Curvelet-Regularized Seismic Deconvolution

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SLIM
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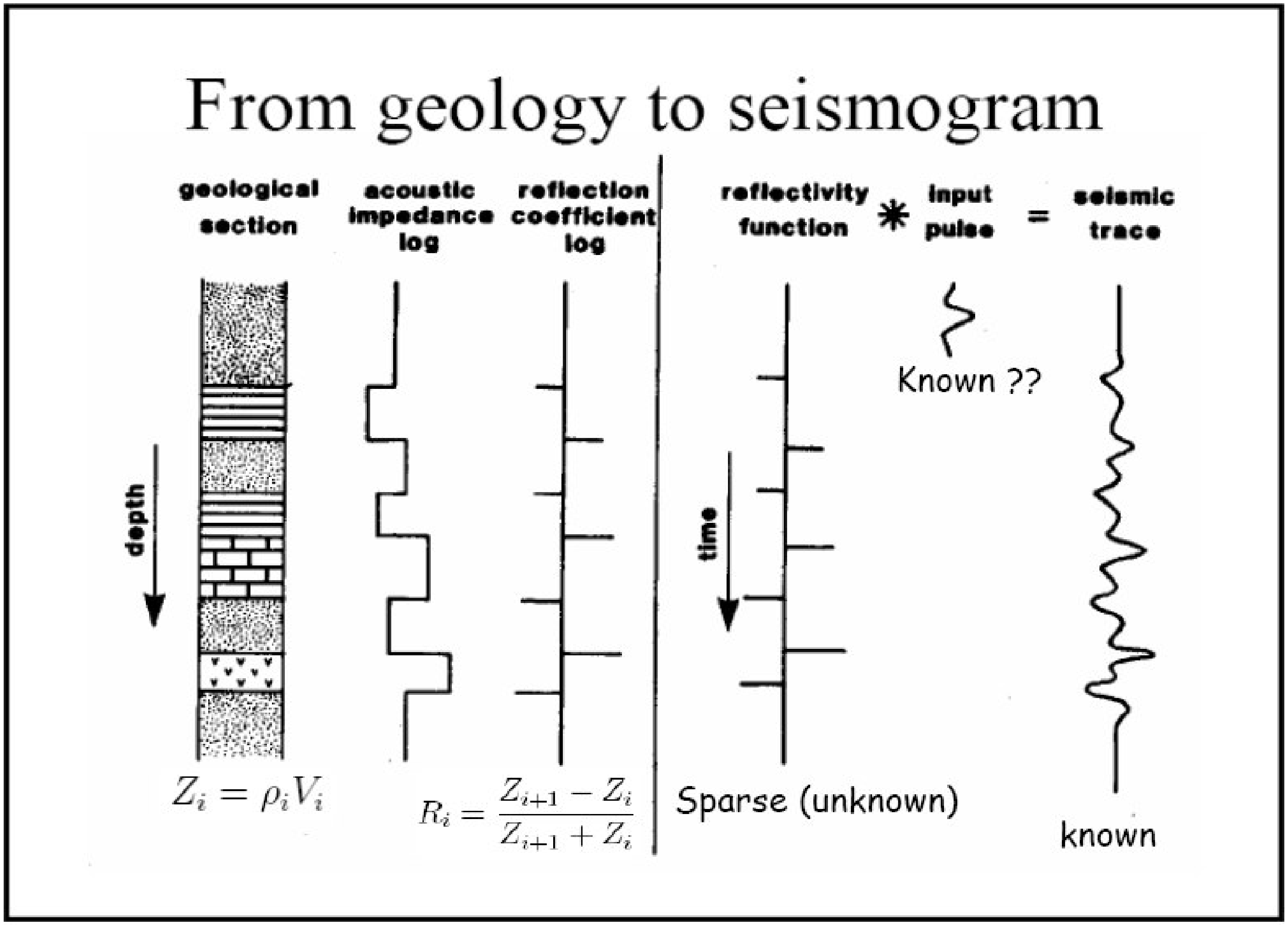
The Premise



From geology to seismogram
geological section
acoustic impedance log
reflection coefficient log
reflectivity function
*
input pulse
=
seismic trace
depth
time
Known ??
$Z_i = \rho_i V_i$
$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}$
Sparse (unknown)
known

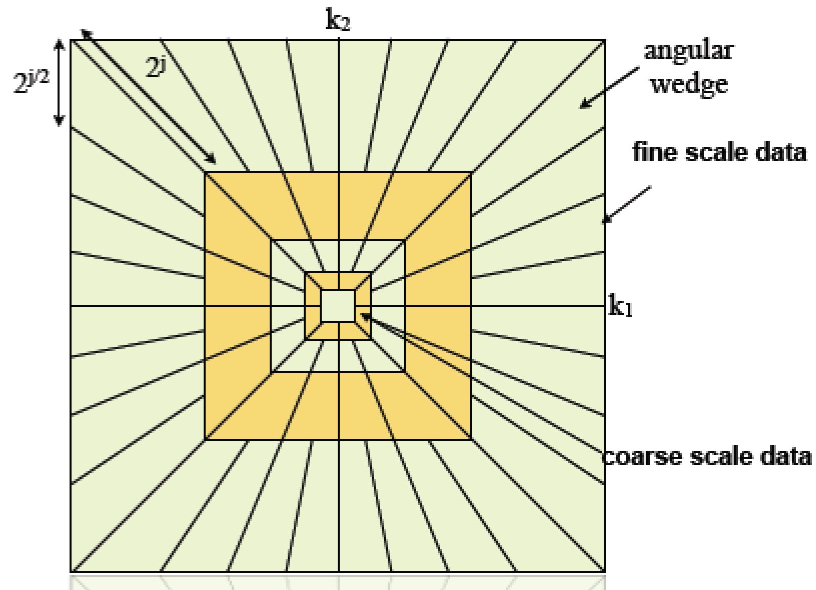
Motivation

- Spiky assumption is too limited to model seismic reflectivity. [Herrmann, 2005]

- There is an inherent continuity along the reflectors. [Hennenfent et al., 2005]

