Curvelet-Regularized Deconvolution

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Motivation

- There is an inherent continuity along reflectors of a seismic image [Hennenfent et al., 2005]
- The wavefront like reflectivity (band-limited) is sparse in curvelet-domain [Hennenfent et al., 2005]
- Assumption of spiky reflectivity is too limited to describe seismic reflectivity [Herrmann and Bernabe, 2004] , [Herrmann, 2005]

Assumptions

- Reflectivity is sparse (coefficients has a rapid decay) in curvelet-domain.
- Source wavelet known.

Forward problem

$$\mathbf{y} = \mathbf{A}\mathbf{m} + \mathbf{n}$$

$\mathbf{y} \rightarrow$ Noisy Signal

$\mathbf{m} \rightarrow$ True signal

$\mathbf{n} \rightarrow$ Noise

$\mathbf{A} \rightarrow$ Convolution Operator

Inversion Problem

$$\min_{x} \quad \|\mathbf{x}\|_1$$

$$s.t \quad \left\|\mathrm{y} - \mathbf{A}\mathbf{C}^{\mathbf{T}}\mathbf{x}\right\|_2 \leq \varepsilon$$

$$\hat{\mathbf{m}} = \mathbf{C}^{\mathbf{T}}\,\hat{\mathbf{x}}$$

$\mathbf{C}^{\mathbf{T}} \rightarrow$ Curvelet Synthesis Operator

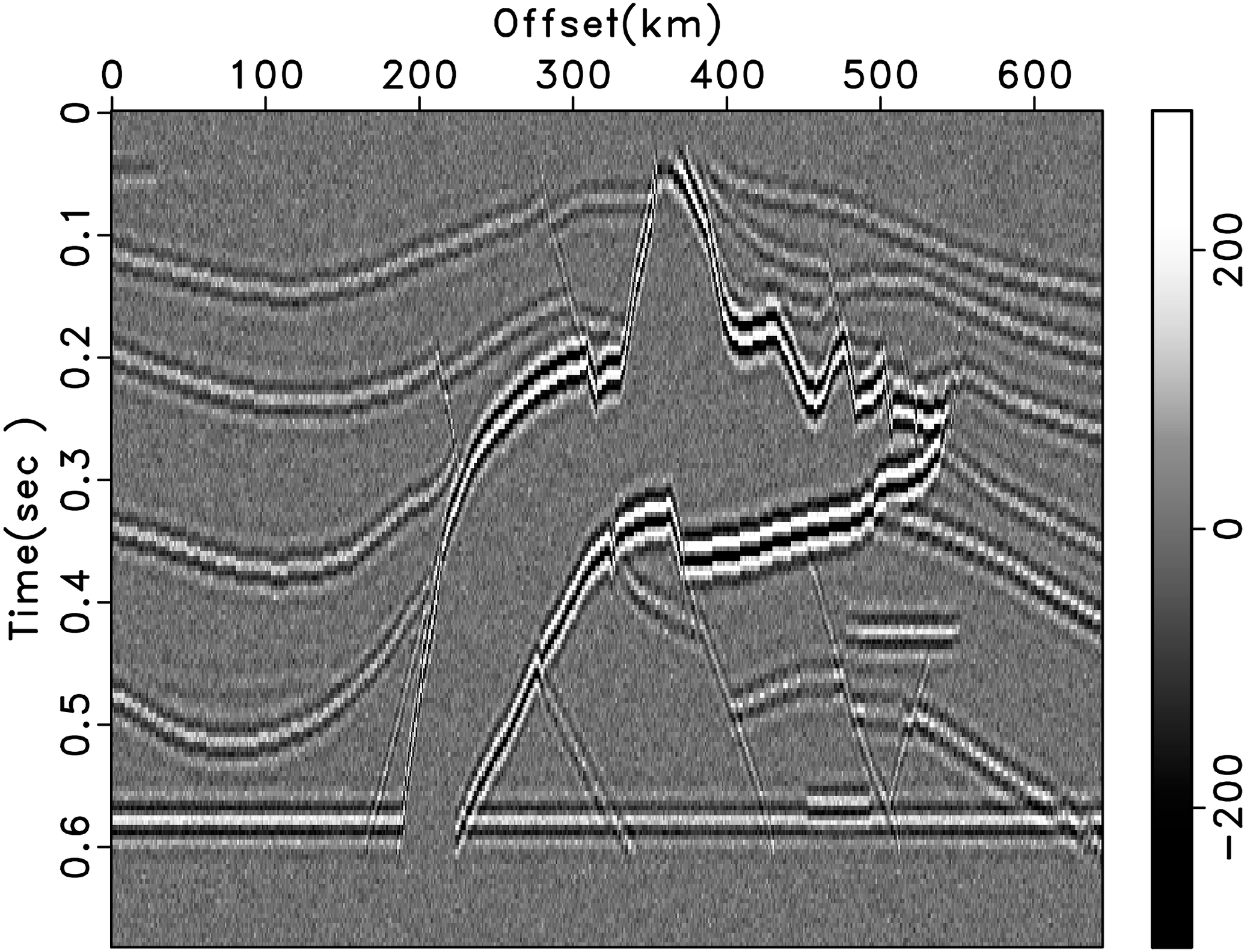
$\mathbf{A} \rightarrow$ Convolution Operator

