Curvelet / Surfacelet comparison

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Outline

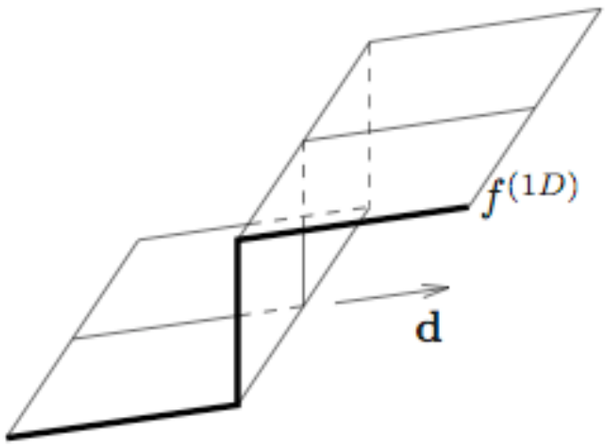
- Properties
- Construction
- Decomposition / Reconstruction example
- Relative decay rates
- Fourier / Space domain comparison

Properties

- Multidimensional signal decomposition ($N \geq 2$)
- decomposition into different scales and directions
- Refinement of angular resolution
- Tree-structured filter bank
- “Same” frequency partitioning as Curvelets
- Perfect reconstruction

Motivation

- Ideal surface discontinuities and curved wavefronts
- locally plane waves
  - A 3-D constant function with a plane discontinuity



- **d:** normal direction to the surface
- Plane: $\{t \in \mathbb{R}^N \quad t \cdot d = 0\}$
- $f^{(1D)}$: a 1-dimensional step function
- Ideal plane discontinuity:

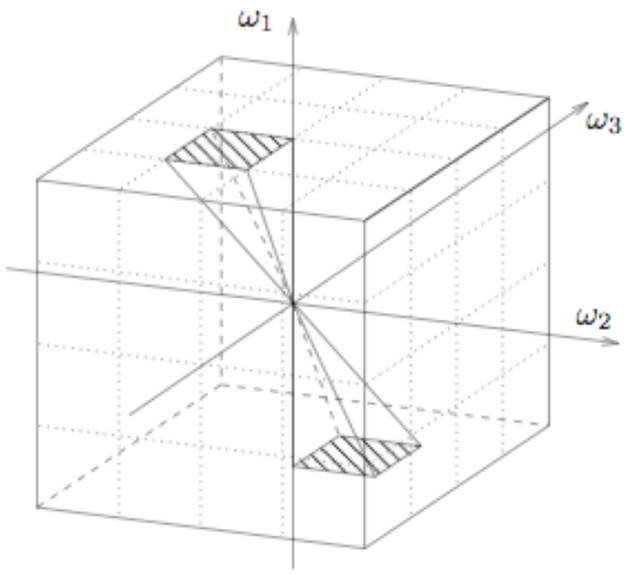
$$y(t) = f^{(1D)}(t \cdot d)$$

Motivation

- Frequency domain support
  - Fourier spectrum of plane waves concentrates on radial lines

$$y(t) = t^{(1D)}(t \cdot d) \quad \overset{\text{FT}}{\Rightarrow} \quad supp \quad \hat{y}(\omega) = \left\{ cd : c \in \mathbb{R} \right\}$$

- Ideal frequency partitioning to analyze plane waves



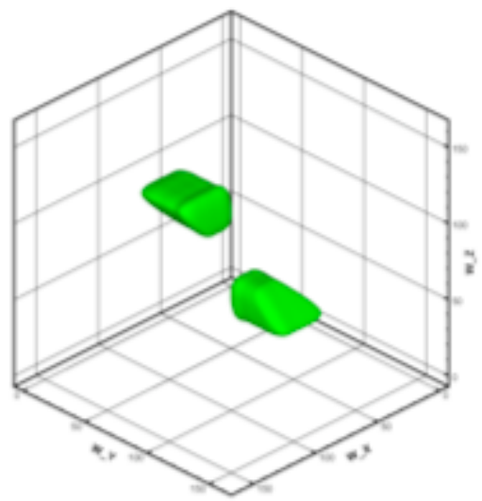
- Rectangular-based pyramids tiling the entire frequency spectrum
- Different frequency bands will capture different plane waves (or singularities) with different orientations

Motivation

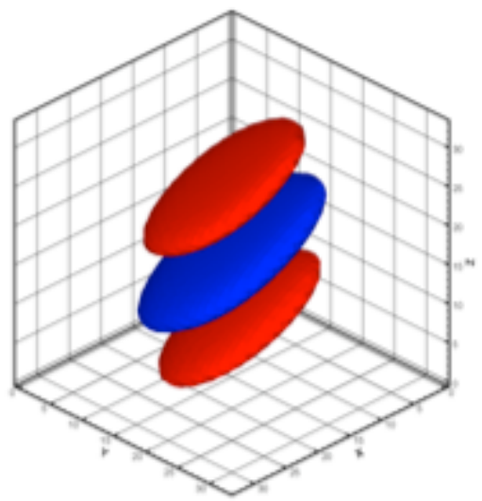
- Resulting goal:
  - A tight frame expansion for functions $f(n)$ in $l^2(\mathbb{Z}^N)$