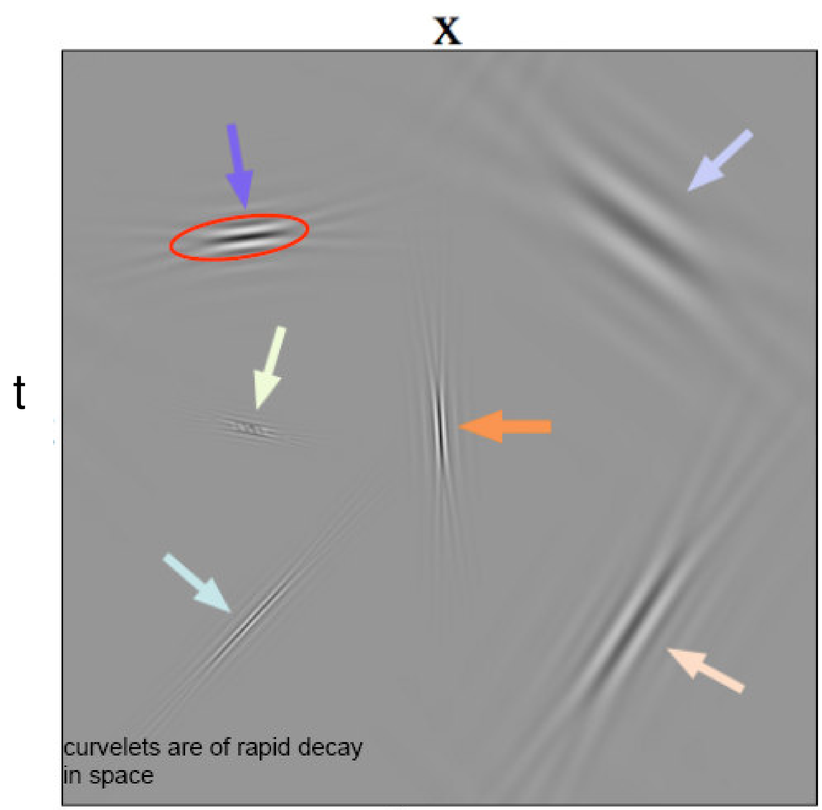
Curvelet transform

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partition into angular wedges
- **anisotropic:** width ≈ length^2
- **pseudo-localized in spatial domain:** rapid decay in space
- **localized in frequency domain**

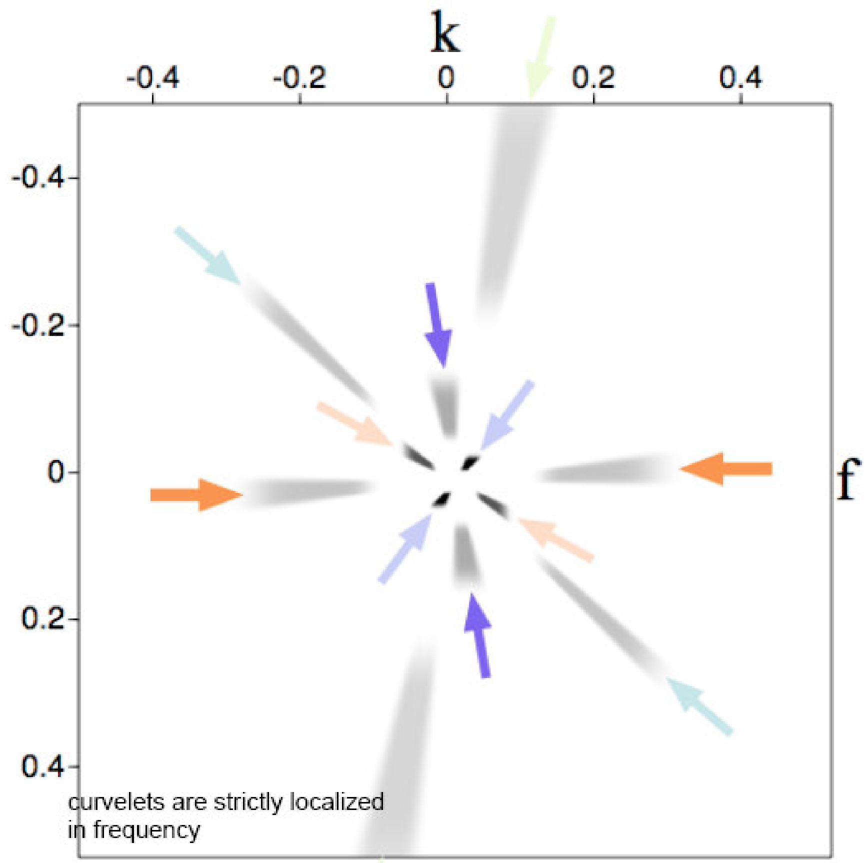


[Candes et al.,2006]