$\epsilon^2 = \sigma^2[N + 2\sqrt{2N}]$ (Chi-square misfit)

Approach

- SEG-AA’ reflectivity model is half differentiated in the frequency domain to obtain non-spiky reflectivity.
- Reflectivity is convolved with Ricker wavelet (central frequency ~30 Hz) and noise is added.
- Operators are formed in SPARCO [Van den Berg et. al]
- The solution is found by SPG/$_1$ [Van den Berg and Friedlander]
- The noise level was estimated by Chi-square misfit criteria.

Now Results!


Offset(km)
0
100
200
300
400
500
600
Time(sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
200
0
−200

Data

SNR~8db

Offset(km)

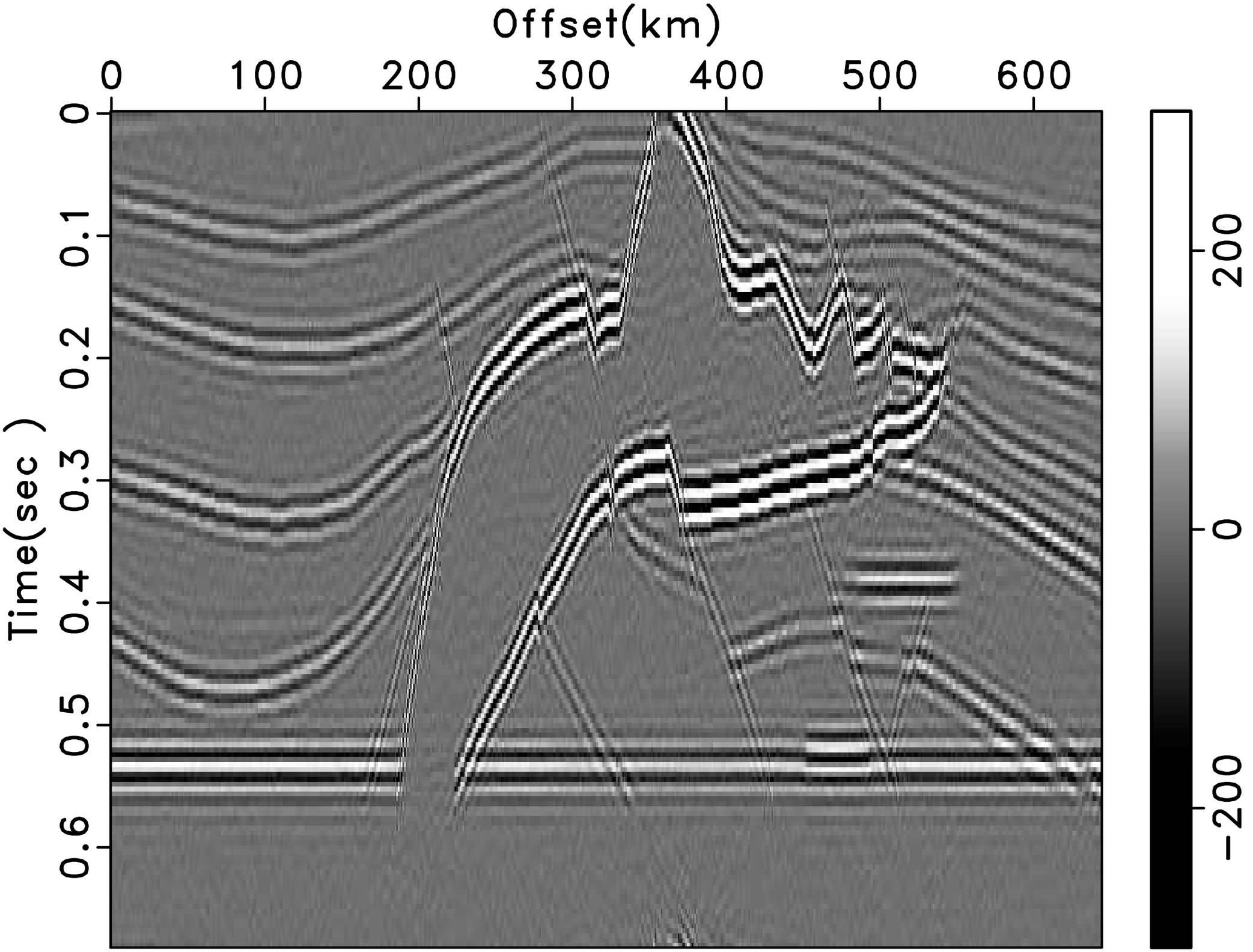
0 100 200 300 400 500 600

Time(sec)

0 0.1 0.2 0.3 0.4 0.5 0.6

200 0 −200

Original Reflectivity



Estimated reflectivity− Curvelets

ε = 9.9332e+03

Offset(km)

