(De)Focused wavefield reconstructions

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joint work with Deli Wang and Gilles Hennenfent

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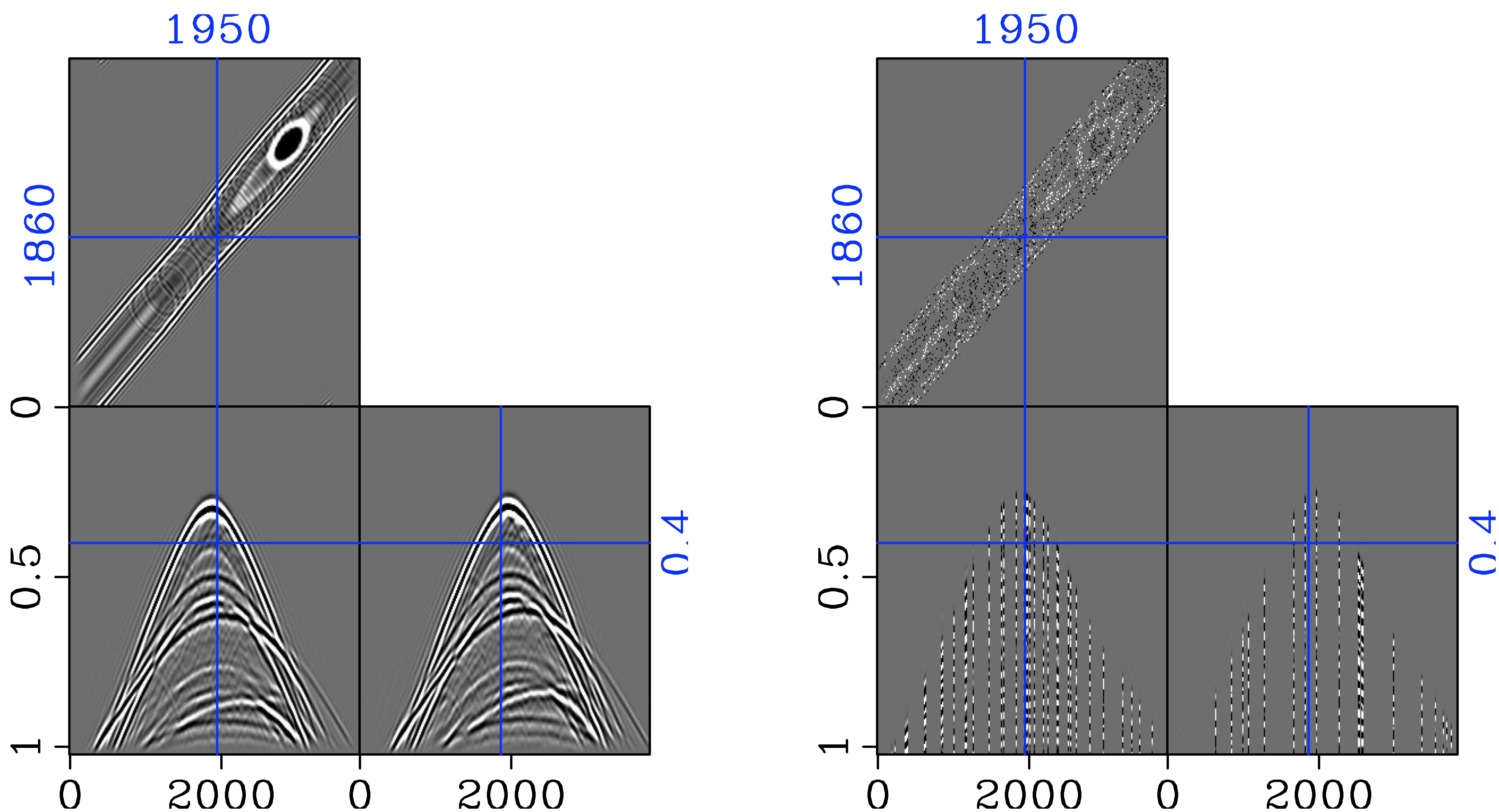
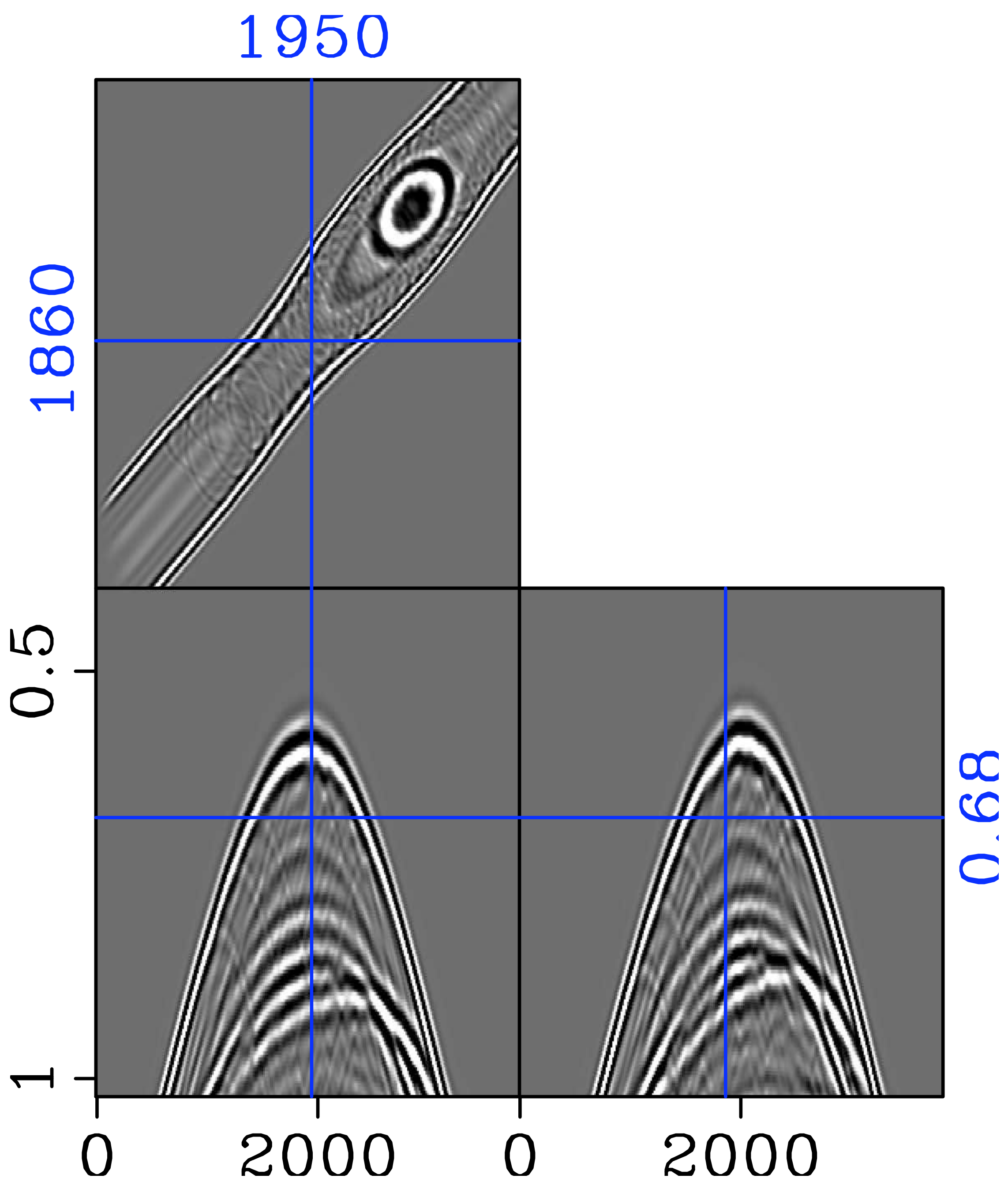
The problem



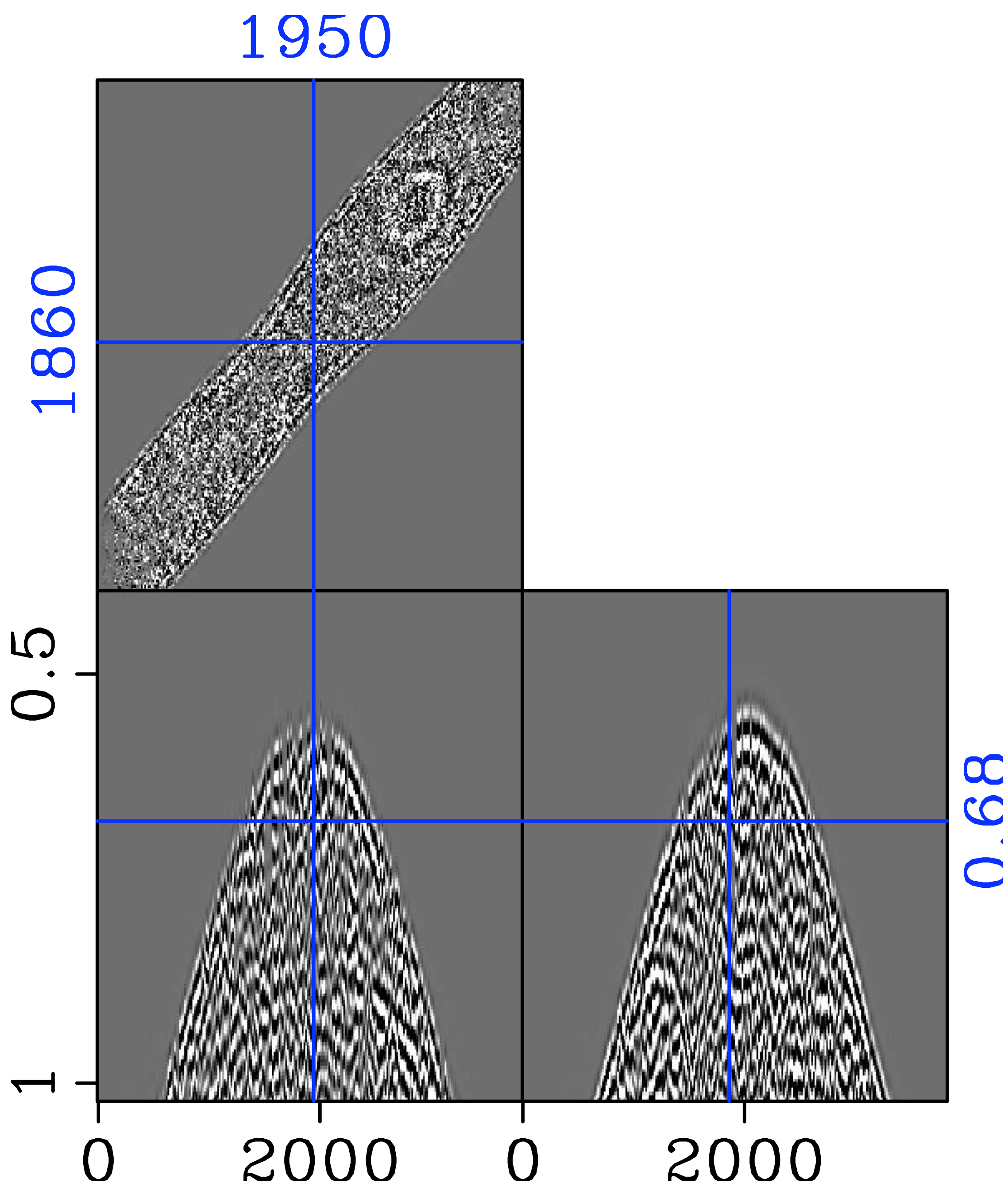
Total data

85 % traces missing

The problem cont'd

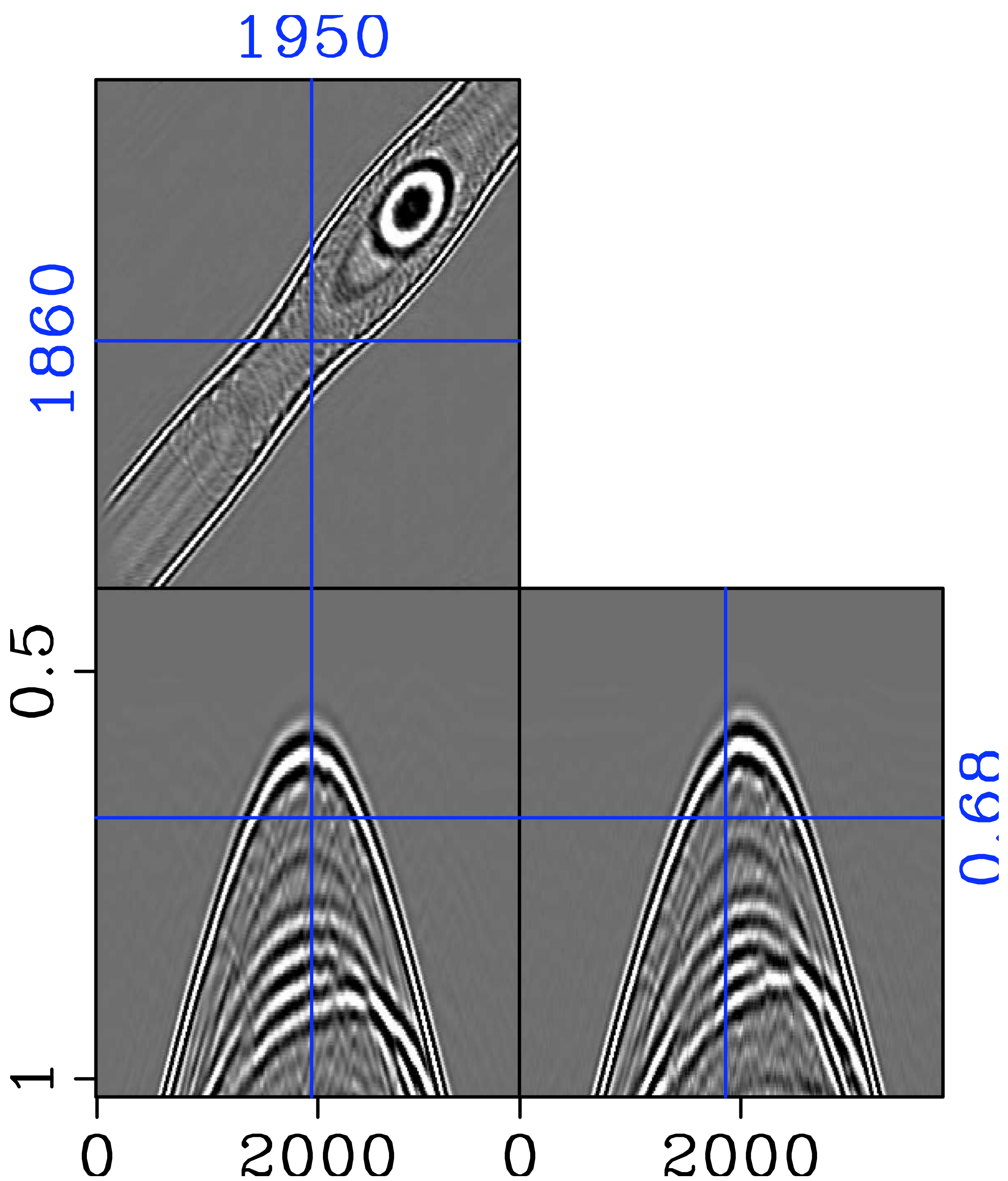


SRME from complete data

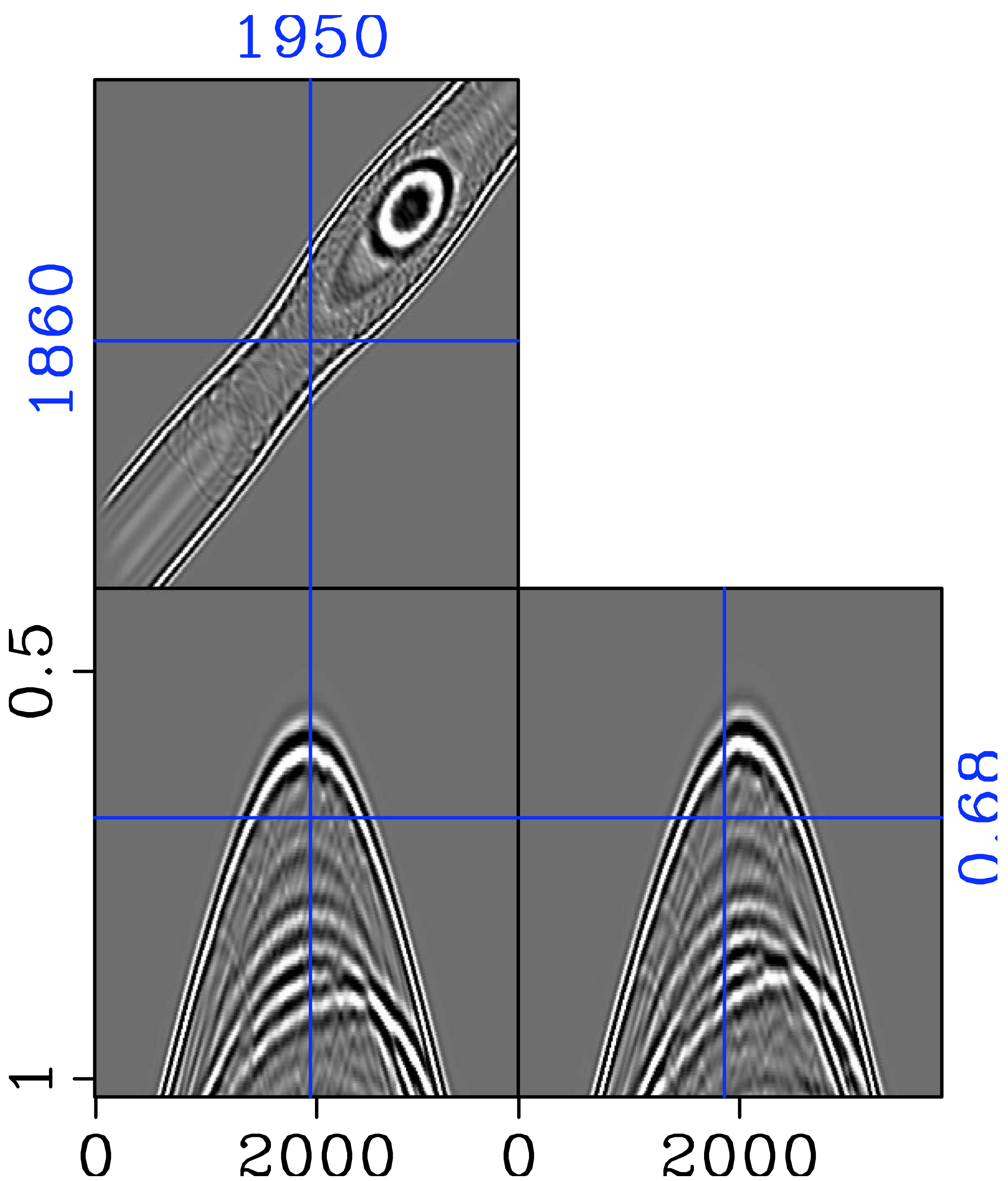


SRME from missing data

Our solution



SRME from recovered data



SRME from original data

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Motivation

Data-driven (SRME) multiple prediction requires *fully* sampled data.

The Focal transform (Berkhout & Verschuur ‘06) allows for

- mapping of multiples => primaries
- incorporation of *prior* information in the recovery

Present a curvelet-based scheme for sparsity-promoting

- recovery of missing data
- prediction of primaries from multiples
- data inverse ... and more ...

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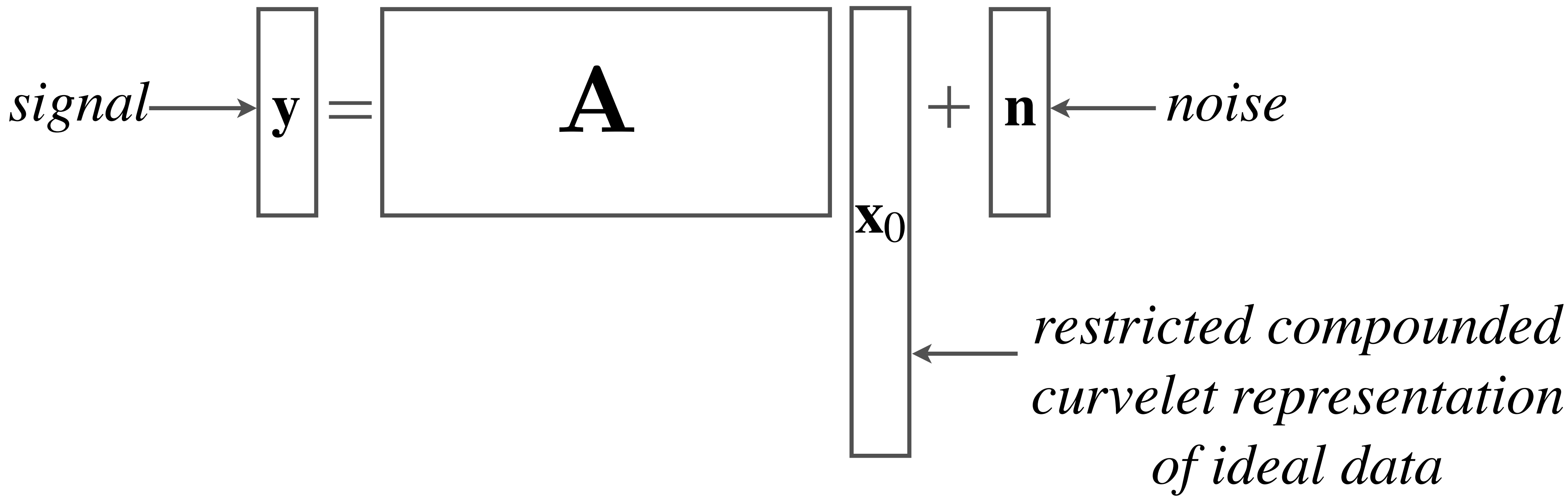
Sparsity-promoting program

Solve for $\mathbf{x}_0$



with

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

- exploit ***sparsity*** in the curvelet domain as a ***prior***.
- find the sparsest set of curvelet coefficients that match the data.
- invert an *underdetermined* system.

Focal transform with curvelets

Focused recovery

Non-data-adaptive Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI) derives from ***curvelet-sparsity*** of seismic data.

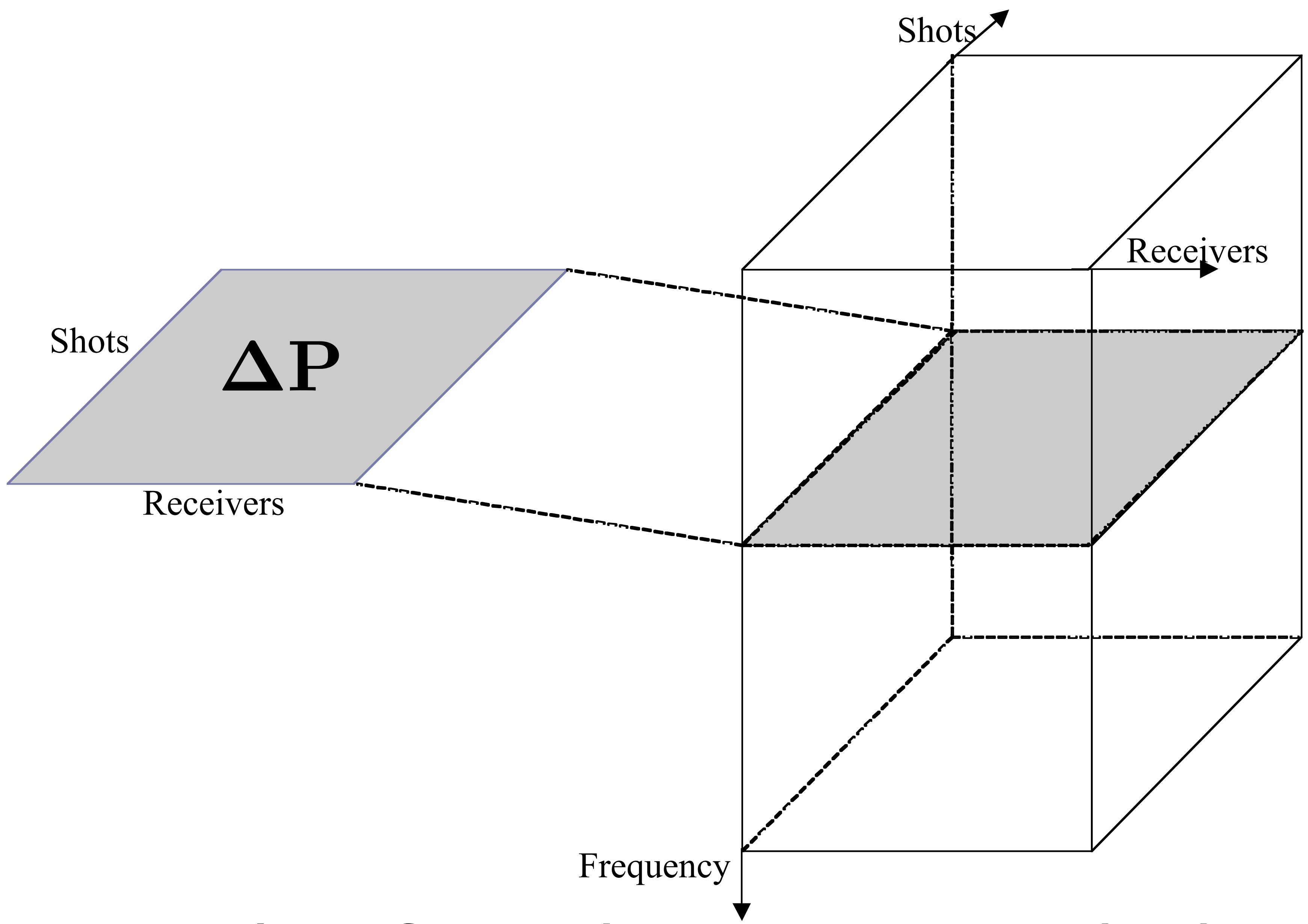
Berkhout and Verschuur's ***data-adaptive*** Focal transform derives from ***focusing*** of seismic data by the ***major*** primaries.