$$f(n) = \sum_{k,m,j} < f, s^j_{k,m} > s^j_{k,m}(n)$$

  - Constructed from a set of basis functions $\{P_k^{(j)}(n)\}$



frequency



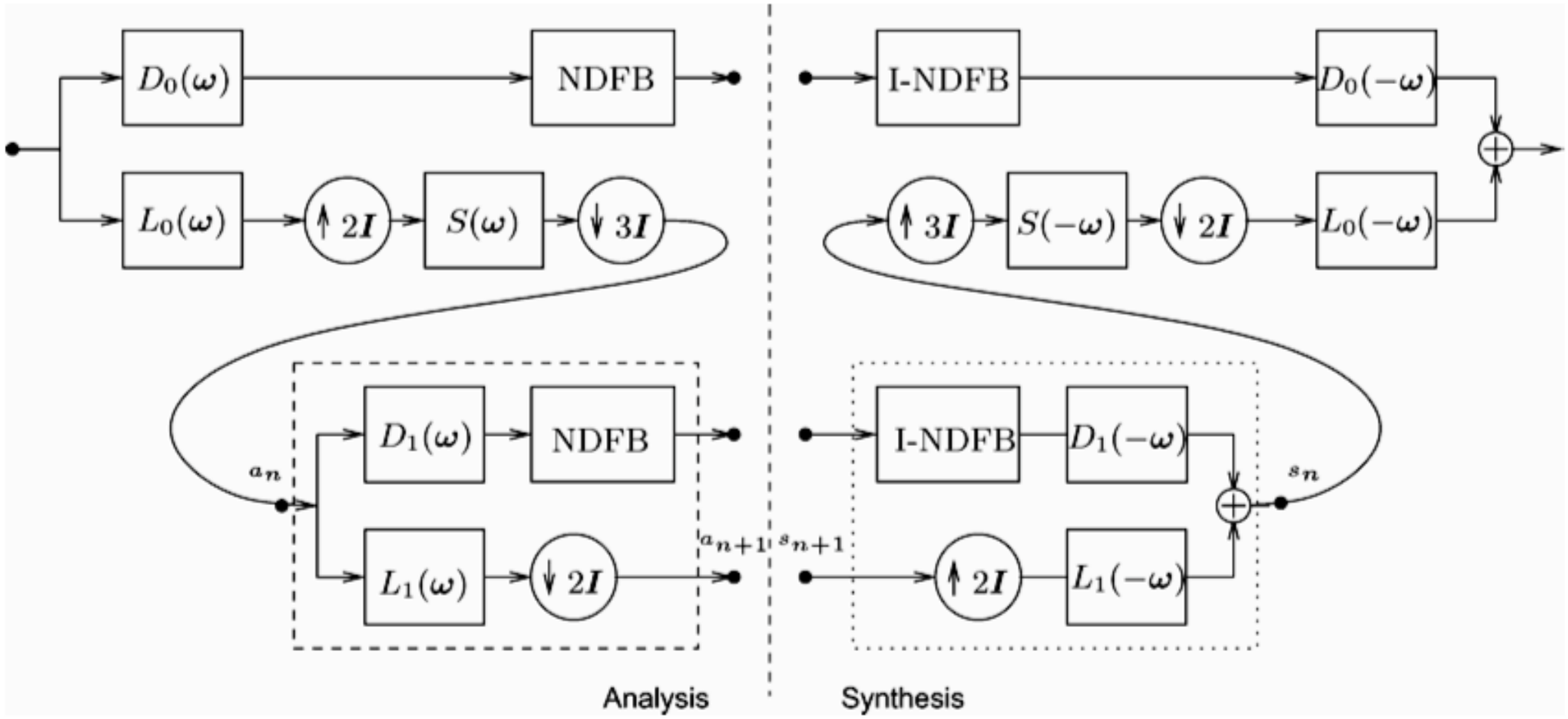
space

Construction

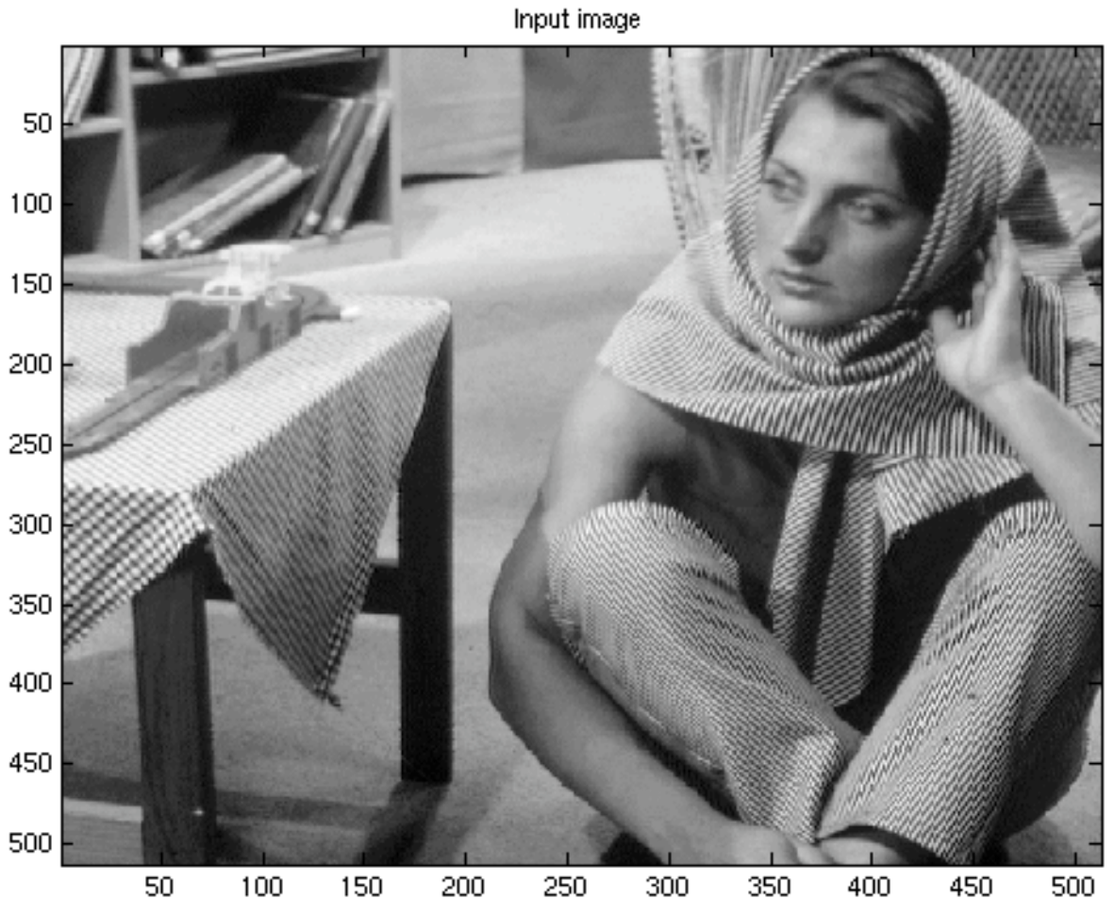
Curvelets	Surfacelets

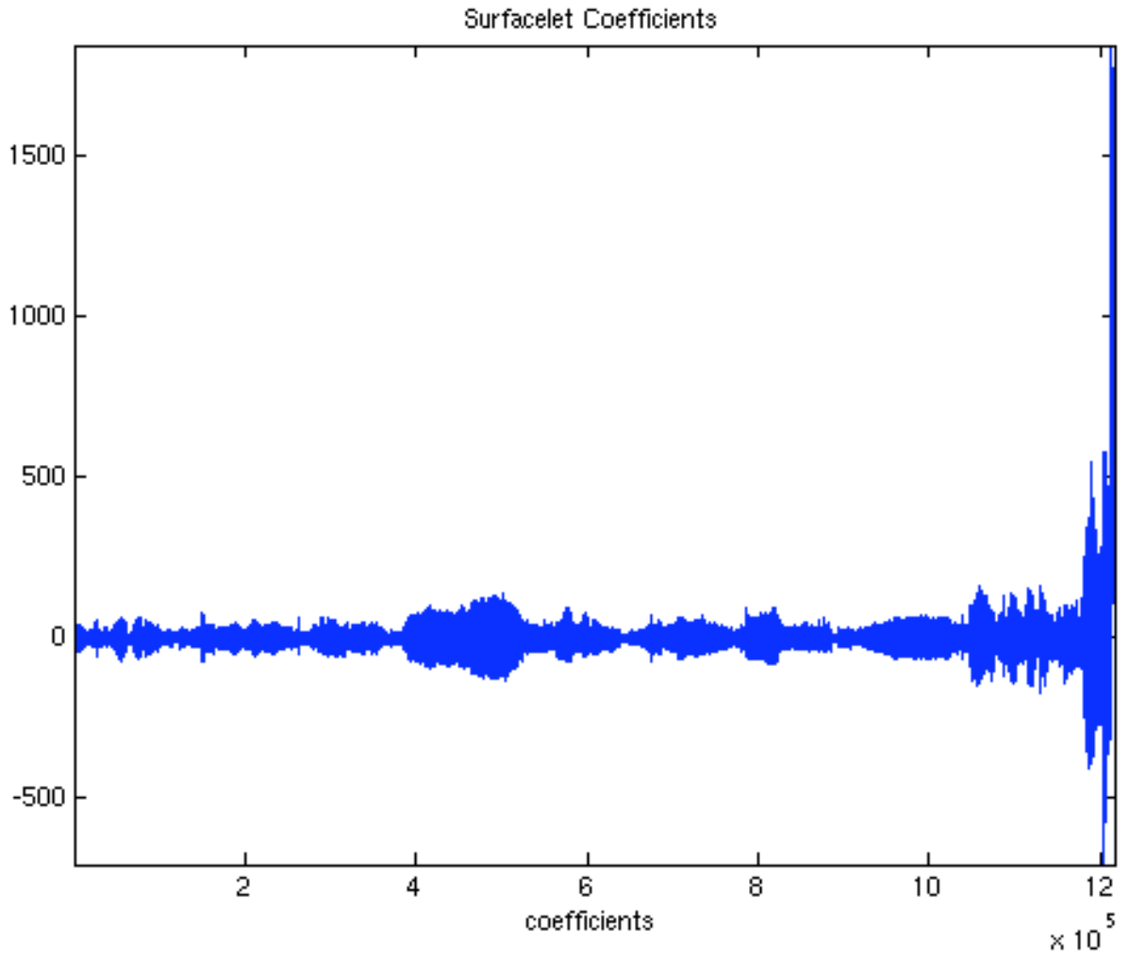
- Define "mother" curvelet
  - Tile frequency space
- Obtain curvelet coefficients
  - Fourier samples X curvelet window functions
- Spacial downsampling
  - alias-free
    - band limited
    - tradeoff in redundancy - spacial localization
      - strictly localized to one angular wedge
  - aliasing allowed to exist
    - canceled by carefully designed filters
    - redundancy factor
    - fast spacial decay
      - filters not strictly bandlimited