2-D Curvelets



t-x domain



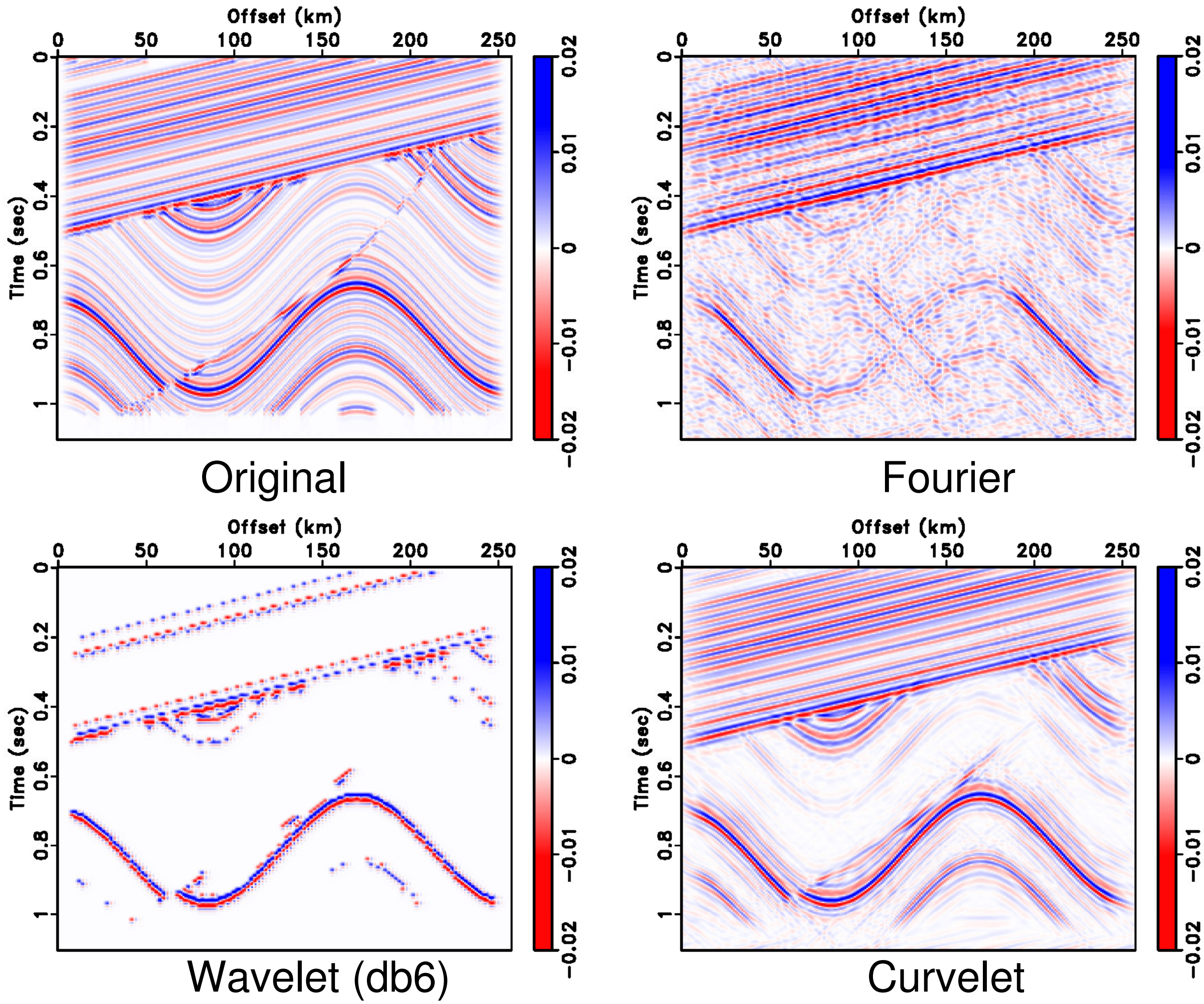
f-k domain

[Adapted from Herrmann and Hennenfent, 2008]

Why curvelets ?

Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Partial Reconstruction of model with 1% Amplitude largest coefficients



Forward problem

$$y = Am + n$$

$y \rightarrow$ Data

$A \rightarrow$ Convolution Operator

$m \rightarrow$ Reflectivity

$n \rightarrow$ Noise

Deconvolution as an optimization problem

$$\min_x \|x\|_1 \quad \text{s.t} \quad \|y - AC^T x\|_2 \le \varepsilon$$

$$\hat{m} = C^T x$$

$$\varepsilon^2 = \sigma^2 \left[N + 2\sqrt{2N} \right]$$

$C^T \rightarrow$ Curvelet synthesis operator,

$\sigma \rightarrow$ Standard deviation of noise,

$N \rightarrow$ No. of data points

Data & Model

- Sigmoid model is half differentiated in the frequency domain to obtain non-spiky reflectivity.

- Reflectivity model is convolved with Ricker wavelet (central frequency = 25 Hz) and random noise (σ=.002) is added.