Both approaches entail the ***inversion*** of a linear operator.

Combination of the two yields

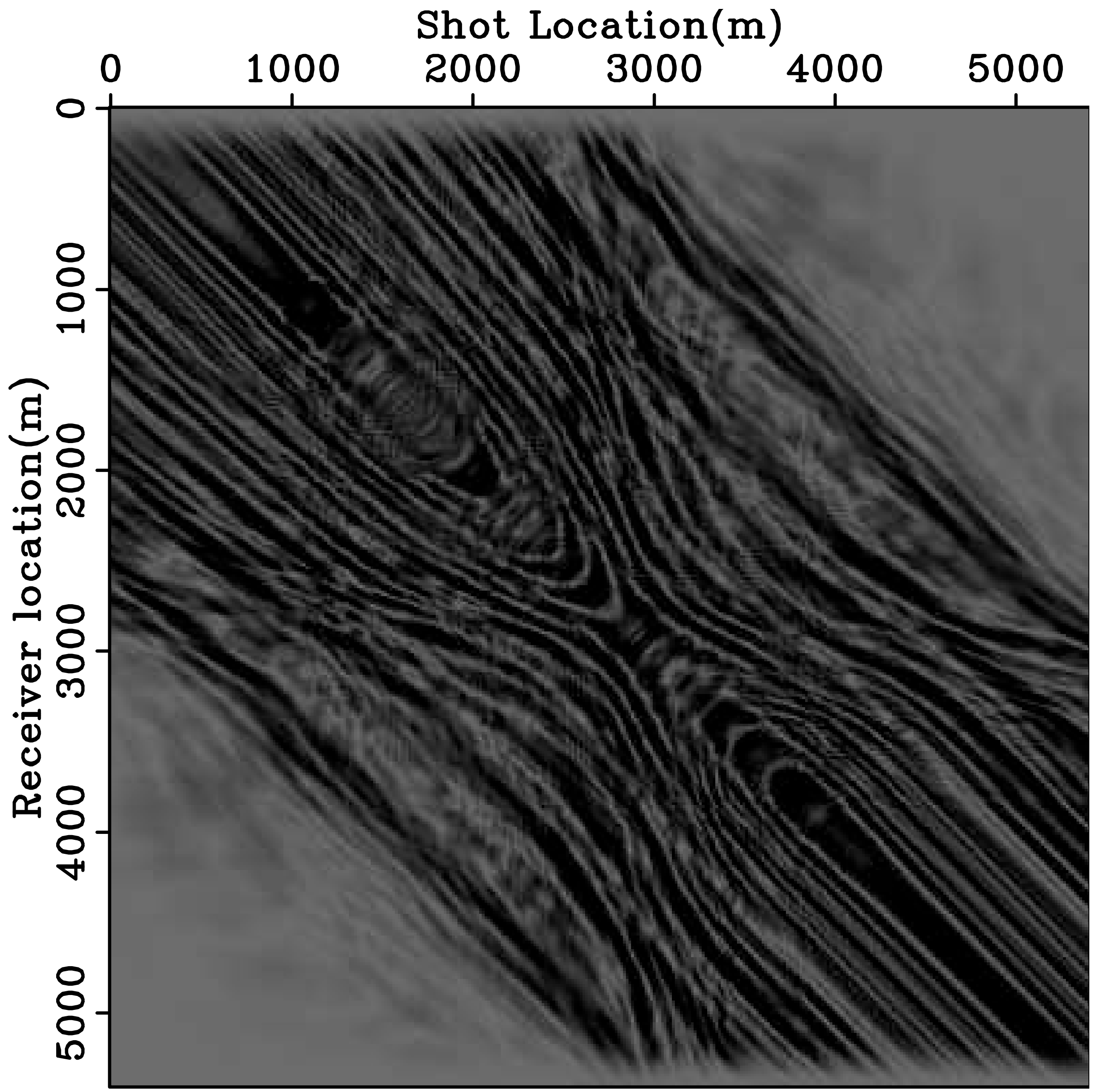
- *improved* focusing => more *sparsity*
- curvelet *sparsity* => better *focusing*

Primary operator



Frequency slice from data matrix with *dominant* primaries.

Primary operator



Frequency Slice (30Hz)

Primary operator

Primaries to first-order multiples:

$$\Delta \mathrm{p} \mapsto \mathrm{m}^1 = \Delta \mathrm{P} \mathcal{A} \Delta \mathrm{p}$$

First-order multiples into primaries:

$$\mathrm{m}^1 \mapsto \Delta \mathrm{p} \approx \Delta \mathrm{P}^T \mathcal{A} \Delta \mathrm{p}$$

with the acquisition matrix

$$\mathcal{A} = \left(\mathcal{S}^\dagger \mathrm{R} \mathcal{D}^\dagger \right)$$

“inverting” for source and receiver wavelet wavelets geometry and surface reflectivity.

Focussed curvelet transform

Solve with 3-D curvelet transform

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{\Delta P C}^T \quad \text{and } \mathbf{\Delta P} := \mathbf{F}^H \text{block diag}\{\hat{\mathbf{\Delta}}\mathbf{p}\}\mathbf{F} \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{p} \\ \mathbf{p} &= \text{total data} \\ \hat{\mathbf{\Delta}}\mathbf{p} &= \mathbf{F}\mathbf{\Delta}\mathbf{p} \\ \mathbf{\Delta}\mathbf{p} &= \text{estimate for the primaries.} \end{aligned}$$

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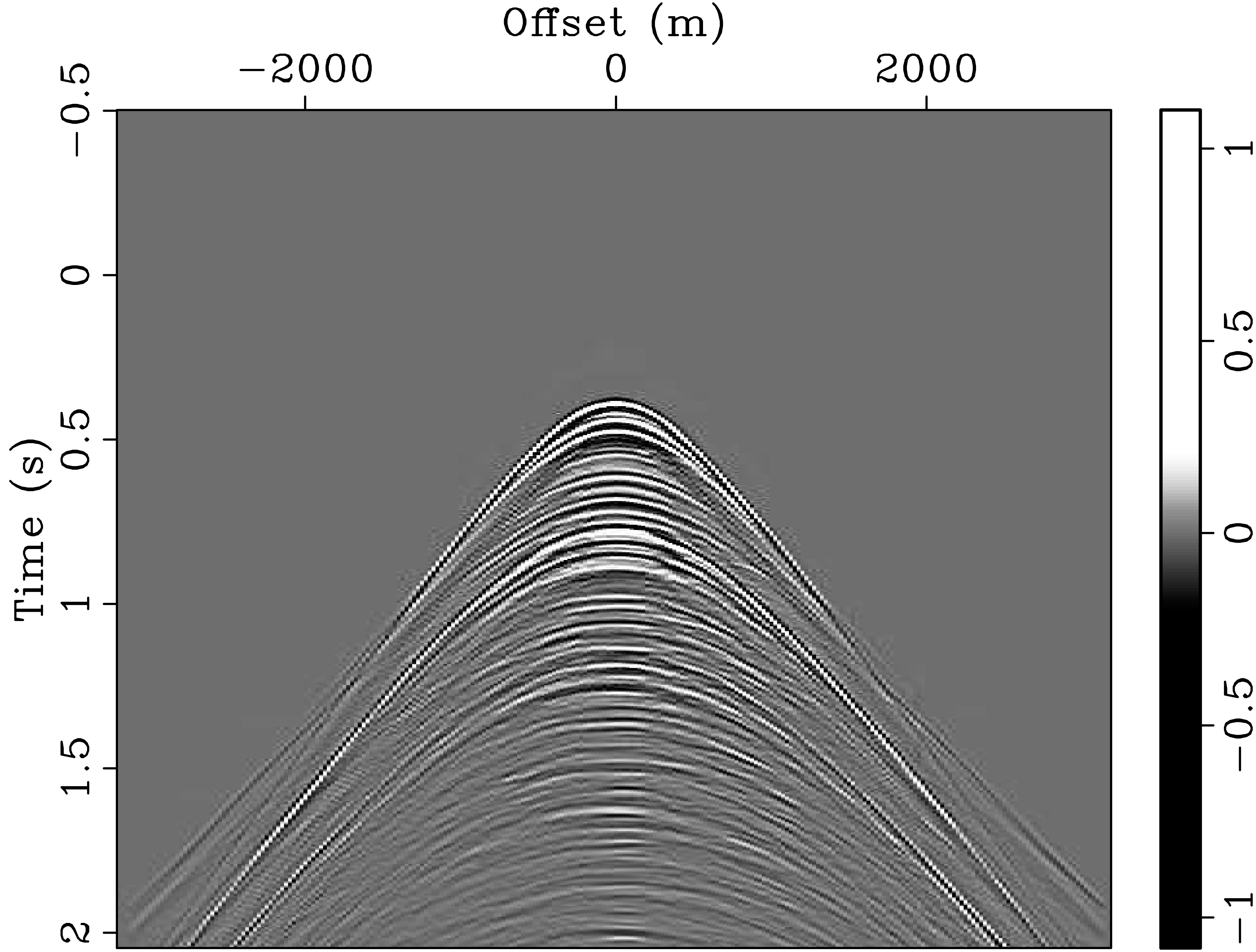
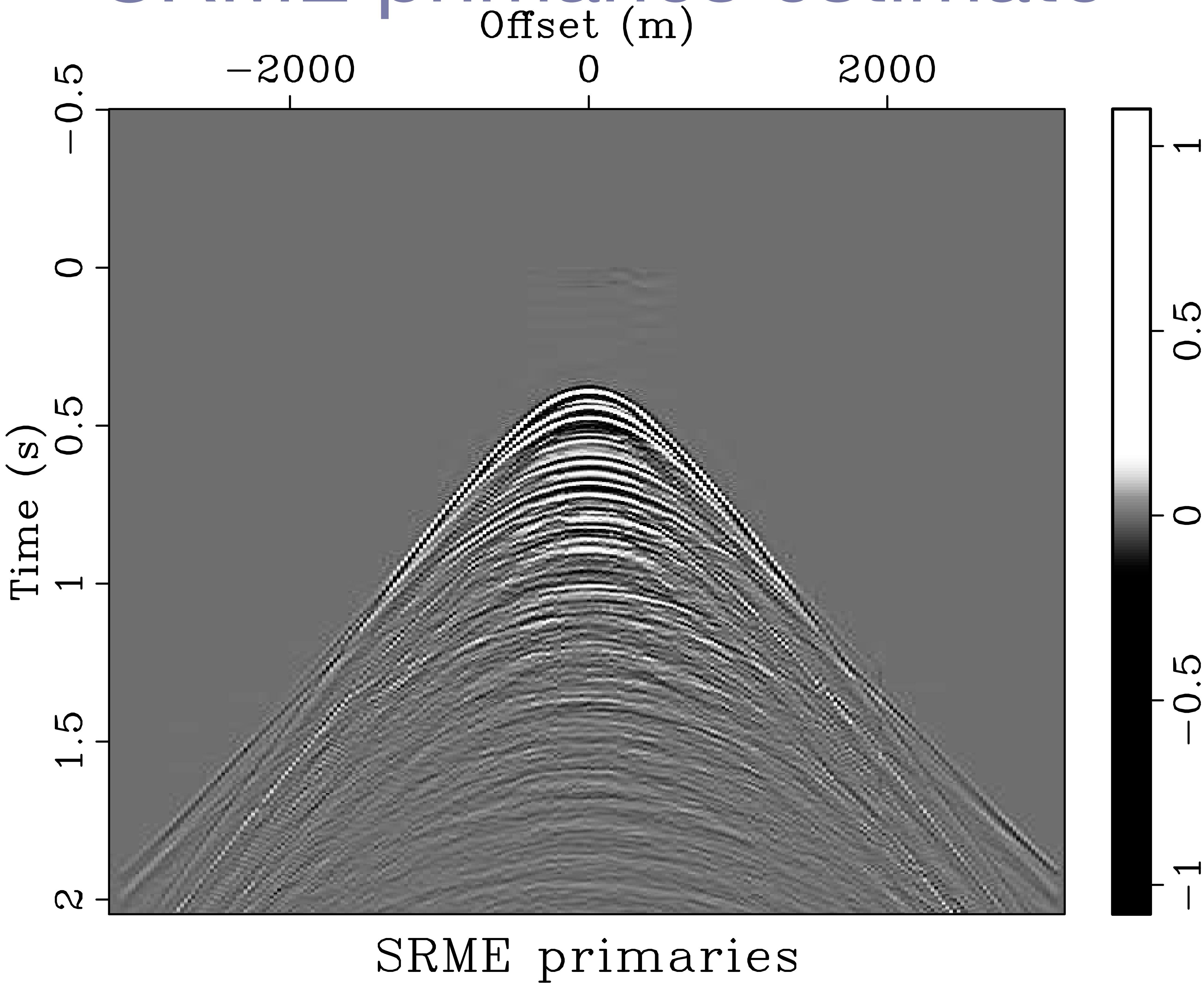
Total data

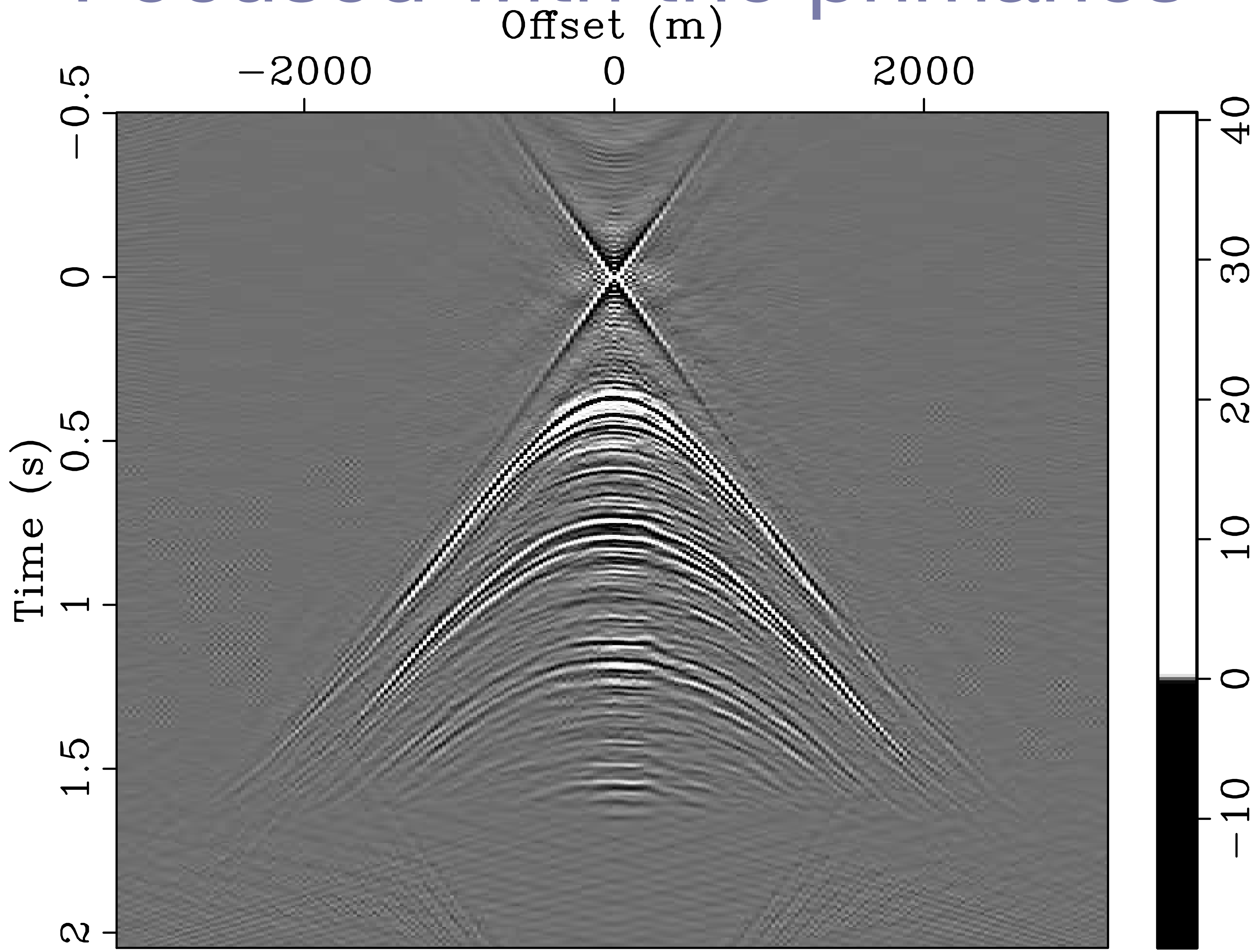


Original data

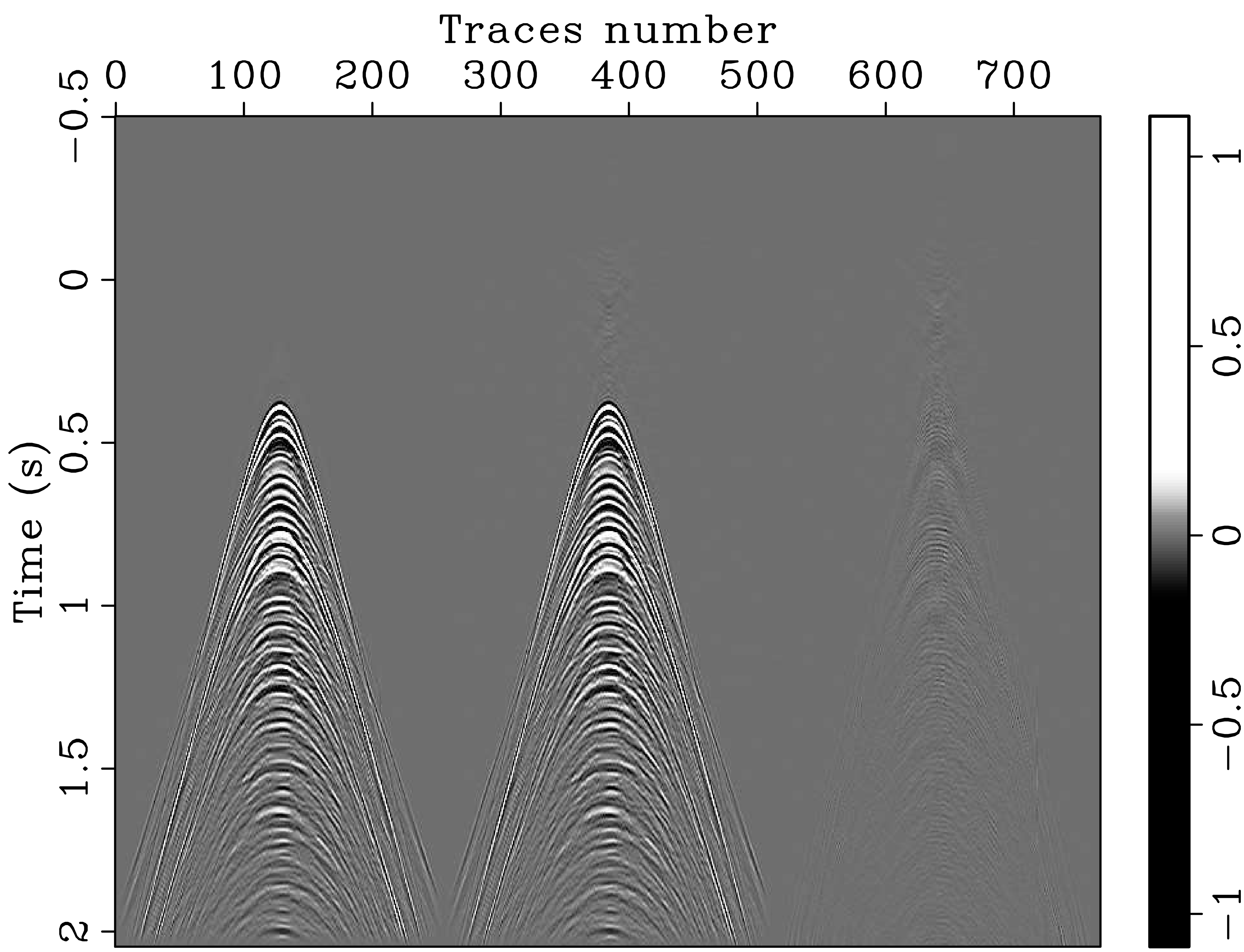
SRME primaries estimate



Focused with the primaries



Difference



Traces number
0
100
200
300
400
500
600
700
Time (s)
−0.5
0
0.5
1
1.5
2
1
0.5
0
−0.5
−1

Focused wavefield reconstruction with curvelets

Recovery with focussing

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{S}^T &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{y} &= \mathbf{R}\mathbf{p} \\ \mathbf{R} &= \text{picking operator.} \end{aligned}$$