Construction

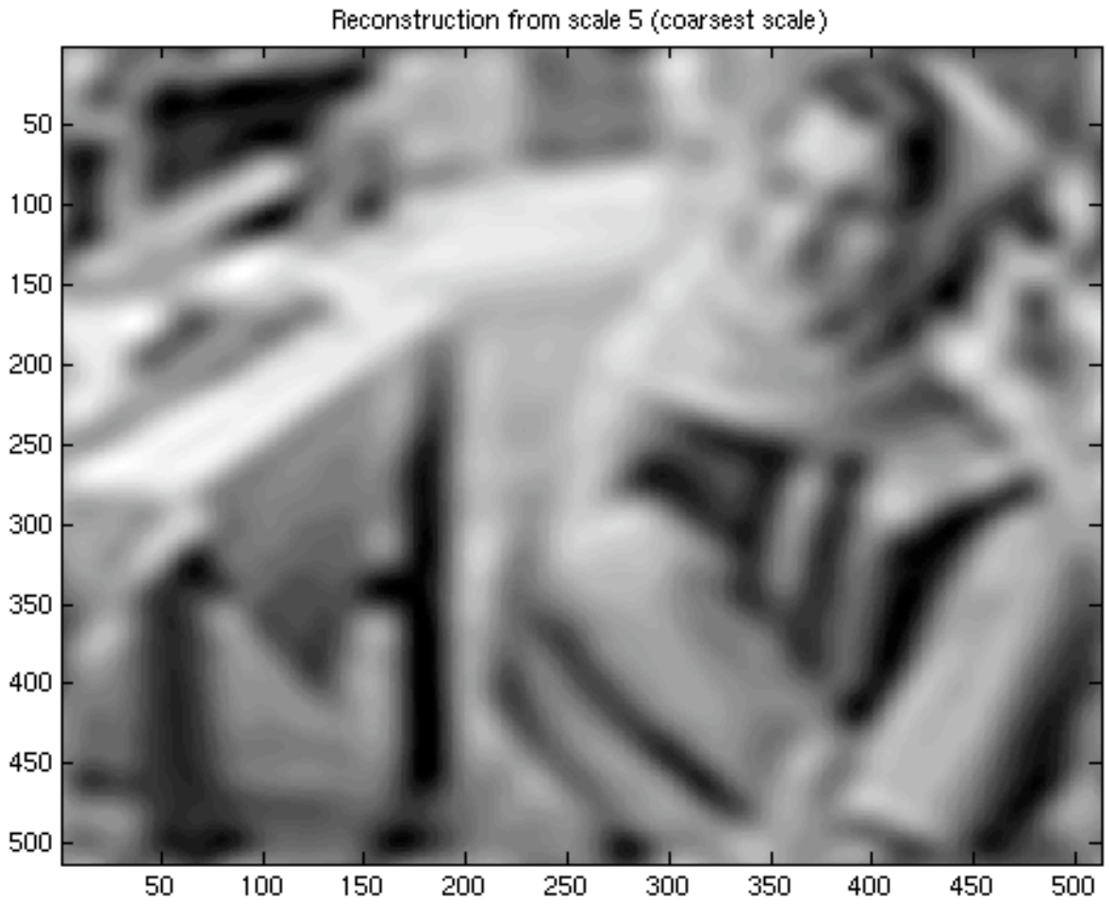


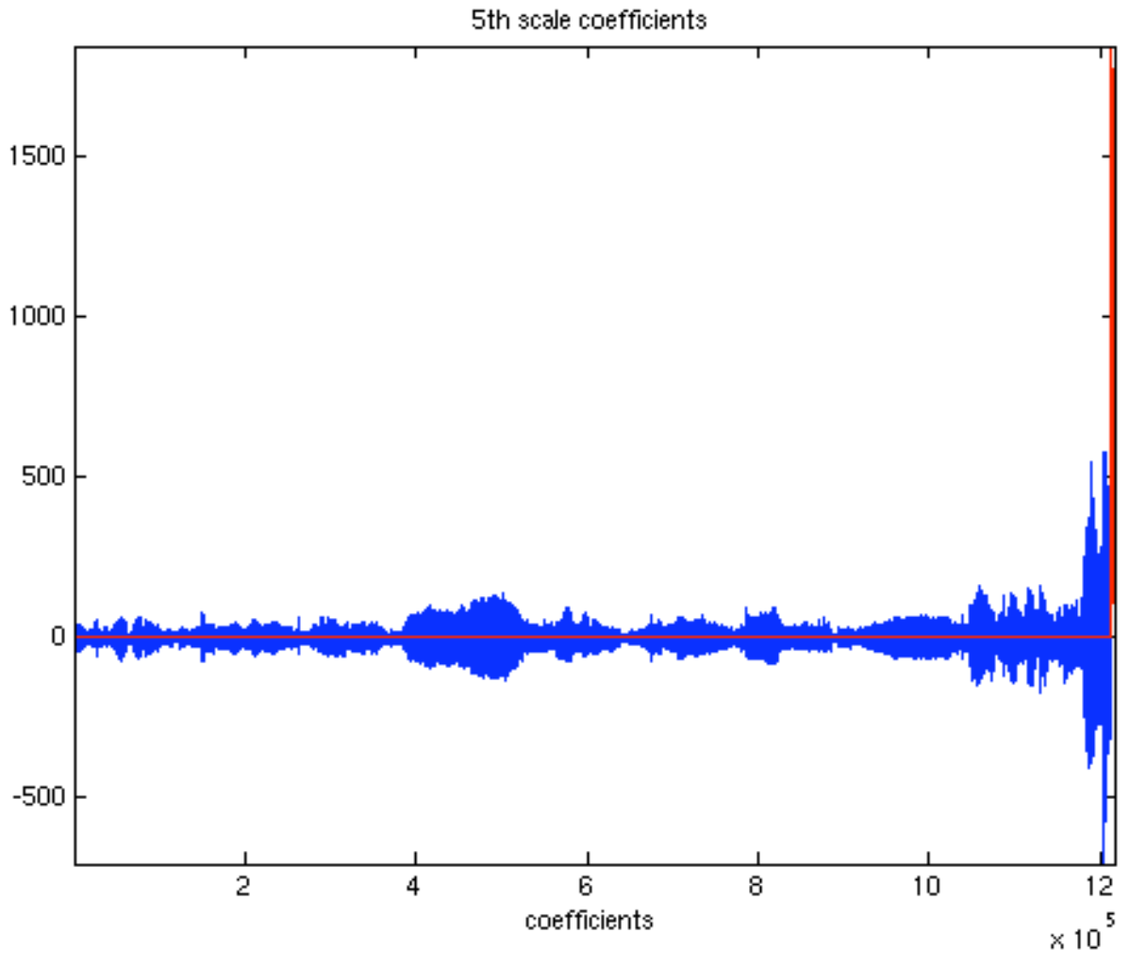
Surfacelet Decomposition/ Reconstruction





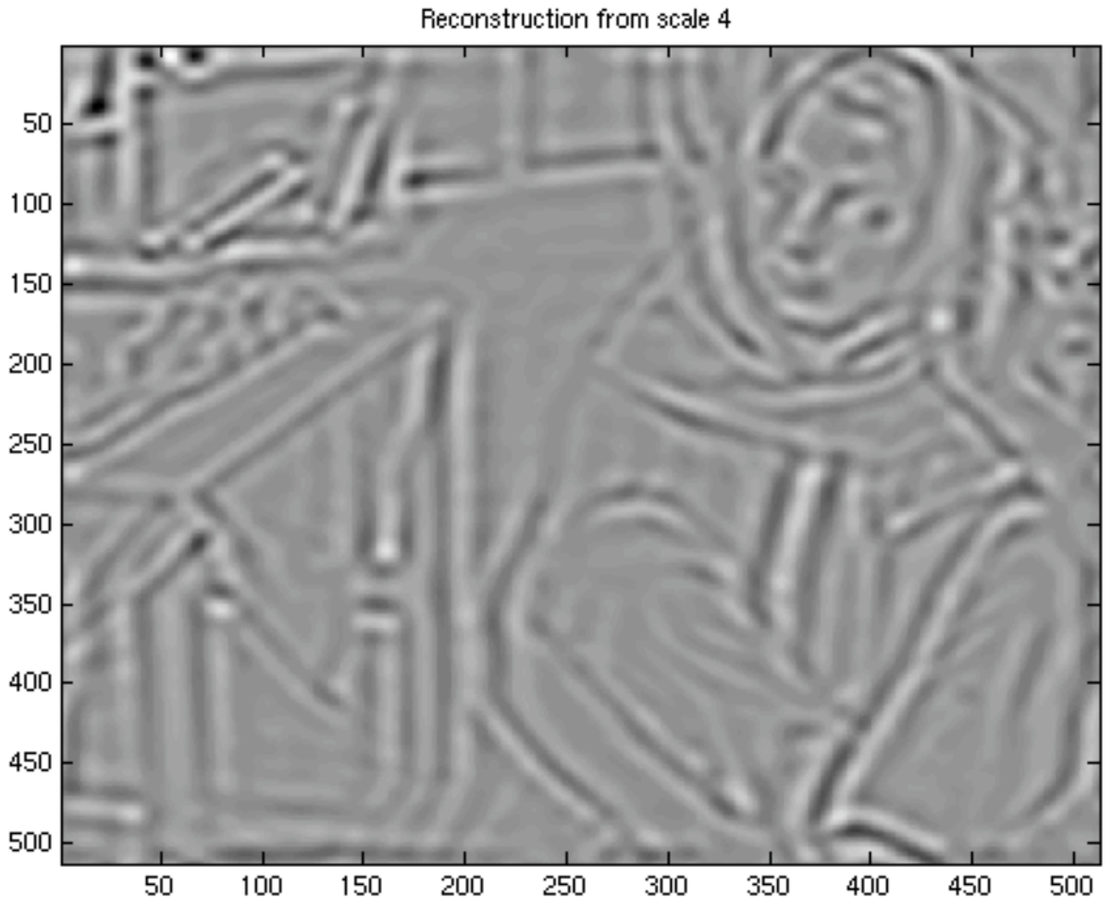
Surfacelet Decomposition/ Reconstruction

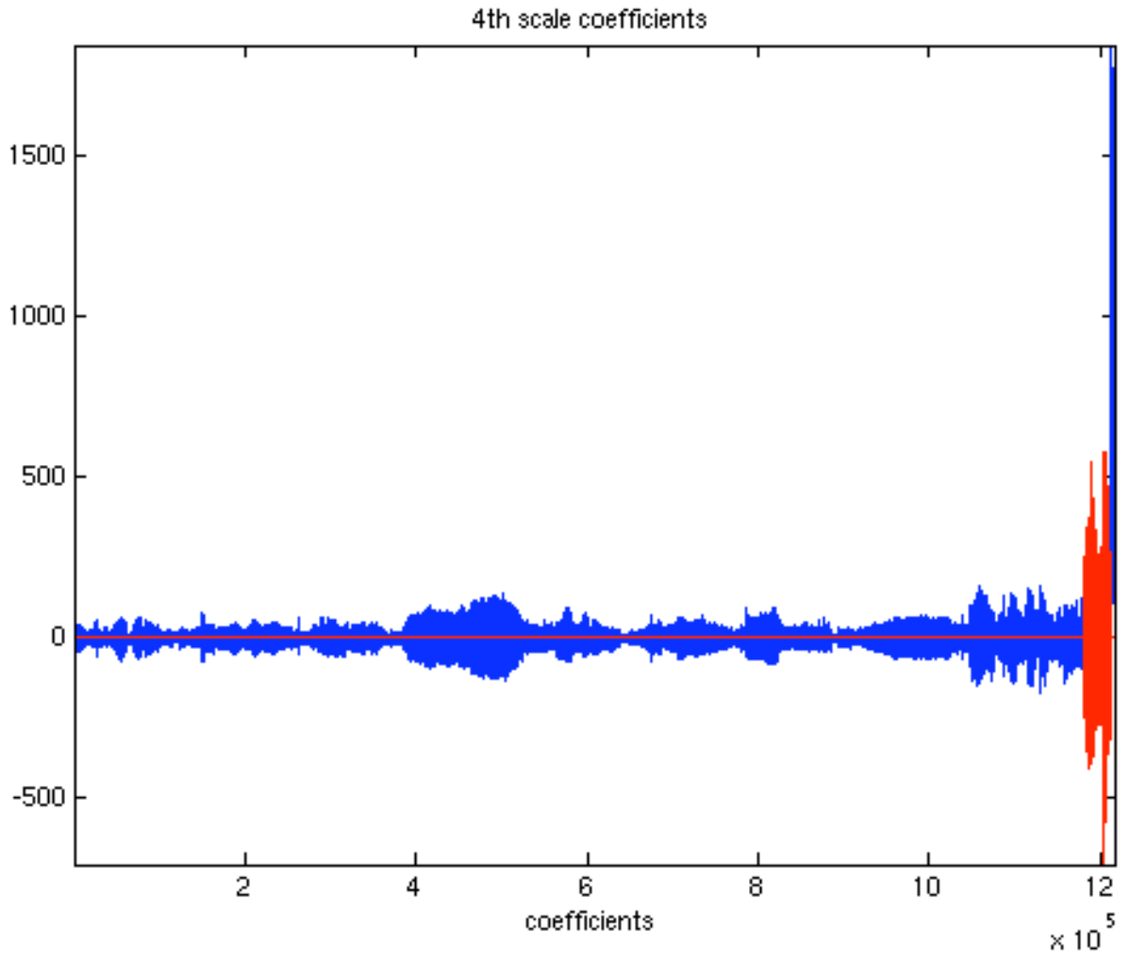




reconstruction from scale 5

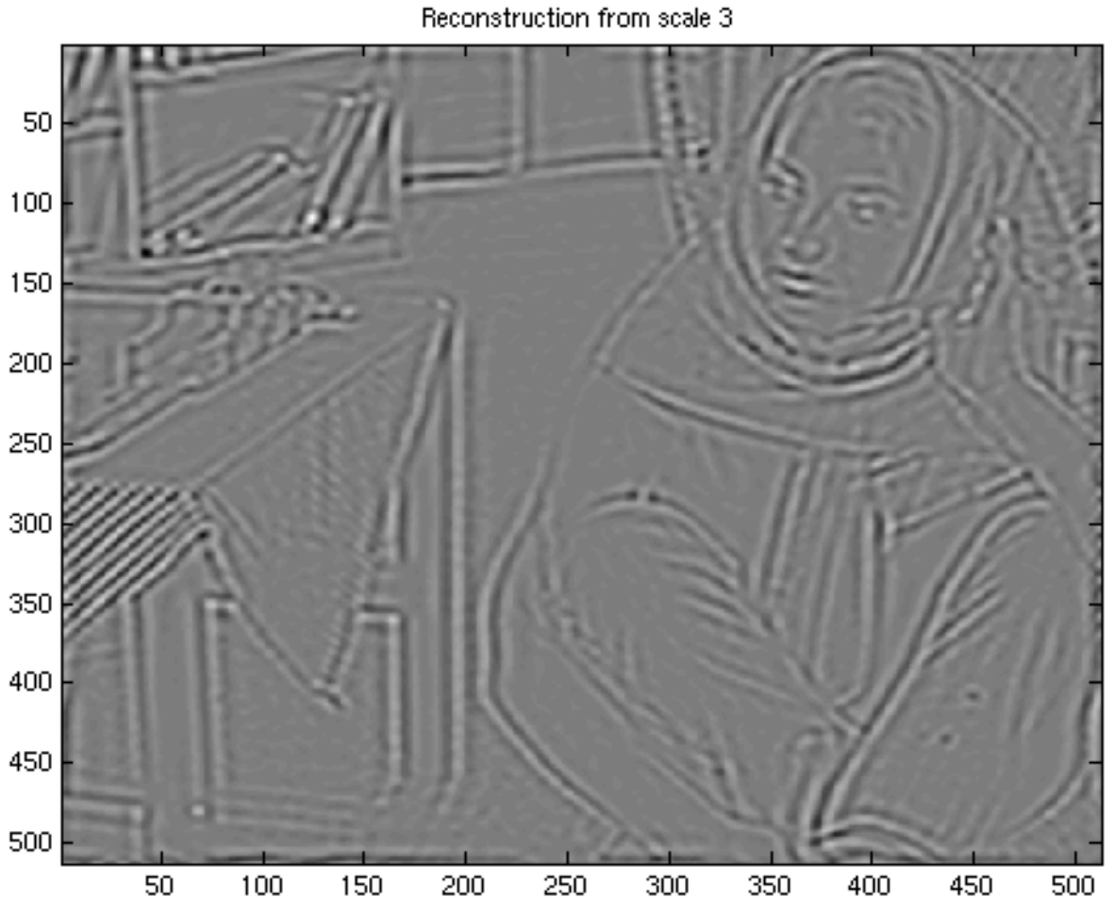
Surfacelet Decomposition/ Reconstruction

