Phase-space matched filtering & migration preconditioning

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Motivation

Migration does not recover the amplitudes.

Least-squares migration is computationally unfeasible.

Lacks robustness w.r.t. noise.

Existing scaling methods

- do not always correct for the order (1 - 2D) of the Hessian (see also Symes '07)
- assume that there are no conflicting dips (conormal)
- do not invert the scaling robustly

Our approach exploits

- invariance of curvelets under the Hessian
- the smoothness of the symbol of the Hessian
- curvelet-domain sparsity

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Existing scaling methods

Methods are based on a diagonal approximation of Ψ.

- Illumination-based normalization (Rickett '02)
- Amplitude preserved migration (Plessix & Mulder '04)
- Amplitude corrections (Guitton '04)
- Amplitude scaling (Symes '07)

We are interested in an 'Operator and image adaptive' scaling method which

- estimates the action of Ψ from a reference vector close to the actual image
- assumes a smooth symbol of Ψ in space and angle
- does not require the reflectors to be conormal <=> allows for conflicting dips
- stably inverts the diagonal

Seismic imaging problem

Forward problem

$$F[c]u := \left(\frac{1}{c^2(x)} \cdot \frac{\partial^2}{\partial t^2} - \sum_{i=1}^{d} \frac{\partial^2}{\partial x_1^2} \right) \mathbf{u}(x,t) = f(x,t)$$

second order hyperbolic PDE

interested in the singularities of

$$m = c - \bar{c}$$

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Inverse problem

Minimization:

$$\widetilde{m} = \arg\min_{m} \|d - F[m]\|_2^2$$

After linearization (Born app.) forward model with noise:

$$d(x_s, x_r, t) = \big(Km\big)(x_s, x_r, t) + n(x_s, x_r, t)$$

Conventional imaging:

$$\begin{aligned} \big(K^T d\big)(x) &= \big(K^T K m\big)(x) + \big(K^T n\big)(x) \\ y(x) &= \big(\mathbf{\Psi} m\big)(x) + e(x) \end{aligned}$$

Ψ **is prohibitively expensive to invert!**

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Normal operator

[Stolk 2002, ten Kroode 1997, de Hoop 2000, 2003]

Alternative to expensive least-squares migration.

In high-frequency limit Ψ is a PsDO

$$\big(\Psi f\big)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} a(x,\xi)\hat{f}(\xi)\mathrm{d}\xi$$

- pseudolocal
- singularities are preserved
- High-frequency argument

Corresponds to a spatially-varying dip filter after appropriate preconditioning.

Our approach

Formulate as a sparsity- and continuity promoting optimization problem

$$\mathbf{P}: \begin{cases} \tilde{\mathbf{x}} = \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \left(\mathbf{A}^T\right)^{\dagger} \tilde{\mathbf{x}}, \end{cases}$$

Based on a diagonal approximation

$$\mathbf{A}\mathbf{A}^T\mathbf{r} \simeq \mathbf{\Psi}\mathbf{r} \text{ with } \mathbf{A} = \mathbf{C}^T\mathbf{D}_{\Psi}^{1/2}$$

with **r** the reference vector.

Estimate $\mathrm{D}_{\Psi}^{1/2}$ using ***smoothness*** of the **symbol**.

Diagonal approximation of the Hessian

Existing scaling methods

Methods are based on a diagonal approximation of Ψ.

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We are interested in an ‘Operator and image adaptive’ scaling method which

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- does not require the reflectors to be conormal <=> allows for conflicting dips
- stably inversion of the diagonal

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Math

"Precondition" the linearized (Born) modeling operator

$$\mathbf{d} = \mathbf{K}\mathbf{m}$$

with

$$K \mapsto K(-\Delta)^{-1/2} \quad \text{or} \quad K \mapsto \partial_t^{-1/2} K$$

$$m \mapsto (-\Delta)^{1/2} m \quad \text{with} \quad ((-\Delta)^{\alpha} f)^{\wedge}(\xi) = |\xi|^{2\alpha} \cdot \hat{f}(\xi).$$

such that the normal equation is near unitary

$$\begin{aligned} y &= K^T K m \\ &= \Psi m \end{aligned}$$

with $\Psi \approx Id$.

Math cont'd

In the high-frequency limit $\Psi = \Psi(x, D)$

- is a pseudodifferential operator of order 0
- has a homogeneous principal symbol $a(x, \xi)$
- acts as a nonstationary dip filter

Lemma 1 *Suppose a is in the symbol class $S^0_{1,0}$, then, with C' some constant, the following holds*

$$\|(\Psi(x, D) - a(x_\nu, \xi_\nu))\varphi_\nu\|_{L^2(\mathbb{R}^n)} \leq C' 2^{-|\nu|/2}. \quad (1)$$

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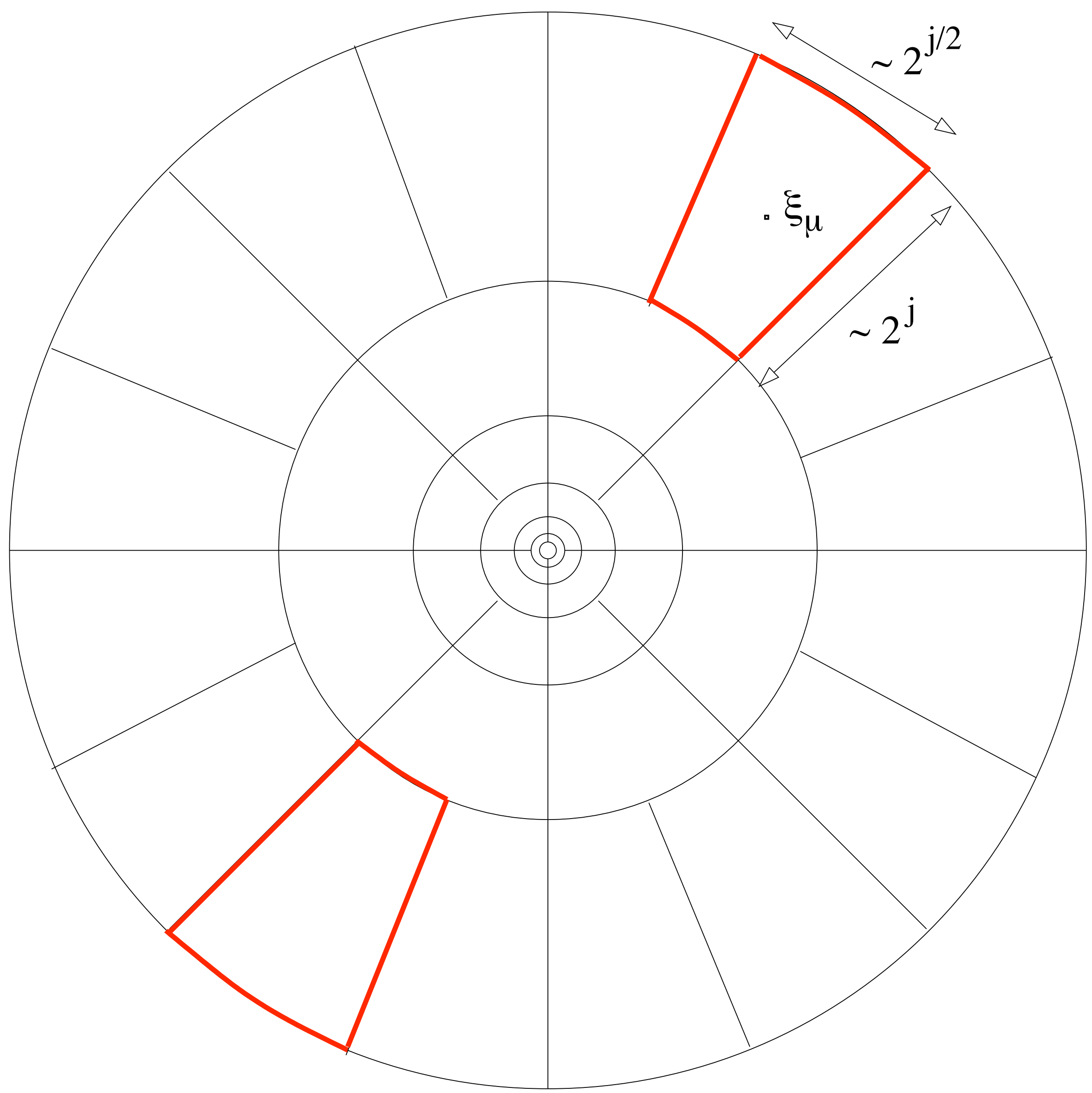
Tiling the ξ space