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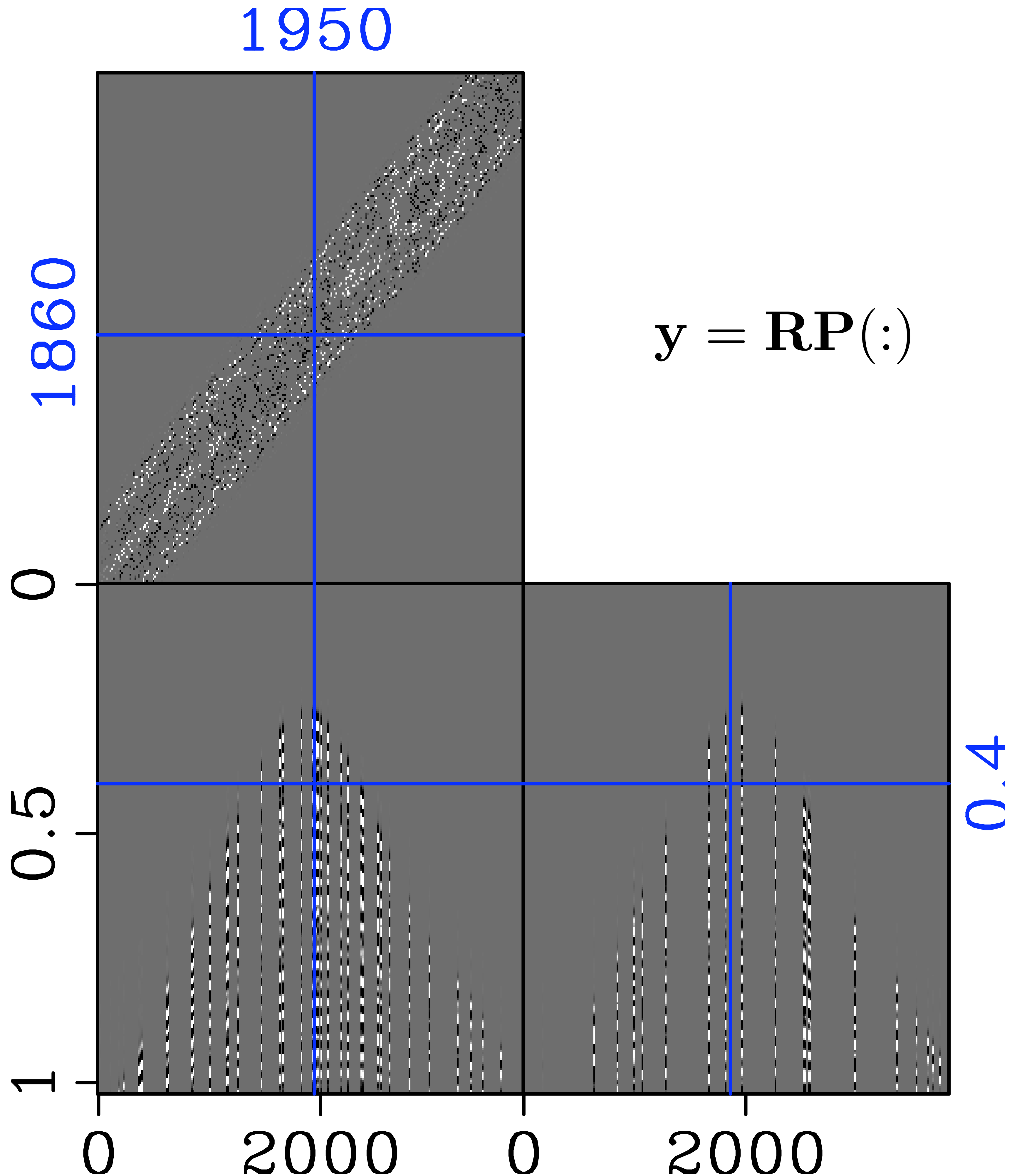
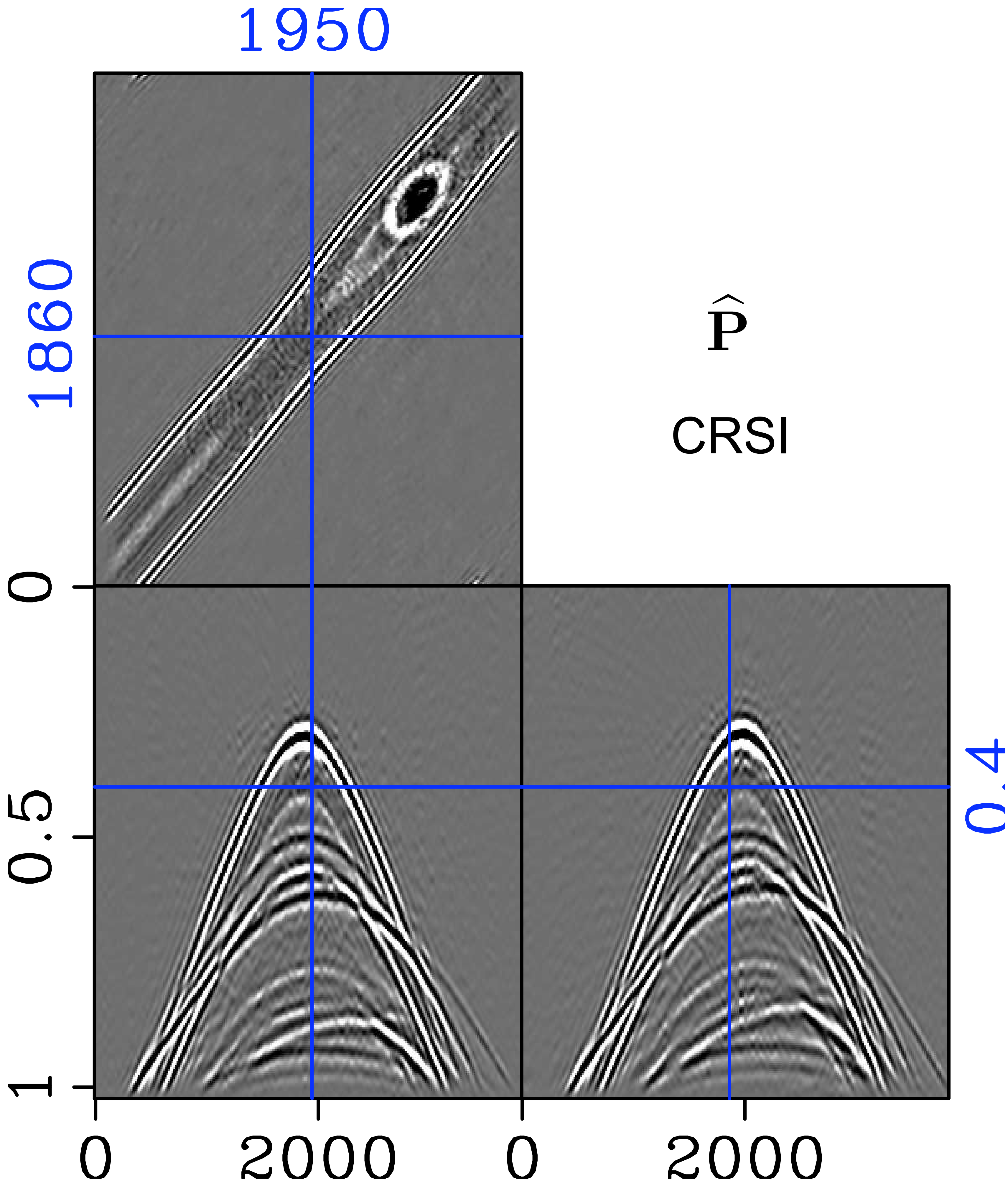
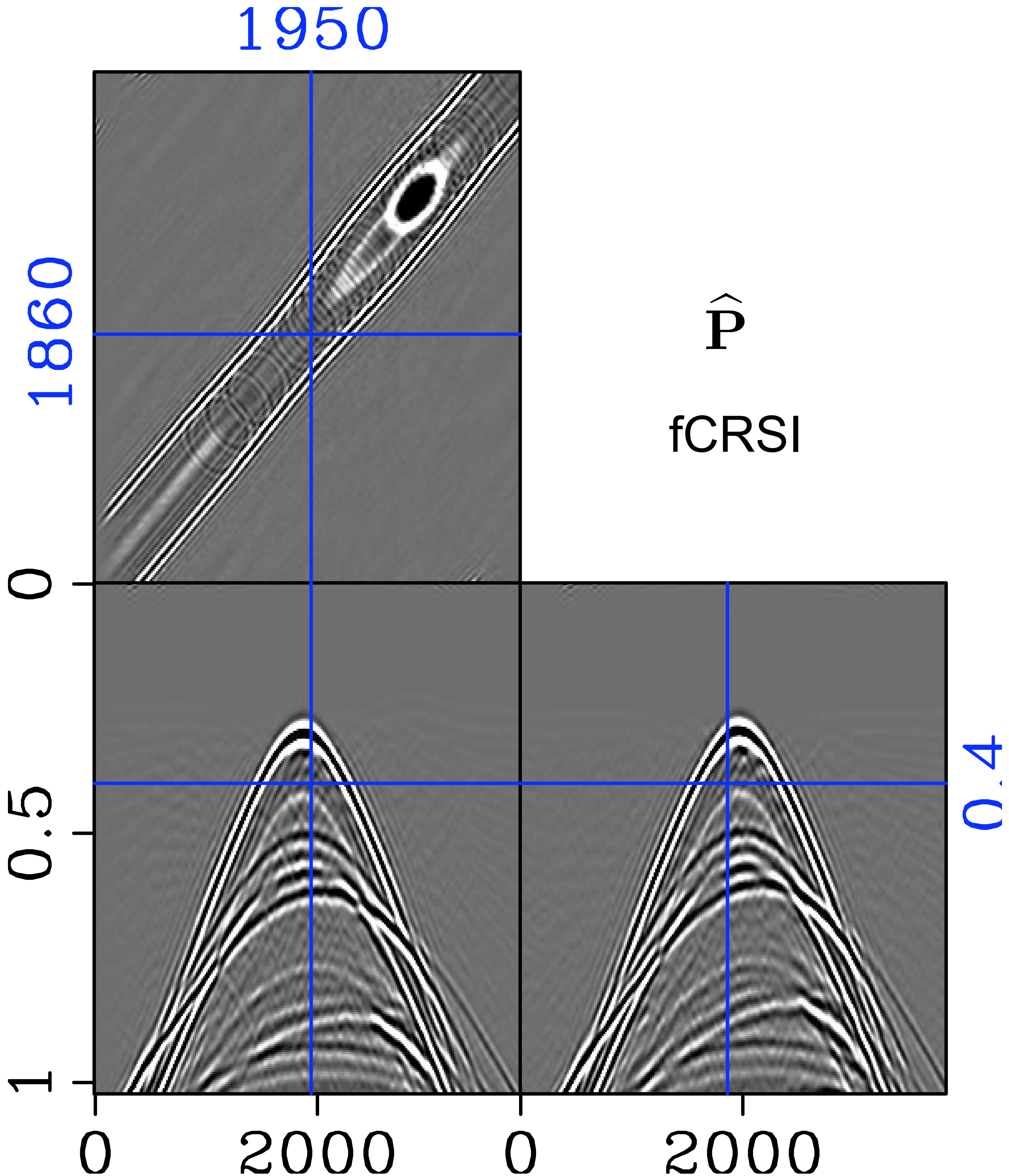
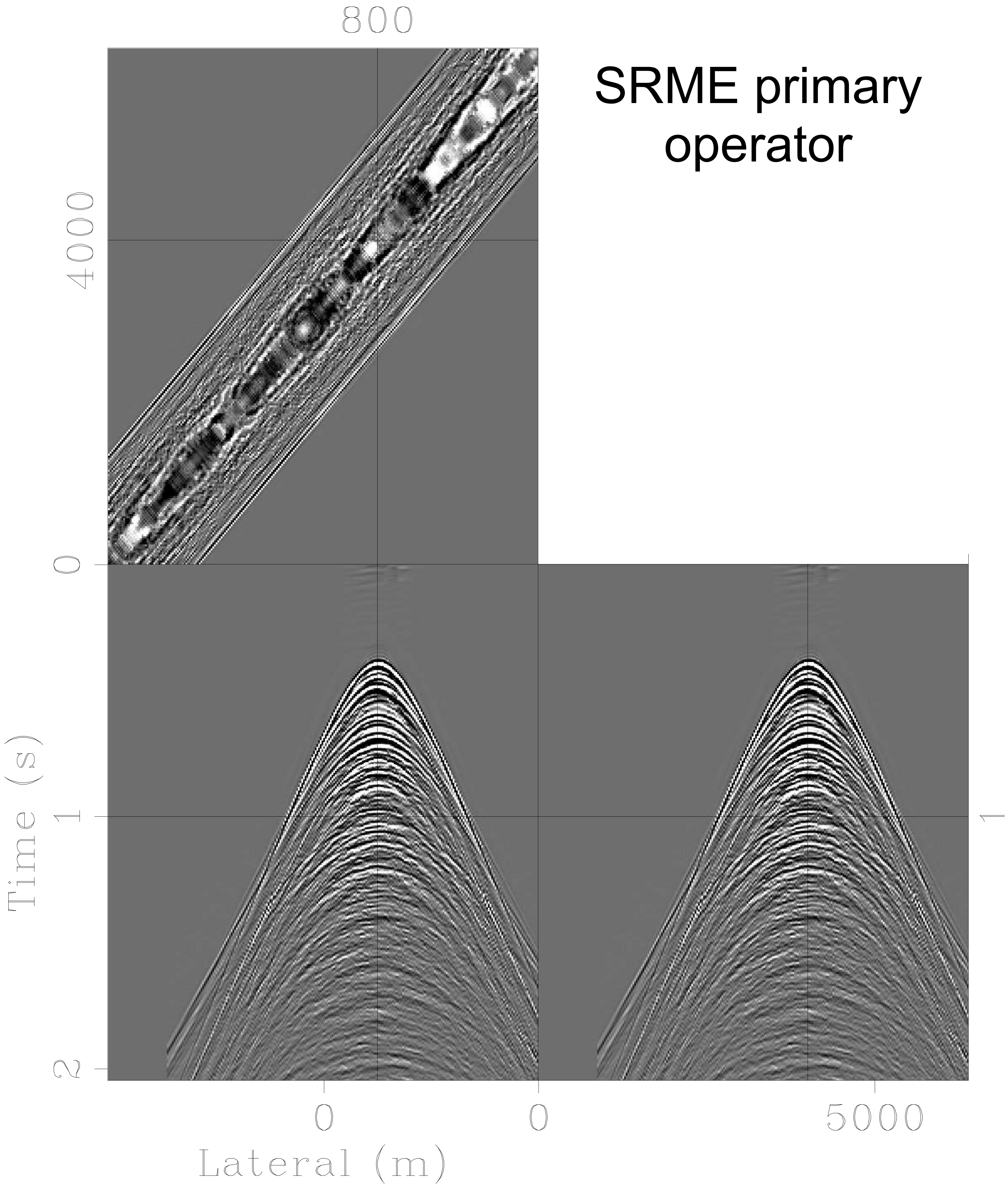


1950
1860
0
0.5
1
0
2000
0
2000
0.4
$\mathbf{y} = \mathbf{RP}(:)$

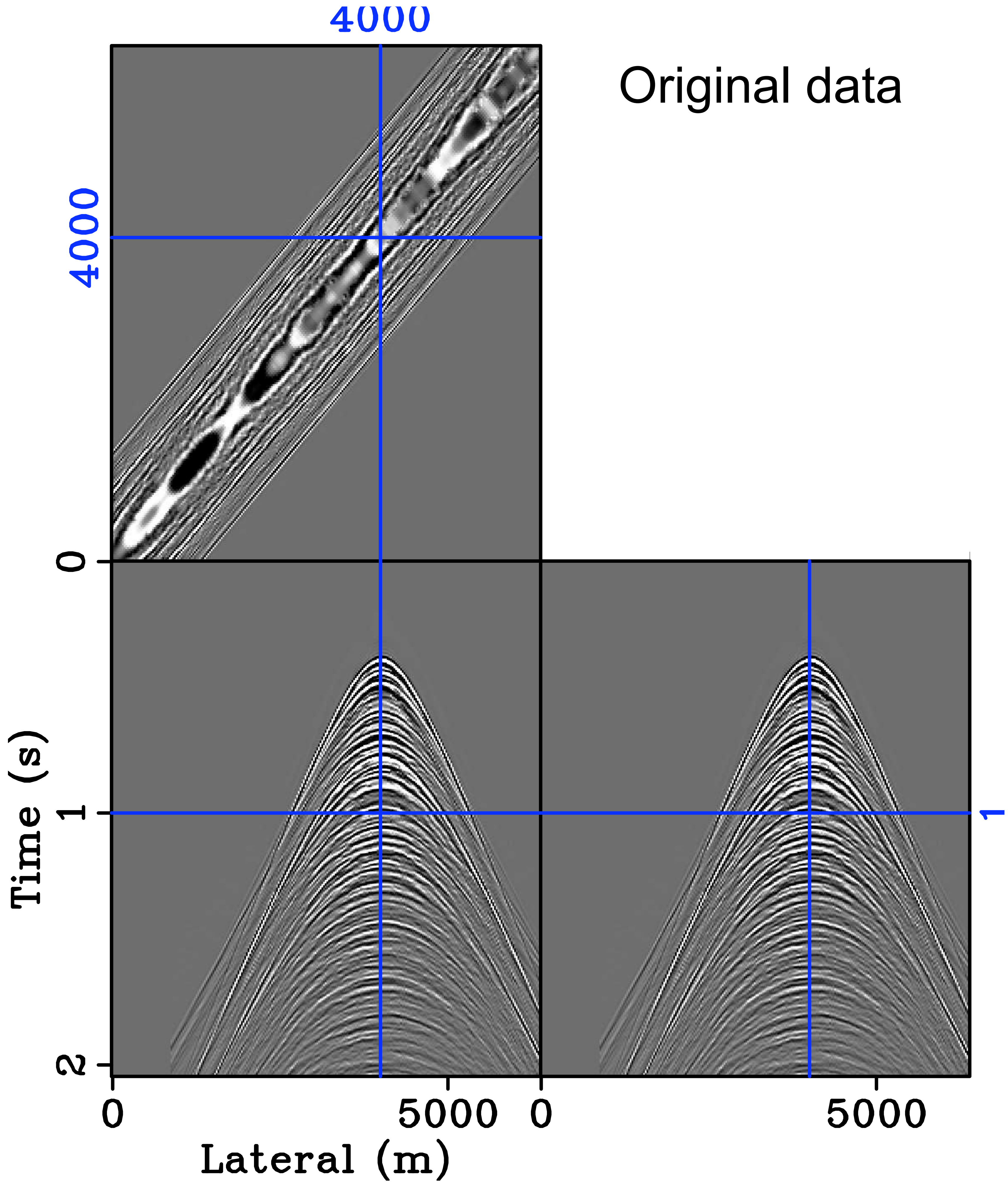

1950
1860
$\widehat{\mathbf{P}}$
CRSI
0
0.5
1
0.4
0
2000
0
2000

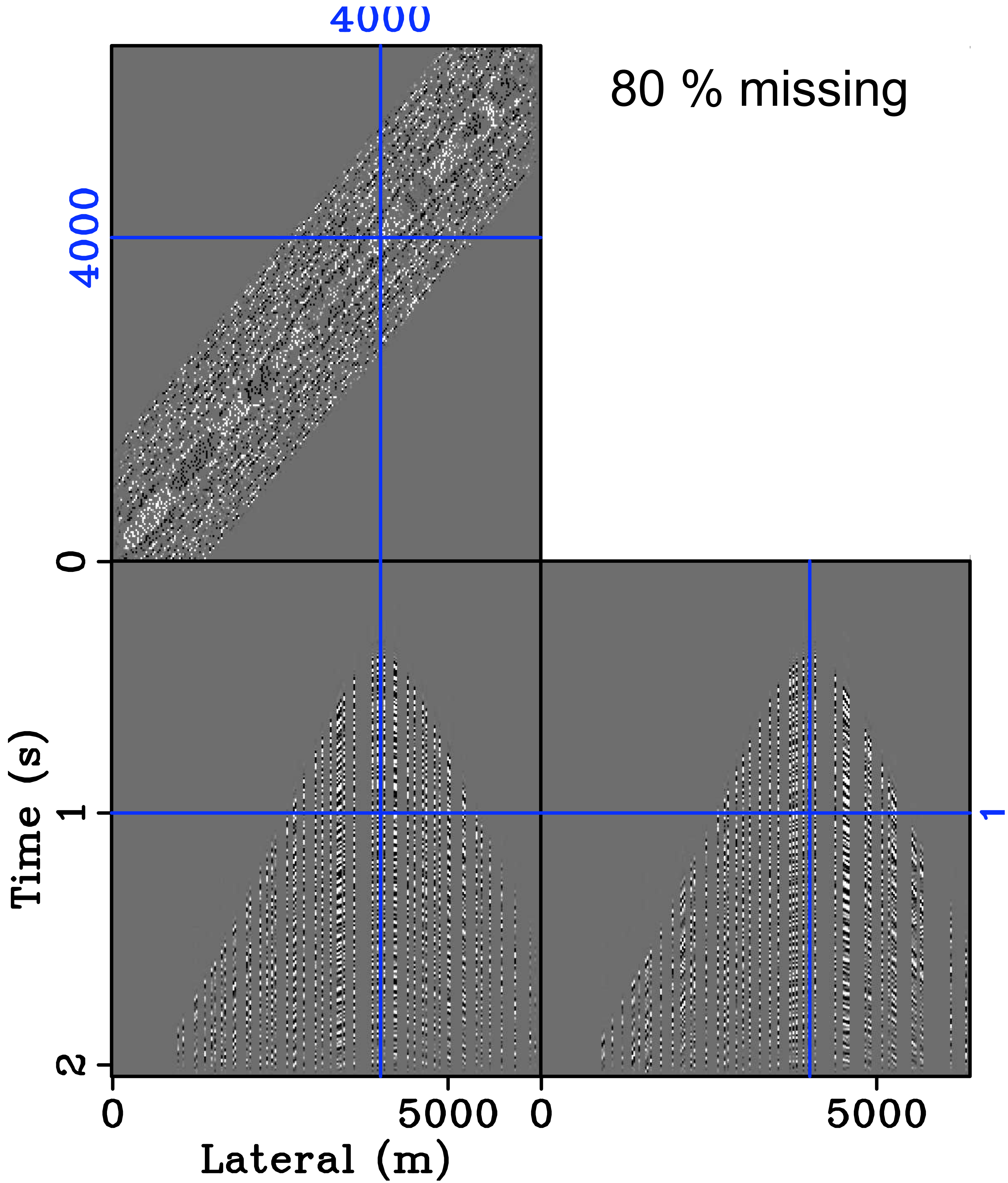

1950
1860
0
0.5
1
0
2000
0
2000
0.4
$\widehat{\mathbf{P}}$
fCRSI

SRME primary operator



Original data





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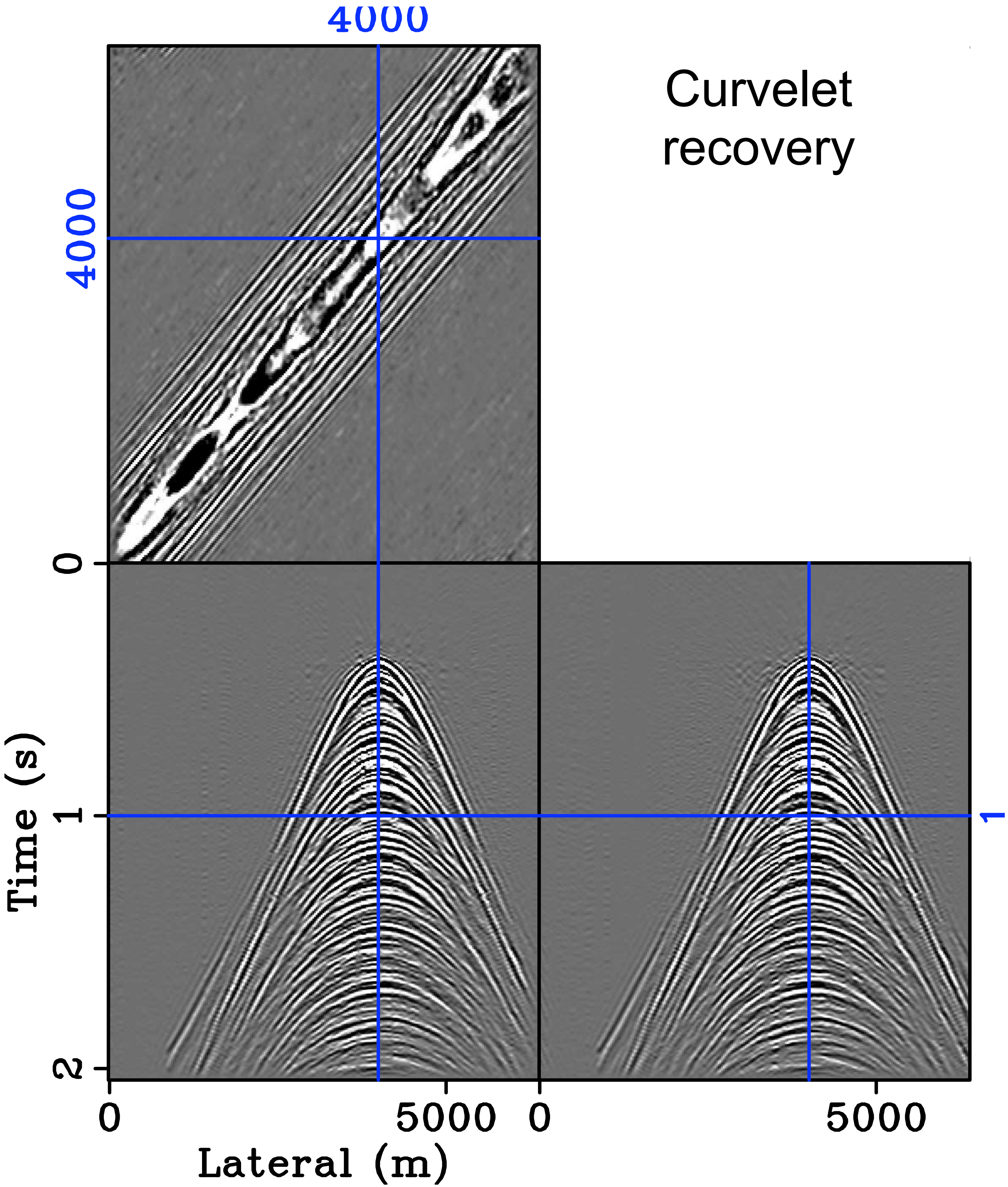