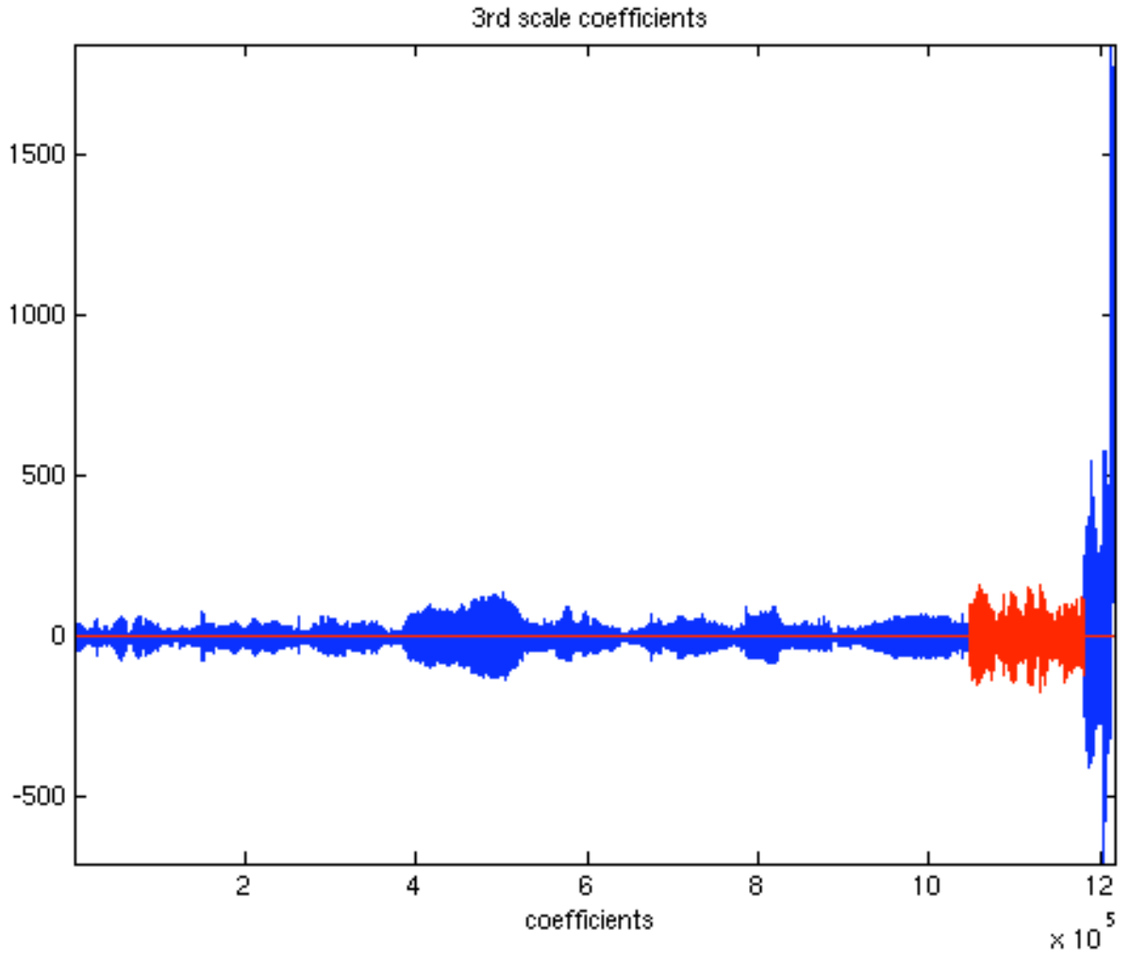




reconstruction from scale 4

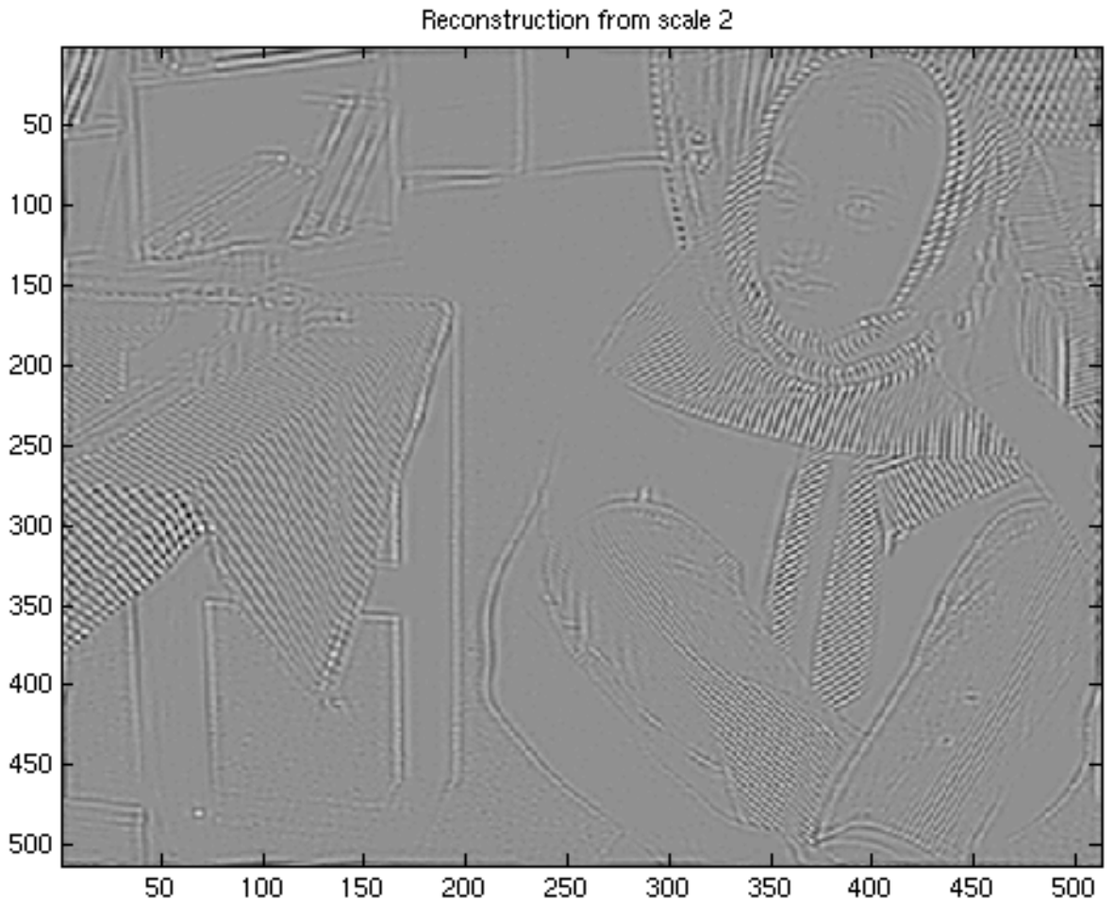
Surfacelet Decomposition/ Reconstruction

