Curvelet Denoising

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Forward Problem

$$\mathbf{y} = \mathbf{m} + \mathbf{n}$$

$\mathbf{y} \rightarrow$ Noisy Signal

$\mathbf{m} \rightarrow$ True signal

$\mathbf{n} \rightarrow$ Noise

Denoising as an Optimization problem

$$\min_{x} \quad \|\mathbf{x}\|_1$$

$$s.t \quad \|\mathbf{y} - \mathbf{C}^{\mathbf{T}}\mathbf{x}\|_2 \leq \varepsilon$$

$$\hat{\mathbf{m}} = \mathbf{C}^{\mathbf{T}}\hat{\mathbf{x}}$$

$\mathbf{C}^{\mathbf{T}} \rightarrow$ Curvelet Synthesis Operator

$\epsilon^2 = \sigma^2[N + 2\sqrt{2N}]$ (Chi-square misfit)

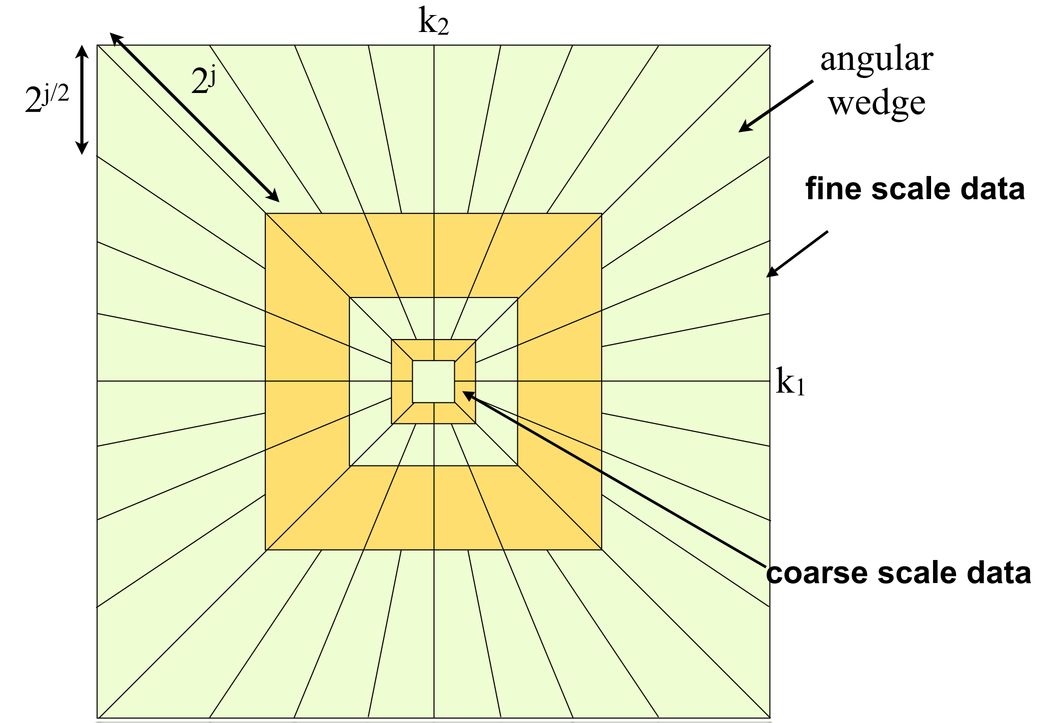
The Curvelet Transform

Representations for seismic data

Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Properties curvelet transform:

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy (8 X in 2-D and 24 X in 3-D)**



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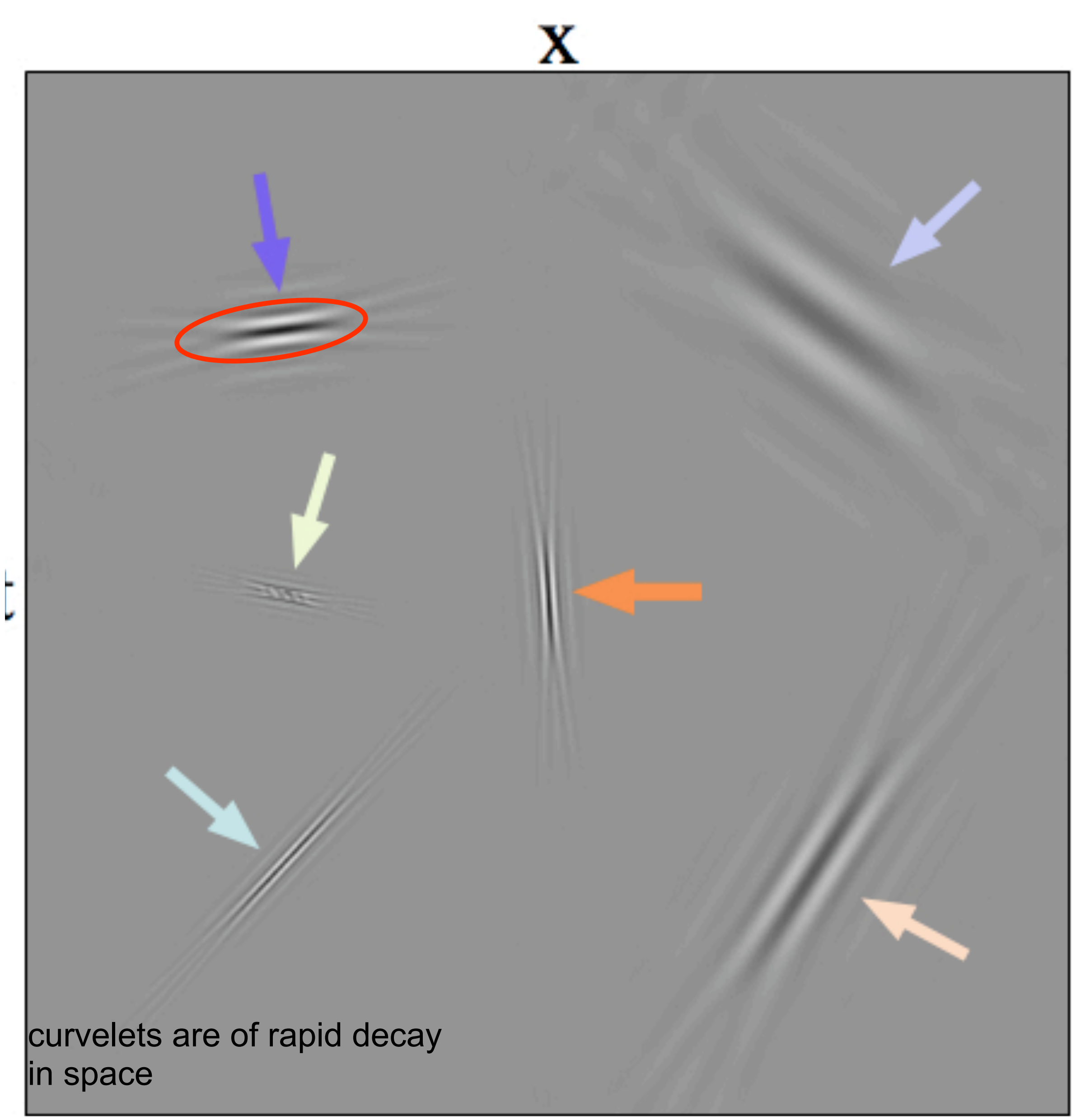
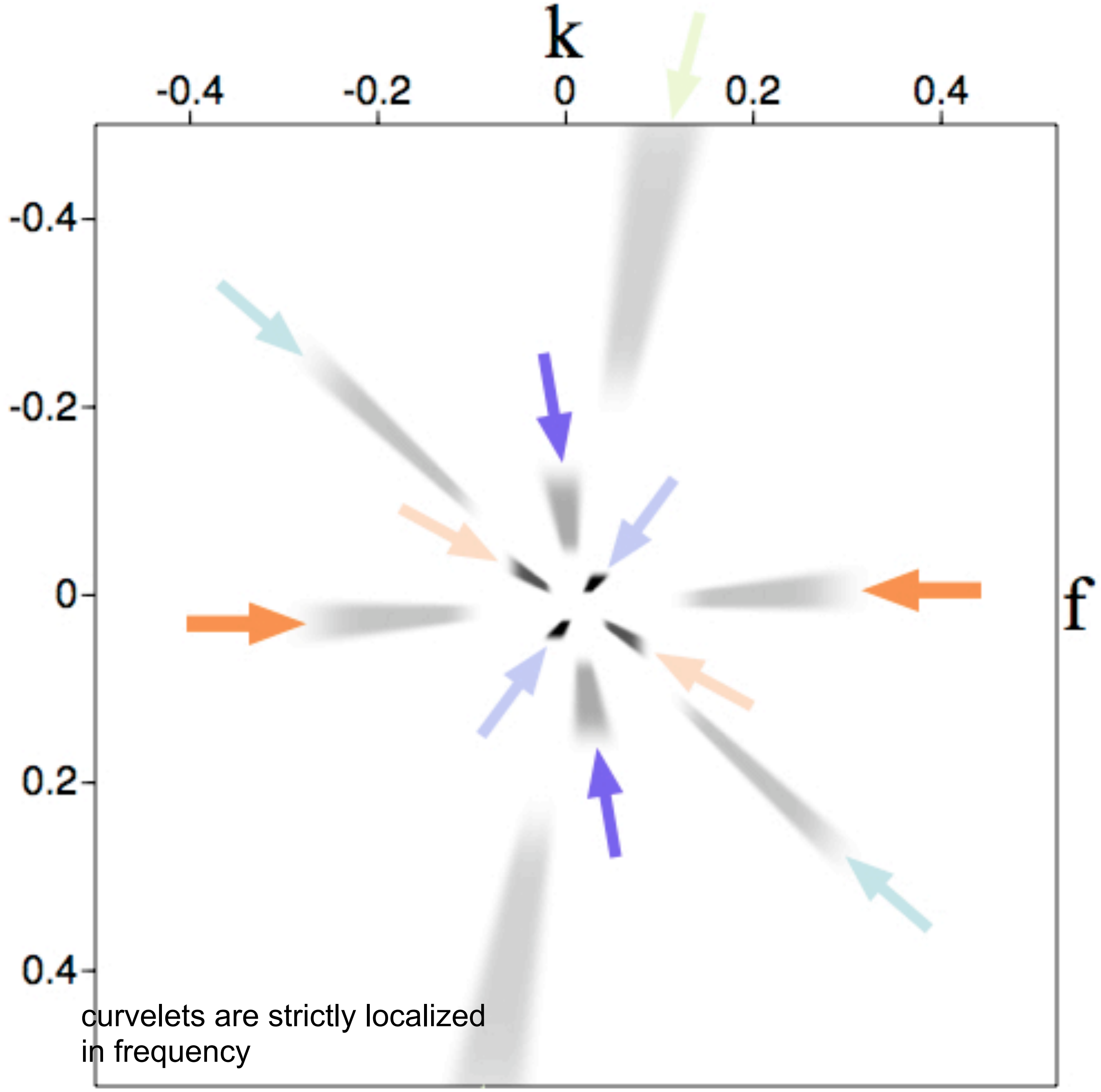
2-D curvelets