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Math cont'd

To approximate Ψ, define the sequence $\mathbf{u} := (u_\mu)_{\mu\in\mathcal{M}} = a(x_\mu, \xi_\mu)$. Let $\mathbf{D}_\Psi$ be the diagonal matrix with entries given by $\mathbf{u}$, i.e.,

$$\mathbf{D}_\Psi := \mathrm{diag}\big(\mathbf{u}\big).$$

Bound for accuracy of the diagonal approximation

$$\Psi \simeq C^T \mathbf{D}_\Psi C.$$

Theorem 1 *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T \mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' is a constant depending on Ψ.

- accuracy improves for higher frequencies
- amenable for sparsity-promoting inversion

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Math cont'd

Allows for an"eigenfunction like" decomposition

$$\begin{aligned}(\Psi\varphi_\mu)(x) &\simeq (C^T\mathbf{D}_\Psi C\varphi_\mu)(x)\\ &= (AA^T\varphi_\mu)(x)\end{aligned}$$

with $A := \sqrt{\mathbf{D}_\Psi}C$ and $A^T := C^T\sqrt{\mathbf{D}_\Psi}$.

Approximation

normal operator

$$
\begin{aligned}
y(x) &= \big(\Psi m\big)(x) + e(x) \\
&\simeq \big(AA^T m\big)(x) + e(x) \\
&= Ax_0 + e,
\end{aligned}
$$

Wavelet-vagulette like

Amenable to nonlinear recovery

Remains to estimate

- scaling coefficients / matched filter coefficients
- use a reference image

Curvelet-domain matched filtering

Matched filtering

Adapt current scaling methodology to phase space.

Exploit smoothness of the symbol valid for smooth velocity models.

Use a reference image sufficiently close to the actual reflectivity.

Generate ‘data’

$$\mathbf{b} = \mathbf{\Psi} \mathbf{r}$$

with

$$\begin{aligned} \mathbf{\Psi} &= \mathbf{K}^T \mathbf{K} \\ \mathbf{K} &= \text{discretized linearized Born modeling operator} \\ \mathbf{r} &= \text{reference vector} \end{aligned}$$

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Original formulation

matched filtering

Find '*positive-entry*' ***scaling*** vector **u** such that

$$\mathbf{b} \approx \mathbf{C}^T \mathbf{D}_\Psi \mathbf{C}\mathbf{r} \quad \text{with} \quad \mathbf{D}_\Psi = \mathrm{diag}(\mathbf{u})$$

by solving the *linear* least-squares problem

$$\tilde{\mathbf{u}} = \arg\min_{\mathbf{u}} \frac{1}{2}\|\mathbf{b} - \mathbf{P}\mathbf{u}\|_2^2 + \eta^2\|\mathbf{L}\mathbf{u}\|_2^2$$

with

$$\mathbf{P} := \mathbf{C}^T \mathrm{diag}(\mathbf{C}\mathbf{r})$$

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Original formulation

matched filtering

Impose *smoothness* in phase space

$$\mathbf{L} = [\mathbf{D}_1 \quad \mathbf{D}_2 \quad \mathbf{D}_\theta]$$

Calculate: $\mathbf{b} = \mathbf{\Psi r}$ and $\mathbf{v} = \mathbf{Cr}$.

Set: $\eta = \eta_{min}$;

while $\exists\, (\tilde{u}_\mu)_{\mu \in \mathcal{M}} < 0$ **do**

Solve

$\widetilde{\mathbf{u}} = \arg\min_{\mathbf{u}} \frac{1}{2}\|\mathbf{b} - \mathbf{Pu}\|_2^2 + \eta^2\|\mathbf{Lu}\|_2^2$

Increase the Lagrange multiplier

$\lambda = \eta + \Delta\eta$

end while

Observations

Computation of matched-filter coefficients *expensive*.

- no 'real' positivity *constraint* while
- $\mathbf{K}^T\mathbf{K}$ is a *positive definite* matrix

In addition, our approach does *not* accommodate

- precise phase-space *smoothness*
- *flexibility* to handle *black-box* implementations
- migration operator *preconditioning*
- *incomplete* data
- seismic *source* function

New formulation

matched filtering

Find *positive-entry scaling* vector **u** such that

$$\mathbf{b} \approx \mathbf{C}^T \mathbf{D}_\Psi \mathbf{C}\mathbf{r} \quad \text{with} \quad \mathbf{D}_\Psi = \mathrm{diag}(\mathbf{u})$$

Translates into minimizing

$$J_\gamma(\mathbf{z}) = \frac{1}{2}\|\mathbf{d} - \mathbf{F}_\gamma \exp(\mathbf{z})\|_2^2 \quad \text{with} \quad \tilde{\mathbf{u}} = \exp(\tilde{\mathbf{z}})$$

involving the following system of equations

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{r}\} \\ \gamma \mathbf{L} \end{bmatrix} \mathbf{w} \qquad \text{or} \qquad \mathbf{d} = \mathbf{F}_\gamma \mathbf{w}$$

with the gradient

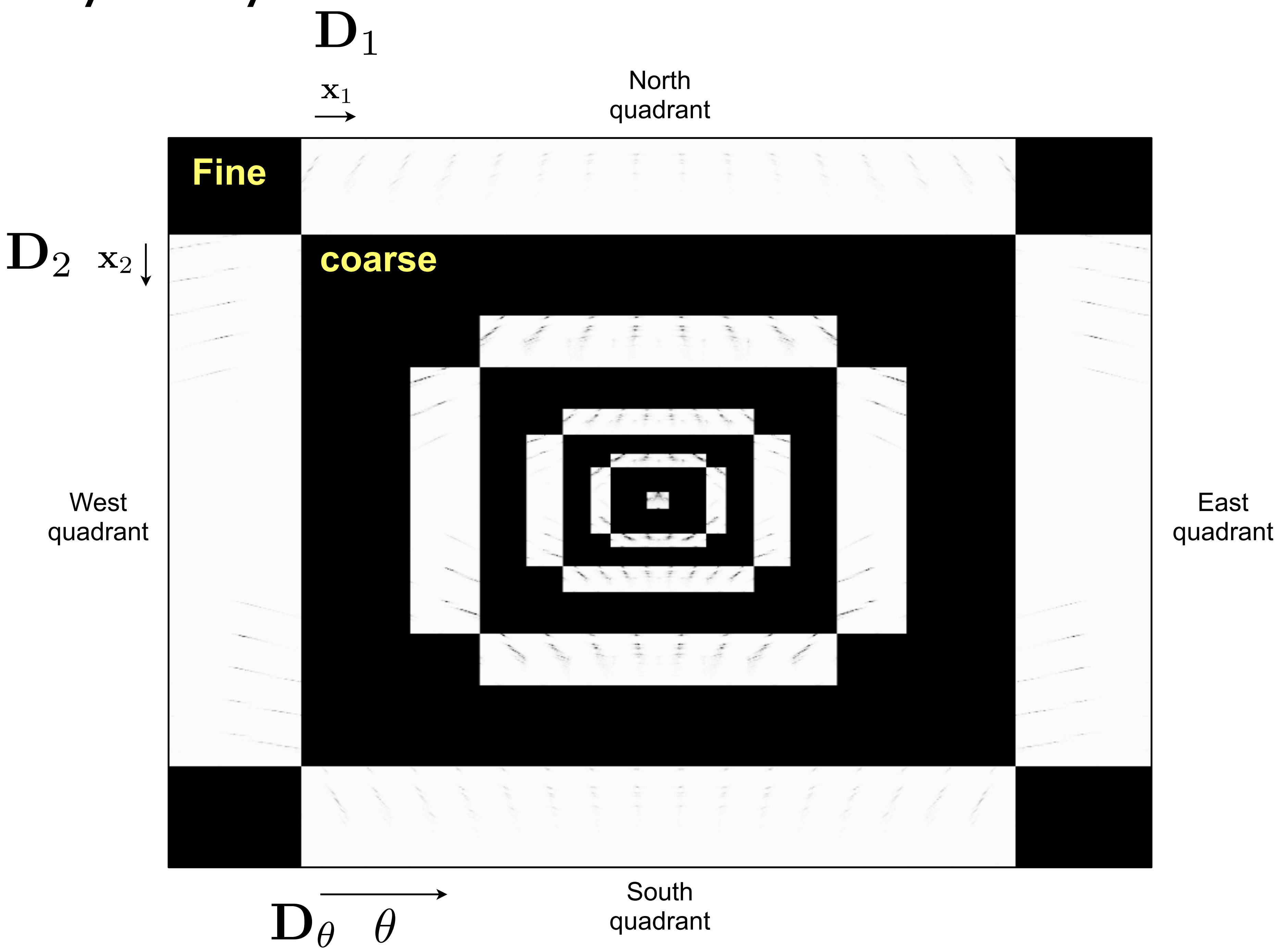
$$\mathrm{grad} J(\mathbf{z}) = \mathrm{diag}\{e^{\mathbf{z}}\}\left[\mathbf{F}^T\left(\mathbf{F}e^{\mathbf{z}} - \mathbf{d}\right)\right]$$