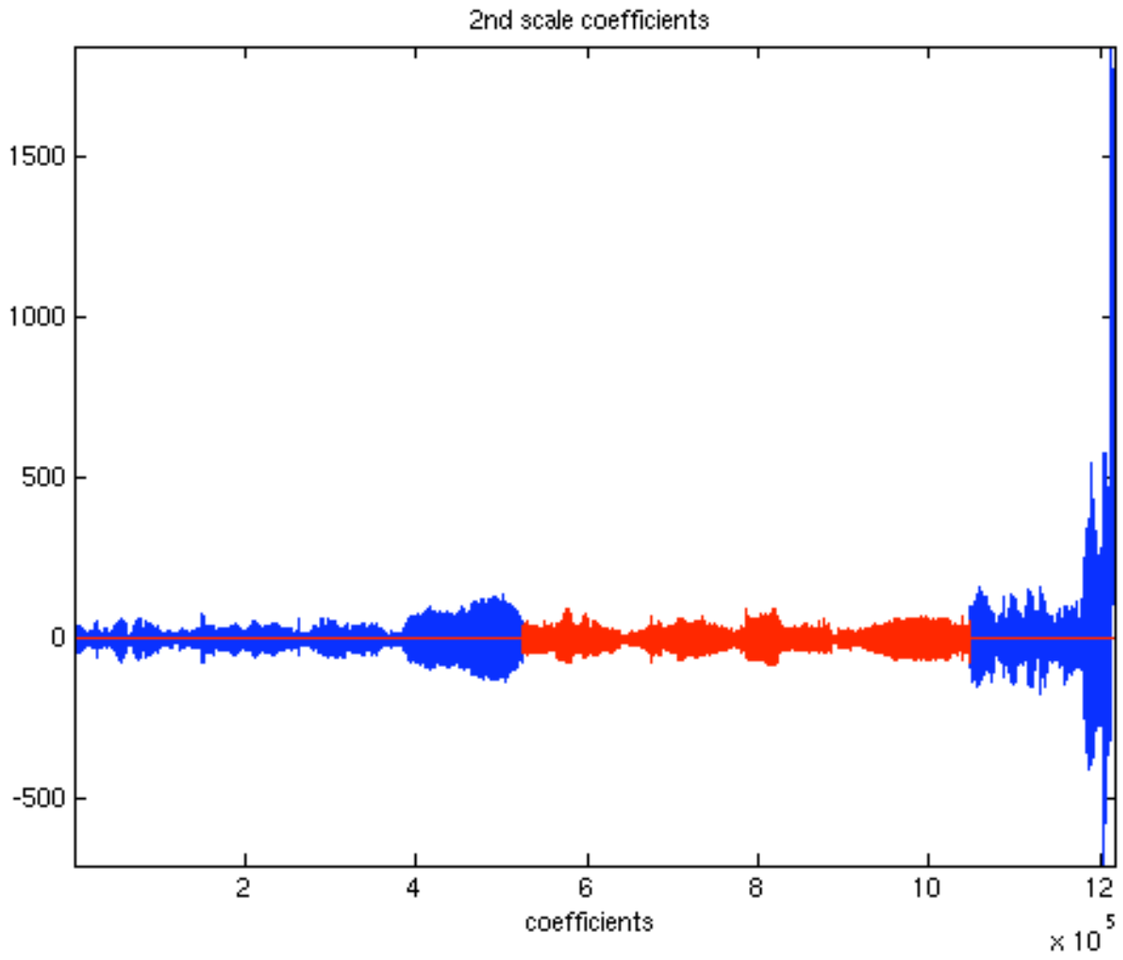




reconstruction from scale 3

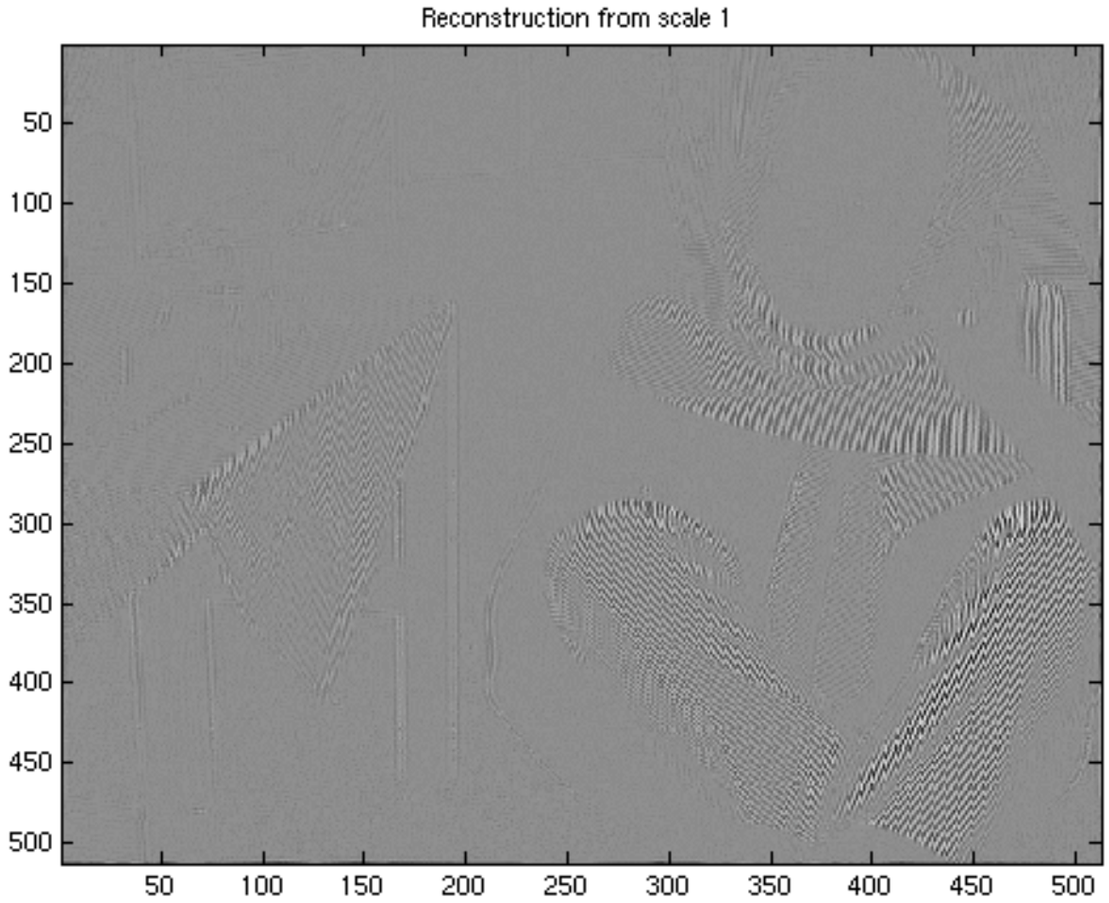
Surfacelet Decomposition/ Reconstruction

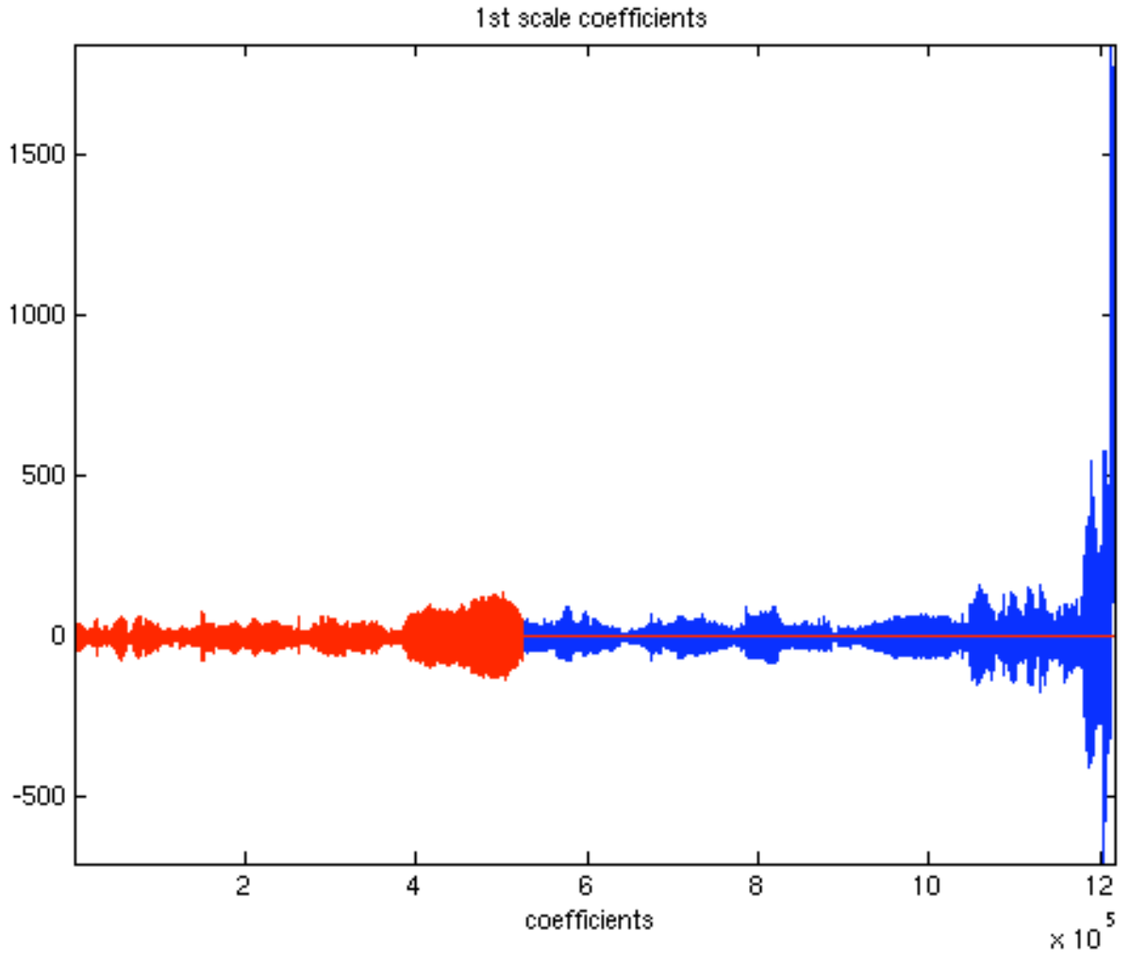




reconstruction from scale 2

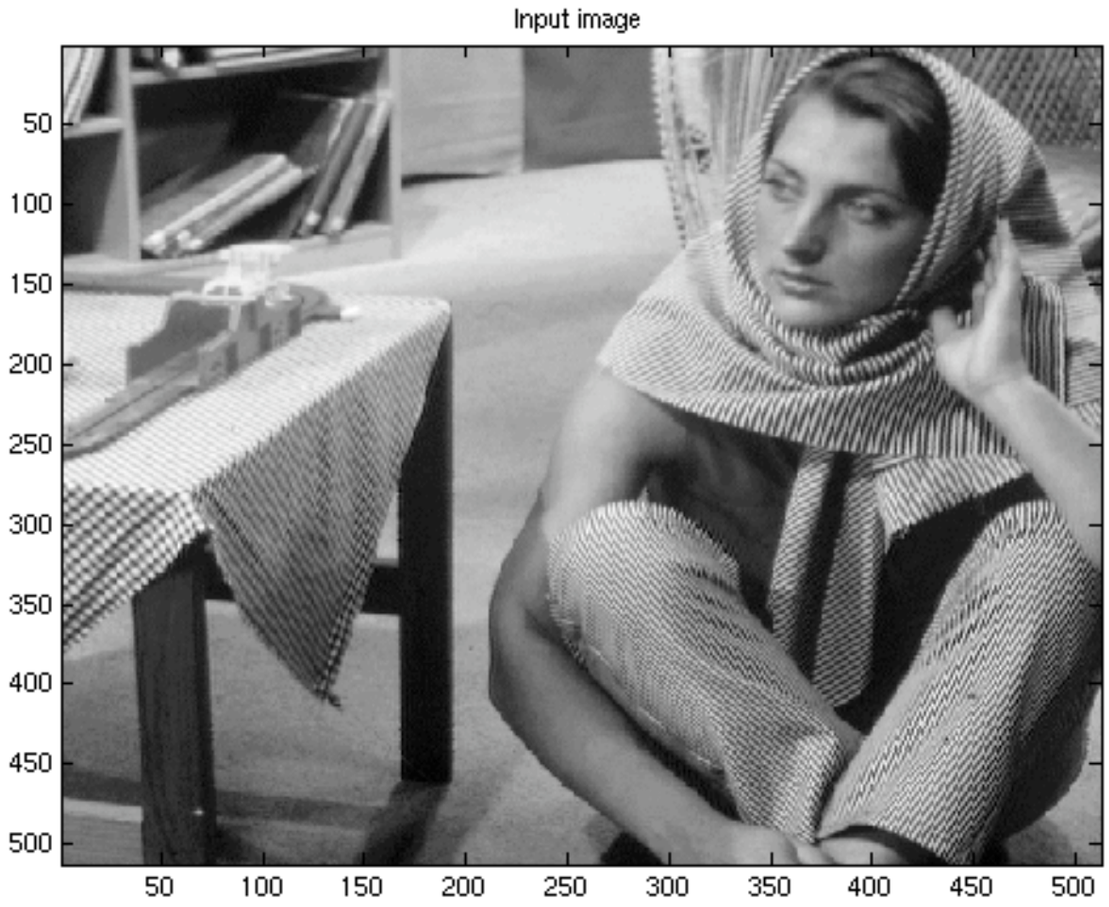
Surfacelet Decomposition/ Reconstruction