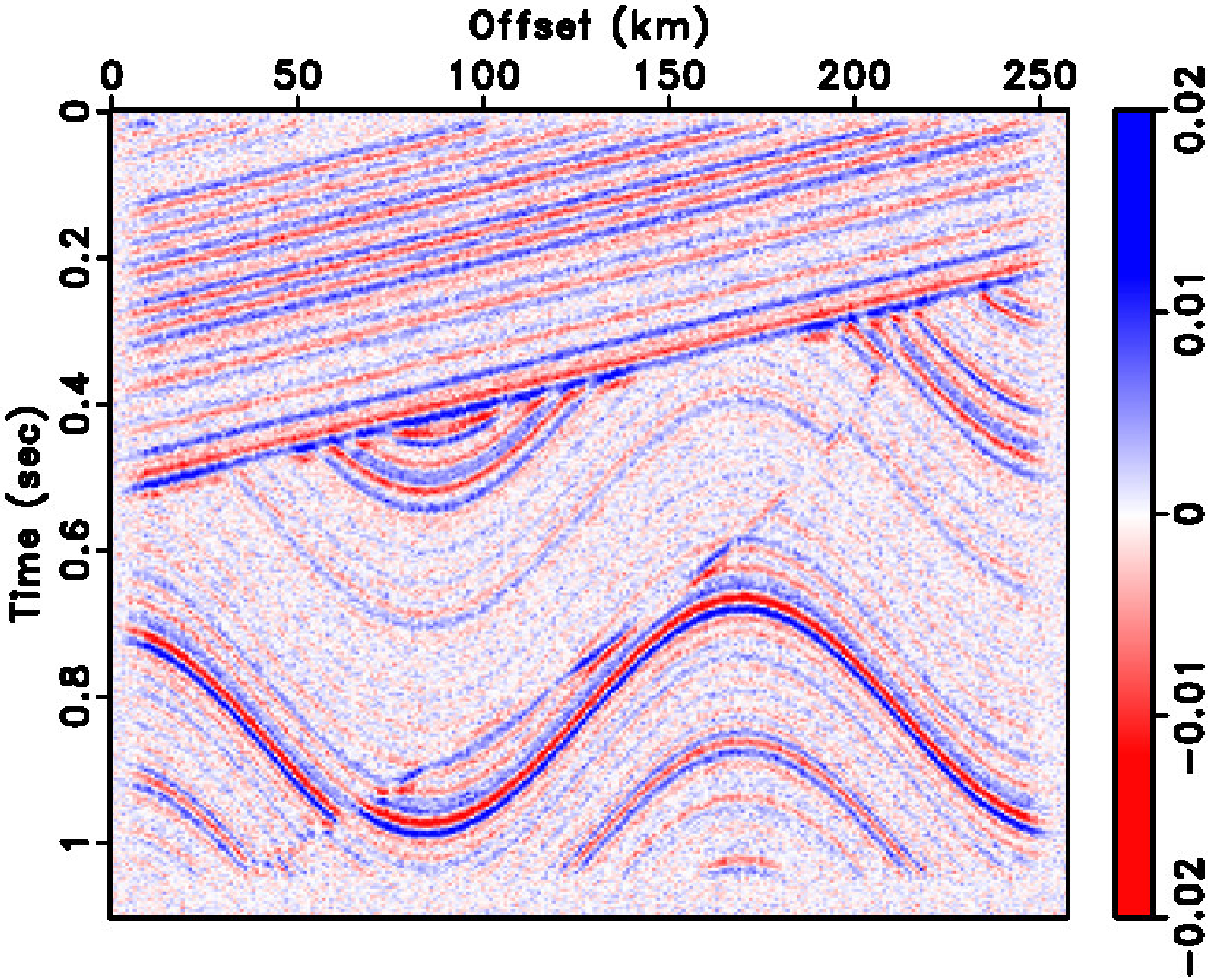
Approach

- Inversion operators were formed in SPARCO. [Van den Berg et. al]
- Solution is found by SPGl_1 algorithm. [Van den Berg and Friedlander]
- Results are compared with those of Spiky Deconvolution defined as:

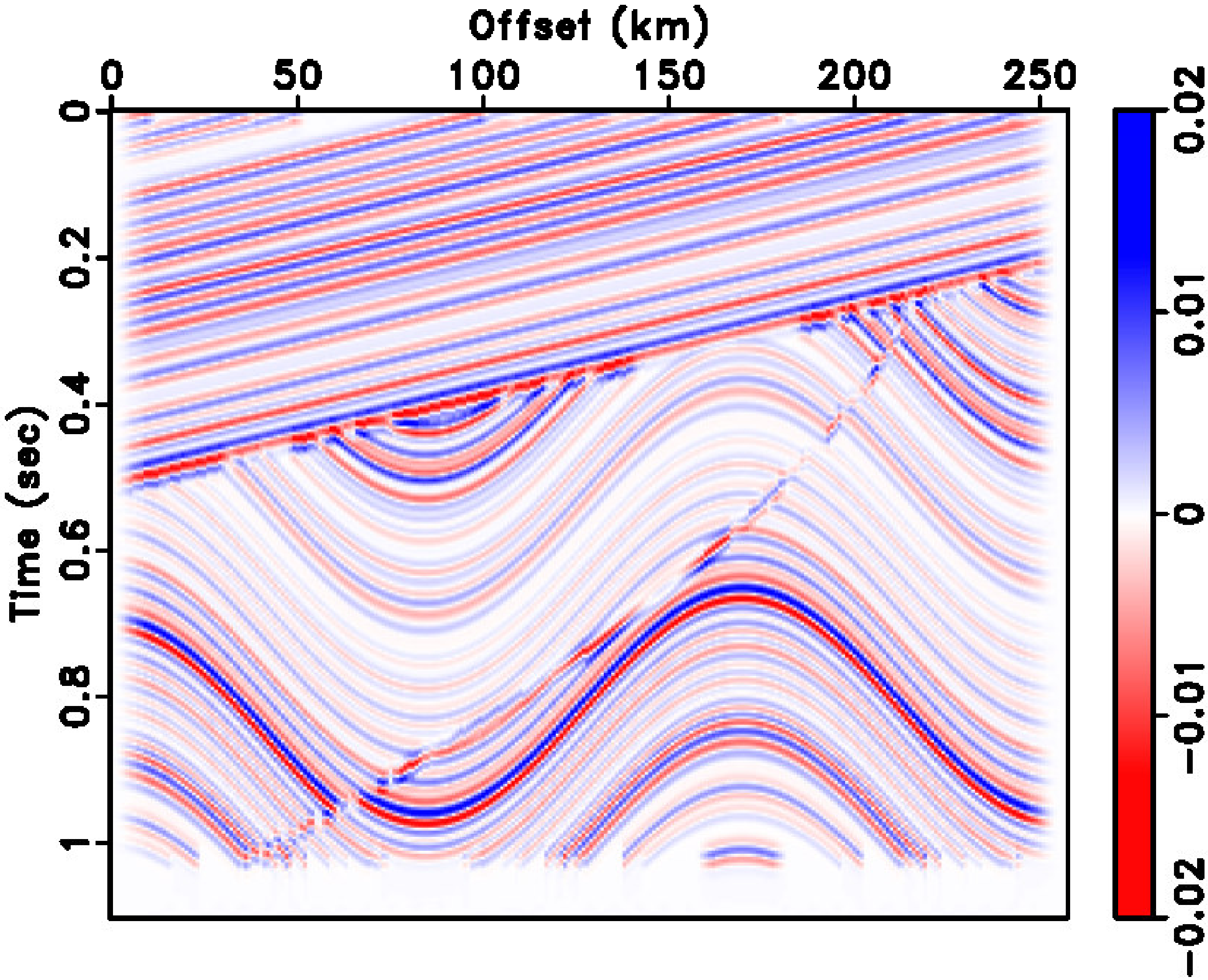
$$\min_{m} \|m\|_1 \ \text{s.t} \ \|y - Am\|_2 \leq \varepsilon$$

[Taylor et al., 1979; Oldenburg et al., 1981]