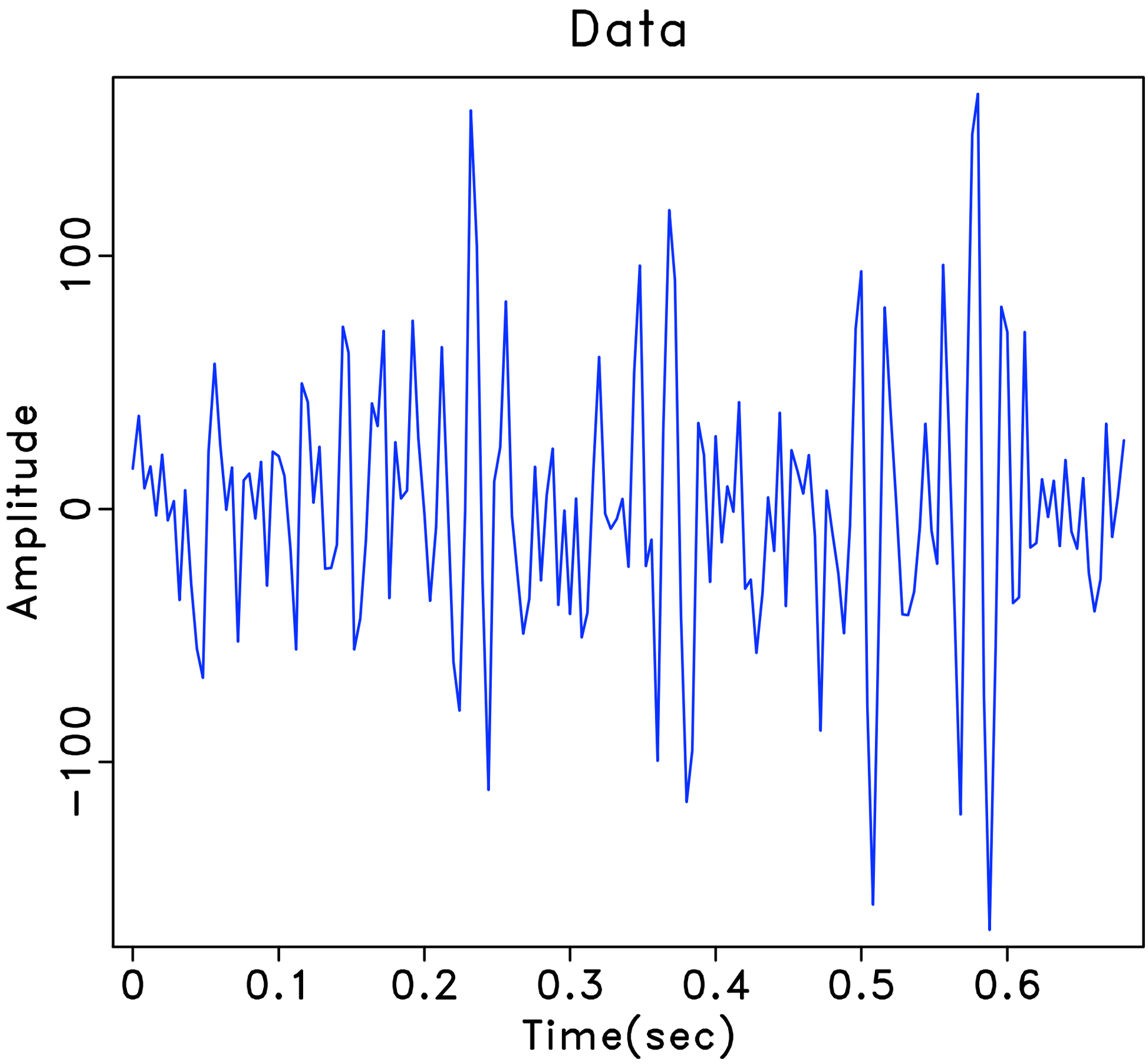
0 100 200 300 400 500 600