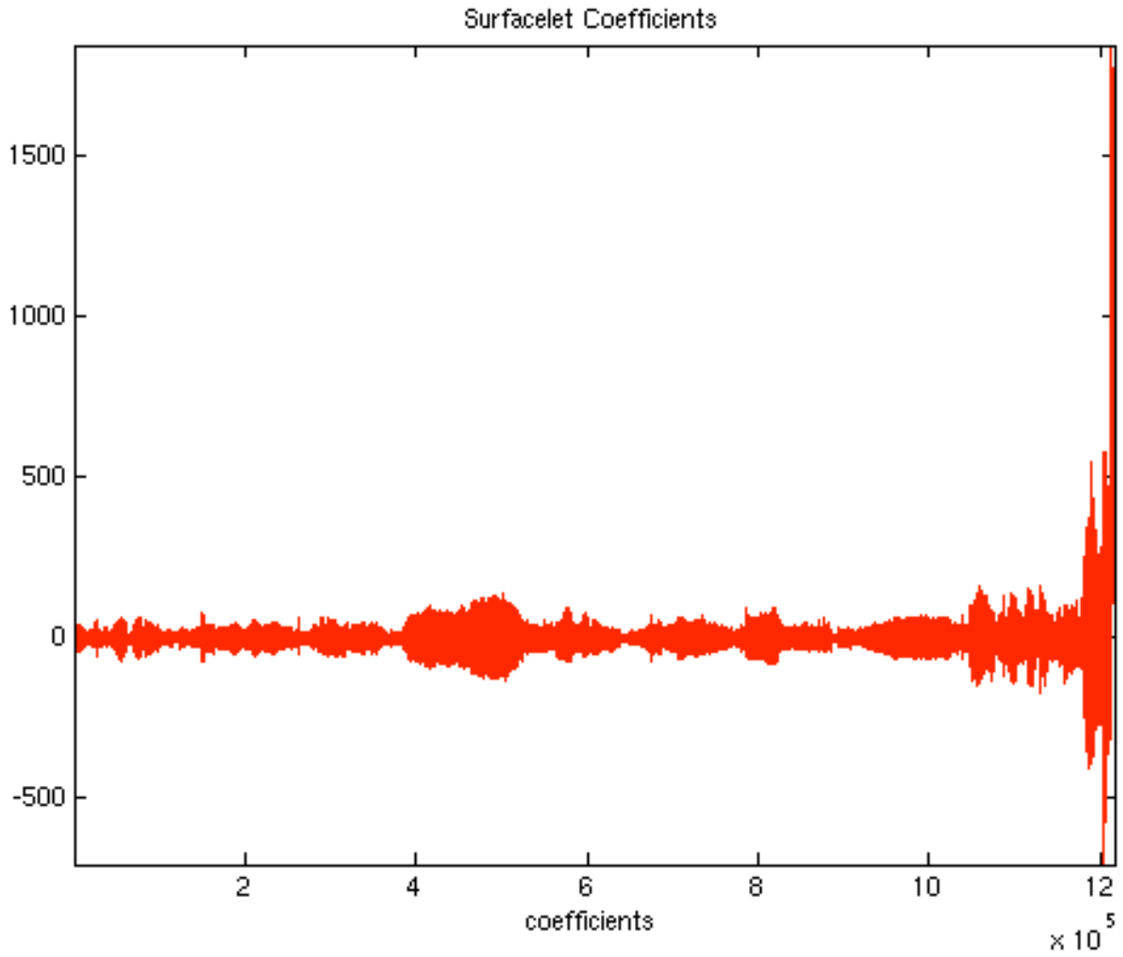




reconstruction from scale 1

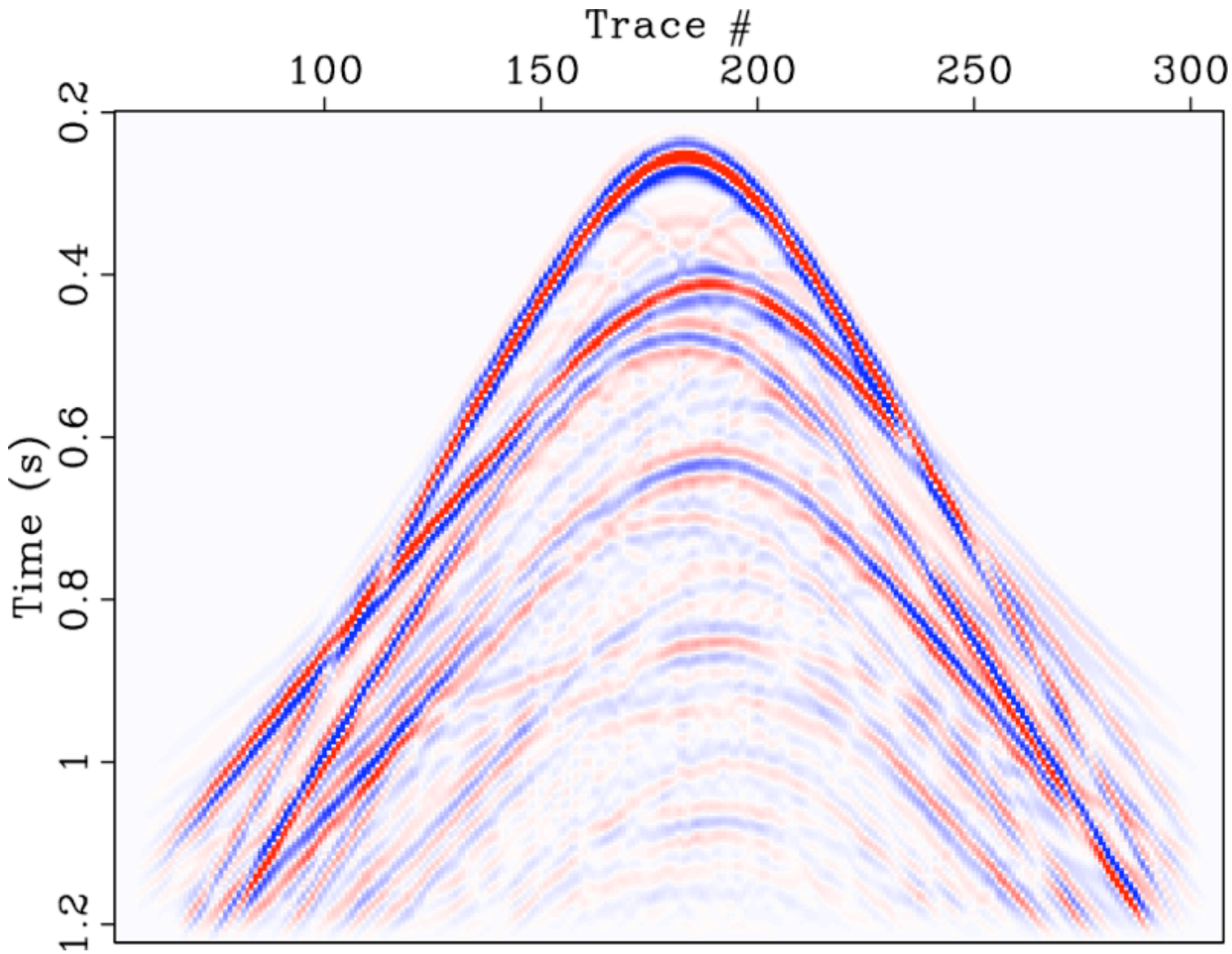
Surfacelet Decomposition/ Reconstruction





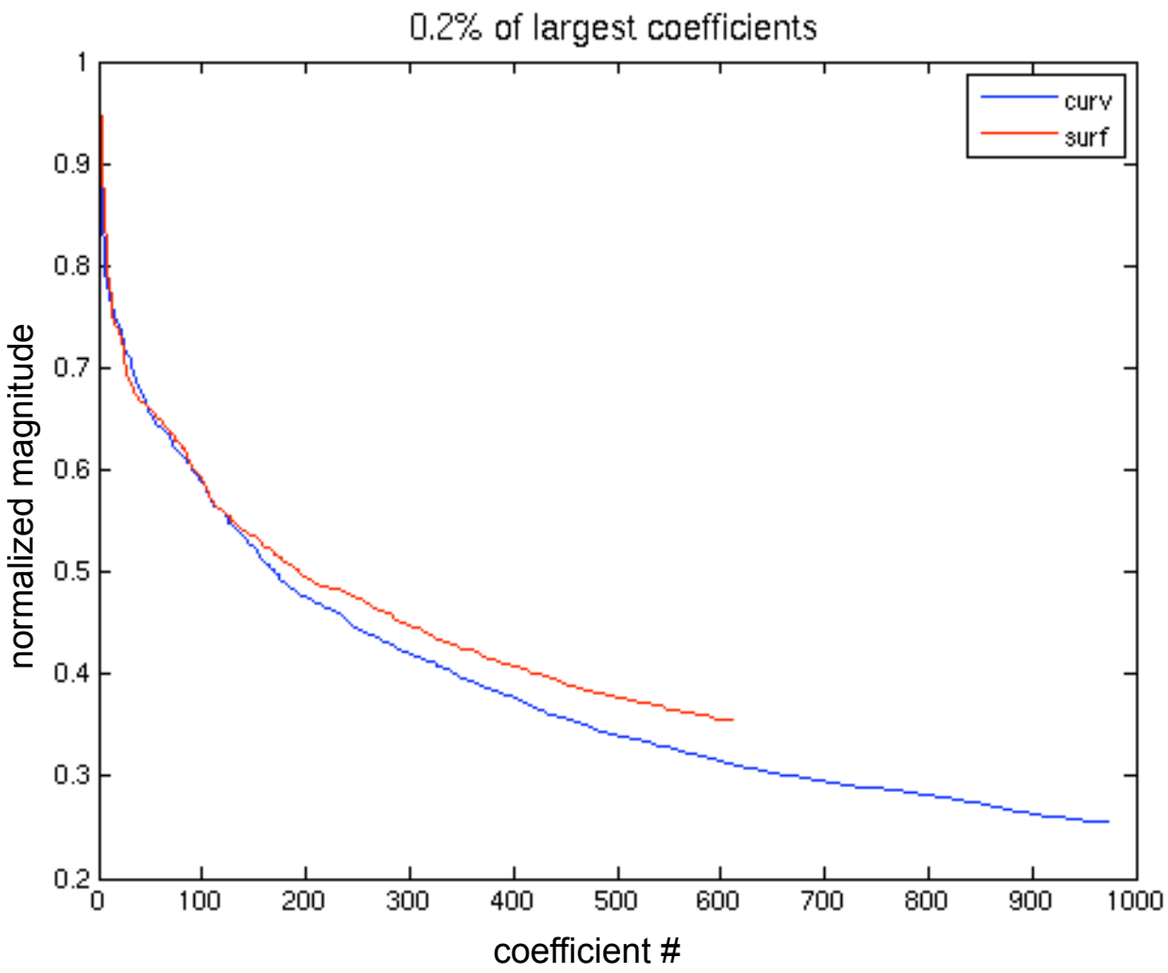
all coefficients = perfect reconstruction