Matched filtering

Impose smoothness in phase space through

$$\mathbf{L} = \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \mathbf{D}_\theta^T \end{bmatrix}^T$$

Positivity of symbol is assured.



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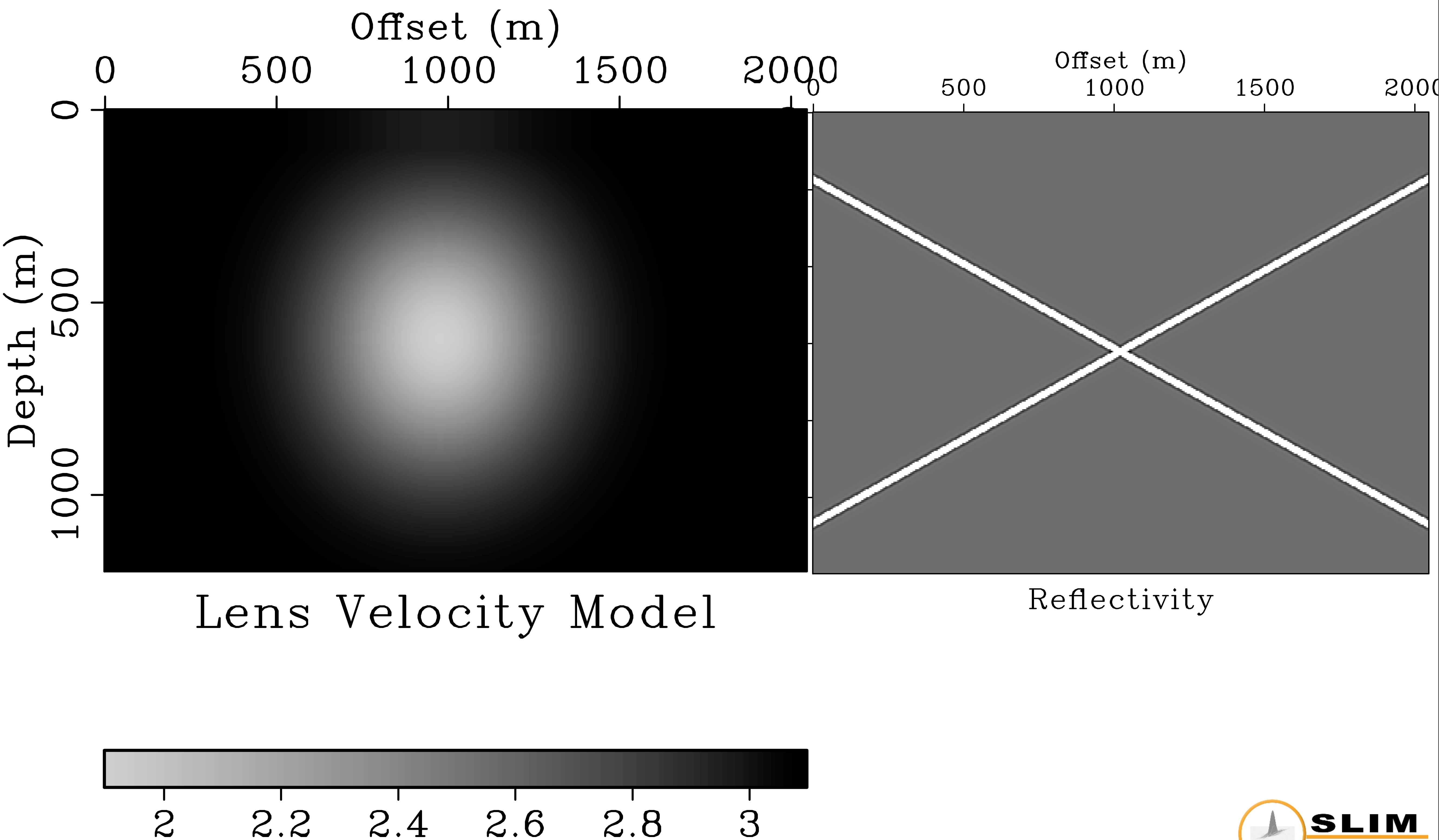
Example