Curvelet recovery
4000
4000
0
1
2
Time (s)
1
0
5000
0
5000
Lateral (m)
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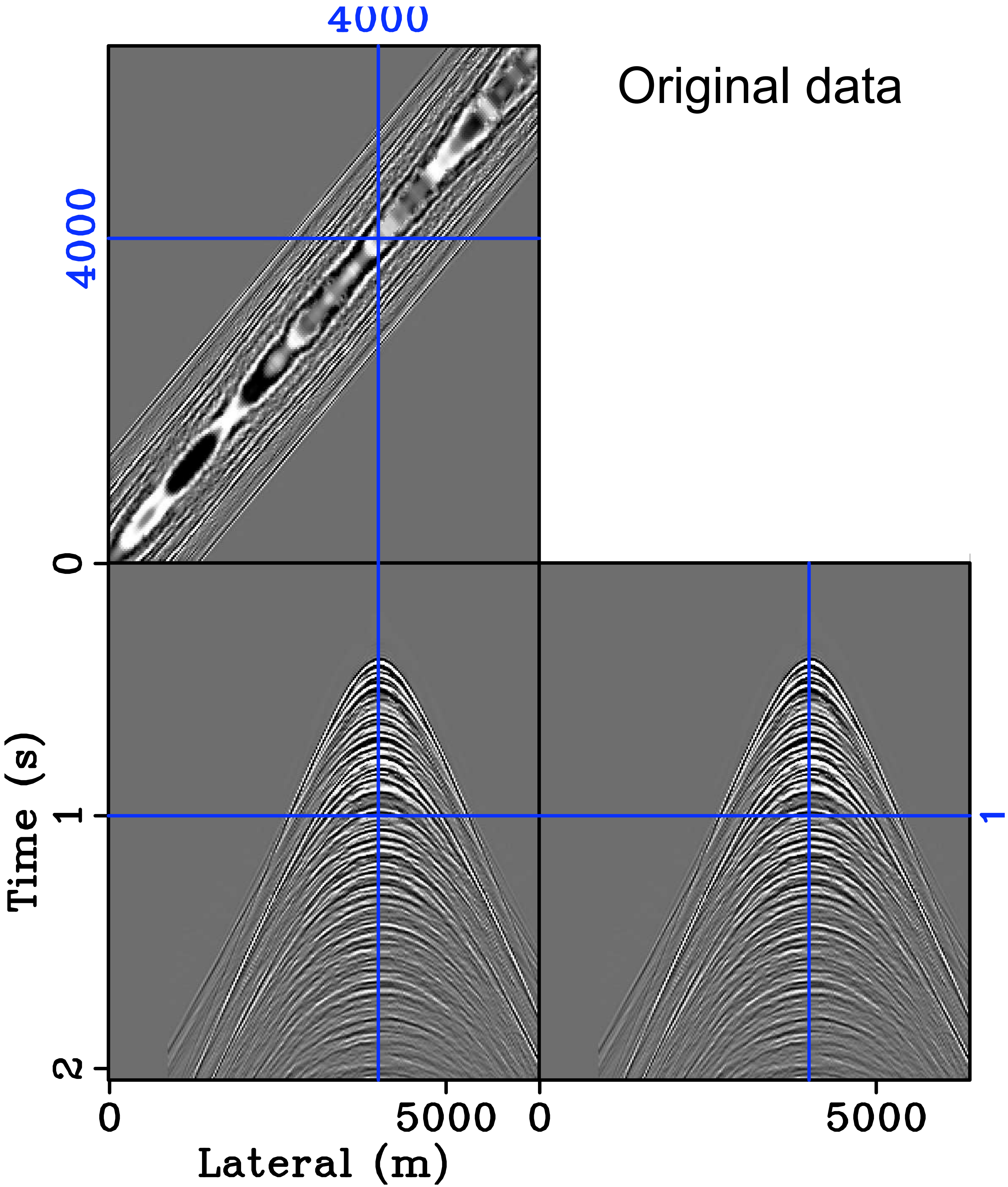
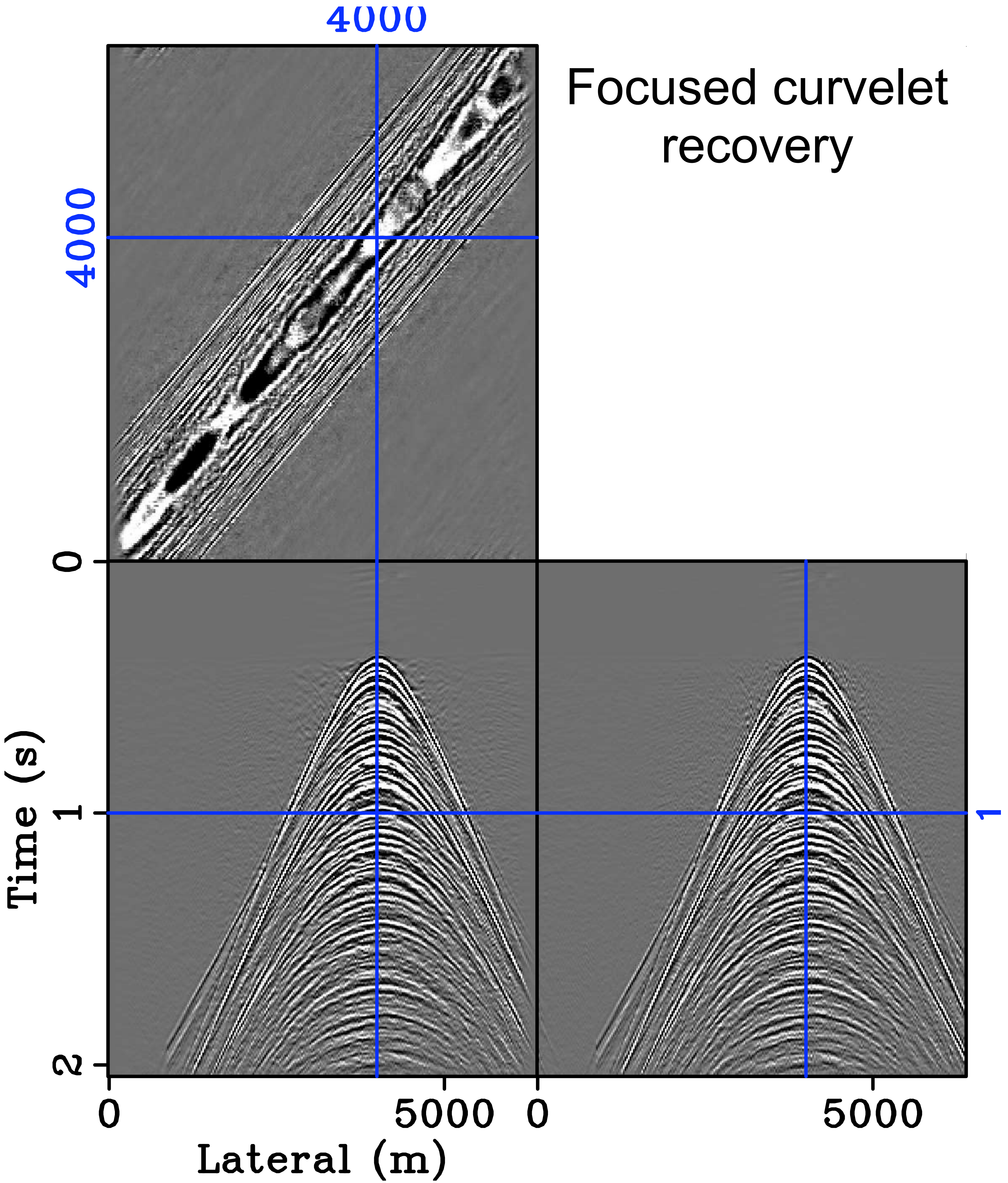
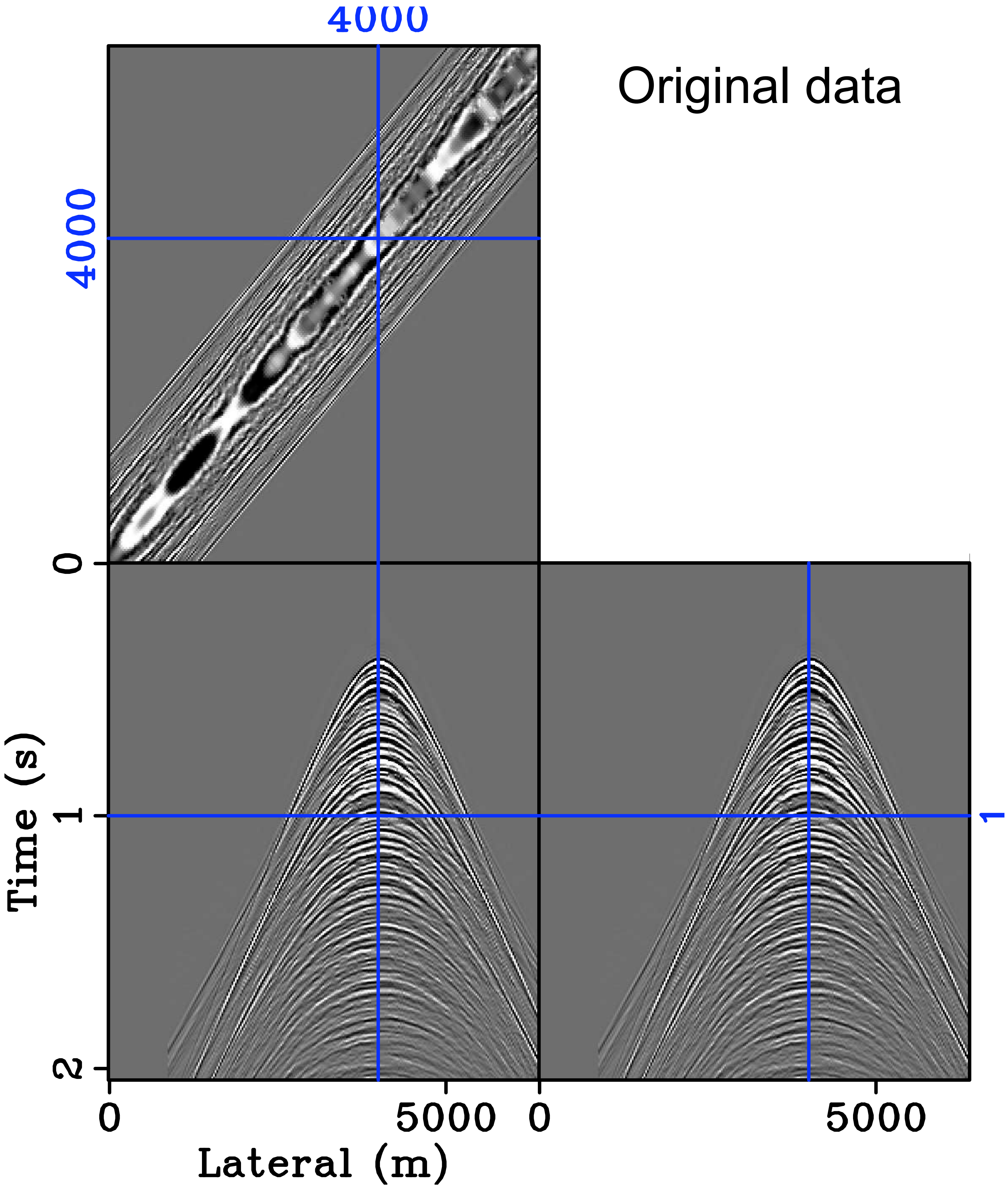
Original data



Focused curvelet recovery

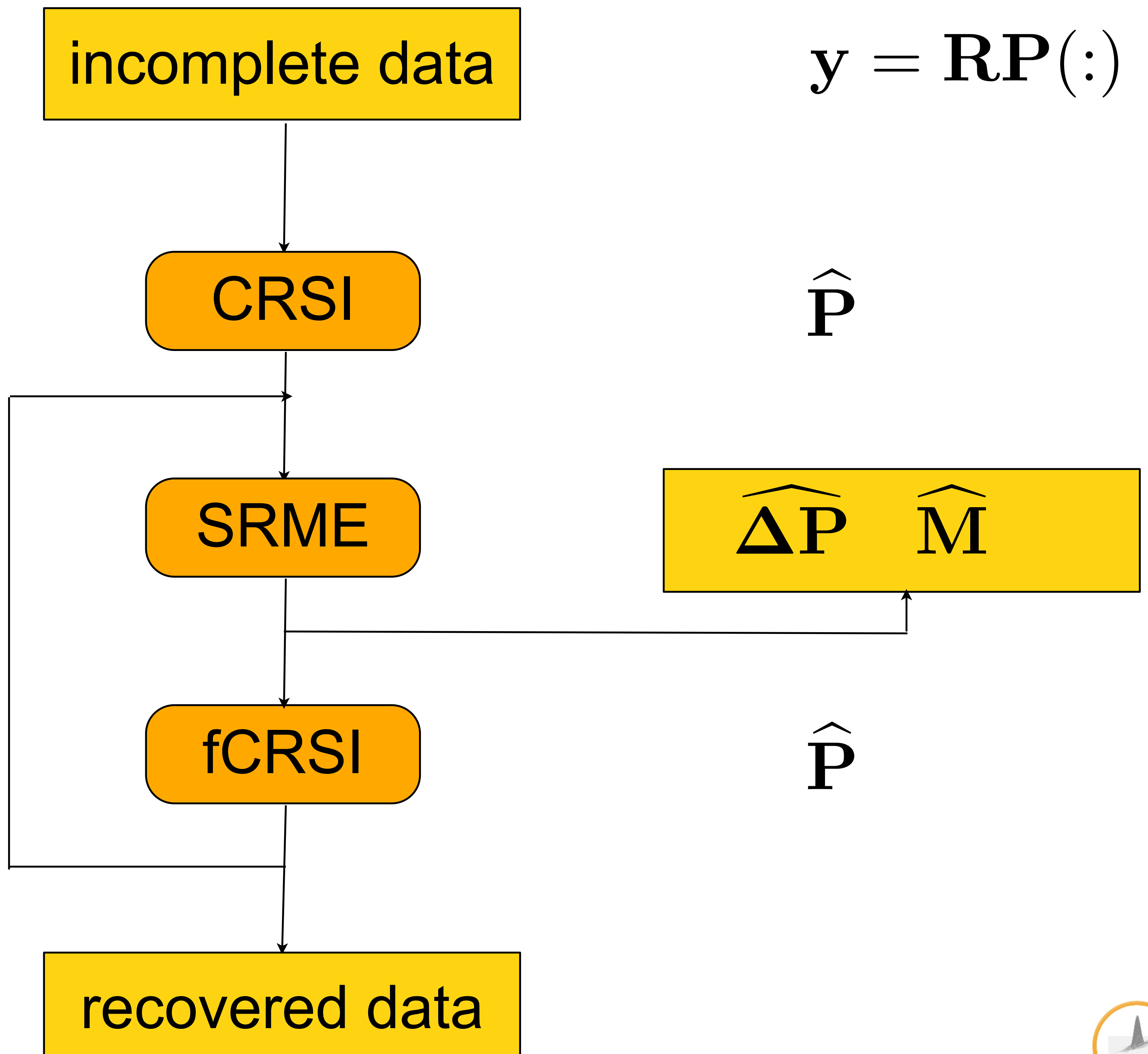


Original data



Multiple prediction with fCRSI

Multiple prediction with fCRSI



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Multiple recovery with fCRSI

Solve

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{p}} = \mathbf{\Delta P C}^T \tilde{\mathbf{x}} \\ \tilde{\mathbf{m}} = \tilde{\mathbf{P}}\tilde{\mathbf{p}} \end{cases}$$

with

$$\begin{aligned} \mathbf{R} &= \text{picking operator} \\ \mathbf{A} &= \mathbf{R\Delta P C}^T \\ \mathbf{S}^T &= \mathbf{\Delta P C}^T \\ \mathbf{y} &= \mathbf{Rp} \\ \tilde{\mathbf{p}} &= \text{recovered data} \\ \tilde{\mathbf{m}} &= \text{recovered multiples.} \end{aligned}$$

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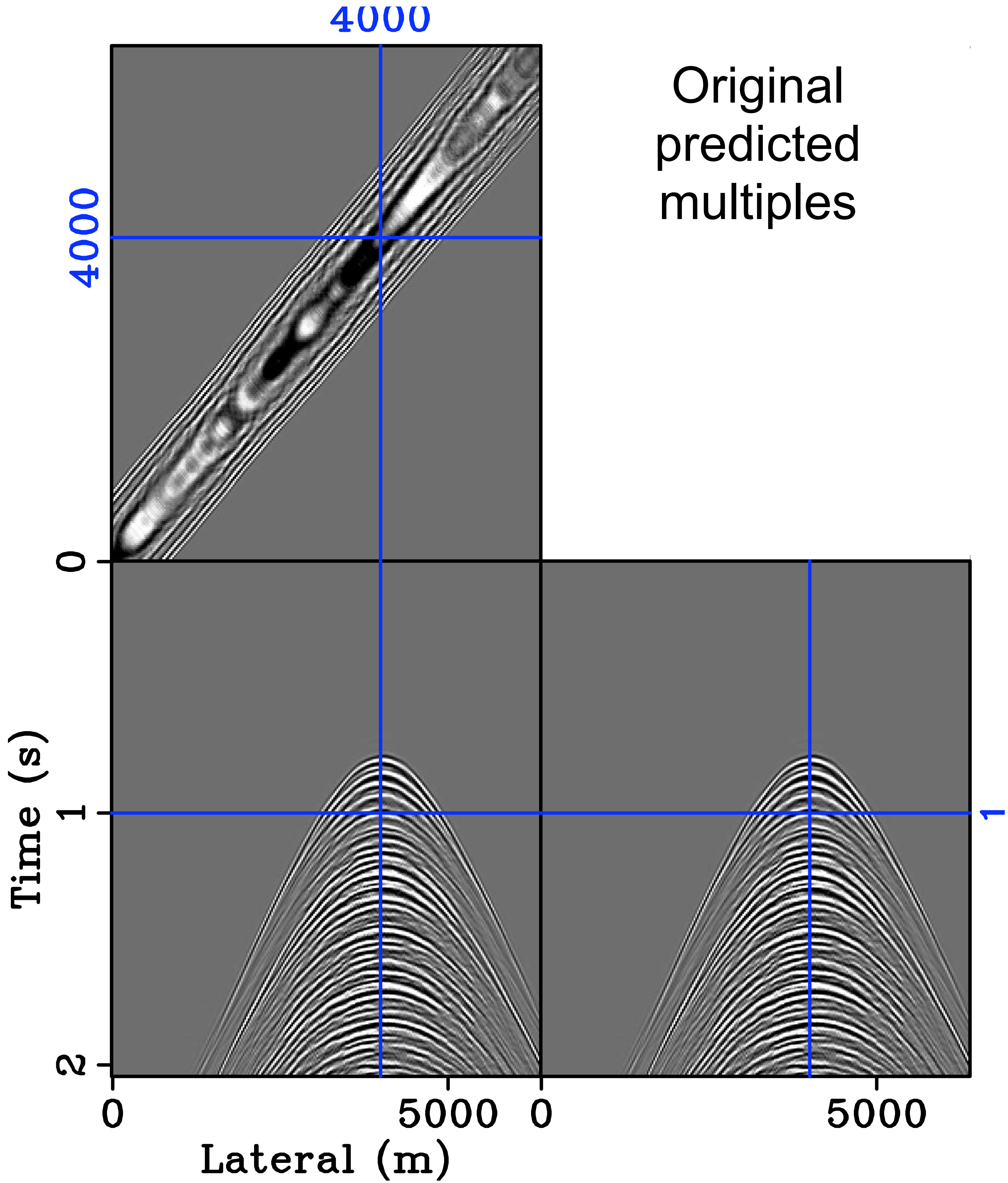
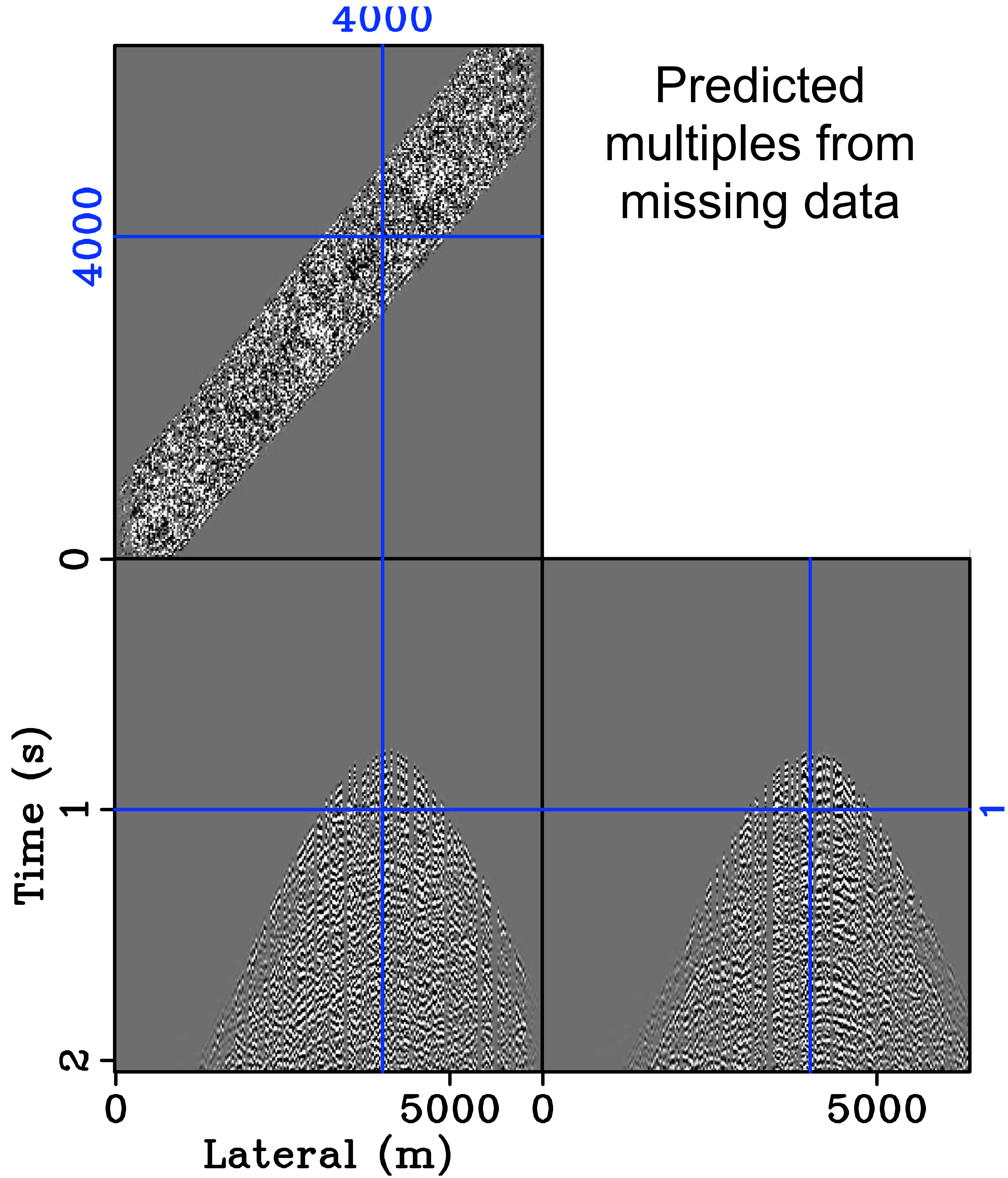
Original predicted multiples