x-t

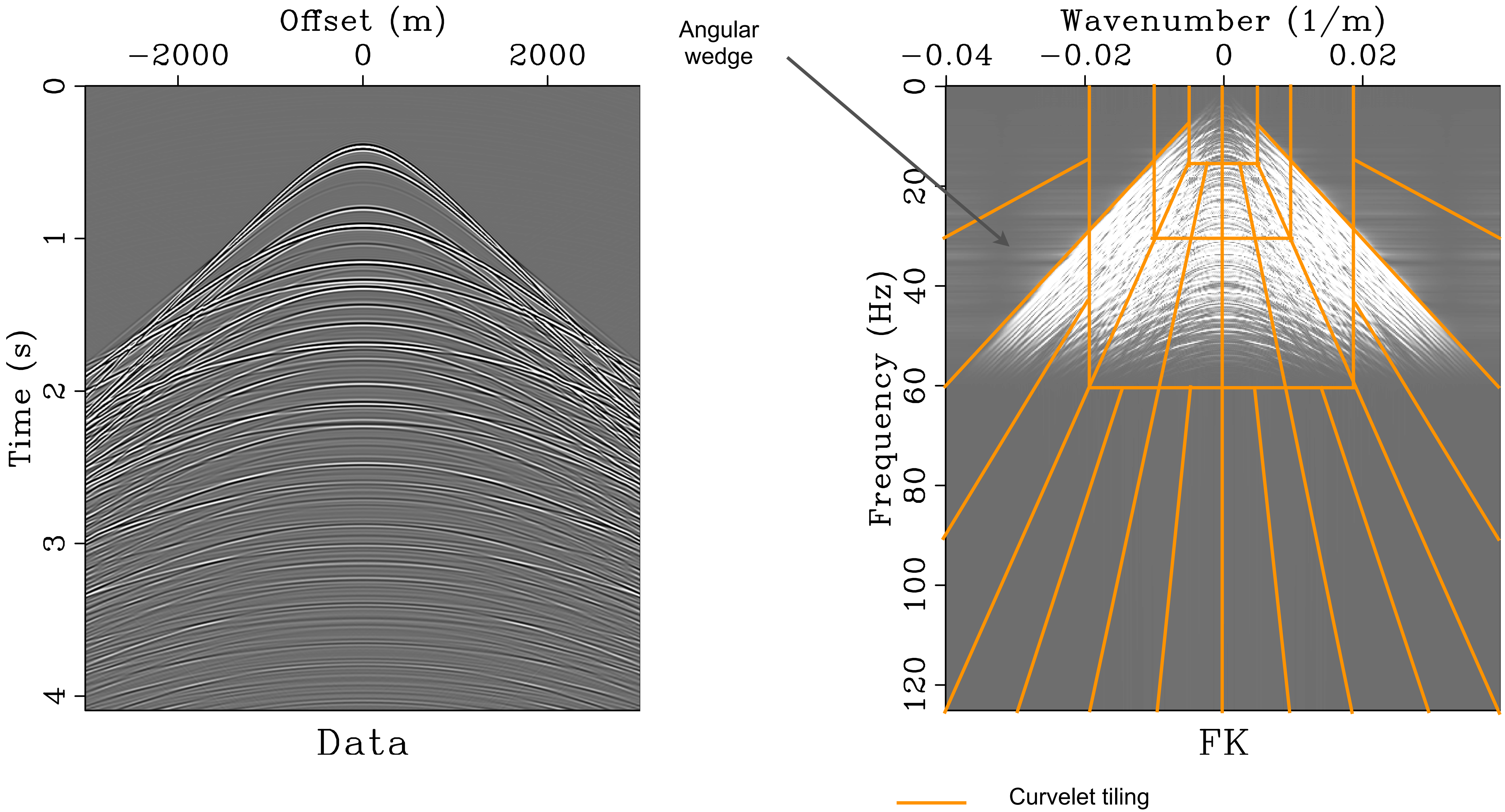


f-k

Oscillatory in one direction and smooth in the others!

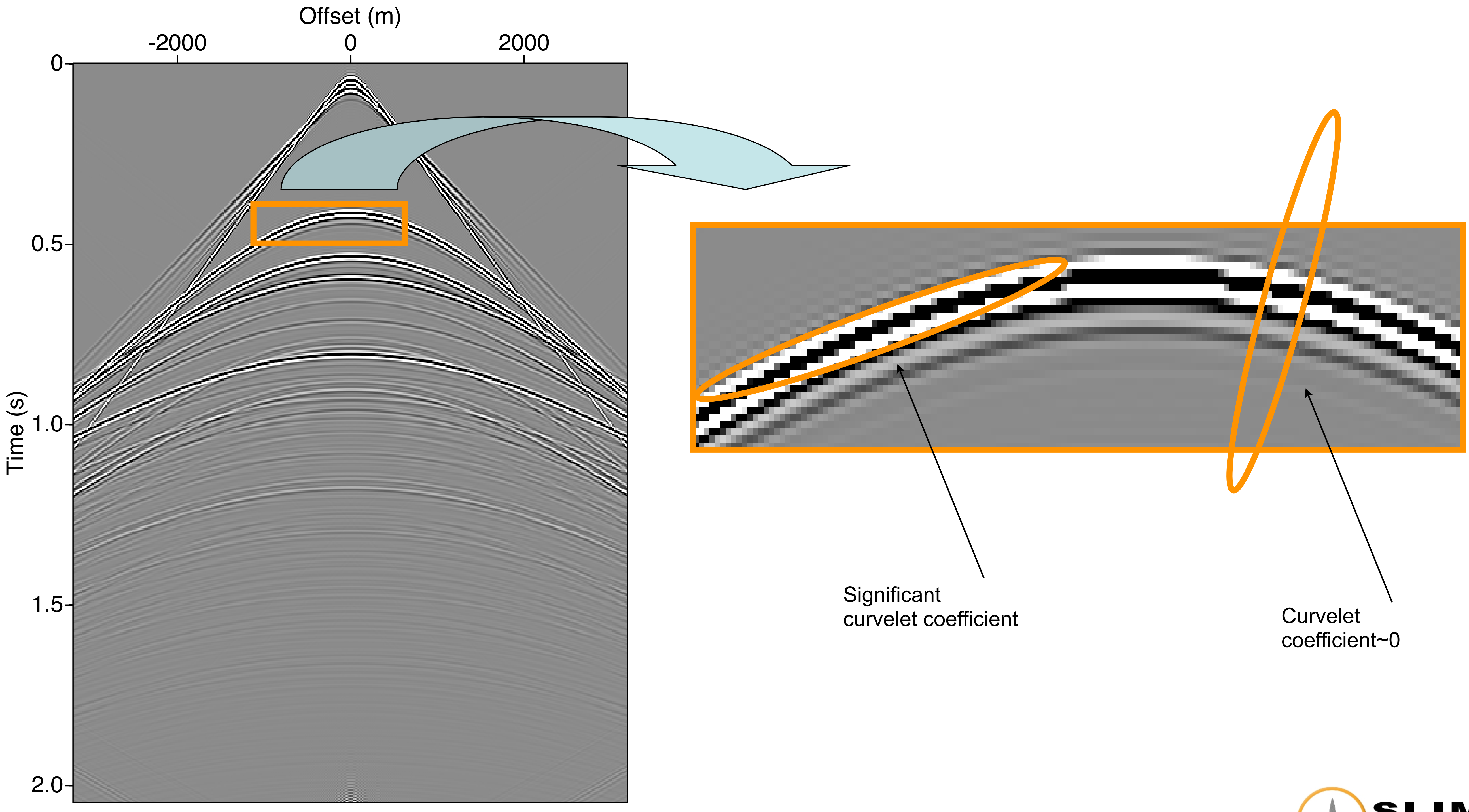
Obey *parabolic* scaling relation $\text{length} \approx \text{width}^2$