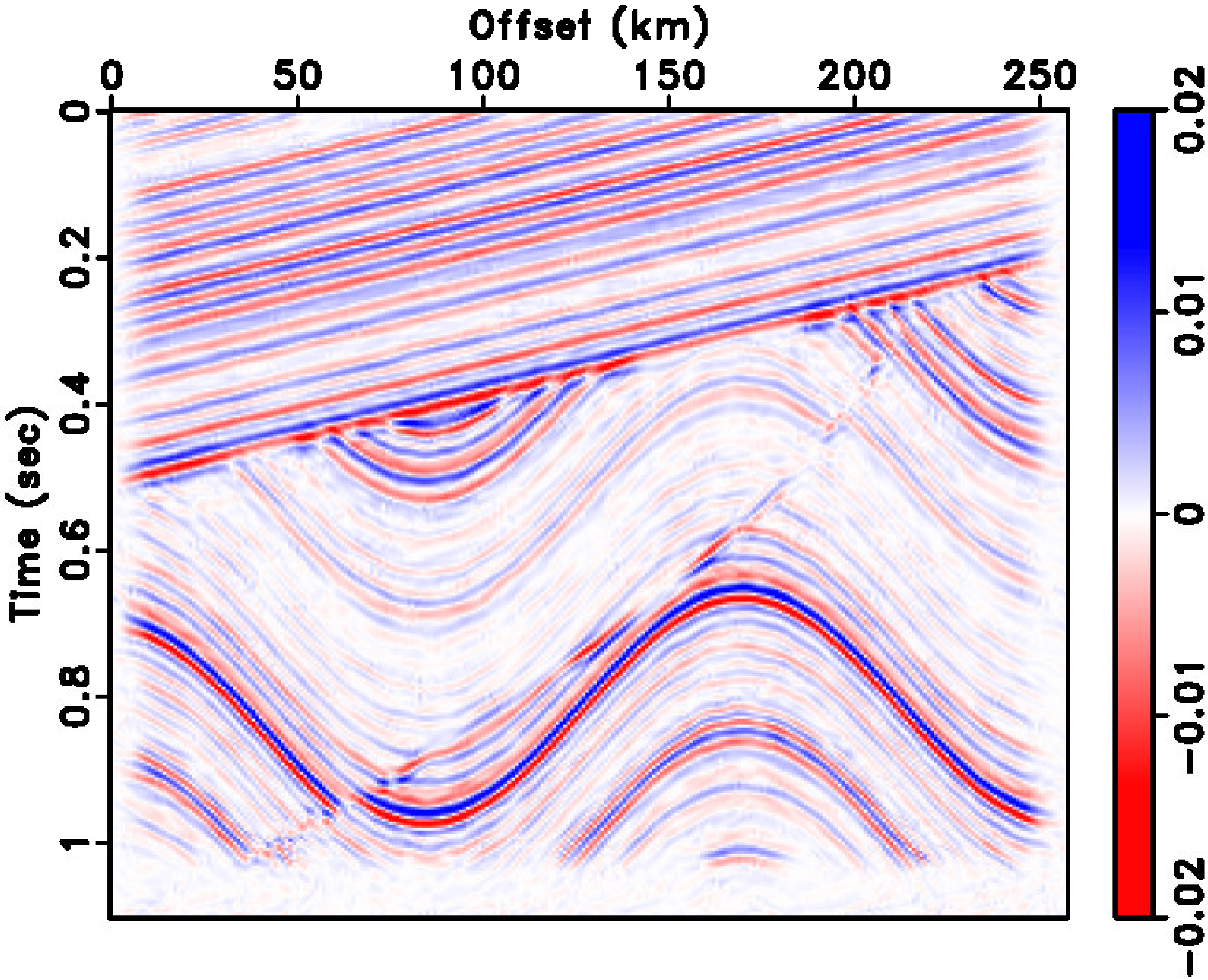
Data


Offset (km)
0
50
100
150
200
250
Time (sec)
0
0.2
0.4
0.6
0.8
1
0.02
0.01
0
−0.01
−0.02