Predicted multiples from missing data

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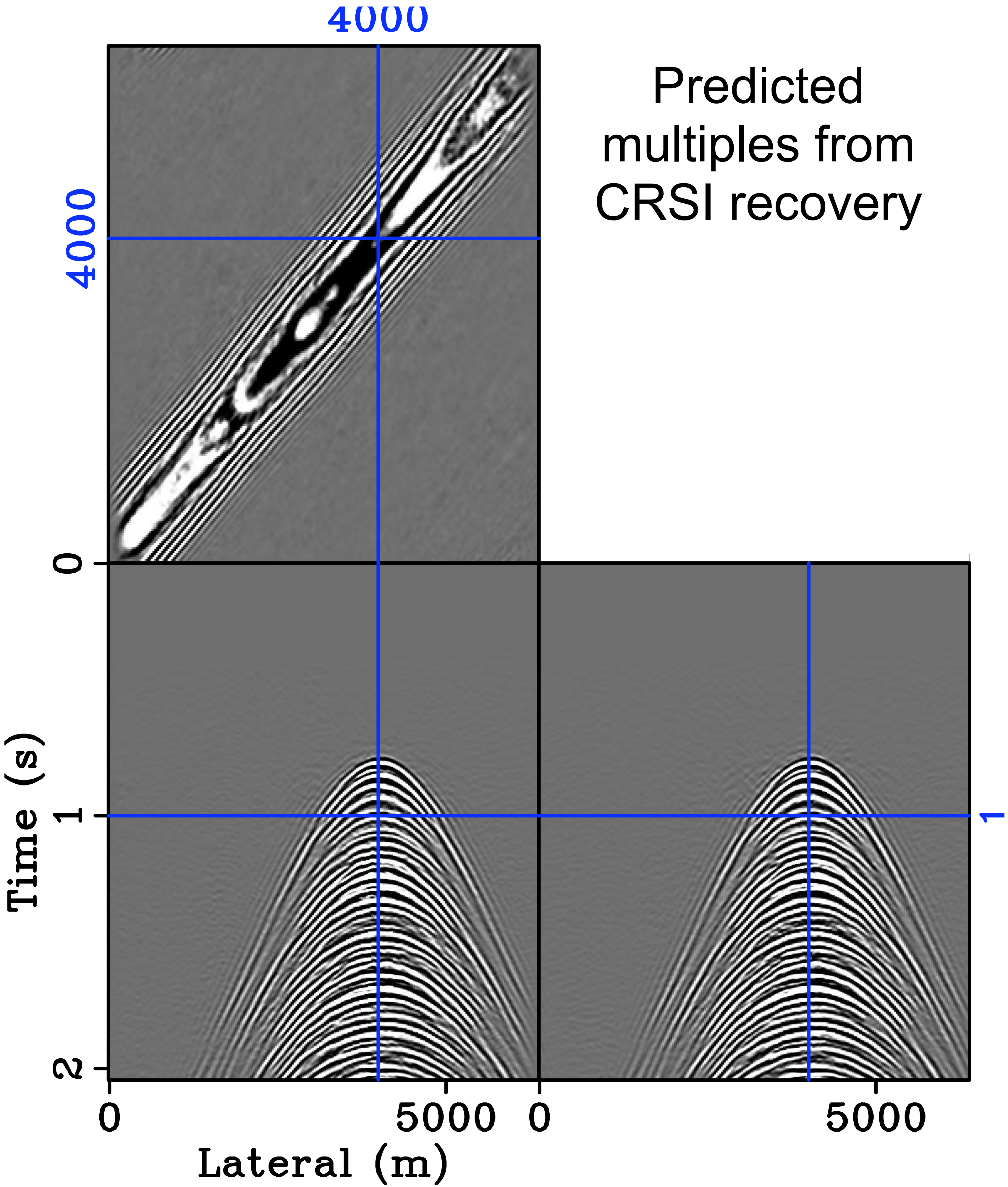
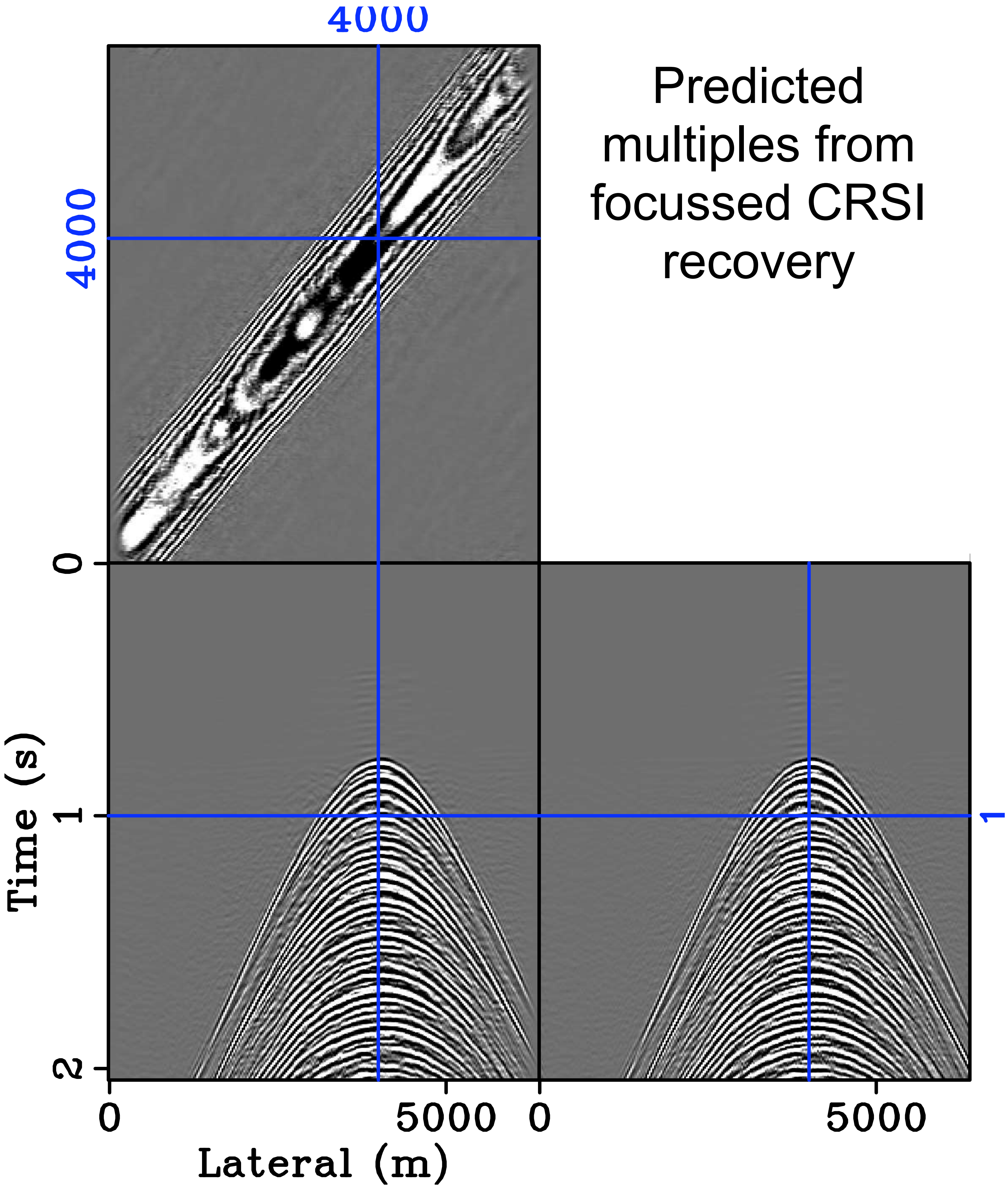
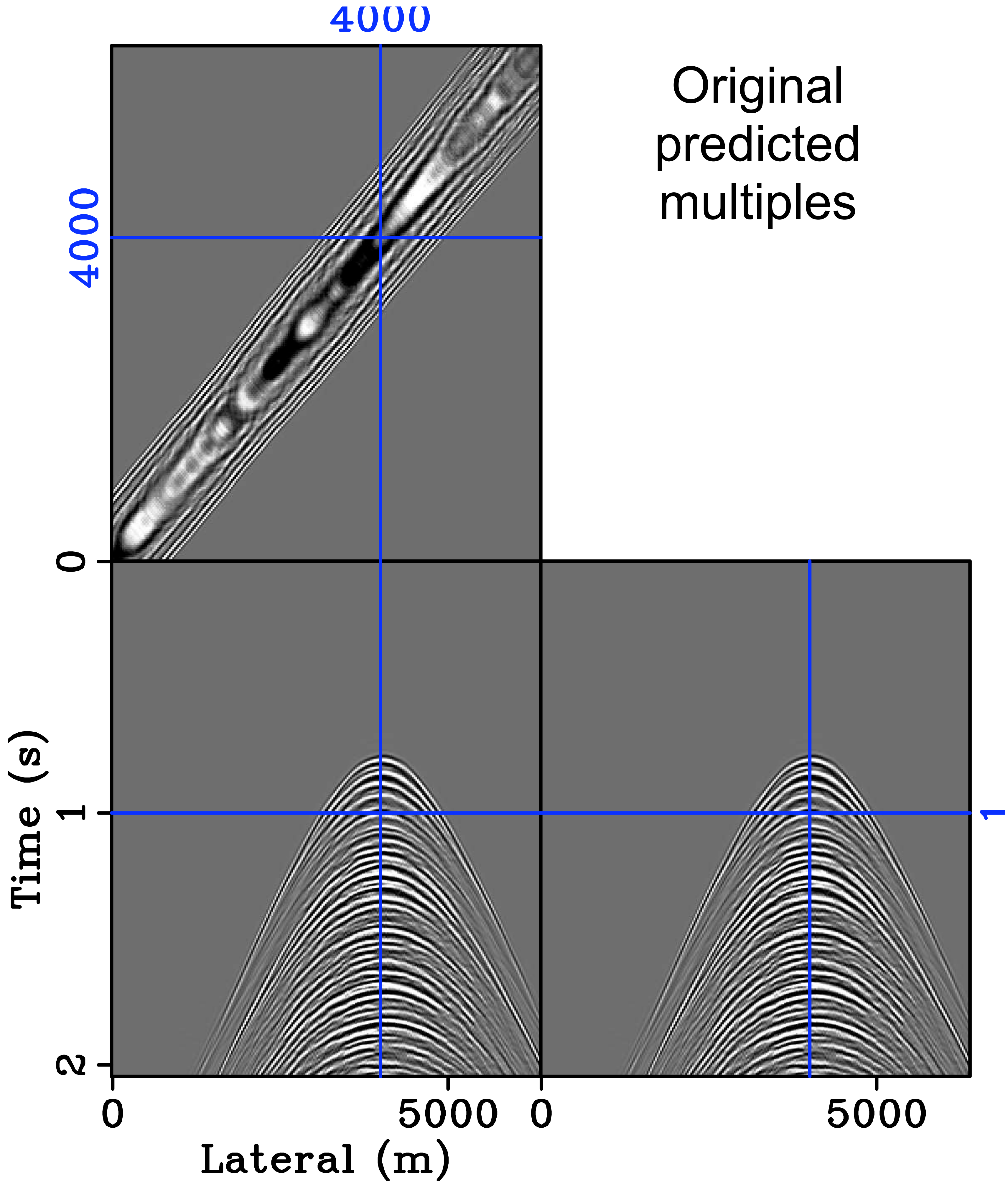
Predicted multiples from CRSI recovery



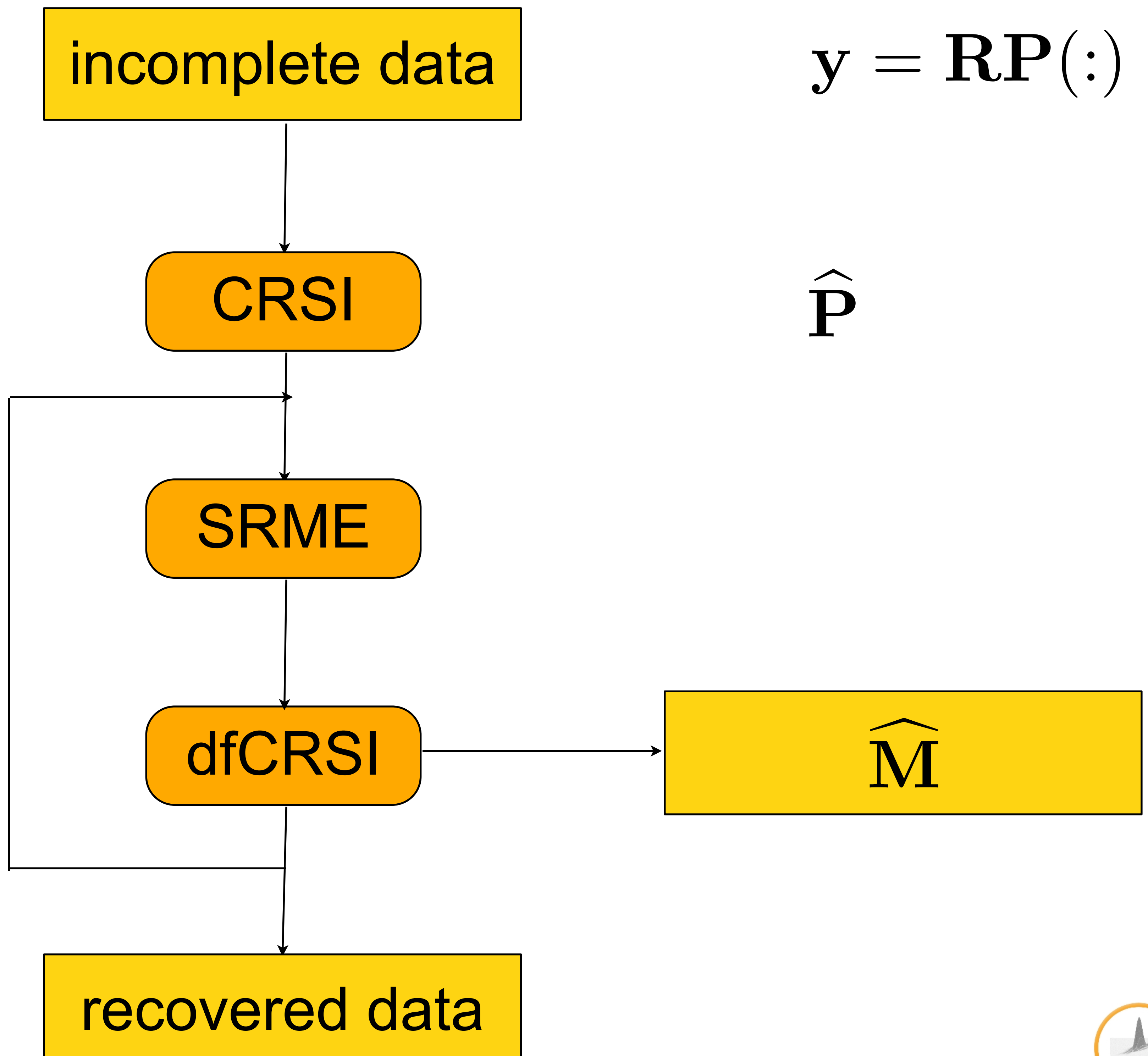
Predicted multiples from focussed CRSI recovery





Original predicted multiples

Multiple prediction with dfCRSI



Multiple prediction with dfCRSI

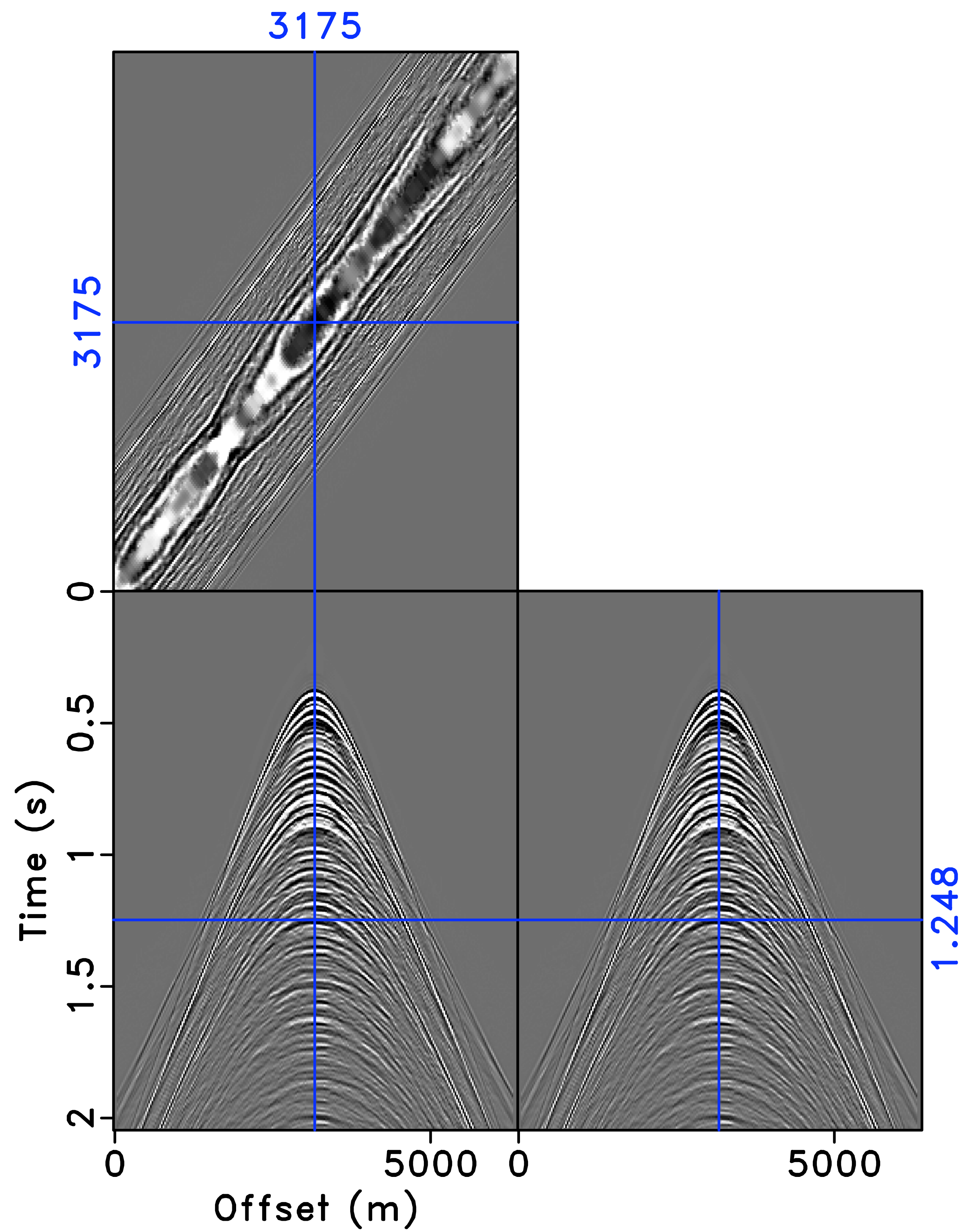
Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^H \tilde{\mathbf{x}} \end{cases}$$

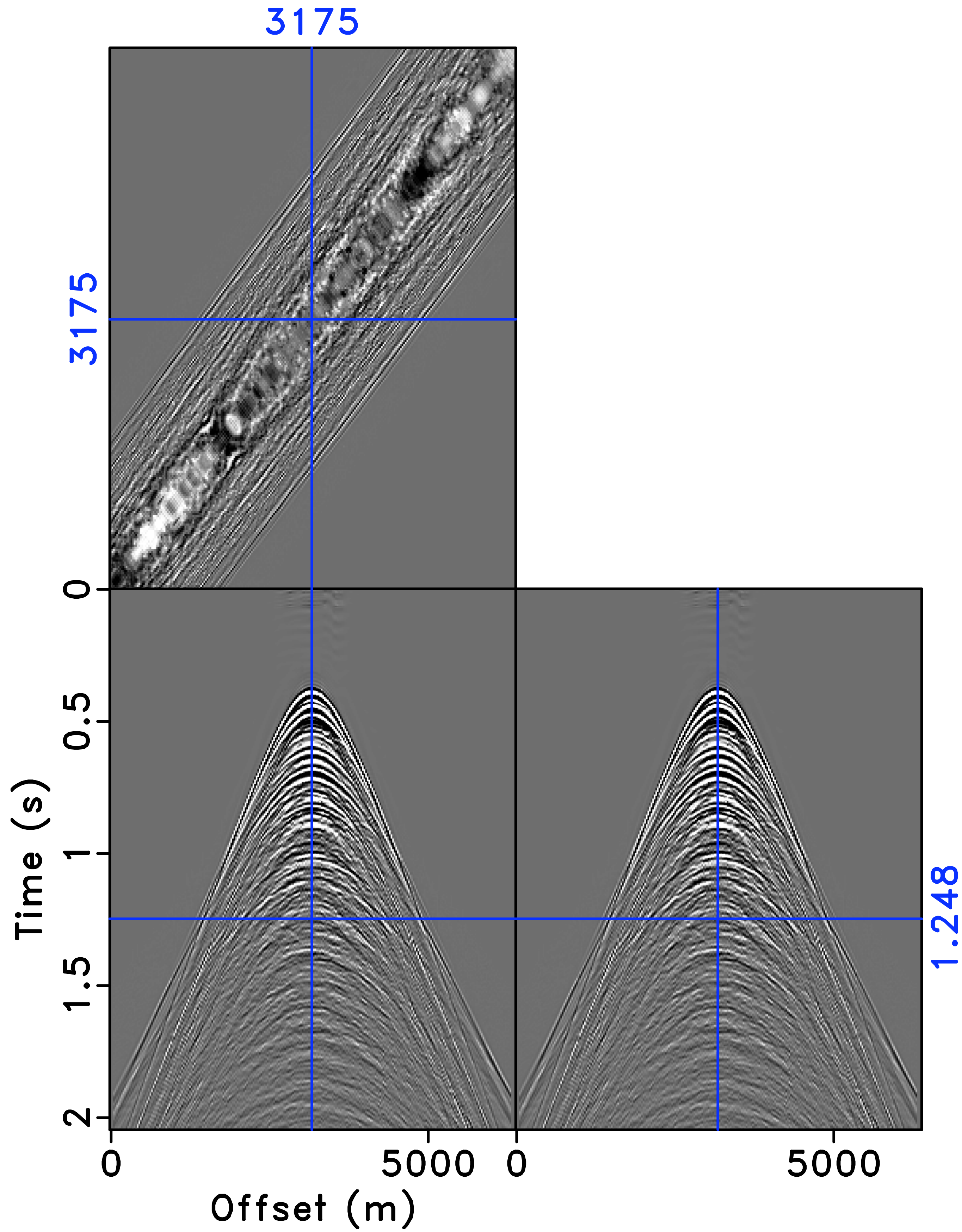
with

$$\begin{aligned} \mathbf{A} &= \boldsymbol{\Delta}\mathbf{P}^H\mathbf{C}^T \\ \mathbf{y} &= \tilde{\mathbf{p}} \\ \tilde{\mathbf{m}} &= \text{predicted multiples.} \end{aligned}$$