Curvelet tiling & seismic data

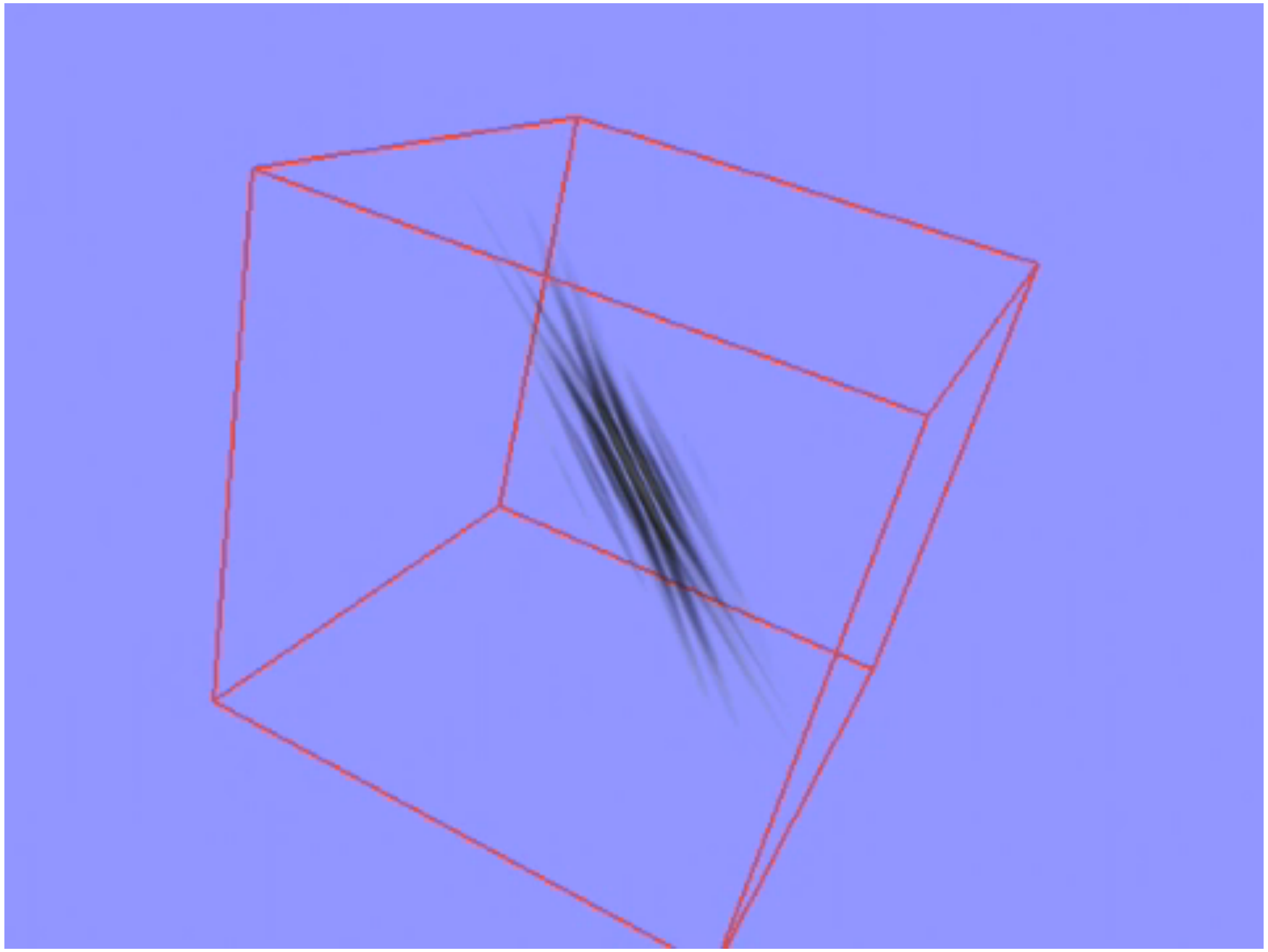


# of angles doubles every other scale doubling!

Wavefront detection

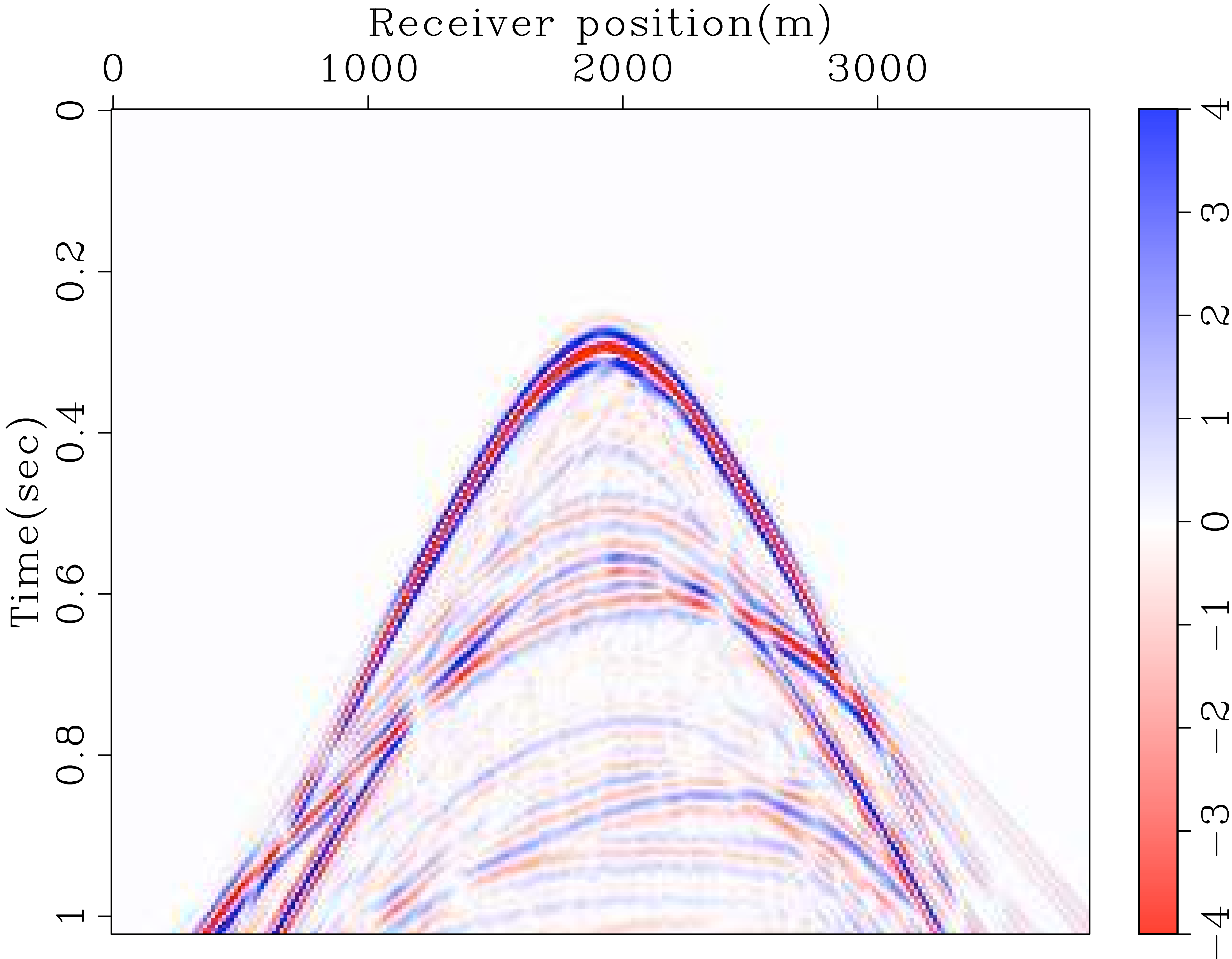


3-D curvelets



Curvelets are oscillatory in one direction and smooth in the others.

Application to synthetic data (2D)



Original Data

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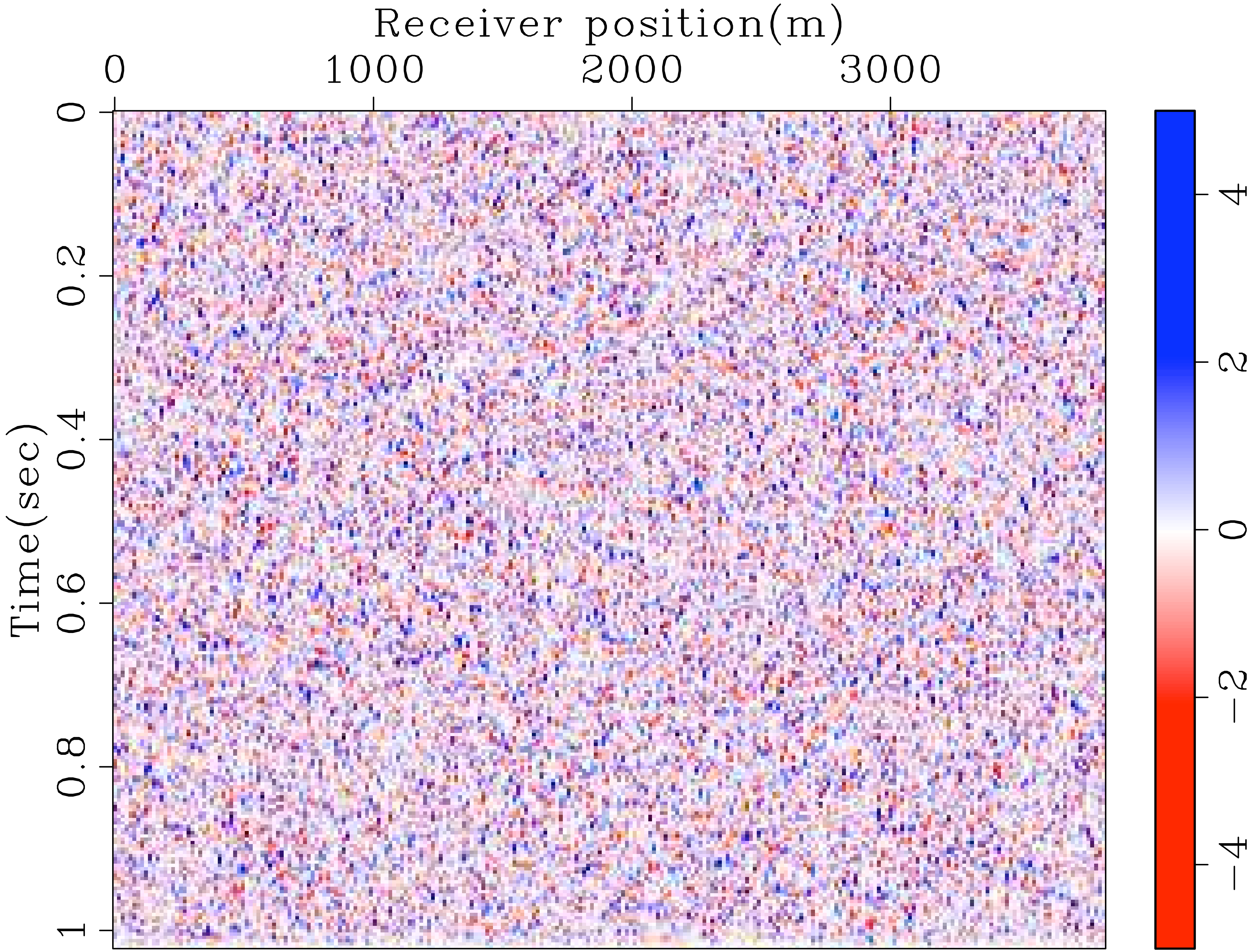
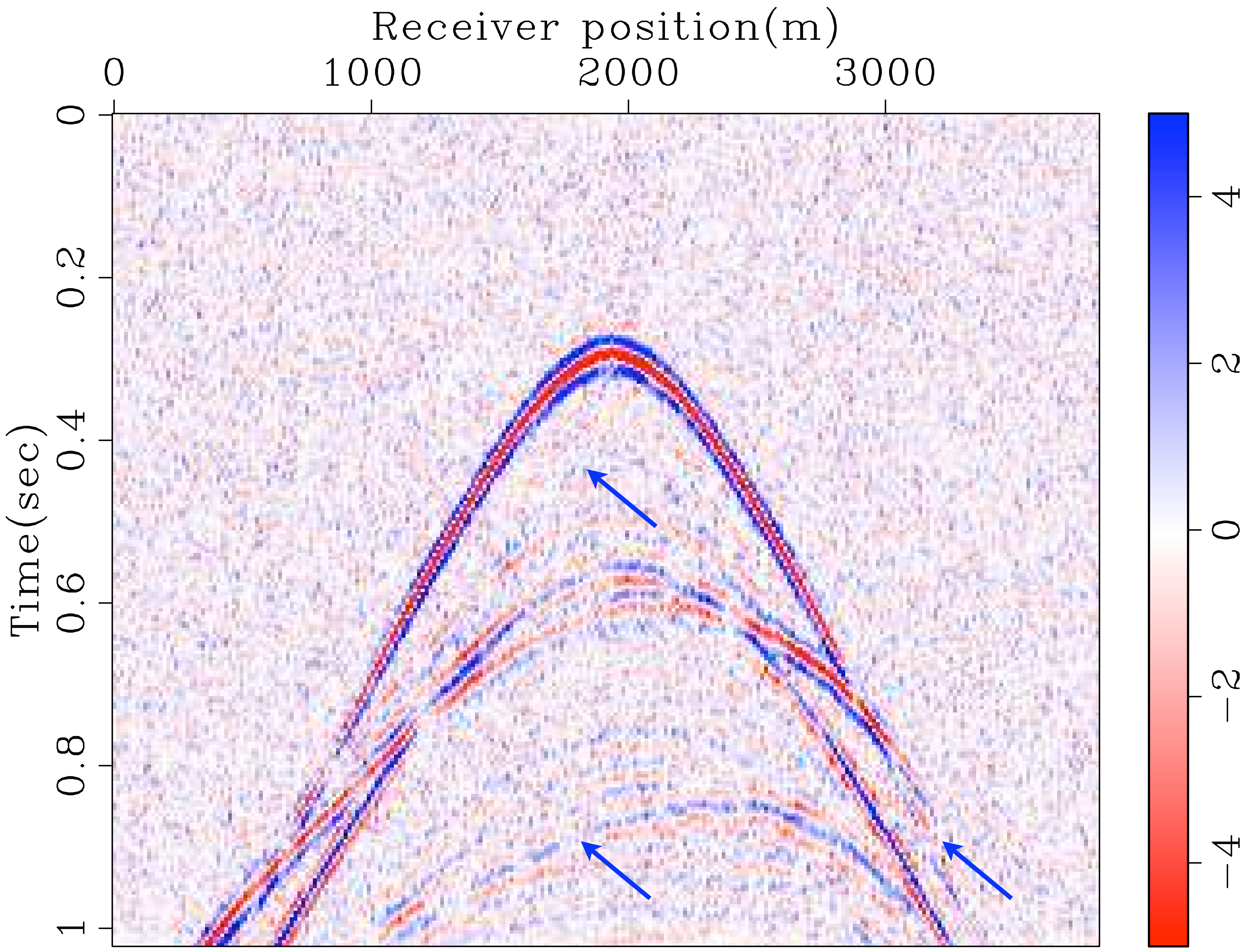
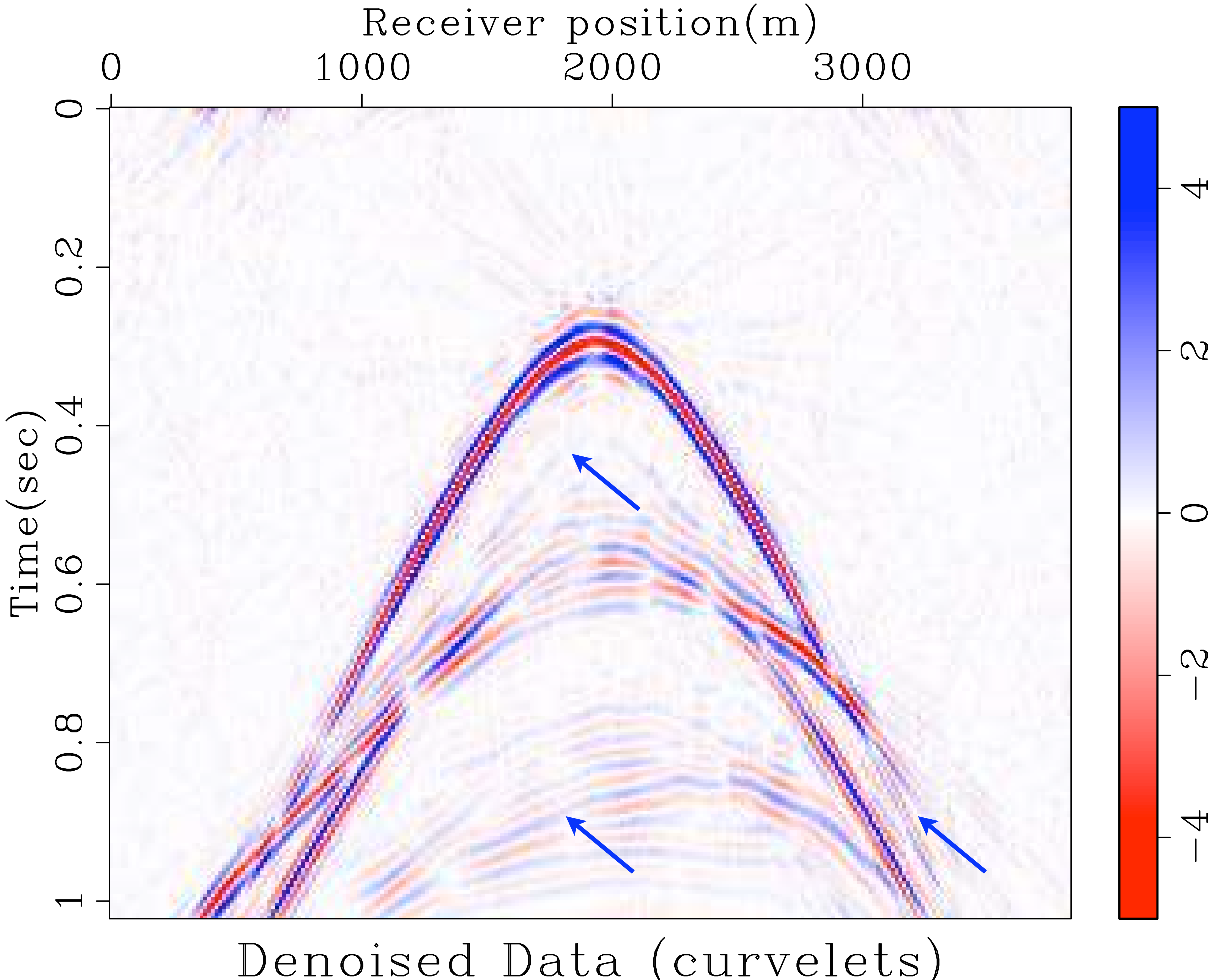