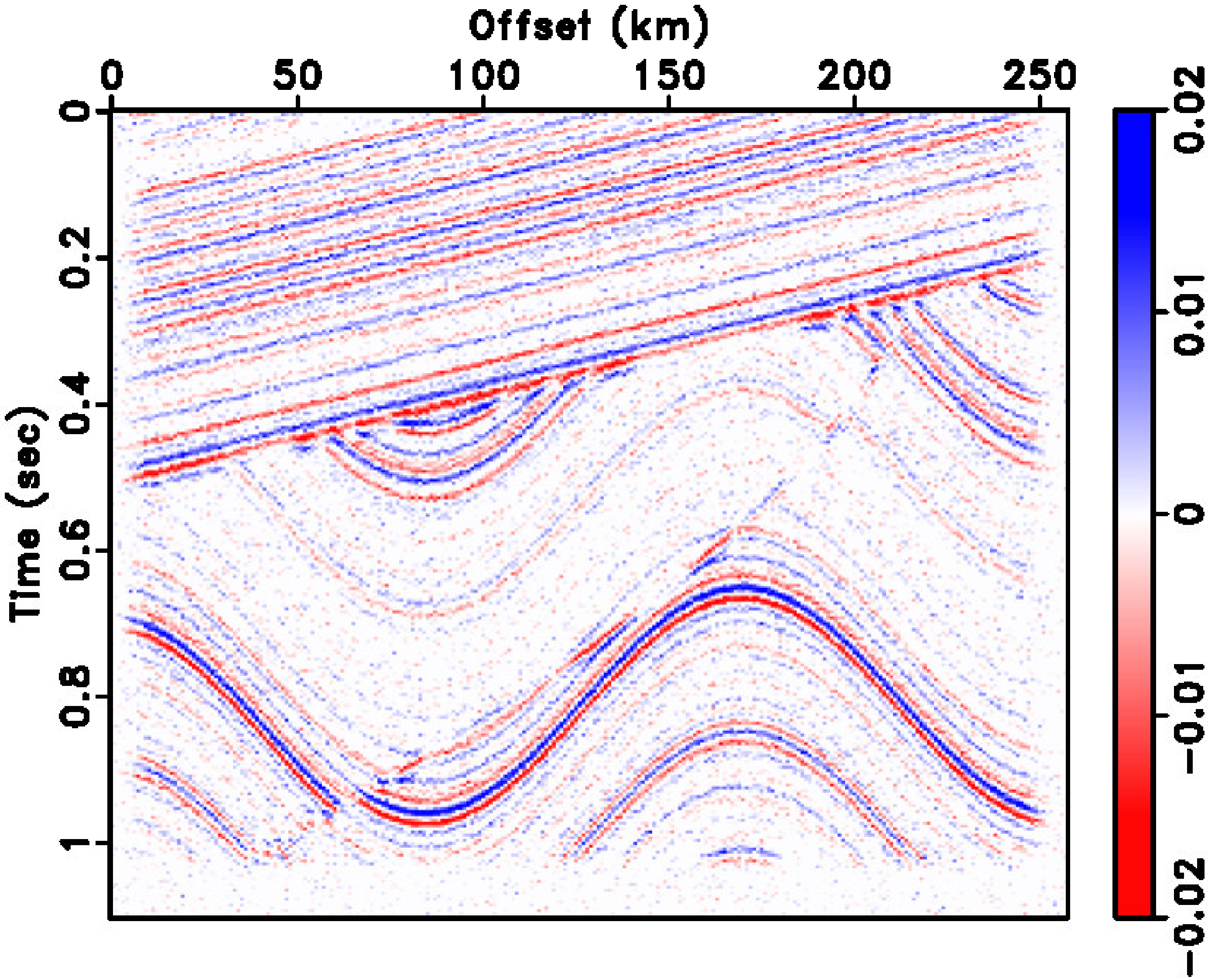
Original Reflectivity



Curvelet Inversion

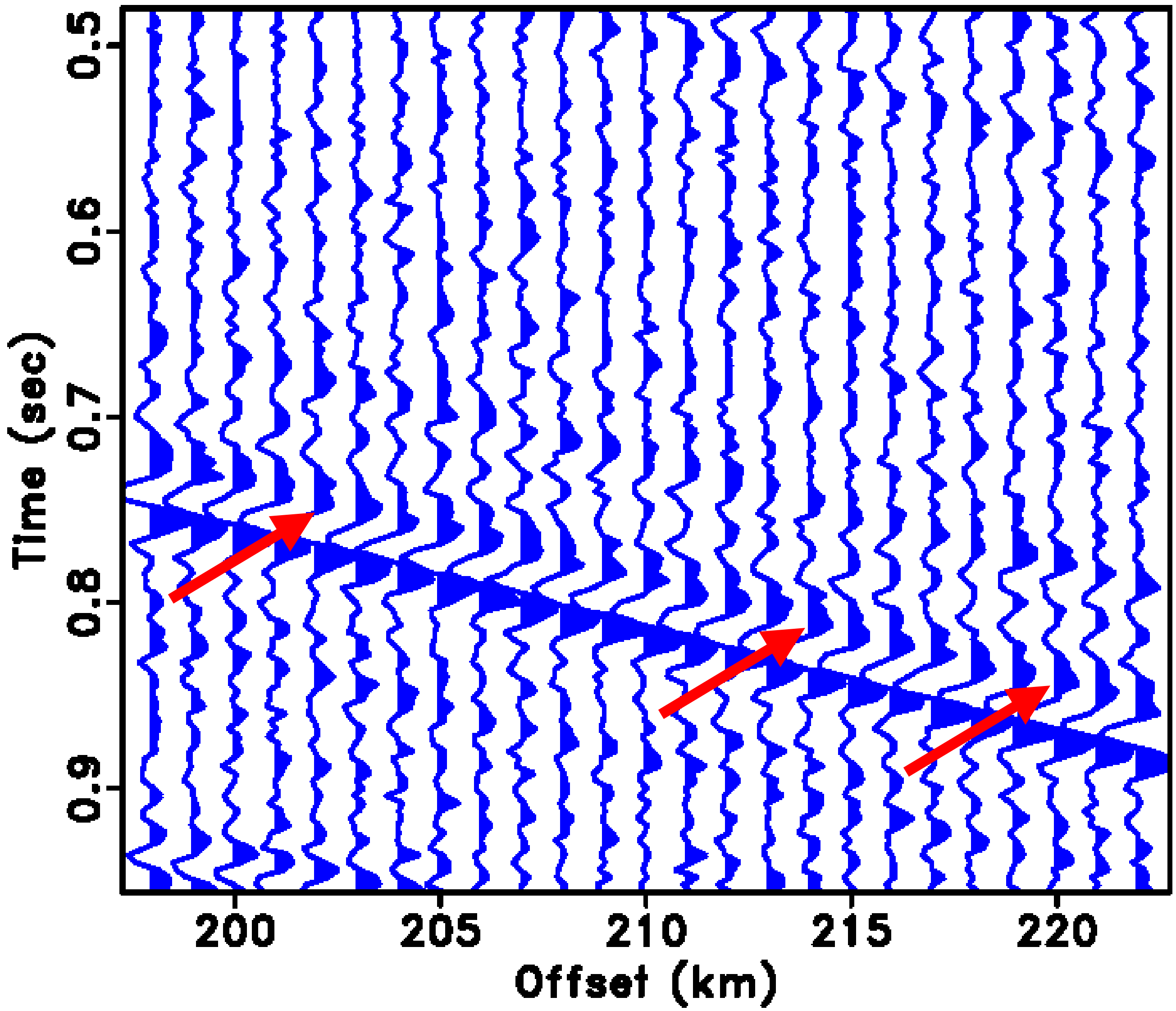