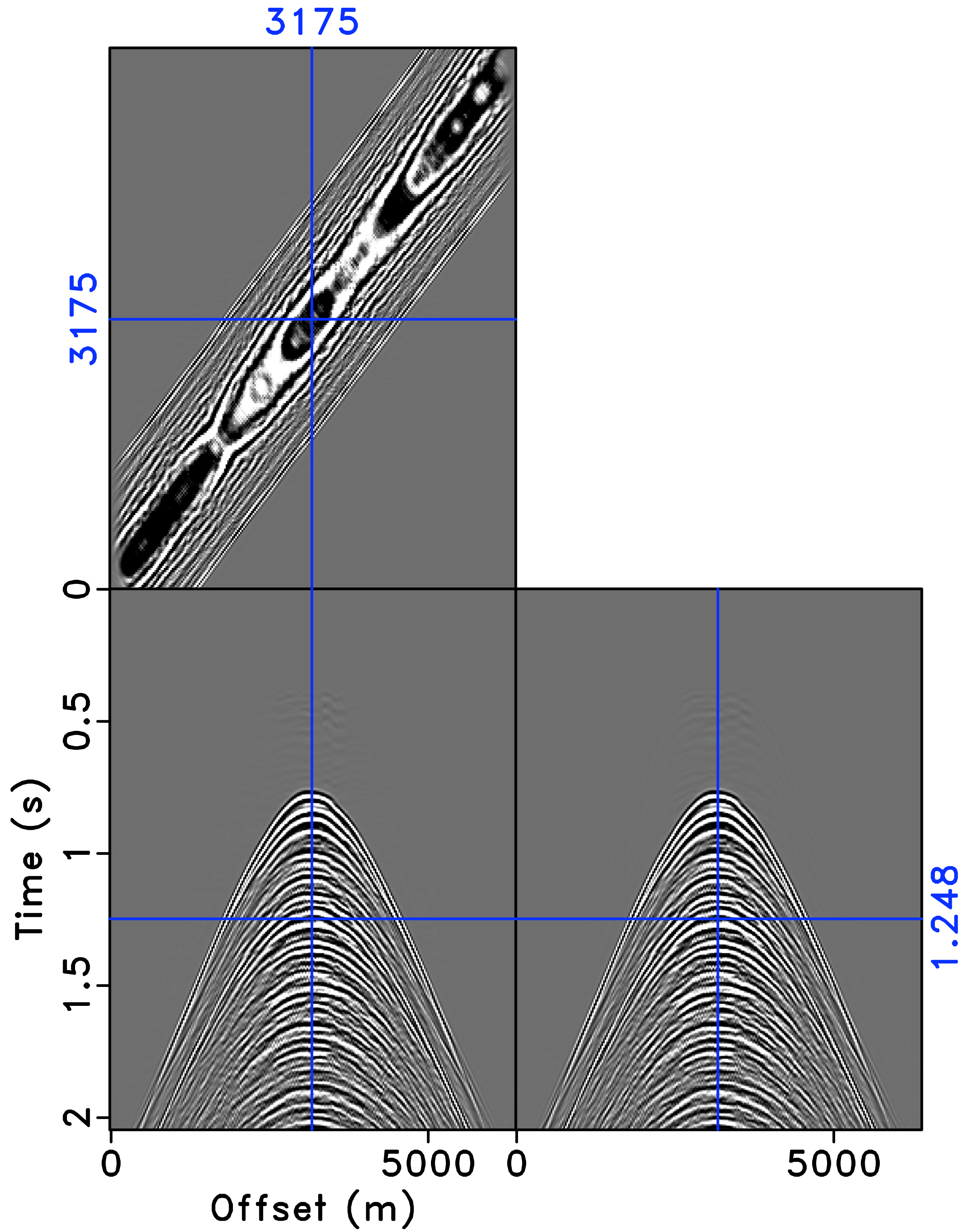
Data

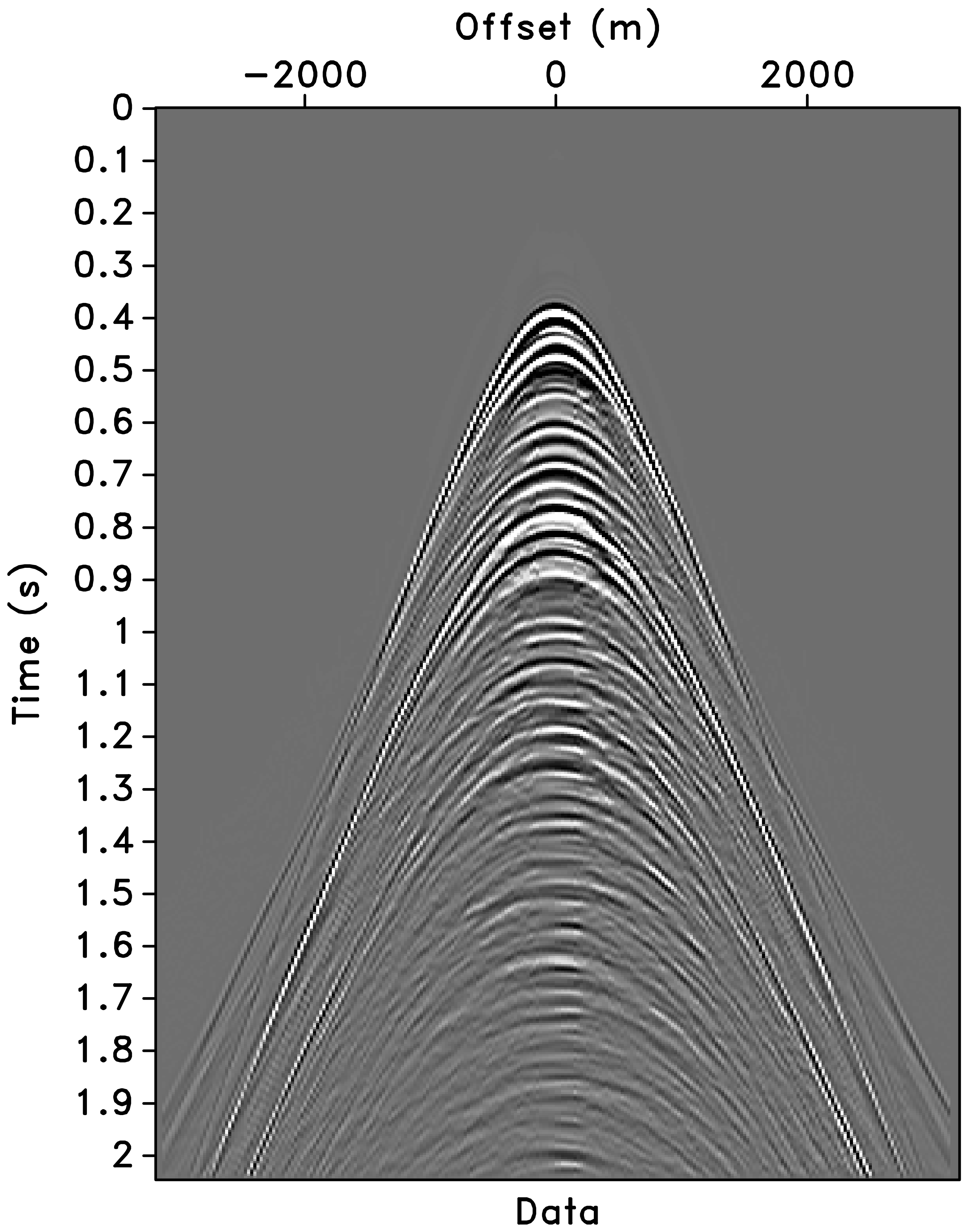


SRME



Predicted multiple





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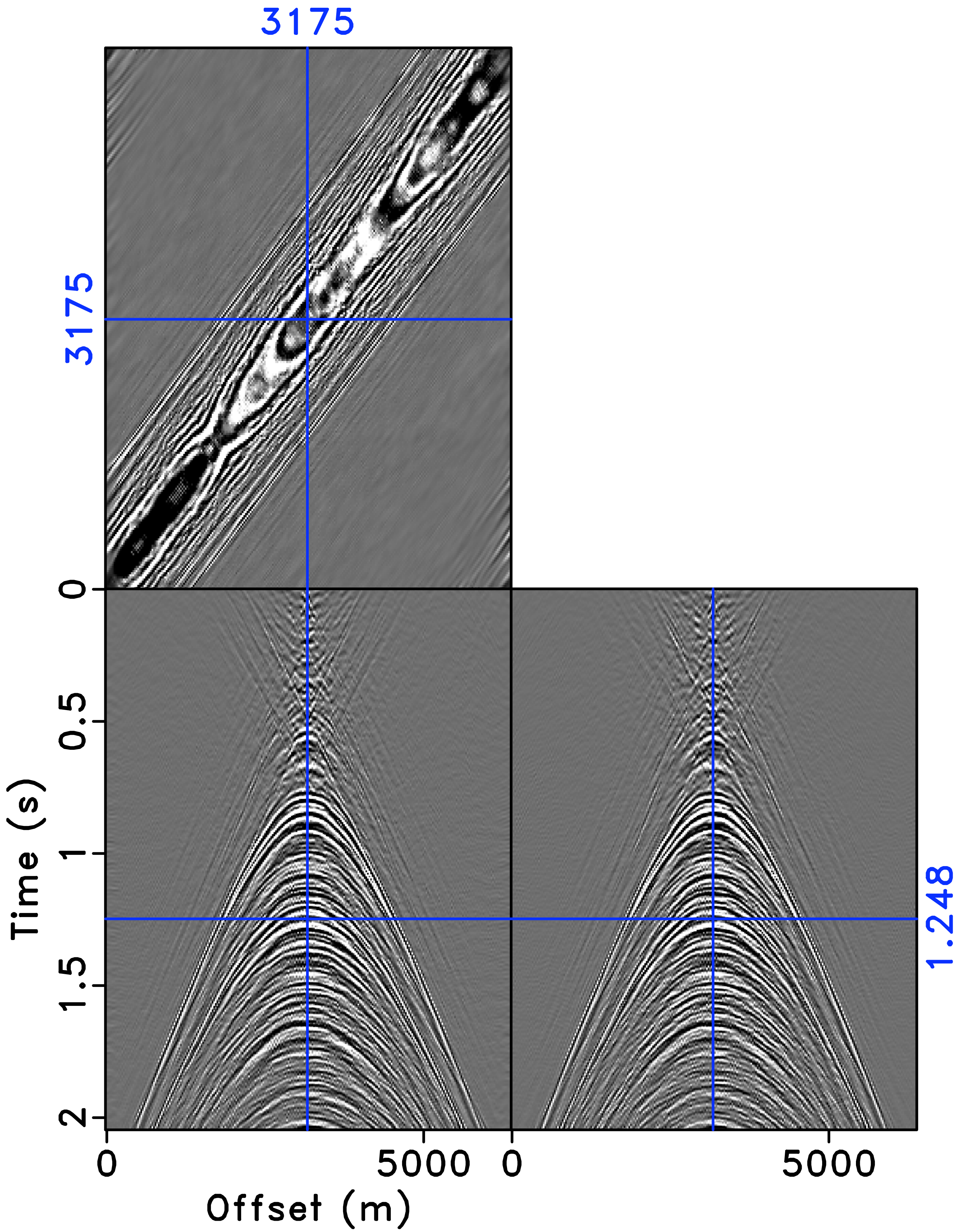
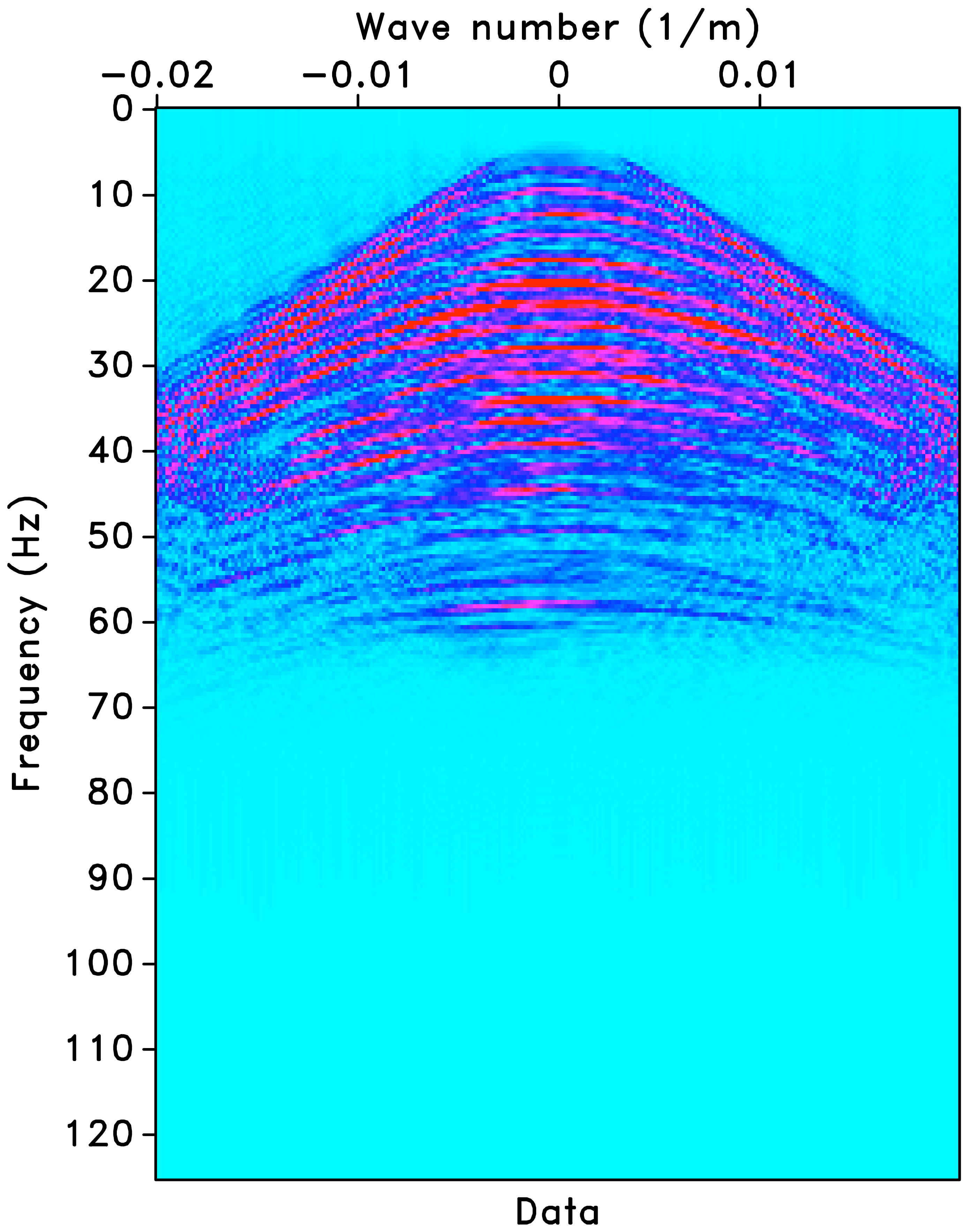
Invert P0





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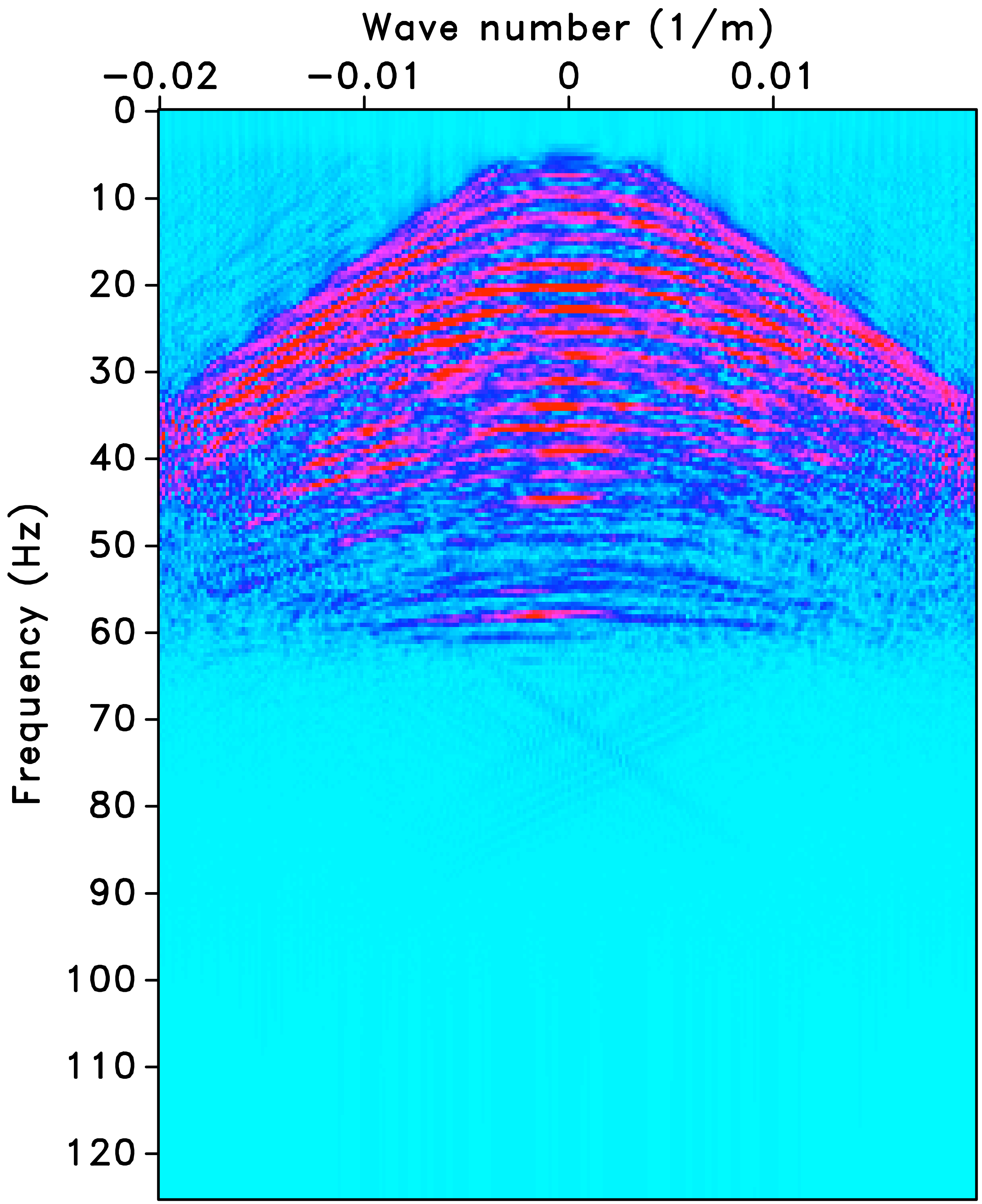
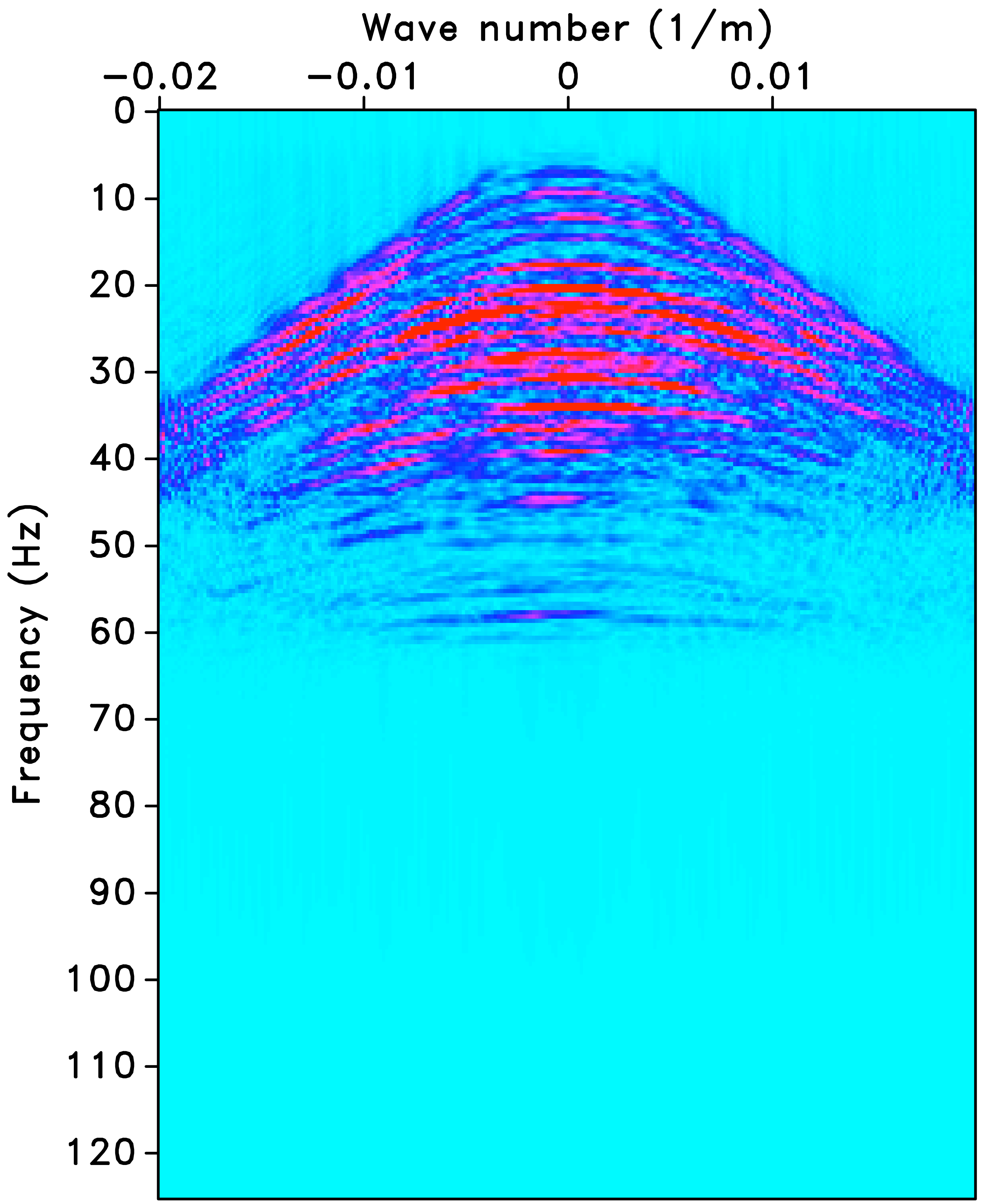
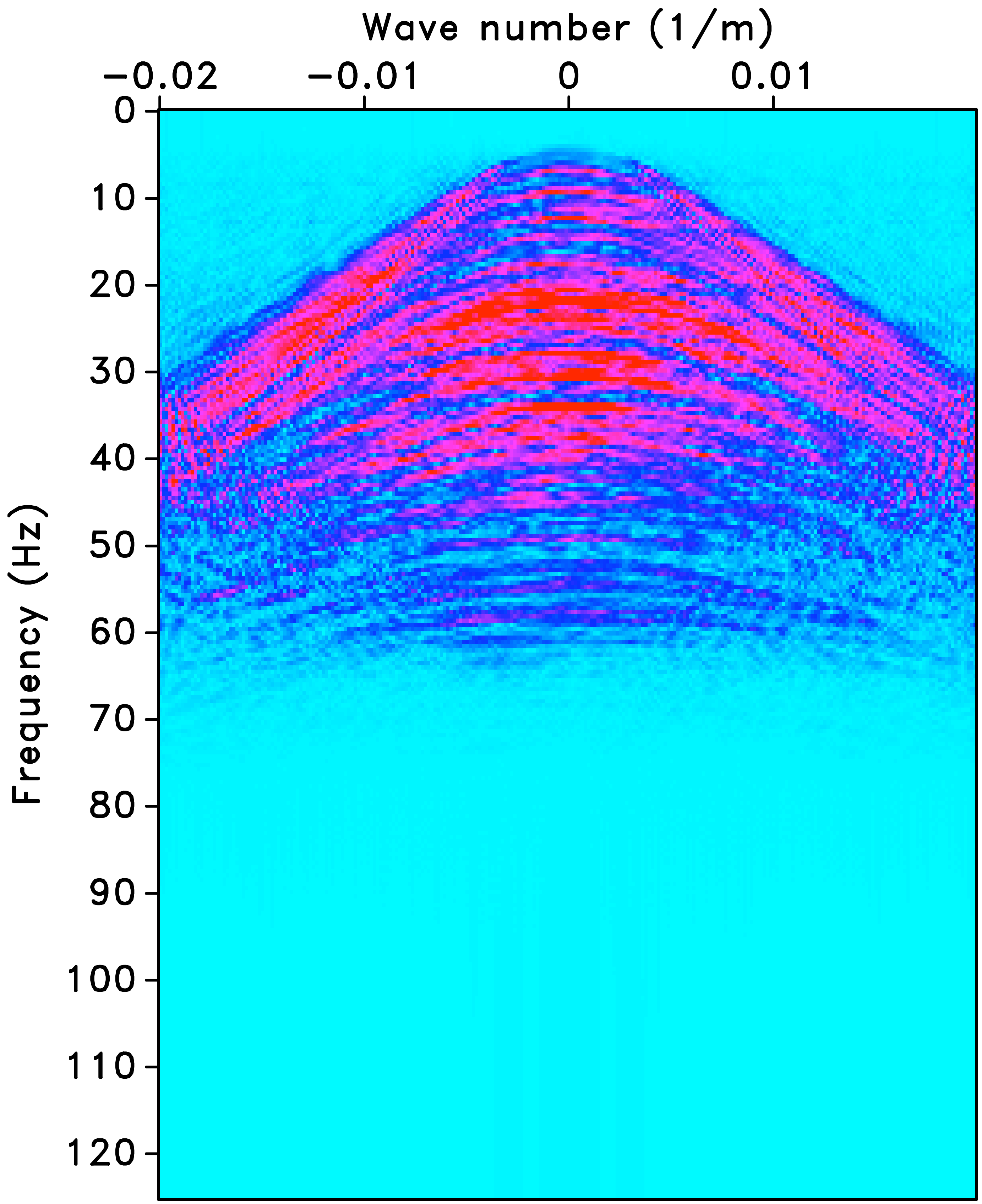


Wave number (1/m)
−0.02
−0.01
0
0.01
Frequency (Hz)
0
10
20
30
40
50
60
70
80
90
100
110
120

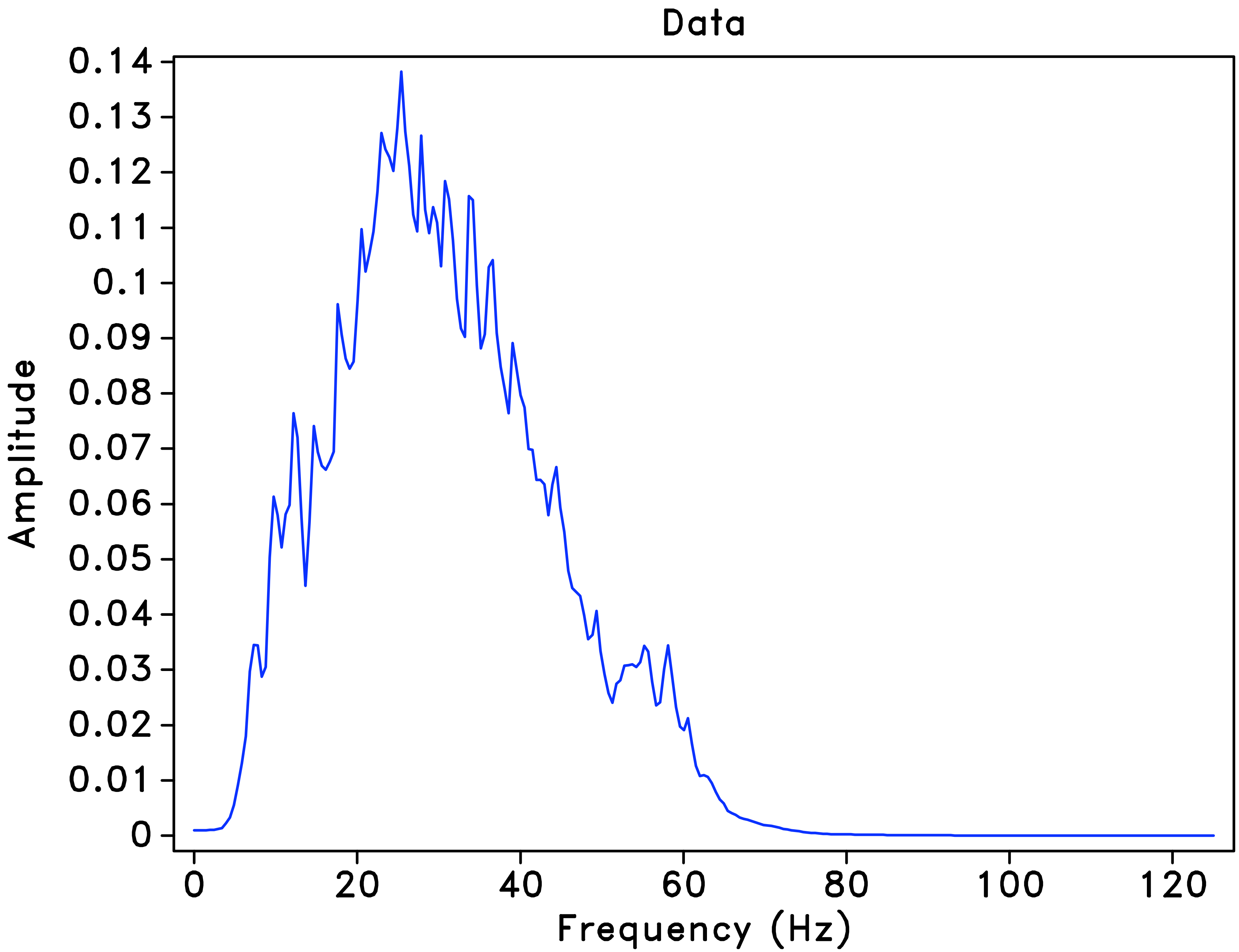
Invert P0



Predicted multiple


Wave number (1/m)
−0.02
−0.01
0
0.01
Frequency (Hz)
0
10
20
30
40
50
60
70
80
90
100
110
120