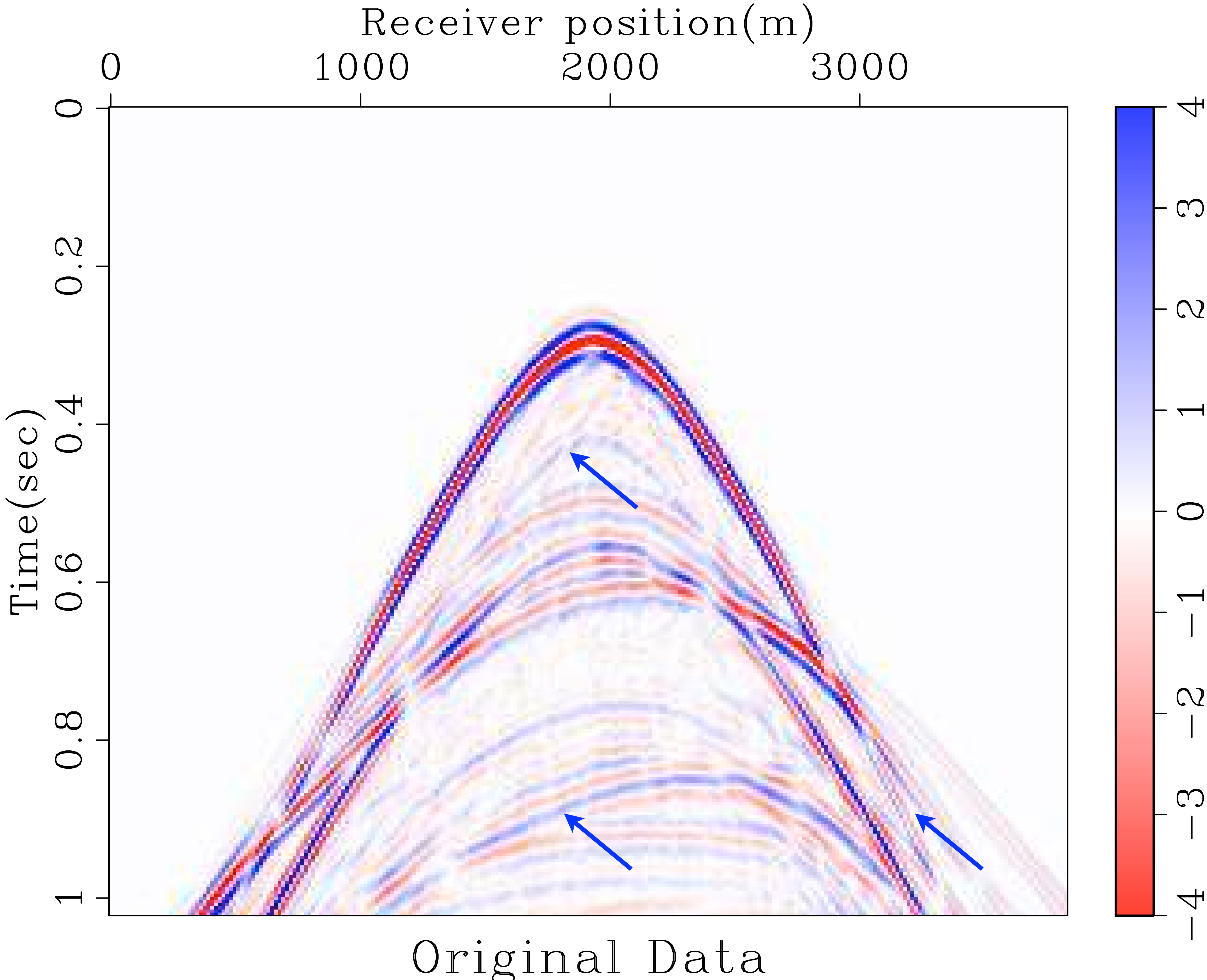
Gaussian Colored Noise(5−60 Hz)



Noisy Data (SNR~1.5 db)



$\varepsilon=200$



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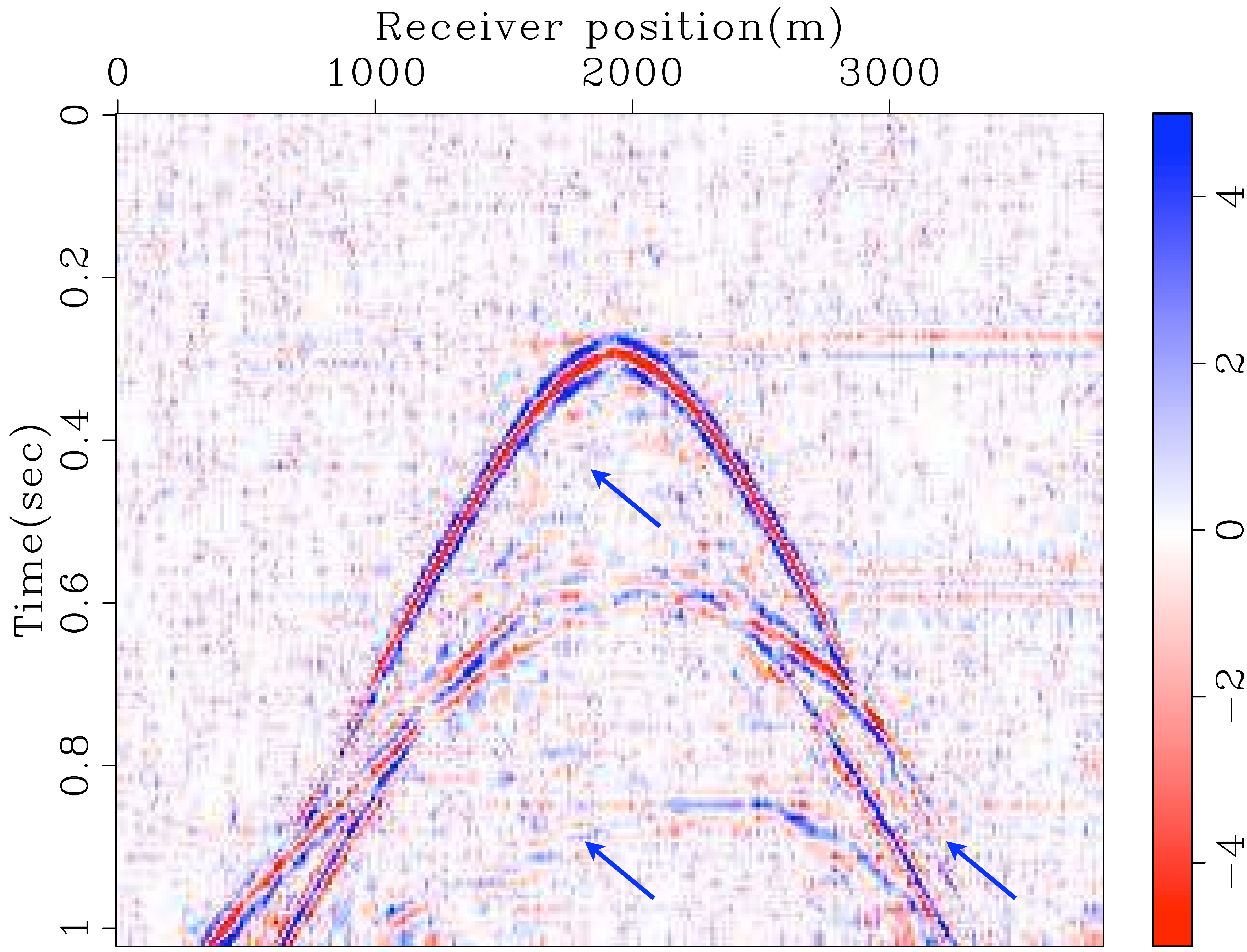
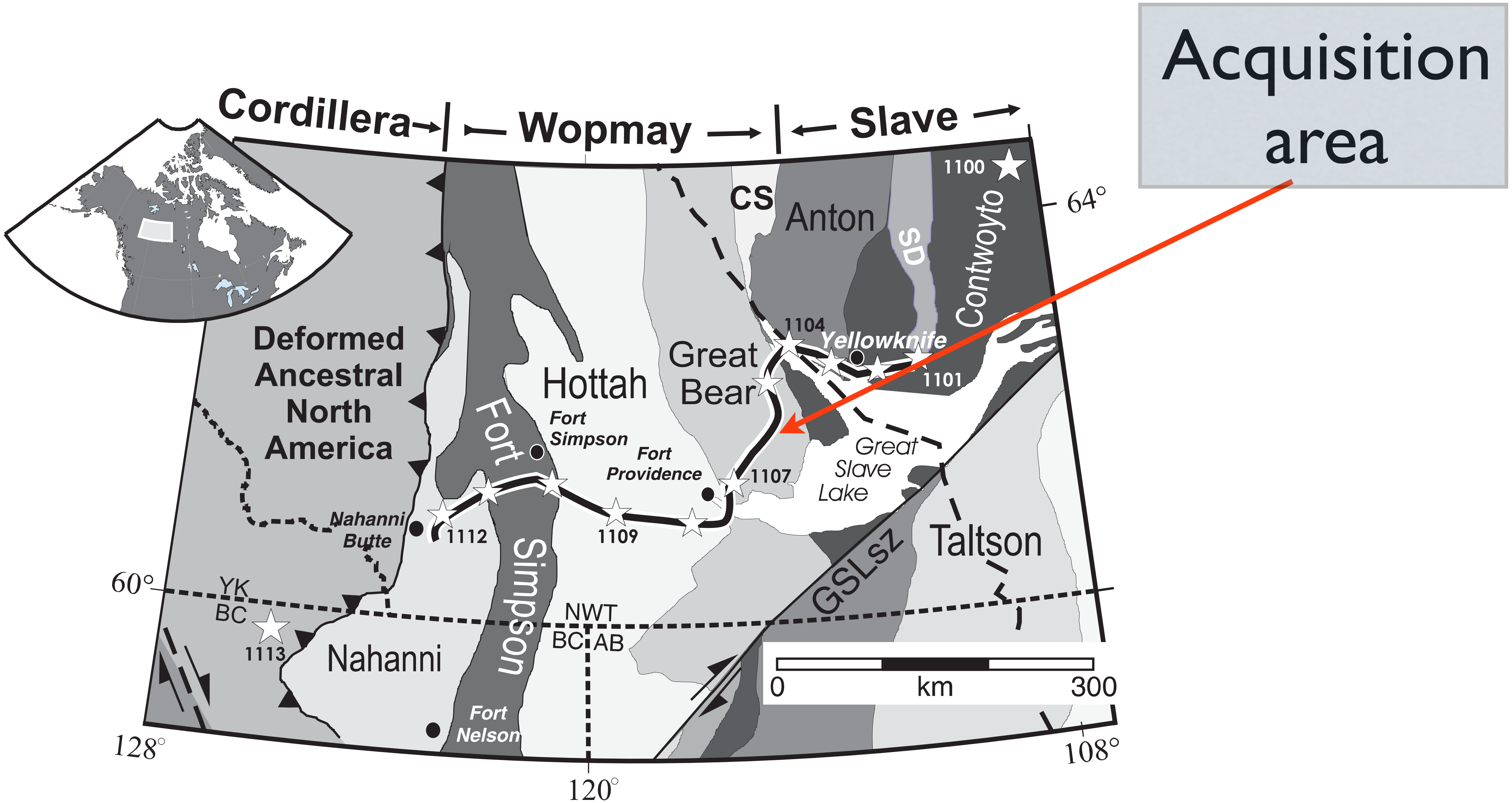