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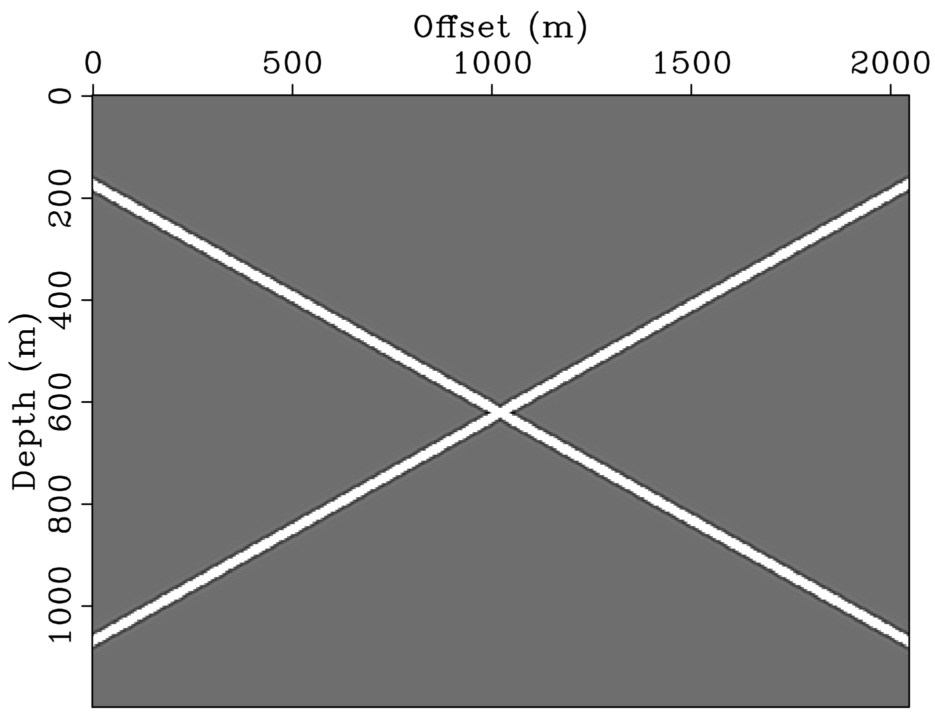
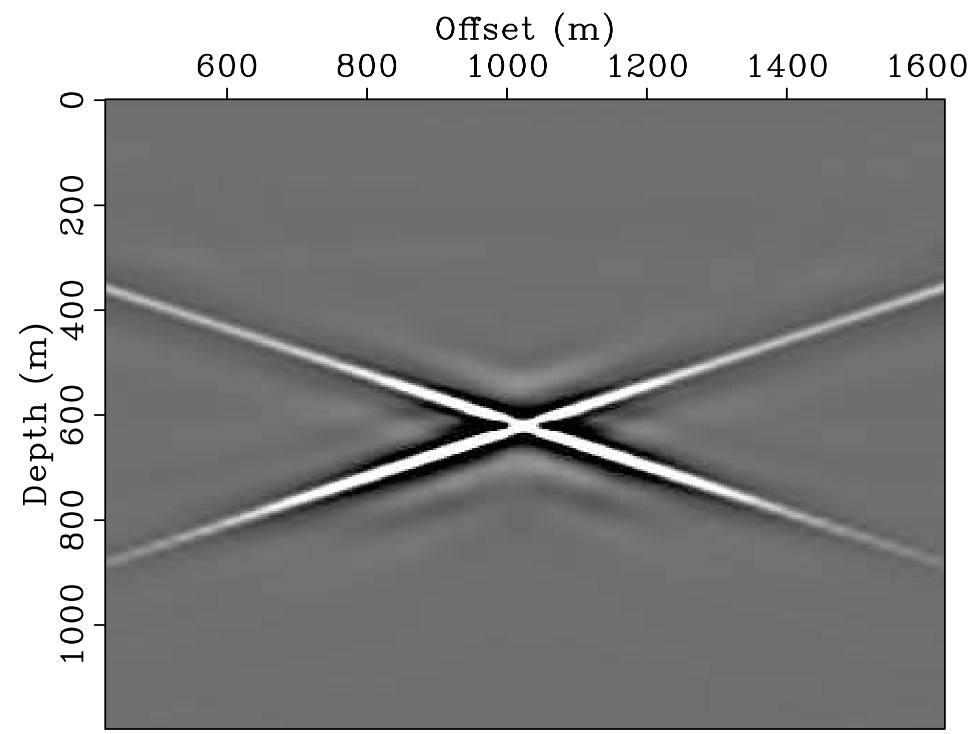
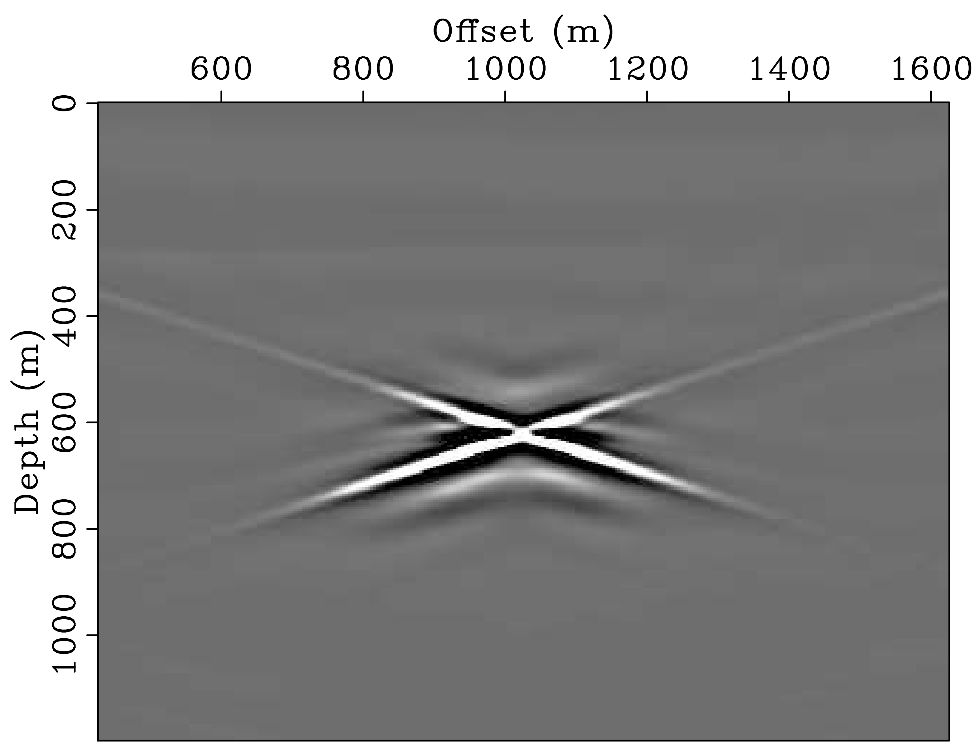
Example