SRME



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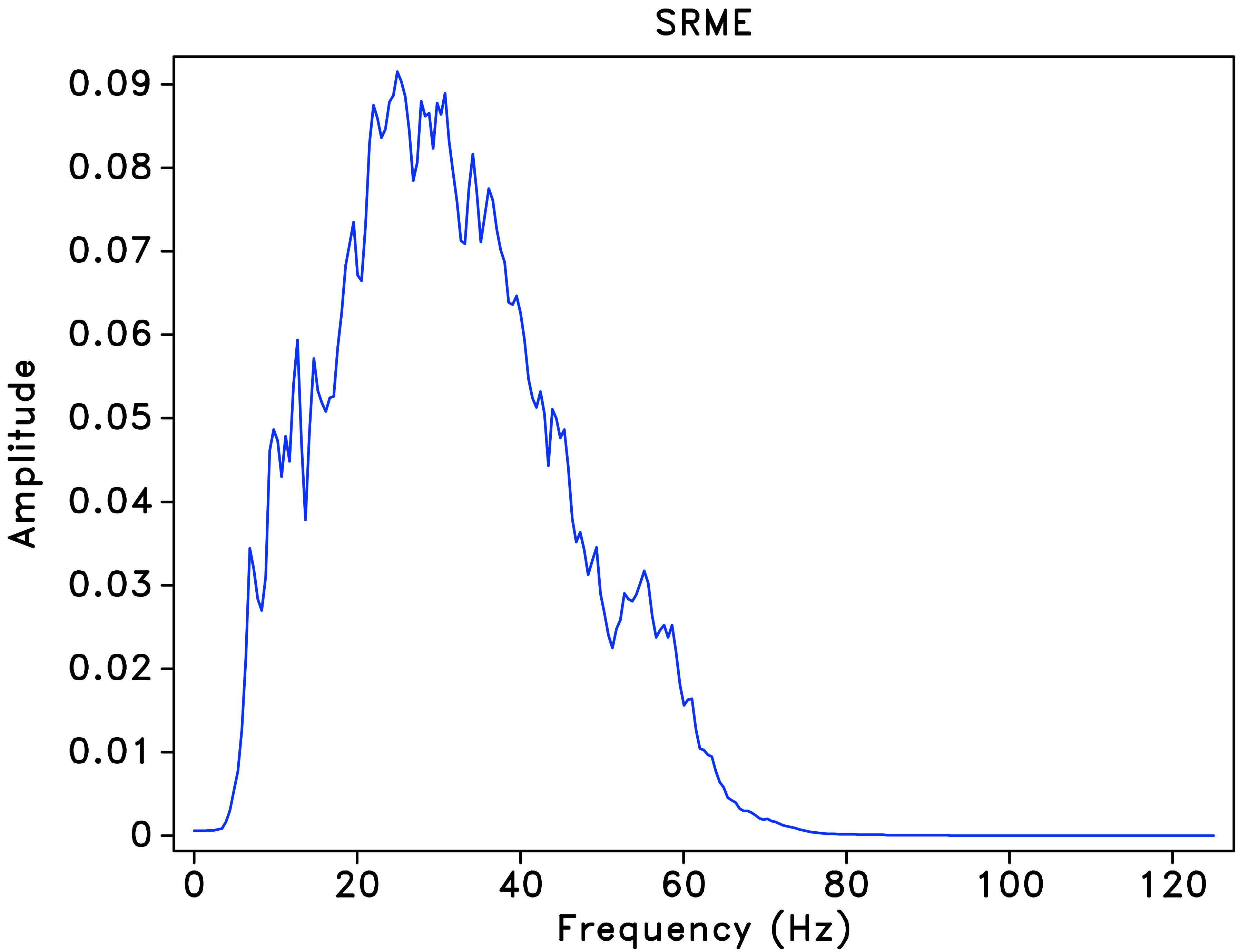

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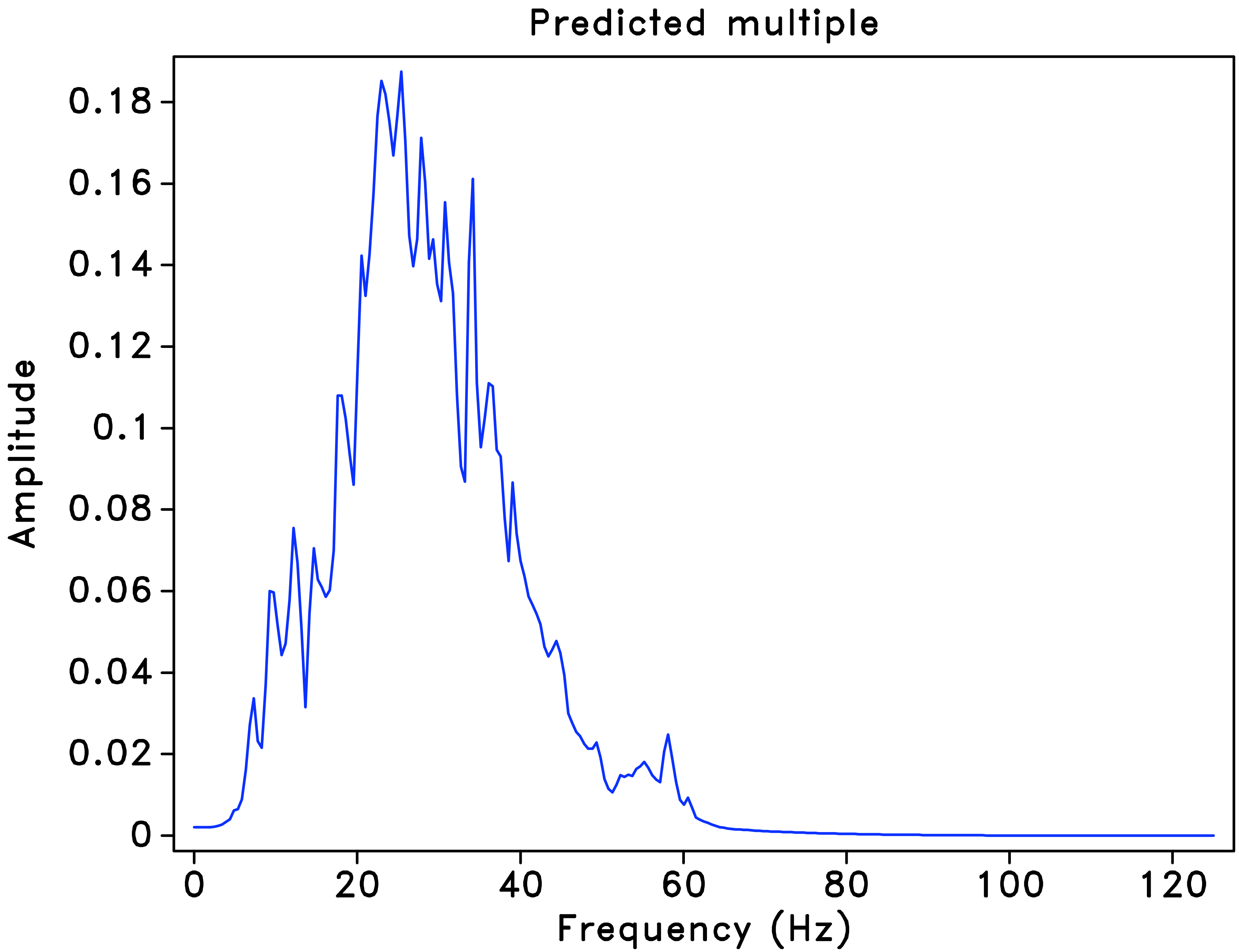

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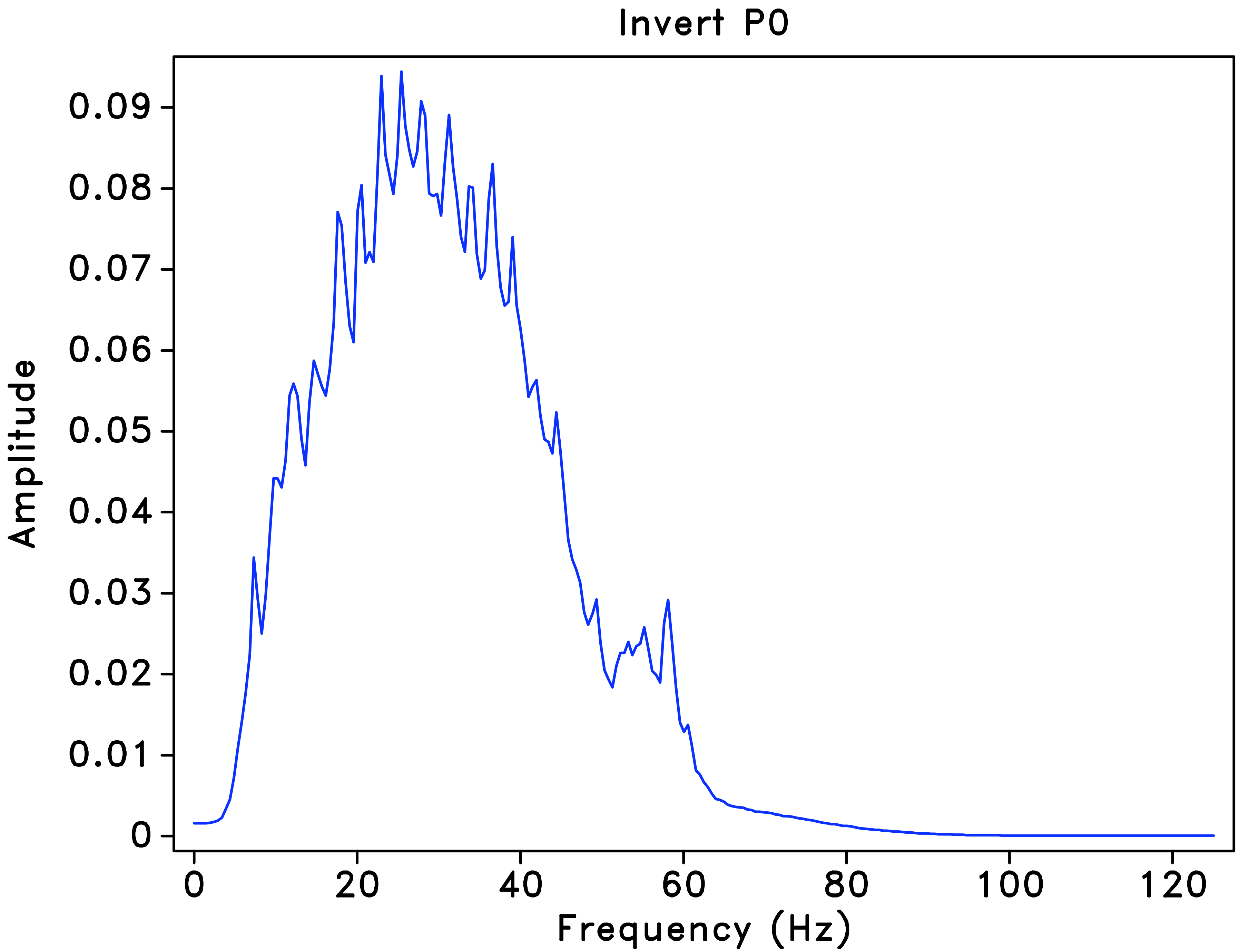
Invert P0



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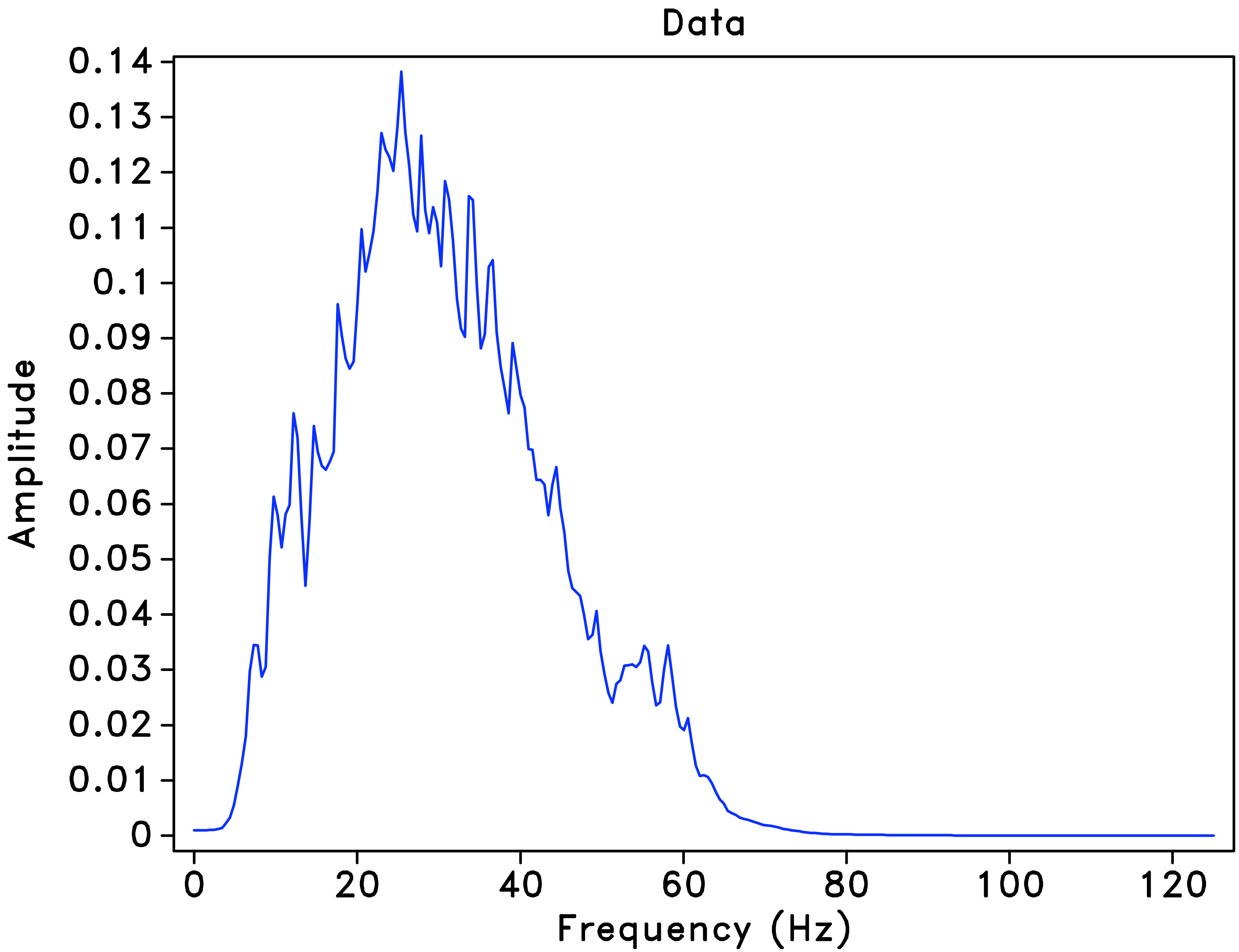

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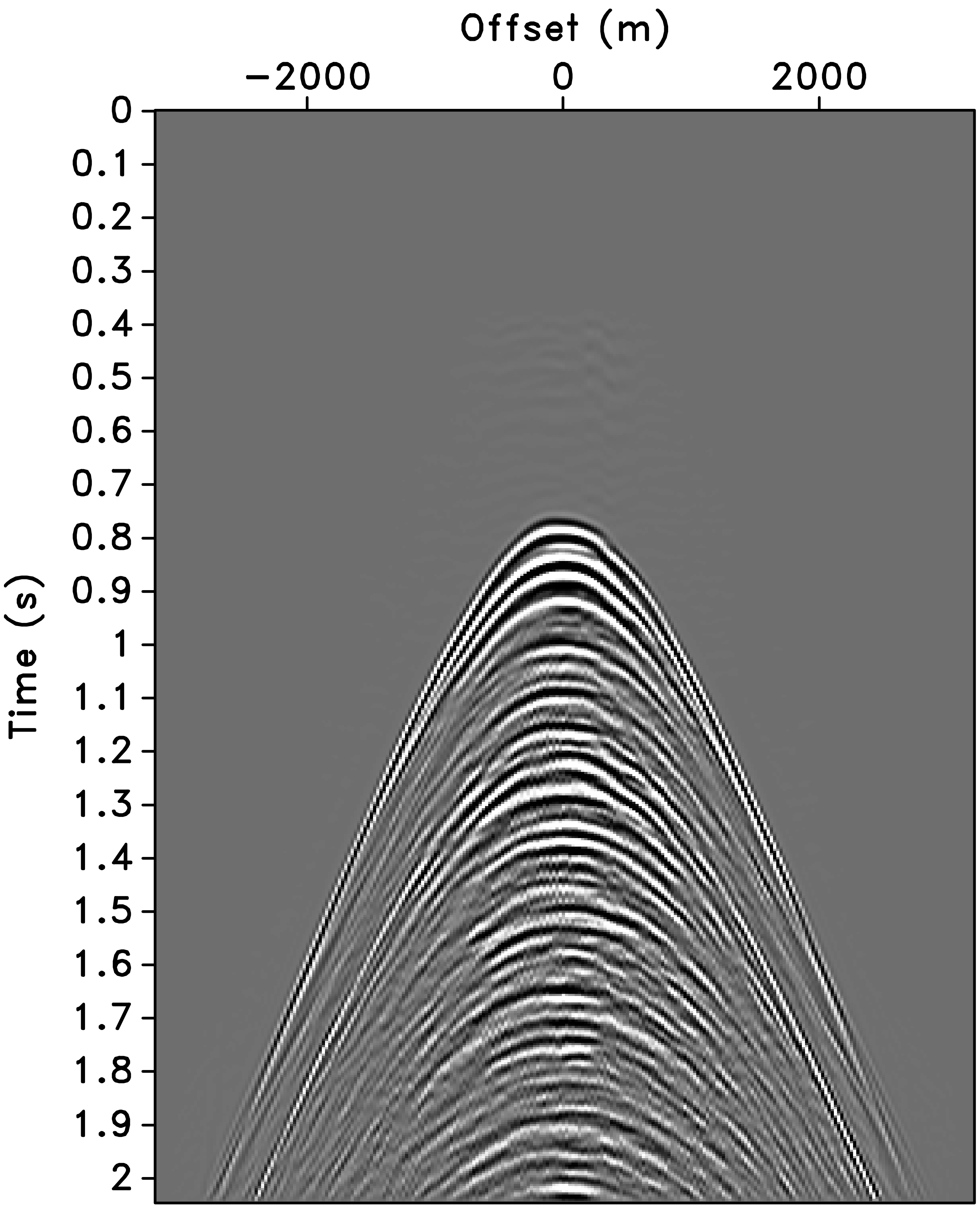
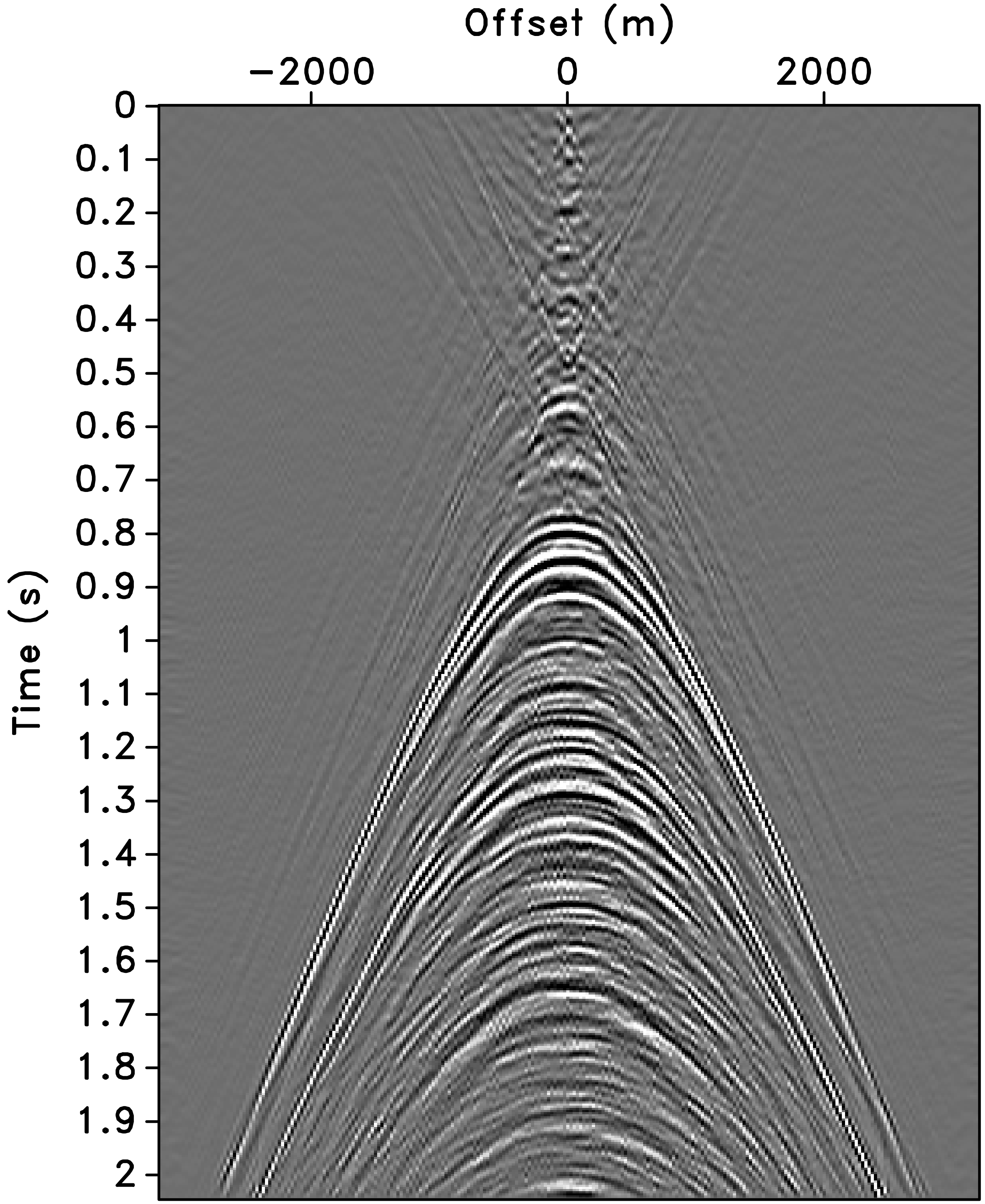


Predicted multiple



Invert P0

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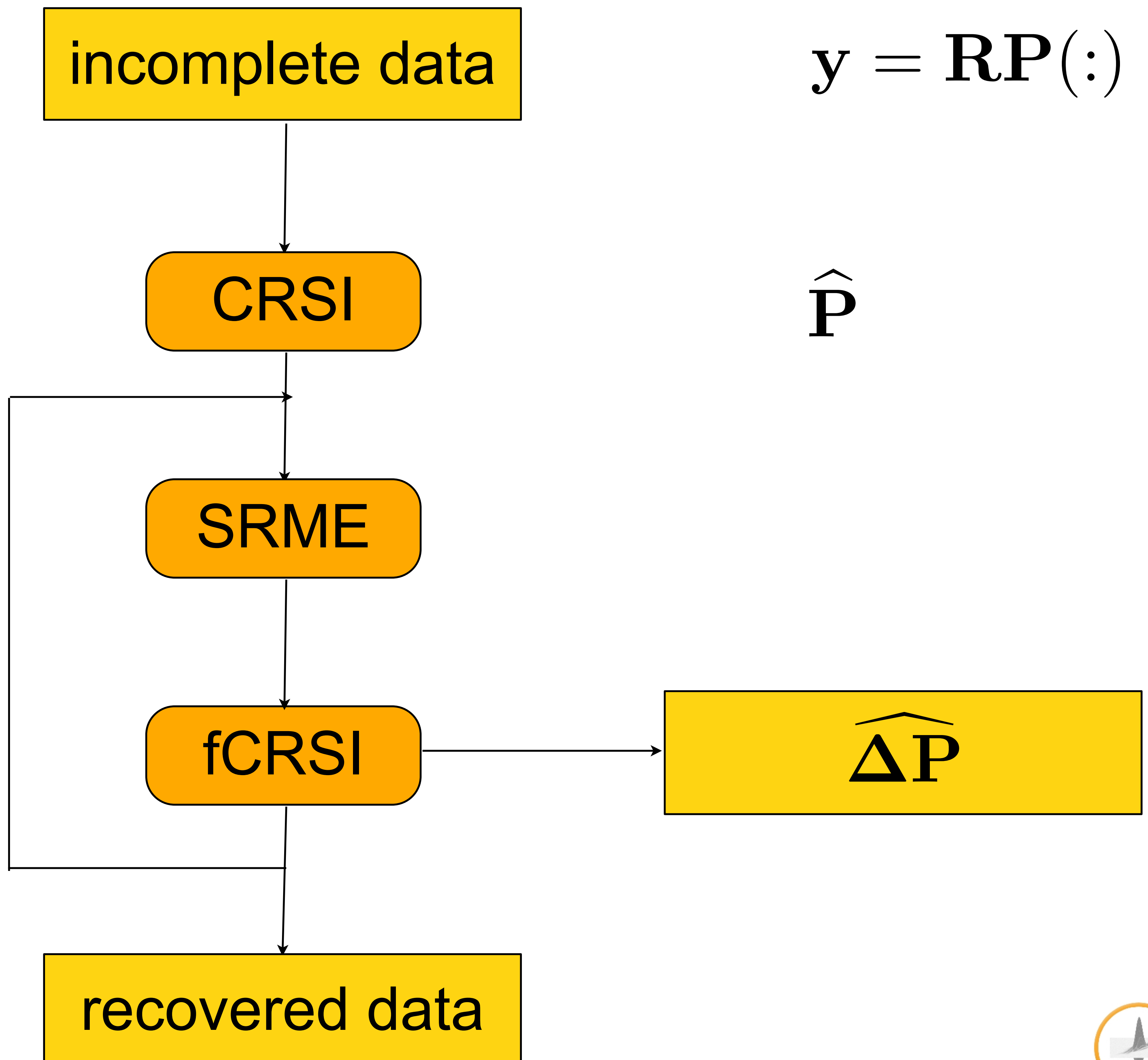
Primary prediction with fCRSI