Denoised image (wavelets)

ε=200

Application on Real post-stack data



Seismic Profile

Extent of Phanerozoic cover

Shot Points

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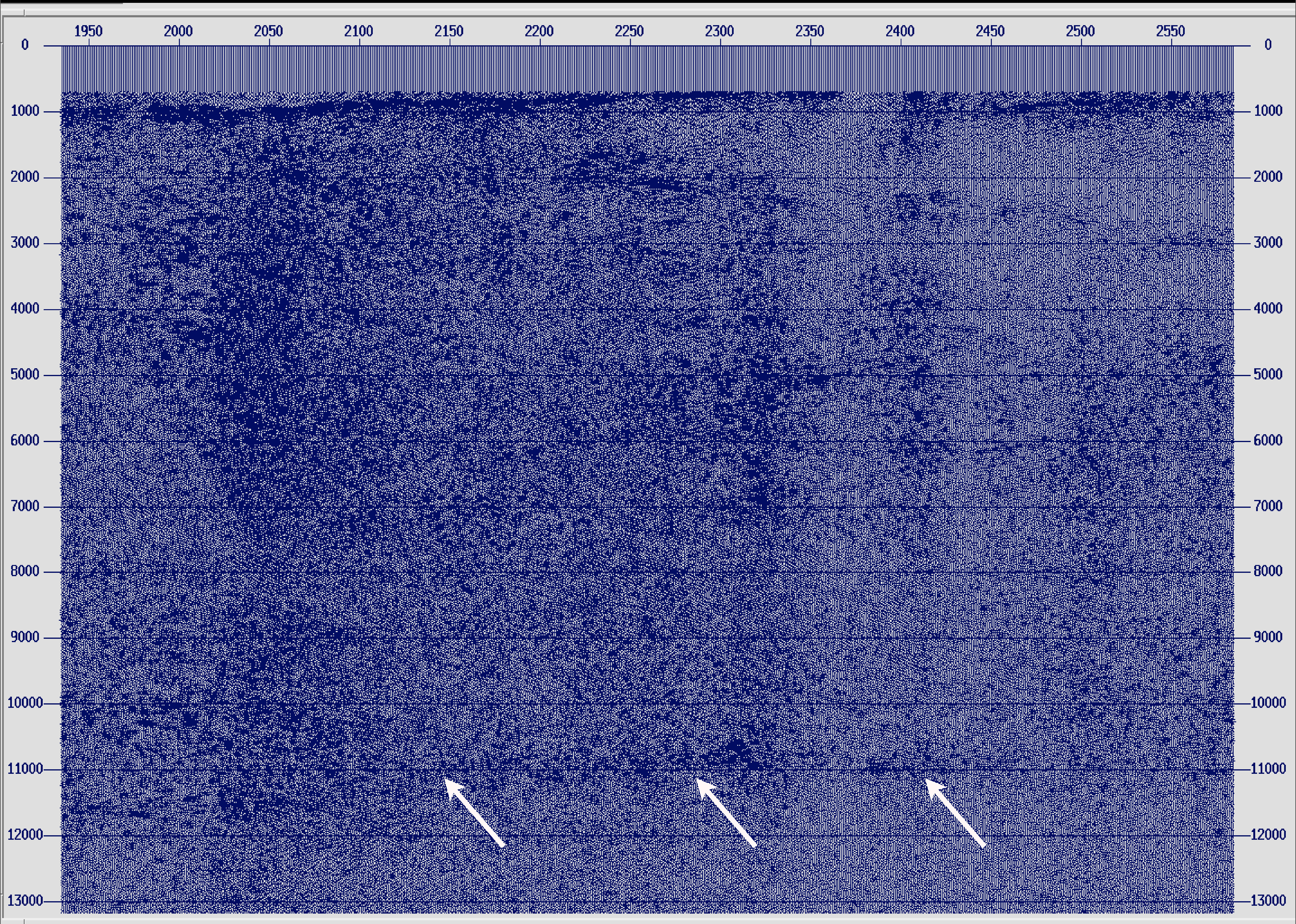
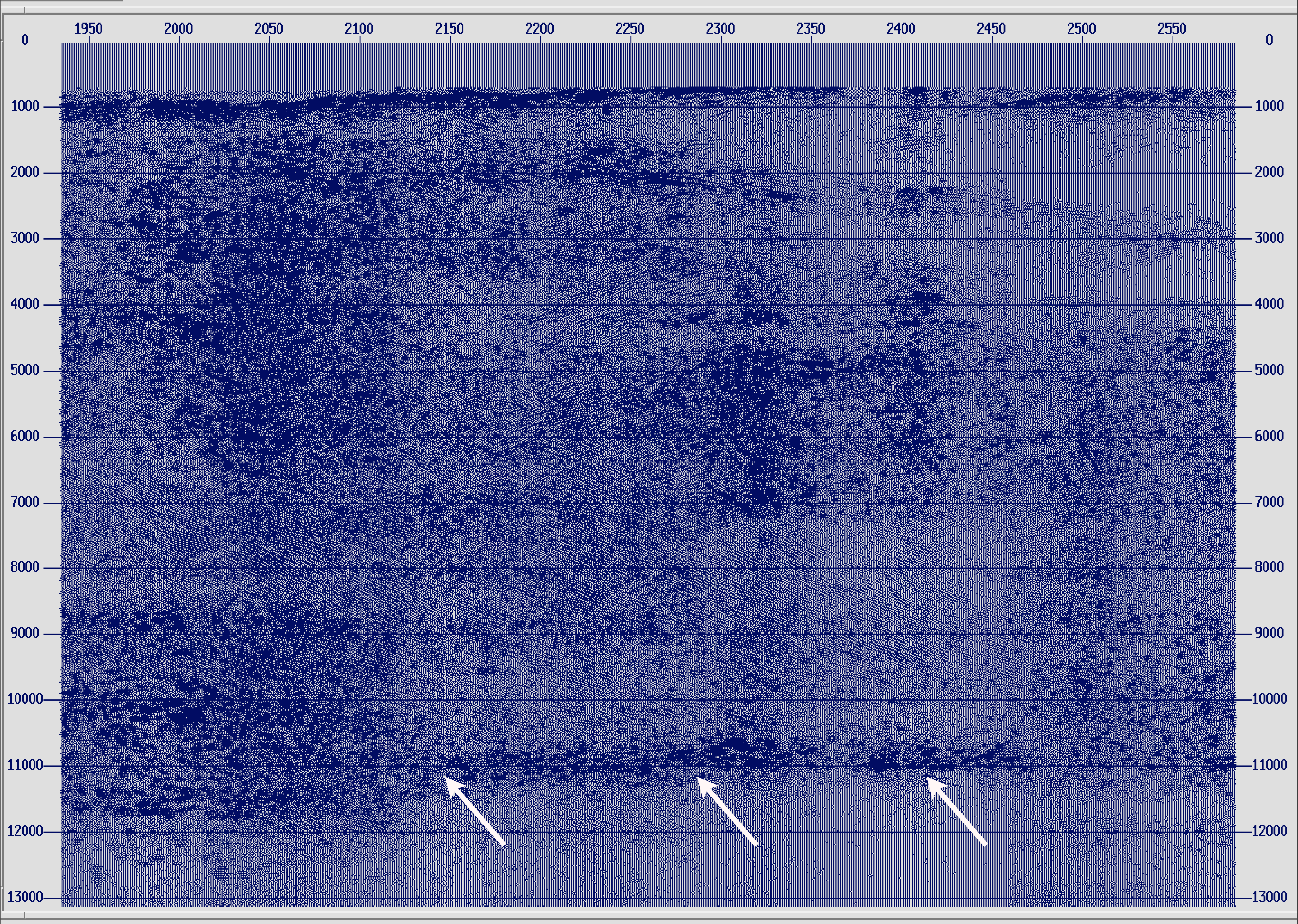
Acquisition Parameters