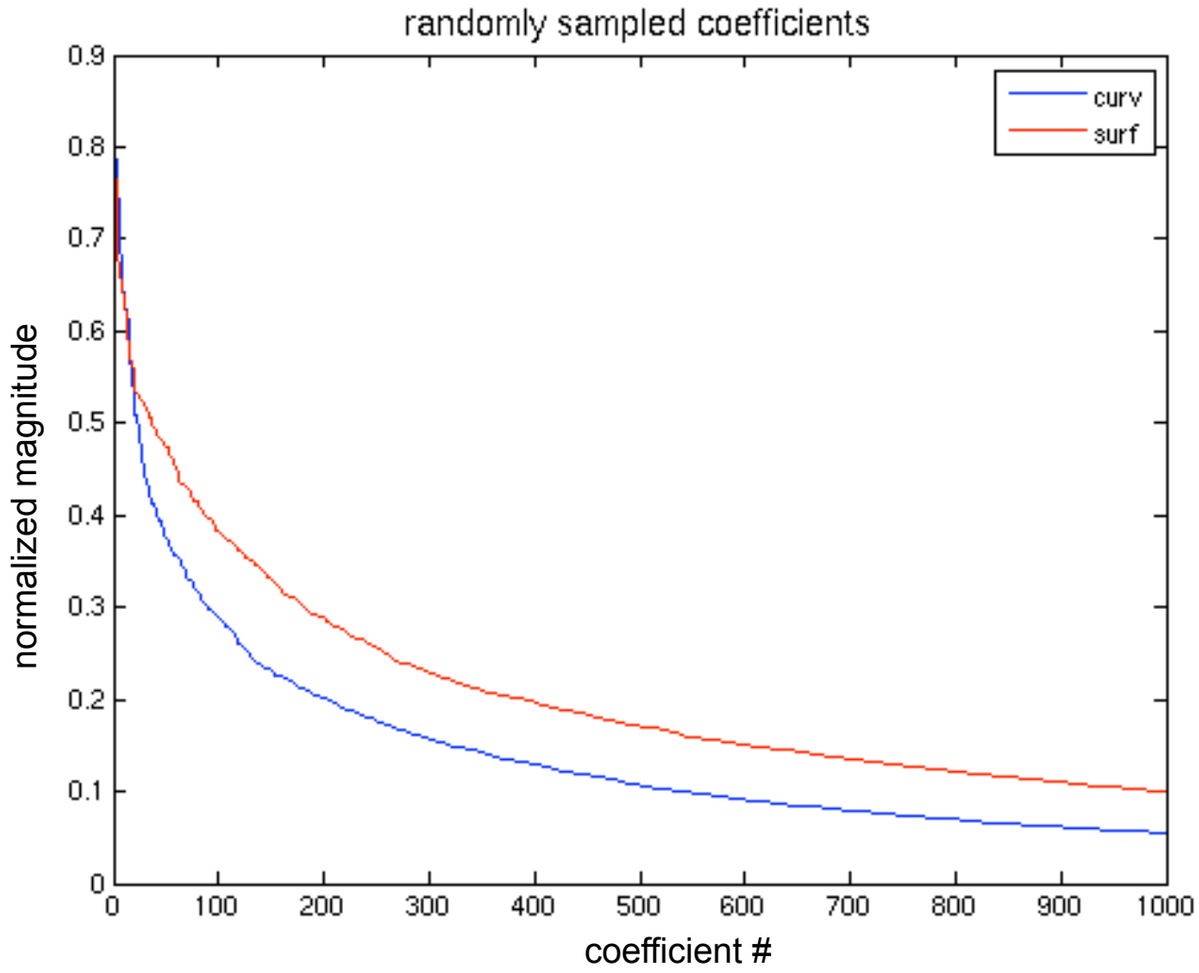
Relative Decay Rates



Relative Decay Rates

- Seismic data example





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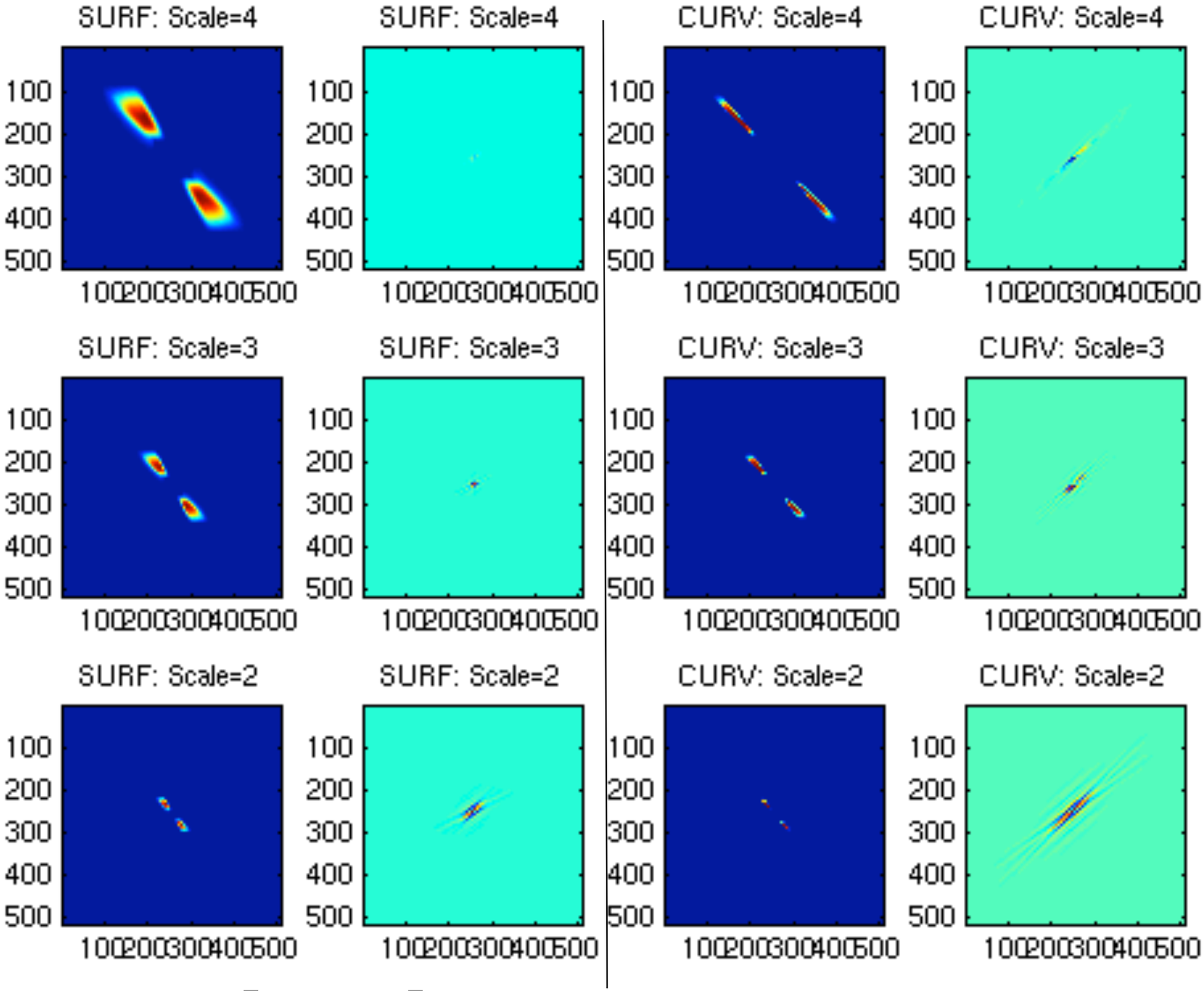
Fourier/Space domain comparison



Surfacelets

Curvelets

Fourier/Space domain comparison

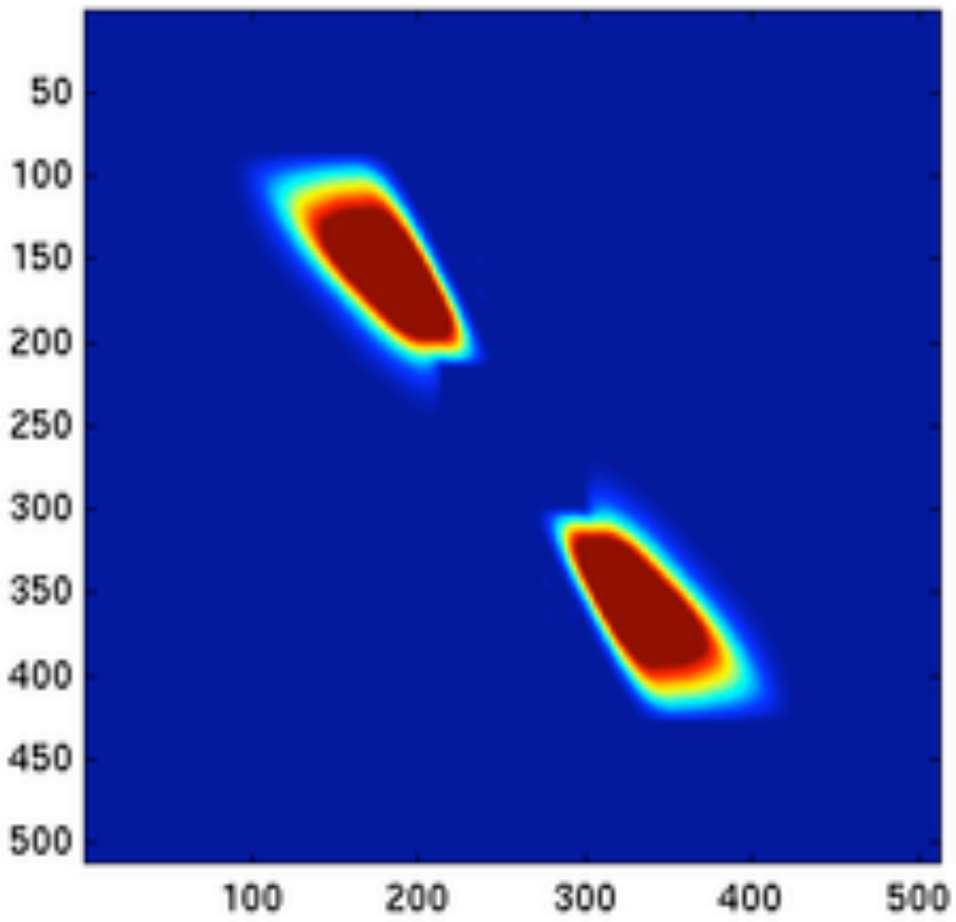
- Fourier domain, a closer look...