Primary prediction with fCRSI



Curvelet-based Focal transform

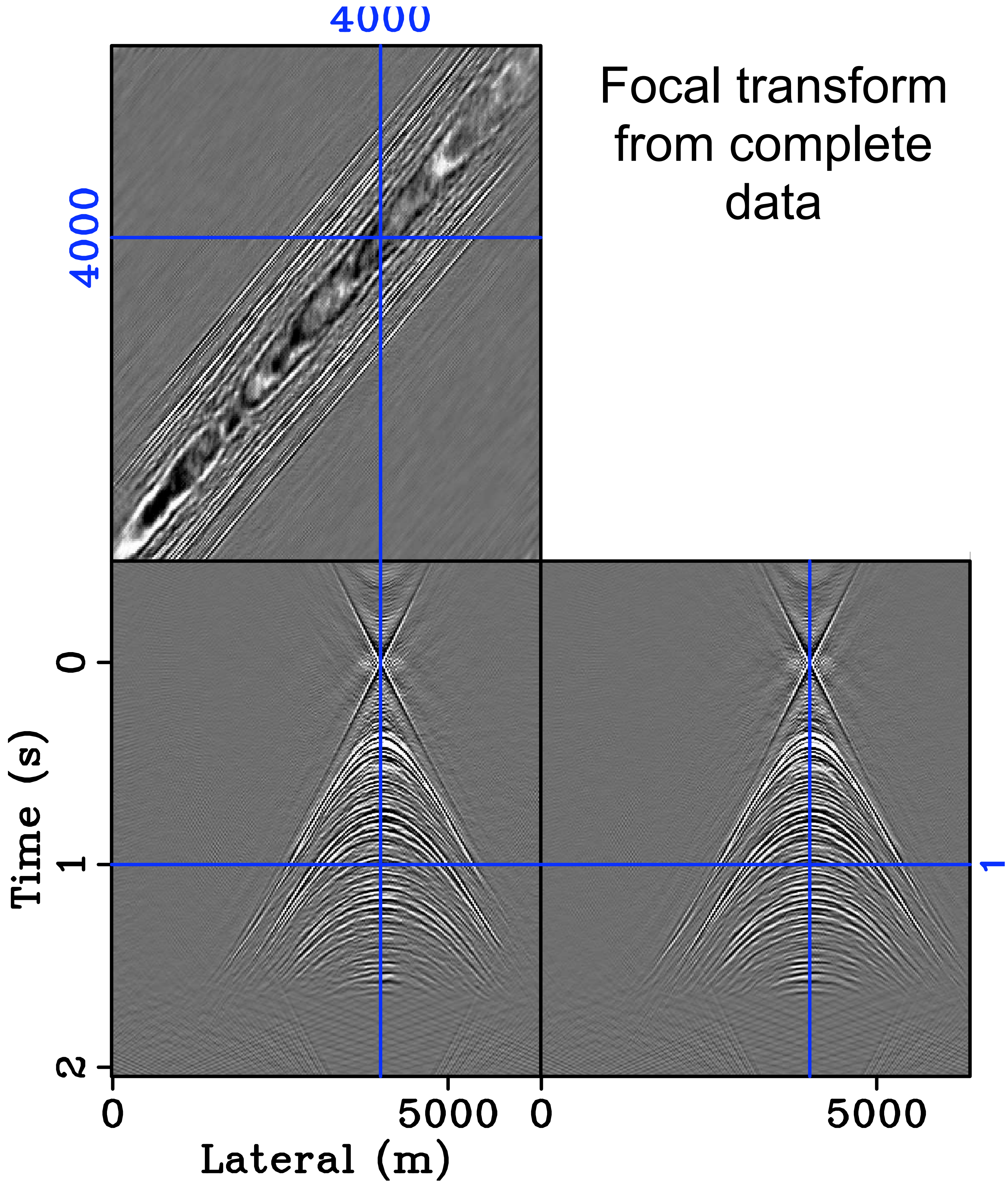
Solve

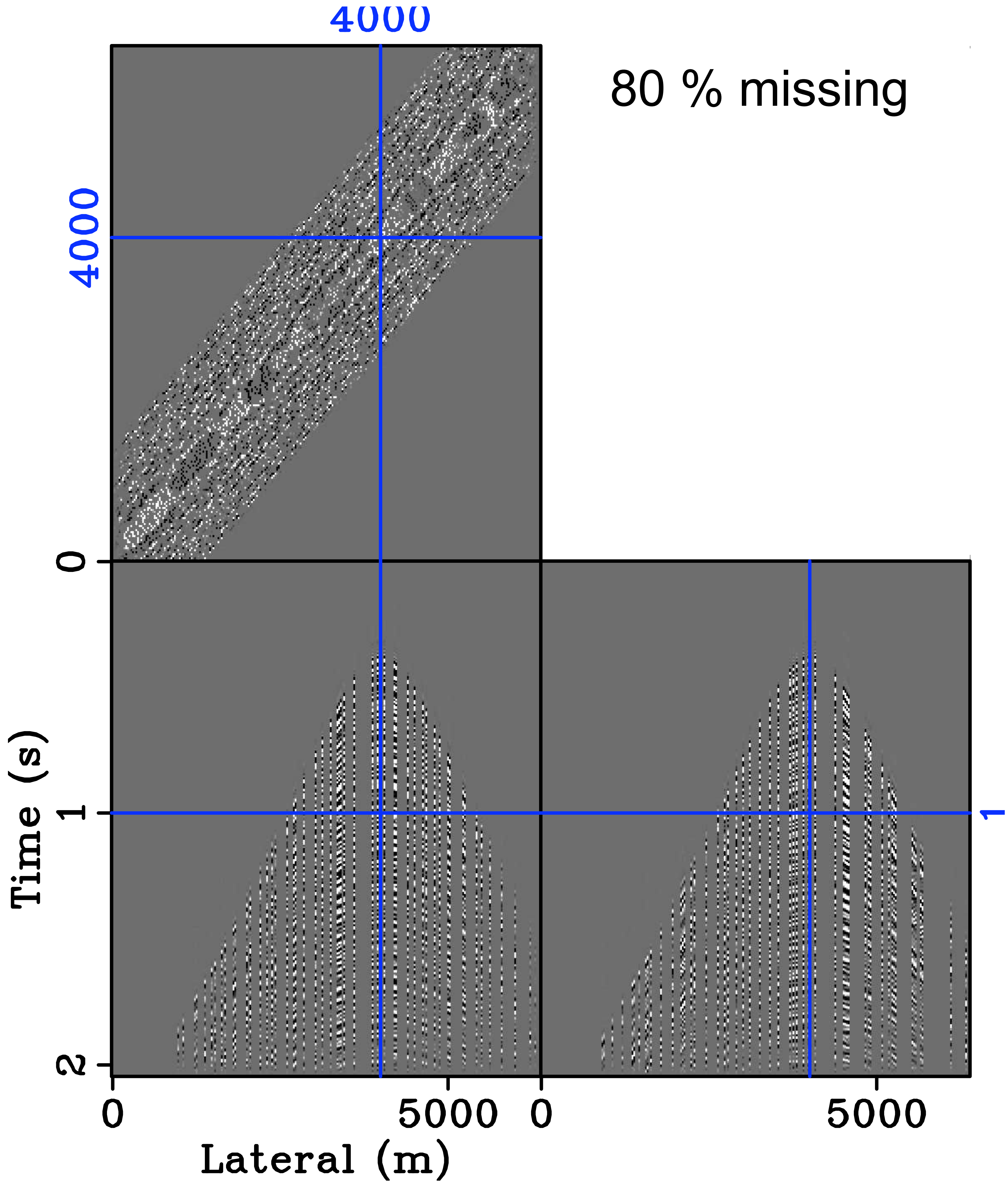
$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{\Delta P C}^T \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data} \\ \tilde{\mathbf{f}} &= \text{focused data.} \end{aligned}$$

Focal transform from complete data





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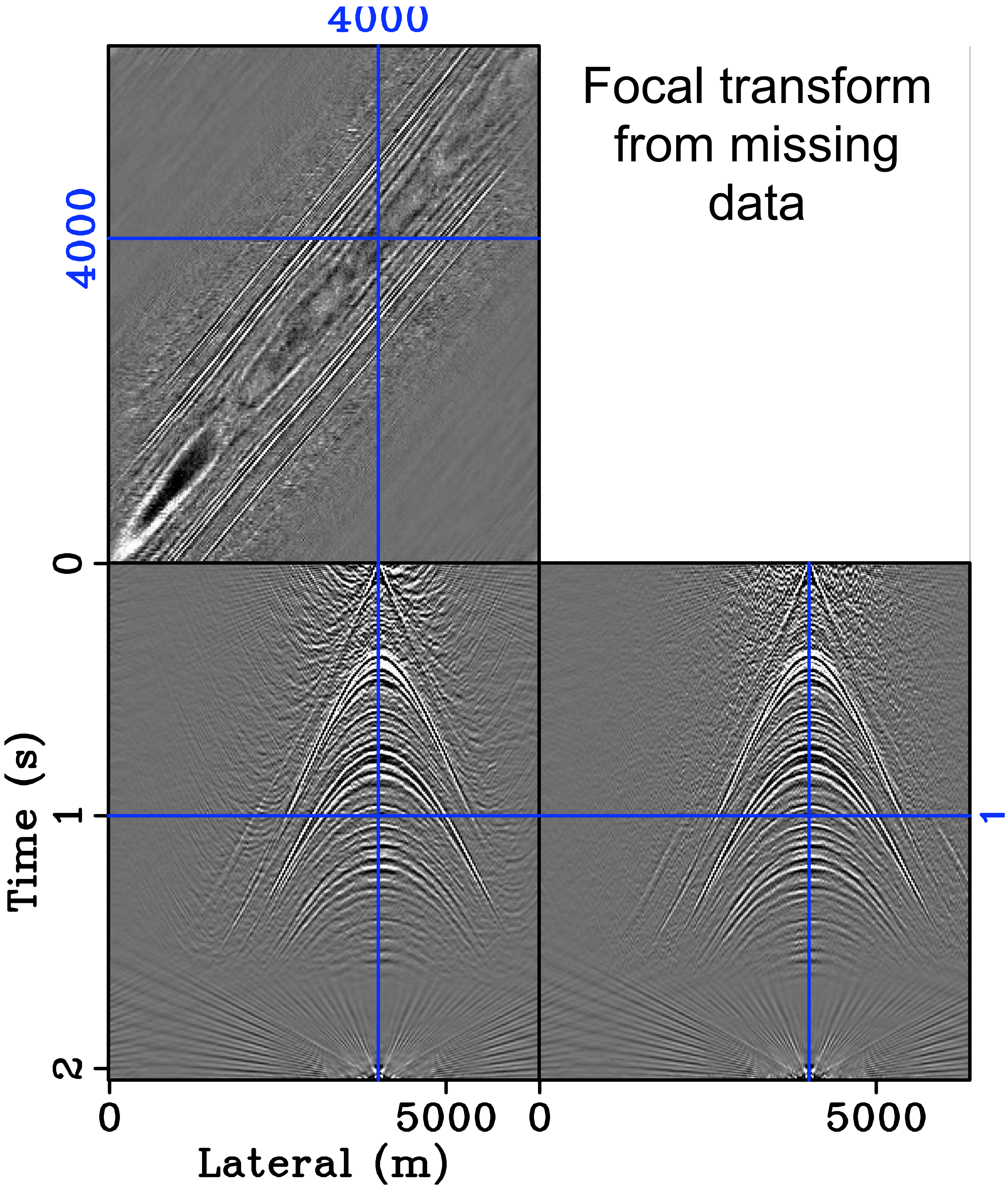


4000
4000
Focal transform from missing data
0
1
2
1
Time (s)
0
5000
0
5000
Lateral (m)

An encore ...
preliminary results for the data inverse

Curvelet-based seismic data inverse

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{p}}^{-1} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with $\mathbf{A} := \mathbf{P}\mathbf{C}^T$

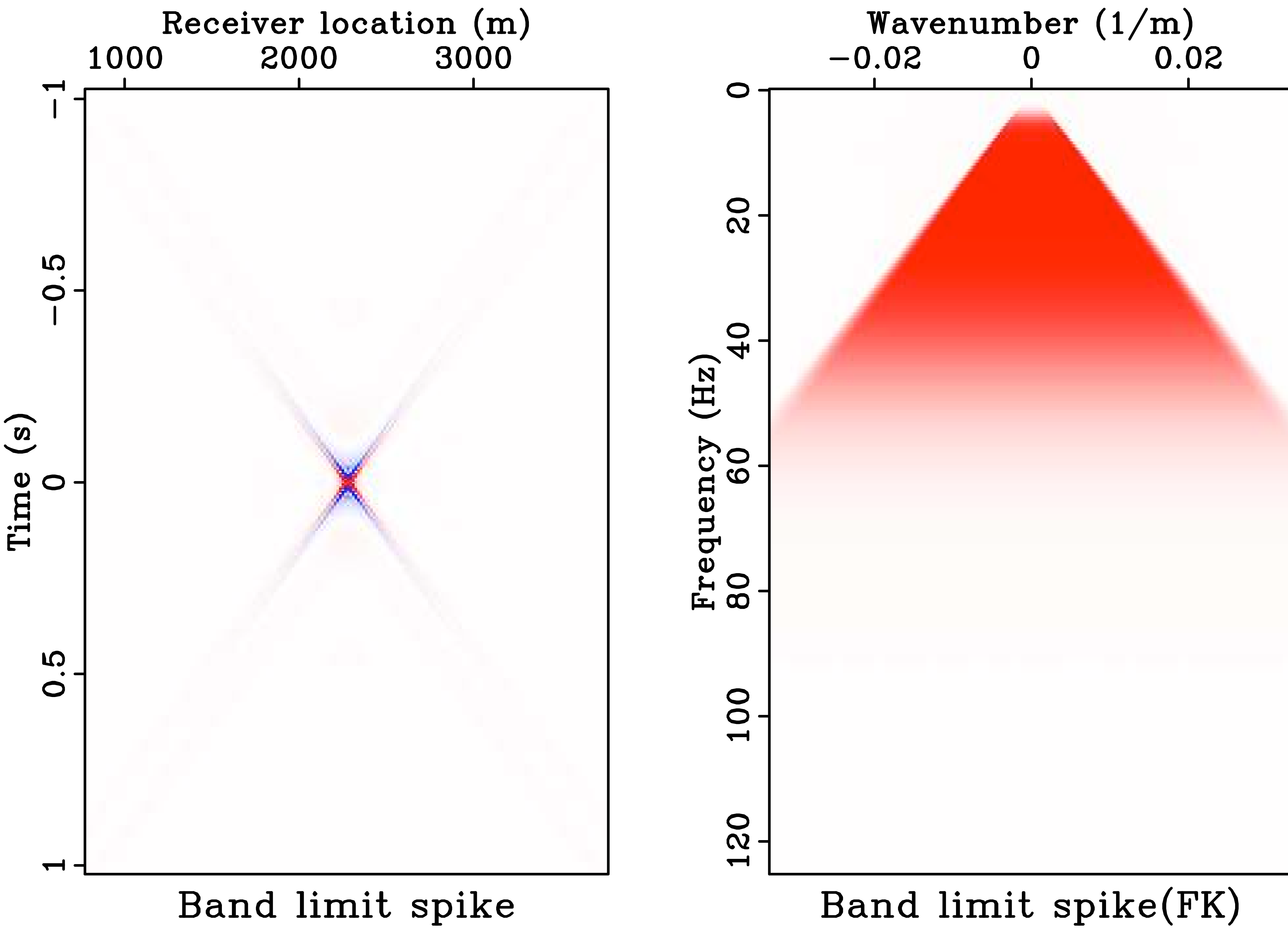
$$\mathbf{S}^T := \mathbf{C}^T$$

$$\mathbf{y} = \hat{\mathbf{I}}$$

$\mathbf{p}$ is the data to be inverted

Curvelet-sparsity regularized *data inverse* computed for the *whole* data volume ......

Curvelet-based seismic data inverse



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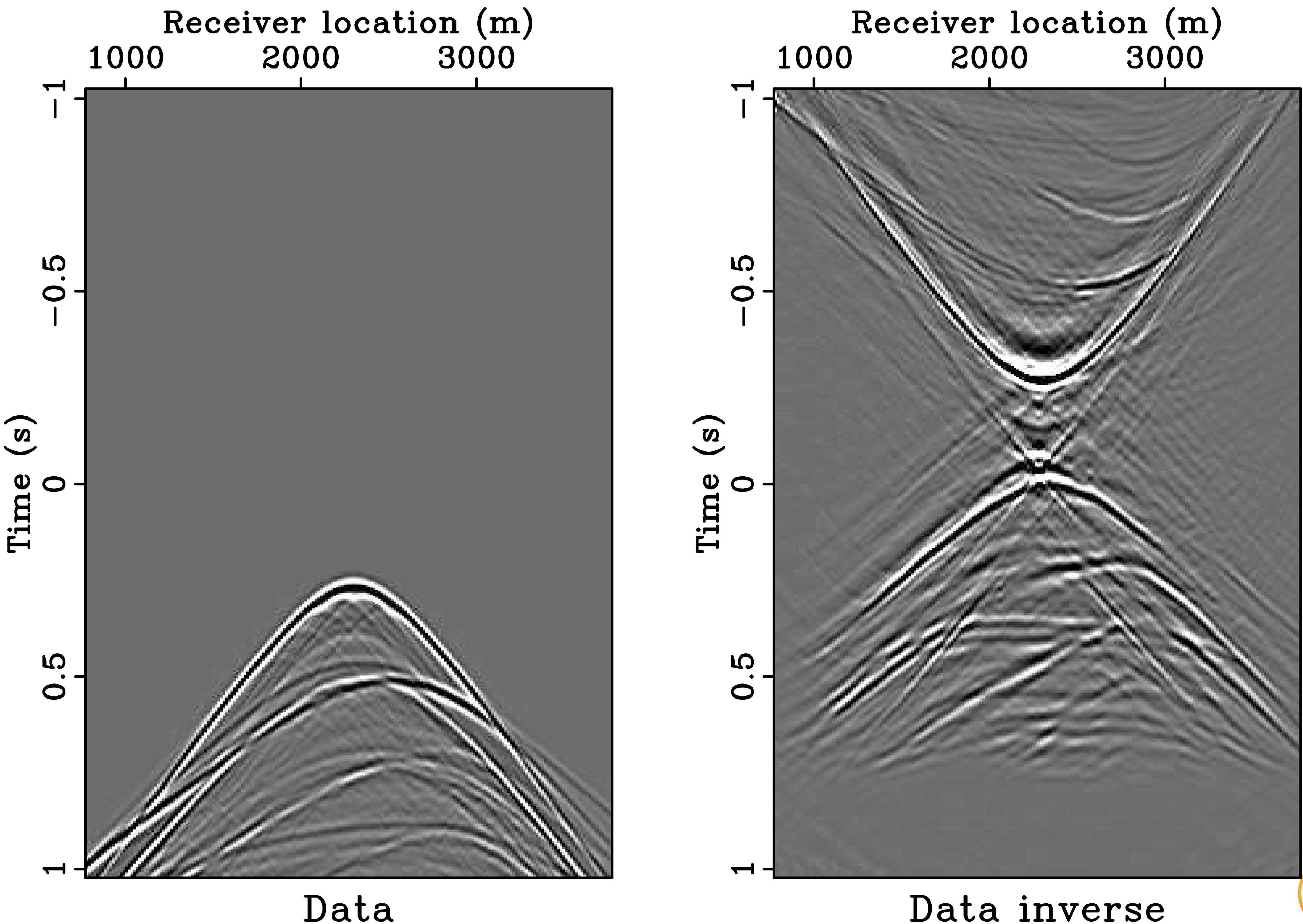
Curvelet-based seismic data inverse



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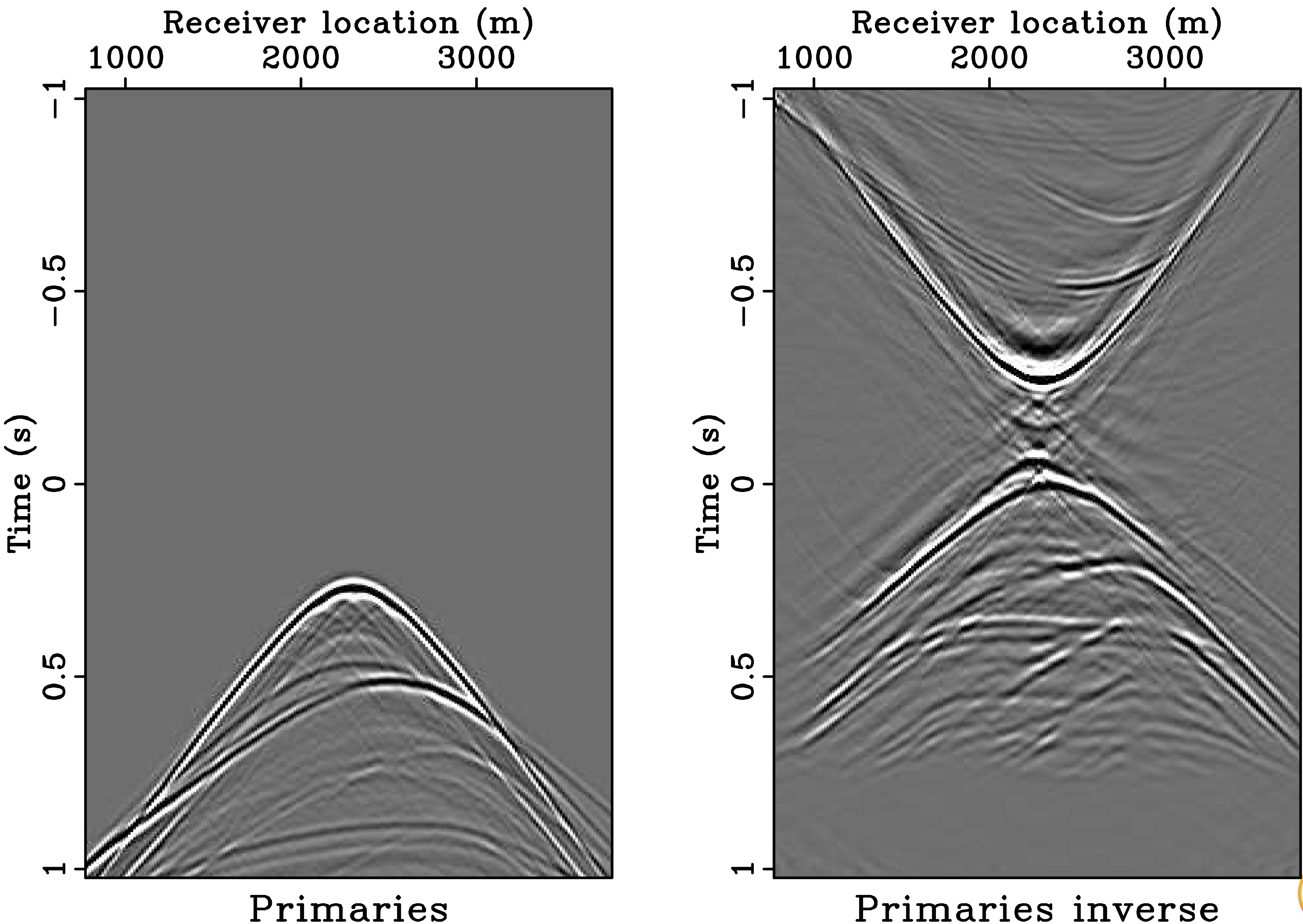

Observations

Focal transform

- allows for incorporation of a priori information
- is reminiscent of an imaging operator
- works because of addition compression and incoherence
- leads to an improved recovery

Outlook

- Restriction corresponds to a compression of the operator
- Opens the way to migration-based recovery
- or a more “blue sky” approach to compressive wavefield extrapolation & imaging

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Conclusions

CRSI

- recovers data by curvelet sparsity promotion
- uses *sparsity* as a *prior*

Focused CRSI

- incorporates additional *prior* information
- strips interaction with the surface <=> more *sparsity*
- improves the recovery and hence predicted multiples
- precursor of migration-based CRSI

Results of curvelet-based computation of the data inverse are encouraging.

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