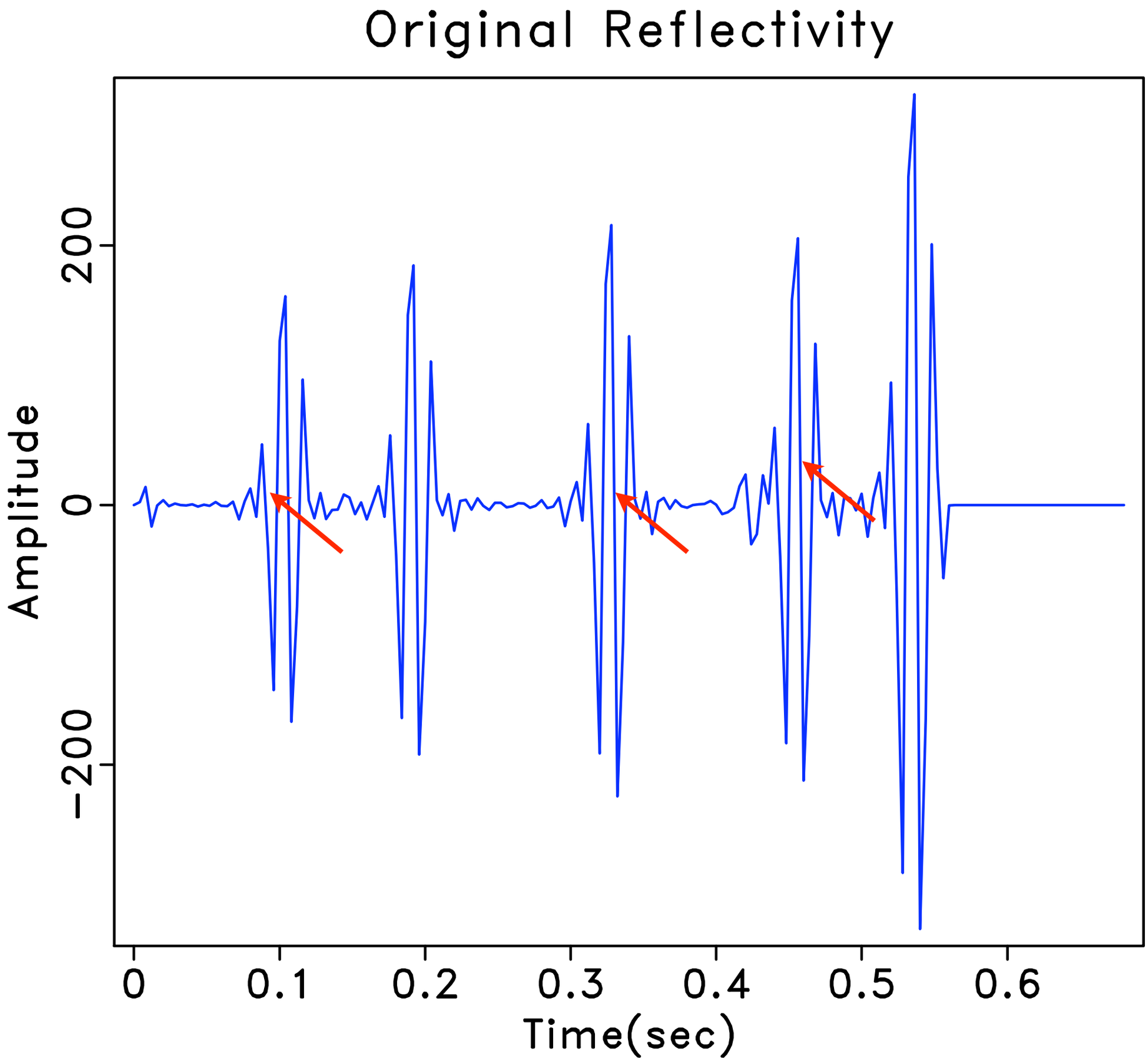
Time(sec)

