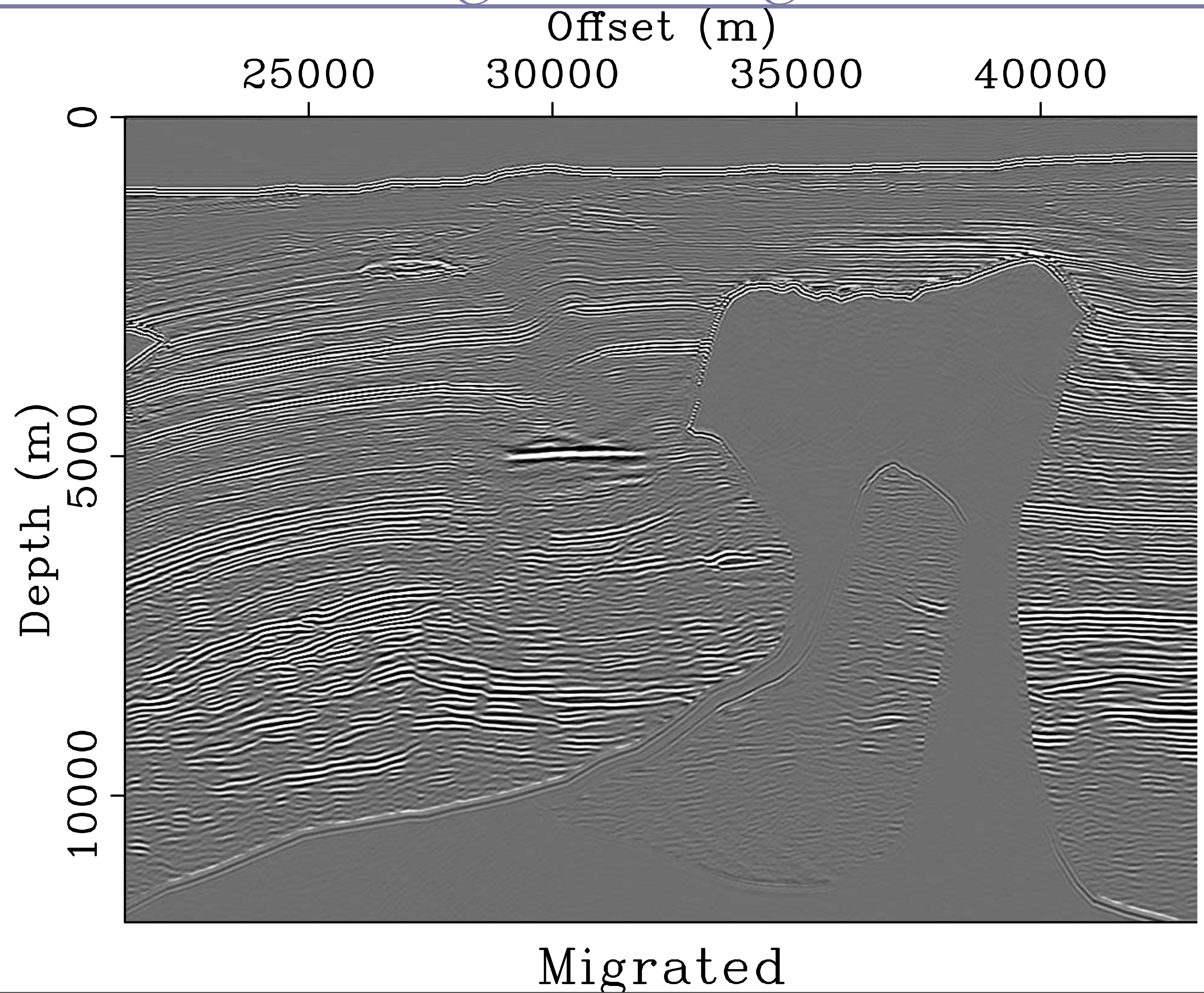
BP-dataset example

Description

- Dataset with
  - ...streamer configuration
  - ...15 (km) streamer and 12.5 (m) group interval and 50 (m) shot interval
  - ...14 (s) of recording time with 6 (ms) sampling interval
  - ...total of 1340 shots and 1201 receivers
- Pre-processing steps:
  - free surface waves (direct arrivals) are removed
  - free surface multiple are removed (using SRME)
  - dipping waves are removed using band pass filtering
  - background velocity is smoothed
  - both migrated and re-migrated images were corrected for illumination map