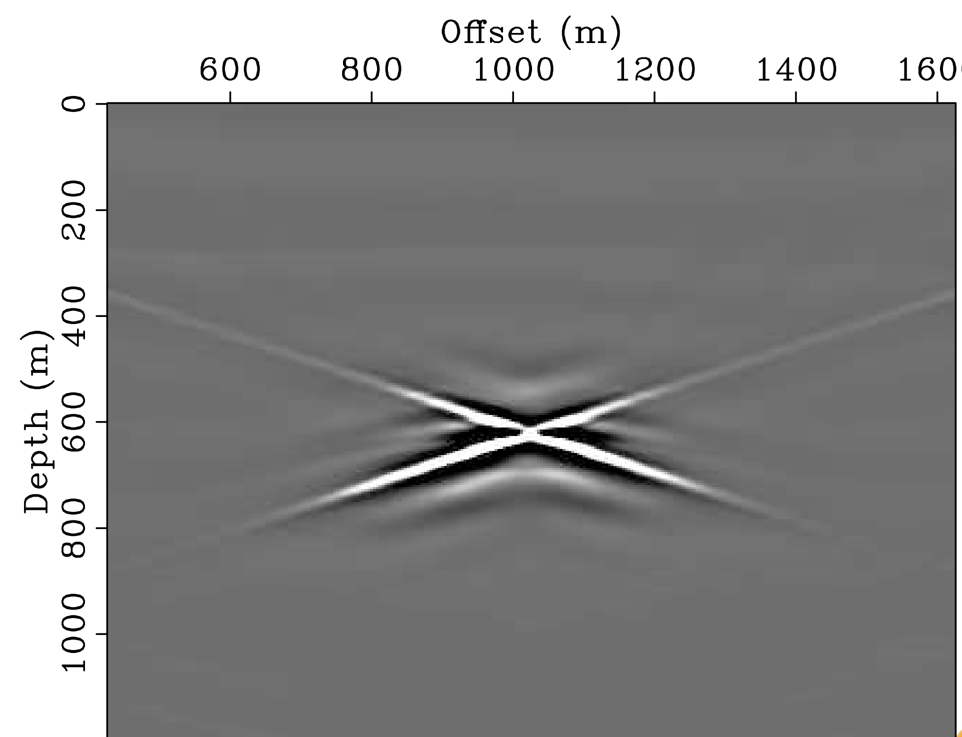
Reflectivity



Migrated Image



Remigrated



Approximate Remigrated

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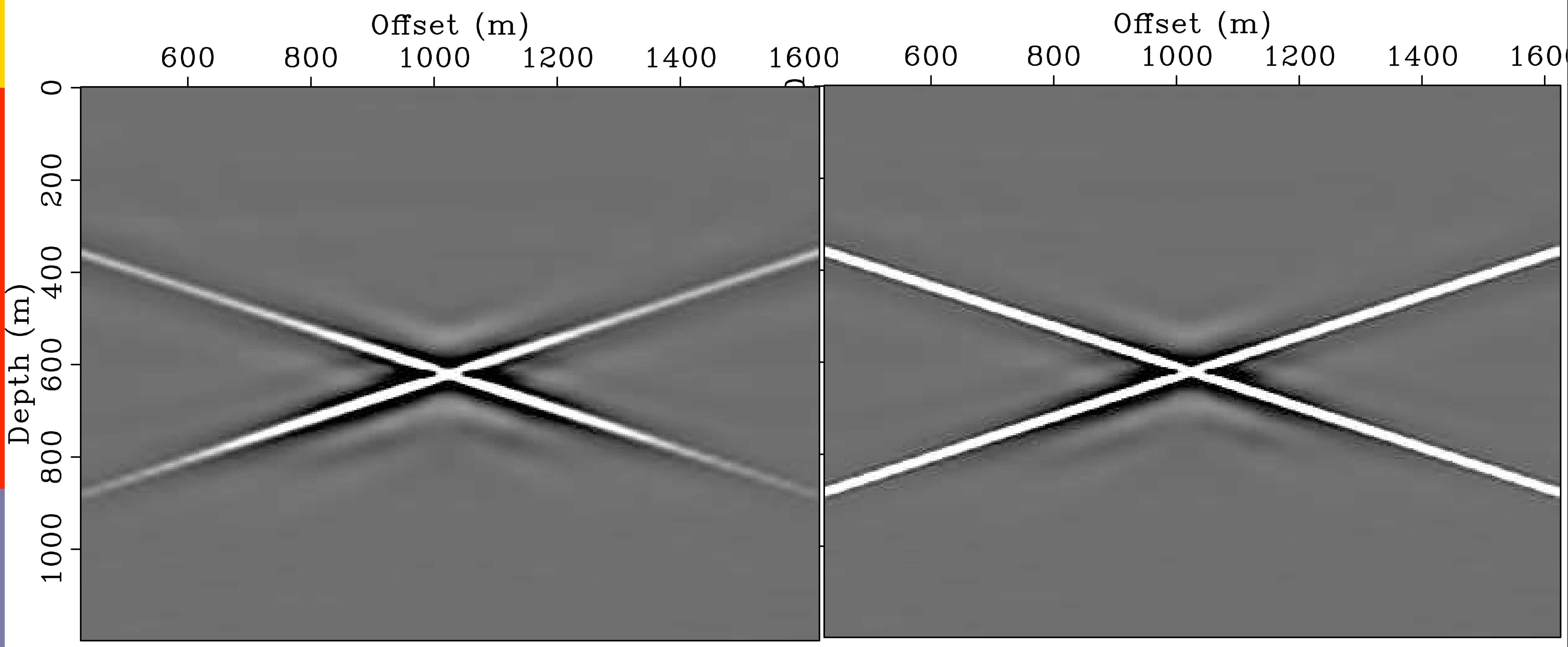
Example



Migrated Image

Enhanced Image

Matched filtering

new parameterization

Problems:

- computation of the matched-filter coefficients expensive (# of unknows = length of curvelet vector)
- Limited smoothness

Parameterize phase space (x, y, θ)

- introduce low-dimensional parameterization phase space
- use B-splines
- define the scaling vector in terms of a spline synthesis

$$\mathbf{u} = \mathbf{B}\boldsymbol{\alpha}$$

with for each scale

$$\mathbf{B} = [\mathbf{B}_1 \quad \mathbf{B}_2 \quad \mathbf{B}_\theta]$$

Matched filtering

new parameterization

Find *positive-entry* scaling vector **u** such that

$$\mathbf{b} \approx \mathbf{C}^T \mathbf{D}_\Psi \mathbf{C} \mathbf{r} \quad \text{with} \quad \mathbf{D}_\Psi = \text{diag}(\mathbf{u})$$

Minimize

$$J_\gamma(\boldsymbol{\alpha}) = \frac{1}{2}\|\mathbf{b} - \mathbf{F}\exp(\mathbf{B}\boldsymbol{\alpha})\|_2^2 + \frac{1}{2}\gamma\|\boldsymbol{\alpha}\|_2^2$$

- with

$$\mathbf{F} = \mathbf{C}^H \text{diag}(\mathbf{C}\mathbf{r}) \text{ and } \tilde{\mathbf{u}} = \exp(\mathbf{B}\boldsymbol{\alpha})$$

- assures positivity (Vogel ’02)
- smoothness depends on # of splines and regularization

With the gradient (Vogel ‘02)

$$\text{grad} J(\boldsymbol{\alpha}) = \text{diag}\{\exp(\mathbf{B}\boldsymbol{\alpha})\}\left[\left(\mathbf{F}^T\mathbf{F} + \gamma\mathbf{I}\right)\exp(\mathbf{B}\mathbf{z}) - \mathbf{F}^T\mathbf{b}\right]$$

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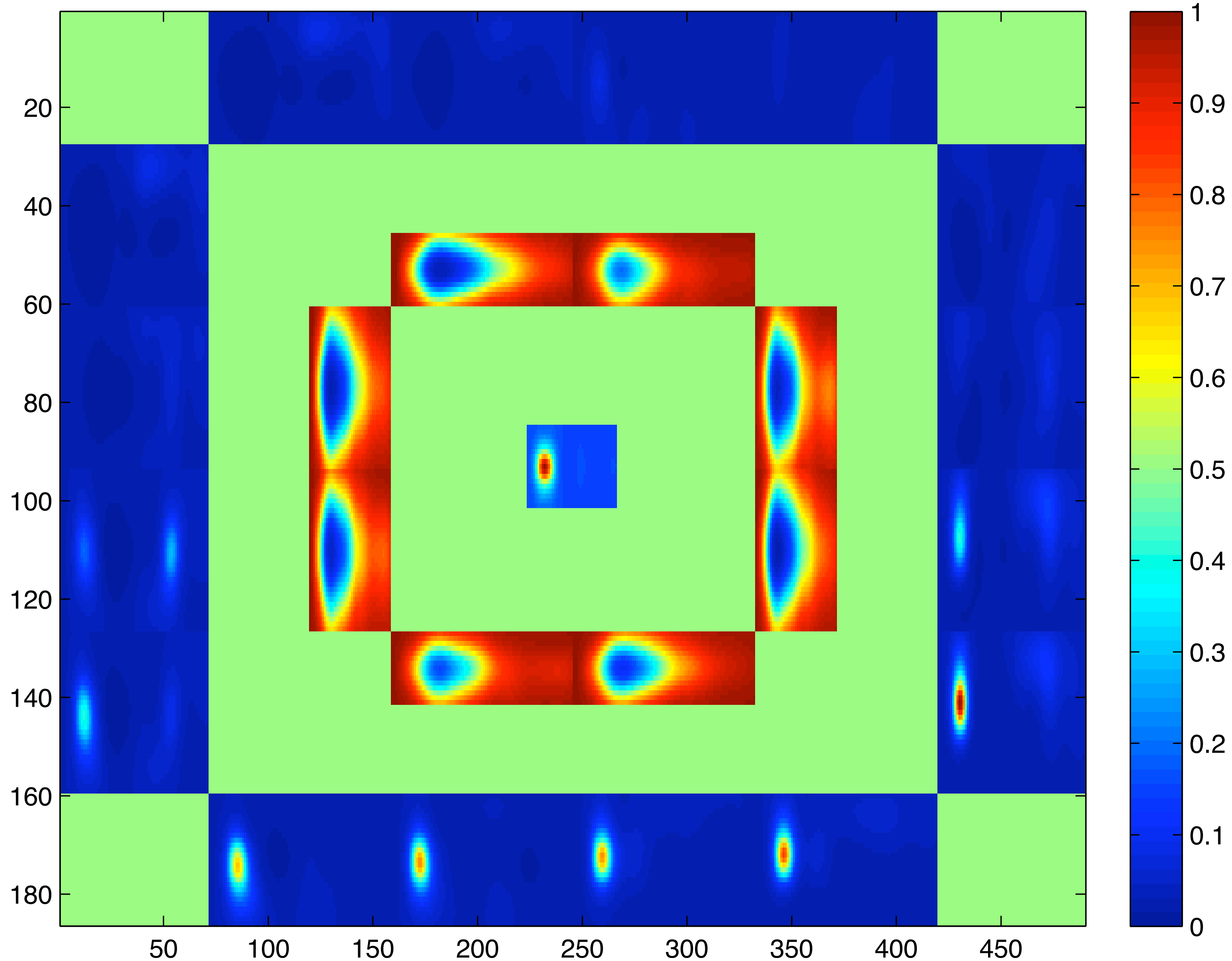
Spline