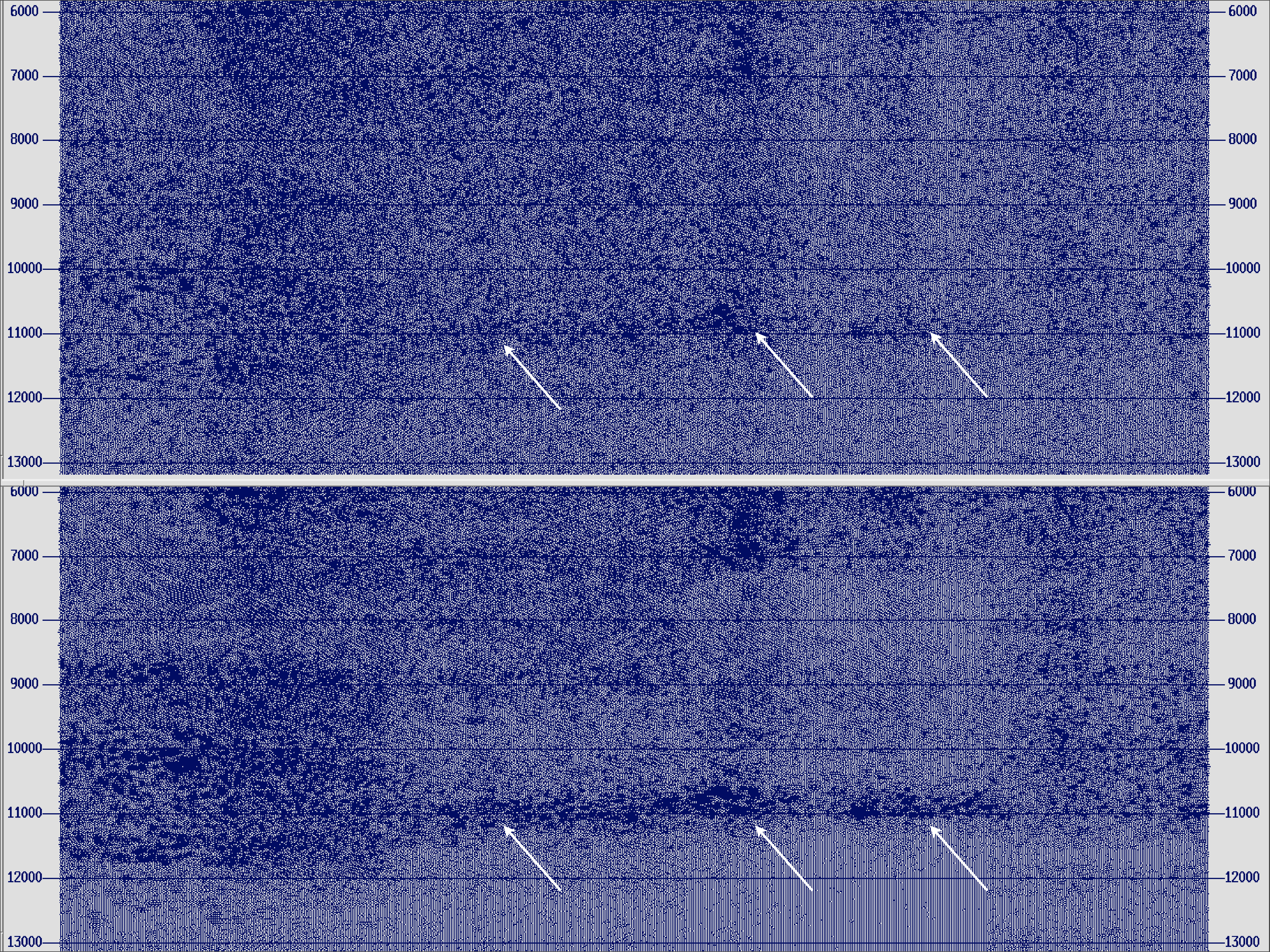
Parameter	Value
Number of vibrators	4 or 5
Number of sweeps	4 or 5
Sweep frequencies	10 - 80 Hz (linear)
Sample rate	4 ms
Sweeplength	20 s
Record length	32 s (uncorrelated)
Number of channels/record	404
Receiver station spacing	60 m
Number of geophones	9 per station
Vibrator point spacing	90 m
Geophone frequency	10 Hz
Nominal	134
Intruments	Sercel 388 (24 bit)

Problems!

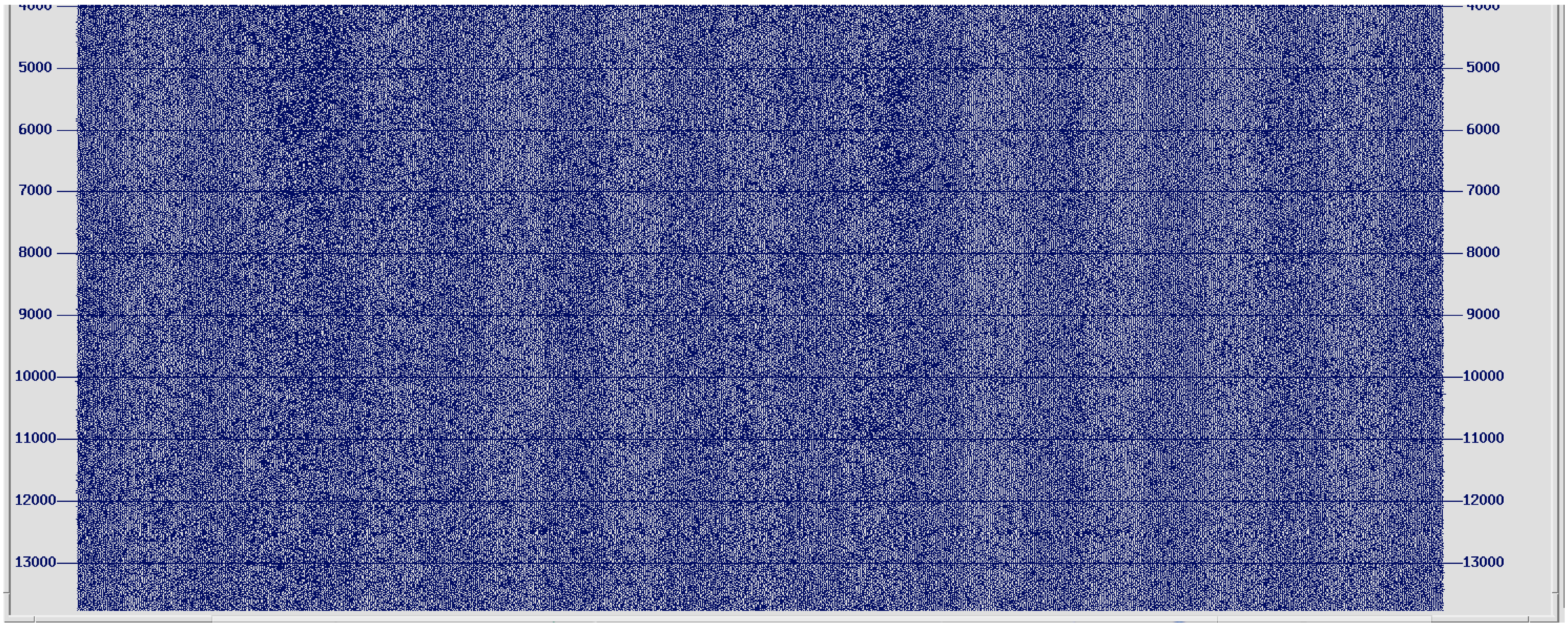
- Very low SNR in deep crustal data
- Careful selection of noise level (ε) is required.
- In our case, We choose the maximum noise level for which the difference between data and estimated model looks like incoherent noise.


1950
2000
2050
2100
2150
2200
2250
2300
2350
2400
2450
2500
2550
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000


1950 2000 2050 2100 2150 2200 2250 2300 2350 2400 2450 2500 2550
0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000


6000
7000
8000
9000
10000
11000
12000
13000

Difference



Note: The difference looks like incoherent noise !

We didn't remove any coherent feature...Hurrah!

3D Curvelet Denoising

Sort the prestack data into Shot-Receiver-Time Volume

$$\min_{x} \|\mathbf{x}\|_1$$

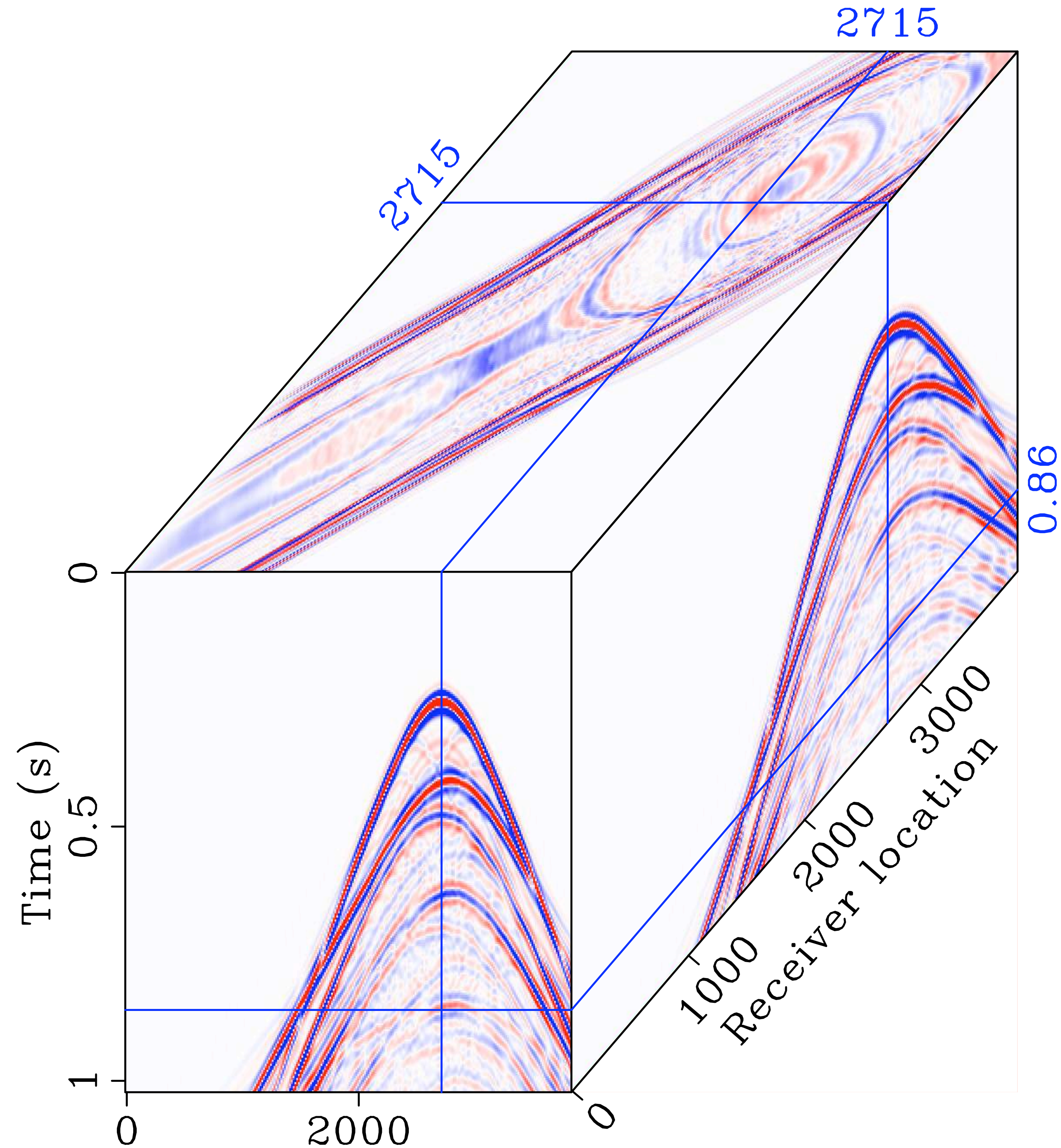
$$s.t \quad \|\mathbf{y} - \mathbf{C}^{\mathrm{T}}\mathbf{x}\|_2 \leq \varepsilon$$