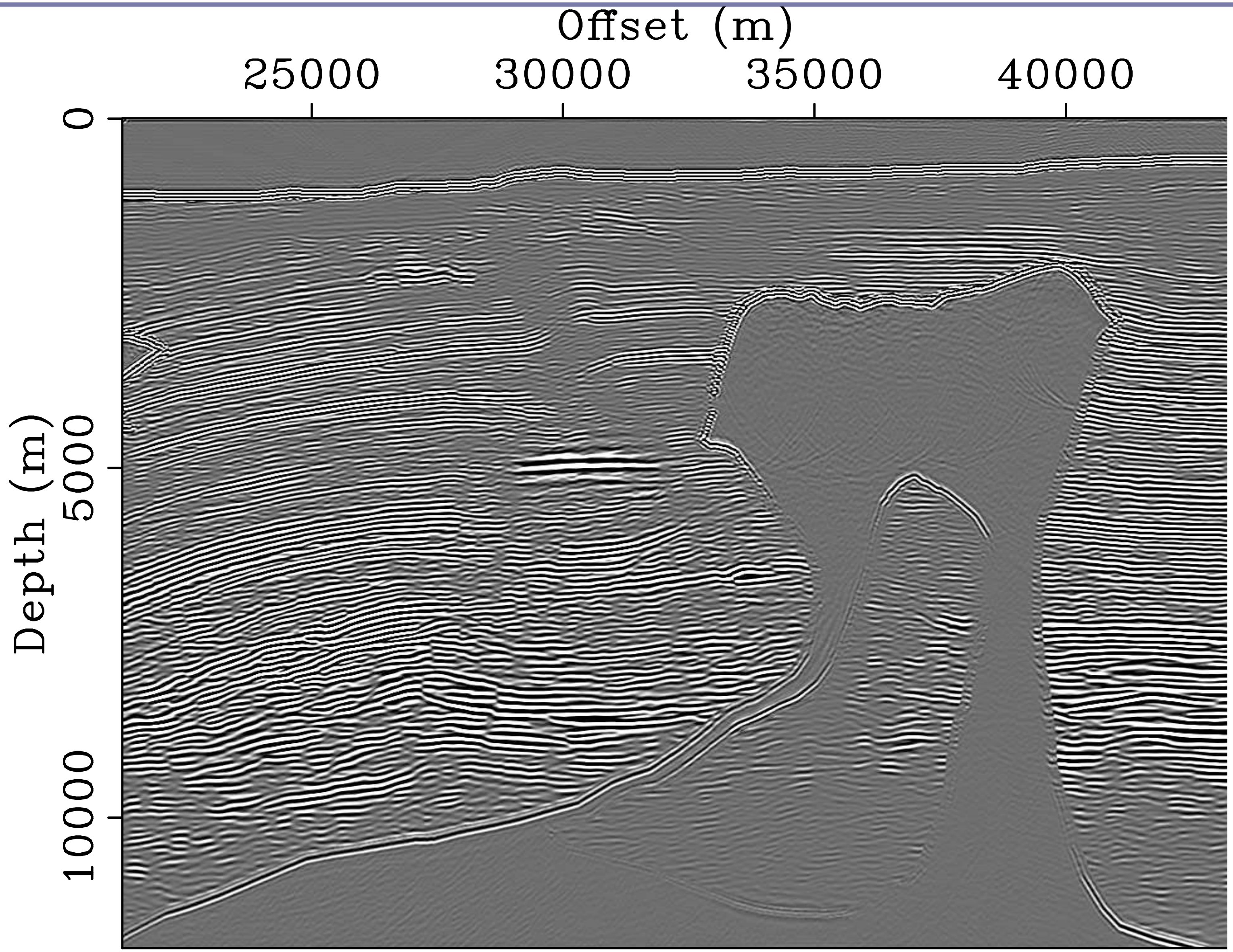
BP-dataset example

migrated image



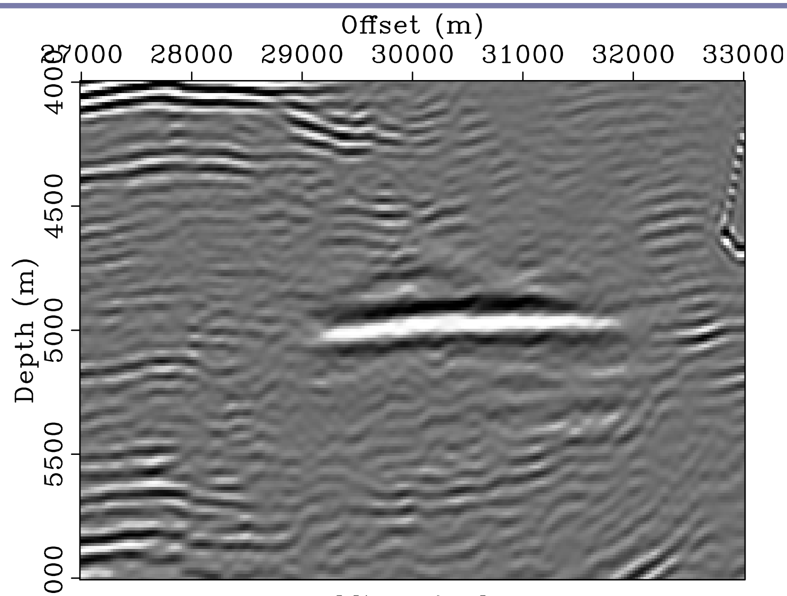
BP-dataset example

amplitude-corrected migrated image

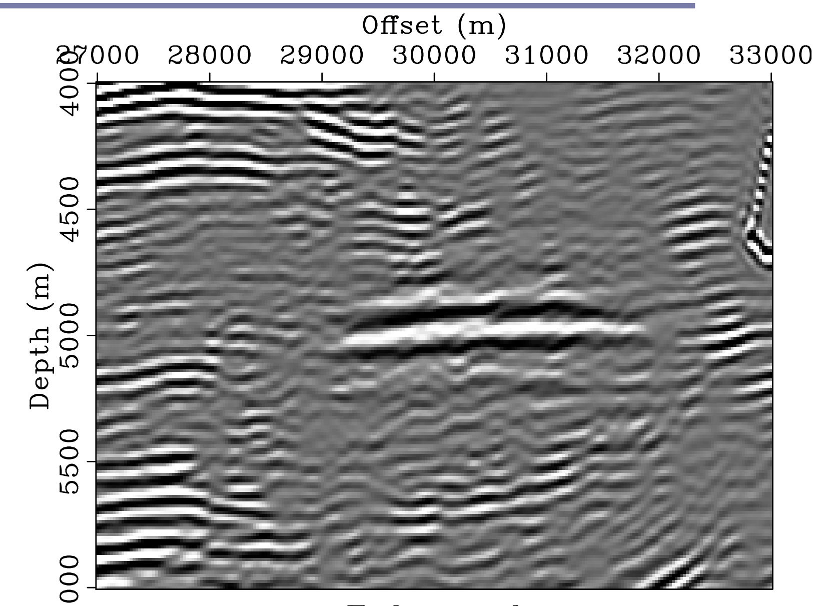


Enhanced

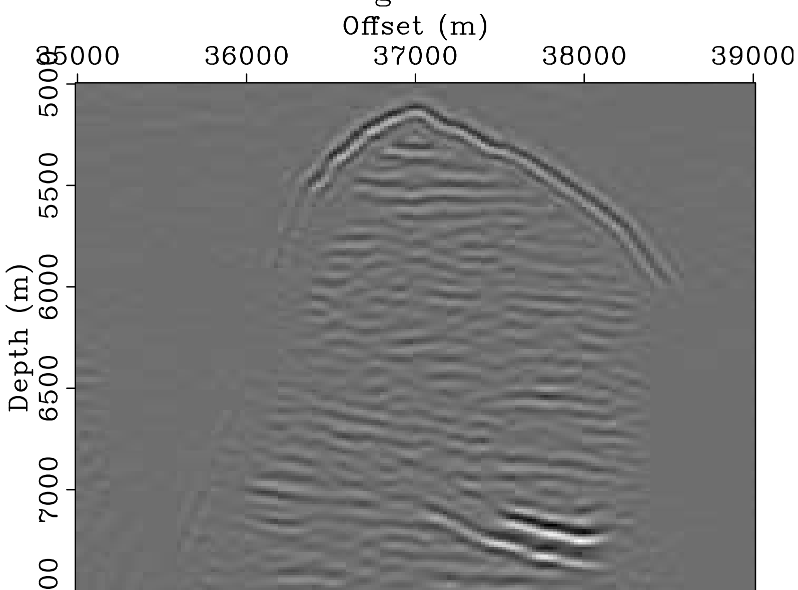
BP-dataset example zoomed comparisons



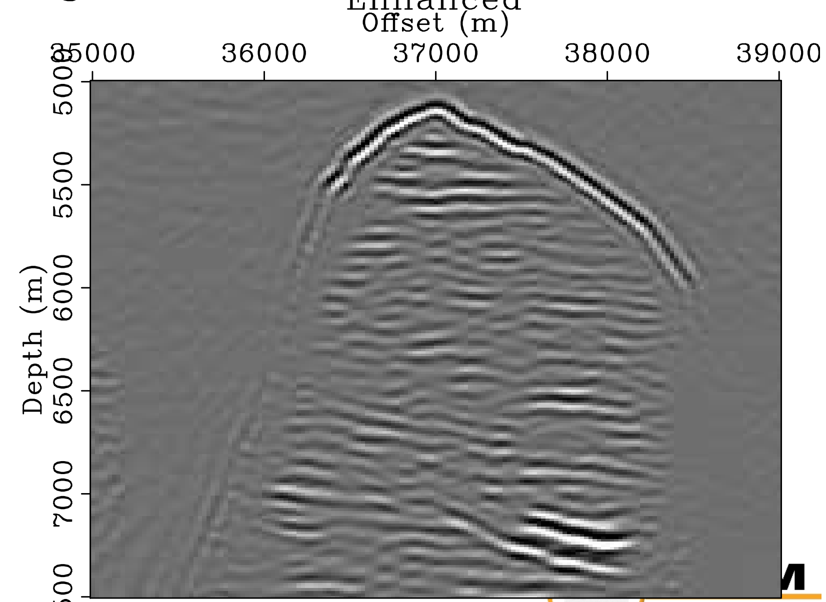
Migrated



Enhanced



Migrated



Enhanced

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Conclusion

This work

- … introduces a novel approach to migration amplitude recovery
- … employs an accurate diagonal decomposition of the expensive normal operator
- … employs curvelets as essential elements in both approximation and estimation
- … can be used instead of illumination map or in conjunction with it

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