Surfacelet

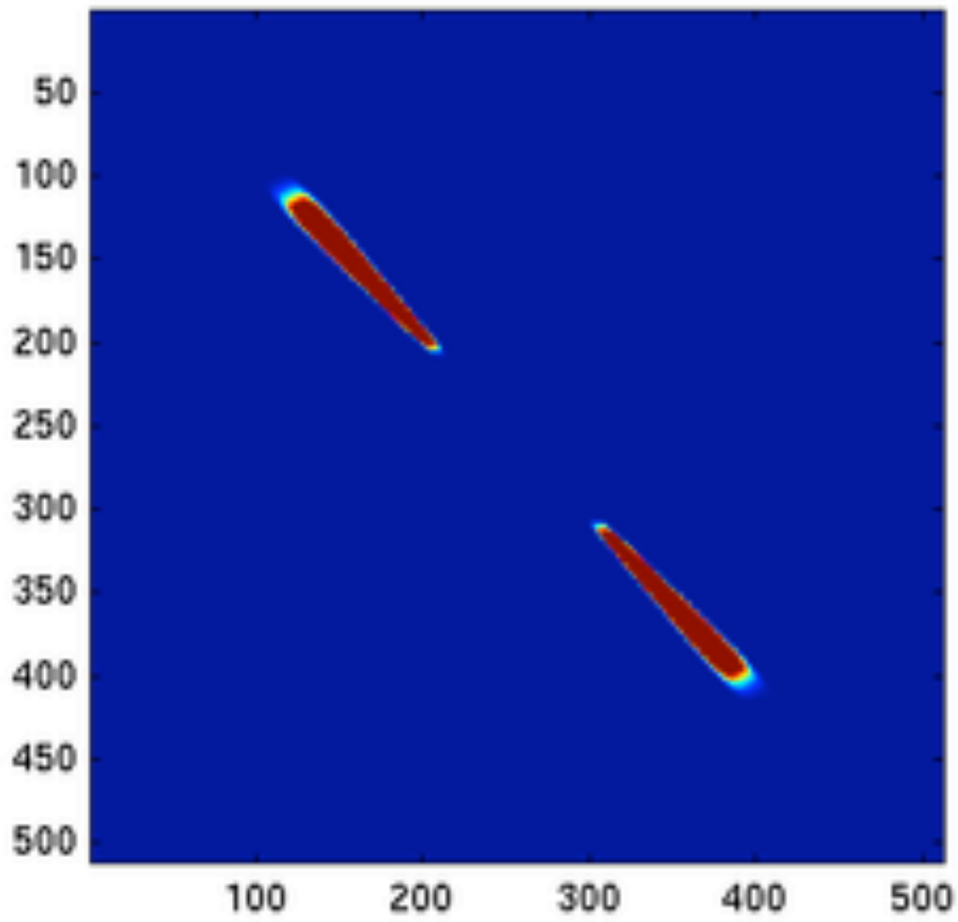
Curvelet

Fourier/Space domain comparison

- Fourier domain, a closer look...



Surfacelet



Curvelet

Fourier/Space domain comparison

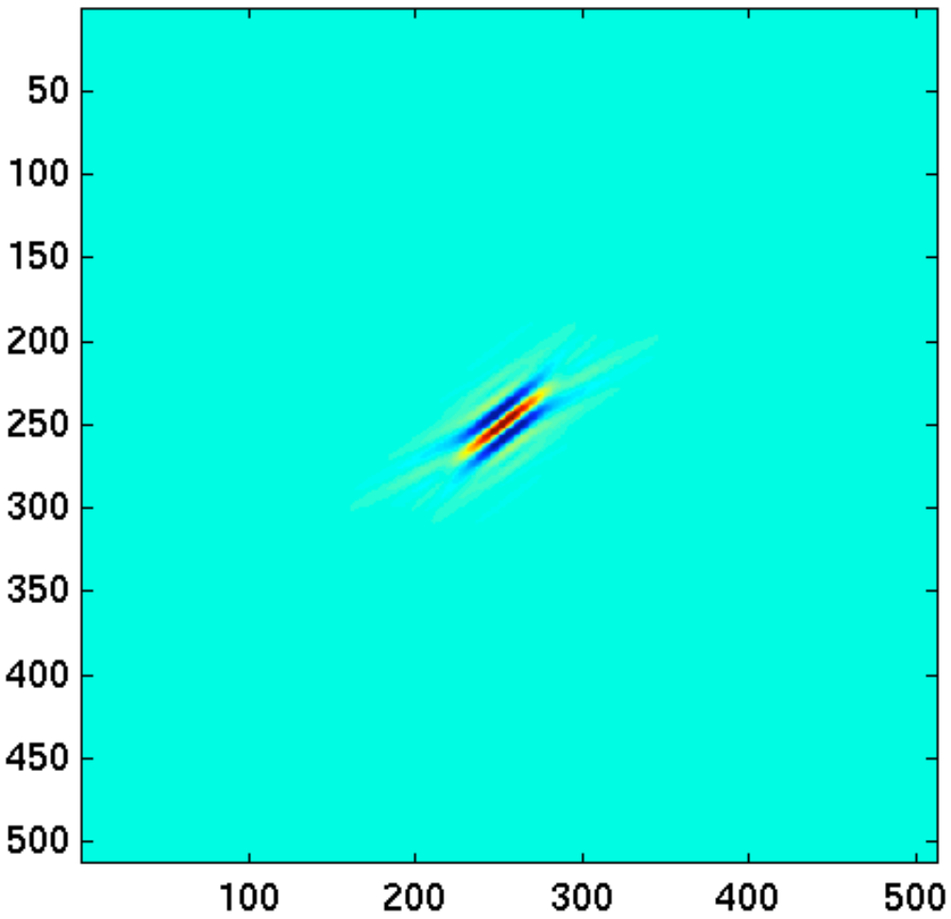
- Space domain, a closer look...

Surfacelet

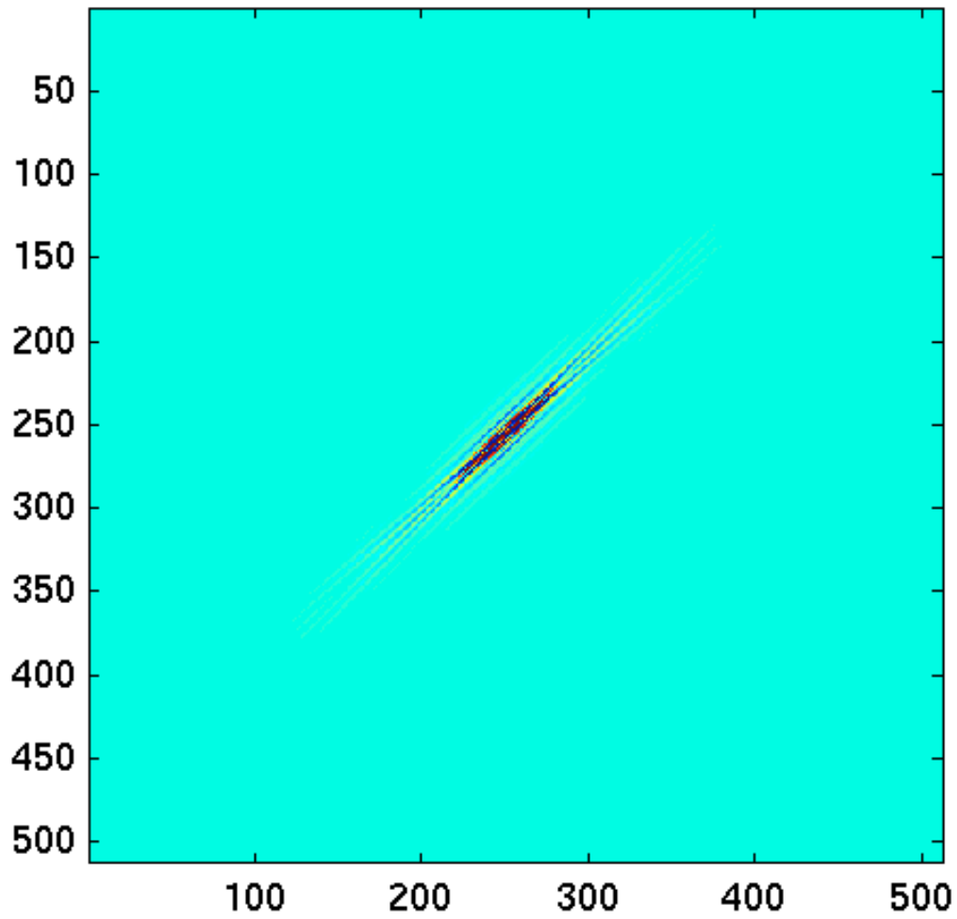
Curvelet

Fourier/Space domain comparison

- Space domain, a closer look...



Surfacelet



Curvelet

Comparison Summary

	Surfacelets		Curvelets	
Redundancy	2D:	~4	2D:	~8
	3D:	~6	3D:	~24
Spacial Localization	Slower spacial decay		Faster spacial decay	
Frequency Localization	Spread over an entire scale		Strict (localized to one wedge)	
Coefficient decay rate	Slower		Faster	

Acknowledgments

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