Lambda=0 perc=1%

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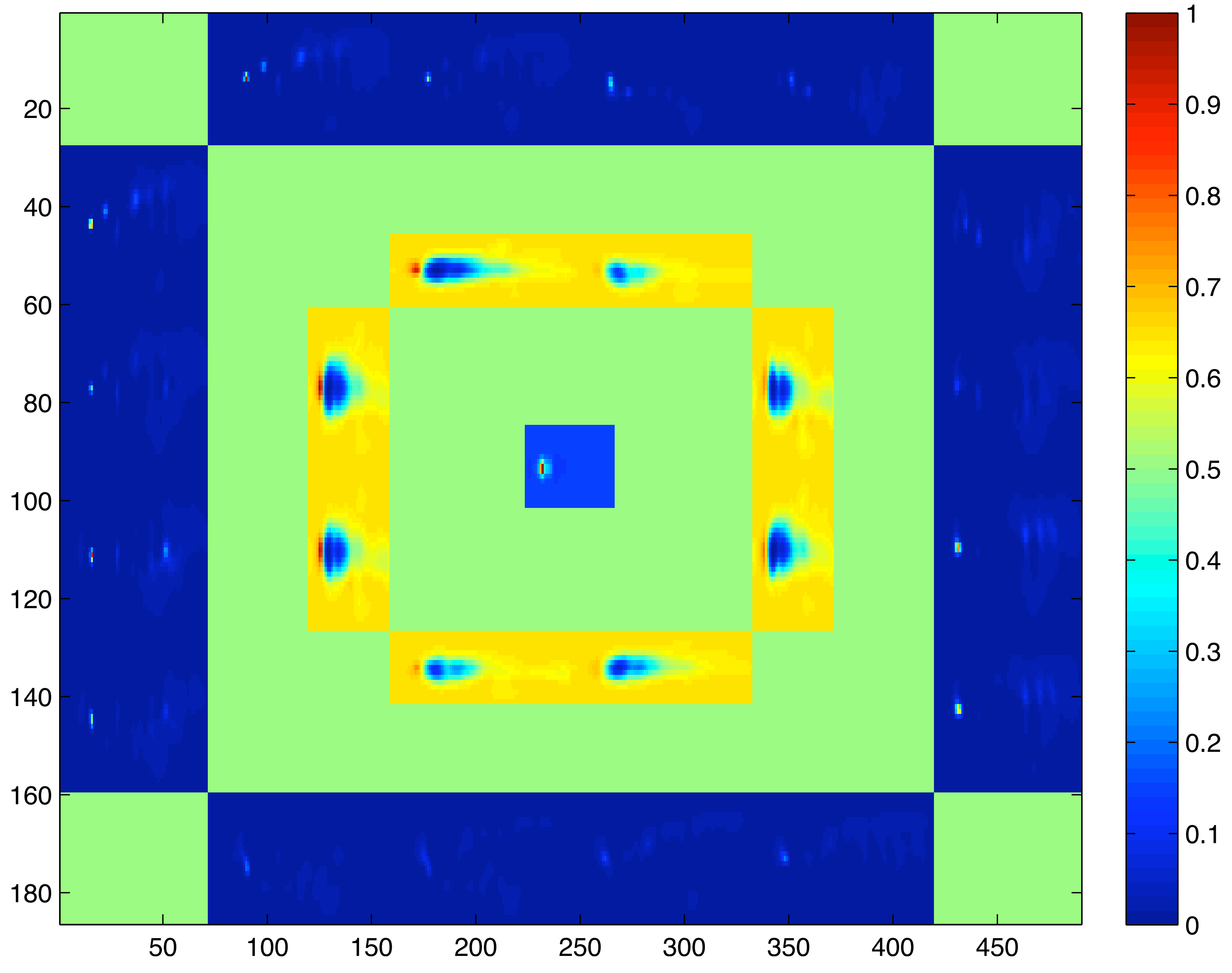
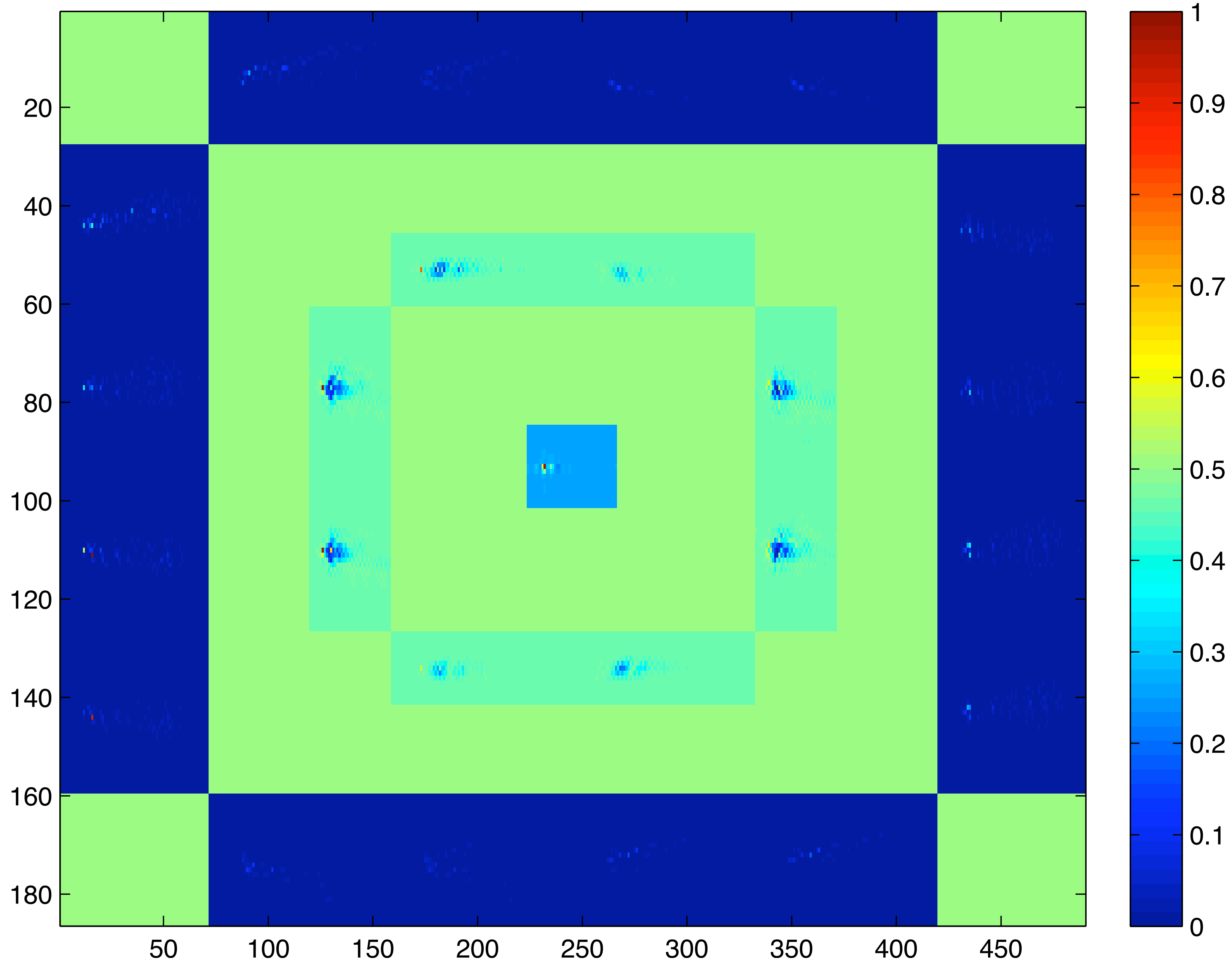
Spline