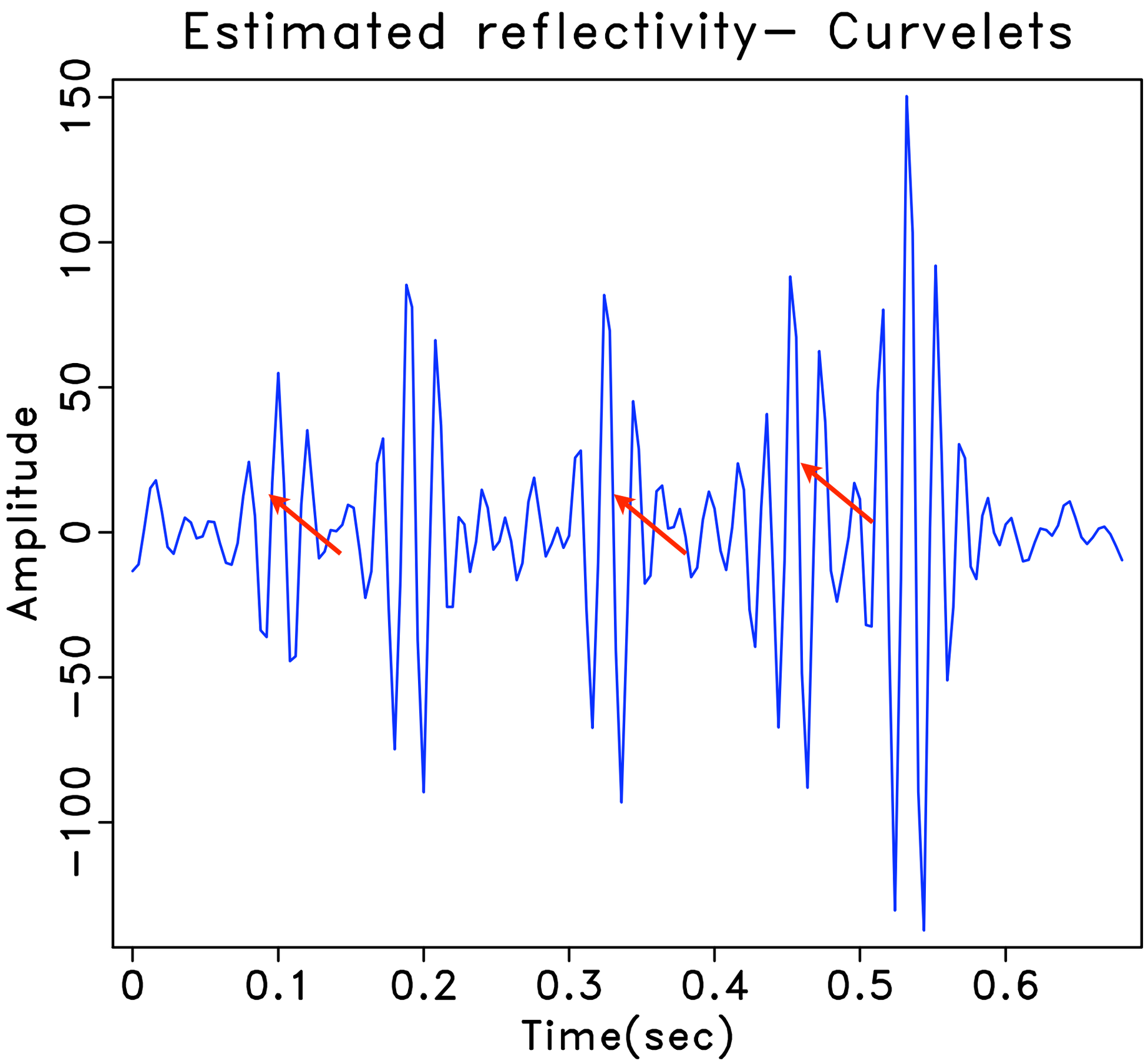
0 0.1 0.2 0.3 0.4 0.5 0.6

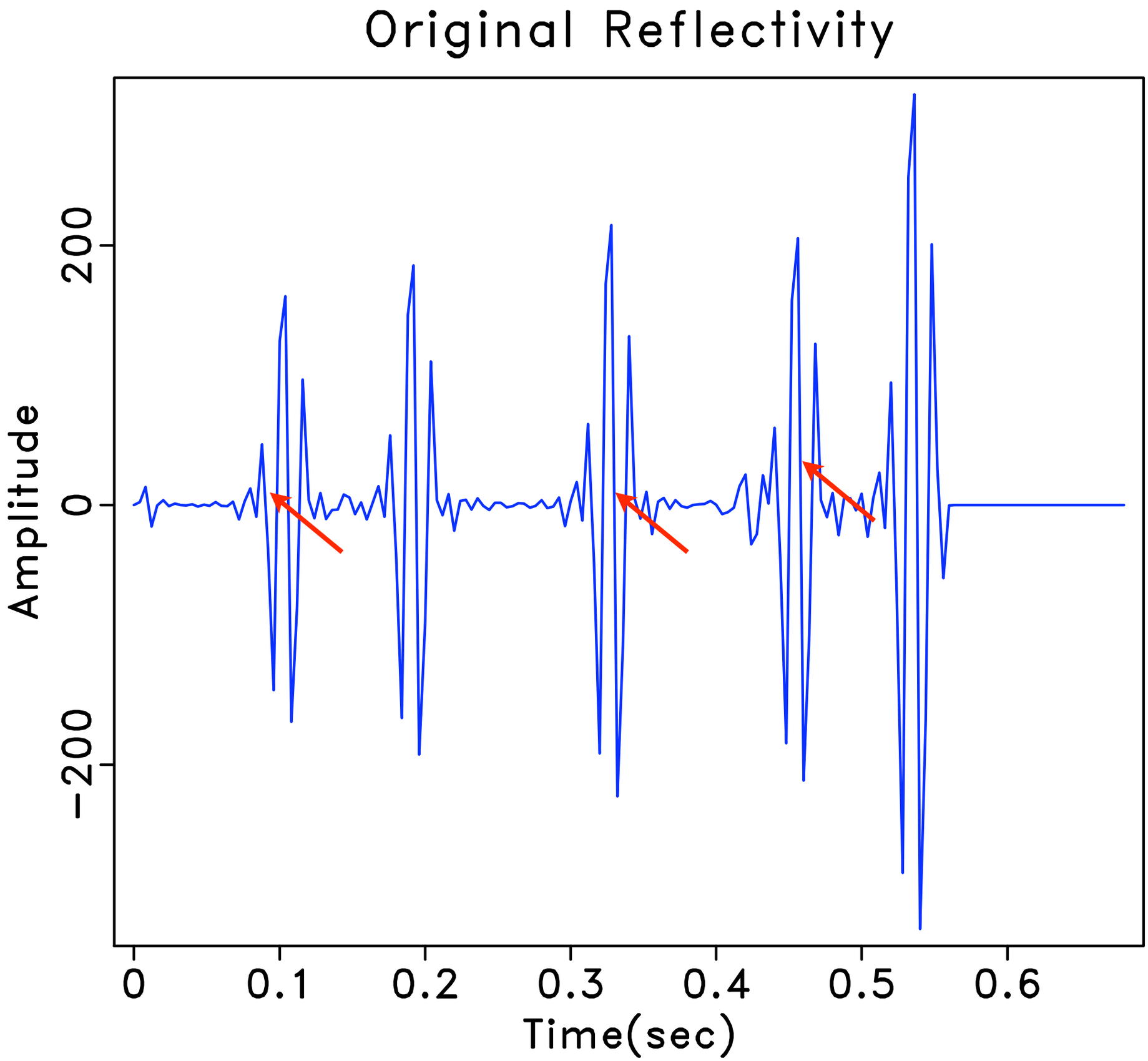
200 0 −200

Estimated reflectivity−Spiky Deconvolution

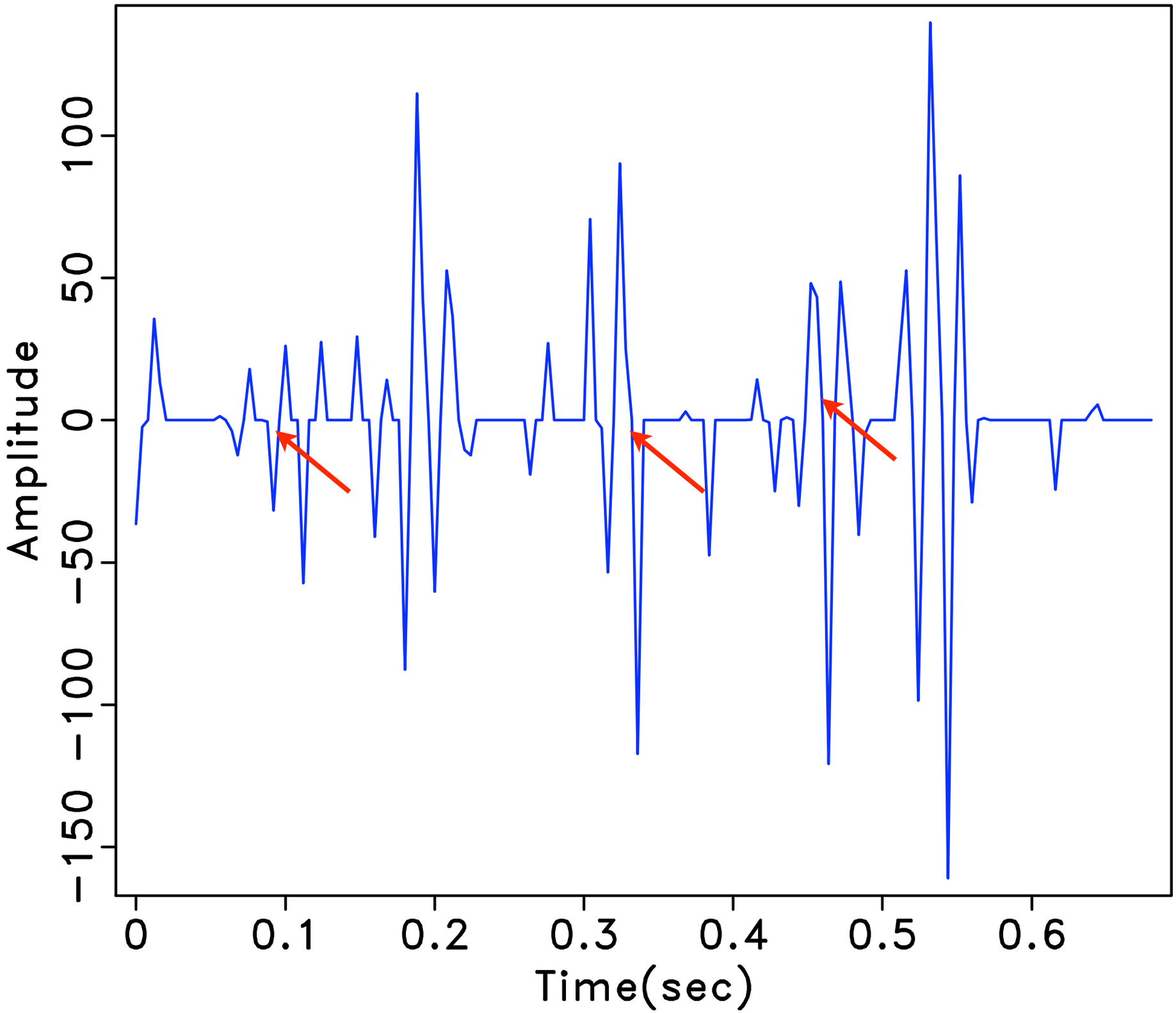

Data
Amplitude
100
0
−100
0
0.1
0.2
0.3
0.4
0.5
0.6
Time(sec)


Original Reflectivity
Amplitude
200
0
−200
0
0.1
0.2
0.3
0.4
0.5
0.6
Time(sec)


Estimated reflectivity– Curvelets
Amplitude
150
100
50
0
−50
−100
0
0.1
0.2
0.3
0.4
0.5
0.6
Time(sec)


Original Reflectivity
Amplitude