Spiky Inversion

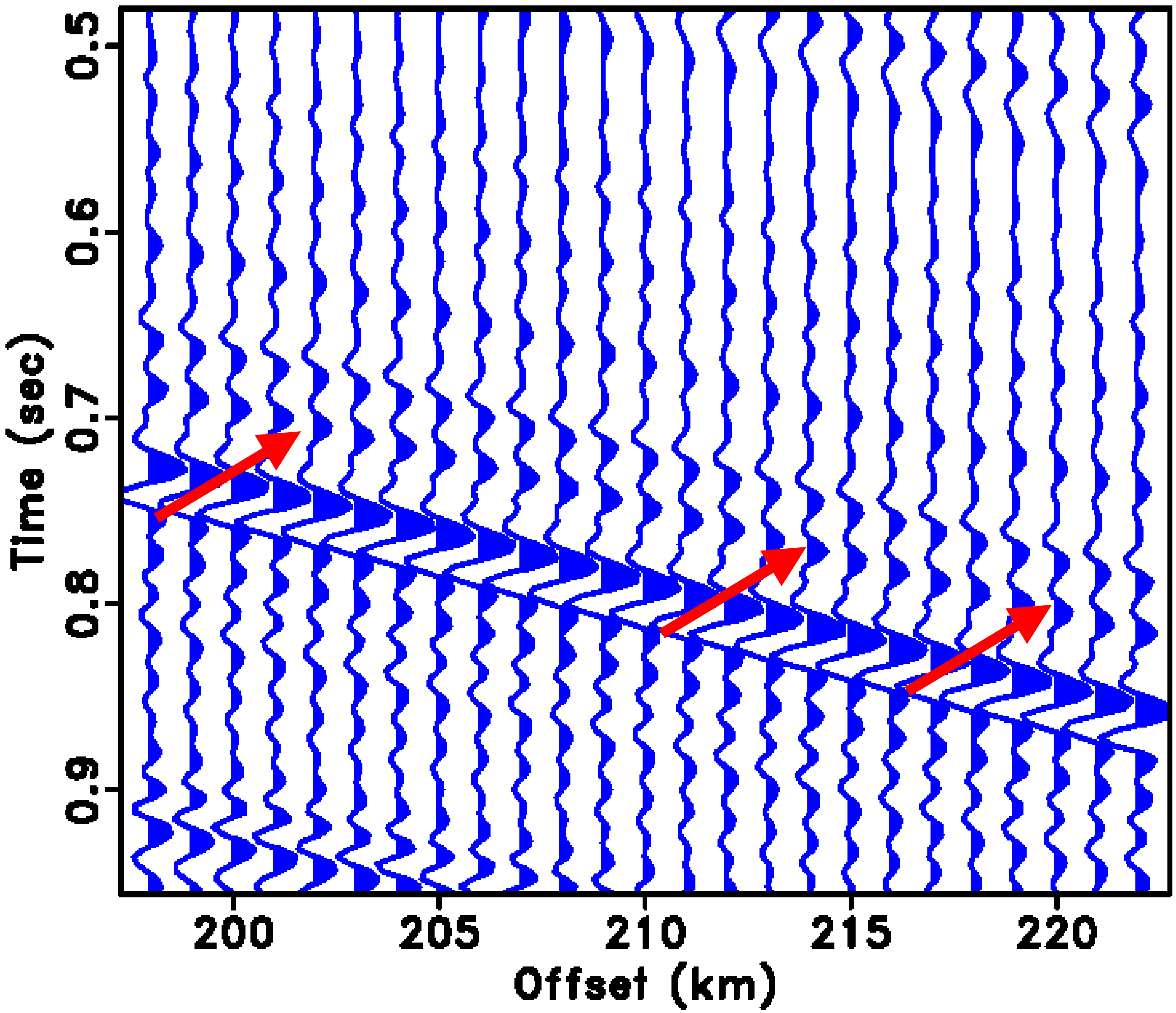


Now Zoom-in wiggle plots!