Lambda=0 perc=3%

Spline



Lambda=0 perc=30%

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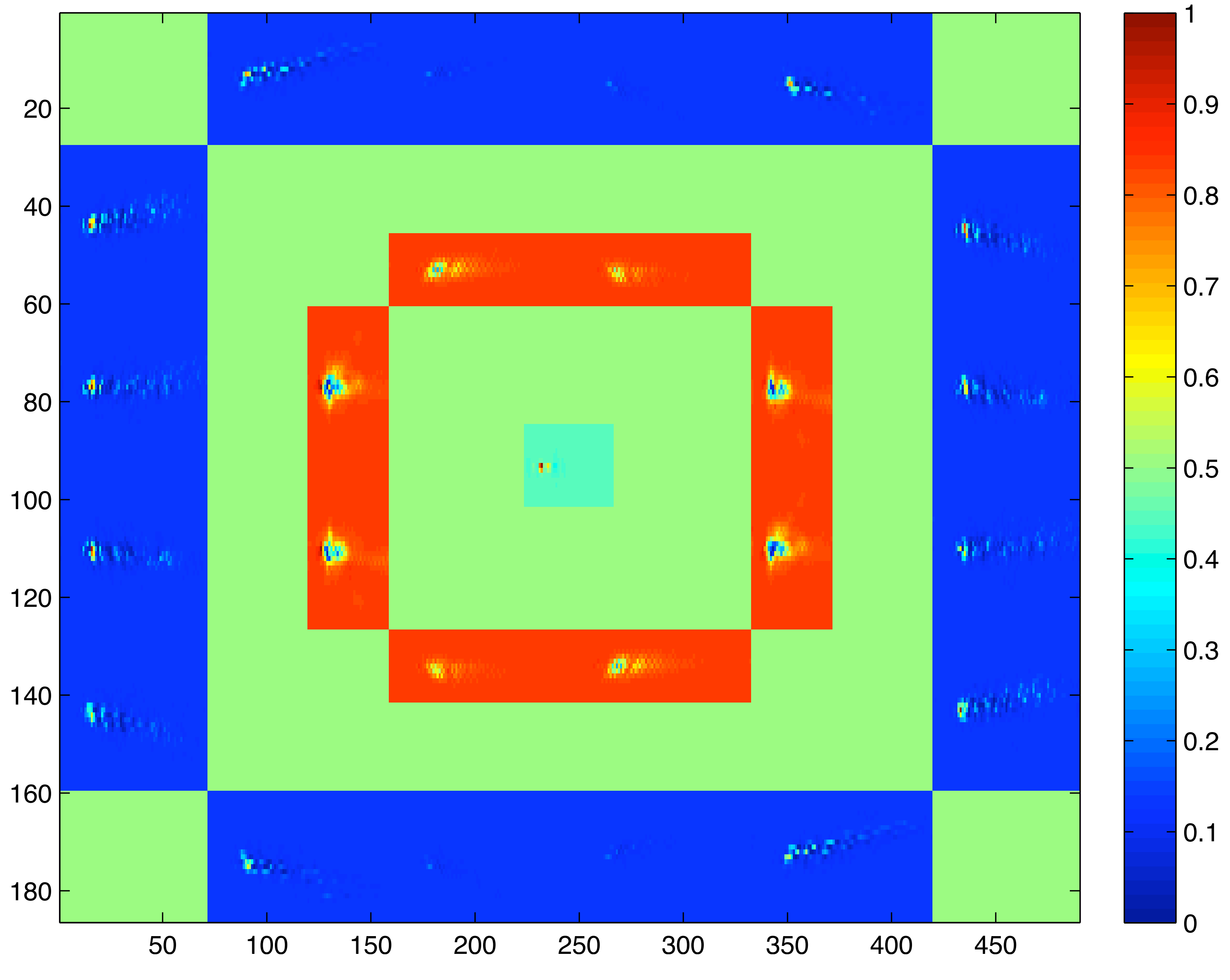
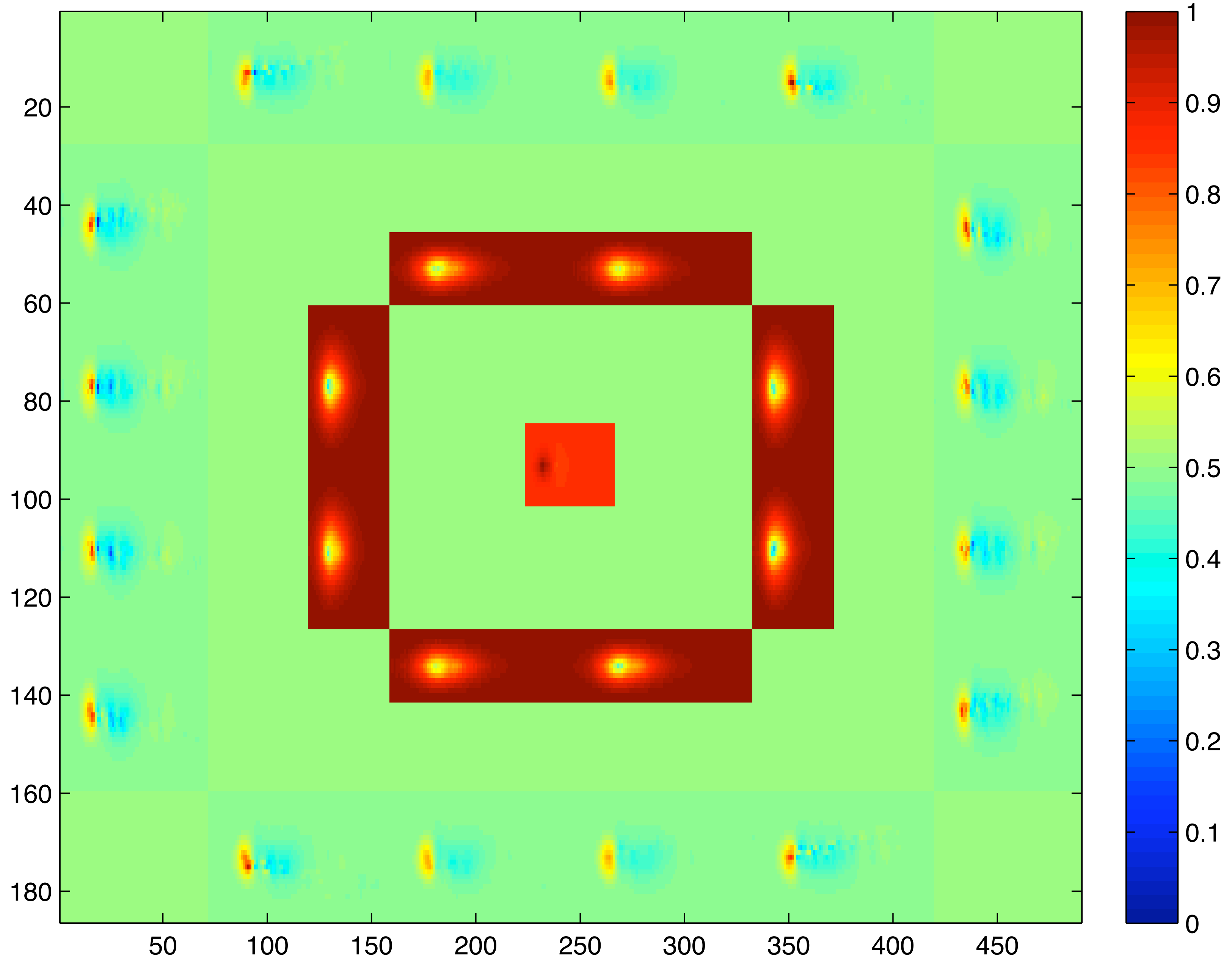
No spline