200
0
−200
0
0.1
0.2
0.3
0.4
0.5
0.6
Time(sec)

Estimated reflectivity−Spiky Deconvolution



Deconvolution Algorithm parameters

ε =9.9332e+03

Curvelet:
Products with A: 22
Products with A': 21

Spiky:
Products with A: 15
Products with A': 14

Conclusion

- We presented a method to deal with Non- spiky reflectivity.
- 2D Deconvolution as opposed to 1D Deconvolution.
- The method uses the multi-dimensional structure of earth model (reflectivity) as opposed to trace by trace deconvolution.

Future work

- The method can be extended to three dimensions to estimate 3D reflectivity which has more structure.
- Try solving Blind Deconvolution problem and wavelet estimation for non spiky reflectivity.

Acknowledgments

- S. Fomel, P. Sava, and the other developers of Madagascar (rsf.sourceforge.net)
- E. van den berg and M. P. Friedlander for SPG/$_1$ (www.cs.ubc.ca/labs/scl/spgl1)
- E. van den Berg, M. P. Friedlander, G. Hennenfent, F. J. Herrmann, R. Saab, and O. Yilmaz for Sparco (www.cs.ubc.ca/labs/scl/sparco/)

This presentation was carried out as part of the SINBAD project with financial support, secured through ITF, from the following organizations: BG, BP, Chevron, ExxonMobil, and Shell. SINBAD is part of the collaborative research & development (CRD) grant number 334810-05 funded by the Natural Science and Engineering Research Council (NSERC).

Thank You!

(Merci Beaucoup)

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