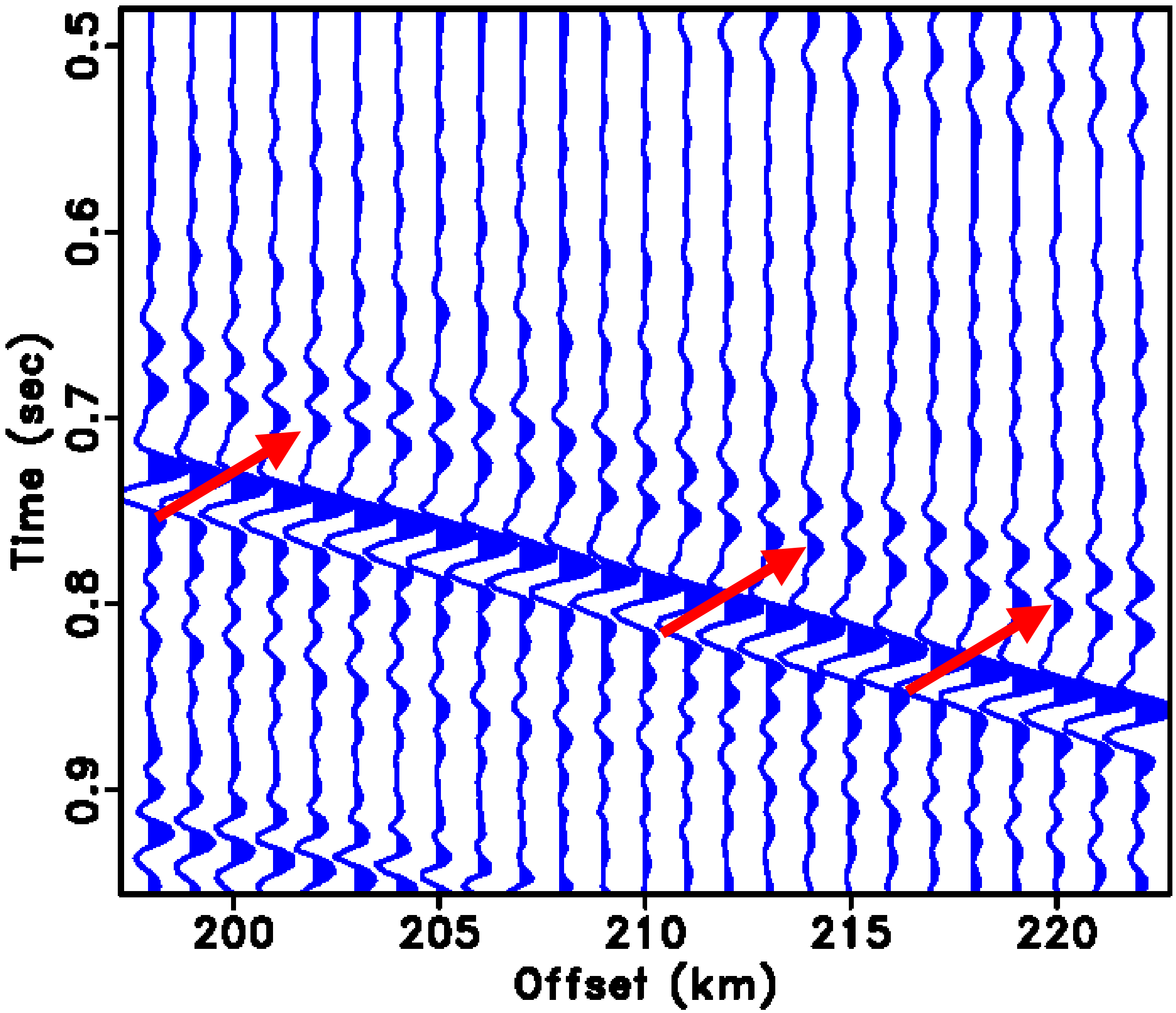
Data



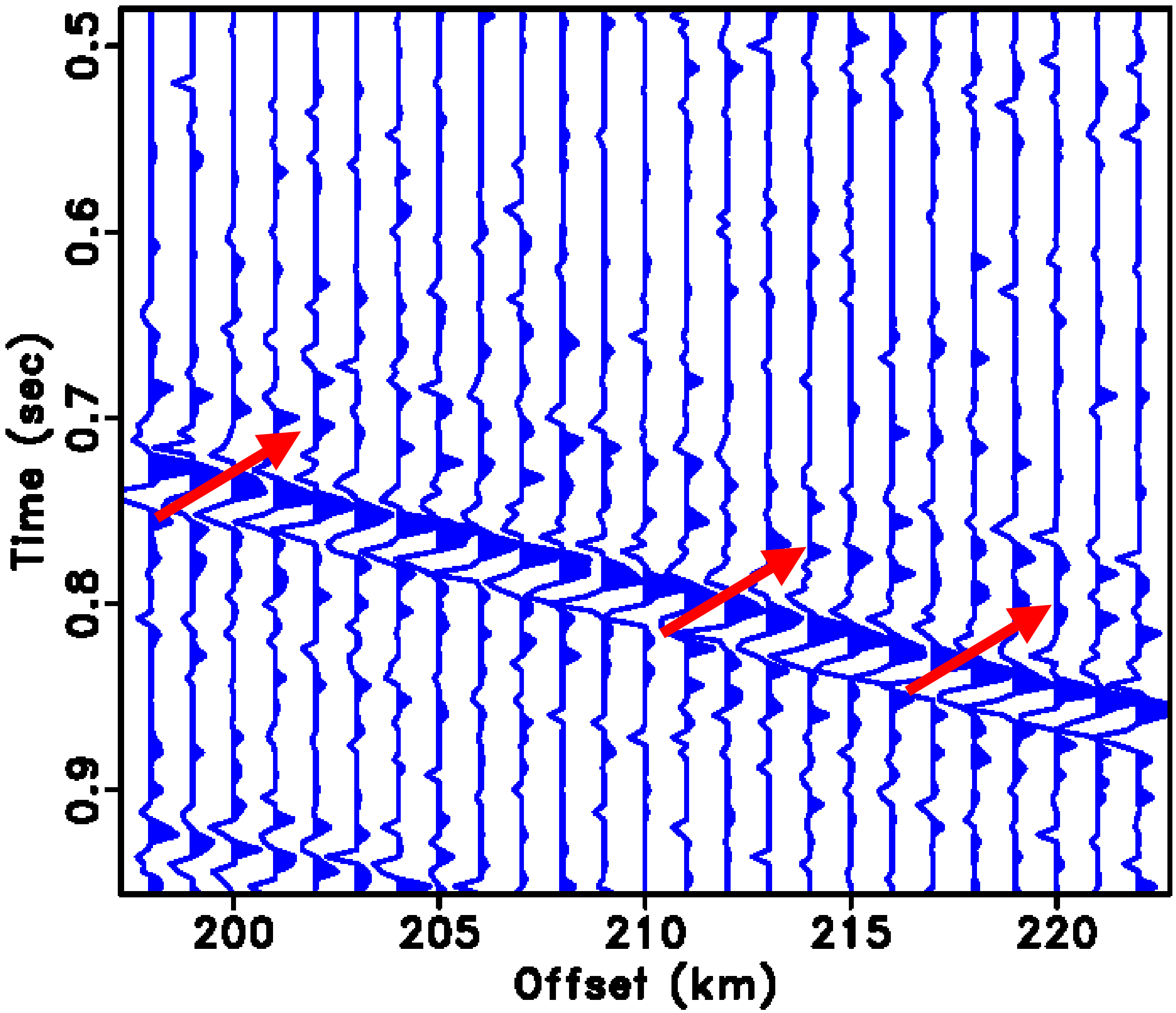
Original Reflectivity



Curvelet Inversion



Spiky Inversion

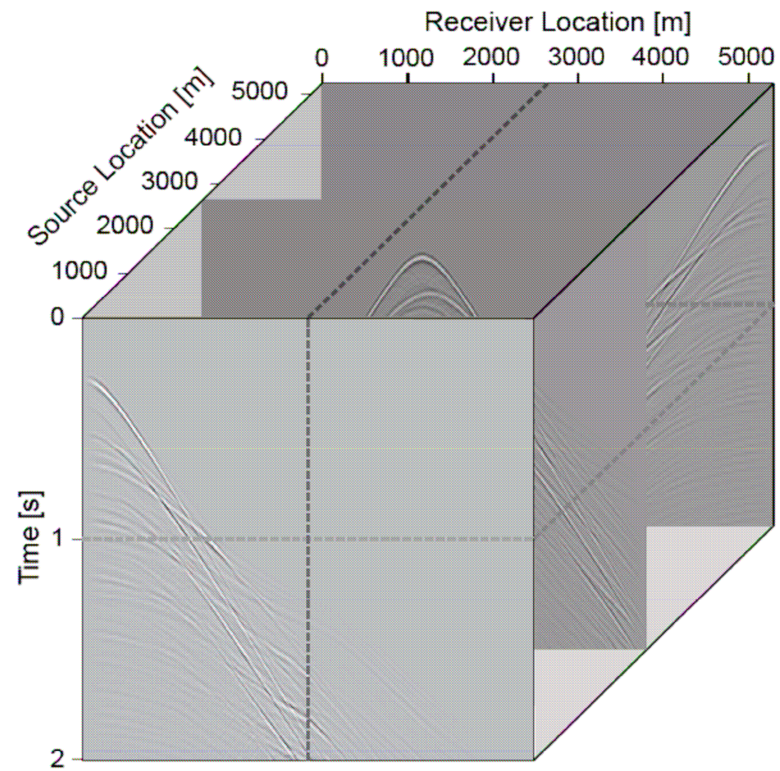


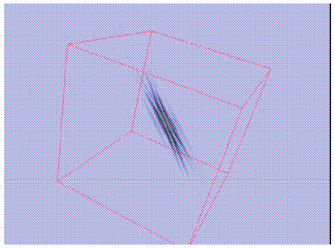
Conclusions

- The algorithm can be applied to estimate non-spiky reflectivity.
- The approach shows promising results by providing better resolution and noise attenuation compared to "Spiky Deconvolution".
- Unlike trace by trace, our approach exploits coherency between the neighborhood traces.

Future work

- Pre-stack 2-D data can be sorted in shot-time-receiver coordinate to output as a 3-D data cube
- 3-D curvelets can then be used for deconvolution
- 3-D curvelets exploit additional coherency among the different shot gathers.





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Thank you!
(Merci Beaucoup)