$$\hat{\mathbf{m}} = \mathbf{C}^{\mathrm{T}}\hat{\mathbf{x}}$$

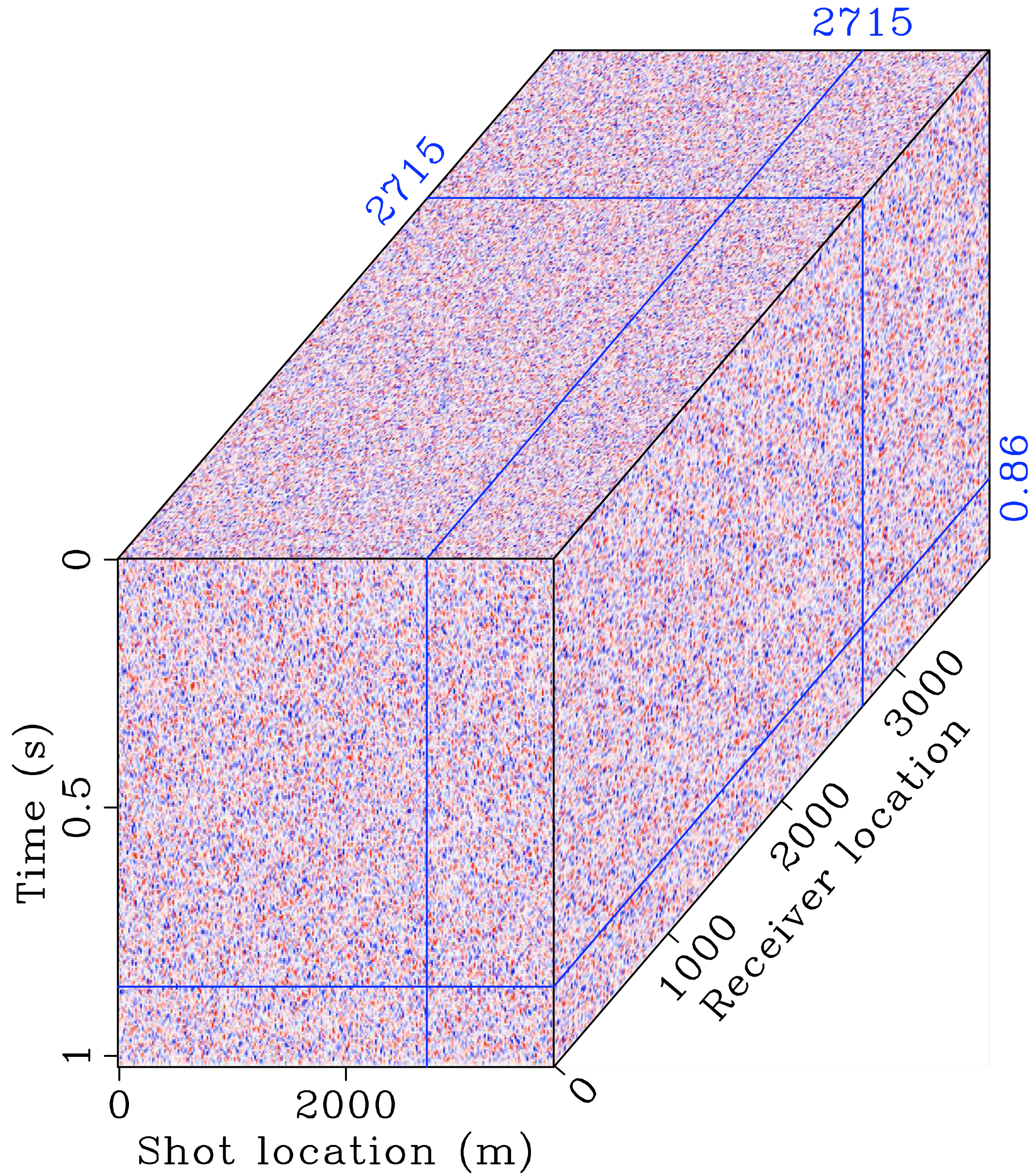
$$\mathbf{C}^{\mathrm{T}} \rightarrow \text{3D} - \text{Curvelet Synthesis Operator}$$

$$\epsilon^2 = \sigma^2[N + 2\sqrt{2N}]$$

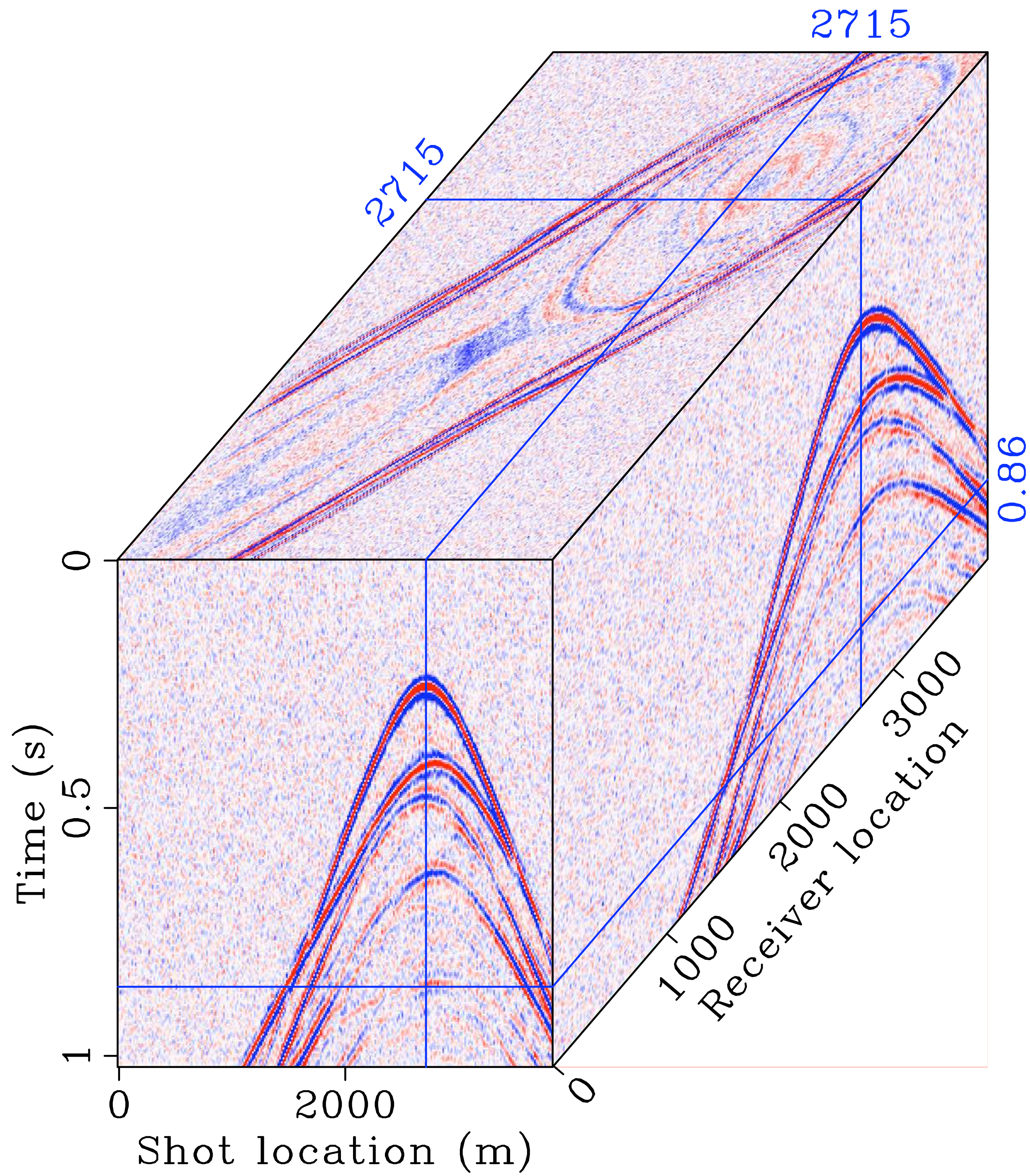
model


2715
2715
0.86
0
0.5
1
Time (s)
0
2000
Shot location (m)
0
1000
2000
3000
Receiver location

Gaussian Colored Noise



Noisy data (SNR~3.3db)



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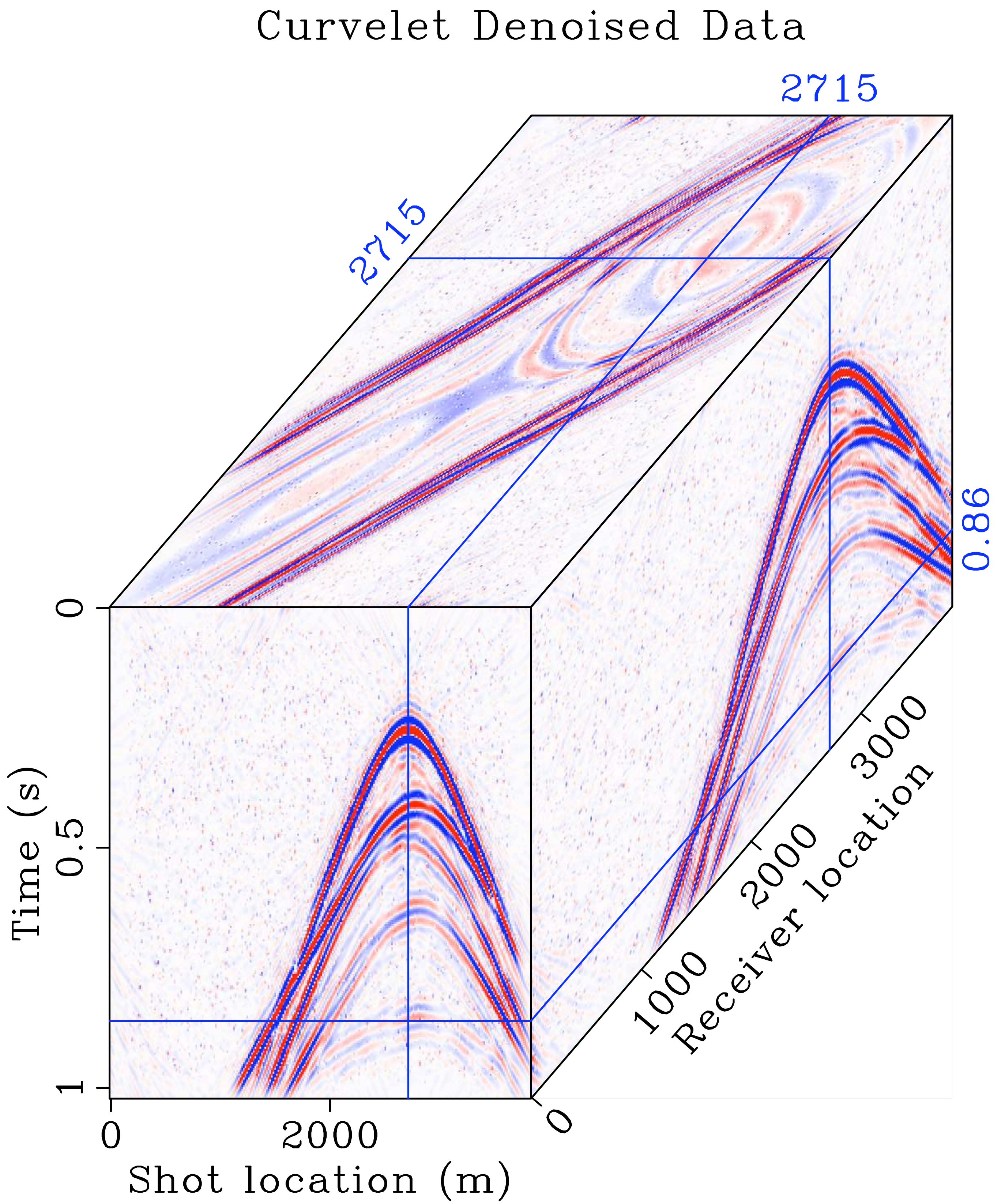