Lambda=0.01

No spline



Lambda=0.05

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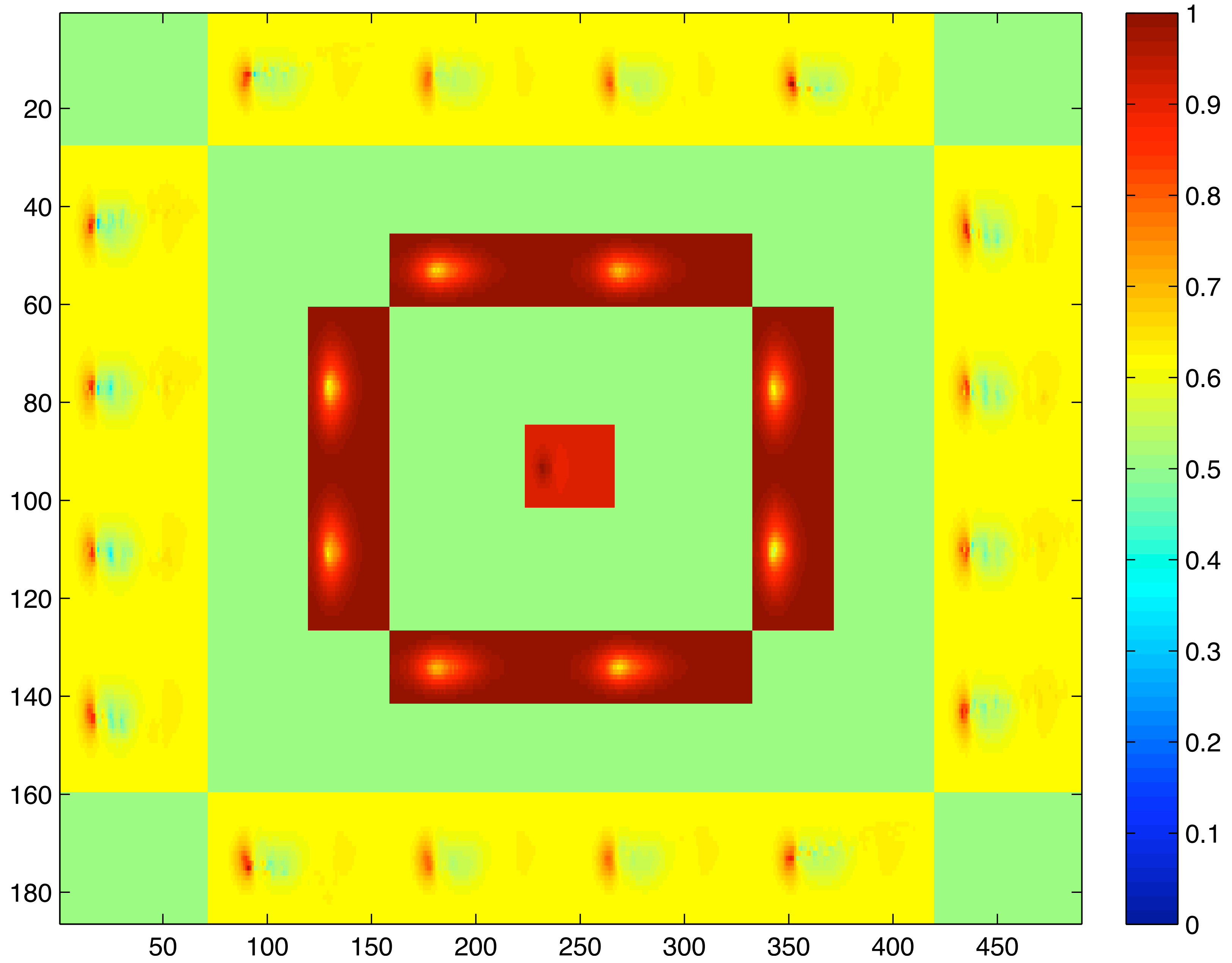
No spline



Lambda=0.07

Observations

Precise smoothness control

- # of nodes
- size of the regularization parameter

Dimensionality reduction should lead to faster convergence.

More work to study the performance.

Extensions

Black-box utility

Approach relied on zero-order operator.

- known when operators are understood exactly
- corrected for in the "migration operator" wavelet

Introduce additional matched filter by 'generating data'

$$\mathbf{f} = \mathbf{K}\mathbf{K}^T\mathbf{d}$$

and minimize

$$J_\eta(\mathbf{z}) = \frac{1}{2}\|\mathbf{f} - \boldsymbol{\mathcal{F}}^H \,\mathrm{diag}(\boldsymbol{\mathcal{F}}\mathbf{d})\exp(2\mathbf{z})\|_2^2 + \frac{1}{2}\eta\|\mathbf{L}\mathbf{z}\|_2^2$$

Redefine

$$\mathbf{K} \mapsto \boldsymbol{\mathcal{F}}^H \,\mathrm{diag}(\tilde{\mathbf{h}})\boldsymbol{\mathcal{F}}\mathbf{K} \quad \text{and} \quad \tilde{\mathbf{h}} = \exp(\tilde{\mathbf{z}})$$

or equivalently replace the source function

$$\boldsymbol{\phi} \mapsto \boldsymbol{\mathcal{F}}^H \,\mathrm{diag}(\tilde{\mathbf{h}})\boldsymbol{\mathcal{F}}\boldsymbol{\phi}$$

Migration preconditioning

Forward model:

$$\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n}$$

Ideal right preconditioning

$$\begin{aligned} \mathbf{K} &\mapsto \mathbf{K}\left(\mathbf{K}^T\mathbf{K}\right)^{-1/2} \\ \mathbf{m} &\mapsto \left(\mathbf{K}^T\mathbf{K}\right)^{1/2}\mathbf{m} \end{aligned}$$

yielding

$$\mathbf{K}^T\mathbf{K} = \mathbf{I}$$

Migration preconditioning

Approximate with curvelet preconditioning.

Define

$$\mathbf{A} := \mathbf{K}\mathbf{C}^T\mathbf{D}_\Psi^{-\frac{1}{2}}$$

$$\mathbf{x} := \mathbf{D}_\Psi^{\frac{1}{2}}\mathbf{C}\mathbf{m}$$

Such that

$$\mathbf{A}^T\mathbf{A} \approx \mathbf{I}$$

by virtue of

$$\mathbf{K}^T\mathbf{K} \simeq \mathbf{C}^H\mathbf{D}_\Psi\mathbf{C}$$

- calculate the diagonal approximation from reference vector and demigrated-migrated reference vector
- estimate the inverse square root directly

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Migration

preconditioning

Minimize

$$J_\gamma(\boldsymbol{\alpha}) = \frac{1}{2}\|\mathbf{r} - \mathbf{F}\exp(2\cdot\mathbf{B}\boldsymbol{\alpha})\|_2^2 + \frac{1}{2}\gamma\|\boldsymbol{\alpha}\|_2^2$$

with

$$\mathbf{F} = \mathbf{C}^H \mathrm{diag}(\mathbf{Cb}) \text{ and } \tilde{\mathbf{u}} = \exp(\mathbf{B}\boldsymbol{\alpha})$$

yielding

$$\mathbf{D}_\psi^{-\frac{1}{2}} = \mathrm{diag}(\tilde{\mathbf{u}})$$

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Seismic data recovery

Migration operator is expensive but the ultimate interpolator.

Solve

$$\mathbf{P}: \quad \begin{cases} \mathbf{y} = \mathbf{R}\mathbf{d} \\ \tilde{\mathbf{x}} = \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \mathbf{A} := \mathbf{R}\mathbf{K}\mathbf{C}^T \\ \tilde{\mathbf{d}} = \mathbf{K}\mathbf{C}^T\tilde{\mathbf{x}} \\ \tilde{\mathbf{x}} = \mathbf{C}^T\tilde{\mathbf{x}} \end{cases}$$

- recovery of data and image from incomplete data
- compression of the operator (e.g. subset of shots or temporal frequencies)
- migration will enhance the recovery
  - increased incoherence
  - additional focusing

Conclusions & future plans

Low-dimensional spline offers more control

Formulation remains to be tested

- for migration-amplitude recovery
- primary-multiple separation

Extensions

- will be reported on during next meeting

Migration based wavefield recovery seems natural but is not the only choice.

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