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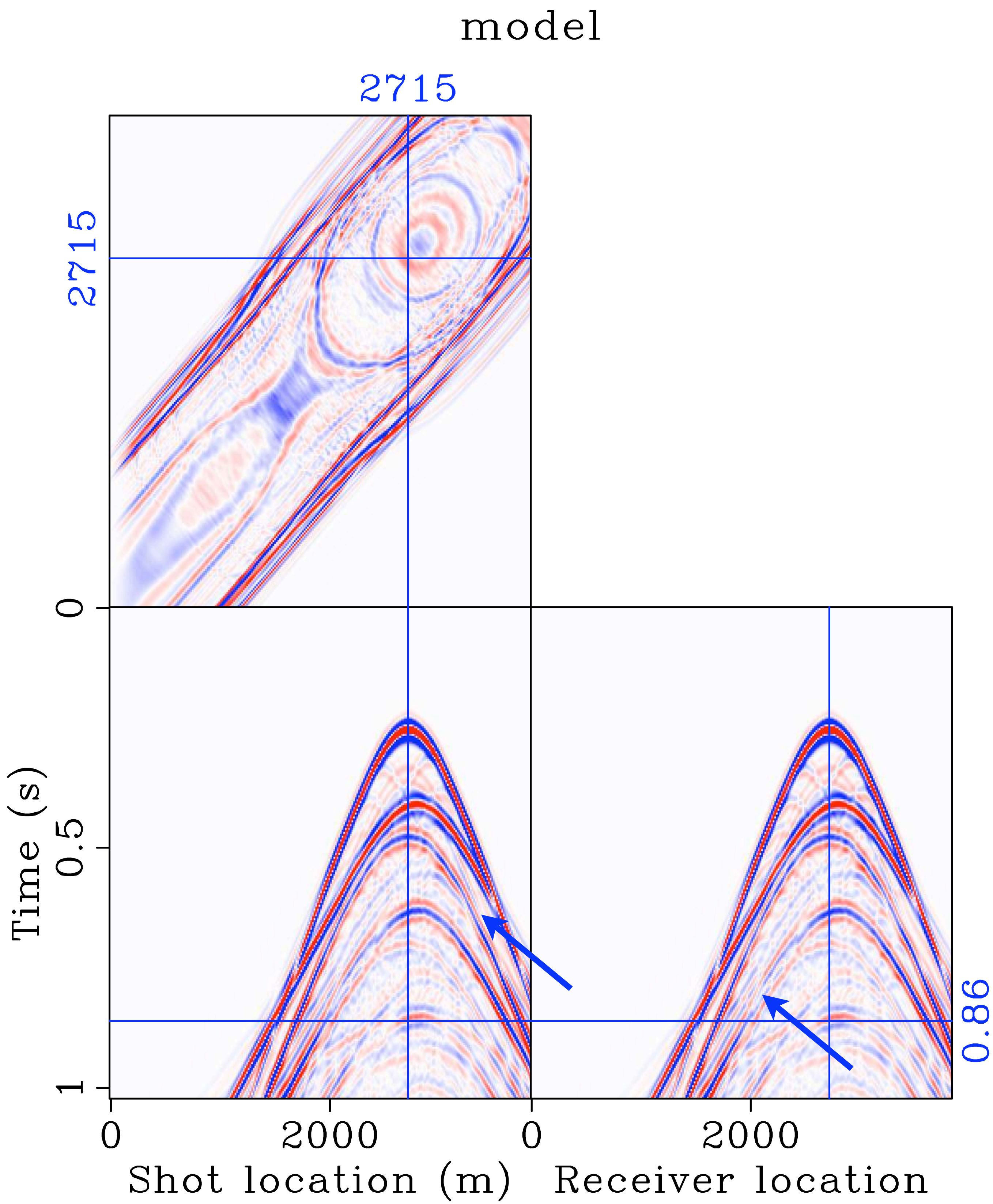

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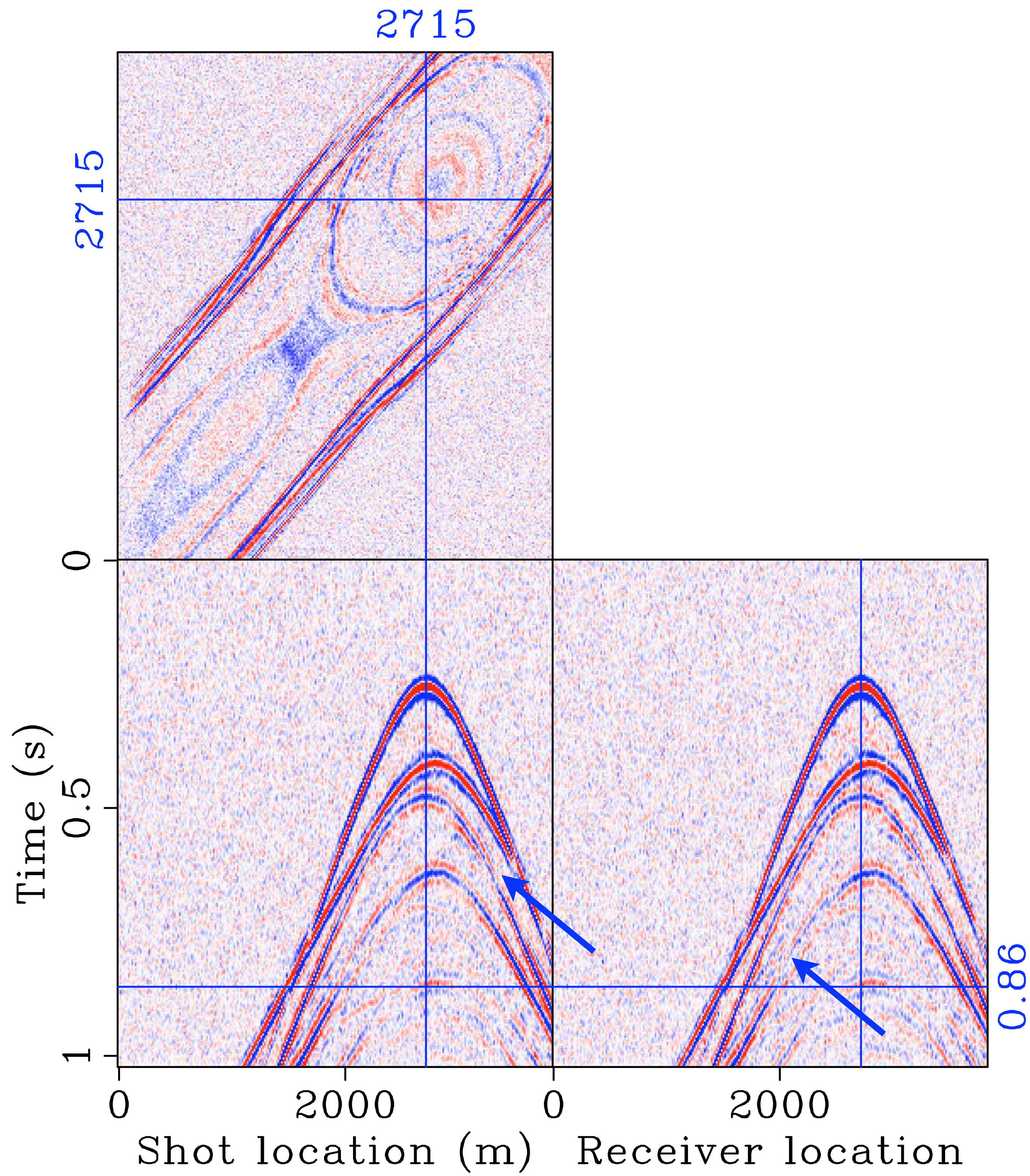
Noisy data (SNR~3.3db)



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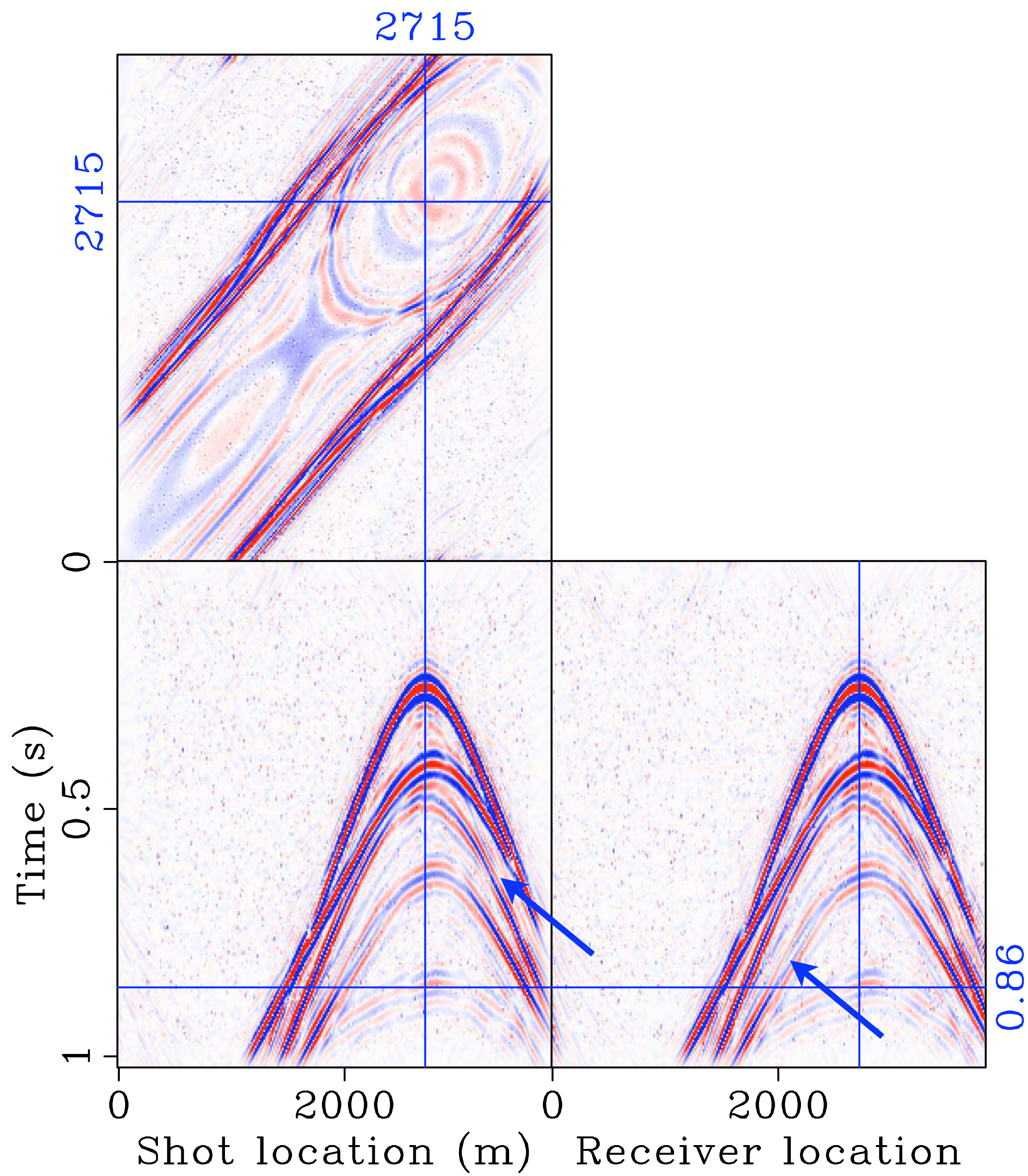
Curvelet Denoised Data



Future work

•For the Prestack data we can distribute each shot gather on different CPU's and solve as an embarrassingly parallel problem.

•Create overlaping windows as three dimensional cubes with tapering such that it has partition of unity and solve on parallel CPU's.

•Careful angular and depth weighting would improve the results

Acknowledgments

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Thank You!
(Merci Beaucoup)

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